Uniform with this volume

A TREATISE ON SURVEYING, VOLUME II (More Advanced Techniques and Developments)

Middleton & Chadwick's

A

TREATISE ON SURVEYING

Sixth Edition

VOLUME ONE

Instruments and Basic Techniques

Revised by

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B. G. M.

PREFACE TO THE SIXTH EDITION

THE BASIC principles and the simpler techniques of surveying have not changed since the previous edition of this work was published, and, in revising the book, it became a question of deciding whether the exposition of fundamentals contained therein had been achieved with the maximum of logic and simplicity.

The present writer would like to take this opportunity of paying a tribute to the late Professor M. T. M. Ormsby, formerly of University College, London, who was responsible for the fifth edition of this work and whose clear and careful teaching has equipped so many engineering students with a knowledge of surveying which has proved of great value in their subsequent careers. It has been considered best to leave his excellent treatment of surveying principles as far as possible in its original form.

Developments have taken place, however, in the design and construction of surveying instruments, and this volume has been brought fully up-to-date in respect of these and all references modernized. All the illustrations have been subject to careful examination and have been re-drawn, and a number of new illustrations have been added.

Some additional notes relating to transition curves for roads have been included, together with a more general discussion of vertical curves. The use of the Beaman arc in conjunction with the tacheometric alidade in plane-tabling has also been explained, and notes on drawing-office procedure and the calculation of areas and volumes have been concentrated in one chapter.

The scope of this volume covers the usual first year syllabus of universities and engineering colleges where the surveying course extends over two years. It is confined to plane surveying but includes the theory and practice of the various types of survey work carried out by engineers employed by railways, drainage boards, county and municipal councils, public works contractors and civil engineering consultants. Much of the contents will also be found useful by technical assistants engaged in town planning, estate development and general constructional work.

B. G. MANTON

IMPERIAL COLLEGE OF SCIENCE AND TECHNOLOGY
LONDON, *October 1954*

EXTRACTS FROM PREFACE TO THE FIRST EDITION

In the United Kingdom, and with Ordnance maps to various scales, and more or less corrected to date, always at hand, the surveyor seldom, if ever, requires to put into practice a knowledge of high-class, or geodetic surveying. The result is, that diplomas are granted to students who possess but a very limited knowledge of this class of work.

In the principal Colonies, however, these conditions do not obtain, and their Governments, not being satisfied with the limited acquirements of many English surveyors, insist on a local training, or apprenticeship and the possession of one of their own diplomas. This restriction has, in many cases, proved somewhat arbitrary.

About three years ago, a number of gentlemen interested in this question, and in the improvement of the standard of qualifications for English diplomas as Surveyors, met at the Surveyors' Institution by kind permission of the Secretary, and formed a committee to consider what steps might best be taken to secure a better position for the English student who might wish to seek employment in one of our Colonies.

The Council of the Surveyors' Institution were approached, and their Secretary, Mr. Julian Rogers, informed the committee that their members were prepared to look favourably on any efforts made in the direction indicated.

It was arranged that the committee should prepare, and submit, a Text-Book, which the Council agreed to adopt, if satisfied with the same. The present Treatise on Surveying is the result. The work has been divided into two Parts, so that problems requiring Geodetic treatment, and Astronomical determinations, as well as the manner of conducting Marine, Route, and other special surveys, are discussed in the Second Part.

The Authors beg to offer the above remarks as their justification for submitting this work to the verdict of public opinion, and whilst making no claim to having compiled a complete treatise on so large a subject as surveying, still it is hoped that all the information necessary to enable a student to acquire the knowledge required of a qualified surveyor is to be found in it.

The following is a list of the Contributors; and it is deeply to be regretted that the sections written by Mr. Leane (who died at sea when on his return from Africa) and by Major-General Woodthorpe, C.B. (appointed Surveyor-General of India just before his death) have had to be produced without the advantage to the Editor of being able to submit proofs for their revision:

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Appointed Surveyor-General of India just before his death

A TREATISE ON SURVEYING

CHAPTER 1

GENERAL SCOPE OF SURVEYING

THE APPLICATIONS of surveying may be divided broadly into two main categories: surveys made for the purpose of obtaining a map of a given area for general use, and surveys made for a specific engineering project. The latter type is usually confined to a more restricted area than those concerned with ordinary mapping, and the information derived from engineering surveys and shown on the plans will normally be more detailed and specialised than that on a general-purpose map. Constructional projects also require 'sections' as well as plans, and the preparation of sections necessitates methods of levelling which would not be used in a mapping survey, although contoured maps are invaluable for the preliminary investigation of many engineering schemes.

PLANE AND GEODETIC SURVEYING. Strictly speaking, a curved surface, such as that of a portion of the earth, cannot be represented without distortion on a plane surface such as a flat sheet of drawing paper. In practice, however, if the area surveyed is sufficiently small no measurable error is introduced if the curvature of the surface, apart from local undulations, is ignored.

The effect of curvature may be seen on the '25-inch' Ordnance maps used so extensively by the technical officers of local authorities, estate agents, town-planners and civil engineers engaged in public works of all descriptions. These maps are drawn to a scale of 1/2500, or 25·344 inches to one mile, and the meridians of longitude are indicated by lines with the appropriate numerals on the top and bottom margins of each sheet. It will be found that the meridians are closer together on the upper, or northern, margin than they are on the lower margin, and this convergence accords with the fact that all the meridians meet at the north pole. The curvature of the earth's surface is thus taken into account in these maps, and distortion is thus reduced.

If curvature is ignored, the survey is known as a 'plane survey', and if it is taken into account the work is known as a 'geodetic survey'. The latter necessitates more elaborate fieldwork procedure, and considerable mathematical computations and surveys of this kind are not discussed in the present volume.

The maximum area which may be treated as a plane surface without undue error depends partly upon the purpose for which the survey is being made and the degree of accuracy required. In geodetic surveys a large area is covered by a network of interconnected triangles, each of which is treated as a spherical, not a plane, triangle. The sides of such triangles are circular arcs, and the summation of the angles, if the sides are very long, will exceed 180° by a measurable amount. If, for example, we consider an equilateral spherical triangle in which each side measures thirteen miles, the summation of its angles exceeds 180° by one second of arc, an amount which is measurable by the most accurate type of theodolite. The area of this triangle would be approximately seventy-six square miles.

The amount by which the sum of the angles of a spherical triangle exceeds 180° is known as 'spherical excess', and, as this quantity could only just be detected in the example quoted, an area of about seventy-six square miles might be regarded as a maximum for the successful application of plane surveying for work of high accuracy. The round figure of a hundred square miles is often accepted as a rough working rule, and in accurate surveys of larger scope geodetic methods are introduced.

Plane triangulation. If the area to be surveyed is of considerable extent, though less than a hundred square miles, a method known as 'plane triangulation' is usually adopted. A typical example of a survey of this kind is that of a large town and its outskirts. The area is covered by a network of connected triangles, the sides of which may be several miles long. A particular side of one triangle is selected for its convenience of access and suitability for accurate measurement, and its length is determined with great precision. All the angles in the network are measured by theodolite, and an instrument reading to one second is commonly employed for this purpose. The lengths of the sides of all the triangles are then computed from the measured length of the chosen side, termed the 'base line', by using the well-known formula

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

In this way, a number of very accurately fixed points known as 'trigonometrical stations' are established. Secondary surveys, which may be chain surveys or traverses, are run between these points for locating detail. Triangulation on a large scale is beyond the scope of this book, but secondary surveys are fully considered.

LINEAR AND ANGULAR MEASUREMENTS. In a very simple survey, such as that for obtaining a plan of a small open field, the entire work may consist of linear measurements only. These surveys are usually called 'chain surveys', even though a steel band or a linen tape is used instead of a chain.

Chain surveys, however, are very limited in their adaptability. Thus if the field mentioned above were covered with scrub and trees it might be difficult to measure the necessary tie-lines across the area. Again, a short survey along a winding country lane can be carried out, after a fashion, by linear measurements alone, but this method is unsatisfactory because of the difficulty of securing good ties to connect the lines. These ties are essential in all chain surveys because the lines must be arranged in the form of triangles to enable the figure to be plotted from lengths only.

If, instead of depending on linear dimensions only, a theodolite is used for the accurate measurement of angles, a much wider variety of surveys can be satisfactorily and quickly carried out. Thus the boundaries of a field can be surveyed accurately and rapidly from a framework of lines forming an irregular polygon with its sides running close to the fences or hedges, and there is no need to measure diagonals or other tie-lines. The same method may be used for surveying a block of buildings, or part of a densely built-up area in a town, types of work in which a chain survey would often be difficult if not impossible.

The combination of linear and angular measurements in these cases is called a 'traverse'. If the framework of the survey lines forms a complete polygon we have a 'closed traverse', but it is not necessary for the lines to form a closed figure. A survey along a winding road, for example, may extend for several miles and consist only of a series of lines meeting at measured angles. This is termed an 'open traverse', and both types of traverse are based on the principle that the survey lines can be plotted if we know both their lengths and the angles between successive lines.

Chain surveys and traverses are the usual methods adopted when plans are required for building development, road improvements, small sewerage schemes and small railway extensions, such as additional sidings. In the case of larger schemes, such as by-pass roads, reservoirs, extensive sewerage works and so on, traverses would be used in preference to chain surveys.

Traverses, in fact, form the everyday job of the assistant civil engineer employed by a local authority or a consultant when a projected constructional scheme is under consideration. The survey is the starting-point of all such schemes, and, since the details of the proposed works are superimposed on the survey plans, the reliability of quantities and estimates and the effectiveness of the design depend to a great extent upon the accuracy and thoroughness with which the original survey is carried out.

LEVELLING. Levelling may be defined as the determination of relative heights, and forms a very important part of all engineering surveys. In planning any constructional work, whether it be the building of a small house or a large power station, a by-pass road or a dam, it is essential to know the depths of excavation for the foundations and for trenches carrying service mains and pipe-lines. In many cases excavation is roughly balanced by filling where high and low places are brought to a uniform level or gradient, and full information as to the relative heights of the ground is obviously necessary if such work is to be planned in a satisfactory way.

It is not only important that the survey work and levelling should be accurate, but sufficient data should be obtained from the fieldwork to enable a true representation of the site conditions to be plotted on the plans and sections. In other words, measurements must not only be correct; they must, in addition, be adequate in number.

SOME OTHER SURVEY METHODS. Although much space is devoted in the present volume to chain surveys, traverses and levelling, there are other methods which are useful in certain circumstances. One of the most important of these is 'tacheometry', a method of determining both distances and heights by observing a graduated staff through the telescope of a specially equipped theodolite. The latter is sometimes given the more distinctive name of 'tacheometer'.

Tacheometry is very useful for locating somewhat indefinite detail to which it would be difficult to take direct measurements by chain or tape owing to its inaccessibility. All that is necessary is a clear line of sight between the instrument and the point to be located and the ability of the staff-holder to reach the point in question.

Tacheometric methods are much used for obtaining data for contoured plans of reservoir sites and building estates. Tacheometric plane-tabling is a modification of the normal method, and enables a contoured plan to be obtained by direct plotting in the field. In this case a drawing-board mounted on a tripod is used in conjunction with a specially designed tacheometer, which rests on the board and is directed to any point whose distance and height are required.

Both methods of tacheometry are dealt with fully in these pages, and their advantages are emphasized since their value does not appear to be widely appreciated.

AIR SURVEYS. The preparation of large-scale plans from air photographs has now become a recognized procedure and, to some extent, is displacing detailed ground surveys in the preliminary study of extensive constructional schemes. It is extremely useful where ground surveys would be difficult and slow. A typical example is the survey of a busy and complicated railway junction, particularly on electrified lines. The accuracy of air surveying has now reached a sufficiently high standard to enable engineering schemes to be based on plans derived from air photographs and plotted to a scale as large as forty feet to an inch.

Air surveys are expensive, but are economically advantageous in the case of large development projects in difficult country and for smaller schemes in exceptional circumstances. In many cases in railway and dock work a plan is urgently needed, and ground methods are impossible except, perhaps, on Sundays or at night when traffic is less heavy. Air surveys are then extremely useful, and may be preferred when time is a factor to be considered, as well as cost. There will always be a vast number of minor projects, however, in which the cost of air photography would be uneconomical in relation to the estimated cost of the proposed work. For this reason it is still essential for the civil and municipal engineer to be familiar with ground-survey methods and to be competent at handling surveying instruments. Furthermore, a most important part of the engineer's duties is the setting-out of new works.

SETTING OUT OF WORKS. Setting out consists of working backwards from the engineer's or architect's drawings to the actual site, thus reversing the process of making the original survey. Before any work can be started the centre lines of future roads and railway tracks, fence lines, corners of proposed buildings, positions of foundations and every other feature at ground level must be accurately marked by pegs. These pegs are set out by linear and angular measurements, and the basic principles of surveying are applied; but the accuracy of the setting-out depends upon the accuracy with which the existing details of the site were surveyed in the first instance, since well-established landmarks and the original survey pegs are used as reference points from which to take measurements locating the position of the new works.

Many kinds of civil-engineering work involve the setting-out of curves, and, although circular arcs suffice in many instances, 'transition' curves are used in railway practice and on modern roads. These curves possess the property that their radius decreases gradually from infinity to a predetermined minimum. The mathematical theory of both circular and transition curves is discussed in the present volume.

SPECIAL TYPES OF SURVEY. Although the broad principles of surveying and the technique of using the normal instruments are applicable to all kinds of constructional projects, there are certain specialized types of survey which require particular notice. One of these is the setting-out of a tunnel, where a surface survey, such as a traverse or a simple triangulation, has to be connected with an underground survey, usually at one or more shafts. This work naturally requires a high degree of accuracy, and special methods are used to overcome such difficulties as those imposed by the restricted dimensions of the shaft and the consequent shortness of the lines of sight. In sea-defence works and river improvements underwater surveys are necessary, and here again special methods are used to obtain the data for such drawings as the cross-sections of a river and contoured plans of beaches carried some distance seawards. Some brief notes relating to these special surveys are included in subsequent chapters.

Chapter 2

CHAIN SURVEYING

Introductory remarks. Land surveying with the chain (with the addition of a cross staff, offset staff, a few poles, and the occasional use of a rough kind of clinometer) was at one time almost the only method of obtaining the necessary data for producing plans, and it still is, and must ever be, an essential part of a surveyor's education.

As a simple example, suppose that a plan is required of the piece of ground shown in Fig. 1.

The surveyor walks over the ground and fixes points, A, D, F, J, etc., called *stations*, and so arranged that the straight lines joining them will run near to the boundaries whose positions are to be shown on the drawing.

These lines are shown on the figure by 'dot and dash' lines.

The stations are marked, on the field, with *ranging rods*, and each of these straight lines is then measured with a chain. At the same time perpendicular measurements called *offsets* are made to the crooked boundaries. The offsets along the line FJ, for example, are shown on the figure by dotted lines. They are taken wherever there is a bend or other special feature in the boundary, and at the end of each chain when the boundary is a smooth curve.

The lengths of the offsets being known, and also their positions along the survey line FJ, it is evident that all points to which they have been measured can be fixed on the drawing. These points are then joined up, according to a sketch, to fix the various boundaries, etc.

Field-book for topography. The book in which the measurements are entered is called the *field-book*, and the various details to which measurements are taken are termed *topography*. The form of this is not always the same, but most commonly it is ruled with two lines down the middle of the page, as in Fig. 2, which is a field-book for the line F to J of Fig. 1.

In writing down the measurements, we begin at the bottom of the page and work upwards. All distances along the straight survey line are entered in the middle column, and the offsets are entered on the *right* or *left* of this column, according to whether they are right or left of the survey line.

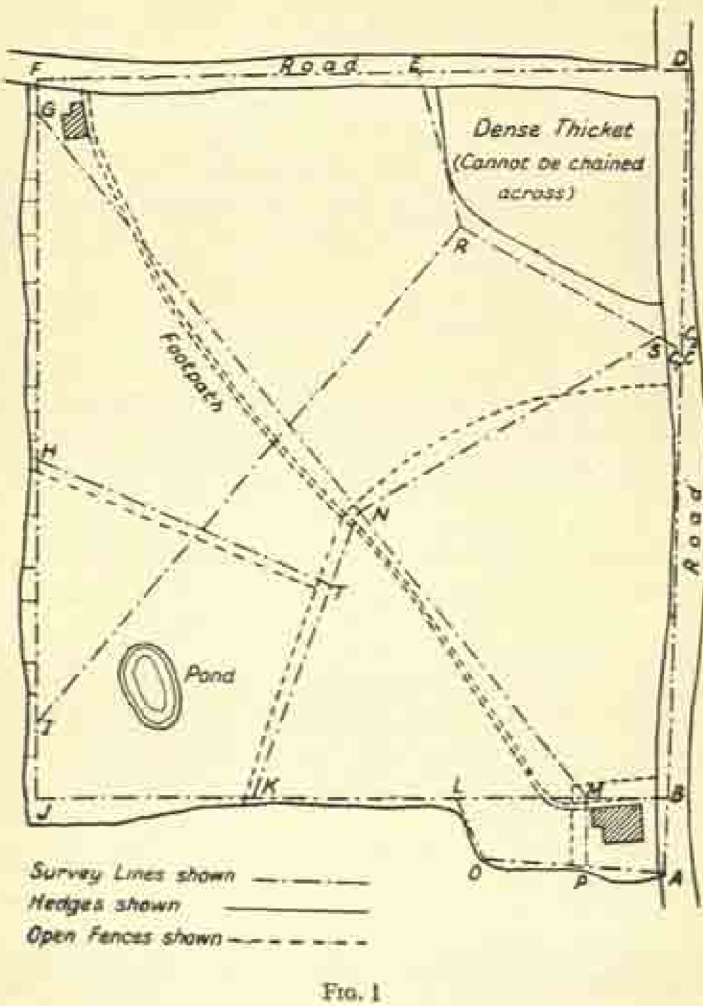

Fig. 1

Thus in the line FJ (Fig. 1), if we start chaining from F, the boundary will lie on the right. All offsets are, therefore, to the right in the field-book, as shown in Fig. 2, and are entered as there illustrated. It will be seen that as the work proceeds a sketch of the boundaries or other objects shown is made in the field-book; this sketch is not necessarily to scale, but is merely to show the draughtsman how to join up the points fixed by the measurements.

The keeping of a neat clear field-book is by no means easy, and it is important that the sketch should never be carried ahead of the measurements, but advanced line by line as the offsets are measured. Special care must be taken at the beginning and end of each line to indicate the connection between the boundaries shown on that line and those adjacent to it.

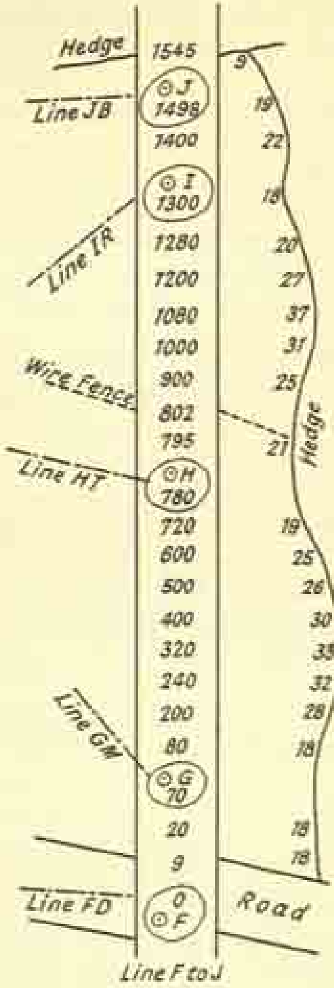

FIG. 2

LINES CROSSING. The manner of dealing with a boundary which crosses the chain line is shown in Fig. 2, by the fence near station H, crossing at chainage 802 feet. The broken line which represents this fence must not be carried across the central column, but, having been drawn to meet, say, the *right*-hand side of this column, it starts afresh at the point directly opposite on the *left*-hand side. The figure (802) between these points shows the chainage at which it crosses.

OTHER FORMS OF FIELD-BOOK. Many surveyors prefer to use a single line down the middle of the page, but in the opinion of the writer a double line, as here shown, is preferable for ordinary work. For complicated large-scale work, where the field-book would better be called a dimensioned sketch plan, the single line may, however, be more convenient, and is much used in railway-maintenance work, especially near stations.

Measuring the offsets. The offsets are measured either with a steel or linen tape, or with an offset rod. The tape is, in general, more convenient. If a large scale is to be used and great accuracy is required, the steel tape is best, as the linen tape is liable to stretch.

If used, the latter should therefore be tested against the chain occasionally to see whether the error is sufficient to be noticeable on the scale of the drawing.

Offset rod. The offset rod is a light rod shod with iron, either ten links in length, marked at each link, or six feet in length, marked at each foot. A link is one-hundredth of a Gunter's chain and measures 7·92 inches. An offset rod is used for taking short offsets and for marking positions, and may be formed at the top into a hook of such construction that the handle of the chain may, with equal facility, be pushed or pulled through a fence with it.

These rods are often made without the hook, and, as they are used for ranging the lines, are called *ranging rods*, or, sometimes, *pickets*.

Continuous offsets. Where more distances than one have to be taken upon the same offset, they may either be taken and booked separately, or in running measure. When the offset rod is being used, the former is the more convenient plan, but when a tape or the chain itself can be stretched, the latter is preferable. Whichever method is adopted, there must be no confusion in the field-book, and no possibility of mistaking the one form of entry for the other when plotting, and it will be found of great service, when re-plotting from old books (when possibly all details of this kind have long since been forgotten), if a uniform system has been adhered to.

For an example of these two methods of booking, see line 16 in the field-book for the survey shown in Fig. 16, where it will be seen that the offsets at 200 and 216 are in accordance with the second method.

Method of taking houses, etc. Sharp corners, such as those of a house, unless they are very near the chain line, may require to be fixed more accurately than by one offset. The left side of Fig. 3 is an enlargement of the house near B in Fig. 1, and the right side is a field-book for a portion of the line BJ, showing how to take the house.

The surveyor gets into line with the sides *ab* and *cd* of the house, standing himself on the chain line, and notes the chainage of the two points *e* and *f* so found – in this case 46 and 163·5 feet. He then measures the distances *eb* and *fd* (viz. 27 and 38 feet) and enters them as shown, the dotted lines in the field-book indicating that they are not perpendicular offsets, but in the direction of the sides of the house produced. He also measures the diagonals *ed* and *fb* and enters them as shown in Fig. 3.

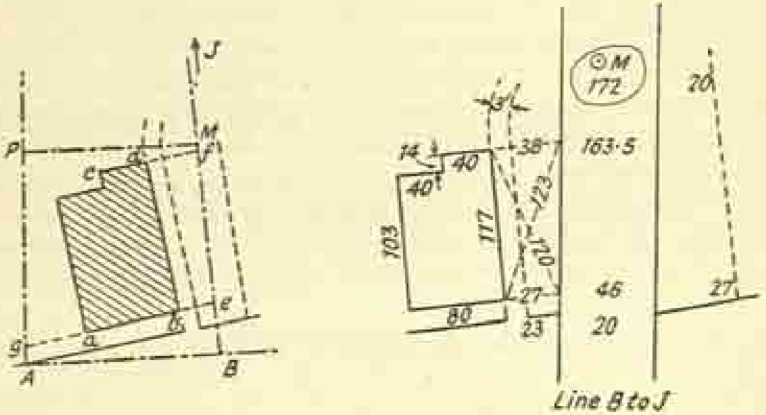

Fig. 3

To plot the figure, the draughtsman marks 46 and 163·5 feet along the chain line. Then circles having these points respectively as centres, and 27 and 123 feet as radii, will meet at the plan of *b*. Similarly *d* is fixed, and so one side of the house. The distance *bd* should then check with its measured length (117 feet). The directions of *ba* and *dc* are then fixed by joining *b* and *d* to the points 46 and 163·5 feet on the chain line; these should give right angles at *b* and *d* if the building is square, and the rest of the building is put in from the dimensions in the field-book.

Alternatively, one side of the house is sometimes produced to meet two chain lines. Thus *ab* produced meets BJ at *e*, and AP at *g*. The positions of these points being known, and also the lengths *eb* and *ga*, *ab* is obviously fixed. If the plan of the house is complicated, details of it can then be given on a separate page of the field-book.

LAYING OUT DIRECTIONS OF OFFSETS. The directions of the perpendicular offsets are usually laid out by eye. The surveyor places himself just behind the chain as it lies on the ground, and lays down the offset rod or tape in approximately the correct direction for the offset. He then looks carefully to see if it is at right angles to the chain as well as he can judge by eye, and, if not, he moves either his own end or that farthest from him according to whether he wants the offset to a fixed point on the boundary or from a fixed point on the chain. Lack of care in this matter is responsible for many bad results obtained by beginners.

If the offset is long or important, its direction should be laid out more exactly. To do this by chain and tape only, suppose an offset is required at a point A on the chain line. (Fig. 4). Set off AB, AC, each, say, 30 feet long. Then if we find a point D such that BD = CD, AD will give the direction of the offset. For this purpose, choose a length for BD and CD, say 50 feet. Then $50 \times 2 = 100$ feet. Now hold the end of the tape or chain at B, and the 100-foot point (that is, the other end in this case) at C. A second assistant then holds an arrow at the 50-foot point of the tape, and, keeping it there, moves until both ends of the line are taut. This fixes D.

With the lengths chosen AD should be 40 feet, as $40^2 = 50^2 - 30^2$. This gives a check on the work, or this relation may itself be used to lay out the direction, in which case only B *or* C will be required.

If the offset is required *from* D, we swing any convenient length of tape with D as centre to meet the chain line at B and C. Then make BA = half of BC, and measure AD.

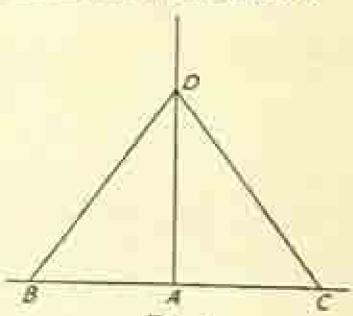

FIG. 4

Many similar methods will suggest themselves. Instrumentally, the offsets are laid out by *cross staff*, or more commonly, by the *optical square*.

THE CROSS STAFF. The old-fashioned cross staff head of wood has been superseded by one of brass (*vide* Fig. 5), with a pair of sights at right angles to each other, and another pair making angles of 45° with the first. Some again have the lower portion divided in a similar manner to a theodolite, and the upper portion made to revolve, thus admitting of angles of any number of degrees being laid off. These heads are adapted to fit on special staves shod with iron, and about 5 feet in length. They are exceedingly useful in small intricate surveys.

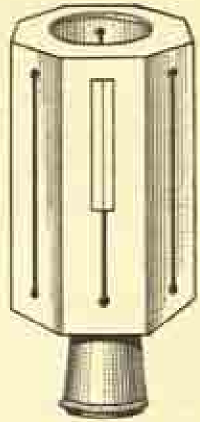

FIG. 5

THE OPTICAL SQUARE. The principle of the optical square is shown in Fig. 6. The instrument consists of a cylindrical metal box from 1½ to 2½ inches in diameter, and ¾ of an inch deep, having two square or oblong apertures, B and D, on its circumference at right angles to each other, and also a small circular eye-hole, A, diametrically opposite to the aperture at B. Two mirrors, E and F, inclined to one another at an angle of 45°, are mounted inside the box with their planes perpendicular to the base. The upper half of the mirror E is silvered, while the lower half is of plain glass. This mirror is placed opposite to the eye-hole A and in a line with the aperture B, about mid-way between the latter and the centre of the instrument, and is inclined to the axis of the instrument drawn through the eyehole and the centre, at an angle of 120°. The second mirror F is silvered all over; it is situated diametrically opposite to the aperture at D, and it is inclined to the axis AB at an angle of 165°. The diagram shows that a ray of light coming from D along DC, will undergo reflection at each of the mirrors F and E, and will finally coincide in plan with the direct ray BA coming through the unsilvered glass.

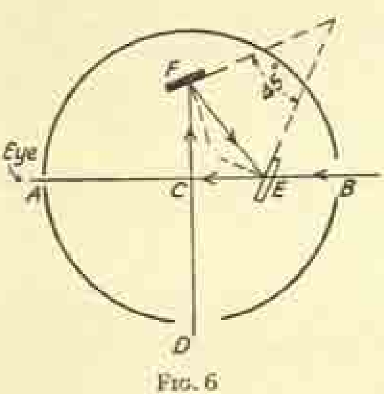

FIG. 6

The instrument is sometimes made without the outer cylindrical box, and with both glasses unsilvered on the upper or lower half. This is rather more convenient for observing offsets on either side, as, with the form shown in Fig. 6, the instrument must be turned upside-down for offsets on one side or the other.

To use the instrument, let A_1B_1 be the survey line (Fig. 7), and suppose an offset is required from a point C on it to a curved boundary. The surveyor stands at C, with the optical square held to his eye, and looks at the ranging rod at B_1 through the apertures A and B (Fig. 6). Objects on the curved boundary will then be visible in the silvered part of the glass E by reflection from F. The offset man is then directed by the surveyor to that point D_1 of the boundary which appears to be exactly above the ranging rod B_1 as seen in the unsilvered part of the glass E. The line CD_1 will then be perpendicular to A_1B_1, and is measured as the offset.

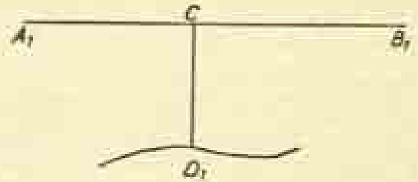

Fig. 7

If D_1 be a corner in the fence (or of a building) the offset will be required *to* it, instead of *from* a fixed point C on the chain line. In this case, the surveyor walks along the chain, holding the square as before, until the corner D_1 as seen in the silvered part of mirror E, appears to coincide with the ranging rod B_1 as seen below it.

Adjustment of the optical square. To adjust the instrument, set out four poles A, A′, B′, B in line (Fig. 8). Stand at any point C,

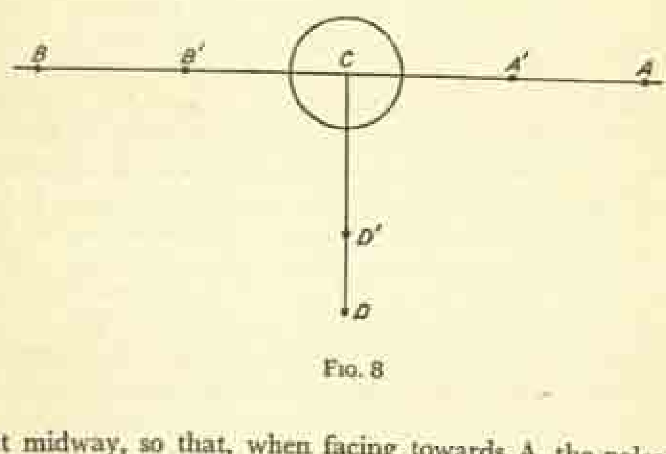

Fig. 8

about midway, so that, when facing towards A, the poles A′, A appear to coincide with one another.

Keeping them so, fix two poles, D′, D, so that they both appear to coincide with A, A′, and hence, of course, are also in line with one another.

Now turn round to face D, D′, keeping those poles in line with one another in one mirror, and move (if necessary) until B and B′ also coincide in the other mirror. Then if the poles D, D′, are exactly in line with B and B′, the adjustment is perfect. But, if not, fix a pole D″, which *is* exactly in line with B and B′; while doing so, be careful to keep D, D′ in line and also B, B′. This use of two poles in each direction is necessary to ensure that the position of the square remains absolutely fixed throughout. It is usual to place a pole at C as a rough guide to position.

Finally, a new pole is placed midway between D″ and D′, and the mirror F (Fig. 6) is adjusted by the screws which fix it to the case until this new pole comes into line with B and B′. D and D′ must still be kept in line with one another.

The whole length AB should not be less than about 200 feet.

Alternatively, D″ may be fixed by facing B, B′, turning the instrument round (or upside-down) so as to see objects on the *left* by reflection.

After adjustment, the test should be repeated.

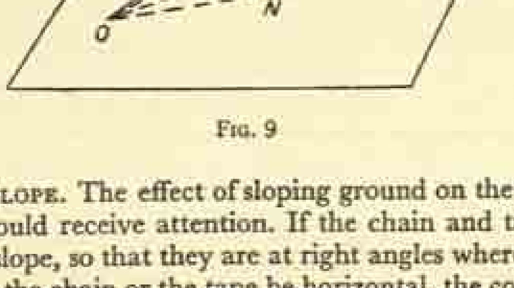

FIG. 9

EFFECT OF SLOPE. The effect of sloping ground on the direction of an offset should receive attention. If the chain and tape be both laid on the slope, so that they are at right angles where they meet, then if *either* the chain or the tape be horizontal, the corresponding directions *in plan* will also be truly at right angles to one another. But if *both* be sloping this will no longer be true. In Fig. 9, let OA be the chain line, at an angle α to the horizontal OB, and let AC be the offset, supposed perpendicular to OA, and inclined at an angle β to the horizontal AE.

OB, BD are the plans of OA, AC, and it is required to find the angle OBD, which the plan of the offset should make with the chain line on paper for a correct result.

The angle is clearly independent of the lengths, so we will assume $OB = AE = l$.

Now $\qquad OA = OB.\sec\alpha = l.\sec\alpha$
and $\qquad AC = AE.\sec\beta = l.\sec\beta$

Put $AOC = \theta$; then as $OAC = 90°$, we have—

$$\tan\theta = \frac{AC}{OA} = \frac{l.\sec\beta}{l.\sec\alpha} = \frac{\cos\alpha}{\cos\beta} \quad . \quad . \quad . \quad (1)$$

Now put $DOC = \phi$, OD being the plan of OC;

then $$\sin\phi = \frac{DC}{OC} = \frac{BA + EC}{OC} = \frac{l\tan\alpha + l\tan\beta}{OA.\sec\theta}$$

But $\qquad OA = l\sec\alpha$

$$\therefore \sin\phi = \frac{\tan\alpha + \tan\beta}{\sec\alpha.\sec\theta} \quad . \quad . \quad . \quad (2)$$

Then $OD = OC\cos\phi = l.\sec\alpha.\sec\theta.\cos\phi$, which gives the length of OD, as θ and ϕ are now known. Lastly, in the triangle OBD, $OB = BD = l$.

Hence, if N be the middle point of OD, $BND = 90°$, and we have—

$$\sin\tfrac{1}{2}OBD = \tfrac{1}{2}OD \div l = \tfrac{1}{2}.\sec\alpha.\sec\theta.\cos\phi \quad . \quad . \quad (3)$$

Hence, knowing the slopes, we find θ from (1), then ϕ from (2), and lastly OBD from (3).

The following table shows the approximate results for different slopes.

VALUES OF ANGLE *OBD* IN FIG. 9

Values of α	Values of β				
	2° 52′	5° 43′	8° 8′	11° 19′	14° 2′
	° ′	° ′	° ′	° ′	° ′
$\frac{1}{20} = 2° 52′$ . . .	89 51	89 43	89 36	89 26	89 17
$\frac{1}{10} = 5° 43′$ . . .	89 43	89 26	89 11	88 51	88 36
$\frac{1}{7} = 8° 8′$ . . .	89 36	89 11	88 50	88 22	87 58
$\frac{1}{5} = 11° 19′$ . . .	89 26	88 51	88 22	87 42	87 8
$\frac{1}{4} = 14° 2′$ . . .	89 17	88 36	87 58	87 8	86 26

If both slopes are of the same sign (both *up* or both *down*), the offset is to be plotted on the paper at this angle with the *back* portion of the chain line. But if the slopes are of *opposite* signs, this angle must be laid off with the *forward* position of the chain line to right or left as the case may be. The lengths must also be corrected for slope, of course.

If *either* the chain line or the offset tape is horizontal there is no error, so that it is usual to hold the tape horizontally and look vertically down upon it. This has the advantage that no correction is necessary to the measured *length* of offset. The right angle cannot, however, be judged with the same accuracy on this system as when chain and tape are in contact and lying on the ground.

LIMITING LENGTH OF OFFSET. Where the direction of the offset is laid out by eye, it is frequently stated that its length should not be more than some arbitrary length. This may be 50 links, 100 links, or 50 feet, for example. Evidently, however, this should depend upon the scale, nature of ground, etc.

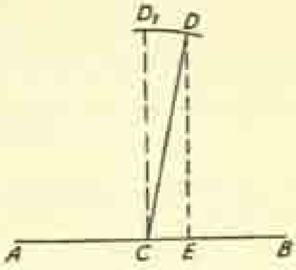

FIG. 10

Thus suppose an offset is measured from a point C on the chain line AB (Fig. 10) to D, and it is laid out so that the angle BCD = 86° instead of 90° (that is, with an error of 4° in direction). The draughtsman plots the length CD at right angles to AB, so that D becomes displaced to D_1. Now $DD_1 = CD \sin 4°$ very nearly $= l \sin \epsilon°$, if l be the length of offset and $\epsilon°$ be the angular error in direction. The object is that this error should not be appreciable on the paper.

Now suppose the scale be n units of length to the inch.

Then the length of CD on paper will be $\frac{l}{n}$ inches if l be the measured length.

Hence DD_1 will be $\frac{l \sin \epsilon°}{n}$ inches.

If 0·01 inch be regarded as the limit of displacement (that is, if we assume that a draughtsman can work to 0·01 inch), we then put

$$\frac{l \sin \epsilon°}{n} = 0{\cdot}01,$$

whence $$l = 0{\cdot}01\, n \operatorname{cosec} \epsilon°$$

the value of ϵ being found by laying out several offsets by eye, and testing them instrumentally to find the maximum error. Then l as calculated above gives the maximum length of offset.

Thus, if the scale be 3 Gunter's chains to 1 inch,

$$l = 0{\cdot}01 \times 3 \times \operatorname{cosec} 4^\circ = 0{\cdot}43 \text{ of a chain, or 43 links.}$$

Similarly, if the scale be 40 feet to 1 inch,

$$\begin{aligned} l &= {\cdot}001 \times 40 \times \operatorname{cosec} 4^\circ \text{ feet} \\ &= 6 \text{ feet approximately} \end{aligned}$$

This indicates the great importance of ensuring that offsets are either measured perpendicular to the chain line or kept strictly within the prescribed limits, which, for large-scale plans, are very short.

If D be a point on a fence nearly parallel to AB, then the offset may be longer. The important displacement in this case becomes the difference between CD_1 and DE, or $l(1 - \cos \epsilon^\circ)$. The fence is shifted *away* from the chain line by an amount given by this formula. In inches on paper this becomes $\frac{l}{n}(1 - \cos \epsilon^\circ)$.

Effect of scale on limit of error. The following table shows the approximate length, in feet or links, which will be represented on paper by 0·01 inch to different scales.

If we assume 0·01 as the limit to which a draughtsman can draw, this means that any shorter length than that given in the table cannot be represented on the paper, and consequently the table gives an indication of the kind of accuracy to be aimed at in measuring the offsets.

Scale	Length shown by 0·01 in.	
	Feet	Links
1 in. to 1 mile	53	80
6 in. to 1 mile	9	13
10 chains to 1 in.	—	10
1 chain to 1 in.	—	1
$\frac{1}{2500}$	2	3
100 feet to 1 in.	1	—
10 feet to 1 in.	0·1	—
20 links to 1 in.	—	0·2
$\frac{1}{500}$	0·4	0·6

The methods of fieldwork described in this chapter would seldom be used for surveys to be plotted on so small a scale as 1 inch to 1 mile.

Errors in length and direction combined. We will next consider the combined effect of errors in the linear measurement of an offset and in laying out the direction thereof.

(1) Given an angular error α in laying out the direction, to find the degree of accuracy with which the length of the offset must be measured, so that the maximum displacement of a point on the paper from one source of error may not exceed that from the other source.

In Fig. 11, let AB be the true distance and direction to B from A.

Let it be measured as AB_1, and plotted as AB_2.

Then BB_1 is the displacement due to the incorrect measurement, and B_1B_2 that due to incorrect direction.

Let $AB_2 = l$, the measured length.

Then $$B_1B_2 = l \, . \, \alpha \text{ if } \alpha \text{ be in radians}$$ $$= l \sin \alpha \text{ nearly}$$

and if the offset be measured with an accuracy of 1 in n,

$$B_1B = \frac{1}{n} \times l = \frac{l}{n}$$

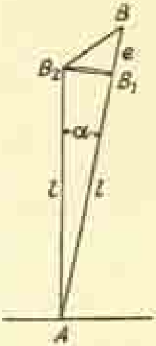

Fig. 11

$$\therefore l \sin \alpha = \frac{l}{n} \text{(as the two errors are assumed equal), or } n = \operatorname{cosec} \alpha$$

Thus if $\alpha = 3°$, cosec $\alpha = 19{\cdot}1$, or the offset must be measured with an accuracy of at least about 1 in 20.

If the offset is measured with an error not exceeding 1 part in 1000, $n = 1000$, $\therefore \alpha =$ about 3 minutes, so that the offset must be very carefully laid out.

(2) Given the scale, and given that the angular error and accuracy of measurement are adjusted so as to give equal maximum displacements, to find the maximum length of offset so that the displacement on the paper from both sources of error may not exceed 0·01 inch.

In Fig. 11, BB_2 is the total displacement, and we may take $BB_1B_2 = 90°$, very nearly.

Hence as $BB_1 = B_1B_2$, we have $BB_2 = \sqrt{2} \times BB_1$. Now $BB_1 = \frac{l}{n}$ where l is the measured length in links or feet, and the accuracy is 1 part in n.

Hence $BB_2 = \sqrt{2} \times \frac{l}{n}$ links or feet, as the case may be, actual length.

Now suppose the scale is 1 inch to m links or feet.

Then the length *on paper* corresponding to BB_2 will be

$$\frac{1}{m} \times \sqrt{2} \times \frac{l}{n} \text{ inches.}$$

But this is to be 0·01 inch.

$$\therefore 0{\cdot}01 = \frac{1}{m} \times \sqrt{2} \times \frac{l}{n}$$

whence
$$l = \frac{m \times n}{100\sqrt{2}} \text{ links or feet}$$

Example:

Accuracy of offsets = 1 part in 400 ($n = 400$)
Scale of plan: 50 ft. to 1 inch ($m = 50$)

Maximum permissible length of offset $= l = \frac{400 \times 50}{100\sqrt{2}}$ feet = 141 feet.

In practice this would be reduced to 100 feet since tapes are not usually made in greater lengths than this and, for convenience in measuring, offsets should be kept within the length of the tape.

(3) If the maximum error in the length of any offset be estimated at a given figure, say e links, the maximum length of offset l links, and the scale 1 inch to m links, find the maximum allowable error in laying off the direction of the offset, so that the maximum displacement may not exceed 0·01 inch on paper.

In Fig. 12, $BB_1 = e$ links, $B_1B_2 = l \sin \alpha$ very nearly,

$$\text{and } BB_2 = \sqrt{e^2 + l^2 \sin^2 \alpha}$$

$$\therefore BB_2 \text{ on paper} = \frac{1}{m}\sqrt{e^2 + l^2 \sin^2 \alpha} = 0{\cdot}01 \text{ inch}$$

$$\therefore \frac{m^2}{100^2} = e^2 + l^2 \sin^2 \alpha$$

or
$$\sin^2 \alpha = \frac{1}{l^2}\left(\frac{m^2}{100^2} - e^2\right)$$

Example. If maximum error in offset = 2 links = e; scale, 1 inch to 400 links; maximum length of offset, 50 links.

Here $\sin^2\alpha = \frac{1}{2500}(16-4)$

or $\sin\alpha = \frac{1}{50} \times \sqrt{12} = \frac{1}{100} \times \sqrt{48} = 0{\cdot}07$

or $\alpha = 4^\circ$ nearly

As a practical exercise the inquiring student may usefully chain a portion of a line, and at four points lay out offsets: (*a*) by eye; (*b*) by optical square; (*c*) by construction (p. 12); (*d*) by construction again or by cross staff.

At each point offsets should be laid out independently on both sides of the chain line, using the same method for both.

The angles should then be measured by theodolite at each point; and the whole should be repeated on sloping ground.

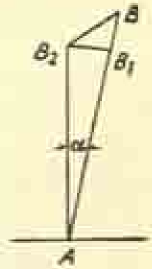

FIG. 12

LENGTHS OF CHAINS. In England two types of chain are commonly used. These are the Gunter's chain, 66 feet long, and the so-called 'Engineer's' chain, 100 feet long. The Gunter's chain is used in railway work; 10 of these chains go to 1 furlong, or 80 to the mile, and the radii of railway curves are invariably measured in chains of this length. The Gunter's chain is also useful in ordinary land-surveying where areas are required in acres, as exactly 10 square chains make one acre, whereby the computation of the area is simplified.

The 100-foot chain is more convenient in surveys for water-supply and sewerage schemes, for dock and harbour work, in road construction, and in civil engineering contracts of all types, the advantages mentioned above being less important in these cases. Its use is now becoming more common for all kinds of surveys.

Abroad, where the metric system is in use, the most common lengths of chain are either 20 or 25 metres, the length of each link being accordingly either 2 or 2·5 decimetres. In the Gunter's chain the length of each link is 0·66 foot, or 7·92 inches; in the 100-foot chain, each link is of course 1 foot long. In the 20-metre chain the link is 7·87 inches long, so that it is liable to be mistaken for a Gunter's chain. Usually in England the word 'chain' is understood to mean a Gunter's chain unless otherwise stated. In America and many of the colonies the 100-foot chain is chiefly used.

Both the Gunter's and the 100-foot chain are divided into 100 sections, each *tenth* section from either end being marked by a brass tag or tablet. The tag which marks the tenth section has one point (or sometimes one hole drilled through it instead); that which marks the twentieth section from either end has two points, or holes, and so on. These tags obviously facilitate the counting of the sections when the length being measured is not an exact number of chains. Constant watchfulness is, however, necessary to avoid confusion, especially where the number of links or feet is between 40 and 60, or 60 and 70, as the 40 tag is the same as that at 60. A folded chain is illustrated in Fig. 13. It is packed in this shape by commencing at the middle point of an outstretched chain and bringing the sections together in pairs to form a double cone. The chain may be either of the ordinary type of iron or steel sections, looped at each end and connected by small rings, or it may be in the form of a continuous steel band when it is called a 'band chain'. The latter is much less liable to stretch in use, and is lighter to handle; but it does not last so long as the ordinary chain, and, unless carefully dried and oiled after getting wet, it is liable to break without notice, and is not easily repaired in the field. Repair outfits, consisting of a combined punch and riveter together with a supply of soft brass rivets, are obtainable, however, but some odd pieces of old tape of the correct section are required, as well, in order to mend a break. Steel bands, oval in section, have been used. These are less liable to break than the flat ones, and easier to handle than the ordinary chain.

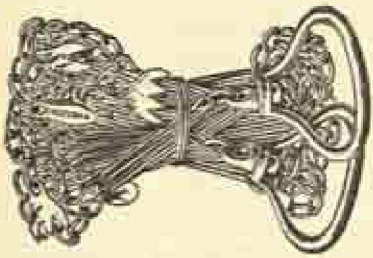

Fig. 13

Steel tapes, graduated to fractions of an inch, should always be used in engineering surveys where great accuracy is required. They must also be used for all accurate setting-out, such as occurs in permanent-way work, steelwork or machinery foundations.

ARROWS, OR PINS. Arrows, or pins, are used to mark the end of each chain, and are made of stout iron or steel wire, about 15 inches in length, one end being formed into a ring for facility of handling, and to which a piece of red tape is conveniently fixed. Ten such arrows are generally used, but this is not always the case, as some surveyors prefer to work with only nine, while others find virtue in using eleven, but in every case the object is to obtain a correct record of the number of chains run, and especially of every tenth chain.

It is convenient to allude to the person drawing the chain as the 'leader', and to the person following and directing it as the 'follower'; the latter is usually the surveyor himself.

Those who work with ten arrows instruct the 'leader' to mark the end of the tenth chain as usual with the tenth pin, and to stretch the eleventh chain without an arrow. The follower then hands over the ten arrows, and the leader sticks one in as usual. It is well to have the end of the tenth chain marked by a peg or stick for future reference. Similar systems may be followed with nine or eleven arrows.

It may, however, be considered as fairly established, that, taken all round, ten is the most convenient number of arrows to use, particularly if the tenth be of special pattern, so that it can be used for marking parts of a chain if desired.

USE OF ARROWS. It will have been gathered that, as the follower picks up the arrows as the chaining proceeds, the number of these held by him at any time indicates how many chains have been 'run' or measured.

If the line is more than ten chains long, the surveyor must personally superintend the changing over of the arrows as already stated. And in any case at the end of each line he must count the number of arrows held by the follower and see that it agrees with the length of the line as entered in his book, and hand the ten arrows to the leader to begin the next line. Neglect of these precautions will very likely lead to the dropping of a chain in one or more lines.

CHAINMEN. Much depends upon the choice of a suitable man to act as 'chainman', or 'leader', and to assist in ranging the lines, etc.

He should be active, intelligent, possessed of good eyesight, and capable of ranging a line with rods or sticks.

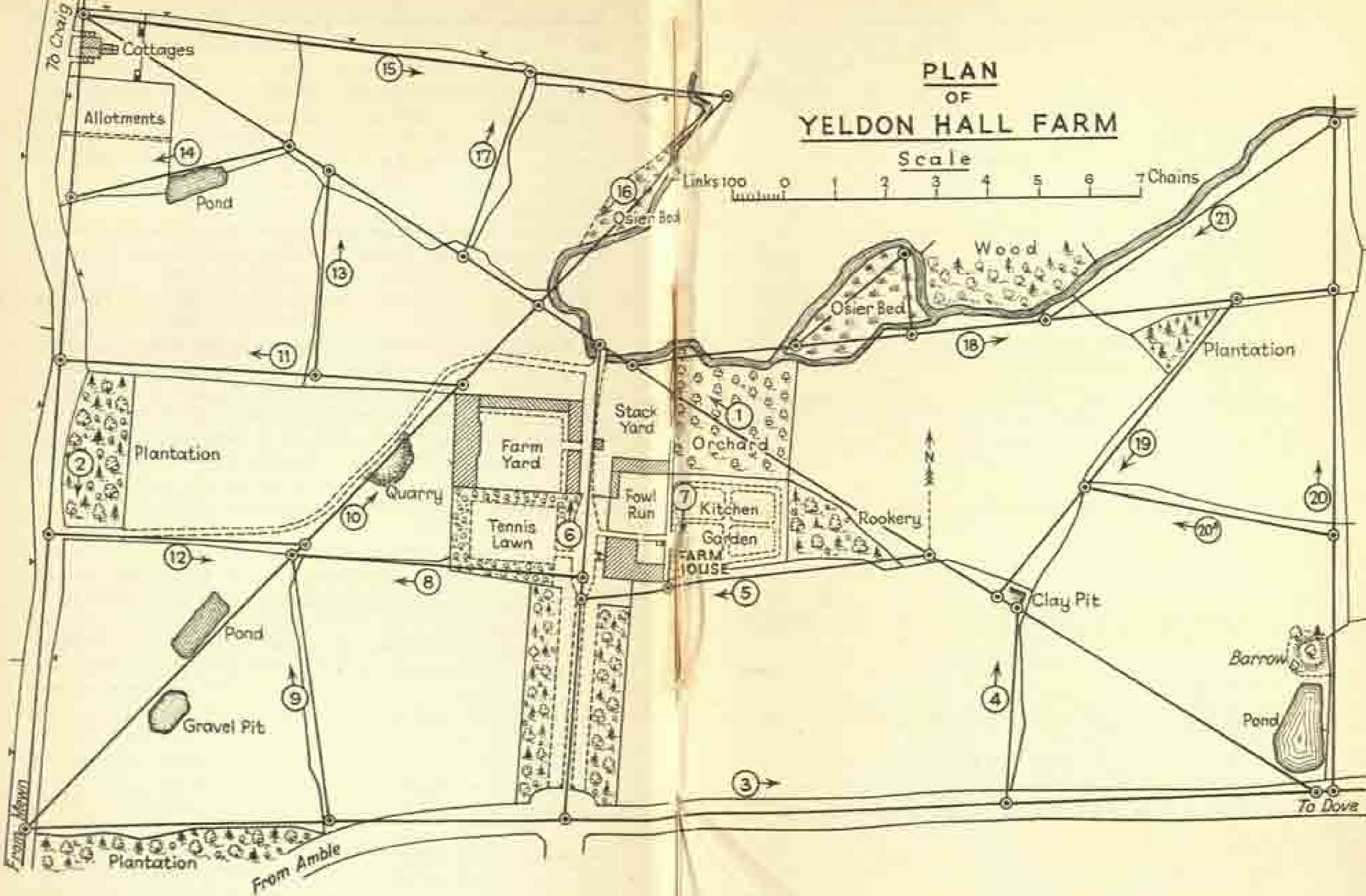
PLAN
OF
YELDON HALL FARM
Scale
Links 100 0 1 2 3 4 5 6 7 Chains
To Craig
Cottages
Allotments
Pond
Osier Bed
Wood
Osier Bed
Plantation
Stack Yard
Orchard
Farm Yard
Plantation
Quarry
Fowl Run
Kitchen Garden
Rookery
Tennis Lawn
FARM HOUSE
Clay Pit
Pond
Barrow
Gravel Pit
Pond
To Dove
From Mawn
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Fig. 14

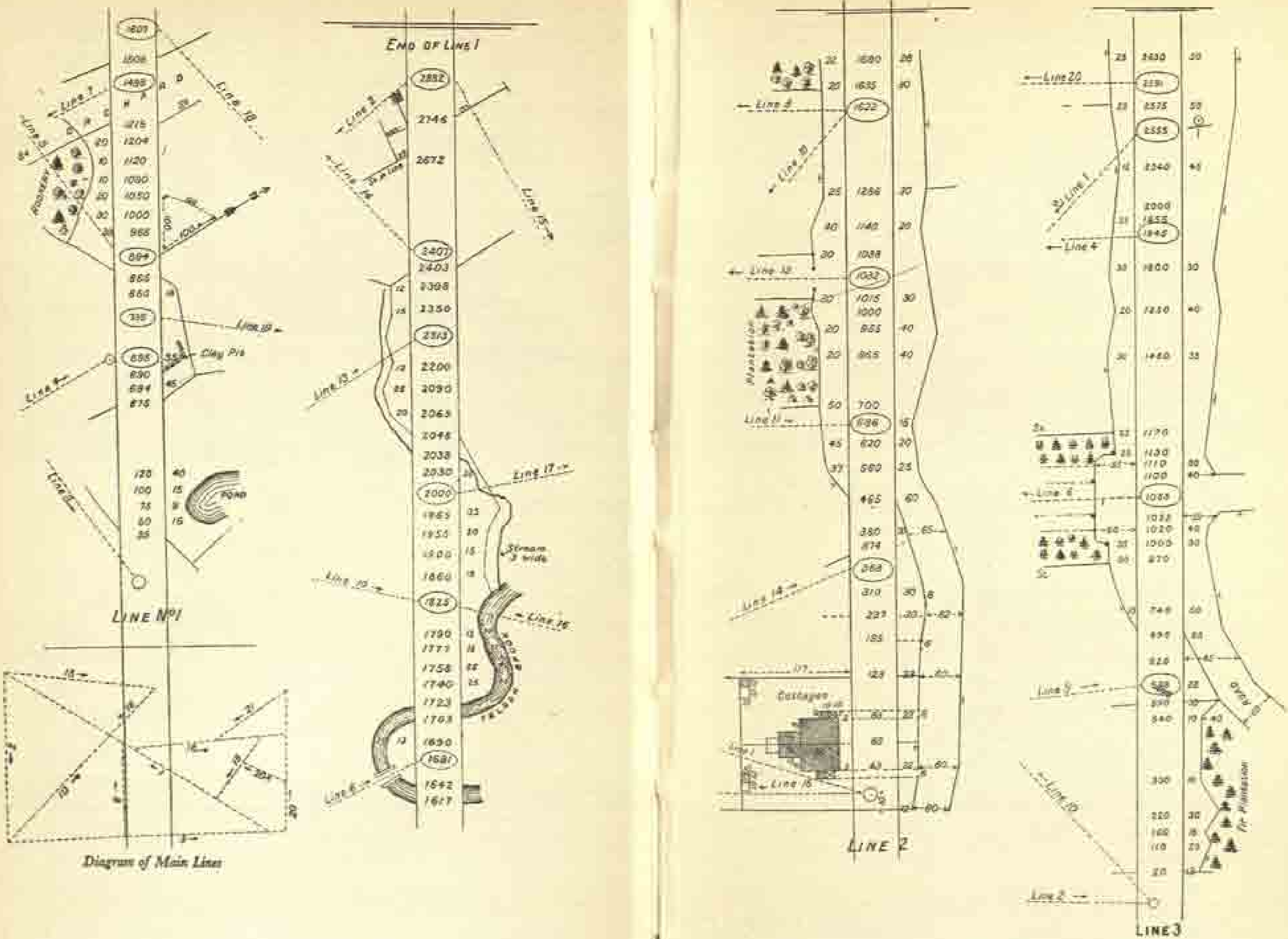

FIG. 14 (*continued*). FIELDWORK NOTES FOR SURVEY OF YELDON HALL FARM

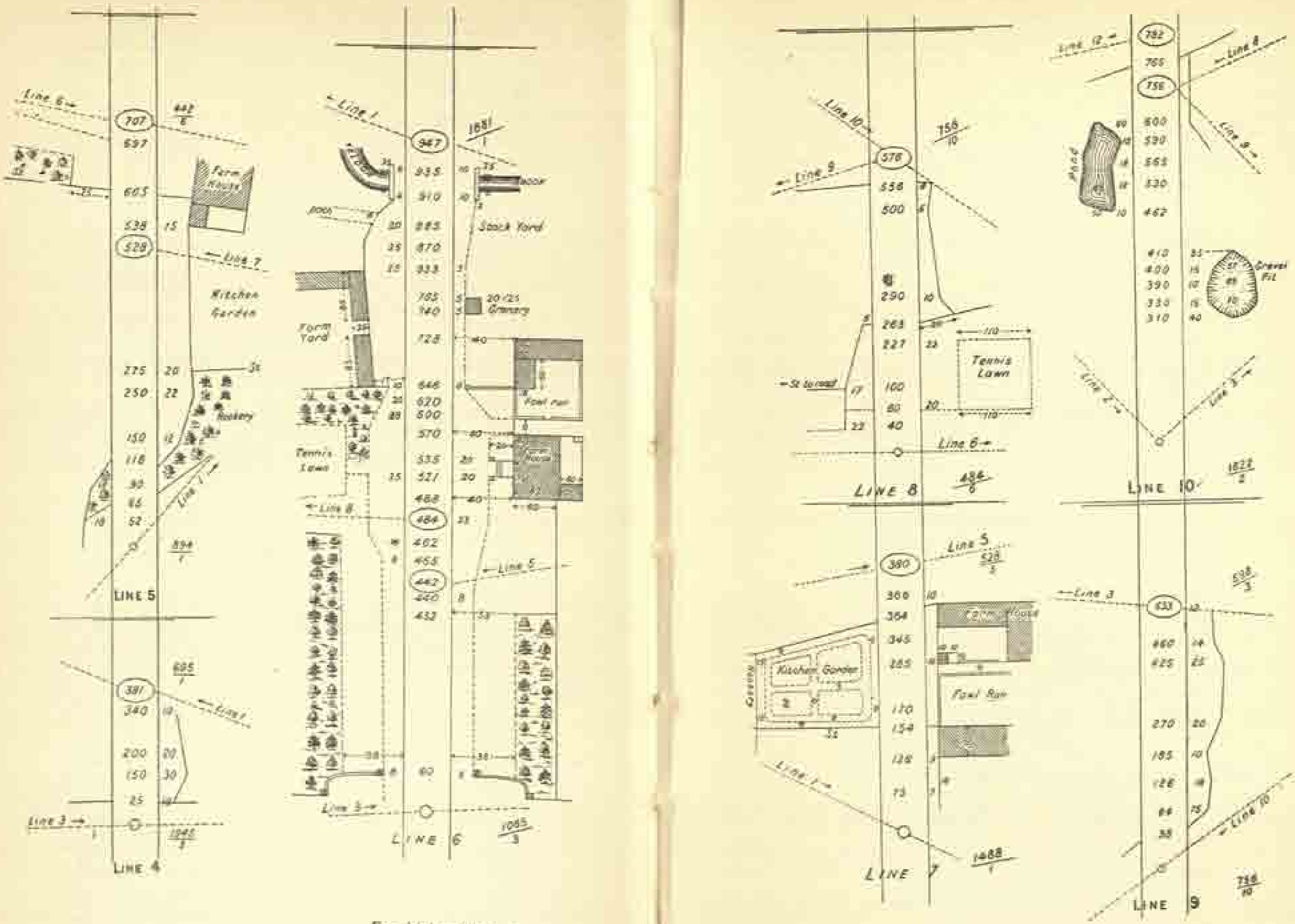

FIG. 14 (*continued*). FIELDWORK NOTES FOR SURVEY OF YELDON HALL FARM

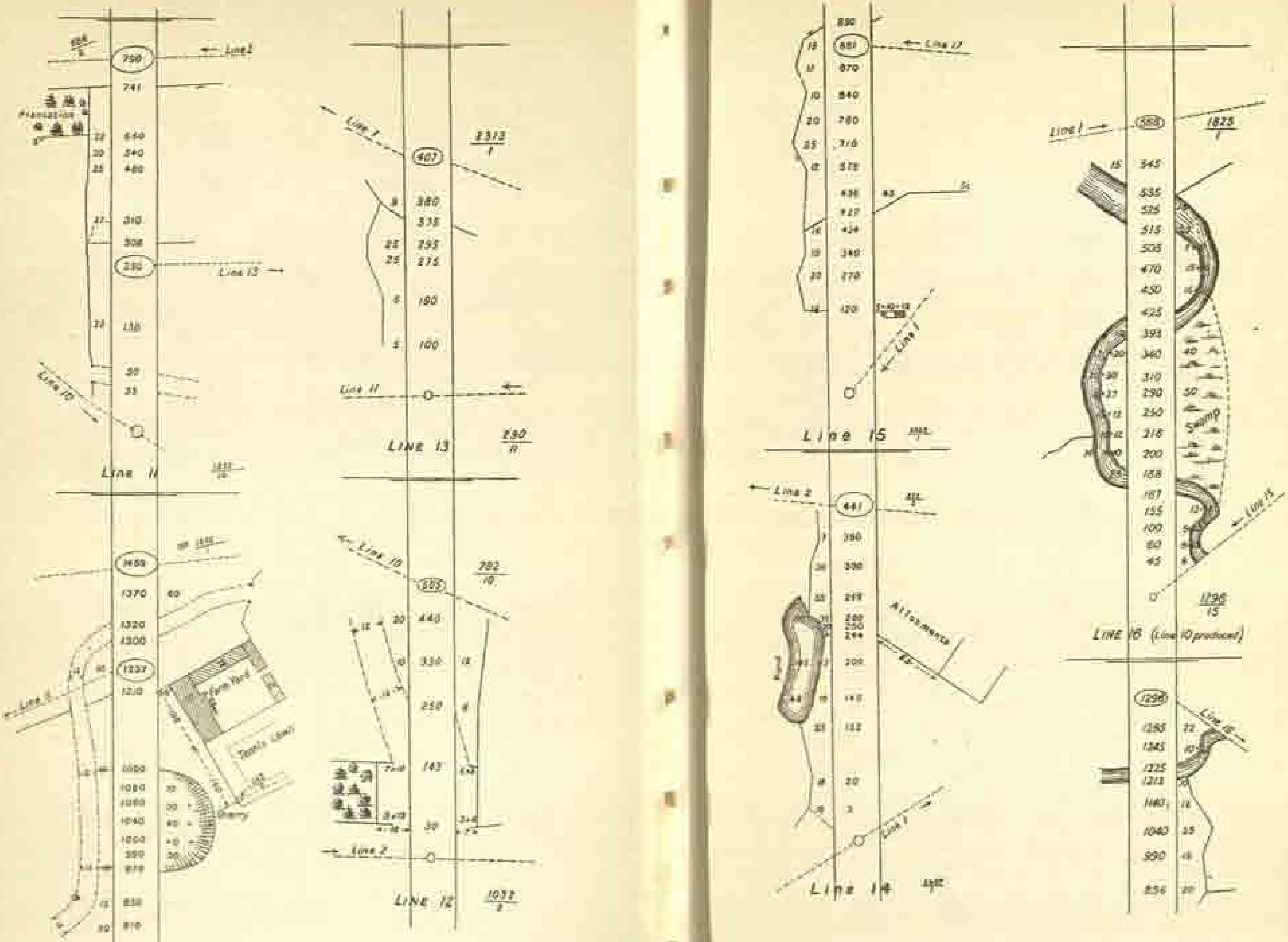

FIG. 14 (*continued*). FIELDWORK NOTES FOR SURVEY OF YELDON HALL FARM

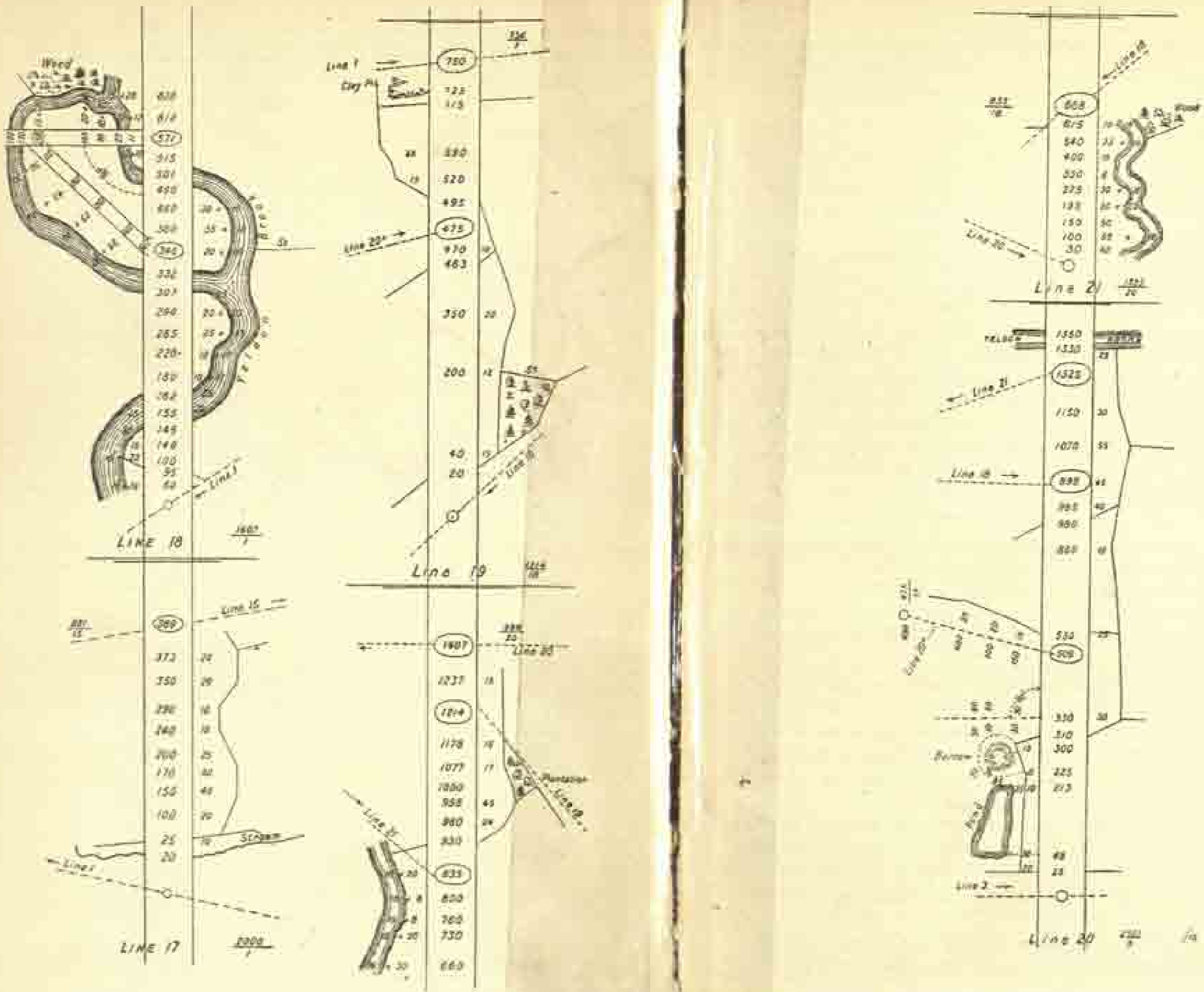

FIG. 14 (*continued*). FIELDWORK NOTES FOR SURVEY OF YELDON HALL FARM

If the survey be one of considerable extent, it is well to train a second man to act as 'follower', and to assist with offsets, so that the surveyor is free to attend to his offsets and booking. In this way the chain can be kept almost continually upon the move.

METHOD OF CHAINING. To chain a given line with accuracy and despatch is not quite so simple a matter as the uninitiated might suppose, and proficiency can only be attained by practice. Accuracy first, despatch second, must be the rule.

The points chiefly to be attended to are that the 'follower', when directing the 'leader' into line, has his eye directly behind the arrow and not on one side, and that the chain is straight.

The 'leader' is so trained that he at once faces the 'follower' and, by the aid of some back object or mark, sets himself approximately in line, the chain being held sufficiently tight to appear straight.

The 'follower' by a suitable signal indicates his wish for the 'leader' to move a little to the right or to the left, as the case may be, and, when satisfied that the direction is perfect, he stoops down, thrusts his thumb through the loop, and brings the handle of the chain into contact with the arrow.

The 'leader' then also stoops, holds an arrow vertically in front of the handle (but touching it), and receives his final direction with the point of the arrow touching the ground. The chain being tight and straight, he knocks in the arrow and is ready to proceed.

The 'leader' must be trained not to proceed, however, till he receives a signal to do so from the surveyor, if offsets are being measured.

The 'follower', as he rises, brings up the arrow upon his thumb, and transfers it to the little finger of his left hand as he walks along.

The 'follower' should avoid all drag upon the chain when it is being pulled along, and be careful to check the 'leader' when he arrives at the proper distance, without jerk, which not only injures the chain but ruffles the 'leader's' temper.

In order to be capable of undertaking large surveys when occasion requires, every surveyor should accustom himself to chain long lines towards a distant object, without much setting out with rods, and with care this can be accomplished as follows:

Before commencing to chain a line the 'follower' should plant his offset rod at its commencement, and, retiring a few yards behind it, examine the ground between it and the distant fore-mark for any well-defined objects that may exist in the exact line.

In cases where no such objects are exactly upon the line, there may be some very near to it, which will be useful to give a general indication of direction, and a little training to the eye will enable him to select and always identify more minute objects. Advantage should be taken of a gap, peculiarly shaped bush or bough in a fence, a conspicuous stone or clod, a thistle, a dock, or a tuft of grass, which may be in the exact line.

On moderately level ground, and with a well-defined fore-mark, such as a corner of a house, a church spire, or a single tree, lines of considerable length, up to half a mile, may be chained with perfect straightness, and without any previous setting out with rods.

The 'leader' must be well trained to select objects behind the starting point, marked by a flag, if necessary, to keep a constant check upon the direction, and should at once remark any accidental deflection from the true line.

The natural tendency is to continually drive the line on one particular side, every man seeming to have a bias for one side or the other. For laying out long lines optical instruments of some kind are now generally used.

With reasonable care, however, upon fairly level ground, and with a carefully tested chain, it should be possible to rely upon distances to within $\frac{1}{1000}$ part of the whole, i.e. one link in 10 chains or about $1\frac{1}{4}$ inches in 100 feet; but when rough or somewhat hilly ground has to be measured it is not easy to chain within $\frac{1}{500}$, i.e. two links in 10 chains or about $2\frac{1}{2}$ inches in 100 feet, while upon very hilly and rocky ground, or on bad boggy land, chaining by ordinary methods becomes unreliable. If great accuracy is necessary, some special means for securing it must be resorted to. In laying down lines which do not fit exactly, though within the margin of permissible error, care should be taken to spread the difference equally throughout the whole line, Thus, if a line measures 10,000 units on the ground, and the space on the plan in which it should fit measures 10,010 units, then it is evident that each 1000 mark should be plotted off as 1001.

INACCURACIES IN CHAINING. Inaccuracies in chaining may result from many causes, some of which are here indicated.

1. Through the chain not being hauled fairly taut at the moment that the 'leader' puts in the arrow.

2. Through the 'leader' not holding his arrow exactly at the chain end, or not having it exactly perpendicular when thrusting it into the ground.

3. Through the 'follower' not bringing his end of the chain fairly to the arrow, or allowing the 'leader' to drag his arrow out of the perpendicular.

4. Through kinks or knots in the chain, especially when it is first opened out.

5. Through inattention to the variations in length to which an ordinary chain is liable, either through wear in the many joints, or through the stretching or opening of the links, and want of care in testing for length.

6. Through inaccurate allowance upon sloping ground, or want of care in chaining over obstacles such as rocks, boulders or fences.

7. Through not keeping in proper alignment.

8. Through variations in temperature from which the chain expands or contracts.

Though this last should not be lost sight of, still it is trifling when compared with the preceding causes of error, as will be seen by the following remarks.

A variation of 80°Fahr., which will more than cover the ordinary range of winter and summer temperature in the British Isles, makes a difference of about $\frac{1}{1800}$ part, or 0·44 of an inch in a 66-foot chain.

Standards of length are accurate at about 60° Fahr., the mean temperature between our extremes of heat and cold, so that errors from this cause can scarcely ever exceed $\frac{1}{4}$ inch per chain of 66 feet.

TESTING CHAINS AND TAPES FOR LENGTH. Chains and linen tapes used for surveying purposes are liable to become inaccurate from various causes. The former sometimes get stretched by rough handling in being pulled through hedges and over broken ground, or they are shortened by links getting trodden on or twisted and thus bent. Tapes may be stretched by continued use in windy weather or shrunk by being soaked with rain. For this reason they should be frequently compared with some standard measure, or the work done becomes unreliable.

In most large towns, standards, such as that at Trafalgar Square, London, are available for this purpose, but in the field recourse must be had to some provisional method. It is then best to make use of a good steel tape, or, failing that, of a levelling staff. The distance being accurately measured off, two stout pegs are driven into the ground, and nails are inserted in their heads marking the exact length required; the chains can be compared with these at regular intervals during the survey, and either lengthened or shortened as required, by means of the adjustable links at the handles, or by removing one or more of the small rings connecting the links.

Tapes not being provided with any means of adjustment, any inaccuracy found to exist in their length must be noted and added to or deducted from measurements taken.

Arranging the lines. Having now seen how the measurements are taken along one line, we must consider the arrangement of the lines, so that the whole can be plotted.

It will be gathered that no angles are measured, so that the lines must be arranged in such a manner that they can be plotted from the lengths alone. Now a triangle is the only irregular figure which can be so plotted; hence the first essential is that there should be one or more main triangles as big as can be obtained on the ground.

In Fig. 1, for instance, GJM is the first main triangle, and would be set out first, JG and JM being arranged if possible so as to point towards gaps or thin places in the hedges, thus enabling them to be easily produced for short lengths GF and MB, to fix stations F and B outside the hedges on the road side. D is fixed next, so that the lines FD and DBA can be measured in the road. These lines would be inside the hedge also, but for the fact that we cannot chain across the bottom right-hand corner of the ground. The remaining stations, for what may be called the subsidiary lines, would not be fixed at once, but as the chaining proceeds.

Thus in chaining along FJ, station H would be fixed near the fence which crosses at 802, its position on the ground marked by ranging rod or otherwise, and the chainage 780 entered in the field-book, as in Fig. 2. The sign ⨀ on the left denotes a station.

Diagram of lines. As soon as the main stations are fixed, a sketch is made of them in the field-book, headed 'diagram of lines', with, of course, the name and date of the survey. For examples of such sketches, see Fig. 16 (p. 39), which is a diagram of lines for the survey in Fig. 1, and the field-book of Yeldon Farm Survey, p. 24.

The stations are lettered or numbered on the sketch. As new stations are fixed they are added on this diagram, and the letters or numbers by which they are to be known are affixed, so that the diagram, when complete, shows all the stations, and these are joined up to show which lines are measured.

This method is, in the opinion of the writer, much the best. But many surveyors number the lines instead of the stations. In this case, the number of the line which branches off at any station must be shown in the field-book, as, for example, lines 4 and 19 in the field-book of line No. 1 for Yeldon Hall Farm, p. 26. This complicates the field-book rather more than the mere booking of a station, whereas the object in view is that it should be as simple as possible. However, numbering the lines in the order in which they are measured facilitates reference to the field-book of any line.

GENERAL CONDITIONS. The general conditions to be kept in view in arranging the lines are:

(1) Arrange one or more main triangles as already stated.

(2) All the lines must be capable of being plotted from their measured lengths alone, by intersecting arcs.

(3) If any lines have to be produced on paper after being plotted, the part produced should be very short in comparison with the part already laid down, because, otherwise, unavoidable small errors in plotting will be magnified when the line is produced (see GF and MB, Fig. 16).

(4) All triangles should be what is called 'well conditioned'. That is, the arcs whose intersection is to fix any station should not meet at a very acute angle. For example, in Fig. 15, if A and B are known on paper, and C is to be fixed from the lengths of AC and BC, the arcs will meet at a good angle, and the intersection is well conditioned. But

FIG. 15

ABD is an ill-conditioned triangle, and D could not be accurately fixed from A and B. This is called a 'bad fix'.

(5) The lines should be as few as convenient, but a line must be run sufficiently near to each object to be fixed to avoid long offsets.

Apart from the limiting lengths of offset to avoid errors (already considered) it is nearly always *quicker* to run two extra lines, say, for a wide bend in a fence, than to survey it by long offsets. For an example of this, see Yeldon Hall Farm Survey (Fig. 14, p. 32), line 18, where two lines are 'built out', for the survey of the osier bed.

(6) In arranging lines which have to pass through hedges, avoid crossing the hedges oftener than necessary. Thus in Figs. 1 and 16 but for the hedge, station S would have been omitted, and the line NS run from N to C. When lines run along roads, it is often impossible to put in ranging rods to mark stations. Thus C might have to be fixed on the grass margin of the road at C_1. In this case its position is best shown by two careful measurements from points such as C and C_2, or by an offset, if short (Fig. 1).

(7) There must be a sufficient number of tie or subsidiary lines to form a check on the accuracy of every part of the chaining when plotted.

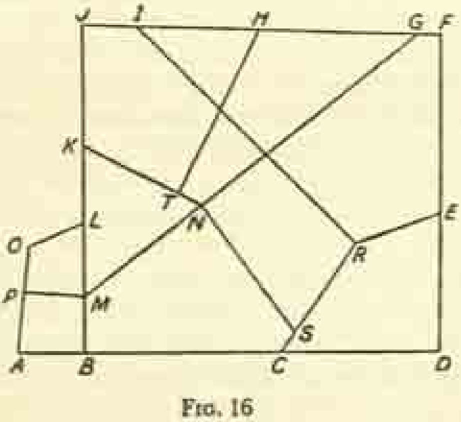

FIG. 16

Thus in Fig. 16, which is the diagram of lines for the survey in Fig. 1, the draughtsman would first study the diagram, and would see from it that he must begin by plotting the triangle GJM.

It would be most accurate to start by drawing GM, and then locate J by intersecting arcs, as the intersection at J is better than at M, but probably he would begin with JGF, so as to fit the map better on the paper. Then locate M by arcs with J and G as centres. The positions of K, L, N, T, I, and H are marked, and the lengths of KN and TH checked so as to see that they agree on paper with their chained lengths. Then JM is produced to fix B and JG is produced to fix F. D is found from B and F as centres and A, C, and E marked. R is found from C and E as centres, and the length RI is checked. S is marked and NS checked. These lines would not fit in properly if there were any error in the chaining. Then O is fixed from A and L, and MP checks this part.

Order of plotting to be studied in field. A lesson which the young surveyor must take well to heart at the outset of his career is that, whilst he is doing the field work, he must, in imagination put himself in the draughtsman's place, and imagine that he is making the drawing from his own notes. Thus the whole order of plotting and checking described above should be thought out by the surveyor *in the field*. And the same mental test must always be applied to the measurements for fixing each individual point.

Use of diagram. The diagram of lines is not only required to show the draughtsman the best order of plotting, but also because most of the points are fixed by intersecting arcs. Now two circles meet in two points. But the diagram will leave no doubt as to *which* intersection is to be taken. Thus all that is necessary in the field-book is that the position of the stations should be marked, as in Fig. 2. When plotting, the diagram should be turned so as to face the observer in the same way as the finished drawing will be before him.

Survey in streets. As an example in the avoidance of long offsets, it may be mentioned that in streets it is often quicker, both for this reason and from considerations of traffic, to run two lines, one down each side of the street, than to use one only.

Fences and boundaries. The boundary fences should in all cases be described, and the side to which the fence belongs noted, and in some cases the positions of any large trees that may happen to stand in it.

It is the custom, now adopted by the Ordnance Survey Department, to survey to the centre of the fences, but it must be borne in mind that the centre of a fence is not always the true boundary of a property.

YELDON HALL FARM SURVEY. As an additional example illustrating the above principles and showing slightly different methods of booking, the Survey of Yeldon Hall Farm, with field-book, is appended (pp. 24 to 33).

RANGING LINES. It frequently happens that, when the two ends of a proposed line have been fixed, it is impossible to range out the line (that is, to fix intermediate points on it) by direct observation from either end. This may be due either to the distance being too great for a ranging rod at one end to be distinctly visible from the other end, or to the occurrence of high ground in between.

In such cases the line may be ranged from the middle as follows: Let A and B, Fig. 17, be the end stations. The surveyor places himself with a ranging rod at C, as near to A as he can go whilst keeping the signal at B distinctly in view, and guessing his position so as to be nearly in the correct line. He then sends an assistant towards B, with instructions to go as far as he can without losing sight of A. The surveyor at C then directs him exactly into line with B, as shown at D. The assistant at D now turns round and directs the surveyor into line with A, as shown at C_1. The surveyor then again directs the assistant into line with B, and so on, until both are in line. The process is expedited by each moving the other *a little too far* in each direction after the first.

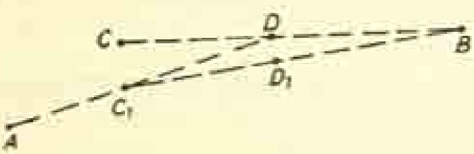

FIG. 17

When lines are to be 'ranged' – or set out – by placing intermediate rods between fixed end points, the surveyor should stand a few yards behind one of the ends, and the ranging should be done *towards* him. As each rod is placed by an assistant, the surveyor must move his eye from left to right, and see that all left and all right edges are in line. Great care must be taken that all rods are vertical.

The rods, when set, can be taken up (being replaced by cleft sticks with pieces of paper inserted in them), and used over and over again, but in no case should the last two or three rods be disturbed until others have been placed in position.

In addition to cutting occasional marks, alluded to under the head of 'stations', the tops of all high fences, whether crossed at right angles or upon the skew, should be cut off so as to appear as a narrow slit in the direction of the line.

THE LINE-RANGER. Instrumentally, the *line-ranger* may be used. This is a reflecting instrument consisting of two right-angled isosceles triangular prisms placed one above the other as shown in Fig. 18. When it is viewed in the manner shown, rays of light

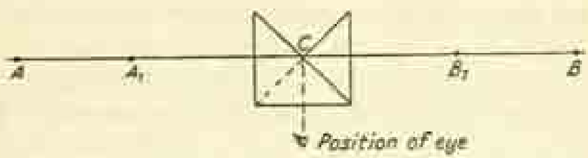

FIG. 18

from A entering the upper prism are totally reflected from the hypotenuse, and enter the eye at right angles to their original direction, so that the signal at A appears directly in front of the observer. Rays from B are similarly reflected from the lower prism.

Hence if it is required to place a signal at C in line with A and B, the observer stations himself near C, brings the instrument to his eye so that he sees the signal at A in the upper prism, then walks backwards or forwards as the case may require (that is to say, about at right angles to the line AB), turning the instrument if necessary so as to keep A in view, until he also sees the signal at B in the lower prism, and gets it to appear as if in exact coincidence with A as seen in the upper prism. The centre of the instrument is then in the required line if the adjustment is good.

TESTING THE ADJUSTMENT. To test the adjustment and the alignment at once, plant a ranging rod at C (Fig. 18) for steadiness, and, keeping A and B carefully in line, direct an assistant to put in rods at A_1 and B_1, so that A, A_1 as seen in the upper prism, and B, B_1 in the lower, will all four appear in coincidence. Then turn round so as to have A, A_1 on the right, and move the instrument till A and A_1 as seen in the lower prism coincide, and B and B_1 also coincide with one another. Then the rods B_1, B should again appear in coincidence with A_1, A. If not, mark a new point B_2 near B_1, which does appear to coincide with A_1, A, then fix B_3 halfway between B_1 and B_2, and adjust the movable prism until B_3 appears to coincide with A_1, A. Then remove all the nearer rods, and refix C, using A and B only.

Or the adjustment may be tested independently by ranging a line AA_1B_1B by eye; then, when A_1, A coincide in one prism and B_1, B in the other, the two former should coincide with the two latter. If not, adjust the movable prism till they do.

CHAINING ON SLOPING GROUND. The distances required in surveying are *horizontal* distances. Hence if the ground slopes this must be corrected in some way. For short steep slopes where great accuracy is not required, the chain is stretched along the slope, say from A to B, Fig. 19. The leader then returns to some convenient link, say the half-chain point, and holds it tight. The follower now puts a ranging rod at A, and raises his end till it appears level with C, as well as can be judged by eye, the ranging rod being held vertically by eye or by the aid of a plumb-bob. The follower then holds tight, and the leader stretches the intermediate part of the chain tight, so that it takes up the line DC. A special arrow is put in at C, the follower goes forward to C with his rod, holds on it the same link which was held by the leader, and the process is repeated till the whole chain is used up. Then an ordinary arrow is inserted, and so on. For falling slopes the process is similar, but the leader has the ranging rod. Downhill chaining by this method is easier than uphill. This is sometimes called 'chaining in steps'.

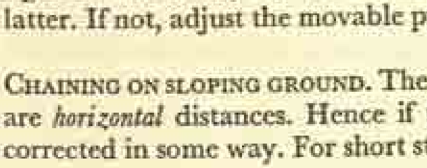

FIG. 19

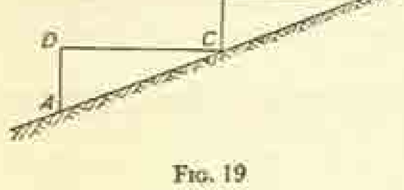

FIG. 20

For longer slopes, or more accurate work, the angle of slope is measured. Let this angle be α (Fig. 20). Now, starting from A, it is desired to place the arrow at B, so that AC may be one chain either 100 links or 100 feet long. Now $AB = AC \sec \alpha$. Hence if $AC = 100$ units, AB will be $100 \sec \alpha$.

Now stretch the chain from A to D, so that $AD = 100$ units. Then $DB = 100 \sec \alpha - 100 = 100 (\sec \alpha - 1)$ units.

Hence the arrow must be placed *in advance* of the end D of the chain by an amount $100 (\sec \alpha - 1)$ units.

This is called the 'hypotenusal allowance'.

Thus if when a 100-foot chain is being used, $\alpha = 4°$, $\sec \alpha = 1{\cdot}00244\ldots$, $\sec \alpha - 1 = 0{\cdot}00244$ and $100 (\sec \alpha - 1) = 0{\cdot}244$ foot, or the arrow must be placed nearly one quarter of a foot ahead of the end of the chain each time.

A table of natural secants should be pasted or written in the field-book, so as to be always available.

The annexed table gives the hypotenusal allowance in different units for various angles, along with the corresponding rise or fall per chain or per 100 feet. These are used to apply the correction in levelling, where we know the rise or fall, but not the angle as a

Angle of slope in degrees	Rise in feet per chain along slope		Hypotenusal allowance per 100-ft. chain		Hypotenusal allowance per Gunter's chain	
	100-ft. chain	66-ft. chain	In feet	In inches	In links	In inches
1	1·7	1·1	0·02	0·2	0·02	0·1
2	3·5	2·3	0·06	0·7	0·06	0·5
3	5·2	3·4	0·14	1·7	0·14	1·1
4	7·0	4·6	0·24	2·9	0·24	1·9
5	8·7	5·7	0·38	4·6	0·38	3·0
6	10·4	6·9	0·55	6·6	0·55	4·4
7	12·2	8·0	0·75	9·0	0·75	5·9
8	13·9	9·2	0·98	11·8	0·98	7·8
9	15·6	10·3	1·25	15·0	1·25	9·9
10	17·4	11·5	1·54	18·5	1·54	12·2
11	19·1	12·6	1·87	22·4	1·87	14·8
12	20·8	13·7	2·23	26·8	2·23	17·7
13	22·5	14·8	2·63	31·6	2·63	20·8
14	24·2	16·0	3·06	36·7	3·06	24·2
15	25·9	17·1	3·53	42·3	3·53	27·9

rule. The student should copy this (or a more extended table) into his level and chain books.

The length could, of course, be chained along the slope, and the corresponding horizontal distance be afterwards calculated, but in this case the corrected distance for each offset would have to be computed. By correcting, as above, in the field, at the end of each chain, no further correction is generally necessary in plotting. If necessary, the positions of intermediate offsets must be corrected by calculation, or by the proportional scale.

THE CLINOMETER. The instrument most commonly used for finding the angle of slope is the clinometer. There are many different forms, of which the most simple consists of a semicircle graduated as shown (Fig. 21), with two pins for sighting at A and B, and a light plumb-bob suspended from the centre. A mark is made on a ranging rod, or level-staff, at the same height above the ground as the observer's eye. An assistant is sent up or down the slope, as the case may be, with this rod, and the surveyor holds the instrument to his eye, with the plane vertical, and aims at the mark by the sighting pins A, B. He then clips the thread of the plumb-bob with his thumb, lowers the instrument and reads the graduation which is under the thread.

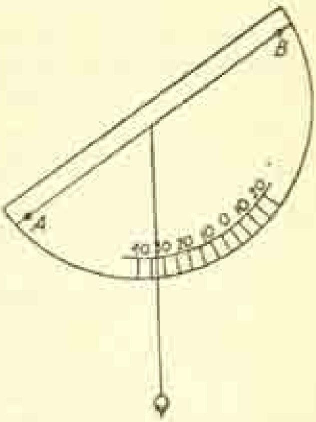

FIG. 21

THE ABNEY LEVEL. More complicated forms are the Abney Level and the clinometer scale. In principle these are similar, consisting of a tube with a sighting aperture E (Fig. 22) at one end, and at the other end a reflector DC, whose lower edge C, is at the middle of the tube. Near this reflector is an opening DF, in the tube, and above this a small level, capable of being rotated in a vertical plane by mechanism not shown in the drawing.

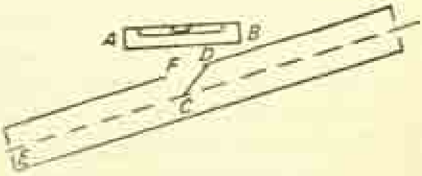

FIG. 22

The tube is placed parallel to the slope by aiming the points E, C, to a mark on a ranging rod at the same height as the observer's eye. It is kept there and the level is turned, and when the bubble becomes central it can be seen in the reflector DC. It is adjusted carefully so that the centre of the bubble appears to coincide with the edge C, and the instrument is lowered and the angle read on a graduated semicircle by means of an index of some kind.

OTHER OBSTACLES IN CHAINING. Other obstacles may be divided into two classes: (1) those which we can see across, such as ponds or rivers; (2) those which we cannot see across, such as houses, hay-stacks, etc.

PONDS, RIVERS, ETC. Let AB (Fig. 23) be the chain line. In these cases it is possible to fix ranging rods C, D in line on the opposite side, by simply sighting across, and all that is required is to find the distance BC. First suppose this is less than one chain. Chain up to B, near the edge, and suppose this is 970 feet from the commencement. Leave a special arrow at B. The rods C and D having been placed, the leader now goes across and stretches a sufficient length of chain to measure the distance BC. Say this is 66 feet. Then the chainage at C will be $970+66=1036$ feet. The follower then goes forward, the thirty-sixth foot point on the chain is held at C, and the rest of the chain stretched towards D to mark the point whose chainage is 1100 feet. One arrow is inserted there, and, the chainage being eleven, the surveyor must see that the follower keeps this one only, handing the rest to the leader.

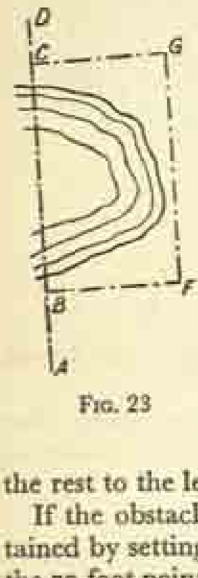

FIG. 23

If the obstacle is a pond, offsets to the right side may be obtained by setting up offsets BF and CG of equal length. Then place the 70-foot point on the chain at F (as the chainage at B is 970 feet), chain along FG, and measure offsets from it, subtracting their lengths from the known length of BF or CG. But the distance BC should still be found by direct chainage if possible.

If longer than one chain, the offsets BF, CG, may be laid out with special care and the length FG measured, but many other methods, with or without the aid of angular measurements, may be devised for finding the distance across. A few are given:

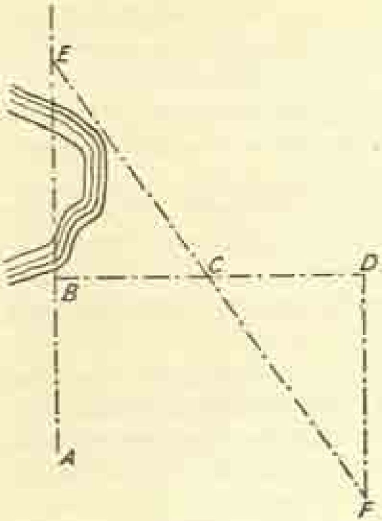

FIG. 24

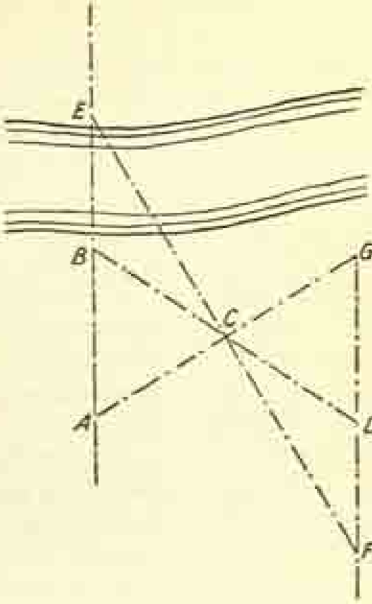

FIG. 25

(1) In Fig. 24, let BE be the distance required. From B set out any line BCD, making BC = CD; then chain along EC, and prolong this line, making CF = EC. Then measure DF, and this will give the length of BE.

This must be added to the chainage at B, and the number of arrows held by leader and follower adjusted as already described.

(2) If the obstacle be a river, we shall be unable to chain EC. Set out BCD (Fig. 24) at right angles to AB by the optical square. Make BC = CD. Then range out DF at right angles to BD, placing two or three ranging rods in a line along DF. Then walk along DF, keeping in line by these rods, until one is also in line with C and E. Then measure DF as before.

Without the optical square: choose convenient points A, B, C (Fig. 25), and range straight lines ACG, BCD, making CG = AC, CD = BC. C, G and D are marked by ranging rods. Now walk along GD produced until one comes in line at F with C and E, a rod having been also placed at E for this purpose. Then DF = BE.

Either of the last two methods may be used to find the distance to an inaccessible point, say *in* the river, provided it be marked by some visible object.

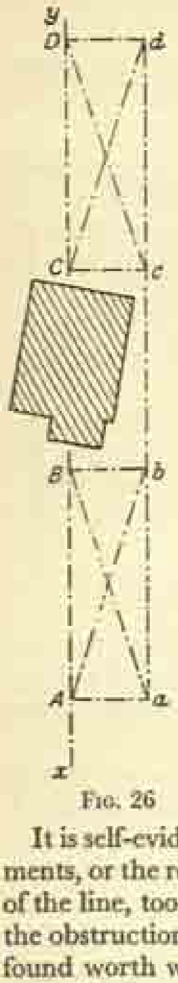

Fig. 26

Ranging past obstructions. In the case of a line being obstructed by buildings or stacks, it is often desirable to range it over them, because had they been visible from the point from which the line was selected, it would in all probability have been so arranged as to avoid such obstructions.

Where, however, it is not easy to range the line over the obstruction, and it is necessary to continue it, this object may be accomplished by means of a parallel diversion in the following manner:

Let *xy* (Fig. 26) be the line, obstructed between the points B and C. Leave rods at A and B and erect the perpendiculars A*a* and B*b*, making them of equal length, and sufficiently long to avoid the obstruction. Produce *ab* to *c* and *d* and let fall the perpendiculars C*c* and D*d*, making them also equal to one another, and to A*a*, B*b*. C and D are then upon the line *xy* and can be produced to the required distance. The lengths AB, CD should never be less than three times that of the perpendicular A*a* and C*c*. It is desirable to chain the diagonals A*b*, *a*B, C*d*, and *c*D, which should be of the same length.

It is self-evident that great care must be exercised in the measurements, or the result will be anything but satisfactory. The direction of the line, too, should be tested (as soon as it is possible to see over the obstructions) by means of the backmarks. Sometimes it will be found worth while to erect long poles to get over an obstacle, but such details must be left to the judgment of the surveyor.

In Fig. 26 it is clear that the distance BC can be found by measuring *bc*. Thus the chainage can be continued.

Marking stations. The positions of all stations on the ground should be marked in some way, so that they can be found again if it is desired to check the work or to continue the survey. In chain surveying this marking frequently consists of cutting out one or more triangular sods of turf, a peg being driven at the apex which marks the station. A heap of stones may also be used but if suitable objects such as gate posts, etc., are available, it is best to make marks upon two or more of these, to measure the distances of the station from these marks, and then to write the distances (in paint or otherwise) beside the marks. Full notes are made in the field-book, so that the point can be readily found.

APPROXIMATE DIRECTION OF NORTH. It is important for referencing purposes that the direction of north should be indicated on all plans. This direction is usually obtained, in surveys of limited extent by means of a prismatic compass, and this instrument, together with its mode of use, is described in Chapter 4. It gives the direction of magnetic north, to which a correction must be added to obtain true north. This correction is known as the 'magnetic variation', or 'declination', and varies from time to time and from place to place. If its value is not known, the direction of magnetic north may be shown on the plan, and the date should be added. The declination for the particular locality and date can then be applied subsequently, if desired. Further notes on this subject are given on p. 214.

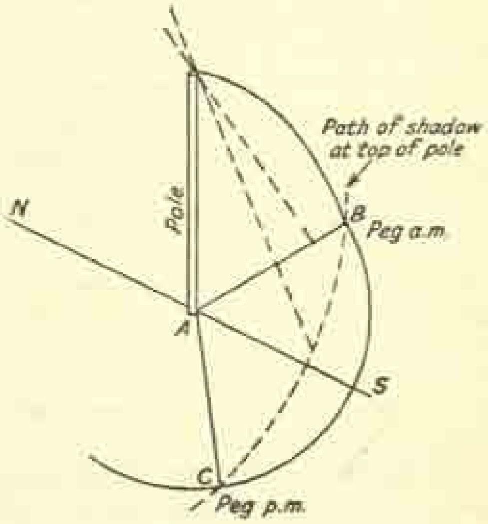

FIG. 27

METHOD OF DETERMINING THE DIRECTION OF TRUE NORTH. The direction of true north, or the meridian line, may be ascertained in the field in the following way:

Select a convenient station, A, upon the main line, where the ground is smooth and level, and plant a pole there firmly and truly vertical (Fig. 27).

About one or two hours before noon, mark, with a small peg, B, the extreme end of the shadow cast by the pole, and, with the foot of the pole as a centre and the distance to the peg as a radius describe a segment of a circle upon the ground in the direction in which the shadow is travelling.

When the sun begins to decline, watch until the extremity of the shadow again coincides with the circle, and drive another peg at that point, C. Bisect the distance between the two pegs, and a line AN, through the station and this point, will be in the direction of the true north. This line should be extended to a sufficient distance to enable it to be accurately surveyed.

In case the sun is likely to be obscured by passing clouds, it is well to take two or more points, at say 10-minute intervals before noon, and strike two or more corresponding arcs.

All these methods are sufficiently near for chain surveying, but, when angular instruments are used, other and more exact methods must be employed.

DETAILS TO BE SHOWN ON PLANS. The details to be shown on plans depend, to some extent, on the purpose for which the plan is required. In general, however, the following points should be noted when a survey is being made:

(1) Nature of ground, e.g. rough pasture, arable, scrub, etc. This information can be shown on a plan, in some cases, by the use of standard Ordnance Survey symbols.

(2) Direction of flow of rivers and streams.

(3) Types of boundaries, e.g. hedge, stone wall, post-and-rail fence, etc.

(4) Varieties of trees, e.g. oak, ash, etc., if sufficiently large to be valuable as timber.

(5) Types of buildings, e.g. brick, timber, corrugated-iron, and the use to which they are put, e.g. dwelling house, store, garage, etc.

(6) Ordnance Bench Marks (see Chapter 3 on levelling).

(7) Signposts and milestones, with any information given thereon.

(8) Names or numbers of all properties, names of streets, Ministry of Transport numbers for classified roads. Directions to neighbouring towns or villages should be indicated on roads, public footpaths and railways.

(9) Hydrants, manholes, P.O. cable markers, lamp-posts, telegraph and telephone poles and any other feature which may require attention during the future constructional work for which a survey is being made.

In general mapping the amount of detail taken in depends upon the scale of the map.

EXAMPLES FOR EXERCISE

(1) Draw out the survey for which the field-book is given in Fig. 16, pp. 24–33.

(2) Criticize the arrangement of lines shown in Fig. 16, p. 24, and sketch a different arrangement of your own.

(3) If in a length of 1 chain (66 feet) along a slope the ground rises 7·8 feet, what is the angle of slope, and what hypotenusal allowance, in inches, should be made? *Ans.* 6° 47′; 5·5 ins.

(4) If an offset be accidentally laid out 3° from its true direction on the field, find the consequent displacement of the plotted point on paper, (*a*) in a direction parallel to the chain line, (*b*) perpendicular thereto, supposing that the offset is 50 links long, and the scale is 1 Gunter's chain to 1 inch. The point is plotted as if the offset had been truly perpendicular to the line.

If the offset is, in addition, measured with a possible error of one part in 250, find the maximum length of offset in order that the resultant displacement of the plotted point from both sources of error may not exceed 0·01 inch, the scale being as before. *Ans.* 0·026 in.; 0·000 7 in.; 19 links.

Chapter 3

LEVELLING AND CONTOURING

Objects in view. In the chapter on chain surveying, it was assumed that a plan of the ground was required, projected, as it were, on to a level surface, and without reference to the relative *heights* of the various points. The determination of these levels or heights, when required, is called levelling.

LEVELLING INSTRUMENTS

The instruments used in levelling are of two main types: the 'dumpy' and the 'tilting telescope' or 'quick-set' patterns. Each type has a telescope which can be so adjusted that a line of sight passing axially through it is horizontal. This horizontal line of sight is directed to a graduated staff held vertically at the point where a height measurement is required.

The dumpy level. A typical dumpy level is illustrated in Fig. 28, which shows the telescope, T, in section. The telescope may be from 6 to 10 inches long, and its optical system consists of an object glass, O, fitted at the front end, and an eye-piece, E, fitted at the observer's end. The position of the eye-piece may be altered slightly in a longitudinal direction by moving it inwards or outwards in its socket. In some instruments the eye-piece is a push fit in an outer sleeve which screws into the telescope end; in others it is threaded into the sleeve and is rotated to effect longitudinal motion.

This adjustment is necessary to meet the requirements of variations in the eyesight of different observers. The eye-piece serves as a magnifier to enlarge the image of the staff produced by the object glass in the plane of the diaphragm, D. The latter consists of a brass ring secured within the telescope by screws, and may be one of three types: (1) The brass ring may serve as the holder for a thin glass plate on which fine horizontal and vertical reference lines are scribed. If three horizontal lines are provided, the centre one is used for determining heights and the outer lines for determining distances by the tacheometric method described in Chapter 6; (2) The brass ring may serve as a frame across which spider-webs are stretched to provide the horizontal and vertical reference lines; (3) The brass ring may be fitted with fine needle-points as horizontal and vertical reference marks. These three types

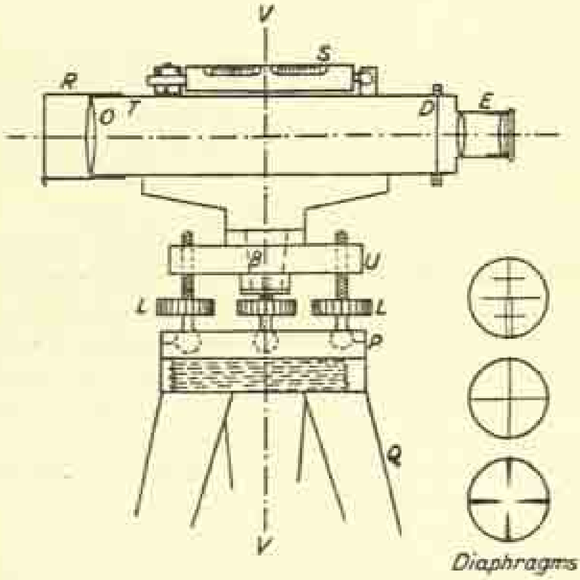

Fig. 28

are shown in Fig. 28. The web diaphragms give the clearest field of view since they contain no glass to collect dust or to mist over, but the glass diaphragms are naturally more robust. In the writer's opinion the third type is the least desirable.

In the case of diaphragms with three horizontal lines it is vitally important to read the central line when determining heights. Although the outer lines are useful for obtaining distances in certain circumstances, it is unfortunately only too easy to read one by mistake when levels are required. No difficulty is found in reading with an inverted image after a little practice.

When the level is to be used the eye-piece is first adjusted so that the reference lines of the diaphragm are sharply in focus. This adjustment is facilitated if the telescope is pointed to the sky or a white wall. The lines in glass diaphragms are sometimes somewhat indistinct, and, if so, they may be rendered more visible by sprinkling a little graphite from a soft black pencil on to the glass and gently rubbing it into the lines with a piece of tissue paper.

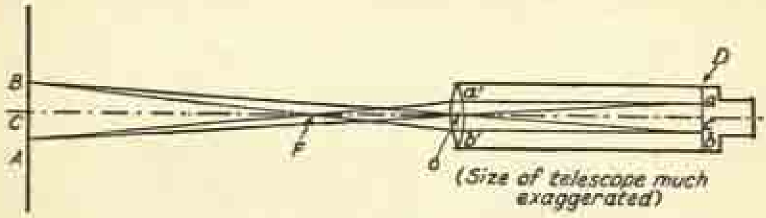

FIG. 29

The optical principle of the telescope is illustrated in Fig. 29. The principal focus, F, of the object glass is outside the telescope; *c* is the centre of the middle horizontal reference line of the diaphragm, D, and *o* is the optical centre of the object glass through which any light ray will pass without deviation. If the line *co* is extended to intersect the staff at C, *co*C is called the 'line of collimation'. If the telescope is rotated about the vertical axis VV in Fig. 28, the line *co*C describes the 'plane of collimation' which, in a correctly adjusted dumpy level, is horizontal.

If we consider two other rays, *aa'* and *bb'* parallel to the optical axis *co*, these will be deviated through the principal focus, F, of the object glass and will intersect the staff at A and B. Oblique rays from *a* and *b* which pass through the optical centre, *o*, will continue undeviated to A and B and when the telescope is correctly focussed an inverted image of the staff will be formed in the plane of the diaphragm. The horizontal reference line will appear superimposed on this image and gives the level reading.

With reference again to Fig. 28, a spirit-level, S, with graduations on the glass, is fitted either to the top or the side of the telescope and a cap, R, is useful for preventing the ingress of the sun's rays or of raindrops, both of which interfere with clear sighting. The telescope is rigidly attached to a tapered spindle, B, which is very accurately fitted into an underframe, U, consisting of a central cylindrical boss with three, or sometimes four, projecting arms. Plate-levelling screws, or 'foot screws', L, are threaded through these arms and are supported in, or on, a base plate, P, which is screwed to the top of the tripod stand, Q.

Four levelling screws give greater steadiness to the instrument, but are considered by many people to be more difficult to adjust. Three screws only are employed in most modern instruments. The methods of adjusting and testing a dumpy level are dealt with later.

THE TILTING TELESCOPE LEVEL. A typical level of this type is shown in Fig. 30. The telescope resembles that of a dumpy level, but, instead of being rigidly supported by the underframe, it is

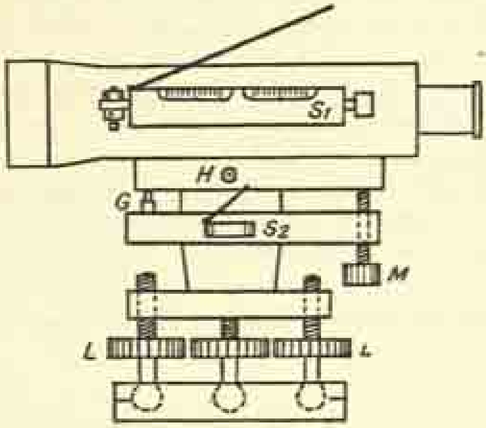

FIG. 30

attached to the latter by a horizontal pivot pin, H. The front of the telescope rests on a spring-loaded plunger, G, and the rear is supported by the micrometer screw, M, which can be turned by a milled head and thus tilts the telescope about the pivot, H. The underframe is usually carried on three levelling screws, L, as in a dumpy, but in some cases a ball-and-socket mounting is used instead. Two spirit-levels are provided: one, S_1, is of the normal sensitive type, generally attached to the side of the telescope, and the other, S_2, is a small circular level used only for rough preliminary levelling by means of the foot screws or the ball-and-socket joint. The sensitive spirit-level is almost invariably provided with a mirror or a prismatic reflecting device so that the observer can see the position of the bubble without leaving his place behind the eye-piece.

The working principles of the dumpy and tilting telescope levels differ somewhat. In the former the setting of the telescope is so adjusted, by means of the levelling screws, that the bubble of the spirit-level remains central when the telescope is rotated about its vertical axis. In other words, this axis is adjusted to be truly vertical.

In the tilting telescope level no attempt is made to adjust the telescope setting so that the sensitive spirit-level remains horizontal if the telescope is turned in different directions. The instrument is roughly levelled for any random position of the telescope, by the use of the levelling screws or the ball-and-socket mounting, in conjunction with the small circular spirit-level. The telescope is then directed to the staff and is accurately levelled by means of the micrometer screw, M, the sensitive spirit-level being used to ascertain when a horizontal setting has been obtained. These instruments are invariably provided with a slow-motion screw which enables the telescope to be turned gradually and precisely, thus facilitating the correct centring of the diaphragm lines on the staff. This is a useful fitment on a dumpy level also, but it is not found universally on levels of this type. Hence the tilting telescope level requires levelling for every reading, while a dumpy in good condition, once levelled perfectly, will remain so until the instrument and its tripod are moved to a new position.

Methods of focussing. The focussing of the diaphragm lines by means of the eye-piece movement has already been mentioned. The focussing of the object glass, to bring the image of the staff clearly into view in the plane of the diaphragm, is accomplished in older instruments by using a sleeved telescope and varying its length by a rack and pinion device, actuated by an external screw.

In modern instruments the telescope is not sleeved, and its length remains constant. Focussing in these instruments is effected by the longitudinal movement of an additional lens mounted within the telescope and moved by a rack and pinion, actuated as before, by an external screw. For the sake of simplicity this lens has been omitted from Fig. 29, but an inverted image is still obtained and the optical principle is merely a modification of that already described.

Size of levels. The size of a level is usually defined by the length of the telescope, e.g., 6, 8 or 10 inches. If the telescope is of the sleeved or draw-tube variety the minimum length is taken, and this occurs when the instrument is focussed on a very distant object.

The improvement in optical craftsmanship which has taken place in recent years has resulted in the production of shorter telescopes which have as great a range and power of definition as the longer and larger telescopes of older levels. Some of the latter were as much as 18 inches long, whereas the telescope of a modern tilting type is normally only some 8 inches in length, even in larger models.

ADDITIONAL FEATURES OF MODERN INSTRUMENTS. Many modern instruments are equipped with minor improvements or additional fitments, and two items in particular are worthy of mention.

One is the prismatic bubble reader which gives an indication of the bubble setting through a small eye-piece located at the side of the main eye-piece. Images of the ends of the bubble are

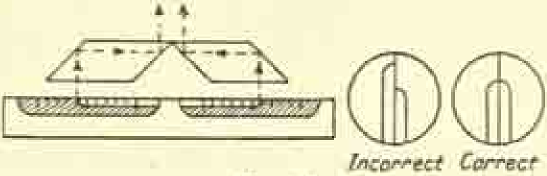

FIG. 31

reflected by 45° prisms in the manner shown in Fig. 31, a further deviation through 90° bringing the images into the eye-piece. The bubble is horizontal when the two ends are in coincidence, as indicated in the diagram.

The other fitting is a graduated circle located beneath the telescope which gives angular readings in conjunction with a reference mark on the telescope body. This enables angles to be measured from any chosen direction reckoned as zero, and is most useful for contouring and for setting out lines at 90° to a main centre-line when cross-sections are being levelled. The accuracy with which angles can be read from these circular scales varies from half-degrees to single minutes according to the size and quality of the instrument.

SELF-ADJUSTING LEVELS. When very accurate levels are required as, for example, in sewerage schemes, a superior type of instrument known as a 'self-adjusting' level is frequently used. One of these instruments is shown in Fig. 32. To some extent it resembles a tilting-telescope level, but the telescope can be rotated through 180° about its longitudinal axis. A spirit-level is mounted at the

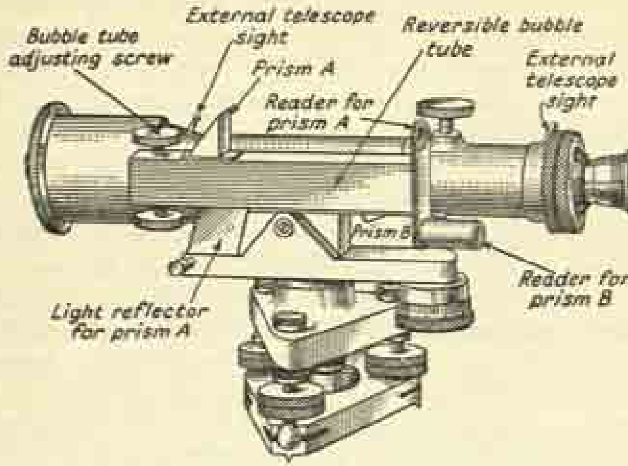

FIG. 32

side of the telescope and is so constructed that it will read whether it is to the left or right of the telescope when viewed from the eye-piece end, rotation through 180° about the longitudinal axis naturally reversing the position of the spirit-level. The latter is correctly set when one end of the reversible bubble is in coincidence with a reference mark on the bubble tube. This makes it essential that the bubble shall maintain a constant length in spite of temperature variations, and this condition is satisfied by a special design of compensating tube. It will be seen from the diagram that the bubble position is viewed, after prismatic reflection, through a small eye-piece at the side of the telescope.

Approximate preliminary levelling is carried out by the foot-screws in conjunction with a small circular spirit-level for any random telescope position. The telescope is then directed to the staff and accurately levelled by the micrometer screw beneath the telescope near the eye-piece. The reading is taken and booked as 'left-' or 'right-face' according to which side of the telescope the spirit-level is situated. The telescope is then rotated through 180° about its longitudinal axis. This will reverse the 'face' of the spirit-level, which is then readjusted, if necessary, by the micrometer screw, and a second staff reading is taken. If the instrument is in good adjustment the two readings will be identical. If not, the mean of the two will be the correct reading.

The spirit-level on this type of instrument is made readily adjustable by means of a milled-headed screw, and a locking screw, located at one end. If the latter is released and the former rotated, a tilting movement is transmitted to the spirit-level. This device, in conjunction with the 180° rotation of the telescope, enables the instrument to be set in perfect adjustment quickly and

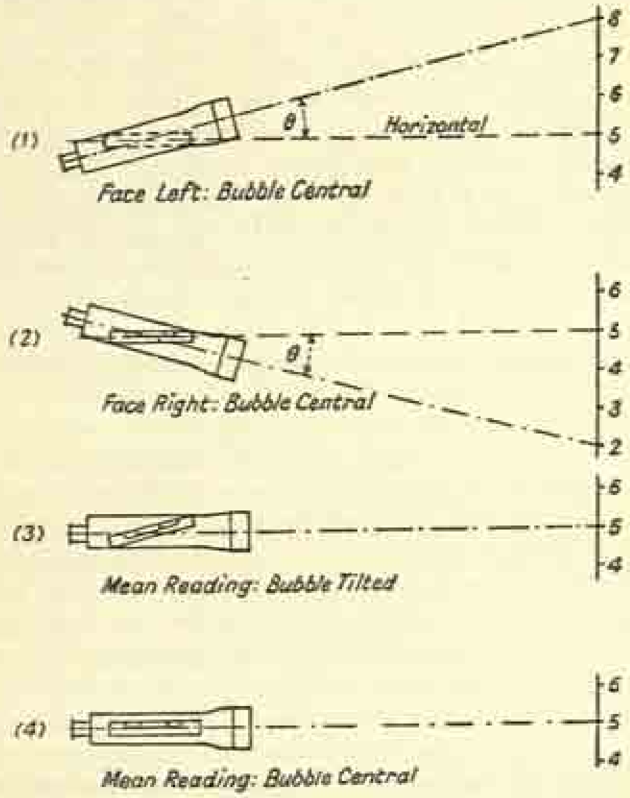

Fig. 33

easily. If the right- and left-face readings on the staff do not agree, the telescope is tilted by the micrometer screw until the mean reading is obtained. This will cause the bubble to run out of centre, and it is brought back to its correct position by means of the adjusting screw on the bubble tube and the setting is locked by the screw provided for this purpose.

The difference in right- and left-face readings is due to the fact that the spirit-level axis and the line of collimation are not parallel, and the above adjustment corrects this discrepancy in the manner shown in Fig. 33, which is self-explanatory.

THE LEVELLING STAFF. The levelling staff in most common use in this country in conjunction with the level is known as a Sopwith staff and is constructed as follows: An upper solid length slides into a central hollow length, which in turn slides into a lower or bottom length, the whole thus collapsing into a portable form (Figs. 34 and 35). These lengths are usually made of well-seasoned mahogany or cedar, fitted and screwed together, and rendered as waterproof as possible. The bottom hollow length is generally about 3¼ inches by 2 inches in cross-section, and 5 feet long, the next length being an easy fit within the first, and the third within the second. The positions can be extended so as to make the total length over all 14, 16 or 18 feet as the case may be, the 14-foot staff with a bottom length of 5 feet being most commonly used. Each length when properly drawn out is held in position by a spring catch A, engaging with the brass rim of the next length below. The bottom is furnished with a brass shoe and the top with a brass cap. The face of each length is figured, either in oil colours or on a paper strip, pasted on and varnished over, the painted figures being naturally the more durable. The division consists of feet, tenths and hundredths of a foot. The actual figuring is done in many ways to suit individual taste, but the pattern known as Sopwith's, illustrated in Fig. 34, is by far the most generally adopted. In this case, the black and white spaces both extend over 0·01 of a foot, and are made of different lengths to facilitate reading. The odd tenths only are numbered in black figures, the top of the figure in every case being in line with the division to which it refers. The figures indicating feet are usually painted red, and are of greater size than the rest. The smaller figures, indicating the tenths, are themselves exactly a tenth of a foot high, so that the bottom of figure 3 represents 0·2 foot, and so on. The figure 6 is generally painted with an open loop, whilst the loop of the 9 is filled up, and the figures 10, 12 and 13 on the top length are

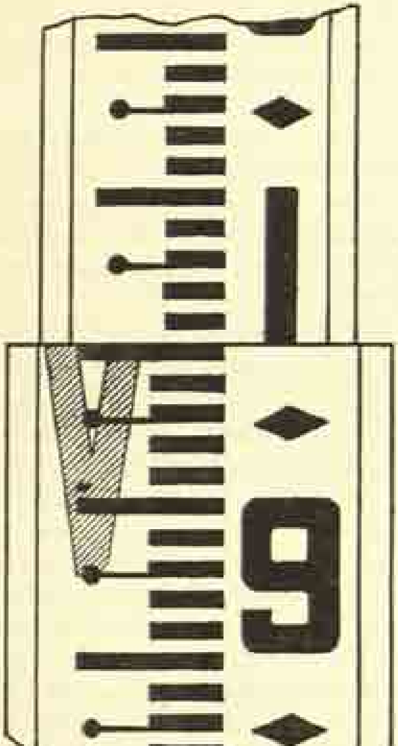

FIG. 34

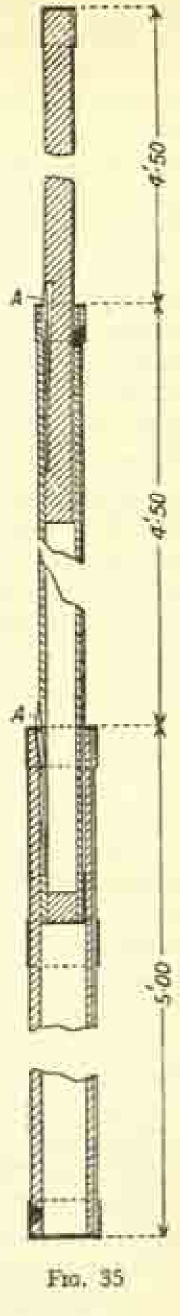

FIG. 35

sometimes painted in Roman numerals or otherwise distinguished. The foot—or other unit—is generally divided decimally. Metres and centimetres are used in countries where the metric system prevails.

The staff is frequently damaged through the telescoping portions being allowed to run down too quickly when the staff is being closed up. The staff-holder should be trained to do this gently.

The telescopic staff is no doubt convenient for travelling by train or car but in the writer's opinion is open to many disadvantages. The woodwork is apt to swell and warp with wet, so as to make it difficult to draw out. It cannot be safely immersed in water, or used as a sounding rod. Errors may be introduced by the failure to extend the staff fully.

The surveyor may direct the staff-holder to pull out the top joint, but if the staff is stiff, he may fail to do so completely, and a serious error may be introduced. Finally, the length, fourteen feet, is too great for general use, especially on hilly ground, for a reason which will be explained later (p. 92). The writer, therefore, prefers a staff ten feet long only, and this length prevents great difference in the length of the 'fore-' and 'back-sight' when levelling up or down hill.

For use in cases where much travelling by rail or car is not required, the writer considers that the staff should be in one single length like the Ordnance Survey ten-foot pattern. The staff should consist of a strip of some well-seasoned light wood about 3 inches wide, and ⅝ inch or ½ inch thick. A strip of wood cut out to give hand-holes for the staff-holder, screwed to the back, serves as a stiffening rib. To each side of the staff small fillets of wood are nailed, projecting about ¼ inch beyond the graduated face of the staff. If the staves are shod at both ends, and made exactly ten feet long, they will be handy for setting out masonry. The writer also prefers a rounded end to the usual square one.

A folding staff ten feet long, with a very stout brass hinge in the middle of its length, is convenient for travelling.

When it is folded, the graduated faces should be inside, and therefore protected.

The staff may be made 2½ inches wide, and about ⅝ inch thick.

The writer disapproves of the ordinary method of numbering the levelling staff, for the following reasons:

A glance at Fig. 36 will show that sometimes the most prominent figure in the middle of the field has to be recorded, and sometimes the said figure diminished by unity.

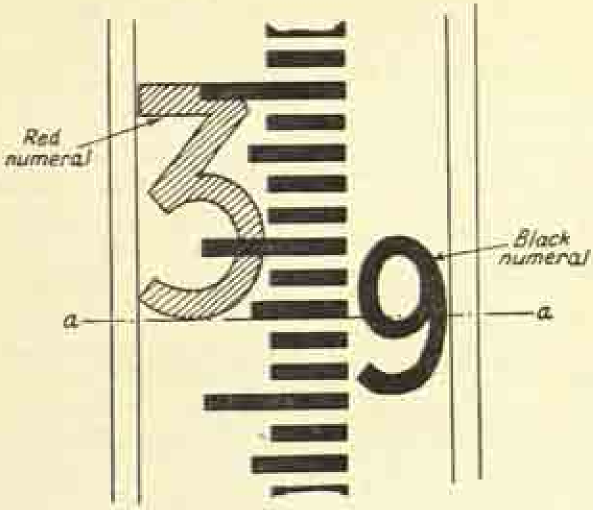

FIG. 36

Thus, if the horizontal wire cuts the staff as at *a*, the figures 3 and 9, so to speak, stare the observer in the face.

Neither of them must be inscribed in his book, for the true reading is 2·85.

The method of graduation shown in Fig. 37 is suggested as a possible alternative.

The feet are marked by large diamonds, half red and half black–red below, and black above.

The tenths are marked by black diamonds, and the half-tenths by small black squares, respectively.

The figures indicating the feet are clearly marked twice on each successive foot, one black and the other red. Thus, when one takes the part between the third and fourth foot-divisions of the staff, it is obvious that any reading between these points is three feet and some decimal of a foot.

Each figure is two-tenths of a foot in height.

The black three, corresponding to the black half of the lozenge, marking the third foot division, extends from 3·10 to 3·30.

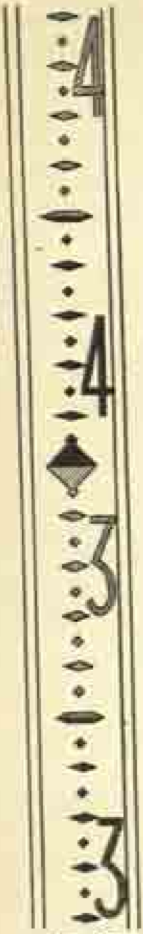

Fig. 37

The red three extends from 3·70 to 3·90.

One foot-numeral is therefore always in view, and the numeral which is nearest to the cross-wire is that to be inscribed in the level-book.

The black and red numerals also assist in enumerating the tenths. Thus:

Bottom of black numeral	0·10
Middle of black numeral	0·20
Top of black numeral	0·30
Intermediate mark	0·40
A long mark	0·50
Intermediate	0·60
Bottom of red numeral	0·70
Middle of red numeral	0·80
Top of red numeral	0·90
Division of red and black lozenge . . .	1·00

The only case in which there can be any hesitation as to the proper foot numeral to inscribe, is when the wire falls exactly on the division between the red and black halves of the large lozenge which indicates the exact termination of the foot.

Then the upper or black figure is to be inscribed.

The diamonds and lozenges, being 0·02 in height, afford the means of estimating the hundredths; 0·02, 0·03, 0·07 and 0·08 alone require to be estimated by eye.

Staves can also be obtained 12 feet long in solid wood, made with a hinge in the centre, and fitted with a stiffening arrangement at the back, to keep the two portions in a straight line when opened out for use. This pattern of staff has the advantages that the face is the same width from top to bottom, and consequently can be read with equal facility at any part, and that the whole arrangement will stand more rough usage than any telescopic staff. On the other hand, it is somewhat heavy.

For Ordnance Survey work a 10-foot solid staff, figured on both sides, is used. One side commences at zero at the bottom, and the other at, say, 1·5 foot; readings are taken front and back, and the staff can be reversed end for end, and read front and back again.

USE OF VANE WITH STAFF. Formerly the staff was provided with a sliding vane which was moved up or down by the staff-holder, according to signals given by the observer, until the centre line of the vane was intersected by the horizontal wire of the level telescope. The height to the centre of the vane was then read off from graduations on the staff. The staff was made telescopic, to facilitate the movement of the vane, when above the reach of the staff-holder. Such a staff is required when very long sights are unavoidable. This may occur, for instance, when readings have to be taken across a river.

When using a vane the levelling instrument is usually entrusted to a subordinate, whose duty it is to adjust it, and to direct the staff-holder to move the vane to its proper position. The chief surveyor reads the staff, and records the reading, as well as the chain measurements made from station to station. He is therefore able to keep with the chain-men, supervise them, and select the alignment of the line of section and the positions of the staff, without the necessity of going backwards and forwards between the instrument and the chain-men. Thus the system has manifest advantages. The observer keeps a subsidiary level-book, which serves to check the surveyor's book.

INVERTED MARKINGS. In an effort to overcome the imagined difficulty of reading the staff with an inverted image, inverted markings for the staff have been introduced. This produces the effect of an upright staff in an inverted field of view, a combination which the writer finds most confusing.

ADJUSTMENTS OF THE LEVEL. The necessary adjustments of the level are divided into (*a*) temporary adjustments, which must be performed at each setting-up or each reading, and (*b*) permanent adjustments, which should require only occasional attention.

TEMPORARY ADJUSTMENTS. The temporary adjustments are: setting-up, levelling, and focussing. In setting-up, the legs must be firmly planted and, before this is done, *one* leg should be lifted and the bubbles roughly adjusted by it. Some surveyors have small brackets or cleats of cast iron or brass screwed to the lower end of each leg of the tripod, so that they can be rammed down with the foot. They are not an unmixed blessing, however.

Levelling-up a dumpy level. In levelling-up, the learner should at once accustom himself to work systematically. If there are four levelling or plate screws, set the telescope parallel to two opposite ones. Then look to see at which end of its run the bubble is, and *slacken the screw under it half a turn.* Note whether, in doing so, you go thumb inwards or thumb outwards, and accordingly proceed *both thumbs inwards* or *outwards* (as the case may be) until the bubble moves over to the other side. Then slowly *tighten* up the screw which you slackened at first, leaving the other one alone. Generally the bubble can thus be brought back to its central position, and the tightening ceases when it reaches there. If it will not come back without forcing, slacken the opposite screw a mere fraction of a turn at a time and continue. If the bubble comes back before the screw which was first slackened becomes tight, then let it go a little too far back, and tighten up the other screw only.

Thus we ensure that all the screws are tight. The turning should be done with thumb and forefinger only, and using only moderate force.

Then turn through 90°, and repeat the process with the other pair. Then turn the telescope back to its *original position* (*not* through 180° therefrom) and adjust again if necessary.

If there is a small cross level, set the telescope parallel to two opposite screws; then adjust the cross level *first* (in the way above described, but using the pair of screws parallel to the cross level), then adjust the main level; then turn through 90°, and adjust the main level in this direction.

If the levels are fairly adjusted, it will then only be necessary to touch the screws a little for back- and fore-sights.

If there are three screws only, set the level parallel to two, and adjust with them, working both thumbs equally, in or out, as the case may require. Then turn through 90°, and adjust with the third screw only. Then back to the old position, and so on.

The beginner is advised to follow these steps carefully, and he should, before turning any screw, *look at the thread* to see which way he must turn it to slacken it.

Parallax test and correction. It has already been mentioned that, when the instrument is correctly focussed and adjusted, the image formed by the object glass should fall exactly in the plane of the diaphragm. To test whether this condition is satisfied, the observer moves his eye up and down while viewing the staff. If the adjustment is correct, there will be no relative movement between the horizontal reference line and the staff readings. If relative motion occurs, the eye-piece is first adjusted by moving it in or out of its sleeve until an extremely sharp view of the diaphragm lines is obtained. The telescope is then re-focussed, if necessary, to give the clearest possible image of the staff and the test is repeated.

Some dumpy levels are provided with a mirror, mounted at an angle of a little less than 45° to the telescope above the level, by means of which the observer can see the level bubble without moving from his position at the eyepiece. The writer cannot, however, say that he has found this to be a very satisfactory arrangement. The mirror is necessarily somewhat long, and is a good deal in the way and apt to get knocked about. It is better to step to the side and look directly at the bubble; the hands will then be in a convenient position for adjusting the levelling screws.

PERMANENT ADJUSTMENTS. It is naturally desirable that the permanent adjustments of the instrument should be as perfect as possible. Of these, the essential one is the parallelism of the level and line of collimation. If care is taken of such matters as the equality of length of fore- and back-sights, any error due to imperfect permanent adjustment is eliminated. The writer, therefore, does not recommend the surveyor to torture his level with the view of obtaining perfect permanent adjustment.

The writer has rarely found that a level would 'traverse', that is, that the bubble would remain stationary in all positions of the telescope correctly after a few days' use. After all, correct traversing is a convenience rather than an essential point. The essential is that the line of collimation should be parallel to the bubble. Then to secure accuracy it is necessary only to bring the bubble to the middle of its run when reading the staff.

At the same time, it is necessary that the surveyor should be able to test, and, if necessary, correct the permanent adjustments of his instruments.

PERMANENT ADJUSTMENTS OF DUMPY LEVEL. In the dumpy level these are three in number:

(*a*) In a sleeved telescope the line of collimation should be parallel to the axis of the draw tube, so that its direction does not alter with the focus.

(*b*) The 'vertical' axis of rotation should be perpendicular to the spirit-level.

(*c*) The line of collimation and the spirit-level should be parallel to one another.

Of these, adjustment (*a*) may generally be neglected.

(*b*) *To make the 'vertical' axis of rotation perpendicular to the spirit-level:* Level up the instrument carefully as described under 'Temporary Adjustments'. Rotate the telescope through 180°. If the bubble runs out of centre, count the number of graduations on the bubble tube indicating the extent of the displacement. Correct half the displacement by the levelling or plate screws, and half, by adjusting the bubble tube itself, by means of the capstan nuts by which it is secured to the telescope. The opposite end of the bubble tube is usually pivoted by a hinge or ball-joint to facilitate this adjustment.

(*c*) *To make the line of collimation parallel to the spirit level.* For adjustment (*c*), choose two firm, smooth points, A and B, on fairly level ground, say 160 to 200 feet apart, and set up the level, if possible, at C, exactly midway between them, or, if more convenient, at any point C_1 such that $AC_1 = BC_1$ as shown in Fig. 38. Level up, and read the staff at A and B, correcting the level for each reading if necessary.

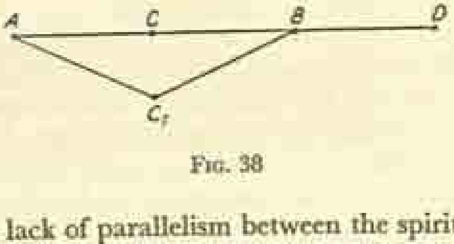

FIG. 38

If there is a lack of parallelism between the spirit-level and the line of collimation, the latter will be tilted when the former is horizontal, and the extent of error resulting from the tilt will be directly proportional to the length of sight. Hence if the lengths of sight are equalized, as in this case, the difference between the readings will give the correct rise or fall, whether there is an error in the line of collimation or not.

Now set up at D, in the line AB produced, so that BD is 80 to 100 feet long, and preferably exactly half of AB. Level up, and read again at A and B.

If the difference is the same as before, the adjustment is correct. But if the line of collimation is *not* parallel to the spirit-level, the reading at A will have a bigger error than that at B, in consequence of the greater distance. Hence, the difference between the readings will *not* be the same as before. This therefore indicates an error, and we may correct it by altering either the line of collimation or the spirit-level.

If there is any reason to suppose that the line of collimation is at fault (for instance, if the surveyor has just re-webbed the instrument, or if the object-glass has been removed without proper precautions), the adjustment should be made with the screws securing the diaphragm to the telescope tube. These may be either two or four in number and are sometimes concealed beneath a covering ring which must be unscrewed. The top and bottom screws, when turned, raise or lower the diaphragm ring. Those at the side, if any, usually work in slotted guides and merely steady the diaphragm. The screws are almost invariably provided with capstan heads, and an accurately fitting tommy-bar must always be used in order to manipulate them without damage.

By trial and error, we work the diaphragm screws a little at a time, and keep on reading at A and B till we get the correct difference, the level remaining central; or we may at once work out the reading which must be obtained at A from D, to bring the line of collimation horizontal, as follows:

Let the reading at A from C − reading at B from C $= d_1$,
And " " " D − " " " D $= d_2$.
These will be ± according to the readings

Then the necessary *increase* in the reading at A is given by the formula $\frac{DA}{BA} \times (d_1 - d_2)$, proper account being taken of all signs.

Example. Suppose the readings from C are, to A 7·63, to B 6·48; and from D, to A 7·21, and to B 6·12.

$$\begin{aligned} \text{Then } d_1 &= 7{\cdot}63 - 6{\cdot}48 = +\,1{\cdot}15 \\ \text{and } d_2 &= 7{\cdot}21 - 6{\cdot}12 = +\,\underline{1{\cdot}09} \\ d_1 - d_2 &= +\,0.06 \end{aligned}$$

$$\therefore \text{ increase in reading at A} = \frac{DA}{BA} \times 0.06$$

If DB = ½ BA, DA will be 1·5 × BA.

Therefore increase = $\frac{3}{2}$ × 0·06 = 0·09, and the diaphragm must be moved till the reading at A becomes 7·21 + 0·09, or 7·30.

Again, if the readings are: from C to A 4·81, and to B 4·83, and from D to A 5·16, and to B 5·02; and supposing that BD $= \frac{1}{2}$ BA,

$$\text{then } d_1 = 4{\cdot}81 - 4{\cdot}83 = -0{\cdot}02$$
$$d_2 = 5{\cdot}16 - 5{\cdot}02 = +0{\cdot}14$$
$$d_1 - d_2 = -0{\cdot}16$$

$$\text{And increase in reading at A} = \frac{\text{DA}}{\text{BA}} \times -0{\cdot}16 = \tfrac{3}{2} \times -{\cdot}016 = -0{\cdot}24$$

$$\therefore \text{ reading at A must become } 5{\cdot}16 - 0{\cdot}24 = 4{\cdot}92$$

Thus, in this last case the reading is to be lowered, and we must raise the diaphragm. To do this, slacken the lower diaphragm screw, then tighten the upper one till the requisite reading is obtained, and then gradually tighten up the lower and upper alternately, keeping the reading the same, till both screws are tight. Use only as much pressure as you can apply with the tip of one finger on the pin, or 'tommy', used for turning. Look to see that the bubble remains central all the time.

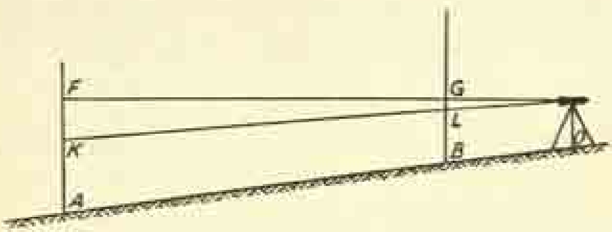

FIG. 39

Proof of formula. As already shown, d_1 will give the correct rise from A to B. Thus, if GF (Fig. 39) be a true horizontal line,

$$d_1 = \text{AF} - \text{BG} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

And if LK be the incorrect line of collimation,

$$d_2 = \text{AK} - \text{BL} \quad . \quad . \quad . \quad . \quad . \quad (2)$$

$$\text{Now FK} - \text{GL} = \text{AF} - \text{BG} - (\text{AK} - \text{BL}) = d_1 - d_2 \quad . \quad (3)$$

$$\text{But GL} = \text{FK}.\frac{\text{DB}}{\text{DA}}$$

$$\therefore \text{FK} - \text{GL} = \text{FK}\left(1 - \frac{\text{DB}}{\text{DA}}\right) = \text{FK}.\frac{\text{BA}}{\text{DA}}$$

$$\therefore \text{ from (3), FK} = \frac{\text{DA}}{\text{BA}}.(d_1 - d_2)$$

But FK is the increase in the reading at A, required to bring the line of collimation up to the true horizontal, that is, parallel to the spirit level.

If there is no reason to suppose that the error is in the line of collimation, adjust the spirit-level. For this purpose we either work out the correct reading at A as before, and then work the plate screws until we obtain this reading; or, by trial and error, work the plate screws a little at a time, and keep on reading at A and B till the correct difference, d_1, with its proper sign, is obtained. This will throw the bubble out. Then adjust the latter by re-setting the bubble tube, using the capstan nuts at one end of the spirit-level as in adjustment (*b*).

The exact method of doing this varies with the manner in which the end of the level is fixed. If there are two capstan-headed nuts, and you wish to *raise* the end you are adjusting, first slacken the upper nut. Then tighten the lower one till the bubble goes a little too far. Then tighten the upper and lower alternately till the bubble is central and both screws tight. See that the reading has not altered before finally tightening up.

ADJUSTMENT BY COLLIMATOR. In adjusting the line of collimation parallel to the spirit-level, opticians generally employ the method, invented by Gauss, by means of a collimating telescope which has previously been carefully adjusted. The method is often useful where it is necessary to re-web and adjust a level after dark, using another level known to be in good adjustment.

The collimating level is set to infinite focus with great care (e.g. by focussing on a star or distant light, parallax being carefully eliminated). It is set up and carefully levelled, and the instrument to be adjusted is then set up a few feet in front of it, and as nearly as possible at the same height, so that when the two telescopes are pointed at one another it is possible to look from one into the other. The two object-glasses are made to face one another, and the level to be tested is carefully levelled.

If now a light be held behind the eyepiece of the collimator (assuming that the work is being done at night) and we look through the other level, careful focussing will reveal an image of the diaphragm lines of the collimator. The horizontal line should coincide in position with that of the other level. If not, the latter must be adjusted by the diaphragm screws to produce coincidence.

Both bubbles must be kept central by the plate screws during the test. It is unnecessary that both levels should be at exactly the same height, as by setting to infinite focus the effect is the same as if we were looking at an infinitely distant object, and the slight difference of level is then negligible.

COLLIMATION TEST AND ADJUSTMENT FOR TILTING-TELESCOPE LEVEL. In this type of instrument it is necessary only that the longitudinal axis of the spirit-level and the line of collimation of the telescope should be parallel. It is convenient to place the instrument and the reference points in one straight line and the test must always be repeated after adjustments are made.

The instrument is set up equidistant from two fixed points and their difference of level obtained, as in the case of the dumpy level, the bubble being brought to the centre of its run for each reading. As the lengths of the sights are equal, this procedure will give the correct difference of level even though the collimation adjustment is wrong.

The instrument is then moved to a position from which the lengths of the sights to the same two points are markedly unequal. If the collimation line and spirit-level axis are parallel the same difference of level will be obtained as before. If not the telescope is tilted up or down as required until the correct difference of level is obtained, ignoring the bubble. When the readings give the correct difference the line of collimation is horizontal and the bubble of the spirit-level is then brought to the centre of its run by careful manipulation of the capstan-nuts at one end of the bubble tube.

FIELD-WORK PROCEDURE. To find the difference of level between two points, either of these instruments is set up and adjusted at a position from which both points can be seen (assuming that this is possible), and a staff-holder is then sent to one of the points with the graduated staff, which he holds vertically, the lower end resting on the point whose level is required.

The surveyor brings the diaphragm lines into focus by pushing or screwing the eye-piece in or out; then directs the telescope towards the staff, sees that the bubble is central, and focusses the staff. He moves his eye about to test for parallax (referred to on p.66), corrects it if it exists, and then reads off the graduation at which the inverted image of the staff appears to be cut by the horizontal reference line. The staff is then sent to the second point and the process repeated, except that it should not be necessary to focus the diaphragm again.

The staff is graduated from the bottom upwards, and the plane of collimation (p. 54) in which the readings are taken, remains horizontal for both points as described above. Hence each reading tells how far the corresponding point is *below* the plane of collimation, and the difference between the readings tells how much one point is below the other. The point at the *lower* level will have the *higher* reading, and vice versa.

DEFINITIONS. Where the levels of many points are required several may be read from one position of the level. The first reading taken from any position is called a *back-sight*. The last reading from any position is a *fore-sight*, and all others are called *intermediates*.

It is usual to assume some arbitrary horizontal plane or level surface lying below all the points. This is called the *datum*. The level of each point is then stated by giving its height *above* this datum. This height is called the *reduced level* of the point.

The reduced level of the collimation plane of the level in any position is called the *collimation level*, or, more loosely, the *height of instrument*. The latter term is vague.

The difference of level from point to point is called a rise or fall, according to whether the second point is higher or lower than the first.

If the levels are along a line (say the centre line of a road or railway) whether straight or curved, the drawing which shows them is called a *section*. The horizontal distances from the starting point to all other points must of course be measured as well as the levels. The section is plotted as if the line were stretched out straight in plan, even if it be actually curved. In the case of a length of road or railway a section of this kind is called a '*longitudinal section*.' *Cross sections* are sections crossing the centre line transversely, usually at right angles. The book in which the readings are entered is called a *level-book* or *field-book*.

PROCEDURE WHEN LEVELLING FOR A SECTION. The line along which the section is to be taken having been marked by ranging rods, a staff is held at the starting point A (Fig. 40), whose level is known or assumed.

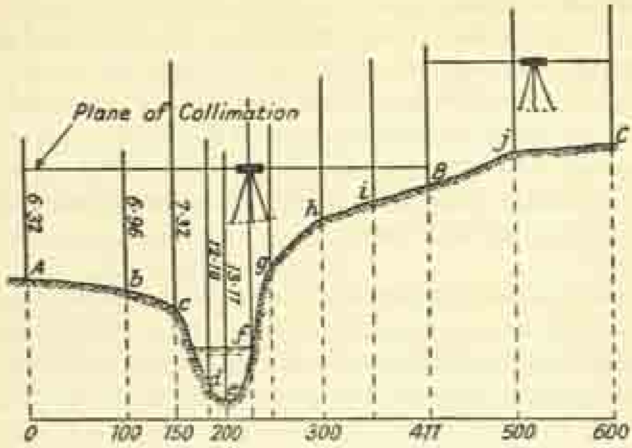

Fig. 40

The instrument is then set up say at X, and adjusted so that the telescope is horizontal. The point X need not be on the line of section, but it may be placed at any convenient point to the right or left thereof. Some care and judgment are required in selecting the position of the instrument. The ground should be firm and fairly level, and the first position should be chosen so that the staff can be read at the starting point for the levels (A in this case), and at as many points after that as possible, and so on for each later position. The sun should be kept, as far as possible, at the observer's back when reading the staff. Thus in levelling along a line running *north*, we would set up on the *east* side in the morning (so as to face *west*), and on the *west* side in the afternoon.

The number of points at which the staff can be read from any position is limited as follows:

(*a*) By the distance becoming so great that the graduations on the staff cannot be read distinctly. This depends upon the optical properties of the telescope.

(*b*) By the ground rising so that the bottom of the staff would be above the plane of collimation, or falling so that the top of the staff would be below it.

(*c*) By the line of sight becoming obstructed by trees or other obstacles.

All these should be considered when choosing the position, and, for accurate work, the level should, if possible, be placed so that it is about equally distant from the points where the back-and fore-sights are to be taken (see p. 92).

The telescope being now directed to A (Fig. 40) the staff is read, and the reading recorded. This reading is a 'back-sight', as before defined (p. 73).

Next, the instrument remaining at the same place, the staff is held at *b*, a point where there is a marked change in the slope of the ground. The staff is then read again and noted.

This reading is called an 'intermediate sight'.

The difference between the readings at A and *b* is clearly the difference between the levels of these two points.

The distance A*b* is then chained and recorded.

The staff is now moved to the point, *c*, in the example the crest of the bank of a brook; it is again read and noted, and the horizontal distance A*c* is measured, by continuous chaining from A.

The staff is next held successively at *d* on the bed, at *e*, also on the bed, at *f* at the water level, at *g* the crest of the bank, at *h* and *i*, and finally at B. The horizontal distances to the several points are also measured.

The reading of the staff at B is called, as before, a 'fore-sight', those at *b*, *c*, *d* . . . *i* having been entered as 'intermediate sights'.

The horizontal distances A*b*, A*c*, A*d*, etc., are determined by continued chaining from A.

FIELD-BOOK FOR SECTION LEVELLING: HEIGHT OF INSTRUMENT METHOD. There are several forms of field-book in use for recording the staff readings and distances. The following is an example, referring to the section in Fig. 40. In this case the reduced level of the starting point A is assumed to be known as 107·45 feet, and this is entered in the first line of the *reduced level* column. It will be obvious from the figure that, given this reduced level for A, and given that the staff at A reads 6·32 feet, the collimation level or height of instrument for the first position will be found by adding these two figures together, and the reduced level of any other point is then found by subtracting the reading at that point from the known height of instrument.

FIELD-BOOK FORM FOR SECTION LEVELLING

Back-sights	Inter-mediate sights	Fore-sights	Height of instru-ment	Reduced levels	Distances (feet)			Remarks
					Left	Centre	Right	
6·32	—	—	113·77	107·45	—	0·00	—	Reduced level of point A
—	6·96	—	—	106·81	—	100	—	Point *b*
—	7·32	—	—	106·45	—	150	—	,, *c* crest of bank of stream
—	12·18	—	—	101·59	—	183	—	,, *d*, bed of stream
—	13·11	—	—	100·66	—	189	—	,, *e*, ,,
—	9·63	—	—	104·14	—	—	—	,, *f*, level of water in stream
—	4·73	—	—	109·04	—	216	—	,, *g*, crest of bank of stream
—	3·13	—	—	110·64	—	293	—	,, *h*
—	2·19	—	—	111·58	—	350	—	,, *i*
—	—	0·73	—	113·04	—	411	—	,, B
9·36	—	—	122·40					
—	8·17	—	—	114·23	—	500	—	,, *j*
—	—	7·42	—	114·98	—	600	—	,, C 
6·32	—	—	121·30	—	—	—	—	Staff on
—	3·11	—	—	118·19	—	—	—	B.M. top of milestone
1·12	—	8·83	—	—	—	—	—	Check levels T.P.
—	—	6·14	—	—	—	—	—	Close on point A
23·12		23·12						

The staff reading at A is entered in the column headed 'back-sights'. The readings taken at *b*, *c*, *d*, etc., *i*, are entered in the column headed 'intermediate sights', whilst the final reading of the set, taken to the staff at B, is entered in the column headed 'fore-sights'. Then a line is drawn across the three columns back, intermediate, and fore, to indicate that the readings taken with the first position of the instrument are terminated.

The computation of the levels is performed as follows:

The reading of the staff when held at A is added to the reduced level of A. This gives the reduced level of the line of collimation of the instrument, and the sum (107·45+6·32=113·77) is entered in the column 'height of instrument'. The readings of the staff at *b*, *c*, *d*, etc., deducted from this number, give the reduced levels of the several points.

For example, the reduced level of *b* is 113·77−6·96=106·81, which is entered in the column 'reduced level', opposite to the reading, and to the appropriate distance 100. Similarly, for the staff readings, at the points *c*, *d*, *e*, *f*, etc. Finally, the 'fore-sight' to staff at B is recorded in the proper column, is deducted from the height of instrument, and the remainder is entered as a 'reduced level'. The computation of the first setting of the instrument is now complete.

Referring again to the figure, to continue the section the staff is kept at B whose reduced level is now known and the level is carried forward, set up at another point which is chosen as before so that we can read the staff at B, and as far as possible onwards from there and adjusted. A new back-sight (in this case 9·36) is then read on the staff at B. The new collimation level (122·40) is found by adding this (9·36) to the known reduced level of B (113·04), and is entered in the appropriate column. The staff is then sent to the points *j*, and C, and so on. The reduced levels of *j* and C are then found by subtracting the readings at those points from the new height of instrument, and so on.

TURNING-POINTS. Points such as B (Fig. 40), where the staff is held while the instrument is moved to a new position, are called *turning-points*, *change sight points*, or simply *change points*. The two readings are often entered in the same line, and such points may be marked 'T.P.' (for turning-point) in the remarks column, as shown in the level book on p. 79.

CHECKING ACCURACY OF FIELD-WORK. Arrived at C, the surveyor is supposed to have completed his work, but he may wish to check its accuracy. To do so, he levels back from C to A, placing the staff and instrument at any convenient points. Between these points, perhaps following some different route, he also wishes to obtain, for future reference, the level of a permanent mark, such as the top of a milestone, near to C. He places the staff on it and reads; then, taking a turning-point, he continues his levelling back to A, by any convenient route, and holding the staff on any desirable points.

When he reaches A, all the 'back-sights' and all the 'fore-sights' are summed up. If the work has been accurately performed, the sum of the 'back-sights' should be equal to the sum of the 'fore-sights'. This is tantamount to the statement that, if all the reduced levels had been worked out, the final reduced level at A should have worked back to the same value as at first. If it be found that this is true, we generally infer that no serious error has been made in the reading of any *back-* or *fore-*sight but it is to be remembered that it affords no check on the accuracy of *intermediate* readings. There is, indeed, no check on such points, except such as is given by a general idea of the contour of the ground. Hence all points of importance should be read twice.

CHECKING ARITHMETIC. To check the arithmetic in working out the levels between, say, A and C, we add up all the back-sights and all the fore-sights, from A to C.

The difference between these sums should be the same as the difference between the first and last reduced levels.

For example, sum of back-sights	$=6{\cdot}32+9{\cdot}36 =$	15·68
" fore-sights	$=0{\cdot}73+7{\cdot}42 =$	8·15
Difference	. . =	7·53
Reduced level of C	. =	114·98
" " A	. =	107·45
Difference	. . =	7·53

The student must remember that this checks the arithmetic only. It does not show that the levels are correct. This can only be shown by levelling back to the starting-point, or by working between two points whose levels were previously known.

FIELD-BOOK FOR 'RISE AND FALL' METHOD. Another form of level book is shown in the annexed table, which refers to levels taken along the centre line of a proposed road, with important additional points.

The 'height of instrument' column is replaced by two, headed 'rise' and 'fall'.

The field-work is the same as before, and the staff readings are entered as already described, and the known (or assumed) reduced level of the first point is entered opposite the first back-sight. In this example the initial back-sight was taken to a point referred to in the 'Remarks' column as 'B.M.' These initials stand for 'bench-mark', described in some detail later.

LEVELS TAKEN FOR PROPOSED NEW ROAD GOING

FROM ____________ TO ____________ *Date* ____________

Back-sight	Inter-medi-ate	Fore-sight	Rise	Fall	Re-duced level	Dis-tance (chains)	Remarks
0·10	—	—	—	—	30·00	—	B.M. on covering flag of drain, C on plan
—	0·40	—	—	0·30	29·70	0·00	Point A on plan Beginning of proposed road
—	5·85	—	—	5·45	24·25	1·00	
—	9·60	—	—	3·75	20·50	2·00	
—	11·35	—	—	1·75	18·75	3·00	
—	12·10	—	—	0·75	18·00	—	Level of highest known flood
—	12·30	—	—	0·20	17·80	4·00	
4·32	—	12·56	—	0·26	17·54	5·00	T.P.
—	7·35	—	—	3·03	14·51	6·00	
—	7·80	—	—	0·45	14·06	7·22	Top of left bank of stream
—	12·14	—	—	4·34	9·72	—	Mean (summer) water level
—	13·54	—	—	1·40	8·32	7·28	Bottom of stream
—	13·28	—	0·26	—	8·58	7·76	" "
—	8·60	—	4·68	—	13·26	7·85	Top of right bank of stream
—	5·35	—	3·25	—	16·51	9·00	
11·78	—	1·27	4·08	—	20·59	10·00	T.P.
—	7·90	—	3·88	—	24·47	11·00	
12·37	—	4·96	2·94	—	27·41	12·00	T.P.
—	9·20	—	3·17	—	30·58	13·00	
—	6·05	—	3·15	—	33·73	14·00	
—	4·80	—	1·25	—	34·98	15·00	
—	3·35	—	1·45	—	36·43	16·00	
—	1·70	—	1·65	—	38·08	17·00	
12·85	—	0·57	1·13	—	39·21	18·00	
—	9·10	—	3·75	—	42·96	19·00	
—	7·60	—	1·50	—	44·46	20·00	
—	4·30	—	3·30	—	47·76	21·00	
13·14	—	2·08	2·22	—	49·98	22·00	
—	11·60	—	1·54	—	51·52	23·00	
—	10·10	—	1·50	—	53·02	24·00	
—	7·55	—	2·55	—	55·57	25·00	
—	5·60	—	1·95	—	57·52	26·00	
—	4·05	—	1·55	—	59·07	27·00	
—	—	2·70	1·35	—	60·42	28·00	Carried forward
54·56		24·14			30·42		
24·14							
30·42					30·00		

LEVELS TAKEN FOR PROPOSED NEW ROAD GOING FROM__________ TO__________ *Date*__________ *Continued.*

Back-sight	Inter-mediate	Fore-sight	Rise	Fall	Reduced level	Distance (chains)	Remarks
2·70	—	—	—	—	60·42	—	Brought forward
—	1·95	—	0·75	—	61·17	29·00	
—	1·35	—	0·60	—	61·77	30·00	
—	0·90	—	0·45	—	62·22	31·00	
—	0·60	—	0·30	—	62·52	32·00	Highest point
—	1·10	—	—	0·50	62·02	33·00	
—	1·75	—	—	0·65	61·37	34·00	
—	2·90	—	—	1·15	60·22	35·00	
—	4·00	—	—	1·10	59·12	36·00	
—	5·50	—	—	1·50	57·62	37·00	
—	7·50	—	—	2·00	55·62	38·00	
—	9·20	—	—	1·70	53·92	39·00	
—	10·00	—	—	0·80	53·12	40·00	
—	11·45	—	—	1·45	51·67	41·00	
1·33	—	12·92	—	1·47	50·20	42·00	
—	3·00	—	—	1·67	48·53	43·00	
—	4·45	—	—	1·45	47·08	44·00	
—	6·20	—	—	1·75	45·33	45·00	
—	7·85	—	—	1·65	43·68	45·83	D on plan. Centre of existing road
—	8·30	—	—	0·45	43·23	1·00	On existing road, to left
—	8·80	—	—	0·50	42·73	2·00	
—	9·20	—	—	0·40	42·33	3·00	
—	7·20	—	2·00	—	44·33	1·00	On existing road, to right
—	6·70	—	0·50	—	44·83	2·00	
—	6·10	—	0·60	—	45·43	47·00	
—	4·45	—	1·65	—	47·08	48·00	
—	3·53	—	0·92	—	48·00	49·23	Point B on plan. End of proposed road
1·18	—	12·44	—	8·91	39·09	—	Check levels
—	—	10·26	—	9·08	30·01	—	Back at A
5·21		35·62			30·41		
		5·21			60·42		
		30·41					

The readings taken from the first position of the level begin with the back-sight 0.10, and end with the fore-sight 12·56, taken at distance 5 chains. The rises and falls are then worked out by taking the difference between each reading *and the next*. The result is a *fall* if the second reading is the *greater*, and a *rise* if the second reading is the *less*.

Falls are then subtracted, and rises added in order to calculate the reduced levels from point to point, starting with the first reduced level (30·00) which has been already entered. The beginner should check this down to the first fore-sight.

When the level is moved, a back-sight is taken as before, the staff remaining at 5 chains. A line may be ruled across the book as already described, but this is really unnecessary, and the new back-sight (4·32) is here shown inscribed on the same line as the fore-sight, both these readings being taken with the staff at 5 chains.

At 6 chains the intermediate is 7·35, and the fall from 5 chains to 6 chains is found by taking the difference between this and the back-sight (7·35−4·32=3·03), and so on.

CARRYING FORWARD. This level book has been extended over two pages to show the method of carrying the figures on.

The last reading on the first page (2·70) is really an intermediate, but it should be written in the fore-sight column on this page, for reasons given below. The same reading is carried forward to the first line of the back-sight column on the next page, the reduced level (60·42) being also carried forward as shown.

When the last point on a page is a turning-point, the fore-sight is entered on that page, and the reduced level is worked out from it.

The new back-sight is entered on the new page, and the reduced level alone is carried forward.

When using the form of field-book shown on p. 76, we carry forward in exactly the same way. The height of instrument should not be carried forward if the last reading is a turning-point, but if an intermediate it may be carried forward.

The arithmetical check already described should be shown at the foot of each page, as in the example.

Thus the whole idea of the above methods of carrying forward is that each page should be treated as a finished section, and should begin with a back-sight and end with a fore-sight, as otherwise the arithmetical check would not work.

COMPARISON OF LEVEL BOOKS. The rise-and-fall method of keeping the level books involves more arithmetic than the height-of-instrument method.

Its advantage is that it affords a more complete check on the arithmetic. Each reduced level is derived from the one immediately preceding. Hence any arithmetical error anywhere is carried forward, and appears at the end, when the arithmetic is checked, so that it can be detected and corrected. In the height-of-instrument method, on the contrary, the reduced level of each intermediate point is obtained independently, so that the arithmetical check proves only that no error has been made in computing the turning-points. Whether this security against arithmetical error is, or is not, worth the extra number of figures is a matter of personal opinion.

BENCH-MARKS. In conducting levelling operations of an extensive character it is often desirable to ascertain the level, not merely of points on the ground along a line of section, or in the neighbourhood thereof, but of some permanent objects or points, for future reference or for the purpose of checking. Such points of reference are called 'bench-marks'.

The Ordnance surveyors of Great Britain incise on walls, gate-posts, etc., the well-known mark, or broad arrow (Fig. 41), indicating the location of an Ordnance bench-mark (O.B.M.). These are shown on certain Ordnance maps with their level inscribed. The maps in common use among surveyors and civil engineers are to the scales of 6 inches to 1 mile, and 1/2500, or 25·344 inches to 1 mile. The latter are extremely useful and are usually spoken of as '25-inch maps'.

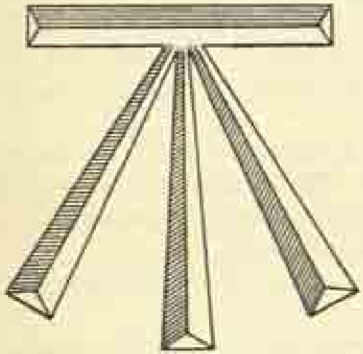

FIG. 41

The levels given on these maps at bench-marks refer to the centre-line of the horizontal V-shaped groove at the top of the mark, so that, when one is using an Ordnance B.M., the heel of the staff should be held level with this line. The common, or inscribed, type of Ordnance mark is not altogether convenient, since the staff is not supported by anything solid, and may therefore slip. In addition to these, however, the Ordnance Survey Department has established two other types, known, respectively, as 'fundamental' and 'flush bracket' bench-marks. The former have been set up in favourable situations on an absolutely firm foundation, and a typical example is shown in Fig. 42. It will be seen that there are two reference marks, one a gunmetal bolt and the other a polished flint embedded in a concrete base within

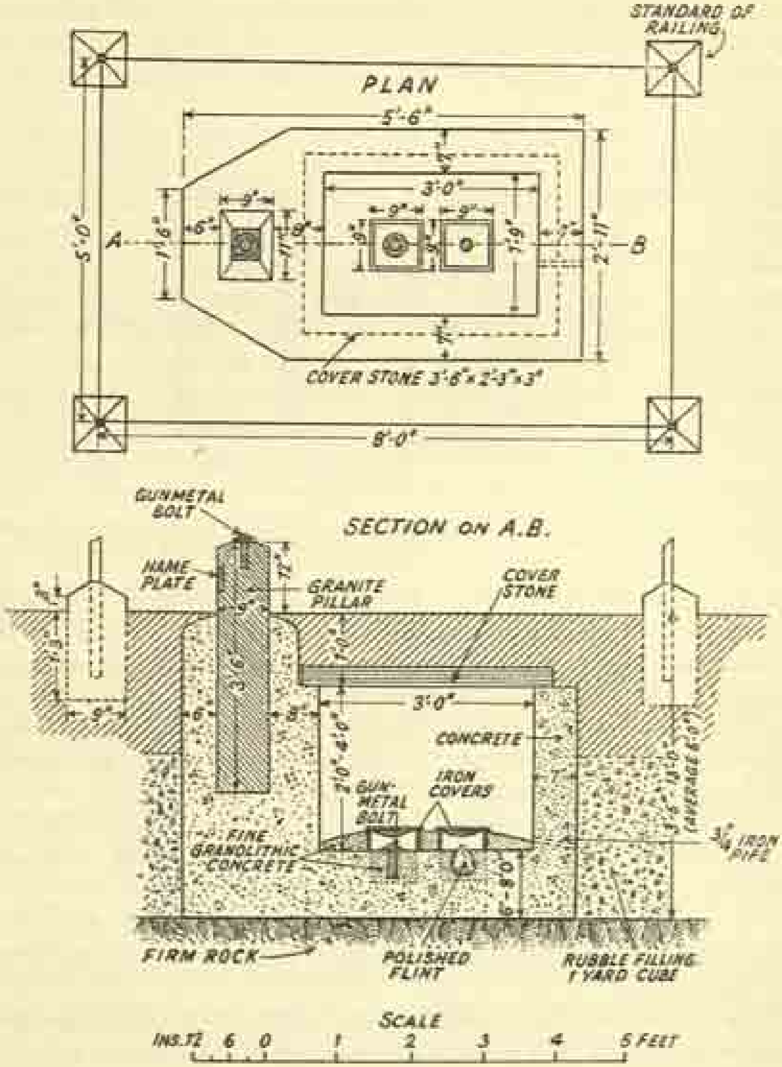

Fig. 42

a concrete pit. These are not accessible for public use, but a third reference mark is provided in the form of a gunmetal bolt set in a granite pillar about one foot high. These fundamental bench-marks, unfortunately, occur only at widely spaced locations and are frequently as much as 30 miles apart.

The flush bracket type is illustrated in Fig. 43. These are set in the walls of suitable buildings, and a special fitting is needed in order to support the staff at the correct height when reading. The

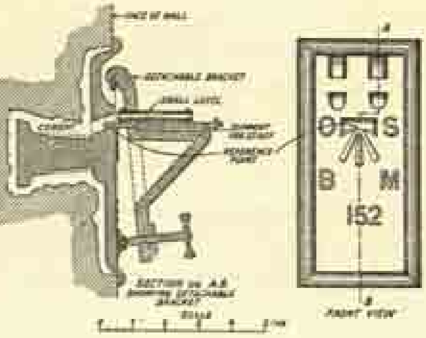

FIG. 43

illustration is self-explanatory. One of these bench-marks has been placed on the wall of the Royal Geographical Society's building in Kensington Gore and another on the wall of Knightsbridge Barracks, about one mile distant. It is intended, ultimately, to establish a complete network of these marks at about this spacing.

Private surveyors will rarely find it convenient to cut bench-marks, since they cannot generally afford the time to do so.

It is better to use existing objects on which the staff can be placed, for example, the tops of milestones, thresholds of doors (close to the jamb where there is no wear), and the like.

A neat sketch of the object forming the bench-mark should be entered in the level-book, with an arrow indicating the position of the staff.

In extensive levelling operations for public works, these sketches should be copied into a reference book, with full descriptions of position, and the reduced levels of each. This record will be of great service in future operations.

If, in a place where the British Ordnance bench-mark is in use, the surveyor desires to incise a bench-mark, he should adopt a different form of mark; for example, an arrow without the horizontal bar, thus ↓, in order to prevent confusion.

In levelling a long line of section, it is well to establish numerous bench-marks in convenient positions, say one at every half-mile, and this will greatly facilitate checking. The surveyor, having completed a line of section, should check his work by levelling back to the B.M. from which he started. If, on closing, a serious error is found, then, by comparing the first and second determinations of the levels of the several B.M.'s en route, he will ascertain the section in which the error has occurred; and he need only verify or re-level that section, and not the whole work.

These bench-marks, however are not only useful for checking the levels. Thus, suppose the levels are for a road or railway. Every point of the original ground along the centre line will probably be removed or banked over in carrying out the work. These fixed points are then of the greatest value as points of reference from which we can decide when cuttings or banks have reached the desired levels, and so on. Hence bench-marks should be chosen in such positions that they will not be interfered with in carrying out the proposed works for which the levels are taken. As a rule the levels are started from a bench-mark near the commencement of the section, not from the first point of the actual section, as shown in the level-book on p. 79.

DATUM FOR LEVELS. In fixing the reduced levels on the Ordnance Survey maps of England, it was desired that the datum should be approximately coincident with the mean level of the sea, this being more constant than low-water level, which was chosen for Ireland. Observations of sea-level at Liverpool were therefore made in 1844, and the mean sea-level as given by these observations was adopted as the datum. It is about 0·65 foot below true mean sea-level at Liverpool. It has since been found however, that this value is not truly representative for the entire country, and in 1921 a more satisfactory datum was established at Newlyn, in Cornwall. The Ordnance surveyors have revised the values of bench-marks based on the old datum, and the more recent editions of the six-inch and twenty-five-inch maps show these revised values. If the levels shown are based on the Newlyn datum, this fact is stated on the map. For most practical purposes it matters little which datum is used, provided that either the old levels or the new levels are adopted throughout the work. When one is referring to Ordnance maps during the course of any levelling, it is very important to see that the old and revised editions are not mixed.

For Ireland, the datum was fixed by the level of the sea at the mouth of the Liffey, at low water of the spring tide on April 8, 1837. It is perpetuated by a mark on the base of Poolbeg lighthouse, and is about 8 feet below sea-level.

For many engineering works the height above sea-level is of little importance, and in such cases the datum is usually taken some round number of feet below an arbitrary bench-mark chosen by the engineer. See the level book on p. 79, for example.

All levels must be referred to that of some permanent and easily recognized mark. The datum-line must be definite, as being so many feet above or below some object, such as the step of a church, etc. It is permissible to use an Ordnance B.M. as the primitive point, and to define the datum-line as being so many feet above or below the same, quoting, if the surveyor sees fit, the level of the B.M. as given by the Ordnance Survey, but he is not permitted to define the datum-line of a Parliamentary plan as 'Ordnance datum' or so many feet above or below it.

Again, in the case of seaport towns, there will be constructions, such as sewers, docks and sea-walls, some parts of which will be above, others below, mean sea-level. If mean sea-level were adopted as datum, much inconvenience would arise owing to the necessity for inscribing + or − before the levels; and to ascertain differences of level, subtraction or addition must be used, as the case may be, causing confusion and increased liability to error.

In such cases, a datum-plane, well below any probable structure, should be assumed. In a survey of the City of Bombay, for example, a plane 100 feet below a certain plate of brass in one of the steps of the Town Hall was adopted as a datum. It was afterwards ascertained, by tidal observation, that mean sea-level was 80·30 feet above that datum-plane. Extreme high water of spring tides was about 89, and extreme low water about 71. All the levels of the dock-works are therefore positive, coping level being 92·00 feet, say, and foundations 52, and so on. Even the levels of the harbour-bottom in the fairway are positive. This proved a most satisfactory arrangement.

EXTRA TURNING-POINTS. In the example on p. 79 the turning-points have all been taken along the centre line of the section. But this is by no means always the case. In levelling along streets on which there is much traffic, for instance, it would be highly inconvenient to take a turning-point in the middle of the carriage-way, as the staff must remain there for some time.

Having read as far as possible along the centre line, and entered the readings as intermediates, the surveyor should direct the staff-holder to some convenient out-of-the-way spot, where the fore-sight should be read, and also the new back-sight, which are entered in their appropriate columns. The distance column has a stroke entered across it, and the letters 'T.P.' are written in the remarks column, to show that these readings were taken for the purpose of carrying on the levels only, and that, once the reduced levels are calculated, no notice is to be taken of this turning-point in plotting. The writer prefers to reserve the use of the letters T.P. in the remarks column for such cases as this.

SOLIDITY OF SUPPORT FOR STAFF. Displacement of the staff, when turning from fore- to back-sight, and taking up a new position of the instrument, is a fertile source of error. The staff should be held for the change on a peg, or if the ground is soft, on a stone firmly stamped down.

The appliance shown in Fig. 44 is useful in such cases. The plate is firmly stamped down, and the staff is held on the spherical boss in the centre.

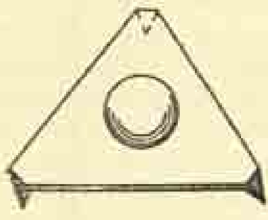

FIG. 44

In hilly countries, if the staff be held on a sloping face of rock, it is apt to slip down when it is turned. It may rest on one angle only, when the act of turning is pretty sure to let it down if the turning is carelessly performed.

Settlement of the staff always tends to make an error in one direction, that is to say, to increase the back-sight unduly. The error is therefore cumulative.

SOLIDITY OF THE GROUND ON WHICH THE INSTRUMENT STANDS. It is obviously desirable that the instrument should be set up on solid ground. If it settles during the interval between the reading of the fore- and back-sights, an error will be introduced. If it is absolutely necessary to erect the instrument on soft ground, then the best plan is to support the legs on strong, firmly driven stakes. If the means of so doing are not available, then the only plan is to read the fore- and back-sights in rapid succession, each time bringing the bubble to the middle of its run, the observer shifting his position as little as possible.

Vertical position of staff. It is obviously essential that the staff be held in a vertical position. The usual plan of attaining this, is to cause the staff-holder to stand to attention, heels together, with the heel of the staff between his toes, and clasping it between the palms of his hands at the height of his face, with the staff touching his nose. In this way a fair approximation to verticality can be attained. It is, however, difficult to induce staff-men to assume this severe attitude.

In windy weather, or on rough ground it is impossible for them to do so. The writer, therefore, recommends the use of some appliance which will enable the staff-holder to hold the staff perpendicular when standing in any attitude. One way of so doing is to attach a small circular bubble-level to the back of the staff, by means of a bracket. The staff-holder then brings the bubble to the middle of the glass-plate, when, if the level is properly adjusted, the staff will be vertical. The drawback to this arrangement is that circular levels are very apt to leak, the spirit in them consequently evaporating. They are also very liable to breakage if the staff is handled carelessly in spite of the protective cover which is usually supplied with these accessories. The level, with its supporting bracket, forms a projection from the staff, and is therefore liable to injury in transport.

The simplest plan is to attach a small pendulum of brass or iron (Fig. 45) to the back of the staff. The point below the pendulum bob fits losely into a ring connected to

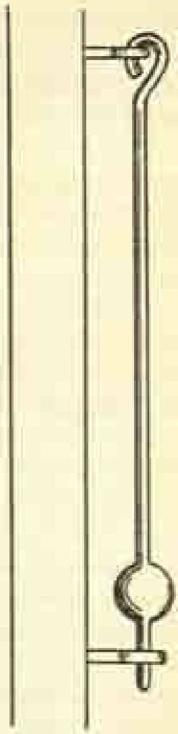

Fig. 45

the staff. The staff-holder has merely to hold the staff so that the pendulum point swings about the middle of the ring, when perpendicularity, in all directions, is secured. The merit of this arrangement is that the staff-holder can see whether the point is in the centre of the ring with greater ease than in the first-mentioned plan. It is not usual to provide levelling-staves with levels, or pendulums, as good work can undoubtedly be performed without them.

The object of the vertical lines in the diaphragm is mainly that, by keeping the staff always between them, the surveyor ensures that he is always using the same portion of the horizontal line. Thus, any slight deviation of this line from a truly horizontal line is rendered harmless. But it is obvious that they also enable the observer to see whether the staff is sloping sideways, so that he may signal to the staff-holder accordingly.

In the absence of a level or pendulum, it is, however, impossible for either the surveyor at the level, or the staff-holder, to see whether the staff is sloping towards or away from the level. Thus, if AB (Fig. 46) be the line of collimation, and OC the sloping staff, the reading obtained will be OC, which is equal to OD. But if the staff were vertical, the correct reading would be OE. Hence, the reading obtained is always too high by the amount ED.

To avoid this, the surveyor should signal the staff-holder to wave the staff slowly to and fro, and the surveyor then watches the line as it moves up and down the staff, and takes the *lowest* reading. The staff should be rocked preferably on a rounded surface, and it must not be tilted about its back edge; otherwise a reading may be obtained which is lower than that in a vertical position. The staff-holder must be trained to take one short step backwards on receiving the signal, as otherwise his body may prevent the staff from being brought far enough back.

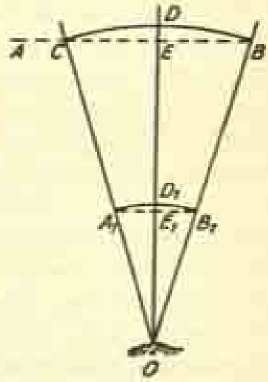

FIG. 46

It is evident that the error will increase with the length of staff OE which is being read; hence this waving is particularly important when the reading is near the top of the staff, and should never be omitted in such cases, especially at turning-points.

SIGNALS TO STAFF-HOLDER. The surveyor will find it exceedingly convenient to arrange beforehand a code of signals with his staff-holder. A list of operations for which signals should be arranged will now be given. Shouting over the work necessarily tends to lower the dignity of the profession, as well as to cause mental irritation, and should be carefully avoided by the young surveyor.

(*a*) To move the staff onwards when the reading is completed. This signal must be given immediately, to avoid delay, and the staff-holder must be taught *not* to move till he receives it.

(*b*) To move the staff bodily to the right or left.

(*c*) To bring the staff vertical when it is sloping sideways. Remember that the telescope reverses things, and you must signal the opposite way to what appears necessary in the telescope.

(*d*) To wave the staff to and fro.

(*e*) For pulling out the upper lengths of the telescoping staff.

(*f*) To bring the staff nearer to the level.

(*g*) To take the staff further away.

(*h*) To note that the reading is to be a *fore-sight*, for example, hold both arms out horizontally (or rather above the horizontal) in front; then give them both a sharp downward movement, so that the hands move through a distance of about a foot.

This signal should always be given so that the staff-holder may take proper precautions as to the spot on which he holds the staff, etc.

It is sometimes necessary that the staff should be slowly raised from the ground to enable the observer to see a figure denoting the whole feet. This never happens, however, unless the staff is so near the level that one can see the whole foot by sighting along the outside of the telescope tube; or verbal instructions can be given at this short distance without shouting.

ADDITIONAL POINTS TO BE NOTED. (1) As already stated, the beginner must be very careful to focus accurately (p. 66) and to eliminate parallax in the eye-piece.

(2) Back-sights and fore-sights must be taken with the greatest care. Hence the bubble must be examined for each such reading, *after* directing the telescope to the staff. If not central, it must be brought so before reading, using the plate screw or screws which are most nearly parallel to the telescope, without setting the latter directly over the screws. In a tilting-telescope level the micrometer screw is used.

(3) Intermediates are often read to the nearest tenth of a foot only.

A line of levels may intersect a brook, a section of which is required. When the staff is held in the bed of the brook, the top may be below the level of the line of collimation. Now, the level of the brook does not demand great accuracy. It would be waste of time to shift the instrument merely for the levels of the brook. To meet such cases a square rod exactly five feet long, with small brackets or cleats nailed to the sides with their tops at one, two, three and four feet from the bottom, is used.

If the staff is held on one of the cleats, it may be brought within range of the instrument, and the proper number of feet can be added to the staff-reading. This arrangement should not be used in the case of fore- or back-sights.

(4) It has been stated (p. 78) that there is no check on the accuracy of intermediate readings, even if we work back to our starting-point; hence such points should be read twice. A sound rule, where accuracy is important, is to read, book the reading, then read again, before signalling the staff-holder to move. This is a wise precaution whether the reading be an intermediate or not.

(5) It sometimes happens (particularly in town work where the choice of positions for the instrument is somewhat restricted) that one desires to take a reading at a point so near to the instrument that the staff cannot be focussed.

In such cases the staff-holder should be directed to slide a piece of white paper up the staff. The paper will be visible, though out of focus, and (although its edge will appear very blurred and indistinct) if the staff-man be directed to stop when this indistinct edge of the paper appears to coincide with the cross line, the reading will then be obtained directly from the staff with only a very small error.

To ensure despatch, the work should be done with as few shifts as possible. Hence the level is often carried so far forward, for the new back-sight, that the top or bottom of the staff (as the case may be) will be just intersected by the line of collimation when the level is adjusted. Practice enables one to judge this fairly well. But, to save time, the surveyor, having chosen his new position, should set the level up so that a pair of opposite plate screws (if there are four) – or one plate screw if there are three – points roughly towards the staff. Then lift *one* leg of the tripod off the ground, and move it sideways or to and fro, so as to bring the main bubble and the small side bubble both roughly central.

Then take aim at the staff *over* the telescope, and bring the main bubble approximately central by the plate screw or screws then most nearly parallel to it. Look through to see if the line of collimation intersects the staff, say near the top. If so, level up properly. If it meets the staff, say, three feet from the top, move it, by guesswork, two feet higher up the hill, then level up without further test. If it is off the staff, the level must, of course, be moved downhill, but be careful to see that the staff is not invisible simply through being out of focus.

EQUALIZATION OF BACK- AND FORE-SIGHTS. With a staff longer than about 10 feet, the method of working just described often leads, however, to inaccuracies.

FIG. 47

It has been stated that the line of collimation should be accurately parallel to the spirit-level. But this can seldom be ensured with absolute exactness, and usually, when the bubble is central, the line of collimation will make a small angle above or below the horizontal, as shown exaggerated in Fig. 47.

Thus if *ceg* be a line parallel to the spirit-level *ll* (and therefore truly horizontal), and *cfh* be the line of collimation (along which we read), it is evident that there will be an error in the reading, and that this will depend upon the distance between the staff and the level. Thus the error *hg* is greater than *ef*.

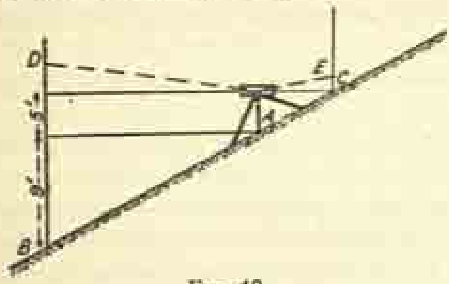

FIG. 48

Now, suppose we are levelling uphill with a 14-foot staff, and we arrange that the back-sight BD (Fig. 48) should be 14 feet, and the fore-sight at C should be roughly zero. The level itself is about 5 feet high.

Hence the distance between A and B will be such as to give a difference of level of $14 - 5 = 9$ feet about, while from A to C the rise will be about 5 feet only. Thus for a nearly uniform slope the distance AB will be greater than AC, so that the error in the reading at B will be greater than that at C. We therefore obtain either too big or too small a rise, according to the direction of the error in the line of collimation. And this will be repeated, in the same direction, for each shift. The level at the top of the hill will be too high or too low accordingly. Now when in checking back we work down the hill, the process will be just reversed. The fore-sights will be at the greater distance, and, if the rises were too great before, the falls will now be too great, and the level at the starting-point may check back correctly in spite of the error at the top.

But if the back- and fore-sights were taken at *equal distances* from the level, both readings would be increased or decreased by the same amount. Hence the difference would be unaffected, and we should obtain the correct rise or fall in spite of the error in each reading. Working in the manner above described, one would obtain approximately this result with a staff whose length is about double the height of the level above the ground, that is, about 10 feet as recommended on p. 62. Reading back- and fore-sights at equal distances is called, somewhat loosely, 'equalization of back- and fore-sights'. Actually, the *distances* are equalized.

Supplementary parts to levels. Supplementary fittings to levels include sight vanes at the side or top of the telescope and reflecting devices which enable the position of the bubble to be seen from the eye-piece end of the telescope. These devices may consist of a hinged mirror, or a prismatic reflecting device already mentioned on page 57.

In addition, levels are often fitted with a compass. This is frequently useful in rapid surveys, and, if combined with additional reference lines on the diaphragm for obtaining distances by tacheometry, affords perhaps the best way of fixing the positions of a number of spot levels, as, for example, in reservoir surveys.

It is desirable here that the levels should be taken at all those spots where there are changes of slope, so as to show the irregularities of the ground. But it is seldom important that the *exact* position of the spot should be located on the plan within a foot. A well-trained staff-holder will pick out the right spots, and their positions are fixed by reading the compass for direction and the appropriate reference lines, known as 'stadia', for distance.

CROSS-SECTIONS. Suppose AB (Fig. 49) is the centre line, in plan, along which a section is required for a proposed work, and that the

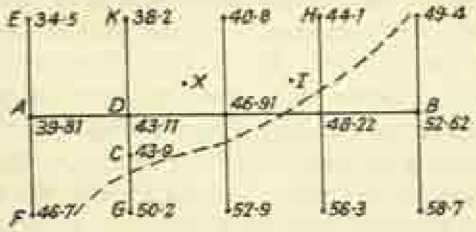

FIG. 49

ground, in addition to rising from A to B, has a transverse slope downwards on the left, as shown by the figures written on the plan. These figures show the reduced levels of the various points, and are called 'spot levels'.

In such cases it is necessary to take cross-sections as well as the longitudinal section, as the transverse slope will not only affect the quantities of earthwork, but may cause modifications in the design of the work from an engineering point of view. These levels may be taken at the same time as the longitudinal section, in which case two or three sections should be set out in advance, and numbered or referenced pegs or laths put in at the points where levels are to be taken.

Thus a peg at K would be marked 100 L. at 1·0, or in some such way, meaning that it is 100 feet to the left at one complete chain length (i.e. 100 feet or 66 feet, as the case may be). Another method of booking the centre line chainage is 1+00 instead of 1·0 along AB. Then, assuming that the levelling starts at A, a position, say X, would be chosen for the instrument, such that the staff at E would be intersected near the top. Assuming that the level were set up, as it usually is, at a height of about 5 feet, the height of collimation would be roughly 48 feet above datum and from this position, with a 14-foot staff, all the points to the left of the broken line could be read. The staff-holder would be sent to E, A, F, in succession; then to G, to see if that could be read. A special signal could be arranged with the staff-holder to pick up the peg, if desired, after the reading is taken. This signal would not be given at G, but, on finding that the point could not be read from X, he would be signalled down the hill to C, D, K, and so on.

After the reading at H no farther-points can be read from X, and the staff-holder is signalled up the hill until the bottom of the staff is just visible, when he gets the fore-sight signal and chooses a convenient spot I for an extra turning-point, as described on p. 87. The level is then moved up, and all the back points like G are read as far as possible.

There are many systems of booking in vogue. If the object is to show spot levels on a plan, it is perhaps most convenient to give each peg a number as it is put in, and to write this number on a sketch plan, together with the position of the peg. Thus Fig. 50 shows the sketch plan for the first two sections, which would be made whilst the latter were being set out.

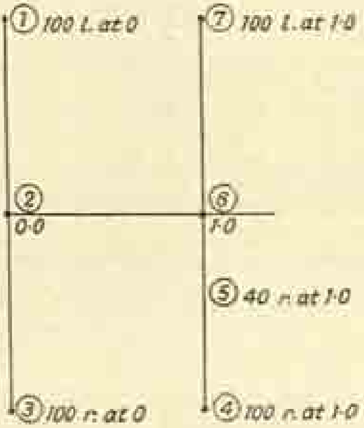

Fig. 50

Then it is necessary only to write the number on the peg, and to put down the number of the peg in the remarks column of the level book as the point is read. If the staff-holder also keeps a book in which he enters the number of each peg as he gets the signal that it has been read, books can be compared every time he comes near the instrument. The convenience of this system in plotting the plan is obvious, but if cross-sections are to be plotted, as is usual in road or railway work or the like, it is more convenient to have the actual positions written in the level book.

For this purpose, the distance column is sometimes divided into three parts headed *left*, *centre* and *right*. Thus, when say No. 7 peg is read, we should enter 100 in the *left* column, and 1·0 or 1+00 in the *centre*, while the *right* column would be blank.

The units of measurement should always be stated.

Level books so ruled can be purchased if desired. If not, the readings are perhaps best entered as in the example below, which is supposed to be the level book for the sections shown in Figs. 49 and 52.

Back-sight	Inter-mediate	Fore-sight	Colli-mation level	Reduced level	Distance	Remarks
13·61	—	—	48·11	34·50	100	L. at 0·0, No. 1
—	8·30	—	—	39·81	0·0	No. 2
—	1·40	—	—	46·71	100	R. at 0, No. 3
—	4·20	—	—	43·91	40	R. at 1·0, No. 5
—	5·00	—	—	43·11	1·0	No. 6
—	9·90	—	—	38·21	100	L. at 1·0, No. 7
—	7·30	—	—	40·81	100	L. at 2·0
—	1·20	—	—	46·91	2·0	—
—	4·00	—	—	44·11	100	L. at 3·0
13·15	—	0·24	61·02	47·87	—	T.P.
—	9.80	—	—	50·22	100	R. at 1·0, No. 4
—	8.10	—	—	59·92	100	R. at 2·0
—	4·70	—	—	56·32	100	R. at 3·0
—	12·80	—	—	48·22	3·0	—
—	11·60	—	—	49·42	100	L. at 4·0
—	8·40	—	—	52·62	4·0	—
—	—	2·30	—	58·72	100	R. at 4·0
26·76		2·54		24·22		(Carried forward)
2·54						
24·22						

This method of booking has the advantage that the position of each point is written as one would naturally describe it, thus 100 feet right at one chain, and so on. In actual use it is as convenient as any other. Any clear system of booking, however, may be used.

Other methods. Cross-sections are often taken independently of the main section, either by clinometer or hand-level. If the former, the surveyor simply stands at each point on the centre line where a level has been taken, and reads the angle of slope on each side of the centre line, noting whether it is up or down.

With a hand-level, the surveyor measures the height of his eye above the ground, stands at the point on the centre line whose level is known from the main section, and reads the staff held at various points on the cross-section.

As a hand-level usually consists only of a sighting tube, without the optical aid provided by lenses, a simplified staff is often used to avoid the difficulty of distinguishing the more complicated graduations of the Sopwith staff beyond a very restricted sighting distance. One simple form of staff, graduated in feet and tenths, is shown in Fig. 51, and a typical form of hand-level is shown in Fig. 60, p. 109.

If all points required in any cross-section cannot be read from the centre line, the surveyor simply moves to one of the points already read (whose level is therefore known) and continues from there.

Perhaps the most satisfactory booking is to make a separate book for each cross-section, taking the observer's height always as back-sight.

These methods have the advantage that the chief surveyor's attention can be concentrated on the accurate levelling of the centre line whilst an assistant obtains the approximate cross-section levels. A slight error in a cross-section is of less importance than an error in centre-line levels.

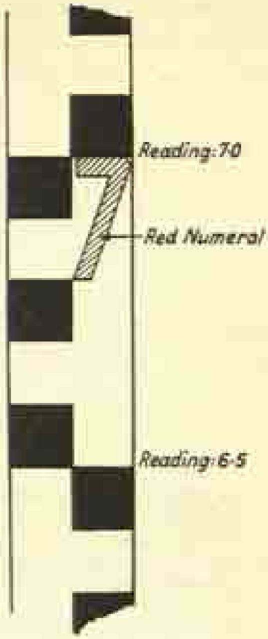

FIG. 51

As an example in the use of cross-sections, the figures given above have been plotted in Fig. 52. It is assumed that the formation line is level with the surface at A, and rises 1 in 60 therefrom along the centre line. The formation width is taken as 40 feet throughout, and the side slopes as $1\frac{1}{2}$ to 1 in cutting, and 2 to 1 in embankment.

The formation line shows the levels to which the earthworks are to be carried before any constructional work is superimposed. Thus in road construction several layers of road material may be placed on the compacted earth and the formation level may be a foot or more below the finished surface level. Again, in railway construction, the formation level is the rail-level minus the depth of rails, sleepers and ballast.

The longitudinal section is shown at the top of the figure. To draw the cross-section say at A (chainage 0·0), we first draw *acb* to represent the formation level, and make it 40 feet wide to scale. This level at A is seen from the longitudinal section to be 39·81.

We therefore set down $cd = 9{\cdot}81$, so as to obtain a line *edf* at some round number of feet (in this case 30·0) above datum.

Then scale off *de* and *df*, the measured distances *left* and *right* at 0·0; at 100 feet *left*, the reduced level is 34·50. Hence we set up $eg = 4{\cdot}5$ feet (note, $34{\cdot}5 - 30{\cdot}0 = 4{\cdot}5$); and in the same way

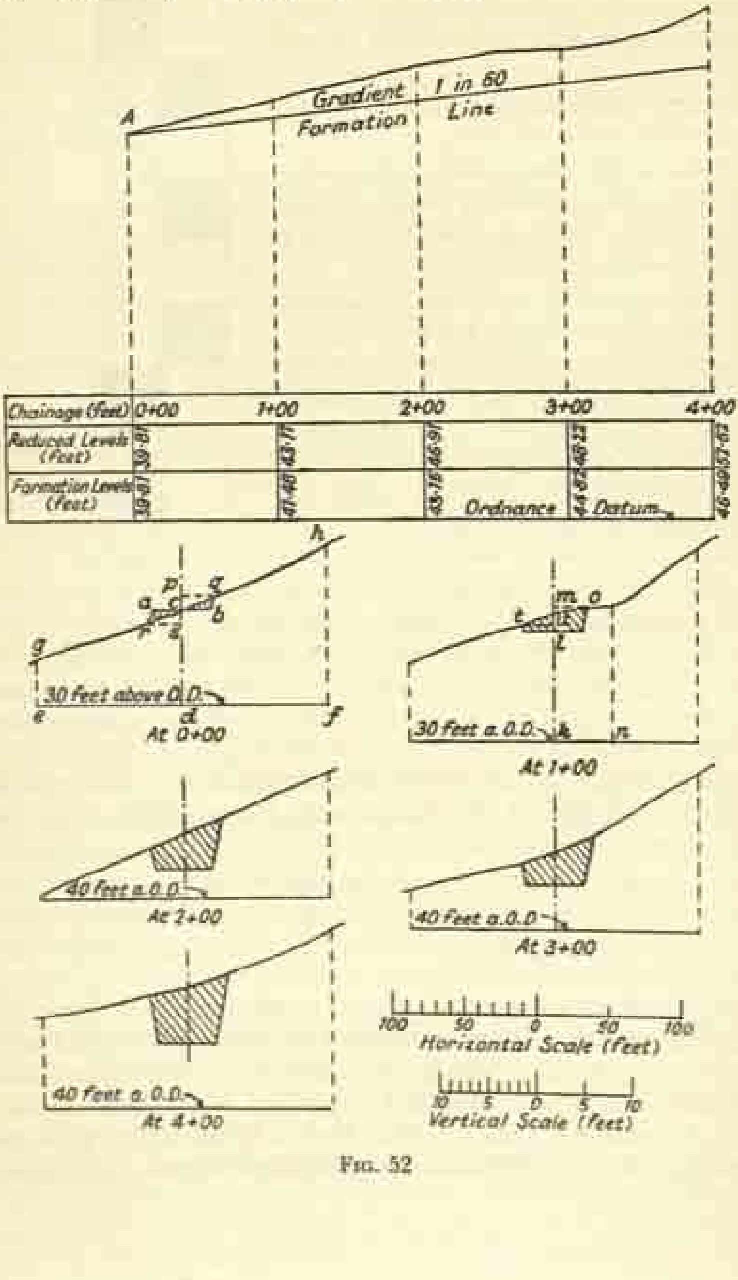

FIG. 52

fh = 16·7 feet, the level at 100 feet *right* being 46·7. Thus we obtain the ground line *gch*. Then from *a* and *b* draw *ar* and *bq* at the proper slopes (2 to 1 or 1½ to 1) according to whether the work at those points is seen to be in bank or cutting. In the drawing of these slopes, the exaggeration of the vertical scale must be taken into account. Thus for 1½ to 1 we take say, 10 units in a vertical direction as measured on the vertical scale, and 15 units horizontally as given by the horizontal scale.

The points *r* and *q* show the intersections of the slopes with the original ground, and the distances *rs* and *pq* show the breadth of ground (to left and right of the centre line respectively) required at A for carrying out the work. This does not include any allowance for fences, catchwater drains at the top of the cutting slopes, or the like, however.

The rest of the cross-sections are all drawn in the same way.

In many cases the cross-sections are drawn to natural scale, i.e. the same scale is used for both horizontal and vertical measurements, but in the example the scales are the same for the longitudinal and cross-sections.

The horizontal line *edf* is frequently omitted, as well as all construction lines, so that the finished cross-section shows only the original ground line and the proposed alterations, the former being shown *black*, and the latter usually *red*.

In engineering drawings it is a common practice to distinguish in this way between the existing and proposed features in the plan or section. Thus in the longitudinal section the formation and finished surface lines would both be red, and the figures giving the formation levels would be written in red ink also.

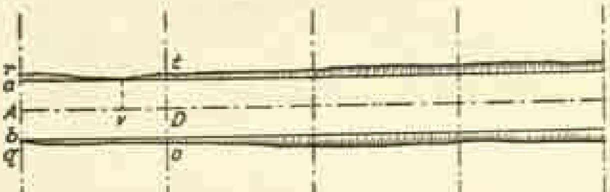

FIG. 53

COMPLETING PLAN. When the cross-sections are complete, the plan of the work can be drawn. Thus Fig. 53 is a reproduction of Fig. 49 without the spot levels. At A we set off A*a* and A*b*, each equal to half the formation width (20 feet in this case), and draw parallels to the centre line. Then set off A*r* equal to *sr* of Fig. 52, and A*q* equal to *pq*. Similarly at D, D*t* is made equal to *ut* of Fig. 52, and D*o* equal to *mo*, and so on.

Then, joining all the points so obtained on the right, we get the outcrop of the cutting in plan. On the left, we are in embankment at A, and cutting at D, as shown by the cross-sections. Somewhere between, there will be no side slope of either kind. The position of this point may be guessed by simply producing the line of the cutting; or we may find it by proportion thus: height of embankment at $a = cs$ (Fig. 52) and depth of cutting at $t = ul$; then $Av : vD = cs : ul$.

It will be apparent from this example that cross-section levels must be taken sufficiently far to the right and left of the centre-line amply to cover the extent of the cuttings and embankments envisaged in the project for which the survey is being made.

Setting out slope pegs. In order that cuttings and embankments may be set out certainly and accurately, the distances from the centre line to the toe (or outcrop) of the slope, both left and right, should not be measured from the drawing, but pegged out directly on the field for each cross-section, with the aid of the level.

For example, if at a certain cross-section (Fig. 54) the formation level is to be 76·87 and the centre line reduced level is 71·04, suppose it is required to place pegs at the points A and B representing the toes of the slopes. Let the formation width be $2b$, and the side slopes n to 1.

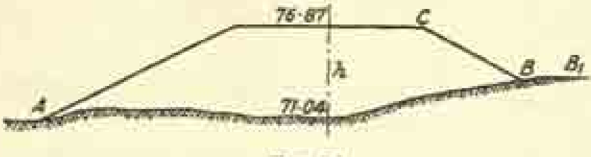

Fig. 54

Then, if the ground were level, the distances of A and B, left and right of centre, would be $b + nh$; a point for B is so guessed to be about this distance away, the nearer or farther, according to the direction of slope. In this case, say $b = 20$ feet, and $n = 2$ to 1. By subtraction of the height of the embankment at the centre-line,

$h = 5{\cdot}83$; $\therefore b + nh = 31{\cdot}66$. Hence, as the ground is rising towards B, we guess a point, B_1, say 28 feet from the centre. Set up the level and take readings at the centre line (marked by a peg) and at B_1; suppose these show a rise of 2·76; $\therefore$ level of $B_1 = 73{\cdot}80$. Now calculate the horizontal distance from the centre line to a point in the slope BC whose level is 73·80; fall from formation $= 76{\cdot}87 - 73{\cdot}80 = 3{\cdot}07$; distance $= 20 + (2 \times 3{\cdot}07) = 26{\cdot}14$.

This is less than the measured distance to B_1; hence we must choose a new point between these two distances, and in two or three tries the measured and calculated distances will agree with sufficient accuracy and the peg will be inserted. It will be clear that this can be done on the ground for any intermediate cross-sections as required, and will greatly facilitate future work.

EFFECT OF EARTH'S CURVATURE. Throughout this chapter it has been tacitly assumed that the datum, or level surface to which the reduced levels are referred, is a plane surface. As a matter of fact, this is untrue, in consequence of the roundness of the earth.

To obtain a correct idea of a level surface at any given height, we must suppose the sea to stand at that height or level, and then to percolate freely through the land, and imagine all flow, and tidal, or other wave movement, to have ceased. The surface of the water will then give a *level surface*. Such a surface would be approximately spherical, though, as is well-known, it would not be truly

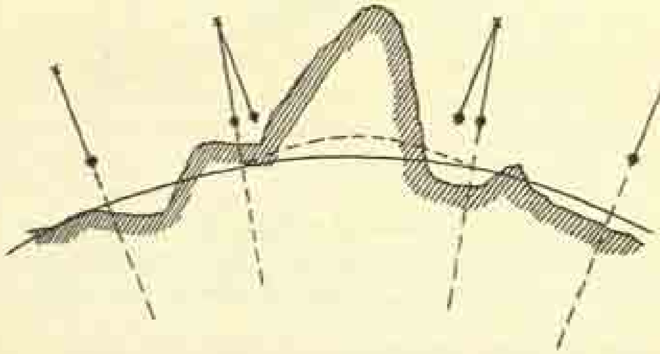

FIG. 55

so, but really spheroidal, having a larger radius of curvature at the poles than at the equator. It would be liable also to small irregularities, due to the local attraction of large mountain masses and the like, as shown by the dotted line in Fig. 55, in which the full curved line shows the direction the level surface would assume without this disturbing influence. The size of the mountain and the amount of displacement of the level surface are, of course, out of all proportion to anything actually existing on the earth.

A plumb-line, suspended above any point on a level surface, would settle in a direction perpendicular to the level surface, as shown by the lines from the points X (Fig. 55). The figure shows how the plumb-line would be deflected by the attraction of the mountain.

Now, in Fig. 56, suppose that a level is set up at A, as shown, and that AC is a level surface through A. The spirit-level, and line of collimation, when adjusted, will be parallel to the direction of this surface at A. Hence, on looking through the telescope, we shall be looking in the direction ID, and D will appear to be on the same level as I.

Now, suppose we take E, so that ED = AI. Then obviously, from level readings, it would appear that A and E were on the same

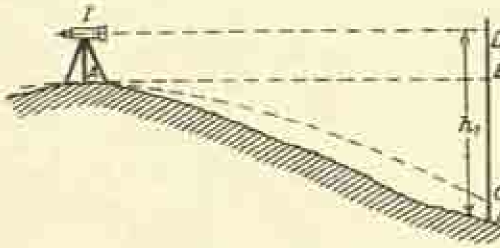

FIG. 56

level, whereas AC is the true level surface. If B be the point on the solid earth at C, the true fall from A to B is given by the drop CB. But, according to the reading of the instrument, the drop would be EB.

The difference, EC, is called the *curvature correction*.

CALCULATION OF AMOUNT. In calculating the amount of the curvature correction it is sufficient, in general, to assume that a level surface, as above defined, is truly spherical.

In Fig. 57, let AB be the level surface through A; let O be the centre of the sphere, and AC a straight line, horizontal at A. Then BC $= c$ is the curvature correction for the distance AB, which may be taken as equal to AC; r is the mean radius of the earth which may be assumed to be 4,000 miles.

Now $\quad OC^2 = OA^2 + AC^2$

i.e. if we put $\quad OB = OA = r$

and $AC = d$

$$(r+c)^2 = r^2 + d^2$$

or $\quad r^2 + 2rc + c^2 = r^2 + d^2$

Whence $\quad c(2r+c) = d^2$

or $$c = \frac{d^2}{2r+c}$$

Now, in the diagram the value of c, as compared with r, is very much exaggerated. Actually c is so small compared with r that we may put $c = \frac{d^2}{2r}$ without practical error.

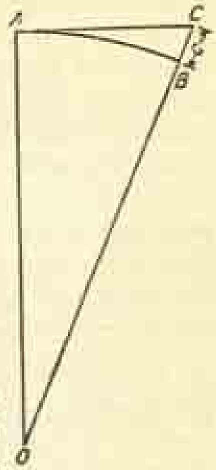

FIG. 57

That is, to find the curvature correction, *divide the square of the length of sight by the earth's diameter*. Both may be taken in the same units, when the answer will be in terms of that unit also.

Taking the earth's diameter as 8,000 miles, for a sight 1 mile long the curvature correction will therefore be

$$1 \div 8{,}000 \text{ miles} = 5280 \times 12 \div 8{,}000 \text{ ins., or about 8 ins.}$$

For one-tenth of a mile $= 528$ feet, which is a long reading with a level, the correction will therefore be $\frac{8}{100}$ in. $= 0{\cdot}007$ foot.

It thus appears that this error is less than 0·01 foot, even for the longest readings in ordinary levelling. Hence it is practically negligible for any single reading.

EFFECT OF EQUALIZATION. In taking a back-sight and fore-sight each reading will be affected by curvature. It is evident, by inspection of Fig. 56, p. 102, that the staff reading is always *too great* in consequence of curvature. Hence each reading will be increased by an amount depending on the distance. The effect is therefore, similar to that of a collimation error, except that it is proportional to the *square* of the distance. It can be avoided, like collimation error, by equalizing the distances of the back- and fore-sights, which will cause both readings to be increased equally, so that the difference will be unaffected.

Effect of refraction. There is another correction to be considered, due to the fact that the ray of light, from the staff to the instrument, does not travel in a straight line, as we have hitherto presumed. Its path is curved in consequence of atmospheric refraction, due to the varying density of the atmosphere at different levels. This effect is not very constant, and is considered more fully in the second volume of this work. In the meantime the student may take it that the *average* effect of refraction is about *one-seventh* that of the curvature, and in the *opposite direction*, so that, to find the combined curvature and refraction correction, we first calculate for curvature, and then take six-sevenths of the result.

Sign of the correction. It has been shown that the effect of the combined error is to *increase* the readings. Hence, in levelling, perhaps the most convenient way to apply the correction, if desired, is to *decrease* each reading by the calculated amount, according to the distance of the staff from the level.

Example. A level is set up and a reading taken to a point A is 4·86 feet. A reading is then taken to B, 600 feet away. Find the corrected rise or fall, (*a*) if the reading be 2·17; (*b*) if it be 6·71 feet. Take the Earth's radius of curvature as 20,930,000 feet.

$$\text{Curvature correction} = \frac{600^2}{2 \times 20{,}930{,}000} = 0{\cdot}0085 \text{ foot}$$

$$\text{Refraction} = \text{one-seventh} = \underline{0{\cdot}0012}$$

$$\text{Total} = 0{\cdot}0073$$

If readings are being taken to the nearest hundredth of a foot, the correction may be assumed to be 0.01 foot.

This is to be *subtracted* from each reading.

$$\therefore \ (a) \ \text{Rise} = 4{\cdot}86 - 2{\cdot}16 = +\ 2.70$$

$$(b) \ \text{Fall} = 6{\cdot}70 - 4{\cdot}86 = 1{\cdot}84$$

If back- and fore-sights are taken at unequal distances, the correction must be calculated for each distance, and *subtracted* from the reading in each case, the corrected rise or fall being then found from the corrected readings.

Reciprocal levelling. It is sometimes necessary to carry a line of levels across a wide river or ravine, AB. Assume that levelling has been brought down to A (Fig. 58), equalizing back- and fore-sights, as already described, so as to eliminate errors of collimation, curvature, and, as nearly as possible, refraction. Thus the level of A is known. We require that of B.

Now if the ground were such as to allow a station to be found for the level from which the distance of the back-sight to A would be the same as that of the fore-sight to B, we could proceed in the

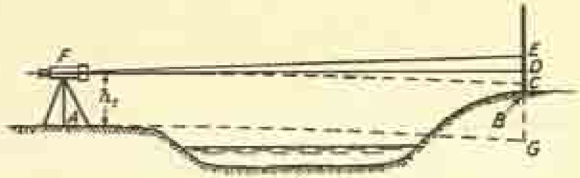

FIG. 58

ordinary way. But in general this will be impossible. Hence for a good result we proceed by reciprocal levelling as follows: set the level up directly over the point A, obtain its collimation height, h_1, above the ground, either by taking a back-sight to a previous point of known level, or by direct measurement with the levelling staff, and take a reading, S_1, on the staff at B.

Now suppose FD is the horizontal straight line through the centre of the telescope, and FC the level surface through the same point. And suppose there is a collimation error, so that the line of collimation, instead of coinciding with FD, is in the direction FE. We will neglect refraction for the moment.

Then draw AG, the level surface at A, parallel to FC, so that $GC = AF = h_1$.

$BE =$ staff reading $= S_1$.
$CD =$ curvature correction $= c$
$DE =$ collimation error $= e$, say.

$$\begin{aligned}\text{Then true rise A to B} &= GB \\ &= GC + CD + DE - BE \\ &= h_1 + c + e - S_1\end{aligned}$$

Now in this expression the collimation error e, though generally small, is unknown; hence we cannot work directly by it.

We now set up at B, therefore, and read back to A.

Let FB (Fig. 59) $= h_2 =$ height of instrument at B
FD = horizontal line
FE = line of collimation
$AE = S_2 =$ staff reading

and let FC and BG be the level surfaces through F and B.

Then true fall, B to A $= GA = EA - ED - DC - CG$

Now the collimation error ED, and the curvature correction DC, will be the same as before, because they both depend upon the distance, and this is practically the same from B to A as from A to B.

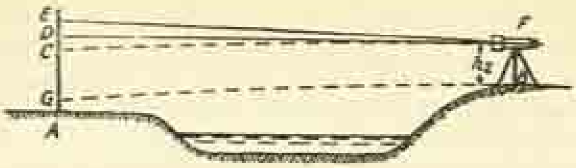

FIG. 59

Hence we may represent them by the same letters e and c as before.

$$\text{Hence true fall, B to A} = S_2 - e - c - h_2$$

Now obviously the true rise from A to B must be the same as the true fall from B to A, and if we denote it by H, we have –

$$H = h_1 + e + c - S_1 \quad . \quad . \quad . \quad . \quad (1)$$

$$H = S_2 - e - c - h_2 \quad . \quad . \quad . \quad . \quad (2)$$

$$\therefore \text{ adding,} \quad 2H = h_1 - h_2 - S_1 + S_2$$

$$= h_1 + S_2 - (h_2 + S_1)$$

$$\text{or} \quad H = \tfrac{1}{2}\{h_1 + S_2 - (h_2 + S_1)\}$$

That is, to find the true rise from A to B, add together the height of instrument at A and the reading of the staff when held on A. Call these plus. Then add together the height of instrument at B and the reading of the staff when held on B, and call these minus. Add algebraically, and take half the result. The answer, *if plus*, means a rise from A to B, and, if minus, fall.

ALLOWANCE FOR REFRACTION. It will be noticed that collimation and curvature errors are eliminated by this method. To allow for refraction, on the average, it is only necessary to take six-sevenths of c in each case, as already stated. Hence clearly refraction would cancel out, too, if we could be sure that it would remain constant. But, as it is liable to vary somewhat with varying atmospheric conditions, it is desirable, if possible, to use two levels, one at A and one at B, the latter being set up over B immediately after taking the reading from A to B, so that there is hardly time for refraction to change appreciably.

Of course the collimation errors for the two levels will be different, so that we must then interchange the levels and repeat the process, the rise being worked out for each level, and the mean taken.

Numerical Example. Reduced level of A = 120·57; heights of instrument: at B 4·92, at A 4·70; staff-readings: at A 2·62, at B 6·81. Find the level at B, and if the distance be 1000 feet and refraction be taken as one-seventh of curvature, find the collimation error of the level for a reading at that distance.

	+		−
Instrument at A . . .	4·70	at B . .	4·92
Staff at A . . .	2·62	at B . .	6·81
	7·32		−11·73
			7·32
		Sum =	− 4·41
		$\frac{1}{2}$ =	− 2·20

Hence from A to B there is a *fall* of 2.20 feet, or level of B = 120·57 − 2·2 = 118·37.

To find the collimation error: the true rise A to B should be given by the expression $h_1 - S_1 + c$ (see p. 105) if there were no collimation error. Hence the difference between this expression and the actual true rise as found above will give the collimation error.

Now c stands for curvature and refraction at a distance of 1000 feet

$$= \left(\tfrac{1000}{5280}\right)^2 \times \tfrac{6}{7} \times 8 \text{ ins.} = 0{\cdot}02 \text{ foot}$$

$$\therefore\ h_1 - S_1 + c = 4{\cdot}70 - 6{\cdot}81 + 0{\cdot}02 = -\ 2{\cdot}09$$

Hence the collimation error converts a true fall of 2·20 into an apparent fall of 2·09, and therefore it makes the staff reading *too small* by 0·11 foot at a distance of 1000 feet.

Note. Most levels will not permit of an ordinary staff being clearly read at a distance of 1000 feet. The reading may, however, be taken by means of a pointer, or a vane slid along the staff by the staff-holder until brought into coincidence with the horizontal reference line in the level diaphragm, at which point the instrument man signals to the staff-holder who thereupon books the reading.

CHECKING LEVELS. When a long line of levels has to be run in haste, it is desirable to have some means of checking which does not involve a second levelling operation.

One method of eliminating mis-readings is to read the staff twice at points of importance, such as change points; firstly, with the staff in the ordinary position, zero downwards; secondly, with the staff inverted, zero upwards; or by figuring the staff on both sides, the zero of one being different from that of the other. This double figuring is used on the 10-foot Ordnance Survey pattern. This staff can be read four times by reversing it and then turning it end for end. This cannot be done with the telescopic staff.

For ordinary purposes check levelling is to be preferred to multiplying the readings, as being the most efficient check. Moreover, it is found that levelling in opposite directions tends to eliminate a slight cumulative error, which is positive when working in one direction, negative in the other, the cause of which is not well understood. When the utmost expedition is required, it will be well to employ a second check-leveller, to level from bench-mark to bench-mark, closely following the chief surveyor.

Degree of accuracy in levelling. The closing error in levelling, with ordinary care, and reasonably favourable conditions as to weather and ground, should not exceed 0·10 feet per mile, increasing as the square root of the distance in miles. That is to say,

		Feet
Permissible error in	1 mile . . .	0·10
" "	4 miles . . .	0·20
" "	9 " . . .	0·30
" "	100 " . . .	1·00

If the surveyor levels a certain distance, and then back again by the same, or even by a different route, bench-mark to bench-mark, the permissible closing error is that due to the distance between the extreme points. If, for example, a distance of 1 mile were levelled, in both directions, the permissible error on closing would be 0·10, although the total distance levelled is 2 miles.

The reason of the above is that certain errors tend to accumulate with opposite signs according to the direction in which the operation is performed; for instance, when levelling uphill the back-sight readings are greater than the fore-sight readings, and any deviation from the vertical position of the staff tends to augment the back-readings far more than the fore-readings. There is, therefore, a tendency to a cumulative error inasmuch as the back-readings are, if anything, in excess of their proper value. When levelling downhill, however, the tendency will be to augment the fore-readings. The result will be that the two errors, being in opposite directions, will tend to compensate each other. The final closing error may be quite permissible, yet the determined height of the hill may be materially too great. In an unclosed circuit there is not necessarily the same regular tendency to compensation, and consequently a greater error per mile is to be expected.

For an example of accurate levelling, the reader is referred to *Notes on the Geodesy of the British Isles*, by Col. Close (1914), where it is stated that in the new Primary Levelling of Great Britain, using a Zeiss level and an invar staff, '1000 miles of levelling have been closed with a probable error of $0{\cdot}0021\sqrt{M}$ feet, where M is the distance in miles. Thus the probable error in 100 miles is $\frac{1}{4}$ inch.'

HAND-LEVELS. Hand-levels are intended for approximate work only, such as preliminary surveys of one kind or another, taking cross-sections in cases where the longitudinal section has been already done, and so on. They are held in the hand in use, and are adjusted by hand alone, being provided with reflectors of some sort so that the bubble can be seen, by reflection, when the instrument is brought to the eye to take the reading.

The most simple form (Fig. 60) consists of a sighting tube with a pin-hole E at one end, a reflector R in the middle (whose lower edge coincides with a diameter of the tube), and a stop F at the other end, whose lower edge also lies in the central plane of the tube. Above the reflector is the bubble L, which is permanently adjusted so that when the bubble is seen by the eye at E, reflected from the edge R, the line ERF is horizontal, and the reading of the staff is taken by using R and F as sights.

The main disadvantage under which the instrument, in this form, suffers is that the bubble and the distant staff are not in focus together. A simplified staff reading direct to one-tenth of a foot is sometimes used in conjunction with the hand-level, as this is easier to read at a distance than the standard Sopwith staff. One type of simplified marking is shown in Fig. 51.

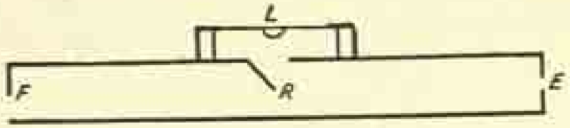

FIG. 60

CONTOURING

CONTOURS. A contour line is a line drawn on the plan, so as to pass through all points which have the same reduced level. Usually the

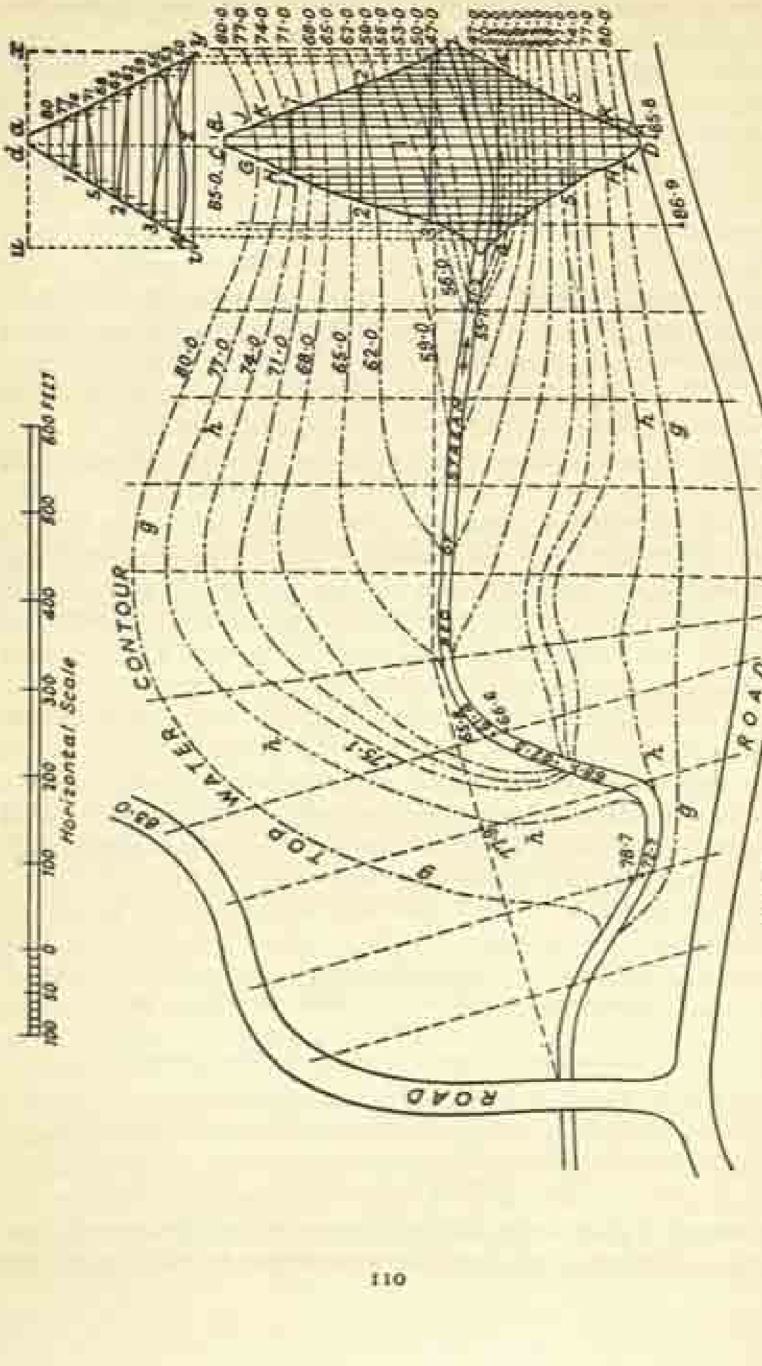

FIG. 61

contours are drawn to show all points whose reduced levels are expressed by some round number of feet.

It is evident that, if two such contour lines be drawn, all points on the one will be the same *vertical* distance above or below those points which lie on the second contour. This vertical distance is called the *vertical interval* between contours, and varies from 1 foot to about 10 feet in large-scale plans intended for building or other engineering work, and from 10 feet to 100 feet or more in small-scale maps. It is evident that when the contours are drawn on a plan, and their levels written upon them, this plan will give a more or less accurate idea of the conformation of the ground as regards levels as well as in plan.

CONTOURING. The method generally adopted for finding the contours in engineering plans is to cover the ground with a series of 'spot levels'. One or more lines are ranged out on the ground in known directions, and cross-sections are taken along lines either at right angles to these, or making known angles with them.

The levels are taken along these lines, and booked in the same manner as already described for cross-sections, thus giving the reduced levels of a number of spots along these cross-sections. Extra points between cross-sections must be taken if there are any irregularities in the ground. The contours are then interpolated on the plan. An example is shown in Fig. 61, which is a plan of a small proposed storage reservoir.

FIG. 62

METHOD OF INTERPOLATION. The first method of interpolation is *by eye*. Thus if A, B, C, D, E (Fig. 62) be five points of known levels, let it be required to interpolate the points whose levels are an even number of feet above datum, viz. 34·0, 36·0, etc.

From A to B the fall is 11·7, or about 12 feet. Hence each foot of fall corresponds with about *one-twelfth* of the distance AB. Now 46·0 is 0·8 foot below 46·8, hence the distance from A to the point whose level is 46·0 will be about 0·8 of one-twelfth, or one-fifteenth of AB. We guess this by eye, and put on the 46·0 mark. Similarly the 36 mark will be about 0·9 of one-twelfth part of BA to the left of B. Actually we guess this amount on a rather liberal scale to allow for the gradual flattening of the slope, and mark the 36 point. We then divide the distance between 36 and 46 into *five* parts (the vertical interval being 10 feet), these parts not being quite equal, but slightly decreasing towards A. The remaining spaces are treated in the same way, and, when all the lines along which levels were taken have been divided in this way, all points having the same level are joined to draw the contour for that level.

When all the contours are drawn they are examined by eye to see if there are any irregularities, such as a much wider interval between two contours than between the next two. Wherever such a state of things is discovered, the levels are examined to see if they justify this sudden change of slope, and if not, the contours are adjusted accordingly.

By sections. In Fig. 63 the same points are shown, but the contour points are interpolated by drawing a section. Take some round figure as the level of AE, usually the nearest round number less than the lowest level along the line. In this case the lowest level is 32·6, and we assume AE = 30·0; then set up A*a* = 16·8 feet on any convenient scale (because the level at A is 46·8, and we are taking AE as 30·0); similarly B*b* = 5·1 feet, and so on.

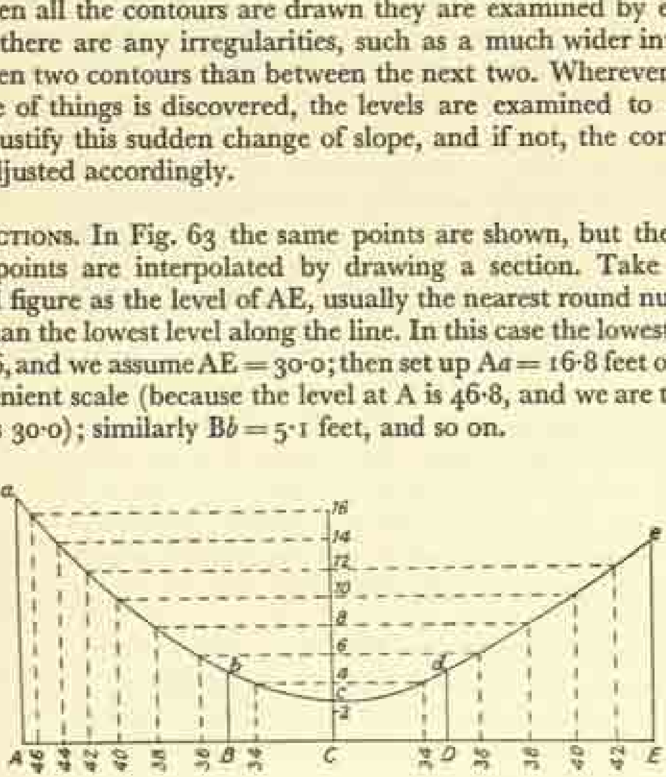

Fig. 63

Join *a*, *b*, etc., to draw the section. Then along any vertical C*c* (preferably near the centre to diminish the effect of any error in drawing the parallels) set up lengths C2, etc., representing, on the scale chosen, the required vertical interval between contours, in this case 2 feet. From the points so obtained lines parallel to AE are drawn to meet the curve. The points of intersection are then projected on to AE as shown.

Obviously they will give the required contour points.

Comparison of methods. In addition, mechanical methods of equal division may be employed. But they lead to undesirable stiffness, and may fail at summits and in hollows. On the whole the method of sections *should* be the best. But to make it so it is necessary to use extraordinary care in the drawing, and if this is done it may be questionable whether the extra accuracy obtained (as compared with skilful division by eye) is worth the extra labour, especially in view of the fact that the contours would probably always be examined and adjusted as already described after being drawn.

Contouring in the field. In some cases it is desired rather to trace or lay out the contour line on the ground itself. Such cases occur, for instance, in dealing with the top water contour in a reservoir scheme, or in maps of large areas, like the Ordnance maps, where a few contours have to be followed for long distances and through many intricate windings.

The paths of these contours are marked out by pegs or pickets driven into the ground, and the positions of these pegs are found by some method of surveying, either at the same time or afterwards. It will be obvious that the windings of any particular contour can be much more accurately followed in this way, on the ground itself, than by interpolation on the plan between points taken along any regular set of lines. But to deal by this method with a large number of contours would involve an expenditure of time which would not, as a rule, be justifiable.

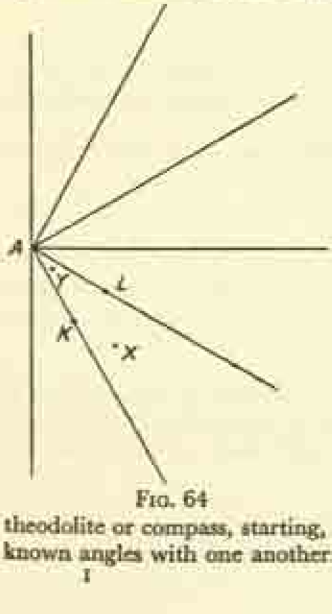

Fig. 64

Examples. As an example we will take the contours round a small hill (Fig. 64).

A point A representing the summit, as nearly as it can be fixed, is taken, and marked on the ground either by a special peg or by measuring its distances from two known points.

Radial lines are then ranged out from A, by theodolite or compass, starting, say from magnetic north as zero, and making known angles with one another.

Then suppose at X there is a bench-mark whose level is known or assumed at 97·7 feet. The level is set up, say at Y, so that it is just possible to see over the summit. A back-sight is taken on the B.M., and the level book worked backwards to find the necessary reading on the contours at 100, 97, 94, etc., the required vertical interval in this case being assumed as 3 feet. Thus if the reading on the B.M. be 4·53 feet, the level book would be made up as follows:

Back-sight	Inter-mediate	Fore-sight	Height of instrument	Reduced level	Remarks
4·53	—	—	102·23	97·7	On B.M.
—	0·37	—	—	101·86	On summit
—	2·23	—	—	100·0	—
—	5·23	—	—	97·0	—
—	8·23	—	—	94·0	—
—	11·23	—	—	91·0	—
0·64	—	13·82	89·05	88·41	T.P.
—	1·05	—	—	88·0	—
		And	so on		

From the back-sight, 4·53, and known level of B.M., we find the collimation level or height of instrument (= 102·23).

The reduced levels of the required contours are then written down, one after the other, as shown, and each is subtracted from the height of instrument to find the reading of the staff for that contour.

Thus for the 100 contour the reading will be 2·23 feet.

The staff-holder would generally be directed to hold the staff on the summit, so as to find the level of that point, the reading being entered as an intermediate.

He is next directed down the hill, along each of the radial lines in turn, until the points K, L, etc., are found by trial and error, the reading at each of these points being 2·23, or nearly so. Usually any reading between 2·20 and 2·30 would be accepted as sufficiently accurate.

These points are marked with pegs to be afterwards surveyed, say by measuring their distances along the radial lines.

For the next three contours, points must be obtained where the readings are 5·23, 8·23, and 11·23 respectively.

It is usual to fix all visible points on one contour before proceeding to the next, not only because it saves climbing up and down the hill, but also because there is less chance of error in taking the same reading several times over, and because the staff-holder can better judge the positions of the points when following one contour.

When the contour at 91·0 has been set out, it is evident, by an inspection of the table above, that the necessary reading for the next contour, if the level remained at X, would be 14·23, whereas the staff would normally be only 14 feet long.

Hence a turning-point must be taken.

The staff-holder is signalled down the hill until a reading near the top of the staff is obtained, and a fore-sight is read, in this case supposed to be 13·82. This subtracted from the height of instrument gives the reduced level of the turning-point. The level is then moved and a back-sight taken, near the bottom of the staff, here supposed to be 0·64. This, added to the reduced level, gives the new height of instrument as 89·05.

The required level of the next contour is then written down, namely 88·0, and this, subtracted from the height of instrument, gives 1·05 as the next reading, and so on.

USES OF CONTOURED PLANS. The uses to which contoured plans are put in engineering are too numerous to be fully described here. It is proposed, however, to deal briefly with a few examples.

CASE OF RESERVOIR. Thus suppose the figure (Fig. 61) represents the plan of a reservoir site, see p. 110.

The broken lines show the survey lines, along which the level and other measurements were taken. The 'dot and dash' lines represent the contours as obtained from the spot levels, only a few of which have been marked on the drawing.

A dam is to be built across the valley, and it has been shown on the drawing as an earth dam, top level 85 feet above datum, top width 10 feet, downstream edge of top along AB.

The downstream slope is taken as $2\frac{1}{2}$ to 1 and the upstream slope as 3 to 1, for purposes of illustration.

The top level to which the water is to be impounded is taken as 80 feet above datum, and the contours have been drawn for every three feet, from that level downwards.

PLAN OF DAM. The first step is to draw the contours on the dam. For this purpose we draw the section. From A and D project up, parallel to AB, to mark the points a and d, so that $ad = 10$ feet, the top width. Then draw the slopes ay, dv.

For this purpose the vertical scale has been taken five times as great as the horizontal. Hence a slope of $2\frac{1}{2}$ horizontal to 1 vertical will be represented on this section by $2\frac{1}{2}$ horizontal to 5 vertical, and we make ax $2\frac{1}{2}$ units long (taking any unit) and then make xy 5 units long.

Similarly for the upstream slope (which is 3 to 1) we make $du = 3$ units, and $uv = 5$ units.

Now, taking the level of ad as 85·0 (top of dam), we set down, along dz, a height of 5 feet. This brings us to a level of 80·0, the required top water-level. After that regular intervals of 3 feet are set down, that being the vertical interval chosen for the contours.

Through the points so found, lines parallel to ad are drawn to meet the slopes, and the points thus obtained are projected down to the plan as shown.

The lines so obtained show, in plan, the horizontal lines or contours (on both surfaces of the proposed dam) at the same levels as the contours on the original ground. Thus FG represents the contour on the upstream slope at the level of 80·0.

It meets the 80·0 contours on the original ground at the points F and G, and these points will therefore lie along the intersection of the slope with the ground. That is, they will be on the 'toe of the slope', as it is called.

All other points on both toes are found in the same way, and joined up to complete the plan of the dam.

The contours, such as FG, could have been drawn without the aid of the section. Thus the level of CD is 85·0, and that of FG is 80·0; hence the vertical interval between them is 5 feet. Now the upstream slope is 3 horizontal to 1 vertical. Hence, for a vertical interval of 5 feet, the corresponding horizontal equivalent will be $3 \times 5 = 15$ feet.

Thus the perpendicular distance between CD and GF must be 15 feet, measured on the horizontal scale. After GF is drawn, the remaining lines must be 9 feet apart, as the vertical interval becomes 3 feet.

Similarly, on the downstream slope, the first contour is 12·5 feet from AB ($12{\cdot}5 = 2{\cdot}5 \times 5$), and the remaining lines are 7·5 feet apart ($7{\cdot}5 = 2{\cdot}5 \times 3$).

The volumes of water in the reservoir when full or when standing at different levels may be computed readily from a contoured plan. Computations of this kind are dealt with in Chapter 8.

Setting out a given gradient. Another use to which contoured maps are often put is for setting out the directions of routes, so as to obtain some definite gradient along the surface of the ground.

Thus in Fig. 65, suppose that the contours are at 10-feet vertical intervals, and that, starting from A, a line is to be set out

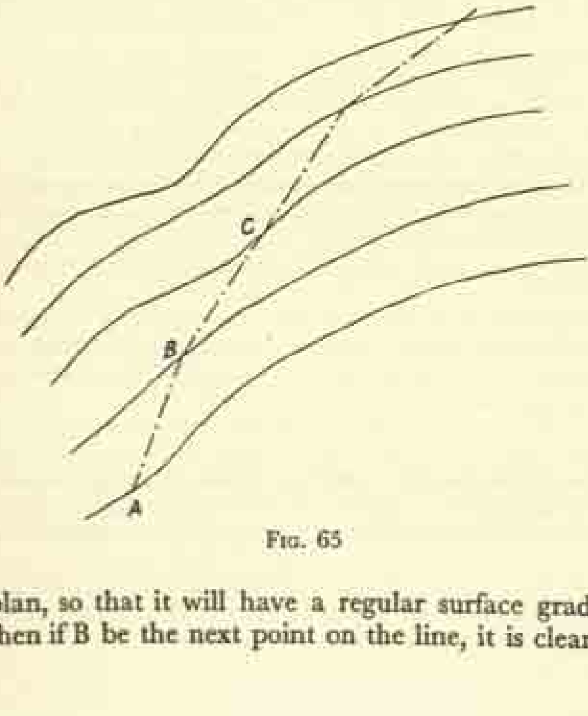

Fig. 65

on the plan, so that it will have a regular surface gradient of 1 in 30. Then if B be the next point on the line, it is clear that the horizontal distance AB must be $30 \times 10 = 300$ feet, as B is 10 feet higher than A, and the gradient is to be 1 in 30 without cutting or filling. Hence we calculate the length in inches on the paper, which represents 300 feet according to the scale; then take this length on the compasses, and, with A as centre, cut the next contour at B. Then use B as centre, and the same radius, to find the next point C, and so on.

Thus, if the scale be 200 feet to 1 inch, 300 feet is represented by 1·5 inches on paper.

This method is, as a rule, used for fixing general directions only, on rather small-scale drawings. A line is then set out to take a mean direction through the points thus found, and will give a general idea of the approximate direction more quickly than it could be otherwise obtained, supposing, of course, that such a contoured map is available.

Setting out line in the field. For larger-scale work, and for fixing directions of irrigation, or other works, aqueducts and the like, so as to have a regular gradient, it is more usual to set out the line at once in the field.

For this purpose (see again Fig. 65), a level is set up at such a point that the top of the staff at A can just be read. Then suppose a 100-foot chain is used. The rise per 100 feet, for a gradient of 1 in 300, for example, will be 0.33 foot. Hence a level book is prepared on the same principles as explained in the setting out of contours, the readings gradually decreasing by 0.33 foot. Thus if the reading at A were 13.64, that at the end of the first 100 feet should be 13.31.

The follower holds the end of the chain at A, and the leader swings his end round, keeping with the staff-holder as the latter is signalled up or down the hill, until the required reading is obtained, say at the point B, the staff being held at the leader's end of the chain. Actually it should be in advance of the end by an amount equal to the hypotenusal allowance for the given slope (p. 44). The arrow is then stuck in there, the follower comes forward to B, and so on.

Underwater contours. In coastal surveys for such projects as sea-defence works and in surveys which include lakes or wide rivers it is frequently necessary to obtain contours of the submerged ground. This may be done by taking soundings from a boat, using the water-level as a datum, and in tidal waters these measurements must be correlated to the readings of a tide gauge. Marine surveys are dealt with in Volume II of this work, but it may be appropriate in the present volume to discuss the procedure by which the position of the boat may be fixed in order to locate the position of each sounding on a plan. One method is to move the boat until it falls into alignment with two pairs of fixed points ashore, the positions of which can be plotted on the plan. The intersection of the rays passing through these pairs of points then fixes the position of the sounding, but a more general method consists in measuring the angles subtended at the boat by three fixed points ashore whose positions are known. This gives rise to the 'three point problem' which may be solved, except in one exceptional case, by geometry, trigonometry or a simple graphical method. It also occurs in plane-tabling and is discussed, in this connection, in Chapter 7.

In Fig. 66 A, B, C, represent the three fixed points and O the position of the boat. The angles AOB and BOC are usually measured by a nautical sextant, or a box sextant, and the angle AOC would be measured, in addition, as a check. The principle of both forms of sextant is shown in Fig. 67. M_1 and M_2 represent in plan view the upper edges of two mirrors. M_1 is fixed and is only silvered over its upper half, the lower half being plain glass. M_2 is entirely silvered and is capable of rotation about a vertical axis when the instrument is held horizontally. M_1 is called the horizon glass and M_2 the index glass, and the extent of movement of M_2 is recorded by an arm which travels over a graduated arc. The arm carries a vernier which enables angles to be read to the nearest minute in the standard type of box sextant and to 10 seconds in the nautical sextant. The zero reading is given when the two mirrors are parallel if the instrument is in good adjustment. Suppose it is required to measure the angle subtended at the instrument by two reference marks P and Q. The surveyor takes

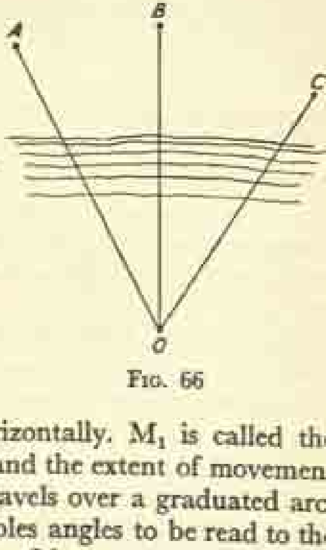

FIG. 66

a direct sight to one of the marks, e.g. Q, through the unsilvered part of mirror M_1 and rotates mirror M_2 by means of a fine adjusting screw until the reflected image of the other mark, P, falls vertically

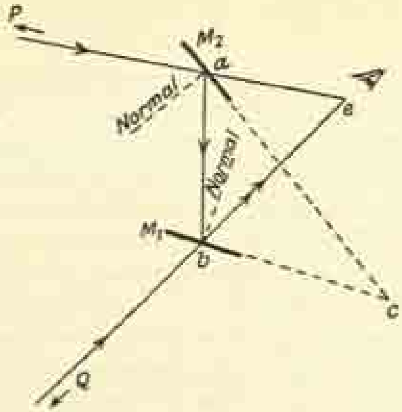

FIG. 67

in line with Q. When this setting is secured the rays from P and Q will take up the positions shown in the figure. The required angle is then twice the angle between the planes of the mirrors. This can be shown by the following simple proof, based on the rule that the angles of incidence and reflection are equal:

The angle between the mirrors

$$= \angle bca = 180° - (\angle cba + \angle bac)$$
$$= 180° - \left\{\left(90° + \frac{\angle abe}{2}\right) + \left(\frac{180° - \angle Pab}{2}\right)\right\}$$
$$= 180° - \left\{90° + \frac{\angle abe}{2} + \frac{180° - (\angle aeb + \angle abe)}{2}\right\}$$
$$= 180° - \left(90° + \frac{\angle abe}{2} + 90° - \frac{\angle aeb}{2} - \frac{\angle abe}{2}\right) = \frac{\angle aeb}{2}$$

In practice, the graduated scale is numbered with the true values of the angles between the mirrors doubled, thus giving a direct reading for the angle subtended by two reference points.

THE THREE POINT PROBLEM. (*a*) *Geometrical solution*. In Fig. 68 let ABC be the three points and O the position of the instrument. Then, since angles in the same segment of a circle are equal, the locus of $\angle$AOB is a circular arc passing through A and B, and the locus of

$\angle$BOC is a circular arc passing through B and C. The point O will be fixed by the intersection of these arcs. Let P and Q be the centres of the arcs. Then $\angle APB = 2\angle AOB$ and $\angle PAB = \angle ABP = 90° - \angle AOB$.

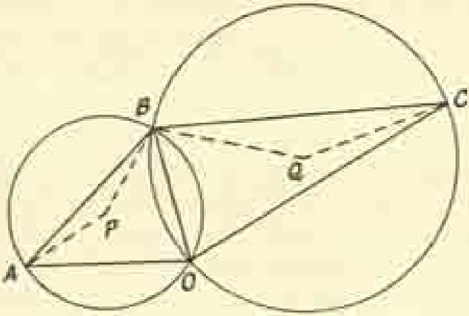

FIG. 68

Thus if the angles PAB and ABP are set off on AB from the known value of $\angle$AOB, P will be the centre of one of the required arcs. Similarly, if angles of $90° - \angle BOC$ are set off from B and C we obtain the centre Q of the second arc. By describing circles from these centres the point O is located at the intersection. This construction, and other methods of solution, fail if A, B, C, and O are concyclic, and a poor intersection is obtained if the points are not well placed and are approaching the concyclic condition.

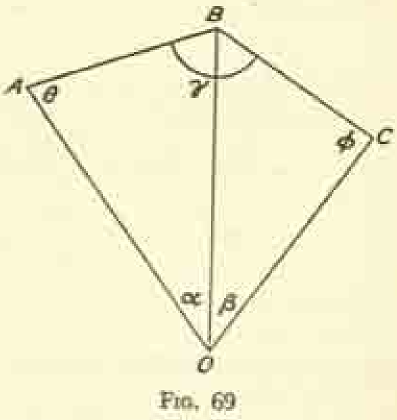

FIG. 69

(*b*) *Trigonometrical solution.* Referring to Fig. 69, the known dimensions are $\angle$AOB, $\angle$BOC, and the distances AB and BC. The angle $\angle$ABC can be computed from the known positions of A, B and C. Let $\angle AOB = \alpha$, $BOC = \beta$, $\angle ABC = \gamma$. Then, if $\angle BAO = \theta$ and $\angle BCO = \phi$:

$$\theta + \phi = 360° - \alpha - \beta - \gamma = \text{a known quantity}, \epsilon.$$

$$\frac{AB}{OB}=\frac{\sin\alpha}{\sin\theta} \text{ and } \frac{BC}{OB}=\frac{\sin\beta}{\sin\phi} \therefore \frac{AB}{BC}=\frac{\sin\alpha.\sin\phi}{\sin\beta.\sin\theta}$$
$$=\frac{\sin\alpha.\sin(\epsilon-\theta)}{\sin\beta.\sin\theta}$$

from which θ can be computed and hence ϕ. We can then find the two components of γ and thus solve Δ^s AOB and BOC by the sine formula to obtain the distances OA, OB, OC.

(*c*) *Graphical solution.* The angles AOB and BOC are carefully set out on a sheet of tracing paper. This may be done from linear dimensions by constructing right-angled triangles to give the correct trigonometrical ratios, a more accurate process than using a protractor. The rays forming the angles are then caused to pass through the appropriate points, A, B, and C, previously plotted on the drawing, by moving the tracing until this condition is satisfied and the position of the point O is then pricked through on to the paper.

EXAMPLES FOR EXERCISE

(1) The following level readings were taken along the centre line of a road: B.S. on point *a*: 9·31, F.S. on point *b*: 8·36, B.S. on point *b*: 7·20, Int. on point *c*: 4·88, Int. on point *d*: 6·66, F.S. on point *e*: 5·37, B.S. on point *e*: 11·23, Int. on point *f*: 10·96 and F.S. 5·04 on the underside of a plate girder bridge crossing the road. The R.L. of point *a* was 100·86. The F.S. 5·04 was taken with the staff held inverted against the underside of the girder and point *f* was on the roadway vertically beneath. Book and reduce the above levels by the Height-of-Instrument method and state the headroom beneath the girder at point *f*. All readings are in feet.

Ans. R.Ls. are: *b*: 101·81, *c*: 104·13, *d*: 102·35, *e*: 103·64, *f*: 103·91, headroom: 16 feet.

(2) The reduced levels of the ground at four points A, B, C, D are 178·32, 178·15, 177·83, and 178·15 ft. respectively. A sewer is to be laid so that its invert is to be 10 ft. below the ground at A, and to fall with a uniform gradient of 1 in 340 to D. The distances AB, AC, AD are 117·6, 264·9, and 441·9 ft. respectively. Find the invert level and depth of trench at B, C, and D.

Ans. 167·97, 167·54, 167·02; 10·18, 10·29, 11·13 ft.

(3) A dumpy level was placed midway between two pegs, A and B, which were 200 feet apart. The reading of the staff on peg A was 5.96 and on peg B 7·16. It was found necessary between these two readings to adjust the bubble reading by means of the foot screws. The level was next moved to a point 50 feet behind peg B but in line with both pegs. The reading on the staff at B was found to be 7·25 and at peg A 6·85. State which of the permanent adjustments was not in order and which may, or may not, have been. If all the adjustments

were correct what staff readings should have been obtained when the level was in the second position?

Ans. Collimation error: 0·40 in 100 feet. Readings should have been: on B: 7·05, on A: 5·85.

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(4) Starting from a B.M. at the level of 176·32, contours are to be laid out on the ground at vertical intervals of 5 ft. from 185 downwards to 160. The initial position of the level is chosen so that the back-sight on B.M. is 9·78 ft. Draw up a field-book showing the readings for the various contours, including one turning-point, for which you may assume your own reading.

Ans. (In the annexed table the fore-sight on T.P. has been assumed at 13·26, and the new B.S. at 1·14.)

B.S.	Inter.	Fore.	R.L.	Remarks
9·78′			176·32	B.M.
	6·10		180·00	
	1·10		185·00	
	11·10		175·00	
1·14		13·26	172·84	T.P.
	3·98		170·00	
	8·98		165·00	
		13·98	160·00	
10·92		27·24	16·32	
		10·92		
		16·32		

Chapter 4

TRAVERSE SURVEYING

Definitions. Traversing differs from chain surveying in that the directions of the survey lines are fixed by angular measurements, instead of being arranged so that they can be plotted from their lengths alone.

Thus in Fig. 70, if AB, BC, CD be three lines of a traverse, following say a road or winding river, we measure *either* (*a*) the angle which

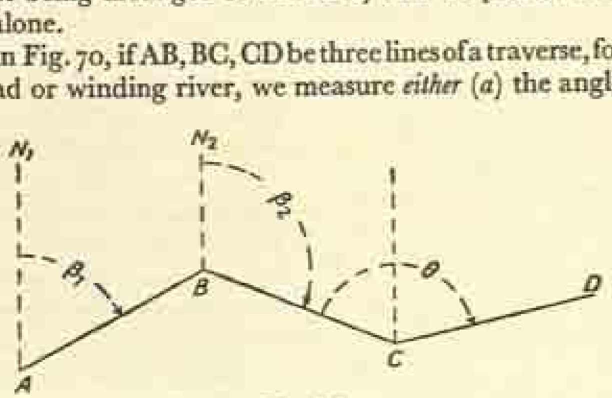


Fig. 70

each line makes with some standard direction, as shown at β_1, β_2, *or* (*b*) the angles between the successive lines, as shown by the angle θ between CB and CD.

In the first case, the standard direction, AN_1 or BN_2, may be magnetic north (as it would be if the angles were measured by compass) or any other suitable direction; and the angles (β_1,β_2) which the lines make with it are called the *bearings* of the lines. The lines are usually chained, and offsets taken, just as in chain surveying.

The expression 'traverse' was doubtless originally borrowed from the art of navigation, in which it signifies 'the method by which the mariner determines the change in the position of his ship', after sailing in various directions, or on different 'courses', for various distances, as estimated by the log.

Open and closed traverses. A traverse is said to be *open* when it consists of a number of lines following some route, as in Fig. 70, and neither returning to its starting point nor beginning and ending at points whose positions on the plan are otherwise known. It is called a *closed* traverse, either if it runs between two points whose positions *are* known, or if it returns to its starting-point, thus forming a closed polygon.

PLOTTING THE LINES. Let ABCDE (Fig. 71) be any polygon. It is clear that if all the sides AB, BC, etc., as well as all the angles ABC, BCD, CDE, DEA, and EAB, are measured, complete data exist for delineating the polygon on paper, and, moreover, it is possible to check the accuracy of the work. Again, if the position of any point, such as A, is already determined and fixed on the plan, and the angle which AB makes with some fixed line, such as the true north and south line SAN drawn through A, is also known, then the whole polygon can be placed in its proper position with regard to the remainder of the survey. One might proceed to lay off the angle NAB, and set off the measured distance AB; then lay off the angle ABC and set off BC, and so on. If the work were accurately performed, both in the field and in the office, then on completing the polygon the starting-point A would be reached, and the polygon would 'close'. This procedure would be both cumbrous and inaccurate. If the polygon did not 'close', absolutely correctly, there would be nothing to show whether the inaccuracy was due to faulty measurements in the field, or inaccuracy in drawing. Moreover, it is quite certain that in laying off each angle (and each distance) there would be *some* error in the drawing.

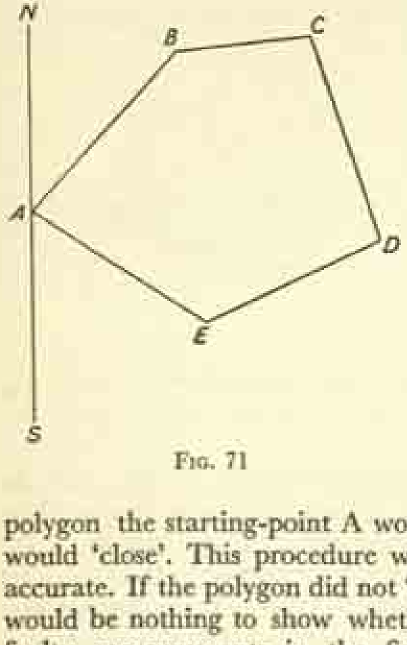


FIG. 71

CO-ORDINATES. For this reason, unless the number of sides is very small, traverse surveys are always plotted by calculating *co-ordinates*, if an accurate result is required.

Thus in Fig. 72, two axes, OP, OQ, are chosen, of which the axis OP is parallel to the 'standard direction', from which bearings are measured, while OQ is perpendicular thereto.

The co-ordinates of each point are the distances of that point from each of these axes. Thus in the figure the co-ordinates of A are ON, which gives the distance of A from the axis OP, and NA, which gives its distance from OQ.

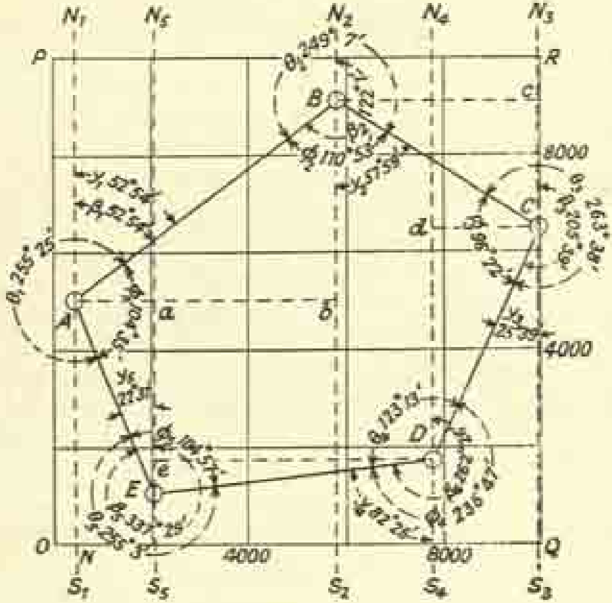


Fig. 72

Choice of standard direction. The standard direction, OP, may be a true north and south line, or a *magnetic* north and south line, as already stated.

In many cases, however, it is chosen arbitrarily so as to make the drawing fit well upon the paper, the direction of a true north and south line on the plan being afterwards shown if desired. Thus in Fig. 72, if there had been an important frontage along ED, we might have chosen that this line should be horizontal along the bottom of the paper. Then the standard direction EN_5 would have been *perpendicular* to ED, or, in other words, we would have started with an *assumed bearing* of 90° for the line ED.

Whole circle bearing. A bearing may be measured in different ways on paper, and expressed in several different notations.

According to the first method, the bearing of each line is measured *clockwise from the standard direction to that of the line*, right round the circle.

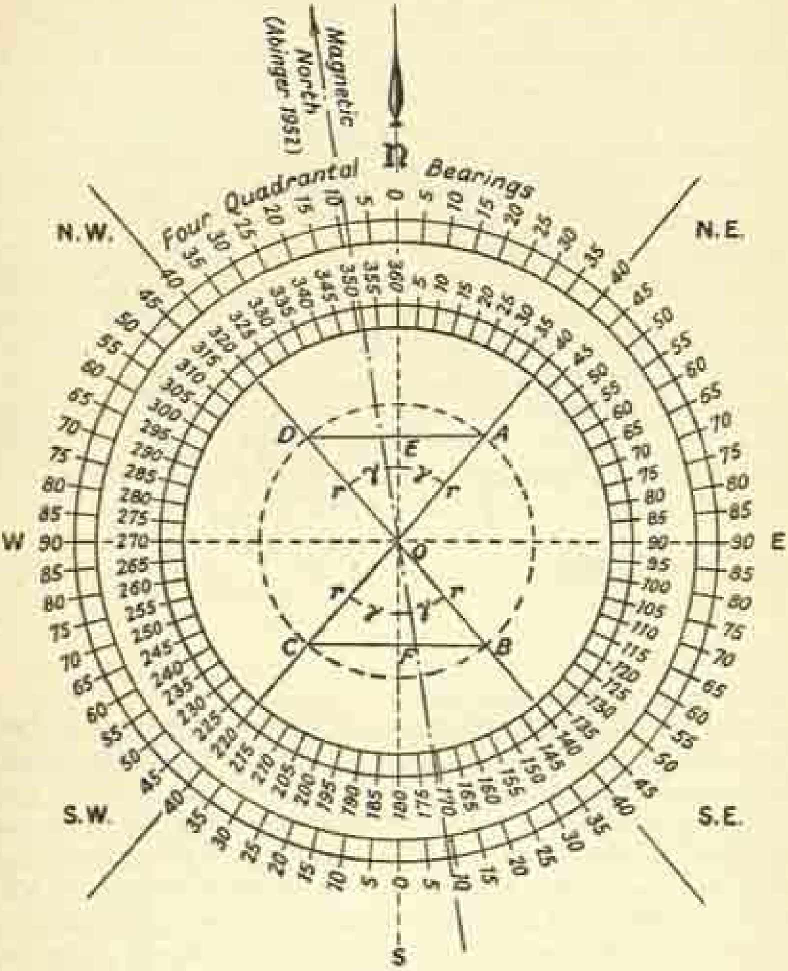


FIG. 73

In English practice this is called the *whole circle* bearing, and is shown, for four lines, by the inner graduated circle on Fig. 73.

ON is the standard direction. Graduations start there and go round clockwise.

The bearing of OA is 40°, that of OB is 140°, of OC 220°, and of OD 320°.

It is to be noticed that the direction of AO is diametrically opposite to that of OA; hence their bearings differ by 180°.

REDUCED BEARING. On the second system, starting with ON (Fig. 73) as before, the standard direction is taken as *either* ON *or* OS, indifferently, and bearings are measured clockwise or anti-clockwise, from whichever of these directions is the nearer.

We thus get the system of graduation shown on the outer circle, which is thereby divided, as shown, into four quadrants, for each of which the zero lies either on ON or on OS.

On this notation, the bearing of *each* of the lines OA, OB, OC, OD is 40°, and we must distinguish between them by stating the quadrant in which each lies.

For the purpose of calculation the standard direction ON is usually regarded as a true north line in English practice, even if it is not really so. Now it is evident that if ON be taken as due north, OE will be due east, and so on. Any line lying between ON and OE is then said to be in the *north-east* quadrant, and so on. Bearings measured in this way are called *reduced bearings*. Thus for the line OD, the reduced bearing is 40°, and the quadrant is north-west. The reduced bearings are marked γ on the figure.

In another notation, often used, the letters showing the quadrant are put before and after the bearing, thus: bearing of OC = S. 40° W., meaning that the bearing of the line OC is 40° measured from south towards west.

It is to be clearly apprehended by the student that, for the purpose of the following calculations, reduced bearings are always to be measured from the *north* or *south* line; never from east or west, and he should practise converting whole circle to reduced bearings, and vice versa, before proceeding.

Thus on Fig. 72 (p. 125) the whole circle bearings are marked β_1, β_2, etc., while the reduced bearings are marked γ_1, γ_2, etc. He should compare these, write down the quadrant for each line, and check by Table A, p. 135.

CALCULATION OF BEARINGS. Now suppose (Fig. 74) that the whole circle bearing is given for the first line, AB, of a traverse survey, and that the various included angles between the successive lines, commencing with AB, have been measured as shown on the figure. It is required to calculate the whole circle bearings of the other lines.

Let AN_1 (Fig. 74) be the standard direction, and let BN_2 be parallel to it.

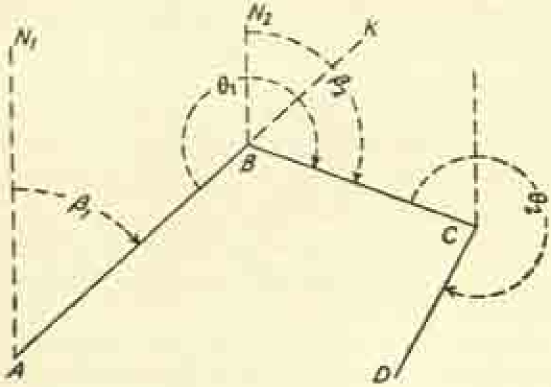


Fig. 74

The whole circle bearing of AB is β_1. Produce AB to K, and let θ_1 be the included angle between AB and BC. Then we are given β_1 and θ_1, and we require to find N_2BC, or β_2, which is the bearing of BC. Now $N_2BK = \beta_1$, because BN_2 is parallel to AN_1; and $KBC = \theta_1 - 180°$.

But
$$\beta_2 = N_2BK + KBC$$
$$\therefore \beta_2 = \beta_1 + \theta_1 - 180°$$

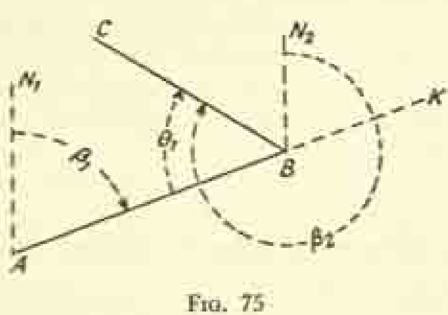


Fig. 75

In some cases, as for example in Fig. 75, it may happen that both β_1 and θ_1 are less than 90°. Hence their sum will be less than 180°.

In the case shown, we have

$$\beta_2 = 360° - \text{CBN}_2$$

But $$\text{CBN}_2 = 180° - \theta_1 - \text{N}_2\text{BK}$$
$$= 180° - \theta_1 - \beta_1$$

Hence $$\beta_2 = 360° - (180° - \theta_1 - \beta_1)$$

or $$\beta_2 = \beta_1 + \theta_1 + 180°$$

This is the rule where $\beta_1 + \theta_1$ is less than 180°.

Hence in all cases we have the following rule:

To the known circle bearing of any line, add the included angle between that line and the next.

If the sum be greater than 180°, *subtract* 180°; *but if the sum be less than* 180°, *add* 180°.

The result will be the whole circle bearing of the next line.

In some cases it may happen that the sum of β_1 and θ_1 is greater than 540°. In this case we subtract 180° as usual, but the result will still be greater than 360°. *We then subtract another* 360°, because the answer after subtracting 180° is quite correct, but a bearing of 380°, for instance, means simply one complete revolution *plus* 20°.

In Table A (p. 135), which refers to Fig. 72, the whole circle bearings are from true north, and that of AB was known to be 52° 54′. This is written in the whole circle bearing column, first line, opposite A.

In column 7 we write AB as the name of line, to make this clear.

Now the angle between AB and the next line is ABC (249° 7′), which is written opposite B (because it is measured at B) in the included angle column. The other angles are tabulated in the same way, each opposite the point where it is measured.

The remaining bearings are then worked out thus:

	°	′
AB =	52	54
ABC	249	7
	302	1
Subtract	180	0
BC =	122	1

	°	′
BC =	122	1
BCD	263	38
	385	39
Subtract	180	0
CD =	205	39

And so on, to –

	°	′
EA =	337	29
EAB	255	25
	592	54
Subtract	180	0
	412	54

	°	′
	412	54
Subtract	360	0
AB =	52	54

It will be seen that this working is according to the rule already given above, and that we check back at the end to the bearing of AB ($=52°\ 54'$), with which we started.

This checks the arithmetic.

Then, but not before, the bearings are written in their proper places: see Table A.

The student should carefully follow all this work, and compare the bearings in the table with Fig. 72, p. 125, where they are marked β_1, β_2, etc.

Measurement of the included angles. It will be evident, by reference to Figs. 74, and 75, that in working out the rule for bearings, we assumed that the included angle was in all cases measured clockwise from the back station. Thus at B (Fig. 74) we are supposed to be working round in the direction ABC, so that A is the back station, and C the forward one. Then θ_1 is marked as if measured clockwise from A. Attention must be paid to this, both in the field and in the office, to see that all included angles are so taken. On Fig. 72 the included angles used in calculating Table A are marked θ_2, θ_3, etc. It should be noted that, when working round a closed traverse from station to station in a clockwise direction, the included angles are exterior to the polygon and, when working in an anti-clockwise direction, they are the interior angles.

Calculation of reduced bearings. The calculation of reduced bearings is the next step, and it has been already referred to. It will

Case	W.C.B. between	Rule for R.B.	Quadrant
I	0° and 90°	= W.C.B.	N.E.
II	90° " 180°	180° − W.C.B.	S.E.
III	180° " 270°	W.C.B. − 180°	S.W.
IV	270° " 360°	360° − W.C.B.	N.W.

be clear that, if it be desired to put this in the form of rules, the table on p. 130 shows how the reduced bearing is found from the whole circle bearing in any case. In the table the whole circle bearing is denoted by W.C.B., and the reduced bearing by R.B.

The student should study these rules carefully along with Fig. 73 (p. 126), to see that he understands them, and should then check the figures in Table A.

DIFFERENCE OF LATITUDE AND DEPARTURE. Now suppose AN (Fig. 76) is the standard direction, and that AB is a line at a bearing β.

In passing from A to B, we go *north* (i.e. in the direction of AN) by an amount AE, and *east* by an amount EB.

The distance AE, by which we go *north* or *south* (as the case may be), from one end to the other of any line, is called the *difference of latitude* for that line.

The amount by which we go *east* or *west* is called the *departure*.

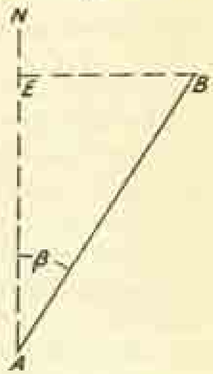


FIG. 76

Now it is clear that β is the angle between AN and AB, and that

$$AE = AB . \cos \beta$$
$$BE = AB . \sin \beta$$

Hence we have the rules:

Difference of latitude = length of line × cosine of R.B.
and *departure = length of line × sine of* R.B.

Simple inspection of Fig. 73 (p. 126) will show that these rules are true in whichever quadrant the line may lie, because our reduced bearings are always between the line and the *north* or *south* direction.

Thus for the line OC,

$$OF = \text{difference of latitude} = OC . \cos \gamma$$
$$FC = \text{departure} \qquad\qquad = OC . \sin \gamma$$

The differences of latitude and departure are computed by these rules.

The difference of latitude for each line is entered *north* or *south* according to whether the first letter in the quadrant for that line is N. or S. (see Table A, p. 135), and the departure is entered in the *east* or *west* column according to the second letter in the name of the quadrant. The calculation of these may be done by slide rule or ordinary arithmetic, but more frequently, perhaps, by logarithm tables. To avoid mistakes in this, care must be taken to see that the distances are used along with the proper bearings. Thus 52° 54′ is the whole circle bearing of AB. Therefore the length of AB (6768·0) must be multiplied by the cosine and sine of that bearing.

NORTHINGS, SOUTHINGS, ETC. A difference of latitude, if north, is called a *northing*; if south, a *southing*. Similarly departures, if east are *eastings*; and, if west, *westings*.

The northings and southings are next summed. If the polygon is closed the results should be very nearly the same. In the example here given (which is hypothetical) they agree exactly; but usually there is a small error, which is distributed over the different lines according to the principles described on p. 183. The eastings and westings are similarly treated. For an example, see the traverse sheet on p. 186. The corrected northings, etc., are used in computing the co-ordinates.

CO-ORDINATES. The next step is to choose co-ordinates for the first point, in this case A. It is, perhaps, most usual to arrange, if convenient, so that the whole survey will lie above the horizontal axis (OQ of Fig. 72, p. 125), and to the right of the vertical axis, OP.

If it be desired to do this, we look at our table of differences of latitude, and we see that from A to B we go *north*, 4082·5. Then we have three lines running *south*, for which the total is 8067·7 south. If we start with 4000 north for A, we could just get this in without negative co-ordinates, but we decide to start with 5000, as some of the survey will lie outside the traverse lines, and we thus have about 1000 feet margin.

For the departures, we see that we begin with 9595·0 *east*. Hence we *could* start with zero, but we choose 400 as allowing a sufficient margin on that side. The remaining co-ordinates are then found by adding northings and eastings, and subtracting southings and westings from point to point.

Thus to find the north co-ordinate of B, we add 4082·5 (which is the *northing* from A to B) to the co-ordinate of A, viz. 5000·0. The result is 9082·5; for the next co-ordinate we *subtract* 2624·2 (the *southing* from B to C) from 9082·5, and so on. The reason for this will be obvious from the diagram, Fig. 72.

The table is usually carried one line farther. Having arrived at the co-ordinates of E, we see that from E to A there is a northing of 3985·2. If this be added to the north co-ordinate of E, we should come back to the co-ordinate with which we started for A. The student should check this for himself, but the check is usually shown in the Table. A similar remark, of course, applies to the east co-ordinates. It will be noticed that the north co-ordinate is sometimes called the 'meridional distance', and the east co-ordinate the 'perpendicular'.

It is further clear that it is not *essential* that all points should come north and east of the original O. Thus, for Table B, the origin has been taken at R (Fig. 72), and all co-ordinates are *south* and *west*. In this case we must add southings and westings, and subtract northings and eastings.

It is usual in plotting traverses first to rule the paper with a 'grid' composed of squares, the sides of which measure a convenient round number of feet, or chains, to the scale of the drawing. If the origin were altered from O to R, the squares would have to be re-numbered starting from R. These squares should be set off with all possible accuracy. They should be drawn at first, not in pencil, but in faint carmine ink, the pen being more accurate than the pencil. They should be carefully checked by stepping and also by diagonals. The diagonal of a rectangle with sides three and four squares should be exactly equal to the length of five squares. When the squares are drawn correctly they may be inked in with Indian ink, any erroneous red lines being easily effaced either with ink-eraser or bleaching liquid. Finally, the traverse points may be plotted by means of a short scale, each within the square in which it occurs.

Check on plotting. The check on the accuracy of the plotting is that the distances between the points on the paper, as plotted, should agree very nearly with the actual measured distances as recorded on the traverse sheet.

Interior and exterior angles. Table A has been drawn up on the assumption that we worked round the traverse in the order ABCDE, so that the clockwise angle from the back station is in each case the *exterior* angle, as will be evident from the figure. In Table B it is assumed that we worked round in the opposite order, so that at B (Fig. 72, p. 125), for instance, C would be the back station, and the clockwise angle from C to A would give the *interior* angle, CBA.

These angles are marked ϕ_1, ϕ_2, etc., in the figure, and are used in Table B. The value of each is, of course, found by subtracting the exterior angle from 360°.

Each line is now taken in the opposite sense to what it was before. Thus we begin with BA instead of AB. Hence, to make the tables agree, we must give to BA a bearing of 232° 54′ (= 52° 54′ + 180°) as explained on p. 127, with reference to OA and AO.

All the rules for calculating the co-ordinates are the same as before. It is not implied, however, that when interior angles are measured the co-ordinates are to be taken as south and west, or vice versa.

x AND *y* CO-ORDINATES. In some countries it is the usual practice to call the horizontal axis (OQ in Fig. 73) the *x* axis, while the vertical axis is called the *y* axis, as in pure mathematics. In this case the co-ordinates are called the *x* and *y* co-ordinates respectively. Bearings may be also measured anti-clockwise from the *x* axis, as in mathematics. In this case it is desirable that angles should be measured anti-clockwise too. To do this with a theodolite graduated *clockwise*, we must take the *forward* station as zero.

AMERICAN NOMENCLATURE. In some parts of the American continent, and possibly elsewhere, the word *bearing* is replaced by *azimuth*; azimuth, moreover, is generally understood to be measured from *south* clockwise, instead of from north.

This does not in any way affect any of the rules for calculating the co-ordinates, except that the name of the quadrant will be altered. Thus for a line whose azimuth, on this system, is between zero and 90°, the quadrant will be S.W. and so on. In English practice the word 'azimuth' is generally used to denote the angle which any line makes with a *true* north and south line, running through the point where the azimuth is measured.

Now lines which are truly north and south lines are *not* parallel to one another anywhere but at the equator, whereas in working

TABLE A REFERRING TO FIG. 72

Points	Exterior angles $\theta_1\,\theta_2\,\theta_3\,\theta_4\,\theta_5$		Whole circle bearings $\beta^1\beta^2\beta^3\beta^4\beta^5$		Reduced bearings $\gamma^1\gamma^2\gamma^3\gamma^4\gamma^5$		Quadrant.	Distances or lengths of sides	Sides
1	2		3		4		5	6	7
	°	′	°	′	°	′			
A	255	25	52	54	52	54	N.E.	6768·0	A B
B	249	07	122	01	57	59	S.E.	4949·8	B C
C	263	38	205	39	25	39	S.W.	5199·8	C D
D	236	47	262	26	82	26	S.W.	5742·0	D E
E	255	03	337	29	22	31	N.W.	4314·1	E A
	1260	00	—	—	—	—	—	—	—

Differences of latitude				Departure				Co-ordinates		Points
N	Line on plan	S	Line on plan	E	Line on plan	W	Line on plan	N Meridional differences	E Perpendiculars	
4082·5	*b* B	—	—	5398·1	A *b*	—	—	5000·0	400·0	A
—	—	2624·2	*c* C	4196·9	B *c*	—	—	9082·5	5798·1	B
—	—	4687·4	D *d*	—	—	2250·9	C *d*	6458·3	9995·0	C
—	—	756·1	*e* E	—	—	5692·0	D *e*	1770·9	7744·1	D
3985·2	E *a*	—	—	—	—	1652·1	*a* A	1014·8	2052·1	E
8067·7	—	8067·7	—	9595·0	—	9595·0	—	—	—	—

TABLE B REFERRING TO FIG. 72

Points	Interior angles		Whole circle bearings		Quadrant bearings		Quadrant.	Distances or lengths of sides	Sides
1	2		3		4		5	6	7
	°	′	°	′	°	′			
B	110	53	232	54	52	54	S.W.	6768·0	B A
A	104	35	157	29	22	31	S.E.	4314·1	A E
E	104	57	82	26	82	26	N.E.	5742·0	E D
D	123	13	25	39	25	39	N.E.	5199·8	D C
C	96	22	302	01	57	59	N.W.	4949·8	C B
	540	00	—	—	—	—	—	—	—

Differences of latitude				Departure				Co-ordinates		Points
N	Line on plan	S	Line on plan	E	Line on plan	W	Line on plan	S Meridional differences	W Perpendiculars	
—	—	4082·5	B *b*	—	—	5398·1	*b* A	0917·5	4201·9	B
—	—	3985·2	*a* E	1652·1	A *a*	—	—	5000·0	9600·0	A
756·1	E *e*	—	—	5692·0	*e* D	—	—	8985·2	7947·9	E
4687·4	D *d*	—	—	2250·9	*d* C	—	—	8229·1	2255·9	D
2624·2	C *c*	—	—	—	—	4196·9	*c* B	3541·7	0005·0	C
8067·7	—	8067·7	—	9595·0	—	9595·0	—	—	—	

the traverse table we assume that the standard direction remains parallel to itself. Thus the calculated bearings (or azimuths in this notation) would not agree with the true azimuths, if each were separately measured from a true north and south line.

We shall return to this subject later, but it certainly seems desirable that some distinction should be made, and it appears clear that it would be a step in the right direction to restrict the meaning of the word 'azimuth' as above suggested, though there are almost equally weighty objections to the use of the word 'bearing' as we have used it.

TRAVERSE SHEET. The table like Table A (p. 135), in which the results obtained in the calculation of co-ordinates are entered is called the *Traverse Sheet*.

CHECKS ON THE CLOSED TRAVERSE. It has been pointed out that in making any traverse we may begin at either end, and work in either direction, and that, accordingly, if the traverse be closed so as to make a complete polygon, we shall measure either *interior* or *exterior* angles.

Then the sum of the included angles should be equal to *twice as many right angles as the figure has sides* plus *or* minus 360°, according to whether *exterior* or *interior* angles are measured. Thus in Table A (p. 135) number of sides $=5$; $2\times5=10$; $10\times90^\circ=900^\circ$; and as *exterior* angles are measured we *add* 360°. Hence the sum should be 1260°.

The angles are summed as shown to check this. In Table B we take $900^\circ-360^\circ=540^\circ$, as interior angles are measured. This checks the measurement of the angles.

CHECK ON CHAINAGE. To check the chainage, it is to be noticed that, if we work back to the starting-point, the sum of the northings must be equal to the sum of the southings, and, similarly the sums must be equal for the eastings and westings. These columns are summed in Tables A and B, to show this check. Any error here, supposing the arithmetic correct, would indicate an error in chaining.

Thus if the northings should sum to about 55 feet, say, more than the southings, and the westings to about 85 feet more than the eastings, we would conclude that there was an error of 100 feet on some line. Most likely the line in question would be measured

100 feet, (i.e. a complete chain length), *too short*; and, as the *southings* and *eastings* are too small, we would expect the error to be on a line running *south-east* (i.e. in the S.E. quadrant) and having its difference of latitude and departure in the ratio of about 55 to 85.

An inspection of Table A would show that the only line satisfying these conditions is BC, and, if it be possible, that line should be re-chained, unless an error can be discovered in taking out its length from the field-book.

DISTRIBUTION OF ERRORS. In Tables A and B, which refer to a fictitious traverse, all measurements are supposed to have been exact. In practice this is, of course, impossible, and there are always *small* errors in these checks. Such errors are distributed first over the various angles, and secondly over the differences of latitude and departure, so as to make them close exactly before calculating the co-ordinates.

The total correction to the angles is frequently divided equally between all the angles; or, if it be desired to work to the nearest minute only, it is applied at the rate of one minute to every third or fourth angle, or whatever the case may require; or it is, perhaps, a sound principle to correct particularly those angles which are adjacent to the shortest sides. Errors are frequently due to slight displacements of the signals, etc.; the effect of this is more marked if the line be short. Thus in Table A, the shortest side is EA. If we had a correction of two minutes to apply, we would apply one minute to the angle A, and one minute to the angle E. At a distance of 300 feet, an error of one inch will make approximately one minute difference in the angle.

In correcting the differences of latitude or departures, we add to one side and subtract from the other, the corrections being roughly proportional to the actual differences of latitude or departures to which they are applied. Thus if the northings add up to 2 feet more than the southings, we *add* 1 foot to the southings and subtract 1 foot from the *northings*, this 1 foot, in each case, being divided up about proportionally to the different northings or southings. For fuller information, see p. 183.

WORKED EXAMPLE. In Fig. 77 is shown the framework of an actual traverse survey. On the traverse sheet, which accompanies the drawing, it will be seen that the small corrections necessary are applied according to the above principles.

The whole circle bearing of HI was assumed, so that the drawing would fit nicely on the paper. The actual observed bearing is given below.

The co-ordinates are calculated with the *corrected* values of difference of latitude and departure, and the check at the end is shown.

SUBSIDIARY TRAVERSE. It will be seen that there is a main traverse round the outside boundary.

From this, at station C, a subsidiary traverse, CK*y*F, branches off, closing on to station F, which is known on the main traverse.

The angles and distances for this are given in the field-book and its traverse sheet is also given.

The whole circle bearing of BC is taken from the main traverse sheet, and the remaining bearings are worked out from the observed angles.

FIELD-WORK DATA FOR TRAVERSE IN FIG. 77.

MAIN TRAVERSE

Point	Observed angle	Whole circle bearing	Distance
H		HI 164° 27′	
I	172° 24′		617·8
A	85° 12′		273·5
B	99° 49′		711·8
C	239° 18′		87·7
D	149° 53′		433·0
E	81° 55′		213·5
F	271° 39′		593·0
G	225° 50′		419·7
M	81° 39′		604·3
N	152° 31′		374·5
O	110° 23′		196·0
O_1	140° 36′		219·7
P	184° 1′		491·0
H	164° 45′		1274·0
I	1259° 55′		

SUBSIDIARY TRAVERSE

Point	Observed angle	Whole circle bearing	Distance
B			
C	165° 39′		114·0
K	208° 42′		279·7
y	187° 7′		126·25
F	181° 19′		
G			

MAIN TRAVERSE (Fig. 77)

Point	Observed Angle	Corr.	Corrected Angle	Whole Circle Bearing (see footnote)	Reduced Bearing	Card. Pts.	Dist. (Feet)	Latitude North		Latitude South		Departure East		Departure West		Co-ordinates North	Co-ordinates East
H																	
I	172° 24′		172° 24′													200·0	2200·0
				74° 57′	74° 57′	N.E.	617·80	160·4				596·6	−·1				
A	85° 12′		85° 12′													360·4	2796·5
				340° 09′	19° 51′	N.W.	273·50	257·3	−·1					92·9			
B	99° 49′	+ 1′	99° 50′													617·6	2703·6
				259° 59′	79° 59′	S.W.	711·80			123·8				701·0	+·1		
C	239° 18′		239° 18′													493·8	2002·5
				319° 17′	40° 43′	N.W.	87·70	66·5						57·2			
D	149° 53′		149° 53′													560·3	1945·3
				289° 10′	70° 50′	N.W.	433·00	142·2						409·0			
E	81° 55′	+ 1′	81° 56′													702·5	1536·3
				191° 06′	11° 06′	S.W.	213·50			209·5				41·1			
F	271° 39′		271° 39′													493·0	1495·2
				282° 45′	77° 15′	N.W.	593·00	130·9						578·4	+·1		
G	225° 50′		225° 50′													623·9	916·7
				328° 35′	31° 25′	N.W.	419·70	358·2	−·2					218·8			
M	81° 39′	+ 1′	81° 40′													981·9	697·9
				230° 15′	50° 15′	S.W.	604·30			386·4	+·2			464·5	+·1		
N	152° 31′	+ 1′	152° 32′													595·3	233·3
				202° 47′	22° 47′	S.W.	374·50			345·2	+·2			145·0			
O	110° 23′		110° 23′													249·9	88·3
				133° 10′	46° 50′	S.E.	196·00			134·1		142·9					
O$_1$	140° 36′		140° 36′													115·8	231·2
				93° 46′	86° 14′	S.E.	219·70			14·4		219·3					
P	184° 01′		184° 01′													101·4	450·5
				97° 47′	82° 13′	S.E.	491·00			66·5		486·5					
H	164° 45′	+ 1′	164° 46′													34·9	937·0
				82° 33′ (Assumed)	82° 33′	N.E.	1274·00	165·2	−·1			1263·2	−·2				
I																200·0	2200·0
	Σ 2159° 55′	+ 5′	2160° 0′					Σ 1280·7	−·4	1279·9	+·4	2708·5	−·3	2707·9	+·3		

Whole Circle Bearing of HI assumed to be 82° 33′ to place drawing conveniently on paper

SUBSIDIARY TRAVERSE (Fig. 77)

Point	Observed Angle	Corr.	Corrected Angle	Whole Circle Bearing	Reduced Bearing	Card. Pts.	Dist. (Feet)	Latitude North		Latitude South		Departure East		Departure West		Co-ordinates North	Co-ordinates East
B																	
				259° 59′	79° 59′	S.W.											
C	165° 39′	—	165° 39′													493·8	2002·5*
				245° 38′	65° 38′	S.W.	114·0			47·04	−·04			103·84	+·3		
K	208° 42′	—	208° 42′													446·8	1898·36
				274° 20′	85° 40′	N.W.	279·7	21·08	+·02					278·20	+·8		
y	187° 07′	—	187° 07′													467·9	1619·36
				281° 27′	78° 33′	N.W.	126·25	25·06	+·04					123·74	+·4		
F	181° 19′	—	181° 19′													493·0	1495·22
				282° 46′			345·0										
Y																	
	Latitude and Departure C to F							Σ:46·14		47·04				505·78			
	ditto from Main Traverse									0·9				505·78			
										0·8				507·30			
								Correction:		−0·1				+ 1·52	(say	+ 1·5)	

* from Main Traverse

When we arrive at the bearing of FG, at the bottom it is seen to be 282° 46′. From the main traverse sheet we see that it should be 282° 45′, showing an error of 1 minute in the angles. This has been left uncorrected, being very small and the lines short.

The differences of latitude and departures are worked out for CK, K*y*, and *y*F. We find–

$$\begin{aligned} \text{Sum of northings} &= 46{\cdot}14 \\ \text{Southing} &= \underline{47{\cdot}04} \\ \text{Total southing} &= \underline{\underline{0{\cdot}90}} \end{aligned}$$

From the main traverse sheet we find–

$$\begin{aligned} \text{N co-ordinate of C} &= 493{\cdot}8 \\ \text{N} \quad ,, \quad ,, \ \text{F} &= \underline{493{\cdot}0} \\ \text{Correct total southing} &= \underline{\underline{0{\cdot}8}} \end{aligned}$$

The southing from the branch traverse is therefore too great by 0·9 − 0·8 = 0·1, and we distribute this amount of correction before calculating co-ordinates.

The same process is applied to the departures.

For the co-ordinates, we take the values from the main sheet for the point C, and check to the co-ordinates of F at the bottom.

This is an example of a traverse between two previously known stations.

It is a good plan to repeat the *point* column at the end of the traverse sheet, so as to have the name of the point near its co-ordinates.

Branch lines. It frequently happens that at or near certain portions of a traverse there is a considerable amount of detail which it is desired to show.

For this purpose it is often possible, and convenient, to take subsidiary chain stations on the traverse lines (like T and X along the line HI, Fig. 77, p. 144) and, according to the position of the detail to be shown, these may either be joined across to similar stations on the other side of the traverse, or the line between them may be used as a base line, from which chain lines can be built out, arranged to be plotted by their lengths alone, as in chain surveying.

No angles need be measured for these lines, thus obviating the necessity of setting up the theodolite at these intermediate stations.

Station *v*, for example, on the figure, is arranged to be so plotted by arcs from *w*, D, and *x*.

To avoid confusion these branch lines have not been shown on the diagram of lines (Fig. 77). But in practice the student must understand that they *should* be shown.

Other forms of traverse sheet. In some traverse sheets the bearing and distance for each line are written opposite the first letter of the line.

Sometimes the points are filled in with blank lines between; the bearings, distance, difference of latitude, and departure, for any line, are then written on the blank line between the two points which mark the ends of the line (see tables referring to Fig. 77).

It will be noticed also that the heading *Quadrant* in Tables A and B (p. 135) has been replaced here by *Cardinal points*. Any line in the north-east quadrant is here said to have the cardinal points

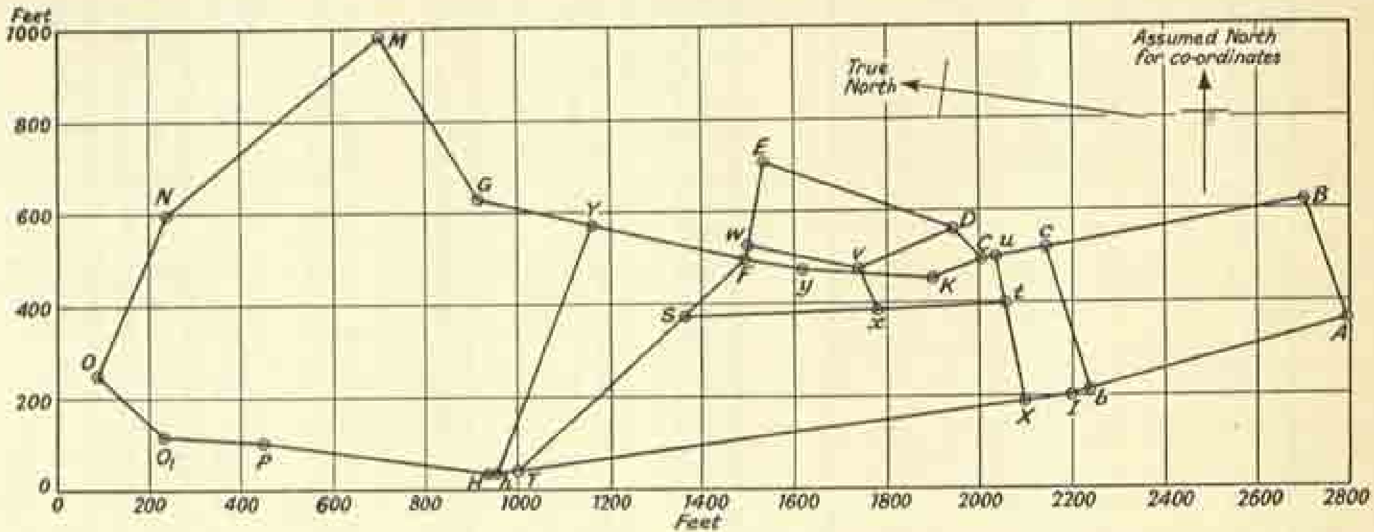

Feet
1000
800
600
400
200
0
0
200
400
600
800
1000
1200
1400
1600
1800
2000
2200
2400
2600
2800
Feet
True North
Assumed North for co-ordinates
M
N
O
O_1
P
G
Y
H
h
T
S
F
W
E
y
V
x
K
D
C
u
t
c
B
A
b
I
X

Fig. 77

N.E., and so on. Such minor modifications are largely a matter of individual taste, and should cause no trouble to the student.

TRAVERSE TABLES. 'Traverse tables' are simply tables of natural sines and cosines, multiplied by numbers from 1 to 10, or from 1 to 100. The computation is thus reduced to addition. Since full directions for use are given in the introductions to these tables, it is unnecessary to describe them here.

FIELD-WORK OF A TRAVERSE

It has already been pointed out that the work of chaining the lines and taking the offsets is the same for a traverse as for a chain survey, and slope is corrected for as in chain surveying. Except in the case of rough approximate traverses for which a prismatic compass may be used for finding the bearings of the lines, traverse angles or whole circle bearings are measured by means of a theodolite, and that instrument will now be described.

THE THEODOLITE. Theodolites are of two main types: those in which the readings are obtained from verniers and those in which micrometers are used. In the latest instruments various forms of optical scales and optical micrometers are incorporated. A standard vernier instrument is shown in Fig. 78.

THE VERNIER THEODOLITE. A three-screw levelling base, similar to that in a dumpy level, is screwed on to the head of the tripod, and a conical bearing is formed in the central boss of this levelling base to receive the hollow axis of the 'lower limb'. Round the edge of the 'limb' or plate attached to this axis is a ring *rr* of silver divided into degrees and half degrees or to 20 minutes. The diameter of this ring is used to indicate the size of the instrument.

Within the hollow axis of the lower limb revolves the solid cone attached to the underside of the 'upper limb'. The upper limb carries the supports *tt* of the telescope (with a magnetic compass sometimes arranged between them), two spirit levels *ll* at right angles to one another, and two verniers *v*, *v*, fixed opposite to each other, and reading on the degrees divided on the adjacent rim of the lower plate. In the ordinary 5-inch instrument with the lower limb divided to half-degrees the verniers are arranged to read down

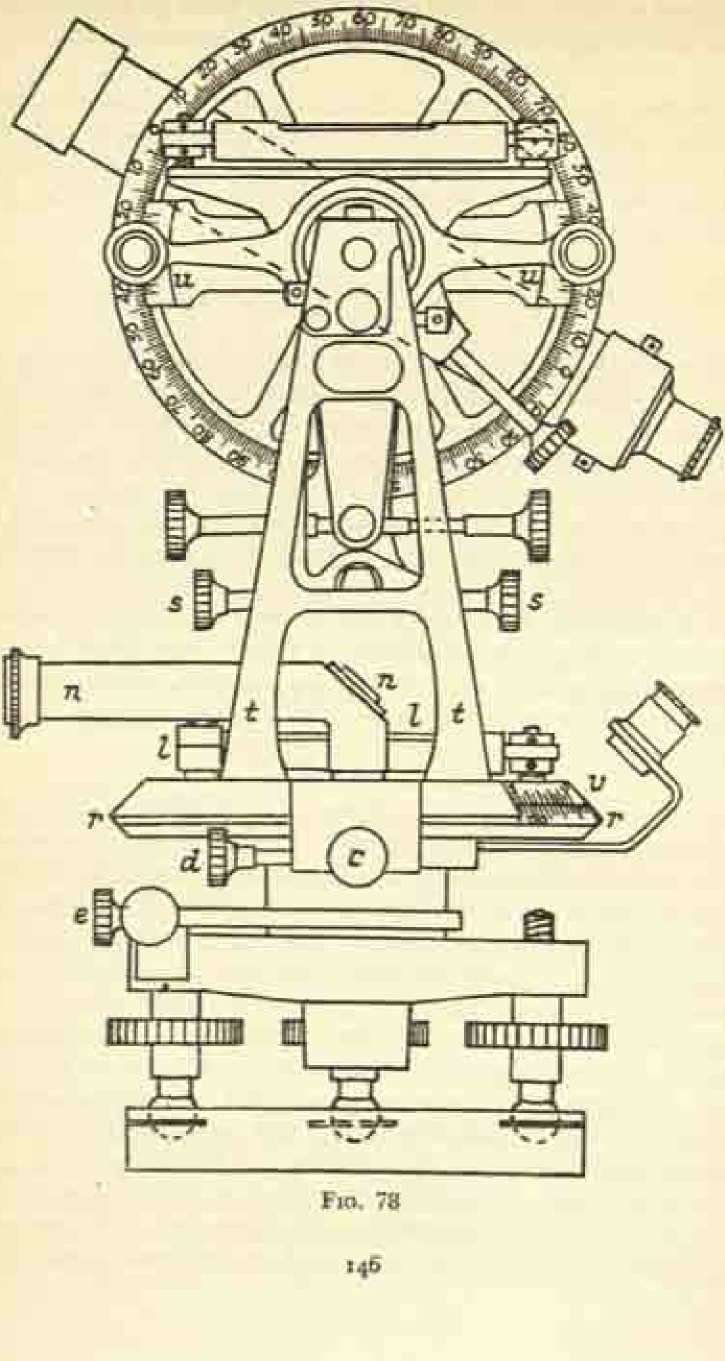


Fig. 78

to single minutes. If the degrees are divided into thirds, the verniers read either to half-minutes or 20 seconds. The principle of the vernier is discussed later. A low-power magnifier is provided for reading the verniers.

A clamp *c* is so placed that when it is tightened the upper and lower limbs are held together by friction, and the only motion possible between them is produced by turning the screw *d* called the *tangent* or *slow motion* screw. A clamp, *e*, similar in principle, is applied to the lower limb and the base of the instrument so that either the upper and lower limbs, or the lower limb and the base, or both pairs, may be clamped together.

In the transit instrument (Fig. 78) the telescope is fixed at right angles to the horizontal axis, which is supported in bearings on the 'A' side supports or frames. The optical principle of the telescope is similar to that of a level, and modern instruments are provided with internal focussing by means of an additional lens. The diaphragms, if of glass, may be ruled in various ways, but usually have two vertical lines closely spaced and three horizontal lines. The purpose of the latter is explained in Chapter 6, dealing with tacheometry. Some diaphragms have two vertical lines halfway across their diameter and a single vertical line mid-way between the other two for the remainder of the diameter. The vertical circle or limb is attached immovably to the telescope. Two verniers, *u*, *u*, reading to 1 minute, 30 seconds, or 20 seconds, are carried on an arm to read on this circle. A clamp and tangent screw are fitted to clamp the vernier arm to the circle, and give slow motion.

The vernier arm is T-shaped, the dropping leg being provided at the lower end with a jaw and two opposing 'clip screws', *s*, *s*, by means of which it may be adjusted on a small bracket projecting from either 'A' support. Sometimes the clip-screws are located on the opposite side of the instrument to the tangent-screw providing slow movement of the telescope in a vertical plane; sometimes the clip-screws and this tangent-screw are located one above the other. When properly adjusted the verniers should read zero when the line of collimation of the telescope is horizontal. A microscope is fitted to each vernier in the same manner as for the horizontal limbs. On the top of the vernier arm a spirit level is provided of greater size and therefore of superior accuracy to the small ones on the upper plate.

Graduation of vertical circle. Vertical circles of theodolites are graduated on several different systems, of which some are shown in Figs. 79 to 81. In Fig. 79 the graduations start from zero on the horizontal, and go opposite ways to 90° on the vertical, in four quadrants.

There are two great disadvantages in this. The first is that *the reading itself* does not tell whether the angle is elevation or depression. This means that the surveyor must be ever on the alert to note the sign of the angle, whereas his great object should be to make all the instrumental work as far as possible automatic, leaving his mind free for the selection of stations and general arrangement of the survey.

The second disadvantage is that two sets of figures are necessary on the vernier, as these figures must go in the same direction as

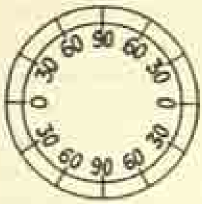


Fig. 79

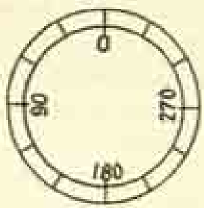


Fig. 80

Fig. 81

those on the main scale. This introduces more mental strain, and there is no doubt that mistakes occur through the reading of the wrong set of figures on the vernier. The advantage is that the actual reading gives the angle of elevation *or* depression, with either face or either vernier, without any subtraction.

In the method shown in Fig. 80, the graduations start at zero on the vertical, and go right round the circle as shown. In Fig. 81, the graduations are in four successive quadrants, all going the same way. With these only one set of figures is necessary on the vernier, and, the face being known, the reading itself is simply booked, and is sufficient afterwards to tell whether the angle was elevation or depression. But, of course, some little arithmetic may be necessary to obtain the actual angle of elevation or depression.

Thus in Fig. 80 the rules would be –

Face left angle of elevation $= 90° -$ reading

" " depression $=$ reading $- 90°$

For the second vernier, read 270° for 90°. For *face right*, these rules would be reversed. The writer personally prefers either of the last two systems described, or something of the same kind, as being less likely to lead to mistakes.

THE VERNIER. Let Fig. 82 represent a scale (straight or curved) and suppose it is desired to take readings along this scale, with an accuracy represented by any given fraction (say one-tenth) of a division on the main scale.

This is generally done by combining with it a shorter scale, on which each division is shorter (or longer) than one on the main scale by the required fraction.

Fig. 82 shows the ordinary direct-reading vernier. In this case each division is shorter than one on the main scale by the required

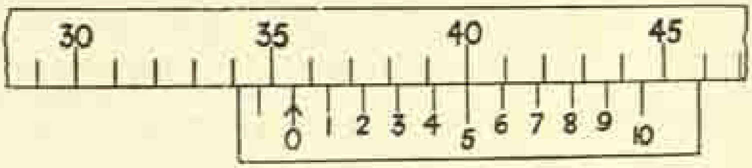


FIG. 82

fraction, and the numbers on the vernier or small scale run in the same direction as those on the main scale.

Fig. 83 shows a backward or retrograde vernier, in which each division on the vernier scale is longer by the required fraction than one on the main scale, and the figures run in this case in opposite directions. This is little used now, though it is the original form.

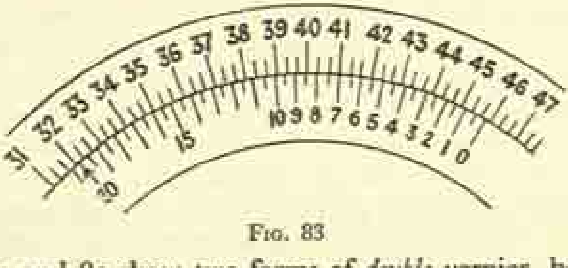


FIG. 83

Figs. 84 and 85 show two forms of *double* vernier, both of the ordinary or direct type. These are required when the graduations on the main scale run in opposite directions from a common zero, as in angles *above* and *below* the horizon. Thus two sets of figures

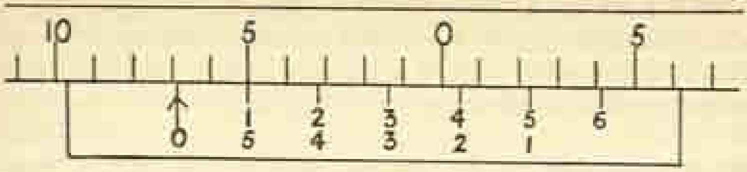


FIG. 84

are required on the vernier, and, for each reading, that set is used which runs in the same direction as the figures on the part of the main scale which is being read.

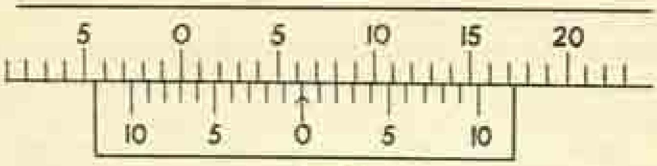


FIG. 85

Generally, if n = number of divisions on the vernier
v = value of 1 division on main scale,

then the vernier reads to $\frac{v}{n}$

The reading is taken by the arrow or zero mark of the vernier. Read the whole divisions first. Then see at what number on the vernier a division line is in exact coincidence with a division line on the main scale. Thus in Fig. 82 the reading is 35·5. The ten divisions on the vernier correspond to nine on the main scale.

In Fig. 83 there are twenty *numbered* divisions on the vernier, but each of these is divided into two. Thus there are forty divisions in all. Hence the vernier reads to one-*fortieth* of a division on the main scale. Supposing the numbered divisions on the main scale to be degrees, each is divided into three parts, so that each part is *twenty minutes*. The numbered divisions on the vernier therefore represent minutes, and the intermediate lines give *half* minutes.

The arrow is between 32° and 32° 20′. Coincidence takes place at 7 on the vernier. The reading is therefore 32° 7′. Had coincidence taken place at the next division on the right, the reading would have been 32° 6′ 30″.

The forty divisions on the vernier correspond to forty-one on the main scale.

Had the arrow been at the other end (that is, at zero on the vernier) the reading would have been 45° 47′.

In closely divided verniers like this, it is not sufficient to look *only* at the one division where the lines appear to coincide.

Thus if coincidence appears to take place at 7, we should look three or four divisions *ahead*, and the same number *back*, where we should note the same amount of discrepancy. For instance, in this case there should be equal discrepancy at *five* and *nine*.

In Fig. 84 there are six divisions on the vernier. Thus if this were applied to a curved scale, on which the divisions were degrees, the vernier would read to one-sixth of a degree, or ten minutes.

This figure shows an artifice sometimes adopted to make a more open vernier. The six divisions on the vernier correspond to *eleven* on the main scale. If it were divided into *twelve* parts, we should obviously have been able to read to *one-twelfth*, but every second division line is omitted; thus the reading is to one-sixth.

The arrow falls on the part of the scale where the numbers run from right to left.

Hence we must use the *lower* set of figures on the vernier, as they run in the same direction.

Thus in the case supposed above, the reading would be 6° 50′, as coincidence takes place at 5 on the vernier, and the arrow is between 6° and 7°.

In Fig. 85 we use the right-hand set of figures on the vernier (as the arrow is to the right of zero), and the reading is 6·2 very nearly.

SUBSTITUTES FOR THE VERNIER. With circles of 6 inches diameter or upwards, readings may be required to ten seconds or less. In

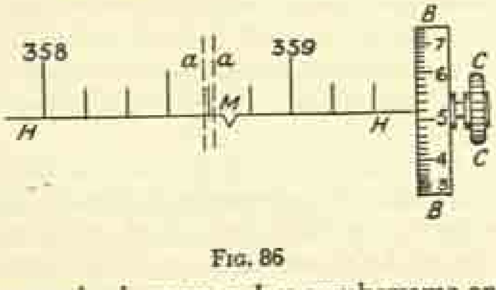


FIG. 86

such cases the vernier is more or less cumbersome and difficult to read exactly, and some form of micrometer is usually substituted. Such micrometers are sometimes applied to smaller circles, but, as all readings are ultimately referred to the graduations on the circle, it is very doubtful (to say the least of it) whether the gain is a real one in these cases.

Fig. 86 shows a typical micrometer. The full vertical lines show the graduations on the circle (here shown as if straight), and the broken lines *a*, *a* represent two spider's webs or other lines, viewed through the eyepiece of the micrometer.

The horizontal line HH travels round the graduated circle, carrying with it the lines *a*, *a*, the micrometers, and an index mark M of some sort (in this case shown by a small V-shaped mark).

The lines *aa* are moved by turning the micrometer head BB by means of the milled head CC, and the whole is so arranged that one revolution of the micrometer moves the lines through one division on the scale.

To take a reading, we first look at the index mark M, and read off its position on the scale – in this case, between 358° 40′ and 358° 50′. We note 358° 40′. Then turn the micrometer until the next *back division* from the mark M lies *midway between* the lines *a*, *a* as shown.

The micrometer head is then read. In this case the reading is 5′ 10″. The whole angle is therefore 358° 45′ 10″.

To set the instrument to zero, first bring the lines into coincidence with the mark M. The micrometer reading should then be nearly zero. Set the micrometer exactly to zero. Then unclamp the upper plate, bring the mark M nearly to 360°, clamp, and adjust by the *tangent screw*, so as to bring the 360° line exactly between the lines. See that the micrometer still reads zero.

If the micrometer reading is not nearly zero when the lines are in coincidence with the mark M, hold the milled head C with one hand, and turn the graduated micrometer wheel (which is only held by friction) with the other hand, so as to bring it to zero; then repeat the adjustment as above.

The mark M on the opposite micrometer (if there are two) should then read 180°, or very nearly so. If not, a small screw will be found on one side of the micrometer box. By turning this the index mark M can be moved until it coincides with the 180° line. Then bring the lines to the mark, and, if the micrometer reading is not zero, hold the milled head C as before, and adjust the graduated wheel as above.

To set to any other reading than zero (say 358° 45′ 10″) set the micrometer wheel to read the odd minutes and seconds (in this case 5′ 10″), but see that the lines are then to the *left* of the mark M (that is, in the *backward* direction from the mark M, according to the graduation of the circle). Also see that they are *less than one whole division* of the main scale away from the mark M.

If they are on the wrong side of the mark, or more than one division away from it, give the micrometer one or more complete revolutions so as to satisfy the above conditions. Then unclamp the plate, bring the lines near the next *back division* (in this case 358° 40′), and then bring them into exact coincidence with it by the tangent screw.

The line HH may be divided on each side of M into spaces each equal to a division on the main scale, or some fraction thereof. It is then called a *comb*.

ERROR OF RUNS. One revolution of the micrometer head should move the lines through exactly one division of the main scale. Otherwise it is clear that the readings on the micrometer cannot correctly give the fractions.

To ensure this *absolutely* is exceedingly difficult in the first place, and, besides, the adjustment is apt to vary in consequence of temperature changes and other causes.

Hence for very exact readings we should read the micrometer when the lines coincide with the *forward* division as well as the back division. If the readings are not the same, the difference is called the 'error of runs', and we must correct for it.

Suppose d be the nominal value of one complete revolution
m_1 the micrometer reading on the back division
m_2 " " forward division

Then m_2-m_1 is the error of the run (that is, in one complete revolution), and the correction to the reading m_1 is therefore

$$\frac{m_1}{d}\times(m_2-m_1)$$

For example, in Fig. 86 suppose we read 5′ 10″ on 358° 40′ and 5′ 2″ (by estimation by eye) on 358° 50′. Then the error of the runs is *minus* 8″, and the correction is

$$-8''\times\frac{5'\ 10''}{10},\text{ or }-4''\text{ nearly.}$$

Hence the reading is 358° 45′ 6″.

Error of runs must be allowed for in this way when accuracy is the chief object.

USING MICROMETERS. With this, as with all other micrometers, it is desirable to make a practice of *always completing* the adjustment of the micrometer by turning the screw in *the same* direction.

Thus, if we choose the positive direction of rotation, and we find it necessary to turn the screw in the negative direction, we should turn it too far, so as to complete the adjustment with the chosen direction of rotation. This avoids errors due to any looseness of the screw or defect in the spring.

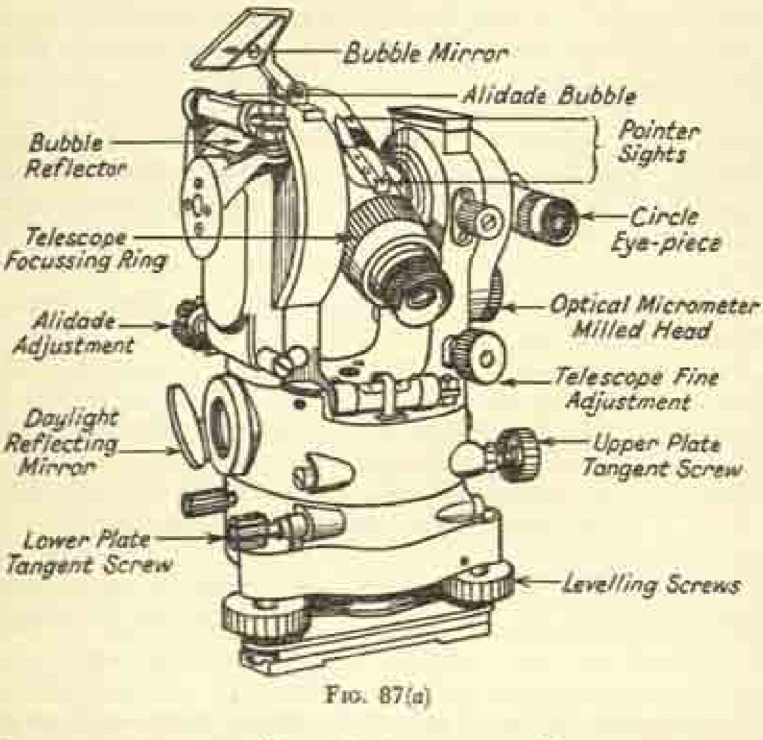


FIG. 87(*a*)

MODERN THEODOLITES. Surveying instruments, if carefully handled, have a long life, and many theodolites of the vernier type already described are still in use and are still in production. Ultra modern theodolites, however, are becoming increasingly common, and these differ in detail from the older vernier models, although in general principle they are much the same. A modern instrument by Hilger and Watts is illustrated in Figs. 87 *a* and *b*, with a diagram of its optical system.

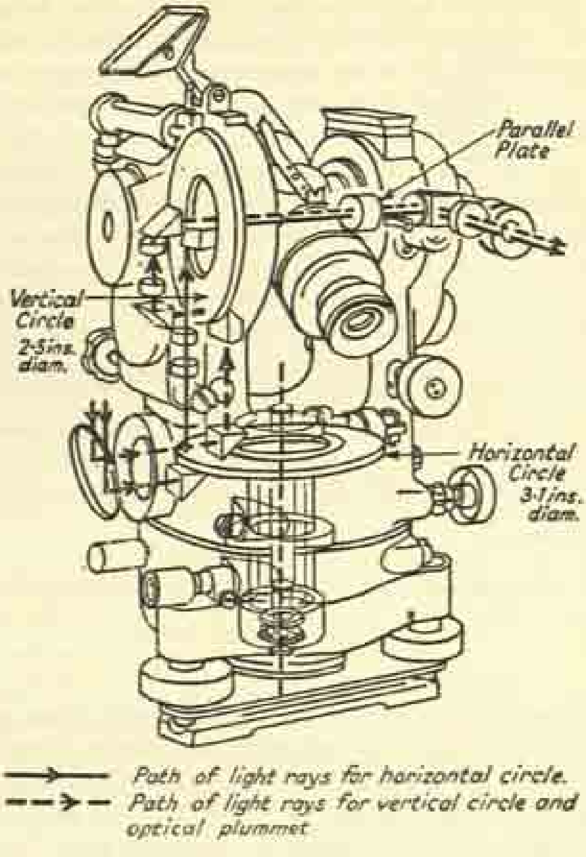


FIG. 87(*b*)

In these modern instruments the circles are of glass, in some cases silvered on the underside, and are totally enclosed. It is claimed that with glass finer lines can be used for the graduations and the optical systems adopted in these instruments enable the scales to be brilliantly illuminated by natural daylight, even in dull weather, while a high degree of magnification is obtained, thus giving increased accuracy. The circle readings, both on the horizontal and vertical scales, are transmitted by prismatic reflection to a small eye-piece mounted alongside the main telescope eye-piece, thus obviating the necessity for walking round to the side of the theodolite to read the verniers or external micrometers.

Some instruments have so-called 'optical scales' in which the graduations on a finely divided graticule are superimposed on the image of the circle, the latter being graduated simply in degrees. In one type of graticule there are two rows of divisions, the upper row being so spaced that they indicate even minutes at two-minute intervals and the lower indicating odd minutes, also at two-minute intervals. The lower set of graduations thus occupy positions midway between those of the upper row and a fine reference line falling across both scales will give a reading to half a minute when symmetrically located between upper and lower graduations. A typical field of view with the reference line at this

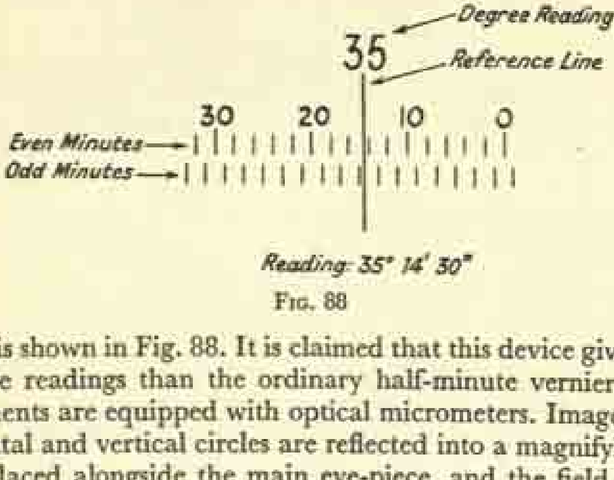


Fig. 88

setting is shown in Fig. 88. It is claimed that this device gives more accurate readings than the ordinary half-minute vernier. Other instruments are equipped with optical micrometers. Images of the horizontal and vertical circles are reflected into a magnifying eye-piece placed alongside the main eye-piece, and the field of view includes a finely divided micrometer scale. In the simpler types the circles are graduated in degrees and the micrometer scale in single minutes, but in more elaborate instruments the circle graduations are sub-divided and the micrometer may give direct readings to ten seconds or even single seconds.

The view in a simple type of micrometer eye-piece is shown in Fig. 89. Here the images of the horizontal and vertical circles are seen simultaneously in separate apertures, and one micrometer scale serves for both. Each scale has its fixed reference mark. Normally a degree graduation on either the horizontal or vertical circle will not appear to fall in perfect coincidence with the corresponding reference mark, but the image of the graduation can be displaced by an optical device so that coincidence is obtained. This optical device, described later, is actuated by a milled-headed screw placed conveniently at the side of the instrument, and the extent of the displacement of the image is recorded on the micrometer scale which is calibrated in appropriate units. These are single minutes in the example illustrated. The reading so obtained is added to the degree reading. The diagram shows the 28° graduation moved into coincidence with the reference mark, and the micrometer scale then reads 14′. The simpler types of instrument reading in this way are suitable for most kinds of engineering survey or for estate surveys, but in some instruments of this type it is not possible to obtain an approximate reading by inspection of the main scale, as in an ordinary vernier or external micrometer theodolite.

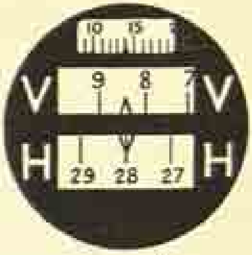


Fig. 89

THE PARALLEL PLATE REFRACTOR. The displacement of the images in optical micrometer instruments is usually effected by the tilting movement of a small glass plate with parallel sides. This simple device is diagrammatically shown in Fig. 90, from which its working principle may be deduced thus:

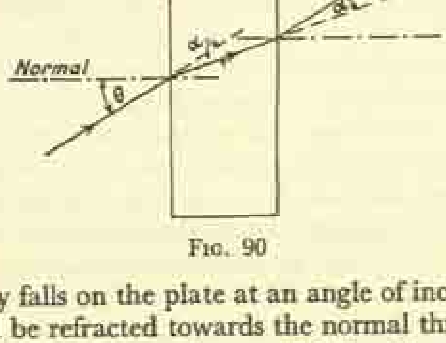


Fig. 90

If a light ray falls on the plate at an angle of incidence θ to the normal, it will be refracted towards the normal through an angle α. On emerging at the opposite parallel face it will be deviated away from the normal again at an angle α, and will thus continue in a direction parallel to its original direction. The extent of the parallel deviation of the ray depends upon the angle of tilt of the glass plate.

Some other features of modern instrument design. There is an increasing tendency to use an optical centring device in place of the plumb-line. This attachment which is located above a central opening in the horizontal plates consists of a diagonal eye-piece fitted with a diaphragm carrying a pair of intersecting reference lines. The intersection of the lines is coincident with the vertical axis of the instrument, and very accurate centring is possible by moving the theodolite laterally until the centre point of the diaphragm coincides with a fine reference point scribed on a peg or other marker. A plumb-line can be most unsatisfactory in windy weather, and an optical plummet is a most useful appliance. An example is shown in Fig. 78. In many up-to-date instruments the telescope focussing is effected by means of a knurled ring concentric with the telescope near the eyepiece end. It is claimed that this type of focussing screw is more convenient and sensitive than the usual screw at the side of the telescope tube. The newer patterns of theodolite are far more compact, lighter and more easily replaced in their cases than older instruments of equivalent accuracy.

The 'Tavistock' theodolite. In 1926 officers of the Ordnance Survey Department and other interested technical officials conferred with representatives of the principal makers of surveying instruments at Tavistock, Devon, with a view to drawing up an approved specification for a precise theodolite suitable for large triangulation surveys. As a result of this conference the 'Tavistock' theodolite was put into production. A full description of this very efficient instrument was given in *Engineering* for May 29th 1931. It gives a direct reading to a second of arc on both horizontal and vertical circles and although rather too elaborate and expensive for general use on engineering surveys, it is an ideal instrument for minor triangulation surveys of extensive areas such as those enclosed within the boundaries of an important city or town. A 'Tavistock' theodolite was used, for example, in the survey of the large township of Dagenham, Essex.

In spite of the high accuracy obtainable, the horizontal circle has a diameter of only 3·5 inches and the vertical circle a diameter of only 2.75 inches, and the instrument weighs only 13 lb. Both circles are glass and readings from opposite ends of a diameter are automatically meaned. All readings, including those of the spirit-levels, are obtained while the surveyor remains at the eye-piece end of the instrument and the adjusting screw for telescope focussing is located at the end of the trunnion axis. Provision is made for the illumination of both circles, the altitude spirit-level, and the diaphragm, thus rendering the instrument suitable for night observations in triangulation and field astronomy.

These processes are referred to in *A Treatise on Surveying*, Vol. II, chapters 3 and 7.

CENTRING OVER STATION. In using the theodolite, the first step is to centre it exactly over the station where the angle is to be measured. In instruments not equipped with the device described above this is done by means of a plumb-bob which hangs from the vertical axis of the theodolite, the string passing through a hole in the tripod head. In this case the instrument is carried by special centring plates, whereby it can be moved, after the tripod is fixed, to the extent of the diameter of the hole, so as to bring it exactly over the centre of the station.

By means of an optical centring attachment the theodolite can be set up with ease over a mark, not less than 9 ins. or more than some 10 ft. beneath it and scribed on the head of a copper nail driven into the station peg. When it is desired to use the plumb-bob the hole in the axis is filled by a long turned plug, with an eye or hook at its lower end.

The theodolite is set up roughly over the station; *two* legs of the tripod are then lifted off the ground together, one in each hand.

It will then be found possible to move the theodolite either towards or away from the observer, or to his right or left, as the plumb-bob may indicate, whilst keeping the horizontal limbs or plates roughly level as well as can be judged by eye.

When the plumb-bob (or telescope *n* in Fig. 78) is nearly central, the lower plate being also about horizontal, the two legs are lowered, *one at a time*, the leg of the tripod being turned down without sensibly moving the instrument at all. The beginner should practise this, following the instructions.

The legs are then slightly adjusted, one at a time, if necessary, to give a better approximation to true centre, and at the same time they are all rammed down tight.

Finally, the centring plates above described are brought into play, so as to bring the point of the plumb-bob exactly over the mark.

Many different forms of these centring plates are on the market.

The end of the bob should not be more than half an inch above the mark, and the centring should be examined from two positions at right angles to one another.

The station mark, for rough work with long lines, consists simply of the hole from which the ranging rod has been removed.

LEVELLING. When horizontal angles only are required, as in a traverse, the levelling is next done, using the two lower levels only. The process is exactly the same as that described for a three-screw dumpy level (page 66) except that it is unnecessary to turn the instrument at all if the two levels on the upper limb, or vernier plate, are in good adjustment and are about equally sensitive. Where extraordinary accuracy is required the theodolite may be finally levelled by the upper bubble.

READING THE ANGLE. The diaphragm lines are now focussed, if necessary, and the vernier is set to read zero on the horizontal scale. For this purpose, set it roughly to zero, then clamp the two plates together, and adjust exactly by the tangent screw, so that the arrow of the vernier comes exactly to 360°. To assist the eye, there is usually an extra division on the vernier, on the other side of the arrow, as shown in Figs. 82 and 83. When the arrow is exactly right, the same amount of discrepancy should be noticed at the divisions right and left of the arrow.

The *lower* plate is now unclamped, and the telescope turned by hand to look at the back station, whichever this may be according to the direction in which the traverse is being run.

In turning, use both hands, and the tips of the fingers only, and turn until the back station appears to be in line with the telescope by taking aim over the top only.

Clamp the lower plate before looking through the telescope, then alter the elevation if necessary, and focus the image. If the aim has been at all well taken, the ranging rod or other signal will appear in the field of view, and the centre of the diaphragm lines is brought to coincide with the lowest visible point of it by using the vertical tangent screw and that of the lower plate.

(*Note*. The lowest visible point of the signal will be the highest visible point of the image.)

Test for eye-piece parallax (p. 66), and correct it if it exists.

The vernier is now checked, to see that it still reads zero.

Next the *upper* plate is unclamped, and the telescope turned to look at the forward station, by taking aim over the top, clamping, focussing, and adjusting with the tangent screws as before, except that this time we must use the *upper* tangent screw instead of the lower one.

Then, as the theodolite is graduated clockwise, it is clear that the angle read off on the vernier will be the clockwise angle from the back station, which is what we required. This angle is read and noted.

READING AN ANGLE BY REPETITION. In order to check the angle, and perhaps obtain a slightly more accurate result, the *lower* plate is now again unclamped, and we direct once more to the back station, in the same way as before, using the lower tangent screw. Look at the vernier to see that the reading is unaltered (that is, it should still read the same angle that is recorded in the book), and then unclamp the *upper* plate and direct again to the forward station.

The vernier should now read exactly double the previous angle; or, if the former were more than 180°, then the new reading *plus* 360° should give the double.

If the theodolite reads, say, to 30", the second reading may differ by this amount from the true double. If so, we divide the new reading (*plus* 360° if the first was over 180°) by two to get the correct value.

If the second value differs by more than the smallest reading of which the theodolite is capable from the true double, the whole process should be repeated till the necessary agreement is obtained, but in many traverses the nearest minute is all that is required, and in that case we may accept a discrepancy of one minute, or even two. It would be absurd to quibble over an error of a minute in the double, for instance, if the traverse is to be plotted with an ordinary protractor. This process is called *repetition*, and it is clear that the angle can be repeated in the same way three or more times.

When carrying the theodolite from station to station, do not leave the plumb-bob hanging. It is best, perhaps, to place the bob in the pocket, without detaching the string from the instrument.

FIELD-BOOK FOR ANGLES MEASURED BY REPETITION. The field-book for the angles may be in any clear and convenient form. Sometimes the angles are simply written on the sketch of lines. The writer prefers a field-book ruled as shown below.

Point	Angle	Remarks
H		Bearing of HI = 164° 27′
I	(1) 172° 24′	
	(2) 344 48	
A	(1) 85° 12′	
	(2) 170 24	
B	(1) 99° 49′	
	(2) 199 38	
C	(1) 239° 18′	BCK (1) 165° 39′
	(2) 118 36	(2) 331 18
D		

In the point column, first and last lines, H and D are filled in, to show that the angles at I and C are the angles HIA and BCD respectively. The double is given in each case, as shown. At C the subsidiary traverse branches off. The angle BCK is read at the same time as the main angle BCD, and written in the remarks column as shown.

DIRECT READING OF BEARINGS. The theodolite may be used to give the bearings directly in the field. Thus in Fig. 91 suppose that A is the first station, and AN_1 the standard direction from which bearings are measured.

Then suppose that the theodolite were set up at A, the vernier brought to zero, and the telescope directed along AN_1 by unclamping the lower plate. Then, if the upper plate were unclamped, and the telescope turned to look at B, the reading of the vernier would give N_1AB, which is the bearing of AB.

Now let the theodolite be taken to B, set up, and the telescope directed back to A by unclamping the *lower* plate. The vernier should still be reading the angle N_1AB as the upper plate has not been touched.

Now let the upper plate be unclamped, and the telescope revolved to look at the next station C. Then obviously the vernier will now read the original N_1AB *plus* the included angle ABC, through which the upper plate has turned.

But we have seen (p. 127) that, to find the bearing of BC, we must take N_1AB plus the included angle ABC, and *plus* or *minus* 180°. Hence all we have to do is to increase (or decrease as the case may be) the vernier reading by 180°, and the result will be the bearing of BC.

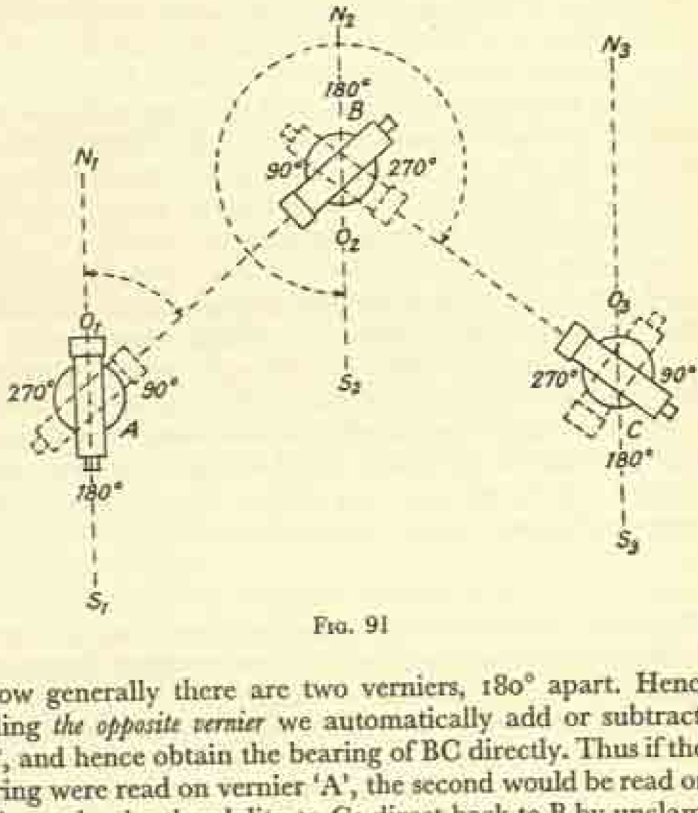


FIG. 91

Now generally there are two verniers, 180° apart. Hence by reading *the opposite vernier* we automatically add or subtract this 180°, and hence obtain the bearing of BC directly. Thus if the first bearing were read on vernier 'A', the second would be read on 'B'.

Then take the theodolite to C; direct back to B by unclamping the lower plate; check vernier B to see that the reading is unaltered; unclamp the upper plate, and direct round to the forward station; then read off the new bearing on vernier A again, and so on. The field-book is ruled in three columns, headed *Line*, *Bearing*, and *Vernier*. The last is filled in each time to avoid any mistakes as to which is to be read.

CHECK ON BEARINGS. If the traverse be a closed one, a check on the work is obtained by working right round the traverse, back again to station A. On setting up there, and looking at B, the bearing of AB should come back to the original value of N_1AB.

To distribute any small discrepancy we add (or subtract) a constantly increasing amount to the successive bearings. Thus if there are, say, seven sides and an error of 2′, we leave the first two bearings, add 1′ to the next two (or three), and 2′ to the remainder.

A rough check may be obtained for each line by the compass, if the latter be of the circular type, often attached to the horizontal plate of the theodolite. With this form of compass, the magnetic bearing can be read at once for each line after directing the telescope along the line. If magnetic north be the standard direction, these bearings should agree with those given by the verniers, as nearly as the compass can be relied upon.

If the standard direction be not magnetic north, then the compass and vernier bearings should differ by a more or less constant amount. With the box compass often found, especially on more modern instruments, this check cannot easily be used if magnetic north be *not* the standard direction. But if it *is* the standard direction, then the compass needle should automatically come to zero at every *second* station, after the lower plate is unclamped and the theodolite directed to the back station. Thus we obtain a check at alternate stations.

Included angles and direct bearings compared. The chief disadvantage of this method for the most accurate work is that the methods of reiteration, described later, and repetition cannot easily be employed.

Moreover, an error in *reading* a bearing is not carried forward. In spite of it, the next bearing may be correctly recorded, and it will not be detected in the check at the end. This error will, however, affect the result of the final calculation of co-ordinates. Now if the side which the error affects is short, the effect on the summation of the 'northings', 'southings', 'eastings', and 'westings', may be within the limits of permissible errors, and will therefore be distributed over the several points, and otherwise good work will be more or less vitiated, on account of an error in recording the angular measurement of one side only.

If the side affected be long, then there will be a large closing error, in the summation of 'northings', 'southings', 'eastings', and 'westings'. As the angles closed correctly, they will not be suspected, and the error will probably be attributed to defective linear measurement, and time may be lost in re-measuring the lines. This cannot take place when 'included angles' are measured. Their summation includes *all errors* in angular measurement, from whatever cause. To avoid this, the reading should always be checked after the instrument is set up at the new station and the telescope is directed to the back station.

If the reading agrees with what is entered in the book, well and good. But, unfortunately, if it does not agree, there is nothing to show whether this discrepancy is due to an error in reading at the back station, or in writing down the reading, or to a slip of the clamping screw whilst moving the instrument from station to station, or to the use of the wrong tangent screw when directing to the back station. There is nothing, that is to say, to show the cause of the discrepancy, unless the latter is so big that it can be detected with certainty by the compass, supposing that we can use the compass.

Hence unless the readings are so carefully taken, and the bookings checked, at each station as to leave no doubt in the surveyor's mind as to their accuracy (and the writer thinks that the more experienced a surveyor is, the less likely he will be to think that there is no doubt about the matter), then the only safe plan, if a discrepancy is found, is to take the theodolite back to the back station, and repeat the work there.

The direct reading of bearings method gives a rough check with the compass as already stated, ensuring the absence of any large error. It saves a small amount of arithmetic in the calculation of bearings. It avoids the accumulation of graduation errors, as each bearing is not measured on the same part of the scale, whereas in reading included angles, we usually set the vernier to zero for each back reading. This result may be also secured, however, by the method of repetition.

An error of *reading*, if undetected in the direct bearing method affects only the side to which it belongs, whereas in included angles it affects all that come after it. This is an advantage where great accuracy is not aimed at. When included angles are measured, *all* errors are carried forward, so that the final error in summation includes them all. Hence it is evident that the closing error in the direct method may be anticipated to be less than in the case of included angles.

If the telescope is transitted, i.e. turned about its horizontal axis, so as to always read the same vernier, it is to be noted that any collimation error which may be present is *doubled*, and may affect the result.

From the above it will be seen, that in the hands of a competent surveyor the 'direct method' is convenient, and may be safely employed. If, on the other hand, as often happens in extensive surveys, the work has to be deputed to less responsible and competent persons, then the measurement of included angles is the most reliable, on account of the complete check which it affords. Finally, if a high degree of accuracy in angular measurements is required, then the 'included angle' system is to be preferred, on account of the difficulty of 'reiteration' or 'repetition' with the 'direct method'.

MEASURING ANGLES BY REITERATION. It has been stated that the method of reiteration should be used in measuring the angles for great accuracy. For this purpose, with a transit theodolite, the angle is first read in the ordinary way, due attention having been paid to correct centring and to ensuring that the signal is vertical.

Both verniers should be read, both to the back and forward stations, and the theodolite should be turned *clockwise* from the back station to the forward; also, on taking aim, the signal should appear to the *left* of the vertical reference line, so as to bring it into coincidence from left to right with the tangent screw.

REVERSING FACE. The surveyor notes the position of the vertical circle relative to the telescope, either left or right, as the case may be, when viewed from the eye-piece end. This determines the 'face' of the instrument, which is booked as 'face left' or 'face right' accordingly. If the telescope is transitted, the object-glass end will occupy the position previously occupied by the eye-piece, and, to bring the latter back to the correct position for taking readings, the instrument must be rotated through 180° about its vertical axis. It will then be found that the vertical circle is on the opposite side of the telescope to its previous position. This procedure is known as 'reversing face'.

After the first set of readings have been taken, the face is reversed, the plates moved to bring the verniers on to a different part of the graduated circle to that previously used and the telescope directed to the forward station.

Read both verniers, as before, turn the telescope this time *anti-clockwise* from the forward to the back station, and aim in each case so as to have the signal to the *right* of the vertical reference line, so that they are brought up from right to left. The angle, with each face, may be doubled or trebled if desired, thus combining repetition with reiteration.

If the angle is *not* repeated, then, with each face, after the reading is taken to the forward station, the upper plate should be unclamped and the back station bisected again (following the direction of rotation, etc., in use with that face), and the reading recorded. It should, of course, come back to its original value; but any small discrepancy may show some slip or torsion of the axis, and in this case the *mean* reading is taken for the back station.

FIELD-BOOK. The field-book for traverse angles thus observed is as follows:

Station	Object	Face Left				Face right				Mean angle	Name of angle
		A ° ′ ″	B ′ ″	Mean ° ′ ″	Diff ° ′ ″	A ° ′ ″	B ′ ″	Mean ° ′ ″	Diff ° ′ ″	° ′ ″	
B	A	0 0 0	0 20	0 0 10		90 0 10	0 20	90 0 15			
	C	27 41 30	41 50	27 41 40	27 41 30	117 41 30	41 50	117 41 40	27 41 25	27 41 31	ABC
	C	55 23 10	23 30	55 23 20	27 41 40	145 23 0	23 20	145 23 10	27 41 30		

It will be observed that the different readings are entered in the appropriate columns, only the minutes and seconds being read on vernier B. The mean of the verniers is taken in each case, and then the differences between these means, which will be the required values of the included angles. The mean of all these values is finally taken.

The field-book is arranged for repetition and reiteration, the second reading to C in each case being the repeated angle. But if repetition were not used, this second reading to C would be replaced by a second reading to A, and, with each face, the mean of the *four* vernier readings to A would be subtracted from the mean of the *two* to C, to get the difference.

Errors in the Vernier Theodolite

Tripod. It is scarcely necessary to point out that the various screws, etc., in the tripod head are sure to work loose thus introducing errors in centring and levelling as the instrument will become unsteady. They must be tightened up from time to time, and in some cases the cap of the tripod is formed into a spanner for this purpose.

Eccentricity. The main reason for reading both verniers is that it eliminates errors of eccentricity.

The vertical axis of the theodolite should absolutely coincide with the centre C of the graduated circle. (Fig. 92).

But suppose it is slightly displaced as shown at C_1, and suppose that, when the instrument is directed to the back station, one vernier is at A on the circle.

Now let the telescope be revolved round C_1 through an angle AC_1B. The vernier will be at B, and the arc AB described on the circle will be a measure of the angle ACB, *not* the true angle AC_1B.

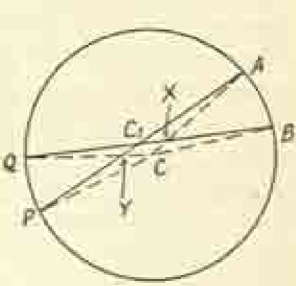


Fig. 92

$$\text{Now } ACB = AXB - C_1BC$$
$$= AC_1B + C_1AC - C_1BC$$

Hence the measured angle is too great by an amount $C_1AC - C_1BC$.

But, meanwhile, the opposite vernier has been moving from P to Q.

The true angle is PC_1Q, while this vernier gives the angle as PCQ.

Now, as before, $PCQ = AYC - C_1PC$
$$= PC_1Q + C_1QC - C_1PC$$

Hence, by addition,

$$ACB + PCQ = AC_1B + PC_1Q + C_1AC - C_1PC + C_1QC - C_1BC$$

But as the triangles CAP and CBQ are isosceles,

$$C_1AC = C_1PC$$
$$\text{and } C_1BC = C_1QC$$

Also $AC_1B = PC_1Q$, each giving the true value of the angle.

$$\therefore \text{ we have } ACB + PCQ = 2AC_1B$$

or $$AC_1B = \tfrac{1}{2}(ACB + PCQ)$$

That is, the mean of the values given by the verniers will give the true angle.

Graduation. The main object of repetition is to eliminate errors of graduation, by carrying on each angle right round the circle. The vernier is set to 90° with reversed face for the same reason, as each angle is thereby obtained, without repetition, on four different parts of the circle. Values other than 90° may also be used.

Personal bias. Each observer, when bringing the vertical reference line into coincidence with the centre of the signal, has a tendency to work the tangent screw either too far or not far enough. Hence the reason for bringing them into contact from opposite sides with different faces.

Torsion. There is always some twist, or lack of absolute rigidity, in the tripod and in the instrument itself. Moreover, it has been found that the sun causes an ordinary tripod to twist slightly in the direction of its motion. Assuming this twist to take place uniformly, its effect is eliminated by reading the different stations in opposite order.

Hence the reason for turning the instrument always one way with each face, but opposite ways with different faces. It is to be observed that, if we are about to turn from *left* to *right*, then we should begin with the telescope on the left of the back station, and bring it up to that station, as well as the forward one, with the left-to-right motion.

PERMANENT ADJUSTMENTS OF THE TRANSIT THEODOLITE. The object of reversing face is to eliminate small errors in the permanent adjustments of the theodolite, other than those above referred to. These errors should in any case be small, and the surveyor should know how to test for, and, if necessary, correct them. There are four adjustments, which we will describe in the proper order; but before the tests are carried out it is as well to ascertain whether the horizontal and vertical lines on the diaphragm are correctly positioned. To do this the telescope should be set as nearly horizontal as possible and a mark made on a wall, 100 feet or more away, in coincidence with one end of the horizontal reference line. The telescope is then moved laterally by means of one or other of the horizontal slow-motion or tangent screws. The mark should remain in coincidence with the full length of the horizontal line. The vertical line may be checked against a plumb-line. In many instruments the diaphragm may be rotated slightly after releasing the screws by which it is attached to the telescope and maladjustment thus corrected. In some modern instruments the ruled graticule fits into an outer ring by a bayonet-type attachment, and this ring may require re-setting if the lines are out of adjustment.

PLATE LEVELS. *The plate levels should be perpendicular to the vertical axis.*

If this adjustment is perfectly made, then when both levels are central the vertical axis will be truly vertical, and the levels will remain central in all positions.

To test it, set one bubble parallel to any two plate screws. Level both levels with the plate screws, the outer axis being firmly clamped. Now turn through 180°, unclamping the inner axis, or upper plate for this. If either bubble moves, correct *half* the movement with the capstan nuts by which one end of the level tube is secured to its supporting screw, and the other half with the plate screws to which it is parallel. Repeat till perfect. The principle is the same as for the similar adjustment in the dumpy level.

When the bubbles both remain central in all positions, unclamp the outer axis and turn the instrument round it. If the bubbles move, the axes are not parallel, and this defect cannot be remedied except by the makers.

When we are using an instrument with this defect, the lower plate must be fixed into position *before* the levelling is done, and must not then be moved (or, at any rate, only slightly moved), so that we must take a random reading on the back station, as well as the forward one, instead of making the former zero. The angles are then got by subtraction, as in the tables on p. 167.

LEVELLING FOR VERTICAL ANGLES. When vertical angles are to be read, the instrument should be levelled by reference to the upper bubble, on the T-piece, which is much more sensitive than the plate levels.

Level up by the plate levels; look at the upper bubble, and if it is not central make it so by the clip screws.

Then turn through 180°, and, if the upper bubble moves, correct *half* the movement by the clip screws, and half by the plate screws to which the bubble is parallel.

Then turn the telescope through 90°, and correct *the whole* of any movement by the *plate screws*.

Then through 180°, and half with each, and through 90°, and the whole with the plate screws, and so on till perfect.

This affords another method of testing the plate levels. For when the levelling is completed as above, both plate levels should be central. If not, they must be brought central by the screws which support them.

The parallelism of the two vertical axes is better tested by the upper level than by those on the plate.

Any error due to a fault in the adjustment of the levels cannot be eliminated by reversing face. But, if the error be small, its effect on horizontal angles is negligible.

HORIZONTAL COLLIMATION. *The line of collimation should be perpendicular to the trunnion or horizontal axis.*

If this adjustment is faulty, the line of collimation will describe a cone (instead of a plane) when the telescope is revolved vertically.

To test it, level up carefully, and look at any well-marked point C (Fig. 93), both horizontal movements being clamped.

Let AB be the plan of the trunnion axis, and FOD the true perpendicular to it, while OC is the line of collimation.

Now transit the telescope (i.e. turn it over vertically round the horizontal axis) and the line of collimation will now be along OE. Mark the point E in line with the central reference point or vertical line of the diaphragm.

Then turn the theodolite round horizontally to look back to C.

If DOC = FOE = α, it is clear that the angle turned through will be $180° - 2\alpha$. The trunnion axis will take up the position A_1B_1, and the perpendicular to it will be D_1OF_1. OC is the line of collimation as before.

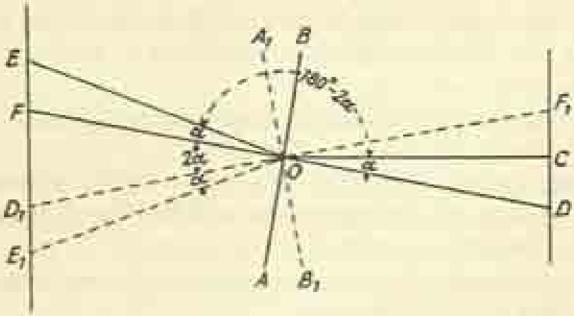


FIG. 93

Lastly transit the telescope again, this time round A_1B_1. The direction of the line of collimation will now be OE_1.

If the reference point of the diaphragm comes back exactly on the old point E, the adjustment is perfect. But, if not, mark the new point E_1. Then mark D_1 so that $E_1D_1 = \frac{1}{4} \times E_1E$, and bring the reference point of the diaphragm into coincidence with the mark D_1 by working the diaphragm screws, one on each side.

The telescope should, preferably, be level for all the readings. But if this be impracticable, the work should be done on uniformly sloping ground, so that the angle of elevation to C is very nearly equal to the angle of depression to E or E_1.

The error caused by maladjustment of the horizontal, or lateral, collimation is negligible if the sights are nearly horizontal, but may be quite appreciable if two successive sights are at widely different altitudes. This is due to the fact that on tilting the telescope the line of collimation describes a cone instead of rotating in a vertical plane. Thus, in Fig. 94, suppose it is required to measure an angle AOB in which the ranging rod at B is much lower than the rod at A. If C is a point on a vertical line through B at the same level as A, the correct value of $\angle$AOB would be given by $\angle$AOC. Let this angle be α. If the line of collimation is not at right angles to the trunnion axis, it will be deviated to points X or Y when the telescope is tilted down to sight B. The direction of deviation will depend upon which face of the instrument is used, and the extent will be the same in both directions. If it were possible to sight on to C the instrument would give the correct reading, α; when the telescope was turned downwards to Y the reading would still be α, but to sight correctly

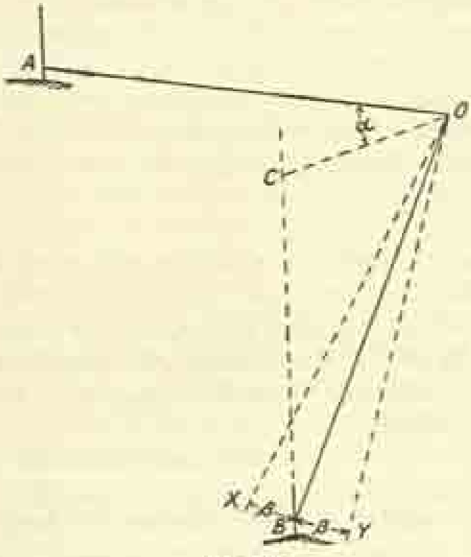


FIG. 94

on to B it would be necessary to turn the upper plate through a further angle β. The apparent value of $\angle$AOB would thus be $\alpha+\beta$. Similarly, on reversing face, it would be necessary to turn the upper plate back through an angle β, and the apparent value would then be $\alpha-\beta$. Hence this error is eliminated by reversing face, as the mean will give the correct reading.

If vertical angles be read, however, it can be shown that any error in a vertical angle due to this cause *cannot* be eliminated by reversing face. But the amount of such error is negligible in all practical cases. Thus, if there be an error of 10 *minutes* in the adjustment of the line of collimation, and we read an angle of elevation of 70°, the error will be less than 3 *seconds*.

For lower angles of elevation it gets rapidly less. The formula is $\sin h = \sin \alpha \cos \theta$, where h is the true altitude, α the observed altitude, and θ the collimation error. The error in the reading of the *horizontal* circle to a point at an altitude of 70° due to ten minutes of error in the line of collimation is, on the contrary, as much as 29 minutes, but this *would* be eliminated by reversing face.

This adjustment may also be tested as follows: Direct on to any point C (Fig. 93) with all motions clamped and the telescope horizontal, and read both verniers on the horizontal circle. Then reverse face, bisect the same point again, and again read both verniers. The mean reading should be the same with each face. If not, *half* the difference gives the collimation error. To correct it, set the verniers to the mean reading, and bisect the mark by the diaphragm screws.

TRUNNION AXIS. *The horizontal or trunnion axis should be perpendicular to the vertical axis.*

If this adjustment is perfect, then when the instrument is carefully levelled the line of collimation will describe a true vertical plane when the telescope is revolved vertically. But, if not, the plane will be inclined.

To test it, level up very carefully, and direct the instrument to any well-marked fixed point preferably at an altitude of not less than 45°, with both horizontal movements clamped. Then, if the adjustment be faulty, the trunnion axis will be inclined, and the line of collimation will describe a sloping instead of a vertical plane.

Turn the telescope down and mark the point in line with the reference point of the diaphragm as low down as possible, and preferably at such a distance that little or no alteration of focus is necessary.

Now reverse face, and direct back to the elevated point. If the adjustment is faulty the slope of the trunnion axis will be reversed and the plane described will now be equally inclined in the opposite direction to its former position. Mark the reference point of the diaphragm as before. If the two marks so obtained coincide, the adjustment is perfect. If not, mark a point mid-way between them. It will be found that one end of the trunnion axis can be raised or lowered by one or more capstan-headed screws. Adjust the movable end accordingly, a little at a time, and keep on observing the elevated point and the centre mark until the diaphragm reference lines traverse correctly from one to the other. If there are two screws, always finish by a *tightening* action so as to avoid leaving the screws slack.

Any error from this source is of about the same order as that due to a collimation error of the same amount. Like that too, it is eliminated by reversing face so far as horizontal angles are concerned, but the very small error in vertical angles is not eliminated.

If the above method is used for testing this adjustment, we must either make the adjustment for horizontal collimation *first*, as here described, or we must ensure that the angle of *elevation* to the high point shall be very nearly equal to the angle of *depression* to the low point. It is difficult to arrange this so as to obtain good angles with fairly distant points, and to avoid altering the focus.

Most books ignore altogether the fact that these two adjustments react upon one another, unless proper precautions are taken. The order here given is recommended, because it is easy to arrange matters as described under the collimation adjustment, and so eliminate the effect of any error in the horizontal axis.

THE STRIDING LEVEL. If, however, the adjustment of the trunnion axis is tested with a striding level, it is, perhaps, better to test it *before* collimation.

The striding level consists simply of an ordinary level tube mounted on two legs the lower ends of which are forked. These forks rest on the ends of the horizontal axis. One end of the level tube is usually marked A, and the other B.

The theodolite is first very carefully levelled, as for vertical angles (p. 171), the legs having been well rammed down, and all being firm and steady. Then, if the striding level is known to be correct, clearly it is only necessary to put it on the horizontal axis. If the bubble takes up a central position, the adjustment is good. If not, the movable end of the axis must be raised or lowered to make it central. But the accuracy of the striding level must not, in fact, be relied upon. To eliminate any error in it, place the level on the axis and read both ends, from the centre outwards, noting which is which, say, left or right of the observer. Then take the level off, reverse it end for end, so that the end which was on the left will now be on the right, and read again.

Next add up the readings at the left end, and those at the right end, and *one quarter* the difference will give the angular error of the horizontal axis, expressed in bubble divisions.

Thus:

$$\begin{array}{lr@{\qquad}lr}
\text{A on left,} & 9{\cdot}4 & \text{B on right,} & 3{\cdot}8 \\
\text{B ,, ,,} & \underline{6{\cdot}8} & \text{A ,, ,,} & \underline{6{\cdot}5} \\
 & 16{\cdot}2 & & 10{\cdot}3 \\
 & \underline{10{\cdot}3} & & \\
 & 4)\ \underline{5{\cdot}9} & & \\
 & 1{\cdot}5 & &
\end{array}$$

The *left* end of the axis, with the above readings, would be *too high* by 1·5 divisions, as the readings on the left are the greater. With B on the left, the reading was 6·8 on that side. Hence to correct the error, we must adjust the movable end of the axis until

$$\text{B on the left reads } 6{\cdot}8 - 1{\cdot}5 = 5{\cdot}3$$
$$\text{and A on the right reads } 6{\cdot}5 + 1{\cdot}5 = 8{\cdot}0$$

After this is done, if it be desired to correct the striding level, it will be found that one end of the bubble tube is adjustable, and we adjust that until both ends read the same. It is not worth while to do this so long as the bubble is readable. To find the reading when the bubble is central, we see that the A and B readings together add up to 13·2 or 13·3 each time. Half this, or 6·6, will give the desired reading. We will call this the central reading.

PROOF OF RULE. To prove the above rule, let c be the central reading, at which the bubble would stand at each end if in perfect adjustment and placed on a truly level axis.

Let the *left* end of the horizontal axis be too high by an amount corresponding to a divisions of the tube. Thus a perfect level being placed on it, the bubble would move a divisions from right to left.

Also let the A end of the striding level be too high by s divisions, so that when placed on a level axis the bubble would move s divisions towards A. Then we shall clearly have the following readings:

$$\begin{array}{ll@{\qquad}ll}
\text{A on left} & c + a + s & \text{B on right} & c - a - s \\
\text{B ,, ,,} & \underline{c + a - s} & \text{A ,, ,,} & \underline{c - a + s} \\
\text{Sums} = & 2c + 2a & & 2c - 2a \\
 & \underline{2c - 2a} & & \\
\text{Diff.} = & 4a & &
\end{array}$$

Hence *one quarter* the difference gives the error a in the axis. Either a or s may, of course, be negative.

VERTICAL COLLIMATION. *The line of collimation should be parallel to the spirit level if the latter be on the telescope.*

If the upper spirit level is on the T-piece, the line of collimation should be parallel to it when the vertical vernier reads zero, or whatever it should read when horizontal according to the system of graduation. It is unusual to find a spirit-level on the telescope except on older instruments.

To test this adjustment, level up very carefully, as for vertical angles (p. 171), and then read the vertical angle to any well-marked fixed point. Suppose that when the levelling was complete and the vernier set to the horizontal reading, the line of collimation was pointing slightly *upwards* instead of being horizontal. Then clearly, when we raise it to look at the object, the reading on the vernier will be *too small*.

Now *reverse face* and read the angle to the same object again. The telescope being now upside down, the line of collimation, which was before pointing upwards, will now point *downwards* when the vernier reads zero. Hence the observed vertical angle to the object will now be *too great*. The mean of the two readings will give the *correct* angle of elevation.

Thus we see that reversing face eliminates the effect of any error in this adjustment, so far as vertical angles are concerned. There is no effect at all on horizontal angles. To correct the error, we may adjust either the line of collimation or the spirit-level.

Correct the former if there is any reason to suppose it is wrong (see adjustment of level, p. 71). For this purpose, set the vertical vernier to read the mean angle of elevation. Then bring the object back to the centre by the upper and lower diaphragm screws. If the bubble is on the T-piece, it should be watched during the test, and if it moves it should be corrected for each reading.

If there is no reason to suppose that the error is in the line of collimation, adjust the spirit level. For this purpose, set the vernier to read the mean angle of elevation, and bring the reference point of the diaphragm back to the object by the *clip screws*. Check the reading.

Then (*a*) if the bubble is on the T-piece, it will now be out of centre, and we bring it back to the centre by the capstan-headed adjusting nuts at one end of the level tube. See that the reference point of the diaphragm is still on the object.

(*b*) If the bubble is on the telescope, unclamp the vertical motion, and set the vernier to the horizontal reading, which is usually zero or 90°, according to the system of graduation.

Look through the telescope, and direct an assistant to make a fine mark on a wall or elsewhere, so as to be bisected by the reference point of the diaphragm. The assistant must, of course, be directed up or down for this, and the telescope must not be moved. The bubble will now not be central if there was an error, and we bring it central by the capstan-headed screws which support it. Look at the mark during the adjustment to see that the telescope itself has not moved. If it *does* move, bring the reference point back to the mark (with any screws) before proceeding. For this final adjustment that face must be used which will cause the level to be right way up.

INDEX ERROR. It is more usual, instead of trying to correct this error (unless it is very great), to note the amount of it, and apply it as a correction to all vertical angles. This correction is called the *index error*, and is clearly *half the difference* between the two observed angles of elevation.

Thus suppose the observed angles are –

$$\begin{array}{rl} \text{Face left,} & \left.\begin{array}{l} 3^\circ\ 45'\ 20'' \\ 3^\circ\ 42'\ 30'' \end{array}\right\} \text{both } \textit{elevations} \\ \text{,, right,} & \\ & 2)\ 2\ \ 50 \\ \text{Index error} = & \ 1'\ 25'' \end{array}$$

to be added to all angles of elevation observed with *face right*, and subtracted for face left.

For depression angles, we must subtract if using the *right* face, and add if left. If read with both faces, the mean is correct.

DEGREE OF ACCURACY IN ANGULAR AND LINEAR MEASUREMENTS COMPARED. In traverse-surveying, angular and linear measurements are combined. The degree of accuracy of the result will depend on that of the less accurate of these two classes of measurement, together fixing points in space.

As far as the theodolite itself is concerned, angular measurements will, with ordinary care, be more accurate than linear measurement, as the following considerations will show.

With the ordinary chain and arrows, the ground not being peculiarly favourable, an error of *plus* or *minus* one part in a thousand would not be excessive.

Now, an angle of 1 minute subtends at 1000 feet, a distance of 0·29 foot. Suppose, to fix our ideas, the line were east and west.

Errors of chaining may displace the terminal point of the line by a foot east or west. An error of one minute in the 'bearing' would only displace the same point by less than one third of a foot north and south.

Now a good 4-inch theodolite, in proper adjustment, should by a single reading determine an angle to within $\pm$ 1 minute. Such an instrument would therefore have a degree of accuracy three times greater than ordinary chaining, and equal to some of the more accurate methods of linear measurement, such as the long steel tape.

EFFECT OF ERRORS OF CENTRING, ETC. Let A, B, and C be three points, for the sake of simplicity assumed to be in a straight line (*vide* Fig. 95). Suppose that AB and BC each measures 100 feet. Now suppose that the surveyor set up his theodolite not exactly over the point B, but at P, one-tenth of a foot from it, and in a

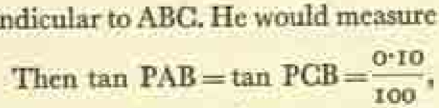


FIG. 95

direction perpendicular to ABC. He would measure the angle APC.

$$\text{Then } \tan \text{PAB} = \tan \text{PCB} = \frac{0{\cdot}10}{100},$$

$$\therefore \angle\text{PAB} = \angle\text{PCB} = 0^\circ\ 3'\ 26'',$$

and the exterior angle APC would read 180° 6′ 52″ instead of 180° 0′ 0″, thus introducing an error of 6′ 52″. This is perhaps an extreme case, but it is one which might, with want of care, easily occur in a complicated survey, such as that of a town.

Next, as to accuracy of bisection of the points observed to. It may not always be possible to see the actual peg marking the station. Supposing that, owing to a bush or an undulation of the ground, the surveyor at P could only see the top of a rod held at A. A careless staff-man might hold it so that its summit was one-tenth of a foot out of plumb, without the surveyor detecting it from the instrument. This would at once introduce an error of 3′ 26″ in the observed angle. Or a similar error might be produced by holding the staff, not on the centre of the peg, but at the side of it. It is therefore desirable that station-points be so placed that the peg itself is visible in the telescope of the theodolite when observing, so that the bottom or point of the ranging rod may be bisected.

If, for any reason, the surveyor has to observe to the top of a rod of considerable length, then he should make sure that it is *securely* and *accurately* fixed, *perpendicularly* above the station-point, and made fast with guys or a pile of stones. To effect this he may use the plumb-line of the theodolite.

In accurate surveys, where the distances are short (town surveys, for example), ordinary rods should not be used. The very thickness of the rod, when a strong sun shines on one side of it, may introduce an appreciable error.

In such cases, the surveyor should observe to the point of a lead pencil, or arrow, held on the peg or bolt, and a pencil mark, or a centre punch mark, should be made at the point at which it is held, to enable the theodolite to be set up exactly over the point observed to.

THE THREE-TRIPOD SYSTEM. This system of traversing is becoming much used for accurate traverses. The equipment required consists of a theodolite, two sighting targets, an optical plumbing device, three levelling bases and three tripods. The theodolite has a special bevel fitting in place of the usual three-arm levelling base, and this fits into any of the levelling bases on the three tripods. The targets and optical plumbing device are interchangeable with the theodolite on these bases. Thus the theodolite is centred precisely over the point previously occupied by the target, and the back and forward targets and the instrument are transferred from tripod to tripod as the work proceeds. The optical plumbing unit enables the levelling bases to be accurately centred over station points such as scribed lines or nail heads on pegs.

For ordinary work, a good plan is to mark the station by *two* pegs, equidistant from it on opposite sides, so that the peg does not interfere with the rod.

SETTING OUT TRAVERSES. The lines should be arranged so that, whilst coinciding (within the limits of moderate offsets) with the boundary of the area to be delineated, they shall be as long as possible. If, for the purpose of determinating a crooked line, short sides are absolutely necessary, it is well to arrange the points so that the bearing of further sides can be determined independently of the short sides.

Thus, if it be necessary to introduce the short sides BC, CD, DE, and EF (Fig. 96), in order to delineate a sharp bend in a road, it will be well to arrange so that the angles ABF and BFG can be observed as well as the angles ABC, BCD, CDE, DEF, and EFG, so that the bearing of FG, and of the sides beyond it, can be determined by angles referring to the relatively long side BF, and therefore free from the possible accumulation of error in the angles at C, D, and E, which affect short sides. Still better if the line BF can be chained, for the polygon BCDEF can then be treated as a subsidiary polygon, or surveyed by the chain alone.

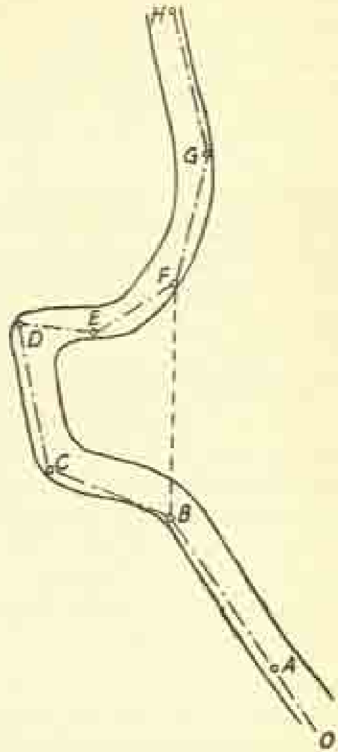


FIG. 96

Station-points should be so placed that the theodolite can be easily set up over them, and so that the surveyor can read his instrument without interruption from traffic. For this reason, the middle of a road is not a desirable station-point.

PERMANENT LOCATION OF POINTS. It is most important to secure the permanency of station-marks, or at least to provide the means of finding them if taken up, or lost. If the surveyor has not the means of marking his points in a permanent manner, for example, with substantial dressed stones, well-bedded, he will do well to establish the position of his station-points by means of measurements to fixed and permanent objects such as gate-posts, corners of buildings, jambs of doors, corners of steps and the like. Each point should be determined by three measurements. In a country where walls abound it is convenient to mark points of reference on the wall with red paint, inscribing the distances on the wall thus (*vide* Fig. 97).

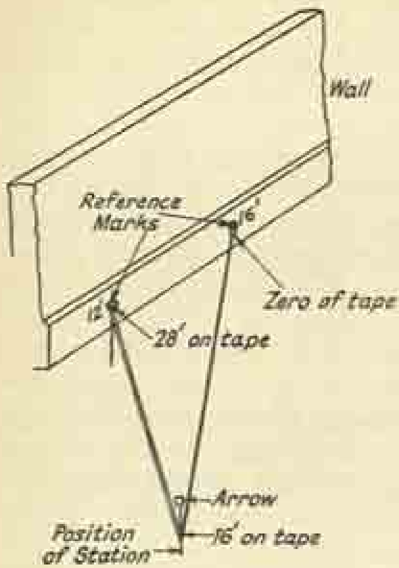


FIG. 97

The painted reference marks are placed at round distances from the station-point. By holding the ring of the tape to the point marked 16, and the graduation $16 + 12 = 28$ to the point marked 12, and stretching the tape with an arrow held at the division 16, the station-point can be accurately recovered. Marks made with iron-oxide paint, such as is used for ships' bottoms, are very permanent. Such measurements should be entered in the field-book with clear sketches, and notes. This information will be found to be of the utmost value, should the surveyor have to verify his work or wish to extend his survey. Pegs and even nails driven into roads are far from permanent, for boys will devote much energy to their removal.

LIMITS OF 'PERMISSIBLE ERRORS' DISCUSSED. Since, unless by accident, there is sure to be some error in the summation of angles, and of 'latitudes' and 'departures', it will be well to examine the amount of error which is unavoidable, and therefore permissible.

The total error of closing, as shown by the summation of 'latitudes' and 'departures' should not exceed the limit of permissible error, proper to the appliances used for measuring, and to the character of the ground. The limits of 'permissible error' in chaining have been discussed under the head of chain-surveying. Some chaining errors are constant in direction, such as an error due to the incorrect length of the chain, but this would have no effect upon the closing of a polygon provided that the same chain were used throughout.

It has been observed that when an extensive closed traverse, including two trigonometrical points, has been 'set up', the closing error of the polygon, treating the trigonometrical points merely as traverse-points, may not be more than 1 in 5000. But when the traverse is divided into two polygons, with the trigonometrical distance as a common side, the error of each (as completed by the trigonometrical side, which must not be altered) is considerably greater, often nearly 1 in 1000.

The difference between absolute 'permissible error' and the 'permissible difference' between two measurements must not be lost sight of, though the ratio is not easy to determine. With fairly careful work, distances being measured with a 66-foot steel band, total closing errors of less than 1 in 2000 can be obtained with ease.

The error in summation of angles, or angular error, must now be considered. The greater part of this will be due to errors of centring and bisection, which the surveyor has it in his power to reduce to a minimum. With an ordinary vernier theodolite it is possible to read an angle to within $\pm 15''$ of arc. If we take therefore $\pm 40''$ as the total error from all causes, a margin of $\pm 25''$ will be allowed for centring and bisection errors, equivalent to a lateral displacement of object of nearly 1 inch in 600 feet, and therefore a liberal allowance for unavoidable errors, exterior to the instrument. The average error of any one observation may be taken at $\pm 40''$. But such errors will not be always in the same direction, but will, to some extent, tend to compensate each other. Consequently the total error must not be proportional to the number of the sides. Probably, as in other matters of measurement, it will increase in proportion to *the square root of the number of sides.*

Thus, the closing error of a polygon of four sides would be

$$40'' \times \sqrt{4} = 1'\ 20''$$

$$\text{Of 9 sides } 40'' \times \sqrt{9} = 2'\ 0''$$

$$\text{Of 100 sides } 40'' \times \sqrt{100} = 6'\ 40''$$

and so on.

ADJUSTMENT OF FINAL ERRORS. The final error in summation in the calculation of co-ordinates may be corrected by a judicious addition to, or deduction from, 'latitudes' and 'departures', the corrections being in proportion to the lengths to which they are applied, as illustrated in the traverse sheet, p. 140.

Cases may, however, arise in which it is necessary to apply some more systematic method of correction.

BOWDITCH'S METHOD. It is capable of mathematical demonstration that, all things being alike subject to error, the most probable correction may be obtained by the following rule, due to Bowditch:

As the sum of all the distances is to each particular distance, so is the total error in 'departure' *to the correction of the corresponding* 'departure', *each correction being so applied as to diminish the whole error in* 'departure'. Proceed in the same way for the correction in 'latitude'.

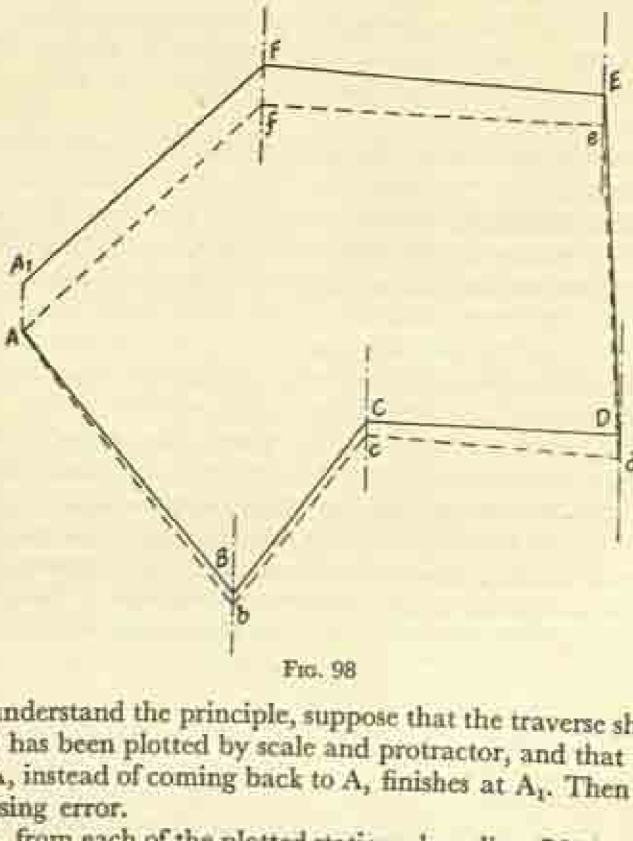


Fig. 98

To understand the principle, suppose that the traverse shown in Fig. 98 has been plotted by scale and protractor, and that the last side FA, instead of coming back to A, finishes at A_1. Then A_1A is the closing error.

Now, from each of the plotted stations draw lines Bb, Cc, etc., all parallel to A_1A, and cut off Bb so that $Bb : A_1A :: AB :$ sum of all the sides. Similarly, $Cc = A_1A \times \dfrac{AB + BC}{\text{sum of sides}}$, and so on. Then if the points A, b, c, etc., be joined as shown by the broken lines, we obtain the traverse as corrected by Bowditch's method.

To obtain the corrections graphically, we set off AB, BC, etc. (Fig. 99), along a horizontal line, equal to the lengths of the respective sides on any scale, usually a much smaller one than that of the main drawing. Then set up A_1a to represent the closing error, either on the same scale as the main drawing, or, usually, on a much bigger scale. Then, on this same scale, Bb will represent the correction at B, and is set off at Bb (Fig. 98) on the scale of the drawing; Cc gives the correction at C, and so on.

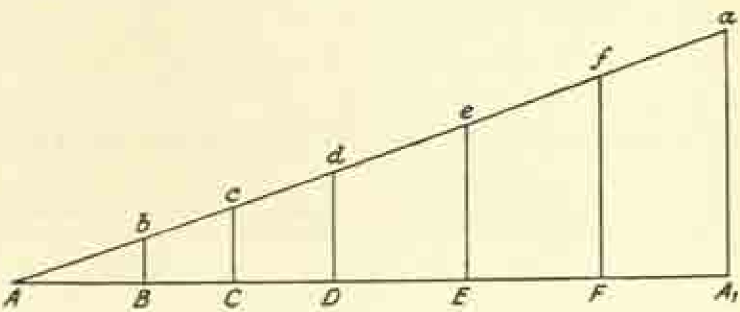


FIG. 99

In Fig. 98 the closing error shown is much greater than would actually be permissible. Exaggerating it still more, as in Fig. 100, let A_1N_1 and BN_2 be the standard direction, and AN_1, bN_2 be perpendicular to it.

Then A_1N_1 will be the total closing error in latitude, and BN_2 will be the correction in latitude for the line AB.

Now, the triangle AA_1N is similar to bBN_2, as Bb is parallel to A_1A.

Hence

$A_1N_1 : BN_2$

$= A_1A : Bb$

$=$ sum of distances : distance AB

by the construction above described. This agrees with the rule on p. 184.

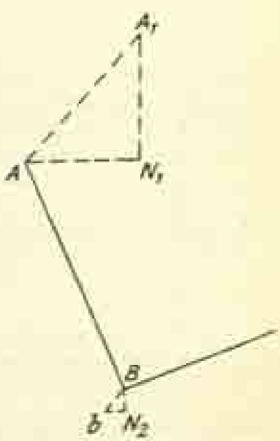


FIG. 100

Worked example

Line	Whole circle bearing	Length (feet)	Difference of latitude									
			N.	Correction				S.	Correction			
				a	b	c	d		a	b	c	d
	° ′											
AB	0 0	400	400·0	+1·0	+0·6	+1·1	+1·1					
BC	282 13	1260	266·6	+0·6	+1·7	+0·7	+0·5					
CD	327 39	1020	861·7	+2·1	+1·4	+2·3	+1·6					
DE	210 42	1690						1453·2	−3·5	−2·3	−3·9	−4·0
EF	110 48	1780						632·1	−1·5	−2·5	−0·5	−1·2
FA	60 46	1120	547·0	+1·3	+1·5	+1·5	+1·6					
		7270	2075·3	5·0	5·2	5·6	4·8	2085·3 2075·3	5·0	4·8	4·4	5·2
			closing error in latitude				=	10·0				

Departure									
E.	Correction				W.	Correction			
	a	b	c	d		a	b	c	d
0·0	0	−0·1	0	0					
					1231·5	+0·3	+0·2	+3·3	+2·2
					545·8	+0·1	+0·1	+1·5	+1·0
					862·8	+0·2	+0·3	−2·3	−2·4
1664·0	−0·4	−0·3	−1·4	−3·1					
977·3	−0·2	−0·2	+2·7	+2·7					
2641·3 2640·1	0·6	0·6	1·3	0·4	2640·1	0·6	0·6	2·5	0·8
1·2	= closing error in departure								

In the table above the differences of latitude and departure have been calculated for a six-sided traverse, whose bearings and distances are given, and the corrections have been worked out by four different methods. The columns headed '*a*' give the corrections by the method used in the traverse sheet on p. 140.

Thus the closing error in latitude is 10·0. Of this one half (= 5·0) is to be *added* to the northings, as the latter are *smaller* than the southings, and the corrections are to be proportional to the amounts corrected.

$$\text{Thus first northing} = 400{\cdot}0$$
$$\text{Sum of northings} = 2075{\cdot}3$$
$$\text{Total correction to northings} = 5{\cdot}0$$
$$\text{Hence correction to first northing} = \frac{5{\cdot}0}{2075{\cdot}3} \times 400 = 1{\cdot}0$$

All corrections are found in this way to one decimal place, and are *plus* for northings and westings, as these were too small.

Actually, the *rate* of correction is 5·0 in about 2000, or 1 in 400 nearly, and the amounts, to one decimal place, can be estimated by mere inspection.

The columns headed '*b*' show the corrections by Bowditch's method. Here the sum of the sides = 7270; closing error in latitude = 10·0; hence the correction to any latitude is $\frac{10{\cdot}0}{7270} \times$ length of corresponding side.

Similarly, the closing error in departure is 1·2; hence correction to any departure $= \frac{1{\cdot}2}{7270} \times$ length of side.

The sign is *plus* for northings and westings, as before. Thus for the line DE –

$$\text{Correction to southing} = -\frac{10{\cdot}0}{7270} \times 1690 = -2{\cdot}3$$

and ,, ,, $\text{westing} = +\frac{1{\cdot}2}{7270} \times 1690 = +0{\cdot}3$

FIG. 101

The corrections have been found by calculation in this way, but they could be found graphically as follows:

Set off OB (Fig. 101) = length of side AB
,, OC ,, = ,, ,, BC
,, OD ,, = ,, ,, CD

and so on, on any scale, and make OA_1 = sum of sides on some scale.

Then set up A_1a = closing error in latitude or departure, as the case may be, on a much bigger scale. Join Oa, and measure the corrections as shown.

This differs from the construction in Fig. 99, p. 185, in that the correction is here required for each separate side, whereas there the cumulative result was required at each station.

This construction could, of course, also be used for method *a*, but in this case we must make A_1a = closing error in latitude or departure; OB, OD, etc., = separate latitudes or departures; OA_1 = sum of northings and southings, or sum of eastings and westings, as the case may be.

COMPARISON OF METHODS. Both these methods of correction alter the included angles and the bearings as well as the lengths of the sides, and may alter the former unduly if they are measured with a theodolite and found to check.

Thus in the table on p. 186 (admitted that the closing error there is somewhat high) we have, after correction, say for the line DE, method (*a*) –

corrected departure = 863·0
and difference of latitude = 1449·7

Hence we can find the corrected bearings.

For DE, we find by method (*a*) 210° 45′ 50″
and by method (*b*) 210° 44′ 40″
against original bearing 210° 42′

showing an alteration of (*a*) 3′ 50″, (*b*) 2′ 40″, both being *increases*.

Similarly for EF the corrected bearings are –

(*a*) 110° 45′ 30″
(*b*) 110° 43′ 40″

giving *decreases* of (*a*) 2′ 30″, (*b*) 4′ 20″.

Hence the included angle DEF is decreased by (*a*) 6′ 20″, (*b*) 7′ 0″.

It will be seen that all these errors are much greater than should be expected for careful theodolite work.

Moreover, clearly there is little to choose between (*a*) and (*b*) as regards alteration to angles.

CORRECTION OF LENGTHS ALONE. Hence clearly it may in some cases be desirable, where the angles are found to check well and are separately corrected, to use a method of correction which will alter the lengths alone.

Let a polygon ABCDEFGH (Fig. 102) have a closing error such that when plotted the second position of A becomes A_1. Join AA_1 and produce it to cut the opposite side of the polygon in X. Bisect A_1A in O. Then we may consider the polygon to be divided into two parts XCBA and $XDEFGHA_1$. If, commencing from X, we replot the part XCBA, enlarging each side in the ratio of XO to XA, without any alteration in the angles, the point O will be reached. In like manner in $XDEFGHA_1$, by drawing a similar figure, each side being diminished in the ratio XO to XA_1, the point O will be reached.

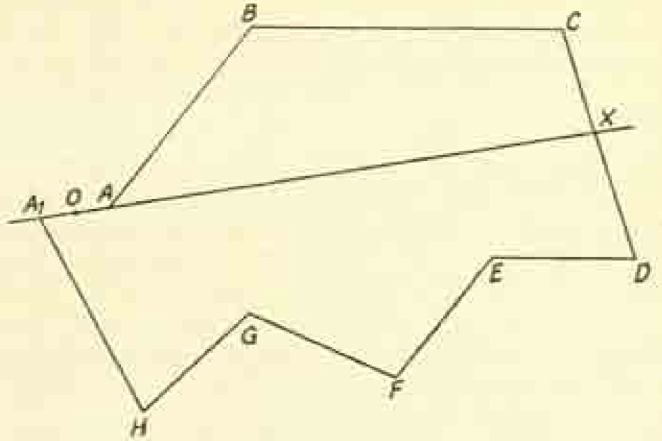


FIG. 102

$$\text{Thus altered length of AB} = AB \times \frac{XO}{XA}$$

$$\text{and } \textit{increase} \text{ in length} = AB \times \frac{XA}{XO} - AB$$

$$= AB \times \frac{AO}{XA}$$

$$\text{similarly } \textit{decrease} \text{ in length of } A_1H = A_1H \times \frac{A_1O}{XA_1}$$

Now, $AO = A_1O = \frac{1}{2}$ closing error; and we may take XA $= XA_1$, the length of the axis of correction, without sensible error. Hence we have the rule as follows:

$$\textit{Correction to any length} = \textit{that length} \times \frac{\frac{1}{2}\textit{ closing error}}{\textit{length of axis}}$$

Some care must be taken in the selection of the line OX, which may be called the 'axis of adjustment' or 'axis of correction'. For instance, the error of closure might place A_1 nearly south of A, in which case the line AA_1 would not cut the polygon at all.

Again, it may be that the axis of adjustment cuts off but a small part of the polygon, and is therefore short, thus necessitating a large percentage of correction.

This can be altered by starting the plotting from a new station, and the object is to arrange so that the maximum length will be obtained for the axis OX, so as to get a minimum amount of

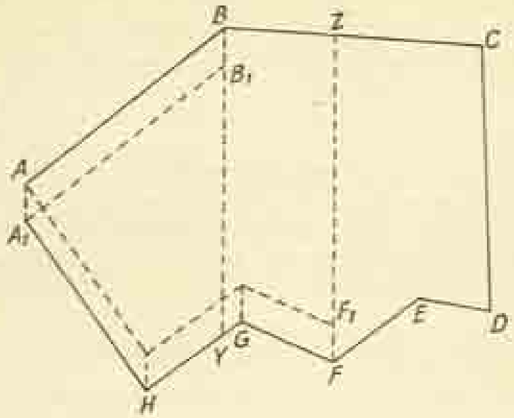


FIG. 103

correction. Thus, in Fig. 103, suppose that, starting from A, we get a closing error A_1A. Then, assuming the drawing correct,

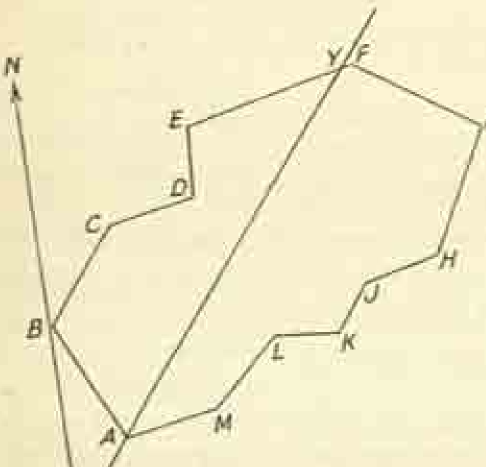


FIG. 104

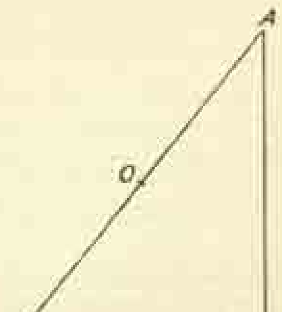


FIG. 105

wherever we start we shall get a closing error parallel and equal to A_1A. Thus if we start at B and go round in the same order as before, B_1B will be the closing error, and BY will be the axis of correction. If we start at F_1, FF_1 will be the closing error, and FZ will be the axis; and it is only necessary to find out which of these is the longer.

To apply this principle practically, plot the polygon approximately with protractor and scale to a small scale, as shown in Fig. 104, ignoring the closing error.

Lay off AB on a separate figure parallel to the meridian (Fig. 105), to represent to some convenient, but much larger scale, the error in latitude, and BC perpendicular to the same, representing error in departure. Join AC and bisect it in O. Then the line AC is parallel to the axis of adjustment, and measured on the scale used to lay off the errors in 'latitude' and 'departure' gives the total closing error, and AO or CO half the closing error.

Next draw the axis of adjustment XY (Fig. 104) parallel to AC, in such a position as to get the greatest length for XY.

Great care must be taken, in drawing the closing error diagram (Fig. 105), to observe the signs.

Thus suppose the *southings* are too *big* compared with the northings, and that we draw AB from *north to south* to represent this difference, as in Fig. 105. Then starting from B, we must go *east to west* for the departure error if the *westings* are too big, but *west to east* if the *eastings* are too big. The figure is intended to show southings and westings too big.

To decide which sides are to be increased and which decreased, we now imagine the plotting to start at A (Fig. 104), and go round in the same order as the calculation was performed, say, ABC, etc. Then clearly when the traverse is plotted to a large scale the finishing point A_1 will be *south-west* of A, as the southings and westings are too big. Both portions of the figure are now to be considered as *starting from* Y, and we want to make them both close on to the mid point of AA_1.

Hence clearly all the sides on the north-west of AY (viz. AB, etc.) are to be *increased* as A is to be brought down while those on the south-east are to be decreased, so as to bring the last side upwards.

The length of YA is measured to scale, and the ratio of correction for each line is $\dfrac{\frac{1}{2}\text{ AC (Fig. 105)}}{\text{AY (Fig. 104)}}$. But, as bearings are unaltered, *both* the difference of latitude and the departure must be increased or decreased in the same ratio as the length of the line.

Hence correction to any difference of latitude

$$= \text{that difference of latitude} \times \frac{\frac{1}{2}\text{ closing error}}{\text{length of axis}}$$

The closing error here is *not* the error in latitude only, but the total amount, found as in Fig. 105.

Similarly, correction to any departure

$$= \text{that departure} \times \frac{\frac{1}{2}\text{ closing error}}{\text{length of axis}}$$

The sign is decided as already described.

As regards the side EF, which is cut by the axis, the difference of latitude and departure should be increased for the part EY, and decreased for YF. Take the departure, for instance.

Then increase in departure E to Y

$$= \text{original departure E to Y} \times \frac{\frac{1}{2}\text{ closing error}}{\text{length of axis}}$$

$$= \frac{\text{EY}}{\text{EF}} \times \text{original departure E to F} \times \frac{\frac{1}{2}\text{ closing error}}{\text{length of axis}}$$

Similarly decrease in departure Y to F

$$= \frac{\text{YF}}{\text{FE}} \times \text{departure E to F} \times \frac{\frac{1}{2}\text{ closing error}}{\text{length of axis}}$$

Hence for the whole line EF, resultant increase in departure

$$= \frac{\text{EY} - \text{YF}}{\text{EF}} \times \text{departure E to F} \times \frac{\frac{1}{2}\text{ closing error}}{\text{length of axis}}$$

That is to say, the corrections for this line must first be calculated as for any other line, and then multiplied by the ratio $\frac{\text{EY} - \text{YF}}{\text{EF}}$.

The final sign of this correction will be *plus* if the greater segment EY is on the side where lengths are to be *increased*, and vice versa.

The lengths of EY and YF are measured from the diagram to give this ratio.

In Fig. 106 is shown the traverse referred to in the table on p. 186, ignoring the closing error.

Here *ab* = 10·0 = error in latitude, plotted from north to south, as southings are too big; and *bc* = 1·2 = error in departure, plotted from west to east, as eastings are too big. The figure *abc* is plotted to an enlarged scale. Hence *ac* is the closing error. It measures 10·1 feet, and DX is the longest line obtainable parallel to it, and measures 1880 feet. Hence the ratio of correction is $\frac{5\cdot05}{1880}$.

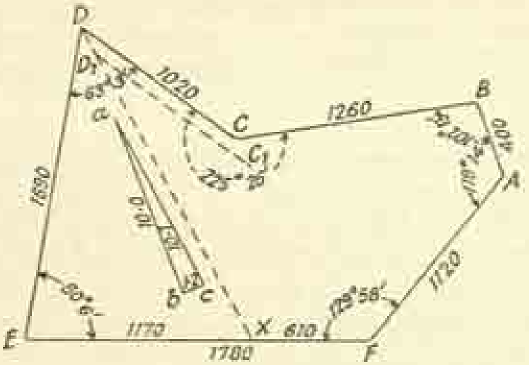


FIG. 106

If we start at D and go round in the order of calculation, viz. D, E, etc., we shall finish up between D and X, as the *southings* and *eastings* are too big (hence we shall finish in the south-east), as shown at C_1D_1.

Hence all sides on the west of DX (viz. DE) must be *decreased* to bring D down to O, the mid-point of DD_1, and all sides on the east *increased*, viz. FA, DD_1, AB, etc.

Thus correction to latitude of AB

$$= + \text{original latitude} \times \frac{5\cdot05}{1880} = + 1\cdot1, \text{ and so on}$$

These corrections are shown in column (*c*), p. 186, and should be carefully checked by the student who wishes to master this part of the work.

For the side EF, EX measures 1170 feet, and XF 610. The difference is 560, the greater segment being on the side with a *minus* correction. The whole length of EF is 1780.

Hence the correction to the departure, for instance, is

$$\text{original departure} \times \frac{5\cdot05}{1880} \times \frac{560}{1780} = - 0\cdot5$$

It will be seen that the corrections by this method are greater in amount than by (*a*) or (*b*), and that the side EF cut by the axis is relatively under-corrected.

SHORTREDE'S METHOD. Shortrede in his traverse tables suggests the following method: calculate the differences of latitude and departures without making any corrections to observed data. Then, by means of a table showing the effect of altering each bearing by one minute, for a given length, the angles are adjusted to sum, and at the same time to give a correction in the right direction. Finally, the difference of latitude and departure for 1 foot distance with each bearing are tabulated, and, using this, the remaining error is eliminated by lengthening or diminishing the sides in due proportion. This method is unnecessarily elaborate in most cases.

ALTERNATIVE TO GRAPHICAL METHOD. Professor Ormsby has suggested the following modification of the graphical method last described when it is desired to correct the sides only. If any line be increased or decreased by a fraction x of itself, the difference of latitude and departure will be altered by the same fraction of themselves, the bearing being unaltered.

Now let L = closing error in latitude
D = " " departure

and suppose that the *northings* and *eastings* are the greater.

Then all lines in the north-east quadrant *and* in the opposite (south-west) quadrant are to be altered by a fraction x, and those in the remaining quadrants by a fraction y.

The differences of latitude and departures of any particular line will be altered by the same fractions of themselves and in forming the equations below, these corrections are all to have the same sign on the side (i.e. latitudes or departures) where the closing error is the greater. But the y corrections are to have the opposite sign on the other side.

Thus in the example shown in Fig. 106 and in the table on p. 186,

L = 10·0, southings greater
D = 1·2, eastings "

Then all lines running north-west or south-east (i.e. BC, CD, EF) are to be altered by a fraction x, and the rest by a fraction y; all signs are to be plus for latitude, and the y signs minus for departure, and we therefore form the equations:

$$400y + 266{\cdot}6x + 861{\cdot}7x + 1453{\cdot}2y + 632{\cdot}1x + 547{\cdot}0y = 10{\cdot}0$$

and $$0{\cdot}0y + 1231{\cdot}5x + 545{\cdot}8x - 862{\cdot}8y + 1664{\cdot}0x - 977{\cdot}3y = 1{\cdot}2$$

where each difference of latitude or departure is multiplied by its proper fraction, the signs being settled as above.

Hence $1760{\cdot}4x + 2400{\cdot}2y = 10{\cdot}0$

and $3441{\cdot}3x - 1840{\cdot}1y = 1{\cdot}2$

whence $x = 0{\cdot}00185;\ y = 0{\cdot}0028$

These are multiplied (by slide rule) by the original differences of latitude and departures to get the corrections.

Thus BC is an x line.

Hence correction to difference of latitude $= 266{\cdot}6x = 0{\cdot}5$

" " " departure $= 1231{\cdot}3x = 2{\cdot}2$

A line like AB may be taken as either x or y, usually the former, though here it is taken as a y line.

The signs of all these corrections are decided by the side where the closing error is the greater. Thus in this case, as L is greater than D, and southings than northings, the corrections are plus for all lines having a *north* difference of latitude, and minus for those running *south*. The signs on the departure side then follow those on the latitude side. In this way the corrections in column *d*, p. 186, were filled in, and the student should check them. The writer believes that this is quicker and better than the graphical method, and probably the best method of correction wherever the lines are fairly long and the angles check well.

Locating errors. If the lengths were chained with what was supposed to be good accuracy, then a closing error such as shown in latitudes in the table on p. 186 would be inadmissible. But if circumstances would not allow the survey to be repeated, and if the angles checked well, then probably the best method of correction would be to locate the error as already described, p. 137. Thus in this case the southings are too big by 10·0, while the departures show very good agreement. The error is probably on a line running nearly due north and south. The only one is AB.

Hence the writer would recommend a bold addition of 10 feet to the length of AB, which runs north; but, of course, *only* on the assumption that the result *must* be achieved by adjustment and not by re-measurement.

Angular errors. Again, suppose that an error of, say, 5° is discovered in the angles when summed. It is probably nearly all in one angle. How can we find out *which*, without going over the survey again?

In Fig. 107, the same traverse as in Fig. 106 has been plotted from the included angles and lengths, with scale and protractor, but with one of the angles increased by 5°. This is shown by the full lines A, B, C, etc., and the closing error is AA_1. Now bisect AA_1 at right angles by the line XY. It will be found that this line passes almost exactly through station C.

This indicates that C was the angle which was wrong, and if we measure $\angle ACA_1$ and find that it is about 5°, we may safely apply

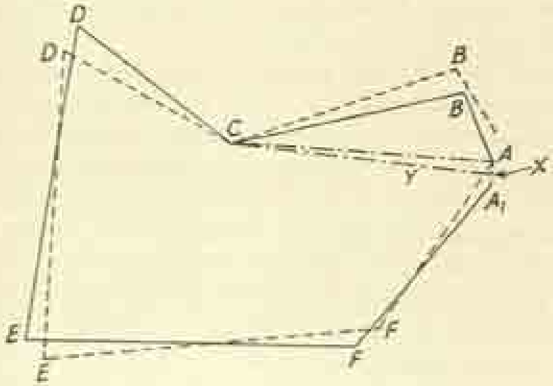


FIG. 107

the correction to that angle, and be sure that we have done the best thing possible under the circumstances. If the line CA_1 were swung upwards through an angle of 5° it would bring the point A_1 to A.

Another construction is as follows: having plotted the figure as above, starting with AB, and working in the direction ABC, etc., start again with BA and plot in the opposite direction, setting off the included angles anti-clockwise. This is shown by the broken lines. Then if the two sets of lines meet at any one station, that station is the one with the incorrect angle. The reason is obvious, because the full lines AB and BC will be correct, but all after that will be incorrect, since the angle at C is wrong. Similarly, plotting backwards, everything will be correct up to station C, and incorrect beyond it. Hence the two positions of C, and of C only, will agree, assuming that C is the incorrect angle.

With a traverse of many short sides the errors cannot, of course, be so certainly located, but at all events they can be *localized*, or shown probably to belong to one or more of only a few lines.

Problems in traversing. In the application of traverse surveying to engineering work in particular many problems arise, most of which can be solved by the use of the following rules:

(1) To find the *back* bearing of a line, given its *forward* bearing.

Rule. Back bearing = forward bearing $\pm$ 180°. Use the *plus* sign if the forward bearing be *less* than 180°, and vice versa.

	(a)	(b)
Example. Bearing of AB =	327° 18′	18° 27′
	180° 0′	180° 0′
Bearing of BA =	147° 18′	198° 27′

(2) To find the included angle between two lines whose whole circle bearings are known.

Rule. Express both bearings as if measured *from* the point where the lines meet. Subtract the smaller from the greater, and the result will be the clockwise angle *from* the line which has the smaller bearing.

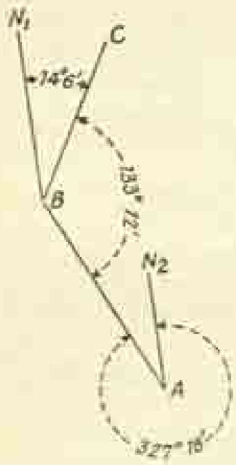


Fig. 108

Example. Bearings of AB, BC = 327° 18′ and 14° 6′, respectively. Find the angle CBA (Fig. 108). Both bearings must be calculated *from* B.

Bearing of AB =	327° 18′
	180° 0′
∴ ,, BA =	147° 18′
and ,, BC =	14° 6′
∴ angle CBA =	133° 12′, clockwise.

If the clockwise angle *from* the line with the bigger bearing be required, then add 360° to the smaller bearing and subtract the bigger.

Thus bearing of BC =	14° 6′
	360° 0′
	374° 6′
Bearing of BA =	147° 18′
Angle ABC =	226° 48′

(3) Given the co-ordinates of two points to find the bearing of the line joining them.

If A and B be the two points, and AN the standard direction, it is clear that, in Fig. 109, AN will be the difference between the north or *y* co-ordinates, and NB will be the difference between the east co-ordinates; and NAB is the reduced bearing.

Hence we have the rule –

tan reduced bearing

$$= \frac{\text{difference between east co-ordinates}}{\text{difference between north co-ordinates}}$$

$$= \frac{\text{departure}}{\text{difference of latitude}}$$

When the reduced bearing is known, it is best to see in which quadrant the line lies, by an inspection of the co-ordinates themselves, and then to work backwards by the rules on p. 131 to find the whole circle bearing.

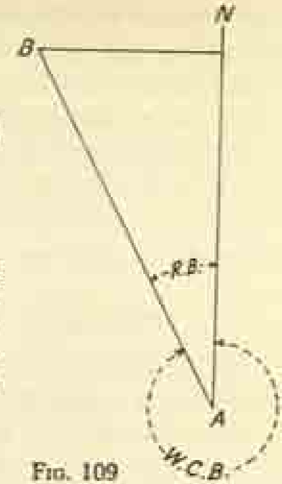


Fig. 109

Example.

Point.	Co-ordinates	
	N	E
A	623·9	916·7
B	981·9	697·9

$$\text{Departure} = 916{\cdot}7 - 697{\cdot}9 = 218{\cdot}8$$
$$\text{Difference of latitude} = 358{\cdot}0$$
$$\therefore \tan \text{R.B.} = 218{\cdot}8 \div 358{\cdot}0$$
$$\text{whence R.B.} = 31^\circ\ 26'$$

and B is north-west of A, because its *north* co-ordinate is *greater* than that of A, and its *east* co-ordinate is *less*. That is, AB lies in the north-west quadrant, and there the whole circle bearing = 360° – R.B. (see Fig. 109); hence bearing of AB = 360° – 31° 26′ = 328° 34′.

(4) To find the length of the line joining two points whose co-ordinates are known.

First rule. Find the reduced bearing as above, then –

(*a*) Length = cosec R.B. × departure

or (*b*) ,, = sec R.B. × difference of latitude

The truth of these rules will be obvious from an inspection of Fig. 109.

Use (*a*) if the departure be greater than difference of latitude, and vice versa, so as to always calculate from the *greater* of the known quantities.

Second rule. Length = $\sqrt{\text{departure}^2 + \text{difference of latitude}^2}$

Example. Find the length of AB in the above case.

First rule. $AB = 358{\cdot}0 \sec 31° 26' = 419{\cdot}6$

Second rule. $AB = \sqrt{218{\cdot}8^2 + 358^2} = 419{\cdot}6$

(5) Given the bearings of two lines AB, BC (Fig. 110), and the co-ordinates of the ends A and C, to find the lengths of AB and BC.

First rule. Find the bearing and length of AC as above, by (3) and (4). Then, knowing all the bearings, find all the angles of the triangle ABC by (2). Lastly, solve the triangle from the three angles and the side AC.

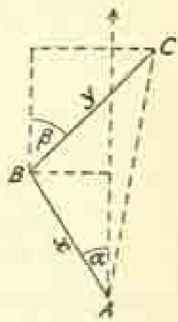


FIG. 110

Second rule. Let x and y be the lengths of the lines and α and β the reduced bearings.

Then departure A to B $= x \sin \alpha$, to be reckoned *plus* if the quadrant is north-east or south-east (that is, if the line AB runs *east*), and *minus* if it is north-west or south-west.

Similarly departure B to C $= y \sin \beta$, the sign to be settled in the same way.

But these together make up the departure from A to C, which is – east co-ordinate of C – east co-ordinate of A, and is *plus* or *minus* accordingly.

Hence

$$x \sin \alpha + y \sin \beta = \text{E. co-ordinate of C} - \text{E. co-ordinate of A}$$

Similarly

$$x \cos \alpha + y \cos \beta = \text{N. co-ordinate of C} - \text{N. co-ordinate of A.}$$

This second equation follows, in the same way as the first, from a consideration of the differences of latitude.

The sign of $x \cos \alpha$ is *plus* if the quadrant for AB is north-east or north-west (that is, if it runs *north*, as we are now considering differences of latitude), and the same for $y \cos \beta$.

From these two equations we can find x and y.

Example. The bearings of AB, CD (Fig. 111) are 112° 15′ and 218° 17′ respectively. The co-ordinates are–

Point	N	E
B	1048·6	382·6
C	687·2	439·4

If AB and DC produced meet at E, find the lengths BE and CE.

The reduced bearing of BE is the same as that of AB, $= 180° - 112° 15' = 67° 45'$, and the quadrant is south-east. This gives the value of α.

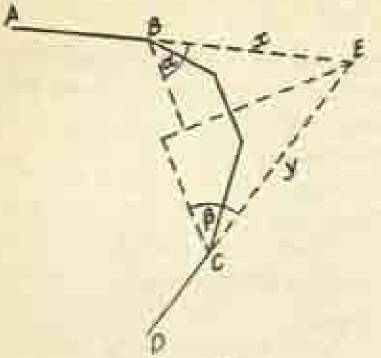


Fig. 111

Similarly β = reduced bearing of $EC = 218° 17' - 180° = 38° 17'$, quadrant south-west.

Let $BE = x$; $EC = y$.

$$\therefore\ x \sin 67° 45' - y \sin 38° 17' = 439{\cdot}4 - 382{\cdot}6$$
$$\text{and} - x \cos 67° 45' - y \cos 38° 17' = 687{\cdot}2 - 1048{\cdot}6$$

$x \sin 67° 45'$ is *plus* because it refers to the departure of BE, and that line runs south-*east*. $y \sin 38° 17'$ is *minus* because the line EC is in the south-*west* quadrant.

$x \cos 67° 45'$ and $y \cos 38° 17'$ are both minus because both lines run *south*.

Hence
$$0{\cdot}92\,554x - 0{\cdot}61\,955y = 56{\cdot}8$$
$$\text{and} - 0{\cdot}37\,865x - 0{\cdot}78\,496y = -361{\cdot}4$$
$$\text{whence } x = 279{\cdot}36,\ y = 325{\cdot}65$$

(6) Given the co-ordinates of A and B (Fig. 112), and given one co-ordinate of a point X on AB, to find the other co-ordinate, and the length of AX.

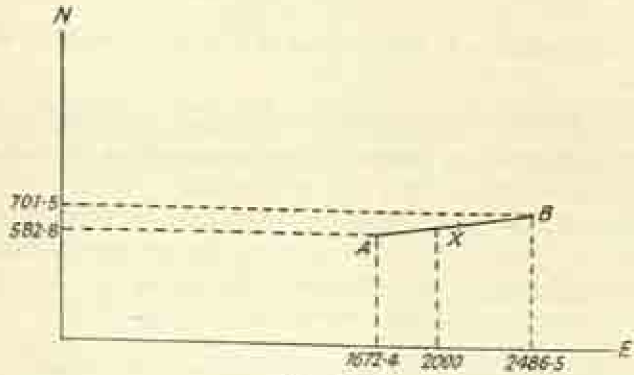


Fig. 112

This problem arises when a traverse has to be plotted on more than one sheet and the points A and B come on different sheets. Say the line crosses the left-hand (or west) margin of sheet No. 2, and the right-hand margin of sheet No. 1, and that the east co-ordinate for either of these margins is 2000·0. Then let the co-ordinates of A and B be–

Point	N	E
A B	582·8 701·5	1672·4 2486·5

It is required to find the north co-ordinate of the point X where the line crosses the 2000 margin. i.e. east co-ordinate of X = 2000·0.

Rule. Departure from A to B : departure from A to X

= difference of latitude from A to B: difference of latitude from A to X.

$$\therefore \frac{2486\cdot5 - 1672\cdot4}{2000\cdot0 - 1672\cdot4} = \frac{701\cdot5 - 582\cdot8}{\text{diff. of lat. A to X}}$$

$$\therefore \text{difference of latitude from A to X} = \frac{118\cdot7 \times 327\cdot6}{814\cdot1} = 47\cdot7$$

$\therefore$ north co-ord. of X = 582·8 + 47·7 = 630·5

For a further example, suppose it is required to find the co-ordinates of the points *h* and Y, Fig. 77, p. 144.

We are given that *h* is 15 feet from H along HI, and the co-ordinates of H and I are–

Point	N	E
H I	34·9 200·0	937·0 2200·0

The length of HI is 1274·0 and the length of H*h* is 15 feet

departure H to I = 1263·0 east
difference of latitude = 165·1 south
$\therefore$ departure H to h = $1263\cdot0 \times \frac{15}{1274}$ = 14·9 feet E.
difference of latitude = $165\cdot1 \times \frac{15}{1274}$ = 1·9 „ N.

Similarly for Y.

FY = 345·0 feet; FG = 593 feet
departure F to G = 578·5 west
difference of latitude = 130·9 north
$\therefore$ departure F to Y = $578\cdot5 \times \frac{345}{593}$ = 336·5 west
difference of latitude = $130\cdot9 \times \frac{345}{593}$ = 76·2 north

Co-ordinates

	N	E
H =	34·9	937·0
	1·9	14·9
h =	36·8	951·9
F =	493·0	1495·2
+	76·2	−336·5
Y =	569·2	1158·7

Negative co-ordinates. In all these cases it is, of course, to be understood that the origin of co-ordinates may be so chosen that, instead of all co-ordinates being north and east, we may have north *and* south co-ordinates, as well as east *and* west. In such cases, if one co-ordinate be north and one south, then to find the difference of latitude between the points we must *add* the numerical values of the co-ordinates, and so on.

Worked example. As a general example showing the use of these rules, we will take the following case:

A railway round a headland is to be in tunnelling between fixed points A and B (Fig. 113), this line being altogether unsuitable for direct measurement of any kind.

At B, which is just beyond the end of the tunnel, a curve is to start, to carry the line round to the line CD. It is impossible, from the nature of the ground, to measure or observe directly between any two points lying respectively on AB and CD.

A very careful traverse is therefore run, as shown, and the angles and distances marked on the figure – here given to the nearest minute and tenth of a link only – are obtained.

A whole circle bearing of 60° 0′ is assumed for the first line AF, and A is taken as the origin of co-ordinates.

The north portion of the traverse is checked by the lines GK and KD.

It is required to find –

(*a*) The angle FAB, to enable the direction of the tunnel to be set out at A.

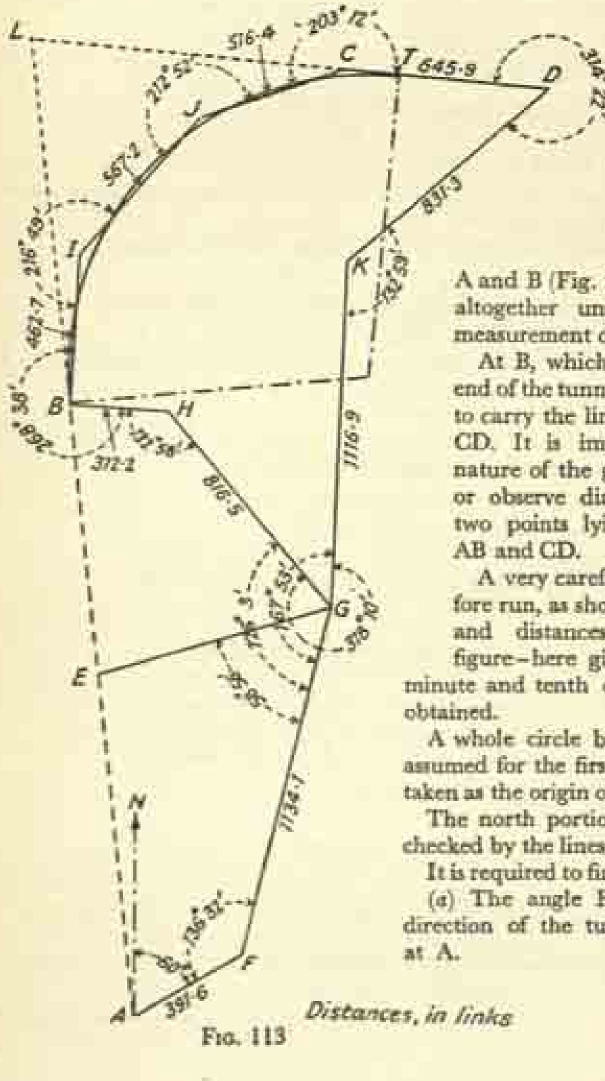


Fig. 113

(*b*) The distance AB.

(*c*) The length BL and angle CLB, which are required for giving the radius of the curve.

(*d*) The angle IBA (or HBA), to enable the curve and the direction of the tunnel to be set out at B.

(*e*) The distance from C to the finishing point T of the curve.

(*f*) Also it is desired to sink a shaft at or about the middle of the tunnel.

From G it is found that a convenient line for measurement can be set out in the direction GE, and the angle FGE is observed as shown. It is required to find the distance GE, from G to the point where this line in plan meets the line of the tunnel, so as to fix the position E of the shaft, and the distance AE.

The first step is to calculate the co-ordinates of the traverse stations. The result is shown in the table on p. 204.

The student should re-work the whole example.

It is obvious that as the first two westings are about 150 links greater than the first two eastings, an east co-ordinate of about 200 should have been chosen for A, to avoid negative or west co-ordinates. It has been purposely taken as zero, however, to show the student that negative co-ordinates are not fatal.

(*a*) To find the angle FAB, first find the bearing of AB.

$$\text{Departure A to B} = -145 - 0 = 145{\cdot}0 \textit{ west}$$
$$\text{Difference of latitude} = 1961{\cdot}6 - 0 = 1961{\cdot}6 \textit{ north}$$
$$\therefore \tan \text{R.B.} = \frac{145}{1961{\cdot}6}, \text{ or R.B.} = 4^\circ\ 13'\ 40'', \text{ nearly}$$

And as the quadrant is north-west–

$$\begin{aligned} \text{W.C.B.} = 360^\circ - \text{R.B.} &= 355^\circ\ 46'\ 20'' \\ \text{and bearing of AF} &= \underline{\ 60^\circ\ \ 0'\ \ 0''} \\ \therefore \text{clockwise angle FAB} &= 295^\circ\ 46'\ 20'' \end{aligned}$$

(*b*) To find the distance AB

$$\begin{aligned} \text{AB} &= 1961{\cdot}6 \sec 4^\circ\ 13'\ 40''\ (\text{Rule 4, p. 198}) \\ &= 1966{\cdot}9 \text{ links} \end{aligned}$$

(*c*) To find the angle CLB

$$\begin{aligned} \text{Bearing of BL} = \text{bearing of AB} &= 355^\circ\ 46'\ 20'' \\ \therefore (\text{Rule 1, p. 197}) \quad ,, \qquad \text{LB} &= 175^\circ\ 46'\ 20'' \\ \text{and bearing of LC} = \quad ,, \qquad \text{CD} &= 97^\circ\ \ 4'\ \ 0'' \\ \therefore (\text{Rule 2, p. 197}) \text{ angle CLB} &= 78^\circ\ 42'\ 20'' \end{aligned}$$

The bearing of CD is taken from the traverse sheet.

To find the distances BL and CL, find the bearing and length of BC.

$$\text{Departure} = 757{\cdot}0 + 145{\cdot}0 = 902{\cdot}0 \text{ east}$$
$$\text{Difference of latitude} = 2994{\cdot}6 - 1961{\cdot}6 = 1033{\cdot}0 \text{ north}$$
$$\therefore \tan \text{R.B.} = \frac{902{\cdot}0}{1033{\cdot}0}, \text{ or R.B.} = 41^\circ\ 7'\ 40''$$

therefore W.C.B. of BC $41^\circ\ 7'\ 40''$, as the quadrant is north-east

$$\begin{aligned} \text{and length} &= 1033 \sec 41^\circ\ 7'\ 40'' \\ &= 1371{\cdot}4 \text{ links} \end{aligned}$$

Point	Observed angle	Whole ⊙ bearing	Distance	Difference of latitude		Departure		Co-ordinates		Point
				N	S	E	W	N	E	
A	° ′	° ′						0	0	A
		60 0	391·6	195·8		339·1				
F	136 32							195·8	339·1	F
		16 32	1134·1	1087·2		322·7				
G	126 3							1283·0	661·8	G
		322 35	816·5	648·5 − 0·1			496·1			
H	132 58							1931·4	165·7	H
		275 33	312·2	30·2			310·7			
B	268 38							1961·6	− 145·0	B
		4 11	462·7	461·5 − 0·1		33·8				
I	216 49							2423·0	− 111·2	I
		41 0	567·2	428·1		372·1				
J	212 52							2851·1	260·9	J
		73 52	516·4	143·5		496·1				
C	203 12							2994·6	757·0	C
		97 4	645·9		79·5	641·0 − 0·1				
D	314 22							2915·1	1397·9	D
		231 26	831·3		518·3		650·0 + 0·1			
K	132 59							2396·8	747·8	K
		184 25	1116·9		1113·6 + 0·2		86·0			
G	318 10							1283·0	661·8	G
H	1800 0			1711·8	1711·4	1543·0	1542·8			

Next find the angles LBC and BCL.

	°	′	″
Bearing of BC =	41	7	40
see Rule 2, p. 197	360	0	0
	401	7	40
Bearing of BL =	355	46	20
Angle LBC =	45	21	20
Bearing of CD =	97	4	0
	180	0	0
,, CL =	277	4	0
,, CB =	221	7	40
Angle LCB =	55	56	20
LBC =	45	21	20
CLB =	78	42	20
Check	180	0	0

$$\text{Then BL} = 1371{\cdot}4 \times \frac{\sin 55^\circ\ 56'\ 20''}{\sin 78^\circ\ 42'\ 20''} = 1158{\cdot}3 \text{ links}$$

$$\text{and CL} = 1371{\cdot}4 \times \frac{\sin 45^\circ\ 21'\ 20''}{\sin 78^\circ\ 42'\ 20''} = 995{\cdot}0 \text{ links}$$

(*d*) To find the angle IBA

$$\begin{aligned} \text{Bearing of BA} &= 175^\circ\ 46'\ 20'' \\ ,,\quad \text{BI} &= \underline{\ \ 4^\circ\ 11'\ \ 0''} \\ \text{Angle IBA} &= 171^\circ\ 35'\ 20'' \end{aligned}$$

The bearing of BA is found from that of AB, and that of BI is taken from the traverse sheet.

(*e*) To find the distance CT

LT = LB, as the tangents from L to the circle are equal

∴ LT = 1158·3 links

and LC = 995·0, as above

∴ CT = 163·3 links, to be set from C, along CD, in order to fix T

(*Note.* If LC were greater than LT, it would have to be set off the opposite way.)

(*f*) To find the distances AE and GE, first find the bearing of GE

	°	′	
Bearing of FG =	16	32	
Included angle =	58	56	(see Fig. 113)
	75	28	
	180	0	
Bearing of GE =	255	28	
∴ ,, EG =	75	28	

∴ R.B. = 75° 28′, quadrant north-east

and R.B. of AE = 4° 13′ 40″, quadrant north-west

Let AE $= x$, EG $= y$ (see Rule 5, p. 199).

$$\therefore x \cos 4^\circ\ 13'\ 40'' + y \cos 75^\circ\ 28' = 1283{\cdot}0$$
$$\text{and } y \sin 75^\circ\ 28' - x \sin 4^\circ\ 13'\ 40'' = 661{\cdot}8$$
$$\text{whence } x = 1083{\cdot}4, \text{ and } y = 767{\cdot}0 \text{ links}$$

Such problems as these are often solved, without the aid of co-ordinates, by a process of repeated solution of triangles, most of the triangles being solved from two known sides and the included angle.

The method is, in the opinion of the writer, more tedious and less accurate, especially as the triangles are often ill-conditioned. The method in the text is therefore recommended in preference.

ACCURATE TRAVERSES.* In some cases, particularly in town surveys, it is desired to obtain as high a degree of accuracy as may be conveniently possible. In such cases a steel band chain is now, perhaps, most frequently used for the lengths. The hypotenusal allowance is not made on the field, nor are arrows used for the chaining. The chain is 'standardized' (that is, its exact length is determined by laboratory tests) when lying flat on the ground at a known temperature and under a known tension.

CHAINING. In use, it should be stretched by a pull of the same amount as that used in standardizing, and the temperature should be taken for each chain length by one or more thermometers laid beside the tape.

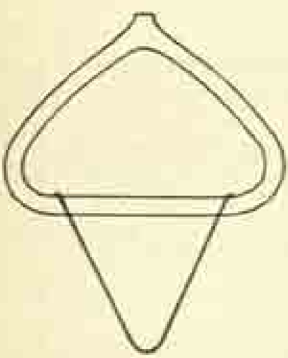


FIG. 114

The most simple method of applying the tension is by a spring balance, hooking into a loop of string attached to the handle of the chain, as shown at A (Fig. 114). This enables the handle of the chain to be firmly pressed down, *almost* into contact with the ground. For better contact, a metal loop may be used instead of the string. Many more elaborate devices have been used, however.

The end of the chain is marked by a fine line ruled on the pavement, or (on soft ground), on a peg or flat piece of board

* See *Precise Traversing* in Volume II

whose top is level with the ground. A good plan is to hold a broad chisel vertically, inside the loop of string (Fig. 114) and with its back touching the handle of the chain. Then slack off the tension, and with a fine pencil rule the line close against the chisel. The *front* of this line marks the end of the chain.

To guard against the error in marking the end, and in adjusting the next chain to the mark, bands up to 10 chains in length are sometimes used.

SAG. If the slope be uniform from end to end of the chain, the band is allowed to lie flat on the ground. But if the ground be undulating or variable in slope, it is obvious that the stretched band will not give a straight line, but will follow the curve of the ground if this be convex, or may hang (for the whole or some portion of its length) clear of the ground altogether, if the latter be sufficiently concave.

This is called *sag*, and, if uncorrected, would lead to a considerable error, and (so that it may be a constant error for each chain) in these cases the chain is usually allowed to hang clear of the ground, being stretched between tripods or pegs which project well above the ground. There may also be intermediate pegs.

A method of using the measuring band, when stretched between tripods, may be described as follows. The back or trailing end of the band is attached to a hook, which can be traversed through a small distance (in the direction of measurement), by means of a screw passing through a lug on a flat plate, which rests on the earth, and is held in position by the foot or knee, or by the use of a lever in the form of a rod, hinged to the above-mentioned plate, and held in position and tightened or slackened by means of a guy-line attached to the rod. The front or leading end of the chain is provided with a similar apparatus, but between this end of the band and the hook a spring balance is introduced.

When the chain is in position (*the two ends being level*), sufficient strain is put on the front end by means of the screw or lever, to show from 10 lb. to 25½ lb. on the spring balance, according to the length and weight of the band used. The screw or lever at the back end is then slackened, or tightened, until the mark on the chain is exactly over the commencement mark on a peg or stone below it. Finally, the strain on the front end is reduced, or increased, until the spring balance records the required strain.

The position of the end of the chain may be transferred to a peg or other mark on the ground by means of a plumb-line, or preferably by the use of a theodolite. A good method of marking the leading end of the tape is by means of a vernier, fixed on the head of a stake, and raised by a screw under the tape. The graduation extends 9 mm. each way, which, being divided into ten parts, reads $\frac{1}{10}$ mm. The screw is provided with a hook, in the centre of its head, to which a plumb-bob may be attached.

The formula for correction for sag is $\frac{L}{24}\left(\frac{wy}{T}\right)^2$ per chain length, where L is the nominal length of the tape in feet, w the weight of the tape per foot length in lb., y distance between the supporting pegs in feet, and T is the tension applied in lb. This formula is arrived at by supposing the tape to hang in a parabola between supports.

Sign. It is obvious that, if any one chain be stretched in this way, the distance between its ends will be less than if it were stretched straight. Hence in measuring a base on this system, *the number of measured* chains will be *too great* in consequence of the sag. Hence to obtain the *correct* number of chains, the correction for sag is always *minus*. The sign of each correction is decided in this way.

Tension. If the pull during use is different from that of standardization, the correction per chain length is

$$(T - T_0) \times \frac{L}{aE}$$

where T is the tension in use, T_0 that of standardization, both in lbs.; a is the sectional area of the tape in square inches. E is the modulus of elasticity of the steel, and may be taken as 30,000,000 lbs. per square inch. If T be greater than T_0, the chain in use will be *longer* than its nominal length; hence the number of measured chains will be *too small*, and the correction is *plus*.

Temperature. In correcting for temperature, it is generally sufficient to take the average of all the observed temperatures (assuming that the system of observation has been uniform throughout) and to correct for this.

Let t_0 = temperature of standardization
t = mean temperature in use
α = coefficient of expansion

Then the correction, per chain length, is $(t-t_0) \times L \cdot \alpha$.

If temperatures are on the Fahrenheit scale, α may be taken as 0·000 007; or 0·000 012 if on the Centigrade scale. This correction is *plus* if t be greater than t_0, for the same reason as before.

INCORRECT LENGTH. If the true length of the chain under standard pull and temperature be not exactly true to the nominal length, it is usual to calculate all the above corrections taking L as the nominal length, which is, of course, a round number.

Then if l be the true length, an additional correction is introduced. The amount, per chain length, is $(l-L)$, *plus* if l be greater than L.

When all these corrections per chain (each with its proper sign) have been summed, the result is multiplied by the number of measured chains to get the whole correction.

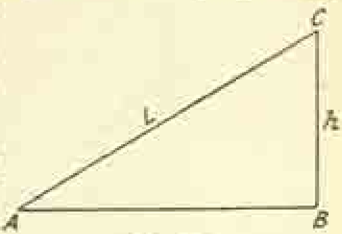


FIG. 115

SLOPE. The level at the end of each chain – or on each peg if there are intermediate supports – is carefully taken.

Then if h be the rise *or* fall in any chain length, to find the true horizontal distance AB (Fig. 115), we have

$$AB^2 = (L^2 - h^2) = L^2\left(1 - \frac{h^2}{L^2}\right)$$

$$\therefore AB = L\left(1 - \frac{h^2}{L^2}\right)^{\frac{1}{2}}$$

Expanding this by the binomial theorem, and neglecting all terms after the second as $\dfrac{h}{L}$ is always a small fraction, we have

$$AB = L\left(1 - \frac{h^2}{2L^2}\right)$$

$$\therefore \text{correction} = AB - AC$$

$$= -\frac{h^2}{2L}, \text{ as } AC = L$$

This correction is always *minus*, because the measured length along the slope is necessarily too great. This result is calculated separately for each chain, by the aid of a table of squares, the results tabulated, and summed to find the correction on the whole length.

DEGREE OF ACCURACY. By working on the above principles it is possible to obtain a result with a probable error of not more than 1 part in 100,000.

COMPASS TRAVERSES. Bearings from magnetic north may, of course, be also measured directly by means of the compass.

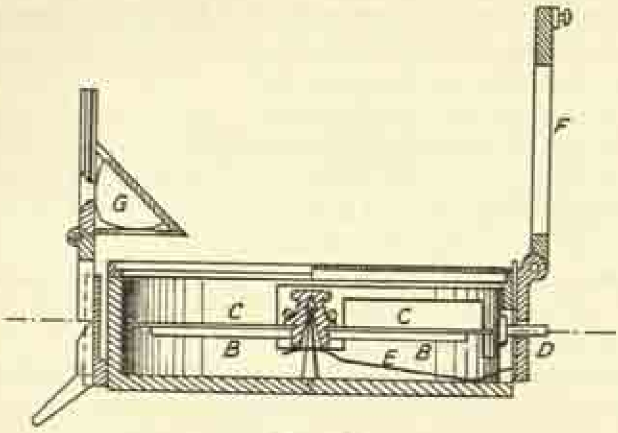


FIG. 116

The most common form is the prismatic compass, of which a section is shown in Fig. 116. It consists of a box from 2 to 4 inches in diameter, with a pivot in the centre, on which the compass needle B revolves. This needle carries a graduated card or metal ring, C, on which the graduations start at zero at the *south* end of the needle, and go thence clockwise, or through west, round to 360°.

At opposite ends of a diameter, the sighting vane F and the prismatic reflector G are fixed to the box. The former consists of a metal frame, in the centre of which is stretched a fine vertical hair or wire.

The prism G acts as a reflector by means of which the graduations on the ring or card below can be read whilst one is looking horizontally into the prism. The faces of the prism are curved as shown, so that it acts as a magnifier as well. Hence the frame carrying it can be raised or lowered so as to bring the graduated scale into exact focus according to the eye of the observer.

Just above the prism, and on the same frame, is a narrow vertical slit, through which the observer looks at the *hair* in the vane F, and then turns the box so as to aim these sights along the line whose bearing is required. For this purpose the instrument must, of course, be set up at a point on the line, usually at the end.

Having taken aim, the observer lowers his eye slightly so as to look into the prism G, and he then reads off the division on the graduated circle which appears to be coincident with the line of the sighting *hair* produced.

It is evident that when we are looking due north, we wish the reading to be zero. But the prism, by which we read, will then be at the *south* end of the needle. Hence the graduations start from zero at that end, and go clockwise, whereby bearings are obtained clockwise from *north*.

The vane F carries a small mirror, which may be used to observe, by reflection, points which are either too high or too low to be observed directly. Both the vane F and the frame G are on hinges, so that they can be turned down when not in use. The former, when pressed right down, say by the cover of the box, automatically actuates a lever E, so as to lift the compass off the pivot, thus avoiding wear of the bearing surfaces. The top of the box, in use, is generally of glass. A metal top fits over this when not in use. At the point D (Fig. 116) is a small brake which is gently pressed to damp the swing of the needle.

CENTRING. The compass, for all but the roughest surveying purposes, is usually mounted on a tripod, so that it can be centred over the station like a theodolite. Being less sensitive, however, it need only be approximately centred.

With six minutes as about the smallest possible reading, this will correspond with an error of about two inches in centring, for a line 100 feet long. With a line 500 feet long, an error of 1 inch in centring could not produce so big an angular error as one minute, and could therefore be altogether neglected.

LEVELLING. The levelling is usually done by eye, by means of a ball and socket joint in the head of the tripod. A clamp is provided to fix the joint, when levelled.

FIELD-BOOK. To use the compass for very rough work, set it up at alternate stations only, the forward bearing being read for one line, and the *back* bearing for the one before.

For plotting the line whose back bearing is measured, we use the measured bearing $\pm 180^\circ$.

For the best work, we set up at each station, and the bearing of *each line* is read both backwards and forwards.

The field-book is then ruled as follows:

Line	Forward bearing	Back bearing	Mean	Remarks
AB	76° 12′	256° 18′	76° 15′	
BC	93° 18′	273° 18′	93° 18′	BX = 185° 48′
CD	167° 42′	345° 12′	167° 42′*	
DE	178° 30′	1° 6′	181° 6′*	
EF	193° 18′	13° 24′	193° 21′	

* see following paragraph

At station A we read the forward bearing of AB. At B we read the back bearing of AB and the forward bearing of BC, and so on.

At B a line BX branches off, it is supposed, and its bearing is entered in the remarks column.

LOCAL ATTRACTIONS. The back and forward bearings should differ by 180° for each line. If they do so nearly, as in the cases of AB and BC, the back bearing is increased or diminished, as the case may require, by 180°, and the mean is then taken between this and the forward bearing.

If, as in the case of CD, a discrepancy be noted when the back bearing is read, this may indicate: (*a*) an error in reading either the forward or back bearing (the latter can be checked at once); or (*b*) some 'local attraction' at D, due to the presence in the neighbourhood of some magnetic substance.

If the latter be the cause, then we would expect about an equal amount of discrepancy for the line DE.

Thus, in the example, the back bearing of CD (in other words, the bearing of DC) *should* be 347° 42′, to agree with the forward bearing. It is measured as 345° 12′, which is about 2¼° too small. Hence, we would expect the forward bearing of DE (which is also measured at D) to be too small by this same amount as compared with the back bearing.

The latter is measured as 1° 6′; hence the forward bearing should be 181° 6′. It is read as 178° 30′, which is too small by 2° 36′, about the required amount.

In this case the readings at D are ignored. We take 167° 42′ for the bearing of CD, while to find that of DE we increase the *back* bearing by 180°, and these results are filled in in the 'Mean' column, from which we plot.

VARIABLE ATTRACTION. If discrepancies of varying amounts be found at several successive stations, probable magnetic attraction at each of them is indicated. In this case it is best to calculate the included angles at each station.

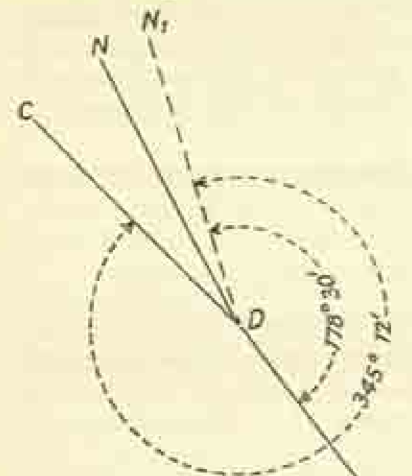


FIG. 117

Thus in Fig. 117, let DN be the true magnetic meridian, DN_1 the direction taken by the needle in consequence of the local attraction. Then the bearings at D are both measured from this line, and, if their values be as shown on the figure, then clearly $CDN_1 = 360° - 345°\ 12'$, and the clockwise angle CDE (*from* the station C)

$$= CDN_1 + N_1DE$$
$$= 360° - 345°\ 12' + 178°\ 30'$$
$$= 193°\ 18'$$

RULE FOR INCLUDED ANGLES. The rule is always the same, namely:

Subtract the *back* bearing of each line from the *forward* bearing of the next, the latter being augmented by 360° if necessary. The included angles being thus known, we can find bearings by the rule on p. 130.

Thus bearing of CD = 167° 42′, and CDE = 193° 18′ as above. Hence bearing of DE = 167° 42′ + 193° 18′ − 180° = 181° 0′, which would be at least as good a result as that given in the table. If the traverse is closed, the included angles as calculated can be summed and corrected before calculating bearings.

The compass, used in this way, is independent of local attractions so long as the latter remain constant at any given station during the period while the instrument is at the station.

MAGNETIC DECLINATION. Information regarding terrestrial magnetism and the average annual value for the magnetic declination at the Royal Observatory, Abinger, is given each year in Whitaker's Almanack from which the following notes have been extracted:

In Great Britain the lines of equal declination, or 'isogonal' lines run approximately from north-east to south-west making an angle of about twenty degrees with the geographical meridians. The estimated average declination at Abinger for 1954 is given as 8° 50′ west and to estimate the value for other localities 15′ are added for every degree of north latitude and 33′ for every degree of west longitude. The average declination at any one place is at present steadily decreasing but the average annual rate of decrease is not constant. The present rate of decrease may be assumed to be 8′.

There are considerable local deviations due to the presence of magnetic minerals and other causes and in addition to the steady decrease there are minor diurnal fluctuations in declination but the latter are too small to be of importance to surveyors.

EXAMPLES FOR EXERCISE

(1) Particulars of a four-sided closed traverse are given in the annexed table.

Line	Length	Difference of latitude		Departure	
		N.	S.	E.	W.
AB	644·3	610·9	—	219·9	—
BC	312·4	82·3	—	—	301·3
CD	576·8	—	438·4	—	374·9
DA	520·4	—	252·6	454·9	—

You are required to compute the corrections for each line–

(*a*) According to Bowditch's method of least distortion (p. 184).

(*b*) Making the correction to any northing, etc., proportional to that northing, etc., on the usual principle.

(*c*) On the assumption that the angles are to be unaltered (p. 188).

Corrections

Ans.

	Difference of latitude				Departure			
	AB	BC	CD	DA	AB	BC	CD	DA
a	−0·7	−0·3	+0·6	+0·6	+0·4	−0·2	−0·4	+0·4
b	−1·0	−0·1	+0·7	+0·4	+0·2	−0·3	−0·4	+0·5
c	−0·8	−0·2	+0·5	+0·7	−0·3	−0·8	+0·4	+1·3

(2) The particulars of a prismatic compass traverse on ground subject to irregular magnetic attraction are given in the table.

Line	Bearing		Length (feet)
	Forward	Back	
	° ′	° ′	
AB	85 24	268 42	536
BC	164 54	340 0	316
CD	261 6	78 42	634
DE	344 24	170 36	140
EA	8 36	186 30	200

Calculate the included angles at each station, check them, distribute the closing error, and, taking the forward bearing of AB as correct, calculate co-ordinates for the traverse, distributing the closing error by method (*b*) of Q. 1. Take the co-ordinates of A as 400 N. and 100 E. to start with.

Ans.

	A	B	C	D	E
N.	400	443	144	64	201
E.	100	634	734	105	77

The corrected bearings are 85° 24′; 161° 37′; 262° 44′; 348° 27′; 6° 29′.

(3) A straight tunnel is to be driven between two points A and B. It is impossible to range out (or measure) this line directly on the surface of the ground in

consequence of obstacles. A very accurate traverse is therefore run, with the results shown in the table.

Line	Point	Length (feet)	Angle
			° ′ ″
AC		681·14	
	C		153 2 15
CD		986·37	
	D		169 46 52
DE		973·24	
	E		149 17 12
EB		765·81	

Taking A as the origin of co-ordinates and the bearing of AC as zero, calculate (*a*) the co-ordinates of C, D, E, and B; (*b*) the length of AB, and the angles CAB, EBA, required for setting out its direction.

It is desired to sink a shaft near the middle of AB. It is found that a suitable line, DF, for measurement can be run from D, and the angle CDF is observed 87° 37′ 20″. If F be the point where DF meets AB in plan, calculate the distances DF and AF, and the angle DFA.

Ans.

	C	D	E	B
N.	681·1	1560·3	2335·7	2623·9
W.	0	447·3	1035·5	1745·3

AB = 3150·8, CAB = 326° 21′ 32″, EBA = 33° 17′ 13″
DF = 493·5, AF = 1583·7, DFA = 85° 41′ 57″

CHAPTER 5

CURVE RANGING

NOMENCLATURE. In England railway curves are defined by the lengths of their radii in chains of 66 feet or 100 links, while road curves are usually defined by their radii in feet, the 100-foot or 'engineer's' chain being used in setting out. In some other countries a similar nomenclature is followed, the chain used being either 20 metres (65·618 feet) or 100 feet in length.

DEGREE CURVES. In the United States, Canada and certain other countries, curves are defined numerically by the magnitude of the angle subtended at the centre either by an arc or a chord 100 feet long. This angle is termed the 'degree of curvature'. Railway engineers generally use the arc definition and road engineers the chord definition, and, in practice, except for curves of short radius, there is little difference between the two. Since the length of an arc, L, is given by $R\theta$, where R is the radius and θ the number of radians subtended at the centre, and since one radian is, to a close approximation, $57{\cdot}3°$, we may write

$$R = \frac{\frac{50}{N}}{57{\cdot}3}$$

where N is the degree of curvature based on an arc 100 feet long. Thus a $3°$ curve, by the arc definition, has a radius of $\frac{5730}{3}$ or 1910 feet. By the chord definition, $R = \frac{50}{\sin\frac{N}{2}}$ and for a $3°$ curve this becomes $\frac{50}{\sin 1°\ 30'}$, or very nearly 1909 feet.

LENGTH OF CHORD. In setting out curves in practice the lengths of the chord and arc are nearly always assumed to be equal, but, for this to be true within the necessary limit of accuracy, the length of the chord must never exceed one-twentieth of the radius. Curves of less than 10 chains radius should be set out with ¼-chain chords, curves of 10 to 20 chains radius with ½-chain chords and curves of over 20 chains with 1-chain chords.

CIRCULAR ARCS

We will first consider simple circular arcs, although, as will be seen later, other forms of curve are used in railway and road construction.

DEFINITIONS. Suppose a line, following originally the straight line AP (Fig. 118) is to be deflected round to follow PB. Then AP, PB (or more strictly the parts of those lines produced backwards, are called the *straights* or *tangents*. These lines would normally form part of the centre line of a road or railway. The point P is called the *apex* of the curve, and the angle BPA is called the *apex angle*. It is usually understood as the *interior* angle between the straights, and hence may be measured *either* clockwise or anti-clockwise from the back station, unlike a traverse angle. It is therefore necessary to state the sense of the angle. It is usual, however, to base all curve calculations on the *deflection* or *deviation* angle, defined below, and not on the apex angle.

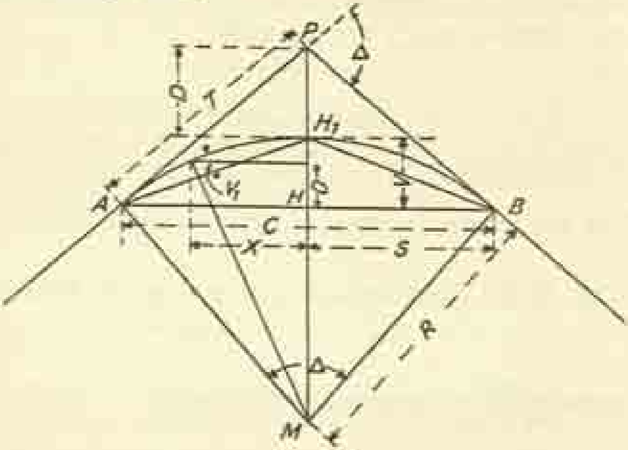


Fig. 118

The points A and B, where the curve begins and ends, are called the *tangent points*; usually in English practice A is called the starting point, and B the finishing point. In America A is often called the *point of curve* and B the *point of tangent*. The distance, PA or PB, from the apex to either tangent point is called the *tangent distance* or *tangent length*.

The angle between one tangent produced and the other tangent is called the *deflection* or *deviation* angle. In Fig. 118 this is the exterior angle APB and is often denoted by Δ. It is clearly equal to the angle AMB subtended at the centre of the curve.

The deflection is *right* or *left*, as follows:

Standing at the centre M, look towards the apex. Then, if the finishing point is on the *right*, the deflection is *right*, and vice versa. Thus in the figure the curve is one of *right* or *left deflection* according to whether B or A is the finishing point respectively.

Thus with right deflection the apex angle is measured *anti-clockwise* from the back station (that is, from the starting point of the curve to the finishing point, or A to B in the figure), and vice versa. Hence the sign of the apex angle is usually stated by naming the direction of the deflection.

If from the original direction, along AP produced, we wish to turn to follow PB, we must clearly turn to the *right*. This is another way of deciding the direction of deflection.

From the figure, *deflection angle* $= 180° -$ *apex angle*.

The chord AB is called the *long chord*.

The angle PAB between the tangent AP and this chord is called the *tangential angle* for the whole curve.

$$\begin{aligned} \angle PAB &= 90° - \angle BAM \\ &= \angle AMP = \tfrac{1}{2}\angle AMB \end{aligned}$$

that is, *total tangential angle* = *half deflection angle*

Similarly, if AH_1 be any other chord, PAH_1 is the tangential angle for that chord.

These tangential angles are sometimes loosely called deflection angles. The student should guard against this practice, as being likely to confuse beginners. It is easy to prove that the tangential angle for any chord is *half* the deflection angle for the same chord.

It must be borne in mind that, before any curve is actually pegged out on the ground, the straights approaching it are set out and cannot, as a rule, be altered in direction without interfering with the junction to some other curve already set out. Under these circumstances it is obvious that, though they may be known approximately, neither the exact point of intersection of the tangents, the lengths of the tangent lines, the positions of the tangent points on the straights nor the radius of the curve has been determined.

As a rule the radii of all curves are selected in the first instance on the plans or maps (of which there may be one or more showing alternative routes), in order to ensure the most favourable location, from the points of view of cost of construction, easy working, and utility of the railway or road.

Speaking generally, in the final location on the ground the radius of the curve as laid down on the plan is adopted, the tangent points being altered to suit. There are, however, cases, in populous districts, in very sidelong ground and in tunnel, where the start and/or the finish of the curve must be adhered to, in which cases it will probably be preferable to alter the radius slightly and to maintain the tangent points as set out.

The apex is usually accessible in open country, but in fully populated or very rugged districts it may be inaccessible, or inconvenient to work from. We must consider these cases separately.

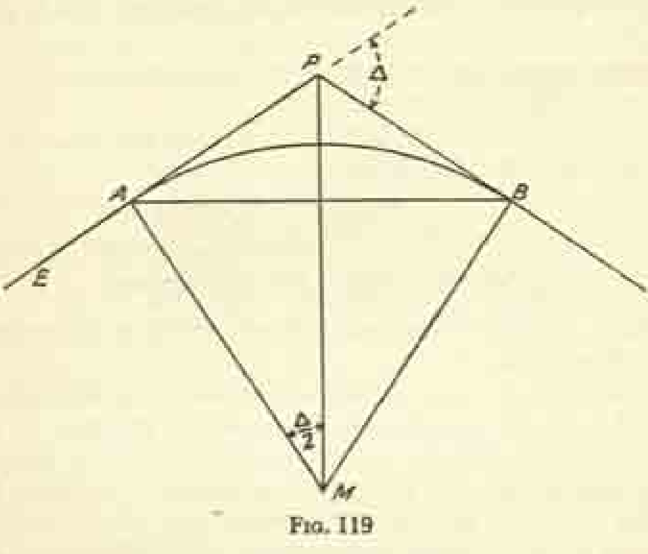


FIG. 119

GENERAL PROBLEM. In any case, therefore, we know either the radius or the starting point on the ground, and the first step is to find whichever of these two is not given.

First suppose the apex is accessible. Set up a theodolite there, and measure the apex angle between the straights already marked out by pegs.

Now in Fig. 119 –

(*a*) Let the radius AM be known.

Then $$PA = AM . \cot APM$$
or *tangent distance = radius × cot half apex angle*
alternatively $$PA = AM . \tan AMP$$
or *tangent distance = radius × tan . half deflection angle*

This is chained off from P to fix A and B

(*b*) Let the starting point, A, be known

Then measure PA, and we have –

$$AM = PA . \tan APM$$
or *radius = tangent distance × tan half apex angle*
alternatively $$AM = PA . \cot AMP$$
or *radius = tangent distance × cot half deflection angle*

The measured length PA is chained off along PB to fix B.

Secondly, suppose the apex is inaccessible or inconvenient to work from.

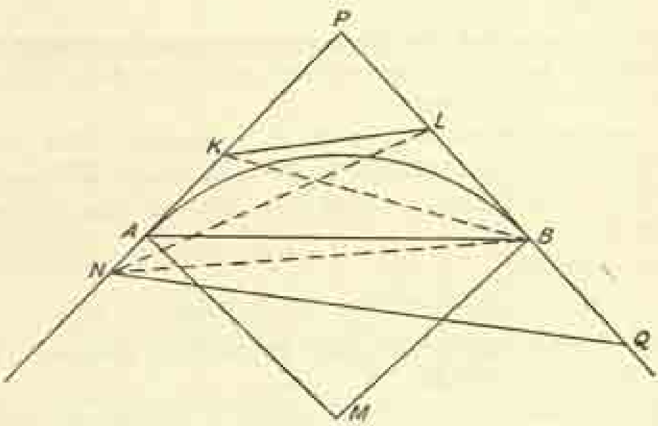


FIG. 120

Then, referring to Fig. 120, the first step is, if possible, to choose two stations, say K, L, one on each straight, visible from one another, and such that the straight line between them can be accurately measured.

The theodolite is set up at K and L, and the angles LKA, BLK are measured. KL is also carefully chained.

Then $\quad PKL = 180° - LKA$

$\quad KLP = 180° - BLK$

and $\quad LPK = 180° - (PKL + KLP)$

Hence we have –

$$\textit{apex angle} = LKA + BLK - 180°$$

Now, knowing KL and the three angles, we can solve the triangle KPL to find PK and KL.

Thus $\quad KP = \dfrac{KL . \sin KLP}{\sin LPK} \qquad PL = \dfrac{KL . \sin PKL}{\sin LPK}$

Then, (*a*) suppose the radius is given.

Find the tangent distance as before, viz.

$$AP = \textit{tangent distance} = \textit{radius} \times \textit{tan half deflection angle}$$

Next take the difference between this result and KP, and set it off from K, towards P if KP be greater than AP, but away from P if KP be less than AP. Thus the starting point A is fixed. The finishing point B is found in the same way, from L, remembering that PB = AP.

Of course the chosen line might have been in the position NQ, or NL, instead of KL, so the direction in which each difference is to be set out must be considered.

(*b*) Suppose the starting point, A, is known. Then measure AK. This is added to or subtracted from the length KP (according to whether K is between A and P or not) to find the tangent distance AP.

Then, as before

$$\textit{radius} = \textit{tangent distance} \times \textit{cot half deflection angle}$$

The difference between PL and the tangent distance is set off from L, as before, to fix B.

If *no* cross line can be found which is suitable for accurate measurement, but the point L (or B) is visible from K and N, we may be able to measure KN, and all the angles of KNL (or KNB), and so find the length KL or KB, indirectly.

In other cases it may be possible to measure lines across with fair accuracy, but impossible to read with the theodolite in consequence of moderately high ground between.

In these cases a straight line can be ranged across, with the theodolite set up on the high ground, as described on p. 226.

Finally, if we can neither see nor measure directly any single cross line, we can generally deal with the problem by traversing, as in the example on p. 202.

Where even this cannot be done, we must resort to tacheometry or triangulation, which latter is described in Vol. II. of this work.

METHOD OF TANGENTIAL ANGLES WITH ONE INSTRUMENT. Assuming that the radius is known and that the starting and finishing points of the curve have been set out, the next step is to locate a series of points round the curve.

Let A (Fig. 121) be the starting point, and AH any chord whose length is preferably less than $\frac{1}{20} \times$ radius.

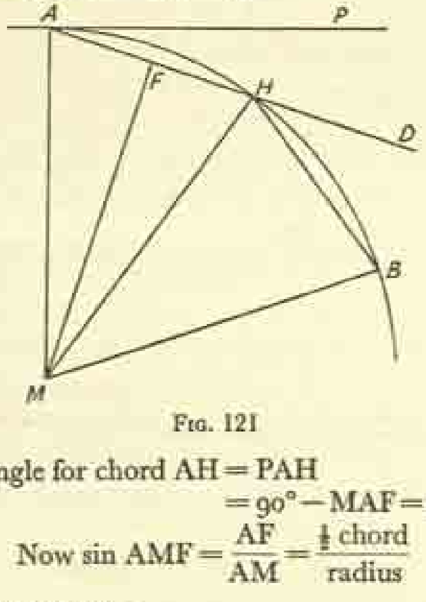


FIG. 121

$$\text{Tangential angle for chord AH} = \text{PAH}$$
$$= 90° - \text{MAF} = \text{AMF}$$
$$\text{Now } \sin \text{AMF} = \frac{\text{AF}}{\text{AM}} = \frac{\frac{1}{2}\text{ chord}}{\text{radius}}$$

Hence we have the rule –

$$\textit{Sin tangential angle} = \frac{\frac{1}{2}\textit{ chord}}{\textit{radius}}$$

This angle can therefore be readily found for the chosen chord.

Now let the theodolite be set up at A, directed along AP with the vernier clamped at zero, and then turned until the same vernier reads the calculated tangential angle. Clearly the telescope is now pointing along AH. If, therefore, the follower be directed to hold one end of a chain or steel tape at A, while the leader swings the chosen chord distance round A as centre (as directed by the surveyor at the instrument) until the arrow held by him comes into line with the reference line of the theodolite, it is obvious that the arrow will then mark the point H on the curve.

Now suppose HB is a second chord of the same length.

$$\text{Clearly } \angle AMB = 2 \times \angle AMH$$
$$\text{But } PAB = \tfrac{1}{2}\,AMB \text{ and } PAH = \tfrac{1}{2}\,AMH$$
$$\therefore PAB = 2 \times PAH$$

In other words, for *two* chords the whole tangential angle will be double that for one chord, and so on.

Thus we set the vernier to read double the original angle (the theodolite of course remaining at A), send the follower to H, and direct the leader to swing his end of the chord as before (round H as centre) to fix B. For the third point, the angle is three times the original angle, and so on.

Length of curve. It has been shown (p. 219) that the tangential angle for the whole curve $= \tfrac{1}{2}$ deflection angle, and that the deflection angle $= 180° -$ apex angle.

$$\text{Hence } \textit{total tangential angle} = 90° - \textit{half apex angle}$$

But each chord set out means one tangential angle as found above. Hence to find the total length of curve, expressed in chords, we have –

$$\text{Length of curve in chords} = \frac{90° - \tfrac{1}{2}\text{ apex angle}}{\text{tangential angle for 1 chord}}$$

That is, the number of chords in any curve is given by the expression

$$\frac{5400 - \tfrac{1}{2}\text{ apex angle in minutes}}{\text{tangential angle in minutes}}$$

We may also express this rule as follows:

$$\text{Length of curve in chords} = \frac{\tfrac{1}{2}\text{ deflection angle}}{\text{tangential angle for 1 chord}}$$

Subchords. Now, it will seldom happen that this result will give an exact number of chords. Hence, to check the curve on to the finishing point B (Fig. 122) already fixed, the last chord must be fractional.

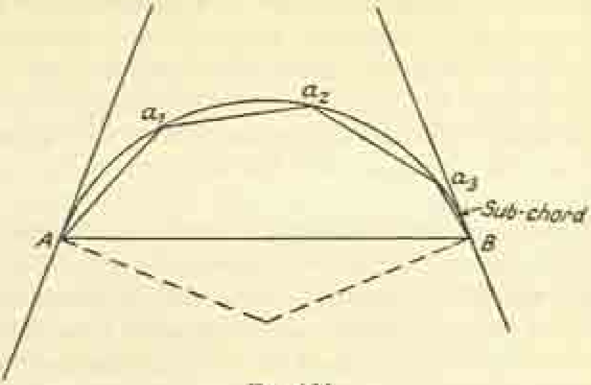


Fig. 122

If a_3 be the end of the last complete chord, the angle to that point is known. Also the whole tangential angle to B is known.

Let the difference be T_1 = tangential angle for chord a_3B.

Also let T = tangential angle for 1 complete chord.

$$\text{Then, very nearly, } a_3B = \text{complete chord} \times \frac{T_1}{T}$$

If the length so found be swung round a_3 as centre until the leader comes into line with the theodolite set to ½ deflection angle, then we should check exactly on to the finishing point, B. Again, it is sometimes desired to insert the pegs at the end of each complete chain from *the commencement of the line*.

Now, the starting point of any individual curve will not generally fall precisely at the end of a chain. Hence the *first* chord may also be fractional, so that the peg a_1 may be placed at the desired point. Thus if the through chainage of A be 10 miles 57·22 chains, and the whole length of chord to be used is ½ chain, then a_1 is to be placed at 57·50 chains. Hence the first chord is 0·28 chain or 28 links.

Now suppose the tangential angle for ½ chain chord is 57·3 minutes (corresponding to a radius of 15 chains), then the angle for 28 links is found by simple proportion, as it is small.

$$\text{Hence first angle} = 57{\cdot}3 \times \tfrac{28}{50} = 32{\cdot}1 \text{ minutes}$$

The first point is fixed by setting this angle, and swinging 28 links round A. The succeeding angles are found by the repeated addition of 57·3 minutes.

PRECAUTIONS. To obtain a good check on to the finishing point, all the preliminary measurements must be made with the greatest care (certainly to a small fraction of a link or foot) the angles must be checked by repetition or reiteration, the starting and finishing points (and other points which may have to be fixed on the straights) must be ranged in with great care, preferably using the theodolite, and the instrument must always be centred exactly, the starting point being marked for this purpose by a peg with a nail in the centre.

As in high winds the plumb-bob is liable to considerable oscillations, and departure from the vertical occurs even when protected by screens, an optical centring device will be found very useful. This consists of a telescope provided with cross lines by the inclusion of a web or glass diaphragm and a diagonal eyepiece, so that the head of the peg and of the central nail driven into it can be observed. The cross lines can be brought to coincide with the central mark and must remain coincident with that mark while the theodolite is rotated about its vertical axis, i.e., the theodolite must be level.

In open country great refinement of position is unnecessary, but in populous districts, in tunnelling and elsewhere, even greater accuracy than that of a peg with a nail driven into it may be required, and the use of a metal plate, finely marked, and set in masonry or concrete, may be necessary.

RANGING LINES WITH THE THEODOLITE. It is clear that if the apex P be inaccessible there may be only a short length of tangent beyond A available for setting to zero when the curve is to be laid out. Hence the back portion of the straight must be utilized.

FIG. 123

Let AN (Fig. 123) be a straight line which is to be prolonged, using a theodolite at N.

Assuming a perfect instrument, it is clear that we can proceed by either of two methods.

(1) Set the theodolite to read zero on A, and then turn the upper plate until the vernier reads 180°, when the telescope will give the requisite direction NP.

If it be desired to read zero along NP (as in the case of curve ranging), then we set to 180° on A.

(2) Set the theodolite to read zero on A, and then transit the telescope; that is, turn it over vertically, leaving the horizontal plates clamped.

Now, in dealing with the adjustment of the theodolite we saw that the result of the first method would be affected by errors of graduation and eccentricity, both of which are small in most modern instruments. These can be eliminated by reading both verniers, as we thereby cover the whole circle. Thus suppose we set vernier No. 1 to read 180° on A from N, and that No. 2 vernier then reads 359° 59′.

Now turn the upper plate until No. 1 vernier reads zero, and say No. 2 gives 180° 0′ 30″. Then vernier No. 1 gives the angle turned through as 180°, while No. 2 gives it as 180° 1′ 30″. We infer that the actual angle has been 180° 0′ 45″, and that to get a true angle of 180° we must set this reading (180° 0′ 45″) on A with vernier No. 1, and then turn the theodolite till the same vernier reads zero. The corresponding readings for No. 2 should be 359° 59′ 45″ and 180° 0′ 30″.

If the angles of depression to A and P from N be the same, any error due to slope of the trunnion axis or faulty collimation will cancel out. If not, the effect of these errors cannot be eliminated by readings with one face only, but the resultant effect will be that due to the *difference* between the angles of depression only, and will probably be very small.

Torsion of the axis or tripod is sometimes given as a disadvantage of this method. But, if the whole be fairly rigid, there is probably no greater torsional effect in rotating the instrument horizontally than in transiting the telescope.

In the second method we are independent of graduation or eccentricity errors, but the effect of any error due to slope of axis or collimation is *doubled*, as will be clear to any one who has mastered the principles of the adjustments of the theodolite. It cannot be eliminated by reading with one face only.

If, however, we set on A, clamp, transit the telescope (i.e. turn it over vertically), and fix the point P_1 apparently in line; then reverse face (see p. 166) and repeat the process, obtaining the point P_2, we may conclude that the point P, midway between these, gives the true prolongation of AN as well as it can be obtained from N. For the best work, such a process should always be used.

If the line is to be prolonged by observations from A, we set on N, clamp, fix P_1 in line; then reverse face and repeat the process. The only difference is that the telescope has not to be transited except in reversing face.

The general conclusion is that if we can use both faces of the instrument, the second method is the more convenient and, perhaps, accurate, and should be used for ranging a line like KL (Fig. 120) over high ground. But if we are restricted to one face (as in the case under consideration) (or in shifting the theodolite as described on p. 231, or wherever we wish to carry straight on with the angular readings from the prolonged line), the first method is likely to give a better result, namely, set to 180° on the back station N, giving zero on P; and check by the second vernier.

Approximate rule for tangential angles. If the chord be not greater than $\frac{1}{20}\times$ radius, it has been stated that it is practically of the same length as the corresponding arc.

Now if c be the angle at the centre subtended by any arc, it is clear that

$$\frac{c}{360^\circ}=\frac{\text{length of arc}}{2\pi\times\text{radius}}$$

$$\therefore c=\frac{180^\circ}{\pi}\times\frac{\text{length of arc}}{\text{radius}}=\frac{180^\circ}{\pi}\times\frac{\text{chord}}{\text{radius}}$$

if we assume chord = arc

Now tangential angle for any chord = half angle subtended at centre by the same chord, as we have shown.

$$\text{Hence tangential angle}=\tfrac{1}{2}c=\frac{90^\circ}{\pi}\times\frac{\text{chord}}{\text{radius}}$$

$$=\frac{5400}{\pi}\times\frac{\text{chord}}{\text{radius}}\text{ in minutes}$$

$$=1719'\times\frac{\text{chord}}{\text{radius}}\text{ nearly}$$

This rule should not be used where the radius is much less than 20 × selected chord, except as a rough check.

EXAMPLE. As an example we will take the following figures. The letters are as shown in Fig. 120 although the values of the angles differ from those indicated in the diagram.

Curve No. 9; radius 25 chains; chainage of K from starting point = 6 miles 67·30 chains; KL = 6·632 chains; angle LKN = 172° 30′ 30″; QLK = 168° 3′ 30″. Deflection right. Chord = 1 chain.

To find the apex and deflection angles.

$$\begin{array}{rrrr} & ° & ' & '' \\ & 172 & 30 & 30 \\ & 168 & 3 & 30 \\ \hline & 340 & 34 & 0 \\ \text{less} & 180 & 0 & 0 \\ \hline \text{Apex angle} = & 160 & 34 & 0 \end{array} \qquad \text{Deflection} = 19°\,26'$$

$$\text{Tangent distance} = 2500 \times \tan \frac{19°\,26'}{2} = 428{\cdot}08 \text{ links}$$

$$\text{KLP} = 180° - 168°\,3'\,30'' = 11°\,56'\,30''$$
$$\text{and PKL} = 7°\,29'\,30''$$

$$\text{KP} = 663{\cdot}2 \times \frac{\sin 11°\,56'\,30''}{\sin 160°\,34'\,0''} = 412{\cdot}45 \text{ links}$$

(*Note.* As the apex angle is over 90°, we look up and use in the calculation the sine of its supplement, namely, 19° 26′.)

$$\text{Similarly LP} = 663{\cdot}2 \times \frac{\sin 7°\,29'\,30''}{\sin 19°\,26'\,0''} = 259{\cdot}9 \text{ links}$$

$$\text{Hence KA} = 428{\cdot}08 - 412{\cdot}45 = 15{\cdot}63 \text{ links}$$
$$\text{and LB} = 428{\cdot}08 - 259{\cdot}90 = 168{\cdot}18 \text{ links}$$

to be set off from K and L, both *away from* P, to fix A and B.

$$\therefore \text{chainage of starting point A} = 6 \text{ mi. } 67{\cdot}30 \text{ ch.} - 15{\cdot}63 \text{ links}$$
$$= 6 \text{ mi. } 67 \text{ ch. } 14{\cdot}37 \text{ links}$$

$$\sin \text{tangential angle} = \frac{\text{half chord}}{\text{radius}} = \tfrac{50}{2500} \text{ for 100 link chord}$$

$$\therefore \text{tangential angle} = 1°\,8'\,46'' \text{ nearly}$$

$$\text{Check by approximate rule} = 1719 \times \tfrac{1}{25} = 68{\cdot}76', \text{ which checks}$$

$$\text{Length of curve} = \frac{\text{half deflection angle}}{\text{tangential angle}} = \frac{9°\,43'}{1°\,8{\cdot}76'}$$
$$= 8{\cdot}479 \text{ chords}$$

The first chord must be 85·63 links to make the chainage up to the next complete chain, viz. 6 mi. 68 ch.

$$\text{Angle for this chord} = \frac{85{\cdot}63}{100} \times 68{\cdot}76 = 58{\cdot}87'$$

The angles are tabulated on p. 230.

Arrived at No. 8 point, we see that one more complete angle would carry us past the final angle, which is to be half deflection angle or 9° 43′.

Hence we find by subtraction the exact angle required to make up 9° 43′, viz. 42·81′ as shown.

The corresponding chord is $100 \times \frac{42 \cdot 81}{68 \cdot 75} = 62 \cdot 3$ links.

This chord is used, with the angle 9° 43′, to check on to the finishing point B.

The sum of the chords is found as shown at the foot of column No. 2. It must check with the length of the curve already found. This checks the arithmetic.

All the angles must be tabulated and checked in this way before *any* are set out.

No.	Chord	Angle	Chainage	Remarks
		° ′		
1	85·6	0 58·87 1 8·76	6 mi. 68 ch.	Curve No. 9
2	100	2 7·63 1 8·76	69	
3	100	3 16·39 1 8·76	70	
4	100	4 25·15 1 8·76	71	
5	100	5 33·91 1 8·76	72	New station
6	100	6 42·67 1 8·76	73	
7	100	7 51·43 1 8·76	74	
8	100	9 0·19 0 42·81	75	
9	62·3 847·9	9 43·0	6 mi. 75 ch. 62·31	

The chainage, as will be clear, is carried round the curve, and the last line gives the chainage of the finishing point, from which we carry on the chainage along the next straight.

The chainmen are instructed as to the lengths of the first and last chords before beginning to set out the curve.

As each point is laid down its angle is ticked off in the table.

CHECKING BY OFFSETS. It is shown later that if the straight line joining any two points on the curve one full chord apart be *prolonged* for another chord's length, the offset inwards to the next point on the curve will be given by the formula $\frac{\text{chord}^2}{\text{radius}}$ (see p. 234).

The leader should be informed of this amount. Thus as soon as two points one full chord apart are fixed, he can get himself in line with them after stretching the chord, and then move inwards the calculated amount. The arrow can thus be put nearly in the correct spot, while the surveyor is changing the setting of his instrument. This not only saves time, but it is clear that, if the surveyor should signal the leader far from the point so fixed, there has probably been an error in the setting of the circle, and the leader should signal to that effect. The formula for offsets for sub-chords is given on p. 236.

MOVING THEODOLITE. Now suppose that, having set out any given point on the curve, say X (Fig. 124), we are unable to see any further from the starting point. The theodolite must then be moved to X to continue the curve.

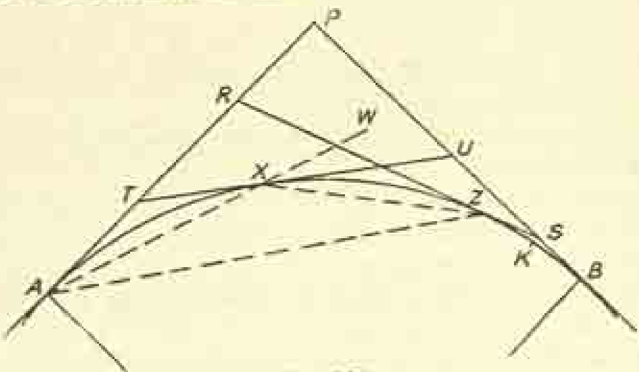


FIG. 124

Let TXU be the tangent at X. Then $\angle$TAX is the tangential angle which was set out to fix X.

$$\text{But } \angle AXT = \angle TAX$$

Hence if X be the nth point on the curve, and t_n be the corresponding angle in the table,

$$\angle AXT = t_n$$

Now adjust the theodolite at X, and set the vernier to read 180° on A. Then turn the upper plate until the same vernier reads the *next angle in the table* (the $(n+1)$th), and the telescope will be in position for fixing the next point, Y, omitted from the diagram for clearness.

Proof:
$$\begin{aligned}\text{reflex } \angle AXU &= \angle AXT + \angle TXU \\ &= t_n + 180^\circ\end{aligned}$$

Hence if the reading along XA be 180°, that along XU will be 360° + t_n, or t_n.

Now as XU is the tangent, and XY the usual chord, we must add one tangential angle to the reading at U to get that at Y.

Clearly the result will be the next angle in the table.

Thus in the table on p. 230, if X be the fifth point, we set to 180° on A, and then bring the vernier to read 6° 42′ 30″ (which is the *sixth* angle to the nearest half-minute) to fix Y. Then continue as before. Note in the remarks column the point at which the instrument was moved, as shown.

The whole angle turned through from A to Y is 186° 42′ 30″. Check this by the two readings of the second vernier.

Alternatively, prolong the line AX to W as described on p. 226 and set to zero on W.

An alternative method is to set to zero on A, and then transit the telescope. This should not be done unless the instrument is known to be in very good adjustment.

Now suppose that, after fixing the *m*th point, Z, a second shift becomes necessary.

Adjust the instrument at Z. Set the vernier to read 180° + t_n (where t_n is the tangential angle to the previous station X) and direct to X by unclamping the lower plate. Then turn the upper plate till the same vernier reads the next angle in the table, viz. the $(m+1)$th, to fix the next point.

Proof: Let RZS be the tangent at Z and let O be the centre of the curve

$$\text{Then } \angle RAX = \tfrac{1}{2}\angle AOX = \text{any angle in segment AXZ}$$
$$\therefore AZX = RAX = t_n.$$

But the reading on X is 180° + t_n

Hence the reading on A will be 180°

Hence, by the same proof as before

$$\text{reading on K} = \text{next angle in table}$$

Alternative methods can be used as before.

LEFT DEFLECTION. If the deflection be *left*, as in Fig. 125, it is clear that the tangential angles must be set off *anti-clockwise* from AP. Hence the readings will gradually decrease, and, in constructing the table of angles, we must begin with 360°, and find the successive readings by repeated subtraction. The final reading will be

360° − half deflection angle. Otherwise the work is the same as before. The preceding formulæ are equally applicable where a 20-metre or a 100-feet chain is used.

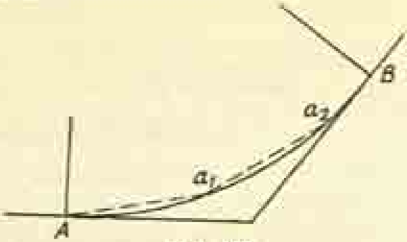


FIG. 125

DEGREE CURVES. A 1° curve is a curve in which a chord of 100 feet subtends an angle of 1° at the centre. The idea is that the tangential angle shall be an exact number of degrees or half degrees. The value of this is largely discounted if it be desired to make the first chord fractional, as described, though it still saves some arithmetic in drawing up the table.

It is quite clear that if the curve be one of $n°$, the tangential angle will be $\frac{1}{2} \times n°$, for a chord of 100 feet.

$$\text{But we have seen sin tangential angle} = \frac{\frac{1}{2}\text{ chord}}{\text{radius}}$$

$$\text{whence } \textit{radius} = 50 \times \textit{cosec}\ \frac{n°}{2}\ \textit{feet}$$

Having found the radius by this rule, we proceed as before, except that the tangential angle is known.

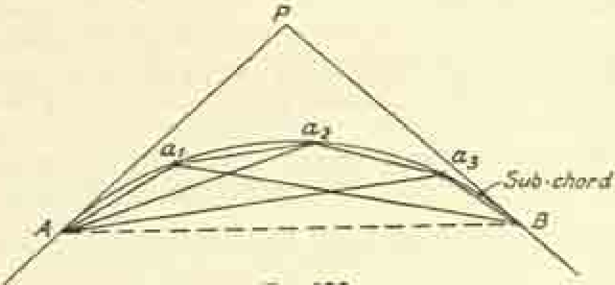


FIG. 126

METHOD OF TANGENTIAL ANGLES WITH TWO INSTRUMENTS WITHOUT CHAIN. Let the two theodolites be set up, one at A and the other at B (Fig. 126). Clamp the upper plates of both instruments to read zero or 360°.

Bring the vertical reference line or intersection of the cross-lines of each to coincide with the respective tangent directions and clamp the lower plates.

Then,

$$\text{angle } PAa_1 = ABa_1$$
$$\therefore PAa_1 + a_1BP = ABP$$
$$= \tfrac{1}{2} \text{ deflection angle}$$

That is, for any point on the curve, the sum of the tangential angles at A and B will be equal to the half deflection angle.

Hence, if the reading at A to a_1 be t_n, and D be the deflection angle, then the reading from B to the same point will be 360° $-(\frac{1}{2}D - t_n)$ or $360° + t_n - \frac{1}{2}D$ as it is set off anti-clockwise.

Hence, having drawn up the table of angles as on p. 230, we insert a new column in which the angles are found by this formula. Thus in the table on p. 230, the half deflection angle is 9° 43′, and the first angle at B will be $360° + 58{\cdot}87' - 9° 43'$, or 351° 15·87′. This would be entered in the new column (which might be headed 'Angle at finishing point') opposite No. 1 point.

The second angle would be $362° 7{\cdot}63' - 9° 43'$, and so on.

Each theodolite is set to its proper reading, and the assistant is directed by both observers until an arrow held by him comes into line with both instruments.

In open country, or where the theodolite must be shifted frequently, the use of one instrument and a chain is to be preferred, but where chaining is inconvenient, the use of two theodolites will generally be preferable.

SETTING OUT CURVES WITH CHAIN AND POLES

Extended chord method. Choose a chord length, l, suited to the radius, R, of the curve, preferably making the chord not more than $\frac{R}{20}$. Then a perpendicular offset from the tangent to the end of this chord will have a length $\frac{l^2}{2.R}$. Thus, for a 20-chain curve a chord length of one chain, or 66 feet, might be used and the length of the offset would be $\frac{66^2}{2640}$ or 1·65 feet. The chord length is measured from A (Fig. 127), and the chain or tape is swung round this point as centre until the offset *ba* measured perpendicular to the tangent has the required value. A folding 5-foot rod will

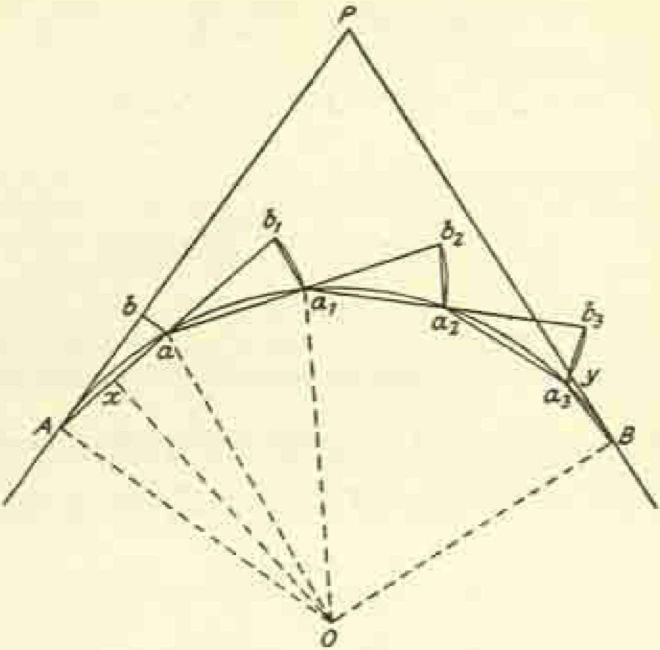


Fig. 127

be found convenient for the offset measurement, and an arrow thus located will be the first point on the curve and must be replaced by a peg, shown at *a* in Fig. 127. Continue the line A*a* to b_1, extending the chord its own length, and insert an arrow. The chord length is now swung about *a* as centre until the displacement b_1a_1, measured as a chord, has a length $\frac{l^2}{R}$, thus locating the point a_1 on the curve. It should be noted that b_1a_1 is *not* perpendicular to ab_1, but ab_1a_1 is an isosceles triangle. Next extend the line aa_1 its own length to b_2 and proceed as before until the last full chain length is reached. The setting-out may be checked by measuring the offset a_3y perpendicular to the tangent PB. This distance should be $\frac{z^2}{2 \cdot R}$ where z is the length of the sub-chord a_3B.

Proof of formulae: Let O be the centre of the curve. Bisect the chord Aa at x. Then $\angle bAa = \angle AOx$. Therefore right-angled triangles AOx, Aba are similar.

Hence
$$\frac{ba}{Aa} = \frac{Ax}{AO}, \text{ or } ba = \frac{l^2}{2.R}$$

Again, in triangles

$$aa_1O,\ ab_1a_1,\ \angle b_1aa_1 = 180° - (\angle a_1aO + \angle AaO)$$
$$= 180° - 2\angle a_1aO = \angle aOa_1.$$

Therefore the triangles are similar.

Hence
$$\frac{a_1b_1}{aa_1} = \frac{aa_1}{aO}, \text{ or } a_1b_1 = \frac{(aa_1)^2}{aO} = \frac{l^2}{R}$$

BY ORDINATES FROM CHORDS. Referring back to Fig. 118, p. 218, if the long chord AB be laid out, and H be its middle point, we can calculate for any abscissa X, the ordinate O to any point on the curve.

Thus if R be the radius, C = length of AB, HH_1 = versed sine, $V = R - \sqrt{R^2 - (\frac{1}{2}C)^2}$.

Then if we join the upper end of the ordinate O to the centre M, we have

$$R^2 = X^2 + (MH + O)^2$$
$$\therefore MH + O = \sqrt{R^2 - X^2}$$
$$\text{or } O = \sqrt{R^2 - X^2} - (R - V)$$

Thus any number of points may be set off by perpendiculars from AB on level open ground.

If the offsets are inconveniently long, we may fix H_1 very accurately (either from AB or from P), then range in the lines AH_1, H_1B, and calculate the abscissæ and ordinates from the middle points of these chords by the same formulæ.

To fix H_1 from P, it is obvious that $PH_1 = R(\text{cosec } \frac{1}{2} \text{ apex angle} - 1)$, and it bisects the apex angle and is thus easily set out with chain and tape. Alternatively, we may set out parallels to AB at known distances from it, and then only the remaining length of offset is to be set off beyond the nearest of these lines (see Fig. 128).

BY OFFSETS INSIDE THE CURVE. Another method which may be adopted where only the ground inside the curve is favourable for setting out is as follows:

Calculate the versed sine of the angle which a chord subtends at the centre of the curve of given degree, or to a given radius.

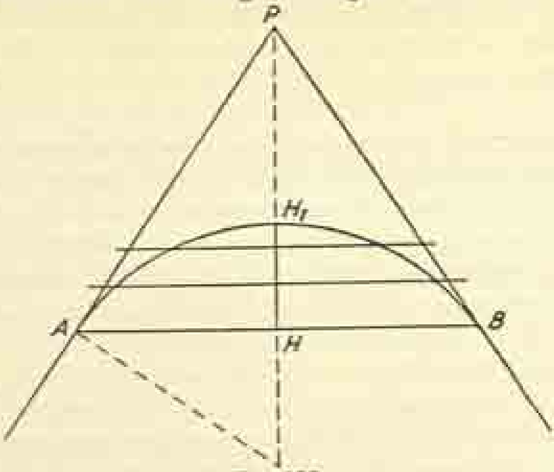


FIG. 128

Then at A (Fig. 129), the point from which the curve springs, set off A*b* equal to half of this versed sine, and in the direction of the

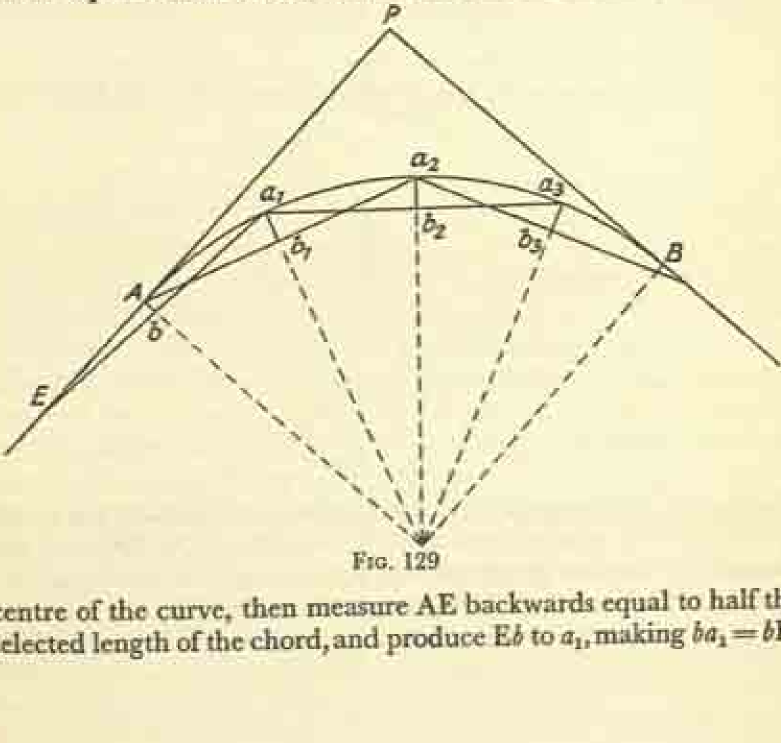


FIG. 129

centre of the curve, then measure AE backwards equal to half the selected length of the chord, and produce E*b* to a_1, making $ba_1 = b$E.

Then a_1 is the first point in the required curve. Now, from a_1 set off the full versed sine towards the centre and range Ab_1a_2, making $b_1a_2 = Ab_1$. This gives a_2 the second point in the curve, and so on till the point of entering the straight is reached. The method is inexact, however.

COMPOUND CURVES. A compound curve is one which is made up of two or more circular arcs of different radii, having common tangent points. The object of a compound curve is to avoid certain points the crossing of which would involve expense, and which cannot be avoided by a simple curve. No part of a compound curve must be of less radius than some fixed minimum depending on the speed at which the curve is to be negotiated.

Taking first the case of two circular arcs only, having a common tangent at T (Fig. 130), the given data for setting out the curves may vary considerably, and in any case there are several methods of solution, while sometimes *no* definite solution can be obtained. Thus all cases cannot be considered here, but a few will be treated.

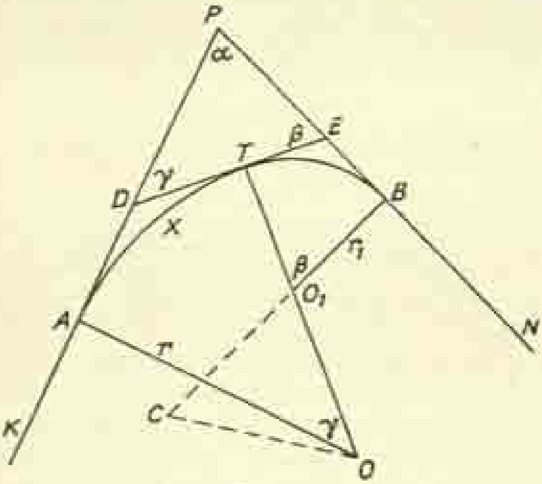


FIG. 130

Suppose, for example, that the starting and finishing points are known on the ground, and also the straights KA, BN, and one radius (r). Then on paper, we can draw the curves as follows:

At A and B, draw AO, BC, perpendicular to KA, BN, and each equal to the given radius r; join CO, and make the angle COT equal to BCO. Then if OT and BC meet at O_1, O and O_1 will be the centres of the separate arcs, BO_1 will be the second radius, and the arcs will touch one another on the line OT which is the common normal.

Proof: If O_1 be the second centre, we must have $OO_1 = r - r_1$.

But as $BC = r$, we also have $O_1C = r - r_1$.

$\therefore$ the triangle O_1CO must be isosceles.

Hence we make it so, in order to find O_1. This will show, on paper, if the radius chosen for r will answer.

Frequently, a third point X, through (or very near to) which the curve must pass, is fixed from the nature of the ground, and r is decided from that condition. It is to be noted that the greater the value of r the greater also will be the value of r_1, within the possible limits.

Now this method of working is, of course, impracticable in the field, and the graphical solution hardly exact enough for a good result in actually setting out the curve.

The following equations may be used for setting out. These are proved in the succeeding pages.

$$2r_1(\text{AP} \sin \alpha - 2r \cos^2 \tfrac{1}{2}\alpha) = \text{AP}^2 + \text{BP}^2 - 2\text{AP}.\text{BP} \cos \alpha - 2\text{BP}.r.\sin \alpha$$

or $$2r_1(\text{AP} \sin \alpha - 2r \cos^2 \tfrac{1}{2}\alpha) = \text{AB}^2 - 2\text{BP}.r.\sin \alpha \quad . \quad (1)$$

where r and r_1 are the radii, and α the apex angle.

Also $$\sin \beta = \frac{r \sin \alpha + \text{AP} \cos \alpha - \text{BP}}{(r - r_1)}$$

where β is the deflection angle for the curve whose radius is r_1 (2)

and $$\gamma + \beta = 180^\circ - \alpha$$

where γ is the deflection angle for the other arc . . (3)

The first of these equations gives r_1, and is not really so complicated as it looks. In using it, either radius may be called r_1, but we must notice that BP is the tangent distance adjacent to the *unknown* radius; AP, that adjacent to the known radius. The deflection angles are known from the second and third equations, and hence all data for setting out the curves.

The tangent distance AD for the first curve is $r \tan \frac{1}{2}\gamma$, and the tangent distance BE for the second curve is $r_1 \tan \frac{1}{2}\beta$. Thus D and E can be fixed. If we measure DE it should be equal to AD + BE, and as DT = AD, we can fix T, the finishing point of one curve and start of the next.

Alternatively, when the graphical solution on paper has been completed, we may measure the value of γ, and take this as our original known quantity instead of r.

Then from Fig. 130 it is clear that $DE = r \tan \frac{\gamma}{2} + r_1 \tan \frac{\beta}{2}$, as already stated.

Also $\quad PDE = 180° - TDA = \gamma$; and $PED = \beta$

Hence $\quad PD = DE . \dfrac{\sin PED}{\sin EPD}$

$$= \left(r \tan \frac{\gamma}{2} + r_1 \tan \frac{\beta}{2}\right) \frac{\sin \beta}{\sin \alpha}$$

$$\text{and } PE = \left(r \tan \frac{\gamma}{2} + r_1 \tan \frac{\beta}{2}\right) . \frac{\sin \gamma}{\sin \alpha}$$

Lastly, $\quad AP = AD + PD$ and $PB = PE + EB$

Hence we have

$$AP = \left(r \tan \frac{\gamma}{2} + r_1 \tan \frac{\beta}{2}\right) \frac{\sin \beta}{\sin \alpha} + r \tan \frac{\gamma}{2} \quad . \quad . \quad (1)$$

$$PB = \left(r \tan \frac{\gamma}{2} + r_1 \tan \frac{\beta}{2}\right) \frac{\sin \gamma}{\sin \alpha} + r_1 \tan \frac{\beta}{2} \quad . \quad . \quad (2)$$

$$\text{and, as before, } \gamma + \beta = 180° - \alpha$$

These are simple equations for finding β, r, and r_1, when γ is known. The value of r should nearly agree with the original value.

AP, PB, and α are, of course, supposed to be measured directly, or obtained indirectly, in the field, as for simple curves.

If, in the process, it has been necessary to find AB, equation No. 1 can be shortened as shown on p. 239.

Proof of formulae. Referring to Fig. 131. Draw QON and AS perpendicular to the second straight PN, and AQ and O_1M parallel to it. Then $QN = AS$ and $NS = AQ$.

Then $OO_1^2 = OM^2 + O_1M^2$,

$$\text{or } (r - r_1)^2 = (QN - QO - MN)^2 + (NS + SP - BP)^2$$
$$= (AP \sin \alpha - r \cos QOA - r_1)^2$$
$$+ (r \sin QOA + AP . \cos \alpha - BP)^2$$

But $QOA = 90° - OAQ = SAO$
$= 90° - PAS = \alpha$

Hence, multiplying out, we get

$$\begin{aligned} r^2 - 2rr_1 + r_1^2 = {} & AP^2 (\sin^2 \alpha + \cos^2 \alpha) + BP^2 \\ & + r^2 (\sin^2 \alpha + \cos^2 \alpha) + r_1^2 \\ & - 2 \sin \alpha \, (AP . r_1 + BP . r) \\ & + 2 \cos \alpha \, (rr_1 - AP . BP) \end{aligned}$$

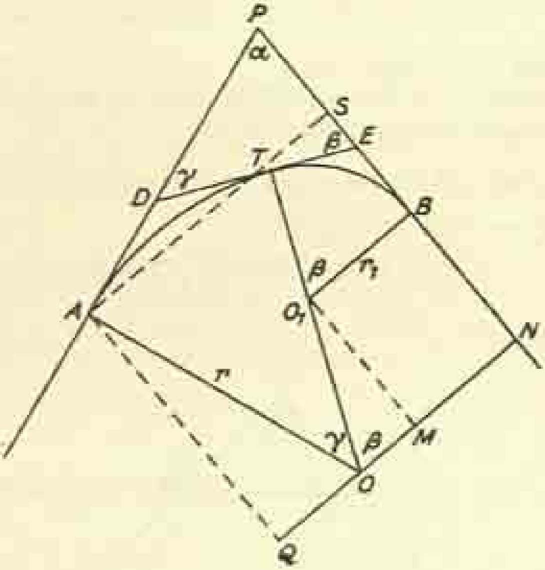


Fig. 131

that is

$$\begin{aligned} 2rr_1 + AP^2 + PB^2 - 2r_1(AP . \sin \alpha - r \cos \alpha) & \\ - 2AP . BP \cos \alpha - 2BP . r \sin \alpha & = 0 \end{aligned}$$

$$\begin{aligned} \text{or } 2r_1(r + r \cos \alpha - AP . \sin \alpha) & \\ + AP^2 + BP^2 - 2AP . BP \cos \alpha - 2BP . r \sin \alpha & = 0 \end{aligned}$$

$$\begin{aligned} \therefore 2r_1 \, (AP . \sin \alpha - r - r \cos \alpha) & \\ = AP^2 + BP^2 - 2AP . BP \cos \alpha - 2BP . r \sin \alpha & \end{aligned}$$

i.e.

$$\begin{aligned} 2r_1 \left[AP . \sin \alpha - r - r \left(\cos^2 \frac{\alpha}{2} - \sin^2 \frac{\alpha}{2} \right) \right] & \\ = AP^2 + BP^2 - 2AP . BP \cos \alpha - 2BP . r \sin \alpha & \end{aligned}$$

i.e. $$2r_1\left[AP.\sin\alpha - r - r\left(\cos^2\frac{\alpha}{2} - 1 + \cos^2\frac{\alpha}{2}\right)\right]$$
$$= AP^2 + BP^2 - 2AP.BP\cos\alpha - 2BP.r\sin\alpha$$

or $$2r_1\left(AP.\sin\alpha - 2r\cos^2\frac{\alpha}{2}\right)$$
$$= AP^2 + BP^2 - 2AP.BP\cos\alpha - 2BP.r\sin\alpha$$

as in equation (1), p. 239.

For equation No. 2, we have $O_1M = (r - r_1)\sin\beta$, and we have seen in the course of the above proof that

$O_1M = r\sin\alpha + AP\cos\alpha - BP$; hence equation No. 2 follows.

Also we have shown that $QOA = \alpha$

$$\text{and } \gamma + \beta = 180° - QOA$$
$$\therefore \gamma + \beta = 180° - \alpha$$

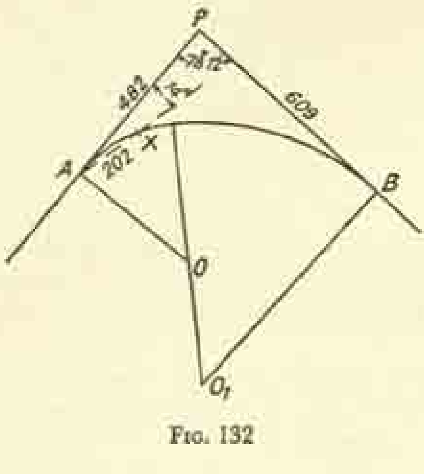


Fig. 132

If the point X (Fig. 132), through which the first curve is to pass, be known, we can of course calculate r, the first radius.

For example, suppose

$$AP = 482 \text{ ft.}, BP = 609 \text{ ft.}, \alpha = 78° 12', AX = 202 \text{ ft.}, PAX = 18°.$$
$$\text{Then } r = \tfrac{1}{2} \times 202.\text{cosec } 18° = 326{\cdot}8 \text{ ft.}$$

Then $2r_1(AP\sin\alpha - 2r\cos^2\tfrac{1}{2}\alpha) = AP^2 + BP^2 - 2AP.BP.\cos\alpha - 2BP.r.\sin\alpha$

We proceed to work out the left-hand side first.

No.	Logm	Antilog.
AP = 482	2·683 05	
Sin α	$\bar{1}$·990 72	
	2·673 77	+ 471·81
2	0·301 03	
r	2·514 28	
Cos $\frac{1}{2}\alpha$	$\bar{1}$·889 89	
	$\bar{1}$·889 89	
	2·595 09	− 393·63
		+ 78·18

2 × 78·18 = 156·36

For the right-hand side—

No.	Logm	Antilog.	Remarks
AP	2·683 05		
	2·683 05		
	5·366 10	+ 232,330	
BP	2·784 62		
	2·784 62		
	5·569 24	+ 370,890	
2	0·301 03		If α be greater than 90°, cos α = − cos (180° − α), and this term becomes positive
AP	2·683 05		
BP	2·784 62		
Cos α	$\bar{1}$·310 68		
	5·079 38	− 120,060	
2	0·301 03		
BP	2·784 62		
r	2·514 28		
Sin α	$\bar{1}$·990 72		
	5·590 65	− 389,630	
		+ 603,220	
		− 509,690	
		+ 93,630	

$$\therefore r_1 = \frac{93{,}630}{156\cdot36} = 598\cdot8 \text{ ft.}$$
$$\underline{326\cdot8 \text{ ,,}}$$
$$r_1 - r = 272\cdot0$$

To find γ and β.

$$\sin\beta = (r \sin\alpha + \text{AP} \cos\alpha - \text{BP}) \div (r - r_1).$$

No.	Logm	Antilog.
r Sin α	2·514 28 $\bar{1}$·990 72	
	2·505 00	319·9
AP Cos α	2·683 05 $\bar{1}$·310 68	
	1·993 73	98·6
		+ 418·5 BP − 609·0
		− 190·5

$$\therefore \sin\beta = \frac{190\cdot5}{272\cdot0} \text{ or } \beta = 44^\circ\ 27'$$
$$\text{and } \gamma + \beta = 180^\circ - 78^\circ\ 12' = 101^\circ\ 48'$$
$$\therefore \gamma = 57^\circ\ 21'$$

Reverse or serpentine curves. In Fig. 133, let PA, PB be the straights to be connected, A and B the tangent points. Graphically, if one radius, r, be chosen, we make $\text{AO} = \text{BC} = r$, both perpendicular to the straights as shown. Join OC, and make the angle $\text{COO}_1 = \text{OCB}$; then O, O_1 are the centres of the arcs, and O_1B is the second radius. Thus a suitable figure for the first radius can be chosen on paper.

To calculate the second radius on the field, we proceed exactly as in the case of the compound curve. The necessary formulæ can be derived from those already given, by making the proper adjustments of signs in accordance with well-known mathematical principles. Thus in Fig. 133, AP is measured *backwards* from the starting point; hence AP in the formula on p. 240, must be reckoned *minus* in this case. The same remark applies to the radius r_1.

Finally, the angle marked α on Fig. 133 is the deflection angle, and therefore the supplement of the apex angle. Hence for α we must now write $(180° - \alpha)$.

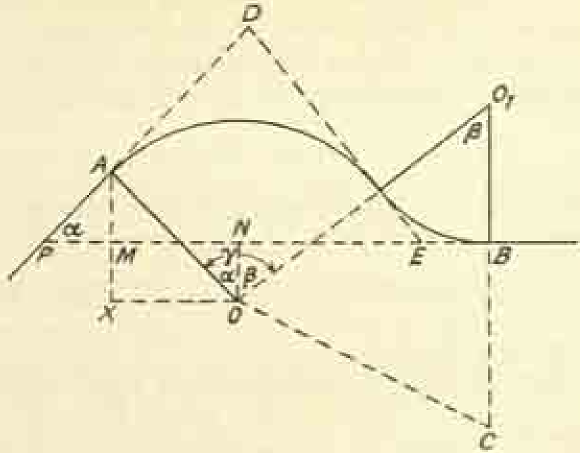


FIG. 133

The result is

$$2r_1(\text{AP}\sin\alpha + 2r\sin^2 \tfrac{1}{2}\alpha) = \text{AP}^2 + \text{BP}^2 - 2\text{BP}.\text{AP}\cos\alpha - 2\text{BP}.r\sin\alpha$$

or

$$2r_1(\text{AP}\sin\alpha + 2r\sin^2 \tfrac{1}{2}\alpha) = \text{AB}^2 - 2\text{BP}.r\sin\alpha$$

If P be between A and D, AP must be reckoned *minus* in this equation. The student is recommended to obtain this equation independently from Fig. 133. If M be the intersection of AX and PN,

$$\text{put } \text{OO}_1^2 = (\text{O}_1\text{B} + \text{AX} - \text{AM})^2 + (\text{PB} - \text{PM} - \text{OX})^2$$

To find the deflection angles for the separate curves, we have

$$\text{OO}_1 \sin\beta = \text{O}_1\text{B} + \text{AX} - \text{AM}$$

$$\text{whence } \sin\beta = \frac{r_1 + r\cos\alpha - \text{AP}\sin\alpha}{r + r_1}$$

$$\text{and } \gamma = \beta + \alpha$$

Tangent distances, etc., can then be found as before. If circumstances permit, it is best to make the radii equal; this can be readily accomplished by putting $r_1 = r$ in the formulæ already given.

INTERVENING STRAIGHTS. Except in crossover roads in railway trackwork the use of reverse curves is to be deprecated, in consequence of the difficulty in adjusting the superelevation and the sudden change in direction which is involved both on railways and roads. The question of superelevation is discussed later.

Straight lines not less than two chains or 132 feet in length should, if possible, always intervene between the curves.

TRANSITION CURVES

(*a*) RAILWAY PRACTICE. On the straight, both lines of a railway track are laid at the same level. On a curve, however, the train is acted on by a horizontal centrifugal force of $\frac{wv^2}{gr}$ units, in addition to the vertical weight w.

The resultant of these forces should be at right angles to the line joining the tops of the rails in cross-section, and this makes it necessary to place the *outer* rail on the curve *higher* than the inner one (Fig. 134).

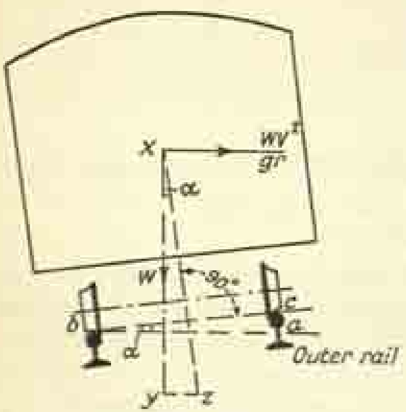


FIG. 134

The difference of level is called the *superelevation of the outer rail*, and it will be apparent from the similar triangles abc, xyz

that $$\frac{h}{b}=\frac{\frac{wv^2}{gr}}{w}, \text{ or}$$

$$h=b\times\frac{v^2}{gr} \quad . \quad . \quad (1)$$

where g = the acceleration due to gravity = 32·2 feet per sec. per sec.

v = velocity in feet per sec.
r = radius of curve in feet
b = gauge of railway in inches
h = superelevation, also in inches

If we pass directly from the straight to the circular arc we must begin to give this superelevation while still on the straight, causing the train to lurch *inwards*. The full superelevation is not attained until some distance beyond the start of the curve, so that, on entering the latter, there is a tendency to lurch *outwards*, causing shaking and uneasy running.

To overcome this it is becoming increasingly the practice to introduce *transition curves*, or spirals, between the two (Fig. 135). These start with a radius approaching infinity on joining the straight, while at the junction with the circular arc the radius of curvature is the same as that of the arc.

In America perhaps the best known forms are Searle's spirals and Crandall's transition curves, both of which are dealt with in special books by their authors. In British railway practice the Glover's spiral and the cubic parabola are most in favour.

LENGTH OF CURVE. The principles on which the length of transition curves should be found have been laid down by W. H. Shortt, in the *Proceedings of the Institution of Civil Engineers*, Vol. 176.

The radial acceleration on the curve is $v^2 \div r$. Now Shortt considers that this, for perfectly easy running, should be acquired at a rate not exceeding 1 foot per sec. per sec. in each second.

If l be the length of curve, t the time taken to pass over it, $t=\frac{l}{v}$; the rate of gain of radial acceleration is, therefore

$$\frac{\frac{v^2}{r}}{\frac{l}{v}}, \text{ or } \frac{v^3}{rl};$$

if this is equated to unity we may put $l=\frac{v^3}{r}$, according to the above rule.

Whence we have, for foot-second units –

$$l=\frac{v^3}{r} \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Now it is practically undesirable to make the superelevation much more than 6 inches with the standard gauge of 4′ 8½″.

If we substitute these values in equation (1), p. 246, we find $v^2 = r \times 3{\cdot}42$.

whence from (2) $\quad l = \sqrt{r} \times (3{\cdot}42)^{\frac{3}{2}}$

$\quad$ or $l = 6{\cdot}3\ \sqrt{r}$, for foot-units $\quad . \quad . \quad . \quad (3)$

This gives the necessary minimum length on the assumption of a maximum superelevation of 6 inches, and a corresponding velocity $1{\cdot}85\sqrt{r}$ foot-second units. It is not, however, advisable that the length should be less than 150 feet (some engineers say 3 chains). This would occur with a smaller radius than 600 feet. Hence below that limit all transition curves should be 150 feet long.

Moreover, with very large radii the velocity $1{\cdot}85\sqrt{r}$ may be greater than that of the fastest train. In this case the length must be found by the formula $v^3 \div r$, where v is the maximum velocity for the line. If we reduce the units to Gunter's chains, we obtain $L = 0{\cdot}77\sqrt{R}$, which Shortt suggests should be increased to $L = \sqrt{R}$, where L = length of curve, R = radius, both in Gunter's chains.

(*b*) Road practice. Modern practice in road design includes the provision of superelevation on curves of 7,500 feet radius, or less. As in railway practice superelevation can be correctly applied only by inserting transition curves connecting the central circular arc with the tangents. The length of these transitions, however, is not determined in accordance with any accepted standard. The Ministry of Transport advises that Shortt's Rule should be used, limiting the rate of gain of radial acceleration to 1 foot per second per second in one second. It is found that this rule, in many cases, gives excessively long transitions, and this may have the effect of leaving insufficient room for an adequate length of circular arc. A wholly transitional curve could then be used, but this is opposed to the views of many road engineers who favour short transitions with an ample length of circular arc.

It has been suggested that to overcome this difficulty a limiting value of 2 feet per second cubed might be substituted for Shortt's figure in the case of road curves, but practical investigations appear to indicate that the rate of gain of radial acceleration is not a decisive factor in normal driving technique, nor does it have a direct bearing on safety.

Another method of determining transition length is by limiting the gradient at which the outer edge of the road is raised in applying superelevation. The latter quantity depends upon the radius of the circular arc, and in British practice the crossfall of the road surface does not normally exceed 1 in 14½. The Ministry of Transport advise that this crossfall should be used on curves of 1,200 feet radius or less.

The superelevation height expressed in this way, and, consequently the transition length, depends upon the width of the carriageway. On a 30-foot carriageway, for example, the maximum superelevation height would be slightly over 2 feet. The longitudinal gradient at which this height is gained should not be steeper

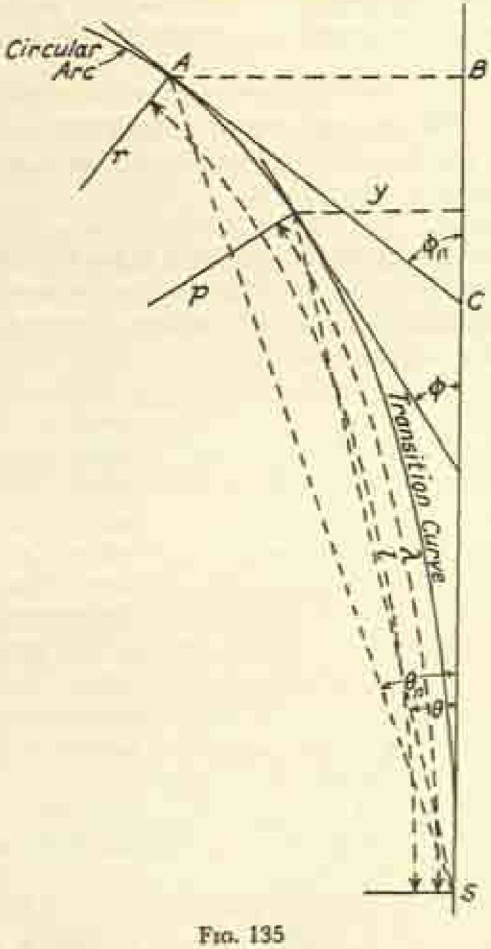


FIG. 135

than 1 in 200 and if the superelevation is applied entirely by raising the outer edge of the road a transition length of 400 feet would be necessary. If this is too long to suit the particular curve, the inner edge may be gradually lowered while the outer edge is raised at an equivalent gradient and the transition length may thus be reduced to 200 feet.

The whole question of transition curves and superelevation has been discussed very fully by the late Professor Royal-Dawson in his books *Elements of Curve Design* and *Road Curves* published by Messrs. E. & F. N. Spon. The mathematical treatment in these books is based on the lemniscate for transition purposes, but this curve, in normal circumstances, coincides very nearly with the spiral.

IDEAL CURVE. The superelevation should be proportional to distance from starting point, so that the rail or outer edge of the road can be raised at a regular rate.

But we have seen (p. 246) that it is inversely proportional to the radius of curvature. Hence in the ideal curve the radius of curvature should be inversely proportional to the distance from starting point of transition curve, or $\rho\lambda = C$, where ρ = radius of curvature, λ = distance from starting point.

This is accurately satisfied in Crandall's curve and in Glover's spiral, the equation of which is $\lambda = m\sqrt{\phi}$, where ϕ = angle between the straight and the tangent to the curve at the point whose distance from the start (measured round the curve) is λ (Fig. 135).

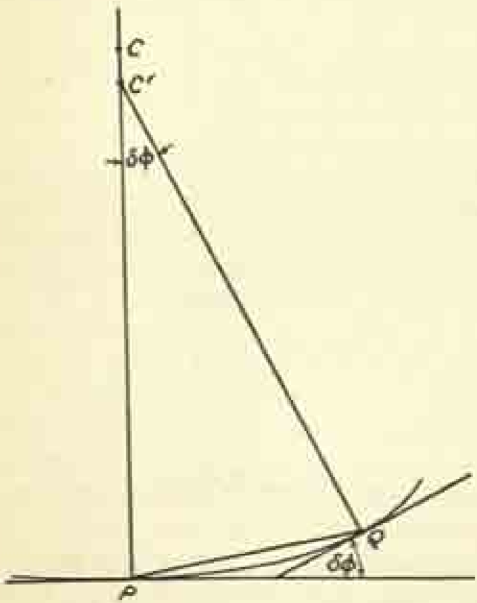


FIG. 136

GLOVER'S SPIRAL. With the above notation–

$$\rho = \frac{d\lambda}{d\phi}$$

This is a fundamental relationship which may be proved thus: Let P and Q in Fig. 136

be two points on a curve and let the length of the arc PQ be a small quantity $\delta\lambda$; also let the angle between the tangents at P and Q be $\delta\phi$ radians. Then, if the perpendiculars to these tangents meet at a point C′, $\angle PC'Q = \delta\phi$. As Q approaches P, the point C′ will change in position according to the variation of the radius of curvature. If the latter increases from Q towards P, let C′ finally occupy a position C when Q and P coincide.

In the triangle PC′Q, $\dfrac{PC'}{\sin PQC'} = \dfrac{\text{chord } PQ}{\sin PC'Q}$

The right-hand side of this equation may be written

$$\frac{\text{chord } PQ}{\sin \delta\phi} \times \frac{\text{arc } PQ}{\text{arc } PQ} \times \frac{\delta\phi}{\delta\phi}$$

Re-arranging:

$$\begin{aligned}\frac{PC'}{\sin PQC'} &= \frac{\text{chord } PQ}{\text{arc } PQ} \times \frac{\text{arc } PQ}{\delta\phi} \times \frac{\delta\phi}{\sin \delta\phi} \\ &= \frac{\text{chord } PQ}{\text{arc } PQ} \times \frac{\delta\lambda}{\delta\phi} \times \frac{\delta\phi}{\sin \delta\phi}\end{aligned}$$

In the limit, when C′ coincides with C,

$$\angle PQC' = 90°, \frac{\text{chord } PQ}{\text{arc } PQ} = 1, \frac{\delta\lambda}{\delta\phi} = \frac{d\lambda}{d\phi}, \text{ and } \frac{\delta\phi \text{ radians}}{\sin \delta\phi} = 1$$

In this case

$$PC = PC' = \sin 90° \times \frac{\text{chord } PQ}{\text{arc } PQ} \times \frac{\delta\lambda}{\delta\phi} \times \frac{\delta\phi}{\sin \delta\phi} = \frac{d\lambda}{d\phi}$$

$$\text{or } \rho = \frac{d\lambda}{d\phi}$$

Hence in the Glover's spiral

$$\rho = \frac{m}{2\sqrt{\phi}} = \frac{m}{2} \times \frac{m}{\lambda} = \frac{m^2}{2\lambda}$$

But at the finishing point A of junction with the circular arc, $\rho = r$ and $\lambda = l$.

$$\therefore r = \frac{m^2}{2l}$$

whence m can be found according to the units and the known length of curve

Thus for Gunter's chains, if we put $L = \sqrt{R}$ we have $m^2 = 2R^{\frac{3}{2}}$.

Then choosing values λ_1, λ_2, etc., etc., for λ (say 1 chain, 2 chains, etc.), we can find the corresponding values of ϕ from the equation $\phi_1=(\lambda_1)^2 \div m^2$, and so on, including the final value ϕ_n when $\lambda=l$. The results are in radians.

Now if x and y be the co-ordinates of any point on the curve, and θ be the tangential angle to the same point, we have

$$dx=d\lambda \cos\phi$$

$$=\frac{m}{2}\cdot\frac{\cos\phi}{\sqrt{\phi}}\cdot d\phi$$

whence, expanding $\cos\phi$ and integrating,

$$x=m\left(\sqrt{\phi}-\frac{\phi^{\frac{5}{2}}}{10}+\frac{\phi^{\frac{9}{2}}}{216}\cdots\right)$$

Similarly, $$dy=\frac{m}{2}\cdot\frac{\sin\phi}{\sqrt{\phi}}d\phi$$

whence $$y=m\left(\frac{\phi^{\frac{3}{2}}}{3}-\frac{\phi^{\frac{7}{2}}}{42}+\frac{\phi^{\frac{11}{2}}}{1320}\cdots\right)$$

Now $$\tan\theta=\frac{y}{x}$$

whence by simple division

$$\tan\theta=\frac{\phi}{3}+\frac{\phi^3}{105}+\frac{\phi^5}{5997}+\ldots\text{or }\theta=\frac{\phi}{3}\text{ nearly}$$

From this the values of θ corresponding to ϕ_1, ϕ_2, etc. (and therefore to the distances λ_1, λ_2, etc.) can be found, and the curve is set out like a circular arc from these tangential angles and distances.

If θ_n be the final value of θ, the point of junction, A, can be fixed from it and the known value of l. Then if CA be the tangent at A, SAC$=\phi_n-\theta_n$. Hence the direction of AC can be set out by a theodolite at A, and produced to give the tangent for setting out the circular arc.

But the position A should first be checked by calculating SB from the above formula for x. Then AB$=$SB $\tan\theta_n$. From these A can be checked.

Then find BC$=$AB $\cot\phi_n$, and mark C, so as to check the direction of AC.

Having completed these calculations, to set out the curve on the ground we must find the starting point S.

Now in Fig. 137, which illustrates another combination of spiral and circular arc, let X be the apex of the curve, i.e. the intersection of the tangents so that half of the circular arc is shown between A and the line OX.

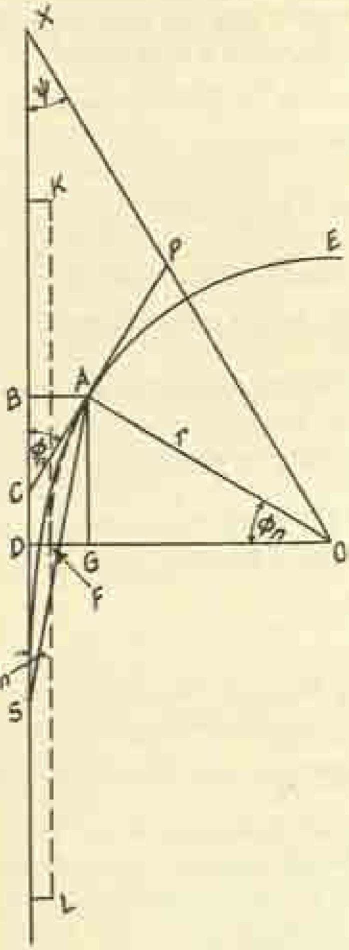


FIG. 137

If CA the tangent at A be produced to meet OX in P, it is clear that we shall have APO = half apex angle for the circular arc.

Also CXO = half apex angle for the whole curve, and we shall denote this by ψ.

But, clearly,

$$APO = CXO + ACX = \psi + \phi_n$$

Now in the triangle PXC –

$AC = AB \operatorname{cosec} \phi_n$; $AP = r \cot APO$

Hence

$$CP = AC + AP$$

and is therefore known

and

$$XC = \frac{CP \sin CPX}{\sin CXP} = \frac{CP \sin APO}{\sin CXP}$$

$$= \frac{CP \sin (\psi + \phi_n)}{\sin \psi}$$

And CS = BS − BC, both of which are known, so that XS can be found, and S fixed by chaining as for a circular arc.

The length of the transition curve is, of course, exaggerated in the figure.

SETTING OUT THE CURVE. By the aid of certain approximations sufficiently exact for most purposes, the calculations can be simplified considerably.

In Fig. 137, let AE be the circular arc (centre O radius r), which is to be connected with the tangent SBX by a Glover's spiral AS. Let OD be drawn perpendicular to the tangent SX and let the circular arc AE be produced backwards to cut OD in F. Then DF is called the *shift* and is the amount by which a plain circular curve would require moving inwards from the tangents in order to make room for the transitions at the entrance and exit to the curve. Also let AG be drawn perpendicular to OD.

As we have agreed, the length $AS = l$, and the total deflection from S to $A = \phi_n$. This is shown by $\angle XCA$ and it is clear that

$$\angle XCA = 90° - \angle CAB = \angle CAG = 90° - \angle GAO = \angle GOA$$

Again, from the equation of the curve,

$$\phi_n = \frac{l^2}{m^2} = \frac{l}{2r}, \text{ since } m^2 = 2rl \text{ (p. 251)}$$

These values are in radians. Now to find DF we have

$$AB = y = m.\phi_n^{\frac{3}{2}} \times \tfrac{1}{3} \text{ very nearly, as } \phi_n \text{ is small (p. 252).}$$

Hence $AB = \dfrac{l.\phi_n}{3}$ as $m.\phi_n^{\frac{1}{2}} = l$, (from the equation of the curve),

or $$y = \frac{l^2}{6r} \text{ as } \phi_n = \frac{l}{2r}, \text{ as shown above.}$$

$$\text{Now } DF = DG + GO - FO = AB + GO - FO = y + r.\cos\phi_n - r$$
$$= y - r(1 - \cos\phi_n) = y - 2r.\sin^2 \tfrac{1}{2}\phi_n.$$

$\therefore$ approximately, $$DF = y - \frac{2r\,\phi_n^2}{4}$$
as ϕ_n is small and is measured in radians.
And substituting for y and ϕ_n, we have

$$DF = \frac{l^2}{6r} - \frac{l^2}{8r} = \frac{l^2}{24} \quad . \quad . \quad . \quad . \quad (1)$$

This is the formula for the shift, which we shall call s.

Now if X be the apex, DOX = half total deflection, and it is clear that a circle with centre O and radius $r + DF$ will touch SX at D.

Hence $$DX = (r + s) \tan \tfrac{1}{2} \text{ deflection}$$
or $$DX = (r + s) \cot \tfrac{1}{2} \text{ apex angle}$$
where s is the shift as above.

Again, $$AF = r \times \phi_n \text{ and } AS = l = 2r\phi_n, \text{ since } \phi_n = \frac{l}{2r}$$
$$\therefore AF = \tfrac{1}{2}l$$

And we may assume that this is practically equal in length to the portion of transition curve between A and DO. Hence, approximately, the length of the transition curve is bisected by DO, and as DS is very nearly equal in length to the curve between S and DO, it follows that SD $= \frac{1}{2}l$, very nearly.

Hence $\quad XS = DX + SD$

$$= (r+s)\cot\tfrac{1}{2}\text{ apex angle} + \tfrac{1}{2}l \quad . \quad . \quad . \quad (2)$$

This fixes the position of the starting point S.

The final deflection angle,

$$\phi_n = \frac{l}{2r} \text{ as above} \quad . \quad . \quad . \quad . \quad (3)$$

The deflection angle ϕ for any other point whose distance from S is λ can then be found by the formula

$$\frac{\lambda^2}{l^2} = \frac{\phi}{\phi_n}$$

whence ϕ_1, ϕ_2, etc. can be calculated for distances λ_1, λ_2, etc., increasing by equal chosen chords.

The corresponding tangential angles θ_1, θ_2, etc. are found by dividing the values of ϕ by *three* (p. 252), and are used for setting out the curve, in the ordinary way, with the chosen chords. It should be noted, however, that the relationship $\theta = \frac{\phi}{3}$ should only be used for practical calculations provided that ϕ does not exceed a limiting value depending upon the accuracy with which the angles can be set out. Normally the tangential angles in curve ranging are sufficiently accurate if set out to the nearest half-minute. In this case the approximate formula $\theta = \frac{\phi}{3}$ can be used provided ϕ does not exceed 20°. If angles are set out to the nearest minute, the limiting value of ϕ may be increased to 25°. This limit would cover the vast majority of cases met with in practice, both in road and railway work.

For setting out the circular arc, we may fix C as previously described, and work from the tangent AC. This length is rather short, however, and it is frequently more satisfactory to fix D at a distance $\frac{l}{2}$ from S, set up DF equal to the shift, very carefully perpendicular to SD, and as far away as possible set up another equal offset as shown at K or L. Then FK (or FL) will give a tangent to the circular arc at F, and we can set out the arc starting from this. It should check up at A, and the part FA is unused.

As an example, suppose a curve of 600 feet radius, deflection angle 50°, is to have a transition curve 150 feet long at each end. The chainage of the apex is 8018·51 feet, and pegs are to be inserted at 50-foot intervals on exact chainages. The calculations would be as follows:

$$\text{Shift} = \frac{l^2}{24r} = \frac{150^2}{24 \times 600} = 1{\cdot}56 \text{ feet}$$

Tangent distance (XS) = 601·56 tan 25° + 75 = 355·51 feet
Hence chainage of starting point = 8018·51 − 355·51 = 7663 feet
Final deflection angle $= \phi_n = \frac{150}{1200} = \cdot 125$ radian = 7° 9′ 44″ nearly
This is for a length of 150 feet
The first chord will be 7700 − 7663 = 37 feet

$$\therefore \phi_1 = 7^\circ\ 9'\ 44'' \times \frac{37^2}{150^2} = 26'\ 9'' \quad \therefore \theta_1 = \frac{\phi_1}{3} = 8'\ 43''$$

For the next point

$$\lambda = 50 + 37 = 87,\ \phi_2 = 7^\circ\ 9'\ 44'' \times \frac{87^2}{150^2} = 2^\circ\ 24'\ 34''$$

$$\therefore \theta_2 = \frac{\phi_2}{3} = 48'\ 11''$$

Similarly, θ_3 is found, and θ_4 by dividing ϕ_n by 3.

The angles are tabulated in the annexed table, and are set out in the usual way from S.

No. of point	Chord	Tangential angle	Chainage
		° ′ ″	
1	37	8 43	7700
2	50	48 11	7750
3	50	1 59 29	7800
4	13	2 23 15	7813
	150		

To set out the circular arc, F and K are fixed as above:

$$\text{Chainage of F} = 7663 + 75 = 7738 \text{ feet}$$

$$\text{For a 50-foot chord, tangential angle} = \frac{1719 \times 50}{600} = 2^\circ\ 23'\ 15''$$

This being set out will have a chainage of 7788, and therefore should fall short of the last point on the transition curve by 25 feet, and the next chord should be taken as 62 feet to bring us to chainage 7850. The corresponding angle will be

$$2^\circ\ 57'\ 36'' + 2^\circ\ 23'\ 15'' = 5^\circ\ 20'\ 51''$$

CUBIC PARABOLA. The cubic parabola is still used by some railways in preference to the Glover's Spiral. But as the two agree closely, it will not be further dealt with. In road work the deflection angles between the tangents are frequently much larger than those encountered in railway practice and the cubic parabola cannot be used in such cases since its radius decreases up to a polar deflection angle of approximately 9° and then increases again.

EXAMPLE OF SURVEY WORK WHEN SETTING OUT A CURVED TUNNEL. The following description of the setting out of a curved tunnel serves as a good example of the survey work involved in an engineering project of this kind. This tunnel is part of the former Glasgow and South Western Railway, and connects a point on the line two miles north of West Kilbride with the north end of the village of Fairlie, where the railway runs on to the pier.

The tunnel is 971 yards long, the greater portion being on a curve, with a radius of 1 mile 6¼ furlongs and a mean gradient of 1 in 100. Two shafts were sunk on the line of the tunnel so as to facilitate progress and to enable the excavation to be carried on from four points simultaneously. These shafts were so placed as to divide the lengths into nearly equal sections, and on the completion of the work they were used as ventilators.

The setting out of a tunnel from open ends only is a matter of little or no difficulty owing to the fact that long base lines from which to work are generally available. When, however, the setting out has to be carried on from shafts as well, the greatest care must

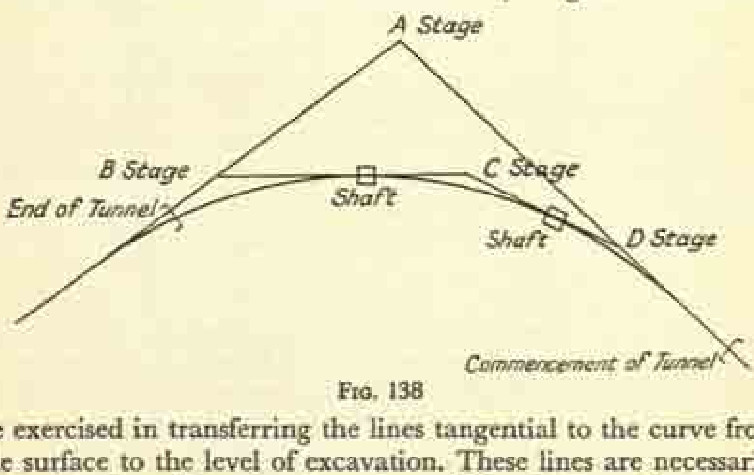


FIG. 138

be exercised in transferring the lines tangential to the curve from the surface to the level of excavation. These lines are necessarily limited in length by the dimensions of the shaft, so that a very slight deviation of one of the points to one side or the other would cause a very considerable error to accumulate in half the length of a section. Fig. 138 shows the arrangement of lines which was adopted for setting out the work in this particular case.

In setting out the centre line on the surface it will be seen from the figure that two intermediate tangent lines, BC and CD, were run in such a manner that their points of contact with the curve were approximately at the centres of the shafts.

In order to adjust these points finally before setting out the tunnel, stages of various heights (according to the nature of the ground), which were visible from one another, were erected over the points of intersection B, C and D. All necessary adjustments to ensure that the lines were tangential to the curve at, or very near to, the centre of length of each shaft were then made to both the angles and distances. The intermediate tangent lines were run across the shafts, and the points at which the lines cut the edges of the shafts were marked by small notches cut in metal plates overhanging the edges of the shafts (*vide* Fig. 139).

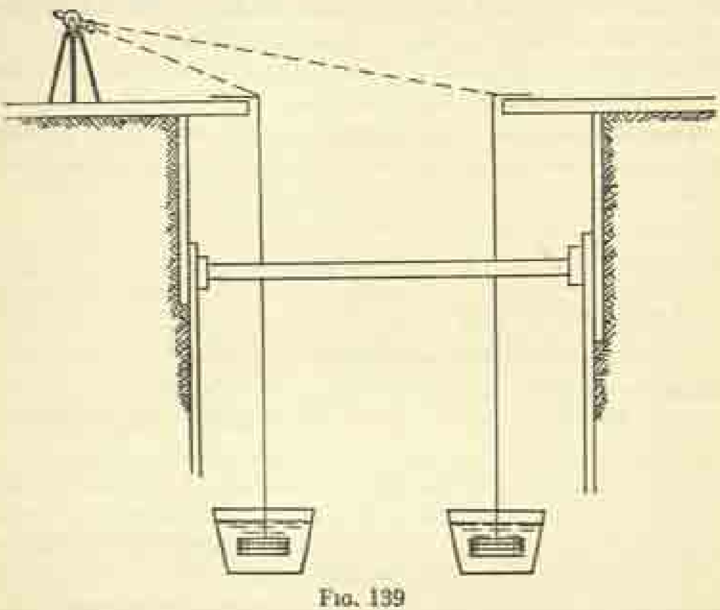


Fig. 139

In order to get these notches very accurately fixed, the theodolite was set up close to the shaft and adjusted exactly in the line of the tangent, by sighting on one stage, transiting, and observing whether the line from the one stage produced exactly cut the other stage, moving the instrument laterally, and repeating until the line from either stage to the theodolite did, when produced, exactly intersect the other stage, and the three points were therefore truly in the same straight line. After this was accomplished, nothing remained but to fix the position of the notches, which could be done very minutely with the instrument close at hand. By this means a line tangent to the curve at the centre of the shaft was fixed.

The transferring of the tangent line to the bottom of the shaft was accomplished by hanging heavy cast-iron weights from the notches overhanging the edges of the shaft by means of copper wires as shown in Fig. 139, the weights being immersed in buckets of water. These weights were of about 40 lbs. each, and were constructed with feathers projecting from the body of the weight to prevent any rotation in the water. In this way the direction of the tangent line was accurately defined at the bottom of the shaft, and the exact tangent point could be found by a measurement between the wires.

Points on the curve in the tunnel could now be easily determined by prolonging this tangent line, and taking offsets calculated according to the distances of the several points from the tangent point at the centre of the shaft. It was, of course, immaterial whether the tangent point was exactly at the centre of the shaft or not, so long as its position, corresponding to that above ground, was accurately known.

The method used for prolonging the tangent lines in the shafts was to fix points ahead, in line with the wires, by eye; for this purpose a small lamp was used, the flame being lined in very carefully with the wires, and an offset measured from it, such offset being calculated according to the distance of the lamp from the tangent point.

The question naturally arises as to the procedure when the points which had to be determined on the centre line were at such a distance from the tangent line that the latter could not be continued without coming into contact with the sides of the tunnel. In the main workings this difficulty did not arise, since, owing to the flatness of the curve, the maximum offset was in no case greater than half the width of the tunnel; consequently the tangent line did not meet the sides. It had, however, to be dealt with in the headings, which were about 6 ft. by 6 ft. In such cases, when the tangent line became exhausted, another tangent was run as shown in Fig. 140, this process being repeated when necessary.

The *modus operandi* was as follows:

When the tangent line approached the side of the tunnel as at B, equal perpendicular offsets were measured from A and B, and the new tangent line DC was run forward in the direction of O. It must be pointed out that the method of setting out the offsets BC

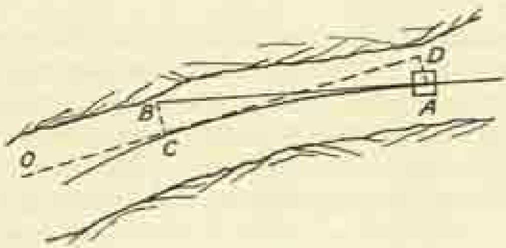


FIG. 140

and AD at right angles to AB is not theoretically correct. To obtain a truly accurate result AB and CD must both be equal to AC. In the case under consideration, however, as in all practical cases of curves in tunnels, the length of chord which can be used is so small compared to the radius (in this case $R = 9570$ ft., and the longest chord which could be got in was only 239 ft.) that the error introduced by setting out BC and AD at right angles to AB is too small to be measured.

Points were fixed by nails in wooden plugs driven into the roof of the heading, and, when any lines required to be extended, strings with weights at their ends, were hung from the nails, and, these being illuminated by lamps with suitable shades, could be sighted and other points fixed ahead.

Greater accuracy can be obtained by setting out the curve with a theodolite, using the method of tangential angles as previously described, and by this means nearly twice the length of curve can be set out from one tangent line, as the instrument does not require to be moved until the line of sight comes into contact with the inner side of the tunnel.

The objection to the use of the theodolite in the case described was the loss of time in adjusting the instrument laterally in prolongation of the wires, as at that period all lateral adjustment had to be made by moving the whole instrument and tripod, which is a tedious process. Since then several improvements have been made in theodolites for use in tunnel work, and they are now almost always provided with an arrangement which permits of a limited lateral movement of the horizontal plates over the legs, so that after the instrument has been set up the final adjustment may be made by means of the arrangement referred to. The objection mentioned above does not therefore hold good at the present time, and the theodolite method is most generally adopted on account of its superior accuracy.

To fix bench-marks for levels at the bottom of the shafts, levels were taken at convenient places on the top of the shafts, and measurements were then made down the shafts by means of a steel tape. Levels can easily be taken underground by the use of lamps, one for shining on the staff, and the other for flashing in front of the instrument to pick up the position of the diaphragm lines. For taking the levels in the headings, tripods with short legs were found most convenient, it being difficult to set up an instrument of the ordinary dimensions, so as to avoid obstacles in the line of sight.

THE WEISBACH TRIANGLE. Instead of aligning the theodolite exactly with the plumbing wires in a shaft, the instrument may be set up at a point remote from the line joining the wires. The length of the offset from the instrument to an extension of this line is then com-

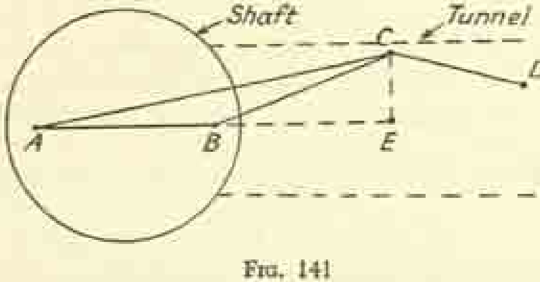


FIG. 141

puted from accurately measured lengths and angles. Thus, in Fig. 141, A and B represent the plumb wires, C the position of the instrument and D a reference point ahead. The angle ACB is carefully measured, and angles ACD and BCD are measured as a check. The lengths AB, AC, and BC are determined very accurately. The length of the perpendicular offset, CE, from C to AB produced, and the value of the angle BCE are computed from the data so obtained.

VERTICAL CURVES

The various gradients at different parts of the line of a road or railway are chosen on the longitudinal section much as the directions on plan. As we replace all angles by curves in plan, so it is essential that all angles or sharp changes of gradient in the section should be replaced by curves in a vertical plane.

It is usual to set such a curve out as an arc of a parabola instead of a circle. There is no practical difference for such flat curves, and the calculations are more simple for parabolic arcs.

PARABOLIC VERTICAL CURVES—GENERAL THEORY. In British practice, gradients are usually defined numerically by stating the horizontal distance in which a rise or fall of one foot occurs. In American practice it is usual to express gradients as percentages. Thus a 1 in 10 gradient would be called a 10% grade, a 1 in 4 gradient a 25% grade, and so on. The latter method is convenient for vertical curve calculations and an upward gradient is considered positive and a downward gradient negative. Thus, if we proceed from A to C in Fig. 142, the gradient AB is positive and the gradient BC negative. The signs would be reversed in proceeding from C to A, but this would not affect the calculations.

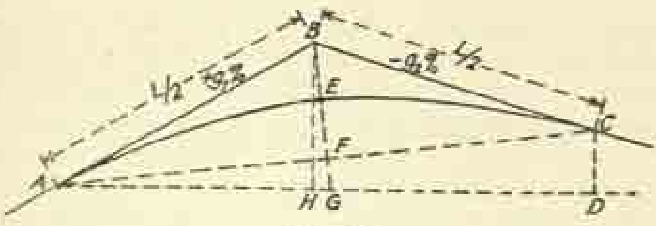


FIG. 142

In order to set out a vertical curve it is necessary to evaluate the vertical offsets from the tangents to the curve at a series of points. Referring to Fig. 142, let the gradient AB be $+g_1\%$ and the gradient BC $-g_2\%$. Let the horizontal line AD be drawn through A and the vertical line CD through C. Let AEC represent a parabolic vertical curve with its mid-point at E, and let F be the mid-point of the chord AC. Then BEF will be a straight line and from the property of a parabola BE = EF.

Gradients on railways and on important roads are normally of such magnitude that for practical purposes it is justifiable to assume that the length of the curve AEC = the length of the chord AFC = the sum of the tangent lengths AB + BC. Furthermore, if BF be produced to cut AD in G and BH is a vertical line, BG may be assumed to coincide with BH. If two gradients as steep as 1 in 10 are drawn to a natural scale, it will be apparent that no very great error is introduced by making these assumptions, and it will be found that they simplify the calculations considerably. The gradients in Fig. 142 are exaggerated.

The length of the curve is first chosen, with due regard to such points as the smooth running of trains or road vehicles and the economics of the project. The latter will control, to some extent, the permissible magnitude of the earthworks involved in excavating through a hill, or building up an embankment. In the case of a vertical curve at a road summit, the question of visibility is an important factor and controls the length in the way discussed later.

Let L be the length of the curve. Usually, though not invariably, the tangents are equal, and this will now be assumed.

VERTICAL CURVE WITH EQUAL TANGENTS.

Referring, again, to Fig. 142: $AB = BC = \frac{L}{2}$

Let the level of A be assumed zero.

Then the level of $B = \frac{L}{200} \cdot g_1$ and the level of $C = \frac{L}{200}(g_1 - g_2)$

$$\text{also F is the mid-point of AC} \therefore \text{level of } F = \frac{L}{400}(g_1 - g_2)$$

The dimension BE is usually denoted by 'e', and H and G are assumed to coincide

$$\therefore e = \frac{FB}{2} = \frac{BG - FG}{2} = \frac{1}{2}\left[\frac{L}{200} \cdot g_1 - \frac{L}{400}(g_1 - g_2)\right]$$

i.e. $$e = \frac{1}{2}\left[\frac{2Lg_1 - Lg_1 + Lg_2}{400}\right] = \frac{L(g_1 + g_2)}{800}$$

$g_1 + g_2$ may be written $g_1 - (-g_2)$, and this expression is the algebraic difference of the gradients which may be represented by

G. Hence $e=\dfrac{LG}{800}$, and this equation will be found applicable to any possible combination of gradients, up or down, provided that the sign convention is observed and that the parabolic vertical curve is symmetrical about the mid-point E.

The value of e gives the dimension of the vertical offset at the mid-point of the curve. For the offset y at any other point on one half of the curve distant x from the tangent point we may use the well-known property of the parabola which is expressed in the relationship

$$\frac{e}{y}=\frac{\left(\frac{L}{2}\right)^2}{x^2},\ \text{or}\ y=\frac{e.x^2}{\left(\frac{L}{2}\right)^2}$$

Numerical example. An up-gradient of 1 in 100 is to be connected with an up-gradient of 1 in 40 by a vertical curve 400 feet long having equal tangents.

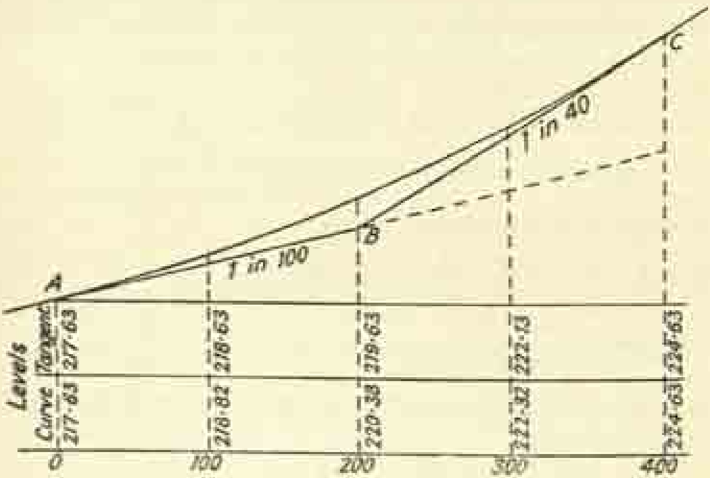


Fig. 143

The reduced level of the lower tangent point is 217·63 feet above datum. Calculate the reduced levels at 100-foot intervals along the curve.

The above conditions are represented in Fig. 143, in which both gradients are exaggerated.

The level at the intersection point of the tangents (B) is the level of the lower tangent (A) $+ 2 \cdot 00 = 219 \cdot 63$ feet above datum

$$g_1 = 1\%,\ g_2 = 2 \cdot 5\%,\ G = g_1 - g_2 = -1 \cdot 5$$

Vertical offset at intersection of tangents

$$= e = \frac{LG}{800} = \frac{400 \times -1 \cdot 5}{800} = -0 \cdot 75$$

(The negative sign indicates that the offset is to be measured upwards instead of downwards).

$$\therefore \text{level at mid-point of curve} = 219 \cdot 63 + 0 \cdot 75 = 220 \cdot 38$$

$$\text{offset at 100 feet from A} = -\frac{1^2}{2^2} \times 0 \cdot 75 = -0 \cdot 19$$

The negative sign is ignored, and the offset added to the level of the tangent 100 feet from A.

This is $217 \cdot 63 + 1 \cdot 00 = 218 \cdot 63$

$\therefore$ level of curve at 100 feet from A $= 218 \cdot 63 + 0 \cdot 19 = 218 \cdot 82$

Level of tangent at 300 feet from A $= 219 \cdot 63 + 2 \cdot 50 = 222 \cdot 13$

Vertical offset will be the same as at 100 feet i.e. $0 \cdot 19$

$\therefore$ level of curve at 300 feet from A $= 222 \cdot 32$ feet above datum

Level of upper tangent point (C) $= 219 \cdot 63 + 5 \cdot 00 = 224 \cdot 63$

The level of the curve at 300 feet from A may be checked by assuming that the lower tangent is produced this distance.

Level of lower tangent at 300 feet $= 217 \cdot 63 + 3 \cdot 00 = 220 \cdot 63$

Vertical offset from this point $= \dfrac{3^2}{2^2} \times 0 \cdot 75 = 1 \cdot 69$

$\therefore$ level of curve at 300 feet from A $222 \cdot 32$, as before.

Vertical curves with unequal tangents. It sometimes happens in road construction that the floor levels of existing structures, the requirements of headroom under bridges and other practical considerations make it necessary to use a vertical curve with tangents of unequal lengths in order to conform with fixed points on the longitudinal section.

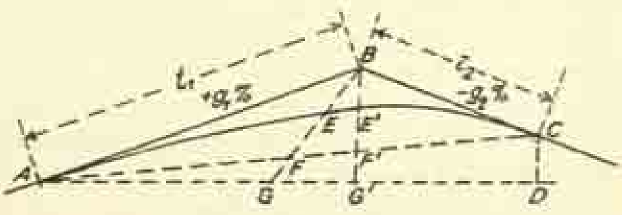


Fig. 144

Let l_1 and l_2 be the lengths of the tangents AB and BC in such a case (Fig. 144) and $+g_1$ and $-g_2$ the percentage gradients.

Let AEC represent the curve, and let F be the mid-point of the chord AC. Then BE = EF = e. Let AD be the horizontal through A and CD the vertical through C, and let BG′ be a vertical line intersecting the curve at E′ and the chord at F′. For normal gradients no serious practical error will occur if it be assumed that

$$BE' = \tfrac{1}{2}\,BF' = e$$

Assuming, as before, that the level of A is zero,

$$\text{the level of B is } \frac{l_1 \cdot g_1}{100}$$

$$\text{and the level of C is } \frac{l_1 \cdot g_1}{100} - \frac{l_2 \cdot g_2}{100} = CD$$

From similar triangles ACD, AF′G′,

$$\frac{F'G'}{CD} = \frac{AF'}{AC} = \frac{l_1}{l_1 + l_2} \text{ very nearly}$$

$$\therefore BF' = \frac{l_1 \cdot g_1}{100} - \frac{l_1}{l_1 + l_2}\left(\frac{l_1 \cdot g_1}{100} - \frac{l_2 \cdot g_2}{100}\right)$$

$$= \frac{l_1^2 . g_1 + l_1 l_2 g_1 - l_1^2 . g_1 + l_1 l_2 g_2}{100\,(l_1 + l_2)}$$

$$= \frac{l_1 . l_2}{100\,(l_1 + l_2)}\Big[g_1 - (-g_2)\Big]$$

$$\therefore e = \frac{BF'}{2} = \frac{l_1 . l_2}{200(l_1 + l_2)} . G$$

The offsets for intermediate points are calculated as before.

Position of highest or lowest point on curve. The highest point of a summit curve or the lowest point of a valley will occur only at the mid-point if the tangent lengths and gradients are equal. When the gradients are unequal the positions of these points may be found thus:

In Fig. 145, let Z be the highest point of the curve AEC, distant x from the tangent point A and let y be the vertical offset from the tangent at this point (FZ).

Then $$\frac{y}{e} = \frac{x^2}{l_1^2}, \text{ or } y = \frac{e . x^2}{l_1^2}$$

$$\therefore \text{height of Z above A} = h = \text{level of F} - y = \frac{x . g_1}{100} - e . \frac{x^2}{l_1^2}$$

As Z is the highest point, the slope of the curve at Z is zero

i.e. $$\frac{dx}{dh}=0$$

i.e. $$\frac{g_1}{100}-2.e.\frac{x}{l_1^2}=0$$

$$\therefore x=\frac{g_1.l_1^2}{200.e}$$

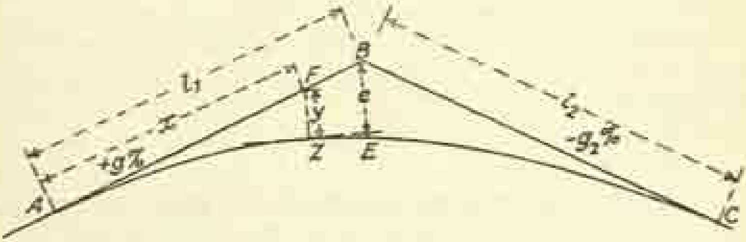


FIG. 145

If x, as given by this equation, is greater than l_1 this indicates that the highest point on the curve is nearer to C than to A and its distance must be measured from C using the formula

$$\frac{g_2.l_2^2}{200.e}$$

Numerical example:

Length of curve: 400 feet. Equal tangents.

Gradient AB: 1 in 400 up i.e. $g_1=0{\cdot}25\%$

Gradient BC: 1 in 100 down i.e. $g_2=-1{\cdot}00\%$

$G=g_1+g_2=1{\cdot}25$

Assume first that the highest point occurs between A and D at a distance x from A.

Then $$x=\frac{g_1.l_1^2}{200.e} \text{ and } e=\frac{LG}{800}=\frac{400\times 1{\cdot}25}{800}=0{\cdot}62 \text{ feet}$$

$$x=\frac{0{\cdot}25\times 200\times 200}{200\times 0{\cdot}62}=80 \text{ feet very nearly}$$

This distance is less than L/2 therefore the assumption is correct. The height of E above A is given by

$$\frac{x.g_1}{100}-e.\frac{x^2}{(L/2)^2}=\frac{80}{100}\times{\cdot}25-0{\cdot}62\times\frac{80^2}{200^2}$$
$$={\cdot}20-{\cdot}10={\cdot}10 \text{ feet}$$

Exactly the same principle may be used for finding the position of the lowest point on a valley curve.

SIGHTING DISTANCE AT ROAD SUMMITS. In order to provide adequate visibility over the summits of vertical curves on roads, minimum sighting distances are specified by the Ministry of Transport in Memorandum No. 575, an official publication giving recommendations for various features in road design. These recommendations are revised from time to time, but the current (1954) minimum sighting distance for single carriageway roads is 1000 feet and for dual carriageway roads 500 feet, the height of the line of sight above the road surface being taken as 3 feet 9 inches.

It is possible from this data to calculate the minimum length of a summit curve to suit any given combination of gradients, but two possibilities have to be considered; the required sighting distance may be obtained while two vehicles are on the tangents and have not reached the vertical curve or, alternatively, the sighting distance is obtained after the vehicles have passed the tangent points and are travelling on the curve. These two conditions are illustrated in Figs. 146 and 147, respectively, and for the sake of simplicity it will be assumed that the two vehicles occupy symmetrical positions in relation to the curve.

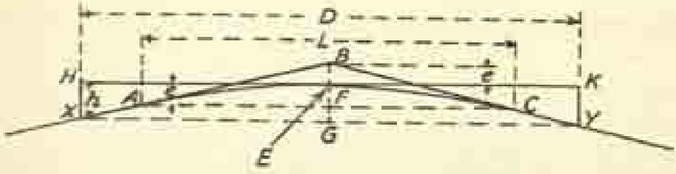


FIG. 146

(1) *Vehicles on tangents* (Fig. 146).

Let AB, BC be the gradients, $+g_1$ and $-g_2$, respectively, AEC the curve of length L and L/2 the tangent length. Let the vehicles occupy positions X and Y such that the line of sight, HK, at a height h above the road surface, is tangential to the summit of the curve when the minimum sighting distance, D, is secured. Let BG be a vertical line intersecting the curve at E, the chord AC at F and the line XY at G. From the similar triangles FAB and GXB:

$$\frac{FB}{BG}=\frac{AF}{XG}.\ \text{But } FB=2e, \text{ very nearly and } BG=e+h$$

Also, $$\frac{AF}{XG}=\frac{L}{D}\therefore L=D\times\frac{2e}{e+h}=\frac{2D\times\frac{LG}{800}}{\frac{LG}{800}+h}$$

$$\text{or } L=\frac{2GD-800\,h}{G}$$

If we put D = 1000 feet and $h=3{\cdot}75$ feet, this equation becomes

$$L=\frac{2000\,G-3000}{G}=2000-\frac{3000}{G} \quad . \quad . \quad (1)$$

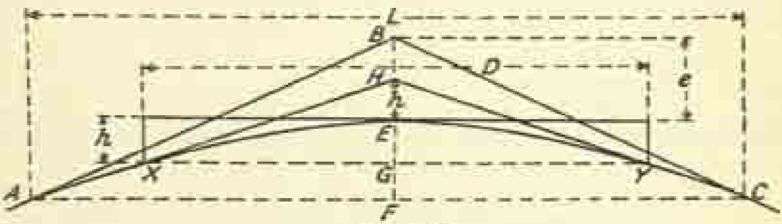


FIG. 147

(2) *Vehicles on curve* (Fig. 147).

Again, let X and Y be the positions of the vehicles with the line of sight tangential to the curve at E. Then XEY is part of the parabola.

Let the tangents at X and Y meet at H

Then, very nearly, HE = EG = h and BE = EF = e

From the property of the parabola: $\dfrac{e}{AB^2}=\dfrac{h}{XH^2}$

i.e. $$\frac{e}{L^2}=\frac{h}{D^2}\text{ or } L^2=\frac{e.D^2}{h}=\frac{\frac{LG.D^2}{800}}{h}\text{ or } L=\frac{GD^2}{800.h}$$

Again putting D = 1,000 and $h=3{\cdot}75$

$$L=\frac{1000\times1000}{3000}.\,G=333{\cdot}3\,G \quad . \quad . \quad (2)$$

If the required sighting distance occurs when the vehicles reach the tangent points,

$$333{\cdot}3\,G=2000-\frac{3000}{G}$$

This equation is satisfied when G = 3,

in which case L = D = 1000 feet.

The assumption made in equation (1) is that D is greater than L, but, if G is greater than 3, L will be greater than 1000 i.e. D is less than L and the assumption is contradicted. Hence when the algebraic difference in gradients exceeds 3% equation (2) must be used. This equation is based on the assumption that D is less than L and, for values of G greater than 3, L is greater than 1000 and the assumption is correct. For different values of h and D, a different value of G must be established which satisfies both equations and gives an indication as to which equation to use for any particular difference of gradients.

EXAMPLES FOR EXERCISE

(1) A circular curve is to touch two lines AB, BC (apex angle 136° 36′, deflection *left*), and to pass accurately through a point X such that BN = 276 links and NX = 88 links, where N is in BA and NX is perpendicular thereto. Calculate the radius, the tangent distance, the length of curve, and tabulate the tangential angles for chain chords, the first chord being a whole chain.

Ans. 22 chains, 64·4 links; 901·1 links; 17·15 chains;
first angle = 358° 44′ 5″; last = 338° 18′.

(2) Make the calculations for a curve of 10 chains radius, with a Glover's spiral at each end, to connect two straights, the apex angle being 120°. Take points 1 Gunter's chain apart, and length of transition curve = $\sqrt{\text{radius}}$, both in chains.

Ans. λ = 100, 200, 300, 316·23 links.
θ = 18′ 10″; 1° 12′ 30″; 2° 43′ 0″; 3° 1′ 10″, to nearest 10″.
ϕ_n = 9° 3′ 30″; AB = 16·64; SB = 315·44; BC = 104·36; XS = 737·76 links;
length of circular arc = 14 half-chain chords and 31·0 links over;
SAP = 186° 2′ 20″.

(Letters refer to Figs. 135 and 137).

(3) A reverse curve is to be run from a point A on the line AC to the point B on the line DB (Fig. 148). AB = 12·42 chains; CAB = 32° 14′; DBA = 16° 48′.

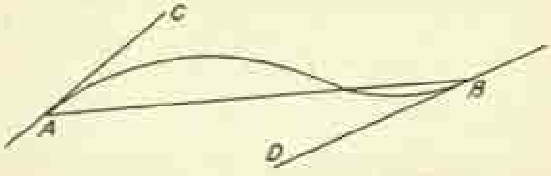


FIG. 148

The radius of the curve connecting A to the common tangent is 8 chains. Find the radius of the second curve, and give general directions for setting out the curve.

Ans. r_2 = 6·23 chains; deflection angles = 58° 50′ 30″ and 43° 24′ 30″.

(4) Two straight lengths in the centre line of a projected road have an intersection angle of 23° 18′, measured from one straight produced to the other straight. They are to be connected by a 6° circular curve. If the chainage of the intersection point is 88 + 96 feet, draw up a table of tangential angles for setting out the curve.

Ans.				
Chainage	88 + 00	89 + 00	90 + 00	90 + 87·3
Tangential angle	3° 2′	6° 2′	9° 2′	11° 39′ (to nearest minute)

(5) As an alternative to the curve in question 4 it is proposed to use two spirals, each 200 feet long, in conjunction with a central circular arc having a curvature of 6°. Calculate the shift and tangent distance for this arrangement. Recognized practical approximations may be used.

(Univ. of Lond. Imp. Coll.)

Ans. Shift: 1·73 feet; tangent distance: 297·3 feet.

(6) The deflection angle between two straights forming the tangents of a highway curve is 48°. The curve is to consist of a central circular arc with two equal transition spirals and the following conditions are to be satisfied:

(*a*) radius of circular arc: 700 feet.

(*b*) external distance not greater than 80 feet.

(*c*) tangent length not greater than 530 feet. It is proposed to adopt a spiral length of 400 feet. Ascertain whether this length is suitable.

(Univ. of Lond. Imp. Coll.)

Ans. Tangent length: 516·2 feet, external distance: 77·7 feet.

(7) At a point on the longitudinal section of a road a rising gradient of 1 in 20 is followed by a falling gradient of 1 in 12½. These gradients are to be connected by an unsymmetrical parabola, the tangent length on the 1 in 20 gradient being 300 feet and the other tangent 400 feet. Calculate (*a*) the offsets from the tangents which would give peg levels at 100-foot intervals for locating the curve, (*b*) the position of the highest point.

(Univ. of Lond. Imp. Coll.)

Ans. If A is the tangent point on the 1 in 20 gradient, offsets are as follows:

Distance from A (feet)	Offset (feet)
100	1·24
200	4·95
300	11·14
400	6·27
500	2·79
600	0·70

Highest point 201·9 feet from A

(8) A rising gradient of 1 in 50 on a proposed road meets a falling gradient of 1 in 40 and the reduced level of the intersection point is found from a longitudinal section to be 207·54 feet above O.D. These gradients are to be connected by a parabolic vertical curve, and a sighting distance of 1000 feet is to be provided

over the summit, assuming that the line of sight is 3 feet 9 ins. above the road surface. Calculate the necessary levels for setting out the vertical curve.

(Univ. of Lond. Imp. Coll.)

Ans. Length of curve required to give 1000 feet sighting distance is 1500 feet.

Distance from tangent point on 1 in 50 gradient (feet)	Reduced level in feet above OD
0	192·54
100	194·39
200	195·94
300	197·19
400	198·14
500	198·79
600	199·14
700	199·19
750	199·10
800	198·94
900	198·39
1000	197·54
1100	196·39
1200	194·94
1300	193·19
1400	191·14
1500	188·79

CHAPTER 6

TACHEOMETRIC SURVEYING

DEFINITION. Tacheometric surveying, or, briefly, 'Tacheometry', is the science of surveying and levelling by means of angular measurements from a known station, combined with determination of distances from the station by the use of webs or lines in the instrument reading on a known base held at the point whose distance is required.

The webs or lines may be *fixed*, so that the distance between them is constant, in which case the length of base must be variable, and the readings are taken on a graduated staff like a levelling staff; or they may be movable, their distance apart being given by a micrometer, in which case a fixed length of base is employed.

The former arrangement is by far the more common, and will be considered first.

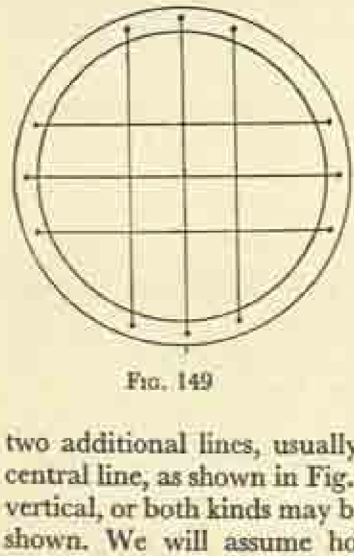


FIG. 149

THE TELESCOPE AS A DISTANCE MEASURER. The only essential difference between a tacheometer (as the instrument to be described is called) and an ordinary theodolite, therefore, is in the diaphragm. In addition to the webs marking the central point, this contains at least two additional lines, usually horizontal, one on each side of the central line, as shown in Fig. 149. The extra lines may, instead, be vertical, or both kinds may be fitted in the same instrument as here shown. We will assume horizontal lines at present. The lines themselves may be spider-webs, lines ruled on glass, or metal points as mentioned already in the description of the level (Chapter 2). In any case they are often called 'stadia' or 'subtense' lines, cross-hairs or cross-wires. If metal points are used, the distance between them is sometimes made adjustable.

Now let O (Fig. 150) be the object-glass, D the diaphragm, with the three hairs a, c, b, where the distance ab is fixed, and c is usually midway between.

Let ss_1 be a graduated staff held vertically at the point whose distance is required.

Then when the telescope is properly focussed, an image of the staff will be formed on the plane of the diaphragm, and the cross-lines a, b will intersect the image at points corresponding with say s, s_1 on the staff. These readings are taken.

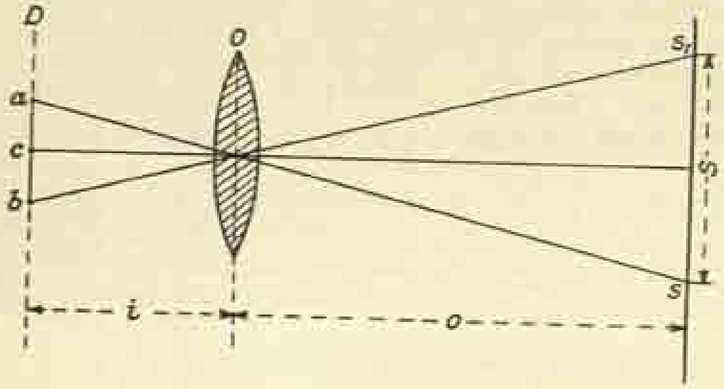


FIG. 150

Put f = focal length of object-glass
S = staff intercept ss_1, the difference between the readings
$\Delta = ab$, the distance between the lines
o = required distance of staff from object-glass
i = distance of diaphragm from object-glass.

Then by the formula for the conjugate foci of a lens $\left(\frac{1}{f}=\frac{1}{u}+\frac{1}{v}\right)$

we have $$\frac{1}{f}=\frac{1}{i}+\frac{1}{o}, \text{ also } \frac{\Delta}{i}=\frac{S}{o}$$

Hence $$\frac{1}{i}=\frac{1}{f}-\frac{1}{o}=\frac{S}{\Delta . o}$$

$$\therefore o = S \times \frac{f}{\Delta}+f$$

The distance Δ between the lines is usually adjusted so that the ratio $\frac{f}{\Delta}$ shall be, as nearly as possible, exactly some round number, usually 100.

The ratio is called the *constant multiplier*, and we will denote it by c.

$$\therefore o = cS + f$$

But the distance actually required is from the *centre of the instrument* not far from the object-glass.

Let d be the distance from the object-glass to the centre of the horizontal axis of the telescope.

D the distance from the centre of the instrument to the staff.

Then clearly

$$\begin{aligned} D &= o + d \\ &= cS + (f + d) \end{aligned}$$

The amount $(f + d)$ is called the *additive constant*.

Its value is usually given by the makers, but if not it is easily found as follows: Focus the telescope on any distant object, not less than say 500 feet away, and measure the distance between the object-glass and the diaphragm screws. This gives the value of f with sufficient accuracy. Then measure d directly, and add.

The formula for distance then becomes

$$D = cS + k$$

where $c = \frac{f}{\Delta}$, and $k = (f + d)$ as above

GEOMETRICAL PROOF. A direct geometrical proof of this may also be given as follows.

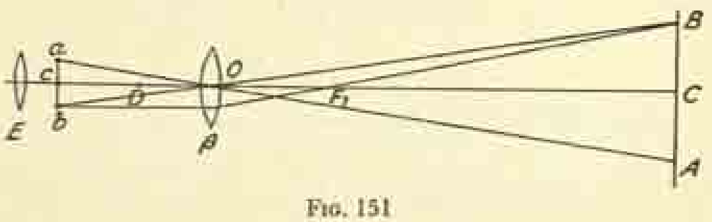


FIG. 151

Let a and b (Fig. 151) be the outer horizontal diaphragm lines as before, and suppose that, when the instrument is focussed on a staff AB, the image of A coincides with the line a, and that of B with the line b. Then all the rays from B will converge to b, but, considering the particular one βb, which, after emerging from the object-glass is parallel to the optical axis, it is clear that in its path it must have passed through F_1, the principal focus of the object-glass, exterior to the telescope.

Now, since when the eyepiece and diaphragm are moved for focussing the staff at different distances, the point b simply travels along the line $b\beta$, these rays will always cut the optical axis produced at the same point F_1, and *at the same angle*. Hence, the distance F_1C of the staff from F_1, is proportional to the length BA intercepted. That is, whatever be the distance,

$$F_1C : BC = F_1O : O\beta$$

But as $BC = \frac{1}{2}BA$, and $O\beta = \frac{1}{2}ab$, we have

$$F_1C : AB = F_1O : ab$$

$$\text{i.e. } F_1C : S = f : \Delta$$

$$\therefore F_1C = S \times \frac{f}{\Delta}$$

and if D be the centre of the instrument, $DO = d$; $OF_1 = f$

$$\therefore DC = F_1C + DF_1$$

$$= S \times \frac{f}{\Delta} + (f + d), \text{ or } cS + k \text{ as before}$$

THE ANALLATIC LENS DISCUSSED. In order to avoid the addition of the constant k at each reading, a third lens, called an 'anallatic lens', was introduced by Porro, of Milan.

Fig. 152 shows the arrangement, and the course of rays from a staff AB. O represents the centre of the object-glass. K is the centre of the anallatic lens, and F_1 is its principal focus.

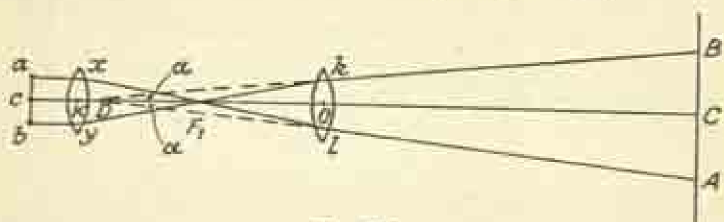


FIG. 152

Let a, c, b be the diaphragm reference lines as before, and A, C, B the corresponding points in the graduated staff. D is the centre of the instrument.

Now, in considering the problem geometrically, we single out those rays of light which, after passing through the anallatic lens, are parallel to the principal axis of the lens, like *ax*, *by*. In modern instruments the anallatic lens serves also as an internal focussing lens and has a small range of movement along the telescope axis.

The paths of the rays are not affected by altering the focus, for this simply moves the anallatic lens parallel to the axis F_1K, so that the paths *ax*, *by* will not be altered.

On the other side of the lens, these paths will always pass through the focus F_1 at a fixed angle.

The object-glass is therefore intersected at points *k*, *l* by these rays, coming from F_1 at a fixed angle. Hence the paths *k*B, *l*A of these rays *outside* the object-glass must be fixed in direction but the intercept *kl* will vary by a very small amount according to the position of the anallatic lens. This variation is so small, however, that it may be neglected.

Let these directions produced backwards meet the axis in D.

Hence D for all practical purposes will be a fixed point, and the angles BDC, CDA will also be fixed. Call these angles α

$$\text{Then clearly } DC = \cot\alpha \times BC = \tfrac{1}{2}\cot\alpha \times AB$$

That is, if, by suitable adjustment of the parts, D be made to coincide with the centre of the instrument, and $\frac{1}{2}\cot\alpha$ be made some round number, say 100, distances will be obtained directly from D, without any correction.

Let f = focal length of object-glass
f_1 = focal length of anallatic lens
$OK = d$ = distance between lenses
$ab = \Delta$; $AB = S$, as before.
$\therefore F_1O = d - f_1$

F_1 and D are conjugate foci for the object-glass

Hence
$$\frac{1}{OD} + \frac{1}{f} = \frac{1}{OF_1} = \frac{1}{d-f_1}$$
$$\therefore \frac{1}{OD} = \frac{1}{d-f_1} - \frac{1}{f} = \frac{f+f_1-d}{f(d-f_1)}$$
$$\therefore OD = \frac{f(d-f_1)}{f+f_1-d}$$

This gives the necessary distance between the centre D and the object-glass.

Again, $\cot\alpha = \dfrac{DO}{Ok}$

But $\dfrac{Ok}{OF_1} = \dfrac{yK}{KF_1}$

$$\therefore Ok = \frac{OF_1 \times yK}{KF_1} = \frac{(d-f_1)\times\frac{1}{2}ab}{f_1}$$

$$\therefore \cot\alpha = \frac{f(d-f_1)}{(f+f_1-d)} \times \frac{f_1}{(d-f_1)\times\frac{1}{2}ab}$$

$$= \frac{ff_1}{(f+f_1-d)} \times \frac{1}{\frac{1}{2}ab}$$

Hence $\Delta = ab = \dfrac{ff_1}{(f+f_1-d)} \times \dfrac{1}{\frac{1}{2}\cot\alpha}$

If we put $\frac{1}{2}\cot\alpha = 100$, this gives the requisite distance between the diaphragm reference lines.

It is clear that f, f_1, and d can be chosen so as to give suitable values both to OD and ab, or Δ.

Then $\frac{1}{2}\cot\alpha = \dfrac{1}{\Delta} \times \dfrac{ff_1}{(f+f_1-d)}$

and distance $=$ DC $=$ S $\times \frac{1}{2}\cot\alpha$

Thus suppose $f = 10{\cdot}5$ ins.; $f_1 = 6$ ins.; $d = 9$ ins.

Then OD $= 4{\cdot}2$ ins.; $ab = 0{\cdot}084$ in., if $\frac{1}{2}\cot\alpha = 100$.

Now suppose ab is accidentally made, say, 0·078 in. instead of 0·084 in. (a big error) in consequence of the difficulty in placing the lines exactly at the right distance apart. Hence $\cot\frac{1}{2}\alpha$ will not be exactly 100. But by slightly altering d, the distance between the lenses, we can bring $\frac{1}{2}\cot\alpha$ back to this exact value.

We have seen that $ab = \dfrac{ff_1}{f+f_1-d} \times \dfrac{1}{\frac{1}{2}\cot\alpha}$

In this equation put $ab = 0{\cdot}078$, $f = 10{\cdot}5$, $f_1 = 6$, $\frac{1}{2}\cot\alpha = 100$

$$\therefore d = f + f_1 - \frac{ff_1}{\frac{1}{2}\cot\alpha \times ab} = 8.42 \text{ ins.}$$

whereas before it was 9·00 ins. Hence d must be decreased 0·58 in.

This will also affect the value of OD, and hence the position of D however.

We have seen that OD $= \dfrac{f(d-f_1)}{f+f_1-d} = 3{\cdot}1$ ins.

Whereas before it was 4·2 ins.

Hence all distances will be in error by 1·1 in.

being obtained from a point that distance in front of the true centre of the instrument.

Now, in order to obtain distances to this accuracy with a tacheometer, we should have to read to the *one thousandth* of a foot on the staff. Hence, as we usually read only the nearest *hundredth*, the error is clearly negligible, even in this extreme case.

TELEMETERS. The general name for an instrument which acts as a distance measurer, without the use of a chain or other direct measuring apparatus, is a *telemeter*. Thus the tacheometer is a particular form of telemeter, and the telescope of it is sometimes called a telemetric telescope.

SLOPING GROUND. Hitherto it has been assumed that the staff is held vertically and that the line of sight is perpendicular to it (in other words, that the telescope is horizontal).

In ordinary tacheometry this will seldom be the case, and it becomes necessary to determine the effect of a departure from these conditions. We will assume that the staff is held vertically.

In Fig. 153, let E be the station on the ground, D the centre of the tacheometer, F the point where the staff FAB is held, and A, C, B the points where the staff appears to be intersected by the three lines. It is required to find the horizontal distance EG, and the difference of level GF. The vertical angle HDC or β must, for this purpose, be read at the same time as the other data.

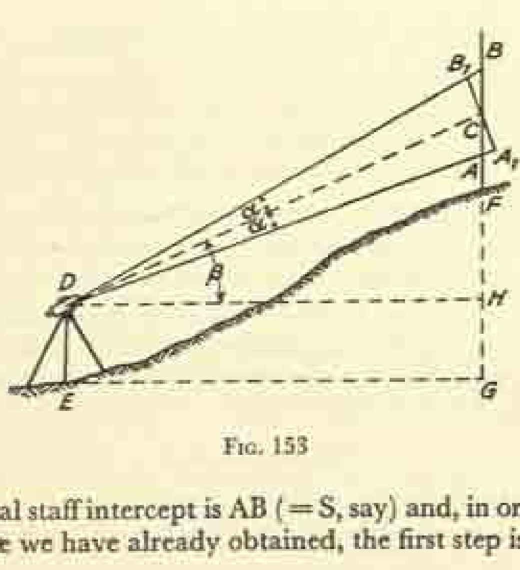


FIG. 153

The actual staff intercept is AB (= S, say) and, in order to apply the formulæ we have already obtained, the first step is to find out what its value *would have been* if the staff had been held at right angles to the line of collimation, which was the condition previously assumed.

Draw A_1CB_1 perpendicular to DC.

Then angle $CDA = \alpha$, where $\cot \alpha$ is usually 200; and angle $ADH = \beta - \alpha$.

$$\therefore CAD = 90^\circ + ADH = 90^\circ + \beta - \alpha$$

$$\therefore \frac{AC}{CD} = \frac{\sin \alpha}{\sin (90^\circ + \beta - \alpha)} = \frac{\sin \alpha}{\cos (\beta - \alpha)}$$

Similarly we can show that

$$\frac{CB}{CD} = \frac{\sin \alpha}{\sin (90^\circ - \beta - \alpha)} = \frac{\sin \alpha}{\cos (\beta + \alpha)}$$

therefore by addition

$$\frac{AB}{CD} = \sin \alpha \frac{\cos (\beta + \alpha) + \cos (\beta - \alpha)}{\cos (\beta - \alpha) . \cos (\beta + \alpha)} = \frac{2 . \sin \alpha . \cos \beta . \cos \alpha}{\cos (\beta - \alpha) . \cos (\beta + \alpha)}$$

But $$A_1C = CB_1 = CD . \tan \alpha$$

$$\therefore \frac{A_1B_1}{CD} = 2 \tan \alpha$$

therefore by division

$$\frac{A_1B_1}{AB} = \frac{\tan \alpha . \cos (\beta - \alpha) . \cos (\beta + \alpha)}{\sin \alpha . \cos \beta . \cos \alpha}$$

$$= \frac{\cos^2 \beta \cos^2 \alpha - \sin^2 \beta \sin^2 \alpha}{\cos^2 \alpha . \cos \beta}$$

$$= \cos \beta - \tan^2 \alpha . \sin \beta . \tan \beta$$

Now $\tan \alpha = \frac{1}{200}$, so that the second term is exceedingly small. In order that it might amount to 1 part in 1000, as compared with $\cos \beta$, it would be necessary that β should be 81°. Such angles of elevation or depression are unknown, and for smaller angles the effect of neglecting this term decreases rapidly. If $\beta = 45^\circ$, the error due to neglecting the second term is 1 part in 40,000; we may therefore ignore it and write

$$A_1B_1 = AB . \cos \beta$$

A more simple approximate proof is arrived at if we put $CA_1A = 90^\circ$, very nearly, and $AB = 2AC$, nearly.

Then $A_1C = AC \cos ACA_1$

But $ACA_1 = 90° - ACD = \beta$

$\therefore A_1C = AC . \cos \beta$

hence $A_1B_1 = AB \cos \beta$, by doubling both sides.

Accepting this formula, therefore, as sufficiently close, we have by the rules already given (p. 275)

$$DC = c \times A_1B_1 + k$$
$$= c \times S . \cos \beta + k$$

where c and k are the constant multiplier and additive constant respectively, and $S = AB$.

Now clearly $DH = DC . \cos \beta$

or $DH = cS . \cos^2 \beta + k \cos \beta$

This gives the horizontal distance, and

$$HC = DC . \sin \beta$$
$$= c . S . \cos \beta . \sin \beta + k \sin \beta$$

Now the true rise $GF = HC + GH - FC$, where GH is the height of the instrument above the ground (which should be measured at each station by the graduated staff itself or by a graduated plumb-bob), and FC is the reading of the *middle line* on the staff.

Hence we have –

rise $= c . S . \cos \beta \sin \beta + k \sin \beta +$ height of instrument
– reading of middle line.

In this formula β is the angle of elevation, and may be *plus* or minus. The signs of the first two terms will vary accordingly.

The whole is summed algebraically, and the result, if *plus*, means a *rise* (and, if *minus*, a *fall*) from the station to the point.

If there is an anallatic lens, k will be zero in both formulæ.

In any case, $k \cos \beta$ is usually taken as equal to k simply, and $k \sin \beta$ is ignored, β not being a very big angle as a rule.

Hence we have the following rules:

Distance $= c . S . \cos^2 \beta + k$

Rise $= c . S . \cos \beta . \sin \beta +$ height of instrument – middle reading.

EFFECT OF SLOPE OF STAFF. Now suppose that the staff, instead of being held truly vertical, like XY, is held at an angle of $\theta°$ to that direction, as shown at FB, Fig. 154. Let C be the middle point of AB.

$$\begin{aligned}
\text{Then the angle } A_1CA &= A_1CX - FCX \\
&= \beta - \theta \\
AA_1C &= 90^\circ - \alpha, \text{ as before} \\
&= 90^\circ \text{ nearly} \\
\therefore A_1C &= AC \cos(\beta - \theta) \\
\text{and } A_1B_1 &= AB \,.\, \cos(\beta - \theta) \\
&= AB(\cos\beta\cos\theta + \sin\beta\sin\theta)
\end{aligned}$$

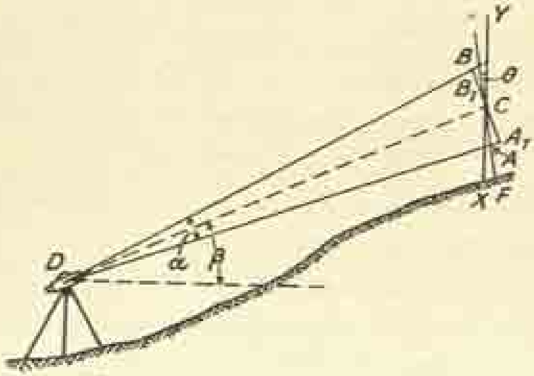


FIG. 154

Since θ will normally be a small angle we may take $\cos\theta$ as unity, so that the result will be $A_1B_1 = AB(\cos\beta + \sin\beta\sin\theta)$, instead of $AB\cos\beta$, which we obtained for the vertical staff.

The error is $AB\sin\beta\sin\theta$, which, expressed as a fraction of $AB \cos\beta$, is $\tan\beta \,.\, \sin\theta$, or, in parts per 1000, $1000\tan\beta\sin\theta$.

Taking $\theta = 1^\circ$, the values of this for different values of β are given in the annexed table, which therefore shows the error, in

β	$1000 \tan\beta \sin\theta$
5°	1·5
10°	3·1
20°	6·3
30°	10·0
40°	14·6

parts per thousand, due to the staff being held 1° out of the true vertical, β being the angle of elevation or depression at which the reading is taken. For other small values of θ (the slope of staff) the error may be taken as proportional to θ. Thus if $\beta = 30°$, and the staff be 30 minutes out of the true vertical, the corresponding error will be 5 parts in 1000.

It is quite clear, therefore, that the effect of even a small slope of the staff may be considerable on steep ground, and special precautions must be taken to ensure verticality. For means of doing this, see the chapter on 'Levelling', p. 88. Waving the staff must not be resorted to, however, as the vertical staff does not give the lowest reading with an inclined telescope.

OTHER METHODS OF HOLDING STAFF. The above source of error has led, together with the arithmetical labour of reduction, to other methods of holding the staff.

Of these, one is to hold it at right angles to the line of sight. The staff is fitted with a pair of sights at right angles to its length, and the staff-holder aims these at the instrument.

Any slight error in the direction of the staff in this position produces little error in the length of the staff intercept.

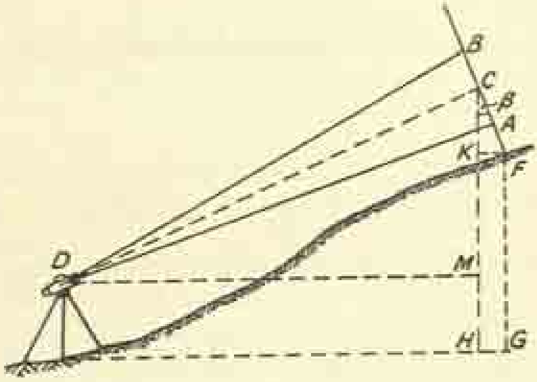


FIG. 155

In Fig. 155, let FB be the staff at right angles to DC

$S =$ staff intercept, AB
$\beta =$ angle of elevation
$c =$ multiplying constant
$k =$ additive constant, if any.

$$
\begin{aligned}
&\text{Then} && DC = cS + k \\
&\text{and} && DM = DC \cos\beta = (c.S + k)\cos\beta \\
&\text{Now angle} && DCH = 90^\circ - \beta \\
&\text{But} && DCH = 90^\circ - KCA \\
& && \therefore KCA = \beta \\
&\text{And } \textit{horizontal distance} && = DM + HG \\
& && = (cS + k)\cos\beta + FC.\sin\beta \\
&\text{and} && \textit{rise} = GF = MC + HM - KC \\
& && = (cS + k)\sin\beta + HM - FC.\cos\beta
\end{aligned}
$$

Here FC is the reading of the middle line, and HM the height of the instrument.

These results may be taken out from traverse tables if desired. If the formulæ be used as here given, however, the arithmetic is scarcely less complicated than with staff vertical, particularly as tacheometric tables are available.

Approximate formulæ. If β be small, we may neglect FC $\sin\beta$ as well as $k\sin\beta$, and put FC $\cos\beta =$ FC, $k\cos\beta = k$.

The formulæ then become –

$$
\begin{aligned}
\text{distance} &= cS\cos\beta + k \\
\text{rise} &= cS\sin\beta + HM - FC
\end{aligned}
$$

With a middle reading of 5 feet, this makes an error of 1 foot in the horizontal distance, when β becomes about $11\tfrac{1}{2}^\circ$.

Angles of depression. If β be an angle of depression, $\sin\beta$ must, of course, be taken as negative in all the formulæ.

Comparison of methods. A disadvantage of this method of holding the staff is that we often wish to take a reading at a place where the staff-holder cannot see the instrument, though the observer there can easily read the top of the staff if held vertically. Again, it is difficult to hold the staff in the proper position on steep slopes, especially if it be at all windy; and, if one is compelled to rely on the staff-holder, it is wise to make the correct attitude as easy as possible.

There is a considerable majority of opinion, however, in favour of the vertical staff, with proper precautions to ensure true verticality.

The staff should be stiffer than an ordinary levelling staff. The latter, particularly if in three sections, bends so much in a wind that the upper sections may easily be more than 1° out of truth, even if the lowest section be vertical.

TACHEOMETER WITH TWO SPIRIT-LEVELS. Some makers supply the tacheometer with the upper level attached directly to the telescope; others fix it to the T-piece; while still other makers supply levels in *both* these positions.

In the opinion of the writer, the level attached to the T-piece is the more useful if readings can be taken with both faces (p. 166), because it can be constantly watched. But for tacheometry we must generally be content (on time-saving grounds) to read, at any rate, all ordinary points (as distinct from proposed new stations) with one face only.

In this case it is, perhaps, safer to use the level on the telescope, as the connection between it and the line of collimation is more rigid and less likely to develop sudden changes in index error. If the level on the T-piece is used, the index error must be frequently tested, say every day, or twice a day (see p. 178).

TESTING INSTRUMENT. In any case, of course, a test for index error is one of the most important tests to be applied to the instrument. All the permanent adjustments should be tested from time to time as for a theodolite.

The additive constant, if any, must be determined as already described (p. 275).

The constant multiplier is found by chaining out, on level ground, lengths of say 200, 300, and 400 feet, or whatever units may be in use, always *plus* k, where k is the additive constant, found as on p. 275.

Then the corresponding staff intercepts, S_2, S_3, S_4, should be exactly 2, 3, and 4 feet respectively, if the multiplier c is exactly 100. For we should have–

$$cS_2 + k = 200 + k, \text{ the chained distance}$$
$$\therefore cS_2 = 200, \text{ and so on.}$$

If S_2 be not 2·00, put $cS_2 = 200$, and find c. Check by the readings at the other distances.

If the results agree, but c is not a round number, its value cannot be corrected, if the lines or subtense wires are fixed and there is no anallatic lens.

If the lines consist of movable metal points, the latter are adjusted to make S_2, say, exactly 3 feet for the 300-foot distance, and then checked at 200 and 400 feet, and perhaps also at 100.

If there is bad agreement between the results for c, this may be due to the fact that there is really an anallatic lens, whereas in the above description we have assumed that there is *not*. If the surveyor does not know whether there is such a lens or not, then he should chain out distances of 200, 300, and 400 feet, find the staff intercepts, and put

$$200 = cS_2 + k$$
$$300 = cS_3 + k, \text{ and so on.}$$

Then find c and k simultaneously.

If k is about zero, there is an anallatic lens.

If a reasonable value for k is found, then find the correct value as before described (p. 275) and proceed as above until good agreement in the value of c is obtained.

Tacheometric tables. Tacheometric tables giving the results of the reduction due to vertical angle can be obtained. Among the best known are those compiled by F. A. Redmond (The Technical Press Ltd.) and Louis and Caunt (Messrs. Edward Arnold).

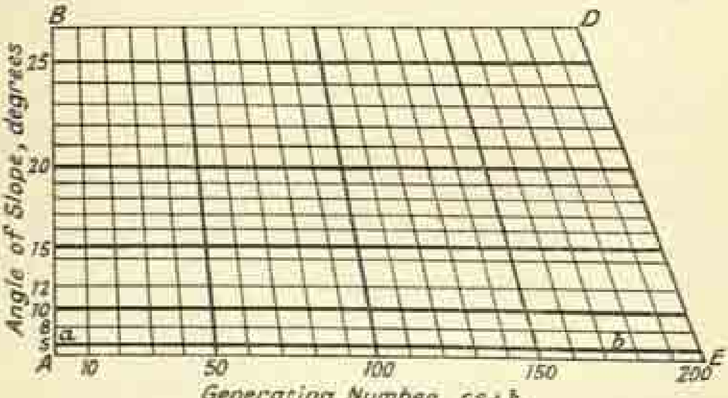


Fig. 156

Direct conversion scale. Fig. 156 shows a conversion scale for obtaining the horizontal distances direct to the scale of the plan. At the foot of the figure a scale of feet is drawn to the scale of the plan which is being made.

Along the vertical line AB distances are set off to a large scale, proportional to the *squares of the sines* of the different angles of elevation. Suppose that a scale of 1 inch = 0·02 was adopted. Thus, to this scale—

$$\text{the length in inches of A.10} = \frac{\sin^2 10°}{0{\cdot}02} = \frac{0{\cdot}0301}{0{\cdot}02} = 1{\cdot}5 \text{ inches.}$$

Through the points of division horizontal lines are drawn.

If now AB be produced until its length = 50 inches, or 1·00 on the scale adopted, the sloping lines in the figure are drawn by joining the points of division on the horizontal scale to the other end of the line so produced.

It is not, however, actually necessary to produce AB for the full length. Suppose AB in figure be made 10 inches long, so as to represent 0·2 to the given scale, then, if BD be drawn horizontally, each division along BD will be 0·8, or four-fifths of those on AE. Hence, the sloping lines can be drawn by setting off a scale on BD

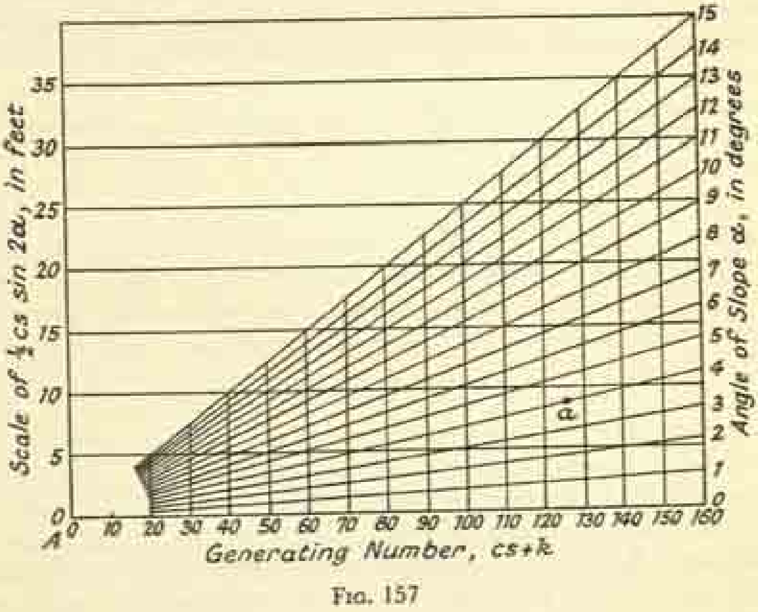


FIG. 157

four-fifths as great as the scale of the plan, and joining corresponding points. In practice we need not go beyond the highest angle of elevation likely to be observed.

To use the scale, suppose the value of $cS + k$ i.e. the uncorrected distance from the tacheometer be 176 feet, and the angle of elevation 8°. The quantity $cS + k$ is sometimes termed the 'Generating Number' and k is zero if, as in most modern instruments, an anallatic lens is provided. Enter the scale with the angle of elevation, and take the distance ab (Fig. 156) along the horizontal line for this angle up to the sloping line drawn from 176 on the scale of feet. This distance ab is simply taken off on the dividers and plotted direct on the plan. It is easy to prove from the construction that $ab = 176(1 - \sin^2 8°) = cs \cos^2 \alpha$.

Such a scale is particularly useful for plane-table work, and may be constructed on metal if desired.

Fig. 157 shows a similar scale for the difference of level. To construct it, if 160 be the highest generating number shown, work out the values of $160 \sin \alpha \cos \alpha$ and set them along the right-hand vertical (to the scale marked on the left), numbering the points so obtained according to the angle. Join to A. Then for a generating number of 125, angle 3° 40′, we obtain point a, which on the left-hand scale gives the difference of level as about 8 feet.

Mechanical contrivances. Several devices have, at different times, been introduced as attachments to the tacheometer, with the object of mechanically performing the reduction of the inclined distance to the horizontal plane, and at the same time obtaining the difference of level between the instrument and the staff station. Instruments with these added complications, however, are not in common use. One form of these instruments, however, is of sufficient interest to be worthy of mention and is briefly referred to below.

Direct-reading tacheometers. In a paper by Prof. Jeffcott (see *Proceedings of the Irish Institution of Civil Engineers* for 1915, vol. xli.) a 'direct-reading tacheometer' is described, in which the readings are taken by metal points. The distance between these points is automatically altered by means of special cams, in accordance with the vertical angle, in such a way that the staff intercept multiplied by a round number (100) *always* gives the horizontal difference directly. A second pair of points, separately worked, gives the rise or fall (subject to correction for height of instrument and middle reading) in the same way.

FIELD-WORK. In describing the field-work of a tacheometric survey, it will be assumed that a contoured plan of the ground is required. The first station for the instrument is chosen in such a position as to give a good view of the surrounding ground. The instrument is here levelled up for reading vertical angles, set to read zero at magnetic north, or along any known line, and its height read with the staff. The staff-holder then goes to all points, which must be shown on the plan in turn. These will include a sufficient number of points along roads, fences, or other boundary-lines; on buildings, ponds, etc.; and, for the contours, all points where marked changes of slope occur, along ridges and hollows, on the sides of rounded eminences, and so on. At each point the horizontal and vertical angles are read, as well as the staff intercepts or micrometer readings (according to the system in use) and the reading of the middle reference line.

The reading of the middle line should be the mean of the other two for small angles of elevation. For steep inclines this check is only approximate, but it always provides a rough check.

All readings should be taken as low down on the staff as is convenient. That part is more steady, and low readings will cause less error from any slope of the staff than high ones.

The distances and rises or falls are then obtained according to the above principles.

The reduced levels are calculated from the known level of the station.

The plan of the point is plotted from the horizontal angle and distance, and the reduced level written beside it for contouring.

In order that the draughtsman may join up his plotted points intelligently, a careful sketch of the ground must be made at the time. Each point is given a number in the field-book, and the position of that point is shown on the sketch.

The drawing is then completed to follow the sketch. New stations are chosen as required, and fixed by readings from the first station.

It is quite clear that the most important work in tacheometry is that of the staff-holder.

If the engineer in charge is at the instrument (as is usual in English practice), it is impossible for him to see that the staff is being held at exactly those points which will best show the true configuration of the ground as regards either plan or levels. He must either leave it to his staff-holder to choose suitable positions for his new stations (which is a very important matter) or he must waste valuable time running backwards and forwards. It is unquestionably easier to train a person of very moderate intelligence to read a levelling staff and a vernier correctly than to train him to select the best points for stations (and for the purposes of the survey generally), and to hold the staff correctly without supervision.

Moreover, if the surveyor himself is with the staff, it is clear that he can make a better sketch, showing more clearly how the features of the ground lie with respect to that spot, than if he saw it only from the distance. In this case he should check his book with that of the observer before moving the instrument.

Sometimes there is a 'booker' who takes down the readings as called out by the observer. This saves time, and often enables two staff-holders to be kept usefully employed, but it is dangerous like all calling out of figures from one person to another, unless the second man can be trusted to repeat the figures as he writes them down.

It is clear that new stations should be fixed with greater care than ordinary points. A full set of readings should always be taken from the new station to the old. This gives two results for bearing (if the compass is used), for distance and for difference of level. It may be desirable to take readings between stations also with both faces of the instrument, entering in the remarks column a note to this effect, and with a statement of which face was used for each reading.

A possible source of error in tacheometry lies in the fact that the lower limb of the tacheometer may move more or less after the readings on the back station have been taken, either through the inadvertent use of the wrong tangent screw, or through insufficiently tight clamping.

This amount of movement may be too small for detection by the compass, and circumstances may make it inconvenient for a permanent mark (visible from a distance) to be left at the back station, to which we could check back.

For this reason it is a good plan, immediately after setting to magnetic north, to take the reading of the horizontal circle on some prominent mark, such as a church steeple, gate-post, or the like. Book this in the remarks column, and, when the work at that station is completed, set on the same mark and check the reading.

Magnetic north should be taken as zero in all cases where no local magnetic attractions are suspected, because a comparison of the back and forward bearings of any line at the time will serve to detect any gross error in reading or booking the horizontal angle to any station.

ARRANGEMENT OF STATIONS. The stations should be arranged in circuits of some kind to check. This may be done either in one big circuit, closing back to the starting point, or in a number of small circuits.

Thus if A (Fig. 158) be the starting point, we may require to fix two stations, B and C, from it. The readings between B and C close the triangle ABC, which is treated as a closed traverse. If D and E be the next two stations, CD and BE may both be too far for clear readings, but we may be able to close the figure BCDE, and so on, thus obviating the necessity for working back to the starting point.

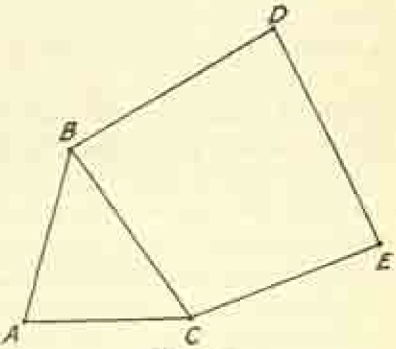


FIG. 158

If the stations are plotted by scale and protractor, each circuit is corrected graphically to close, as described in 'Traverse Surveying', Chapter 4.

If a triangulation or trigonometrical survey has previously been made (see Volume II), the stations of this survey give a valuable check, as described later.

CHECKING RISES AND FALLS. The rises and falls between the stations in each circuit must also close correctly, and they are corrected, if necessary, by small amounts to make them do so (exactly like differences of latitude or departures) before calculating any reduced levels.

The reduced levels of the stations are found from these corrected rises and falls (that of the first being known or assumed).

The reduced level of each station is then written in the proper place in the field-book, and the levels of the remaining points calculated from it.

PLOTTING. The positions of these points are plotted by scale and protractor (i.e. by *polar* co-ordinates).

For this purpose, say from station C the observed bearing to A was 276° 12′.

We take a circular protractor, put its centre over station C on the paper, and set it to read this figure on the line CA. The protractor is then held or weighted down; all the remaining bearings observed from C are marked round its edge by needle dots, with numbers of points attached; and finally the protractor is removed, and the proper distances scaled off along the rays through these points.

To avoid confusion of numbers, the numbering should be continuous from 1 to 100. It is generally possible to begin again from 1 without confusion. Some surveyors prefer to begin from 1 at each station, but give letters to the stations, and call the points $A_1, A_2; B_1, B_2$, etc.

Special scales and protractors combined have been used for plotting. But in plotting for the best result we should work out the included angles from the horizontal circle readings, and recompute the bearings from these as described under traversing (p. 213).

The distances and bearings being known, we can either calculate co-ordinates so far as the stations are concerned, or plot with scale and protractor. The former is the more accurate method. For the rules, see 'Traverse Surveying'.

FIELD-BOOK. There is no standard form of field-book. That used by the writer is shown on p. 296, with some figures filled in as an example. It is intended for use with a tacheometer which reads zero in the zenith, so that the reading of the vertical circle tells the angular distance from the zenith to the line of collimation. This is referred to as the 'zenith distance' in the table.

The 'generating number' is the name given to the value of $cS+k$.

In this case $k=0$, and $c=100$.

The column headed 'difference of level', refers to the calculated difference before correcting for height of instrument and middle reading.

The book shown is intended for use with fixed diaphragm lines and vertical staff. It must be modified for other systems, of course.

The field-book is interleaved with blank pages for the sketches, unless the sketch is made by the staff-holder, in which case it may be kept in a separate book. Stations should, of course, be shown in the sketches, with their letters or numbers attached.

NUMBER OF POINTS. A fair day's work would probably be about 60 to 80 points from 6 or 8 stations. Professor Ormsby has done 44 points from 11 stations (acting as his own observer and booker, and including location of stations, etc.) in the course of a morning, but this was in a survey where all the readings were short.

SPECIAL ADVANTAGES. The special advantages of tacheometry are that roughness of ground is no disadvantage (except to the staff-holders); that no points need be fixed except those which are necessary for the survey; that plan and levels are obtained in one operation; that the party need not necessarily consist of more than two persons; and that, as no chaining has to be done, ponds, shrubs and undergrowth, gardens, etc., do not interfere with the work so long as it is possible to find stations from which we can read over or across them.

DEGREE OF ACCURACY. The accuracy of the measurements made with the telemetric telescope depends largely on the power and optical precision of the instrument, on the distinct marking of the divisions on the staff, and to some extent also on the vision of the observer. The distance at which the single division of the staff can be read with certainty is limited. It requires a powerful and excellent telescope to bisect the hundredths of a foot at a distance of 400 feet. Much, however, depends upon whether the sun is in front of or behind the observer.

Experience shows that, with the telemetric telescope, the probable error in measurement is not proportional to the distance measured. Up to a certain distance, dependent on the optical perfection of the instrument, the probable error is nearly constant. Beyond that distance, the errors become suddenly large and erratic.

For example, with a small and not very good theodolite telescope, it was found that measurement could be obtained to the nearest foot, up to 330 feet, but beyond that distance errors of two, three, and even four feet occurred. With a more powerful and better telescope, the limit of accuracy appeared to occur at 800 feet.

With a telescope of about 12" focal length, of fair quality, distances may be obtained to about $\frac{1}{500}$th of the truth, up to about 400 to 600 feet. With a somewhat larger telescope, of very superior quality, the error should not exceed $\frac{1}{1000}$th up to even 700 feet. Telemetric measurement, therefore, compares favourably with all but the best chaining.

Tacheometry with theodolite or level. Tacheometric surveying may be effected by means of any theodolite (which has a good and powerful telescope), merely by providing subtense reference lines in its diaphragm. When ruled on glass, in the manner described, subtense lines are always a useful addition to level or theodolite. In all stadia diaphragms the middle reference line should be distinguished from the others in some way to prevent an outer line being read in error when a level reading is required.

A level fitted with stadia wires and compass affords a rapid method of tracing on a plan the contour line at any given level, for instance the top water line of a storage reservoir. The staff-holder is moved till the middle line gives the proper reading for the contour (see Chapter 3), when the stadia lines give the distance and the compass the direction at once.

Subtense measurement. There is, finally, to be considered the method in which the length of the staff is fixed, and the angle subtended by this fixed length at the instrument station is measured by the theodolite. In this case a horizontal 'subtense bar' is usually employed instead of a graduated staff, the fixed distance being indicated by vanes accurately positioned on the bar. If the angle subtended by a fixed length d is θ, the distance from the subtense bar to the instrument is given by

$$\frac{d}{2} \cdot \cot \frac{\theta}{2}$$

and if the telescope is horizontal this would be the plotted distance. If not, a correction must be applied to reduce the inclined distance to the horizontal. A modification of this procedure is discussed below.

Trigonometric determination of heights and distances. An ordinary theodolite, without additions of any kind, can be used in this method. Angles of elevation or depression from the horizontal are observed to two marks on a staff held vertically at the point observed to. In Fig. 159 α and β are the vertical angles of the upper and lower marks respectively, whilst l is the distance apart of these marks and L and H the horizontal or vertical distances

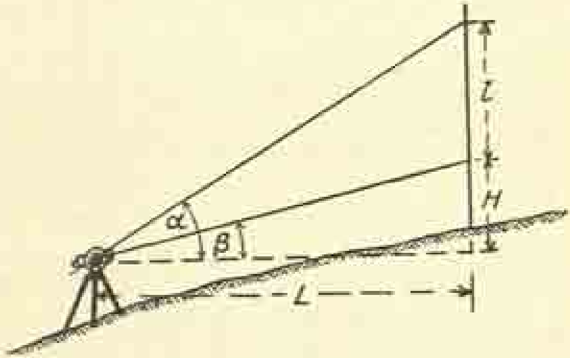


FIG. 159

respectively, of the *lower* mark measured from the centre of the instrument. It will be seen from Fig. 159 that

$$\frac{l+H}{L} = \tan\alpha \text{ and } \frac{H}{L} = \tan\beta$$

By division
$$\frac{l+H}{H} = \frac{\tan\alpha}{\tan\beta}$$

$$\therefore H = \frac{l\tan\beta}{\tan\alpha - \tan\beta}$$

$$\text{and } L = \frac{H}{\tan\beta} = \frac{l}{\tan\alpha - \tan\beta}$$

These expressions apply when both α and β are angles of elevation When both are angles of depression $\tan\alpha - \tan\beta$ becomes $\tan\beta - \tan\alpha$, whilst when α alone is an angle of elevation, the denominator is $\tan\alpha + \tan\beta$. The required factor can readily be worked out by the use of a table of 'natural tangents'.

This is an excellent and simple method of determining heights and distances, and capable of producing fairly accurate results, dependent, obviously, on the minuteness of the subdivisions of a degree which can be read on the vertical arc of the theodolite.

EXAMPLE OF A TACHEOMETRIC FIELD-BOOK

Tacheometric Survey at . Date . Instrument .

Station	Point	Bearing	Readings	Generating number	Zenith distance	Angle of elevation	Difference of level	Instrument and middle reading	Difference	Rise	Fall	Reduced level	Distance	Remarks
K		° ′			° ′	° ′		4·82				256·8		Mag. north as zero. 323° 18′ on spire of Bruton Church
	J	97 12	9·76 8·32	144	86 3	+3 57	+9·90	9·03	−4·21	5·7		262·5	143·3	
	12	175 6	3·48 0·26	322	89 2	+0 58	+5·43	1·87	+2·95	8·4		265·2	321·9	
	13	209 4	4·83 1·64	319	95 6	−5 6	−28·24	3·23	+1·59		26·6	230·2	316·5	
	L	268 5	7·28 2·85	443	93 8	−3 8	−24·18	5·06	−0·24		24·4	232·4	441·7	
L								4·90				232·7		Mag. north as zero. 331° 42′ on spire
	K	88 13	8·73 4·31	442	86 40	+3 20	−25·66	6·52	−1·62	24·0		256·7	440·5	
	14	196 0	12·86 9·72	314	86 45	+3 15	+17·77	11·28	−6·38	11·4		244·1	313·0	

The 'readings' are those of the top and bottom lines. The difference between them = S. Generating number = $cS + k$.

The top line of each station is blank, except for the name (or number) of station (col. No. 1), the height of the instrument (col. No. 9), and reduced level of station (col. No. 13). The reading of middle line for each point is given in col. No. 9.

Angle of elevation = $\beta = 90° -$ zenith distance, and is $\pm$ accordingly.

'Difference of level' = gen. no. $\times \sin\beta \cos\beta$, and is $\pm$ according to the sign of β.

Distance = gen. no. $\times \cos^2\beta$. Both these are obtained from tables or slide rule.

Difference = height of instrument − middle reading ($= 4{\cdot}82 - 9{\cdot}03 = -4{\cdot}21$ for No. 1 pt.).

The 'difference of level' and 'difference' are summed algebraically. The result, if *plus* is a rise, and if *minus* a fall, always from the station to the point.

If the staff be held in a horizontal position and perpendicular to a line passing through one of the marks and the theodolite, the angles required may be read off the horizontal limb, but in this way distances only can be determined.

With a theodolite, if the angles are small, they can be magnified by the method of repetition (see Chapter 4) so as to obtain a more correct mean value.

EXAMPLES FOR EXERCISE

(1) It was required to determine the value of the multiplying constant of a tacheometric theodolite, and the following distances were chained out from the instrument and readings were taken by the tacheometer. It was not known whether the instrument had an anallatic lens or not. Angle of elevation was zero.

Distance chained in feet	Stadia readings r_1	r_2
50	4·32	5·30
100	4·48	6·46
200	4·90	8·88
300	6·08	12·06

What was the multiplying constant, and had it an anallatic lens or not?

Ans. Multiplying constant: 50. No anallatic lens. Additive constant: *1* foot.

(2) A theodolite with a single horizontal wire was set up at a station A and sighted to a vertical staff held at a station B. With an angle of elevation of 10° 42′ the staff reading was 10·48 and with an angle of elevation of 10° 09′ the staff reading was 6·15. A tacheometric theodolite was then set up at station A in place of the first theodolite and with an angle of elevation of 12° 06′ the readings of the upper and lower stadia wires were 13·85 and 9·30. If the multiplying constant of the tacheometric theodolite was 100, what was the additive constant?

(Univ. of Lond., Imp. Coll.)

Ans. 1·24 feet (to two places of decimals).

(3) (*a*) A tacheometric theodolite was used to determine the distance away and reduced level of a distant point X. The angle of elevation was $\theta°$. The staff was held at X in a position normal to the line of sight of the telescope. The additive constant was $f+c$ and the multiplying constant k. Deduce an expression for the value of the reduced level of X.

(*b*) A theodolite with a single horizontal wire was sighted to a staff held vertically at a point X. The angle of elevation was $\theta°$ and the staff reading r_1. The telescope was then depressed, and the staff reading was r_2 with an angle of elevation $\phi°$. Show that the horizontal distance of X from the theodolite was $(r_1-r_2).\cos\theta.\cos\phi.\operatorname{cosec}(\theta-\phi)$.

(Univ. of Lond., Imp. Coll.)

$$Ans.\ [ks+(f+c)]\sin\theta-\left(\frac{r_1-r_2}{2}\right)\cos\theta+\text{ht. of inst.}$$

Chapter 7

ON THE PLANE-TABLE AND METHOD OF USING IT

Preliminary remarks. The plane-table is usually regarded in England as an instrument of secondary accuracy, ranking little above the prismatic compass, and suitable only for topographical work on a small scale. As usually constructed and described, it is not adapted to the preparation of accurate and large-scale plans. On the Continent, however, it has received many developments, and is classed as an instrument of precision, only second to the small theodolite. The uncertain climate of England is not favourable to the use of the plane-table, hence its neglect. In the tropics,

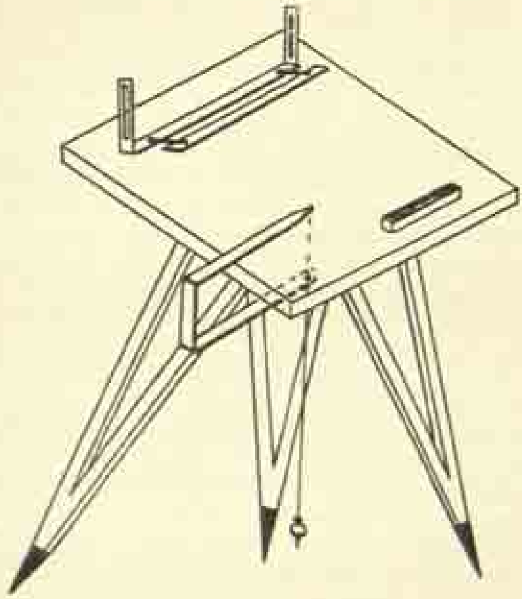


Fig. 160

on the other hand, and in continental climates generally, it is of the utmost use, and for purposes of instruction in surveying it is invaluable.

CONSTRUCTION OF A SIMPLE FORM OF PLANE-TABLE. The construction of the plane-table, in its simplest form, is so well known as scarcely to need description. A board about 1′ 6″ or 2′ 0″ square (Fig. 160), made of well-seasoned wood, is supported upon a light but firm tripod stand, like that of a camera, but firmer. It is

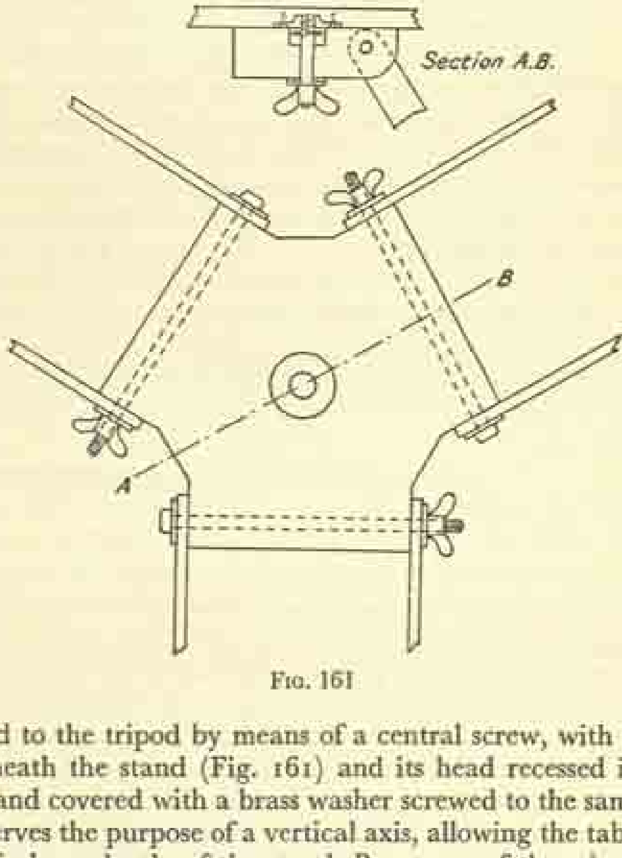


FIG. 161

attached to the tripod by means of a central screw, with a wing-nut beneath the stand (Fig. 161) and its head recessed into the board, and covered with a brass washer screwed to the same. This screw serves the purpose of a vertical axis, allowing the table to be turned independently of the stand. By means of the wing-nut, it may be clamped in any desired position.

The table is levelled (by moving the legs) by means of the long compass used with it, or by two small spirit-levels fixed at right angles to each other in a block of wood.

Rays are drawn to the various objects by means of a sight-rule or alidade, which consists of a plain ruler about 1′ 6″ long, with a fiducial edge, and provided at each end with an ordinary sight-vane, like that of a compass or circumferentor. It is a great convenience, however, to attach the fiducial edge to the body of the rule, after the manner of a sliding parallel ruler (*see* Fig. 177, p. 318).

When working to a large scale, and when, consequently, the rays are short, a metal plumbing-fork is necessary (Fig. 162). The opening of the fork is sufficiently wide to take in the table and the paper on it. The lower leg of the fork is provided with a hook immediately below the point of the upper leg. The plumb-bob is suspended from the hook. When the point of the upper leg coincides with any point on the paper, the plumb-bob hangs immediately below it, so that it may be brought vertically above the point which it represents on the ground. Or a point may be found on the paper which is vertically above the station point at which the table is set up.

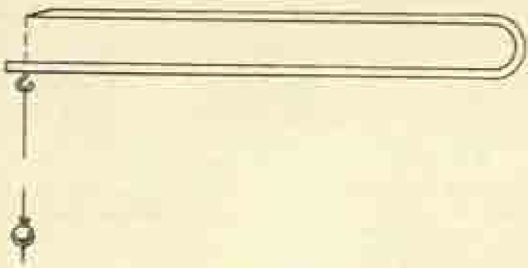


Fig. 162

A compass is a very necessary adjunct to the plane-table. The best form is a long (5 or 6 inch) bar needle enclosed in a rectangular box, which allows it to play about 5° or 10° on either side of the meridional line. When the table has been placed in its proper orientation, the compass-box is placed upon it, and moved about until the ends of the needle come to rest at zero: a line is then drawn round the box. By replacing the box within this rectangle, and turning the table till the ends of the needle come to rest at zero, the table will be approximately oriented.

The paper must not be fixed to the table by means of paste or glue. The expansion and contraction of wood and paper are not the same, nor is that of wood the same 'with' and 'across' the grain. The paper used should be of the very best quality, well stretched. It must never be rolled or folded, but must be carried flat in a portfolio.

MODE OF ATTACHING THE PAPER. To attach the paper to the table by means of ordinary drawing-pins is inconvenient. The heads of the pins get in the way of the sight-rule. Unless the pins are put in a new position every time the paper is attached to the table, thus making a number of holes, the perforations become enlarged, and the paper is apt to shift.

A better plan is to use screw clamps; if one of these gets in the way of the sight-rule it can be shifted. Four of these are necessary, but a set of six is convenient, so that one of the spare clamps may be put on before the one that is in the way is taken off. Another merit of screw clamps is that a final adjustment in azimuth, i.e. in rotary movement about the vertical axis, may be made by slacking three of the clamps judiciously, and moving the paper on the table.

A good plan for small-scale surveys is to mount the board with linen pasted round the margins, and then to mount the paper on to the linen, which spans over cracks in the wood, and a nice workable surface is obtained. Good millboard is in every way preferable to paper. It expands and contracts regularly, and wind does not cause it to flutter, as paper is apt to do. The surface should be of the best possible quality, so as to stand the rubbing and scrubbing which it will have to receive.

White paper is painful to the eyes; pale drab, green or pale slate-colour is more comfortable to work upon in bright sunlight. A pale sepia or burnt-umber colour is, according to the writer's experience, generally found to be the most pleasant.

SURVEYING BETWEEN KNOWN POINTS PROJECTED ON THE PAPER. With the enumerated appliances, it is possible to produce a complete survey, the position of any two points being previously known. Usually, these will be previously determined trigonometrical points, projected on the paper by co-ordinates in the manner already described, or they may be the extremities of a base line measured with a chain. If trigonometrical points are available, it will be well to project three at least, and preferably more, so as to serve as checks to the work.

Let A and B (Fig. 163) be the two known points on the ground, *a* and *b* their projections upon the plane-table sheet. At A the table is set up and levelled. The fiducial edge of the sight-rule is applied to the line *ab* on the sheet. The table is then turned in azimuth till the line of sight of the rule intersects the signal at B. Strictly speaking, when the intersection is made, the point *a* on the sheet should be vertically above the point A on the ground. The effect of neglecting this condition will be discussed later on. Suffice it for the present to say that when one is working to a small scale, the error due to defective centring will be negligible.

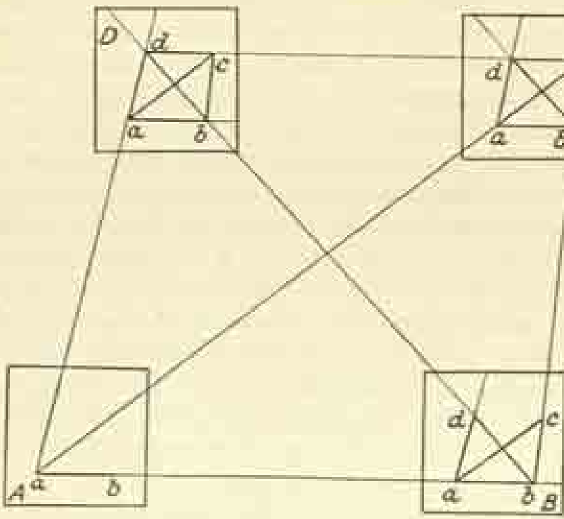


FIG. 163

When the fiducial edge of the sight-rule coincides with the line *ab* on the sheet, and its line of sight intersects the point B, the table is correctly 'oriented'. The relative bearings of the points on the plan should then coincide with the points on the earth which they represent.

At this stage of the proceedings the compass-box should be placed on the table, moved until the needle comes to rest at zero, and its position marked as described. When the table is set up at another station, it may be oriented, with the degree of accuracy of which the compass is capable (*vide* 'Traverse Surveying'), by replacing the box within the rectangle described, and turning the table till the needle comes to zero.

Now, to fix some other new point such as C: If an ordinary sight-rule be used, the leg of a pair of dividers, or a pin, or the point of a hard pencil is planted at *a* on the sheet to serve as a pivot, and the sight-rule is turned until the line of sight intersects the point C.

The ray *ac* is then drawn, a note or reference, or sometimes a rough sketch of the object, being made in the margin, to ensure identification. Rays are also drawn to other objects which are to be fixed.

The table is now transported to B, set up and levelled. It is then oriented, by placing the fiducial edge along *ab*, and turning the table in azimuth, till the line of sight intersects station A and is clamped fast. Then the sight-rule is turned to C and the ray thereto drawn. The intersection of the rays *ac* and *bc* is the position of the point C on the plan. In like manner the positions of other points are fixed by intersecting the rays drawn at A from *a* on plan, with rays through B from *b* on plan. Finally, a ray is drawn to some new point D destined to serve as a future station for the instrument.

The instrument is next set up at C, oriented on A, and the orientation checked by directing the sight-rule to B, and the ray *cd* is drawn to D, cutting the ray *bd* and thus fixing D.

Thus, a system of triangulation may be carried on graphically. The three angles of the major triangles will be protracted directly on the table, analogous to the principal triangles of a trigonometrical survey, each point being observed to and from. Intermediate points will be determined of course by observations from each of these stations just as in tacheometry, except that each is fixed by the intersection of two rays from different stations.

If the object be to produce a topographical map on a small scale, say $\frac{1}{60{,}000}$, or one inch to the mile, or less, the intermediate points will be so numerous that the eye may be trusted for sketching in intermediate detail, such as the courses of rivers or roads, the position and outlines of villages, the approximate configuration of the ground by eye-contours, or even more precisely by the use of a clinometer, such as Abney's level, in the manner practised in military reconnaissance.

If the map is ultimately to be published, on a scale of $\frac{1}{60{,}000}$ or less, it will be well to draw the original sheets to a larger scale, say 6 inches to the mile, and reduce by photography. It is not easy to produce in the field the fine drawing that the small-scale map requires. In this manner the topographical map of India was produced. The parallel ruler arrangement with the sight-vane greatly facilitates the intersection of points and the drawing of rays. It is unnecessary to use a needle or pencil-point as a pivot for the sight-rule, making holes or marks on the paper. It is only necessary to place the sight-rule, so that when the intersection is made its fiducial edge is near to the station point on the paper. When the line of sight intersects the objective point, the station point on the table can be intersected easily by extending the parallel bar.

ERRORS DUE TO INACCURACY IN 'CENTRING' EXAMINED. It has been stated that the point on the paper should be vertically above the point on the ground which it represents. It is not always easy to satisfy this condition perfectly, and at the same time level and orient the table. To do so might necessitate several shifts, and occasion much loss of time. It is therefore well to examine the nature and extent of the errors due to defective centring. Suppose that rays are to be drawn from some point *a* near the corner of the sheet, a second point *b* being already determined.

In what follows capital letters refer to points on the ground, the corresponding small letters to the points on the plan.

The table is set up with its centre over the station A (Fig. 164). The angle CAB is that subtended at A by the rays to the station B and C. If rays were drawn with the sight-rule through *a* on the plan, they would include the angle C*a*B, materially different from ∠CAB. The error caused by defective centring depends upon the length of the ray and on the scale of the plan.

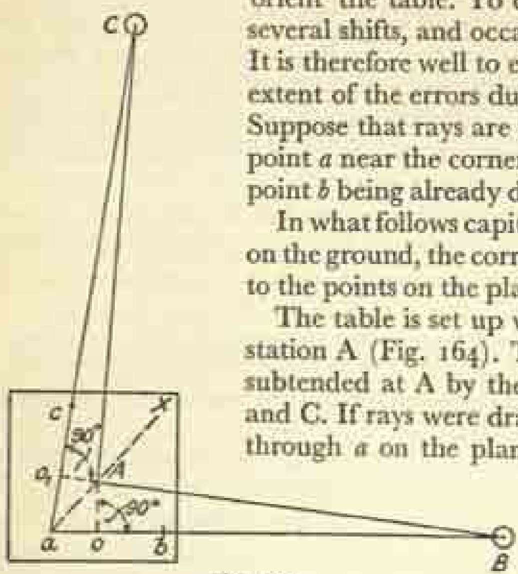


FIG. 164

Produce *a*A to *x*

Then $\angle CAX = \angle CaA + \angle aCA$

and $\angle BAX = \angle AaB + \angle ABa$

$\therefore \angle CAB = \angle CaB + \angle aCA + \angle ABa$

or $\angle CAB - \angle CaB = \angle aCA + \angle ABa$

or the error in the angle at a = sum of angles subtended at the points B and C by the centring error aA.

With a table 2 feet square it will be scarcely possible to be so far out of the centre that the perpendicular Aa exceeds 1 foot. If the point observed to were distant 5000 feet, ∠ABa would be 41 secs. A similar error might occur in the ray aC, so that the angle CaB might be in error by 1′ 22″. Both rays being 5000 feet long, if the scale of the plan is $\frac{1}{10000}$ (nearly 6 inches to 1 mile), the rays as drawn would be 6 inches long on paper. Now an error in the angle cab of 1′ 22″ would displace the point c by ·0002 foot, a quantity quite inappreciable. Even if the rays were only 500 feet long, other conditions being the same, though the actual error in angle would be nearly ten times greater (nearly 14 mins. and therefore appreciable), still the displacement of the point c would be inappreciable owing to the shortness of the rays as drawn, which would then be but ·05 foot in length.

Great care in centring necessary for scales larger than $\frac{1}{2500}$. So we see that in surveys to a small scale, even as great as $\frac{1}{2500}$, extreme accuracy in centring the point on the paper over the station is not essential, *except for very short rays*. It is, however, desirable that centring should be performed as accurately as possible (without great waste of time), for fear of accumulation of error.

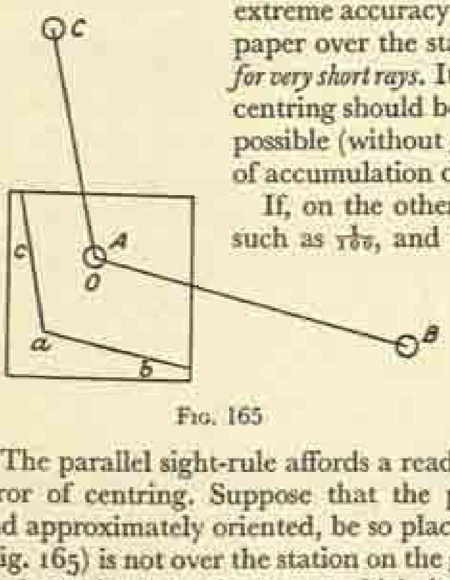


Fig. 165

If, on the other hand, the scale be large, such as $\frac{1}{100}$, and the station on paper is 2 inches away from the vertical through A, the corresponding error on paper is $\frac{1}{50}$ inch, and is hence quite measurable.

The parallel sight-rule affords a ready means of eliminating the error of centring. Suppose that the plane-table, when levelled and approximately oriented, be so placed that the point a on plan (Fig. 165) is not over the station on the ground which it represents. Find a point o on the paper which *is* vertically over A. Lay the sight-rule with its fiducial edge intersecting the point o exactly parallel to the ray ab. With the sight-rule in this position, turn the table in azimuth until the line of sight intersects the station. The table is now correctly oriented, though not correctly centred. Now with the fiducial edge intersecting *o* direct the sight-rule to C. extend the parallel bar, and without moving the sight-rule, draw the ray *ac* through *a*. The angle *cab* is equal to CAB and is therefore correct. In correcting the orientation, the position of *o* with regard to A may be slightly altered. Usually the movement will be so slight as to have no effect on the orientation. If it does affect it, find a new position for *o* and proceed as before. The question as to whether any error of centring is negligible or not may be solved at once, by considering whether the actual horizontal distance, as shown by the plumbing-fork between the point on the plan and the station point, is an amount which would be appreciable when drawn to the scale of the plan. Suppose the actual error in centring were 1 foot, and the scale of the plan $\frac{1}{10000}$, then the actual error on the plan would be $\frac{1}{10000}$ of a foot, and therefore wholly inappreciable. If the scale were $\frac{1}{500}$, the error would be just appreciable. If the scale were $\frac{1}{100}$, it would be on paper 0·01 foot, and therefore, too serious to neglect.

FIXING INTERSECTED POINTS. In the course of plane-table surveying, it will often be convenient to fix points such as chimneys, steeples, trees, or corners of buildings, over which the plane-table cannot be set up. Nevertheless such points may be useful for the further development of the plan.

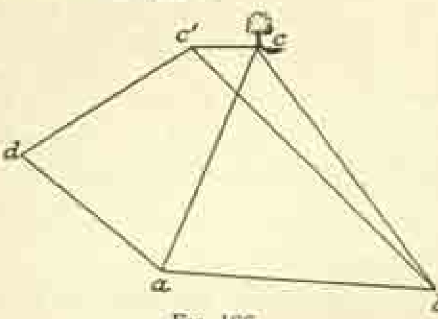


FIG. 166

Let *c* (Fig. 166) be a conspicuous tree, whose position has been determined by the intersection of rays drawn from *a* and *b*. To make use of this station for further work, the plane-table is set up at a point *c*′ near to the tree. It is then oriented carefully by means of the compass. Then the ray *cc'* is drawn, and the distance *cc'* is measured and set off along *cc'* thus fixing with a close approximation to accuracy the position of the table. It is evident that as the distance of *c'c* is small, an error in orientation by compass, of even a degree or two, will not affect the position of *c* on the plan. The point *c'* may now be taken as fixed with nearly as much accuracy as *c*. There is, however, uncertainty as to the correct orientation of the table. Lay the sight-rule with its fiducial edge intersecting *c'* and *b*. Turn it till the line of sight cuts B. Then check by placing the sight-rule along *c'a*, when the line of sight should cut A, if *c'* be correctly determined. Rays may now be drawn from *c'* intersecting different points, such as *d*. If from *a* a ray *ad* has been drawn, *d* is determined.

THE 'THREE-POINT PROBLEM'. The three-point problem sometimes occurs in plane-tabling. It consists of fixing the position of a point on the ground, not previously located on the plan, from three known points whose positions have already been plotted. These points must be visible from the point whose position is to be fixed on the paper. The problem may be solved graphically with the plane-table, as follows:

Let A, B and C (Fig. 167) be the three known points the positions of which have been accurately located on the plane-table drawing. Let O be the unknown point at which the table is set up. It is required to fix the position of O on paper by observing *from* it to A, B, C. Let A be on the left, B on the right, and C in the middle or behind the observer's back, as he regards A and B, and let *a*, *b*, *c* be the plans of A, B, C.

Lay the sight-rule through *b* and *a* on table, and direct on A (*a* being towards A). Through *b* draw a ray X*b*X towards C as in Fig. 167 (i).

Lay the sight-rule again through *a* and *b*, and direct on B, by rotating the table to the position shown in Fig. 167 (ii) *b* being towards B. Through *a* draw a ray Y*a*Y towards C.

Join the intersection *d* of these two rays so drawn to *c*. Lay the sight-rule along this line. Turn the table till the line of sight cuts C, *c* being towards C as shown in Fig. 167 (iii). The table is now oriented. Draw rays through *a* and *b* directed to A and B respectively. Their intersection fixes *o*, the point of observation on plan. These rays should intersect on the ray *dc* already drawn.

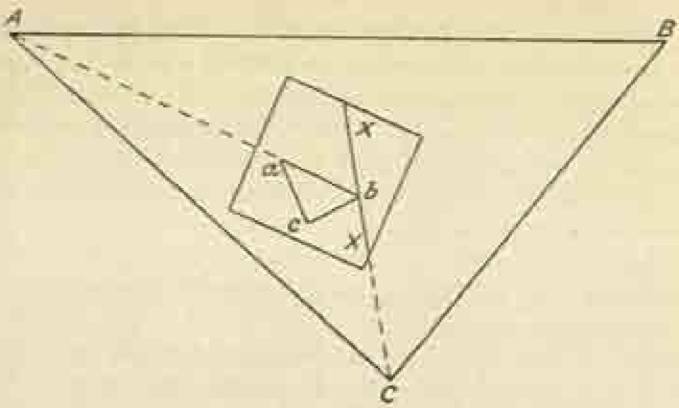


FIG. 167 (i)

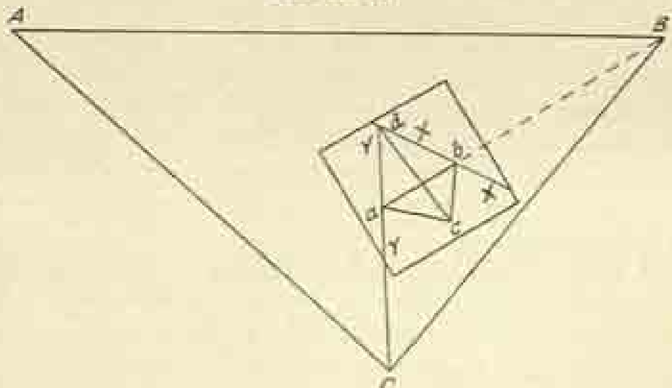


FIG. 167 (ii)

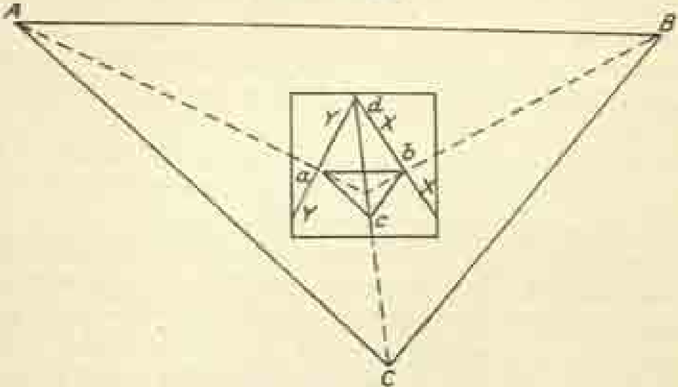


FIG. 167 (iii)

The centre of rotation of the table has been displaced in these diagrams. This is because the size of the table has been greatly exaggerated and, if the centre of rotation were drawn as a fixed position, as it actually is in practice, the intersection of X*b*X and Y*a*Y would fall beyond the boundaries of the drawing-board.

The accuracy of this solution depends upon the length of the line from *c* to the intersection of the two auxiliary rays drawn from *a* and *b* to C. If this line is very short or if the angle of intersection is bad, the conditions are unfavourable to accuracy, and some other points must be used.

Strictly, the table should be moved at each operation so as to bring the point at which the angle is drawn over the station point. This precision is rarely necessary, and any error may be eliminated by means of the parallel sight-rule as already described.

Other methods of resection. Resection is the general name given to the processes such as that just described whereby the

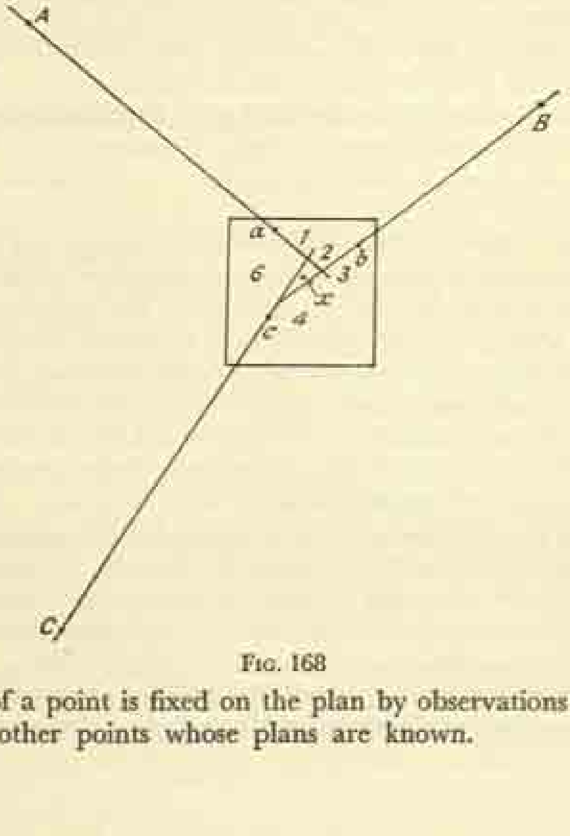


Fig. 168

position of a point is fixed on the plan by observations *from* that point to other points whose plans are known.

In *Topographical Surveying* by Colonel Close (H.M.S.O.), the following solution is described.

Let a, b, c (Figs. 168 and 169) be the plans of three points A, B, C, all visible from the unknown station where the board is set up. It is required to mark on the plan the position of this station. (*Note.* The relative size of the board is much exaggerated in the figures.)

Roughly orient the board by means of the compass, and draw rays through a, b, c to the corresponding points on the ground, producing them backwards to meet as shown.

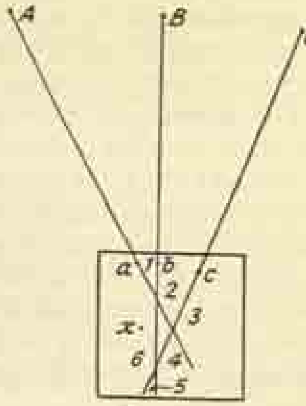


FIG. 169

If these rays meet in one point, that point is the required position of the station.

But, in general, they will not do so, and a small triangle, called the 'triangle of error' will be formed as shown.

The true position of the station is then found as follows:

(*a*) If the triangle of error is *inside* the triangle (*abc*) formed by the three fixed points, then the required station is also *inside* the triangle of error (Fig. 168).

(*b*) If the triangle of error is *outside* the triangle *abc* (Fig. 169), the true position of the station must be such that, when facing the fixed points, it is *either* to the right or to the left of *all* the rays drawn as above.

Thus in Fig. 169 the station must be in section 6, to the left of all rays, or in section 3, to the right.

(*c*) In either case the position is found by the fact that its distance from each of the three rays must be proportional to the lengths of the corresponding rays.

Thus in Fig. 169 the rays to a, b, and c are all about equal; hence the true position must be in section 6, say at x, because it is clear that any point in section 3 would be considerably nearer to the rays drawn to A and C than to that drawn to B, whereas it should be equally distant from all.

The exact position can usually be fixed by eye, as accurately as by construction, at the first try. Having guessed the exact spot x, set the alidade along *xa* (supposing that A is the most distant point), and orient on A. Then draw rays through *b* and *c* to B and C. If these pass through *x*, well and good. But if we make a bad guess, we shall obtain a much smaller triangle of error. Repeat the above process till perfect.

Proof. An explanation, rather than a formal proof, of the correctness of the method may be given as follows:

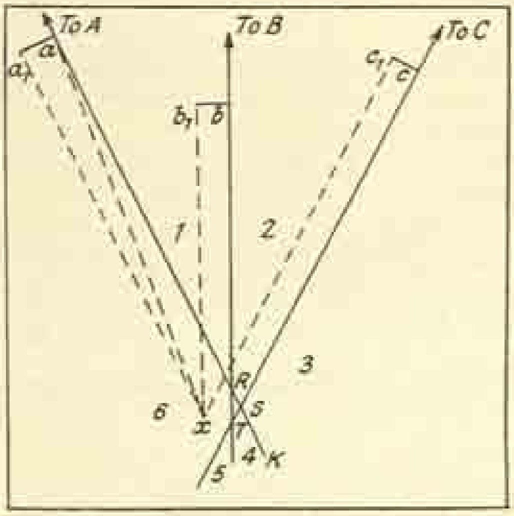


FIG. 170

In Fig. 170, for instance, suppose *x* is the correct point. Then, in the original position of the board, the ray K*a* was drawn towards A. Now, if the alidade be set along the correct ray *xa*, it is clear that, to bring the sights back to A, we must rotate the table through the angle *xa*K anti-clockwise, so that the ray *xa* will take up the position xa_1 (practically) parallel to K*a*, or *a* will move to a_1.

Now, clearly, in this rotation of the board *b* and *c* must also move to the left (that is, in the same direction as *a*), and the *distance* moved by each of them will be proportional to the length of the corresponding ray. The new positions of *b* and *c* are therefore at b_1 and c_1, and parallels to the original rays through these points must meet at *x*; hence the distance of *x* from each ray must be proportional to its length. And if *x* were taken inside the triangle of error, or in the sections 1, 2, 4 or 5, it would be found, in the case shown in Fig. 170, that, if we draw rays from any point so chosen parallel to the original rays, some of these will lie to the right and some to the left of the original rays. This would mean that in rotating the board we move some points clockwise and some anti-clockwise, which is absurd. For simplicity, it has been assumed that the board turns round x as centre. It can easily be shown that, for any other centre, the *relative* motion of x and a is the same as if x were the centre.

CONSTRUCTION. The exact point x may be found by construction if desired. First guess it roughly, and measure the lengths (which will be nearly correct) xa, xb, xc.

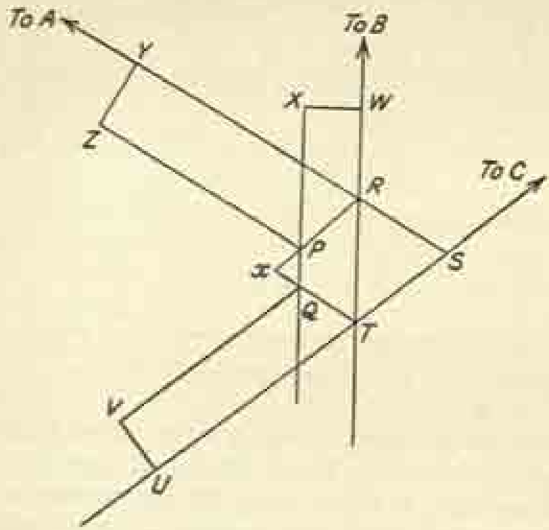


FIG. 171 (*a*)

In Fig. 171(*a*) let RST be the original triangle of error. Set up UV, WX, and YZ to represent xc, xb, and xa on any scale, and draw parallels to the original rays, meeting in pairs at P and Q as shown. Join RP, TQ to meet in x, the required point.

The proof of this construction is as follows:

Draw xm, xn, xo perpendicular to the rays to A, B, C respectively (Fig. 171(*b*))

Then the condition to be satisfied is:

$$xm : xn : xo = xa : xb : xc$$
$$= tm : un : vo$$

From the figure:

$$xm : tm = xR : PR = xn : un$$
$$= xT : QT = xo : vo$$

i.e. $$xm : xn : xo = xa : xb : xc$$

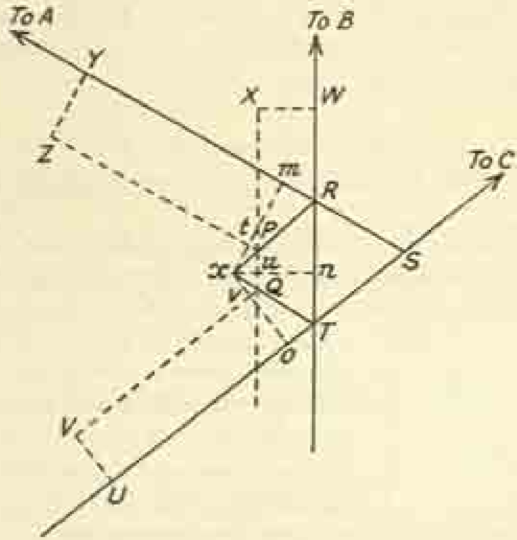


FIG. 171 (*b*)

SOLUTION BY TRACING PAPER METHOD. Another solution is as follows:

Place a piece of tracing paper on the board, and on it draw rays from any point to A, B and C. Then move the tracing paper about over the drawing until these rays pass through the points *a*, *b*, *c* respectively, and prick through the common point of intersection on to the drawing. This will be the required position of the station if the points have been suitably chosen.

FAILURE OF SOLUTION OF 'THREE-POINT PROBLEM'. Resection (using three known points), as a method, depends upon the fact that the position of the station can, in general, be fixed when the angles *axb*, *bxc* (subtended at the station by the known lines *ab*, *bc*) are known. Clearly, however, if the points A, B, C are so arranged with respect to the station that a circle drawn through *a*, *b*, *c* on the paper would pass through the true position of *x*, there are an infinite number of positions for *x* on the circumference of this circle which would give the correct values to both these angles.

In such cases the method fails altogether (that is, it may lead us to adopt a totally incorrect position for the station), and it is unreliable if the circle *abc* passes anywhere *near* *x*. Hence care must be taken to choose the fixed points so that this does not happen. This remark applies to the method of resection generally, not only to this particular solution.

USING TWO POINTS ONLY. If only two fixed points are visible, we may fix the position of *x* from them if we can trust the compass for orientation.

Place the compass-box in the rectangle which was ruled round it at the previous station, and turn the board till the needle comes to zero. The board is then oriented approximately correctly. If *a* and *b* be the two known points, we draw rays through them to A and B, the points on the ground, and their intersection fixes the required station. A station so fixed should not, preferably, be used to fix new stations for carrying on.

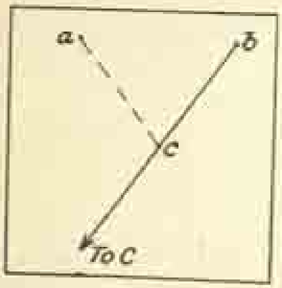


FIG. 172

WITH ONE BACK RAY. Sometimes, with two known stations A and B, we finish the work from A without noticing that a station will be required at another point C. When at B this fact is discovered. We draw a ray from *b* (Fig. 172) towards C, therefore.

On setting up at C, we set the alidade along this ray and orient on to B. This ray is shown by a full line in Fig. 172. Now draw a ray through *a* towards A, to meet this ray in the required position of *c*.

This is not a 'resection' method.

RESECTION FROM TWO KNOWN POINTS. If only two known points are visible, it is still possible to fix the positions of two unknown stations. In Fig. 173, let A, B be the known stations, and suppose it is required to fix the positions of X and Y.

This is best done with the aid of tracing paper. Set up at X, mark the point vertically over X on the tracing paper and draw

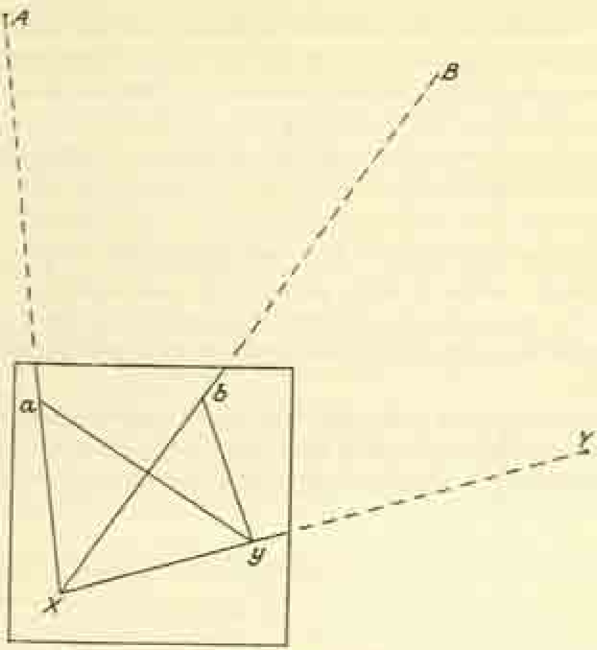


FIG. 173

rays to A, B, and Y. If XY can be measured conveniently, it is scaled off along the last ray to fix the position of Y.

If now the board be taken to Y, oriented back to X, and rays drawn to A and B, the intersections of these rays with those from X fix the positions of A and B. If now the tracing paper be placed over the drawing on which A and B are plotted, it should be found that both A and B on the tracing can be made to coincide with A and B on the drawing, and X and Y are then pricked through.

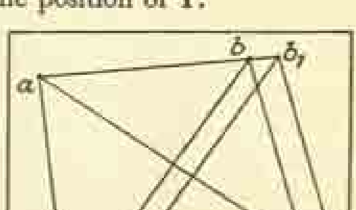


FIG. 174

But, if we cannot well measure

XY, we *guess* its length and scale it off. Suppose x_1, y_1 (Fig. 174) are the positions of X and Y as marked on the tracing paper, and a, b_1 are the positions found for A and B.

Then if the tracing be placed over the drawing so that a coincides with the plan of A, and ab_1 with the line ab (b being the plan of B on the drawing), then b_1 will not coincide with b. Now from b (as seen through the tracing) draw bx, by parallel to b_1x_1, b_1y_1, meeting ax_1 and ay_1 at x and y. Then these are the corrected plans of X and Y, and can be pricked through.

Plane-tabling with the aid of chain or tape. Up to this point it has been assumed that plane-table surveying is conducted entirely by triangulation, without using any appliance for linear measurement, except perhaps a 50-foot tape for measuring short distances.

For topographical surveying on a small scale this method suffices. But when the survey is made to a large scale it would be laborious.

Suppose that a tortuous road had to be surveyed. Having fixed

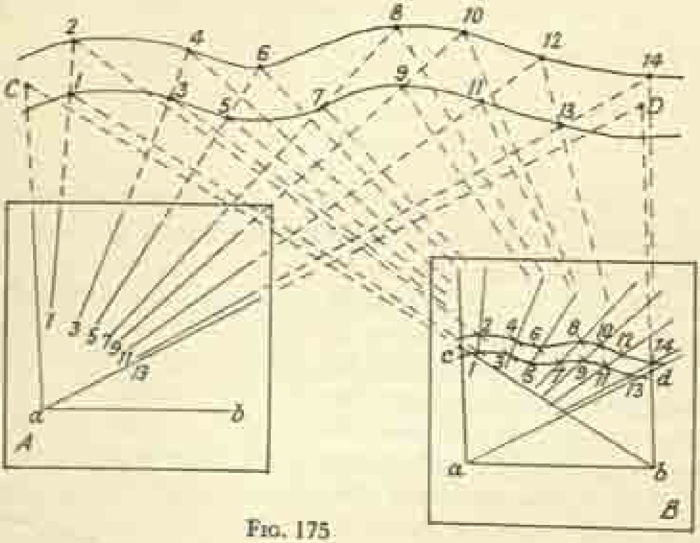


Fig. 175

two points a and b (Fig. 175), rays would be drawn from a to a number of leading points in the boundaries of the road. These points would be fixed by the intersection of rays drawn from b.

Now every point on the ground must be carefully marked and numbered in the first instance, so that they may be identified when drawing the intersecting rays from *b*. The staff-holder must be careful not to miss a point when the second set of rays is being drawn, otherwise hopeless confusion, and loss of time and temper, would result. The sheet will be covered with a maze of rays.

If a chain or steel tape were added to the equipment of the plane-tabler, his procedure would be obviously simplified. With their aid traversing can be carried out with the plane-table quite as accurately, or more so than would be the case if made with a theodolite and chain and plotted with a protractor.

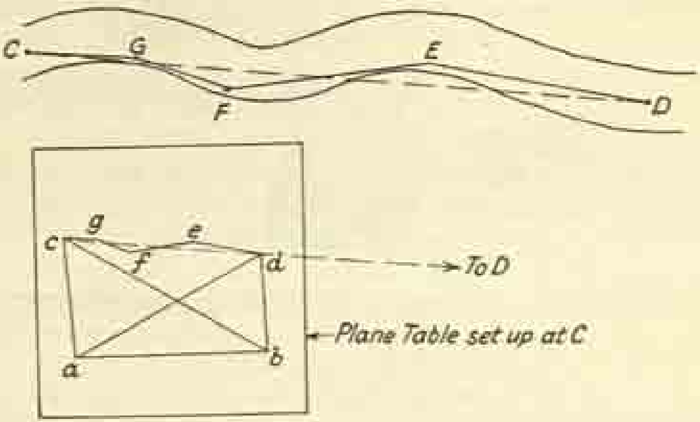


Fig. 176

Two points C and D (Fig. 176) in the road would be determined by the intersection of rays from *a* and *b* as before. Putting up the table at D, and orienting on A and verifying on B, the position of *d* would be checked. The ray *de* would then be drawn and the distance DE measured and plotted. Intermediate distances and offsets would be taken as in chain-surveying. They might be plotted at once, or better, noted on a slip of paper, so that they could be plotted when the traverse is closed on C. Having laid off the distance *de*, set up the table at E, orient on D, and check the orientation by rays to A and B, if these points be visible. The ray *ef* is drawn, and so on until closure is effected at C, this stage of the work being shown in Fig. 176. This done, the offsets might be plotted, and the work, so far, would be complete. In this way the streets of a village might be easily surveyed.

THE TACHEOMETRIC PLANE TABLE. The most useful appliance for linear measurement that can be used with the plane-table is the tacheometric alidade. Fig. 177 shows a modern instrument of this type (Hilger and Watts pattern).

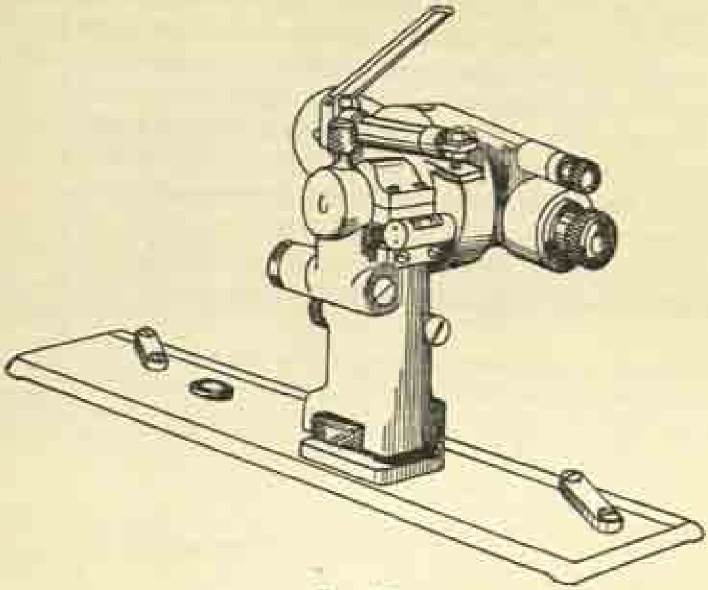


FIG. 177

The alidade consists of a tacheometric telescope mounted on a rigid vertical support attached to a metal straight edge which is placed on the drawing board. The straight edge is usually about 15 inches long and has a hinged strip with a bevelled edge for ruling which is capable of a limited parallel movement. The longitudinal axis of the telescope is vertically above the ruling edge when the latter is in the closed position, but the full parallel movement may be used without introducing appreciable errors in the drawing. The telescope can be rotated in a vertical plane and is fitted with a graduated vertical circle or a graduated arc so that its angle of tilt can be measured. The usual clamp and slow-motion screws are provided as in a theodolite.

The index for reading vertical angles is connected to an arm which supports a spirit-level, and when this is central and the reading zero the telescope will give a horizontal line of sight if correctly collimated. A small circular spirit-level is usually provided on the straight edge, and levelling is achieved in the simpler instruments by moving the tripod legs. In more elaborate instruments the board is attached to a three-arm levelling base which is screwed on to the tripod head. It is difficult, however, to maintain the board in a perfectly level position because of the lack of rigidity of most plane-tables. The Beaman arc is a useful refinement which is frequently used in place of the normal vertical circle and is described in the following paragraph.

THE BEAMAN ARC. When reading with a tacheometric telescope fitted with an anallatic lens and inclined at an angle α to the horizontal, the distance, D, of the staff from the instrument is $c.S.\cos^2\alpha$ and the height of the middle reading of the staff above the trunnion axis of the telescope is $c.S.\cos\alpha.\sin\alpha$ where c is the constant multiplier (almost invariably 100) and S is the staff intercept.

To simplify the calculations for distance and height when using a tacheometric plane-table alidade, and to obviate the use of trigonometrical tables, the Beaman arc will be found very useful. It is attached to the telescope and rotates with it but is graduated on a broadened rim instead of on the face. There are three sets of graduations and, as shown in Fig. 178, they consist of (1) the customary scale of degrees and fractions on which the angle of tilt of the telescope (α) can be read in the ordinary way (2) irregularly spaced graduations giving the whole number percentage corrections for distance corresponding to various values of α. These corrections may be expressed as $100(1-\cos^2\alpha)$, and the 3% mark, for example, is opposite the reading 10° on scale No. 1 since this value of α satisfies the equation $100(1-\cos^2\alpha)=3$. (3) graduations giving $100\sin\alpha.\cos\alpha$ for various values of α.

FIG. 178

To use this device the T-piece bubble, which is located on an extension of the index arm of the Beaman arc, must be carefully levelled. When the bubble is central a zero reading on scale No. 1 indicates that the line of collimation is horizontal, provided that the instrument is in good adjustment. When taking a staff reading the telescope may be tilted to give a whole number reading on the scale for the height multiplier.

For instance, if this scale reads 10, α is approximately 6° and the percentage distance correction for this angle as read from the middle scale is 1%. Suppose the readings of the stadia at this setting were 3·84, 5·07, and 6·30. Then $S = 2{\cdot}46$.

Let height of alidade axis be 4·25 feet and let the constant multiplier be 100.

Then difference in height between ground at staff and ground at instrument $= 10 \times 2{\cdot}46 - 5{\cdot}07 + 4{\cdot}25 = 23{\cdot}78$ feet.

The horizontal distance is

$$246 - \frac{246 \times 1}{100}, \text{ or } 243{\cdot}5 \text{ feet.}$$

A superior type of alidade is now obtainable in which the readings of the Beaman arc are reflected into a magnifying eye-piece. The circle is glass, following modern theodolite practice. This type of alidade is shown in Fig. 177. The vertical circle, with Beaman arc graduations, is entirely enclosed, and the eye-piece for reading it will be seen above the telescope eye-piece.

Thus equipped, the plane-table becomes an instrument of considerable precision, useful for filling in detail, or even for making large-scale surveys of considerable extent, working from a carefully measured base, or a control network.

General rules to be observed when using the tacheometric alidade. The following general rules should be adhered to, in surveying with a plane-table and tacheometric alidade.

(*a*) No new station of the table should be taken up by a ray and a distance only.

Every new station should be fixed either by one ray and a measured distance, and a ray drawn from an already determined station, verified by a back ray drawn at the new station to the second station, or by a ray and distance from the station left, verified by back rays to two other fixed stations.

(*b*) Only as much ground should be surveyed from one position of the table as will bring the longest distance well within the range corresponding to the permissible error. In short, there should be no hesitation to shift the table.

(*c*) In surveying a crooked boundary or fence, take up a position for the table that will make the rays approximately tangents to the curves, rather than normals.

(*d*) The orientation of the table should be frequently checked by referring back to already fixed points. A slight pressure on the margin of the table produces considerable twist on the vertical axis. The clamp may slip slightly, unperceived, and the consequence will be that an area will be surveyed correctly in itself, but displaced with regard to the rest of the plan.

In the preparation of an extensive survey, on the scale of $\frac{1}{2500}$, it is both desirable and economical to traverse the roads and leading boundaries, and to plot them on the sheet before commencing to use the plane-table. This gives a larger number of points to check upon, for, if performed properly, traversing is only second in accuracy to minor triangulation. The chances of distortion referred to in the last paragraph are therefore materially reduced.

Plane-table surveying cannot be performed with economy, accuracy and dispatch during a high wind. The telescope vibrates too much, time is lost in waiting for a lull, and the staff is read hurriedly. In short, a high wind, even without rain, stops plane-table work. Traversing, on the other hand, can be carried on in windy or even moderately rainy weather.

Plane-table work, even if the sheets are tinted as recommended, is trying to the eyes, and telescope readings are fatiguing. It is desirable to rest occasionally when the weather is unpropitious for plane-table work, and the surveyor can then occupy himself with traversing, or perhaps measuring main lines and angles, only leaving the re-measurement and offset detail work to a subordinate (*vide* 'Traverse Surveying'). Each surveyor should have two sheets in hand, one in the traverse stage, and the other being completed with the table. The computation and plotting of the traverses will be performed at the head office.

Measuring differences of level with the telescope. With the tacheometric alidade differences of level may be determined from the vertical angles, and the staff-readings of the three horizontal diaphragm lines.

The apparent distance (a) is the difference of the readings of the top and bottom lines multiplied by 100, assuming an anallatic lens.

The difference of level $\triangle h$ between the instrument axis and the staff reading of the middle reference line is given by the expression

$$\triangle h = a \sin\theta \cos\theta$$

An 'eye and object correction' must be applied as in tacheometry.

Call rises +
Falls −
Height of instrument +
Staff reading of middle reference line −

Sum algebraically, and the result is the difference of levels of the instrument and staff positions.

A slide rule is sometimes provided for making this calculation in the field. It usually forms part of the rule used for reducing the apparent to the horizontal distance.

The vertical angles must be read to minutes, and the level should be on the T-piece, so that it can be brought central by the clip-screws for each reading, as in the theodolite. Index error should be tested frequently by reading a vertical angle with both faces, as in the theodolite (see p. 178).

Complete topographical map prepared with plane-table. With the tacheometric alidade and plane-table a complete topographical map may be prepared. The levels of a number of points may be fixed from each station and entered on the plan. Contours may then be sketched in. The vertical angles between two successive positions of the table should be taken both forward and backwards, and the mean used, in order to eliminate the effects of curvature and refraction. It would probably be advisable to book the main forward and backward readings, and work them out accurately at leisure. To facilitate this, the group of level points taken at each station of the table might be enclosed with a pencil line, and the levels of the points inscribed on the plan; correction would then be easy.

Desirability or otherwise of taking vertical angles with the plane-table. If the country is bold and rugged, and the main object is to produce a topographical plan showing the configuration of the ground, no better procedure than vertical angles and deduced difference of levels can be adopted.

If, on the other hand, the country is level and the main object is to produce a plan recording correctly roads, fences, buildings, etc., then the surveyors will have quite enough to do without determining heights. Levelling will be performed more cheaply, expeditiously, and accurately, as a separate operation after the plan is finished.

CONDITIONS UNDER WHICH THE PLANE-TABLE MAY BE PROFITABLY EMPLOYED. The conditions under which the plane-table may be profitably employed may be discussed as follows.

The first question is that of climate. It would be absurd to use the plane-table in a persistently wet climate. It is a fine weather instrument. Yet even in the British Isles it is useful for verifying and bringing to date Ordnance maps in connection with Parliamentary plans, or for filling in topographical detail in conjunction with traverses for a variety of engineering projects.

If a cadastral map of a country much intersected with fences has to be produced, it is invaluable, as a secondary instrument, for filling in detail. It has the advantage when used with the tacheometric alidade that it involves no damage to crops. The staff-holder naturally walks round the margins of the fields. There is no chaining through standing crops, no smashing fences, and no annoying claims for compensation.

In the open country, especially if the survey is carried on by means of an organized party of staff surveyors, computers and draughtsmen, the plane-table is not so valuable as it would be with a less complete organization.

To the engineer who has to study an unsurveyed country for the purpose of preparing a project for a road, railway, or other public work, the plane-table is invaluable. He can, unassisted except by a few labourers, rapidly produce a contoured plan amply sufficient to locate a line of communication, in a far more satisfactory manner than if he proceeded at once with theodolite and level.

EXAMPLES OF GREAT PLANE-TABLE SURVEYS. The preliminary survey of the St. Gothard railway was entirely made with the plane-table. A contoured plan of the valley was first made to a small scale. On this, the line, with its numerous tunnels, was studied and located. The magnificent contoured map of Switzerland was produced by means of the plane-table, with triangulation as a basis. Sketching from nature, as it were, the plane-table surveyor can give expression to features better than if the drawing were done at the office. The engineer can note many points, such as the nature of the ground and the manner of overcoming difficulties.

The plane-table has been found useful for filling in the details of a town survey to a scale as large as $\frac{1}{100}$ (approximately $\frac{1}{8}$ of an inch to the foot), such as that of Kingston, Jamaica. Bolts were fixed in the ground at intersections of streets, and at intervals of 200 feet. The exact positions of these were determined accurately. Two such points were projected on each sheet: starting from one the section was surveyed, checking to the second. The streets of the town in question are straight and approximately at right angles to each other. The bolts in the north and south streets were ranged out with the theodolite, and the distances were measured with a long steel band, 10 chains in length. The plane-table was of the simple type described and figured at the commencement of this chapter. The sight-rule as figured was designed for use with it.

The various objects, corners, doors, veranda-posts, steps, etc., within 50 feet of the table were determined by rays and tape measurements; the distances from place to place of the table were measured with a good chain. As soon as finished the sheet was traced. In this manner a plan of each street was produced, suitable for the design of sewers and drains, and for their record after construction.

It was found that the detail could be drawn in to scale on the ground in little longer time than would be taken to make sketches and measurements. There was less chance of omissions and error than with the usual plan.

THE PLANE-TABLE IN CONTOURING. The plane-table, with the tacheometric alidade lends itself admirably to contouring, especially when used in conjunction with the level. The telescope of the plane-table may be used to fix the contour points. This is not, however, a convenient plan, for the sight-rule does not traverse easily on the paper. In the opinion of the writer, it is better to use a level to determine the contour points, leaving to the plane-table the duty of registering their position on the plan. The work is much facilitated by the employment of two observers, one for the level, and the other for the plane-table. The leveller fixes the points on the contour by a procedure similar to that described (see 'Setting out contours in the field', Chapter 3), with the exception that there is no necessity for planting pickets. The leveller determines the contour points, and the plane-tabler fixes them at once on the plan by drawing a ray and measuring the subtense on the same staff as is used by the leveller. The work proceeds with great rapidity, and this system has the great merit that the contours are drawn, as it were, from nature. Minor accidents of the ground can be sketched in. It is also possible for the plane-tabler to sketch in by eye, with no small accuracy, contours intermediate to those determined instrumentally, thus preparing a very fine topographical plan, with great rapidity and precision.

CHAPTER 8

DRAWING OFFICE PRACTICE: COMPUTATION OF AREAS AND VOLUMES

PAPER FOR PLANS. All original plans should be plotted on best hand-made drawing paper, mounted on brown canvas and well seasoned. If the plan is a very large one, it should have a thin paper mounted between the drawing paper and canvas, or holland backing to give it extra substance. Machine-made papers should be avoided, as they are more liable to become distorted by contraction than hand-made, and, in the case of alterations being necessary, are rendered rough and unsightly by erasures, while the abraded surface takes up dirt very rapidly, and cannot be effectually cleaned without injury to the plan.

Canvas- or linen-backed paper is obtainable in long rolls and is useful for plans of roads and railways which take the form of a lengthy ribbon. It usually happens that engineering surveys are used as the basis for working drawings, plans and sections of the projected work being superimposed on the original plotting. In such cases the final drawings are almost invariably traced, since a number of copies will be needed by the engineers and contractors, and sometimes the original drawings are finished in pencil. Many kinds of tracing paper, tracing cloth and new transparent plastics are now available but most of them are subject to expansion and contraction.

Copies are obtained from the tracing by photographic processes, the type of print depending on the kind of printing paper used. Thus, ferro-prussiate paper gives white lines on a blue background (hence the term 'blue-print'); ferro-gallic or dye-line paper gives dark purple lines on a white background. Both these papers are developed in a water-bath, and the consequent wetting and drying may cause much distortion. For this reason important dimensions should never be scaled from prints which have been submitted to this treatment, and it follows that the original drawings should be fully and clearly dimensioned. This rule applies not only to the measurements needed for the actual construction but also to those necessary for setting-out the work. It is, however, possible to obtain 'true-to-scale' copies by processes which do not involve wetting and drying.

USE OF LOOSE SHEETS, AND NOT MOUNTED PAPER. Surveys should not be plotted on paper mounted on a board, for the distortion which takes place on cutting off is very great. As many kinds of good paper, such as Whatman's, are, in their original condition, so much buckled as to make accurate work difficult, the writer has found it convenient to flatten the paper before using it. To this end a number of sheets are mounted on a drawing-board, one above the other, by moistening and glueing down the edges in the usual manner. When dry, but not too dry, they are cut off and laid flat in a drawer and allowed to remain there for some weeks, till they have attained the average hygrometric condition of the air of the office. They remain smooth and flat, and are therefore pleasant to work upon, and expand and contract in a fairly regular manner.

PLOTTING ON MILLBOARD FOR PLANE-TABLE. If the details of a survey are to be filled in by the plane-table, the traverse or trigonometrical work should be plotted on millboard, which should be clamped to the table, and not attached by glue or gum. Millboard expands and contracts fairly uniformly in all directions, so that if the distances are laid off from a scale of the same material, no inconvenience is experienced, even if the sheet be exposed to varying atmospheric conditions for several weeks. For plane-table work, the millboard should be of a grey or drab colour to avoid the fatigue to the eye which white paper causes.

PLOTTING A CHAIN SURVEY

Scales. Boxwood, ivory and plastic scales can be obtained of any desired pattern, and to any proportion. Those for plotting a survey made with a Gunter's chain are usually 12 inches in length, marked on one edge with chains and decimals of a chain and on the other edge with an equivalent scale of feet. This is useful in case it is necessary for measurements to be scaled or plotted in feet, as in the case of buildings and building land.

Offset scales (Fig. 179) are similar to the above, but only about 2 or 3 inches in length, and frequently without the 'feet equal' side. It is very desirable that the zero of the offset scale should be at the centre of its length, and not at one end.

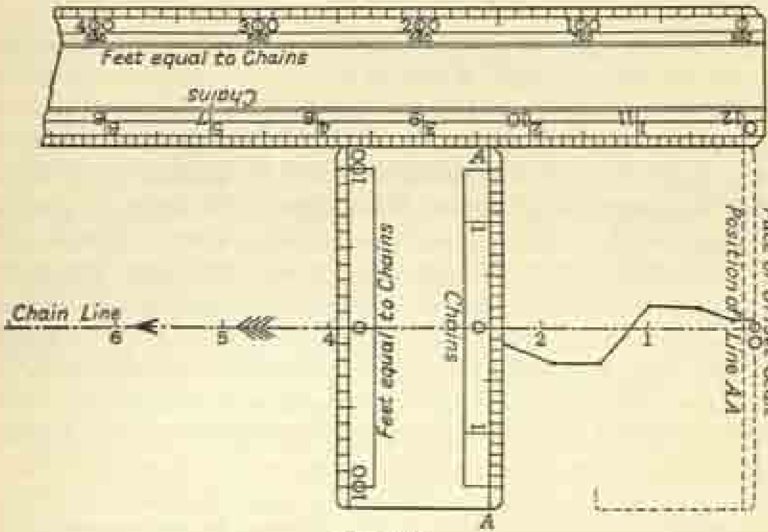


FIG. 179

LAYING DOWN LINES ON PLAN. The position of the first, or main, line should be carefully chosen, and it should be so ruled in that the whole plan, when completed, will be well placed upon the paper. If true north and south runs vertically, as is the case with Ordnance maps, so much the better, but this is not essential. A steel straight edge is the most suitable kind of ruler, and may be obtained of any desired length. Some surveyors use a hard, fine-pointed pencil for ruling in the lines, and afterwards ink them in with faint blue, while others use the eye end of a needle, which makes a fine indented line without scratching the paper. A fine needle-point should be used for pricking off the stations and for marking chainages at convenient intervals according to the scale of the drawing. This latter is a practice greatly to be commended, as it saves much time in the adjustment of new stations upon any of the main lines, after a possible expansion or contraction of the paper. The radii for the intersecting arcs are usually struck by means of a beam compass.

When the survey lines are laid down and proved, for the whole or for a portion of the survey, the details can be proceeded with, and here again the precaution taken to prick off marks at convenient intervals upon the main lines will come in very useful.

As offsets have sometimes been recorded upon one side of the chain line, and sometimes upon the other, it will be found convenient and expeditious to lay the long scale parallel to the line in question, and at such a distance that the zero of the offset scale shall coincide with the survey line.

The scale should then be weighted down at both ends, and the offset scale left free for the work. This method of plotting, especially if the details are very numerous, will be found to afford facilities for ruling in from point to point, and drawing in details, which would not exist were the long scale laid close to the survey line.

In plotting, the points should be marked off either with a very hard finely-pointed pencil or be pricked in with a fine needle.

The '*ruling in*' should be done with a medium pencil, which can be cut to a fine chisel-shaped point, and will admit of the lines drawn being erased with india-rubber when necessary.

The surveyor should acquire the habit of reading his 'field-book' when it lies in any position upon the table, so that the line in the book may always be placed before him in the same direction as the line he is plotting.

This will save much confusion, especially in complicated work.

It may not always be possible to remain in the neighbourhood of the survey until all the detail is plotted, but, at all events, it is exceedingly unwise to leave it until every line has been laid down and proved to be correct, either upon the plan itself or upon a diagram drawn to scale.

Most surveyors manage to keep their plotting closely up to the field-work. The first wet day affords an opportunity of getting the main lines laid down, after which it is easy as a rule to keep the plotting well in hand, either by working in the evening, or by rising earlier in the morning.

LIST OF SCALES IN COMMON USE

$\frac{1}{792}$ or 1 chain to an inch		For small estates and single fields
$\frac{1}{1584}$,, 2 chains ,,		Ditto and for working surveys
$\frac{1}{2376}$,, 3 ,, ,,		For first-class tithe maps of England and for large estates
$\frac{1}{3168}$,, 4 ,, ,,		Ditto
$\frac{1}{3960}$,, 5 ,, ,,		Ditto
$\frac{1}{4752}$,, 6 ,, ,,		Ditto
$\frac{1}{6336}$,, 8 ,, ,,		Ditto
$\frac{1}{1200}$,, 100 feet to an inch		Building and engineers' plans
$\frac{1}{2400}$,, 200 ,, ,,		Drainage schemes, etc.
$\frac{1}{4800}$,, 400 ,, ,,		Ditto
$\frac{1}{120}$,, 10 ,, ,,		Surveys on which working drawings are designed
$\frac{1}{240}$,, 20 ,, ,,		Ditto
$\frac{1}{360}$,, 30 ,, ,,		Ditto
$\frac{1}{480}$,, 40 ,, ,,		Ditto
$\frac{1}{500}$,, 41·667 ,, ,,		Ditto
Medium Ord. of England	$\frac{1}{10560}$ or $\frac{1}{6}$ mile to an inch	= 6 ins. to a mile
Minimum scale of deposited plans	$\frac{1}{15840}$ or $\frac{1}{4}$,, ,,	= 4 ins. ,, ,,
Small Ord. map of England	$\frac{1}{63360}$ or 1 ,, ,,	= 1 inch to a mile
Large Ord. map of England	$\frac{1}{2500}$ or 0·03946 ,, ,,	= 25·344 inches or 2·112 feet to a mile
'New' Ord. scale for towns	$\frac{1}{500}$ or 41·667 ft. to an inch	= 10·56ft. to a mile
'Old' ditto	$\frac{1}{528}$ or 44 ,,	= 10 ,, ,,

The student should compare this with the table on p. 18.

CONSTRUCTION OF SCALES. In the table above each scale is represented in the first place by a fraction, which is called the representative fraction. Thus a scale of 100 feet to 1 inch has a representative fraction of $\frac{1}{1200}$, and, to that scale, *any* length on the ground is represented by that fraction of itself.

Thus 12 metres would be shown as 1 centimetre on the drawing, and the scale might be described with equal truth as 12 metres to 1 centimetre.

THE DIAGONAL SCALE. The diagonal scale provides a useful means of obtaining a measurement to a finer degree of accuracy than that given by a scale of comparatively coarse graduations. Thus, if a scale reads only to one-tenth of an inch, the simply constructed diagonal scale illustrated in Fig. 180 enables measurements to be

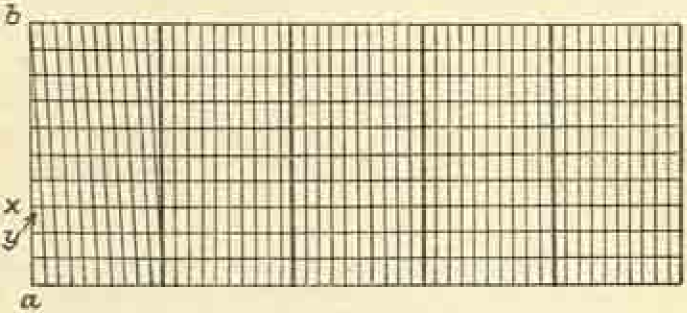


FIG. 180

taken to one-hundredth of an inch. The horizontal base-line is marked off in inches and tenths, and the vertical line, *ab*, is made of any convenient length and sub-divided into ten parts. The intercept *xy*, for example, is thus 0·07 inch.

The same principle may be used, for example, to construct a scale of metres for a plan whose scale is given as 100 feet to 1 inch, and it is not necessary to know the relation between the foot and the metre, if we possess a scale of centimetres.

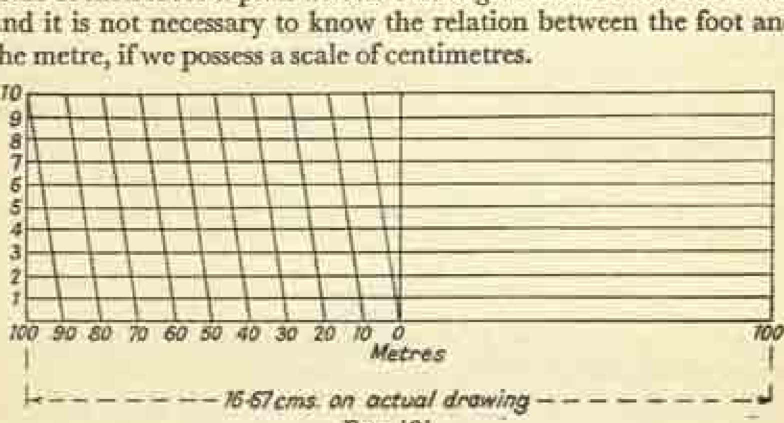


FIG. 181

On a scale of $\frac{1}{1200}$, 12 metres are represented by 1 centimetre, and therefore 200 metres by 16·67 centimetres, which is about a convenient length of scale. We set this off, and subdivide as shown in Fig. 181, which thus gives readings down to 1 metre.

FINISHING THE PLAN, AND TITLE. After the work is all plotted, and any errors or doubtful points examined and corrected, the next thing is to ink it in.

This must be done with a drawing pen, and waterproof Indian ink of good quality should be used, as it is practically everlasting while other inks sooner or later either fade away or injure the paper. It is usual to show by characteristic signs all leading features, such as woods, plantations, gardens, quarries, sand, gravel, or clay pits, tips, spoil banks, railway banks and cuttings, and such like; also to colour all dwelling-houses carmine or lake, all other buildings Payne's grey or neutral tint, rivers and other water blue, etc.

For these and the printing, unless a highly ornamental finish is desired, the characteristics published for the 6-inch, 25-inch, and other Ordnance maps of England may be taken as a guide, except that more detail is put upon these maps than is desirable or necessary for estate plans. Lithographed sheets illustrating these conventional signs and types of lettering are published by the Ordnance Survey Department.

CALCULATION OF AREAS

Areas of land shown on plans and the areas of cross-sections showing the extent of excavations and embankments are frequently required. Figures of irregular shape may be resolved into triangles, rectangles, parallelograms or trapezoids and the areas of these figures are found thus:

1. The area of a triangle is found by multiplying its base by half its perpendicular, or vice versa. It may also be found from the lengths of the three sides, thus:

$$\text{Area} = \sqrt{S \times (S-a) \times (S-b) \times (S-c)}$$

a, b and c being the three sides and S their half sum.

Or, putting it into words:

From half the sum of the sides, subtract each side separately, and multiply together the half sum and the three remainders. The square root of the product will be the area required. This method is rarely used.

2. The area of a rectangle or parallelogram is found by multiplying the length by the perpendicular height.

3. The area of a trapezoid is obtained by multiplying the perpendicular distance between the parallel sides, by half the sum of the parallel sides.

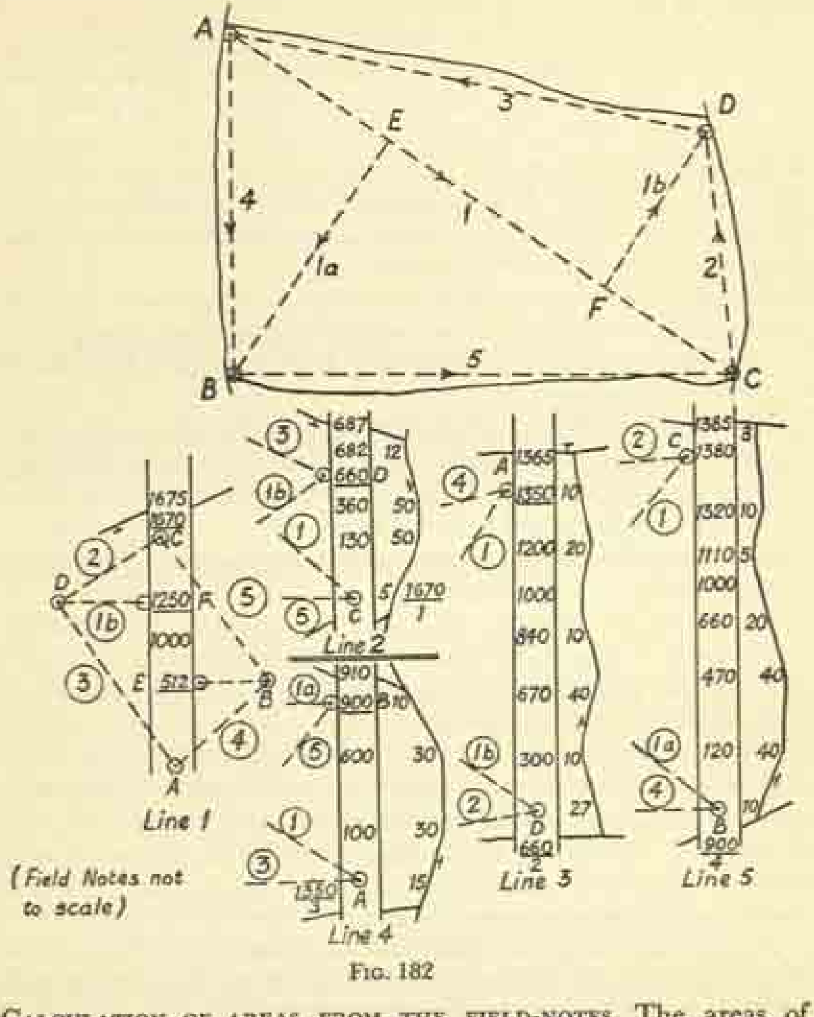


Fig. 182

Calculation of areas from the field-notes. The areas of enclosures may be obtained from the field-notes direct, without reference to a plotted plan, but when it is desirable to do so the survey lines must be specially arranged for the purpose.

In the case of the 'single field' survey annexed the survey lines were arranged as in Fig. 182.

The line AC formed the common base for the triangles ABC and ADC, while BE and FD were purposely set out to form the perpendiculars, and these two triangles embrace the bulk of the area of the field.

Considering the portion between the survey lines and the boundary, the lengths of any two adjacent offsets are known from the field-notes. These, together with the intermediate portion of the chain line (whose length is found by subtracting the chainage of the first offset from that of the second) and the intermediate portion of the fence, form a trapezoidal figure. The area is calculated by multiplying the length along the chain line by *half* the sum of the perpendicular offsets.

The intermediate portion of fence may be regarded as straight, as offsets are taken to all bends, or close together on curves.

The details of the working for the single field are given in the annexed table (p. 336).

The result of the calculation is recorded in square links, the chain used being the ordinary one of 66 feet. Since there are 100,000 square links in an acre, it is only necessary to mark off, by means of a decimal point, five figures from the right, when the figures to the left of the decimal point represent the acres, and those to the right decimals of an acre.

The quantities are given in these units upon the 25-inch Ordnance maps of England. When an apparent division occurs, such as a road or pathway, the sign ∽ is used to indicate that this is included in the total area given.

If the result is required in acres, roods, and perches, it is necessary to multiply the decimal portion by 4 (the number of roods in an acre), and again cut off five figures, when the figure on the left of the point will be roods, multiply again by 40 (the number of perches in one rood), and again cut off five figures, the result is in perches and decimals of a pole or perch.

By an inspection of the field-notes, or plan, it will be seen that along the lines Nos. 2 and 3 the marks ⌶ are *inside* the fences. This conventional sign, so placed, indicates that the true boundary is outside the line of hedge. On the lines 4 and 5 the reverse is the case. It is customary when the legal boundary is outside the enclosure to *add* a strip 4 feet wide, whose length in this case may be taken as 687+1365, or say 2050 links (the combined lengths of lines 2 and 3). Similarly, when the legal boundary is within the hedge line, we *deduct* a strip whose length in this case is $910 + 1385$, or say 2300 links.

The width of 4 feet may be taken as 6 links without sensible error. Hence the resultant correction is $(2300 - 2050) \times 6$, or 1500 square links to be *deducted*.

All the lengths for each line should be summed (as a check on the arithmetic) to see that the sum agrees with the total length of the line. This check may be shown in the table.

If the chain used had been one of 100 feet in length, the result would of course have been in square feet, of which there are 43,560 in an acre.

The foregoing method gives the area with the greatest possible accuracy, but when it is determined to make use of the 'field-notes' for this purpose, great care must be exercised so to arrange the survey lines in the fields that no piece of ground be taken into account twice over and that no piece be omitted altogether.

Such cases are likely to occur at the corners of fields, unless proper precautions be taken.

The best method of avoiding such double measurements is to commence each line from a station upon an existing line, and to produce it through the point from which the next line will start, until it strikes the boundary.

Thus line 2 commences from 1670 line 1, and is continued beyond 660, the actual closing point from which the next line starts, until the boundary fence is reached at 687. This enables the surveyor to run an offset to the corner by which to obtain the area of that small piece, which otherwise would be omitted from the calculations. Likewise line 3 commences from 660 line 2 and is run past 1350, the starting point of next line, and lines 4 and 5 are treated in like manner.

Areas by the planimeter. Where the area is to be obtained from the plan, the Amsler planimeter is probably the most expeditious and accurate instrument for the purpose. Its form is well known, and, as it is not strictly a surveying instrument, no description is here given. Full instructions for use are supplied with the instrument.

In its improved form the bar is graduated to give the area either in square inches or in acres and decimals to a number of definite scales.

No.	Offsets $(h + h_1)$	$\frac{h+h_1}{2}$	Length	Area	Remarks
1	15 30	22·5	100	2,250	
2	30 30	30	500	15,000	Line AB
3	30 10	20	300	6,000	
4	10 0	5	10	50	
5	10 40	25	120	3,000	
6	40 40	40	350	14,000	
7	40 20	30	190	5,700	
8	20 5	12·5	450	5,625	Line BC
9	5 10	7·5	210	1,575	
10	10 3	6·5	65	422	
11	5 50	27·5	130	3,575	
12	50 50	50	230	11,500	
13	50 12	31	322	9,982	Line CD
14	12 0	6	5	30	
15	27 10	18·5	300	5,550	
16	10 40	25	370	9,250	
17	40 10	25	170	4,250	
18	10 20	15	360	5,400	Line DA
19	20 10	15	150	2,250	
20	10 5	7·5	15	113	
21	743 509	626	1670	1,045,420	ABCD, line AC

Deduction for fences = 250 × 6
= 1500 sq. links
= 0·015 acre
= 2·4 perches

Final area, 11·494 acres

1,150,942 sq. links = sum

11·509 42 acres
4

2·037 68 roods
40

1·507 20 perches

Area = 11·509 acres, or 11 acres 2 roods 1·5 perches

The needle point round which the instrument revolves should be *outside* the area if possible. If it is necessary that it should be inside, an addition has to be made to the reading, the amount of this addition being stamped on the bar above the mark to which the index has been set. But it will probably be more accurate to divide the area into parts, and planimeter each part with the needle point outside it.

For accurate work the setting of the planimeter should not be trusted.

It is best to rule out with the utmost care at least two rectangles of about the same area as the figure to be dealt with. Planimeter these with any setting of the planimeter, and see that the results agree.

Then planimeter the unknown area at least twice. The results should agree within 0·1 per cent.

The reading of the two areas establish a ratio between the known rectangular area and the unknown area, from which the latter can be found.

For example:

1st rectangle	8 ins. by 3 ins.	Planimeter area = 36·15
2nd ,,	6 ins. by 4 ins.	,, ,, = 36·17

The mean reading is 36·16; the true area in each case = 24 sq. ins. Unknown area, planimetered; 1st result 41·17, 2nd 41·19, mean = 41·18.

$$\text{Then true area in sq. ins.} = 24 \times \frac{41 \cdot 18}{36 \cdot 16} = 27 \cdot 33 \text{ ins.}$$

If the scale is 6 chains to 1 inch, this will be 36 × 27·33 or 983·95 sq. chains = 98·395 acres, etc.

'EQUALIZATION OF BOUNDARIES.' In the absence of a planimeter the writer has found this the most accurate method. Referring to Fig. 183, the irregular boundary of the area is replaced by straight lines following it very closely, but arranged so that whenever a small piece is cut off the area, an equal piece (as well as can be judged by eye) is added on outside. The lines are best drawn by the aid of a transparent protractor or set square with a fine line ruled right across it, the side on which the line is drawn being turned downwards. The square or protractor is moved till this line appears to occupy the right position for equalization. It is held there, the ends of the line are marked by fine needle dots, the square removed, and the line ruled in in pencil, and so on. The lines should be ruled not to *meet* only, but to *cut* one another. We then go all round the figure and mark the points of intersection with fine needle dots, as at *a*, *b*, *c*, *d*, etc. If these dots be joined up in whatever order may seem most convenient, we get a set of triangles, which are numbered as shown. These are taken in pairs, having a common base. Thus for Nos. 1 and 2, the common base

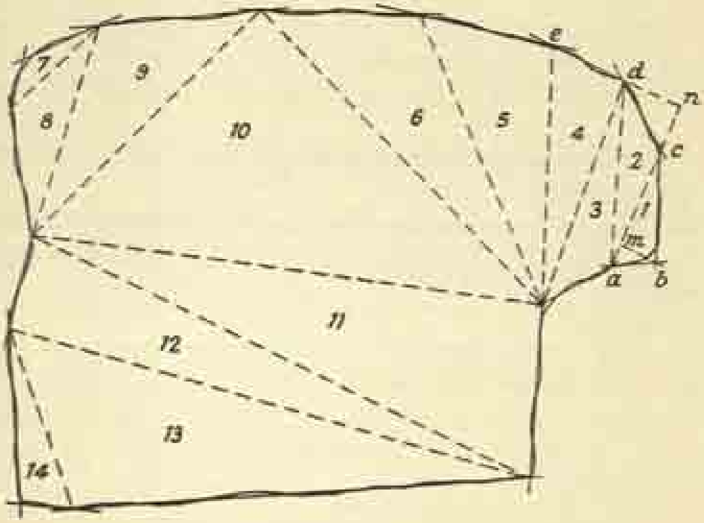


FIG. 183

is *ac*. Perpendiculars *bm* and *dn* are drawn to it, and give the *heights* of the triangles. The common base and these heights are measured, and half the product of the base into the sum of the heights will give the area of the two triangles. And so on.

Tabulate as follows:

Nos.	Heights	Sum	Base	Product
1 and 2	$\begin{Bmatrix}2{\cdot}55\\4{\cdot}59\end{Bmatrix}$	7·14	7·92	56·549
	And so on			

In some cases (for instance, in No. 13 here) only one triangle may be taken at a time.

All measurements are taken in feet or chains to scale. When the table is completed, the last column is summed, and *half* the result will give the area in square feet or chains. The writer has always obtained results by this method within 0·1 per cent., but the straight lines must follow the curved boundary very closely.

SIMPSON'S RULE. In the case of a strip of ground, say that required for a portion of cutting or embankment, we may know the breadths

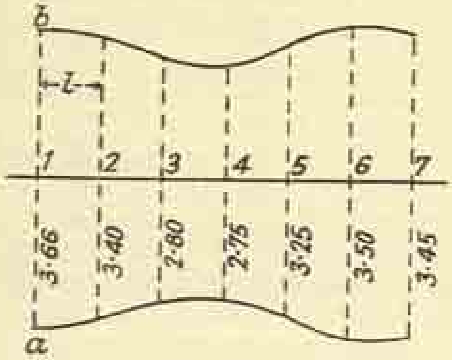


FIG. 184

at a number of points, 1, 2, etc. (Fig. 184), at equal distances along a centre line. These breadths, *ab*, etc., are called *ordinates*, and those to which *even* numbers are attached (as for example those at 2, 4, etc.) are called *even* ordinates, the others being *odd*.

Let the distance between the ordinates be *l*. Then the rule for the area is:

$$\frac{l}{3}(4 \times \text{sum of even ordinates} + 2 \times \text{sum of odd ordinates} + \text{sum of first and last ordinates})$$

The first and last ordinates are not included in the sum of the odd ordinates. The first ordinate is always numbered one.

The rule applies only when the *last* ordinate is odd. If the number attached to the last ordinate is even, the figure is usually either re-divided into an *even* number of parts (so as to get an *odd* number of ordinates) or the area is calculated by the ordinary trapezoidal rule: take *half* the sum of the first and last ordinates + the sum of all intermediate ordinates, and multiply by the distance, *l*, between ordinates.

The rule is stated rather differently in different books. The difficulty is to remember which are even ordinates and which odd. It is best to remember that the first and last ordinates are not multiplied by any factor. Those next to the first and last are multiplied by *four*, and then the successive ordinates by 2 and 4 respectively. The calculation should be tabulated as here shown, the table referring to Fig. 184.

No.	Ordinate	Do. × factor
1	3·66	3·66
2	3·40	13·60
3	2·80	5·60
4	2·75	11·00
5	3·25	6·50
6	3·50	14·00
7	3·45	3·45
		57·81

The last column contains the ordinates multiplied by one, two, or four according to position. It is summed, and the sum multiplied by one-third the distance between ordinates. Thus if the ordinates are one chain apart, the area will be $57{\cdot}81 \times \frac{1}{3} = 19{\cdot}27$ square chains, or if, as commonly occurs, 100 feet apart, the area will be $57{\cdot}81 \times \dfrac{100}{3}$, or 1927 square feet.

Area by computing scale. The method of arriving at the area by means of the 'computing scale' must now be considered. This appliance is used by the Ordnance Survey for computing the area of fields and other enclosures shown on 25-inch maps.

The 'computing scale' can be obtained to any scale, and to give results in acres and decimals, or in acres, roods, and perches, as desired. It is simply a device by which the number of chains passed over by the sliding vane is recorded, and to this end it is used in conjunction with a piece of transparent paper, ruled with parallel lines exactly 1 chain apart to the scale of the drawing. This is placed on the irregular area so that the uppermost line just touches the top of the area, as in Fig. 185.

The instrument consists of two parts, the scale itself, and a small sliding frame, carrying a fine wire or hair at right angles to the scale. The latter is graduated so that a length which represents

10 chains to scale is numbered 1 acre, and each acre is divided into 4 roods, that is, assuming that the area is required in acres.

To use the scale: set the centre of the sliding frame to zero, placing the scale upon the paper in the same direction as the lines, and with the wire so adjusted that the portion marked *a* in the following diagram, Fig. 185, is equal to the portion marked *b*; then holding the scale firmly, slide the frame forward until the wire is in such a position as to make the portion marked *c* equal to that marked *d*; then lift the scale bodily, taking care not to disturb the sliding frame, and commence upon the next line, making *e* equal to *f*; then slide the frame on till *g* equals *h*, and so on until the whole length of the scale is used. If the field is not then completed, make a small mark or dot immediately under the wire, and commence from the right hand and work *backwards* to the left. It will be seen that the scale is figured from left to right upon the upper side, and continued from right to left upon the lower side; for a 3-chain or 25·344-inch scale it is usually 5 acres in length, so that double its length, or 10 acres, can be run off before making any special note.

It is clear that the reading of the scale at any time will give the area down to that point at once.

The degree of accuracy attainable by the use of this instrument is proportionate to the skill of the operator in the equalization of the boundaries.

A 'universal computer' is made in which the scales are interchangeable, and its case usually contains scales of 1, 2, 3, 4, 5 and 6 chains to an inch, as well as 6 inches and 5 feet to a mile. Ruled transparent papers or parchments to match, are also obtainable.

It is manifest that any required scale could be fitted to such a sliding frame.

ENLARGING AND REDUCING PLANS. The enlargement of plans is to be avoided if possible. It is better to plot to a larger scale. Reductions (or enlargements when unavoidable) may be performed by the pantograph or eidograph, illustrations of which will be found in all the catalogues, or by photography.

Enlarging or reducing may also be done by covering the plan with numbered squares, other squares similarly numbered being drawn on the new sheet, but bigger or smaller according to the desired degree of alteration of scale. The points where the various

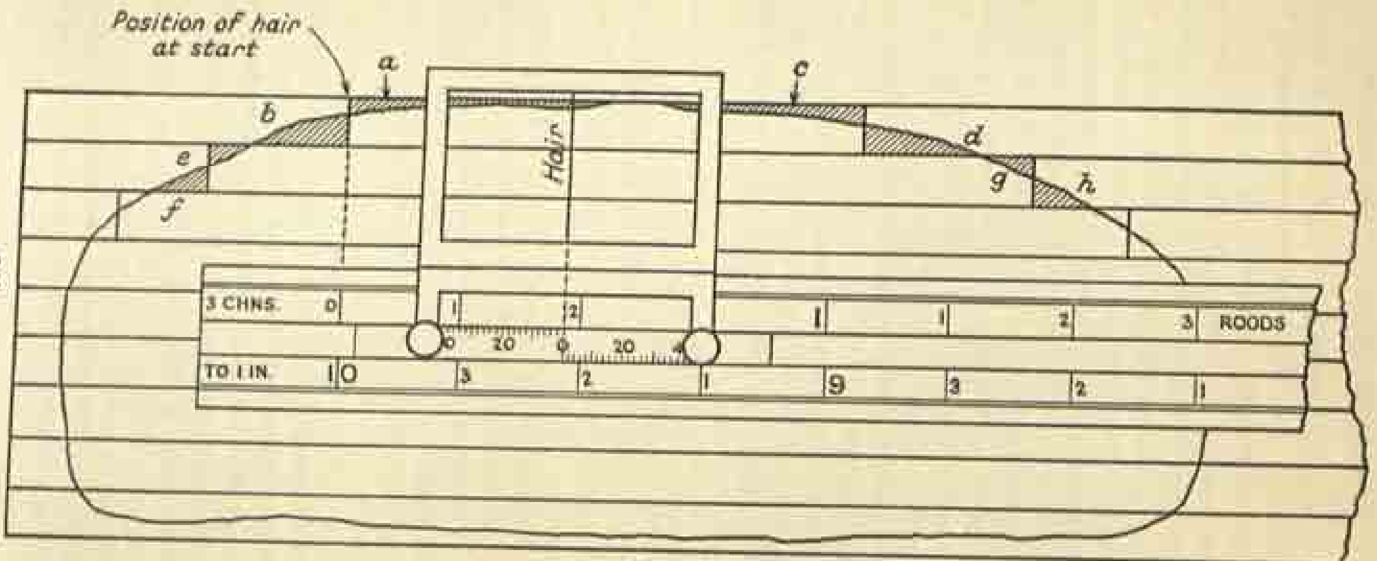


Fig. 185

boundaries cut the sides of these squares are then transferred to the corresponding points on the new squares by direct measurement or by proportional compasses.

PLOTTING TRAVERSES

PLOTTING BY MEANS OF A PROTRACTOR. Traverse-surveys may be plotted by means of a protractor, direct from the field-book. If, however, included angles are measured, the bearings should always be calculated and the lines plotted from those to avoid accumulation of drawing errors.

The most efficient protractor for the purpose is one in which the graduated arc is combined with a parallel ruler and is fitted with a vernier.

Another method is to fix to the paper (with its zero and 180° points, accurately in the meridian), a circular protractor with two arms, carrying pricking points placed at opposite extremities of a diameter; then to prick off and number several pairs of points, the lines joining each pair, representing one bearing. These lines all pass through the centre of the protractor. After a sufficient number of pairs of points have been pricked off, the protractor is removed, and the bearings are then transferred to their proper positions by means of a parallel ruler, and each distance is laid off.

These circular protractors are usually provided with verniers, so that 'bearings' may be set off to single minutes or even less.

A somewhat similar process may be carried out by means of a cardboard (or other) protractor with the central portion cut away to an internal diameter of 9 or 12 inches. This being placed on the drawing with zero and 180° north and south, it is secured by weights. The starting point of the survey must be inside the ring, to left or right according to the direction of working. A long parallel ruler is now placed so that its edge passes through the bearing of the line to be plotted, and through the opposite point in the graduated circle. It is then run along to pass through the starting-point, and the line ruled in and scaled off; and so on, for as many lines as can be plotted inside the ring, after which the protractor must be moved. The method is convenient for small scale work.

COMPARISON OF METHODS OF PLOTTING. Good work can no doubt be done with the protractor, if very carefully used, with the disadvantage, however, that a closing error which may be due to either incorrect field work or to errors in draughtsmanship cannot be located without going over the whole work again.

The method of co-ordinates admits of positive check, at every stage of the work, even before plotting is commenced.

Another advantage of the method of co-ordinates is that several draughtsmen can be employed simultaneously, in plotting and filling in.

The traverse sheets and field-books can be distributed amongst several persons, who can work each on a separate sheet of paper.

If, on the other hand, the protractor be used, it is convenient, and almost necessary, to have the whole plan laid down at once on a single sheet of paper.

Although the protractor cannot compare with co-ordinates as to accuracy and convenience of plotting, and should never be used on the main lines of important work, still it is a most useful appliance. It is often desirable to make a preliminary plot, as work progresses, to see how the work comes in.

The division of the paper into squares practically renders the surveyor independent of the expansion and contraction due to alterations of temperature, and of the moisture of the air. Paper absorbs moisture freely, expanding with moisture, and contracting on drying very considerably. Moreover, the rate of expansion or contraction is not by any means constant in every direction.

If, at any time, serious distortion takes place, perhaps owing to exposure to sun in checking on the ground, or in filling-in detail with the plane-table, the squares afford a ready means of restoring the plan to correctness. Lastly, the squares add materially to the value of the finished plan, for they afford a means of taking out areas and measuring long distances, with greater accuracy than is possible with the short scales usually inscribed on plans.

AREAS IN TRAVERSING

The squares drawn on the paper in plotting by co-ordinates afford a means of estimating the area of any piece of ground, by counting the whole squares, and finding the areas of the fractional parts of squares by equalizing the boundaries, so as to form triangles or trapezia, and measuring the dimensions of these.

The area of a field surveyed by a closed traverse may be calculated by this method. An example is given in Fig. 186, in which the side of each square represents 200 feet. The dotted lines show

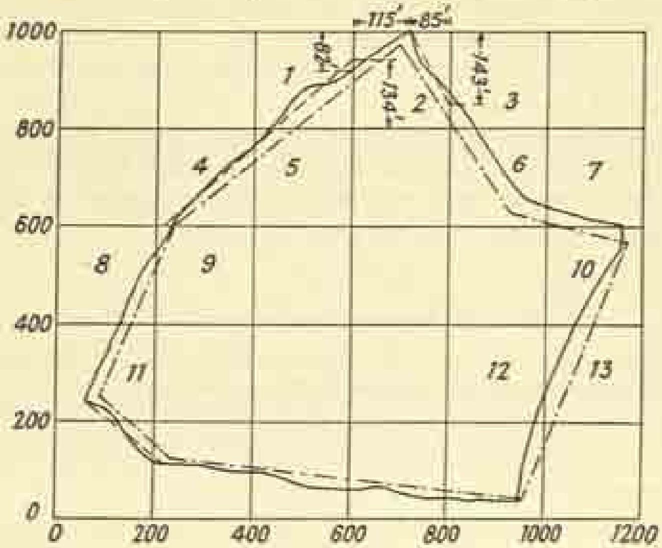


Fig. 186

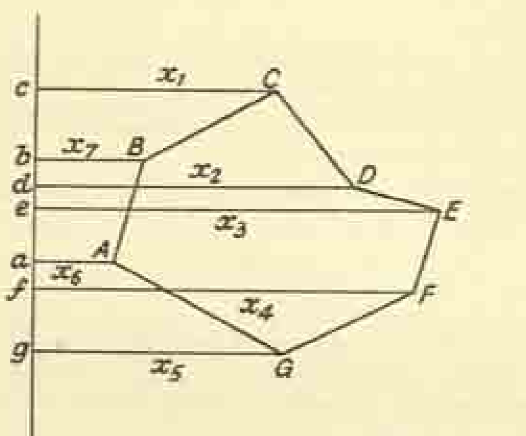


Fig. 187

where the boundaries have been 'equalized', whilst some of the dimensions measured have been marked on the figure for reference.

The complete working is left as an exercise for the student. The result obtained should be 638,296 square feet.

AREAS FROM CO-ORDINATES. The area of any closed polygon may be determined direct from the traverse sheet, without plotting, and therefore free from all errors in scaling off distances, and from those due to shrinkage of paper.

It is evident that the area of the polygon ABCDEFG (Fig. 187) is equal to that of the figure *c*CDEFG*g* less the figures *c*CBAG*g*. Now each of these figures is composed of a number of trapezia *c*CD*d*, DE*ed*, and so on.

Hence the area of the original figure

$$= c\mathrm{CD}d + d\mathrm{DE}e + \ldots - g\mathrm{GA}a - a\mathrm{AB}b - \ldots$$

Now *c*C, *b*B, etc., are the east co-ordinates of the different points, and we will denote them by x_1, x_2, etc.

$$\therefore \text{area} = \frac{x_1+x_2}{2} \times cd + \frac{x_2+x_3}{2} \times de + \ldots$$

$$- \frac{x_5+x_6}{2} \times ag - \frac{x_6+x_7}{2} \times ba - \ldots$$

Again O*c*, O*d*, etc., are the north co-ordinates, and we will denote them by y_1, y_2, etc.

$$\therefore cd = y_1 - y_2;\ de = y_2 - y_3$$
$$ag = y_6 - y_5 = -(y_5 - y_6)$$
$$ba = y_7 - y_6 = -(y_6 - y_7), \text{ and so on}$$

Hence

$$\text{area} = \frac{x_1+x_2}{2}(y_1-y_2) + \frac{x_2+x_3}{2}(y_2-y_3) + \frac{x_3+x_4}{2}(y_3-y_4)$$
$$+ \frac{x_4+x_5}{2}(y_4-y_5) + \frac{x_5+x_6}{2}(y_5-y_6)$$
$$+ \frac{x_6+x_7}{2}(y_6-y_7) + \frac{x_7+x_1}{2}(y_7-y_1)$$

If we multiply this out, after writing down the whole series, certain terms cancel, and we find the result–

$$\tfrac{1}{2}[x_2(y_1-y_3) + x_3(y_2-y_4) + \ldots + x_7(y_6-y_1) + x_1(y_7-y_2)]$$

Put into words, this rule reads as follows: Take each east co-ordinate in turn, and multiply it by the difference between the north co-ordinates of the *two adjacent points*.

Sum the results algebraically, and take *half* the sum.

In finding the differences, the north co-ordinates must be taken in the same order all round the polygon, so that each difference may be either plus or minus, and attention must be paid to the sign.

Thus x_2 ($=dD$) is the east co-ordinate of D (see Fig. 187).

The adjacent points are E and C.

Suppose we take it that E is the forward point, and C the back.

Then, if we take north co-ordinate of E − north co-ordinate of C, we must take the differences in the same order all round, namely, *plus* the north co-ordinate of the forward point and *minus* the

Pt.	E. co-ord.	Adjacent N. co-ords.	Difference	Product +	Product −
M	697·9	623·9 595·3	+ 28·6	19,960	
G	916·7	569·2 981·9	− 412·7		378,322
Y	1158·7	36·8 623·9	− 587·1		680,273
k	951·9	34·9 569·2	− 534·3		508,600
H	937·0	101·4 36·8	+ 64·6	60,530	
P	450·5	115·8 34·9	+ 80·9	36,445	
O_1	231·2	249·9 101·4	+ 148·5	34,333	
O	88·3	595·3 115·8	+ 479·5	42,340	
N	233·6	981·9 249·9	+ 732·0	170,775	
				364,383	1,567,195 364,383
					2)1,202,812
					601,406

north co-ordinate of the back point. But we could equally well take them in the reverse order, provided we take *all* in the same order.

If the final result is *minus*, simply change its sign to plus. If the co-ordinates are some north and east, and others south and west, we simply treat those which are south and west as *minus*, and work algebraically. This rule gives the area within the traverse lines only. The area between these lines and the boundary can then be found from the field-books directly as described under chain surveying, p. 333.

The area of the field in Fig. 186 has been worked out by this method.

The co-ordinates of all points, except *h* and Y, are from the traverse sheet, Fig. 77, p. 140; those of *h* and Y were found on p. 201.

It will be noticed that, starting from M, the adjacent north co-ordinates are those of G and N, of which the former is regarded as the forward station, and its co-ordinate is put uppermost. Hence for G the co-ordinate of Y is uppermost, and so on. The difference is found in each case by subtracting the lower from the upper.

The area of the part between the traverse lines and the boundaries is worked out and tabulated as on p. 336, except that in the actual working, in this case, many of the boundaries were equalized before calculation.

$$\begin{aligned} \text{The result was} \quad & \ 37{,}004 \text{ square feet} \\ \text{Area of polygon} & = \underline{601{,}406} \\ \text{Total} & = 638{,}410 \end{aligned}$$

The area as found on p. 346 was 638,296 square feet, showing a discrepancy of about one part in 6000, which is equivalent to an error in linear measurement of about one part in 12,000.

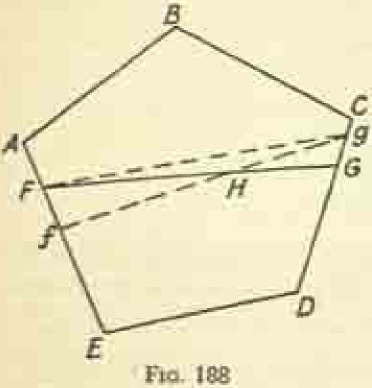


Fig. 188

Cutting off given areas. A problem which may occasionally arise, and is often set at examinations, is that of cutting off a given fraction of the area of a closed figure by a line satisfying some definite condition. A few examples are given.

(*a*) Where the line is to start from a given point F on the boundary (Fig. 188).

First obtain an approximate direction for the line FG, by guessing or by using squared paper, and measure the length DG.

From this calculate the co-ordinates of G (knowing those of D and C), then the length and bearing of FG, and hence the angle FGC. Also calculate the area FEDG from the known co-ordinates.

Now, if Fg be the corrected line, the area of the triangle FGg is the difference between the area FEDG and the area to be cut off, and is therefore known.

But $\quad\quad$ area of $FGg = \frac{1}{2} FG.Gg.\sin FGg$

Hence gG is known as FG and FGg are known.

(*b*) When the bearing of the dividing line is given, or the angle it makes with one side.

First obtain an approximate line as before, say FG. Then measure EF, and find co-ordinates for F; we know the bearings of GF and GD, and the co-ordinates of D and F, whence we may find FG and GD (p. 199); hence co-ordinates of G, and area of FEDG, and therefore the necessary correction in area. The line FG is to be moved parallel to itself through a perpendicular distance h, so as to produce this correction. Say the necessary direction of movement is *towards A and C*. Then let α_1, α_2, be the angles AFG, FGC, which are both known. Then it is easy to show that the area of the trapezium formed by moving FG is

$$\left[FG - \frac{h}{2}(\cot\alpha_1 + \cot\alpha_2)\right]h$$

Proper attention must, of course, be paid to the signs of $\cot\alpha_1$ and $\cot\alpha_2$. They will be negative if AFG and FGC are obtuse.

Equating this to the required correction in area we have a quadratic in h, but it is usually sufficient to find an approximate value, h_1, by putting $FG \times h_1 =$ required area.

Then write this value h_1, instead of h in the term inside the bracket, and recompute h. When the perpendicular distance h is known, the distances along FA and GC can be reckoned as $h \operatorname{cosec} \alpha_1$ and $h \operatorname{cosec} \alpha_2$.

(*c*) If the line is to pass through a given point inside the area, say H, of which the co-ordinates are also known.

Choose an approximate line as before; fix it, say, by assuming a bearing for it. Then from the known bearings of FG, EF and DG, together with the co-ordinates of D, E and H, we can find EF, FH, HG, GD, and hence the co-ordinates of F and G, and the area of either section of the figure. The difference between this and the required area gives the necessary correction.

Now, the line FG is to be turned through an angle α, say, as shown by the dotted line fg, to give this correction. The alteration produced is the area of $FfH - GgH$.

The angles HFf and HGg are known. Call them θ and ϕ respectively.

$$\text{Then } Ff = \frac{HF \sin \alpha}{\sin(\alpha + \theta)}$$

$$\therefore \text{area of } HFf = \frac{\frac{1}{2} . HF^2 \sin\theta \sin\alpha}{\sin(\alpha + \theta)}$$

$$= \frac{HF^2}{2(\cot\alpha + \cot\theta)}$$

$$\text{Similarly area of } HGg = \frac{HG^2}{2(\cot\alpha + \cot\phi)}$$

$\therefore$ if A = necessary correction in area

$$A + \frac{HG^2}{2(\cot\alpha + \cot\phi)} = \frac{HF^2}{2(\cot\alpha + \cot\theta)}$$

This is a quadratic in $\cot\alpha$, from which α can be found.

The directions of Ff and Gg are decided according to whether the trial area was too great or too small. Proper attention must then be paid to the signs of $\cot\theta$ (i.e. HFf) and $\cot\phi$ (i.e. HGg). The positive value must be taken for α.

Generally it is near enough to assume that

$$Ff : Gg :: HF : HG \quad \therefore Gg = \frac{Ff.HG}{HF}$$

$$\therefore A = \tfrac{1}{2}HF \times Ff \sin\theta - \tfrac{1}{2}HG . \frac{HG . Ff}{HF} . \sin\phi$$

$$\text{whence } Ff = \frac{2HF \times A}{HF^2 \sin\theta - HG^2 \sin\phi}$$

From this we can find Ff, and therefore the co-ordinates of f and bearing of fH.

The formula is exactly true if AE and CD are parallel.

In other cases, unless the movement be very small, we must either use the first formula or make a second approximation starting with *fg*.

Plotting Sections

Longitudinal and cross sections. A 'longitudinal section' represents the undulations of the ground and the constructional levels along the centre-line of a road, a railway or a sewer or other pipe-line.

A 'cross section' provides similar data along lines at right-angles to the centre line.

Many methods are in vogue for plotting sections. The form shown in Fig. 189 is one that is often used, and is plotted from the field-book on p. 79. The base line at the bottom is the datum line or a line at some round number of feet above it.

Exaggeration of vertical scales. It is usual to plot the vertical heights to a larger scale than the horizontal distance. The object of this is to give a clear idea of the variations of level and slope of the ground. Another reason for increasing the vertical scale is that the measurement of the verticals is more important than that of the horizontal dimensions. For example, in the case of an embankment with sloping sides, an error of one foot in scaling a length of one hundred feet, only affects the contents of the bank by 1 per cent. But the area of the cross-section of a bank varies roughly as the square of its height, consequently if the bank were ten feet high, an error of one foot in scaling this dimension would introduce an error of about 20 per cent. in its area. The ratio of vertical exaggeration must depend upon the formation of the ground, and on the purpose for which the section is taken. For sewer sections an exaggeration of 10 to 1 is convenient.

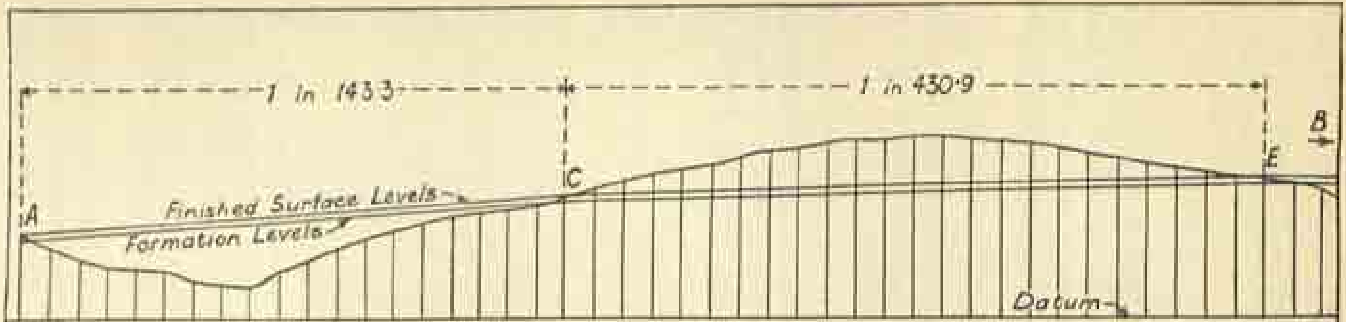
In the example shown in Fig. 189, the vertical scale has been exaggerated ten times, as compared with the horizontal.

If a Gunter's chain is used, it is usual to take the horizontal scale as 1, 2, or some other exact number of chains to 1 inch, and the vertical scale as 10, 20, or other round number of *feet* to 1 inch.

Thus a horizontal scale of 1 chain (=66 feet) to 1 inch, with a vertical scale of 10 feet to 1 inch, would give an exaggeration of 6·6 to one.

The scales should, of course, be always stated on the drawing.

Beneath the datum line in the section are other lines forming columns in which are inscribed the following numbers (Fig. 189):



1	2	3	4	5
0	29·70	28·45	1·25	
1	24·25	29·15		4·90
2	20·50	29·85		9·35
3	18·75	30·54		11·79
4	17·80	31·24		13·44
5	17·34	31·94		14·60
6	14·49	32·64		18·15
7	14·17	33·34		19·17
8	13·68	34·03		20·35
9	16·51	34·73		18·22
10	20·59	35·43		14·84
11	24·47	36·13		11·66
12	27·41	36·82		9·41
13	30·58	37·52		6·94
14	33·73	38·22		4·49
15	34·98	38·92		3·94
16	36·43	39·62		3·19
17	38·08	40·31		2·23
18	39·21	41·01		1·80
19	42·96	41·71	1·25	
20	44·46	41·94	2·52	
21	47·76	42·17	5·59	
22	49·98	42·41	7·57	
23	51·52	42·64	8·88	
24	53·02	42·87	10·15	
25	55·57	43·10	12·47	
26	57·52	43·33	14·19	
27	59·07	43·57	15·50	
28	60·42	43·80	16·62	
29	61·17	44·03	17·14	
30	61·71	44·26	17·45	
31	62·22	44·49	17·73	
32	62·52	44·73	17·79	
33	62·02	44·96	17·06	
34	61·37	45·19	16·18	
35	60·22	45·42	14·80	
36	59·12	45·66	13·46	
37	57·62	45·89	11·73	
38	55·62	46·12	9·50	
39	53·92	46·35	7·57	
40	53·12	46·58	6·54	
41	51·67	46·82	4·85	
42	50·20	47·05	3·15	
43	48·53	47·28	1·25	
44	47·08	47·19		0·11
45	45·33	47·11		1·78

Reference :

Column 1 : Distances
" 2 : Ground Levels
" 3 : Formation Levels
" 4 : Depth of Excavation
" 5 : Depth of Filling
} in feet

Fig. 189

Distances, reduced levels: copied from field-book.

Formation level in the case of a road or railway, or invert level in the case of a sewer, calculated from the known points A, C, etc.

Height of bank, or depth of cutting.

The formation level is put on after the section is plotted, and the heights of bank and cutting are then computed and inscribed, and are available for the computation of quantities.

Choosing gradients. In the example, the thick black line represents the original ground levels, along, say, the centre line of a part of a proposed road or railway. When the section is plotted the proposed gradients are shown on it, as indicated by the lines marked 'Finished surface levels', and 'Formation levels'. These indicate that it is proposed to bring the finished surface of the work to the line ACE and beyond to a point B, not shown, thus necessitating embankment from A to C and beyond E, and cutting from C to E. The formation line represents the levels to which the earthwork will have to be carried for this purpose, and the vertical distance between the two lines is everywhere the same, and equal to the depth of material used in forming the road or railway. In the example the formation is taken as 15 inches (1·25 feet) below the finished surface, this being a depth of broken stone, etc., sometimes adopted in country roads.

In choosing the gradients there are several guiding principles such as the following: (1) the gradients should be as gentle as possible; (2) no work should be dead level in cutting, as it is difficult to drain; (3) the cost of the work should not be greater than is justified by its importance.

The cost can be reduced by sacrificing the gradients, so as to reduce the amounts of cutting and embankment, and also by arranging that the volume of material taken from the cuttings shall be just sufficient to make the embankments, thus avoiding the necessity either of obtaining extra earth or of carting the extra material into 'spoil heaps'. Appropriate factors for the expansion and compaction of the soil must be applied when making calculations for balanced earthworks.

Other considerations may of course arise in many cases, but it is unnecessary to deal with them here.

The finished surface of the proposed road is to be level with the ground at A, C and E.

CALCULATION OF FORMATION LEVELS. In road work, and very often in railway work, it is the usual practice to rule in the finished surface and formation lines as chosen, and to *measure* the formation levels from the latter. (*Note.*–The formation levels are required, because, of course, the earthwork must be equalized by the formation line, and not by the finished surface.)

But in sewer work, or wherever the exact alignment is important, the intermediate levels should be *calculated* from those of the known points, such as A, B, etc.

It is proposed to give here a worked example to show how such calculations are performed.

The first step is to find the rise (or fall) per 100 feet or per chain. In this case the formation levels at A and C (chainage 19·0) are first found by subtracting 1·25 feet from the ground levels at those points.

	At A	At C
Reduced levels	29·70	42·96
	1·25	1·25
Formation levels	28·45	41·71

$\therefore$ rise in 1900 feet $= 41{\cdot}71 - 28{\cdot}45 = 13{\cdot}26$ feet

$\therefore$ „ 100 feet $= 13{\cdot}26 \div 19 = 0{\cdot}698$ foot

Starting with the formation level at A, this rise is then repeatedly added to find the formation levels at the end of each hundred foot length.

The calculation is carried to three decimal places if the results are required to two, to avoid accumulation of error. Thus:

at A,	28·450	and so on to	
	0·698	at 1800 feet,	41·014
at 100 feet,	29·148		0·698
	0·698	at 1900 feet,	41·712
at 200 feet,	29·846		

It will be seen that the level at 1900 feet comes back (to two decimal places) to the correct value as previously found, namely, 41·71. This checks the arithmetic.

Any formation levels at intermediate points can then be found.

It will be noticed that the figures are written to two decimal places only on the drawing.

The length from C to E is treated in the same way, and is left as an exercise for the student.

The formation levels give the heights above datum of the formation line everywhere. And as the reduced levels give the heights of the original ground, it is clear that the difference between the two at any point will give the total depth of earthwork at that point. This will be *cutting* if the formation level is *less* than the reduced, and embankment if greater.

The depths marked on the figure have been worked out in this way.

DETERMINATION OF VOLUMES

Assuming that the ground is level in cross-section, the shape of a portion of embankment will be as shown in Fig. 190. Here AD and GJ represent the original ground surfaces at two points, say one chain apart. BF and HL show the heights of the embankment at the same points, and BC or HI gives the top width of the bank, or 'formation width' as it is called.

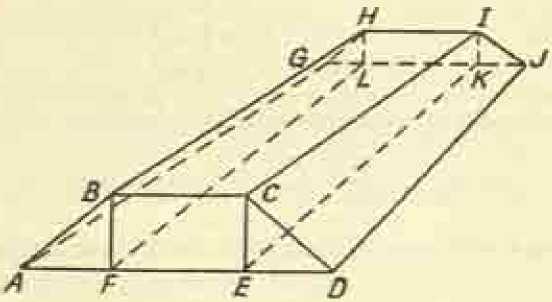


FIG. 190

CD, AB, etc., represent the side slopes, which are more or less steep according to the material of which the bank is made. The exact slope is expressed in any case by stating the ratio of the horizontal distance (between any two points on the slope) to the vertical difference of level between the same points. Thus, if we say that the slope is 2 to 1, we mean that the horizontal distance, ED, is *twice* the vertical height, EC.

For calculating the volume of the bank, it will be observed that the central portion BCEFHIKL is *wedge-shaped*.

Let $BF = h_1$, and $HL = h_2$, the end heights
$BC = b$, the formation breadth
$BH = l$, the length between ends

$$\text{Then vol. of wedge} = bl\left(\frac{h_1 + h_2}{2}\right) \quad . \quad . \quad (1)$$

The side slopes are frustra of pyramids. That is, when the height IK becomes zero, the breadth KJ also vanishes. The volume, therefore, cannot be *correctly* found by taking the mean of the end sections and multiplying by the length.

The correct formula is as follows:

Let A_1 = area of triangle ECD

A_2 = area of triangle IKJ

$$\text{Then vol. of each slope} = \frac{l}{3}(A_1 + \sqrt{A_1A_2} + A_2)$$

If the side slopes are n to 1, then $ED = n \times EC = n \times h_1$.

$$\begin{aligned}\text{But area of ECD} &= \tfrac{1}{2}ED \times EC \\ &= \tfrac{1}{2}nh_1 \times h_1 \\ \therefore A_1 &= \tfrac{1}{2}nh_1^2 \\ \text{Similarly } A_2 &= \tfrac{1}{2}nh_2^2\end{aligned}$$

$$\therefore \text{vol. of each slope} = \frac{l}{6}(nh_1^2 + nh_1h_2 + nh_2^2)$$

$$\text{and for both side slopes, } \frac{l}{3} \times n(h_1^2 + h_1h_2 + h_2^2) \quad . \quad (2)$$

Many tables have been calculated from these formulæ and are obtainable in book form. Among these may be mentioned those by Grace (published by E. & F. N. Spon), and Crandall (Chapman & Hall).

The formulæ are too cumbersome for use without tables, and, in view of the irregularity of the ground, it is probable that in very few cases will any important error be introduced by using an approximate formula in which the volume is taken as the length multiplied by the mean of the cross-sectional areas of the two ends.

$$\text{Now, area of BCEF} = BC \times FB = b \times h_1$$

$$\text{and area of ECD} = \tfrac{1}{2}n \, . \, h_1^2 \text{ as above}$$

$$\therefore \text{for the two side slopes, area} = nh_1^2$$

$$\text{and total area of end} = bh_1 + nh_1^2$$

$$\text{and for the other end, } bh_2 + nh_2^2$$

$$\begin{aligned}\therefore \text{volume} &= l \times \left(\frac{bh_1 + nh_1^2 + bh_2 + nh_2^2}{2}\right) \\ &= l \times \left[b\left(\frac{h_1 + h_2}{2}\right) + n\left(\frac{h_1^2 + h_2^2}{2}\right)\right]\end{aligned}$$

If there is another length, l, with a height h_3 at the further end, the volume of this length will be

$$l \times \left[b\left(\frac{h_2 + h_3}{2}\right) + n\left(\frac{h_2^2 + h_3^2}{2}\right)\right]$$

and the whole volume of the two lengths becomes

$$l \times [b(\tfrac{1}{2}h_1 + h_2 + \tfrac{1}{2}h_3) + n(\tfrac{1}{2}h_1^2 + h_2^2 + \tfrac{1}{2}h_3^2)]$$

and so on for any number of lengths.

As an example, the calculation of the volume of embankment from A to C (Fig. 189) is here tabulated. It will be observed that for the *first* and *last*, we must take the *half* depth and the *half square*, as indicated by the formula.

Chainage	Depth	Square	Remarks
1	2·45	12·00	halves
2	9·35	87·42	
3	11·79	139·00	
4	13·44	180·63	
5	14·60	213·16	
6	18·15	329·42	
7	19·17	367·49	
8	20·35	414·12	
9	18·22	331·97	
10	14·84	220·23	
11	11·66	135·96	
12	9·41	88·55	
13	6·94	48·16	
14	4·49	20·16	
15	3·94	15·52	
16	3·19	10·18	
17	2·23	4·97	
18	0·90	1·62	halves
	185·12	2620·56	
	40	2	
	7404·80	5241·12	
	5241·12		
	12645·92		

Thus the first actual depth is 4·90, of which the square is 24·0. We take 2·45 and 12·0, and write *halves* in the remarks column. The formation width is taken as 40 feet, and the side slope as 2 horizontal to 1 vertical.

It will be seen that the second and third columns are summed; the former is multiplied by the formation width (b in the formula), or 40 feet, and the latter by the ratio of the slopes (n in the formula), or 2 in this case. The sum of these results must then be multiplied by l, which in this case is 100 feet. The volume would then be obtained in cubic feet, and to bring it to cubic yards we divide by 27.

$$\therefore \text{volume in cubic yards} = 12{,}645{\cdot}92 \times 100$$
$$= 46{,}837 \text{ nearly}$$

The calculation has been carried to a greater degree of accuracy than would generally be necessary, or indeed justifiable.

The volume has been obtained from 100 feet to 1800 feet. The small end pieces may be allowed for, if desired.

From the figure, the embankment is seen to begin at 20 feet, and end at 1860 feet. Hence between 0 and 100 feet we have to allow for 80 feet length of embankment, with a height of zero at one end, and 4·90 at the other.

$$\text{The volume} = \frac{80}{27}\left[40 \times \frac{4{\cdot}90}{2} + 2 \times \frac{(4{\cdot}90)^2}{2}\right] \quad . \quad . \quad . \quad (1)$$

Similarly for the piece between 1800 and 1860 feet the height at 1800 feet is 1·8 feet, hence

$$\text{volume} = \frac{60}{27}\left[40 \times \frac{1{\cdot}80}{2} + 2 \times \frac{(1{\cdot}80)^2}{2}\right] \quad . \quad . \quad . \quad (2)$$

Working out these expressions, we obtain—

From (1)	361·5	cubic yards
From (2)	86·5	,,
Sum	448·0	,,
And volume from table . .	46,837·0	,,
Total	47,285·0	,,

GRADIENTS WITH EXAGGERATED VERTICAL SCALE. In drawings such as that in the foregoing example (Fig. 189) it must be remembered that we must allow for the exaggeration of the vertical scale.

If, for example, the ratio of exaggeration is 10 to 1 (p. 351), a gradient of 1 in 40 would appear on the drawing as *ten* vertical to forty horizontal.

BREADTHS AND AREAS BY CALCULATION. Where the transverse slope of the ground is uniform, it is not difficult to find formulæ for the sectional area and horizontal breadth of the earthwork in an excavation or on an embankment.

In Fig. 191, let AB = formation width = $2b$

$$EL = x_1 \qquad FM = x_2$$
$$EH = y_1 \qquad FG = y_2$$
$$CD = \text{depth at centre} = h$$

Also suppose the side slopes are n horizontal to 1 vertical, and the transverse slope of FE is k to 1.

Then $\qquad BH = x_1 - b = ny_1$

and $\qquad x_1 = k \times DL = k(h - y_1)$

By subtraction, $-b = y_1(n+k) - kh$

whence $\qquad y_1 = \dfrac{kh-b}{k+n}$ and $x_1 = b + ny_1 = \dfrac{k(b+hn)}{k+n}$

Similarly $\qquad y_2 = \dfrac{kh-b}{k-n}$ and $x_2 = \dfrac{k(b+hn)}{k-n}$

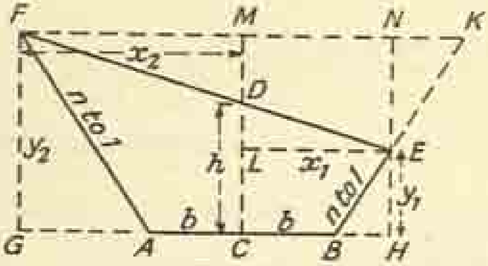


FIG. 191

Now to find the area, we subtract the triangle FKE from the trapezium FKBA.

$$MC = FG = \frac{AG}{n} = \frac{x_2 - b}{n}$$

Similarly $\quad EH = \dfrac{x_1 - b}{n}$; and $EN = MC - EH = \dfrac{x_2 - x_1}{n}$

$$\therefore \text{area} = \frac{FK + AB}{2} \times MC - \frac{FK}{2} \times EN$$

$$= (x_2 + b)\left(\frac{x_2 - b}{n}\right) - x_2\left(\frac{x_2 - x_1}{n}\right)$$

$$= \frac{x_1 x_2 - b^2}{n}$$

This is a simple formula where the side slopes are equal.

Where the section is partly filling and partly cutting, as in Fig. 192, it can easily be shown by similar reasoning that the greater width

$$x_2 = \frac{k(b+hn)}{k-n}$$

as before, and the smaller

$$= \frac{k(b-hn)}{k-n}$$

while the area $= \text{LAF}$ on the greater side

$$= \frac{b(x_2-b)^2}{n(x_1+x_2-2b)}$$

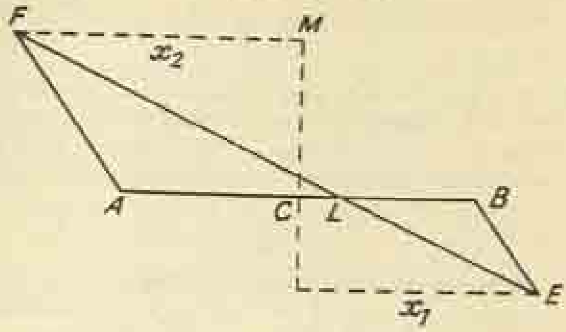


FIG. 192

and on the smaller side LBE

$$= \frac{b(x_1-b)^2}{n(x_1+x_2-2b)}$$

For the method of fixing E and F on the field, see p. 100.

PLAN WITH NO TRANSVERSE SLOPE. Let AB (Fig. 193) be a portion of the centre line in plan, and suppose *ac* and *bd* are parallel lines showing the formation width.

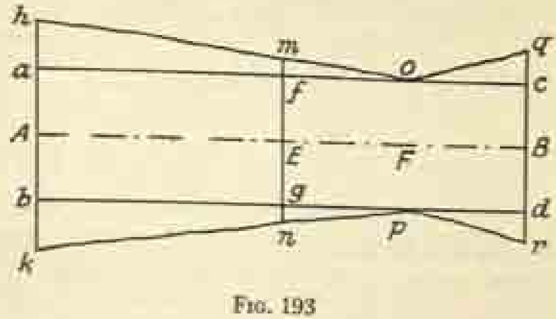


FIG. 193

Then suppose the depth of embankment at A, as determined from a longitudinal section, is 10 feet, and the side slopes 2 to 1. Then the *breadth* of the slope in plan will be $2 \times 10 = 20$ feet. Hence we must make $ah = bk = 20$ feet. Similarly if the depth of embankment at E be 4 feet, then $fm = gn = 2 \times 4 = 8$ feet to scale.

The lines *hm*, *kn*, will then show the bottom (or *toe*) of the embankment slopes in plan. Now suppose that at B we are in cutting, the depth of cutting being 8 feet, say. Assume that the side slope in cutting is $1\frac{1}{2}$ to 1. Then make $cq = dr = 1\frac{1}{2} \times 8 = 12$ feet. At the point F, where the longitudinal section shows that the formation line meets the ground, there will be no side slopes, as shown at *o* and *p*. Then join *oq* and *pr* to draw the plan of the top (or outcrop) of the cutting slopes, and so on.

VOLUME OF A RESERVOIR FROM CONTOURS. To find the volume of water which a reservoir will contain, we take out the areas covered by the water when it stands at different levels. A contoured plan is invaluable for this purpose.

Referring, again to Fig. 61, p. 110, when the reservoir is full to the top, it will cover the area F*gg*G, and this area is obtained by planimeter or otherwise.

At the level of 77·0, the water will cover the area H*hh*H, and so on.

Now the vertical interval between these levels is 3 feet. Hence, if we take the *mean* of the areas F*gg*G and H*hh*H, in square feet, and multiply by this vertical interval of 3 feet, we obtain, with fair accuracy, the volume of the horizontal slice of water, as it were, between the levels of 80·0 and 77·0.

This is repeated for each pair of successive contours.

In practice we work from the bottom *upwards*, beginning therefore in this case with the area covered when the water stands at the level of 50·0.

The very small amount of water below this level is neglected.

The whole is tabulated as in the table below.

It will be evident that the actual calculation could have been shortened had the object been to find the volume with a *full* reservoir only. But it is clearly useful to know approximately the volume of water in the reservoir when it stands at *any* level.

It is for this reason that we work upwards, and that the table is worked out fully as shown.

In taking out the areas, the river-bed has been neglected.

They are taken out in square inches, and are reduced to square feet according to scale. Thus the scale of the original drawing was 100 feet to 1 inch. Hence each square inch was $100 \times 100 = 10{,}000$ sq. ft. The successive means are then taken, and each mean is multiplied by the vertical interval (in this case uniformly 3 feet) to obtain the volumes of the successive slices. Up to the level of 53·0, the total volume will be the same as that of the first slice. At the level of 56·0 it will be the first two volumes added together. At the level of 59·0, the third must be added to the previous total, and so on.

Level	Area (sq. ins.)	Area (sq. ft.)	Mean	Volume (cub. ft.)	Total volume	Level
50·0	0·17	1,700				
53·0	0·33	3,300	2,500	7,500	7,500	53·0
56·0	0·58	5,800	4,550	13,650	21,150	56·0
59·0	1·95	19,500	12,650	37,950	59,100	59·0
62·0	5·33	53,300	36,400	109,200	168,300	62·0
65·0	10·07	100,700	77,000	231,000	399,300	65·0
68·0	17·57	175,700	138,200	414,600	813,900	68·0
71·0	21·37	213,700	194,700	584,100	1,398,000	71·0
74·0	25·50	255,000	234,350	703,050	2,101,050	74·0
77·0	33·31	333,100	294,050	882,150	2,983,200	77·0
80·0	44·46	444,600	388,850	1,166,550	4,149,750	80·0
				4,149,750		

To check the arithmetic, the volume column is summed, and the result should agree with the total volume at the level of 80·0.

VOLUME OF DAM. The total volume of the dam, from ground level upwards, may be found on a similar principle.

Thus the horizontal section area of the dam at the level of 85·0 is represented by ABCD. At the level of 80·0, it is GJJF. The mean of these areas multiplied by the vertical interval of 5 feet will give the volume of the dam between those levels, and so on.

From 80·0 down to 47·0 (which may be taken as the bottom of the dam) the vertical interval is uniformly 3 feet, and the calculations can be shortened, as the *total volume* only is required in this case.

Thus let the area at the level of 47·0 be called A_1; that at 50·0, A_2, and so on, finishing with the areas HKKH at 77·0 (which we will call A_{n-1}, and GJJF at 80·0, which we will call A_n. Then, from 47·0 to 80·0

$$\text{volume} = 3 \times (\tfrac{1}{2}A_1 + A_2 + A_3 + \ldots + A_{n-1} + \tfrac{1}{2}A_n).$$

The tabulation is shown in the annexed table.

Level	Area (sq. in.)	Area × factor
47·0	0·36	0·18
50·0	1·16	1·16
53·0	1·57	1·57
56·0	1·92	1·92
59·0	2·32	2·32
62·0	2·59	2·59
65·0	2·73	2·73
68·0	2·77	2·77
71·0	2·69	2·69
74·0	2·42	2·42
77·0	2·04	2·04
80·0	1·54	0·77
		23·16
		3
		69·48
80	1·54	0·77
85	0·46	0·23
		1·00
		5
		5·00
	Total =	74·48
Volume $= 100^2 \times 74{\cdot}48$ $= 744{,}800$ cub. ft.		

In column No. 3 we take the *half* areas at the levels of 47·0 and 80·0, and the whole areas at the other levels.

This column is summed up the level of 80·0, and the result is multiplied by the vertical interval, 3 feet.

The piece between 80 and 85 is worked out separately and the two results added to find the total.

As the areas are in sq. inches this must be multiplied by 100^2 to reduce to cubic feet, as the scale is 100 feet to the inch.

By sections. Alternatively, we may divide the length CD of the dam into any even number of equal parts (in this case we have taken *six* parts) by lines 11, 22, etc.

Sections of the original ground are then drawn along these lines, using the contours for reading off the levels at different points on the lines.

Thus the line 11 in plan meets the upstream toe at the level of 75·0, as well as can be determined. It cuts the 74·0 contour at a point which is evident on inspection, and meets the down stream toe at or about the level of 73·0. These three points are projected up to the corresponding levels in the section to draw the ground line numbered 1. If, now, we take the area between this ground line and the top level, *da*, of the dam, we have the cross-sectional area of the dam, down to ground level, along the line 11 of the plan.

This is repeated for each of the lines, and the volume can then be determined from these areas, say by Simpson's rule.

The results are as follows, the area at each end being obviously zero:

Level	Area (sq. in.)	Area × factor
0	0·00	0·00
1	0·21	0·84
2	0·77	1·54
3	1·56	6·24
4	1·75	3·50
5	0·37	1·48
6	0·00	0·00
		13·60

The whole length CD of the dam is scaled as 472 feet. Hence the distance between the sections is 78·7 feet (= 472 ÷ 6).

The areas are in square inches. Now 1 square inch on the section = 100 × 20 = 2000 square feet, as the original scales were 100 feet to the inch horizontally, and 20 feet to the inch vertically.

Therefore the volume is $\frac{78 \cdot 7}{3} \times 2000 \times 13 \cdot 6 = 713{,}500$ cubic feet

This method is probably more convenient than the first in dealing with the dam, especially if the volume of the puddle wall or central part is separately required, as it can be readily shown on the sections and the results separated.

But it suffers from the disadvantage that, quite possibly, no section may pass through the deepest part of the dam.

Thus, in the figure, the stream comes entirely between the sections 33 and 44, and thus the deepest part of the dam is missed. In the example, the result by this method is, in consequence, too small by about four per cent.

This can, however, be overcome if necessary by taking the sections closer together, or by dividing into an *odd* number of parts and using the trapezoidal rule (p. 333) instead of Simpson's rule.

VOLUME OF EXCAVATION FROM CONTOURS. As another example, we will suppose that Fig. 194 represents the contours on a piece of ground, on which a building is to be erected, the overall outside dimensions to be 120 feet by 80 feet, and the whole area to be excavated down to the level of 17·0 feet above datum for basements, etc. It is required to find the volume of the excavation.

In the figure, ABCD is the plan of the outside of the building.

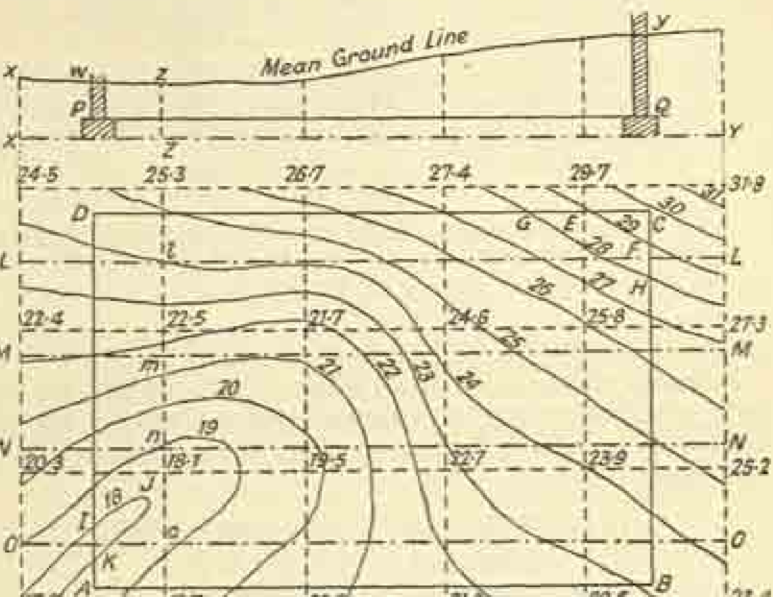


FIG. 194

DIRECT METHOD BY CONTOURS. One method of working is similar to that already described. Thus ECF represents the horizontal area of the excavation at the level of 29·0; GCH represents the area at the level of 28·0, and so on, to IDCBAKJ at the level of 18·0, and the whole area ABCD at the level of 17·0.

These various areas are planimetered; half the first and last and the whole of the rest are summed and multiplied by the vertical interval, in this case 1 foot.

Level	Area (sq. in.)	Area × factor
29·0	0·11	0·05
28·0	0·59	0·59
27·0	1·28	1·28
26·0	2·45	2·45
25·0	4·70	4·70
24·0	8·09	8·09
23·0	11·83	11·83
22·0	14·91	14·91
21·0	17·54	17·54
20·0	19·77	19·77
19·0	22·42	22·42
18·0	23·90	23·90
17·0	24·00	12·00
		139·53

In the original drawing the scale was 20 feet to 1 inch; hence the sum of the last column must be multiplied by 400×1.

$$\text{Hence total volume} = 139{\cdot}53 \times 400 = 55{,}812 \text{ cubic feet.}$$

This neglects the corner ECF, which is above 29 feet. To allow for it, if desired, we may take ground level at C as 29·9. Then the corner may be taken as a pyramid whose base = ECF, and height = 0·9 foot.

$$\begin{aligned} \text{Area of ECF} &= 0{\cdot}11 \times 400 = 44 \text{ square feet} \\ \therefore \text{volume} &= 44 \times 0{\cdot}3 = 13 \text{ cubic feet} \\ \text{or total volume} &= 55{,}825 \text{ cubic feet} \end{aligned}$$

METHOD OF MEAN SECTIONS. In another method of working, the length AD is divided into any number of equal parts – in this case four – and through the middle points of these parts lines are drawn parallel to AB or DC. These are shown in the figure by 'dot and dash' lines, LL, MM, etc.

If now we draw a section representing the *mean section* of the ground along these four lines, the area between this line and the foundation level will give the mean sectional area of the excavation.

To draw the mean ground line, we choose some line like LMNO, and read off, from the contours, the levels at which this line is cut by each of the four dot and dash lines.

Thus for the left-hand end we have

$$\begin{array}{rr} \text{level at} & L=23{\cdot}4 \\ ,, & M=22{\cdot}1 \\ ,, & N=20{\cdot}7 \\ ,, & O=\underline{19{\cdot}0} \\ & 4)85{\cdot}2 \\ & \text{Mean}=21{\cdot}3 \end{array}$$

Now, in the section, we choose a line XYZ and fix its level usually at some round number of feet above datum, in this case 15·0. Hence the level of 21·3 will be 6·3 feet above X, and we set up Xx to represent this on any convenient scale. On the original drawing 10 feet to 1 inch was used, but generally the vertical scale is the same as the horizontal in such cases.

For the next line, the levels at o, n, m, and l are seen to be respectively 18·8, 19·0, 21·6, and 24·0, giving a mean of 20·85, and we set up Zz, therefore, to represent 5·85 feet. This is done for all the lines, and we thus obtain the 'mean ground line' as marked.

Now draw PQ at the level of 17·0 (i.e. 2 feet above XY) to represent the bottom of the excavation.

The area wyQP is now planimetered and found to be 3·48 square inches.

Each square inch represents $20 \times 10 = 200$ feet, according to the scales given, and length of AD = 80 feet.

Hence volume of excavation $= 3{\cdot}48 \times 200 \times 80 = 55{,}680$ cubic feet, which agrees closely with the previous result.

This method is probably more convenient, and more generally used in practice, than the first, in all cases such as this, where the work is rectangular is shape.

The mean ground line should always be shown, and so marked, on the drawing.

If there are foundations for the walls, below the level of 17·0, as shown, these are best allowed for separately, perhaps, being simply the sectional area of the footings multiplied by their total length.

The spot levels shown on the drawing are 30 feet apart each way. The student is advised to copy these on a larger scale, and to interpolate the contours and re-work the whole exercise for himself.

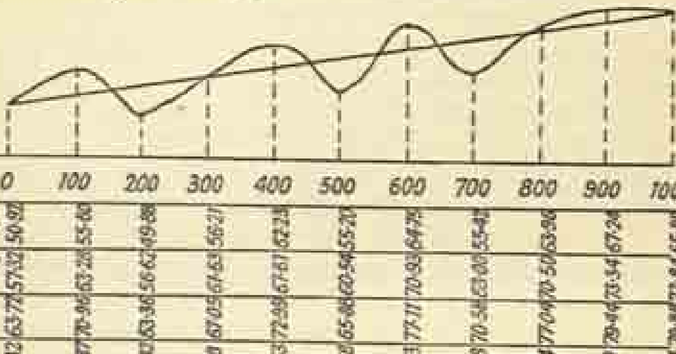
Haulage or mass-haul curves. In the construction of lines of communication involving cuttings and banks, extra expense is often incurred through material being carted the wrong way, and it is not easy to estimate the cost of cartage at all accurately.

In such cases, mass-haul curves are distinctly helpful. In drawing them, cuttings are taken as *plus*, and fillings *minus* (or vice versa), and the curve shows the accumulated volume – algebraically – from the start to any point on the line.

For example, suppose we have a longitudinal road section such as that shown in Fig. 195, which is an artificial example intended only to show the principle in a small space. Fig. 196 (*a*), (*b*), and (*c*) show the first three cross-sections, the vertical scale being exaggerated five times.

The student is advised to draw out the whole example, using scales of 50 feet to 1 inch horizontal and 10 feet vertical for the longitudinal section, and 10 feet and 2 feet respectively for the cross-sections.

The formation width is here taken as 30 feet–horizontal–throughout, and the side slopes as 2 horizontal to 1 vertical both in cutting and banks. A more detailed cross-section may be used, as in Fig. 196 (*d*) showing the formation trimmed to the road camber, areas being found by planimeter.



Distances	0	100	200	300	400	500	600	700	800	900	1000
Reduced Levels: 50′ Left	50·92	55·60	49·88	56·21	62·23	55·20	64·75	55·42	63·96	67·24	65·80
Reduced Levels: Centre Line	57·32	63·28	56·62	61·63	67·61	60·54	70·93	63·00	70·50	73·34	72·84
Reduced Levels: 50′ Rt.	63·72	70·96	63·36	67·05	72·99	65·88	77·11	70·58	77·04	79·44	79·88
Formation Levels	57·32	58·87	60·42	61·98	63·53	65·08	66·63	68·18	69·74	71·29	72·84

Fig. 195

To draw the mass-haul curve, areas of cutting and fill at each cross-section must be determined and tabulated as on p. 357; these are done at every 100 feet by drawing or calculation, as may be more convenient in each case.

Between these, several intermediate points may be required, namely: (1) points at which cuttings or fillings end. For example the cross-section at zero includes both cutting and filling, while at 100 there is cutting only. Somewhere between, the filling must end, and up to that point some material must be transferred across from the cutting on one side to the bank on the other. Though this does not enter into the *haulage*, it must be paid for, and it is necessary to estimate the quantity. The filling will be zero when the ground line BC (Fig. 196 (*a*), (*b*)) passes through the end A of the formation width, that is when AD becomes zero. Now in Fig. 196 (*a*), $AD = -1{\cdot}94$ feet, and in (*b*) $AD = +2{\cdot}2$. Hence, by interpolation, it will be zero

at chainage $100 \times \dfrac{1{\cdot}94}{1{\cdot}94 + 2{\cdot}2} = 47$

feet as shown on p. 371, and as the areas of cutting and filling at zero are each 19·5 square feet, the volume of filling between zero and

47 feet will be $47 \times \dfrac{19{\cdot}5}{2} = 458$ cubic

feet, to be tipped across.

At 100, the area of cutting is 197 square feet. Hence the volume of cutting between 0 and

100 is $\dfrac{197 + 19{\cdot}5}{2} \times 100 = 10{,}820$

cubic feet, of which 458 are to be tipped across as above, while the difference, or 10,362, is to be hauled to form a bank elsewhere.

Similarly between 100 and 200, filling begins at

$$100 + 100 \times \frac{2{\cdot}20}{8{\cdot}20} = 127$$

and cutting ends at

$$100 + 100 \times \frac{7{\cdot}0}{8{\cdot}8} = 180$$

and these volumes are dealt with in the same way.

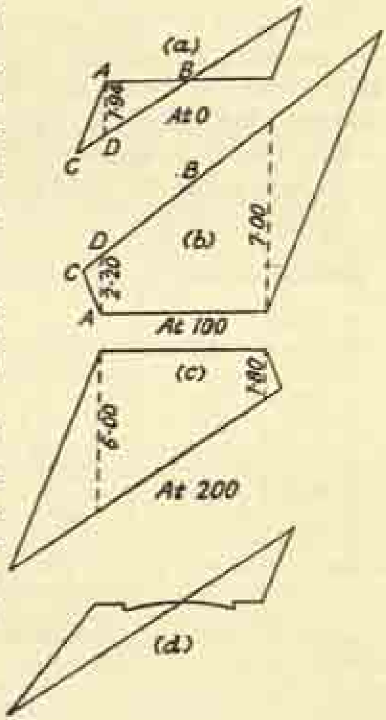


FIG. 196

(2) Other extra points are required whenever the section changes from excess of cutting to excess of bank (or vice versa) to find the chainage of that point at which cutting and bank are *equal* in area.

These are necessary, because at these points the accumulated volume will be at a maximum or minimum, and it is clearly necessary to find these points to estimate the volumes correctly.

If the ground is either level or at a uniform slope, transversely, and if the formation is symmetrical as in this example, such points will occur when the depth on the centre line is zero, and the required chainage is found from the longitudinal section, as at 154.

Usually, however, these conditions are not satisfied, and the chainage must be found from interpolated cross-sections or from the figures themselves as well as may be.

The actual sectional areas at these points in this case are found by assuming a transverse slope for the ground intermediate between those on either side. For instance the slopes at 100 and 200 are 1 in 6·5 and 1 in 7·4 respectively. Hence at 154 we assume a slope of 1 in 7, and hence find a sectional area of 22. In other cases they would be found from the interpolated sections.

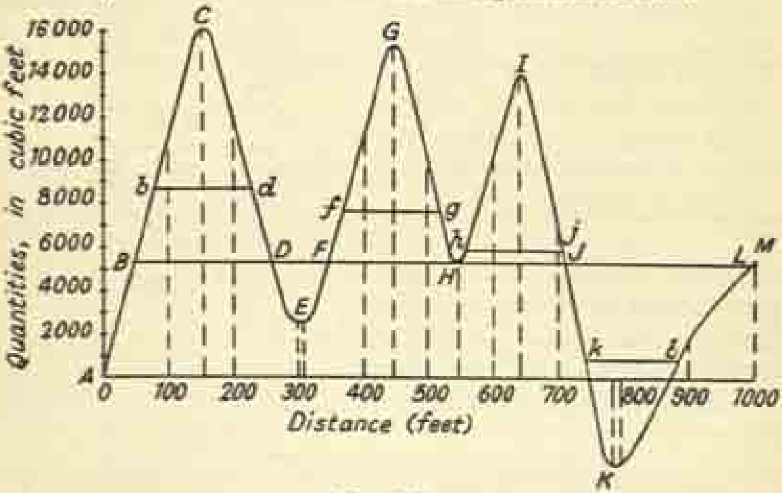


Fig. 197

In this way the table on p. 371 was prepared, and the last two columns are plotted in Fig. 197, which is the mass-haul curve. There is an excess cutting of 5,547 cubic feet to be disposed of somewhere.

Wherever the curve is *rising* from left to right, it means *cutting*; a *falling* curve means bank.

Chainage	Area in square feet		Volume in cubic feet		Accumulated volume	Corresponding Chainage
	Cutting	Bank	Cutting (+)	Bank (−)		
0	19·5	19·5			0	0
				458		
47		0	10,820			
100	197·0				10,362	100
127		0	5,913			
				297		
154	22·0	22·0			15,978	154
			286			
180	0			4,370		
200		168·0			11,894	200
257	0			9,500		
			226			
300	10·5	21·5			2,620	300
			61	83		
304	20·0	20·0			2,598	304
				410		
345		0	9,048			
400	168·5				11,236	400
420		0	4,524			
				280		
448	20·0	20·0			15,480	448
			190			
467	0			5,460		
500		190·0			10,210	500
531	0			5,040		
			170			
548	20·0	20·0			5,340	548
				230		
571		0	5,250			
600	182·5				10,360	600
625		0	4,040			
				150		
640	20·0	20·0			14,250	640
			300			
670	0			7,800		
700		240·0			6,750	700
750	0			11,180		
			360			
786	20·0	20·0			− 4,070	786
			406	189		
800	38·0	7·0			− 3,853	800
			5,825	350		
900	78·5	0			1,622	900
			5,025	1,100		
1000	22·0	22·0			5,547	1000

Inspection of the curve makes it quite clear that we shall obtain the minimum haulage if the excess cutting from A to B (chainage 0 to 52) be disposed of in a 'spoil heap' near A; the cutting from B to C (52 to 154) should be carted forward to form the excess of embankment between C and D (154 and 270), while the cutting from F to E (304 to 345) should be carted backwards to form the bank between D and E; FG and HI forwards to bank GH and IJ respectively; LK backwards to form the bank JK; and ML disposed of in a spoil heap near M. Thus work can be commenced with confidence at many points.

No allowance has here been made for 'soil expansion.' It is assumed that 1 cubic yard of cutting will go into 1 cubic yard of bank. Experience on this point varies somewhat, but to allow for it all volumes of cuttings may be multiplied by a factor representing the expected ratio of expansion before using them in the table.

A certain distance may be agreed on as a maximum up to which haulage is to be paid for at a fixed rate. Suppose in this example we take 150 feet, which is represented in Fig. 196 by *bd*, *fg*, *hj*, and *kl*.

Then no extra haulage will be paid for the portion *b*C. But the cutting B*b* has to be carted to *d*D. The quantity is about 3300 cubic feet to scale, and the average distance should be measured approximately through the centre of gravity of B*bd*D, say 180 feet, to estimate the extra cost. Other portions of the curve are dealt with in the same way.

EXAMPLES FOR EXERCISE

(1) Draw a scale of (*a*) yards, (*b*) metres for a map whose scale is 1 mile to 1 inch.

Ans. 10,000 yds. = 5·68 ins.; 10,000 m. = 15·78 cms.

(2) If the scale of a map is to be converted from 6 ins. to 1 mile to 100 m. to 1 cm. by the method of squares, and if the original map be covered with squares of $\frac{1}{4}$-inch size, what must be the side of the squares on the new map?

Ans. The rep. fractions are $\frac{1}{10560}$ and $\frac{1}{10000}$ respectively. The squares must be in the same ratio, those on the new map being the larger as the scale is larger. Hence side of square $= \frac{1}{4} \times \frac{10560}{10000} = 0{\cdot}264$ in. Thus we would set off 13·2 ins. and divide into 50 parts.

(3) The original scale of an old map is obsolete from shrinkage and other causes. But it is found by trial that, on an average, a length of 682 yds. is represented by 4·89 ins. (*a*) Draw scales of yards and metres for the map. (*b*) If it be desired to convert the scale to $\frac{1}{5000}$, using $\frac{1}{4}$-inch squares on the original, how big must the squares be on the new map?

Ans. (*a*) 1000 yds. = 7·17 ins.; 1000 m. = 19·91 cms.
(*b*) 12·55 ins. to 50 squares.

(4) A road is to be made on ground with a transverse slope of n to 1. The formation is to be level transversely, of total width b, and to be intersected by the ground slope so that at any cross-section the road is in cutting on one side and in embankment on the other. Side slopes are k to 1 in cutting, and l to 1 in filling. Show that, in order to equalize the cross-sectional areas of cutting and embankment at each cross-section, the formation width must be divided so that

$b\dfrac{\sqrt{n-k}}{\sqrt{n-k}+\sqrt{n-l}}$ is in cutting, and $b\dfrac{\sqrt{n-l}}{\sqrt{n-k}+\sqrt{n-l}}$ is in filling.

Find the results when $b=100$ ft., $n=5$, $k=1\cdot5$, $l=2$, and the sectional area of cutting or embankment.

Ans. 52 ft. in cutting, 48 ft. in filling, 385 sq. ft. about.

(5) A straight line ABC is ranged along the bottom of a valley, roughly parallel to the stream which is there. The distances AB, BC are each 100 ft. At A, B, C cross-sections of the valley are ranged out at right angles to ABC, and levels are taken along these cross-sections. The results are given in the annexed table, where all distances are in feet, left or right (as stated) of ABC. The direction of flow is from A to C. The stream may be taken as everywhere 15 ft. wide and 2 ft. deep.

R.L.	Distance.	Remarks
260·3	300	Left at C
265·6	300	,, B
258·7	300	,, A
257·1	200	Right at B
250·6	200	,, A
238·2	100	,, A
245·0	100	,, B
237·7	100	Right, midway between B and C
237·5	100	Right at C
250·6	200	,, C
238·7	200	Left at C
248·1	200	,, B
242·3	200	,, A
228·1	100	,, A
226·9	100	,, B
224·7	100	,, C
224·4	—	At C
226·8	—	,, B
227·1	—	,, A
224·3	21	Left at A } in centre of stream
223·0	46	,, B } in centre of stream
221·9	18	,, C } in centre of stream

(*a*) Plot the levels and the stream on a plan, and draw the contours at vertical intervals of 3 ft., from 256·0 downwards to 226·0.

(*b*) A dam is to be built across the valley, 10ft. wide at the top, upstream slope 3 horizontal to 1 vertical, downstream slope $2\frac{1}{2}$ to 1, top level of dam

256·0. The upstream edge of the top to pass through B in plan and be perpendicular to ABC, so that it coincides in plan with the middle cross-section.

Show the plan of the dam, and the contours on both faces.

(*c*) Divide the length of the dam into six equal parts, and draw vertical cross-sections at the five intermediate points of division. Use these to find the volume of the bank above ground level.

Check the result by dividing the dam into horizontal slices 3 ft. high, using the contours to find the volume.

(*d*) Find the area of ground, in plan, covered by the dam.

(*e*) The dam is to be faced, on its upstream slope with concrete slabs from the top down to the level of 240·0.

Find the actual area of face to be thus covered, and draw the complete elevation of the upstream slope.

(*f*) Draw up a field-book giving imaginary, but possible, readings for the original levels with a 14-ft. staff, and make up the book fully showing all necessary checks.

Ans. Vol. by cross-sections, 22,300 cub. yds.; from plan, 22,150 cub. yds.
Area of dam = 49,800 sq. ft.; area of slope to be faced, 16,900 sq. ft.

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