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THE BRAIN AND SKULL OF A PALEONISCID FISH FROM
THE PENNSYLVANIAN OF WESTERN MISSOURI

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(Read April 22, 1937)

ABSTRACT

Describes a cast of the brain of a Paleoischid fish from the upper Pennsylvanian beds near Kansas City, Mo. The cast is peculiar in that it is an intra-meningeal cast, not a cast of the endocranial cavity, and is therefore an accurate replica of the brain. The filling material is Colophonite which reproduces the most minute details. The cerebral portion of the brain is very small but the optic lobes very large. The cerebellum and the otic region are preserved entire. The semicircular canals, the sacculus and utriculus of both sides are reproduced completely. The cast is described in detail and compared with previous descriptions of a perfect material by Eastman and Moodie. The skull is described and compared with previous descriptions of similar material by Watson.

The preservation of the remains of fish and other forms of life in concretions recovered from shales of Mississippian and Pennsylvanian ages has been known for some time. In certain shale layers of the Oread limestone, Upper Pennsylvanian, near Lawrence, Kansas, the concretions are highly phosphatic, and may be in part coprolitic. The nodules from the Chanute Shale, Iola limestone, at Kansas City, Missouri, are more calcareous in appearance but contain a large content of phosphorus. The preservation of the bones and of the form of the brain as casts is singularly perfect in some of the concretions but in others there remains only the impression of the outer surface of the neurocranium. The casts were formed by the removal of material and subsequent filling by a form of phosphatic calcite called Colophonite.

The most important accounts of such remains of fish have been made by Eastman, Moodie and Watson.¹

¹ Eastman, Charles R., "The Devonian Fishes of Iowa." Iowa Geological Survey, XVIII, pp. 264–271, Fig. 40, 1908.

Moodie, Roy L., "A New Fish Brain from the Coal Measures of Kansas, With a Review of Other Fossil Brains." Journal of Comparative Neurology, XXV, No. 2, pp. 135–181, Fig. 19, 1915.


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Recently the author received for study a considerable number of concretions from the Chanute Shale, a division of the Iola limestone, at Kansas City, Missouri, through the kindness of Mr. M. B. Kleihege, the collector and owner. They contain remains of Arthropoda which can be identified as Malacostracan (Schizopoda and Ceratiocarina) with some that cannot be identified; sharks of genera related to Cladodus, Orodus and Periplectrodus; Paleoniscid fish of undetermined genus but apparently close to Rhadianichthys. The remains of fish correspond so closely with the casts of brains described by Eastman and Moodie and the neurocrania described by Watson that there can be little doubt that they belong to identical or closely related forms, certainly within the same family. Eastman described the impression of a skull and a cast of the brain from Mississippian beds at the base of the Waverly formation in Boyle County, Kentucky, which he identified as Rhadianichthys and gave it the new specific name, deani. The brain was further described by Moodie. Both Moodie and Watson in their descriptions of similar material were content to refer to them as Paleoniscids without attempting generic or specific identification. The specimens here described confirm in large part the work of these authors but permit emendation and considerable addition.

The following descriptions are based on remains from Kansas City and primarily on two specimens, one a nearly perfect replica of the brain, the other a nearly complete neurocranium. All references to other specimens will be indicated.

The peculiarity of the brain casts is that they are evidently replicas of the brain itself and not endocranial casts. This is a most important difference as the brains of many of the fish, amphibia and reptiles did not fill the cavity but were surrounded by a large amount of fatty and supporting tissue. Endocranial casts of such forms give a very inadequate picture of the brain. Excellent illustrations of the difference between a cast of the brain and a cast of the cavity in which the brain lies are given by Doctor Tilly Edinger in her book, *Die fossilen Gehirne*, page 31, figure 6.
It is difficult to understand how such perfect replicas of the brain were formed, especially as the brain in the living lower Teleostomi and presumably the related extinct forms is not covered by such perfect meninges as the brain in the higher vertebrates. Instead of the several meninges there is in the living forms a single meninx (see Kappers, Huber and Crosby, *Comparative Anatomy of the Nervous System of the Vertebrates Including Man*, p. 56). Evidently such covering as did exist was sufficiently resistant to remain after the brain substance had disappeared and permit the infiltration of waters bearing the calcite and colophonite which form the intramembranous cast. It is to be noted, however, that the filling material lies close to the bone of the neurocranium which suggest that the brain far more nearly filled the cavity than might have been expected. This point is confirmed from several specimens in which the brain cast and bones of the neurocranium are more or less nearly complete.

The brain cast which forms the basis of this description is very nearly perfect, lacking only the structures in the region of the choroid plexus over the mid-brain and the medulla, the infundibulum and hypophysis (in part supplied from another specimen), and a small section from each posterior semicircular canal. Otherwise the otic region is perfect, showing all details of the canals and the vestibular portions. In general the brain is much like the one described by Moodie from a similar cast found in the Oread limestone, somewhat higher in the Pennsylvanian series, from near Lawrence, Kansas. (See Moodie, cited above, Figs. 15, 16 and 17.)

The olfactory nerve is elongate with a slight swelling marking the position of the olfactory bulb. Immediately behind this there is a sharp depression of the upper surface, followed by a second, larger swelling and elevation of the upper surface. This swelling probably contains the olfactory lobes but so merges into the parts which must be regarded as the little developed diencephalic region that the two are indistinguishable. The olfactory lobes join the diencephalon and the optic lobes but not in the manner figured by Moodie; a portion of
the optic lobes extends forward of the point of juncture so there is a decided re-entrant on each side which marks the diencephalic region. This indicates a greater development of the mid-brain than in Moodie's specimen, if as seems probable the specimens are undistorted and show the shape as it was in life. It is probable that the larger part of the fore-brain is olfactory and that the lower part of the mid-brain is occupied by fibre tracts associated with the olfactory region. Herrick (Neurological Factors of Animal Behavior, Fig. 73) shows how large a portion of the anterior brain segments is occupied by such fibres in some of lower fish, as the dog-fish, Squalus acantbias. The relatively large olfactory capsules of the skull indicate a considerable power of smell though less than that of sight, as shown by the large orbits and the great development of optic lobes.

Watson in his description of the neurocranium of an evidently similar fish from the same geological horizon and locality calls attention to the presence of two fontanelles in the upper surface. Many of the concretions are hollow showing impressions of the outer surface only, all trace of bone and brain having been lost. In such cases there are commonly two irregular pillars of matrix extending across the cavity from top to bottom. These were evidently formed by the penetration of the matrix through the fontanelles and the delicate membranes of the choroid plexus in the diencephalic and medullar regions, but not in sufficient quantity to fill the cavity. The anterior fontanelle is in the region marked by Moodie as the thalamus and just where the epithalamic structures would be located. In none of the casts is there any trace of these structures preserved.

The anterior end of the olfactory tract is broken across in two specimens, but in neither is there any trace of the rhinencele described and figured by Moodie. Anterior to the first olfactory expansion the tract retains a fixed diameter until it divides sharply into two rami leading to the nearly terminal olfactory chambers of the neurocranium; the division is shown by impression in several specimens which preserve the anterior
end of the skull. On the under side of olfactory tract just posterior to the break there are distinct grooves curving outward and backward which probably mark the origin of the division into two lateral rami.

The optic lobes are, in correlation with the great size of the orbits, very large. As shown in Figs. 1 and 2, Plate I, they are very similar to those in the specimen figured by Moodie. The median portion of the upper surface of the cast as far back as the posterior edge of the cerebellum is obscured by the invasion of matrix. Moodie's specimen was more perfect in this region and shows a median elongate structure which he interpreted as the cast of a plexiform vessel. The upper surface of both optic lobes is marked by several small papilliform elevations which at first inspection seem to be significant, but as they are not symmetrical on the two sides they are probably adventitious. Very possibly the small points marked by Moodie, in his Fig. 17, as nuclei on the lower half of the thalamus are of this character; as he saw only one side of his cast he was unable to check the symmetrical arrangement of the structures.

The hypothalamic region is obscured by the adhesion of matrix but shows the beginning of a decided downward extension. On another specimen the posterior portion of a large lower diverticulum is preserved; it extends backward and downward, the apex reaching so far back as to lie below the ampulla of the anterior semicircular canal. This structure is sharply separated from the optic lobe by a strong transverse groove. The apex is somewhat obscured by adherent matrix but seems complex as if there had been a distinct hypophysis. Just in front of the main process there is a bit broken off as if there had been an anterior portion, but it is possible that only a scale from the surface is missing.

There is a depression on the outer side of each optic lobe suggesting a division into separate regions. This depression is not described by Moodie and the author has found no figure or description of it in the literature which he has consulted. Its significance is unknown. On the lower surface of each
optic lobe there is a clearly marked, slender, longitudinal elevation which is obviously a part of the brain and not a trace of an external vessel as its anterior part becomes more distinct and prominent and its posterior end shades into the lobe. This cannot be the IIInd nerve as it is too small and too lateral in position; it is too far forward to be the VIth, possibly it is a branch of the Vth. Underlying the posterior outer portion of the optic lobe and lying between it and the ampulla of the anterior semicircular canal on each side are the lobi inferiores. They are apparently continuous with the cerebellum but the exposure between the anterior semicircular canal and the optic lobe is very narrow, making it difficult to determine this point. It is possible that these prominences are auricular extensions of the acustico-lateralis lobes of the medulla from which the cerebellum arose phylogenetically (Herrick, *Neurological Foundations of Animal Behavior*, p. 189), or auricular extensions of the cerebellum. At the inner posterior edge of each optic lobe there is evidence, on the lower surface, of a large process which though obscured by matrix is still determinable; this is in the position of the Vth nerve, or of the Vth plus associated nerves, perhaps the VIth and VIIth. The origin of IIInd nerve must be here, but cannot be certainly identified.

The upper surface of the cerebellum is as figured and described by Moodie, but with less perfect exposure. The median portions of each side are partly obscured by matrix but it is evident that they were distinct and that there was no crista. The sides are largely covered by the anterior semicircular canals which were so pressed against them as to form the grooves figured by Moodie, in whose specimen the canals were not preserved. The medial portion of the cerebellum exposed is not so much expanded and altogether gives the impression of not having been so large as in Moodie's specimen; certainly there was no approximation to the size attained in the teleost brain.

Posterior to the cerebellum the upper surface is exposed for a short distance anterior to the matrix covered area marked
by Moodie as the site of the vagal lobes and by Edinger as
the glandula myelencephalica. The area directly between
the points of juncture of the anterior and posterior canals of
the two sides is described by Moodie as occupied by the tuber-
cula acoustica, which are figured by him from the specimen
described by Eastman as Rhadianichthys deani. In the spec-
imen described here there is no indication of such structures.
Doctor Elizabeth Crosby, who failed to find any evidence of
their presence, expressed doubt of their occurrence in a primiti-
tive ganoid. As the fibres forming the tubercula are in
modern forms largely from the cochlear nerve, and as there
is no cochlea present, or even the beginning of a lagena, the
presence of the tubercula is not to be expected. It is more
probable that the anterior pair of the prominences lying near
the mid-line are the facialis lobes of the VIIth nerve and the
posterior pair are the lobi vagi. The region is very close to
the posterior fontanelle and the surface may have been dis-
torted or obscured in Moodie’s specimen. The same area is
preserved in two less nearly perfect specimens and neither
shows the presence of the tubercula.

There is no expansion of the region immediately behind
the posterior fontanelle such as is called by Moodie the lobus
vagi but posterior to and below this region there is a promo-
nence, partly surrounded by the posterior semicircular canal,
which may mark the presence of fibres from the anterior
lateral line nerves.

Near the posterior end of the cast, and probably of the
medulla, there are two small prominences symmetrically
placed on the lower part of the sides which are regarded as
marking the position of the IXth and Xth nerves.

Posterior to the suggested location of the Vth nerve the
lower surface of the cast is much contracted by the close
approximation of the large dependent sacculi. The narrow
surface exposed is marked by a low and broad but very
apparent ridge. Near the posterior edge of the sacculi the
lower surface rises vertically to the under side of the medulla;
this surface is continued upward on either side of the medulla
as a narrow groove which is separated from the prominence referred to above as the possible lobus vagi by a very thin but prominent wall. The lower surface of the medulla is marked by a longitudinal depression which continues backward to the broken edge of the cast.

The preservation of the otic region is most striking in its perfection. This region is complete and undistorted on both sides, except for the loss of a small section from each posterior semicircular canal. The broken surfaces for the missing parts are clearly shown so there can be no doubt as to their contour and extent. The figures given by Moodie correspond in a general way with the present specimen but there is much difference in detail (compare Moodie's Fig. 10). The form and relations of the parts are shown in Pl. I. The two vertical canals meet at a decided angle above and enter the utriculus by a common opening but without a common sinus. The anterior canal descends sharply outward and downward and lies in a depression on the posterior surface of the cerebellum. It terminates in a prominent ampullar extension. Immediately behind and within this extension is the enlargement of the anterior end of the horizontal canal, called by Moodie the "enlargement of the anterior ampulla"; the form and relations of these to expansions are shown in Pl. I, Figs. 2 and 3. The posterior canal extends outward and then sharply downward, enlarging to a small ampulla before entering the utriculus by the sinus.

The horizontal canal stands well out from the others; there is little or no expansion of the posterior end which enters the utriculus independently but close to the point of entry of the posterior vertical canal.

The shape of the saccular is very different from that suggested by Moodie (compare his Fig. 10). It is much deeper and more sharply marked off from the utriculus. The lower face on each side is formed by matrix as if the membrane had been broken, but the faces are symmetrical in position and inclination on both sides, looking outward and downward. A slight excavation on the left side revealed a bit of hard
yellowish material which may be a portion of the otolith. The inner wall of the dependent portion of the sacculus is convex antero-posteriorly and slopes upward and inward, approximating the corresponding face of the other side and narrowing the exposed lower face of the cast, as described above. The outer wall rises nearly vertically, a little concave if anything. On both sides the outer face is marked just below the level of the horizontal canal by an irregular prominence. These are possibly accidental but are singularly symmetrical in position; they may mark the origin of the ductus endolymphaticus, but this is very uncertain. The utriculus rises directly from the sacculus with little if any contraction at the point of union. It has nothing of the prominence suggested by Moodie's figure. In general, the form of the otic structures closely resembles that of a modern teleost, such as the cod.

In 1915 Watson described the neurocrania of two Paleoniscid fish from the same geological horizon as those here described, at Kansas City. It is a pleasure to record how accurate are his determinations. Plate II, 1–3, of a very perfect neurocranium, are given as, in part, confirmatory of his work. There are differences in minor details, as might be expected, but the agreement is so close, especially with the one Watson called "Paleoniscid A," that there can be no doubt of the close specific relationship, or identity, of the two. He notes the presence of a well developed hypodome, an unexpected occurrence. In two of the nodules casts of this structure are preserved. The cavity was heart shaped with the lower surface smooth and the upper surface marked by a deep median groove which extends from the broad, deeply indented anterior end almost to the pointed, posterior extremity.

In only one major character does this specimen fail to agree with Dr. Watson's description. It has been impossible to locate and confirm the excavation and foramen at the sides of the posterior end of the parasphenoid, labelled by him as the exit of the palatine branch of the facialis nerve. The surface of the bone is apparently unbroken at this point.
There are several nodules which show impressions of the dermal bones of the roof of the skull and the opercular region, but they are badly disturbed and broken. No continuity can be made out. These appear to have been badly decayed specimens, or perhaps the material is coprolitic and the bones have been partly destroyed in their passage through the body of the captor.

One or two nodules show typical ganoid scales, with a sculpture of oblique, approximately parallel lines, peg and socket articulation, and fringed posterior edges. They resemble the scales of *Rhadinichthys* but are insufficient for satisfactory determination.

In conclusion the author desires to express his appreciation of the skill of Miss Grace Eager, artist in the Museum of Zoology of the University of Michigan, for the care and skill which has produced the beautiful drawings of the brain cast, and to Dr. Elizabeth Crosby who has aided him in its interpretation.
Fig. 1. Upper view of neurocranium of Paleoniscid fish. AVC, anterior vertical canal; HC, horizontal canal; PC, posterior vertical canal; AF, anterior fontanelle; SC, spiracular canal. × 2.

Fig. 2. Lower surface of same specimen. BPT, basipterygoid process; CC, foramen for common carotid; EBA, foramina for epibranchial arteries; EMA, foramen for efferent mandibular artery; IC, foramen for internal carotid artery; PS, ascending process of parasphenoid; SC, spiracular canal; VF, vestibular fontanelle; IX and X, ninth and tenth nerves. × 2.

Fig. 3. Left side of same specimen. BPT, basipterygoid process; EBA, foramina for epibranchial arteries; EMA, foramen for efferent mandibular artery; IC, foramen for internal carotid artery; PTP, postorbital process; SC, spiracular canal; VF, vestibular fontanelle; IX and X, ninth and tenth nerves. × 2.
Drawings of Brain Cast of a Paleoniscid Fish.
1, upper view; 2, lower view; 3, lateral view, right side; all $\times 3$. 
QUANTUM-COUNTER AMPLIFIERS FOR GAMMA RAY DETECTION, AND APPLICATIONS TO STUDIES IN RADIUM POISONING *

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ABSTRACT

A portable, rugged amplifier, with a direct reading counting-rate-meter, has been developed for the study of feeble gamma rays. The instrument has been simplified so as to permit successful operation for those not specifically trained in experimental physics. Quantities of radium of the order of $5 \times 10^4$ gm. or more may be determined in a few minutes. Calibrations have been made for determining the amount of radium contained in living victims of radium poisoning.

INTRODUCTION

In 1928 Geiger and Müller described a cylindrical cathode, wire anode discharge tube (tube-counter) which is sensitive to gamma rays as well as to any ionizing radiation. Many workers have developed modifications of this instrument, each designed to meet particular detection requirements. As a sensitive detector of feeble gamma rays, tube-counters of this type have few peers. The reason is simple. For every gamma ray quantum, producing one or more secondary electrons inside the cathode cylinder, the instrument discharges once. A vacuum-tube amplifier and recording circuit automatically add up the total number of such pulses. Thus there is an observed count for each gamma ray quantum favorably absorbed. But every detecting instrument has a background, or zero effect which it exhibits in the absence of the source of radiation to be measured. The magnitude of the statistical fluctuation in the background determines how feeble a source of radiation can be measured with any particular instrument. In a tube counter each cosmic ray, contamination alpha ray

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and beta ray also produces one count and thus each absorbed gamma ray is on a par statistically with these much more heavily ionizing rays. It is just because the tube counter counts particles, and is oblivious of the relative ionization produced by these particles, that it is a favored instrument for the detection of very feeble gamma rays.

We had previously developed tube counters having a particularly high sensitivity to gamma ray quanta. These counters have fine mesh screen cathodes made of heavy elements in order to increase the efficiency of production of countable secondary electrons. To obtain the highest sensitivity optimum values of the operating voltage and the pressure of the gas used to fill the counter were found empirically. Such a counter will be called a quantum counter for brevity and to emphasize these special properties.

A number of types of vacuum tube amplifiers and recorders had been developed by various workers. Several of these were satisfactory for use in a research laboratory by a suitably trained physicist. They did not lend themselves to portable use, nor were they sufficiently reliable in the hands of a novice. Several problems in connection with our studies of radium poisoning demanded the development of an instrument without their bulk and multiplicity of controls. With the aid of a grant from the Penrose Fund, such an instrument has now been developed. It is rugged, small and portable, operates on 110 volt a.c., has but one control knob, and is direct reading.

The technical details have been published and the present report will only summarize the main points and comment on a few matters which await further examination.

**High Voltage Supply**

Each tube counter has an operating voltage range and a particular voltage at which it exhibits optimum sensitivity. This voltage is generally somewhere between 1200 and 2000 volts for air filled quantum counters, and depends
on the size of the counter and the pressure of the enclosed gas. The breadth of the range of permissable voltages is mainly governed by the type of high resistor used in coupling the counter to the high voltage supply and to the amplifier. If this resistance is as high as $10^{10}$ to $10^{11}$ ohms the operating range may be several hundred volts wide. At lower values the operating range narrows markedly, becoming only a few tens of volts wide at resistance values of $10^9$ ohms. The most satisfactory resistor seems to be in the neighborhood of $5 \times 10^9$ ohms, where the operating range will be about one hundred volts.

Most counters also show a slight dependence on voltage even within this allowed range. It is therefore necessary to provide a very stable high voltage supply for the counter, the variations being held below 1 part in 1000 if possible. Cosyns has described counters which are nearly independent of applied voltage over a considerable range. But the experience of other workers indicates that this characteristic is to be attributed not to the counter tube itself but to his use of an electronic device in place of the more usual and much more compact high resistance.

The two most satisfactory forms of laboratory voltage stabilizer included the use of dry batteries. Moreover each required the adjustment of two controls to permit the selection of any arbitrary output voltage as well as the stabilization of this voltage. Unfortunately these two controls are mutually interacting, hence some familiarity with the instrument is necessary for proper adjustment.

The first difficulty, that of the dry battery, was removed in one of these circuits by replacing it with a small neon lamp, thus considerably lightening the weight and decreasing the volume of the stabilizer. However, this neon-pentode stabilizer still required the readjustment of both controls whenever the output voltage is to be changed by much more than a hundred volts.

To circumvent this difficulty we developed also a stabilizer composed entirely of neon lamps. A rectifier and
filter condenser supplies about 3000 volts to a series load containing a metallized resistance of several megohms and about 35 small G.E. neon glow lamps (type C D 1002). Each lamp then exhibits a voltage drop of about 60 volts, which is independent of fluctuations in the rectifier voltage. By means of a well insulated rotary switch any voltage up to 2100 may then be tapped from the series-bank of neon lamps. For vernier adjustment of the voltage within the 60 volt steps thus provided, a high resistance potentiometer may be used across the two end lamps in the series-bank. Thus compactness and simplicity are given the stabilizer.

A word of warning is in order concerning the neon lamps. As supplied by the manufacturer, a batch of lamps may contain individuals having operating voltages as low as 50 volts and as high as 105 volts. From a box of 100 lamps, however, a group of 35 can usually be selected which have suitable properties. Moreover the starting characteristics of each individual lamp may vary with operating history. Careful selection, and exclusion of imperfect lamps, will usually result in an excellent stabilizer, capable of maintaining an output voltage constant to less than 0.1 per cent for input a.c. line voltage fluctuations of 7 per cent, and with a negligible drift after the first few minutes of operation.

Amplifier

Each discharge of the quantum-counter delivers a very short pulse to the first grid of a vacuum tube amplifier. The voltage peak of this pulse is adequately large to permit easy amplification. The amplified pulses actuate a magnetic message register for adding up the total number of pulses in any measured unit of time. The operation of a message register however, requires not a voltage surge alone but rather the passage of a given number of coulombs of electricity through its coils. It is therefore often desirable to employ a pulse lengthening stage in the amplifier. The simplest device of this type is again a small neon glow lamp.
Any amplifier requires an auxiliary timing device, since the rate of counting is always the desired result of the measurement. The auxiliary clock, as well as the computations of the differences and rates could be eliminated by incorporating a time scale in the amplifier. This was done by developing an amplifier and output device which reads directly in counting rate.

**Counting Rate Meter**

Hunt had described a frequency meter for use primarily on periodic audio frequency vibrations. His general method has recently been used by Neher and Harper for very high speed counting of random pulses. We were, however, concerned with much lower counting rates, and in addition sought to avoid batteries and moving parts such as the magnetic relay employed in Cosyns meter.

After tests on a number of types of circuits we settled on the one which showed the most promise and developed it into the circuit finally described. The first-stage amplifier is a '57 in conventional pentode hook-up. It feeds into a double triode ('53) back coupled relaxation oscillator, developed from a circuit employed by Edgerton in the strobotac. This stage generates a pulse of definite time duration which is independent of the voltage and duration of the (shorter) pulse fed it from the counter and first stage.

Finally these pulses now of definite and uniform duration are fed to an output pentode ('57) the plate current of which is swung from cut-off to saturation and held there during each pulse. This pentode thus passes a definite quantity of electricity into a large condenser for every discharge of the quantum counter. Shunting the condenser is a resistance in series with a micrometer. The electrical time constant of the output circuit is 30 seconds, and we have shown that the statistical fluctuations observed in the output current correspond to the natural fluctuations expected in 60 second intervals if the counting were done by message register.
A selector switch permits the use of four sizes of coupling condenser between the second and third stages, thus allowing the instrument to cover four overlapping ranges of counting rates from about 50 to 2000 per minute, and at the same time preserve accuracy of reading.

The output current at high counting rates is not quite linear with counting rate. This non-linearity enhances the accuracy of reading at low counting rates but does make scale calibrations necessary. We hope to modify the output stage that this slight non-linearity can be circumvented without sacrificing the large output current now available. At present it seems more worth while to preserve this large output current, which permits the use of a rugged output meter.

A variable bucking current is provided with which the output meter can be reset to zero while the background counting rate is being measured. Thereafter the activity of any source of gamma rays or x-rays is read directly from the meter, the principal corrections and computation having been automatically accomplished electrically by the instrument.

Figure 1 shows two of the instruments. On the right is a portable unit with its carrying case cover removed. The first stage amplifiers and the quantum counter are in the aluminum housing seen in the foreground. The flexible cable connecting this with the amplifier permits placing the counter in any desired position with respect to a fixed source of radiation. At the left is an amplifier with cover removed; in the foreground is a screen-cathode quantum-counter tube such as is used in all the instruments.

**Determination of the Radioactivity of a Closed System**

Through the use of the quantum counter and the counting-rate-meter we have developed a precision method for determining the total gamma ray activity of any source even though it is contained in an inaccessible and absorbing medium. This general method was particularly applied
QUANTUM-COUNTER AMPLIFIERS

Fig. 1. Portable type of counting rate meters.
to the problem of measuring the total radium C content of living victims of radium poisoning. Here the non-uniform distribution of radium in the skeleton, and the internal ab-

![Image](image_url)

**Fig. 2.** The numbers show the relative intensity of gamma radiation at five points near a seated victim of chronic radium poisoning. They also indicate the absolute intensities due to 1 microgram of radium (as RaC) distributed characteristically in the subject's skeleton. The unit of absolute intensity is the gamma ray intensity observed on the same instrument at a distance of 1 meter from 1 microgram of unshielded radium.

sorption and scattering of the gamma rays emitted within the body offer complications which had defeated previous attempts at direct measurements of this type.
Fig. 3. Laboratory type counting-rate-meter for routine examination of radioactive products
As a result of a large number of such tests we have now determined the absolute gamma ray activity to be expected at eleven points near the body of a victim of radium poisoning. This multiplicity of calibration positions makes it possible to perform satisfactory measurements even on hospitalized patients who are immobilized by braces. For patients who are able to sit up a number of check determinations are made possible by using several of the positions indicated in Fig. 2. For example in this way, routine examinations of workers in radium industries may be made. The high sensitivity and stability of the quantum counter instruments described above makes it possible to reduce the time required for such measurements to less than a tenth of that required with earlier instruments. Moreover the detection instrument is now simple enough to be successfully operated by those not specifically trained in experimental physics.

In the routine measurement of small quantities of packaged radium (down to $5 \times 10^{-8}$ gm.) such as are used by some physicians, the quantum-counter method is generally superior to others. Figure 3 shows a laboratory type of instrument designed specifically for this purpose. Here the quantum-counter is mounted in the vertical sleeve at the back of the amplifier.

For larger quantities of radium (above $10^{-4}$ gm.) a gamma ray ionization method is usually simpler, although the quantum-counter can be used if the source is kept several meters from the detector. For minute quantities of radon or radium ($10^{-8}$ to $10^{-14}$ gm.) such as are found in the breath and excreta of victims of radium poisoning, the more elaborate emanation technique is demanded.

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SOME CHYTRIDIACEOUS INHABITANTS OF SUBMERGED INSECT EXUVIAE

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ABSTRACT

In the heterogeneous groups of plant-like organisms which at present constitute the fungi, none is perhaps more peculiar or more obscure in its relationships than the Chytridiales, an order of the Phycomycesetes ("Algal-like Fungi"). The "chytrids," as they are commonly called, occur mainly in fresh water, where they may often choose strange and unusual substrata for their source of organic material.

The chytridaceaeous fungi described in this paper, which were collected in Denmark, England and in several localities in the United States, are remarkable in that all of them were found living in the submerged and empty exuviae, or cast-off integuments, of the larval, pupal and nymph stages of certain insects which pass through their immature stages in fresh water. Furthermore, so far as is known at present, all but one of these fungi (Rhizidium mycophilum A. Br.) appear to be entirely confined to this most extraordinary habitat.

While over three decades have passed since any of the hitherto known species described in this paper have been reported, they have been found by the author to occur in abundance in exuviae, particularly those of members of the Chironomidae (midges), Ephemera (mayflies), Odonata (Anisoptera, dragonflies) and Phryganeidae (caddisflies).

In their method of development and general body plan the species exhibit a striking degree of similarity. Briefly, the posteriorly uniciliate free-swimming zoospore, which is the active agent of non-sexual propagation, comes to rest, loses its cillum, and from the body of the spore a system of delicate, branching rhizoids emerges. These elongate and ramify extensively within the empty exuvia, absorbing material which is transported to the developing spore body. Gradually the latter, which is also entirely within the cavity of the insect integument, enlarges and eventually becomes transformed into a sporangium, the contents of which become divided into numerous zoospores. These escape through a pore formed in the wall of the sporangium. A unique type of sexual reproduction has also been observed in several species. In this process two or three young thalli become connected by an anastomosing of their rhizoidal systems and the contents of one or more plants pass through the rhizoids into a single thallus. The main body of the receptive plant (formed originally by the body of a quiescent zoospore), after increasing in size, surrounds itself with a thick wall and becomes transformed into a resting structure. This, upon germination, probably functions as a sporangium and gives rise to free-swimming spores.

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Of the fungi described, Rhizoclosmatium aurantiacum, Rhizidium ramosum, Rhizophlyctis Petersenii and Oboleidium hamatum are new species. None of the others has been heretofore collected in the United States nor have they apparently been observed in Europe, so far as the records indicate, during the past 30 years.

**INTRODUCTION**

Although it is generally appreciated that chytridiaceous fungi may be remarkable and oftentimes bizarre in their body structure and reproductive activities, it is no less true that in their efforts to obtain nourishment they may also choose strange and unusual substrata. Of such, none is perhaps more extraordinary than the submerged, empty larval, pupal, and nymph cases of certain insects which pass through their immature stages in fresh water. Just what nutriment there is in the exuviae which is attractive to the swimming spores of these phycomycetous fungi is not at present known. It seems reasonable to suppose, however, that in the complex series of chemical processes which precede the emergence of the adult insects, substances are produced, particularly by the molting glands, which are readily available to the fungi. That it is not the chitinous material of the integument which is fed upon seems indicated by the fact that the fungi may disappear after a relatively short period of growth, whereas the case itself may remain apparently unchanged many months in water before disintegrating. When integuments are examined directly after collection fungi are ordinarily not present in any great variety or number. However, if these exuviae are kept in shallow bowls of distilled water for a few days, a remarkable growth of phycomycetous fungi ensues.

It would appear that not all insect groups are suitable for the needs of the fungi, although not many have as yet been explored. For example, the exuviae of mosquitoes and crane-flies seem almost devoid of chytridiaceous fungi, whereas the larval cases of the Chironomidae (midges), the nymph cases of the Ephemerida (mayflies), the Odonata (Anisoptera) (dragonflies), and particularly the pupal
cases of the *Phryganeidae* (caddisflies) are unusually favorable substrata.

Nearly all our knowledge of integument fungi we owe to the work of Docent Dr. H. E. Petersen of Copenhagen who, over 30 years ago (1903), called attention to the fact that in this habitat there existed a remarkable and characteristic phycomycetous flora. In the summer of 1933 it was the writer’s good fortune to have the opportunity of collecting and observing these fungi under Dr. Petersen’s guidance, and to that indefatigable and gracious investigator the writer wishes to express at this time his most grateful appreciation. As a result of the information thus obtained, it was possible not only to observe a number of Danish species but to extend these observations to material collected in the United States. The following account is based mainly on specimens obtained from these countries, and, to a lesser degree, on a few from the vicinity of Cambridge, England.

**Methods of Collection**

In the Danish lakes the *exuviae* of caddisflies are found in June and July in great numbers floating among the reeds near the leeward shore and in the debris piled up by the wind at the water’s edge. While one may pick many of these almost invisible cases directly out of the lake and put them into small bottles of water, this method requires a considerable amount of exploration. A more efficient method, at least in American waters, is to gather a large quantity of the wet debris blown up along the edge of the lake, wrap it in newspapers or put it in metal containers, and bring it back to the laboratory. There, small amounts are put in shallow glass dishes, against a dark background, and the material is well mixed with a little water. Each dish is then rocked gently back and forth and the integuments picked out as they move toward the surface. Subsequently, these are placed in shallow bowls of distilled, or filtered lake water and left in a cool laboratory room (17–
20° C.). Usually within 12 hours some fungi will have appeared and during the next 3–4 days they will achieve their maximum period of development. In the following few days their numbers rapidly decline—the available nutrient probably being nearly exhausted—and only the resting spores and a few sporangia of certain species will persist.

In addition to the chytridiaceous fungi, to be described in this paper, various species of Aphanomyces always appear. These have been referred by Petersen (1903, 1905) to A. lavis deB., A. scaber deB. and A. coniger, a new species. A further study of them is highly desirable, particularly the deBaryan species.

In Denmark, material was obtained principally from the Esrom Sö, Fure Sö, Sönder Sö, and a small pond in Dyrehaven, all localities in Sealand. In Jutland collections were made in a few small ponds among the dunes at Raab-jaerg Milen near Skagen. The American collections came from three localities: a small pond near Hyannis, one at Nonamesset Beach on Cape Cod, Massachusetts, and Clark’s Pond, Canaan, New Hampshire (N. H.). The writer is greatly indebted to Professor Leland Griggs of the Department of Biology of Dartmouth College for the opportunity of making a number of collections at the latter site.

**Fungi Collected**

1. Asterophlyctis sarcopoides H. E. Petersen

This is one of the most striking of integument-inhabiting chytrids. The zoöspore, after swarming for about two hours, comes to rest and retracts its cillum which deliquesces into a droplet and is apparently absorbed into the body of the spore (Fig. 1, a). A short germ tube is then produced which almost immediately forms two oppositely-directed branches (Fig. 1, b). As enlargement of the spore body proceeds, the branches elongate and rebranch, generally dichotomously. A marked inflation of the whole region
of juncture of the primary germ tube and the two branches is soon perceptible (Fig. 1, c–e), and this ultimately forms the broadly fusiform or spherical sub-sporangial apophysis, so conspicuous a feature of the mature thallus (Fig. 1, j).

![Diagram of Asterophlyctis sarcoptoides](image-url)

**Fig. 1.** *Asterophlyctis sarcoptoides*, all X 1160. Figs. a–j, thalli in various stages of development; fig. a, zoöspore with deliquescing cillum attached to it; fig. b, very young stage, deliquesced cillum still present, rhizoidal system established; fig. c, later stage, showing beginning of irregular shape assumed by spore body and broadening of region of apophysis; fig. d, another, showing beginning of fusiform apophysis; figs. e–g, slightly later stages; fig. h, young thallus with one of the spines already solid, resembling *Obelidium mucronatum*; figs. i, j, nearly mature thalli; fig. k, sporangium with bifurcated spines.
The spore body as it enlarges may assume various un-symmetrical shapes (Fig. 1, f, g, i), caused chiefly by the formation of broad lobes. As the sporangium increases in size, the irregular lobations become more pronounced and are responsible for the strikingly stellate appearance of the mature sporangium (Pl. 1, fig. 13, Pl. 4, fig. F). Occasionally, very young thalli are formed which resemble *Obelidium mucronatum* to a marked degree (Fig. 1, h).

During growth of the fungus, the content of the enlarging body of the zoöspore, which will become the sporangium, undergoes a series of changes. These were not followed in great detail. The hyaline plasma of the zoöspore contains a single oil droplet which very soon after the spore comes to rest fragments into two parts. As germination proceeds, the droplets break up further. The hitherto relatively clear protoplasm soon becomes distinctly granular, vacuoles may appear, and the whole may become charged with numerous small globules and minute, very refractive bodies. The globules, as the thallus assumes its maximum size, become collected into groups (Fig. 1, j), and probably fuse to form the "reserve bodies" of the zoöspores.

The mature sporangium is irregularly stellate (Pl. 1, fig. 13, Pl. 4, fig. F), and possesses a varying number of rather blunt, often thick-walled, refractive protuberances, distributed haphazardly over its surface. Occasionally, sporangia similar to those shown in Fig. 1, k, Pl. 1, fig. 19, are found, the surfaces of which are covered with great numbers of small, thin-walled, generally bifurcated spines. Such sporangia may possibly be variations of the usual type, or they may belong to another, as yet unnamed species. The typical sporangia are variable in size; in the Danish material they were 17–23 μ in width, the nearly spherical apophysis varying from 6–11 μ; in the American specimens they were generally 20–28 μ in width, the apophysis being 8–10 μ. Occasionally, giant sporangia were found, one in a dragonfly from Clark's Pond measuring 52 μ in diameter by 33 μ high, the apophysis being 13 μ. However, the zo-
ospores of this large specimen were only 2.6 \( \mu \) in diameter. From the sub-sporangial apophysis there emerges—generally from two oppositely-placed main branches—the rhizoidal system. This consists in mature plants of many gradually tapering, profusely branched rhizoids. Due to the tenuity of its more distal parts it is impossible to obtain any precise idea of the extent of this nutrient-gathering system. While its components may generally be followed as much as 100 \( \mu \) on either side of the sporangium, they probably go considerably farther.

The zoöspores are cleaved out simultaneously in the sporangium. During maturation, and sometimes earlier, there is formed, generally on the lower third of the sporangium, a single discharge papilla about 4–5 \( \mu \) in diameter. Upon its deliquescence, the zoöspores emerge in a compact mass, devoid of individual ciliary activity, through a pore formed in the wall of the sporangium (Pl. 1, fig. 14). At the mouth of the pore they remain in a motionless, sub-spherical cluster for about two minutes. Then a few spores on the periphery initiate individual motion; this ordinarily starts as a slight "trembling" of the spore body which increases in its intensity and culminates in a violent lashing of the spore from side to side. In a short time all are in motion and the group now becomes a writhing mass of tugging individuals endeavoring to pull their cilia free from some apparently confining substance. After a few minutes of this wild activity, occasional spores dart from the mass and swim away, quickly followed by others, until all have been dispersed. The presence of a confining vesicle during this period of marked activity at the mouth of the pore could not be demonstrated. Indeed, from the fact that the spores escaped from the mass at different points of the periphery, it seems very probable that none was present and by what method they were kept within a definite, circumscribed region during the period of intense activity preceding escape is not known. However, it did seem that in the late stages the lower extremities of the cilia were
held to the mouth of the pore, or perhaps within the sporangium. The significance of this swarming at the orifice of the sporangium—characteristic of many other chytrids as well as Asterophyctis—is not clear. To the writer, it seems somehow to be connected with the final fashioning of the cilia. It is possible that these are unformed or only partially formed at the time of emergence and may require further maturation before the spore can swim away.

Occasional variations in the method of spore discharge are found. The most common of these is the possession by the spores of a slight degree of individual movement immediately after discharge. However, they have never been observed to be discharged directly from the sporangium as motile, free-swimming bodies. In some cases, the discharge papilla had bored through the wall of the integument and the spores discharged out into the water (Pl. 1, fig. 17).

The zoöspore (Pl. 1, fig. 15) is of the usual chytridiaceous type, and possesses a broadly ellipsoidal to nearly spherical body, 5 μ long by 3 μ in diameter, within the clear plasma of which is imbedded either centrally or somewhat laterally, a refractive globule. Trailing behind is a single cilium about 35 μ in length. During the period of swarming, which is, under the conditions of observation, up to about two hours, the spore darts swiftly throughout the integument, eventually coming to rest, giving a few feeble hops, and then retracting its cilium. No amoeboid motion was observed, although it may possibly occur.

Resting spores have been found in both the Danish and American material. These are essentially similar to the sporangia (Pl. 1, fig. 16, Pl. 4, fig. E) but possess much thicker walls—the spines in particular being nearly solid. Further, the rhizoidal system is generally not so regularly spaced as that of the sporangia. The resting bodies are usually about 20–30 μ in greatest diameter. Petersen (l.c.) has figured these spores as "sporanges du second type"
and has inferred that they may possibly be resting structures.

It will be recalled that, in connection with the formation of resting spores in a related form, *Siphonaria*, Petersen (1903) showed that an anastomosing of the rhizoidal systems of two (or 3) plants occurred and that the contents of one of these (a smaller thallus) probably passed into the other, which became the resting spore. In a previous paper the present writer (1935) mentioned his confirmation of Petersen's observations on this peculiar type of sexual process and extended it to another genus, *Rhizoclossmiatium*. While the evidence is as yet scanty that there is a sexual process preceding resting spore formation in *Asterophlyctis*, there seem to be some grounds for believing that here, too, it exists. In Pl. 1, fig. 18, is shown a resting spore, found in a midge integument from New Hampshire, to the delicate rhizoidal system of which are attached two undeveloped thalli of *Asterophlyctis*. This is not an isolated instance, but one of a number of such cases observed. It seems probable that these small structures are in reality "male" thalli which have transferred some of their contents by means of their rhizoidal systems into the other plant. However, in spite of the numerous instances observed of this association of small and large thalli, the writer is not as yet prepared to say that in *Asterophlyctis* sexuality is definitely known, even though the evidence at hand for it is very strong. Another type of sexuality may be indicated by the young plant shown in Pl. 4, fig. D. Here, attached by a tube to the thallus, is a knob-like body, possibly a cystospore; the significance of this is not clear.

In very old midge integuments, maintained in the laboratory for some months, empty thick-walled structures were found, which were probably germinated resting spores. While the process of germination was not observed, from the presence of a prominent, basal pore in the wall, it is probable that they liberated zoöspores in the same manner as sporangia.
Collections. **Denmark:** (caddisflies); Dyrehaven, 20. VI. 33; Gribskov, 24. VI. 33; Raabjaerg Milen, Jutland, 20. VII. 33. **United States:** Clark's Pond, N. H., 10. IX. 34, 8. X. 35 (midges), 15. X. 35 (dragonfly), 31. V. 36 (mayflies); Cape Cod, Mass., 23. VII. 34 (caddisfly).

2. *Siphonaria variabilis* H. E. Petersen

In the Danish collections, this remarkable fungus was most often found in caddisfly exuviae which had been about 6-7 days in the laboratory. Apparently it reached its greatest development after the first "wave" of fungus invasion had passed. It has been observed only twice in the New Hampshire collections, once in dragonfly and once in mayfly integments. The following description is drawn entirely from Danish material.

The method of development of *Siphonaria*, while in the main like that of *Asterophlyctis*, presents certain peculiar features which are difficult to visualize and illustrate. Further, the small size of the fungus renders interpretation of the details very susceptible to error, and the following account may, no doubt, be amended by subsequent observations.

The zoöspore upon coming to rest, loses its cillum in the same manner as *Asterophlyctis* (Fig. 2, a). After retraction, the spore may undergo a limited period of feeble amoeboid movement (Fig. 2, b). Upon germination, it enlarges and there is formed at the narrower end of the broad, somewhat irregularly shaped pyriform body, a cup-like thickening of the membrane, which generally appears as a distinct, crescent-shaped black line (Fig. 2, c, d). From the opposite, broader end there is produced a germ tube which soon divides into two opposed branches (Fig. 2, d). There is reason for believing that this tube is actually produced sub-basally, rather than basally (Fig. 2, e). The body of the spore enlarges and the extension and branching of the rhizoidal system continues. For the sake of clarity, the shape of the young sporangium may perhaps be compared
with that of a pear; the narrower end, possessing the cup-like membrane, becomes tilted downward as growth proceeds (Fig. 2, f), and in the furrow thus formed on the "underside" the rhizoidal system has its origin. As this system becomes more elaborate the primary germ tube in-

Fig. 2. Figs. a–j, Siphonaria variabilis; figs. k–o, Rhizoclosmatium globosum; all × 1160. Fig. a, zoospore with deliquescent cillum; fig. b, zoospore devoid of cillum, undergoing ameboid motion; figs. c–f, various stages in the establishment of the thallus, the discharge papilla already clearly evident; fig. g, immature thallus, front view; fig. h, immature thallus, side view, showing the relation of the papilla and the swollen point of origin of the rhizoids; fig. i, rear view of young thallus, showing relationships of apophysis and papilla; fig. j, front view of young thallus. Figs. k–o, Rhizoclosmatium globosum; figs. k, l, earliest stages seen; figs. m–o, young, developing thalli, showing apophysis and rudiments of rhizoidal systems.
flates and forms a spherical or subspherical apophysis. In mature sporangia, this structure is often difficult to see, for in “front-view” (Fig. 2, g, j) it is covered by the downward-tilted prominent “nose” of the sporangium, whereas in “rear-view” it is almost invisible against the background of the sporangial protoplasm (Fig. 2, i, Pl. 1, figs. 1, 2). However, in “side-view” (Fig. 2, h) and in discharged sporangia it is very prominent (Pl. 1, fig. 9).

Two characteristics of the rhizoids of *Siphonaria* are the thickness of their walls, which may appear as double lines for a considerable distance from the sporangium, and the tendency in many cases for the rhizoidal branches to arch above the sporangium and nearly encircle it (Pl. 1, figs. 1–3).

As increase in the size of the sporangium proceeds, the narrower end becomes more conspicuous and it is soon evident that it will become the discharge papilla.

During development, changes take place in the character of the protoplasm. In the quiescent zoöspore, there is found a single minute, rust-colored droplet, and, usually in addition, a somewhat larger hyaline body, possibly the nucleus. The latter is soon lost sight of as germination takes place, but the globule is observed to fragment into several smaller bodies. These lose their color and there is left no trace of pigmentation in the protoplasm. One or more vacuoles may appear as growth continues, as well as colorless, refractive globules and minute, irregularly shaped bodies. When the sporangia have attained a diameter of about 20–25 μ, the protoplasm assumes a faint rust tinge, possibly due to an even dispersion of minute globules of that color. These probably fuse to form the larger, regularly spaced globules which characterize the sporangium as it reaches maturity (Pl. 1, fig. 3). What causes the reappearance at this time of the distinctive coloration which was lost during early stages in the formation of the thallus, is not known. At maturity, the rhizoids are drained of their contents which pass into the sporangium.
The mature sporangium, while somewhat variable in form, is usually sub-globose or broadly reniform. Its precise shape is difficult to describe, but can be visualized, in part, from the drawings. Wide variations in size were found. It was noted, however, that in small integuments, the majority of the sporangia were much smaller and less variable in size than in larger ones. Thus, in the former instance, most of the sporangia varied from 15.6–20.8 μ in diameter by 15.6–23.4 μ in height, the breadth generally exceeding the height, whereas in the latter case, most of them varied from 31.2–39.0 μ in diameter by 31.2–36.4 μ in height (Pl. 1, fig. 8). However, in these same integuments, where, incidently, sexual reproduction was occurring in abundance, there were mature sporangia which did not exceed 15.6 × 15.6 μ (Pl. 1, fig. 7). In dragonfly integuments from New Hampshire the sporangia were about 42 μ high by 38 μ in diameter.

The cleavage of the zoöspores was no different from the simultaneous process observed in related forms. The papilla of discharge, which in this unusual case was recognizable from the onset of germination of the spore and which was clearly distinguishable throughout the development of the thallus, became very prominent in the mature sporangium because of the formation of a refractive glistening material within it. However, if one compares the size of the mature structure with that laid down by the spore at its germination, it is evident that there has been very little increase in its diameter. While the spores were completely cleaved out in the sporangia, it seems probable as in Astero-phlyctis, that final maturation of the cilia occurred after discharge. Upon the deliquescence of the tip of the protruding papilla, the majority of the spores flowed out (Pl. 1, fig. 4) and formed a motionless, nearly spherical or columnar mass at the mouth of the short discharge tube (Pl. 1, fig. 5). After a few minutes the remaining spores emerged and joined the main body which was still intact, but now more irregular in shape. The last zoöspores to
emerge were seen to possess feebly developed cilia. After another minute, during which time the even contour of the mass was noticeably crenulated by the spherical shape of the now individualized spores (Pl. 1, fig. 7), independent motion was initiated. During the brief period of "group swarming" which followed, the mass of spores became very elongate. Finally, the individuals broke away and darted about aimlessly in the integument. There seems to be no evidence for the presence of a vesicle, and, as in Asterophlyctis, the method whereby the spores are kept together after discharge is unknown. As has been suggested, the whole process may be concerned with the maturation of the cilia. Minor variations in the process of discharge were frequent, but in the majority of cases observed, it followed the sequence just outlined.

The spore body, which is 5.0 μ long × 2.5 μ diameter, is somewhat pyriform, with the narrow part directed forward (Pl. 1, fig. 6). In the rear there trails a single cilium about 4–5 times that of the spore body in length. Within the body of the moving spore are generally two conspicuous structures, a minute, laterally placed rust-colored globule, and a larger, hyaline, spherical body, possibly the nucleus.

The zoospores dart rapidly about and come to quick stops for brief periods before resuming their activity. Immediately after liberation from the discharged mass they may hop about, flexing their cilia before assuming the darting type of movement. They seem capable of only feeble amœboid motion and this is of very limited duration. However, interestingly enough, this usually occurs after the partial or whole retraction of the cilium, a feature, which the writer has never observed in any other chytrid.

In establishing the genus Siphonaria, Petersen (1903) indicated that the formation of resting spores was preceded by a type of conjugation of a very unusual character. In this process, a rhizoid from one thallus becomes fused with that of another. After fusion, further growth of one plant is stopped and it discharges its contents into the other,
which continues to increase in size, surrounds itself with a thick membrane, and becomes the resting spore.

The present writer can confirm unqualifiedly Petersen's observations on the method of conjugation in Siphonaria. In many cases actual contact of the two bodies appeared to have been accomplished solely by the efforts of the thallus which was later to become the resting spore (Pl. 1, fig. 10). The rhizoids of this plant seemed to seek out the other and often, instead of fusing with the vegetative system, made direct contact with the main body. While, because of the small size and inconspicuousness of the rhizoids the early stages in this fusion were not observed, the writer has followed the subsequent fairly rapid transportation of materials from the smaller to the larger body (Pl. 1, figs. 10, 11).

The "male" structures, one or two of which (Pl. 4, figs. A–C) were connected to a single receptive body, were similar in shape to that of the very young thalli. Whether they were produced only by the small sporangia noted previously (Pl. 1, fig. 7) is not known. They varied from 7–11 μ in length by 5–8 μ in diameter and possessed a rhizoidal system of limited extent. The mature resting spore (Pl. 1, fig. 12) was broadly ellipsoidal or somewhat reniform, the majority 13.0–15.6 μ high by 15.6–18.2 μ in diameter, and possessed a brown, smooth or slightly crenulated wall, 2.0–2.5 μ thick. The rhizoidal system was, in most cases, stout and well-developed, but not so extensive as that formed by the sporangia. The content of the spore appeared hyaline, coarsely granular, and guttulate. Germination of the resting spore was not observed.

3. *Rhizoclosmatium globosum* H. E. Petersen

This is the most ubiquitous chyrid found in exuviae where it may occur in great numbers. It has been collected by me in New Hampshire, on Cape Cod, Massachusetts, and at Cambridge, England, as well as from localities in Sealand and Jutland, Denmark.

In its method of development *Rhizoclosmatium* is, so far as can be determined, almost exactly like that of *Asterophlyctis*, save that the spherical or somewhat ellipsoidal body of the germinating spore retains its shape throughout its subsequent growth and transformation into a sporangium (Fig. 2, k-o).

Considerable variation in the shape of the sub-sporangial apophysis was noted. In practically all of the Danish specimens this was broadly fusiform, as in *Asterophlyctis*, and 5 μ broad by 3 μ high. In the American and British material, however, it was more variable in this respect, the apophysis frequently being sub-spherical, pyramidal or clavate (Pl. 2, fig. 18), and occasionally possessing a basal stalk from which the rhizoids arose. The fusiform shape was, however, predominate (Pl. 2, fig. 20, Pl. 4, fig. H).

The mature sporangia were spherical or sub-spherical, smooth-walled, and 13–22 μ in diameter, although occasional dwarf specimens, 9–10 μ in diameter were observed (Pl. 2, fig. 21). The very delicate rhizoidal system was much branched and often exceedingly extensive (Pl. 4, fig. H).

The zoöspores were formed within the sporangium and were liberated after the formation of a discharge pore, in most cases situated near the basal apophysis (Pl. 2, fig. 19). However, in material from midge exuviae collected in New Hampshire, this was sub-apical, or occasionally apical (Pl. 2, fig. 21). The zoöspores were discharged in a compact mass, and underwent the same period of swarming before escape as was noted in *Asterophlyctis*. In *Rhizoclosmatium*, however, there seemed to be a vesicle formed which surrounded the emerged spore mass. This structure could occasionally be detected under particularly favorable con-
ditions; additional proof of its presence was supplied by *Spirilla* which gathered around the ejected spores. During later phases of the swarming of the zoöspores within the vesicle, these bacteria could be seen unsuccessfully attempting to bore through the wall and after the escape of the spores, which took place at one spot on the vesicle, they lost all interest in the membrane and moved away.

The zoöspores were somewhat ellipsoidal, 3–4 µ long by 2.5 µ in diameter, possessed a colorless, centrally or laterally placed oil globule and a posterior cilium about 20 µ long (Pl. 2, fig. 24).

After a week or more of non-sexual reproductive activity, the formation of sporangia generally ceased and the fungus produced only resting spores. The small size of the organism and particularly the tenuity of the rhizoidal system made observations on the formation of these resting spores unusually difficult. However, the writer feels that further work by other investigators will confirm the fact that they are produced after a sexual act essentially like that found in *Siphonaria*.

A considerable number of immature resting bodies both from Danish and American material were examined under particularly favorable optical conditions, and in almost every case it was possible to trace a connection between these structures and one, or occasionally two, smaller thalli which were universally present in the immediate vicinity (Pl. 2, figs. 22, 23, Pl. 4, fig. G). The only indications that could be detected of the passage of material from the smaller to the larger thallus were slight irregularities in the diameter of the rhizoids (Pl. 2, fig. 22). Observations seemed to show also that the two thalli were at first of the same size, but that the receptive one soon increased markedly, whereas the other remained practically unchanged.

The mature resting spore (Pl. 2, fig. 23) is generally spherical or broadly spheroidal with a smooth, faintly brown wall about 2 µ thick. Like the sporangia, there is a broad fusiform basal apophysis from either tip of which a rhi-
zoidal system emerges. However, this system appears more limited in extent than that of the sporangia. The germination of these spores has not been witnessed.

Collections. Denmark: (caddisflies); Sønder Sø, 18. VI. 33; Dyrehaven, 20. VI. 33; Gribskov, 24. VI. 33; Esrom Sø, 24. VI. 33; Raabjaerg Milen, Jutland, 20. VII. 33. United States: Clark’s Pond, N. H., 10. IX. 34, 20. IX. 34, 8. X. 35 (midge); 15. X. 35 (dragonfly); 31. V. 36 (mayflies); Cape Cod, Massachusetts, 23. VII. 34 (caddisfly). England: Cambridge, 10. VIII. 35 (midge).

4. Rhizoclosmatium aurantiacum n. sp.

Under this name the writer segregates a form mentioned by Petersen in his discussion of Rhizoclosmatium globosum, and observed by the writer several times in Danish and American collections. The fungus occurs in company with R. globosum but differs from it in several particulars. As the two fungi generally occur in the same integument it does not seem probable that one is simply a physiological variety of the other.

The sporangia (Pl. 2, figs. 14, 15) are spherical, and 27–38 μ in diameter—larger than those of R. globosum. Further, the smaller, ellipsoid zoospores (2.5 × 2.0 μ), are provided at maturity with a minute, rusty-orange globule, rather than a larger, colorless one (Pl. 2, fig. 16).

The method of spore discharge, the basal position of the discharge pore (Pl. 2, fig. 17), the apophysis, and the rhizoidal system are similar to those of R. globosum.

R. aurantiacum n. sp.

Sporangia globose, smooth-walled, 27–38 μ in diameter, generally with a broadly fusiform subsporangial apophysis from the narrower ends of which the branched rhizoidal system arises; zoospores somewhat elliptical, 2.5 μ long by 2.0 μ in diameter, provided with a minute rusty-orange globule, escaping by a basal pore formed near the apophysis. Resting spores not observed. Saprophytic in the
pupal cases of Phryganeidiae, Denmark; in nymph cases of the Odonata, New Hampshire, U. S. A.

Rhizoclosmatium aurantiacum n. sp.¹

Sporangia globosa, pariete laevi, 27–38 μ dia, sub-sporangiali apophysi late fusiformi plerumque praedita, cuius ab extremis angustioribus systema rhizoideum ramosum enascitur; zoospora paenulum elliptice, 2.5 μ long × 2.0 μ dia., globulo minuto luteofusco praeditæ, ex ostiolo basali iuxta apophysim formato emergentes. Sporæ perdurantes non observatae.

Saprophyticum in integumentis puparum Phryganeidiae, in Dania; in integumentis nympharum Odonatorum, in New Hampshire, Americæ borealis.

Collections. Denmark: (caddisflies), Gribskov, 24. VI. 33; United States: Clark’s Pond, New Hampshire, 20. IX. 34; 15. X. 35 (midges); 31. V. 36 (mayflies).

5. Rhizidium mycophilum A. Braun

This fungus has been collected by me only in New Hampshire, and at Cambridge, England.

While the development of the sporangia was not followed in any detail, it was evident that the zoöspore upon germination produced a single stout rhizoid which, while it occasionally gave rise to secondary branches, always remained the most conspicuous component of the nutrient-gathering system (Pl. 2, figs. 1–3). The body of the spore itself enlarged and ultimately formed a spherical or sub-spherical sporangium, 28–50 μ in diameter (Pl. 2, fig. 4). Generally, though not universally, the proximal portion of the rhizoid became slightly inflated, forming a sub-sporangial apophysis (Pl. 2, fig. 6).

After the deliquescence of a sub-apical or lateral pore, the zoöspores, which were clearly delimited within the sporangium, emerged in a compact sub-spherical mass (Pl. 2, fig. 5). Subsequently, the sporangium and the apophysis,

¹ I am greatly indebted to Prof. R. C. Nemiah of Dartmouth College for the Latin descriptions contained in this paper.
which at this time could be seen to be separated by cross-walls from both the sporangium and the rhizoid, partially collapsed rendering the empty rhizoidal system nearly invisible. Outside the sporangium the mass of zoöspores gradually fell apart into separate units, each with its conspicuous refractive globule. After about a minute the spores assumed individual movement and there was initiated a very remarkable period of swarming. At first the writhing mass, which by this time was ten or more my distant from the orifice, assumed a broadly reniform shape (Pl. 2, fig. 7, a). After several minutes, during which time the spores continued to swarm violently, the shape of the mass changed and became broadly sagittate (Pl. 2, fig. 7, b). This swarming was maintained for some minutes until finally a few spores darted from the apex of one of the lobes, soon followed by the remainder, which escaped by the same route, or from the tip of the opposite lobe.

In its main aspects this type of discharge resembles that found in the other species described in this paper. However, in two features it differs. The first of these is the change in shape that the ejected mass of swarmers undergoes, and the second the fact that, so far as could be determined, the spores moved away from the orifice and had no connection with the sporangium. As in *Rhizoclosmatum*, a vesicle seems to be formed which confines the spores and against which they beat in an effort to escape. Further evidence for the presence of a vesicle is to be found in the action of bacteria which rush to the spore mass and form a boundary around it. Then, too, when escape is finally achieved by the swarmers, they emerge from two definite spots, the tips of the lobes, not indiscriminately over the surface as one would expect if no confining membrane were present.

The zoöspores are spherical and 4 μ in diameter or broadly ellipsoidal and 5 μ long by 3 μ in diameter; they possess a single (rarely 2) conspicuous, laterally-placed,
colorless, refractive globule. From the rear of the body trails a cilium, 30–33 μ in length.

No resting spores were formed in any of the material.


6. Rhizidium ramosum n. sp.

In exuviae from both Denmark and New Hampshire there was occasionally observed in limited numbers, a species of Rhizidium which differed in several respects from R. mycohilum and other members of the genus.

The sporangium is globose or rarely ellipsoidal, and 20–45 μ in diameter. From the tip of a short basal main axis there is produced a widespread system of stout, branching rhizoids (Pl. 2, figs. 8–10, Pl. 4, fig. 7). A feature of this fungus, particularly in the American specimens, is the formation on the sporangium as it approaches maturity of large irregular protuberances of highly refractive material (Pl. 2, figs. 12, 13). These may possibly be discharge papillae; if so, only one of them is functional. The exit pore may be formed at any point on the surface of the sporangium (Pl. 2, fig. 12) but is usually opposite the point of attachment of the rhizoids (Pl. 2, fig. 8). Upon the deliquescence of the papilla the zoospores, which are cleaved out within the sporangium, emerge in a compact column (Pl. 2, fig. 9) which quickly becomes expanded into a globular mass (Pl. 2, fig. 10). They lie in a motionless cluster at the orifice of the pore for several minutes until those near the periphery suddenly dart off. Eventually the others disentangle themselves and pull away until all are dispersed. The spores (Pl. 2, fig. 11) are elliptical, 6 μ long by 4 μ in diameter, and possess a minute, laterally placed, refractive, colorless globule and a long posterior cilium.

As shown in the figures, there are certain slight differences between the Danish (Pl. 2, figs. 8–10) and Ameri-
can material (Pl. 2, figs. 12, 13), which at this time the writer does not feel are of sufficient import to separate the two as distinct species. *Rhizidium ramosum* differs from *R. mycophilum* in possessing a very short sub-sporangial trunk terminating in a profusely branched rhizoidal system rather than a single elongated main axis. A further difference is found in the peculiar refractive nodules which occur on its sporangia.

*Rhizidium ramosum* n. sp.

Sporangium globose or rarely ellipsoidal, the surface often bearing several irregular, refractive protuberances, 20–45 μ in diameter, with a short, basal main axis from which arises a system of stout branching rhizoids; zoospores ellipsoidal, 6 μ long by 4 μ in diameter, with a lateral colorless globule and a single posterior cilium, emerging from the sporangium in a compact column after the deliquescence of a single, usually terminal, discharge papilla. Resting spores not observed.

Saprophytic in integuments of *Phryganeidae*, Denmark; *Chironomidae*, *Ephemeridae*, United States (N. H.).

*Rhizidium ramosum* n. sp.

Sporangium globosum aut raro ellipticum, superficie aliquot protuberationibus irregularibus refractivis sepe praedita, 20–45 μ dia., habens primarium axem basalem brevem e quo systema rhizoidorum validorum ramosorum enascitur; zoosporae ellipticae, 6 μ long × 4 μ dia., globulo laterali inolorato atque uno cilio posteriore praedita, e sporangio velut columna compacta, papilla dimensionis singula plerumque terminali dissoluta, emergentes. Sporae perdurantes non observatae.

Saprophyticum in integumentis *Phryganeidiarum*, in Dania; *Chironomidarum*, *Ephemeridarum*, in America boreali.

7. *Rhizophlyctis Petersenii* n. sp.

What was without doubt the most striking and certainly the largest of these integument-inhabiting chytrids was found in the empty cases of midges and dragonflies from Clark’s Pond, N. H., in the autumns of 1934 and 1935.

The developing thallus consists of a "Centralblase," which, while it is generally spherical, sub-spherical, or el-

![Diagram of Rhizophlyctis Petersenii](image)

**Fig. 3.** Irregularly-shaped thallus of *Rhizophlyctis Petersenii* n. sp.; in some rhizoids the content has migrated into the central-body and a cross-wall has been formed. \( \times 260 \).

lipoidal, and free in the cavity of the integument (Pl. 3, fig. 4), may occasionally be irregularly peltate and appressed to the inner surface of the substratum (Figs. 3, 4). From this main body, which may be regarded as the fundament of the sporangium, there arise 1–10 stout rhizoids, up to 10–15 \( \mu \) in diameter, which branch profusely and ramify extensively in the integument (Pl. 4, fig. 7). The "Centralblase" of the mature thallus varies from 50–170 \( \mu \).
in diameter while the rhizoids may extend as much as 500 μ or more from it. In most cases the sporangial fundament is immersed in debris and it is usually only by tracing the stout rhizoids to their source that the main body can be found.

Early stages in the development of the thallus were not observed. In growing plants the protoplasm, which contains numerous, irregularly-shaped, colorless refractive and orange-brown bodies, may undergo a period of strong vacuolization. As they approach maturity this vacuolization is lost, and the rhizoids are drained of their contents, which become accumulated in the "Centralblase," now considered the sporangium. The protoplasm becomes very dense, assumes a peculiar "fuzzy" appearance, and the pigmentation becomes more intense, especially near the center (Pl. 3, fig. 4). At this time a prominent papilla is formed on
the surface. This elongates and within 48 hours has become a tube about 50 μ long by 20 μ in diameter with a refractive tip (Pl. 3, fig. 1). The protoplasm at this stage becomes homogeneous, colorless, and shot with minute, evenly-spaced globules of the pigmented material.

The zoosporangia are formed within the sporangium and are liberated upon the deliquescence of the refractive tip of the discharge tube. Hundreds of spores emerge in a mass and form a large, spherical, possibly hollow, motionless cluster at the orifice of the tube (Pl. 3, fig. 2). Shortly thereafter they initiate individual movement and swim away. Their progress through the water is a swift, even one interspersed with frequent sudden changes of direction. Prolonged periods of ameboid movement are occasionally resorted to. The body of the spore is nearly spherical, 5.2 μ in diameter, and contains several colorless refractive granules and a small orange-brown globule generally near the point of attachment of the single long posterior cilium (Pl. 3, figs. 5, 6).

Resting spores were found in several instances. These were intercalary or terminal, spherical, ellipsoidal or irregularly shaped (Pl. 3, figs. 3, 7), and had a thickened wall and densely granular, dark, orange-brown contents. The spherical ones were about 25 μ in diameter, the elliptical, 23 × 27 μ and the irregularly shaped ones 23–35 μ × 17–20 μ. No evidence of any sexual process preceding their formation was observed.

This fungus, in its size, pigmentation, and the unusual shapes assumed by its sporangia and resting spores, is strikingly distinct from other described species of the genus and it is, therefore, termed *Rhizophlyctis Petersenii* n. sp., in honor of Docent Dr. Henning E. Petersen of Copenhagen, who first called attention to the peculiar and characteristic chytridiaceous flora found in submerged insect integuments.
Rhizophyctis Petersenii n. sp.

Sporangium spherical or irregularly shaped, smooth-walled, the spherical specimens, 50–75 μ in diameter, the irregular ones 60–170 μ × 40–140 μ; rhizoids arising from 1–10 places on the sporangium, stout, up to 15.6 μ in diameter near the point of origin, extensive, much branched, 500 μ or more in length; zoosporangia posteriorly uniciliate, very numerous, emerging through a discharge tube up to 50 μ long by 20 μ in diameter and forming a motionless globular cluster at the orifice before dispersing, nearly spherical, 5.2 μ in diameter, with a minute orange-brown globule and several refractive granules; resting spores intercalary or rarely terminal, relatively thick-walled, spherical, elliptical or irregular, with a densely granular, orange-brown content.

In empty larval cases of the Chironomidae, Odonata, N. H., United States.

Rhizophyctis Petersenii n. sp.

Sporangium pariete lævi, globosum aut forma irregulare, illud 50–75 μ dia., hoc 60–170 μ × 40–140 μ; rhizoidea ex 1–10 locis sporangiis nascentia, valida, usque ad 15.6 μ dia. prope locum originis, late patentia, ramosissima, 500 μ aut plus long.; zoosporae postice uniciliate, plurimae, ex tubulo dimensionis usque ad 50 μ long. × 20 μ dia. emergentes, et ante dispersionem cumulum globosum immobilem in ore facientes; pæne spheroidea, 5.2 μ dia., globulo minuto, luteo-fusco aliquotque granulis refractivis praeditæ; sporae perdurantes intercalares aut rarum terminales, parietibus crassiusculis, spheroidea, ellipticae aut irregulares, protoplastmate luteo-fusco denseque granuloso completae.

In integumentis larvarum vacuis Chironomidarum, Odonatorum, in New Hampshire Americae Borealis.

Collections. United States: Clark’s Pond (N. H.), 10. IX. 34 (midge); 5. X. 35 (dragonflies).

8. Obelidium (?) mucronatum Nowak.

A limited amount of material of Obelidium (?) mucronatum was found in a caddis fly integument from a pond near Hyannis, Cape Cod, Massachusetts.
The youngest thalli observed were exceedingly minute, being scarcely more than 5 μ high by 3 μ in diameter (Fig. 5, a–c). At this stage a delicate rhizoidal system emerged from either side of the narrow base of the pyriform body within which could be seen a refractive globule. As the plant grew it increased in height, the body became fusiform (Fig. 5, e–g) and the rhizoidal system extended and branched. The basal globule broke up into a number of smaller units, and transient vacuoles appeared in the now more granular protoplasm. Early in the formation of the thallus it could be seen that the basal portion, from which the rhizoids arose, was differentiated as a bulbous or cup-like part from the rest of the body (Fig. 5, d). As growth continued, this distinction became even more evident.

The mature sporangium (Fig. 5, h) was broadly fusiform, 20–23 μ long by 7–8 μ in diameter, and rested on the cup-like base which was 3–4 μ in diameter. In some specimens there was detected at the narrow apex a clear space which the writer interprets as a vacuole (Fig. 5, g). This region remained hyaline in contrast to the slightly granular content of the main body and base, within which were 20 or more evenly spaced, refractive globules. The rhizoidal system was delicate and much branched, radiating in all directions from a single point on the base. A minute refractive nodule could generally be seen on discharged sporangia at the point of emergence of the vegetative system (Fig. 5, i); this was probably a plug separating the two parts. Discharge of the zoöspores was not witnessed, although empty, somewhat collapsed sporangia were found.

These plants differ in certain important respects from Nowakowski’s species and it is possible that they should be segregated from it. The most frequently illustrated figure of *O. mucronatum* (Nowakowski, 1876, fig. 1) shows a sporangium with a broadly elliptical main body, possessing a solid apical spike, all resting on a short, thick-walled narrow stalk which terminates basally in a contiguous spherical, thick-walled structure from which rhizoids
emerge. The present fungus possesses no such spike, although what has been interpreted as a terminal vacuole may possibly be it; nor is there any pronounced stalk aris-

![Diagram of Figs. a–i, Obelidium (?) mucronatum; Figs. j–m, Obelidium hamatum n. sp.; all × 1060. Figs. a–c, very early stages in development of thallus; Figs. d–f, immature thalli showing thick-walled basal region; Fig. g, thallus having an apical vacuole; Fig. h, mature sporangium; Fig. i, discharged sporangium, showing contrast in thickness of walls of upper and lower parts of sporangium. Figs. j–m, Obelidium hamatum n. sp., Fig. j, empty sporangium intact in host, showing intramatrial barbs beneath outer host wall; Figs. k, l, empty sporangia; in Fig. l, the opening in the stalk for emission of the zoospores may be seen; Fig. m, zoospores emerging by amoeboid crawling from the sporangium.](image-url)
ing from the basal part. In these features, however, it resembles very markedly Nowakowski's (loc. cit.) figure 3, and it is to be supposed from this that his fungus was somewhat variable with respect to stalk formation. Nevertheless, in the total absence on the American material of a pronounced apical spine—so characteristic a feature of the European fungus—a difference is found, the full taxonomic significance of which must await further investigation.

Collections. United States: (caddisflies) near Hyannis, Cape Cod, Massachusetts, 23. VI. 34.

9. Obelidium hamatum n. sp.

What appears to be another, hitherto undescribed species of Obelidium was found on larval cases of midges in New Hampshire. The sporangia rested on the surface of the substratum, and, like a species of Rhizophidium, sent their rhizoidal systems into the interior of the integument (Fig. 5, j).

The main body of the sporangium was broadly ovoid, 8–9 μ in diameter by 10–12 μ high, and possessed a distinctly thickened wall (Figs. 5, k, l). Its lower part was prolonged into a broad thin-walled stalk about 4 μ in diameter by 8–12 μ in length which was continuous with the main body of the sporangium. At about the middle of this stalk there were formed two oppositely-placed spines or barbs which were usually tilted upward toward the sporangial body. These spines appeared to be just beneath the surface of the exuviae and were obviously a mechanism for holding the fungus fast to the substratum. Within the integument, the stalk abruptly terminated in a delicate, branched, or more often unbranched, rhizoid of limited extent.

In all save one instance, the sporangia had been completely discharged before the fungus was discovered. However, a plant was found from which minute ellipsoidal zoospores were emerging through an opening at the base of the main body of the sporangium where it joins the stalk

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(Fig. 5, m). These were about 4 μ long by 2 μ in diameter, posteriorly uniciliate, uniguttulate, and at the moment were creeping in an ameboid manner out of the sporangium and over its surface. From the small number (four) which were found, it seemed very probable that the main discharge had occurred earlier and that these spores were merely stragglers.

In the shape of its sporangium, the lack of an apical spine, the possession of two barbs on the stalk, the thickening of the sporangial wall rather than that of the stalk, and in its smaller size this interesting chytrid differs from *Obelidium mucronatum* which is considered its closest relative. For these reasons it is termed *Obelidium hamatum* n. sp.

*Obelidium hamatum* n. sp.

Main body of the sporangium extramatrical, broadly ovoid, thick-walled, 8–9 μ in diameter by 8–12 μ in length, possessing a basal, thin-walled stalk about 4 μ in diameter by 8–12 μ in length, continuous with it, and on which are two oppositely-placed, intramatrical spines; rhizoidal system intramatrical, feebly developed, branched or unbranched, emerging from the abruptly tapering tip of the intramatrical part of the stalk; zoosporae ellipsoidal, 4 μ long by 2 μ in diameter, posteriorly uniciliate, uniguttulate, escaping by an opening at the base of the main body of the sporangium. Resting spores not observed.

On larval cases of Midges (*Chironomidae*), United States (N. H.).

*O. hamatum* sp. nov.

Corpus ipsum sporangii extramatricale, late ovatum, 8–9 μ dia. × 8–12 μ long., basi in stipitem parietibus tenuibus productum, c. 4 μ dia. × 8–12 μ long., in quo 2 aculei intramatricales oppositi stant; systema rhizoidenum vix effectum, ramosum aut non ramosum, ab extremo abrupte angustato partis intramatricalis stipitis enasceens; zoosporae ellipsoidae, 4 μ long. × 2 μ dia., postice uniciliatae, uniguttulatae, e foramine in basi corporis proprii sporangii emergentes. Sporae perdurantes non observatae.
In integumentis larvarum Chironomidarum, in New Hampshire, Americæ borealis.

Collections. United States: (midges) Clark's Pond (N. H.), 8. IX. 34.

SUMMARY

In the exuviae of the immature stage of midges, dragonflies, caddisflies, and mayflies, which are submerged in fresh water, there has been found to develop a remarkable and peculiar fungous flora. These fungi, in the main, belong to the order Chytridiales of the Phycomycetes, although representatives of the Saprolegniales—notably species of Aphanomyces—may also appear.

Nine species of chytrids, collected in Denmark, England, and the United States, are described in some detail, and of these, Rhizoclosmatium aurantiacum, Rhizidium ramosum, Rhizophlyctis Petersenii and Obelidium hamatum appear to be new species.

BIBLIOGRAPHY


EXPLANATION OF PLATES

All drawings were made with the aid of the camera lucida from living material.

PLATE 1

Figs. 1-12. Siphonaria variabilis, from Danish collections. All × 530.
Fig. 1. Immature, colorless thallus.
Fig. 2. More mature thallus, showing return of orange-brown pigmentation.
Fig. 3. Back view of mature sporangium, showing point of attachment of rhizoids.
Figs. 4, 5. Two stages in the discharge of the spores through the basal pore.
Fig. 6. Zoöspores.
Figs. 7, 8. Sporangia of two sizes found in abundance in integument containing resting spores. Gametangia?
Fig. 9. Empty sporangium showing basal pore, thick-walled apophysis and three zoöspores which have germinated in the sporangium.
Fig. 10. Conjugation of two plants of unequal size, by means of their rhizoids. The content of the smaller, "male," plant is steadily passing into the rhizoid of the larger, receptive thallus.
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Figs. 13-19. Asterophyctis sarcocptoides. All × 530.
Fig. 13. Mature sporangium of Danish material showing basal apophysis. Economy of space has prevented the drawing of the whole rhizoidal system.
Fig. 14. Same sporangium discharging its spores through sub-apical pore.
Fig. 15. Zoöspore.
Fig. 16. Mature resting spore, from Danish material.
Fig. 17. Sporangium of American material discharging its spores through the wall of the midge integument to the outside. Spores violently swarming and beginning to disperse.
Fig. 18. Resting spore from American material, showing two immature thalli associated with it ("males")?
Fig. 19. Sporangium from Danish material, showing regularly bifurcated spines. Another species?
PLATE 2

Figs. 1–7. Rhexidiun mycophilum. Figs. 1–5, 7, × 530; fig. 6, × 325.
Figs. 1–3. Thalli in various stages of maturity.
Fig. 4. Mature sporangium, showing apophysis, basal plug and discharge papilla.
Fig. 5. Emergence of zoosporangia.
Fig. 6. Apophysate sporangium, from British collection.
Fig. 7. Changes in shape assumed by the swarming spore mass of fig. 5, after discharge; (a) first shape, (b) final shape.
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Fig. 8. Mature sporangium from Danish material.
Figs. 9–10. Stages in the discharge of the zoosporangia.
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Fig. 12. Sporangium from American material, showing basal pore and nodules of refractive material.
Fig. 13. Immature sporangium showing refractive material within the sporangium. In the possession of a large, unbranched rhizoid, this plant approaches R. mycophilum.
Figs. 14–17. Rhexociosmatium aurantiacum n. sp. All × 530.
Fig. 14. Mature sporangium with basal discharge papilla, from Danish material.
Fig. 15. Mature sporangium from American material.
Fig. 16. Zoosporangium.
Fig. 17. Empty sporangium from Danish material. Two laggard zoosporangia escaping.
Figs. 18–24. Rhexociosmatium globosum. Figs. 18–21, × 530; figs. 22–23, × 1000; fig. 24, × 530.
Fig. 18. Immature sporangium, showing clavate apophysis. American material.
Fig. 19. Sporangium discharging its zoosporangia through a basal pore into a vesicle. Danish material.
Fig. 20. Immature sporangium from American material, showing the more typical fusiform apophysis.
Fig. 21. Dwarf sporangium with apical discharge pore. American material.
Fig. 22. Early stage in conjugation of two plants. The fusiform enlargements in the rhizoids may possibly be due to material from the smaller plant migrating to the larger, receptive thallus.
Fig. 23. Another case of rhizoidal anastomosis. The receptive thallus has been transformed into a resting spore.
Fig. 24. Zoosporangium.
PLATE 3. *Ehizophyctis Petersenii* n. sp.

**Fig. 1.** Mature sporangium with discharge tube. × 260.

**Fig. 2.** Same immediately after discharge, the spores forming a motionless spherical cluster at the tip of the discharge tube. × 260.

**Fig. 3.** Irregularly-shaped resting spore. × 530.

**Fig. 4.** Mature thallus; discharge tube beginning to form. × 260.

**Fig. 5.** Zoöspores; the one on the left showing the shape while actively swimming, the other the shape during amoeboid movement (freehand).

**Fig. 6.** Zoöspore drawn to scale. × 260.

**Fig. 7.** Ellipsoidal resting spore formed just beneath wall of integument. × 530.
PLATE 4

Figs. A-C. *Siphonaria variabilis.* (Danish material, stained with eosin.) × 640.

Fig. A. Resting spore with two "male" thalli (indicated by arrows) attached.

Fig. B. Resting spore with male thallus (indicated by arrow) in focus.

Fig. C. Same at slightly different focus showing continuity of rhizoids of resting spore and male cell.

Figs. D–J, from living material, American collections.

Fig. D. Young thallus of *Asterophlyctis sarcoptoides* with attached tube terminated by a cyst. × 640.

Fig. E. Resting spore of *Asterophlyctis*, the rhizoidal system not shown. × 640.

Fig. F. Sporangium of *Asterophlyctis*. × 400.

Fig. G. Resting spore of *Rhizoclosumatium globosum*, showing empty male thallus (indicated by arrow). × 640.

Fig. H. Immature sporangium of *Rhizoclosumatium globosum*. × 640.

Fig. I. Mature sporangium of *Rhizodium ramosum* n. sp.; the rhizoidal system just visible, radiating from the base of the sporangium. × 400.

Fig. J. Thallus of *Rhizophlyctis Peterseni* n. sp., showing the spherical "central body" (the sporangial fundament) and the stout rhizoids radiating from it. × 250.
POSTNATAL GROWTH OF THE EXTERNAL NOSE

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ABSTRACT

To account for the variety in racial and familial form of noses the hypothesis is tested that they develop along different lines. To find out how they develop repeated measurements have been made on babies and older children over a period of five to twelve years. The results show that, indeed, the course of development of children with unlike noses is different and some cases are cited showing in detail the variation in growth of the various nasal dimensions. In some cases the individual idiosyncrasies are foreshadowed at birth; in other cases the nasal proportions may become quite altered by excessive growth of some one dimension. The closeness of the course of development in identical twins is illustrated. The conclusion is drawn that the development of the nose is under the influence of genes; but the active genes may differ at different stages in ontogeny quantitatively (in functional output) or qualitatively due to the activity of new genes like those that determine the activity of endocrine glands. When the total gene complex is the same, as in identical twins, the result is practically identity of nose form.

I. STATEMENT OF PROBLEM

Man's outer nose is unique. In other animals, like tapirs and elephants the elongated nose is a proboscis with the nostrils at the tip, and this is true even of the proboscis monkey—Nasalis larvatus; so our outer nose deserves study. The anthropoid apes show a rudimentary outer nose, Figs. 1, 2, 3, 4.

So also in ontogeny, the face is at first without protuberance (Fig. 5); acquires a slight papilla by the 4th month; is still devoid of a well marked elevated bridge at birth. The nostrils which at first look forward as they do in the gorilla come, before birth, to look downward.

The form of the nose varies in adult humans between the different races. It varies in height (from nasion, just above the root, to the subnasion); in width, as between the rabbi of Tunis and the Tasmanian woman, Fig. 6 (above); in relation of width to height as between the Sakoi of the Malay peninsula and the Bannock Indian chief, Fig. 6 (be-
Fig. 1. Full face of young male gorilla. (Courtesy of New York Zoological Society.)
Fig. 2. Profile of the young male gorilla shown in Fig. 1. (Courtesy of New York Zoological Society.)
Fig. 3. Profile of young chimpanzee. (Courtesy of New York Zoological Society.)
Fig. 4. Full face of orang utan. (Courtesy of New York Zoological Society.)
low). It varies in profile (Fig. 7) as between the man from Java with nose only a little more prominent than that of a Gorilla or 5th month fetus to that of the aquiline nose of the Bannock chief. Indeed, one can find a great variety of nose form in the adult profile (Fig. 8)—the almost fetal nose of the Australian (a); the Negro nose (b) the projection of which is almost buried in the thick upper lip; the Syrian nose (c) shown in the outline of an Armenian’s face; the various Nordic noses (d–g), with nose bridge ranging from convex to concave (d, g), in depth (f and g), and

![Profile of face of a series of human embryos two to five months old and a newborn child. (From K. Peter, 1913.)](image)

in the method of union of nasal septum with upper lip (d and f). In the Mongoloid dwarf of European stock, at the extreme right, the outer nose is slightly developed.

The problem is what determines these differences? We may test the hypothesis that noses are different because they develop differently in different families under the influence of different developmental factors.

II. Methods

The opportunity was offered to follow the postnatal development of the nose in a series of babies from birth to three years; by means of measurements taken repeatedly
first at fortnightly, later monthly and finally quarterly intervals. These were taken as part of the Normal Child

![Fig. 6. Full face views of noses of representatives of peoples of different races of mankind showing variation in form.](image)

Development Study of Columbia University (Dr. Myrtle B. McGraw, Assoc. Director). Also, I have followed during a
period of five to twelve years duration the changes in nose
dimensions in a number of normally or nearly normally de-
veloped children in the Orphan Asylum of Brooklyn and at
Letchworth Village. Dimensions measured were height,
width, depth, salience and depth of nasal bridge at root of

Fig. 7. Left, man from Java. Nyessen, 1929, p. 64. Right,
Bannock Indian Chief.

nose. The growth curves were plotted for individuals and
in some cases *en masse*.

Acknowledgment is gratefully made to the cooperation
of Dr. McGraw, Dr. E. J. Humphreys of Letchworth Village
and Dr. W. R. Blair Director of the New York Zoological
Park.
Results

First, I may point out that the hypothesis that the nasal form is determined by the course of development is supported by the observation that in the case of children with unlike noses the course of development is different. This is the case of two boys, E. H. and C. D., whose profiles are shown (Fig. 9). One notes the difference in height of nose and in depth of nose bridge at root. There is also a difference in depth of nose at apex in favor of E. H. Figure

![Fig. 8. Profiles of face of various races as described in text, being camera drawings of photographs from various sources.]

10 gives the curve of development of the nasal dimensions in the two boys from 6 to 18 years in one case and 8 (or 11) to 22 years in another. One notes that the nose of C. D. which is now lower than that of E. H. has been lower from the beginning of observations and that the bridge of the nose at root of C. D. is now shallower than that of E. H. and has been so from the beginning of observations, 8 years back; though the growth curves of this dimension are not strictly parallel and the bridge at root of the smaller nose seems to have stopped growing at 16 years while the deeper nose bridge is still getting deeper at 19 years.

The forms of these growth curves are of interest. They
are not straight lines over the whole extent. In most cases growth of the nasal dimensions is relatively slow at 10 to 14 years. From 14 to 18 years the speed of growth is much increased. The beginning of the period is about the time when the growth of the body as a whole is most accelerated. But the high growth speed of nasal dimensions generally continues after the age of the spurt of growth of
the body as a whole usually slows up. However, these particular two boys were both still increasing in stature at 18 years. The adolescent spurt of growth in the body size affects even facial features.

![Graph showing nasal dimensions growth](image)

**Fig. 10.** Curves of growth of nasal dimensions in E. Ham. and C. Del. (Cf. Fig. 9.)

Figure 11 shows the absolute changes in outline of the facial feature of those two boys from juvenality to maturity. The root of the nose has grown much higher in E. H.: very little in C. D. (These profiles are centered at the subnasale.)

It is, however, not always true that the relative adult
nasal dimensions of a number of unrelated persons are all predetermined at birth. Figure 12 indicates clearly that the rate of growth of a dimension is in some children rapid, in others slow, so that their comparative sizes may change during development. Thus there was a time when the nose of Hun. No. 83 was higher than that of any of the others on the chart, but it is lower now than in three of them. It is now a low concave nose. Conditions external to the gene may have affected its growth. But it seems probable that in the complex interplay of developmental forces it shall come about that the functioning of now this gene (or gene

![Diagram of nose changes](image-url)
complex) now that shall assume the upper hand in the process.

Second, a more convincing fact bearing on the hypothesis of genic control of nasal form is its resemblance in identical twins (Fig. 13). Such twins show a great resemblance in most somatic and psychic qualities which leads to

![Graph of nasal growth](image)

**Fig. 12.** Curves of absolute growth of nasal height and nasal width in five children, to show diversity in the rate of development in different individuals and at different ages.

the conclusion that their development is directed by the same genes, and this conclusion is strongly supported by this resemblance in minute details, as of nose form. We suspect that from the beginning the noses of such twins have undergone an identical development and in so far as it has been possible to follow the growth of the nose of the G twins (Fig. 14) this is the case (Fig. 15). In depth and
salience the measurements of the twins from 10 to 15 years are identical. In nasal width there is a possible difference at 12 years of 1 mm.; in nasal height of 2 mm.; and thus in the nasal index of 4 per cent. In view of the great difficulty

![Fig. 13. Profiles of a pair of identical twin girls, Angelina G. and Mary G. (Courtesy of Letchworth Village.)](image)

of determining the nasion (from which nasal height is measured) even this small difference may be illusory. In the ratio of nasal depth to nasal height the difference is less than 2 per cent.
Discussion

The postnatal growth of the nose is controlled by one or more genes which, working together, direct each nose along a definite path. These dissimilar gene complexes show their specific effects in the dissimilar forms of the nose—a difference which is most marked in the case of the different human races, but which is marked even inside the same race; i.e. there are family differences. The form is determined by the size and form of the nasal bones and of the four large cartilages: lateral nasal, septal and R. and L. alar.

The course of development is not for long a straight line. The activity of the primary genic influence may become weakened so that growth slows up. A genic imbalance may become established so that a third factor which was more or less inhibited in early stages comes to play an important role. Thus under the influence of general growth-promoting hormones at 15 to 17 years many nasal dimensions make a spurt in their growth as the body as a whole does.
The sequential action of form-developing factors of the nose is illustrated in Fig. 15. This diagram indicates growth of nose form by the oblique line the position of which is a resultant of the factors that determine the relation of breadth to height. In the negro the breadth factors

Sequential action of nose-developing factors.

![Diagram showing sequential action of nose-developing factors](image)

**Fig. 15.** Diagram illustrating the hypothesis of sequential action of nose developing factors in the nose of Negroes and Whites respectively. The oblique lines are the resultants of the height factors represented in the vertical, and breadth factors in the horizontal line. In the case of the special size factor an oblique is added to the vertical and horizontal components, the resultant growth line being the resultant of all three.

(relative to height factors) are stronger than in the white man. Still later the contrast is intensified though the intensity of the growth as a whole is reduced. A new factor, promoting size, probably of endocrine origin, now becomes
active jointly with the other two, height and breadth; the resulting direction of nose development is now the resultant of these three factors. This is of course a mere schema, doubtless greatly simplified.

Conclusion

The development of the nose is under the influence of genes; but the active genes may differ at different stages in ontogeny quantitatively (in functional output) or qualitatively due to the activity of new genes like those that determine the activity of endocrine glands. When the total gene complex is the same, as in identical twins, the result is practically identity of nose form.

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ABSTRACT

One of the basic phenomena in human morphology on which now for the first time are possible some generalizations, will briefly be dealt with in this paper. It is the curious phenomenon that deserves the term of human typogeny. It consists of the fact that every large part of the human body and probably every larger organ tends to develop, aside from many intermediates, a number of well-defined characteristic forms or types, which tend to recur in the same racial group, under like conditions, in similar proportions. These types are realized only in the form of the parts, and have but secondary relations to size. They are more than mere oscillations about the average or extremes of the range of normal variation. They appear somewhat like so many morphologic aggregates or "crystallizations" in the ranges of variation, regardless of the averages, modes and extremes of such ranges.

Empirical observations of some of the results of this process, dating from far back in anthropology, will be discussed.

It is concluded that, although much additional work remains to be done in this field, there is involved evidently a general and already hereditary tendency toward form differentiation.

One of the major realities in nature with which man is still but very imperfectly acquainted is his own body. It may seem incredible, yet it is true that not a single organ or part of our body is as yet completely known and understood; and this applies in even a greater degree to our functions and our endowments, and above all to the laws that underlie and govern everything in the body.

The main reasons for this are the great organic as well as functional complexity of our system; and its variability with age, sex, individual activities, admixtures, environmental conditions, and hidden inherent causes. The result is that the human body, living and even dead, cannot be learned from any individual specimen, nor even from a few individuals or bodies, but requires mass studies and long-lasting observations. Moreover, in man, there enters into the matter also the widespread and influential factor of

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pathology. And in addition to all this the human subject, due to its long period of development and life course, is unfit for experimentation; besides which any experimentation, even if possible, under our religious views and sentimental feelings would be frowned upon or forbidden. There are still many and potent obstacles to post-mortems, dissection, exhumation; and not seldom even to collection of the skeletal remains of native peoples.

Notwithstanding these difficulties, studies on man are progressing in every civilized country; and in more favored centers, where there are large collections and other facilities, they are gradually reaching facts and laws that hitherto were barely if at all suspected. Some of these, due particularly to work in connection with our own institutions, have already begun to show fairly the laws of growth in childhood; evidences of growth of the body and its different parts during adult life; similarities in the range of variation in the same dimensions in different human groups; separate ontogenetic life courses of different features and organs; fundamental physiological similarities in wide-apart groups of whites, American aborigines, and other races; and in other directions.

Physical anthropology thus, through intensive studies on large living materials and human remains, is determining the full ranges of conditions, with their separate life courses, and beginning to touch the basic processes and laws of the human organism. All of this opens a vast field of future endeavors, which are bound sooner or later to be not merely of academic interest but also of substantial human utility.

One of the basic phenomena in human morphology on which now for the first time are possible some generalizations, will briefly be dealt with in this paper. It is the curious phenomenon that deserves the term of human typogeny. It consists of the fact that every large part of the human body and probably every larger organ tends to develop, aside from many intermediates, a number of well-
defined characteristic forms or types, which tend to recur in the same racial group, under like conditions, in similar proportions. These types are realized only in the form of the parts, and have but secondary relations to size. They are more than mere oscillations about the average or extremes of the range of normal variation. They appear somewhat like so many morphologic aggregates or "crystalizations" in the ranges of variation, regardless of the averages, modes and extremes of such ranges.

Every known measurement on the human body and every relative dimension or index, if secured on an adequate number of individuals of the same sex, age-period, and race, gives a frequency distribution which, when properly represented graphically, shows an approach to a regular frequency curve. But when we take a similarly large number of observations on the shape or form of a part, these observations, even if reduced to numerical values, do not all give a single curve but rather a polygon consisting of three to seven connecting curves or waves or hills, the summits of which regardless of their heights remain always well apart from each other. There is in such cases no more a single curve, but a chain with so many connected densities or aggregations. We are confronted therefore not anymore by the simpler phenomenon of variability, but by one of a tendency at a differentiation within that range into several distinct forms, or a typogeny.

Empirical observations of some of the results of this process, without efforts at or possibility of understanding their meaning, date from far back in anthropology. They extended in a measure to types of the body as a whole, but principally to the shapes of the head or skull, of the face, and of the nose.

Since the time of Galen, and probably before, there were recognized more or less hazily several "temperaments," or as they came to be called later, "constitutions," which meant so many distinct types of the human body. The matter of these types is extremely complex, involv...
phology, physiology and even pathology, and though it has received a great deal of attention within the last three decades it is still far from well defined or understood. There is no question about the tendency toward a recurrence of several such general body types, but this is largely true, it seems, only in the white race, is greatly complicated through age, and is frequently connected with and altered by disturbances of glands of internal secretion. Just what the residuum of normal bodily types may be is as yet uncertain.

Since the nineties of the last century there has also grown in anthropology an appreciation of two other, and in this case entirely normal and fairly universal, types of the body, the brachymorph and the dolichomorph. In the first of these types the general tendency in the body and its parts is to relative shortness and broadness, in the second to relative length and narrowness. In the first type the head, face, neck, trunk, limbs, hands and feet are all relatively short, broad and inclined to stockiness; in the second type they are relatively long, narrow and inclined to slenderness. The first type is more frequent among markedly brachycephalic, the second among markedly dolichocephalic populations, but in a pronounced form both are rather rare, and there are many intermediate conditions. While well known to anthropology these two types have not yet received as comprehensive attention as they require. It is possible that an intermediate or mesomorph type will deserve to be established.

During the earlier parts of the present century Robert Bennett Bean, more especially, called attention to still other body types, based principally on the differing relative proportions of the trunk and the lower limbs. There are bodies with relatively long trunk and short legs, others with relatively short trunk and long legs, and still others with relatively medium proportions in both trunk and legs; either may be of phylogenetic or ontogenetic causation, and the marked cases in each are accompanied by differences in
other parts and structures. Regrettably, though the main facts are substantial enough, the subject thus far was found to be too involved for any clear and definite conclusions.

There are still other types of bodies, of racial character. The *predominant* male and especially female bodies of the White, the Mongoloid (including the Eskimo and the American Indian), and the Negro and Negrito, differ markedly in the shape of the neck, trunk and lower limbs, though all the types may occasionally be found in members of any of these races.

Thus there are several classes of types of the body as a whole, namely the normal "constitutional" or basically physiological; the general morphogenic, with its two extremes of brachy- and dolichomorphy; the compensatory, manifested principally in varied relative proportions of the trunk and limbs; and the racial.

Let us now proceed to the separate body constituents.

Different shapes of the head and skull, of the face, the forehead, the nose, lips, and chin, had been noticed probably since the time when man began to observe. They received attention particularly by the artists, from the first decade of the nineteenth century also by the phrenologists, and since 1842, or nearly one hundred years ago, when Anders Retzius called attention to dolicho- and brachycephaly, by the professional anthropologists. In the eighties and nineties and for a time thereafter the subject received considerable attention by the Lombroso school in criminology and especially by Giuseppe Sergi in racial studies. Sergi studied in particular the shapes of the skull as viewed from above, regardless of its relative dimensions, and in time reached a detailed classification of these shapes, without seriously attempting to solve their meaning or genesis.

Sergi recognized five main shape-types of the skull and head, and each of these types presented, in his view, several subvarieties. Later Sergi left out one of these, the "trapezoid," and on the rest expressed himself thus: ¹ "Through-

out this immense (Mediterranean) stock, from the equator in Africa to the Arctic circle in Scandinavia, we have found four characteristic and constant cranial forms, always found together in every region and in every clime, with whatever variations in external characters; these are the pentagonal (Pentagonoides), ellipsoidal (Ellipsoides), ovoid (Ovoides), and the arrow-shaped (Beloides). These four forms represent four cranial varieties, each with a series of variations which constitute sub-varieties or sub-forms, corresponding to the type to which they belong, and which are distributed in different proportions in the groups of populations."

Notwithstanding Sergi's intensive attention to the subject, but little progress was made in understanding its significance, and even the classification, partly because of secondary details and nomenclature, partly perhaps because the time was not ripe, has not found a regular place in anthropometric and cranioscopic procedures.¹

Sergi attempted also to classify the forms of the face in the living. He recognized in the Mediterraneans also four main types, namely the ellipsoid, ovoid, triangular, and the parallelogrammatic. These in 1916 were increased by Pöch to 10,² the differences between some of which were so small that the scheme became impracticable and was not followed. Here too the efforts were largely limited to registration of the types and the frequency of their occurrence in some human groups, without much inquiry as to their meaning and causation.

So far as the individual parts of the face are concerned, their forms or types were largely shown already in 1864, by Broca. They are presented effectively by Topinard in 1885,³ and since then are given more or less adequately in all the textbooks on anthropology or anthropometry. They need not be enumerated in this place. The types of the dorsal border of the nose were found to be particularly well

² Given also in Martin, p. 220.
³ Éléments d'anthropologie gén., 8o, Paris.
defined; and those of the eyes, nose, lips, malars, chin, and ears were learned to have interesting racial distribution and peculiarities.

In 1897 I began to work on the already respectable Huntington collection of bones of the skeleton in the old College of Physicians and Surgeons, New York City. Up to that time—and in fact to this day—such bones are described in the anatomical textbooks as if they each presented but a single type. This very soon I found to be erroneous. Before many months passed it was seen that all the long bones and most of the larger parts of the skeleton presented each a number of normal types, some of which were very markedly different from others. In the long bones these types were limited largely to the shaft; in the scapula, sternum, first rib, and the sacrum, they involved parts to all of the bone. As studies progressed the different types were seen to have differing incidence in the various racial groups represented in the collection, and there seemed to appear some occupational correlations.¹ There could be found no former records of such findings, and so in 1898 and again in 1900 and 1901, they were brought to the attention of the Association of American Anatomists.

Since then the subject has never lost in interest to me and has been followed not only on the much richer eventual Huntington collection, which since has passed into my care in Washington, but also on extensive Indian, Eskimo and other skeletal materials of my division in the U. S. National Museum. The results are still largely unpublished, but partial accounts have appeared in several of my reports. They substantiate and extend the earlier findings, and they have led to the recognition that we are confronted here with a series of related phenomena in human developments and differentiation that have a broad common basis and that deserve to be grouped together under the general term of

¹ There were records on most of the subjects from which the collection proceeded as to nationality, sex, age, occupation, and cause of death.
"human typogeny." The qualifying "human" is necessary, for while there are indications that the phenomenon of typogeny is also in evidence more or less in other living forms, our knowledge about these, in this respect, is still rudimentary.

After the foregoing remarks we may now approach the facts, as seen at this moment. A final elucidation of the phenomena involved, however, is not yet possible.

**Shapes of the Head**

There are three more or less related yet distinct categories of these shapes or types. The first of these is due to simple mechanical causes. The skull has three main dimensions, namely length, breadth and height. These dimensions during growth stand together in compensatory relation, so that a material change in any one affects the two others. Under average circumstances, in any human group, the hereditarily conditioned mechanical influences that act on the skull result in the development of a vault of close to the average relative proportions for that group. But if these mechanical influences functionally, incidentally, or even artificially be altered, the shape of the skull will be altered. If a greater pressure during the developmental period is brought to bear on the sides of the skull, the vault will grow narrower and therefore longer and higher; if there be less bilateral pressure, or more pressure on the back, the vault will grow broader and shorter, the height being less affected. In artificial deformations and in result of premature occlusion of one or another of the cranial sutures, the compensatory developments lead to grotesque forms. Excluding all the artificial and the more or less pathological conditions, there still remains here a class of purely ontogenetic, yet not hereditary, differences in head form which differ etiologically and thus are apart from others, even though they cannot be clearly recognized and separated in practice.
The second class of skull shapes or types is much like the preceding, but the forms are *hereditary*. The most widely known of these forms are those that depend on ratios of the breadth and length of the vault and are expressed by the so-called cephalic or cranial index. There are individual normal human skulls in which this index \( \left( \frac{\text{breadth} \times 100}{\text{length}} \right) \) is as low as 63, others in which it rises to well over 90, and there appears to be a fairly fixed intermediary condition. In any single limited racial group the range of these shapes behaves as a simple dimension, giving a regular curve from the minimum in the group to the maximum. It is only when large races or humanity as a whole are taken that the picture approaches a curve with so many nodes or that of genuine typogeny.

However, the relative ratios of the breadth and length of the vault are only a part of the picture in this class of form differences. The height of the vault plays also a role, and that not merely in compensation with the breadth and length. This adds some complexity to the case. Thus, there are types of skull and head that average, in given groups, broad, short, and low; or broad, short, medium high; or broad, short, high. Others are narrow, long, medium; or narrow, long, high; while still others are medium, medium, medium; or medium, medium, low; or medium, medium, high; and there are still other combinations. The fundamental feature of all the types of this class is that they, with their individual oscillations, are hereditary.

The third class of skull shapes shows *typogeny* most clearly. It includes the types of the contours of the vault, more particularly those that appear when the vault is viewed from above. It is these types to which Sergi devoted so much attention. There are, as now seen, four main types, each with several variants and numerous interconnections. They are the pentagonal, the ovoid, the elliptic, and the spheroidal. Except the last, all may be long, medium, or short. They show considerable individual dif-
ferences of frequency in different races and groups. They are essentially hereditary. Their representation is richest in the white and the yellow-brown races, and limited in the negro. Their range forms no curve or line with a mini-

![Diagram of head and skull contours](image)

Main Types of Head and Skull Contours

mum, mean, and maximum, but rather a ring with so many interconnected nodes or differentiations. The various shapes are more or less connected with or influenced by the relative proportions of the vault and hence with types of Class 2, but they are also more or less independent of these.
In their optimum development they are very striking and very definitely apart from each other. They characteristically represent the process of human typogeny. They have family, group, and racial values.

**Types of Face**

The face presents a number of contour types both frontally and in profile. These types are connected with the dimensions of the bony parts and largely also with sex and especially age, but they have also more or less of individuality and fall within the scope of human typogeny.

The main frontal facial types, in adults and before senility has set in, are the elliptical; the ovoid; the parallelogramatic (parallel sides, with evenly rounded top and base); the rounded or spheroid, to which may perhaps be added the triangular (tapering toward a pointed chin); and the angular (angles of lower jaw prominent); but these last two depend largely on the size and form of the lower jaw.

The facial types, like those of the vault, are especially represented in the white and the yellow-brown races, much less so in the negroid peoples. While much affected by ontogenetic conditions they are substantially hereditary, and they fall within the expressions of human typogeny.

**The Nose**

The ridge of the nose presents in the adult several distinct shapes which are among the best manifestations of human type formation. These shapes are the concave, the straight, the wavy (or concavo-convex), and the convex (or aquiline). As in all other cases these forms, except the straight, occur in various degrees and interconnect with each other, but when well developed they are characteristic and stand well apart. The concave form (as the pentagonal in the vault) is connected directly with the infantile.
Main Face Contours

1. Rounded
2. Elliptical
3. Parallelogrammatic
4. Pointed
5. Angular
The Chin

There are three types of the chin—the pointed, the rounded, and the square. They are not merely the productions of musculature or of massiveness of the bone, but are in the main hereditary and connect with human typogeny.

Body Parts: The Skeleton

The neck, the thorax, the breasts, the lumbar region, the pelvis, the genitals, the buttocks, thighs, calves, fingers and toes, all present more or less well defined different shapes, expressions of human typogeny. But some of the clearest examples of this phenomenon are encountered in various parts of the skeleton. Some details on the latter, based on extensive original observations, follow:

The First Rib

The first rib, while the shortest of the fixed ribs, has more individuality than any other of the series. It presents three normal interconnecting types, namely the uniformly curved or semilunar, the near straight-bodied or pistol-shaped; and the angular or trisegmental.

The Sternum

The types of the human sternum are not yet definitely classified, but several are fairly outstanding. There is what may be called the blunt cruciform type, a nicely sculpted form with a moderate narrowing below the manubrium followed by gradual broadening toward the lower end. There are the chimpanzoid and the orangoid sterna; the first long, narrow, with nearly straight parallel sides; the second broad, with manubrium mostly attached and body, though broad above, enlarging moderately further toward the lower end. There is also the wedge sternum, broad above and tapering like a wedge downward; and there may be still another type.
The Scapula

The body of the scapula presents three main types. These are determined by the shape of the vertebral border of the bone. This border may be concave, straight, or convex. These types in a measure represent the minimum, medium and maximum development of the vertebral border, and thus behave like a simple dimension or quantity, but they also occur genetically in bones of various dimensions, and to that extent fall within the scope of human typogeny.

In addition the inferior angle with the lower portion of the axillary border, the supraspinous part of the axillary border, and especially the superior border of the bone, tend each to assume several shapes which modify the aspect of the whole scapula. The greatest variety of these shapes is seen in the superior border, and some of these are clearly of hereditary character and hence of anthropological value. This border may be practically horizontal, or of any slope from slight to steep, or nicely curved, or again characteristically angular, like a cross-cut of a square basin. Typogeny is clearly involved in these connections.

The Sacrum

Typogeny in the sacrum is partly obscured by the marked sexual differences in the bone; nevertheless there are instances in which it is quite evident.

The main variant is the relatively long and narrow anthropoid-like sacrum; but there is also a broad and short sacrum that is outside of the bounds of mere sex variation. And there are sacra that anteriorly, from above downward, are practically straight, while others show from slight to pronounced curvatures; but in these cases it is difficult to separate ontogenetic and even semipathological from hereditary conditions and it is not easily determinable therefore what may be of true typogenetic causation.
HUMAN TYPOGENY

THE LONG BONES

In the shafts of these bones, especially the humerus, ulna, femur, tibia and fibula, typogeny is marked and received my particular attention. It begins to manifest itself before birth but does not reach full expression until well in adult life. The results in the tibia and the fibula are very striking. The various types can best be seen on sections of the shafts at the middle of the bone. Such sections in some cases show such differences that, when viewed by themselves, it is difficult to believe they come from the same bones of the skeleton. They present marked group and racial differences. One of the types in each bone connects with the general fetal form of the same in its particular ethnic aggregate. Some of the types are influenced and perhaps even brought about ontogenetically, but the majority are true expressions of hereditary human typogeny.

In the humerus the main types are the cylindrical (juvenile form), the triangular, two (opposite) lateral prismatic, and the quadrilateral.

In the ulna there is the prismatic form, the lateral prismatic, and the fluted (anterior surface hollowed out); this last being probably of ontogenetic origin.

The femur may be cylindrical (juvenile form), elliptic, prismatic, or plano-convex, with some tendency in cases toward quadrilateral (anterior surface divided into two by a vertical ridge proceeding from above).

The types of the shaft of the tibia are the prismatic, the lateral prismatic (semi-lozenge), the fluted (antero-external surface hollowed out), the quadrilateral (posterior surface divided by a marked vertical ridge into two), and the gorilloid (shaft almost plano-convex, anterior border stout and very dull). There is also the pear-shaped form, in which the posterior surface is convex with the postero-external border very feebly developed, but this is probably a subdeveloped shaft of ontogenetic causation.

The fibula is the most variable. Its adult shaft pre-
sents the following types: prismatic; quadrilateral; pentagonal; and several deeply fluted varieties (either one surface, or two, or all three surfaces, hollowed out).

**Other Skeletal Parts**

Some typogenetic manifestations are also observable in the clavicle, the atlas, the pelvic bones, the radius, the patella, the calcaneus; but the results are seldom sufficiently clear cut and hence are difficult of classification.

**Brain and Internal Organs**

Form differences certainly exist in all these, but due to many difficulties it is wholly uncertain as yet how far these variations are hereditary and may be due to the process of typogeny.

**Discussion**

The object of this communication is to call attention to as yet but imperfectly known although plainly a potent factor in human morphogeny, a factor that, mainly on hereditary basis, tends toward the production, in different parts of the body and skeleton, of forms or types which are different from, and more than mere grades of, any one condition.

The production of these types must be connected with differences in innervation, and of musculature where such exists, but there are very substantial indications that it is not due to mere accidental conditions in these, nor to their ordinary variability. There is involved evidently a general and already hereditary tendency toward form differentiation, which deserves to be called human typogeny.

That this tendency is hereditary is shown by the fact that, according to accumulating data on the subject, the proportion of the different types of any part in any one racial group remains fairly constant, but that this proportion differs materially between unrelated groups or races.
The biological significance of these typogenetic variants can only be an increase, to the limits of possibilities under given conditions, in morphological diversity—which facilitates group and racial differentiation.

Much additional work remains to be done in this field. The factual evidence is still far from fully determined on many parts of the body; the segregation of the results of typogeny alone from those that appear related but are due to other causes, is as yet nowhere-near exact or complete; the presence and workings of the factor of typogeny in mammals other than man and in other forms of organisms, is to be ascertained; the direct ways and means by which the diverse types are produced are still unknown; the racial distribution of the various types is still known but very inadequately; and the deep causation as well as the full meaning of the phenomenon will need much further work and thought. The present effort must of necessity be limited to the calling of attention to this evidently generalized, and thus far but very imperfectly realized, typogenetic tendency in the human body and its constituents.
A. Group of trees (taken as type), cultivated in the Borda Garden, Cuernavaca—showing habit of growth.
A MEXICAN PUZZLE (YUCCA HOWARD-SMITHII)

WILLIAM TRELEASE
Professor Emeritus of Botany, University of Illinois
(Read by title April 22, 1937)

ABSTRACT

An account of the most-planted tree-yucca of Mexico City and the Western Cordillera, with clustered trunks, weak-pointed, finely toothed leaves, hanging inflorescence, and occasional fleshy fruit—which has not been identified with any adequately described species.

In January of this year, Professor Howard L. Smith, of the University of Wisconsin Law Faculty, asked my opinion on a tree-Yucca cultivated at the famed Borda Garden in Cuernavaca, where we were passing the winter. Professor Smith has taken an interest in the Yuccas for many years, and his attention was called to these trees because the panicle, instead of being erect or continuing the direction of the leafy axis, was gracefully pendent, as it is in a few other Mexican species. Though the flowers had passed when we looked at the trees, the inflorescence-remnants were still present and unquestionably reflexed from the ends of the branches.

The same species is planted more or less commonly in and around Cuernavaca either in hedge-rows or within patios and other gardens; and it is represented by fine old specimens here and there in the Alameda and Chapultepec Park of Mexico City—where it has been used somewhat extensively in the newer boulevard plantings.

Though differing considerably in stoutness of trunk and branches, this is one of the trees characteristically producing several trunks from a common base; its leaves are rather flexible, only slightly pungent at end, and minutely denticulate on the margin.

These patent features differentiate it, even superficially, from all other recognized cogeners except the little-known
Yucca yucatana and the much cultivated Yucca elephantipes (or guatemalensis). Neither of these has pendent panicles and the former has downy flower-clusters while the latter characteristically has a single stout trunk usually greatly dilated near the ground. Though they flower freely, the species of this group, as commonly known, are out of the geographic range of Pronuba and bear only an occasional fruit obviously due to self-pollination and, of course, lacking the deformities caused by the moth in oviposition, and without larve in the seeds. Such fruits as are formed are of the baccate type with yellowish-white flesh and a papery core surrounding the thick ruminated seeds.

These characters mark this Yucca as of the group Sarcoyucca, in which I have been able to recognize only two species with merely denticulate and not filiferous leaves: Yucca aloifolia, with freely produced coreless purple-fleshed fruit; and Yucca guatemalensis, with sparingly produced, whitish-fleshed fruit with a core.

On these very evident differentials, Yucca Howard-Smithii is definitely excluded from Y. aloifolia, but differs from Y. elephantipes only in its multiple trunk and, especially, in its hanging inflorescence—the feature that particularly attracted Professor Smith’s attention.

It is quite possible that one or more of the garden-names now referred to Y. elephantipes may have been intended for this tree; but if so I find it impossible now to say which may have been so applied, or to find an adequate published description of it.

A more puzzling possibility however exists, that some one of the many names collected under Yucca aloifolia may belong to Smith’s Yucca, notwithstanding the certainty that the latter is by no possibility really an aloifolia form. Of these questionable names, Yucca conspicua of Haworth, placed under Y. aloifolia by Engelmann, may prove ultimately to pertain to this species, but in the absence of definite information that it has coreless pale-fleshed fruit and pendent panicles, this reasonable supposition must be
regarded as unproven, though Cuernavaca specimens collected by Bourgeois have been so-referred.

Perhaps the most surprising thing about Smith’s Yucca is that thus far no clue has been obtained as to its exact nativity. Mexican gardeners commonly know it by the generic native name “izote,” but I have failed to find a person who can tell me that he has seen this particular kind of izote growing in a state of nature—which, so far as my own knowledge goes, is equally true of Y. elephantipes.

Technically, Yucca Howard-Smithi, n. sp., may be characterized as follows:

A small tree some 5–7 m. tall, branching from the base; leaves rather flexible but not recurving, dagger-shaped, 3 or 4 × some 50 cm., at most slightly pungent, the margin roughened by small prominences; panicles very short-stalked, hanging from the ends of the leafy branches, oblong, nearly glabrous; flowers some 6 cm. in diameter; ovary sessile, with very short style; fruit pendent, fleshy, oblong, some 5 × 10 cm. with yellow-white pulp and papery core about the thick ruminated black seeds.

Type locality and geographic range unknown, but frequently cultivated in and about Cuernavaca, Morelos, and in Mexico City, and probably native to the middle altitudes of the Central Cordillera of Mexico.

Closely related to Y. elephantipes and Y. yucatana, which have ovoid erect panicles; and apparently to Y. duraconis and Y. conspicua of gardens, which have broader and softer recurving leaves.
B. A more robust specimen. Cultivated in the Plaza de la Constitucion, Cuernavaca.
C. Portion of the same, showing pendent panicle with two immature fruits.
RACE: A FACTOR IN HUMAN METABOLISM

FRANCIS G. BENEDICT

Director Nutrition Laboratory, Carnegie Institution of Washington, 1907–1937

(Read April 25, 1937)

ABSTRACT

A survey of the basal metabolism of numerous races of humans has been made in the past ten years by the Nutrition Laboratory and its various collaborators, with emphasis upon Chinese (both those born in the United States and those in China), South Indian women, Maya Indians in Yucatan, and mixed races in Hawaii. This paper summarizes the findings, with special reference to two recent studies on Chinese, the observations of Professor Leslie G. Kilborn in Szechwan and the 4-year research of Dean Stanley D. Wilson of Yenching University in Peiping, China. As the Nutrition Laboratory's survey was based upon the use of a single technique, carried out by well-trained investigators, the results are strictly comparable. Even when allowances are made for differences in temperament (Oriental placidity versus Occidental tenseness) and for inexperience in serving as subjects of metabolism measurements, it is clearly shown that Oriental races in general have a metabolism somewhat lower than that of Caucasians in the United States. The South Indian women in Madras have a very low metabolism (17 per cent below the Caucasian prediction standards), and this is further depressed about 10 per cent during deep sleep. Researches from other laboratories have given varying results. In striking contrast to the low metabolism of the majority of the Oriental races is the high metabolism of the Maya, established in three expeditions to Yucatan. The Indians in Chile have likewise been found by Dr. Pi-Suñer to have a high metabolism. Twenty-four Miao males in Szechwan, China, have been demonstrated by Dr. Kilborn to have a metabolism 16 per cent above the Caucasian prediction standards, accompanied by a low pulse rate of 55 beats per minute. To the well-known factors affecting basal metabolism—weight, height, age, and sex—must now be added a fifth factor, race.

For many years physiologists were dominated by the thought that the heat production of every warm-blooded animal is controlled by the surface area and that for each square meter of surface area the heat lost from the body amounts to 1000 calories per 24 hours. When this law was first enunciated by Rubner,1 half a century ago, no recognition was given to the possibility that there might be differences in the heat production per square meter of surface


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area caused, for example, by differences in sex, age, and race. Rubner apparently believed that age played no role, for he applied his law to young infants. The racial factor likewise was ignored, for Rubner's dogs (the measurements on which formed the basis of his enunciation of the surface area law) ranged in adult weight from 3 to 31 kg. and hence obviously represented pronounced differences in race. With the development of more accurate technique for measurement of the metabolism and with the completion of more careful surveys of the metabolism not only of humans but of many animal species, it has become apparent and today is accepted without question that the energy metabolism of an animal or man is closely associated with certain physiological conditions, particularly muscular activity and digestive activity. The influence of these two factors can be ruled out in the case of humans by studying the individual while lying quietly, 12 hours after the last meal. Under these conditions and provided the body temperature is normal, the metabolism is considered to be the basal metabolism. As data accumulated and more attention was paid to the betterment of technique, it became evident that weight, height, age, and sex also influence the basal metabolism. With advancing age, for example, the heat production of humans decreases, and from the standpoint of sex women have a metabolism about 7 per cent less than that of men of the same size and age. Hence the age, sex, weight, and height of the person whose basal metabolism is measured must be known, in order to compare his metabolism with the normal metabolism standards that have been established for Caucasians.

In a comprehensive survey made with standardized technique, trained observers, and close attention to all the requirements known to be essential for accurate metabolism measurements, the Nutrition Laboratory has studied in cooperation with numerous institutions the basal metabolism of a number of human races, as follows:
Observations have been made on Araucanian Mapuches in the Chilean mountains by Dr. J. Pi-Suñer, who had previously received training at the Nutrition Laboratory and hence measured his subjects with the Nutrition Laboratory technique. Other racial studies by wholly independent investigators have been made since our program was begun, and a considerable amount of experimental material is now available. The results of these investigations have been excellently summarized by Dr. E. F. Du Bois in the third edition of his unique book on basal metabolism.

As several of the Nutrition Laboratory's researches on racial metabolism have been completed in the past few years and the manuscripts reporting the results are now in the hands of the publishers, a summary of the survey made

by this Laboratory is justifiable at this time. In the interest of economy of space we will omit discussion of the relative merits of the several existing methods of comparing the metabolism of individuals of different sizes and will compare our findings only with the Harris-Benedict prediction standard.¹

It early appeared evident that the metabolism of certain races was markedly different from this Caucasian standard. The deviations were by no means in one direction, although in most instances they were below the predictions. The first evidence, in the Nutrition Laboratory's series, of an apparent racial factor in human metabolism was noted with Chinese women studied in the United States who, whether born in China or born of Chinese parents in the United States, had a low heat production compared with Caucasian women. A most careful series of observations made by Professor Eleanor D. Mason shows that young South Indian women (Tamilis and Malayalis) in Madras have a heat production averaging 17 per cent below that of Caucasian women. On the other hand, three metabolism surveys made in Yucatan on the occasion of expeditions of the Carnegie Institution of Washington to Chichen Itza leave no doubt but that male Mayas have a basal metabolism on the average 8 per cent higher than that of white men of the same physical characteristics. Although a deviation of 8 per cent would not be significant for an individual, it is certainly statistically significant when noted with a large group of individuals. An even higher metabolism was shown by the Araucanian Mapuches studied by Dr. Pi-Suñer. With hardly an exception his subjects had a metabolism pronouncedly above the prediction standard, averaging +10 per cent with the 31 men and +15 per cent with the 14 women.

Investigations by other laboratories, especially on the Chinese and Japanese, have shown divergent values, and

further coöperative studies on Chinese with a carefully controlled technique identical with that employed in our observations on other races were instituted by the Nutrition Laboratory. One of our coöperating investigators, Professor H. S. D. Garven, found that 20 Chinese males in Moukden, Manchuria, had a basal metabolism only 3.5 per cent below the Harris-Benedict standard, that is, hardly different from the metabolism of Caucasians. Similarly Professor L. G. Kilborn has found that Chinese in Szechwan (54 males and 14 females) have a metabolism reasonably close to the prediction standards and differing but little from that of a relatively small group of Westerners living in the same region.\(^1\) On the other hand, Kilborn’s observations on 24 male Miao show that this Asiatic race has a strikingly high metabolism, averaging 16 per cent above the Harris-Benedict standards. (See table 1.)

### TABLE 1

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Average for all subjects ........................................... +15.8

In the studies of Garven and Kilborn, owing to the exigencies of what might properly be termed “field work”

\(^1\) Efforts have been made by some investigators to establish norms for Caucasians living in countries other than their native homes. Such studies are complicated by the factor of acclimatization, and further observations are needed on this particular problem.
(the strangeness of the experimental procedures to inexperienced subjects, the lack of time for training the subjects, and the consequent possibility of apprehension on their part), the majority of the subjects were by necessity measured on but one day. For the establishment of the basal metabolism of an individual it is recommended in the United States that measurements be made on at least two and preferably three days, not farther apart than one week but not necessarily consecutive. Experience with most Caucasians and with many individuals of other races has indicated that the metabolism tends to be somewhat higher on the first day of experimenting than on subsequent days when the subject has become more accustomed to the somewhat unusual, although thoroughly comfortable conditions of metabolism measurement. Hence one might think that values obtained with subjects on one day only might give a picture of a basal metabolism somewhat above the true basal level that would be found if the observations were repeated on subsequent days. On the other hand, the phlegmatic calm characteristic of the Oriental may well be expected to play a role even on the first day. In certain of the observations made by Kilborn there was a hint of this. In the research with Dean Stanley D. Wilson of Yenching University, Peiping, China, special attention was given to this particular problem. Each of his 120 subjects was measured on at least three days. On the first day the metabolism was more often lower, rather than higher, than it was on the following days, indicating that the subjects were rapidly conditioned to the experimental procedure and that the measurements were uncomplicated by emotional disturbance. Hence the absence of observations on the second and third days in the series of Garven and Kilborn need not be considered to weaken the interpretation of the results.

Dean Wilson’s subjects comprised 65 men and 55 women, chiefly Chinese and Manchus, and represented both the sedentary and the laboring classes. Their urinary
nitrogen excretion was essentially the same as that noted with Caucasians in the United States, which indicates that these Chinese were not on a low protein diet. The metabolism decreased with age, the average rate of decrease in total 24-hour heat production per year increase in age being about the same for the Chinese males (7 calories per year) as that noted with Caucasian males (7.15 calories). The Chinese women, on the other hand, showed an average rate of decrease in metabolism (5 calories per year) about twice as fast as that noted with Caucasian women (2.29 calories per year). This comparison may be taken as evidence that the Chinese woman ages more rapidly than her Caucasian sister.

The sex difference apparently is more pronounced among Chinese, at least those of sedentary occupation, than among Caucasians. The Chinese women of the sedentary class, studied by Dean Wilson, had a total 24-hour heat production 17 per cent lower than that of sedentary Chinese men of the same weight and age. (See Fig. 1.) This difference in sex among those of the sedentary class is likewise apparent when the results are expressed per kilogram of body weight or per square meter of body surface. With the laboring class, however, the sex difference is more of the order of that found with Caucasians.

With the Manchus of the laboring class, both males and females, lower values for total heat production were found than with Chinese laborers of the same weight, age, and sex. This finding suggests that there may be a racial difference in metabolism within the Oriental race itself, a suggestion likewise offered by Kilborn's observation that the Miaoos have a very high metabolism. There were no marked differences in the average ages, weights, heights, pulse rates, respiration rates, and pelidisi (index of nutrition) of the Chinese and Manchus, and thus far there is no factor to explain the lower metabolism of the Manchus other than race. The Chinese of the laboring class, of both
sexes, had a metabolism higher than did those of the sedentary class.

In all instances the metabolism of Dean Wilson's Chi-

![Graph showing heat production per 24 hours referred to body weight for Chinese men and women of the sedentary class. The plotted points are for Chinese women of the sedentary class, those 40 years of age and over being indicated by the ages in years written against the plotted points. The two full-line curves represent the trend of the total heat production with increasing weight for the two groups of women under and over 40 years of age. The two broken-line curves indicate the trend of metabolism with increasing weight for Chinese men of the sedentary class.]

nese subjects was lower than that of Caucasians of corresponding physical characteristics. Furthermore the racial difference was greater with the male than with the
female Chinese, in contrast to most of the findings on Japanese.¹

Another striking feature of these several racial studies is that with some of the groups of subjects the pulse rates were abnormally low when compared with those noted with Caucasians, and yet these low rates were not invariably accompanied by low metabolism. In fact, in some instances low pulse rates were associated with unusually high metabolism. Different levels for the normal pulse rates of the average American man and woman have been recorded in text books. The 136 Caucasian men studied by the Nutrition Laboratory had an average pulse rate of 61 beats and the 103 women, 69 beats per minute. Wilson's groups of Chinese, having low basal metabolism, had pulse rates not far different from these for Caucasians, the average pulse rate of the Chinese men being 61 beats and of the Chinese women, 63 beats. Among the 65 males, however, there were 4 individuals who had heart rates lower than 50 beats per minute; among the 136 Caucasian men in the Nutrition Laboratory series four also had this low rate. The male Chinese in the Kilborn series had an average rate of 60 beats or essentially the same as the Caucasian rate, and their metabolism was much the same. Seven of these males, however, had pulse rates below 50 beats. The 14 Chinese women studied by Kilborn had an average pulse rate of 66 beats per minute or lower than the average shown by the Caucasian women, and three of these Chinese women had rates of 58 beats or below, which is a very low rate for women. Yet their metabolism was reasonably close to the prediction standards. Similarly the male Chinese in the Garven series, whose metabolism did not differ greatly from prediction, had an average pulse rate of 53 beats or lower than that of the 136 Caucasian men. Among the racial groups showing metabolism pronouncedly higher

¹ Some Japanese investigators have interpreted the finding of a low basal metabolism in their race as an index of inferiority in the race. The Nutrition Laboratory believes that such an interpretation is not correct.
than the Caucasian standards, the pulse rates were either unusually low or no higher than the Caucasian rate. Thus the 24 male Miaoos in the Kilborn series had an average pulse rate of 55 beats and seven in this group had rates below 50 beats. The Maya Indians likewise had low heart rates, in many instances as low as 40 or 45 beats per minute, notwithstanding their very high metabolism. The high metabolism of the Indians studied by Pi-Suñer was not associated either with a high or a low pulse rate, for the average rate of his male subjects was 60 beats and of his female subjects 67 beats.

The Nutrition Laboratory's racial survey has clearly established that there are marked differences in the basal metabolism of human races. The importance of studying the causes of these metabolic differences cannot be overestimated. The absence of perfect correlation between the pulse rate and the metabolism is challenging. Factors of climate and diet are, for the most part, ruled out, and there are no apparent external conditions that seem to be causative. There remains to be studied, however, the relation between the basal metabolism, the blood chemistry, and particularly the endocrine balance. Perhaps one of the most outstanding features of the racial survey thus far is that there seems to be little relationship between the basal metabolic level and the intellectual or physical ability.
ON THE SIGNIFICANCE OF CERTAIN ORIENTAL PLANT NAMES IN RELATION TO INTRODUCED SPECIES

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Director of the Arnold Arboretum

(Read April 22, 1937)

ABSTRACT

Following a brief consideration of the significance of the use of plant names in the Indo-Malaysian-Polynesian regions attention is called to the fact that the comparative philologists have made little or no use of the extensive data that are available, although it is manifest that in many cases the name of a particular plant has been transmitted with the plant itself from one people to another, while in other cases early expanding peoples have apparently carried the name and applied it to indigenous species of wide natural distribution. Extensive data are presented on the significance of the name in the case of two pan-tropic manifestly man-distributed species, the coconut and the frangipanni, and one pan-tropic species, Hibiscus tiliaceus, of natural distribution, to emphasize the points mentioned above. In the case of the coconut the conclusions are diametrically opposed to Cook's theory of an American origin, in that apparently the spread of this economic palm was from west to east rather than from east to west, and that it probably did not occur in America until after it was introduced first by the Portuguese and a little later by the Spaniards. The great center of development in its local names is in the Old World tropics, with the niu series in use from Madagascar, through Malaysia to the eastern limits of Polynesia. It is reasonably apparent that here the name was spread with the plant itself. The frangipanni is native of Mexico. One of the crude plant-forms depicted on the 12th century temple of Borobudur, Java, has been identified as representing this plant; if this identification be correct it would mean its presence in Java in the 12th century. The philological, historical, and botanical evidence is wholly opposed to this assumption, and the conclusion drawn is that the crude Borobudur figure does not represent the frangipanni, and that the plant was first introduced into the Philippines by the Spaniards late in the sixteenth or early in the seventeenth century, and from the Philippines was soon transmitted to other parts of the Old World tropics, and with it, to a certain degree went its Nahautl name. Hibiscus tiliaceus was selected as a plant of natural pan-tropic distribution. A study of its local names indicates that those in the hao-bao-bago series were transmitted by an expanding people from Madagascar and India through Malaysia and Polynesia; and as new lands were occupied the name was apparently applied to the plant there found as an indigenous species. It is noted further that this same series of names is applied over the entire region to a number of utterly different and botanically not closely allied spe-
cies, but in each case plants that have strong bast fibers as does the Hibiscus. The assumption is that in this case the original form meant bast, and was thus applied to unrelated plants that produced strong bast fibers.

Following these three cases is a brief discussion of the significance of the Sanskrit, Chinese, and Nahautl plant names in Malaysia and in the Philippines. The conclusions in reference to the first two are that they were introduced into the Archipelago, with the plants themselves, by Indian and Chinese colonizing or trading peoples beginning perhaps 2000 to 2500 years ago, and that the names have persisted practically unchanged since their introduction. The Nahautl series represent plants introduced from Mexico by the Spaniards following 1565 via the old Manila-Acapulco galleon route that persisted for about 250 years. The Sanskrit-Chinese series represent sub-prehistoric introductions from Asia, the Nahautl series introductions from America within the period of modern history, the one paralleling the other.

There is a great wealth of philological material in the aboriginal plant names recorded in association with the Latin binomials used under the binary system to designate individual plant species. A high percentage of such names, within limited geographic areas, particularly in such parts of the world as India, Malaysia, including the Philippines, and Polynesia, are really safe guides to the identification of genera, species, and even varieties. Very many of them are even more fixed, as designating certain definite units, than are many of our Latin binomials. They have been used for many centuries to indicate definite species and will be used for many centuries to come, regardless of the vagaries of the binominal system. They are not

1 Some of the sources of information of Indian, Malaysian, and Philippine plant names are given below. Most of the Polynesian and Micronesian names cited were compiled from my manuscript card catalogue of Polynesian plants.


changed because of priority, because of varying conceptions of what constitutes a genus or a species, or because of the personal idiosyncrasies of this or that botanist, but persist generation after generation as definite designations for definite plant forms; some are of very local application, others are applied to the same species over a very wide geographic range.

Comparative philologists have made little use of these data, probably for the reason that the average worker in that field has lacked the intensive botanical knowledge properly to select significant species for study among the tens of thousands of described ones, and perhaps also because they have been repelled by the very large number of recorded native plant names. In the general field of philology, for the purposes of comparison, it has been simpler, and perhaps just as effective from the standpoint of those interested in the relationships of languages, to compile lists of conspicuous objects such as the sun, moon, stars, water, parts of the human body, numerals, certain verbs and other words.

In this day of specialization, it is scarcely to be expected that many botanists will delve into the intricacies of anthropology, archeology, and comparative philology, or that many specialists in these fields will master even the rudiments of systematic botany. Yet individuals working in these diverse fields can be of mutual assistance to each other. Cases may be cited where taxonomists have misinterpreted the botanical evidence as to the place of origin of cultivated species, such as in accrediting the cultivated cucurbitas to Asia; yet the archeological data absolutely supports the phytogeographical evidence that these plants originated in America. One erroneously assumed that tobacco must have been native of Africa and "proved" that this was the case, in spite of the botanical evidence that is overwhelmingly in support of its American origin. Another insisted that the smoking of tobacco originated independently in New Guinea on the basis of a native species
of tobacco, yet there is no botanical evidence that any tobacco species occurred in New Guinea before the period of European contacts, while the one species that does grow there, *Nicotiana tabacum*, is of hybrid and indubitably of American origin. One "proves" that the coconut was a native of the New World and that it must have been transmitted across the Pacific at a very early time by man, and another definitely shows that the whole concept and argument was erroneous and that in all probability the coconut did not even occur in tropical America in pre-Columbian times, and that it was first introduced by the Portuguese into Brazil and a little later by the Spaniards into Mexico.

The average botanist usually has no knowledge of and little interest in comparative philology, or is too busy with the multitudinous problems within his own field to master the rudiments of it or to assemble and to attempt to interpret the significance of this or that series of cognate forms even among those names applied to the basic cultivated species. He may realize that in all countries where agriculture is practiced that many of the cultivated species, and the weeds of cultivation, are exotics. He may realize further that some of these plants were disseminated at a relatively early date in the history of the human race, many of them in prehistoric times, others within the historic period, particularly following the expansion of the European colonizing nations after the middle ages. He often does not fully realize that the boundaries of the one hemisphere or the other were the actual limits to the diffusion of cultivated plants originating in America or in Eurasia and that a general interchange of these economic plants between the two hemispheres did not occur until after the first circumnavigation of the globe by Magellan in 1520.

With no claims to other than a very superficial knowledge of comparative philology and with no training whatever in this field, I have had the temerity to compile certain lists of native plant names with the conviction that in this field we have a source of information that will yield impor-
tant results to both the philologist and the botanist, and more particularly to those individuals who are interested in the origin, history, and the approximate time of dispersal, and the early limits to dissemination of cultivated plants. I have done this with the full realization that I may be accused of selecting a subject regarding which I know nothing, and again because I may err in drawing conclusions that perhaps a professional comparative philologist might not accept. In the utilization of the philological data one may misinterpret the significance of this or that name, and occasionally one may include words or forms of words that perhaps should not be cited.

If one examines the long lists of plant names recorded from India, Malaysia, the Philippines, Micronesia, and Polynesia, one is impressed with the fact that for the endemic species and those of limited geographic distribution the local names are mostly what we may designate as autochthonous; that is, names characteristic of local languages or dialects and used specifically to designate this or that natural group, usually a species, sometimes applied in a generic sense, sometimes even used in a varietal sense. Sometimes one finds cognate forms of a single word, not infrequently with myriads of variants, very widely used to designate species of manifestly natural distribution. Again one notes that another term, or cognate forms of it, may be used over vast areas—and we may cite Madagascar, across Malaysia and Polynesia to Hawaii and the Marquesas Islands—to designate certain universally distributed cultivated species, the plant manifestly for the most part man-distributed. In these two cases it seems only reasonable to assume that for the non-cultivated plant of natural wide distribution, the name was carried by an early expanding people and automatically applied to the species, known to them in their original home land, as it was observed in the new lands occupied by them. With the man-distributed cultivated plants it seems to be likewise logical to assume that the plant itself, with its original name, was actually
distributed by an expanding people, or the plant and its name was passed from one group to another by diffusion.

In addition to the three categories mentioned above one notes still another, that is, a series of borrowed or adapted names applied to introduced species. And when a cultivated or naturalized plant is found in a region remote from its original home such names become distinctly significant as to when, by whom, and how such plants were originally introduced. Borrowed names again fall into two categories, those actually taken from strictly foreign languages, some in historic, some in pre-historic times, and those adapted from the name of some other species, usually by the addition of modifying words, because of some similarity or assumed similarity between the introduced plant and a native or earlier introduced one.

To illustrate these points data have been compiled on the local names of the coconut (Cocos nucifera Linn.) from Madagascar to Hawaii, Hibiscus tiliaceus Linn., over the same range, and the frangipanni (Plumeria acuminata Ait.) from Mexico, its original home, and from the Indo-Malaysian region where it is a common introduced plant. Supplementing these data and further to support the idea that native plant names are worthy of a more intensive study than they have hitherto received, I have compiled lists of Sanskritic, Chinese, and Nahautl (Mexican) names currently used in the Philippines and in Malaysia to designate certain definite and for the most part introduced and cultivated species.

To explain the occurrence of these names thousands of miles away from their places of origin certain historical data are given appertaining to contacts in ancient and in comparatively recent times. These data are presented for what they are worth. The conclusions are my own, subject to corrections and extensions that others may suggest. Too frequently comparative data, of the type under discussion, have been used to bolster up often illogical pre-conceived theories. Merely because there are Sanskritic,
Chinese, and Nahautl plant names in current use in Mal- 

diasia and in the Philippines is no evidence whatever of 
affinities between the languages of India, China, and Mex- 
ico, with those of Malaysia, for there are no genetic rela-
tionships between the diverse languages involved. Such 
names in the insular area discussed are merely borrowed 
one, or introduced with the plants themselves. It is, 
however, important that we should know approximately 
when they were introduced. Again merely because there 
may be occasional similarities between certain plant names 
used in America with some used in the Old World is, in 
most cases, little reason for concluding that there were 
ancient contacts between the early peoples in these widely 
separated regions; sometimes similarities are merely acci-
dental, sometimes they may be explained by historical con-
tacts.

As evidence the comparative philology of plant names 
must be used with restraint and caution. When, however, 
there is a general agreement in plant names with other 
data, historical, biological, and otherwise, then we can ac-
cept the evidence as supporting this or that conclusion. 
Factors to be considered are what is known regarding ex-
panding and colonizing peoples in ancient and in relatively 
modern times, what light modern archeology throws on the 
problem or problems, what we may learn from the annals 
of history and exploration, and what the taxonomy and 
phytogeography of plants in general teaches us. We are 
not justified when we merely select certain facts that sup-
port a preconceived theory and ignore those data that are 
opposed to it. To be effective and convincing conclusions 
must be logical ones. When illogical arguments are sup-
ported by citing only those data favoring the contention or 
contentions and ignoring the known facts that are opposed 
to the theory, the well known method of half truths, then 
we should look on such papers as contributions to the field 
of imaginative fiction, perhaps interesting as fiction but 
not to be considered seriously. In general philological evi-
dence is of distinct value when the conclusions that we may draw from a consideration of such data are in conformity with the known biological facts of plant distribution.

The Coconut

The actual place of origin of this plant is somewhat of a mystery although it is indubitably native of some part of the Old World tropics. The tribe or subfamily, Cocoinae, to which Cocos belongs, is predominantly characteristic of tropical America, with the genera Attalea, Maximiliana, Diplothemium, Jubaea, Orbignya, and Arecastrum, Butia, and Glaziovia, narrow segregates from Cocos in tropical America, Eleis in tropical Africa, Jubaeopsis in southeastern Africa, and Cocos, now pantropic in cultivation. The reference of numerous native tropical American species of Arecastrum, Butia, and Glaziovia to Cocos has confused the issue. Assuming that Bentham and Hooker’s and Drude’s concept of Cocos as a genus, sensu latoire, be correct, then it would be surprising if all the wild species of a considerable genus should be natives of tropical America, and the one widely cultivated species dominant in the Old World and not an indigenous plant in America. This classification and this distribution was apparently the chief basis of Cook’s illogical attempt to prove the American origin of Cocos nucifera Linn. It is apparent that he accepted current classification as correct.

Beccari called attention to the fact that Jubaeopsis caf- fra Beec. of southeastern Africa has many more affinities with Cocos nucifera Linn. than has any other known palm among those hitherto referred by authors to the genus Cocos. After carefully examining the characters of the groups involved he expressed the opinion that it would be better to regard Arecastrum, Butia, and Glaziovia as of


generic rank, rather than as subgenera of Cocos; Arecas-
trum with two or, at most, few species, Butia with ten to
twelve or more species, and Glaziova with more than forty
species, all in tropical America. With this disposition of
the American so-called species of Cocos, the latter genus
stands as monotypic, with a single species, Cocos nucifera
Linn., and with its closest ally the monotypic Jubaopsis
confined to southeastern Africa. In favor of an Old World
origin of Cocos is also Hill's record of fossil Cocos nuts
from New Zealand. For an extensive and critical con-
sideration of the problem of American versus Old World
origin of the coconut see Chiovenda.2

Cook's argument in general was that the coconut origin-
ated in some part of tropical America; that it was an
inland species, which is incredible; and that it was an early
introduction into Polynesia by prehistoric voyagers from
America, which is also incredible and for which absolutely
no proof exists. His general statement is a convincing one
in support of his contention, but unfortunately he over-
stressed the factors that tended to support his preconceived
theory and understressed or ignored those that were op-
posed to it.

Without dissecting Cook's argument in detail, I wish to
discuss two points other than the philological one. As
against possible dissemination of the coconut by ocean cur-
rents he argues that the chances are hundreds to one that
coconuts falling into the water will be thrown back imme-
diately upon their own coast like other objects floating in
the surf and definitely says: 'High waves or tides, instead
of floating shore debris away, merely carries it farther in-
land, as everybody familiar with seacoasts know' (italics
mine). What about an off-shore wind and an outflowing
tide? What about the situation at the ends or on the lee
of small islands? What about estuarine conditions with

1 Hill, A. W. The Original Home and Mode of Dispersal of the Coconut.

2 Chiovenda, E. La culla del Cocco. Contributo alla ricerca della patria
a strong outflowing current? On the basis of Cook's statement how could one possibly account for the wide distribution of strand plants that are adapted to dissemination by floating seeds or fruits? How could seeds and fruits, including coconuts, and floating debris in abundance, reach isolated sand bars and slightly raised reefs that support no vegetation because at times they are entirely flooded or swept by heavy waves? How about floating debris in abundance that one notes at certain times far from land in such a region as the Malay Archipelago?

The categorical statement is made that numerous economic plants including the sweet potato, the bottle gourd (Lagenaria), the true gourd (Cucurbita), cowhage (Mucuna), yam bean (Pachyrhizus), and one or more species of yams (Dioscorea), all probably of American origin, existed in Polynesia and in the Malay region in prehistoric times. For the sake or argument we may admit that these plants were native of some part of America, but only one, the unimportant bottle gourd (Lagenaria), seems definitely to have been of pantropic distribution in prehistoric times. It is probable, or at least possible, that the sweet potato reached Polynesia from America in pre-Magellan times. Burkill, however, effectively shows that on the basis of its Malay names, which are all borrowed ones, that it is a late introduction into the Malay Archipelago. The Old World true gourd records were undoubtedly based on erroneous determinations or confusion as between the true gourd and some other cucurbitaceous plant. The cowhage, i.e., the form with stinging hairs on its pods, is not an edible or even a cultivated plant. I know of no records that would indicate the occurrence of the yam bean in the Old World until after it was introduced there by the Europeans, nor do I know of any data that would prove that any cultivated yam had this distribution in prehistoric times. The "nu-

merous economic plants” of prehistoric pantropic distribution, stressed by Cook, seem to be conspicuous by their absence. In this connection Cook states that the banana was introduced into America in prehistoric times, but the bulk of the evidence is utterly opposed to this assumption, and the probability is that it was first introduced by the Portuguese via the Cape Verde Islands.

It is admitted that some botanists argue for at least a limited interchange of neotropic and palæotropic plants across the Pacific in pre-Magellan times. However, it is notable that the arguments supporting the contention that the banana and the coconut were of pre-Columbian occurrence in America are of the most nebulous character and are far from convincing, and the suggested pre-Columbian occurrence of Old World species in America generally stop with these two plants. If two species were so introduced, why not many more?

It has been suggested that various plants of American origin also occurred in the Polynesian-Malaysian region in pre-Magellan times, but here again there is little tangible evidence to support such contentions. In general in reference to these supposedly early plant immigrants from America to Malaysia one is impressed by the relative unimportance of the species for which such claims are made. Why should a tree useful only for ornamental purposes, such as the Plumeria, or one of very slight economic importance, such as the Ceiba, be selected for dissemination by early or even fairly late civilized man when one would naturally expect to find important food plants preferred, such as maize, the garden, field, and lima beans, yautia, tomato, peanut, cassava, squash, pumpkin, pepper, and others? These in competition with the suggested non-economics failed to make the passage.

It is difficult to prove just when certain neotropical plants reached the palæotropical regions and vice versa. It is, however, manifest that most if not all of the really important species failed to make the passage until after the
Europeans commenced to make trans-Pacific voyages. I know of no single case of supposedly pre-Magellan introduction one way or the other that is susceptible of definite proof. Conclusions based on early published fragmentary records are often of doubtful significance because of the simple fact that positive identification is usually impossible from the totally inadequate data recorded. Misidentification is more apt to be the explanation of the supposed occurrence of this or that cultivated plant far “out of range” at an early date.

Cook, apparently to support a preconceived theory of American origin of agriculture and a very early dissemination of economic plants across Polynesia by ocean-going peoples sailing from America, claims that a considerable number of American plants in the Old World support this idea, but my general conclusion is that American plants in the Polynesian-Malaysian regions in pre-Magellan times were more conspicuous by their absence than by their presence in the Old World tropics.

Cook’s philological argument supporting an assumed American origin of the coconut is exceedingly weak. How should we interpret these data? He admitted that the lack of native American names had been accepted as proof that the palm could not have existed in America before the arrival of the Spaniards, and then proceeds to set up the remarkable theory that probably the word “coco” itself is of American origin. It is true that the word is used in America for designating certain plants, but for nothing in aboriginal languages at all resembling the coconut. That the word is a favorite plant name in several Central American languages applied particularly to plants that have bulbs or bulbous roots, is no argument in support of this name being applied to such a totally different plant as the coconut palm. What weight should be placed on the suggestion that a Tupi Indian name in Brazil, nha, there applied to the Brazil nut tree, Bertholetia excelsa, has anything whatever to do with the Polynesian name niu applied
to the coconut? And yet Cook makes the most extraordinary statement: "The agreement and distribution of the Polynesian, Melanesian, Malayan and other Oriental names of the coconut indicates a westward migration for the race which introduced it in the western Pacific."

Contrasted to the situation in America the names of the coconut in the Old World tropics are myriad, but I see no reason for considering that any of them are to be associated in any way with vernacular names in America, except as the word coco itself has been adopted in most parts of America, introduced by the Portuguese and the Spaniards with the plant itself. In this connection Bartlett gives excellent reasons for believing that the word coco is not, as generally supposed, a word of European origin from the Portuguese coco (a bugbear or an ugly mask), but that it was perhaps adopted by the early Portuguese explorers in the Orient from its Turkish name cock-indi more or less spread in Malaysia by the Arab traders, quoting Rumphius' opinion, written 1670, or somewhat earlier, in support of his contention. Forms actually cited in the early European literature appertaining to the coconut in the Old World tropics include coquos and quoquos, both plural, coco, and coquo. Forms actually used in various parts of Malaysia at an early date include kokær, coker, igo, calucu, and lalucco, the last three from Rumphius; and I am inclined to agree with Bartlett in his expressed opinion that these last Indonesian names are not modern adaptations of the Europeanized coco, for they were recorded not later than 1670.

The various words used to designate the coconut in India and Malaysia are very numerous, but fall into several distinct categories of apparently unrelated names. It is entirely reasonable to suppose that the most widely dispersed one, the niue-nia-niu-niog series, is the oldest. In various forms this extends from Madagascar to parts of India, through Malaysia and the Philippines, Micronesia,

and Melanesia, to the extreme eastern limits of Polynesia, Hawaii and the Marquesas Islands. Without attempting to list all the variants, or to indicate always where the several forms are used, the following list is impressive. Nyiur, nyor, niyu, nia (Malay Peninsula); nuir, niue (Sumatra); njior, njior, njor, ijer, enhor (Java); niel (Ceram); njæ (Bali), and numerous other variants used in various parts of the Malay Archipelago such as njær, niæ, njær, njol, nikwel, ninel, ninelo, nio, niæ, niæra, niæi, niwe, niwel, niwer, njejong, njejor, nhir, nihwe, njæh, njor, noa, næra, newolo, noö, noör, noöra, noro, æ, ær, ohi, onjæ, etc. Going farther afield we find wau-niù and vau-niù in Madagascar (the name probably introduced with the plant itself by invading Indonesian peoples). In India narel, nariyal, na-riel, narikel, nariyaland, narikelala, and numerous other forms apparently derived from the Sanskritic narjil. In New Guinea naæ ajo is recorded, in the New Hebrides maru, and in New Caledonia nu. In the Philippines the coconut is universally known from north to south as niog. And in various parts of Polynesia we find again the constant occurrence of this same root in such names as niu (Fiji, Samoa, Tahiti, Hawaii, Yap, Makatea), nia (Tahiti), nu (Truk, Rarotonga), ni (Ponape), nius (Palau), niyog (Guam), indicating a reasonably universal use of slight variants of one name all over Micronesia and Polynesia. I have no records of radically different names for this palm from any part of the Micronesian-Polynesian region.

There are several other series of coconut names that apparently have nothing to do with the niue-nia-niu-niog series, which it does not seem necessary to discuss in detail here. They are the halambir-kelambir series with such variants as karambil, krambel, krambil, karambie, ketjambil in the Malay Peninsula, Sumatra, Java, and neighboring small islands. The kalapa series with such variants as klapa, kelapa, kelapo, is largely used in Java. Other series could be built up from the Persian pol, the Singalese ong, and the pankoi-peirol-pænæ series in Celebes and neighbor-
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ing islands. In all these, other than the niue-nia-niu-niog series, the use of the names is invariably restricted to limited geographic areas, rather clearly indicating that these names became current later than the niu series which extends from Madagascar to India through Malaysia and the Philippines to the eastern limits of Polynesia. Bartlett’s conclusion is that in Malaysia the njijær series is the oldest, spread by early Indonesian peoples, that the halambir-kelambir series is next, having come into use much later than the njijær series, and that the kalapa series is still younger than the halambir-kelambir one.

In this discussion I have not considered it necessary to list all the recorded oriental names of the coconut. De Clercq\(^1\) records about 115 for the Netherlands East Indies with about 125 additional ones for recognized varieties, Heyne\(^2\) in 1927 lists about 180 names from the same region, while Watt\(^3\) records nearly 100 different names for British India. For tropical America Cook was able to record only ko-ko (which of course is Sp. coco), sura no and sia ua in Costa Rica, other than the ubiquitous coco introduced by the Spaniards and the Portuguese, while its even fewer Brazilian aboriginal names appear merely to be transfers of names of native palms to the coconut palm. Were the plant native of any part of tropical America or an early introduction from the Old World we might logically expect to find a very considerable number of vernacular names actually in use for it.

In view of the great paucity of colloquial names of the coconut in tropical America, and the enormous number in the Indo-Malaysian region, from the standpoint of comparative philology alone, ignoring other factors, the only conclusion that I can draw is diametrically opposed to that of Cook. It seems clear that the distribution of the coconut

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\(^3\) Watt, G. *A Dictionary of the Plant Products of India*, 2: 415. 1889.
was not from east to west, but definitely from west to east; that the name was disseminated with the plant itself, and that early migratory peoples were largely responsible for the spread of this palm in the Old World tropics, even as modern man was probably responsible for its introduction into tropical America, or at least for its wide distribution here. Ocean dispersal has also been a factor. The palm can maintain itself in favorable places without the aid of man. We may even admit its possible arrival on the west coast of tropical America in pre-Columbian times, but this does not alter the picture. If it were introduced there by natural means or by man, its introduction must have been consummated not long before the arrival of the Europeans; for if of ancient introduction, it would certainly have been quickly distributed by the natives by diffusion even as other economic plants of American origin were widely distributed within America in prehistoric times. One should not forget that invariably a plant of great economic importance, once introduced into a new region to which it is adapted, spreads with remarkable rapidity as witnessed by the fact that within twenty-five years after its introduction into Spain maize or Indian corn had reached western China via the then little travelled overland trade route from Asia Minor, doubtless the one actually travelled by Marco Polo.

The center of the great variation of the coconut, with the establishment of numerous recognizable varieties, and the center of the great development of its specific and varietal colloquial names is the Indo-Malaysian region of the Old World tropics.

*Plumeria acuminata* Aiton

The genus *Plumeria*, with about 100 recognized species, is a group strictly confined to tropical America except as certain species have been introduced into other tropical countries for ornamental purposes. Among the plants figured by Plete from the bas reliefs of the twelfth century temple of Borobudur, Java, is one that Bakhuizen van den
Brink\(^1\) in 1931 took to represent a *Plumeria*, although Cammerloher\(^2\) does not include it in his slightly earlier consideration of the plant forms there depicted. Two years later Bakhuizen van den Brink\(^3\) positively identified Plete's figure of the plant as the species now commonly known in many parts of Malaysia as *kambodja = Plumeria acuminata* Ait. If his identification be correct, all my argument is of no value for this would mean that this American plant occurred in Java in the twelfth century. His argument, other than his positive identification of the crude, greatly conventionalized bas relief on the Borobudur temple, is in general that the Malay name *kambodja* and those in the *tjempaka* series are Sanskritic in origin, which is manifestly true, and that such names as *bonga gulong tsjutsju, culong tsjutsju* and *culu tsjutsju* are of Malay origin, not corruptions of its Nahautl name *kalachuche* as modified in the Philippines. *Bonga* is of course the Malay name for flower. The other parts of these words he thinks may have been derived from the Malay *gelæng =* head dress or wreath, or *gælæng =* to twist or roll up, or *golong =* roll, tube, and *sæsæ =* milk!

Intrigued by this explanation, I attempted to confirm its possibility by scanning all the entries in Heyne's work wherein are listed approximately 20,000 Malaysian plant names. There are many of these wherein the word *sæsæ* forms a part of the word, but in all of these I have detected no variants even approaching *tsjutsju*. In general the form *sæsæ* remains unchanged whether used as the first or the second part of a plant name. In the first case we find *sæsæ lopek* (*Codium tomentosum* Stackh.), *sæsæ mænding*

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(Campananæa javanica Blume), sæsæ perada (Globba sp.), sæsæ pæka (Fatoua pilosa Gaudich.), and sæsæ rimæ (Polystictus sacer Fries). In the second case, for plants mostly with milky sap, we find geteh sæsæ, djintahan sæsæ, gelang sæsæ and gitan sæsæ (Willughbeia firma Blume), kolonsæsæ (Calotropis gigantea R. Br.), kalansæsæan (Plumeria acuminata Ait.), balam sæsæ (Palaquium acuminatum Burck), kajæ sæsæ (Cerbera manghas Linn.), kambang sæsæ (Tabernemontana divaricata Linn.), sira- wan sæsæ (Arcangelisæa flava Merr.), and madang sæsæ (Ficus alba Blume). Applied to plants without milky sap, but here in the sense of sæsæ = breast or nipple, as the same root susu is applied in the Philippines in such plant names as susong kalabao, suso-susoyan, suso koyi, suson damulag, and susong kabayo, we find in Malaysia arenj sæsæ mendeng (Eleagnus javanica Blume), pasang sæsæ (Quercus teysmannii Blume), djambæ sæsæ (Eugenia malaccensis Linn., Psidium guajava Linn.), bawali sæsæ (Passiflora laurifolia Linn.) and sirikadja sæsæ (Annona muricata Linn.). Variants as single words are sæsæan, sæsæadæ, sæsæak, sæsæen, sæsæakan, and sæsæerae, and sæsong. No better success is met in an attempt to locate gelæng, gælæng, and golong, and the supposed derivatives of these words culong (kulung) and culu (kulu), as parts of plant names in Heyne's long list. In view of this evidence I am unable to accept Bakhuizen van den Brink's suggested origin of gulong tsjutsju, culong tsjutsju and culu tsjutsju as corruptions of pure Malay words (p. 127), but rather consider that they are merely corruptions of the Mexican-Philippine kalachuche.

The frangipanni, temple flower, or graveyard flower, Plumeria acutifolia Ait., is now universally distributed in cultivation in the tropics of both hemispheres. It is indubitably a native of Mexico. What light does comparative philology throw on its introduction into the Old World?

In Java the tree is known by various local names including kambodja, sambodja, sembodja, tjampaka bakul, tjam-
paka maldja, tjampaka sabakul, tjompaka bakul, tjompaka maldja, and tjompaka sabakul. Kambodja as a plant name was derived from Cambodia, a place name, the latter derived from Kambu, mythical founder of the Khmer race; it is certainly Sanskritic in origin. It seems to be evident that it was applied to the Plumeria in various parts of Malaysia under the impression that the plant came from Cambodia; and indeed it may well have been introduced into some parts of Malaysia from that country although it is not native there, nor is there any evidence of its ancient cultivation in Cambodia; the only recorded names I have been able to locate for Indo-china are co koc don, cay dai, and tiampa (the latter a variant of champaka). In the Philippines we find a parallel in the application of the names akapulko and kapurko to Cassia alata Linn., the name derived from the Mexican city of Acapulco whence the plant was introduced into the Philippines. Similarly, such Philippine plant names as kauayan sina, bunga china, and kapas sanglai (from China, or of the Chinese) indicate the belief, not always true, that the plants to which they appertain came from China. Such names are common in Malaysia, Heyne listing perhaps one hundred. Thus for plants of strictly American origin we find manila and katjang manila (Arachis hypogaea Linn.), sawo manila (Achras zapota Linn.), nangka manila (Annona muricata Linn.), ki manila (Cassia alata Linn.); kalè djawa (djawa = Java), øenti djawa, mærø djawa (Carica papaya Linn.), øbi djawa, lame djawa (Ipomœa batatas Poir.), panda djawa (Ananas comosa Merr.), pandang djawa (Agave cantala Roxb.), katabi djawa, sabrang djawa, kentang djawa (Solanum tuberosum Linn.), srikaja djawa (Annona muricata Linn.), tere djawa (Zea mays Linn.), kænjit djawa (Bixa orellana Linn.); ketepang tjina (Cassia alata Linn.), lehiø tjina, njihø tjina (Arachis hypogæa Linn.), øbi tjina (Ipomœa batatas Poir.), medjawik tjina (Maranta arundinacea Linn.), and petenj tjina (Leuœena glauca Benth.). Not one of these names indicates the true geo-
graphic origin of the plant concerned. Other geographic names used as parts of plant names include malaka (Malacca), timor (Timor), bali (Bali), bandaeng (Bandong), ambon (Amboina), etc., as well as the words kastela, kastila, kastera, katila, and hale kastila (Castile), indicating some connection or assumed connection with the Spaniards.

It seems entirely safe to conclude that the application of the name kambodja to Plumeria in Malaysia only indicates a plant of exotic origin that was merely thought to have come from Cambodia, but not necessarily one of ancient introduction. It is interesting in this connection to note that slight variants of this word are used for other introduced species wholly unrelated to Plumeria, such as kambodjo, karambodja, and karambodjo (Citrullus vulgaris Schrad.), and kampadja (Carica papaya Linn.).

The tjampaka series, in my opinion, merely represent borrowed names adopted, with modifiers, from the Sanskritic champaka, as was the case in India (p. 133). There is no evidence that kambodja was ever used by Sanskritic peoples as a plant name.

In the Philippines where the Plumeria was introduced from Mexico by the Spaniards in the sixteenth or early in the seventeenth century, it is very widely known as kalachuche; some of the Philippine variants are kalachuchi, kalasusi, kalasutsi, kalanoche, karachucha, kalasasi, kalatsutsi, and karatuche. These are all slightly modified forms of the ancient Nahauatl name kalachuchl, in Mexico. The name was introduced into the Philippines by the Spaniards with the plant itself. Among the forms used in Mexico today are such variants as cacaloxochitl, cacalosuchil, cacaloxochipe, jacaclosuchil, suchlacahue, and tizaxochitl, all cognate forms of its Nahauatl name, even as the lesser variants in the Philippines were derived from the same source.

Mercado,1 writing in the last third of the seventeenth century, categorically states under calachuche "Aunque

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este árbol no es de esta tierra, sino traido de la Nueva España, hay ya tanto aqui que no hay pueblo donde no le haya." This was perhaps written not more than a hundred years after the plant was introduced into the Philippines and by an individual born in the Philippines in 1640 and who doubtless was familiar with the plant from his early youth. The Spaniards commenced active colonization of the Philippines, operating from Mexico, in 1565. Camell 1 slightly later merely describes it as Cacaloxochitl Mexicana, but does not definitely state that it was Philippine although we may infer that this was the case, otherwise he hardly would have described it in a work on Philippine trees. Rumphius 2 states regarding this plant as it occurred in Amboina: "Videtur primum ex Cambodja fuisse deductus, unde & nomen obtinuit, ac forte primum in Ternatam per Sinenses mercatores, unde ante paucos quosdam annos in Amboinam delatus est, in Java tamen alisque locis itidem obcurrir. Alii dicunt in Manilhis quoque repe-riri, atque inde cum subsequenti flore [Flos manilhanus = Tabernaemontana divaricata R. Br.] in Ternatam fuisse deductum." This is a most significant statement. It should be remembered that the Spaniards captured Ternate and Tidore in 1609 and maintained control of these islands over fifty years, operating from Manila. Compare also Rumphius’ statement that the plant was then (about 1670) a recent introduction into Amboina with Mercado’s statement that in the last half of this same century there was scarcely a town in the Philippines where the plant did not occur.

Heyne records about forty different names used in Malaysia for this species, many of them in the kambodja and tjempaka series, and all or mostly borrowed ones. It is significant that in Celebes, Ternate, and in Ceram, islands

1 Camell, G. J. Descriptiones fruticum & arborum Luzonis. In Ray, J. Hist. Pl., 3: App. 79. 1704. (This manuscript was sent to Petiver in 1701, and was probably written in the last decade of the seventeenth century.)

2 Rumphius, G. E. Herbarium Amboinense, 4: 86. t. 38. 1743. (The manuscript must have been written before 1670, the date when Rumphius became blind; final copy of this part of it was made in 1695.)
to the south of the Philippines (and Ternate and Tidore were actually occupied by Spain following 1609) we find such names as the following in actual use today: karasæsi, kolo-tjætjæ, kalasesæ, kalansæsen, kalongsæsæ, and saja kolo-tjætjæ. These are apparently all derived from the Mexican-Philippine kalachuche, the name transmitted southward from the Philippines with the plant itself. One can scarcely credit Bakhuizen van den Brink’s belief (see p. 127), in view of this evidence, that the cognate forms gulong tsjutsju, kulutsjutsju and kulongtsjutsju mentioned by him are of pure Malay origin.

It is improbable that the Portuguese introduced this plant into India direct, first for the reason that they had no Mexican contacts, and second because Rheede tot Draakestein does not describe it in his monumental work on the plants of Malabar (1678–1703). Had the species been of twelfth century, or earlier, introduction into any part of tropical Asia, it is certain that it would have been generally disseminated in India shortly after such introduction. Surely if it occurred in Cambodia or in Java at that time, it certainly would also have been introduced into India and into Amboina, yet we have Rumphius’ statement that it was only introduced into Amboina shortly before 1670. The fact that Rheede tot Draakestein did not illustrate and describe it is distinctly conclusive evidence against the occurrence of Plumeria in India until after its introduction by the Europeans sometime after 1700. On the whole it seems to be highly probable that it reached India from the Philippines, having been transported by some returning traveller or merchant.

Such Indian names as gulachin, gutachin, and golainchi are suspiciously like its modified Nahautl kalachuche as used in the Philippines, and I suspect are but corruptions of it. This then would be merely another case of a plant introduced into a new region with its original name somewhat modified. The longer series of names used for this plant in India exactly parallel the Javan series derived
from champaka, and include such forms as china champac, gorur champa, catchampa, champa pungar, khair champa, dolochampa, dolochapa, goburchamp, khad-champo, rhurachapa, and rhadachampo. The "champa" and cognate forms of this part of the names is from the vernacular champa or chambac (Sanskritic champaka), properly the name of the magnoliaceous Michelia champaca Linn. In India and in Java, this local name, with modifiers, was apparently applied to the introduced frangipanni because of one similar character common to the two totally different plants, the very fragrant flowers. The names used for this plant in most parts of the Old World, other than the corruptions of its Nahautli kalachuche, are merely borrowed ones, undoubtedly indicating an introduced plant for which it was desirable to coin a new name. The evidence from early European accounts, early European trans-Pacific contacts, and that of its local names in the Old World, to me point unequivocally to the introduction of this plant into the Orient by the Spaniards via the old Acapulco-Manila trade route, sometime after 1565, and its dissemination into other parts of the Old World tropics from the Philippines.

My response to Bakhuisen van den Brink's positive identification of the crude twelfth century Borobudur relief as representing kambodja = Plumeria acuminata Ait. is that this greatly conventionalized figure does not represent a Plumeria, no other tangible evidence existing that this plant occurred in Java before it was introduced by the Europeans not earlier than the seventeenth century, and that the figure may in all probability be merely a crude and greatly conventionalized attempt to depict Tabernamon- tana divaricata (Linn.) R. Br., a species which probably was cultivated in Java when the Borobudur temple was constructed, or even a more crude representation of the very common Murraya paniculata Jack. As I see the picture the botanical, historical, and philological evidence is wholly opposed to the introduction of Plumeria into the Old
World before the latter part of the sixteenth or the early part of the seventeenth century.

Hibiscus tiliaceus Linn.

Conclusions drawn from partial evidence are frequently untrustworthy. As an illustration of this point the case of Hibiscus tiliaceus Linn. is illuminating. In 1918 O. F. and R. C. Cook¹ extensively discussed this species making the remarkable statement: "as with the coconut palm and the sweet potato the maho figures more prominently among the Polynesians than among the natives of tropical America, although the American origin of the plant is even more clearly indicated" (italics mine). They erroneously assumed that it was a cultivated plant in the Old World and a wild one in America. As a matter of fact it occurs naturally along the strand throughout the tropics of both hemispheres and unquestionably attained its world-wide distribution some millions of years before man became a factor in plant distribution. Its seeds are ideally adapted to dissemination by floating in salt water, as the hard seed coat is impervious and the seeds float indefinitely. Guppy² includes it in the list of those littoral plants whose seeds "float for months." I know of no records that indicate just how long they will float and still retain their viability. In some regions it occurs in dense thickets inland, and in parts of Polynesia it was formerly planted. Its relative prominence as an economic plant in Polynesia was due to the fact that in many islands it was the only or chief source of fibres for nets and cordage.

The basis of Cook's argument was apparently the similarity between its vernacular names maho and mahagua in tropical America, and its Polynesian names in the hao, mao, mau, vao series. They erroneously assumed that it was a wild plant in tropical America and a cultivated one in the

Old World. They were intrigued to find support to a general thesis of an early westward migration of peoples and cultures into Polynesia from tropical America. As with O. F. Cook's earlier argument based on the origin and dispersal of the coconut, his contentions and conclusions fail to convince the investigator who considers both sides of the argument, and can only be classed as a failure whether judged by the points that were assumed to support the theory, or by those data opposed to it that were not cited.

What is the contribution to the problem from comparative philology? We admit at the outset a similarity between the American and the Polynesian names, but is this similarity more than accidental? What do we find as between the Indo-Malaysian and Polynesian names? It was assumed by Dr. Cook that the similarities between the American and the Polynesian names were conclusive, but that certain names used in Fiji, Guam, and the Philippines might not belong in the maho series. I believe both assumptions to be wrong. The numerous Malaysian names were largely ignored but it was admitted that those used in Madagascar and in neighboring islands appeared to be connected with the Malaysian and Polynesian series.

The significant recorded Philippine names for this plant are bago, balibago, balobago, malabago, malabagu, malambago, mayambago, mulabago. Of these malabago and balibago are the most commonly used ones. Significant names in Malaysia are balebirang, balæh, balo, baæ, baøk, bayæ, bahæ, baræ, bajaran, baræ bhender, baræch, boc, faæ, hæla, hæwi ai, haræ, kabaæ, kalimbæan, kawaæan, kelambæoen, lago, lamææ, molombææ, molumbagæ, molowahæ, molowagæ, baæk, siræn, wahæ, waæ, waræ, waræ laæt, waræ lenga, and waræ lengis; in Madagascar and neighboring islands baro, var, varo, vau, vaæ; and in India bania, baria, bola. In various parts of Micronesia and Polynesia, we find the following: hau (Hawaii), au (Rarotonga), fau, buraæ, au (Makatea), fou fafinæ (Funafuti), fau, fau to (Samoa), purau fau (Tahiti), vau, vau dina (Fiji), karao (Ponape), fau
(Rapa), *hau, fau* (Marquesas), *purau* (Tuamotus), and *pago* (Guam).

The only conclusion that I can draw is that the whole series of Indian, Mascarene, Malaysian, Philippine, Micronesian, and Polynesian names for this ubiquitous pantropic strand tree are cognate forms of one word. I suspect that the root of this word perhaps originally appertained to bast fibres, or to a plant that produced strong bast fibres, and that its application was gradually extended not so much to those species that resembled the original, but to unrelated forms that produced bast fibres. In support of this contention *bago* is a Philippine name very widely applied to *Gnetum gnemon* Linn., a plant that produces one of the strongest bast fibres; *malabago* as applied to *Hibiscus tiliaceus* Linn. is literally "false bago." In various parts of Malaysia *Gnetum gnemon* Linn. is known under such names as *ai howa, ai sowa, ambong bagæ, bagæ, bakæ-bakæ, bangæ, and blinago, wahaæ, wagæ, wa sowa, awali, wali, ta amæ, sawa, loi*, with numerous other local names unrelated to this series.

Again in the Philippines we find the same root in widely used words for certain thymeleaceous representatives of *Wickstræmia* and *Phaleria* that have strong bast fibres but which otherwise do not in the slightest resemble *Gnetum gnemon* Linn., or *Hibiscus tiliaceus* Linn.; such names include *bago, bari, salago, baleo, and palupo*. In Polynesia we find *Wickstræmia* known as *o'ovau, ooaæ and oao-oao* in Tahiti, *auro-era* in the Marquesas Islands, and a *Phaleria* known in Samoa as *suni vao*. In other parts of Polynesia quite different names are used, such as *akia, akai, akia kaule, tao wap* (the first part of which I suppose may be assigned to the *vao* series) and in Palau as *ongæl*. And it is perhaps significant that in India certain names of *Daphne*, another thymeleaceous plant with strong bast fibres, are *setburuwa, satpura, shedbarwa, balwa, bhalua,* and *barua.* Is not our common American name for *Tilia,* bass wood = bast wood, a parallel case? I believe that the recurrence
of this form to represent plants wholly dissimilar, except in producing strong bast fibres, is supporting evidence for the contention that the original word appertained essentially to bast fibres and became applied to various plants that produced strong bast fibres.

As *Hibiscus tiliaceus* is essentially a plant of natural distribution, as opposed to one distributed primarily by man, the only logical conclusion that one can draw from the evidence of comparative philology is that the name, often more or less modified, was carried from west to east by an early migrating or expanding people of Indonesian stock and applied to the plant found growing naturally in all or most lands that these people reached. These migrating peoples in many cases found in their new homes no better fibre material than that yielded by this *Hibiscus*, and continued to call the plant by the name with which they were familiar in the lands whence they came. Forms of the same name were also applied to *Gnetum*, *Daphne*, and *Wickstræmia*, not because these resembled each other but merely because they produced strong bast fibres.

In the Philippines the g is the same as the r of the RGH law \(^1\) which is as well established for these languages as is Grimm’s law for the Indo-European ones. This law applies to certain consonants in Malaysian and Philippine languages, in initial, medial, and final positions where with a reasonable degree of regularity one consonant is substituted for another, and at times certain ones are eliminated. Indonesian linguistics postulate the existence in the archaic mother language of a sound which was pronounced in various ways and which accordingly now appears in different dialects in various forms. These are R, G, H, Y, and L. This is illustrated by the Indonesian word for vein or root, appearing in different dialects as *urat*, *ugat*, *uhat*, *uyat*, *ulat*, *uwat* and *uat*. This chameleon of sound appears to be present in the word *bago* and nobody should be surprised

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to find it recurring in such forms as *baro, baho, bayo, balo, bawa, bao,* etc.

From a purely philological standpoint a very weak spot in Cook's argument is the expressed doubt that the Fijian *vahu* (it should be *vau* fide Mr. Sydney Ray), the Philippine *balibago,* and the Guam *pago* belong to the *mao* series. The elimination of the consonant in the Polynesian forms is exactly what we might expect to find if the theories of comparative philologists be reasonably correct, and I see no reason for doubting the validity of their conclusions. One does not have to be an accomplished comparative philologist to realize that the Malaysian and Polynesian languages are closely allied, perhaps the most noticeable differences being in the elimination of certain consonants in Polynesian speech.

**Sanskritic Plant Names in Malaysia and the Philippines**

A considerable number of plants are today commonly known in Malaysia, including the Philippines, by but slightly modified Sanskritic names. In view of the fact that the Malaysian languages have no general relationships with ancient Sanskrit, what is the significance of these names? Most of the plants to which they are applied are exotic, as far as the archipelago is concerned, but at least two species are indigenous. Among these plant names that I consider must be interpreted as of Sanskritic origin are the following:

*Sulasi* (Sansk. *tulasi*) a name widely applied to several species of *Ocimum* in the Philippines and in Malaysia. *Lasona* (Sansk. *lasuma, rasona*) in the Philippines applied to the common onion (*Allium cepa* Linn.), with many cognate forms in Malaysia. *Malisa* (Sansk. *maricha*) applied to the black pepper (*Piper nigrum* Linn.) in the Philippines, with numerous cognate forms in Malaysia. *Kachumba* and *kasobha* (Sansk. *kasumbha*) names of the safflower (*Carthamus tinctorius* Linn.) used in the Philippines with
cognate forms in Malaysia for this introduced and cultivated plant. *Malunggai* (Sansk. *marungi*) the common Philippine name for the horse-radish tree (*Moringa oleifera* Lam.) with cognate forms in Malaysia. *Kastuli* (Sansk. *latakasturika*) a Philippine name for *Abelmoschus moschatus* Medic. (*Hibiscus abelmoschus* Linn.), with cognate forms in Malaysia. *Mutha* (Sansk. *mustaka*, Hindu *mutha*) used in the Philippines for a ubiquitous weedy sedge (*Cyperus rotundus* Linn.), with *mota* in use in Java and *wêta* in Celebes. *Patola* (Sansk. *patola*) is the Philippine and Malaysian name for two species of the dishcloth gourd (*Luffa*); the Sanskritic name belongs with *Trichosanthes*, another genus of the same family. *Champaka* (Sansk. *champaka*) is a common Philippine and Malaysian name for the introduced and cultivated *Michelia champaca* Linn. *Dansuli* (Sansk. *gandasuli*) is used in Mindanao for the introduced *Hedychium coronarium* Koenig, with cognate forms in Malaysia. *Lagundi* (Sansk. *nirgundi*) in various forms is commonly applied in the Philippines and in Malaysia to both *Vitex negundo* Linn. and *V. trifolia* Linn. *Daua* (Sansk. *yava* = barley, or perhaps originally “cereal”) is the usual name in the Philippines for the cultivated Italian millet (*Setaria italica* Beauv.) with cognate forms in Malaysia; in Malaysia it is also applied to other cultivated cereals such as *Andropogon sorghum* Brot., and in Sumatra to *Eleusine coracana* Gaertn. as *djaba*. This root is apparently the source of the place name Java, indicating a rich agricultural island. The list of Sanskritic plant names in Malaysia could be considerably extended.

As these species occur in the Philippines and in Malaysia, at least six are normally found only in cultivation, the onion, black pepper, safflower, patola (as to the cultivated forms, but forms of *Luffa cylindrica* M. Roem., commonly occur as wild plants), champaka and Italian millet. The *Moringa* is semicultivated, rarely, perhaps never, truly wild. The *Hedychium* is both cultivated and naturalized. Various forms of *Ocimum* are planted and also occur as
casuals in and about towns and settlements. The *Cyperus* and the two species of *Vitex* are true natives, or possibly one, *Vitex negundo* Linn. is an introduced plant. The *Abelmoschus* is everywhere naturalized, but is manifestly an introduced plant. We are thus, for the most part, dealing with a series of introduced plants, several of which are entirely dependent on man for their continued existence in the Archipelago.

The most logical explanation of the occurrence of these plant names in the Archipelago is that these exotic species were introduced from India with their original Sanskritic names at the time of extensive contacts between India and Malaysia. It is definitely known that advanced Indian peoples visited the Malay Archipelago in the fourth and fifth centuries B.C. Later they colonized the region, this colonial venture eventually culminating in the establishment of the empire of Sri-Vishaya or Sri-Vijaya in Sumatra, dominant from the seventh to the twelfth century. This Sumatran empire, at the peak of its power, embraced much of the Malay Archipelago, except parts of Java, the Lesser Sunda Islands, and New Guinea. It colonized or controlled the Philippines and had outposts in Formosa and in Hainan, even collecting tribute from Ceylon and from southern India.

This Sumatran state was gradually undermined and finally superseded in the twelfth century by the growing Javan state widely known as the Empire of Madjapahit. This Empire dominated the entire Malay Archipelago, including the Philippines, and even extended its influence to New Guinea. Its power was broken by the introduction of the Mohammedan religion and its rapid spread in the fifteenth century. It was during the period of dominance of this old Javan state or empire that the great temple at Borobudur was constructed, this dating from the twelfth century.

SIGNIFICANCE OF ORIENTAL PLANT NAMES

We thus have a logical explanation of the occurrence of borrowed Sanskritic names among an alien people. There is no reason to suppose that these plants, so far as they are introduced ones in Malaysia, were introduced from India with their Sanskritic names by the Malay peoples. Rather it seems logical to explain them by assuming that the expanding and colonizing peoples of India, extending their sway to the Malay Archipelago, took with them their own cultivated plants, thus introducing their own names with the plants themselves; and the names have persisted throughout the centuries that have elapsed since these old empires were destroyed, although practically all knowledge of the ancient domination of the Archipelago by alien Indian peoples has been lost to the present-day natives of Malaysia. To a certain degree, we are able to reconstruct the picture of colonization, conquest, and control of a vast archipelago by an alien people, although the extant historical data are scanty. Combining what is known of the history of the region from 500 B.C. to about 1500 A.D. with certain botanical and philological data, we can, I believe, assume that these early Indian colonizing peoples did just what our own ancestors did when they colonized the eastern part of North America; they naturally brought with them their own culture, and to support this culture in the new environment they brought with them the cultivated plants and domesticated animals on which they were dependent for food in their old home. Where in Malaysia they found growing plants identical with those they knew in India, they naturally applied their own names to these plants, and these names, such as those for *Vitex* and *Cyperus*, exceptionally were adopted for these plants by the alien peoples that the invaders controlled. In some cases the invaders undoubtedly adopted Malaysian plant names, but this would probably be confined to those cases where species were represented with which they were not familiar in India. We here have an excellent example of diffusion, not only of the civilizing influences of the advanced Indian peo-
ples, who in expanding colonized not only Indochina but the Malay Archipelago as well, but who in their colonial expansion also took with them the cultivated plants with which they were familiar at home.

**Chinese Plant Names in the Philippines**

Like the Indian peoples the Chinese have had long continued contacts with the Philippines, undoubtedly exceeding 2000 years. While a considerable number of Chinese words have been incorporated in the Philippine languages, the number of plant names is relatively few; and they are all names of plants that occur only in cultivation, although one species (*Dolichos*) is naturalized in certain regions. This paucity of Chinese plant names is probably explainable by the fact that most of the Chinese visiting the islands have been traders rather than colonists.

Plant names of Chinese origin, all applied to cultivated plants of exotic origin, are as follows: *tunghao* (*Chrysanthemum coronarium* Linn.), *ungsoi* (*Apium graveolens* Linn.), *batau* (*Dolichos lablab* Linn.), *sitau* (*Vigna sinensis* Savi), *pechai* (*Brassica pekinensis* Ruhr.), and *kuchai* (*Allium uliginosum* G. Don). The Chinese word *cha* = tea, is locally applied to *Ehretia microphylla* Lam., sometimes used as a substitute for tea, and in such compounds as *chaang-bundok*, and *chaang-gubat*, literally wild tea. The occurrence of these Chinese names in the Philippine languages to a certain degree parallels the Sanskritic names discussed above, clearly indicating certain early contacts and, of course, a certain diffusion of useful plants and their names. Some of these plants, such as *Dolichos lablab* Linn., may have been introduced into the archipelago by other than the Chinese, but most of the plants mentioned are peculiarly characteristic of Chinese agriculture.

**Mexican Plant Names in the Philippines**

The Philippine archipelago was discovered by Magellan in 1521. About forty-three years later the Spaniards com-
menced active colonization from Mexico as a base. The first settlement was made at Cebu in 1564, but six years later Manila was selected as the center of operations. To maintain contacts the Manila galleon route was established, the ships being dispatched annually from the west coast of Mexico, first from Navidad and later from Acapulco, to Manila, and vice versa. For about 250 years this trade route was maintained, being terminated by Mexican independence about 1820. This broke the lines of communication between Spain and the Philippines via Acapulco, Mexico City, and Vera Cruz, after which the islands were opened to foreign trade. For much of the preceding 250 years the trans-Pacific Philippine-Mexican trade had been largely in the nature of a government monopoly. As throughout this entire period the Philippines were governed more or less as a dependency of New Spain, we should naturally expect to find certain Mexican influences in the Philippines.

In plant names of introduced species we find a beautiful parallel in the Philippines as between the somewhat shadowy Asiatic contacts through Malaysia up to the close of the fifteenth century, and the more sharply defined Mexican contacts after the Acapulco-Manila galleon route was established. Naturally the colonizing Spaniards introduced a large number of Mexican species into the Philippines, and the archipelago thus, to a certain degree, became a center in the Old World for the dissemination of American plants, both economic species that were purposely and weeds that were accidentally introduced. Essentially the process was the same as that which I have assumed to be the case in pre-Spanish times when the Islands were controlled and colonized by Indian peoples.

Most of the plant names that the Spaniards introduced from Mexico with the plants were naturally of Mexican and chiefly Nahautl origin. Among them are *kolites*, applied to certain edible species of *Amaranthus*, but here the name was probably merely applied to the plants in the Philip-
pines; this does not predicate the actual introduction of *Amaranthus* from Mexico, although some forms may have thus been introduced. *Abutra* is a somewhat similar case. This name is applied to an *Arcangelisia* in the Philippines, but the word is of American origin, appertaining to the genus *Abuta* of tropical America. In the Philippines the name was applied by the Spaniards to a native species that was supposed to have the same medicinal properties as its American congener; it has several native names of Malaysian origin, such as *lagtang*, *suma* and *uplig*. More closely associated with introduced plants of indubitably American and chiefly of Mexican origin may be mentioned the following: *kakauate* (*Gliricidia sepium* (Jaq.) Steud.); *kalachuchi* (*Plumeria acuminata* Ait.); *kamanchile* (*Pithecelllobium dulce* (Roxb.) Benth.); *kamote*, the sweet potato (*Ipomoea batatas* Poir.); *chiko* (*Achras zapota* Linn.); *chiko-mamei* (*Calocarpum sapota* (Jaq.) Merr.); *sapote* (*Diospyros ebenaster* Retz.); *aposotis* (*Chenopodium ambrosioides* Linn.); *kamate* or *tomate*, the tomato (*Lycopersicum esculentum* Linn.); *cacao* or *chocolate* (*Theobroma cacao* Linn.); *ayapana* (*Eupatorium triplinerve* Vahl.); *mani*, the peanut (*Arachis hypogea* Linn.); *mana* (*Jatropha multifida* Linn.); and *maguey* (*Agave cantala* Roxb.). *Akapulko* or *kapurko* (*Cassia alata* Linn.) probably indicates the place of origin, Acapulco, as far as the Philippine occurrence of this species is concerned, quite as *savo manila* and other similar names (see p. 129) in Java indicates the place whence *Achras zapota* was at an early date introduced into that island, and as the Manila tamarind, in the eighteenth century became a somewhat accepted name in India for *Pithecelllobium dulce* Benth., which was introduced into India from the Philippines by the captors of Manila in 1762.

In some cases names introduced from Mexico with the plants have undergone great modifications in the past two or three hundred years, as illustrated by the word *kamanchile* (*Pithecelllobium dulce* Benth.). While this form is
even now the most generally used one in the Philippines, it has varied from the original to such an extreme as *damortis*, the series running *kamanchile, kamansile, kamarsilis, kamunsil, kamontres*, etc., to *chamultis, damulkis*, and *damortis*. In this series of Philippine names fifteen variants are now recorded. As such a sharply differentiated and easily pronounced word as *kamanchile* has run this gamut of change in less than 300 years, among a people for most of this period predominantly illiterate, it is perhaps understandable how we may expect other words and other plant names to vary to confound the comparative philologist. The species in question is a very sharply defined one, with no near allies in the Philippines, and is a tree of very considerable economic importance, partly because of its edible fruit, but chiefly because it yields the most used tan bark in the Archipelago.

Most of the introduced plant names have varied but slightly or not at all. With such variations as these indicated above from *kamachile* to *chamultis, damulkis*, and *damortis* in 300 years or less, we can more readily appreciate the reasons for variations in the series of names for the coconut and for *Hibiscus tiliaceus*, discussed above, and those for very numerous other cultivated and wild species in Indo-Malaysia, for with these some thousands of years are involved. And we should remember that in the Malay Archipelago and the Philippines about 400 languages or dialects are spoken.

As the Sanskritic and Chinese plant names in Malaysia indicate early contacts and a certain diffusion, so also do the names of American origin indicate contacts and diffusion, but the latter was strictly within the historical period of the last 350 years or so. Attention has been repeatedly called to the fact, for the benefit of the extreme diffusionists in anthropology, that no important cultivated food plant or domesticated animal, other than the dog, in pre-Columbian times transcended the limits of the eastern or the western hemisphere. As the Eurasian civilizations were based on
an agriculture founded on strictly Eurasian plants and animals, and as the pre-Columbian American civilizations were based on a strictly American agriculture in turn based on native American plants and animals, it seems to be evident that the Atlantic and the Pacific Oceans formed practically impassable barriers to early and late civilized man up to the close of the fifteenth century. Had there been contacts by which the civilizations of one hemisphere influenced those of the other, it is inevitable that contacting peoples would have transmitted from one hemisphere to the other some of the plants and perhaps some of the animals on which their civilizations were based and by which they were maintained. There is thus a limit to diffusion that the extreme diffusionists have not recognized, and this limit is a geographical one, in general the boundaries of the eastern and of the western hemispheres. Most of the basic cultivated food plants were widely distributed within the limits of one hemisphere or the other long in advance of the spread of the advanced cultures developed in favored localities in Eurasia and in America; in other words the diffusion of culture failed to keep pace with the diffusion of agriculture, and in the extension or diffusion of both agriculture and the cultures based upon it there were definite geographic limits that were not transcended until the close of the fifteenth century.
A BIOLOGICAL STUDY OF THE SUB-ARCTIC MOLLUSCA

ALAN MOZLEY

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ABSTRACT

A study of the fresh water and terrestrial mollusca as a representative group of animals of the Sub-Arctic Region, with the object of gaining insight into the history in nature of such a group which has recently migrated into virgin territory. This study has been based on field work by the author in Canada during 1924–31, in northern Asia in 1932 and 1933, and in Fennoscandia during 1934, supplemented by examination of the collections in many European and American museums. The main results and conclusions are as follows:

1. The molluscan fauna of the Sub-Arctic Region is made up of two hundred and twenty-five species and varieties. One hundred and seventy of these inhabit fresh water, and the remaining fifty-five are terrestrial. These species are drawn from a wide variety of different phylogenetic groups. It appears that the fauna is made up to a considerable extent of especially hardy representatives of groups which are found in greater variety in the territories lying further to the south. There are in addition a very few families, such as the Lymnaeidae, which appear to find conditions in this region specially favorable, and occur in great abundance.

2. The geographical affinities of the Sub-Arctic fauna are as follows: in both Eurasia and North America the Sub-Arctic fauna is closely akin to that which inhabits the temperate zone portions of each continent respectively; in addition to this there are a number of species which are characteristic of and common to all parts of the Sub-Arctic Region. It appears therefore that there have been two series of migrations in this region, viz. (i) from south to north in both Eurasia and North America; and (ii) in a circumboreal direction, by way of an isthmus in the neighborhood of the Behring Strait from North America to Eurasia, and vice versa, and also possibly by way of an isthmus in the neighborhood of Greenland.

3. The distribution of both fresh water and terrestrial species within the Sub-Arctic Region is governed largely by climate and the character of the vegetation. Both of these factors have far-reaching direct and indirect effects.

4. There has been some modification of the original molluscan stock since its migration into this region, but the amount of change is small in comparison with that which has been observed in other parts of the world which have a warmer climate than that of the Sub-Arctic. Some species have undergone little change, while others have been profoundly modified. In certain instances the change appears to have resulted from a rearrangement or segregation of genetic factors which are present in the bulk of the population, rather than a change of a fundamental character. Certain species which attain a considerable size in the temperate zone are represented in the Sub-Arctic by dwarf...
varieties. A very few totally distinct species appear to have had their origin in the Sub-Arctic Region.

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I. INTRODUCTION

The Sub-Arctic Region lies between what are commonly known as the Temperate and Frigid Zones. More specifically it may be defined for the purposes of this account as the territory lying between the fiftieth parallel of north latitude and the Arctic Circle. It thus includes parts of northern Asia, North America, Europe, and Greenland. Obviously this definition cannot be applied too literally, since in certain places near the sea coast, notably in northern Europe, the Temperate Zone extends much further northward than in the interior of the great continental land masses.

Some question may arise as to whether this territory possesses any noteworthy features which are peculiar to itself. A consideration of the matter, however, must lead to the conclusion that such a condition does exist. As far as the fauna is concerned, there is a point in the neighborhood of the fiftieth parallel of north latitude at which a relatively rapid change takes place. This may be regarded as the southern boundary of the Sub-Arctic Region. Here within a few degrees of latitude many species of animals which have a wide range in the southern and central parts of their respective continents disappear from the fauna, and their places are taken by other forms, which in many instances have only a limited distribution in the south, or do not occur there at all. This change from a southern and central fauna to a northern one appears to be a more or less direct reaction to the severity of the climate, and in some instances at least, is believed to be brought about by
a restriction in the types of habitat available. Largely as a result of this condition, the outstanding characteristic of the Sub-Arctic fauna is the predominance of a small number of species of each of the classes of animals which are represented in the fauna. These species appear to be well suited to life under a wide range of conditions within the region; occur in large numbers of individuals; and in different parts of their range may split up into distinct local races.

For several years a study has been made of the fresh water and terrestrial mollusea as a representative group of animals in the Sub-Arctic Region, with the object of gaining some insight into the history in nature of such a group which has recently migrated into virgin territory. In this connection it may be mentioned that the greater part of the Sub-Arctic Region has been subjected to severe glaciation within relatively recent times, so that the previously existing fauna was exterminated. The questions to which attention was directed in the course of this investigation were as follows:

1. What is the constitution of the Sub-Arctic molluse fauna?

2. From what geographical and phylogenetic sources has it been derived?

3. What environmental factors govern the distribution of the species within the region?

4. To what extent has the original immigrant stock been modified since its migration into the Sub-Arctic Region?

Studies along these lines were made in Canada during the period 1924–31; in northern Asia in 1932 and 1933; and in Fennoscandia during 1934. In addition to this work in nature, the collections in certain museums were examined, particularly those in Washington, London, Manchester, Copenhagen, Stockholm, Helsingfors, Leningrad, and Moscow. Thanks are due to the curators of these museums, and to many individuals in the several countries in which field work was carried out, for their aid in diverse ways.
The grant of the Walter Rathbone Bacon Scholarship of the Smithsonian Institution for the period 1932–34 made it possible to carry out the essential investigations in northern Eurasia, and special acknowledgment is due to the officers of that Institution, and of the United States National Museum for their hearty coöperation. The author is particularly indebted to Doctor Nils Hj. Odhner of the Naturhistoriska Riksmuseet in Stockholm for assistance while studying the Scandinavian fauna; to Professor A. Luther of Helsingfors for help during the stay in Finland; to Doctor Chas. H. O'Donoghue of the University of Edinburgh for his continued friendly advice and encouragement; and to Professor E. A. Andrews of the Johns Hopkins University, whose assistance in many ways has made possible the completion of this work.

II. The Constitution of the Fauna

The total number of species and varieties of non-marine mollusca which are known to inhabit the Sub-Arctic Region is two hundred and thirty-five. One hundred and seventy of these are fresh water forms, and the remaining fifty-five are terrestrial. This number does not include Theodoxus fluviatilis and Clausilia bidentata which barely enter the Sub-Arctic Region.

The most striking feature of the composition of the molluscan fauna of this region is that while the species represent a variety of phylogenetic groups, many of the genera (even according to a conservative nomenclature) are represented in any given area by a very few species. It may be that these are protean forms, and that their offshoots in more southerly latitudes are too specialized to survive in the north. In any event it appears that this fauna is made up to a considerable extent of especially hardy representatives of groups which are found in greater variety in the territories lying to the south. This fact, which implies a severe struggle with environmental conditions, and the elimination of many non-hardy species, is
one of the sources of interest in this fauna. It should be pointed out however, that this does not apply to the fresh water snails of the families Lymnaeidae and Planorbidae which appear to be especially well fitted for life under subarctic conditions, and are found in greater variety in this region than elsewhere.

The lists of species which are given in this chapter are believed to be reasonably complete. No species of doubtful validity are included. With the exception of members of the family Sphæriidae, actual specimens of each of the forms listed have been examined by the author.

The Interrelationship of the European, Northern Asiatic and North American Faunas

Sixteen species of non-marine mollusca (10 terrestrial and 6 fresh water) have a circumboreal range, being found in Europe, northern Asia, and North America. The majority of the species which inhabit the northern part of Asia are identical with those found in Europe. There are a few endemic species and varieties in the northern part of Asia (e.g., Lymnaea zazurnensis, L. palustris saridalensis, and Valvata antiquilina) but most of these are closely related to well known European forms. Apart from the circumboreal species mentioned above, the molluscan fauna of the far northeastern part of Asia has distinct affinities with that of North America. Examples of this are as follows:

(i) The occurrence of Planorbis trivolvis, a common North American snail, as a fossil in the Chuckchee Peninsula, Siberia (see Mozley 1935).

(ii) The occurrence of a species of Ancylus (A. coloradensis) in the Rocky Mountain region of North America. Apart from this the genus Ancylus is known only from the Palæarctic Region.

(iii) A number of species of land snails are said to occur on both sides of the Behring Strait (Dall 1905). The
series of specimens from this region which have been examined in the course of this investigation have been too small to serve as a basis for a critical opinion.

The molluscan fauna of North America is characterized by the presence of numerous species and varieties which do not occur in Europe and northern Asia. There is comparatively little evidence of a direct relationship between the fauna of North America and that of Europe. However, a common European land snail, *Helix hortensis*, is found in Labrador, Newfoundland, and also on the North American mainland for some distance to the south. It is known from Pleistocene deposits in the State of Maine. As this species is not known to occur in northern Asia, and would hardly have been overlooked if it did occur there, its presence in North America is difficult to account for except on the basis of a former land connection between Europe and eastern North America. The existence of a land bridge connecting North America and Asia in the neighborhood of the Behring Strait on one or more occasions in the past may be regarded as extremely probable.

*Catalogue of Sub-Arctic Mollusca*

I. TERRESTRIAL SPECIES

Class Gastropoda

Order Pulmonata

Family Pupillidae

1. *Strobilops affinis* Pilsbry, North America
2. *Pupilla muscorum* (Linné), Europe, northern Africa, Central and Northern Asia, Northern North America
3. *Pupilla muscorum lundströmi* (Westerlund), Northern Europe and Asia
4. *Gastrocopta holzingeri* Sterki, North America
Family Vertiginidae

5. *Vertigo modesta* (Say), Europe, North America, Northern Asia
6. *Vertigo modesta parietalis* (Aney), North America
7. *Vertigo Gouldii* (Binney), North America
8. *Vertigo ovata* Say, North America
9. *Vertigo alpestris* Alder, Europe
10. *Vertigo arctica* Wallenberg, Europe
11. *Vertigo liljeborgi* Westerlund, Europe
12. *Columella edentula* (Draparnaud), Europe, Northern Asia (including Japan), North America

Family Valloniidae

13. *Acanthinula harpa* Say, Europe, Northern Asia, North America
14. *Vallonia costata* (Müller), Europe, Northern Africa, Northern Asia, North America
15. *Vallonia pulchella* (Müller), Europe, Northern Africa, Northern Asia, North America
16. *Vallonia tenuilabris* (Braun), Northern Asia (also known as a fossil from Germany)
17. *Vallonia albula* Sterki, North America
18. *Vallonia parvula* Sterki, North America

Family Cochlicopidae

19. *Cochlicopa lubrica* (Müller), Europe, Northern Africa, Northern Asia, North America

Family Endodontidae

20. *Gonyodiscus ruderatus* (Ferussac), Europe, Northern Asia
21. *Gonyodiscus angulosus* (Mousson), Northern Asia
22. *Gonyodiscus anthonyi* (Pilsbry), North America
23. *Punctum pygmaeum* (Draparnaud), Europe, North America
24. *Oreohelix stantoni* Dall, North America
Family Arionidae

25. *Arion subfuscus* (Draparnaud), Europe

Family Zonitidae

26. *Zonitoides nitidus* (Müller), Europe, Northern Africa, Northern Asia, North America
27. *Zonitoides arboreus* (Say), North America
28. *Zonitoides exigua* (Stimp.), North America
29. *Retinella radiatula* (Alder), Europe, Northern Asia, North America
30. *Retinella binneyana* (Morse), North America
31. *Euconulus fulvus* (Müller), Europe, Northern Asia, North America
32. *Euconulus chersinus polygyratus* (Pilsbry), North America
33. *Vitrina pellucida* (Müller), Europe, Northern Asia
34. *Vitrina limpida* Gould, North America

Family Limacidae

35. *Agriolimax lavis* (Müller), Europe, Northern Asia, North America
36. *Agriolimax agrestis* (Linné), Europe

Family Eulotidae

37. *Eulota fruticum* (Müller), Europe, Northern Asia
38. *Eulota schrenkii* (Middendorf), Northeastern Europe
39. *Eulota nordenskiöldi* (Westerlund), Northern Asia

Family Helicidae

40. *Helix hortensis* Müller, Europe, North America
41. *Fruticicola bicallosa* (Frivaldsky), Northern Asia
42. *Arianta arbustorum* (Linné), Europe

Family Succineidae

43. *Succinea pfeifferi* Rossmässler, Europe, Northern Africa, Northern Asia
44. *Succinea putris* (Linné), Europe, Northern Asia
45. *Succinea oblonga* Draparnaud, Europe
46. *Succinea turgida* Linné, Northern Asia
47. *Succinea altaica* Martens, Northern Asia
48. *Succinea retusa* Lea, North America
49. *Succinea ovalis* Say, North America
50. *Succinea grosvenori* Lea, North America
51. *Succinea avara* Say, North America
52. *Succinea avara vermeta* Say, North America
53. *Succinea hawkinsi* Baird, North America
54. *Succinea chrysis* Westerlund, North America

Family Auriculidæ

55. *Carychium exile* Lea, North America

II. AQUATIC SPECIES

Class GASTROPODA

Order Pectinibranchia

Family Viviparidæ

1. *Viviparus fasciatus* (Müller), Europe, Northern Asia
2. *Campeloma decisum* Say, North America

Family Paludestrinidæ

3. *Bithynia tentaculata* (Linné), Europe, Northern Asia, North America
4. *Bithynia leachi inflata* (Hansén), Europe, Northern Asia
5. *Hydrobia ventrosa* (Montagu), Europe, Northern Africa, Central and Northern Asia

Family Valvatidæ

6. *Valvata piscinalis* (Müller), Europe, Asia Minor, Northern Asia
7. *Valvata antiquilina* Mozley, Northern Asia
8. *Valvata aliena* Westerlund, Northern Asia
9. Valvata macrostoma Mörch, Europe, Northern Asia
10. Valvata siberica Middendorf, Northern Asia
11. Valvata siberica frigida Westerlund, Europe, Northern Asia
12. Valvata tricarinata (Say), North America
13. Valvata lewisi Currier, North America
14. Valvata lewisi helicoidea Dall, North America

Family Amnicolidae

15. Amnicola limosa (Say), North America
16. Amnicola limosa porata (Say), North America
17. Amnicola walkerii Pilsbry, North America
18. Amnicola emarginata (Küster), North America
19. Amnicola winkleyi mozleyi Walker, North America

Order Pulmonata

Family Lymnaeidae

20. Lymnaea stagnalis (Linné), Europe, Northern Asia
21. Lymnaea stagnalis jugularis Say, North America
22. Lymnaea stagnalis sanctamariæ Walker, North America
23. Lymnaea stagnalis lilliane Baker, North America
24. Lymnaea stagnalis wasatchensis Baker, North America
25. Lymnaea columella casta (Lea), North America
26. Lymnaea megasoma (Say), North America
27. Lymnaea auricularia (Linné), Europe, Northern Asia
28. Lymnaea pereger (Müller), Europe, Northern Asia
29. Lymnaea pereger ovata (Draparnaud), Europe, Northern Asia
30. Lymnaea zazurnensis Mozley, Northern Asia
31. Lymnaea palustris (Müller), Europe, Northern Africa, Northern Asia, North America
32. Lymnaea palustris corvus Gmelin, Europe
33. Lymnaea palustris kolguevensis Smith, Europe
34. Lymnaea palustris saridalensis Mozley, Northern Asia
35. Lymnaea palustris kazakensis Mozley, Northern Asia
36. Lymnaea palustris draverti Mozley, Northern Asia
37. Lymnaea palustris bolotensis Mozley, Northern Asia
38. Lymnaea palustris castorensis Mozley, North America
39. Lymnaea vahlii (Möller), Greenland
40. Lymnaea vahlii holbolli Möller, Greenland
41. Lymnaea traskii (Tryon), North America
42. Lymnaea lagotis (Schrank), Europe, Northern Asia
43. Lymnaea truncatula (Müller), Europe, Northern Asia
44. Lymnaea caperata Say, North America
45. Lymnaea umbilicata (C. B. Adams), North America
46. Lymnaea parva sterki B. Baker, North America
47. Lymnaea dalli Baker, North America
48. Lymnaea modicella (Say), North America
49. Lymnaea obrussa decampi (Streng), North America
50. Lymnaea exigua (Lea), North America
51. Lymnaea hedleyi Baker, North America
52. Lymnaea lanceata (Gould), North America
53. Lymnaea catascopium Say, North America
54. Lymnaea emarginata (Say), North America
55. Lymnaea emarginata angulata (Sowerby), North America
56. Lymnaea emarginata canadensis (Sowerby), North America
57. Lymnaea walkeriana (Baker), North America
58. Lymnaea preblei Dall, North America
59. Lymnaea randolphi Baker, North America
60. Lymnaea atkaensis (Dall), North America
61. Lymnaea glabra (Müller), Europe, Northern Asia
   (Interglacial species—Lymnaea saskatchewananensis Mozley, North America)
62. Amphipelea glutinosa (Müller), Europe, Syria, Northern Asia

Family Planorbidae

63. Planorbis planorbis (Linné), Europe, Northern Africa, Northern Asia
64. Planorbis corneus (Linné), Europe, Northern Asia
65. Planorbis antrosus Conrad, North America
66. Planorbis antrosus sayi (Baker), North America
67. Planorbis antrosus royalensis Walker, North America
68. Planorbis trivolvis Say, North America
69. Planorbis trivolvis pilshyi Baker, North America
70. Planorbis trivolvis kolymense Lindholm, Far Northeastern Asia
71. Planorbis corpulentus Say, North America
72. Planorbis corpulentus multistatus (Baker), North America
73. Planorbis infracarinatum (Baker), North America
74. Planorbis campanulatus wisconsinensis Winslow, North America
75. Planorbis campanulatus rudentis Dall, North America
76. Planorbis campanulatus davisi Winslow, North America
77. Planorbis exacuous Say, North America
78. Planorbis exacuous megas Dall, North America
79. Planorbis deflectus Say, North America
80. Planorbis arcticus Möller, Europe, Northern Asia, North America
81. Planorbis umbilicellatus Cockerell, North America
82. Planorbis crista (Linné), Europe, Northern Africa, Northern Asia, North America
83. Planorbis acronicus Ferussac, Europe, Northern Asia
84. Planorbis borealis Loven, Europe, Northern Asia
85. Planorbis vortex Linné, Europe
86. Planorbis compressus Michaud, Europe, Northern Asia
87. Planorbis spirorbis (Linné), Europe, Northern Africa, Northern Asia
88. Planorbis johanseni Mozley, Northern Asia
89. Planorbis leucostoma Millet, Europe, Northern Asia
90. Planorbis contortus (Linné), Europe, Northern Asia
91. Planorbis complanatus (Linné), Europe, Northern Asia
92. Planorbula armigera (Say), North America
93. Planorbula crassilabris (Walker), North America
94. Planorbula campestris Dawson, North America
95. Segmentina nitida (Müller), Europe, Northern Asia

Family Physidae

96. Physa fontinalis (Linné), Europe, Northern Asia
97. Physa sartlandinensis Mozley, Northern Asia
98. Physa gyrina Say, North America
99. Physa gyrina hildrethiana Lea, North America
100. Physa ancillaria Say, North America
101. Physa integra Haldeman, North America
102. Aplexa hypnorum (Linné), Europe, Northern Asia, North America

Family Ancylidae

103. Ancylus lacustris (Linné), Europe, Northern Africa, Northern Asia
104. Ancylus coloradensis Henderson, North America
105. Ferrissia parallela (Haldeman), North America
106. Ferrissia rivularis (Say), North America

Class Pelecypoda

Order Eulamellibranchia

Family Unionidae

107. Margaritana margaritifera Linné, Europe, Northern Asia? (specimens not examined), North America
108. Unio pictorum Linné, Europe, Northern Asia
109. Quadrula quadrula (Rafinesque), North America
110. Amblema costata Rafinesque, North America
111. Fusconaia flava (Rafinesque), North America
112. Strophitus rugosus (Swainson), North America
113. Anodonta cygnea (Linné), Europe, Northern Asia
114. Anodonta anatina (Linné), Europe, Northern Asia
115. Anodonta grandis Say, North America
116. Anodontia grandis footiana Lea, North America
117. Anodontia kennicotti Lea, North America
   ?Anodontia beringiana, Northern Asia, North America (no adequate series of specimens available)
118. Anodontoides ferussacianus (Lea), North America
119. Lasmodona complanata katherinae (Lea), North America
120. Lasmodona compressa (Lea), North America
121. Proptera alata megaptera (Rafinesque), North America
122. Actinonaias carinata (Barnes), North America
123. Ligumia recta latissima (Rafinesque), North America
124. Lampsilis ventricosa (Barnes), North America
125. Lampsilis siliquoidea rosacea (De Kay), North America
126. Lampsilis superiorensis (Marsh), North America

Family Sphæriidæ

127. Sphærium corneum (Linné), Europe, Northern Asia
128. Sphærium sulcatum (Lamarck), North America
129. Sphærium crassum Sterki, North America
130. Sphærium solidulum Prime, North America
131. Sphærium stamineum (Conrad), North America
132. Sphærium emarginatum (Prime), North America
133. Sphærium torsum Sterki, North America
134. Sphærium acuminatum (Prime), North America
135. Sphærium vermontanum Prime, North America
136. Sphærium striatum m (Lamarck), North America
137. Sphærium occidentale Prime, North America
138. Sphærium tenue (Prime), North America
139. Sphærium notatum Sterki, North America
140. Musculium lacustre (Müller), Europe, Northern Africa, Northern Asia
141. Musculium transversum (Say), North America
142. Musculium truncatum (Linsley), North America
143. Musculium rosaceum (Prime), North America
144. *Musculium rycholiti* (Normand), North America
145. *Musculium securis* (Prime), North America
146. *Pisidium annicum* (Müller), Europe, Northern Africa, Northern Asia
147. *Pisidium astartoides* Sandberger, Europe, Northern Asia
148. *Pisidium casertanum* (Poli), Europe, Northern Asia
149. *Pisidium nitidum* Jenyns, Europe, Northern Asia
150. *Pisidium subtruncatum* Malm, Europe, Northern Asia
151. *Pisidium henslowanum* (Sheppard), Europe, Northern Asia
152. *Pisidium supinum* A. Schmidt, Europe, Northern Asia
153. *Pisidium parvulum* Clessin, Europe, Northern Asia
154. *Pisidium steenbuchii* (Möller), Greenland, Europe, Northern Asia
155. *Pisidium lilljeborgii* Clessin, Europe, Northern Asia
156. *Pisidium compressum* Prime, North America
157. *Pisidium fallaz* Sterki, North America
158. *Pisidium punctatum* Sterki, North America
159. *Pisidium variabile* Prime, North America
160. *Pisidium variabile brevius* Sterki, North America
161. *Pisidium minusculum* Sterki, North America
162. *Pisidium adamsii* Prime, North America
163. *Pisidium neglectum* Sterki, North America
164. *Pisidium scutellatum* Sterki, North America
165. *Pisidium roperi* Sterki, North America
166. *Pisidium subrotundum* Sterki, North America
167. *Pisidium splendidulum* Sterki, North America
168. *Pisidium tenuissimum* Sterki, North America
169. *Pisidium rotundatum* Prime, North America
170. *Pisidium ferrugineum* Prime, North America

III. The Distribution of the Mollusca Within the Sub-Arctic Region

The range of habitats within a given territory in which an organism can survive is to some extent a key to the
geographical distribution of that organism. Nevertheless such a principle as "Local distribution is the function of geographical distribution," cannot be successfully applied in an arbitrary manner, for apart from the possibility of the presence of physical barriers, it sometimes happens that the habits of an animal differ in diverse parts of its geographical range. Unfortunately the information at present available which relates to the distribution of the mollusca is not sufficient to make it possible to follow up this line of thought.

In the following account the local distribution of the land and fresh water mollusces in the Sub-Arctic Region is first discussed from the standpoint of general geographical conditions as they find expression in the landscape types. Then a brief description of the kinds of places in which these animals live is given. Finally the most important environmental factors which govern the occurrence of these animals in any particular habitat are summarized.

The Landscape Types

The primary types of landscape met with in the Sub-Arctic Region are the Tundra, Taiga, Forest-Steppe, Steppe, and Steppe-Desert. These may be regarded as representing fundamental natural regions in that they possess distinctive features in the nature of the substratum, relief, climate, soils and vegetation. Well defined examples of each of these landscapes are to be found commonly in the northern part of Asia and in North America, but in Europe, as a result of long-continued human occupation they are often difficult to recognize. The species and genera of plants and animals which characterize each of these great natural regions differ considerably in northern Asia and North America. Some diversity, involving the replacement of one species by another is also to be found in different parts of each of these continents. Nevertheless as far as the form of the landscapes is con-
cerned, there is a marked similarity, or even identity between northern Asia and North America. A question may arise as to whether the principal agencies which contribute to the genesis and morphology of these landscapes are not to be found outside the living organism, and to a great extent in the climate of the region. A case in point is to be found in the form and distribution of certain terrestrial and aquatic habitats on the Forest-Steppe of Canada and Siberia. On both continents the Forest-Steppe is a border-line territory in which the available moisture is barely sufficient to support tree growth even in the most favorable localities. The chief source of water supply is the snow which accumulates during the winter. During the thaw in spring ponds are formed in the vicinity of the largest snowdrifts which generally occur near groves of trees. It is not clear whether the occurrence of a grove of trees, with its associated terrestrial fauna, in a particular place is due to the presence of a pond during the spring, or whether on the other hand the initial factor is the presence of the trees which act as snow collectors, and so result in the presence of the snowdrift. Since the associated terrestrial and aquatic habitats are inter-dependent, and their development is simultaneous it is conceivable that there is a tendency in the natural economy of the region which governs their occurrence. Whether or not this view is a valid one, and can be extended to other natural regions remains to be seen, but it suggests interesting possibilities.

The Tundra or Barren Ground is a belt of treeless land which occupies the extreme northern part of Europe, Asia, and North America. Part of this territory is low-lying and water-logged, but there are also considerable areas of dry sandy and rocky country. As far as the climate is concerned, the outstanding characteristic of this region is the low prevailing temperature, and the shortness of the warm season. In all probability this latter condition is an important one in limiting the variety of the fauna. The fact that part of the country is water-logged has a similar
effect, particularly since a considerable proportion of the remainder of the territory consists of lichen tundra and bare rock, which are equally unsuitable for many organisms. In the southern part of this region, where groups of semi-prostrate trees are occasionally to be found, there may be a few species of terrestrial mollusca, as for example, *Vertigo* sp. near Fort Churchill on the shores of Hudson Bay, but for the most part the land snail fauna of the Tundra is a meager one. The most important aquatic habitats on the Tundra are ponds. Although these may have a considerable surface area they are generally shallow, so that the temperature of their waters during the long days of summer may be relatively high. The fauna of such ponds however, is meager in comparison with that of ponds on the forest-steppe and grassland.

The Taiga, or Northern Coniferous Forest, forms a broad belt to the south of the Tundra on all the continental land areas of the Sub-Arctic Region. The rich growth of vegetation, which in some respects dominates the animal life of the region, is made possible by the high temperature and long growing season in comparison with the Tundra, and by the abundance of moisture as compared with the grasslands to the south. The vegetative cover over most of this territory consists of continuous forest, composed in the main of species of *Pinus*, *Picea*, or *Larix*. Places in which there is too much standing water to permit of the growth of trees are invaded by shrubs such as *Ledum* and *Vaccinium*, together with grasses (*Festuca*, et al.), and mosses including *Sphagnum* and *Polytrichum*. In many instances the occurrence of terrestrial mollusca within this territory is profoundly affected by the character of the vegetation. Pure stands of needle-leaved trees, whether *Pinus*, *Picea*, or *Larix*, are not suitable habitats for land snails, and as a rule only scattered individuals of two or three species are to be found. Naturalists working in different parts of this region have been inclined to attribute this condition to the presence of resinous ex-
tracts of decaying pine needles, etc., but it is possible that the absence of suitable food is an equally important factor. The fact that the presence of a relatively small number of birch or poplar among the conifers generally results in a considerable increase in the abundance and variety of the molluscan fauna, lends some support to this view. Nevertheless it must be admitted that the fallen and decaying logs of birch (Betula spp.) support a much richer fauna than do those of the needle-leaved trees. The characteristics of the aquatic habitats of this region may also be considerably affected by the vegetation. Many of the ponds and lakes have been invaded by Sphagnum, grasses and shrubs to such an extent that they have ceased to be suitable habitats for fresh water molluscs. This is the principal reason for the rarity of temporary ponds in the Taiga. In addition to the purely physical action of the semi-aquatic plants in crowding out the aquatic fauna, it is probable that Sphagnum has an inimical effect upon many animals owing to extracts which arise from it or at least are generally associated with its growth. This explanation of the poverty of fauna in peaty waters has been put forward by a number of writers, and is almost proverbial among field collectors. West (1910) has advanced the view that peat extracts play an important part in governing the character of the vegetation in many Scottish lochs. It may be worth noting that when such waters are aerated, as occurs in streams with falls and rapids, the flora and fauna becomes much richer. Many of the larger lakes of the Taiga region occupy more or less deep depressions in the bed rock which thus lies exposed along their shores. Such shores are subject to severe wave action, which renders them unsuitable habitats for the majority of fresh water molluscs. A few species however, have become adapted to life under such conditions, and may be found in certain localities in considerable numbers of individuals. In instances where the slope of the shores of a lake is sufficiently gradual, and conditions of exposure and
bottom are suitable, a number of species of aquatic and semi-aquatic plants may be found forming colonies of some size. For the most part these plants are species of *Phragmites*, *Typha*, *Scirpus*, *Sparganium*, *Carex*, *Juncus*, or *Equisetum*. Such plant associations provide food and shelter from the waves and so support a large and varied fauna of mollusca.

The Forest-Steppe is a narrow belt of groves and glades which is situated between the forested territory described above and the grassland. In the northern part of the Forest-Steppe the groves become gradually larger until they fuse to form a continuous forest, while along the southern border they become gradually smaller and finally disappear altogether, leaving open grassland. In North America the most common tree on the Forest-Steppe is the Trembling Aspen (*Populus tremuloides*), while over the greater part of the northern Asiatic Forest-Steppe the prevailing tree species are Birches (*Betula verrucosa* var. *latifolia*, and *B. pubescens* var. *allapica*). The Forest-Steppe presents a particular combination of conditions which is distinctly favorable for such species of fresh water and terrestrial mollusca as are able to endure the severe climate which prevails in this northerly situation. Moisture is abundant at certain seasons, but a sodden condition of the soil and vegetation is seldom met with. The growth of trees, while sufficient to cast an effective shade, and thus to result in some moisture being retained in the forest litter, is not so dense as to prevent the growth of herbs, especially near the borders of the groves. In addition, the leaves of the trees, when they fall to the ground, form the basis of an excellent food supply for land snails. As regards moisture and food therefore, life conditions are suitable for molluscs, but although the number of individuals may be very considerable, there is no great variety in the species which occur in such habitats. This is a characteristic feature of the Sub-Arctic fauna. Similarly favorable conditions are met with in the aquatic habitats.
*Sphagnum* bogs are rare, but on the other hand, a very high concentration of mineral salts in solution, such as is of frequent occurrence in the waters of the Steppe and Steppe-Desert, is not commonly found on the Forest-Steppe. The marshes of this region are particularly rich, and in many instances have the most varied fauna of any habitats in this Region. The borders of semi-stagnant meandering streams also support a rich fauna, the species for the most part being the same as those found in marshes. It is in the Forest-Steppe that the temporary ponds attain their greatest development, and gave their most varied and abundant fauna. This is due largely to the reliability of their water supply as a result of the snow in winter collecting about the groves of trees as mentioned in a previous paragraph. In consequence of this the ponds have a longer annual aquatic phase than do those in the other natural regions lying to the south, and are more regular in their occurrence year after year. The intermittent streams on the Forest-Steppe may also support a small fauna of mollusca, as for example *Planorbis umbilicatellus* in an intermittent stream lying to the north of Birtle, Manitoba; and *Lymnaea palustris* et al. in Paskwegin Brook, Saskatchewan.

The Steppe or grassland, known in Canada as the Prairie, is characterized by the absence of trees, and the considerably greater concentration of mineral salts in solution in many of the natural waters as compared with those of the Tundra, Taiga, and Forest-Steppe. Both of these conditions are attributable to the occurrence both periodically and at irregular intervals of a serious deficiency of precipitation, and also to the high evaporating power of the air. Except in saline localities there is generally a luxuriant growth of grasses on the Steppe, and these form a thick and continuous turf which reduces to a minimum the erosive action of the floods which follow the thaw in spring. In valleys, or on hillsides, when the turf is once broken deep ravines may be formed, and these are
barren of life. One of the notable features of the Steppe is a marked restriction in the number of different kinds of habitats which are available for settlement by aquatic and terrestrial animals as compared with the Forest-Steppe. With this is correlated a diminution in the number of species which occur in a given individual habitat. These conditions become more pronounced in a southerly direction in continental Asia, and towards the western interior of North America. The periodical shortage of moisture during the summer, combined with the low winter temperatures have the effect of limiting the number of habitats in which terrestrial mollusca are found. For the most part the only places in which these animals occur on the Steppe are marshes, along the borders of streams, and occasionally under shrubs on the flood plains. In a few places on the Kirghiz Steppe of northern Asia empty and bleached shells of land snails have been found in debris carried down by intermittent streams the drainage basins of which are entirely devoid of trees and shrubs. The habitat of these shells is not known, but it is possible that during moist seasons a few terrestrial mollusces may live on the grasses and herbage. An alternative explanation is that the shells found were fossils washed out of some Post-Pleistocene deposit. The marshes of the Steppe region have not as rich a fauna as have those on the Forest-Steppe. In many instances this is due to the presence of a considerable concentration of mineral salts in the waters, and is also to some extent due to the fluctuations in water level in such lakes which results in the destruction of many of the plants (see Mozley 1935, p. 673). Temporary ponds are of common occurrence on the Steppe, but as the duration of their aquatic phase is not as long as is that of those on the Forest-Steppe their fauna does not include as many different species. Some of the Steppe ponds, especially those which occur in valley bottoms, and old lake beds, are saline, and this has the effect of still further reducing their fauna. Similar conditions considerably intensified, are
met with in the intermittent streams. Most of the permanent streams of the grassland regions of both northern Asia and North America draw their water supply from neighboring mountainous areas, so that they cannot be regarded as Steppe streams in the strict sense of the term. The fauna of ponds situated on the flood plains of these streams is generally a rich one, and in all probability these habitats serve as natural reservoirs from which much of the neighboring territory is populated with aquatic animals during years of unusually abundant moisture.

The Steppe-Desert is situated to the south of the Steppe proper. The most characteristic feature of this region is the growth of the grasses in compact clumps with bare spaces intervening. This is the result of the much lower precipitation as compared with the Steppe. The deficiency of moisture has an unfavourable effect upon the aquatic fauna, and the majority of the bodies of standing water are saline. The temporary ponds are an exception to this general rule, their waters almost invariably being fresh (except under the conditions noted above—see under Steppe). The prolonged severe droughts which frequently occur in this region however, result in many of the temporary ponds being without water over a period of two years or more. This results in their fauna being somewhat meager. Marsh development is at a minimum, and so the rich association of species which are found in those situations in other natural regions is generally absent from the Steppe-Desert. The only exceptions to this which have been observed have been along the margins of streams which rise in the neighboring mountainous regions. Most of the other streams in the Steppe-Desert are of the intermittent type, and rarely if ever contain molluscs. The next natural region lying to the south, the Desert, is situated outside the Sub-Arctic Region.
The Molluscan Habitats

The habitats of non-marine molluscs in the Sub-Arctic fall into two obvious classes, those of the land areas, and those of fresh water. There is little overlapping between these two types of station except for the occurrence of a few fresh water species such as *Lymnea truncatula* (in Eurasia) and *L. parva sterki* (in North America) on mud flats along the borders of streams, and of members of the terrestrial genus *Succinea* in similar situations and in marshes. Specific examples of the fauna of representative series of mollusc habitats have been given in a previous paper (Mozley 1935).

The terrestrial mollusca of the Sub-Arctic Region are found occupying a somewhat limited range of habitats. With few exceptions the land snails which inhabit this territory are forest dwellers. The high degree of uniformity which is to be found in the fauna of widely separated areas may to some extent be correlated with this uniformity of habitat. While it is true that there is some diversity in the species of trees which make up the forests, this appears to have but little effect upon the character of the fauna. Provided that food of some sort is available, it appears that the trees are of importance principally as providing shade and thus checking evaporation. The Temperate (as distinct from Sub-Arctic) parts of Europe, Asia, and North America each have a distinct land snail fauna which is made up of a large number of characteristic species. Very few of these invade the northern parts of their respective continents, the fauna of those regions being composed of other species which in most instances either have a circumboreal range, or are represented in Eurasia and North America by closely related and ecologically equivalent forms. It is probable that the widely variable conditions of moisture, and the low temperature in winter combine to exclude the majority of the species which are found in the neighboring regions to the south. The most favorable
habitats for terrestrial molluscs in the Sub-Arctic Region are moist (but not water-logged) situations in forests of Aspen (*Populus tremuloides*) in North America, and Birch (*Betula* spp.) in northern Asia.

There is much greater diversity in the habitats of the fresh water mollusca in the Sub-Arctic Region than in those of the terrestrial species. In this account the habitats are discussed under several headings, namely: the temporary ponds, permanent ponds and small lakes, large lakes, and streams. It should not be assumed however, that these are absolutely distinct types. There are numerous intermediate examples which are difficult to place definitely in one class or another. As a rule the quiet-water habitats have a richer fauna of mollusca than streams.

Temporary ponds which are formed by the melting snow in spring are among the most characteristic aquatic habitats in the Sub-Arctic Region. These ponds have a short period of submersion in the spring, which is followed by progressively drier conditions during the summer and autumn, and by low temperatures during the winter. The period of activity of the aquatic animals which are permanent residents of such pools, and not merely migrants is about two months in each year. It is obvious therefore that the conditions of life for aquatic animals in these habitats are severe; nevertheless a large number of animals belonging to diverse phylogenetic groups are able to maintain themselves in them. Some species in fact are to be found only in temporary ponds. This is true of certain phyllopod crustacea such as *Apus*, *Lepidurus*, and *Branchipus*, and also of certain mollusces (see Mozley 1932). It is probable that the richness in species of the temporary pond fauna in the Sub-Arctic Region is to some extent due to the regularity of their occurrence in the spring of each year. This is the result of their being fed by snow water, and not being dependent upon the occurrence of seasonal rains. Another important factor may be the absence of winter thaws. In this connection it may be worth noting
that Shelford (1919) states that the occurrence of Branchipus in the Chicago area of North America is extremely sporadic, and indicates that in all probability this is due to the occasional winter thaws during which these temporary pond crustacea commence their development, only to be killed by subsequent severe frosts before they have reached maturity. In North America and northern Asia the most common temporary pond molluscs are Lymnaea palustris and Aplexa hypnorum. In the course of field work covering an extensive area in Scandinavia only one typical temporary pond was found (near Strömsund, Jämtland), and this contained only Aplexa hypnorum. No temporary ponds were found in Finland, and Professor A. Luther of Helsingfors states that to his knowledge no specimen of Branchipus, or any closely related form (Eubranchipus, etc.) has ever been collected in that country.

Ponds which contain water permanently, and small lakes, are of particular importance as habitats in the Sub-Arctic Region, since they are of common occurrence, and in most instances present conditions which are favorable for molluscan life. For the most part these are shallow bodies of water of such small area that great waves are not formed on the surface by the wind. Their essential characteristic is the quietness of their waters and the abundance of food. This renders possible a greater abundance of living organisms per volume of water than in any other type of aquatic habitat in this region. Submerged vegetation, including such plants as Potamogeton, Elodea, and Utricularia, is usually common, and provides a site for the growth of microscopic organisms which serve as food, either directly or indirectly, of many of the larger animals. This is also true of the rooted plants which have leaves floating upon the surface, e.g. Nymphaæ, Polygonum, and Zizania. Snails and small fresh water mussels are frequently to be found upon such plants in large numbers. The shores of many ponds and small lakes are occupied by marsh plants such as Typha, Phragmites, Scirpus, and
Sparganium which generally support a varied molluscan fauna. As has been mentioned in a previous paragraph, the presence of Sphagnum in a body of water results in a much reduced molluse fauna. On the Steppe and Forest-Steppe the fauna of ponds closely resembles that of marshes. In the Taiga the pond fauna is usually meager owing to the presence of Sphagnum, while on the Tundra of Eurasia a limited number of observations indicate that Lymnaea pereger, Planorbis acronicus, Aplexa hypnorum and Pisidium spp. are the most common molluses. Ponds on the North American Tundra have been found to contain Lymnaea palustris, Planorbis arcticus, Physa gyrina, and Aplexa hypnorum. Ponds and small lakes in several parts of Greenland are inhabited by Lymnaea vahlii and L. vahlii holbolli. Lymnaea palustris kolguevensis inhabits Kolguev Island.

The large lakes of the Sub-Arctic Region fall into two classes, namely those which have outlet streams and those which have not. The molluscan fauna of lakes of the latter type has been discussed in previous papers (Mozley 1930, 1935). The characteristic features of the lakes of the first group are the possession of a moderate or great depth (5 meters or more), and a considerable expanse of open water which results in the shores being subject to wave action. The most striking example of a body of water of this kind in the Sub-Arctic Region is Lake Baikal (depth 1741 m.). Conditions in this lake however, are so extreme that the great majority of characteristic Sub-Arctic animals are unable to exist there. In many of the lesser, though still large lakes of this region, and particularly in North America, three distinct molluscan associations are to be found, namely:

(i) The exposed shore association: Lymnaea stagnalis lacustris in Europe; Lymnaea emarginata with L. catastropium, or else either L. stagnalis sanctemariæ or L. stagnalis lillianæ in North America.
(ii) The bottom association: *Lymnaea pereger* in Europe; *Amnicola limosa, A. walkeri, Valvata tricarinata, Lampsilis siliquoidea* and *Anodonta grandis footiana* (or *Anodonta kenneicottii*) in North America.


The molluscan fauna of streams in the Sub-Arctic Region varies widely with the character of the habitat. Sluggish meandering streams have a fauna which closely resembles that of marshes. In hilly or mountainous country the high inclination of the stream beds, the presence of scree and silt, and the low temperature may act singly or collectively to limit the variety and abundance of the molluscan fauna (see Mozley 1933). There are numerous streams of large size in the Sub-Arctic Region. Such rivers as the Pitea and Kalix (Sweden), the Kemi (Finland), Ob, Irtish and Yenissei (northern Asia), and the upper parts of the Athabasca (North America) have a very meager molluscan fauna in their main channels. It is only in the side waters of most of the great rivers of this region that fresh water molluscs are found in any numbers. It is worthy of note however, that several species of Unionidae are found in considerable abundance in the Nelson River and the Red River of the North (not the Arctic Red River). Ponds on the flood plains of large streams are important as habitats for molluscs in some parts of this region (see Mozley 1935).

*Important Environmental Factors Governing the Distribution of Molluscs within the Sub-Arctic Region*

Two general classes of environmental factors play a predominant part in governing the distribution of both fresh water and terrestrial molluscs in this territory, namely those relating to the climate of the region and those con-
nected with the vegetation. Not infrequently it is difficult to distinguish between these two classes. It may be well to point out that within a single geographical area there may be significant local differences which play an important part in governing the occurrence of molluscs and other animals. On the Forest-Steppe of both Asia and North America for example, the presence of groves of trees has a profound effect upon the fauna by providing food (dead leaves and twigs), and in hindering evaporation from the surface of the soil. As a result of these conditions a number of species of mollusca are able to live in these situations. In comparison with the territories lying to the south, the presence or absence of lime appears to be of minor importance in influencing the distribution of land snails in any particular part of the Sub-Arctic Region. This may be because a certain minimum of lime, sufficient for the needs of the molluscs, is present almost everywhere in the Region. The critical role in determining the presence or absence of these animals therefore passes to other environmental factors.

As far as the fresh water molluscs are concerned, the effect of climate and vegetation upon distribution in the Sub-Arctic Region appears principally in five ways, namely: (i) through the mineral salts and other materials in solution; (ii) through the action of waves in beating upon the shores of bodies of standing water; (iii) through marsh development; (iv) through the force of the current in streams; (v) through desiccation. The action of these factors may be summarized as follows:

(i) Mineral Salts in Solution.

The changes in climate which have taken place in the Sub-Arctic Region since the end of the last period of glaciation have resulted in certain of the lakes in this area losing their connection with the sea. Owing to the absence of an outlet and continued evaporation there is subsequently a progressive increase in the concentration of min-
eral salts in solution in the waters of such a lake. This process culminates in a saturated solution and the deposition of salt crystals around the shores as the lake dwindles in size and finally becomes a salt flat. A high concentration of mineral salts has an unfavorable effect upon the molluscan fauna. This is manifested in two ways, namely, in a direct physiological manner, and through the destruction of the aquatic and semi-aquatic vegetation upon which many molluscs are dependent for food and shelter.

(ii) Wave Action.

The action of waves in breaking upon the shores of lakes plays an important part in determining the suitability of such situations for occupation by molluscs. Most of the gastropods which inhabit the Sub-Arctic Region live in shallow water, and apparently find it difficult to exist in large bodies of water where the shore line and neighboring bottom are frequently disturbed by wave action. For the most part any stragglers which find their way into situations of this sort are either broken to pieces or carried into deep water where they are likely to be devoured by fishes. There are, however, several species and varieties of Lymnaea which are especially adapted for life in habitats which are subject to severe wave action and are unable to exist elsewhere (e.g. Lymnaea stagnalis sanctaemariae).

(iii) Marsh Development.

The occurrence of groups of aquatic and semi-aquatic plants in and around ponds, small lakes, and streams has a profound effect upon their suitability for molluscan life. The swamps of the Sub-Arctic Region are dominated by Larch (Larix spp.), and with the trees Sphagnum is usually found in some quantity. This moss also occurs in great abundance in the Sub-Arctic bogs, and as has been mentioned elsewhere in this account, has an inimical effect upon the molluscan fauna. Marshes however provide both food and shelter for many species.
(iv) *Force of the Current in Streams.*

As has been mentioned under (ii) above the majority of the fresh water mollusces which inhabit the Sub-Arctic Region are adapted for life only in quiet waters. Any considerable increase in the current of a quiet stream results in the elimination of many species. While it is true that the inclination of the stream bed plays an important part in determining the force of the current in many streams, the climate and vegetation are significant modifying factors. It frequently happens that the backwaters of streams which have a swift current support a rich fauna of mollusces.

(v) *Desiccation.*

The occurrence of severe droughts, both periodically or at irregular intervals, is one of the characteristic features of the climate of the Sub-Arctic Region. In many instances these droughts have the effect of reducing the molluscan fauna to a minimum. A number of aquatic species however, are able to withstand a considerable degree of desiccation (see Mozley 1932).

**IV. THE VARIATION OF SOME SUB-ARCTIC MOLLUSCA**

The repeated glaciation to which the greater part of the Sub-Arctic Region was subject during the Pleistocene period may be considered, for practical purposes, to have resulted in the extermination of the whole of the molluscan fauna which inhabited the area previous to that time (see Mozley 1934). It may be of interest to enquire what modification (if any) the species which migrated into this territory in Post-Pleistocene times have undergone since their immigration. Even a cursory review of the fauna reveals that considerable diversity exists in the degree, and possibly also in the nature, of the change which has taken place.

Bearing in mind the observations of Crampton (1916), Sumner (1932), and other students in diverse countries, it cannot be said that profound modifications of the original
immigrant stock have taken place in the Sub-Arctic Region. The relatively short period of time which has elapsed since these animals invaded this territory is probably a contribu-
tory factor in the small amount of change which has taken
place, and it is possible that the low temperatures which
prevail over the whole of the Sub-Arctic Region through-
out most of the year may also have played a part. What-
ever the nature of the process may be it does appear that
evolutionary change takes place with greater frequency in
countries which have a high prevailing temperature than
in those which are subject to cold weather during a consid-
erable part of each year. It is conceivable that this may
be a simple physical effect of low temperature. In most
instances the diversities which appear to have arisen since
these animals invaded this territory in post-glacial times
involve a change in the relative proportions of the parts of
the shell. Except in certain lacustrine varieties these
changes are not supposed to have any survival value. It
is interesting to observe that within certain limits each
family of fresh water molluses inhabiting this region tends
to vary in a characteristic manner. The Valvatidæ form a
good example of this (Mozley 1935).

*Aplexa hypnorum* (Linné)

*Aplexa hypnorum* is a fresh water gastropod which is
found in considerable numbers throughout the Sub-Arctic
Region. In northern Asia this species has been found as
far north as N.L. 73° 30'. In the region lying to the south
of the fiftieth parallel it is seldom found in abundance, and
what individuals do occur are of small size. In spite of
the immense geographical range of this snail it varies be-
tween somewhat narrow limits. The principal diversity
in the extreme variants, either within a series of shells from
a single habitat, or between individuals from far-distant
localities, is found in the length of the spire. Even in this
character however the extent of variation in *A. hypnorum*
is much smaller than in many other fresh water pulmonates
which inhabit this region. Supposed sub-species, *e.g.* *A. hypnorum tryoni*, have been described on the basis of shells with elongated spires, but none of these appear to possess really distinctive features. Long-spired individuals are almost equally likely to be met with as isolated individuals, or as forming the bulk of a local population. It should be noted however, that while this conclusion is based upon the examination of numerous specimens, both in their natural habitat and in museum collections, it has not yet been statistically demonstrated. It appears therefore, that *Aplexa hypnorum* is a species which varies between narrow limits, and what variation does occur is as likely to be met with in one locality as in another. No tendency towards the accumulation of diversities, or segregation into local races has been observed.

**Unionidae**

The unionid fauna of Sub-Arctic America has been derived from the valley of the Mississippi River. It is a matter of some interest to observe that several different species of these fresh water mussels appear to have been modified to an approximately equal extent by the severe conditions of life in this northerly situation. That is to say, mussels which attain a large size in the drainage of the Mississippi River are represented in the northern part of the continent by dwarf forms which in some instances are sufficiently distinct to have been given specific or varietal names. The degree of dwarfing is approximately equal in each instance (Pls. I and II). The species in which this is true are the following:

<table>
<thead>
<tr>
<th>Mississippi Form</th>
<th>Sub-Arctic Form</th>
</tr>
</thead>
<tbody>
<tr>
<td><em>Quadrula quadrula</em></td>
<td>Unnamed dwarf variety</td>
</tr>
<tr>
<td><em>Anodonta grandis footiana</em></td>
<td><em>A. kennicotti</em></td>
</tr>
<tr>
<td><em>Lasigmone complanata</em></td>
<td><em>L. complanata katharina</em></td>
</tr>
<tr>
<td><em>Ligumia recta latissima</em></td>
<td>Unnamed dwarf form</td>
</tr>
<tr>
<td><em>Lampsilis radiata</em></td>
<td><em>L. borealis</em></td>
</tr>
</tbody>
</table>
| *Lampsilis siliquoidea* | *L. siliquoidea rosacea* and *
| | *L. superioresis* |
| *Lampsilis ventricosa* | Dwarf variety (unnamed)* |
It seems reasonable to suppose that the cause of this dwarfting lies in the general geographical conditions of Sub-Arctic America. It is probable that the great length of the winter is an important factor. This may affect the fresh water mussels both directly through the long period during which they are in a partly dormant condition, and also indirectly through the food supply which must be similarly responsive to the low temperature and darkness of the winter months. To these influences the several species listed above have reacted in a similar way, and to an approximately equal extent.

The structural modifications which have resulted may not in a sense, be evolutionary changes. It is conceivable that they are more of the nature of simple physiological results of the unfavorable conditions which these animals have to endure in the northerly territory. If this be true it is not surprising that the morphological changes have taken place to an approximately equal extent in diverse species. Whether or not such reactions as this have any evolutionary significance is difficult to say, but it is evident that at least as long as the environmental conditions remain as at present, the mussels will remain dwarf forms, with characteristics which mark them off from their southern relatives. What would be the result of transferring the northern mussels to the Mississippi River, it is not possible to predict. Such an experiment might not be relevant to the main fact, namely, that in their natural habitat these animals are structurally distinct from the related forms in more southerly latitudes. The above remarks notwithstanding, it is possible that these diversities may differ from some types of evolutionary change in degree only.

It is of some importance to note that not all of the fresh water mussels which inhabit Sub-Arctic America are dwarf forms of southern species. In some instances the Canadian shells are of a size equal to that of average specimens of the same species in the Mississippi drainage. This is known to be true of *Amblema costata* (specimens taken
about four miles below the Englishman’s Rapid on the Berens River, Manitoba), and also of *Anodontaoides ferussacianus* (specimens taken in the Whitesand River, near Theodore, Saskatchewan). These two species however, are not among the most common mussels of this region, and for the most part have a rather limited or sporadic distribution within the territory. From this it might appear that they are unable to tolerate the unfavorable conditions which are found over the greater part of this region, and so occur only in especially favorable habitats, in which they attain a size equal to that of individuals in the Mississippi drainage.

**Lymnaeidae**

A greater degree of variation is to be found in the fresh water gastropods belonging to the family *Lymnaeidae* (and the closely related *Planorbidae*) than in any other group of molluses in the Sub-Arctic Region. Although these two families have a wide range of occurrence over the earth’s surface, being found even in Australia, they are nowhere represented by such large numbers of individuals, or by so many diverse types as in the northern part of the northern hemisphere.

**Lymnaea palustris and L. emarginata**

Two of the most common members of this family in North America are *Lymnaea palustris* and *L. emarginata*. A comparison of the local and geographical variation of these species in the central part of Canada and the United States (Mozley 1935a) showed that while in *L. palustris* the range of variation in several shell characters in one locality approximates that which occurs over the greater part of the geographical range of this species in North America, in *L. emarginata* there is a well marked tendency for the range of variation in the same characters to constitute only a small part of that found over the whole geographical area inhabited by the species. That is to say,
in the one instance local variation approximates geographical variation, while in the other it covers only a small part of it. In other words *L. emarginata* tends to split up into local races, whereas *L. palustris* does not. It is important to note that this difference in the type of variation is so strongly marked that it can be recognized whenever a large series of shells is examined, even in a casual manner. Students of the systematics of mollusca have bestowed numerous specific and varietal names upon diverse races and extreme variants of each of these two groups. A statistical demonstration of the essential facts has been given in another paper (Mozley, *loc. cit.*).

The explanation of this diversity in type of variation is believed to lie in the local distribution of the species. Within the territory in which the statistical study was carried out, *L. palustris* occurs in a high proportion of the very numerous ponds and small lakes; for example, it was found in 238 of a series of 315 such habitats examined. These ponds and small lakes are often situated close together, and it is easy to understand that there are frequent opportunities for the mixture of their fauna. Under these conditions the appearance of recessive characters would be sporadic, the variation in all localities would be expected to be very similar, and its range wide.

In contrast to this continuous type of variation is that of *L. emarginata*, which is found as a rule only on the rocky or bouldery shores of large lakes, a type of habitat which is of relatively rare occurrence in the area where the work was done. Such habitats are often situated one or two hundred kilometers apart. There is thus little opportunity for genetic intermixture between the members of different colonies. Under these conditions recessive characters might be expected to appear in the bulk of the population in certain localities, and thus to form the basis of local races.

The conclusions noted in the paragraphs above were drawn from a study of these species in central Canada and
parts of the northern United States. When the scope of the investigation was extended to include the northern Rocky Mountain area it was found that the Lymnaeae for the most part occupied isolated habitats in that region and there tended to split up into local races some of which have been given specific names, e.g. *Lymnaea traskii* Tryon (Mozley 1930a).

*Lymnaea emarginata* is a purely North American species. *L. palustris* however, is also found in Europe and northern Asia. Several distinct races of *L. palustris* were discovered in the latter territory and were described as subspecies (Mozley 1934a). In connection with the appearance of local races of this species in northern Asia the following remarks regarding the local distribution of *L. palustris* in that region as compared with North America may be of significance, "In collecting over any extensive area in Siberia and northern Kazakstan one is impressed by the fact that this species is less common there than in the comparable regions of North America. In parts of Canada *Lymnaea palustris* (or closely related and homologous forms the systematic position of which is not yet clear) is extremely common, and is to be found in abundance in almost any pond, small lake, and marshy area. . . . This is not true in Northern Asia, and though to the casual observer conditions may appear to be very similar, the proportion of ponds and lakes which is populated by *Lymnaea palustris* is much smaller—about one in four or five." Obviously if the explanation of the difference in type of variation observed in *L. emarginata* and *L. palustris* in central Canada is valid, it may also apply to the diversity of the variation of the latter species in the Rocky Mountain area and in northern Asia described above.

*Lymnaea stagnalis* Linné

*Lymnaea stagnalis* is a large fresh water gastropod which is to be found in abundance in most parts of the Sub-
Arctic Region. The geographical range of this species includes the northern part of North America, northern Asia from Kashmir to Siberia, and very nearly the whole of Europe as well as that part of Africa which lies to the north of the Sahara, and Asia Minor. The northern limit of this species in Great Britain is Edinburgh, while in Sweden it is found as far north as Gellivare (N.Lat. 67° 7', E.Long. 21° 40' approx.).

In North America the most common form of *Lymnaea stagnalis* is known as *L. stagnalis jugularis*. This variety is found over the greater part of the continent. Several sub-species have been described from North America the most important of which are *L. stagnalis sanctaemarieae*, *L. stagnalis lillianae*, and *L. stagnalis wasatchensis*. Each of the three latter forms appears to be specially adapted to the special conditions under which it lives and there is little or no tendency for their areas of distribution (either local or geographical) to overlap. In northern Asia there is one generally prevailing and widely variable form which has been referred to under the name *Lymnaea stagnalis* Linné (Mozley 1933). In the course of extensive collecting in this area no other race of this species was found. In northern Europe there is one common form which is very similar to that found in northern Asia, and in addition numerous more or less distinct races which have been described as sub-species. Westerlund (1886) lists twenty-eight of these from the Palæarctic Region, while Paiget (1929) recognizes twelve varieties in addition to the typical form in Switzerland alone. The characteristic ratios of the length of the shells to the length of the aperture in these varieties are stated by Paiget to be as follows:
According to the evidence advanced by Paiget, some at least of these races breed true for several generations.

A study of the races of *L. stagnalis* in other parts of Europe indicates that these or comparable varieties are not always as well defined as they are reported to be in Switzerland. It appears that the condition described by Paiget, although probably correct as far as the observations of fact are concerned, is only a special instance of the extremely wide variation exhibited by this species. In northern Europe it frequently happens that one body of water is inhabited by what might be regarded as essentially one race of *L. stagnalis*. In other instances however, the range of variation among the individuals found in a single lake or pond may be very wide, and as far as certain of the shell characters are concerned, may approach the range of variation which is to be found over the whole of the geographical range of the species.

The condition which exists in Finland is of special interest. Many different bodies of water in that country were examined in the course of this investigation, and a marked tendency was found for the *L. stagnalis* in any single lake or pond to be more or less alike, and recognizably different from those in any other Finnish lake. It therefore appears that there are many local races. It is important to note however, that the intergradation of these
races is much finer than is reported to be the case in Switzerland, and also that the diverse forms do not appear to be specially adapted to the local conditions under which they live.

It is not yet possible to advance a full explanation of the diversity in type variation described above, but since this account is based upon a great many observations there can be little doubt that the general nature of the situation is as stated. It seems to be reasonable to suppose that the root of the diversity lies in multiple factor inheritance and that in some manner not at present understood, a considerable degree of segregation has occurred in some parts of the Sub-Arctic Region, while in others there has been little or none. This matter requires further investigation.

*Lymnaeidae in Isolated Mountain Lakes*

In several different parts of the Sub-Arctic Region there are mountain lakes which by reason of the ruggedness of the surrounding country, and the torrential character of their outlet streams, are isolated from other bodies of water. That is to say there is little or no opportunity of an intermixture of their fauna with that of neighboring lakes. For the most part the fauna of such lakes is meager. In many instances in fact, no mollusca whatever are to be found in them. It is interesting to observe that when these animals do occur in such situations, there is a marked tendency for the inhabitants of each such body of water to resemble closely one another, and to be recognizably different from all the related forms in the surrounding territory. In other words, they form local races. This is true of the *Lymnaea* in Lake Zazurnia situated in the mountain range known as Khamar Daban, to the east of Lake Baikal. The form which inhabits this lake is closely related to *Lymnaea pereger*, but is distinct from it, and has been named *L. zazurnensis*. Similar instances are to be found in the northern Rocky Mountain region of North America. In Amethyst Lake, Alberta, for example, there is an unique
Lymnaea, as yet undescribed, but related to L. palustris and L. traskii.

It is unlikely that any changes of a fundamental nature are involved in the production of "new" forms such as these. It is probable that the particular combination of characters which mark off these races has arisen through the segregation in these isolated localities of multiple genetic factors which are present in the bulk of the population of related forms in other areas but are not much in evidence.

It is possible to place a different interpretation upon the facts outlined here on the basis of a supposed direct influence of the environment upon the form of the shell, etc. On the whole, bearing in mind the wide range of variation of related forms in other localities, and a certain amount of experimental evidence relating to other species (Paigct, loc. cit.), this explanation appears to be an unlikely one.

V. Conclusions

It will be evident from the foregoing account that much remains to be learned about this fauna. Nevertheless sufficient information is available to justify tentative answers being given to the questions mentioned in the Introduction.

1. The molluscan fauna of the Sub-Arctic Region is made of two hundred and twenty-five species and varieties. One hundred and seventy of these inhabit fresh water, and the remaining fifty-five are terrestrial. These species are drawn from a wide variety of different phylogenetic groups. It appears that the fauna is made up to a considerable extent of especially hardy representatives of groups which are found in greater variety in the territories lying further to the south. There are in addition a very few families, such as the Lymnaeidae, which appear to find conditions in this region specially favorable, and occur in great abundance.

2. The geographical affinitics of the Sub-Arctic fauna are as follows: in both Eurasia and North America the
Sub-Arctic fauna is closely akin to that which inhabits the temperate zone portions of each continent respectively; in addition to this there are a number of species which are characteristic of and common to all parts of the Sub-Arctic Region. It appears therefore, that there have been two series of migrations in this region, viz. (i) from south to north in both Eurasia and North America; and (ii) in a circumboreal direction, by way of an isthmus in the neighborhood of the Behring Strait from North America to Eurasia, and vice versa, and also possibly by way of an isthmus in the neighborhood of Greenland.

3. The distribution of both fresh water and terrestrial species within the Sub-Arctic Region is governed largely by climate and the character of the vegetation. Both of these factors have far-reaching direct and indirect effects.

4. There has been some modification of the original molluscan stock since its migration into this region, but the amount of change is small in comparison with that which has been observed in other parts of the world which have a warmer climate than the Sub-Arctic. Some species have undergone little change, while others have been profoundly modified. In certain instances the change appears to have resulted from a rearrangement or segregation of genetic factors which are present in the bulk of the population, rather than a change of a fundamental character. Certain species which attain a considerable size in the temperate zone are represented in the Sub-Arctic by dwarf varieties. A very few totally distinct species appear to have had their origin in the Sub-Arctic Region.

List of References


Large valve Lamigona complanata, Cincinnati, Ohio. Small valve Lamigona complanata, Birdtail Creek, near Birtle, Manitoba. × 1.
Small valve Lampsis ventricosa var. Lake Winnipesaukee, near Melbock, Minnesota. X 1.
Photographs made by the Photographer of the Smithsonian Institution under the direction of the author.
THE DISTRIBUTION OF ELECTRICITY ON TWO CYLINDERS

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Abstract

The distribution of electricity on two infinitely long conducting strips with their edges parallel and lying in the same plane, and on two infinitely long strips in parallel planes at a finite distance apart when there is symmetry about the plane perpendicular to and bisecting them, is determined in this paper. The strips may be of equal or unequal breadths. They may be insulated with any assigned charges on unit lengths and their difference of potential is determined, or they may be at the same potential and the ratio in which a given charge divides between them is found. For strips of equal breadths the force between unit lengths is found when the strips have any charges. The distribution of electricity on the strips when they are placed in a uniform electric field is also determined.

If the two strips have equal and opposite charges the equipotential surfaces will consist of two systems of cylinders, one system surrounding each of the strips. If any pair of these cylinders is taken, one from each of the two systems, they may be considered as conducting cylinders and the distribution of electricity is found on them when they are charged in any manner. There is one system of pairs of cylinders that is of special interest as it is a fair approximation to the pairs of cylinders obtained by dividing a circular cylinder by any plane parallel to the axis and separating the two portions a short distance. The force between two such equal cylinders when they have any charges is found. Finally it is shown how the distribution of electricity on any such pair of cylinders when placed in a uniform electric field may be determined.

1. The capacity of unit length of a condenser consisting of two infinitely long strips of equal breadths in the same plane was found by J. J. Thomson in his Recent Researches. The corresponding problem for two infinitely long strips of the same or different breadths lying in parallel planes at a finite distance apart was solved by Love. The present paper deals with the more general problem when the two strips have any charges; the condenser problem is then a special case where the two charges are equal and opposite. The distribution of the charges when the strips are placed in a uniform electric field is also determined. Finally, a method is given by means


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of which the distribution of electricity on a wide variety of pairs of cylinders may be found. The method here employed is one that was described by Richmond,\textsuperscript{1} and which has recently been applied to a number of problems.\textsuperscript{2}

2. Let there be two infinitely long plane conducting strips in the same plane with their sides parallel to each other. These strips intersect a \( z \)-plane, perpendicular to their lengths, in the lines \( AB \) and \( DC \) (Fig. 1). The breadths of the strips

\[
\begin{array}{c}
\text{y} \\
\hline
D \quad C \quad O \quad A \quad P \quad B \\
\hline \text{x}
\end{array}
\]

Fig. 1

are \( AB = a_1 \), \( DC = a_2 \). \( O \) is the origin of coördinates in the \( z \)-plane. Let \( CA = d \). On account of symmetry we need consider only the upper half of the \( z \)-plane. In order to solve the electrical problems connected with these conductors we shall need to make use of the Schwarz-Christoffel transformation by means of which the interior of any polygon in the plane of a complex variable is transformed to the upper half of a \( t \)-plane so that the contour of the polygon transforms into the real axis of the \( t \)-plane. In the present case it is evident that the relation between the \( z \)- and \( t \)-planes is

\[ z = ft, \]

where \( f \) is a constant giving the scale of the diagram.

The solution of our problems requires the use of elliptic functions. We shall make use of the substitution of the first order,

\[
t = \frac{sn \lambda}{1 - k \, sn \delta \, sn \lambda}, \tag{1}
\]

\textsuperscript{1} Proceedings London Mathematical Society, 22, p. 483, 1923.
\textsuperscript{2} Proceedings American Philosophical Society, LXXV, pp. 11, 549, 1935; LXXXVI, pp. 125, 251, 269, 1936.
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where \( k \) is the modulus of the elliptic functions and \( \delta \) a real
constant lying between \(-K\) and \(+K\), \( K \) being the complete
elliptic integral of the first kind. We can now write

\[
z = \frac{sn \lambda}{1 - k \sn \delta \sn \lambda}.
\]

(2)

Let \( t = -r, -p, 0, n, m \) correspond to the points \( D, C, O, A, B \) in Fig. 1. With the corresponding values,

\[
\begin{align*}
t &= m \\
n &= 0 \\
\lambda &= K + iK' \\
K &= 0 \\
- K &= K + iK'
\end{align*}
\]

we find

\[
\begin{align*}
OB &= f_m = \frac{f}{k(1 - \sn \delta)}, & OC &= f_p = \frac{f}{1 + k \sn \delta}, \\
OA &= f_n = \frac{f}{1 - k \sn \delta}, & OD &= fr = \frac{f}{k(1 + \sn \delta)}.
\end{align*}
\]

(3)

The infinitely distant point of the \( z \)-plane corresponds to
\( t = \infty \), \( \lambda = iK' + \delta \), and the origin to \( t = \lambda = 0 \). We find
at once

\[
\begin{align*}
a_2 &= \frac{1 - \sn \delta}{1 + k \sn \delta} \frac{1 - \sn \delta}{1 + k \sn \delta}, \\
ad &= \frac{2k}{1 - k} \frac{1 - \sn \delta}{1 + k \sn \delta}, \\
a_1 &= \frac{2k}{1 - k} \frac{1 + \sn \delta}{1 - k \sn \delta}, \\
d &= \frac{2k}{1 - k} \frac{1 + \sn \delta}{1 - k \sn \delta}
\end{align*}
\]

(4)

and

\[
\frac{DB \cdot CA}{DA \cdot CB} = \frac{4k}{(1 + k)^2}.
\]

If \( \delta = 0 \), the two strips have equal breadths.

3. Let \( Q_1 \) and \( Q_2 \) be the charges on unit lengths of the
strips \( AB \) and \( CD \) and \( V \) the difference of potential between
them. If the complex potential be

\[
\chi = \varphi + i\psi,
\]

with \( \varphi \) as the real potential, the \( \chi \)-plane is shown in Fig. 2,
where we assume that $AB$ is at the higher potential. $t = -s$ corresponds to the point of equilibrium. It is shown as being on $CD$, but of course its position depends upon the relative charges on the two conductors. The differential equation for transforming from the $\chi$- to the $t$-plane is

$$\frac{d\chi}{dt} = \frac{C(t + s)}{|t + r \cdot t + p \cdot t - n \cdot t - m|^3}. \quad (5)$$

There is a jump of $-2\pi i (Q_1 + Q_2)$ in passing from $t = +\infty$ to $t = -\infty$, and so we get

$$C = -2(Q_1 + Q_2).$$

Using the substitution (1), we find the indefinite integrals,

$$\int \frac{dt}{|t + r \cdot t + p \cdot t - n \cdot t - m|^3} = -k \frac{cn \delta \ dn \delta \lambda}{}, \quad (6)$$

$$\int \frac{tdt}{|t + r \cdot t + p \cdot t - n \cdot t - m|^3}$$

$$= -\Pi(\lambda, \delta) + i \sin^{-1} \left( \frac{dn \delta \ dn \lambda}{k'[1 - k^2 sn^2 \delta sn^2 \lambda]} \right), \quad (7)$$

where $\Pi(\lambda, \delta)$ is Jacobi's form of the third elliptic integral,

$$\Pi(\lambda, \delta) = k^2 \ sn \delta \ cn \delta \ dn \delta \int_0^1 \frac{sn^2 \lambda d\lambda}{1 - k^2 \ sn^2 \delta sn^2 \lambda}.$$
The solution of (5) satisfying the conditions
\[
  t = \frac{m}{n} - p - r \\
  \lambda = K + iK' - K - K + iK' \\
  \chi = V + 2\pi i(Q_1 + Q_2) - V - 2\pi iQ_2 - 2\pi iQ_2
\]
is
\[
  \chi = 2(Q_1 + Q_2) \left\{ \Pi(\lambda, \delta) - Z(\delta) \cdot \lambda - \frac{\pi \delta}{2KK'} \lambda - \frac{\pi \delta}{2K'} \right. \\
  \left. - i \sin^{-1} \left[ \frac{dn \delta dn \lambda}{k'(1 - k^2 sn^2 \delta sn^2 \lambda)} \right] \right\} + \frac{\pi}{K'} (Q_1 - Q_2) \lambda + i\pi (3Q_2 + Q_1) + \frac{\pi K}{K'} (Q_1 - Q_2).
\]

The difference of potential between the two strips is given by
\[
  V = \frac{2\pi K}{K'} (Q_1 - Q_2) - 2(Q_1 + Q_2) \frac{\pi \delta}{K'}.
\]
The point of equilibrium, \( t = -s \), is given by
\[
  s = \frac{1}{k cn \delta dn \delta} \left\{ \frac{\pi (Q_1 - Q_2)}{2K'(Q_1 + Q_2)} - Z(\delta) - \frac{\pi \delta}{2KK'} \right\}.
\]

If the two strips are at the same potential, \( V = 0 \), and we get from (9),
\[
  \frac{Q_1 - Q_2}{Q_1 + Q_2} = \delta
\]

or the charges are in the ratio,
\[
  \frac{Q_1}{Q_2} = \frac{1 + \frac{\delta}{K}}{1 - \frac{\delta}{K}}.
\]

If the two strips form a condenser so that \( Q_1 = -Q_2 = Q \), the difference of potential is
\[
  V = \frac{4\pi K}{K'} Q.
\]
and so the capacity of unit length of the condenser is

\[ S = \frac{K'}{4\pi K}. \tag{12} \]

In this case the complex potential (8) becomes

\[ \chi = \frac{2\pi Q}{K'} \lambda + \frac{2\pi K}{K'} Q - 2\pi iQ. \tag{13} \]

4. In order to find the surface density of the distribution of electricity on the conductors we need to evaluate

\[ \frac{d\chi}{dz} = \frac{d\chi}{dt} \frac{dt}{dz} = -X + iY, \tag{14} \]

\(X, Y\) being the components of the electric intensity. Now we have

\[
\frac{d\chi}{dz} = -\frac{2(Q_1 + Q_2)}{f} \left( \frac{t + s}{t + r \cdot t + p \cdot t - n \cdot t - m} \right)^i \\
= \frac{2(Q_1 + Q_2)}{f \cdot cn \cdot \lambda \cdot dn \cdot \lambda} \left( 1 - k \cdot sn \cdot \delta \cdot sn \cdot \lambda \right) \left\{ k \cdot cn \cdot \delta \cdot dn \cdot \delta \cdot sn \cdot \lambda \\
+ (1 - k \cdot sn \cdot \delta \cdot sn \cdot \lambda) \left[ \frac{\pi(Q_1 - Q_2)}{2(Q_1 + Q_2)K'} - Z(\delta) - \frac{\pi \delta}{2KK'} \right] \right\}. \tag{15}
\]

On the strip \(AB\), \(\lambda = K + i\nu\), where \(\nu\) varies from \(O\) at \(A\) to \(K'\) at \(B\), and

\[ sn \cdot \lambda = \frac{1}{dn' \cdot \nu}, \quad cn \cdot \lambda = -\frac{i k' \cdot sn' \cdot \nu}{dn' \cdot \nu}, \quad dn \cdot \lambda = \frac{k' \cdot cn' \cdot \nu}{dn' \cdot \nu}. \]

The accents on the elliptic functions mean that the complementary modulus, \(k'\), is to be used. So we get \(X = 0\), and as the surface density, \(\sigma\), is given by \(Y = 4\pi \sigma\), we find on \(AB\),

\[ \sigma = \frac{(Q_1 + Q_2)(dn' \cdot \nu - k \cdot sn \cdot \delta)}{2\pi f k'^2 sn' \cdot \nu \cdot cn' \cdot \nu} \left\{ k \cdot cn \cdot \delta \cdot dn \cdot \delta \\
+ (dn' \cdot \nu - k \cdot sn \cdot \delta) \left[ \frac{\pi(Q_1 - Q_2)}{2K'(Q_1 + Q_2)} - Z(\delta) - \frac{\pi \delta}{2KK'} \right] \right\}. \]

If \(Q_1 + Q_2 = 0\), so that the two strips form a condenser, the
surface density on $AB$ is given by
\[ \sigma = \frac{Q(dn'v - k\ sn\ \delta)^2}{2fk'K' \ sn'v \ cn'v}. \]

The relation between $v$ and $x$, the abscissa of any point on the strip, is given by
\[ x = \frac{f}{dn'v - k\ sn\ \delta} = \frac{ka_1(1 - sn\ \delta)(1 - k\ sn\ \delta)}{(1 - k)(dn'v - k\ sn\ \delta)}. \]

When $\delta = 0$, so that the two strips have equal breadths, we find
\[ \sigma = \frac{Qd}{4kJ'(CP \cdot AP \cdot BP \cdot DP)^{1/3}}. \]

This agrees with the result given in the Recent Researches.

5. We shall next find the force between unit lengths of the two strips in the same plane. We see that by symmetry this force is along the $x$-axis. If we regard this force as resulting from the stress, $2\pi\sigma^2$, integrated over one of the strips, it is evident that the only contribution comes from the edges where the surface density is infinite. But we can get over this difficulty by calculating the surface integral of the Maxwellian stress $(X^2 - Y^2)/8\pi$, taken over unit length of the plane of which the $y$-axis is the trace in the $z$-plane. For strips of equal breadths, $\delta = 0$, and the calculation is very simple. Writing $\lambda = u + iv$, we have, when $x = 0$,
\[ y = f \frac{sn'v}{cn'v} \]
and $v = 0$ for $y = 0$, $v = K'$ for $y = \infty$. Further,
\[ dy = f \frac{dn'v}{cn'^2v} dv. \]

Now from (14) and (15) we find for $\delta = 0$, $\lambda = iv$,
\[ X = -\frac{(Q_1 - Q_2)\pi \ cn'^2v}{K'f \ dn'v}, \]
\[ Y = \frac{2(Q_1 + Q_2)k \ sn'v \ cn'v}{f}. \]
The force between the two equal strips in the same plane is therefore

\[ R = \frac{\pi (Q_1 - Q_2)^2}{4fK' \sqrt{2}} \int_{0}^{K'} \frac{cn^2 v}{dn'} v \, dv - \frac{(Q_1 + Q_2)^2 k^2}{\pi f} \int_{0}^{K'} \frac{sn^2 v}{dn'} v \, dv. \]

Now

\[ \int_{0}^{K'} \frac{cn^2 v}{dn'} v \, dv = \frac{\pi}{2(1 + k)}, \]

\[ \int_{0}^{K'} \frac{sn^2 v}{dn'} v \, dv = \frac{\pi}{2k(1 + k)}, \]

and so

\[ R = \frac{\pi^2 (Q_1 - Q_2)^2}{8fK'^2(1 + k)} - \frac{k(Q_1 + Q_2)^2}{2f(1 + k)}. \]

If \( a \) is the breadth of each strip and \( d \) the distance between their adjacent edges, we have from (3)

\[ d = 2f, \quad k = \frac{d}{2a + d}, \]

so that

\[ R = \frac{\pi^2 (Q_1 - Q_2)^2}{8K'^2} \left(2a + d\right) - \frac{(Q_1 + Q_2)^2}{2(a + d)}. \]

If \( a \) is very small compared with \( d \), the two strips approximate to parallel line charges, \( Q_1 \) and \( Q_2 \) per unit length, at a distance \( d \) apart. In this case \( k \) approaches unity and \( K' \) approaches \( \pi/2 \). Therefore \( R \) approaches the value

\[ R = -\frac{2Q_1Q_2}{d}, \]

a known result.

6. Let now the two unequal strips in the same plane be placed in a uniform electric field along the \( x \)-axis; we shall find the charges induced on them, and their difference of potential, when they are insulated. The \( x \)-plane is shown in Fig. 3, and the differential equation for transforming to the \( t \)-plane is

\[ \frac{dX}{dt} = \frac{C(t + s')(t - s')}{\{t + r \cdot t + p \cdot t - n \cdot t - m\}^3}. \quad (16) \]
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In order to integrate this equation we use the substitution (1), and we have, in addition to the integrals (6) and (7),

\[ \int \frac{t^{2} dt}{\left[ t + r \cdot t + p \cdot t - n \cdot t - m \right]^3} \]

\[ = - \frac{1}{k \ cn \ \delta \ dn \ \delta} \left( \frac{dn \ \delta}{dn \ \delta} - \frac{E}{K} \right) \lambda \]

\[ - \frac{sn \ \delta}{k \ cn \ \delta \ dn \ \delta} \left( k \ cn \ \delta + dn \ \delta \right) \]

\[ \times \left\{ \Pi(\lambda, \ \delta) - i \ sin^{-1} \left( \frac{dn \ \delta}{dn \ \delta} \right) \right\} \]

\[ + \frac{1}{k \ cn \ \delta \ dn \ \delta} \left\{ Z(\lambda) - \frac{k \ sn \ \delta \ cn \ \lambda \ dn \ \lambda}{1 - k \ sn \ \delta \ sn \ \lambda} \right\}. \quad (17) \]

The solution of (16) satisfying the conditions,

\[ \lambda = K + iK' \quad K \quad -K \quad -K + iK' \]

\[ \chi = \begin{bmatrix} V \\ V \\ 0 \\ 0 \end{bmatrix} \]

is

\[ \chi = \frac{C}{k \ cn \ \delta \ dn \ \delta} \left\{ \frac{\pi}{2KK'}(K + \lambda) + Z(\lambda) - \frac{k \ sn \ \delta \ cn \ \lambda \ dn \ \lambda}{1 - k \ sn \ \delta \ sn \ \lambda} \right\}. \quad (18) \]

The two points of equilibrium, one on each conductor, correspond to \( t = s' \) and \( t = -s'' \), and \( s', s'' \) are determined by the two equations,

\[ s' - s'' = \frac{sn \ \delta \left( k^2 \ cn^2 \ \delta + dn^2 \ \delta \right)}{k \ cn^2 \ \delta \ dn^2 \ \delta}, \]

\[ s's'' = \frac{E' - k^2K' \ sn^2 \ \delta}{k^2K' \ cn^2 \ \delta \ dn^2 \ \delta}. \]

(19)
If \( \delta = 0 \) the two strips are of equal breadth and
\[
s' = s'' = \frac{1}{k} \left( \frac{E'}{K'} \right)^i.
\]
(20)

The difference of potential between the two strips is determined by
\[
V = \frac{\pi C}{kK' \text{cn} \delta \text{dn} \delta}.
\]

7. The constant \( C \) must be determined so as to make the potential at infinity, \( \varphi = Fx + \text{const.} \), where \( F \) is the intensity of the uniform field. At the point \( z = \infty \), \( \lambda = iK' + \delta \). Put
\[
\lambda = iK' + \delta + \epsilon.
\]
Then we find from (18) that \( \chi/C \) approaches the value, when \( \epsilon \) approaches zero,
\[
\frac{\chi}{C} = \frac{1}{k \epsilon \text{cn} \delta \text{dn} \delta} + \text{const.}
\]
From (2) we find that when \( \epsilon \) approaches zero,
\[
x = \frac{1}{k \epsilon \text{cn} \delta \text{dn} \delta}.
\]
Hence it follows that
\[
C = Ff
\]
for a field in the direction of the negative \( x \)-axis. The difference of potential between the two strips may now be put in the form
\[
V = \frac{\pi F d}{2kK' \text{cn} \delta} = \frac{\pi F}{K'(1 - k)} (a_1 a_2)^i,
\]
where \( d \) is the distance between the adjacent edges of the two strips of breadths \( a_1 \) and \( a_2 \).

8. If the two strips are at the same potential in the field along the \( x \)-axis the charges induced on them will be equal and opposite. Let these be \( \pm Q \) per unit length. The \( x \)-plane is now different from that shown in Fig. 3, but the
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differential equation for transforming it to the $t$-plane is the same as (16), with of course different values for $s'$ and $s''$. The solution of (16) subject to the conditions

$$\lambda = K + iK' \quad K \quad - K \quad - K + iK'$$

$$\chi = 0 \quad 2\pi iQ \quad 2\pi iQ \quad 0$$

is

$$\chi = \frac{C}{k \, cn \, \delta \, dn \, \delta} \left[ Z(\lambda) - \frac{k \, sn \, \delta \, cn \, \lambda \, dn \, \lambda}{1 - k \, sn \, \delta \, sn \, \lambda} \right] + 2\pi iQ, \quad (21)$$

with

$$Q = \frac{C}{4kk \, cn \, \delta \, dn \, \delta}.$$ 

and

$$s' - s'' = \frac{sn \, \delta}{k \, cn^2 \, \delta \, dn^2 \, \delta} \left( k^2 \, cn^2 \, \delta + dn^2 \, \delta \right),$$

$$s's'' = \frac{1}{k^2 \, cn^2 \, \delta \, dn^2 \, \delta} \left( dn^2 \, \delta - \frac{E}{K} \right). \quad (22)$$

The constant $C$, found in the same way as before, is $C = Ff$. The equal and opposite charges induced on unit lengths of the two strips are now given by $\pm \, Q$, where

$$Q = \frac{Ff}{4kK \, cn \, \delta \, dn \, \delta} = \frac{Fd}{8kK \, cn \, \delta} = \frac{F}{4K} \frac{(a_1 a_2)^3}{(1 - k)}.$$ 

The electric intensity at any point of the $z$-plane and the surface density of the distribution of the induced charges on the conductors are found just as in Art. 4. We now have

$$\frac{dx}{dz} = - X + iY = - \frac{Fk \, cn \, \delta \, dn \, \delta}{cn \, \lambda \, dn \, \lambda} \left\{ sn^2 \, \lambda - (s' - s'') \right\} \times (1 - k \, sn \, \delta \, sn \, \lambda) \,

If the two strips are insulated the values of $s'$, $s''$ given by (19) are to be used, while if the two strips are at the same potential we use (22). If the breadths of the two strips are equal we find for the surface density on $AB$, in the case of the insulated strips,

$$\sigma = \frac{Fk}{4\pi k'^2 \, sn' \, v \, cn' \, v} \left[ 1 - \frac{E'}{k'^2 K \, dn'^2 \, v} \right].$$
If the two strips are at the same potential we find

\[ \sigma = \frac{F}{4\pi kk'^2 sn' v cn' v} \left\{ \frac{E}{K} dn'^2 v - k'^2 cn'^2 v \right\}. \]

The coordinate \( x \) of any point on the strip is given by

\[ x = \frac{f}{dn' v} = \frac{ka_1}{(1 - k)dn' v}. \]

9. In the second problem we shall take the two unequal, infinitely long parallel strips in planes at a distance \( d \) apart. The \( z \)-plane is shown in Fig. 4. Let \( OE = d_1 \), \( OF = d_2 \),

\[ FE = d = d_1 + d_2, \quad EA = EB = a_1, \quad FC = FD = a_2. \]

The upper half of the \( z \)-plane is transformed to the \( t \)-plane by means of the differential equation,

\[ \frac{dz}{dt} = \frac{A(t + s')(t - s')}{\{t + r\cdot t + p \cdot t - n \cdot t - m\}^i}. \tag{23} \]

This equation is of the same form as (16), and its solution
satisfying the conditions
\[ \lambda = K + iK' \quad K \quad -K \quad -K + iK' \]
\[ z = \frac{d}{2K} \lambda + \frac{K'd}{\pi} \left\{ Z(\lambda) - \frac{k \text{sn} \delta \text{cn} \lambda \text{dn} \lambda}{1 - k \text{sn} \delta \text{sn} \lambda} + \frac{d_1 - d_2}{2} \right\} \]
with
\[ A = \frac{K'kd \text{cn} \delta \text{dn} \delta}{\pi} \]
s' and s'' are again given by (19).

On EA, \( \lambda = K + iv \), \( z = d_1 + iv_1 \), and on FC, \( \lambda = -K + iv \), \( z = -d_2 + iv_2 \). We therefore get from (24)
\[
\begin{align*}
y_1 &= \frac{K'd}{\pi} \left\{ \frac{k^2 \text{sn}' v \text{cn}' v}{dn' v - k \text{sn} \delta} - Z(v, k') \right\}, \\
y_2 &= \frac{K'd}{\pi} \left\{ \frac{k^2 \text{sn}' v \text{cn}' v}{dn' v + k \text{sn} \delta} - Z(v, k') \right\}.
\end{align*}
\]
Let \( v_1 \) and \( v_2 \) be the values of \( v \) corresponding to the edges A and C respectively. \( v_1 \) and \( v_2 \) may be determined by the conditions that \( y_1 \) and \( y_2 \), given by (25), be maxima. Or we may use the values of s' and s'' given by (19). Putting
\[
\begin{align*}
A &= \frac{\text{sn} \delta}{k \text{cn}^2 \delta \text{dn}^2 \delta} (k^2 \text{cn}^2 \delta + \text{dn}^2 \delta), \\
B &= \frac{E' - k^2K' \text{sn}^2 \delta}{k^2K' \text{cn}^2 \delta \text{dn}^2 \delta'}
\end{align*}
\]
we find
\[
\begin{align*}
\frac{1}{s'} &= \frac{(A^2 + 4B)^{\frac{1}{2}} - A}{2B}, \\
\frac{1}{s''} &= \frac{(A^2 + 4B)^{\frac{1}{2}} + A}{2B}.
\end{align*}
\]
The signs of the radicals are chosen so as to make, as they must be, s' and s'' positive. Since s' and -s'' are the values of t at A and C, we get from (1),
\[
\begin{align*}
dn' v_1 &= c_1 = k \text{sn} \delta + \frac{(A^2 + 4B)^{\frac{1}{2}} - A}{2B}, \\
dn' v_2 &= c_2 = k \text{sn} \delta + \frac{(A^2 + 4B)^{\frac{1}{2}} + A}{2B}.
\end{align*}
\]
If $\delta = 0$, the two strips have equal breadths, $a$. Then we have $A = 0$, $B = E'/k^2K'$, $v_1 = v_2 = \alpha$,

\[ dn' \alpha = k' \left( \frac{K'}{E'} \right)^i, \]  
and

\[ a = \frac{K'd}{\pi} \left\{ \frac{k'^2 \sin' \alpha}{dn' \alpha} - \frac{Z(\alpha, k')}{} \right\}. \]  

For large values of $a/d$ the modular angle is very small and rapidly converging series may be used. We may write (29).

\[ \frac{\pi}{K'} \frac{a}{d} = \frac{\pi \alpha}{2KK'} - iZ(K + i\alpha). \]

Using the Fourier expansion for the Zeta function, this gives

\[ \frac{a}{d} = \frac{\alpha}{2K} + \frac{2K'}{K} \sum_{1}^{\infty} (-1)^n \frac{q^n}{1 - q^{2n}} \sinh \frac{n\pi \alpha}{K}. \]

Similarly, (28) gives

\[ 1 = -\frac{4\pi K'}{K} \sum_{1}^{\infty} (-1)^n \frac{nq^n}{1 - q^{2n}} \cosh \frac{n\pi \alpha}{K}. \]

For very small modular angles, with $k = 10^{-3}$ and less, we may put

\[ K = \frac{\pi}{2}, \]
\log K' = \log \left( \log \frac{4}{K} \right) + 0.3622157 \ldots

(the logarithms are to the base 10), and

\[ q = \frac{k^2}{16}. \]

For these small modular angles it will be found that $2\alpha$ is so large that we may put

\[ \cosh 2\alpha = \sinh 2\alpha = \frac{1}{2} e^{2\alpha}. \]

Assuming this for the present, and since $q^{2n}$ is negligible in comparison with unity, we have as the equation, for de-
termining $\alpha,$

$$\frac{1}{4K'} = \frac{qe^{2a}}{(1 + qe^{2a})^2}.$$

Solving this quadratic equation, remembering that $\alpha$ must be less than $K',$ we get

$$qe^{2a} = 2K' - 1 - [4K'(K' - 1)]^\frac{1}{2}.$$  \hfill (32)

Equation (30) may now be written

$$\frac{a}{d} = \frac{\alpha \pi}{\pi} - \frac{2K'}{1 + qe^{2a}}.$$  \hfill (33)

The following table gives the results of the calculation for values of $k$ ranging from $10^{-3}$ to $10^{-12}.$

<table>
<thead>
<tr>
<th>$k$</th>
<th>$\log K'$</th>
<th>$qe^{2a}$</th>
<th>$\alpha$</th>
<th>$a/d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^{-3}$</td>
<td>0.9187667</td>
<td>0.0321054</td>
<td>6.574670</td>
<td>1.9285</td>
</tr>
<tr>
<td>$10^{-4}$</td>
<td>1.0251680</td>
<td>0.0247732</td>
<td>8.747522</td>
<td>2.6213</td>
</tr>
<tr>
<td>$10^{-5}$</td>
<td>1.1105635</td>
<td>0.0201668</td>
<td>10.947360</td>
<td>3.3223</td>
</tr>
<tr>
<td>$10^{-6}$</td>
<td>1.1818952</td>
<td>0.0170127</td>
<td>13.164909</td>
<td>4.0286</td>
</tr>
<tr>
<td>$10^{-7}$</td>
<td>1.2431470</td>
<td>0.0147035</td>
<td>15.394557</td>
<td>4.7387</td>
</tr>
<tr>
<td>$10^{-8}$</td>
<td>1.2968182</td>
<td>0.0129542</td>
<td>17.638209</td>
<td>5.4518</td>
</tr>
<tr>
<td>$10^{-9}$</td>
<td>1.3445801</td>
<td>0.0115680</td>
<td>19.879803</td>
<td>6.1670</td>
</tr>
<tr>
<td>$10^{-10}$</td>
<td>1.3876060</td>
<td>0.0104570</td>
<td>22.131898</td>
<td>6.8840</td>
</tr>
<tr>
<td>$10^{-11}$</td>
<td>1.4267508</td>
<td>0.0095325</td>
<td>24.388208</td>
<td>7.6024</td>
</tr>
<tr>
<td>$10^{-12}$</td>
<td>1.4626573</td>
<td>0.0087584</td>
<td>26.644842</td>
<td>8.3221</td>
</tr>
</tbody>
</table>

Returning to the general case when $\delta$ is not zero and so the two strips have unequal breadths, we may assume values for $\delta$ and the modular angle and so calculate $A$ and $B$ by means of the equations (26). The equations (27) then give the values of $dn' v_1$ and $dn' v_2,$ and so $v_1$ and $v_2$ may be found. The values of the ratios $a_1/d$ and $a_2/d$ are now given by using (25). For very small modular angles the Fourier expansions of the elliptic functions lead to comparatively simple results. Since

$$dn' v = \frac{1}{sn(K + iv)}.$$
we get, from the known expansion of $\eta s u$,

$$dn' v = \frac{\pi}{2K} \text{sech} \frac{\pi v}{2K} - \frac{2\pi}{K} \sum_{n=1}^{\infty} (-1)^n \frac{q^{2n} - 1}{1 - q^{2n} - 1} \cosh (2n - 1) \frac{\pi v}{2K}.$$ 

Assuming that $v_1$ and $v_2$ are so large that we may write

$$\cosh v = \frac{1}{2} e^v,$$

we get, with $K = \frac{\pi}{2}$,

$$c_1 = dn' v_1 = 2e^{-v_1} + \frac{2qe^{v_1}}{1 + q^2e^{2v_1}}.$$ 

Now $v_1$ and $v_2$ are each less than $K'$ so that $q^2e^{2v_1}$ and $q^2e^{2v_2}$ are negligible compared with unity. So we find

$$e^{v_1} = \frac{4c_1}{k^2} \left\{ 1 - \left(1 - \frac{k^2}{c_1^2}\right)^{\frac{1}{2}} \right\},$$

$$e^{v_2} = \frac{4c_2}{k^2} \left\{ 1 - \left(1 - \frac{k^2}{c_2^2}\right)^{\frac{1}{2}} \right\}. \quad (34)$$

In equations (25) for very small modular angles we may put

$$dn' v_1 = c_1, \quad sn' v_1 = 1, \quad cn' v_1 = c_1 \left(1 - \frac{k^2}{c_1^2}\right)^{\frac{1}{2}},$$

$$dn' v_2 = c_2, \quad sn' v_2 = 1, \quad cn' v_2 = c_2 \left(1 - \frac{k^2}{c_2^2}\right)^{\frac{1}{2}}.$$ 

The Fourier expansion for $Z(v, k')$ is

$$Z(v, k') = \frac{\pi}{2K} \left\{ \tanh \frac{\pi v}{2K} - \frac{v}{K'} - 4 \sum_{n=1}^{\infty} (-1)^n \frac{q^{2n}}{1 - q^{2n}} \sinh \frac{n\pi v}{K} \right\}.$$ 

For very small modular angles, and for large values of $v$, this reduces to

$$Z(v, k') = 1 - \frac{v}{K'}.$$
So we have

\[
\begin{align*}
\frac{a_1}{d} &= \frac{K'}{\pi} \left\{ \frac{v_1}{K'} - 1 + \left(1 - \frac{k^2}{c_1^2}\right)^{\frac{1}{2}} \left(1 - \frac{k \cdot \text{sn} \delta}{c_1}\right)^{-1}\right\}, \\
\frac{d}{a_2} &= \frac{K'}{\pi} \left\{ \frac{v_2}{K'} - 1 + \left(1 - \frac{k^2}{c_2^2}\right)^{\frac{1}{2}} \left(1 + \frac{k \cdot \text{sn} \delta}{c_2}\right)^{-1}\right\}.
\end{align*}
\] (35)

If values of \(k\) and \(\delta\) are assumed, (26) and (27) determine \(c_1\) and \(c_2\). \(v_1\) and \(v_2\) are then determined by (34), after which (35) gives the ratios \(a_1/d\) and \(a_2/d\).

As an example, take \(\text{sn} \delta = 1/\sqrt{2}\), or, practically, \(\delta = K/2\), with \(k = 10^{-12}\). We find

\[\begin{align*}
\log K' &= 1.4626573 \\
\log c_1 &= 12.1434500 \\
\log c_2 &= 11.3406371 \\
\log v_1 &= 1.4496194 \\
\log v_2 &= 1.4020502
\end{align*}\]

and

\[\begin{align*}
a_1/d &= 12.786 \\
a_2/d &= 7.726
\end{align*}\]

and therefore,

\[a_1/a_2 = 1.658.\]

Larger values of \(\delta\) increase this ratio while smaller values bring it closer to unity.

10. When the two strips in parallel planes have charges \(Q_1\) and \(Q_2\) on unit lengths, at a difference of potential \(V\), the \(X\)-plane is the same as in Art. 3, and all of the results given in that article are valid in this case. The electric intensity is given by (14). We now get, combining (5) and (16),

\[- X + iY = - \frac{2(Q_1 + Q_2)\pi}{K'kd \ 	ext{cn} \ \delta \ \text{dn} \ \delta \ l^2 - (s' - s'')t - s's''}.\] (36)

\(s\) is given by (10) and \(s', s''\) by (19).

In the particular case \(\delta = 0\), the breadths of the two strips are equal; and if \(Q_1 + Q_2 = 0\), so that the strips form a con-
denser, we find on \( E'AE \) (Fig. 4) where \( \lambda = K + iv \),
\[
X = \frac{2Q\pi}{K'k^2d} \cdot \frac{dn'^2v}{1 - \frac{E'}{k^2K'}dn'^2v}.
\]
The surface density on this strip is given by
\[
\sigma = \frac{1}{4\pi} \mid X \mid.
\]
The capacity of unit length of a condenser consisting of two strips in parallel planes is given by (12) and depends therefore only upon the modular angle. (12) may be written
\[
S = \frac{1}{4\pi^2} \log \frac{1}{q},
\]
and for very small modular angles, when the ratio of the breadths to the distance apart is large, this is
\[
S = \frac{1}{2\pi^2} \log \frac{4}{k},
\]
where the logarithm is to the base \( e \). We observe from the table given in Art. 9, and from the calculation at the end of that article, that the capacity of a condenser consisting of two equal strips such that the ratio of their breadths to the distance apart is 16.323 is the same as that of a condenser consisting of two unequal strips such that these ratios are
\[
\frac{2a_1}{d} = 25.572, \quad \frac{2a_2}{d} = 15.452.
\]
This capacity is
\[
S = \frac{1}{2\pi^2} \log 4 \cdot 10^{12} = 1.470.
\]
When the two equal strips are at the same potential, so that \( Q_1 = Q_2 = Q \), we find on \( E'AE \),
\[
X = \frac{4Q\pi}{K'kd} \cdot \frac{dn'v}{1 - \frac{E'}{k^2K'}dn'^2v}.
\]
It follows that the surface densities at the points \( E' \) and \( E \) are in the ratio,

\[
\frac{k(K' - E')}{(E' - k^2K')}.
\]

which is very small for strips near together in comparison with their breadths. In the case of the condenser the corresponding ratio is

\[
\frac{K' - E'}{E' - k^2K'}.
\]

11. The force between unit lengths of two strips in parallel planes with any charges may be found by the same method as that used in the case of two strips in the same plane (Art. 5). When the two strips have equal breadths, \( \delta = 0 \), and we get from (36) for the components of the electric intensity in the plane \( x = 0 \), where \( \lambda = iv \),

\[
X = - \frac{\pi^2(Q_1 - Q_2)}{k^2K'^2d} \frac{cn'^2 v}{sn'^2 v + c^2 cn'^2 v'}
\]

\[
Y = \frac{2\pi(Q_1 + Q_2)}{kK'd} \frac{sn' v cn' v}{sn'^2 v + c^2 cn'^2 v}
\]

with

\[
c^2 = \frac{E'}{k^2K'}.
\]

From (24) when \( \lambda = iv \),

\[
y = \frac{K'd}{\pi} \left\{ \frac{sn' v dn' v}{cn' v} - Z(v, k') \right\}
\]

and

\[
dy = \frac{k^2K'd sn'^2 v + c^2 cn'^2 v}{cn'^2 v} dv.
\]

Then the force between the two strips is

\[
R = \frac{1}{4\pi} \int_{-\infty}^{\infty} (X^2 - Y^2) dy,
\]
or
\[
R = \frac{\pi^2(Q_1 - Q_2)^2}{4k^2K'^2d} \int_0^{K'} \frac{cn^2 v dv}{c^2 - (c^2 - 1) sn^2 v} - \frac{(Q_1 + Q_2)^2}{K'd} \int_0^{K'} \frac{sn^2 v dv}{c^2 - (c^2 - 1) sn^2 v}
\]

We find
\[
\int_0^{K'} \frac{cn^2 v}{c^2 - (c^2 - 1) sn^2 v} dv = K' dn'^2 \alpha \left\{ 1 - \frac{dn' \alpha}{k'^2 sn' \alpha cn' \alpha} Z(\alpha, k') \right\},
\]
\[
\int_0^{K'} \frac{sn^2 v}{c^2 - (c^2 - 1) sn^2 v} dv = \frac{K'}{k'^2} \frac{dn' \alpha}{sn' \alpha cn' \alpha} Z(\alpha, k').
\]

By (28) we have
\[
dn' \alpha = \frac{1}{c} = k \left( \frac{K'}{E'} \right)^4.
\]

Accordingly, the force between unit lengths of the two strips is
\[
R = \frac{\pi^2(Q_1 - Q_2)^2}{4k^2K'^2d} \ dn'^2 \alpha \left\{ 1 - \frac{dn' \alpha}{k'^2 sn' \alpha cn' \alpha} Z(\alpha, k') \right\}
- \frac{(Q_1 + Q_2)^2}{k'^2d} \frac{dn' \alpha}{sn' \alpha cn' \alpha} Z(\alpha, k').
\]

For large modular angles the ratio \(a/d\) is very small so that in the limit when \(k\) approaches unity the two strips approach two parallel line charges, \(Q_1\) and \(Q_2\), per unit length, at a distance \(d\) apart. Now we have
\[
Z(\alpha, k') = \frac{2\pi}{K'} \sum_i \frac{q'^n}{1 - q'^n \sin \frac{n\pi\alpha}{K'}}.
\]

When \(k\) approaches unity, \(q'\) approaches zero and \(K'\) approaches \(\pi/2\). So keeping terms in \(q'\) only we get
\[
Z(\alpha, k') = 8q' \sin \alpha \cos \alpha.
\]
We can put in this case,

$$dn'\alpha = 1, \quad sn'\alpha = \sin \alpha, \quad cn'\alpha = \cos \alpha, \quad \text{and} \quad q' = k'^2/16.$$ 

So we get

$$\frac{dn'\alpha}{k'^2 \cdot sn'\alpha \cdot cn'\alpha} Z(\alpha, k') = \frac{1}{2},$$

and therefore,

$$R = -\frac{2Q_1Q_2}{d}.$$ 

This is the known attraction between two line charges $Q_1$ and $Q_2$ at a distance $d$ apart as we found from the case of going to the limit from two strips in the same plane.

For very small modular angles the ratio $a/d$ is large. If we have a parallel plate condenser and neglect end effects, assuming that the charges are uniformly distributed over the adjacent surfaces of the plates, the attraction between two plates of breadth $2a$ is, per unit length,

$$R' = \frac{\pi Q^2}{a},$$

$Q$ being the charge on a unit length of the positive plate. For very small modular angles we may put

$$Z(\alpha, k') = \tanh \alpha - \frac{\alpha}{K'}$$

and as we have seen in Art. 6 that $\alpha$ approaches $K'$ as $k$ is diminished, it follows that $Z(\alpha, k')$ approaches zero. And as $a/d$ approaches the value $\alpha/\pi$ we see that the force between the two equal strips when $Q_1 = -Q_2 = Q$ tends to the limit,

$$R = \frac{\pi Q^2}{a} = R'.$$

12. In a uniform electric field along the $x$-axis all planes perpendicular to this axis are equipotential surfaces, and so any portions of them may be made conducting surfaces. Therefore the difference of potential between the two uncharged strips will be $V = Fd$, where $F$ is the constant
intensity of the electric field; the surface density of the charges
induced on the strips is \( \sigma = \pm F/4\pi \), being equal and opposite
on the two sides of each strip.

But if the two strips are at the same potential, the complex
potential is given by (21) but of course with a different value
for the constant \( C \) than in Art. 8. We now have, combining
(16) and (23),

\[
\frac{dX}{dz} = \frac{C\pi}{K'kd} \frac{t + s_2 \cdot t - s_1}{\text{cn} \, \delta \, \text{dn} \, \delta \, \text{t} + s'' \cdot t - s'},
\]

where \( s_1 \) and \( s_2 \) are written for the \( s', s'' \) of (22); \( s' \) and \( s'' \) are
given by (19). When \( z = \infty, t = \infty \), and \( dx/dz = F \). Hence

\[
C = \frac{FK'kd \, \text{cn} \, \delta \, \text{dn} \, \delta}{\pi}.
\]

Accordingly we have in this case,

\[
X = \frac{FK'd}{\pi} \left[ Z(\lambda) - \frac{k \, \text{sn} \, \delta \, \text{cn} \, \lambda \, \text{dn} \, \lambda}{1 - k \, \text{sn} \, \delta \, \text{sn} \, \lambda} \right] + 2\pi i Q,
\]

with

\[
Q = \frac{FK'd}{4\pi K'}.
\]

The charges induced on the two strips are \( \pm Q \), the whole
induced charge being zero.

If the breadths of the two strips be equal, we find

\[
-X + iY = F \frac{k^2 \, \text{sn}^2 \lambda - \left(1 - \frac{E}{K}\right)}{k^2 \, \text{sn}^2 \lambda - \frac{E'}{K'}}.
\]

Along the \( y \)-axis,

\[
Y = 0,
\]

and

\[
X = -F \frac{k^2 \, \text{sn}^2 v + \left(1 - \frac{E}{K}\right) \text{cn}^2 v}{k^2 \, \text{sn}^2 v + \frac{E'}{K'} \text{cn}^2 v}.
\]
This gives a measure of the shielding effect produced by the two equal strips at the same potential.

On the strip $E'AE$, where $\lambda = K = iv$, we find for the surface density,

$$\sigma = \frac{F}{4\pi} \left( 1 - \frac{E}{K} \right) \frac{dn^2}{E' \frac{E'}{K} dv} - k^2$$

$v = 0$ at $E'$, and $v = K'$ at $E$.

When the two uncharged strips at the same potential are placed in a uniform electric field along the $y$-axis, the $\chi$-plane is the $t$-plane turned through a right angle. So we have,

$$\chi = iC't = iC' \frac{sn \lambda}{1 - k \frac{sn}{\delta} \frac{sn}{\lambda}}.$$

We now have

$$\frac{dx}{dz} = -X + iY = iC' \frac{dt}{dz}.$$

Since $Y = F$ when $t = \infty$, we get from (23)

$$C' = \frac{FK'kd \frac{cn}{\delta} \frac{dn}{\delta}}{\pi}.$$

The charge induced on the upper half of the strip $AB$ is

$$Q_1 = \frac{C'}{4\pi} (m - n),$$

and the charge induced on the upper half of the strip $CD$ is

$$Q_2 = \frac{C'}{4\pi} (r - p).$$

We therefore get, using (3),

$$Q_1 = \frac{FK'd(1 - k)cn \delta \frac{dn}{\delta}}{4\pi^2(1 - sn \delta)(1 - k \frac{sn}{\delta})},$$
$$Q_2 = \frac{FK'd(1 - k)cn \delta \frac{dn}{\delta}}{4\pi^2(1 + sn \delta)(1 + k \frac{sn}{\delta})}.$$
Accordingly,
\[ \frac{Q_1}{Q_2} = \frac{a_1}{a_2}. \]

When the strips have equal breadths, \( \delta = 0 \), and the surface density of the charge induced on \( E'AE \), where \( \lambda = K + iv \), is
\[ \sigma = \frac{Fk' r^2}{4\pi k} \frac{sn' v \ cn' v}{1 - \frac{E'}{k'^2K'^2} dn'^2 v}. \]

13. We shall now show how the results that have been obtained may be applied to get the distribution of electricity on pairs of cylinders, with any charges, of a wide variety of form of cross-section. From Equation (13) we see that the potential, the real part of \( \chi \), has a constant value for constant values of the real part of \( \lambda \). Put
\[ \lambda = w = u + iv. \] (37)

If we take \( u = \alpha K \), where \( \alpha \) is a positive constant, less than unity, we shall get an equipotential surface which, for \( \alpha = 1 \), becomes one of the two strips that we have considered. Similarly, if we take \( u = -\beta K \), where \( \beta \) is a positive constant less than unity, we get another equipotential surface. For \( \beta = 1 \) we get the other one of our pair of strips. If we now substitute (37) in (2) or in (24) we shall get equations of the form
\[ z = F(w). \] (38)

Separating the real and the imaginary parts of (38) we get
\[ x = f_1(u, v), \]
\[ y = f_2(u, v). \]

These are the equations in the parametric form of the cylinders \( u = \text{const.} \) The variable \( v \), lying between 0 and \( K' \) determines the position of a point on one of the cylinders. We shall consider only the case \( \delta = 0 \), which means that the two strips from which we begin have equal breadths.
Taking first equation (2), we find
\[
\begin{align*}
x &= f \frac{sn \ u \ dn' \ v}{1 - dn^2 \ u \ sn^2 \ v}, \\
y &= f \frac{cn \ u \ dn \ u \ sn' \ v \ cn' \ v}{1 - dn^2 \ u \ sn^2 \ v}.
\end{align*}
\]

We see that \( y \) vanishes for \( v = 0 \) and for \( v = K' \), and has a maximum for a value of \( v \) determined by the equation
\[
sn'^2 \ v = \frac{1}{1 + k^2 \ sn^2 \ u}.
\]

This maximum value of \( y \) is at a distance from the origin
\[
x_0 = \frac{f}{2k \ sn \ u} (1 + sn^2 \ u)^4 (1 + k^2 sn^2 u)^4,
\]
and its value is
\[
b = \frac{f}{2k} \frac{cn \ u \ dn \ u}{sn \ u}.
\]

The breadth of the cylinder is given by the difference between the values of \( x \) for \( v = K' \) and \( v = 0 \). It is
\[
a = \frac{f}{k} \frac{1 - k sn^2 u}{sn u},
\]
and the distance of the nearer vertex of the cylinder to the origin is
\[
d_1 = f \ sn \ u.
\]

Let us now consider the cylinder for which
\[
u = (1 - \epsilon)K.
\]

We have
\[
\begin{align*}
sn \ u &= \frac{cn \ eK}{dn \ eK}, \\
\frac{cn \ u}{k' \ sn \ eK} &= \frac{dn \ u}{dn \ eK}.
\end{align*}
\]

Let \( \epsilon \) be so small that we keep only first powers of \( \epsilon \). Then we have to a first approximation
\[
\begin{align*}
\frac{sn \ u}{1}, \\
\frac{cn \ u}{ek'K}, \\
\frac{dn \ u}{k'},
\end{align*}
\]
and (39) gives

\[
\begin{align*}
x &= \frac{f}{dn' v}, \\
y &= f e k'^2 K \frac{sn' v cn' v}{dn' v}.
\end{align*}
\]

(40)

The cylinder represented by (4) has a breadth along the \(x\)-axis

\[a = f \frac{1 - k}{k}.\]

The maximum value of \(y\) is

\[b = f e k'^2 K \frac{1}{2k},\]

at a distance from the origin

\[x_0 = f \frac{1}{2k} 2^4 (1 + k^2)^4,\]

and the distance of the nearer vertex to the origin is

\[d_1 = f.\]

In order to show the nature of the cylinders obtained in this way Fig. 5 has been plotted to scale. The cylinder on the right is that for which \(u = K/2\), and is plotted from (39). The cylinder on the left is that for which \(u = -(1 - e)K\), with \(e = 1/20\), and is plotted from (40). The modular angle
is 5°. The Smithsonian Tables of Elliptic Functions were used in plotting these curves.

The area of the cross-section of any of these cylinders is given by

\[ A = \int \int \frac{dudv}{h^2} , \]  

(41)

where

\[ \frac{1}{h^2} = \left( \frac{\partial x}{\partial u} \right)^2 + \left( \frac{\partial x}{\partial v} \right)^2 = \left( \frac{\partial y}{\partial u} \right)^2 + \left( \frac{\partial y}{\partial v} \right)^2 . \]  

(42)

From (39) we find

\[ \frac{\partial x}{\partial u} = \frac{\partial y}{\partial v} = f \; cn \; u \; dn \; u \; dn'v \; \frac{cn^2 v - k^2 \; sn^2 u \; sn'^2 v}{(1 - dn^2 u \; sn^2 v)^2} , \]  

\[ \frac{\partial x}{\partial v} = -\frac{\partial y}{\partial u} = f \; sn \; u \; sn'v \; cn'v \; \frac{k^2 \; cn^2 u + dn^2 u \; dn'^2 v}{(1 - dn^2 u \; sn^2 v)^2} . \]  

(43)

For the cylinder defined by \( u = \pm (1 - e)K \), if we neglect powers of \( e \) higher than the second, we get

\[ A = \frac{2f^2 eK}{3k^2} \{(1 + k^2)E' - 2k^2K'\} . \]

14. We next consider equation (24) for the two strips in parallel planes; with \( \delta = 0 \), we have

\[ z = \frac{d}{2K} w + K'd \frac{\pi}{\pi} Z(w) . \]  

(44)

Separating the real and the imaginary parts we get

\[ \frac{x}{f} = Z(u) + \frac{\pi u}{2KK'} + \frac{k^2 \; sn \; u \; cn \; u \; dn \; u \; sn'^2 v}{1 - dn^2 u \; sn^2 v} , \]  

\[ \frac{y}{f} = -Z(v, k') + \frac{dn^2 u \; sn'v \; cn'v \; dn'v}{1 - dn^2 u \; sn^2 v} , \]  

(45)

where we have put

\[ f = K'd \frac{\pi}{\pi} . \]
From (44) we find

\[
\frac{1}{f} \frac{dz}{dw} = \frac{1}{f} \left( \frac{\partial y}{\partial v} - i \frac{\partial x}{\partial v} \right) = \frac{E'}{K'} - k^2 sn^2 w. \tag{46}
\]

From this, or by differentiating the equations (45), we have

\[
\frac{1}{f} \frac{\partial y}{\partial v} = \frac{1}{f} \frac{\partial x}{\partial u} = \frac{E'}{K'} \left( \frac{k^2 (sn^2 u \ dn^2 v - cn^2 u \ dn^2 u \ sn^2 v \ cn^2 v)}{(1 - dn^2 u \ sn^2 v)^2} \right), \tag{47}
\]

\[
\frac{1}{f} \frac{\partial x}{\partial v} = -\frac{1}{f} \frac{\partial y}{\partial u} = \frac{2k^2 \ sn \ u \ cn \ u \ dn \ u \ sn' \ v \ cn' \ v \ dn' \ v}{(1 - dn^2 u \ sn^2 v)^2}.
\]

It is seen that \( y \) vanishes for \( v = 0 \) and for \( v = K' \); it has a maximum for a value of \( v \) given by equating the first of (47) to zero. If \( v_0 \) be the value of \( v \) corresponding to the maximum of \( y \), and we put

\[ sn^2 v_0 = \xi, \]

\( \xi \) is given as the smaller of the two roots of the equation

\[
\left( \frac{E'}{k^2 K'} \ n^2 u - cn^2 u \right) dn^2 u \cdot \xi^2 - \left( \frac{2E'}{k^2 K'} dn^2 u - k^2 sn^2 u - cn^2 u \ dn^2 u \right) \xi + \frac{E'}{k^2 K'} - sn^2 u = 0.
\]

If \( u = \pm K \), so that the cylinders reduce to two equal parallel strips in planes at a distance \( d \) apart, this equation is equivalent to (28).

The maximum thickness of these cylinders along the \( x \)-axis is given by the difference between the values of \( x \) for \( v = K' \) and \( v = 0 \). So, calling this thickness \( b \), we get from the first of (45)

\[
\frac{b}{f} = \frac{cn \ u \ dn \ u}{sn \ u}.
\]

Let now \( u = (1 - \epsilon)K \). Then keeping only first powers
of we find

\[
\begin{align*}
\frac{x}{f} &= -Z(\varepsilon K) + \frac{\pi}{2K'} (1 - \varepsilon) + \frac{k'^2 \varepsilon K}{dn^2 v'}, \\
\frac{y}{f} &= -Z(v, k') + \frac{k'^2 sn' v}{dn' v}.
\end{align*}
\tag{49}
\]

To this approximation the maximum value of \( y \), denoted by \( a \), is the same as in the case of the two equal parallel strips, determined by (28) and (29). The maximum thickness of the cylinder \( u = (1 - \varepsilon)K \) is given by

\[
\frac{b}{f} = ek'^2 K.
\]

Figure 6 shows the cross-sections of two of these cylinders for a modular angle of 5°. For the cylinder on the right, \( u = K/2 \), and for the cylinder on the left \( u = -K \) so that it reduces to a strip. In plotting the cylinder \( u = K/2 \) equations (45) were used. These are now,

\[
\frac{x - x_0}{f} = \frac{k'^2 k'}{1 + k' 1 - k' sn'^2 v},
\]

where

\[
\frac{x_0}{f} = \frac{1}{2} (1 - k') + \frac{\pi}{4K'},
\]

and

\[
\frac{y}{f} = \frac{k' sn' v}{1 - k' sn'^2 v} - Z(v, k').
\]

The Smithsonian Tables of Elliptic Functions were used in plotting Fig. 6.

For very small modular angles we may use the Fourier expansions of the elliptic functions. The expansion of the Zeta function in (44) leads to

\[
\begin{align*}
\frac{x}{f} &= \frac{\pi u}{2KK'} + \frac{2\pi}{K} \sum_i \frac{q^n}{1 - q^n} \sin\frac{n\pi u}{K} \cosh\frac{n\pi v}{K}, \\
\frac{y}{f} &= \frac{\pi v}{2KK'} + \frac{2\pi}{K} \sum_i \frac{q^n}{1 - q^n} \cos\frac{n\pi u}{K} \sinh\frac{n\pi v}{K}.
\end{align*}
\]
For modular angles for which $k = 10^{-3}$ or less we can take with sufficient approximation,

$$q = k^2/16, \quad K = \pi/2.$$ 

The infinite series will begin to contribute appreciably only for values of $v$ that are large enough so that we can put

$$\cosh 2v = \sinh 2v = \frac{1}{2} e^{2v}.$$ 

Now $q^2$ is negligible compared with unity. Therefore if we put

$$V = q e^{2v},$$

we shall have

$$\frac{x - x_0}{f} = 2 \sum_{i=1} V^n \sin \frac{n\pi u}{K},$$

$$\frac{y}{\dot{f}} = \frac{v}{K'} + 2 \sum_{i=1} V^n \cos \frac{n\pi u}{K}$$

with

$$\frac{x_0}{\dot{f}} = \frac{\pi u}{2KK'}.$$ 

Making use of the two known series \(^1\)

\(^1\) Bromwich, Infinite Series, second edition, p. 186.
\[
\sum r^n \sin nx = \frac{r \sin x}{1 - 2r \cos x + r^2}, \\
\sum r^n \cos nx = \frac{r \cos x - r^2}{1 - 2r \cos x + r^2},
\]
we get
\[
\frac{x - x_0}{f} = \left( \frac{2V \sin \frac{\pi u}{K}}{1 + V^2 - 2V \cos \frac{\pi u}{K}} \right)
\]
\[
\frac{y}{f} = \frac{v}{K'} - \frac{2V (V - \cos \frac{\pi u}{K})}{1 + V^2 - 2V \cos \frac{\pi u}{K}}.
\]

If \( u = (1 - \epsilon)K \), with \( \epsilon \) small, we may write
\[
\frac{x - x_0}{f} = \frac{2\pi \epsilon V}{(1 + V)^2},
\]
\[
\frac{y}{f} = \frac{v}{K'} - \frac{2V}{1 + V},
\]
with
\[
\frac{x_0}{f} = \frac{\pi (1 - \epsilon)}{2K'}.
\]

The expressions that we have obtained are not quite accurate for small values of \( v \). For \( v = 0 \) the true values are
\[
x - x_0 = y = 0.
\]

Now for \( v = 0 \), \( V = q \), and since for small modular angles \( q = k^2/16 \) it is obvious that the error made is insignificant. Even for values of \( v \) as large as \( K'/2 \), \( V = q^1 \), so that if \( k \) is less than \( 10^{-3} \) the expressions that have been derived will lead to results of sufficient accuracy.

Figure 7 shows the cross-sections of two cylinders, plotted to scale. The cylinder on the right is given by \( u = \alpha K \), with \( \alpha = 1/2 \); that on the left by \( u = -\beta K \), with \( \beta = (1 - \epsilon) \). The modulus is \( k = 10^{-6} \), and \( \epsilon \) has been taken as \( 1/20 \).
Figures 8–10 show the cross-sections of three pairs of cylinders for which \( \alpha + \beta = 1 \). The modulus is \( k = 10^{-12} \). In Fig. 8 the two cylinders are equal, with \( \alpha = \beta = 1/2 \). In Fig. 9, \( \alpha = 1/3 \) and \( \beta = 2/3 \), while in Fig. 10, \( \alpha = 1/4 \) and \( \beta = 3/4 \). It will be observed from Fig. 8 that we have here an approximation to two equal semi-circular cylinders separated by a small distance. And it also appears from Figs. 9 and 10 that all pairs of cylinders for which \( \alpha + \beta = 1 \) approximate to what we should get if a solid circular cylinder were cut by a plane parallel to its axis and the two portions separated by a distance which decreases as the modulus is diminished.

Equation (46) may be developed into the Fourier series,

\[
\frac{1}{f} \frac{dz}{dw} = \frac{\pi}{2KK'} + \frac{2\pi^2}{K^2} \sum \frac{nq^n}{1 - q^{2n}} \cos \frac{n\pi w}{K}.
\]

From this, or by differentiating (50), with the same approximations that we have used, we get

\[
\frac{1}{f} \frac{\partial x}{\partial u} = \frac{1}{f} \frac{\partial y}{\partial v} = \frac{1}{K'} + \frac{4V}{K'} \left\{ \begin{array}{c} (1 + V^2) \cos \frac{\pi u}{K} - 2V \\ + V^2 - 2V \cos \frac{\pi u}{K} \end{array} \right\} 
\]

\[
\frac{1}{f} \frac{\partial x}{\partial v} = -\frac{1}{f} \frac{\partial y}{\partial u} = \frac{4V(1 - V^2) \sin \frac{\pi u}{K}}{1 + V^2 - 2V \cos \frac{\pi u}{K}}.
\]

(51)

For the special case \( u = K/2 \), as in Fig. 8, we find that \( y \) is a maximum for a value \( v_0 \) of \( v \) determined by

\[
V_0 = qe^{-2v_0} = (2K')^\frac{1}{v} - (2K' - 1)^\frac{1}{v}.
\]

The maximum value of \( y \), as well as the corresponding value of \( x \), may therefore be determined from (50).

The area of the cross-section of any of these cylinders is given by (41) and \( h^2 \) is determined by (42). From (51) we
find
\[
\frac{1}{f^2h^2} = \frac{1}{K'^2} + \frac{8V^2}{V'(K' - 1) + (1 + V^2) \cos \frac{\pi u}{K}} \left[ \frac{1 + V^2}{z} \right] \left( \frac{\pi u}{z} \right)^2.
\]

For the cylinder defined by \( u = K/2 \), the limits of integration for \( u \) are \( K/2 \) and \( K \); for \( v \) they are 0 and \( K' \) and the result must be doubled to take in the lower half of the cylinder. The result of the integrations gives
\[
A = f^2K \left( 1 - \frac{1}{K'} \right) = \frac{\pi f^2}{2} \left( 1 - \frac{1}{K'} \right),
\]
with \( K = \pi/2 \) for small modular angles. Thus we see that the cylinders illustrated in Fig. 8 have a cross-sectional area,
\[
A = \frac{\pi f^2}{2} 0.9655.
\]

This shows how closely they approach to semi-circular cylinders of radius \( f \). For infinitely small modular angles \( K' \) approaches infinity and the area therefore approaches the value \( \pi f^2/2 \).

15. Equations (39) and (45) are derived from transformations of the form of (38) such that if constant values, \( u = \alpha K \) and \( u = -\beta K \), be given to \( u \), where \( \alpha \) and \( \beta \) lie between 0 and 1, we obtain a pair of cylinders of the types illustrated in Figs. 5–8. The upper halves of the \( z \)-planes, external to the cylinders, correspond to the interior of a rectangle in the \( w \)-plane which is formed of the \( u \)-axis, a line parallel to the \( u \)-axis at a distance \( K' \) from it, and two lines parallel to the \( v \)-axis, one at a distance \( \alpha K \) to the right and the other at a distance \( \beta K \) to the left of the origin. The infinitely distant point of the \( z \)-plane corresponds to \( u = 0, v = K' \); the origin of the \( z \)-plane corresponds to \( u = v = 0 \). The interior of this rectangle may be transformed to the upper half of a \( t_1 \)-plane by means of elliptic functions to a different modulus from that which has been used in defining the cylinders. We shall dis-
tistinguish the new elliptic functions by the subscript 1. Thus, 
k_1, k_1' are the two complementary moduli used in the transform-
tion to the \( t_1 \)-plane; \( K_1, E_1 \) are the complete elliptic
integrals of the first and second kinds to the modulus \( k_1 \), and
\( K_1', E_1' \) to the complementary modulus \( k_1' \).

The Schwarz-Christoffel differential equation for transform-
ring from the \( w \)- to the \( t_1 \)-plane is

\[
\frac{dw}{dt_1} = \frac{A}{|t_1 + r_1 \cdot t_1 + p_1 \cdot t_1 - n_1 \cdot t_1 - m_1|^4}.
\] (58)

As we pass along the \( x \)-axis in the \( z \)-plane in the positive
direction from \(-\infty\), the points at which the axis meets the
cylinders correspond to \( t_1 = -r_1, -p_1, n_1, m_1 \) in order. We
use the substitution obtained from (1) by changing to the
modulus \( k_1 \) and writing \( \delta_1 \) for \( \delta \); with the aid of (6)
we get for the solution of (58) that satisfies the conditions,

\[
\begin{align*}
t_1 &= m_1 & n_1 &= 0 & -p_1 & -r_1 & \infty \\
\lambda_1 &= K_1 + iK_1' & K_1 &= 0 & -K_1 & -K_1 + iK_1' & iK_1' + \delta_1 \\
w &= \alpha K + iK' & \alpha K &= 0 & -\beta K & -\beta K + iK' & iK'
\end{align*}
\]

\[
w = \frac{K'}{K_1'} (\lambda_1 - \delta_1),
\] (59)

where

\[
\delta_1 = K_1 \frac{\beta - \alpha}{\beta + \alpha},
\] (60)

\[
Ak_1 \text{cn} \delta_1 \text{dn} \delta_1 = -\frac{K'}{K_1'}
\]

and

\[
\frac{K}{K'} = \frac{2K_1}{K_1' (\alpha + \beta)}.
\] (62)

A given pair of cylinders is completely defined when \( \alpha, \beta \) and
the modulus \( k \) are given. Then equation (62) determines the
ratio \( K_1/K_1' \), and therefore the modulus \( k_1 \) may be found.
Equation (60) then determines \( \delta_1 \).

16. If the cylinder \( u = \alpha K \) has a charge \( Q_1 \) per unit length,
at potential \( V \), and the cylinder \( u = -\beta K \) has a charge \( Q_2 \).
per unit length at potential 0, the \( \chi \)-plane is the same as in Art. 3, and we can use all of the results obtained there if the modulus of the elliptic functions be taken as \( k_1 \) instead of \( k \), and \( \delta_1 \), given by (60), be written for \( \delta \). In particular, the capacity of a condenser formed of unit lengths of the two cylinders is given by

\[
S = \frac{K_1'}{4\pi K_1}.
\]

Using (62) this may be written

\[
S = \frac{K'}{2\pi(\alpha + \beta)K}.
\]  
(63)

It follows that all pairs of cylinders for which \( \alpha + \beta = 1 \) have a capacity which is just twice that of the pair of strips from which they were derived. Thus the cylinders illustrated in Figs. 8–10 all have the same capacity per unit length. This capacity is, with \( k = 10^{-12} \),

\[
S = 2.9401.
\]

When the cylinders have any charges, \( Q_1 \) and \( Q_2 \), the difference of potential between them is given by (9) in which the modulus \( k_1 \) instead of \( k \), and \( \delta_1 \) instead of \( \delta \), is to be used. Thus we find for the difference of potential,

\[
V = 2\pi \frac{K}{K'} (\alpha Q_1 - \beta Q_2).
\]

The point of equilibrium, \( t_1 = -s_1 \), is determined by (10) which now becomes

\[
S_1 = \frac{1}{k_1 \ cn \ \delta_1 \ dn \ \delta_1 \ 2K_1'(Q_1 + Q_2)} \left\{ \pi (Q_1 - Q_2) - \frac{\pi \delta_1}{2K_1 K_1'} - Z(\delta_1) - \frac{\pi \delta_1}{2K_1 K_1'} \right\} \text{ (mod. } k_1). \]

The electric intensity at any point of the \( z \)-plane is given by

\[
-X + iY = \frac{2(Q_1 + Q_2)K_1'}{K'} \left\{ k_1 \ \cn \ \delta_1 \ \dn \ \delta_1 \ \sn(\lambda_1, k_1) \right. \\
\left. + \frac{\pi (Q_1 - Q_2)}{2K_1'(Q_1 + Q_2)} - Z(\delta_1) - \frac{\pi \delta_1}{2K_1 K_1'} \right\} \frac{dw}{dz}. \]  
(64)
For the elliptic functions inside the brackets the modulus is \( k_1 \), while in \( dw/dz \) the modulus is \( k \). Now

\[
\frac{dw}{dz} = \left( \frac{\partial y}{\partial v} - i \frac{\partial y}{\partial u} \right) h^2,
\]

where

\[
\frac{1}{h^2} = \left( \frac{\partial y}{\partial v} \right)^2 + \left( \frac{y \partial}{\partial u} \right)^2 = \left( \frac{\partial x}{\partial v} \right)^2 + \left( \frac{\partial x}{\partial u} \right)^2
\]

and

\[
\left| \frac{dw}{dz} \right| = h. \tag{65}
\]

We can therefore determine \( dw/dz \) for pairs of cylinders derived from equal strips in the same plane (Fig. 5) by means of the equations (43), and for pairs of cylinders derived from equal strips in parallel planes (Figs. 7–10) from the equations (47). For very small modular angles we may use (51) in place of (47).

In the condenser problem \( Q_1 + Q_2 = 0 \), and the surface density at any point of the cylinders is given by

\[
\sigma = \frac{Q}{2K'} h, \tag{66}
\]

\( Q \) being the charge on unit length of the positive cylinder and \( h \) is to be evaluated for the given point. Thus the surface density at any point of the conductors may readily be found.

But in the general case when the cylinders have any charges, \( Q_1 \) and \( Q_2 \), we must evaluate \( sn(\lambda_1, k_1) \) in (64). By (59),

\[
\lambda_1 = \frac{K_1'}{K} w + \delta_1
\]

and on the cylinder defined by \( u = \alpha K \) we have

\[
\lambda_1 = K_1 + i \frac{K_1'}{K} v.
\]
Therefore

\[ sn(\lambda_1, k_1) = \frac{cn \left( \frac{iK_1'}{K} v \right)}{dn \left( \frac{iK_1'}{K} v \right)} \text{ (mod. } k_1) \].

The case \( \alpha + \beta = 1 \), illustrated in Figs. 8–10, is of special interest. By (62) we have

\[ \frac{2K'}{K} = \frac{K_1'}{K_1} \]

and it follows from this that

\[ q_1 = q^2 \].

We therefore have an application of Landen’s quadratic transformation with the results,

\[ k_1 = \frac{1 - k'}{1 + k'} = \frac{k^2}{(1 + k')^2}, \]

\[ k_1' = \frac{2k^4}{1 + k'}, \]

\[ K' = \frac{1}{2}(1 + k')K, \]

\[ K_1' = (1 + k')K', \]

and

\[ cn[ (1 + k')iv, k_1] = \frac{1 + k' sn'^2 v}{cn' v \ dn' v}, \]

\[ dn[ (1 + k')iv, k_1] = \frac{1 - k' sn'^2 v}{cn' v \ dn' v}, \]

and therefore,

\[ sn(\lambda_1, k_1) = \frac{1 + k' sn'^2 v}{1 - k' sn'^2 v} \text{ (mod. } k') \]

on the cylinder \( u = \alpha K \). On the cylinder \( u = -\beta K \) we merely change the sign of this expression.

The surface density on the cylinder \( u = \alpha K \), in the case
\[ \alpha + \beta = 1, \] may now be expressed in the form,

\[
\sigma = \frac{(Q_1 + Q_2)(1 + k')}{2\pi} \left[ \frac{k_1 \, cn \, \delta_1 \, dn \, \delta_1(1 + k' \, sn'^2 \, v)}{1 - k_1 \, sn \, \delta_1 - (1 + k_1 \, sn \, \delta_1)k' \, sn'^2 \, v} \right. \\
+ \frac{\pi (Q_1 - Q_2)}{2(1 + k')K'(Q_1 + Q_2)} - Z(\delta_1, k_1) \\
- \frac{\pi}{2} \frac{\beta - \alpha}{(1 + k')K'} h, \quad (67)
\]

where \( h \) is to be evaluated at the given point by means of (43) or (47). In this expression for the surface density the elliptic functions \( sn \, \delta_1, cn \, \delta_1, dn \, \delta_1 \) and \( Z(\delta_1) \) are to be formed with the modulus \( k_1 \). As they are constants for any given pair of cylinders it is more convenient to leave them in this form than to transform them to the modulus \( k \).

If the two cylinders are at the same potential we have

\[ Q_1 = \alpha Q, \quad Q_2 = \beta Q, \]

where \( Q \) is the sum of the charges on unit lengths of the two cylinders. We then find for the surface density,

\[
\sigma = \frac{Q(1 + k')}{2\pi} \left[ \frac{k_1 \, cn \, \delta_1 \, dn \, \delta_1(1 + k' \, sn'^2 \, v)}{1 - k_1 \, sn \, \delta_1 - (1 + k_1 \, sn \, \delta_1)k' \, sn'^2 \, v} \right. \\
- Z(\delta_1) \right] h, \quad (68)
\]

and for equal cylinders, as in Fig. 8,

\[
\sigma = \frac{Q(1 - k')}{2\pi} \frac{1 + k' \, sn'^2 \, v}{1 - k' \, sn'^2 \, v} h. \quad (69)
\]

17. The force between unit lengths of the cylinders \( \alpha = \beta = 1/2 \) when they have any charges, \( Q_1 \) and \( Q_2 \), may easily be found. In the plane \( x = 0, w = i v \), and by (59)

\[ \lambda_1 = i \frac{K_1'}{K'} v = (1 + k')i v. \]

Then by Landen’s transformation

\[ sn(\lambda_1, k_1) = i(1 + k') \frac{sn' \, v}{cn' \, v \, dn' \, v} \]
in the plane \( x = 0 \). Furthermore, by (46) we find

\[
\frac{dw}{dz} = \frac{1}{f k^2 c^2 - (c^2 - 1) s n^2 v} = h,
\]

where

\[
\frac{1}{c^2} = d n^2 \alpha = \frac{k^2 K'}{E'},
\]

as in (28). Then we get from (64)

\[
X = -\frac{\pi (Q_1 - Q_2)}{f k^2 K'} \frac{c n^2 v}{c^2 - (c^2 - 1) s n^2 v},
\]

\[
Y = \frac{2(Q_1 + Q_2)}{f} \frac{c n' v}{s n' v c n' v} \frac{d n' v [c^2 - (c^2 - 1) s n^2 v]}{c^2 - (c^2 - 1) s n^2 v}.
\]

By integrating the Maxwellian stress over the plane \( x = 0 \) we get for the force between unit lengths of the two cylinders,

\[
R = \frac{1}{4\pi} \int_0^{\infty} (X^2 - Y^2) dy.
\]

Now

\[
dy = k^2 f \frac{c^2 - (c^2 - 1) s n^2 v}{c n^2 v},
\]

so that we can write

\[
R = \frac{\pi (Q_1 - Q_2)^2}{4 f k^2 K'} \int_0^{K'} \frac{c n^2 v d v}{c^2 - (c^2 - 1) s n^2 v} - \frac{k^2 (Q_1 + Q_2)^2}{\pi f} \int_0^{K'} \frac{s n^2 v d v}{d n' v [c^2 - (c^2 - 1) s n^2 v]}
\]

The first integral we have already had in Art. 11. For the second integral we find

\[
\int_0^{K'} \frac{c n^2 v d v}{d n' v [c^2 - (c^2 - 1) s n^2 v]} = \frac{K' s n' \alpha}{k^2 c n^2 \alpha} \frac{d n' \alpha}{dn' \alpha} - Z(\alpha, k').
\]
Therefore,
\[
R = \frac{dn' \alpha}{\int_{k'^2} \frac{\pi (Q_1 - Q_2)^2}{4k'^2 K'} \frac{dn' \alpha}{\alpha} \left[ - \frac{k'^2 (Q_1 + Q_2)^2 K' \frac{sn' \alpha \alpha}{cn' \alpha}}{\pi cn'^2 \alpha} \right] \times \left[ \frac{k'^2 \frac{sn' \alpha \alpha \alpha}{cn' \alpha}}{dn' \alpha} - Z(\alpha, k') \right].
\]

If \( Q_1 + Q_2 = 0 \), the force between the two cylinders is of course the same as the force between the two equal strips at a distance apart given by
\[
d = \frac{\pi f}{K'}.\]

For the two cylinders are equipotential surfaces, one surrounding each of the strips when they have equal and opposite charges.

For modular angles that are so small that we can put
\[
E' = 1, K = \pi/2, k' = 1,
\]
we have
\[
\begin{align*}
dn' \alpha &= k K' \alpha, \\
\alpha &= k(K' - 1) \alpha, \\
\alpha &= 1,
\end{align*}
\]
\[
Z(\alpha, k') = 1 - \frac{\alpha}{K'}.
\]

Then we find
\[
R = \frac{1}{\int} \left( \frac{K'}{K' - 1} \right)^4 \left[ \frac{\pi (Q_1 - Q_2)^2}{4} \right. \\
&\left. - \frac{(Q_1 + Q_2)^2}{\pi} \frac{K'}{K'} \right] \left[ \left( \frac{K' - 1}{K'} \right)^4 - 1 + \frac{\alpha}{K'} \right].
\]

\( K' \) and \( \alpha \) are given in the table on page 205. The ratio of the repulsion between the two cylinders when they have equal charges to their attraction when they have opposite charges is therefore
\[
\frac{4}{\pi^2 K' - 1},
\]
and for very small modular angles this is simply \( 4/\pi^2 \). In the limit when \( k \) approaches 0, the two cylinders merge into a
complete circular cylinder. The force tending to separate the two halves is then, since $\alpha/K'$ approaches unity,

$$\frac{4Q^2}{f\pi}.$$

$2Q$ is now the charge on the complete cylinder and $f$ its radius, and so our results agree with this known value.

18. If any pair of cylinders of the types illustrated in Figs. 5–10 be placed in a uniform electric field along the $x$-axis, the complex potential when the cylinders are insulated and uncharged is given by an equation derived from (18) by changing to the modulus $k_1$. The constant $C$ in this equation must be determined so that at infinity the potential shall be $\varphi = Fx + \text{const.}$, where $F$ is the intensity of the uniform field. Now when $z = \infty$, $\lambda_1 = iK_1' + \delta_1$. Put

$$\lambda_1 = iK_1' + \delta_1 + \epsilon_1. \quad (70)$$

Then we get from (18)

$$X = \frac{C}{k_1\epsilon_1 \cn \delta_1 \dn \delta_1} + \text{const.},$$

in the limit when $\epsilon_1$ approaches zero. By (59) when $\lambda_1$ has the value given by (70) $w$ has the value given by

$$w = iK' + \epsilon, \quad (71)$$

where

$$\epsilon = \frac{K'}{K_1'} \epsilon_1.$$

For the cylinders derived from two equal strips in the same plane, we find by using (71) in $z = f \sn w$,

$$x = \frac{f}{k\epsilon} = \frac{iK_1'}{K_1' \epsilon_1}.$$  

Hence it follows that the constant $C$ for pairs of cylinders derived from two equal strips in the same plane is given by

$$C = \frac{Ff k_1 K_1'}{k K_1'} \cn \delta_1 \dn \delta_1.$$
Similarly, if we substitute (71) in (44), we find
\[ x = \frac{fK_1'}{K'e_1}. \]

So it follows that for pairs of cylinders derived from two equal strips in parallel planes the constant \( C \) is given by
\[ C = \frac{FjK_1'k_1}{K'} cn \delta_1 dn \delta_1. \]

The difference of potential between the two uncharged insulated cylinders in a field \( F \) along the \( x \)-axis is given by
\[ V = \frac{\pi C}{k_1K_1' cn \delta_1 dn \delta_1}. \]

It therefore follows that the difference of potential between any two cylinders that are derived from two equal strips in the same plane is
\[ V = \frac{\pi Fj}{K'e_1}. \]

Since \( f = 2d \), where \( d \) is the distance apart between the adjacent edges of the two strips, the difference of potential between the two cylinders in a field along the \( x \)-axis is the same as that between the two strips placed in the same field.

The difference of potential between any two cylinders derived from equal strips in parallel planes is
\[ V = \frac{\pi Fj}{K'} = Fd, \]

where \( d' \) is now the distance between the two strips. So it follows that the difference of potential between the cylinders is the same as that between the strips in the same field. Thus all the pairs of cylinders shown in Figs. 8–10 will have the same difference of potential when placed in a uniform electric field along the \( x \)-axis provided the scale constant \( f \) is the same for all.

If the two cylinders are at the same potential in the field along the \( x \)-axis, equal and opposite charges, \( \pm Q \), will be
induced upon them. The complex potential is given by changing to the modulus $k_1$ in equation (21) and $Q$ is given by

$$Q = \frac{C}{4K_1 k_1 \text{cn } \delta_1 \text{dn } \delta_1}.$$

The constant $C$ is found to have the same values for the two types of cylinders as when the cylinders were insulated and uncharged. Thus we find for all pairs of cylinders derived from two equal strips in the same plane,

$$Q = \frac{Ff}{2kK(\alpha + \beta)},$$

using (62). For all pairs of cylinders derived from equal strips in parallel planes, we have

$$Q = \frac{Ff}{2k(\alpha + \beta)}.$$

The ratio $Q/V$ in both cases gives the capacity $S$ determined by (63).

19. In a uniform field along the $y$-axis the complex potential is

$$X = iCt_1 = iC \frac{\text{sn } \lambda_1}{1 - k \text{sn } \delta_1 \text{sn } \lambda_1} \text{(mod. } k_1).$$

The constant $C$, determined so that at infinity

$$\varphi = Fy + \text{const.},$$

has the same values as in the preceding article. We can therefore determine at once the charges induced on the upper and lower halves of the two cylinders. For pairs of cylinders derived from two equal strips in the same plane, the charge induced on the upper half of the cylinder $u = aK$ is

$$Q_1 = \frac{FfK_1'(1 - k_1)\text{cn } \delta_1 \text{dn } \delta_1}{4\pi kK'(1 - \text{sn } \delta_1)(1 - k_1 \text{sn } \delta_1)}.$$

For the charge $Q_2$ induced on the upper half of the cylinder $u = -\beta K$ we merely need to change the sign of $\text{sn } \delta_1$. 
For pairs of cylinders derived from equal strips in parallel planes we find for the charge induced on the upper half of the cylinder \( u = \alpha K \),

\[
Q = \frac{F f K'(1 - k_1)cn \delta_1 dn \delta_1}{4\pi K'(1 - sn \delta_1)(1 - k_1 sn \delta_1)}
\]

and by changing the sign of \( \delta_1 \) we get the charge induced on the upper half of the cylinder \( u = -\beta K \).

The expressions for the surface density of the induced charges when these cylinders are placed in a uniform electric field may easily be found by the same methods that have previously been used and the details of the calculation will not be given.

20. The referee to whom this paper was sent asked that a final section be added to give practical details for carrying out the computations in order to get numerical results. As the problem of the two half-cylinders, illustrated in section in Fig. 8, is one that may have some applications, and as one can show the methods better by applying them to a definite case, this problem will be taken to show how numerical results may be obtained.

It is only for very small modular angles that our transformations give even approximate semi-circular cylinders. For these small modular angles we can take, with sufficient approximation,

\[
K = \pi/2,
\]

\[
\log K' = \log \log \frac{4}{k} + \log \frac{1}{M}.
\]

(72)

Logarithms to the base 10 are denoted by Log. \( k \) is the sine of the modular angle, and

\[
\log \frac{1}{M} = 0.3622157 \ldots.
\]

Tables of elliptic functions are not available for the small modular angles we shall use. The smallest modular angle in Legendre's Tables is \( \theta = 0^\circ.1 \). For this angle,

\[
k = \sin \theta = 1.74533 \ldots \times 10^{-3}.
\]
Our approximations give
\[
\log K = 0.1961199, \\
\log K' = 0.8885786.
\]
Legendre's Tables, which are to at least twelve places of decimals, give
\[
\log K = 0.1961202 \ldots, \\
\log K' = 0.8885789 \ldots.
\]
If we take values of the modulus, \(k = 10^{-3}\) and less, our approximations are sufficient if we work with seven-place logarithms. We shall also require the quantity,
\[
q = e^{-x^{K'/K}} = e^{-2K'}.
\]
To a sufficient approximation this is
\[
q = \frac{k^2}{16}.
\]
If the two equal cylinders be obtained by dividing a circular cylinder of radius \(f\) by a plane through its axis, and then separating the two halves a short distance, \(2x_0\), this distance is given by
\[
\frac{2x_0}{f} = \frac{\pi}{2K'}.
\]
For the largest modulus we can use, that is, \(k = 10^{-3}\), this gives
\[
\frac{2x_0}{f} = 0.18939.
\]
For smaller distances than this we get better approximations to semi-circular cylinders. So if the ratio \(2x_0/f\) is given, \(K'\) is known by (75); (73) then gives \(q\), and (74) gets the value of \(k\).

In order to plot the sections of these cylinders so as to show how much the edges are rounded off, we use the equations,
\[
\begin{align*}
\frac{x - x_0}{f} &= \frac{2V}{1 + V^2}, \\
\frac{y}{f} &= \frac{v}{K'} - \frac{2V^2}{1 + V^2},
\end{align*}
\]
where \( V \) is defined by
\[
V = q e^{2v}.
\] (77)

\( v \) is the parameter that defines the position of a point on the cylinder and varies from 0 at the point nearest the origin to \( K' \) at the most distant point. Equations (76) are valid only when \( v \) is large enough so that we can put
\[
\sinh 2v = \cosh 2v = \frac{1}{2} e^{2v}.
\] (78)

For small values of \( v \) we have
\[
\frac{x - x_0}{f} = 4q \cosh 2v - \cdots,
\]
\[
\frac{y}{f} = \frac{v}{K'} - 4q^2 \sinh 4v + \cdots.
\]

But since the largest value of \( q \), that for \( k = 10^{-8} \), is \( 6 \cdot 10^{-8} \), the error made in using (76) for all values of \( v \) is insignificant. The maximum value of \( y \) corresponds to a value \( v_0 \) of \( v \) given by
\[
V_0 = q e^{2v_0} = (2K')^2 - (2K' - 1)^2.
\] (79)

In order to compute the components of the electric intensity at any point in the plane we need the relations between the rectangular coordinates, \( x, y \), and the curvilinear coordinates, \( u, v \). These relations are given by
\[
\begin{align*}
\frac{x}{f} &= \frac{u}{K'} + \frac{2V \sin 2u}{1 - 2V \cos 2u + V^2}, \\
\frac{y}{f} &= \frac{v}{K'} - \frac{2V(V - \cos 2u)}{1 - 2V \cos 2u + V^2}.
\end{align*}
\] (80)

where \( V \) is given by (77). For all points in the first quadrant outside the cylinder \( u \) lies between 0 and \( \pi/4 \) and \( v \) lies between 0 and \( K' \). For values of \( v \) that are too small for (78) to be valid we should take
\[
\begin{align*}
\frac{x}{f} &= \frac{u}{K'} + 4q \sin 2u \cosh 2v, \\
\frac{y}{f} &= \frac{v}{K'} + 4q \cos 2u \sinh 2v.
\end{align*}
\]
DISTRIBUTION OF ELECTRICITY ON TWO CYLINDERS

But here again the error made by using (80) for all values of \( v \) is insignificant because \( q \), and therefore \( V \) for small values of \( v \), is negligible. The origin of coördinates is given by \( u = v = 0 \), and the infinite point by \( u = 0, v = K' \) or \( V = 1 \). Along the \( x \)-axis from the origin to where it meets the cylinder, \( v = 0 \), and \( u \) increases from 0 to \( \pi/4 \). Along the \( x \)-axis from where it cuts out of the cylinder, \( v = K' \), \( V = 1 \), and \( u \) diminishes from \( \pi/4 \) to 0 at infinity. Along the \( y \)-axis, \( u = 0 \) and \( v \) increases from 0 at the origin to \( K' \) at infinity.

If the two cylinders are freely charged, with equal and opposite charges, \( Q \) being the charge on unit length of the cylinder on the right, the components of the electric intensity at any point are given by

\[
\begin{align*}
X = & -\frac{2Q}{K' A^2 + B^2} A \\
Y = & \frac{2Q}{K' A^2 + B^2} B,
\end{align*}
\]

(81)

where

\[
A = \frac{\partial y}{\partial v} = \frac{\partial x}{\partial u}, \quad B = \frac{\partial x}{\partial v} = -\frac{\partial y}{\partial u}
\]

and \( A \) and \( B \) are given by

\[
\begin{align*}
\frac{A}{f} = & \frac{1}{K'} + 4V \left\{ (1 + V^2) \cos 2u - 2V \right\} \\
\frac{B}{f} = & \frac{4V (1 - V^2) \sin 2u}{(1 - 2V \cos 2u + V^2)^2}.
\end{align*}
\]

(82)

For values of \( v \) that are too small for the relations (78) to be valid we should use

\[
\begin{align*}
\frac{A}{f} = & \frac{1}{K'} + 8q \cos 2u \cosh 2v, \\
\frac{B}{f} = & 8q \sin 2u \sinh 2v.
\end{align*}
\]

(83)

The surface density at any point on the cylinder with a charge + \( Q \) is given by

\[
\sigma = \frac{Q}{2K'} h,
\]

(84)
where $h$ may be computed from

$$\frac{1}{J^2h^2} = \frac{1}{K'^2} + \frac{16(K' - 1)V^2}{K'(1 + V^2)^2}. \quad (85)$$

For small values of $v$ we should use

$$\frac{1}{J^2h^2} = \frac{1}{K'^2} + 64q^2 \sinh^2 2v. \quad (86)$$

The capacity of the condenser formed of unit lengths of the two cylinders is simply

$$S = \frac{K'}{\pi^2}. \quad (87)$$

When the two cylinders are at the same potential their charges are equal. The components of the electric intensity at any point in the plane are now given by

$$X = \frac{16QV}{A^2 + B^2} \left[\frac{A(1 + V^2) \sin 2u - B(1 - V^2) \cos 2u}{(1 + V^2)^2 - 4V^2 \cos^2 2u}\right],$$

$$Y = \frac{16QV}{A^2 + B^2} \left[\frac{A(1 - V^2) \cos 2u + B(1 + V^2) \sin 2u}{(1 + V^2)^2 - 4V^2 \cos^2 2u}\right], \quad (87)$$

where the values of $A$ and $B$ given by (82) are to be used. For small values of $v$ we may compute the components of the electric intensity from the equations,

$$X = \frac{32Qq}{A^2 + B^2} \left\{A \sin 2u \cosh 2v - B \cos 2u \sinh 2v\right\},$$

$$Y = \frac{32Qq}{A^2 + B^2} \left\{A \cos 2u \sinh 2v + B \sin 2u \cosh 2v\right\}. \quad (88)$$

In (88) the values of $A$ and $B$ given by (83) are to be used. The surface density of the distribution on each cylinder is given by

$$\sigma = \frac{4QV}{\pi(1 + V^2)} h, \quad (89)$$

with the value of $h$ given by (85). For small values of $v$ the
surface density is given by

$$\sigma = \frac{8Qq}{\pi} \cosh 2v \cdot h,$$

with $h$ determined by (86).

If the two equal, nearly semi-circular cylinders are placed in a uniform electric field along the $x$-axis the surface density of the charge induced on them when they are insulated from each other may be computed from

$$\sigma = \frac{Ff}{2\pi K'} \left\{ \frac{8K'V^2}{(1 + V^2)^2} - 1 \right\} h,$$

(90)

where $F$ is the intensity of the uniform field and $h$ is given by (86). The point of equilibrium where $\sigma$ changes in sign is thus given for a value of $v$ determined by (79) and it therefore coincides with the maximum value of $y$ on the cylinder. It may be shown that this result does not depend upon the approximations we have used, but is a perfectly general one. The hydrodynamical interpretation is that if two equal cylinders of the type we are considering be placed in a stream of fluid flowing at infinity along the $y$-axis, the stream will divide on each cylinder at the point where $y$ is a maximum. For values of $v$ too small for (78) to hold the surface density may be computed from

$$\sigma = -\frac{Ff}{2\pi K'} \left\{ 1 - 16K'q^2 \cosh 4v \right\} h,$$

(91)

with $h$ given by (86).

If the two cylinders are at the same potential in a field along the $x$-axis the surface density may be computed from

$$\sigma = \frac{4Ff}{\pi} \frac{V^2}{(1 + V^2)^2} h,$$

(92)

with $h$ determined by (85). For small values of $v$ we should use

$$\sigma = \frac{8Ff}{\pi} q^2 \cosh 4v \cdot h,$$

(93)

with $h$ given by (86).
In a field along the $y$-axis the surface density of the charge induced on each cylinder may be computed from

$$\sigma = \frac{2Ff}{\pi} \frac{V(1 - V^2)}{(1 + V^2)^2} h,$$

with $h$ given by (85). For small values of $v$ we should use

$$\sigma = \frac{4Ff}{\pi} q \sinh 2v \cdot h,$$

with $h$ given by (86).

The expressions for the components of the electric intensity at any point in the plane when the cylinders are placed in a uniform field are a little more complicated. When the two cylinders are insulated from each other and in a field along the $x$-axis, the components of the electric intensity may be computed from

$$X = \frac{2Ff}{C^2(A^2 + B^2)} \left[ A \left\{ \frac{C^2}{K} + 8V^2 \left[ (1 + V^4) \cos 4u - 2V^2 \right] \right. \right.$$

$$+ 8BV^2(1 - V^4) \sin 4u \right\},$$

$$Y = \frac{2Ff}{C^2(A^2 + B^2)} \left[ 8AV^2(1 - V^4) \sin 4u \right.$$

$$- B \left\{ \frac{C^2}{K} + 8V^2 \left[ (1 + V^4) \cos 4u - 2V^2 \right] \right\},$$

where

$$C = 1 + V^4 - 2V^2 \cos 4u,$$

and the values of $A$ and $B$ given by (82) are to be used. For small values of $v$ we should use the equations

$$X = \frac{2Ff}{A^2 + B^2} \left[ A \left\{ \frac{1}{K} + 16q^2 \cos 4u \cosh 4v \right\} \right.$$

$$\left. + 16Bq^2 \sin 4u \sinh 4v \right\},$$

$$Y = \frac{2Ff}{A^2 + B^2} \left[ 16Aq^2 \sin 4u \sinh 4v \right.$$

$$- B \left\{ \frac{1}{K} + 16q^2 \cos 4u \cosh 4v \right\} \right].$$
with $A$ and $B$ given by (83). When the two cylinders are at the same potential in a field along the $x$-axis the components of the electric intensity may be computed from

$$X = \frac{16FfV^2}{C^2(A^2 + B^2)} \left[ A \{(1 + V^2) \cos 4u - 2V^2\} + B(1 - V^4) \sin 4u \right],$$

$$Y = \frac{16FfV^2}{C^2(A^2 + B^2)} \left[ A(1 - V^4) \sin 4u - B\{(1 + V^4) \cos 4u - 2V^2\} \right].$$

For small values of $v$ we use the equations

$$X = \frac{32Ffgq}{A^2 + B^2} \left[ A \cosh 4v \cos 4u + B \sinh 4v \sin 4u \right],$$

$$Y = \frac{32Ffgq}{A^2 + B^2} \left[ A \sinh 4v \sin 4u - B \cosh 4v \cos 4u \right],$$

with $A$ and $B$ given by (83). At infinity, where $u = 0, v = K'$, both solutions give $X = F, Y = 0$.

When the two equal cylinders are placed in a uniform field along the $y$-axis the components of the electric intensity at any point in the plane may be computed from

$$-X = \frac{8FfV}{C^2(A^2 + B^2)} \left[ A \{(1 - V^2)\{(1 + V^2)^2 + 4V^2 \cos^2 2u\} \sin 2u - B\{(1 + V^2)^2 - 4V^2 \sin^2 2u\} \cos 2u\right],$$

$$Y = \frac{8FfV}{C^2(A^2 + B^2)} \left[ A\{(1 + V^2)\{(1 - V^2)^2 - 4V^2 \sin^2 2u\} \cos 2u + B\{(1 - V^2)^2 + 4V^2 \cos^2 2u\} \sin 2u\right].$$

At infinity, these give $X = 0, Y = F$. For small values of $v$ we may compute the components of the electric intensity from the equations

$$-X = \frac{16Ffg}{A^2 + B^2} \left[ A \cos 2u \cosh 2v + B \sin 2u \sinh 2v \right],$$

$$Y = \frac{16Ffg}{A^2 + B^2} \left[ A \sin 2u \sinh 2v - B \cos 2u \cosh 2v \right],$$

with $A$ and $B$ given by (83).
ELECTROSTATIC PROBLEMS CONNECTED WITH THICK CYLINDRICAL SHELLS

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ABSTRACT

The problem of determining the distribution of electricity on an infinitely long cylindrical shell, the cross-section of which is an arc of a circle, was solved by Bickley in 1926. In the present paper the solution is given for a shell of finite thickness, that is, an infinitely long cylinder the cross-section of which is bounded by two concentric circular arcs and two radii. The distribution of electricity on the shell is determined when it is freely charged, and the distribution of the induced charge is found when the shell is placed in a uniform electric field.

The solution of the problem for a single thick shell is then modified so as to apply to two equal, symmetrically placed thick shells. This is one of the very few problems for which an exact solution has been obtained by means of which the effect of finite thickness of the plates of a cylindrical condenser can be evaluated. A table is given to show the amount of this effect in a number of condensers of different dimensions.

1. The problem of the distribution of electricity on an infinitely thin cylindrical shell, the cross-section of which is an arc of a circle, was solved by Bickley. The problem of a thick cylindrical shell, that is, an infinitely long cylinder, the cross-section of which is bounded by two concentric circular arcs and two radii, is solved in this paper. The solution obtained for this problem is then modified so as to apply to the problem of two equal symmetrically placed cylindrical shells of finite thickness.

Fig. 1 shows the cross-section of the shell in the $z$-plane. On account of symmetry it is necessary to consider only the upper half of the plane. The radii of the two arcs are $a$ and $b$ with $b > a$, and $2\alpha$ is the angle they subtend at the center 0. Taking

$$w = \log \frac{z}{c} = \log \frac{r}{c} + i\theta,$$

1 Philosophical Magazine, 35, p. 396, 1918.

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the \( w \)-plane is shown in Fig. 2. The \( w \)-plane is transformed to the upper half of the \( t \)-plane by means of the Schwarz-Christoffel method. With

\[
0 < r < p < n < m < \infty,
\]

the differential equation for the transformation is

\[
\frac{dw}{dt} = \frac{A}{t} \left( \frac{t - p \cdot t - n}{t - r \cdot t - m} \right)^{\frac{1}{3}}. \tag{1}
\]

There is a jump in \( w \) of \( i\pi \) when \( t \) passes from \(+\infty\) to \(-\infty\), and so \( A = 1 \). There is a jump in \( w \) of \(-i\pi\) when \( t \) passes through 0, and therefore

\[
pn = mr. \tag{2}
\]

Equation (1) is integrated by means of the substitution,

\[
t = \frac{m(n - r) - n(m - r) \, sn^2 \lambda}{(n - r) - (m - r) \, sn^2 \lambda}, \tag{3}
\]
with

\[ k^2 = \frac{m-r}{n-r} \frac{n-p}{m-p}, \quad k'^2 = \frac{p-r}{n-r} \frac{m-n}{p-n} \]  

(4)

It follows that corresponding values of \( t \) and \( \lambda \) are given by

\[
\begin{align*}
 t &= r & p \\
 \lambda &= K & K + iK' & iK' & 0
\end{align*}
\]

If we put

\[
\begin{align*}
 sn^2 \delta &= \frac{n-r}{m-r}, \\
 sn^2 \gamma &= \frac{m-n-r}{n m - r},
\end{align*}
\]

we see that \( \delta \) and \( \gamma \) are real, with \( \gamma > \delta \). The origin in the \( z \)-plane corresponds to \( t = 0 \), and for this value \( \lambda = \gamma \). The point at infinity in the \( z \)-plane corresponds to \( t = \infty \), and for this value \( \lambda = \delta \). The relation (2) now gives

\[
\frac{sn \gamma cn \gamma}{dn \gamma} = \frac{sn \delta cn \delta}{dn \delta},
\]

and since \( \gamma \) must be greater than \( \delta \) it follows that

\[ \gamma = K - \delta, \]

with \( \delta \) less than \( K/2 \). The following relations, derived from the substitution (3) when \( \delta + \gamma = K \), will be found useful in making the required transformations in the sequel.

\[
\begin{align*}
 \frac{t}{n} &= \frac{cn^2 \delta - dn^2 \delta}{dn^2 \delta (sn^2 \delta - sn^2 \lambda)}, \\
 \frac{m}{n} &= \frac{cn^2 \delta}{sn^2 \delta dn^2 \delta}, \\
 \frac{p}{n} &= \frac{k'^2}{dn^4 \delta}, \\
 \frac{r}{n} &= \frac{k'^2 sn^2 \delta}{cn^2 \delta dn^2 \delta}
\end{align*}
\]

(5)
Equation (1) may now be written
\[
\frac{dw}{d\lambda} = \frac{4 \, sn^2 \delta \, cn^2 \delta \, cn \, 2\delta}{sn \, 2\delta} \times \frac{dn^2 \lambda}{(sn^2 \delta - sn^2 \lambda)(cn^2 \delta - dn^2 \delta \, sn^2 \lambda)},
\]
and its integral is
\[
w = 2\Pi_1(\lambda, \delta) - 2\Pi_1(\lambda, K - \delta) + B,
\]
where \(B\) is a constant, and
\[
\Pi_1(\lambda, \delta) = sn \delta \, cn \delta \, dn \delta \int_0^\lambda \frac{d\lambda}{sn^2 \delta - sn^2 \lambda}.
\]
In terms of the Jacobian functions, we have
\[
\Pi_1(\lambda, \delta) = -\frac{1}{2} \log \frac{H(\delta - \lambda)}{H(\delta + \lambda)} - \lambda Z(\delta).
\]
The conditions to be satisfied are given by the scheme
\[
t = m \, \quad m \, \quad n \, \quad p \, \quad r \, \quad r \, \quad r \, \quad r
\]
\[
\lambda = 0 \, \quad iK' \, \quad K + iK' \, \quad K
\]
\[
w = \log \frac{b}{c} \, \quad \log \frac{b}{c} + i\alpha \, \quad \log \frac{a}{c} + i\alpha \, \quad \log \frac{a}{c}
\]
For these four values of \(\lambda\) the elliptic integral of the third kind, \(\Pi_1(\lambda, \delta)\), has the values
\[
\Pi_1(0, \delta) = 0,
\]
\[
\Pi_1(K, \delta) = -KZ(\delta) + \frac{i\pi}{2},
\]
\[
\Pi_1(K + iK', \delta) = -(K + iK')Z(\delta) - \frac{i\pi\delta}{2K} + i\frac{\pi}{2},
\]
\[
\Pi_1(iK', \delta) = -iK'Z(\delta) - \frac{i\pi\delta}{2K} + i\frac{\pi}{2}.
\]
We therefore find that the constant \(b\) has the value \(\log b/c\), and the dimensions of the shell are given by the equations
\[
\log \frac{b}{a} = 2K\{Z(\delta) - Z(K - \delta)\}, \quad \text{(7)}
\]
\[
\alpha = \pi \left(1 - \frac{2\delta}{K}\right) - \frac{K'}{K} \log \frac{b}{a}. \quad \text{(8)}
\]
With the proper choice of the modular angle and with values of \( \delta \) lying between 0 and \( K/2 \), it is possible to give the ratio \( b/a \) any value between unity and infinity, and the angle \( \alpha \) any value between zero and \( \pi \). For large values of \( b/a \) the modular angle must be large, and for large values of \( \alpha \), \( \delta \) must be small. The Smithsonian Tables of Elliptic Functions may be used to evaluate (7), and then \( \alpha \) is given by (8). Or we can use the Fourier series for the Zeta function so that (7) may be calculated from

\[
\log \frac{b}{a} = 8\pi \sum_{1}^{\infty} \frac{q^{2n}}{1 - q^{4n}} \sin \frac{2n\pi \delta}{K},
\]

where

\[
q = e^{-\frac{\pi K}{K^*}}.
\]

The quadratic substitution (3) that has been used is suitable for moderately thick shells. For very thick shells the modular angle is large and the series in (9) does not converge rapidly. We can make use of a different quadratic substitution that will give thick shells for small modular angles, or we may transform the elliptic functions to the complementary modulus. For large modular angles the ratio \( b/a \) may be determined from the equation

\[
\log \frac{b}{a} = \frac{\pi K}{K^*} \left[ 1 - 2\beta + 2 \frac{q^{2(1-\beta)} - q'^{2\beta}}{(1 + q'^{\beta})(1 + q^{2(1-\beta)})} \right. \\
- \left. 2 \sum_{1}^{\infty} (-1)^{n} \frac{q^{2n}}{1 - q'^{n}} \left[ \frac{1}{q'^{n\beta}} - \frac{1}{q^{n(1-\beta)}} \right] \right],
\]

where

\[
\beta = \delta/K
\]

and

\[
q' = e^{-\frac{\pi K}{K^*}}.
\]

Since \( \beta \) is less than \( \frac{1}{2} \) the infinite series in this expression will not contribute appreciably to the result if the modular angle is greater than \( 89^\circ \). These problems may be solved in a little simpler way by using a linear instead of a quadratic substitu-
tion. For example, the linear substitution
\[ t = \frac{sn \lambda + sn \delta}{sn \lambda - sn \delta}, \]
with \( \delta \) lying between zero and \( K \), will be found to solve the problems. The advantage of the quadratic substitutions is that they lead directly to more rapidly converging series.

We can now write the integral of (1), that satisfies the given conditions, in terms of the Jacobian Theta functions,
\[ \log \frac{z}{b} = \log \frac{H_1(\delta + \lambda)}{H_1(\delta - \lambda)} + \log \frac{H(\delta + \lambda)}{H(\delta - \lambda)} - \frac{\lambda}{K} \log \frac{b}{a}, \quad (10) \]

or
\[ \frac{z}{b} = \frac{H_1(\delta + \lambda)H(\delta + \lambda)}{H_1(\delta - \lambda)H(\delta - \lambda)} \left( \frac{b}{a} \right)^{-\frac{\lambda}{K}}. \quad (11) \]

At infinity, \( z = \infty, \lambda = \delta \). Put \( \lambda = \delta - \epsilon \). Then as \( \epsilon \) approaches zero we have
\[ H(\epsilon) = \epsilon \pi H'(0) = \epsilon \left( \frac{2Kk'}{\pi} \right)^{\frac{1}{2}}, \]
\[ H_1(\epsilon) = H_1(0) = \left( \frac{2Kk}{\pi} \right)^{\frac{1}{2}}. \]

It therefore follows from (11) that
\[ \text{Limit } z \epsilon = \lim_{\substack{z \to \infty \atop \epsilon \to 0}} \frac{\pi b H_1(2\delta)H(2\delta)}{2Kk k'} \left( \frac{b}{a} \right)^{-\frac{\delta}{K}}. \quad (12) \]

At the origin, \( z = 0 \), and \( \lambda = K - \delta \). Put \( \lambda = K - \delta + \epsilon \). We now get from (11)
\[ \text{Limit } z \epsilon = \lim_{\substack{z \to 0 \atop \epsilon \to 0}} \frac{2a K k k'}{\pi H_1(2\delta)H(2\delta)} \left( \frac{b}{a} \right)^{\frac{\delta}{K}}. \quad (13) \]

Along the convex surface, \( DC \), in Fig. 1, \( \lambda = iv \), where \( v \) varies from 0 at \( D \) to \( K' \) at \( C \). With
\[ z = be^{i\theta} \]
we find from (10) that \( v \) and \( \theta \) are connected by the relation
\[
\theta = \frac{1}{i} \log \frac{H_1(\delta + iv)H(\delta + iv)}{H_1(\delta - iv)H(\delta - iv)} - \frac{v}{K} \log \frac{b}{a}.
\] (14)

Along the concave surface, \( AB, r = a \), and \( \lambda = K + iv \), where \( v \) varies from 0 at \( A \) to \( K' \) at \( B \). The relation between \( v \) and \( \theta \) on this surface also is given by (14). It follows that any radius of angle \( \theta \) meets the concave and convex surfaces at the same value of \( v \). Along the end, \( BC \),

\[
\lambda = iK' + u,
\]

and

\[
z = re^{iu},
\]

and therefore along the end the relation between \( r \) and \( u \) is given by
\[
\frac{r}{b} = \frac{\Theta_1(\delta + u) \Theta(\delta + u)}{\Theta_1(\delta - u) \Theta(\delta - u)} \left( \frac{b}{a} \right)^{-\frac{u}{K}},
\] (15)

where \( u \) varies from 0 at \( C \) to \( K \) at \( B \), and \( r \) from \( b \) to \( a \).

For numerical computation we may make use of the developments of the Theta functions in series. By the transformation theory of the Theta functions we have
\[
H(u)H_1(u) = \left( \frac{2K'}{\pi} \right)^{\frac{1}{i}} H(2u, q^2),
\] (16)
in which the Eta function on the right is to be formed with \( q^2 \) replacing the \( q \) that is used in the Eta functions on the left, but keeping \( K \) the same. Similarly,
\[
\Theta(u) \Theta_1(u) = \left( \frac{2K'}{\pi} \right)^{\frac{1}{i}} \Theta(2u, q^2).
\] (17)

We therefore get from (14)
\[
\theta = 2 \tan^{-1} L - \frac{v}{K} \log \frac{b}{a},
\] (18)

where \( L \) is given as the quotient of two rapidly converging
series,
\[
L = \frac{\sum_{n=0}^{\infty} (-1)^n q^{2n(n+1)} \cos (2n + 1) \frac{\pi \delta}{K} \sinh (2n + 1) \frac{\pi v}{K}}{\sum_{n=0}^{\infty} (-1)^n q^{2n(n+1)} \sin (2n + 1) \frac{\pi \delta}{K} \cosh (2n + 1) \frac{\pi v}{K}}.
\]
(19)

From (13) we find, by using (17),
\[
\frac{r}{b} = \left(\frac{b}{a}\right)^{-\frac{u}{K}} M,
\]
(20)

where \(M\) is given by
\[
M = \frac{1 + \sum_{n=1}^{\infty} (-1)^n q^{2n^2} \cos \frac{2n \pi}{K} (\delta + u)}{1 + \sum_{n=1}^{\infty} (-1)^n q^{2n^2} \cos \frac{2n \pi}{K} (\delta - u)}.
\]
(21)

In order to use equations (12) and (13) we have the expansion, obtained by the aid of (16),
\[
H(2\delta)H_1(2\delta)
\]
\[
= \left(\frac{2Kk^{-1}}{\pi}\right)^i 2q^i \sum_{n=0}^{\infty} (-1)^n q^{2n(n+1)} \sin (2n + 1) \frac{2\pi \delta}{K}.
\]
(22)

For the infinitely thin shell, \(a = b\), and the points \(B\) and \(C\) in Figs. 1 and 2 coincide. Therefore \(n = p\), and from (4),
\[
k = 0, \quad k' = 1.
\]

It follows that for the infinitely thin shell,
\[
K = \frac{\pi}{2}, \quad K' = \infty, \quad q = 0.
\]

From (8) we get
\[
\delta = \frac{\pi - \alpha}{4},
\]
(23)

and from (18) and (19),
\[
\tan \frac{\theta}{2} = \tan \frac{\alpha}{2} \tanh 2v,
\]
(24)
with \( v \) varying from 0 at \( A \) to \( \infty \) at \( B = C \). For the infinitely thin shell (22) reduces to

\[
\frac{H(2\delta)H_1(2\delta)}{k} = \frac{1}{2} \sin \alpha. \tag{25}
\]

2. Let now the thick cylindrical shell have a charge \( Q \) on unit length. Taking

\[
\chi = \phi + i\psi,
\]

with \( \phi \) as the real potential, the \( \chi \)-plane is shown in Fig. 3, and to transform it to the \( t \)-plane we have

\[
\frac{d\chi}{dt} = \frac{C}{(t - r \cdot t - m)^i}. \tag{26}
\]

As there is a jump in \( \chi \) of \(-2\pi iQ\) when \( t \) passes from \(+\infty\) to \(-\infty\), the constant \( C = -2Q \). A solution of (26) that satisfies the conditions,

\[
\chi = 0 \text{ for } t = r,
\]

\[
\chi = 2\pi iQ \text{ for } t = m,
\]

may be written

\[
\chi = 4iQ \cos^{-1} \left( \frac{m - t}{m - r} \right)^i. \tag{27}
\]

Using the substitution (3) this may be put in the form,

\[
\chi = 4iQ \cos^{-1} \frac{cn \delta \ sn \lambda}{(sn^2 \lambda - sn^2 \delta)^i}. \tag{28}
\]
The values of \( \psi \) at the four points, \( D, C, B \) and \( A \), are given by

\[
\psi = 2\pi Q \quad 4Q \cos^{-1} cn \delta \quad 4Q \cos^{-1} \frac{cn \delta}{dn \delta} \quad 0
\]

If \( Q_1 \) be the charge on unit length of the convex surface, \( Q_2 \) on the concave surface, and \( Q_3 \) on both ends of the shell, these charges are given by

\[
\begin{align*}
Q_1 &= Q \left( 1 - \frac{2}{\pi} \cos^{-1} cn \delta \right), \\
Q_2 &= \frac{2Q}{\pi} \cos^{-1} \frac{cn \delta}{dn \delta}, \\
Q_3 &= \frac{2Q}{\pi} \cos^{-1} \frac{cn^2 \delta + k' sn^2 \delta}{dn \delta}.
\end{align*}
\]

(29)

The electric intensity at any point of the \( z \)-plane may be obtained from (1) and (26). These give

\[
\frac{dX}{dz} = -X + iY = -\frac{2Q}{z} \frac{t}{(t - p \cdot t - n)}. 
\]

(30)

By the use of the substitution (3) this becomes

\[
\frac{dX}{dz} = -\frac{2Q}{z} \frac{dn \delta \cdot cn \delta \cdot \delta - dn^2 \delta \cdot sn \delta \cdot \lambda}{z \cdot dn \lambda \cdot cn^2 \delta - dn^2 \delta \cdot sn^2 \delta}.
\]

(31)

At \( z = \infty \), where \( \lambda = \delta \), this approaches the correct value,

\[
\frac{dX}{dz} = -\frac{2Q}{z}.
\]

If \( \sigma_1 \) be the surface density on the convex surface where \( \lambda = iv \), \( \sigma_2 \) on the concave surface, where \( \lambda = K + iv \), and \( \sigma_3 \) the surface density on the end where \( \lambda = u + iK' \), we find

\[
\begin{align*}
\sigma_1 &= \frac{Q}{4\pi b cn 2\delta} \frac{sn 2\delta}{sn \delta} \frac{1 - sn^2 \delta \cdot dn_r^2 \cdot v}{cn' v \cdot dn' v}, \\
\sigma_2 &= \frac{Q}{4\pi a cn 2\delta} \frac{sn 2\delta}{sn \delta} \frac{dn^2 \delta - cn^2 \delta \cdot dn_r^2 \cdot v}{k' \cdot cn' \cdot v \cdot dn' \cdot v}, \\
\sigma_3 &= \frac{Q}{4\pi r cn 2\delta} \frac{sn 2\delta}{sn \delta} \frac{dn^2 \delta - k^2 \cdot cn^2 \delta \cdot sn^2 \cdot u}{k^2 \cdot sn \cdot u \cdot cn \cdot u}.
\end{align*}
\]

(32)
THICK CYLINDRICAL SHELLS

The accents on the elliptic functions indicate that they are to be formed with the complementary modulus, $k'$.

From the results just obtained those for the infinitely thin shell may be derived by going to the degenerate case, $k = 0$. We must replace $sn$, $cn$ by $sin$, $cos$; $dn$ by 1; $sn'$ by tanh, and $cn'$ and $dn'$ by sech. $v$ is given in terms of $\theta$ by (24) and $\delta$ is defined by (23). The surface density on the infinitely thin shell of radius $b$ is then given by

$$\sigma = \frac{Q}{4\pi b} \cos \frac{\theta}{2} \pm \left( \cos^2 \frac{\theta}{2} - \cos^2 \frac{\alpha}{2} \right)^\frac{1}{i},$$

where the upper sign refers to the convex and the lower sign to the concave surface. The given charge, $Q$, divides between these two surfaces in the ratio, obtained from (24),

$$\frac{Q_1}{Q_2} = \frac{\pi + \alpha}{\pi - \alpha}.$$

These results agree with those found by Bickley for the infinitely thin shell.

3. When the thick shell is placed in a uniform electric field along the $x$-axis the $x$-plane is shown in Fig. 4. The

![Fig. 4.](image)

point of equilibrium upon the shell is given by $t = s$. To transform the $x$- to the $t$-plane we have the differential equation,

$$\frac{d\chi}{dt} = \frac{C(t - s)}{(t - r \cdot t - m)^i}. \tag{33}$$
A solution of this equation that satisfies the conditions,

\[ x = 0 \quad \text{for} \quad t = r \quad \text{and for} \quad t = m, \]

is given by

\[ x = C(t - m \cdot t - r)^i \quad (34) \]

together with

\[ s = \frac{m + r}{2}. \quad (35) \]

Using the substitution (3) we may write (34)

\[ x = 2nC \frac{cn 2\delta}{dn \delta} \frac{sn \lambda}{sn 2\delta \lambda} \frac{cn \lambda}{sn 2\delta \lambda - sn^2 \lambda}. \quad (36) \]

The constant \( C \) must be determined so that the potential at infinity shall be \( \varphi = -Fx + \text{const.} \), for a field \( F \) in the direction of the positive \( x \)-axis. If we put \( \lambda = \delta - \epsilon \) in (36), and let \( \epsilon \) approach zero, we find, with the aid of (12),

\[ nC = -\frac{\pi bFH_1(2\delta)H(2\delta) dn^2 \delta sn 2\delta}{2Kkk'\frac{b}{a}} \left( \frac{b}{a} \right)^{-\frac{\delta^2}{a^2}}. \quad (37) \]

The values of \( \psi \) at the four points \( D, C, B \) and \( A \) are now given by

\[ t = m \quad n \quad p \quad r \]

\[ \psi = 0 \quad 2nC \frac{cn 2\delta}{dn \delta} \frac{sn 2\delta}{dn 2\delta} \quad 2nC \frac{k' \cdot cn 2\delta}{dn^2 \delta sn 2\delta} \quad 0 \]

The charges induced on unit lengths of the convex and concave surfaces and the two ends may therefore be written

\[ 2\pi Q_1 = -2nC \frac{cn 2\delta}{dn \delta} \frac{sn 2\delta'}{sn 2\delta}, \]

\[ 2\pi Q_2 = 2nC \frac{cn 2\delta}{dn^3 \delta sn 2\delta'}, \quad (38) \]

\[ 2\pi Q_3 = 2nC \frac{cn 2\delta(dn^2 \delta - k')}{dn^3 \delta sn 2\delta}. \]

For the position of the point of equilibrium we find from
(5) and (35)

\[ sn^2 \lambda = \frac{sn^2 \delta}{sn^2 \delta - cn^2 \delta}. \]

On the convex surface, \( \lambda = iv \), and so if the point of equilibrium is on this surface we must have

\[ sn' v = \frac{sn \delta}{cn \delta}. \]

This equation can be satisfied for a value of \( v \) between 0 and \( K' \) if \( sn \delta < cn \delta \). On the end, \( \lambda = iK' + u \), so that if the point of equilibrium lies upon the end we must have

\[ sn u = \frac{cn \delta}{sn \delta}. \]

This equation can be satisfied for a value of \( u \) between 0 and \( K \) if \( cn \delta < sn \delta \). In the limiting case, \( sn \delta = cn \delta \), or \( sn^2 \delta = \frac{1}{2} \), and the edge \( C \) is the point of equilibrium. For every modular angle there is one value of \( \delta \) less than \( K/2 \) that satisfies the condition \( sn \delta = cn \delta \); with this value of \( \delta \) and the given modular angle, equations (7) and (8) determine the dimensions of the shell for which the edge \( C \) is the point of equilibrium in a field along the \( x \)-axis. The hydrodynamical interpretation of this is that if the shell is placed in an infinite fluid flowing at infinity along the \( y \)-axis, the stream will divide at the edge \( C \). The point of equilibrium can never lie upon the concave surface. For an infinitely thin shell the point of equilibrium lies upon the convex surface at an angle \( \theta \) determined by

\[ \tan \frac{\theta}{2} = \sin \frac{\alpha}{2}. \]

The electric intensity at any point in the \( z \)-plane is determined by

\[ \frac{d \chi}{dz} = \frac{nC (sn^2 \delta \ cn^2 \lambda + cn^2 \delta \ sn^2 \lambda)(cn^2 \delta - dn^2 \delta \ sn^2 \lambda)}{2z \ sn^2 \delta \ cn^2 \delta \ dn \delta \ dn \lambda (sn^2 \delta - sn^2 \lambda)}. \]  

(39)

By giving \( \lambda \) its appropriate values on the three surfaces, the
surface density of the induced charge may be found from this expression. Since at \( z = \infty \) we must have

\[
\frac{d\chi}{dz} = -F,
\]

the constant \( nC \), given by (37), may also be found by putting \( \lambda = \delta - \epsilon \) in (39), and taking the limiting value, with the aid of (12) when \( \epsilon = 0 \).

4. When the thick cylindrical shell is placed in a uniform electric field along the \( y \)-axis the complex potential is given by

\[
\chi = -iCt.
\]

The constant \( C \) must be determined so that the potential at infinity shall be \( \varphi = -Fy + \text{const} \). Proceeding just as before, we find

\[
\chi = -iCn \frac{cn^2 \delta - dn^2 \delta \ sn^2 \lambda}{dn^2 \delta \ (sn^2 \delta - sn^2 \lambda)}, \tag{40}
\]

where \( Cn \) is again given by (37).

The charges induced on unit lengths of the upper halves of the three surfaces are now given by

\[
4\pi Q_1 = -2Cn \frac{cn \ 2\delta \ cn \ \delta}{sn \ 2\delta \ sn \ \delta \ dn \ \delta},
\]

\[
4\pi Q_2 = -2Cn \frac{k^2 \ cn \ 2\delta \ sn \ \delta}{sn \ 2\delta \ cn \ \delta \ dn^2 \ \delta}, \tag{41}
\]

\[
4\pi Q_3 = -2Cn \frac{k^2 \ cn \ 2\delta \ sn \ \delta \ cn \ \delta}{sn \ 2\delta \ dn^3 \ \delta}.
\]

These charges are all positive for a field in the direction of the positive \( y \)-axis. The electric intensity at any point in the \( z \)-plane is determined by

\[
\frac{d\chi}{dz} = -\frac{iCn}{z} \frac{sn \ \lambda \ cn \ \lambda}{sn \ \delta \ cn \ \delta \ dn \ \delta \ dn \ \lambda} \frac{cn^2 \ \delta - dn^2 \ \delta \ sn^2 \ \lambda}{sn^2 \ \delta - sn^2 \ \lambda}. \tag{42}
\]

Since at \( z = \infty \) the electric intensity is given by

\[
\frac{d\chi}{dz} = iF,
\]
the constant \( nC \) in (42) may be determined by putting \( \lambda = \delta - \epsilon \) and taking the limiting value, given by (12), when \( \epsilon = 0 \). The surface densities of the induced charge on the three surfaces may be expressed by substituting the appropriate values for \( \lambda \) in (42). The points \( A \) and \( D \), where \( \lambda = K \) and 0, are the only points of equilibrium.

5. For the infinitely thin shell corresponding results are given by going to the degenerate case, \( k = 0 \). The constant \( nC \), given by (37), becomes, with the use of (23) and (25),

\[
nC = - bF \cos^2 \frac{\alpha}{2}.
\]

Then the charge induced on unit length of the convex surface of the infinitely thin shell in a field along the \( x \)-axis is, from (38), given by

\[
Q = \frac{bF}{2\pi} \sin \alpha,
\]

with an equal and opposite charge induced on the concave surface. The surface density of the induced charge is, from (39), given by

\[
\sigma = \pm \frac{F}{4\pi \cos^2 \frac{\alpha}{2}} \left( A \mp \sin^2 \frac{\alpha}{2} \cos^2 \frac{\theta}{2} \right) \left( A \pm \cos \frac{\theta}{2} \right)^2,
\]

where the upper sign refers to the convex and the lower to the concave surface, and \( A \) is an abbreviation for

\[
A = \left( \sin^2 \frac{\alpha}{2} - \sin^2 \frac{\theta}{2} \right)^{\frac{1}{3}}.
\]

The surface density vanishes at the point of equilibrium, determined by

\[
\tan \frac{\theta}{2} = \sin \frac{\alpha}{2}.
\]

In a field along the \( y \)-axis the charge induced on unit length of the upper half of the infinitely thin shell is given by

\[
Q = \frac{bF}{2\pi} \sin \frac{\alpha}{2} \left( 1 \pm \sin \frac{\alpha}{2} \right),
\]
where the upper sign refers to the convex and the lower to the concave surface. The surface density of the induced charge is given by

\[ \sigma = \frac{F}{4\pi} \frac{\left( \cos \frac{\theta}{2} \pm A \right)^2}{A} \sin \frac{\theta}{2}. \]

6. We shall next consider the problem of two equal thick cylindrical shells when they are placed symmetrically as shown in Fig. 5. The \( y \)-axis is now an axis of symmetry, as well as the \( x \)-axis, and so we need consider only the region of the \( z \)-plane enclosed within the dotted lines. If the dotted lines consist entirely of equipotentials and lines of force we can solve the electrostatic problems connected with the two shells. This will be the case when the two shells have equal and opposite charges and also when they have equal charges at the same potential.

The transformation of the \( w \)- to the \( t \)-plane is given by equation (11), but as the jump in \( w \) at \( 0 \) and \( \infty \) is now \( \pm i\pi/2 \) instead of \( \pm i\pi \), the constant \( A \) in that equation has the value
\( \frac{1}{2} \) instead of 1. The relation (2) still holds. Instead of (7) and (8) we now have
\[
\log_a b = K \{ Z(\delta) - Z(K - \delta) \} \tag{43}
\]
and
\[
\alpha = \frac{\pi}{2} \left( 1 - \frac{2\delta}{K} \right) - \frac{K'}{K} \log_a b . \tag{44}
\]
Accordingly, in order to calculate \( \log b/a \) from the series (9) the right-hand side must be divided by 2. Instead of (11) we now have
\[
\left( \frac{z}{b} \right)^2 = \frac{H_1(\delta + \lambda)H(\delta + \lambda)}{H_1(\delta - \lambda)H(\delta - \lambda)} \left( \frac{b}{a} \right)^{-2\lambda/K}. \tag{45}
\]
Putting \( \lambda = \delta - \epsilon \), we find
\[
\text{Limit } \lim_{\substack{z \to 0 \\ \epsilon \to 0}} \epsilon \left( \frac{z}{b} \right)^2 = \frac{\pi b^2 H_1(2\delta)H(2\delta)}{2Kkk'} \left( \frac{b}{a} \right)^{-2\lambda/K}, \tag{46}
\]
and putting \( \lambda = K - \delta + \epsilon \), we find
\[
\text{Limit } \lim_{\substack{z \to 0 \\ \epsilon \to 0}} \epsilon \left( \frac{z}{b} \right)^2 = \frac{2a^2Kkk'}{\pi H_1(2\delta)H(2\delta)} \left( \frac{b}{a} \right)^{-2\lambda/K}. \tag{47}
\]
These must now be used in place of (12) and (13).

On the convex and concave surfaces the relation between \( \theta \) and \( v \), taking the place of (14), is
\[
\theta = \frac{1}{2i} \log \frac{H(\delta + iv)H_1(\delta + iv)}{H(\delta - iv)H_1(\delta - iv)} - \frac{v}{K} \log_a b . \tag{48}
\]
This may be computed from
\[
\theta = \tan^{-1} L - \frac{v}{K} \log_a b , \tag{49}
\]
where \( L \) is given by (19). In place of (15) and (20), on the end, we have
\[
\frac{r^2}{b^2} = \frac{\Theta(\delta + u)\Theta_1(\delta + u)}{\Theta(\delta - u)\Theta_1(\delta - u)} \left( \frac{b}{a} \right)^{-2\mu/K} = M \left( \frac{b}{a} \right)^{-2\mu/K} . \tag{50}
\]
where \( M \) is given by (21).
The electrostatic problems connected with two equal infinitely thin shells were solved in a recent paper in these Proceedings, and the results there obtained may be derived from those in this paper by going to the degenerate case, \( k = 0 \). For the infinitely thin shells (44) gives

\[ 2\delta = \frac{\pi}{2} - \alpha, \]

(49) and (19) give

\[ \tan \theta = \tan \alpha \tanh 2v, \]

while (22) is now

\[ \frac{H(2\delta)H_1(2\delta)}{k} = \frac{1}{2} \sin 2\alpha. \]

7. We shall first suppose that the shell on the right has a charge \( Q \) per unit length, at potential \( V \), and the shell on the left a charge \(-Q\) at potential \(-V\). The \( y\)-axis is now the equipotential \( \varphi = 0 \), and the \( x\)-axis is a line of force. We shall take \( \psi = 0 \) from 0 to \( A \), and \( \psi = 2\pi Q \) from \( D \) to \( \infty \). The \( x\)-plane is shown in Fig. 6, and to transform it to the

![Fig. 6.](image)

\( t\)-plane we have the differential equation

\[ \frac{dX}{dt} = \frac{C}{(t \cdot t - r \cdot t - m)^3}. \]  

(51)

In order to integrate this equation we shall use the substitution

\[ t = r \sin^2 \lambda_1, \]  

(52)

\[ ^1 \text{Proceedings American Philosophical Society, LXXVI, p. 251, 1936.} \]
with a modulus \( k_1 \) determined by
\[ k_1^2 = \frac{r}{m}. \]  
(53)

The subscript 1 is used to indicate that the elliptic functions are to the modulus \( k_1 \) in order to distinguish them from those to the modulus \( k \) that are used in defining the shells. A solution of (51) satisfying the conditions

\[
\begin{align*}
t &= m & r &= 0 & \infty \\
\lambda_1 &= K_1 + iK_1' & K_1 &= 0 & iK_1' \\
\chi &= V + 2\pi iQ & V &= 0 & 2\pi iQ
\end{align*}
\]
is
\[ \chi = \frac{2\pi Q}{K_1'} \lambda_1. \]

The constant \( C \) has the value given by
\[ C = -m^i \frac{\pi Q}{K_1'} \]
and
\[ \frac{Q}{V} = \frac{K_1'}{2\pi K_1}. \]

Since the difference of potential between the two shells is \( 2V \) the capacity of unit length of the condenser formed by them is
\[ S = \frac{K_1'}{4\pi K_1} = \frac{1}{4\pi^2} \log \frac{1}{q_1}, \]
(55)
where
\[ q_1 = e^{-\frac{\pi K_1'}{K_1}}. \]

From (54) and (55) we find for the relation between the modulus \( k_1 \) and the complementary modulus, \( k' \),
\[ k_1 = k' \frac{8n^2 \delta}{cn^2 \delta}. \]
(56)

For two infinitely thin shells this gives
\[ k_1 = \frac{1 - \sin \alpha}{1 + \sin \alpha}, \]
which, in the previous paper, was denoted by \( k \).
The following table shows the effect of finite thickness on the capacity of a condenser consisting of two equal shells. The first and second columns give the modular angle and the value of $\delta/K$ used in defining the shells. The third and fourth columns give the ratio $b/a$ and the angle $\alpha$. These are computed from equations (43) and (44). The fifth column gives the capacity of unit length of the condenser formed of the thick shells, and the last column that for infinitely thin shells.

<table>
<thead>
<tr>
<th>$\theta$</th>
<th>$b/K$</th>
<th>$b/a$</th>
<th>$\alpha$</th>
<th>$S$</th>
<th>$S'$</th>
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<tr>
<td>$89^\circ$</td>
<td>1/90</td>
<td>1.2300</td>
<td>84$^\circ$ 34'.3</td>
<td>0.5597</td>
<td>0.3792</td>
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<td>79$^\circ$ 10'.9</td>
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<td>0.3091</td>
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<td>0.0638</td>
</tr>
<tr>
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<td>1.1533</td>
<td>85$^\circ$ 15'.6</td>
<td>0.5384</td>
<td>0.3929</td>
</tr>
<tr>
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<td>1.3328</td>
<td>80$^\circ$ 32'.8</td>
<td>0.4680</td>
<td>0.3228</td>
</tr>
<tr>
<td>$87^\circ$</td>
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<td>1.1192</td>
<td>85$^\circ$ 39'.6</td>
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<tr>
<td>$87^\circ$</td>
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<td>1.2515</td>
<td>81$^\circ$ 20'.6</td>
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</table>

On all the surfaces of the shell the value of $\lambda_1$ is

$$\lambda_1 = K_1 + iv_1,$$

where $v_1 = 0$ at $A$ and $v_1 = K_1'$ at $D$. Let $v_1'$ be the value of $v_1$ at $C$, where $t = n$, and $v_1''$ its value at $B$ where $t = p$. Then we get, from (53) and (5),

$$dn' v_1' = \left(\frac{r}{n}\right)^i = \frac{k' sn \delta}{cn \delta dn \delta'},$$

$$dn' v_1'' = \left(\frac{r}{p}\right)^i = \frac{sn \delta}{cn \delta}.$$ It follows that

$$v_1' + v_1'' = K_1',$$
and since \( v_1'' \) is less than \( v_1' \) we must have \( v_1'' \) less than \( K_1'/2 \). We now find for the four points, \( D, C, B \) and \( A \), in Fig. 5

\[
\begin{align*}
t &= m \\
\lambda_1 &= K_1 + iK_1' \\
\lambda_2 &= K_1 + iv_1' \\
\lambda_3 &= K_1 + iv_1'' \\
\psi &= 2\pi Q \\
\psi_1' &= 2\pi Q \frac{v_1'}{K_1'} \\
\psi_1'' &= 2\pi Q \frac{v_1''}{K_1'} \\
\psi_3 &= 0
\end{align*}
\]

If we call the charges on unit lengths of the convex and concave surfaces, \( Q_1 \) and \( Q_2 \), and on the two ends, \( Q_3 \), we find

\[
\begin{align*}
Q_1 &= Q \left( 1 - \frac{v_1'}{K_1'} \right), \\
Q_2 &= Q \frac{v_1''}{K_1'} = Q_1, \\
Q_3 &= Q \frac{v_1' - v_1''}{K_1'}.
\end{align*}
\]

When the two thick shells form a condenser the charges on the convex and concave surfaces are equal; this same property was found for the infinitely thin shells.

In order to calculate \( v_1''' \) we may use the series

\[
dn' v_1 = \frac{\pi}{2K_1} \text{sech} \frac{\pi v_1}{2K_1} \\
- \frac{2\pi}{K_1} \sum_{1} (-1)^n \frac{q_1^{2n-1} \cosh (2n-1)}{1 - q_1^{2n-1}} \frac{\pi v_1}{2K_1}.
\]

By writing

\[
dn' v_1''' = c_1,
\]

we get as a first approximation

\[
\cosh v_1'' = \frac{1}{c_1}.
\]

Since \( v_1'' \) is less than \( K_1'/2 \) this will be a good approximation if the modulus \( k_1 \) is very small. For larger values of the modulus \( k_1 \) the method of successive approximations may be used.

The following table shows the distribution of the charges on the three surfaces for some of the shells used in the former table.
The electric intensity at any point of the z-plane is given by
\[
\frac{d\chi}{dz} = -X + iY = \frac{2C}{z} \left( \frac{t}{t - p \cdot t - m} \right)^{1/3}.
\]
This may be written, using the substitution (3) and the relations given in (5),
\[
\frac{d\chi}{dz} = -\frac{\pi Q}{zK_1'(sn^2 \delta \ cn 2\delta)} \frac{sn 2\delta}{\delta \ cn 2\delta} \times \frac{(cn^2 \delta - dn^2 \delta \ sn^2 \lambda)^{(sn^2 \delta - sn^2 \lambda)^{1/3}}}{dn \lambda}.
\]
On the convex surface, \(\lambda = iv\); on the concave surface, \(\lambda = K + iv\); and on the end, \(\lambda = iK' + u\). The electric intensity vanishes only at \(z = \infty\), where \(\lambda = \delta\). The surface density at any point of the shell may be found from this expression for the electric intensity. At the points where any radius meets the concave and convex surfaces the surface densities are inversely proportional to the radii of these surfaces.

On the end, \(BC\), of the shell, \(\lambda = iK' + u\). From (50) we find that the point on the end corresponding to \(u = K/2\) is that for which \(r = (ab)^{1/2}\), the geometric mean of the radii of the two surfaces. The surface density at this point is given by
\[
\sigma = \frac{Q}{4K'_1'(ab)^{1/2} \ sn^2 \delta \ cn 2\delta} \frac{1 - (1 - k') \ sn^2 \delta}{(1 - k')^{1/3}}.
\]
This becomes infinite only for \(k' = 1\), that is, for an infinitely thin shell. At the edges, \(B\) and \(C\), the surface density is infinite.
When the two shells are at the same potential they have equal charges, \( Q \), of the same sign. The \( y \)-axis is now a line of force, as is the \( x \)-axis. The transformation of the \( x \)-to the \( t \)-plane is given by (26), with the same value, \( C = -2Q \), for the constant. The solution of this equation is given by (27). Equations (29) give the distribution of the charges on the three surfaces.

In place of (31) the electric intensity is now given by

\[
\frac{dx}{dz} = -\frac{2Q}{z} \frac{sn\ 2\delta}{sn\ \delta\ cn\ \delta\ cn\ 2\delta} \frac{cn^2\ \delta - dn^2\ \delta\ sn^2\ \lambda}{dn\ \lambda}.
\] (58)

At \( z = \infty \), where \( \lambda = \delta \), this gives

\[
\frac{dx}{dz} = -\frac{4Q}{z},
\]

which is the right value for a charge \( 2Q \) within a finite distance of the origin. The electric intensity vanishes at the origin where \( \lambda = K - \delta \).

For any charges, \( Q_1 \) and \( Q_2 \), on the two shells the solution may be obtained by addition of the two solutions that have been given. We merely need to add the solution for the case of two equal and opposite charges, ±(\( Q_1 - Q_2 \))/2, to that for two equal charges of the same sign, (\( Q_1 + Q_2 \))/2. The difference of potential between the two shells is now \( 2V \), the same as that in the first case.

8. When the two thick shells are placed in a uniform electric field along the \( x \)-axis there are two cases to consider. We shall first suppose that the two shells are insulated from each other and that \( 2V \) is the difference of potential between them. We consider only the induced charges. The \( x \)-plane is similar to that shown in Fig. 4 except that we have only one quadrant of the whole plane. To transform the \( x \)-to the \( t \)-plane we have the equation

\[
\frac{dx}{dt} = \frac{C(t - s)}{(t - t - r \cdot t - m)^4}.
\] (59)

Using the substitution (52) the solution that satisfies the
conditions

\[ t = 0 \quad r \quad m \]
\[ \lambda_1 = 0 \quad K_1 \quad K_1 + iK' \]
\[ \chi = 0 \quad -V \quad -V \]

is

\[ \chi = 2Cm^1 \left[ \frac{\pi}{2K_1K_1'} \lambda_1 - Z(\lambda_1) \right]. \tag{60} \]

For the point of equilibrium, \( t = s \), we find

\[ \frac{s}{m} = \frac{E_1'}{K_1'}, \]

and the difference of potential between the two shells is given by

\[ 2V = -2Cm^1 \frac{\pi}{K_1'}. \]

The constant \( C \), determined so that the potential at infinity shall be \( \varphi = -Fx + \text{const.} \), is given by

\[ 2Cn^1 = -bF \, dn \delta \left( \frac{sn}{cn} \frac{2\delta}{2\delta} \right)^i \times \left( \frac{\pi H_1(2\delta)H(2\delta)}{2Kk'k'^1} \right)^{\frac{1}{i}} \left( \frac{b}{a} \right)^{-\frac{\delta}{K}}. \tag{61} \]

The electric intensity is given by

\[
\frac{d\chi}{dz} = \frac{2Cn^1}{zK_1' sn^2 \delta} \left( cn^2 \delta - dn^2 \delta sn^2 \lambda \right)^i \times \left[ (K_1' - E_1') sn^2 \delta cn^2 \delta - (K_1' sn^2 \delta dn^2 \delta - E_1' cn^2 \delta) sn^2 \lambda \right],
\]

and from this the surface density of the induced charge upon the shell may be determined.

For the point of equilibrium, we can write

\[ \frac{s}{n} = \frac{E_1'}{K_1' sn^2 \delta dn^2 \delta}. \]

\( s/n \) may be equal to, greater than or less than unity. If it be equal to unity then the edge \( C \), where \( \lambda = iK' \), is the point of
equilibrium. If \( s/n \) be less than unity the point of equilibrium lies on the end, \( BC \); and if it be greater than unity it lies on the convex surface, \( CD \). Since

\[
\frac{s}{p} = \frac{E_1'cn^2 \delta dn^2 \delta}{K_1'k'^2 sn^2 \delta},
\]

and this is always greater than unity, the point of equilibrium can never lie upon the concave surface nor at the edge \( B \).

In order to find the charges induced on the three surfaces from \( \psi \), the imaginary part of \( \chi \), in (60), we have, on the shell,

\[
\lambda_1 = K_1 + iv_1,
\]

and \( v_1 \) varies from 0 at \( A \) to \( K_1' \) at \( D \). Its values at \( B \) and \( C \) have been found in Art. 7. In order to evaluate \( Z(\lambda_1) \) on the shell with the aid of the tables, we write

\[
Z(K_1 + iv_1) = iZ(K_1' - v_1, k_1') - \frac{i\pi v_1}{2K_1K_1'}.
\]

Or we may use the series

\[
Z(K_1 + iv_1) = \frac{2\pi i}{K_1} \sum_1^{\infty} (-1)^n \frac{q_1^n}{1 - q_1^{2n}} \sinh \frac{n\pi v_1}{K_1}
\]

to compute the value of \( Z(\lambda_1) \).

We next assume that the two shells are at the same potential in the field along the \( x \)-axis. The equation for transforming the \( \chi \)- to the \( t \)-plane is again (59), but the conditions to be satisfied are now

\[
\begin{align*}
t &= m & r &= 0 \\
\lambda_1 &= K_1 + iK_1' & K_1 &= 0 \\
\chi &= 0 & -2\pi iQ &= -2\pi iQ
\end{align*}
\]

The solution is given by

\[
\chi = -2\pi iQ - 4K_1QZ(\lambda_1).
\]

\( Q \) is the charge induced on unit length of the right-hand shell and an equal and opposite charge is induced on the other shell. \( Q \) is given by the equation

\[
Q = -\frac{Cm^4}{2K_1}.
\]
The constant $C$ is given by (61). The point of equilibrium, $t = s$, is given by

$$\frac{s}{m} = 1 - \frac{E_1}{K_1}.$$ 

This may be written

$$\frac{s}{r} = \left(1 - \frac{E_1}{K_1}\right) \frac{1}{k_1^2}.$$ 

Since $s/r$ is always less than unity, the point of equilibrium lies on the $x$-axis between the origin and the point $A$. There is another point of equilibrium at an equal distance from the origin on the negative $x$-axis.

The electric intensity when the two shells are at the same potential is given by

$$\frac{d\chi}{dz} = \frac{2Cn^4}{zK_1 sn^2 \delta (cn^2 \delta - sn^2 \delta sn^2 \lambda)^i} \times \left[\frac{E_1 sn^2 \delta cn^2 \delta}{dn \lambda} - \frac{K_1 sn^2 \delta dn^2 \delta}{(sn^2 \delta - sn^2 \lambda)^i} - (K_1 - E_1) cn^2 \delta \right] sn^2 \lambda.$$ 

From this the surface density of the induced charges may be determined.

The complex potential for a field along the $y$-axis is given by

$$\chi = -2iCt^i = -\frac{2iCn^4}{dn \delta} \frac{(cn^2 \delta - dn^2 \delta sn^2 \lambda)^i}{(sn^2 \delta - sn^2 \lambda)^i}.$$ 

In order that the potential at infinity shall be $\varphi = -Fy + \text{const.}$ the constant $C$ must have the value given by (61). The electric intensity is now given by

$$\frac{d\chi}{dz} = \frac{iCn^4}{z sn \delta cn \delta} \frac{(cn^2 \delta - dn^2 \delta sn^2 \lambda)^i sn \lambda cn \lambda}{(sn^2 \delta - sn^2 \lambda)^i dn \lambda}.$$ 

The points $A$ and $D$ are the only points of equilibrium.
THE RESISTANCE OF CYLINDRICAL CONDUCTORS AT HIGH FREQUENCIES

E. P. ADAMS

Henry Professor of Physics, Princeton University

(Read by title, April 22, 1937)

ABSTRACT

When plane electric waves of very high frequency are guided by any system of cylindrical conductors, the axes of the cylinders being normal to the plane of the waves, an approximate value for the resistance of unit length of one of the cylinders is given by the expression,

\[ R = \frac{(2\pi \mu \tau p)^\frac{1}{2}}{\left( \int \sigma^2 \, ds \right)^\frac{1}{2}} \]

\( \sigma \) is the surface density of the charge on the conductor in the corresponding problem of the distribution of electricity on the cylinders, the integrals being taken around the contour of the cylinder in the plane of the waves. \( \tau \) is the specific resistance of the conductor, \( \mu \) its magnetic permeability, and \( p \) is equal to \( 2\pi \) times the number of vibrations per second.

The only problems for which a solution has been obtained for any frequency are those of two coaxial circular cylinders and two parallel circular cylinders external to each other. It is shown that the limiting values for high frequencies deduced from these solutions are the same as those given by the expression quoted above. This is then applied to some problems for which the general solution has not been obtained, but for which the corresponding electrostatic problems have been solved. These problems are those of two circular cylinders, one inside the other, but placed eccentrically; two semi-circular cylinders separated a short distance; and two thick circular cylindrical shells.

The self-inductance and capacity of unit length of circuits formed of two of these cylinders are also given.

1. If plane electric waves are guided by any cylinders of perfect conductivity, the axes of the cylinders being perpendicular to the plane of the waves, the electric force terminates normally upon the cylindrical surfaces; there is no component of electric force in the axial direction. Under these conditions we can speak of a charge on the surface of the cylinders and a surface current upon them. If \( v \) be the velocity of propaga-
tion, and $\sigma$ the surface density of the charge, the surface current density is given by $v\sigma$. The component of magnetic force in the plane of the waves tangential to the cylinders is discontinuous at their surfaces; just outside the surface the tangential component of the magnetic force is given by

$$H = 4\pi v\sigma.$$  

According to Poynting's theorem there is no flow of energy into the conductors and so there is no dissipation.

If the conductivity of the conductors is not perfect, but the periodic time very small, there is an effective thickness, $h$, just within the surface of the conductors, in which the whole current may be supposed to flow. This thickness is given by

$$h = \left\{ \frac{\tau}{2\pi \mu p} \right\}^{\frac{1}{2}},$$  

(1)

where $\tau$ is the specific resistance, $\mu$ the permeability and $2\pi/p$ the periodic time. Inside this layer there is therefore an axial electric force; and as the latter must be continuous at the surface, just outside the cylinder it is given by

$$E = \frac{\tau \sigma v}{h}.$$  

By Poynting's theorem there is now a flow of energy per unit of time and unit of length into the conductor given by

$$W = \frac{1}{4\pi} \int EH \, ds.$$  

$ds$ is an element of length of the contour of the cylinder and the integration is taken completely around it in the plane of the wave. But we have also,

$$W = RC^2,$$

where $R$ is the resistance of unit length of the conductor and $C$ the current. Since

$$C = \int \sigma v \, ds = vQ,$$

$1$ Jeans, *Electricity and Magnetism*, Art. 537.
Q being the charge per unit length, we get

\[ R = (2\pi \mu \tau p)^{1/2} \frac{\int \sigma^2 ds}{Q^2}. \]  

(2)

If, therefore, the electrostatic problem of determining the surface density of the charge on a cylinder has been solved, equation (2) may be used to determine its resistance to currents of high frequency.

Equation (2) has been used by Cockcroft \(^1\) to calculate the high-frequency resistance of a single conductor of rectangular cross-section. The same method was used by Butterworth \(^2\) to get the high-frequency resistance of two parallel circular cylinders with the currents in the same direction. And Strutt \(^3\) used this method to find the high-frequency resistance of a single cylinder with an elliptical cross-section.

If the electric waves are guided by two cylinders, one of which forms the return for the other, the capacity, \(S\), and the self-inductance, \(L_1\), both per unit length of the circuit, are, in the case of perfect conductivity, related by

\[ L_1S = c\mu, \]  

(3)

c being the dielectric constant and \(\mu\) the permeability of the medium. The solution of the electrostatic problem gives \(S\), and therefore \(L_1\) may be found. But if the conductivity be not perfect, there will be a small addition to the magnetic energy contributed by the field inside the conductor. Inside the conductor the magnetic force may be taken to be

\[ H = H_0 e^{-z/h}, \]

where \(z\) is measured normally inwards from the boundary, and

\[ H_0 = 4\pi v\sigma, \]

the magnetic force at the surface. The magnetic energy in-

\(^1\) Proceedings of the Royal Society, 122, p. 533, 1928.

\(^2\) Proceedings of the Royal Society, 107, p. 706, 1925.

\(^3\) Annalen der Physik, 85, p. 781, 1928.
side unit length of the cylinder of permeability \( \mu_1 \) is therefore

\[
2\pi\mu_1v^2 \int_0^\infty \sigma^2 ds \int_0^\infty e^{-2s/b}dz = \frac{1}{2} \frac{v^2 Q^2 R_1}{p}.
\]

In the return conductor there is a similar amount of energy with \( R_2 \) replacing \( R_1 \). The magnetic energy in the conductors contributes a term \( L_2 v^2 Q^2/2 \) to the whole magnetic energy, and therefore,

\[
L_2 = \frac{R_1 + R_2}{p}.
\]  \( \text{(4)} \)

The self-inductance of unit length of the circuit is now

\[
L = L_1 + L_2,
\]  \( \text{(5)} \)

where \( L_1 \) is given by (3) and \( L_2 \) by (4).

**Two Parallel Circular Cylinders, External to Each Other, Carrying Equal and Opposite Currents**

2. Let \( a, b \) be the radii of the two cylinders and \( 2d \) the distance between their axes so that \( 2d > a + b \). The transformation,

\[
z = \frac{1}{2} \csc \, \frac{1}{2} w,
\]  \( \text{(6)} \)

where \( w = u + iv \), with \( v = \alpha \) and \( v = -\beta \), yields two circles of radii

\[
a = f \csc \alpha,
\]

\[
b = f \csc \beta.
\]  \( \text{(7)} \)

Their centers are at a distance \( 2d \) apart, where

\[
2d = f(\coth \alpha + \coth \beta).
\]

If, therefore, we take the complex potential,

\[
\chi = \phi + i \psi = iAw,
\]

\( \psi \) will have constant values on each of the two circles. The difference of potential between them is

\[
\varphi_a - \varphi_b = -A(\alpha + \beta),
\]
and the charge on unit length of the cylinder of radius $a$ is

$$Q = -\frac{1}{2}A,$$

with an equal and opposite charge on unit length of the cylinder of radius $b$. Thus the capacity of unit length of the pair of cylinders is

$$S = \frac{c}{2(\alpha + \beta)} = \frac{1}{2}c \left[ \cosh^{-1} \left( \frac{4d^2 - a^2 - b^2}{2ab} \right) \right]^{-1},$$

where $c$ is the dielectric constant of the medium between the cylinders.

The electric intensity is given by

$$\frac{dX}{dz} = \frac{2Q}{f} (1 - \cos w),$$

and so the surface density on the cylinder of radius $a$ is

$$\sigma = \frac{Q}{2\pi f} (\cosh \alpha - \cos u).$$

As the element of arc is

$$ds = \frac{f \, du}{\cosh \alpha - \cos u},$$

we find

$$\int \sigma^2 \, ds = \frac{Q^2}{2\pi a} \coth \alpha.$$ 

Therefore by (2) the resistance of this cylinder per unit of length is given by

$$R_1 = \left( 2\pi \mu_1 \tau_1 p \right)^\frac{1}{2} \coth \alpha \frac{2\pi a}{2\pi a}, \quad (8)$$

where the subscript 1 refers to the cylinder of radius $a$. Similarly, the resistance of unit length of the cylinder of radius $b$ is given by

$$R_2 = \left( 2\pi \mu_2 \tau_2 p \right)^\frac{1}{2} \coth \beta \frac{2\pi b}{2\pi b}. \quad (9)$$
In terms of the dimensions of the cylinders we find,

\[
\text{coth } \alpha = \frac{4d^2 + (a^2 - b^2)}{\left[16d^4 - 8d^2(a^2 + b^2) + (a^2 - b^2)^2\right]^{1/4}}
\]

\[
\text{coth } \beta = \frac{4d^2 - (a^2 - b^2)}{\left[16d^4 - 8d^2(a^2 + b^2) + (a^2 - b^2)^2\right]^{1/4}}
\]  

(10)

The resistance of the circuit consisting of unit lengths of the two cylinders is now,

\[
R = R_1 + R_2.
\]  

(11)

If \( a = b, \mu_1 = \mu_2, \) and \( \tau_1 = \tau_2, \) so that the two cylinders have equal radii and are made of similar materials, the resistance of the circuit given by (11) is the same as the first and most important term in the result given by Mie.\(^1\) Snow,\(^2\) in a wholly different way, obtained (11) for wires of equal radii but different materials.

Our expression for the self-inductance of unit length of the circuit, (5), gives for conductors of equal radii and similar materials the first two terms of Mie's result. Mie gives a third term which, for sufficiently high frequencies, is negligible. Snow (l.c., Eq. (141)) obtained the same result as (5) when applied to cylinders of equal radii.

A Circular Cylinder Parallel to an Infinite Plane Conductor

3. If, in the case just considered, the two cylinders have equal radii and equal and opposite charges, the infinite plane perpendicular to and bisecting the line of centers is an equipotential surface. On this plane \( w = u, \) and \( u \) varies from \( \pi \) at the origin to 0 at infinity. The difference of potential between the cylinder and the plane is \( -A\alpha, \) and the charge on unit length of the cylinder is \( -A/2, \) with an equal and opposite charge on unit length of the plane. Hence the capacity

\(^1\) Annalen der Physik, 2, p. 230, 1900. Eq. (62).

of unit length of the circuit consisting of cylinder and plane is

\[ S = \frac{c}{2\alpha}, \]

c being the dielectric constant of the medium, and

\[ \cosh \alpha = \frac{d}{a}, \]

where \( d \) is the distance of the axis of the cylinder of radius \( a \) from the plane. The surface density on the plane is given by

\[ \sigma = \frac{Q}{2\pi f}(1 - \cos u), \]

and the element of length by

\[ ds = \frac{f \, du}{1 - \cos u}. \]

Without changing the solution on the side of the plane containing the cylinder we may suppose the plane to be the bounding surface of a semi-infinite conductor. So we have or the plane,

\[ \int \sigma^2 \, ds = \frac{Q^2}{2\pi d} \coth \alpha. \]

The resistance of unit length of the plane is accordingly,

\[ R_2 = (2\pi \mu \tau \tau_p)^{\frac{1}{2}} \frac{\coth \alpha}{2\pi d}, \]

where

\[ \coth \alpha = \frac{d}{(d^2 - a^2)^{\frac{1}{2}}}. \]

The resistance of unit length of the wire, \( R_1 \), is given by (8), and then \( R_1 + R_2 \) gives the resistance of unit length of the circuit. If the radius of the wire, \( a \), is small compared with its distance from the plane, \( d \), \( \coth \alpha \) is nearly unity, and we get the same result that was obtained by Snow \(^1\) for the problem of a horizontal antenna above the conducting earth.

\(^1\) L.c., Equation (143).
The self-inductance of unit length of the circuit is given by

\[ L = 2\mu \alpha + \frac{R_1 + R_2}{p}, \]

where \( \mu \) is the permeability of the medium. We can also write,

\[ \alpha = \log \frac{d + (d^2 - a^2)^{\frac{1}{2}}}{a}. \]

**Two Circular Cylinders, One Inside the Other, Carrying Equal and Opposite Currents**

4. The transformation (6), with \( v = \alpha \) and \( v = \beta \), gives two circles of radii \( a \) and \( b \), given by (7), with their centers at a distance \( d \), given by

\[ d = f(\coth \beta - \coth \alpha). \]

This is the case of a cylindrical core, of radius \( a \), surrounded by a cylindrical sheath, of internal radius \( b \), with their axes at a distance \( d \) apart. The thickness of the sheath is immaterial provided it be greater than the thickness of the skin given by (1). The resistances of the core, per unit length, \( R_1 \), and of the sheath, \( R_2 \), are given by (8) and (9), but we now have in place of (10),

\[ \coth \alpha = \frac{b^2 - a^2 - d^2}{(b^2 - a^2)^2 + d^4 - 2d^2(a^2 + b^2)^{\frac{3}{2}}}. \]

\[ \coth \beta = \frac{b^2 - a^2 + d^2}{(b^2 - a^2)^2 + d^4 - 2d^2(a^2 + b^2)^{\frac{3}{2}}}. \]

The capacity of unit length, \( S \), is given by

\[ S = \frac{c}{2(\alpha - \beta)}, \]

where

\[ \cosh (\alpha - \beta) = \frac{a^2 + b^2 - d^2}{2ab}. \]

Therefore the self-inductance of unit length of the circuit
formed of core and sheath is given by

\[ L = 2\mu(\alpha - \beta) + \frac{R_1 + R_2}{p}. \]

If \( d = 0 \), the core and sheath are coaxial and the problem reduces to that of the submarine cable. When \( d = 0 \), \( \alpha \) and \( \beta \) are infinite, \( \coth \alpha = \coth \beta = 1 \), while

\[ \cosh(\alpha - \beta) = \frac{a^2 + b^2}{2ab}. \]

The latter equation may be written,

\[ \alpha - \beta = \log \frac{b}{a}. \]

Therefore for the coaxial core and sheath the resistance of unit length of the circuit is

\[ R = \frac{(2\pi \mu_1 \tau_1 p)^{\frac{1}{2}}}{2\pi a} + \frac{(2\pi \mu_2 \tau_2 p)^{\frac{1}{2}}}{2\pi b} = R_1 + R_2, \quad (12) \]

and the self-inductance is

\[ L = 2\mu \log \frac{b}{a} + \frac{R}{p}. \quad (13) \]

Equations (12) and (13) are the limiting values for high frequencies that come from the complete solution of the problem of the cable.\(^1\)

The effect of increasing \( d \), the distance between the axes of the core and sheath, is to increase the resistance and capacity and to decrease the self-inductance of the circuit.

**Two Separated Semi-Circular Cylinders**

5. In a paper just preceding this one in these *Proceedings*,\(^2\) the distribution of electricity on two cylinders that have approximately semi-circular cross-sections, and are separated a


\(^2\) "The Distribution of Electricity on Two Cylinders." This paper is referred to as I.
short distance, was found. Figure 8, in I, is a fair representation of what would be obtained by cutting a circular cylinder by a plane passing through its axis, separating the two halves a short distance, and rounding off the sharp corners. For two such cylinders with equal and opposite charges the surface density was found in I (66) to be given by

$$\sigma = \frac{Q}{2K'}, h,$$

where $Q$ is the charge on unit length of the positive cylinders, $K'$ the complete elliptic integral of the first kind to the modulus $k'$, and $h$ is given, for very small modular angles, by the equation,

$$\frac{1}{f^2h^2} = \frac{1}{K'^2} + \frac{8V}{K'(1 + V^2)^2}.$$

In this,

$$V = qe^{2v},$$

where $v$ defines the position of a point on the cylinder and varies from 0 to $K'$. For very small modular angles we may use the approximations,

$$q = e^{-\pi K'/K} = e^{-2K'} = \frac{k^2}{16}.$$

The radius of the circular cylinder is $f$, and the separation, $2x_0$, between the two halves, is given by

$$\frac{2x_0}{f} = \frac{\pi}{2K'}.$$

The element of arc is $ds = dv/h$, and so we find,

$$\int \sigma^2 ds = \frac{Q^2}{4fK'} \int_0^1 \frac{(1 + V^2)}{V(1 + 2bV^2 + V^4)} dV,$$

in which $b$ is an abbreviation for

$$b = 1 + 8K'(K' - 1).$$

By the change of variable, $V^2 = x$, the integral becomes a standard form; since $q$ and $qK'$ are negligible in comparison
with unity, we get

\[ \int \sigma^2 \, ds = \frac{Q^2}{8fK} \log \frac{2}{q^2(1 + b)}. \]

The high-frequency resistance of unit length of a circuit consisting of two such approximately semi-circular cylinders, when one of them is the return for the other, is therefore, by (2),

\[ R = \left\{ (2\pi \mu_1 \tau_1 p) \log \frac{2}{q^2(1 + b)} \right\} \frac{B}{f}, \]

where

\[ B = \frac{1}{8K} \log \frac{2}{q^2(1 + b)}. \]

The expression for the resistance has been written as the sum of two terms to allow for making the two cylinders of different materials.

The largest modulus that can be used with our approximations is \( k = 10^{-3} \). The following table gives the values of the factor \( B \), as well as the relative separation, \( 2x_0/f \), of the two cylinders. The resistance of the circuit is therefore increased by bringing the two cylinders nearer together.

The capacity of unit length of a condenser consisting of the two cylinders was found in I (63), and is, with \( \alpha = \beta = 1/2 \),

\[ S = \frac{cK'}{2\pi K}, \]

putting in \( c \), the dielectric constant of the medium. Therefore, by (3) the high-frequency self-inductance of unit length of the circuit is given by

\[ L = \frac{2\pi \mu K}{K'} + \frac{R}{p} = \frac{2\pi^2 \mu}{\log \frac{1}{q}} + \frac{R}{p}. \]
If instead of a circuit consisting of two equal, nearly semi-circular cylinders with equal and opposite currents flowing in them, the currents are in the same direction in both cylinders, the electrostatic surface-density is given by \( I \) (69), in which \( Q/2 \) is the charge on unit length of either cylinder. With the same approximations for small modular angles, we now find,

\[
\int \sigma^2 \ ds = 2Q^2 \cdot \pi^2 f \left( 1 - \frac{1}{K'} \right)^{-\frac{1}{4}} \sin^{-1} \left[ \frac{4K'(K' - 1)}{1 + 4K'(K' - 1)} \right].
\]

The resistance per unit length of the conductor consisting of the two separated approximately semi-circular cylinders is therefore given by

\[
R = (2\pi \mu \tau p)^{\frac{1}{4}} \cdot \frac{1}{\pi^2 f} \left( 1 - \frac{1}{K'} \right)^{-\frac{1}{4}} \sin^{-1} \left[ \frac{4K'(K' - 1)}{1 + 4K'(K' - 1)} \right].
\]

If \( k = 0 \), this becomes

\[
R = (2\pi \mu \tau p)^{\frac{1}{4}} \cdot \frac{1}{2\pi f},
\]

the high-frequency resistance of a single circular cylinder of radius \( f \).

**Two Thick Cylindrical Shells**

6. As a final example we shall consider the two cylinders obtained by cutting a tube, the external and internal radii of which are \( b \) and \( a \), by two planes passing through the axis of the tube and inclined to each other at an angle \( \pi - 2\alpha \). If the portions of the tube included within the two planes be removed, the remaining portions form two thick cylindrical shells, each subtending an angle \( 2\alpha \) at the axis. The electrostatic problem of the distribution of electricity on these shells was solved in a recent paper.\(^1\) When the two shells have equal and opposite charges, \( \pm Q \), the electric intensity is given by \( \Pi \) (57),

\[
\frac{dx}{dz} = \frac{\pi Q}{zK_1'} \frac{\sin 2\delta}{\sin^2 \delta} \frac{(\sin^2 \delta - \sin^2 \lambda)^{\frac{1}{2}}(\cos^2 \delta - \cos^2 \delta \sin^2 \lambda)}{\cos \lambda}.
\]

\(^1\) "Electrostatic Problems Connected with Thick Cylindrical Shells." This paper will be referred to as \( \Pi \).
From II (6) we get, dividing by 2 because that equation refers to a single shell,
\[ dw = \frac{dr}{r} + i d \theta = \frac{2 \, sn^2 \delta \, cn^2 \delta \, cn \, 2 \delta \, \lambda \, d \lambda}{sn \, 2 \delta (sn^2 \delta - sn^2 \lambda)(cn^2 \delta - dn^2 \delta \, sn^2 \lambda)}. \]
On the convex surface, \( r = b \), and \( \lambda = iv \), where \( v \) varies from 0 to \( K' \). On the concave surface, \( r = a \), and \( \lambda = K + iv \), where \( v \) varies from 0 to \( K' \). So we find,
\[ \int \sigma^2 \, ds = \frac{Q^2 K'}{4bK_1^2 \, sn^2 \delta \, cn \, 2 \delta} \]
where the integration is extended over the whole convex surface of one of the shells. We get the same result for the integral taken over the concave surface except that \( b \) is replaced by \( a \).

Over the ends, \( \lambda = iK' + u \), where \( u \) varies from 0 to \( K \). Integrating over the two ends we find,
\[ \int \sigma^2 \, ds = \frac{Q^2}{4K_1^2 \, sn^2 \delta \, cn \, 2 \delta} \int^K_0 \frac{dn}{r} \]
If the shell is not too thick, the error made by substituting for \( r \) in the integral its value for \( u = K/2 \) will be small. This value is \( r = (ab)^{1/2} \). The high-frequency resistance of unit length of the circuit consisting of the two shells when one forms the return for the other is therefore given by
\[ R = \frac{(2\pi\mu \tau \rho)^{1/2} k' \, sn \, 2 \delta}{2bK_1^2 k_1 \, cn \, 2 \delta} \left\{ K' \left( 1 + \frac{b}{a} \right) + K \left( \frac{b}{a} \right)^{1/4} \right\}. \]
The resistance is infinite for infinitely thin shells for which \( k = 0 \), \( K' = \infty \). The modulus \( k_1 \) is given by
\[ k_1 = k' \frac{sn^2 \delta}{cn^2 \delta}. \]
The following table shows how the high-frequency resistance varies for circuits formed of shells of different dimensions. The first column gives the ratio \( b/a \) of the external to
the internal radii of the shells; the second column gives the angle $\alpha$ subtended by half a shell at the axis; the last column gives the quantity,

$$R' = \frac{bR}{(2\pi\mu\tau p)^{1/4}}$$

$R$ being the high-frequency resistance of unit length of the circuit.

<table>
<thead>
<tr>
<th>$b/a$</th>
<th>$\alpha$</th>
<th>$R'$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.5094</td>
<td>$79^\circ 10'.9$</td>
<td>0.9473</td>
</tr>
<tr>
<td>1.2300</td>
<td>$84^\circ 34'.3$</td>
<td>1.2948</td>
</tr>
<tr>
<td>1.1553</td>
<td>$85^\circ 15'.6$</td>
<td>1.4269</td>
</tr>
<tr>
<td>1.0812</td>
<td>$86^\circ 9'.8$</td>
<td>1.6598</td>
</tr>
<tr>
<td>1.0249</td>
<td>$87^\circ 11'.2$</td>
<td>2.0869</td>
</tr>
</tbody>
</table>

For currents flowing in the same direction along the two cylindrical shells, the solution of the electrostatic problem is given by II (58). The electric intensity is now

$$\frac{dx}{dz} = -\frac{2Q \sin 2\delta (cn^2 \delta - dn^2 \delta \sin^2 \lambda)}{z \sin \delta \ cn \ delta \ cn \ 2\delta \ dn \ lambda}.$$  

The evaluation of the integral in (2) for the convex and concave surfaces and for the ends requires the three complete elliptic integrals of the third kind, given by,

$$\int_0^{K'} \frac{dv}{1 - sn^2 \delta \ dn^2 v} = K' + \frac{sn \ delta}{cn \ delta \ dn \ delta} \left\{ \frac{\pi \ delta}{2K} + K' \ Z(\delta) \right\},$$

$$\int_0^{K'} \frac{dv}{sn^2 \ delta + cn^2 \ delta \ sn^2 v} = K' + \frac{sn \ delta}{cn \ delta \ dn \ delta} \left\{ \frac{1}{2} \pi - \frac{\pi \ delta}{2K} - K' \ Z(\delta) \right\},$$

$$\int_0^{K} \frac{du}{1 - k^2 \ sn^2 \ delta \ sn^2 u} = K \left\{ 1 + \frac{sn \ delta}{cn \ delta \ dn \ delta} \ Z(\delta) \right\}.$$

The accents on the elliptic functions denote that the complementary modulus, $k'$, is to be used. For the high-frequency resistance, $R$, of unit length of the two cylindrical shells,
with the currents in the same direction, we find,

\[
\frac{bR}{(2\pi \mu_{\tau}p)^{i}} = \frac{1}{2\pi} \left\{ 1 - \left( \frac{1 + \frac{b}{a}}{K} \right) \delta \right\} \\
+ \frac{1}{2\pi^{2}} \left\{ \frac{dn^{2} \delta sn 2\delta}{cn 2\delta} - 2Z(\delta) \right\} \left\{ \left( 1 + \frac{b}{a} \right) K' + \left( \frac{b}{a} \right)^{i} K \right\}.
\]

If \( \delta = 0, \alpha = \frac{1}{2}\pi, \) and the two shells form a complete circular cylinder of radius \( b; \) its high-frequency resistance accordingly is given by

\[
R = \frac{(2\pi \mu_{\tau}p)^{i}}{2\pi b}.
\]
ILLUSIONS
HARRISON S. MORRIS
Formerly Director Pennsylvania Academy of the Fine Arts
(Read April 24, 1937)

ABSTRACT

The paper follows Poor Richard in rallying some of the follies of the day.

If we could reincarnate Benjamin Franklin in this age—and almost anything is possible with him—it is conceivable that he would use his inspired sense on our illusions.

He had none of his own, though his age had its quota; as when a lady should encase her form divine in a tent of crinoline, and tea should be drunk from a slippery saucer.

But we may conjecture what he would say now, to our crowding illusions that, in the words of the song of his day, fill "a world turned upside-down."

Our dinners are served to us in the dark. To be sure, four or more candles shed an illusive light on the darkened dishes and the shadowy faces; but the light is not vouchsafed for distinguishing the—I had almost said, "funeral baked meats." This it may be is what a hostess designs. Critical or invidious views are thus turned aside and you eat often without distinguishing between the soup and the salad. Conversation assumes the hush of decorum, and the inarticulate are thus placed on a polite plane with the nimble-tongued.

But in the railway diner there is no lack of light or of information. The bill-of-fare is blatant with its offerings. You may have milk-fed chicken or you may choose sugar-cured ham. Nothing is what it seems. You conjecture, but are never told, whether chickens really prefer to be fed on milk, though you have never known one to be so fed; or whether feeding milk to chickens improves them as morsels of diet; or whether, in fact, you are at all sure your appetite is stimulated by your being informed of the kind of nourishment chickens take.
And the same is true, I suppose, of sugar-cured ham. If you are skeptical at all about the cure of ham from some unfortunate malady, you may rest at ease, for the cure apparently concerns its seasoning, not its health.

But why you should be allured to eat by adjectives which you can not verify, and of which the virtue is not apparent, only illusion can answer.

And as you try to eat to the rhythm of the wheels, you are also suffering another illusion, which has, perhaps, some very unfavorable effects. The air from conditioning gadgets is blowing on your neck and giving you a congested head and nose. The stream of conditioned air cannot be cut off because convinced victims believe in it. And with your milk-fed chicken and sugar-cured ham you imbibe a more realistic effect called the flu.

So, too, it is with the air-conditioned building, made usually of glass. Whether the pipes inside are concealed, as in all practical houses so far known, is left to the caprice of an architect who insists on being original and independent.

We may conceive of such a structure using exposed glass water and waste pipes through which the man on the floor below may gauge the health of his associate of the floor above—a convenience, but not so friendly as the accustomed hand-shake and inquiry.

And returning to the table of daily meals, it seems that knives which we used to associate with cutting, are now an ornament of the table, dissociated from any such function, which may be in the nature of guarding a guest from a bleeding finger, but is, in reality, only another way of deceiving ourselves with gentle untruth.

All this would of course have engaged the penetrating Poor Richard had his day gone as has ours. He would have inquired why it is that dogs don’t bite. Like some of the incredulous amongst us, he would have supposed they did bite. But he would have been assured that nobody with any mental equipment—or a household pet—believes that dogs do bite.

The little dears or animals as big as a cow, are “man’s
best friends," and unless they are goaded by inhuman treatment they are as sweet as the lambs they slaughter.

Those who don’t like dogs are the only people who are bitten. They, of course, deserve to be. As for rabies, nobody has actually experienced it, and thus there is nothing to tell. Lovable little dogs could not, and would not afflict others with rabies, whatever that may be.

I approached a happy lady coming from the door of an apartment house. She had a little black dog on a string. I ventured to say, "Is he dangerous?"

"Dangerous," she laughed, "all she wants is to be loved."

No, dogs do not bite, and the raising of sheep with us has disappeared. The caresses of harmless roving dogs, the masters of which love them and allow others to share their love, have devastated our mutton.

And it is conceivable what the many-sided philosopher would have written in his day if the English language, which he adorned, had sunk into capital letters. He would probably have asked why the practice should not be extended for the compression of dictionaries, and the language of scientists. P.W.A. and W.P.A. would have confounded him as they do us. But our dismay is turned into incredulity when our Government compresses speech on the one hand and elongates it on the other. Mark this vital collocation of words given to the victim of the surtax as directions for performing that painful act:

"The surtax on any amount of surtax net income not shown in the table below is computed by adding to the surtax for the largest amount shown which is less than the income, the surtax upon the excess over that amount at the rate indicated in the table."

As F. P. A., himself guilty of being alphabetical, says:

"Disrespect for decent English is growing so, that we are astonished that the editorials weren’t headed, ‘Like Maine goes.’"

So, too, in our careless use of a rich inheritance: the language of Shakespeare and Wordsworth, do we assume that anybody ought to guess what a moron of the press means, when he uses "peek" for "peep," or when the caricaturist expects us to divine which one of his figures is speaking.
All this is probably due to the importations we have admitted from Europe, not to speak of Asia and Africa. One of our illusions, the fantastic one that we bar from our shores the undesirables of other and darker lands, is heart-breaking. Anybody can lift the latch and walk in, whether clean or criminal, whether on foot or sewed up in a gunny sack. And thus we are giving away our heritage of righteous character, and are sinking into sentimentality.

Evidence of this faces us in every newspaper. The act of sentimental clemency by judges and juries has brought into our courts the illusion of parole and probation. Almost any criminal can be let loose on these excuses. They are an encouragement to crime, which sticks its finger at its nose and says, "Why worry, in again, out again," and thus plots another crime against you and me.

All of which is the brood of Illusion. We are living on the assumption that man is perfectible, the assumption that has brought Europe to desperation, and is invading us with its heresies and hypocrisies.

The perfectibility of man is the chief of our Illusions. Labor and sorrow are wasted on him, and he isn't much changed since the Stone Age.

The League of Nations was wrecked on the theory of the man perfectible. But when that hopeful illusion was brought down to practice, it was found that to prevent wars, it must wage them. And it was without power, arms, unity. The man perfect had not arrived.

What then must we do?

Trust in Nature and imbibe its Beauty and Truth, and rest on the logic of the Universe.

But still, I find myself wondering if the perfect man—not to say lady—would be as lovable, as worthy of friendship, as possessed of a sense of humor, as the purified one without blemish.

Charles Lamb, you know, said,

"Give me man as he is not to be."
A PALEOCENE RODENT, PARAMYS ATAVUS

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(Read April 22, 1937)

ABSTRACT

The geological range of the Rodentia is extended backward into the Paleocene Epoch by the discovery of several teeth in the Ft. Union formation at Bear Creek, southern Montana. This identification of a lower molar and a few incisors of the oldest known true rodents contributes but little information about the morphogeny and the origin of the group, because the teeth so closely resemble those of the well known Eocene rodents, hitherto the earliest representatives of the order. For several reasons, however, the Paleocene specimens are recorded and discussed. Rodent incisors are distinguishable by growth habit and microstructure from those of other mammals.

Investigation supported by William Berryman Scott Research Fund.

INTRODUCTION

RODENTS are the most successful and numerous of living mammals. They exceed all other mammalian orders combined in the number of genera and species and are, in their ability to flourish in diverse environments, the most triumphant. Although they constitute more than half of the census of mammals living today, the fossil types are progressively less abundant as the ancestry of the order is traced backward in time. From rocks of the early Eocene epoch only one group is known, the paramyids, which, in dental and other structures, could have been ancestral to all later rodents. Most of the early Eocene specimens have been referred to the genus Paramys.

Many authors have already suggested that the differentiation of the rodents from other mammals, presumably from insectivores, occurred at least as early as Cretaceous time. The Paleocene teeth herein described support this conjecture because they are as specialized as homologous teeth of some
early Eocene paramyids, having no "more primitive" characteristics that furnish clues to the derivation of the rodents from another order.

Another frequently expressed belief is that, although the simplexidentates (squirrels, beavers, rats, mice, porcupines, guinea pigs, and other groups) and the duplicidentates (hares, rabbits, and their close relatives) are often classified as suborders of the order Rodentia, the two groups are so remotely related that they should be ranked as distinct orders. This conception is likewise sustained by the Fort Union teeth because they show no evidence that the two groups were more similar in early Tertiary time than they are today. In tooth pattern and structure the Paleocene rodents were as different from the earliest known late Eocene hares as the latter were from their contemporary simplexidentates. From known specimens, even by the dubious aid of evolutionary extrapolation, the phylogenetic line of the lagomorphs and that of the true rodents cannot be made to converge toward a common ancestry in the early Tertiary period. When more complete specimens of Paleocene rodents and earlier lagomorphs are found they may indicate the origin and antiquity of the two groups, but the present material is inadequate for additional conjecture.

Eurymylus \(^1\) from the Gashato of Mongolia is the only other genus which has recently been referred to the Paleocene Rodentia, but its specialized structures exclude it from a position ancestral to later simplexidentates. Superficially, and in some details, its jaw and maxilla resemble similar parts of lagomorphs. Although Matthew, Granger, and Simpson have stated that it probably does not belong among the duplicidentates, and Dr. A. E. Wood says that this genus cannot be a rodent and probably is not a lagomorph,\(^2\) the specimens need further consideration.


PALEOCENE RODENT, Paramys Atavus

The curious characteristics of Eurymylus make its classification difficult and, at present, arbitrary. It shares this quality of unique structures with several other Gashato genera. When compared with any typical European or American Paleocene fauna, that of the Gashato is peculiar, and its relationship to standard European and New World early Tertiary faunas presents problems even more puzzling than most long distance correlations. Contrary to expectation before its discovery, the Gashato fauna is not composed of forms ancestral to later European and American mammals and, although it is probably of Paleocene age, it consists mainly of specialized and unique genera which appear to be taxonomically isolated when compared with better known types.

Asia was long regarded as an area from which various orders and families of mammals deployed to America and Europe during the Tertiary. As an evolutionary incubator, however, Asia has been disappointing because few, if any, of its known fossils clearly represent forms ancestral to those of other regions. In many cases the correlation of Asiatic Tertiary horizons with others elsewhere is very indefinite, and the direction of proposed migrations, whether to or from Asia, is determined by such vague estimates as the "degree of evolution" of the respective faunas.

Another explanation, alternative to migration, to account for the appearance of a new mammalian group at a certain stratigraphic level is that, although ancestral forms lived earlier in the same area, their remains have not been discovered. This absence may be laid to the fortunes of collecting or to facies differences.

A few years ago no Paleocene Perissodactyla, Artiodactyla, Rodentia, Primates or Dinocerata were known, and all of these orders were believed to have evolved in some unknown area and then to have migrated suddenly to the regions where their remains have been found in early Eocene sediments. Individuals representing the latter three orders, the rodents, primates and Dinocerata, have since been found in Paleocene strata, and some teeth of condylarths from this epoch are so
similar to those of Eocene dichobunid artiodactyls that the derivation of the latter order from the former seems very probable. Individual lower molar teeth of the two groups are, in some instances, indistinguishable. The extent of this resemblance was not realized when a Paleocene genus, Phenacodaptes,\(^1\) was described and tentatively and questionably referred to the primitive artiodactyls. Since then many more specimens have been collected and, as recorded by Simpson,\(^2\) the genus may be more acceptably regarded as a condylarth, at least until diagnostic skeletal material is found.

Many postulated migrations have been proved imaginary by the discovery of ancestral forms in older rocks in the same area; and the use of negative evidence, that is, the absence of a group, as a device for correlation is repeatedly demonstrated to be treacherous. Even yet, however, nomadic habits are attributed to many unknown ancestral strains as an explanation where facts are absent. Recently Simpson, in writing about the "progressive diversification" of the mammalian fauna during the American Paleocene, says: "In greater part it is probably caused by the immigration of new types of mammals from some as yet unknown center or centers of evolution."\(^3\)

The age relationship of the Bear Creek Paleocene mammals to other faunas has been discussed by Simpson,\(^4\) who noted the rodent incisors but believed that they pertained to some kind of a multituberculate. A revised list of the Bear Creek mammals follows:

**INSECTIVORA**

*Leptictidae*

*Leptacodon siegfriedi*

*Nyctitheriidae*

Protentomodon ursirivalis
?Pantolestidae
?Pentacodon cf. inversus

?Dermaptera
Plagiomenidae
Planetetherium mirabile

Primates
Plesiadapidae
Plesiadapis sp.
Apatemyidae
Labidolemur kayi
Carpolestidae
Carpolesistes nigridens
C. aquilae

Tæniodonta
Stylinodontidae
?Psittacotherium lobdelli

Rodentia
Paramyidae
Paramys atavus, new species.

Carnivora
Arctocyonidae
Thryptacodon pseudarctos
Mesonychidae
Dissacus cf. navajovius

Condylarthra
Phenacodontidae
Phenacodus cf. primævus

Description of New Species

Classification:¹

Order Rodentia
Suborder Protrogomorpha
Superfamily Ischyromyoidae
Family Paramyidae
Genus Paramyidae
New species, P. atavus

¹As suggested by Dr. A. E. Wood.
HOLOTYPE—Princeton No. 14,200, right lower molar, probably the second, Figs. C and D, Pl. I.

PARATYPE—Princeton No. 14,200a, left upper incisor, Fig. A, Pl. I. Gift of Dr. J. C. F. Siegfriedt.

REFERRED SPECIMENS—Princeton No. 14,200c, fragment of left lower incisor, Am. Mus. No. 22,155, left lower incisor lacking anterior tip; ¹ No. 22,156, fragment of right upper incisor; ² No. 22,166, part of right upper incisor and anterior tip of left lower incisor; No. 22,163, almost complete right lower incisor; ³ No. 22,199, a fragment of a right upper incisor and two pieces of left lower incisors.

HORIZON—Bear Creek Paleocene; about equivalent in age to the Tiffany of Colorado, the "Princeton Quarry" of Park County, Wyoming, and the Scarritt Quarry in Sweetgrass County, Montana.

FORMATION—Fort Union.

LOCALITY—Eagle Coal Mine, Bear Creek, Carbon County, Montana.

GENERIC ANALYSIS—Paramys, hitherto reported from the Eocene, and ranging from the Gray Bull to the Uinta in North America, has been defined and figured by several authors since the type description by Leidy in 1871. Loomis in 1907 and Matthew in 1910 and 1915 cited the generic distinctions. The incisors have been described as "broad or of moderate width with convex anterior face" and the lower molars have "low marginal cusps, little or not at all crested, entoconid a distinct cusp instead of a marginal ledge, broad shallow basined heels." Many other skeletal structures have been noted but are useless in the present analysis of incisors and lower molars. There can be little doubt about the reference of the specimens to Paramys until

¹ This specimen was figured by Simpson, Fig. 10A, page 14, in "A new mammalian fauna from the Fort Union of Southern Montana," Am. Mus. Novitates, No. 297, 1928, when he regarded these incisors as belonging to multituberculates.

² Fig. 10B, ibid.

³ Fig. 11, ibid.
*Paramys atavus*

Enlarged about 10×

Fig. A. Paratype, No. 14,200a, broken left upper incisor, external view, anterior tip below.

Fig. B. No. 14,200b, cross section of upper incisor, anterior edge at top, external side on left. Lines at intervals indicate attitudes of dentinal tubules.

Fig. C. Holotype. No. 14,200, right $M_2$, crown view; $en^d =$ entoconid, $hy^d =$ hypoconid, $me^d =$ metaconid, $pr^d =$ protoconid.

Fig. D. External view of same.
the genus undergoes a restudy and revision which it greatly needs because it probably includes several groups that should have generic rank.

**Specific Analysis**—Paleocene age. Incisors and lower molar small. Anterior-posterior diameter of upper incisor—2.9 mm., medial-lateral diameter—1.4 mm. Width of $M_2$—1.75 mm., length (anterior-posterior)—1.9 mm. All molar cusps comparatively elevated and rounded. Enamel smooth. "Trigonid" high, well above heel. Metaconid higher and more robust than protoconid. Interior basin broad and smooth. Entoconid small, marginal. Hypoconid small, labially situated. $M_2$ slightly longer in proportion to its width than the molars of Eocene paramyids.

Two small ridges connect the protoconid with the metaconid, enclosing between them a diminutive perched basin. The anterior ridge mounts almost to the summit of the metaconid, adding to the apparent size of the latter. Postero-internally the slope of the metaconid is concave but the cusp itself shows no more doubling than do Eocene specimens.

The outward (labial) position of the hypoconid makes the posterior part of the heel basin wider than the anterior. Loomis' mesostylid (median external cusp) has no internal spur such as can be seen in many small specimens of *Paramys* from the Wind River. Grooves on the anterior and the posterior sides of this cuspule, between it and the protoconid and the hypoconid respectively, are shallow, with rounded floors.

There are many minor features in which the species differs from all available specimens of the most similar early Eocene species of *Paramys*. Some of the latter are almost as small and many have several of the above-named characteristics, which have not however been found combined in any other teeth. Most of these features are subject to great variation in *Paramys*, but in general, *P. atavus* more closely resembles, in incisors and lower molars, the early Eocene (Gray Bull)
small species like *P. murinus* than it does the larger Gray Bull species or any specimens from the Wind River. More detailed discussion for speciation seems futile until additional parts of Paleocene rodents are discovered.

**Rodent Incisors**

Like the upper incisor of most rodents, that of *P. atavus* is a larger sector of a smaller ring than the lower. It is also less twisted so that, if continued to complete several circles, it would form a more tightly coiled spiral.

Enamel is confined to the rounded anterior-lateral surfaces of the incisors. On the upper incisors it extends around the external side for a distance a little less than one third of the antero-posterior diameter of the tooth, and on the lower incisors the enamel covers less of the lateral surface, about one fourth of the diameter. The tip of the upper incisor, as usual among rodents, wears to a blunter chisel-shaped edge than the lower. The roots of the Paleocene rodent incisors are unclosed and unconstricted, having grown from persistent pulps.

Skiagrams aided in showing some of the incisor characteristics mentioned above. Thin sections were made and compared with those of other teeth which are sometimes confused with rodent incisors. The results corroborate in part the conclusion of Tomes in 1850,¹ and numerous authors since, that in conjunction with other features, the microstructure of dentine and enamel in rodent teeth is useful in distinguishing them from the teeth of many other mammals.

Figure B on the accompanying plate is a drawing of a section of an upper incisor of *P. atavus*. This shows the direction of the dentine tubules, their attitude being emphasized on the illustration by lines drawn at intervals where the curvature varies. This seems to be a characteristic pattern for the incisors of *Paramys, Ischyromys*, and several other rodent genera, and is distinct from the tubule form of other mammalian orders with which comparisons were made.

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At the start of this examination of various incisors it was believed that there would be great difficulty in distinguishing rodent incisors from other small teeth which are frequently referred to as "gliriform." Soon, however, the fact became clear that the incisors of rodents are very different from all other teeth. As far as can be ascertained at present, only a few other small mammals have an incisor that grows throughout life from an open root, with enamel confined to a continuous band on the antero-external surface of the tooth from tip to root end. The wombat, *Phascolomys*, the aye-aye, *Daubentonia*, the hyrax, *Procavia*, and the lagomorphs have incisors, upper or lower or both, that fulfill these specifications in varying degrees; but they differ in many respects, such as shape, wear habit, distribution of cement, and dentine pattern, from those of rodents.

Incisors of the multituberculate, *Eucosmodon*, have been described as rodent-like several times by various authors because the enamel is limited to the antero-lateral borders, but this adjective is misleading. The incisors of all multituberculates are very easy to distinguish from those of rodents, especially when the whole teeth are examined and not merely the parts protruding from the alveoli. Multituberculate incisors formed closed roots after a period of eruption that varied in different genera. Even on relatively young individuals of *Eucosmodon* the terminal constriction is obvious. The same has been noted of *Microcosmodon*, a related genus whose lower incisor was erroneously described as gliriform.

In multituberculate incisors the enamel never extends to the end of the root but terminates at various distances short of it. Enamel deposition may have continued for a while as the tooth gradually grew to keep pace with the wear on its tip, but in old animals the enameled portion of the tooth grew

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completely out of the socket. Most multituberculates had enamel coating the entire tip of the incisor and ending outside of the alveolus or descending into it but a short distance.

Other groups of small Tertiary and later mammals, whose long and procumbent scalpriform incisor teeth have been, or might easily be, confused with those of rodents, are the Cœnolestidae, Polydolopidae, and Phalangeridae (marsupials); the Plesiadapidae, Carpolestidae, and *Phenacolemur* (primates); and *Picrodus* (of uncertain order). Enamel covers the entire tip of the incisors of most of these, and although it occurs in some as bands restricted to the anterior border and the sides, the ends of the root are in each very constricted or closed.

Not only is the enamel of *P. atavus* distinctive among early Tertiary mammals by virtue of its unique distribution on the incisors, but it possesses features in its microstructure which assist in the identification of fragmentary teeth. The incisors herein referred to *P. atavus* were identified as pertaining to rodents because of their characteristic microstructure, even before the molar was discovered; but this evidence alone was regarded as insufficient for taxonomic security.

Tomes, Carter¹ and others have described how tubules or fibrils in the dentine are continued into the enamel in the teeth of several groups of mammals. This penetration of the dentinal fibrils into the enamel is apparently a characteristic of all multituberculates, all living lemurs except *Daubentonia* (*Cheiromys*), all marsupials except *Phascolomys*, many insectivores, and all hyracoids, but has been reported among rodents only in jerboas.

It is interesting to note that the sole exception among the lemurs and that among the marsupials are two of the three genera mentioned above as having limited bands of enamel on incisors that grow from persistent pulps. The third genus, *Procavia*, has these characteristics confined to the upper incisors, the lowers having enamel-covered crowns and closed roots. It would thus seem that in construction and habit of

incisors, the wombat and the aye-aye simulate rodents more closely than do any other mammals.

Characteristic structures and arrangements of the prisms in the enamel of rodent teeth aid in their identification. Tomes ¹ pointed out that in simple forms of enamel, prisms pass from the dentine to the periphery of the enamel and are, with slight exception, exactly parallel, but in rodents contiguous layers of prisms start off from the dentine at different angles, alternating in this respect, so that if the section embraces more than one layer, patterns are produced by the crossing of the prisms.

Sections of the rodent teeth from Bear Creek show this twisting of bundles of prisms in the enamel, giving it the typical crosshatched or gnarled rodent pattern. Buest ² suggests that the enamel prisms or rods are sometimes spirally arranged in the inner regions of the enamel of human teeth, but this is not a consistent or characteristic pattern for the primates.

In summary, it may be said that the Rodentia are among the most ancient orders of mammals and that the teeth of the earliest known, Paleocene, members of the group had already developed many of the details of enamel and dentine structure that are characteristic of living rodents.

² Buest, T. B., Dental histology and embryology, p. 18, 1934.
A NOTE ON SOME SPECIES OF MARINE MOLLUSKS
OCcurring IN BOTH POLYNESIA AND THE
WESTERN AMERICAS

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ABSTRACT

This paper contains a brief discussion of similarities and possible means of migration observed in the molluscan fauna of the Indo-Pacific region and the western Americas and their offshore islands. A map is included showing the general relations of the land masses and islands. A list of species from the eastern Pacific is given containing some species which are identical and others which are analogous to those in the Indo-Pacific. It is pointed out that in the species herein cited the Gastropoda are more common than the Pelecypoda.

One of the fascinating aspects of the study of Conchology is the mystery of the occurrence of certain species of mollusks in widely separated regions, species which appear like wanderers from foreign lands. For instance, there are a number which apparently have reached the western Americas from such distant regions as the Caribbean, Japan and Polynesia.

Dall \textsuperscript{1} mentioned that a decidedly Indo-Pacific element is present in the marine molluscan fauna of Clarion Island, Revillagigedo Group, but Strong and Hanna \textsuperscript{2} have stated that only two Indo-Pacific species were present in a collection containing 61 species from that locality studied by them.

In the present paper the writer proposes to place on record some of the shallow-water species which occur both in Polynesia and in the tropical and subtropical waters of western North and Central America, including the islands

Map showing position of Pacific island groups (modified after Loomis, 1918). Homolographic projection.
south to the Galapagos. Some other forms, different but analogous, are also worthy of record. The consideration of the identity, or similarities of species from localities so widely separated raises the question as to whether the separation is biologic or merely nomenclatorial.

Many of these occurrences have been brought to light by various expeditions, namely, the Expedition of the California Academy of Sciences to the Galapagos Islands, 1905–1906, which also visited Clipperton Island; those of the same Academy to the Gulf of California in 1921 and to the Revillagigedo and Tres Marias Islands in 1925; the G. Allan Hancock Expedition to the Galapagos Islands in 1931–1932; the Templeton Crocker Expedition to the Galapagos Islands in 1931–1932; also by the various collecting trips of H. N. Lowe along the tropical shores of western North America. The writer here acknowledges helpful suggestions received from Dr. G. D. Hanna, Mr. A. M. Strong, Mr. I. McGuire and Dr. H. G. Schenck, in the preparation of this paper. Acknowledgement is also due Mr. C. M. Cross for assistance in the preparation of the map and to Mr. Frank L. Rogers who has furnished the photographs included in this paper. These photographs are the result of work accomplished during a government Works Progress Administration project.

The following species known from western North and Central America are only the most striking examples of forms which appear to be identical with or analogous to Polynesian or Indo-Pacific ones. Further study will probably reveal additional species which are common to the two regions.

*Pinctada mazatlanica* Hanley. Tres Marias Islands; Isabel Island between Mazatlan and the Tres Marias Islands; the Gulf of California to Panama and the Galapagos Islands. This pearl oyster was considered by Jameson (*Proc. Zool. Soc. London*, 1901, p. 376) and Lamy (*Journ. de Conchyl.*, Vol. 57, 1909, p. 227), to be
merely a subspecies of the Indo-Pacific *Pinctada margaritifera* Linnaeus.

*Acmæa striata* Quoy & Gaimard. Cited by von Martens (*Sitz. Gesell. Naturf. Freunde zu Berlin, Jahrg. 1902, no. 6, p. 137*) from Cocos Island and the Galapagos Islands; also cited by Stearns from the Galapagos Islands. This is an East Indian species.

*Aspella pyramidalis* Broderip. Mazatlan, Sinaloa, Mexico to Panama and Galapagos Islands. This is close to the Polynesian and East Indian *A. anceps* Lamarck.

*Cassis vibex mexicana* Stearns. Tres Marias Islands, and La Paz, Lower California (Stearns); San José Island, Gulf of California (Lowe). This is close to *C. vibex* Linnaeus, a Polynesian species.

*Cheilea equestris* Linnaeus. San Juan del Sur, Nicaragua; Galapagos Islands. The species is also known in Polynesia and the East Indies.

*Colubraria jordani* Strong (MS.). Socorro Island, Revillagigedo Group; Galapagos Islands. This is close to "*Triton* " nitidulus" Sowerby from Polynesia and the East Indies.

*Colubraria lucasensis* Strong & Hertlein. Cape San Lucas, Lower California; Galapagos Islands. This is similar to a species figured by Chemnitz (*Neues Syst. Conch.-Cab.*, Bd. 10, 1788, Pl. 162, figs. 1552, 1553) from the East Indies and Island of St. Maurice.

*Conus dalli* Stearns. La Paz, Lower California; Tres Marias Islands; Cocos Island; Costa Rica; Panama. This species is very close to certain variants of *Conus textile* Linnaeus, a well known species in Polynesia and the East Indies.

*Conus ebræus* Linnaeus. Galapagos Islands and Cliperton Island. The specimens are identical with others from Polynesia and the Philippines.

*Conus ebræus vermiculatus* Lamarck. The remarks accompanying the preceding species apply also to this form.
Crepidula aculeata Gmelin. Magdalena Bay and the Gulf of California; Tres Marias Islands; Revillagigedo Islands; Central America south to Panama, and the Galapagos Islands. This species has been recorded from both the Atlantic and Indo-Pacific regions.

Cymatium amictum Reeve. Tres Marias and Revillagigedo Islands; from the Gulf of California to Panama, and the Galapagos Islands. There seems at the present time no way to separate the west American and Indo-Pacific forms.

Cymatium vestitum Hinds. Gulf of California; Tres Marias and Revillagigedo Islands, south to Panama and the Galapagos Islands and Clipperton Island. This species is very close to the Indo-Pacific C. pileare.


Cypraea isabella-mexicana Stearns. Tres Marias and Revillagigedo Islands; also Gulf of California and Cape San Lucas, Lower California; Clipperton Island. This form differs but slightly from the well known Indo-Pacific C. isabella Linnaeus.

Cypraea moneta Linnaeus. Cocos Island; Galapagos Islands. This is a widespread Indo-Pacific species.

Cypraea scurra Chemnitz. Clipperton Island. Recorded from the Hawaiian Islands (Baldwin, Nautilus, Vol. 11, no. 11, 1898, p. 123; Ingram, Nautilus, 50, no. 3, 1937, p. 81), and the East Indies.


Hipponix antiquatus Linnaeus. San Benito Islands, San Roque Island, and Guadelupe Island, off the west coast of Lower California; Tres Marias Islands; Gulf of California and Galapagos Islands. This is a wide-spread Indo-Pacific species.

Hipponix barbatus Sowerby. (? = pilosus Deshayes). Cape San Lucas, Tres Marias and Revillagigedo Islands south to Oaxaca, Mexico and Galapagos Islands. Also Polynesian.

Mitra mitra papalis Linnaeus. Clipperton Island. A well known Indo-Pacific species.

Murex (Chicoreus) palmae-rosae mexicana Stearns. Tres Marias Islands (Stearns). Considered by Stearns to be a subspecies of the Indo-Pacific M. palmae-rosae Lamarck.


Voluta deshayesii Reeve. Clipperton Island (Dall, Nautilus, Vol. 24, no. 10, 1911, p. 112; Lowe, Nautilus, Vol. 46, no. 4, 1933, p. 112). A New Caledonian species. Voluta ancilla Solander described from Patagonia was also recorded from Clipperton Island by Lowe. If this

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1 See the remarks of Pilbry and Bryan concerning this species (Nautilus, 31, no. 3, 1918, p. 102, pl. 9, Fig. 11. Hawaii). Dautzenberg and Bouge (Journ. de Conchyl., 77, no. 2, 1933, p. 246) have considered nodus Lamarck to be the correct specific name for this species.
species really inhabits Clipperton Island it is an anomalous occurrence.

The echinoids *Brissus latecarinatus* Leske and *Metalia spatagus* Linnaeus occur both in the Gulf of California and in Polynesia and the East Indies.

Such wide dispersals of species across vast areas of open ocean as are cited in the foregoing list may be favored by a long free-swimming stage, considered as a factor linked with those of ocean currents and attachment to drifting objects, to pelagic fishes and mammals or to the feet of birds. There seems to be no evidence to indicate that these forms have been distributed to the localities indicated through any agency of man. Whatever the means of dispersal, it may be significant that the shallow-water molluscan species considered in the present paper with one exception, are all marine gastropods. This would seem to imply that the pelecypods find it less easy to reach widely scattered localities and that they are essentially dwellers on the borders of continents or major land masses. It is true that there are species of pelecypods from western North and Central America which do resemble rather closely others from Polynesia, but the similarity appears to the writer as less certain than that of the gastropods listed above.

Another striking fact is that certain of these species cited from the eastern border of the Pacific occur only in the islands far off the coast, namely the Galapagos Islands, Cocos Island, Clipperton Island, and the Tres Marias and Revillagigedo Islands. Also interesting is the fact that many of the species cited in the present paper belong to the families Conidae, Cymatidae, and Cypræidae.

Carpenter (*Rept. Brit. Assoc. Adv. Sci.* for 1856 [issued 1857], p. 346) recognized this fact. In the following beautiful words, he stated what every student of geographical distribution of mollusks must remark in a comparison of these Polynesian and west American shells:
"... having, when examining the shells of the Marquesas in the center of the Pacific, found several conspicuous and well-known forms of the Asiatic Seas, in spite of (in parts) the profound depth of ocean that lies between; he will naturally expect, as he reaches the American shores, to find also not a little in common with the opposite shores. He crosses the vast unbroken expanse of the West Pacific; one flank of the hemisphere of waters, which of itself almost rivals the Atlantic in extent. He pauses at the solitary Archipelago of the Galapagos, in the very longitude of the Gulf of Mexico, guarding (as it were) the great bay of Central America, and within 600 miles of its shores. Even here his eye rests with pleasure on a few well-known Cones and other forms, which have crossed the fathomless depths and come to claim kindred with their molluscan brotherhood of the New World. But here they stop. They could traverse half a world of waters. The human spirit that gives them understanding and a voice, beholds them on the very threshold of the promised continent, in whose bays and harbours, protected by the chain of everlasting mountains, they shall find the goal of their long pilgrimage. But the Word of the unknown Power has gone forth; and the last narrow channel they attempt to cross in vain."

To summarize, there are certain west American species which can be recognized as identical with and others that are very similar to those of Polynesia, while the results of recent studies of the Tertiary and Recent molluscan fauna of western North and Central America seem to indicate that the number of additions from Polynesia to this region have been small in comparison to the number received from the Caribbean and Japanese regions.

PLATE I

Figures of marine mollusks from Polynesia and the Western Americas, illustrating species which are identical or similar.

Fig. 1. Conus ebraeus Linnaeus. Length 35 mm., maximum diameter 21 mm. Plesiotype, No. 7055 (Calif. Acad. Sci. Paleo. Type Coll.), from Loc. 1243 (C.A.S., H. Hemphill), Island of Mauritius.

Fig. 2. Conus ebraeus Linnaeus. Length 32 mm. (apex incomplete), maximum diameter 20.5 mm. Plesiotype, No. 7056 (Calif. Acad. Sci. Paleo. Type Coll.), from Loc. 23,000 (C.A.S.), Cliperton Island.

Fig. 3. Cypraea isabella-mexicana Stearns. Length 40.3 mm., maximum diameter 22 mm. Plesiotype, No. 7068 (Calif. Acad. Sci. Paleo. Type Coll.), from Loc. 27,577 (C.A.S.), Maria Madre Island, Tres Marias Group, Mexico.
Fig. 4. *Cypraea isabella* Linnaeus. Length 30.4 mm., maximum diameter 16.2 mm. Plesiotype, No. 7067 (Calif. Acad. Sci. Paleo. Type Coll.), from Loc. 28,330 (C.A.S.), Rimitera Island, Austral Group.

Fig. 5. *Drupa vicinus* Linnaeus. Length 22.4 mm., maximum diameter 20 mm. (shell not complete). Plesiotype, No. 7059 (Calif. Acad. Sci. Paleo. Type Coll.), from Loc. 25,562 (C.A.S.), Kealakekua Bay, Hawaii.

Fig. 6. *Drupa vicinus* Linnaeus. Length 25.1 mm., maximum diameter 23.5 mm. Plesiotype, No. 7060 (Calif. Acad. Sci. Paleo. Type Coll.), from Loc. 23,036 (C.A.S.), Clipperton Island.

Fig. 7. *Conus ebrus vermiculatus* Lamarck. Length 24.7 mm., maximum diameter 16.5 mm. Plesiotype, No. 7058 (Calif. Acad. Sci. Paleo. Type Coll.), from Loc. 23,001 (C.A.S.), Clipperton Island.

Fig. 8. *Conus ebrus vermiculatus* Lamarck. Length 23.8 mm., maximum diameter 15 mm. Plesiotype, No. 7057 (Calif. Acad. Sci. Paleo. Type Coll.), from Loc. 1244 (C.A.S., H. Hemphill), Island of Mauritius.

Fig. 9. *Cypraea gilleti* Jousseaume. Length 40 mm., maximum diameter 31 mm. Plesiotype, No. 7066 (Calif. Acad. Sci. Paleo. Type Coll.), from Loc. 23,047 (C.A.S.), Clipperton Island.

Fig. 10. *Cypraea gilleti* Jousseaume. Length 38.3 mm., maximum diameter 27.8 mm. Plesiotype, No. 7065 (Calif. Acad. Sci. Paleo. Type Coll.), from Loc. 28,311 (C.A.S.), Fanning Island. Ted Dranga collector.

Fig. 11. *Cypraea moneta* Linnaeus. Length 30.9 mm., maximum diameter 23.1 mm. Plesiotype, No. 7064 (Calif. Acad. Sci. Paleo. Type Coll.), from Loc. 23,042 (C.A.S.), Hood Island, Galapagos Islands.

Fig. 12. *Cypraea moneta* Linnaeus. Length 27.4 mm., maximum diameter 20.4 mm. Plesiotype No. 7063 (Calif. Acad. Sci. Paleo. Type Coll.), from Loc. 27,987 (C.A.S.), Palawan, Philippine Islands.

Fig. 13. *Quoyia madreporarum* Sowerby. Length 14.2 mm., maximum diameter 9.1 mm. Plesiotype, No. 7072 (Calif. Acad. Sci. Paleo. Type Coll.), from Loc. 13,069 (C.A.S., H. Hemphill), Hilo, Hawaii.

Fig. 14. *Quoyia madreporarum* Sowerby. Length 13.1 mm., maximum diameter 10 mm. Plesiotype, No. 7073 (Calif. Acad. Sci. Paleo. Type Coll.), from Loc. 12,518 (C.A.S., H. Hemphill), Hawaiian Islands.

Fig. 15. *Quoyia madreporarum* Sowerby. Length 13.5 mm., maximum diameter 9.9 mm. Plesiotype, No. 7074 (Calif. Acad. Sci. Paleo. Type Coll.), from Loc. 23,779 (C.A.S.), Maria Madre Island, Tres Marias Group, Mexico.

Fig. 16. *Drupa morus* Lamarck. Length 19.6 mm., maximum diameter 14.3 mm. Plesiotype, No. 7062 (Calif. Acad. Sci. Paleo. Type Coll.), from Loc. 23,036 (C.A.S.), Clipperton Island.

Fig. 17. *Drupa morus* Lamarck. Length 20.9 mm., maximum diameter 13.1 mm. Plesiotype, No. 7061 (Calif. Acad. Sci. Paleo. Type Coll.), from Loc. 315 (C.A.S., H. Hemphill), Tahiti.

Fig. 18. *Conus daili* Stearns. Length 72 mm., maximum diameter 36 mm. Plesiotype, No. 7053 (Calif. Acad. Sci. Paleo. Type Coll.), from Loc. 27,578 (C.A.S.), Juanito Island, Tres Marias Group, Mexico.

Fig. 19. *Conus textile* Linnaeus. Length 64.4 mm., maximum diameter 32 mm. Plesiotype, No. 7054 (Calif. Acad. Sci. Paleo. Type Coll.), from Loc. 27,986 (C.A.S.), Island of Guam.

Fig. 20. *Asella pyramidalis* Broderip. Length 14.5 mm., maximum diameter 7.2 mm. Plesiotype, No. 7070 (Calif. Acad. Sci. Paleo. Type Coll.), from Loc. 27,232 (C.A.S.), Conway Bay, Indefatigable Island, Galapagos Islands.
Fig. 21. *Aspella anceps* Lamarck. Length 16.8 mm., maximum diameter 7.4 mm. Plesiotype, No. 7069 (Calif. Acad. Sci. Paleo. Type Coll.), from Loc. 451 (C.A.S., H. Hemphill), Samoa.

Fig. 22. *Aspella pyramidalis* Broderip. Length 6.6 mm., maximum diameter 5.3 mm. Plesiotype, No. 7071 (Calif. Acad. Sci. Paleo. Type Coll.), from Loc. 27,224 (C.A.S.), Tangola Tangola, Oaxaca, Mexico.

Fig. 23. *Mitra mitra papalis* Linnaeus. Length 115 mm., maximum diameter 35 mm. Plesiotype, No. 7052 (Calif. Acad. Sci. Paleo. Type Coll.), from Loc. 23,131 (C.A.S.), Clipperton Island.
FRANÇOIS ANDRÉ MICHAUX, THE BOTANIST
AND EXPLORER

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(Read April 24, 1937)

ABSTRACT

This paper sketches in outline the chief events in the life of André Michaux and that of his son, the subject of the sketch. The two are considered because their work was so very thoroughly intertwined.

The chief botanical explorations of François André Michaux are outlined, and something of the botanical accomplishment of both the father and the son stated.

The relation of the son to the American Philosophical Society is pointed out, and the story of the Michaux Memorial Grove in Fairmount Park has been brought up to date.

The person who finds himself called upon to write on Michaux, the botanist and explorer, may well find himself perplexed as to the course to be pursued. There were two Michauxs, both eminent in our early history for their botanical accomplishment and for their wide-ranging explorations. Father and son, they have left records of achievement that will make the name ever illustrious in the annals of American botany.

André Michaux, the father, is the man who comes first to the mind of the botanist. His name was rendered memorable by his discovery and description of a host of plants that had not been brought to the attention of science before his time. By virtue of close observation and by his clear descriptions of the new plants found, he is now credited with the discovery of over three hundred species of plants.

I have looked through the 7th edition of Asa Gray’s Manual of Botany for plants credited to Michaux, senior, in an area bounded on the north by the 48th parallel from the Gulf of St. Lawrence to Lake Superior and the international...
boundary, thence to the northwest corner of Minnesota; on
the west by the western boundary of Minnesota and north-
western Iowa, then southward along the 96th meridian; and
on the south by the southern boundaries of Kansas, Missouri,
Kentucky and Virginia. This area was but part of the
country traversed by him. He traveled from Hudson Bay to
Florida, along the eastern side of the continent, and penetrated
westward to the Mississippi River.

Accredited to André Michaux in that portion of the coun-
try seen by him and included in the narrower range of the
Manual, I find 24 new genera of flowering plants. He
discovered and first described 293 species of flowering plants,
of which 91 have since been referred to genera other than
those to which Michaux assigned them, leaving 202 species
unchanged in the present botanical record, in spite of over a
century of the most critical study. The persistence of the
name of Michaux in textbooks now in daily use is due almost
entirely to the explorations, critical study and accurate
description of material collected by the father, André
Michaux.

His son, François André Michaux, however, is the main
subject of my sketch, although their lives and work were closely
interwoven until the death of the father.

André Michaux, like his son, François André Michaux,
was born on a royal estate at Satory, near Versailles, France.
The Michauxs for generations had been in charge of this
estate. André, père, with whom our interest begins, was
born here in 1746, and grew up on the estate under his father's
instruction. His marriage in 1769 to Cecil Claye, the daugh-
ter of a wealthy farmer, was followed after eleven months by
her death, leaving André with the infant son. It is said that
as a relief to despondency he turned to an intensive study of
botany under the eminent French botanist, Bernard de
Jussieu. This brought him nearer to the Royal Gardens of
Paris. He soon began actively to collect plants, visiting
England, Auvergne and the Pyrenees. A diplomatic appoint-
ment took him to Persia and Mesopotamia, where he collected
seeds and plants for introduction into France. After three years in the Near East (1782 to 1785), André returned to France, and was immediately sent by the Government to North America, then little known to science, to study the forest trees, with the object in view of introducing into France such species as might prove useful in naval and other construction work.

On October 1, 1785, André Michaux, his fifteen-year-old son, François André, and a practical gardener, Paul Saulnier, landed in New York, and proceeded to establish a nursery near Hackensack, New Jersey, in which American trees were to be grown for shipment to France. The interest in applied science at that time centered in Rambouillet, and to the Rambouillett nurseries André Michaux’s trees, to the number of 60,000, were sent. After a year and a half spent in botanical explorations and in establishing the northern nursery near New York, father and son went to Charleston, South Carolina, leaving Saulnier in charge of the first nursery. The year 1787 saw the purchase of 220 acres of land about ten miles from Charleston, on which a second nursery was to be developed, and on which trees of more southern origin were to be grown for shipment from the port of Charleston.

In the year 1787, exploring expeditions were made into Florida and to the Bahamas, also a strenuous excursion up the Savannah River to the head waters, over the Appalachians into the Tennessee Valley and then the return to Charleston. A visit was paid to the northern nursery in that busy year.

The son was retained at Charleston, probably much of the summer, either because of ill health or to manage the new nursery in his father’s absence. His troubles were increased in 1789 when he was shot by a companion hunter, the projectile injuring his left eye. This accident seems to have been much above the horizon during the year and longer.

Troubles at home in France began to interfere with plans in America. Remittances ceased, the faraway scientists were out of sight, and much trouble was near at hand. Michaux,
père, advanced what money he could to continue the work, and pledged his property in France to secure loans.

Early in 1790, François, by this time a grown-up young man of twenty years, returned to France, now in a state of revolution. He espoused the popular cause, although the family had for generations been closely attached to the Royal interests, and his own childhood had been spent on a part of the Royal domain. Perhaps his course of action was somewhat influenced by what he saw in the nurseries at Rambouillet. He found there but a few of the trees that his father had striven so earnestly to secure for France. Half the total number had been given by the Queen to her imperial father in Austria, others had been given to members of the Court to embellish their private estates, and those that remained in the Royal Nurseries had been shamefully neglected.

François seems to have spent several succeeding years in the study of medicine, with the plan in mind of returning to America to practice. This plan was given up, however, when in 1796 his resources were exhausted. After years of strenuous exploration following his son’s return to France, André too sought the homeland. He sailed from Charleston in August, with the botanical materials and records that have made his name imperishable in the annals of American botanical science, but he was financially exhausted.

To add to his troubles, his boat was wrecked off the coast of Holland. He was rescued after floating on a board to which he had been tied, and in time much of his water-soaked botanical material also was rescued and dried out. Unfortunately, packages of priceless notes and records were lost, leaving gaps in the story of his strenuous life in the New World. He arrived in Paris after a time, where he was most favorably received, but the government was not interested in his plans for further explorations.

With the help of François, the year 1796 and several following years were devoted to the cultivation of the material from America, and in the preparation for publication of his luxuriously illustrated volume on the North American Oaks
and his volumes of the "Flora Boreali-Americana." These publications set up a landmark in American botanical history.

André, still strenuous for the bright reward of additional scientific knowledge, through ministers and through the Central Society of Agriculture, tried to persuade Napoleon, then First Consul of the French Republic, to authorize further explorations in remote lands. Only after great effort was André appointed naturalist in a scientific exploration to Australian seas, under the command of Captain Baudin, it being agreed that Michaux should be permitted to remain at the Isle de France if he desired. After the freedom the uncontrolled explorer had enjoyed in his American years, he was disgusted with the haughty and discourteous manners of Captain Baudin toward the scientists associated with the expedition, and abandoned his commission at Mauritius. After six months, he decided that Madagascar offered a better chance to accomplish something for France and for Science, and set about establishing a nursery at Tametave for trees and other plants. He died there of fever in December, 1803.

François, now arrived at vigorous young manhood, sharing his father’s objectives and feeling strongly as André had done that there was still much unfinished business in America, and relying on his experiences and acquaintanceships there, solicited a commission to resume work in the United States. De Chaptal, the Minister of the Interior, dissatisfied with the course of things in the nurseries still lingering near New York and near Charleston, gave François charge of them, with orders to ship to France such trees as still remained in them and to sell the land.

He set out on October 10, 1801, from Bordeaux for Charleston in the same boat, in charge of the same captain that had brought him home to France years before.

Thus, a rather remarkable thing was happening, the father who had rendered his name and that of France illustrious in the history of botany, was denied the opportunity to return to continue this work, being rather sent to his death in Madagascan wilds, while his son, with whom he had begun
scientific work in America, was sent back to carry on his father's work. As one sailed eastward in the naval vessel, the other took familiar passage to the west.

François arrived at Charleston on October 9, 1801, and spent much of the winter shipping stock from the Charleston nursery. He also visited Paul Saulnier, left in charge of the Hackensack nursery near New York. Saulnier, as a matter of fact, spent the rest of his life at this place, content to believe that he was still serving his master "Le Roi." He is said to have introduced the Lombardy poplar into America. If so, it remains a living reminder of that rare thing, a soul content to remain faithful over a few things.

François had accompanied his father on many long, wearisome and exciting journeys. Now he was to plan his own course and to travel much. He began botanizing in New Jersey and along the Hudson River. He discovered new oaks and hickories and sent the seeds to France. He did something that his father had done little of, in that he gave minute attention to the useful properties of the trees that his father was content to describe accurately and to name scientifically. He studied especially the oaks with tall trunks, good wood, and, in some cases, barks useful for tanning or dyeing. The lofty trunks characteristic of the American trees were greatly lacking in France. Of the forest trees of France, only 36 kinds reached a height of 30 feet. Of this number, 18 made up the bulk of the forests, and only 7 kinds were highly useful for civil and naval architecture. Among the American forest trees, 140 species reached the height required and many sorts offered the desired kinds of utility.

In June, 1802, he visited Philadelphia, meeting among others Collin, B. S. Barton, Vaughan, Peale and William Bartram. He is said to have enjoyed Bartram's Garden and William Hamilton's "Woodlands."

Having a few months at his disposal, he decided to go to Kentucky and Tennessee, regions that had won such high praise from his father. He set out on June 27, 1802, on his long journey. Going by way of Lancaster, Columbia, York,
Carlisle and Shippensburg, he crossed the Alleghenies, arriving in Pittsburgh in ten days, having traveled by stage, horseback and on foot. On July 14th, he set out for Wheeling, where he bought a canoe in which he descended the Ohio River in company with an American officer named Craff. On July 17th he was at Marietta, and on July 24th he landed at Limestone (probably now Maysville) in Mason County, Kentucky. From here he walked alone to Lexington, arriving there on August 10th. He continued on to Nashville, where he spent four weeks collecting in the vicinity or along the banks of the Cumberland River.

His return trip was to take him to Charleston. Starting on September 5, 1802, he entered upon a more interesting route than that followed in going westward. Fort Blunt, West Point and Knoxville, Falls of Roaring River and Jonesboro, in Tennessee, brought him to the mountains on the North Carolina boundary line. Soon he reached the farm of old Davenport, who had guided his father in those mountains in former years. He continued on by way of Morgantown, Winnsboro and Columbus, finally reaching Charleston, South Carolina, after a journey of 1800 miles, made in three and a half months. This trip furnished material for a book *Voyage à l'Ouest des Montes Allegheny*, published in 1804, and translated into English soon after. In this work one has an account not only of the vegetation, but comments on the conditions and methods of agriculture, on the nature of the soils, particular plant products, and the commercial relations between these backwoods regions and the seacoast cities.

On March 1, 1803, he sailed for France, landing at Bordeaux on March 26th. On going to Paris he found the publication of his father's volume on the oaks delayed. Although the text had been printed in 1801, the plates were still lacking. He also attended to the publication of his father's great "*Flora Boreali-Americana*." He associated the botanist, Claude Richard, with the work in a literary capacity. Michaux's efforts at this period also resulted in the publication in 1804 of his own "*Voyage.*"
While in France, he took occasion to urge the planting of trees especially suitable to areas of that country lacking good native sorts. A communication to the Central Agricultural Society of France called attention to the magnificent timber trees found in the swampy lands of parts of America, and contrasted the willow oak, white cedar and cypress trees with the short, scrubby growth found in the wet lands of France. Sandy areas of America with the live oak so highly valued in naval construction were contrasted with unproductive sands of France. His communication in 1805 on the desirability of naturalization of forest trees of North America was warmly supported in a report to the Central Society of Agriculture by a committee consisting of De Pertuis, Cels and Correa de Serra, a member of this Society, at one time Portugese minister to the United States and temporarily at the head of the Department of Botany at the University following the death of Benjamin S. Barton. The plan proposed that a naturalist be sent to the United States to collect seeds and young trees of desirable sorts, to be sent to the national nurseries of France. The minister of finance, Duke de Gaete, favored the project, and François Michaux was entrusted with the responsibility of trying again to bring American trees to France.

He sailed from Bordeaux to Charleston on February 5, 1806, but after three days at sea, his boat was picked up by the British man-of-war "Leander" and Michaux was taken on board. After forty-three days afloat, he was landed on the Bermuda Islands, where he was allowed to go ashore. This detour in his route gave him a chance to study new plants and new scenes. These he described in a report "Notice sur les Isles Bermudes." He was finally released and permitted to resume his journey. He was again in the United States near the end of May.

Now began a series of explorations covering much of the Eastern United States. Starting in the District of Maine, he traversed the states of the Atlantic coast as far south as Georgia, covering a distance of 1800 miles in this survey from
northeast to southwest. Along the coast he visited shipyards to see the timbers and methods used in American ship construction. He talked less with botanists and other scientists and more with artisans, and sought to meet the most skillful workmen in wood. He made five explorations into the interior.

He penetrated Maine along the Kennebec and the Sandy Rivers, perhaps to the Rangeley Lakes. He crossed New Hampshire and Vermont, going from Boston to Lake Champlain. Farther south he traversed the present state of New York from the city of that name to Lakes Erie and Ontario. Proceeding from Philadelphia, he reached the Monongahela, Allegheny and Ohio Rivers, and again following a route that must have been better known to him from his earlier travel, he went from Charleston to the sources of the Savannah and Oconee Rivers. In these journeys into the interior, he studied the trees making up the bulk of the forests, with special reference to the nature and uses of their woods, their commercial values in the exchanges between the states and in their relation to demands for exportation. Barks used in tanning, qualities and prices of woods, values for fuel, cabinet work and otherwise, were objects of his inquiry. A complete collection of polished specimens of different woods was made.

He remained in America about three years on this quest, making many new acquaintances among the prominent men of the day, Muhlenberg, William Hamilton, Barton, Hosack and Alexander Wilson being among the number. This may have aided in bringing about American recognition.

On April 14, 1809, Michaux was made an honorary member of the Philadelphia Society for Promoting Agriculture, a Society organized in 1785 to advance agricultural interests. The membership of this Society overlapped that of the American Philosophical Society in many instances, and for years the two organizations met in the same rooms. In the minutes of the meeting of the Philadelphia Society for Promoting Agriculture, held on January 10, 1809, Dr. James Mease, of Philadelphia, a member of this Society, proposed the name of
M. André Michaux, of Paris, as an honorary member, his election following at the meeting held on February 14th of the same year. Although he later lived near Paris, his interest in the affairs of the Society continued. Repeated references to communications received from him, pamphlets, books, and other publications, now on the shelves of the Society’s library, offer substantial evidence of this interest.

On April 21st, of the same year, Michaux was elected to membership in the American Philosophical Society, an honor highly esteemed during the rest of his life. The Michaux Fund gives eloquent testimony on this point.

He now returned to France, presented himself to the Central Society for Agriculture that had sponsored his enterprise, and reported what he had done. The results of his efforts in America had been reaching France in shipments from time to time in the form of seeds and young trees. In this effort he had better success than his father. From the seeds sent over by the son, about 250,000 trees were obtained.

A committee was appointed with Corra de Serra as chairman to report to the Central Society on his effort. The report was most flattering. Michaux was complimented on the faithful execution of his trust, and a vote of thanks was passed in recognition of the important service rendered. With his great task of travel and physical hardship in a new and untamed world successfully accomplished and appreciated by his sponsors, Michaux, now about forty years old, still had long years before him in which to aid through efforts in France to bring to the homeland the solid result toward which the efforts of both his father and himself had been directed.

For two years following his return to France, he was busy in securing the publication of his great work "Histoire des Arbres Forestiers de l’Amérique du Nord." Volume I appeared in 1810, volume II in 1812, and the third volume in 1813. The text was illustrated with 144 copper plates, designed by the eminent French artists, the Redoutés. In 1817, an English translation in four volumes by Augustus L.
Hillhouse was published in Paris under the title of "North American Sylva." William Maclure bought the plates in Paris and brought them to the United States. American editions followed, the first brought out in New Harmony, Indiana, in 1842; a second appeared in 1852 in Philadelphia with notes by J. J. Smith.

After the appearance of his great work, Michaux devoted his attention to the cultivation and propagation of trees for public use in France. The Central Society of Agriculture still supported him by entrusting the use of a large estate to his care for experimental purposes. On his own country place near Pontoise and on the extensive lands of M. Delamarre, he carried on work of this character until his death. After a strenuous young manhood, the quieter life of the experimentalist must have had its rewards. He had the time and opportunity to carry to a successful conclusion the projects that had sent him to the wilds. He continued to take a special interest in the use of the so-called "heaths," of which France had at that time two million unproductive acres. As a result of forty years of experimentation by Michaux and the Central Agricultural Society, it was shown that these lands could be improved and used productively by the growing of "certain resinous trees," undoubtedly pines. This seems to be indicated by him in a letter written in 1852 to Isaac Lea, then President of the American Philosophical Society, in which he mentioned the Russian pine (*Pinus sylvestris*), known to us as the Scotch pine. He recommended this tree for special consideration in the northern and middle states of this country. It may seem somewhat strange that one who had studied the trees of this country and the conditions prevailing here, as Michaux had done, should have found any considerable use for this inferior species in the regions named.

In his letter to Isaac Lea, just referred to, written at the ripe old age of 82 years, he glances over the past. Speaking of his *Sylva*, he says: "The science of botany was the principal object of my father's explorations in North America and the
'Flora Boreali-Americana' was the result of these explorations. As for me, I took another view of the vegetable kingdom whilst in our country—a view more limited and less scientific, it is true, but perhaps more generally profitable to the farmer and the landholder, as well as to that class of society, so numerous in the Northern States of the Union, who employ wood in so many different ways. I do not consider my Sylva Americana as complete as it might be; thus, for instance, I have omitted several species which grow in lower Louisiana and in the two Floridas. In the second place, I have described and figured some trees that are deficient in the flowers and in the fruits. Had circumstances permitted, I would have returned to the United States, and, in a new edition, have corrected the errors and filled up the omissions. I would thus have been able to present to the American nation a work worthy of her great name, but now that I have arrived at a very advanced age—nearly 83 years—I can do nothing in this respect other than to express my regrets and the hope that some native arboriculturist may complete my researches on the plan which I have adopted. The publication of such a work would be attended with much benefit to the country, and afford particular honor to him who would undertake it.

Although he would be hardly called an arboriculturist, an English-born botanist, who also attained scientific fame in America, may have come in answer to Michaux's prayer. Thomas Nuttall, a botanical disciple of Benjamin S. Barton, of the Botanical Department of the University of Pennsylvania, later himself professor of botany at Harvard University, brought out a book in three volumes dealing with about 120 kinds of trees, mainly from the far west, a region not visited by Michaux. The two works supplemented each other in the way that Michaux had hoped, and the set of volumes makes a very important contribution to our knowledge of the forests of the United States a hundred and twenty-five years ago.

Michaux's interest in America dealt principally with trees,
but he concerned himself with other matters also. He happened to be planning a trip along the Hudson River in 1807, and hearing that a steamboat was to make its trial trip soon, found his interest aroused. He was pleased that he and another Frenchman were passengers when Fulton made the first trial trip of the "Clermont." That they were the only passengers may also have added to his satisfaction.

Not long before his death (1852), he added his contribution to the bulky literature that grew out of the terrible yellow fever epidemic that visited Philadelphia (in 1821) under the title of "Causes of Yellow Fever in the United States."

Michaux's life as a bachelor ended in a rather sudden decision at an advanced age to marry his housekeeper, a remote relative who had long managed his household. Elias Durand, a Frenchman residing for a time in America, who had known Michaux in his later years, and who had aided him by collecting and forwarding American tree seeds to France, writes in his account of Michaux, published in the Transactions of the American Philosophical Society in 1890, that he was carried off with frightful suddenness by a stroke of apoplexy on the 23rd of October, 1855, "after a busy day among his American trees."

He was in comfortable financial circumstances, and deposited a will with Isaac Lea four years before his demise, providing for the now well-known Michaux Fund, to be used by the American Philosophical Society after his wife's future had been provided for.

After the death of this lady, the fund was used in part for the creation of a monument to the generous donor, and this monument still exists as the Michaux Memorial Grove of oaks in Fairmount Park. This grove was planted near Horticultural Hall at the opening of the Centennial Exhibition in 1876, when Fairmount Park was in the earliest stage of its formation. The Chairman of the Committee on Trees and Nurseries in the Fairmount Park Commission, Mr. Eli Kirk Price, who was also chairman of the Committee on the Michaux Fund in this Society, reported to the Society on
June 16, 1876, that "he had visited today the Fairmount Park nursery and found the ground well taken care of, and the large stock of trees in a flourishing condition. Of the acorns planted last winter, nearly all have grown except those of the 'Bartram Oak,' and of the fifty or sixty of those planted not one has grown. So far the proof is in favor of that oak being a hybrid."

It seemed to me a matter of interest at this time to see what has happened to this Grove in the sixty or more years since it was established.

Accordingly, the area indicated to be that occupied by the Grove on a map of the Park prepared for the Centennial Exhibition of 1876, shown us by Mr. S. N. Baxter, the present Arboriculturist of the Commission, was visited and the ages of a number of trees now present determined by the use of an increment borer. Mr. R. D. Forbes and his assistant, Mr. O. M. Wood, both of the Allegheny Forest Experiment Station, made the test on trees selected by Mr. Baxter as representative specimens. Out of 7 tested, 6 showed an age estimated to be 60 years. These trees undoubtedly belong to the Michaux Grove. From this examination it seems clear that the Memorial Grove is still in large part made up of the oaks reported on by Mr. Price in 1876 and, judging from the uniformity of the Grove, relatively few of the original trees have had to be replaced.

Michaux Avenue now passes through the Grove and presents one of the most beautiful spots in the Park. Thus, the living memorials of these friends of trees are still in their prime, and, if spared by weather, disease and other hostile influences, should delight generations still to come before old age in the end brings them down.

In closing this sketch, I will take the liberty of making some suggestions regarding ways in which the Society may continue to carry out the will of the donor, and at the same time advance the significance of the organization.

Michaux was an ardent advocate of a public policy of support for the scientific study of wild plant life. He studied
it with equal interest with reference to human utility. Probably influenced to a considerable extent by the needs of France at that time, his special efforts were centered on trees in nature and under cultivation. Something perhaps remains in the forests of France of the trees introduced by the Michauxs. How much they actually did for their country in this respect I do not know. For us and for the rest of the world, their greatest accomplishment is seen in the writings they have left. With the world changing as rapidly as it is, authentic records of the past should be sought out and held with great care by repositories like this Society. Hence, accounts of explorations and records of earlier days in the fields and forests are basic material. As the scene changes, succeeding accounts will continue the story.

For example, the question of soil conservation, a very old one and now much in the public mind, is presenting itself as a series of problems in agriculture, forestry, wild life preservation and human welfare in all of their many aspects. Old records tell of nature and of early human life. We should have those early records. The story should be continued by the important writings of later days. Thus, the early contributions of the Michauxs, father and son, remaining in our library, become a priceless possession as the record continues to unfold.

Memorial trees in time die and their places are forgotten. The practical accomplishment of the past must be sought in books and other lasting sources of information. Exploration, forestry, botany, plant introduction and plant utilization, the problem of the soil that supports all life, are subjects that may fittingly claim the earnest attention of the Society in building up and preserving its collection of records. Utility may be immediate. Certainly there will be a priceless value for the future if civilization persists and present ideas of worth continue to be held.
NEOLITHIC SITES IN THE YUGOSLAVIAN PORTION OF THE LOWER DANUBIAN VALLEY

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ABSTRACT

The Yugoslavian portion of the lower Danubian Valley is adjacent to the south side of the river from the Iron Gate to the mouth of the Timok. Recent field reconnoitering in this area resulted in the recording of fresh data on a score of sites. Among them are five extensive settlements with rich Neolithic deposits. And it seems likely that some of the other sites, all, thus far, revealing only later deposits will, on excavation, disclose Neolithic occupation as well. The Neolithic material remains from these sites comprise varieties quite characteristic of the western half of the lower Danubian Valley at large. They contain, among other things, the barbotine class of pottery which has a wide distribution in the Danubian area. This pottery provides a very useful means for comparative studies and chronological reconstructions, as it is particularly characteristic, locally, of the initial Neolithic occupation. Technologically, the barbotine ware is a well distinguished class. (The Appendix deals with some qualitative aspects of a sample series from Starčevo.) While the observations thus far gathered in the field and in the laboratory are quite suggestive, no definite conclusions regarding the Neolithic culture history of the area can be drawn until adequate excavations are made. Although the data thus far gathered have only a provisional value, it is obvious that certain dependable leads can now be followed. The Yugoslavian portion of the lower Danubian Valley offers splendid opportunities for archaeological field work. The Neolithic sites, although damaged by Danubian erosion, are especially attractive in this regard since they seem to be potential sources of highly desirable data.

The Yugoslavian portion of the lower Danubian valley is confined to the right bank of the Danube between the Iron Gate¹ and the mouth of the Timok.² Its interior extent is

*With an Appendix by Donald Horton, entitled "Note on a microscopic study of a sample group of barbotine sherds with positive applique from 'Grad,' Starčevo."

¹ Geotectonically, the Iron Gate is identified by the approximately 130 km. long cleft which the Danube follows in its passage through the Balkan-Carpathian mountain system; cf. Cvijić, 1908, pp. 1 ff.

²The corresponding course of the Danube provides a part of the boundary between Yugoslavia and Roumania; the islands in this section of the river belong to Roumania.
defined by the Miroč, Veliki Greben, and Deli Jovan ranges in the west, by the foothills of the Svrljiška Planina in the south, and by the Yugoslav-Bulgarian boundary line in the southeast. This boundary joins the right bank of the Timok at a point approximately 8 km. above the estuary and coincides with the river thereafter.

At the present time the prehistory of this area is very unevenly and indeed incompletely understood. Systematic excavations have not been attempted yet and reconnoitering efforts have been concentrated chiefly along the Danube. Although certain stray finds as well as several sites have been recorded deep in the interior, our knowledge of prehistoric conditions aside from the bank of the Danube is decidedly inadequate. Yet it is quite apparent that as early as the Neolithic Age, some of even the most outlying districts were reached. For this reason it is imperative to deal with the region as a whole, despite the circumstance that the bulk of positive occupational evidence is concentrated in the immediate riparian zone of the Danube. It is possible, of course, that some of the advantages proffered by the river may well have been a discriminating factor in this respect especially during the initial implantation of Neolithic economy. Nevertheless, an early penetration inland is supported by distributional factors which clearly reflect genetic affinities with the cultural status common to the river loci.

From the eastern edge of the Iron Gate to the mouth of the Timok, the Danube pursues an undulating course with intermittent straight stretches, and its channel shifts frequently. The banks are subject to seasonal inundation and there is considerable erosion which often changes their contours. Ice flows and atmospheric weathering also cause pronounced damage. Owing to these actions, sites in vulnerable locations are now more or less exposed and vertical profiles of their deposits are thereby brought to view. The task of finding archaeological stations is then conveniently facilitated and simplified. The bank varies in height from place to place. A minimum of 1 m. or even less is to be
found between the villages of Kostol and Mala Vrbica; and this is true of only a short distance, for the bank attains a much greater height as either of the two communities are approached. At Korbovo, Brza Palanka, and Velesnica, the average is approximately 3 m. to 4 m. While at Prahovo and Radujevac a maximum of 12 m. to 15 m. is reached. In the last named instance, dunes are present and add to the normal height of the bank. At the mouth of the Timok the Danubian bank is 2.5 m. to 3 m. high. The repeatedly active water and ice erosion finds little resistance in the composition of the banks, which consists of sand, gravel, loess (probably rather altered loess), and humus. Cvijić identified this formation as the last (i.e. contemporary) or seventh Danubian terrace in the lower valley of that river.\(^1\) At low state of water, a shelf of packed sand and gravel, varying in width from a few centimeters to as much as 100 m., extends on an inclined plane (in the manner of a beach) from the base of the bank to the edge of the stream. (Pl. VIII: 1–5.—However, this “flood plain” does not confine the high water of the Danube which, in floods, overflows the banks as well.) In places containing archaeological deposits embedded in the terrace (i.e. the immediate bank, the seventh terrace), the surface of the shelf is usually strewn with dislodged cultural remains.\(^2\) These peculiar circumstances, apalling as they certainly are, nevertheless aid an archaeological survey.\(^3\) The exposed

\(^1\) Cvijić, 1908, p. 16.

\(^2\) The natives seem to be well acquainted with locations of sites and often possess valuable relics. Vandalism, especially in Roman ruins, is not uncommon, and certain amount of selective collecting for commercial purposes is likewise on record. At times falsified oddities are offered for sale, such as, for example, a tile with incised depiction of, presumably, Trajan’s Danube bridge, said to have been found “in digging” near the remains of this structure below the village of Kostol. (In general effect and in several details this portrayal closely approaches the freak executed upon a fragment of a capitol, allegedly “discovered” in the same locality, and published by Vasić, 1907a.) In my personal experiences I have found the natives most obliging in rendering useful information and guidance.

\(^3\) The prehistoric sites are characterized by a level surface plane which, unless disturbed by cultivation or other artificial incursions, does not necessarily disclose the presence of archaeological deposits. However, the Roman ruins, which are especially numerous in this region, are usually distinguishable by a mound-shaped accumulation surmounting their masonry remains. The series of
sections are particularly useful in examining structural features and stratigraphic data. It thus becomes plausible to pursue there a speedy investigation for the purpose of a general orientation. With the valuable evidence in situ so abundantly at hand, the scattered material assumes a much greater significance than might be the case in a strictly surface-collecting survey. In certain instances the exposed deposits can be followed uninterruptedly for a distance of several hundred meters. Various details of tectonical features and a truly remarkable amount of cultural material (especially sherds) are in view at most of the sites. Under such circumstances correlation of stray finds with reliable depositional evidence is frequently attainable. It is obvious, of course, that even these conditions do not diminish the necessarily superficial nature of the resulting impressions. Only systematic excavations, conducted on a large scale, can lead to conclusive interpretations and deductions. Until that is done no more than a provisional description can be presented. It now becomes my task to attempt such a description and I do so in full cognizance of the many inevitable shortcomings.

The initial archaeological survey concerned primarily with the prehistory of the area under discussion was undertaken by Professor Vasić in 1907. He then collected various material in ten localities situated on the bank of the Danube between Kladovo and Raduževac and summarily ascribed the total to "the Iron Age." Having found mostly pottery analogous with that from "Žuto Brdo," Vasić, despite rectangular embankment-enclosures, each with an outside most, particularly frequent in the neighborhood of Negotin, and invariably called meteris (i.e. bulwark), are unmistakably identifiable by their well preserved features. These earth works, often erected upon ancient sites, are unquestionably quite recent in origin.

1 Vasić, 1910, pp. 2 ff.
2 Nine of these lie directly on the bank of the Danube, and one, i.e. "Pesak," Kladovo, on the bank of an old, now dry (yet seasonally flooded) arm of the river; op. cit., pp. 5 ff.
4 Ibid., pp. 1 ff.
5 A site previously explored by Vasić (1907, pp. 1 ff.) and relegated by him to the Iron Age. This interpretation, however, is not in agreement with the cultural position of the site which falls within the Bronze Age proper (cf. Hoernes,
several examples of clearly Neolithic ceramics and stone artifacts,¹ did not recognize or allow the existence of this earlier period in any of the ten sites. Rather he attributed the older (i.e. Neolithic) elements to survivals persisting in considerably later contexts (i.e. the misconceived Iron Age).²

More recent reconnoitering in the same region has established fresh evidence of Neolithic occupation. This consists of the following: (1) In situ deposits of settlements revealing abundant material and valuable structural features, particularly profiles of dwelling pits as well as of floors. (2) Secondarily placed material found in the immediate vicinity of such deposits and fully correlated therewith. (3) Secondarily placed material found in localities which, as far as has been possible to ascertain by examination of their exposed profiles, do not reveal correspondingly datable deposits. (4) Stray material, obtained from private hands, and said to have been found in deposits in which Neolithic occupation is indubitably proved. Moreover, additional data on later periods have also been secured. Among these are sherd collections duplicating Vasić’s finds of 1907,³ i.e. Bronze Age material, a few examples suggestive of La Tène pottery, numerous Roman antiquities, and Early Slavic (mediæval) sherds, as well as sundry details regarding the nature and structure of such later deposits. It is the purpose of this writing to summarize those findings which pertain to the

² Cf. Vasić, ibid., p. 12, where the reference to “Gradac,” near Leskovac (Vasić, 1911, pp. 97 ff.), makes the case clear. The erroneous interpretation and dating of “Gradac” (op. cit.) has recently been emphatically reiterated by Vasić (1936, pp. 137 and 154).
³ For illustrations of the typical pieces, cf. Vasić, 1910, Pls. II–IX, to which may probably be added also the following: Pl. IV: 18, 19; V: 40; VI: 41 (?); VIII: 59. Vasić’s collection of 1907 was originally housed in the National Museum at Belgrade which was destroyed by bombardment during the World War; cf. Petković, 1921, pp. 205 ff., and Vasić, 1932, p. ix.
Neolithic Age. While my chief objective is concerned with sites containing actual Neolithic deposits which I have personally examined I am also drawing from other pertinent sources. Thus I include localities in which Neolithic material appears in a secondary position but not in situ, as well as chance finds as yet perhaps totally undocumented by depositional evidence. Some of these instances may lead to additional discoveries and certainly provide potential guidance for further field work.

No traces of human occupation antedating the Neolithic Age have yet been noted anywhere in the Yugoslavian portion of lower Danubian valley. The nearest occurrences, geographically, of Paleolithic existence are reported from Bulgaria where several caves with Aurignacian deposits have been explored, and from Little Walachia where one station, presumably of a Lower Aurignacian phase, has been investigated. There are no indications of Mesolithic economy anywhere in the right-bank sector of lower Danubian valley. On the opposite side, according to Nestor “echte mesolithische

1 The field work from which this task arises was sponsored by the Fogg Art Museum, Harvard University (season 1933), and the American School of Prehistoric Research (seasons 1933 and 1934); the National Museum of Belgrade co-operated in the program of 1933 and the Negotin Museum in that of 1934; cf. Fewkes, 1934, pp. 24 ff., 1934a, pp. 33 ff., and 1935, pp. 16 ff.

Most of the material collected during these investigations is deposited in the Negotin Museum. The National Museum at Belgrade has a representative series of the 1933 finds as well as certain specimens previously purchased from private hands.

2 With the kind permission of the Negotin Museum, which is hereby gratefully acknowledged, it becomes possible, on this occasion, to incorporate certain of its independent finds and general records made available to me through the courtesy of Mr. S. Stefanović.


4 Cf. Nestor, 1932, p. 22. This author questions the designation of the point from Sâgletu as of the Abri-Audit type (i.e., note 61), and speaks of yet other, unpublished, traces of Palaeolithic sites in Little Walachia (ibid.). See also his comments regarding the dubious Mousterian (?) or Aurignacian bone and stone pieces from a now destroyed cave near Topalu on the Danube in Dobrogea. However, Morosan, 1928/1929, pp. 1 ff., had previously interpreted the Topalu finds as Aurignacian. And more recently Morosan, 1936, p. 1242, expressly states: “L’industrie paléolithique de Topalu en Dobrogea semble aussi appartenir à un tel [i.e. Middle Aurignacian] facies.”
Funde sind bisher nur in der Kleinwalachai zu verzeichnen." 1 The material in question is said to consist of "einige Mikrolithen, die eine der Chwalibogovice-Kultur Polens entsprechende Facies aufweisen sollen." 2 With respect to the Neolithic Age, the vestiges of which are clearly recognized more or less throughout the lower valley of the Danube, there can be no doubt that even the apparently "oldest" remains bespeak a well developed cultural complexity. Although regional differences are distinguishable, a common fundamental basis of economy is demonstrated. This reflects a communal mode of life, sessile conditions, agriculture and stock raising—in other words controlled economy. Local parental antecedents from which the determining criteria of such an innovation might be considered to have germinated are certainly totally unsubstantiated. On the contrary, the very nature of the Neolithic cultural status, collectively viewed, indubitably bespeaks derivation from external sources. Moreover, it seems obvious that not only fresh cultural inspirations, but also a fresh influx of people must be accounted for, at least at the beginning of the Neolithic manifestations. It would be futile to speculate on such aspects as place of origin, motive, velocity, route, etc., of the forces involved. The process occurred subsequent to the accumulation of the quarternary loess mantle which covers most of the valley area. However, it does not seem to have taken place immediately after that deposition was completed. That an interval of time lapsed between the two events is indubitably proved, at least in some localities, by an aboriginal humus layer which overlies the loess. 3 In such instances the Neolithic

1 Nestor, 1932, p. 29.
2 Ibid., based on the work of Nicolescu-Plospor who, states Nestor, "unterscheidet zwei mikrolithenführende Kulturen, die auch geographisch auseinander gehalten worden können." It is patent, of course, that such meager and one-sided evidence does not lend itself to conclusive interpretation. The plausibility of recognizing cultures on the basis of a few microliths is, obviously, open to question.

3 In the instances specifically noted in the region under discussion the maximum accumulation of this humus did not exceed 0.4 m. It is interesting to recall that at "Vinča" the thickness of a similar aboriginal humus layer has been ascertained by Vasić to measure 0.5 m. (Vasić, 1910a, p. 23) and even 0.7 m.—0.75 m. (Vasić, 1936, p. 8).
deposits pierce the ancient humus and penetrate farther below into the loess. We have no objectively ascertained data as to the nature of living conditions prevailing during the acute period of the loess formation. The impression one gathers from examining the exposed profiles of the mantle along the Danube is that they were unfavorable to man; such an impression is inconclusive and cannot claim recognition. At any rate positive traces of contemporary occupation have not been ascertained therein. The Neolithic evidence is invariably clearly intrusive. In view of these considerations it seems imperative to regard the initial Neolithic settlers as pioneering colonists. Their socio-economic status reveals a well rounded cultural foundation rather than an experimental stage. This interesting circumstance is particularly significant with respect to the inseparable ethno-cultural duality which immediately implies a process of primary diffusion as the most plausible rationalization of its existence. It also mitigates against an autochthonous development and strengthens the recognition of a hiatus.

In dealing with the individual sites within the Yugoslavian portion of lower Danubian valley I use predominantly the present tense although I realize that some of the conditions here described may have been altered since 1934. The sites fall into the following three categories:

(A) Definitely Neolithic settlements unquestionably identified by deposits in situ.

(B) Likely (?) Neolithic settlements, i.e. those in which secondarily placed Neolithic material appears without positive proof of correspondingly datable deposits.

It is commonly assumed and frequently asserted (by archaeologists rather than by palaeobotanists) that the loess zone of the Danubian area was originally covered by primeval forest as the Neolithic Age ensued and that the initial settlements required substantial clearing of the growth in order to be more or less permanently allocated; cf., e.g., Childe, 1929, pp. 26 ff. (general application to middle Danube and lower Danube valley), and Vasić, 1936, p. 8 (specified for "Vinča"). In either instance analytical proof is not cited.

As a matter of fact in 1934 I noted several changes in site profiles at Kusjak, Korbovo, and Kostol as compared with conditions existing there in 1933. While some of these were attributable to natural erosion others were plainly due to artificial causes.
(C) Miscellaneous sites from which sundry Neolithic (and later) material has been secured, but in which depositional circumstances have not been adequately identified as yet.

Inasmuch as the material (ceramic, bone, stone, etc.) from the several localities is quite homogeneous in character and falls into a uniform classificational scheme, I shall enumerate the individual occurrences at each site and detail the total collectively under a separate heading. This will avoid repetition of descriptive data.

(A) DEFINITELY NEOLITHIC SETTLEMENTS

These include five sites, all situated upon the immediate bank of the Danube. Their positions are here given in relation to modern communities; the sequence of their numbers (1–5) is correspondingly marked in the accompanying sketch map (Fig. 1). The natives refer to these sites simply as obala, i.e. river bank, although in certain cases specific names prevail. Whatever the individual usages they are here given in transliteration. In the instance of the two sites at the village of Korbovo, however, I have arbitrarily employed geographic directions to clarify the distinction. Thus I call the site situated upstream from the village “Obala Northeast,” Korbovo, in contrast to “Obala Southwest,” Korbovo, which identifies the site situated downstream from the village. And I shall, hereafter, give the name of each site in quotation marks.

1. “Obala” at Kostol

The village of Kostol lies 2.5 km. east of Kladovo. The site of “Obala” is situated within the land belonging to this community, about 2.5 km. eastward from its center towards Mala Vrbica (Fig. 1, site number 1), and approximately 0.8 km. downstream from the ruins of the Moesian abutment of Trajan’s Danube bridge (Pl. VII: 11). The height of the bank bearing the deposits varies from about 3 m. (upstream
Fig. 1. Sketch map showing the course of the Danube between Veliko Gradište, Yugoslavia, and Novo Selo, Bulgaria, based on Cvijić, 1908, and on the pertinent section sheets of the Royal Military Institute of Yugoslavia, issues 1925–1929. The definitely Neolithic sites are numbered 1–5, and the (?) likely Neolithic sites are lettered a–h, while the miscellaneous sites are numbered i–v. (Miscellaneous sites vi and vii fall outside the area embraced by the sketch map.) The insert, showing the course of the Danube from Novi Sad, Yugoslavia, to the Black Sea, is a free sketch.
margin) to about 0.5 m. (downstream margin), and is only partially exposed by erosion. The visible profile of the culture level, consisting of pits and superimposed debris, appears within a length of approximately 200 m. In the corn fields upon the horizontal plane of the bank we collected surface sherds over an area considerably exceeding the stretch of the exposed bank and reaching inland as much as 300 m. This, of course, does not provide any dependable means with which to judge the probable interior extent of the site, because the material was placed in its secondary position in the process of geoponic activities. Similarly the relics which we found at the base of the exposed profile and on the adjacent shelf had been secondarily deposited. However, in the culture deposits themselves, both in pits and in the immediately superposed debris, plentiful evidence of Neolithic occupation exists in situ. The character of this material is identical with that represented in the surface lot. Moreover, the deposits themselves did not reveal (1933 and 1934) anything but Neolithic relics. Both the pits and the debris surmounting them contained, as much as has been possible to ascertain by observation, uniform structural nature and contents. Although the revealed face of the bank appears to mark only a portion of the site it is noteworthy that its entire length demonstrates homogeneous depositional circumstances and like material. The maximum vertical span of the exposed deposits measured 1.8 m. (approximately 50 m. downstream from the western edge), whereas the minimum (sampled at the downstream margin, where a natural depression extends from the Danube inland) amounted to 0.4 m. The reduced thickness off the edge of this depression is attributable to water erosion as well as to recent ploughing. The total observations and the material remains definitely establish this locality as a settlement site. Its full extent, as well as the question of whether or not it belongs exclusively to the Neolithic Age cannot be determined without proper excavation. The material includes: Barbotine, incised, fluted, burnished, and hand-smoothed pottery; flint blades, slate
shoe-last celts, and milling stones and querns of sandstone; and fragments of wall plaster.  

2. “Obala Northeast” at Korbovo

Korbovo, the southern-most community in the region of Ključ, is situated on the bank of the Danube opposite the northwestern point of the Roumanian island of Ostrovul Corbului (Fig. 1, site number 2). The site of “Obala Northeast” lies immediately upstream from the northern limit of Korbovo. Its exposed deposits are first noticeable at a distance of about 1 km. from the boat station and extend for approximately 0.6 km. Within this stretch the height of the Danubian bank fluctuates from 2 m. to 4 m., and its surface plane rises gently inland to meet the higher elevation of the background (Pl. VIII: 3). The eroded profile contains sharply distinguishable culture pits cut through a 0.3 m. to 0.4 m. thick layer of aboriginal humus into the loess mantle and often penetrating to the sand niveau. The upper horizontal limit of the pits is surmounted by a zone of debris which terminates with the contemporary humus mantle averaging 0.2 m. in thickness. The maximum depth of the culture bearing deposits, taken vertically from the bottom of the deepest pit to the base of the top soil, measured (in 1933) 2.6 m. The pits are definitely of the habitation type as proved by the presence of fired wall plaster, their size and distribution, floors, nature of accumulations, and contents. In several instances superimposed floors, interspaced by a fill rich in sherds, ashes, etc., are visible; the side walls of these pits, as much as revealed in the exposed profile, are either vertical or gently sloping inward toward the bottom. A truly amazing abundance of ceramic remains was noted in 1933 and 1934 both in the pits and in the superimposed debris. The downstream margin of the bank terminates quite abruptly on an acute angle (Pl. VIII: 1) with which the distribution of the exposed deposits ends; there is a marked dent, about

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1 This material, not fully illustrated here, is duplicated in the corresponding categories found at “Obala Northeast,” Korbovo, and “Obala,” Kusjak; vide infra.
30 m. wide, below which the bank is again aligned with that of "Obala Northeast" and its height increases toward the village. Owing to lack of exposure we were not able to ascertain the demarcation of the site on its northern side. However, the site, again a settlement, seems to continue uninterruptedly throughout the observed distance of 0.6 km. The surface plane of the culture bearing bank is not under cultivation but is used as pasture land.\(^1\) In addition to the damage caused by seasonal floods, the site suffers considerably from illicit digging pursued by youngsters tending live stock and occasionally, it seems, also by profit-seeking pot hunters.\(^2\) While the inclined shelf, which resembles a beach, is strewn with quantities of dislodged material, the surface plane of the bank, since it is not being cultivated, gives a very poor reward to the collector. The site appears to have the thickest and (in contents) richest Neolithic deposits on the Yugoslavian bank of the lower Danube. Although an indeterminate portion of its original deposits is now gone, a systematic excavation is highly desirable. The Neolithic material, found either \textit{in situ} or in secondary position, comprises the following: The barbotine, incised, fluted, burnished, burnish-decorated, and hand-smoothed classes of pottery; ceramic human figurines with incised decoration; ceramic altars; flint blades and chips; milling stones and querns of sandstone; slate shoe-last celts; bone awls; fired wall plaster; animal bones (among which domestic cattle and pig, dog, roe deer, red deer, and sturgeon [vertebræ] have been identified by me in the field).\(^3\)

\(^1\) It is, incidentally, communal land and readily available for excavation.

\(^2\) Fortunately at least some of the material so plundered has been obtained either by the National Museum at Belgrade or, more recently, by the Negotin Museum.

\(^3\) Vasić, 1908, Fig. 11 (text pp. 99 ff.), and 1910, Pl. V: 77 (text p. 12), illustrates an axe-shaped specimen of white marble from "Korbovo"; its dimensions are given as "4.5 by 3.4 by 1.6 cm." (\textit{op. cit.}, p. 12). Vasić, 1910, pp. 9–12, recorded only one site at Korbovo, but did not detail its location. As has been indicated, there are two separate sites within the land of this village, here distinguished, respectively, as "Obala Northeast" (now under consideration) and "Obala Southwest" (to be described later). It is, therefore, rather difficult to assign the marble pendant (?) in question to either one of the two sites with dependability. It seems to me, however, that in view of the circumstance that
In addition to the Neolithic material, we also collected later remains in secondary position, but did not note any *in situ*. We recognized sherds of the Late Bronze Age (Vatin phase), as well as fragments of Roman brick, amphorae, and pyramidal weights. The site of "Smradila," which lies on the bank about 1 km. upstream from "Obala Northeast," contains Roman ruins (probably those of a castellum); immediately southward of the prominent mound at "Smradila" we noted a series of skeletal graves either in plain pits or in tile cists, plainly revealed in the profile of the bank, but without any furniture in view. While the tiles suggest Roman provenience, and perhaps an association with the military station (?), the plain graves offer no clue as to their probable period.

From Ostrovul Corbului Neolithic, Bronze Age, and Roman remains are reported.

3. "Obala" at Velesnica

The village of Velesnica lies 12.5 km. due east of Korbovo (measured on a direct aerial line). The site is situated on the bank of the Danube, partially within the village proper and

Vasić, *op. cit.*, p. 9, specifically referred to "a clearly recognizable [culture] level," our "Obala Northeast" is perhaps the more likely place. For at "Obala Southwest" the deposits are not nearly so clearly visible. Moreover, Vasić (*ibid.*) remarked that he collected his material at "Korbovo"—including, presumably, the marble piece—"on the [Danubian] strand subjected to inundation" (free translation). This also favors "Obala Northeast" where the pertinent, seasonally flooded shelf has an average width (at low water) of at least 25 m. as compared with the maximum of 8 m. (minimum zero) noted at "Obala Southwest" during the reconnaissance trips here described. It is to be remembered, of course, that the lapse of time between Professor Vasić's investigations and our first trip to this region amounts to exactly twenty-six years; in so pronounced an interval considerable erosion may have taken place and changed the condition of the bank at both sites. It is for this very reason that I abstain from venturing on a conclusive decision with respect to the marble piece. And it is worthy of note in this connection that we have found secondarily placed Neolithic material (and the marble pendant certainly is of a Neolithic type) at "Obala Southwest," Korbovo, although our search for corresponding deposits in the exposed profile of that site did not meet with positive results.

1 *I.e.* "the Žuto Brdo type" of Vasić, 1907, and 1910; cf., however, note 10, *supra*.

2 Bârcáci, 1924, pp. 283 ff.
partially below its southern limit; its vertical profile is especially prominently exposed along its southern margin. The deposits revealed in 1933 were concentrated within a distance of approximately 300 m. and the culture level with pits amounted to as much as 2.5 m. in maximum vertical span. We ascertained Neolithic material in situ only in scattered instances, but always in pits. On the other hand the surface finds from the adjacent sloping shelf comprised predominantly later remains, especially those of the Bronze Age, Iron Age (?La Tène), and the Roman and Early Slavic periods. And we also recognized Bronze Age (Vatin phase) and Roman sherds in the level surmounting the pits. The site is a settlement containing, in addition to Neolithic, later vestiges as evidenced by the mixed and complicated stratigraphic features. In certain pits suggestive of a Neolithic origin we noted intrusive later material, while other pits disclosed exclusively Bronze Age remains. Being located largely under a portion of the village, the site is not available for excavation except in its southern section. The Neolithic material consists of the barbotine, fluted, burnished, and hand-smoothed classes of pottery, flint blades and milling stones. We did not notice association of wall plaster with the apparently Neolithic pits; wherever it was seen in situ it could not be clearly assigned to a specific provenience because of complicated stratigraphic conditions and mixture of sherds reflecting two or more periods. The question must be settled by excavation. Similarly, it would be difficult to place some of the animal bones (domestic pig and cattle, wild boar, and ulna of a red (?) deer—the last named specimen judged by its size) which we recorded.

Two additional localities both on the bank of the Danube, each presumably with similar deposits and material remains, are said to be situated near Velesnica. The one situated about

1 It was first recorded by Vasić, 1910, pp. 13–14, who also excavated there in 1910 (cf. ibid., pp. 1–2); as far as I am aware no results of this excavation have been published. (Professor Vasić kindly informed me, in a personal interview during the season of 1932, that his field notes antedating the World War had been lost.)
1 km. upstream from the village is called "Jelaš"; the other, lying about 1.5 km. downstream from the village is known as "Biljevina." It was not opportune to visit either of these places during our field work, and I am at a loss to say whether they contain Neolithic remains or not. On information obtained from seemingly dependable local informants it would seem that the two localities mark rather extensive settlements and that they apparently both contain Roman relics (tile, amphorae, pithoi).

4. "Glamija-Obala" at Ljubičevac

The village of Ljubičevac is located 3.7 km. southward of Velesnica and the site lies an equal distance farther downstream opposite Grabovica Island. The term Glamija, sometimes Glameja, is a corruption of mogila, the Slavic equivalent for mound. A Roman ruin, apparently the remnant of a castellum, with a surface appearance of an oval tumulus, is still partially preserved there. Its name is Glamija and after it, it seems, the bank containing the deposits with which we are here concerned, is named "Glamija-Obala." This is again a settlement characterized by similar features as in the instances already stated; it is, however, more extensive (viewed from the river) than the other sites here detailed, for it is about 1 km. long. The deposits form a culture level with pits penetrating below it into the loess, the total vertical span ranging from 2 m. to 2.5 m. The average height of the bank itself varies from 3.5 m. to 4.5 m. In several pits we noted rather excessively fired wall plaster which seemed to indicate conflagration of daubed superstructures of dwellings. Artificially fired floors, such as occur at "Vinča,"¹ may possibly exist at this site; however, positive deductions in this regard must await excavation. The pits observed during our reconnoitering in 1933 revealed an equilateral, vertical profile, a rounded or flattened bottom, and a rather shallow depth (not exceeding 1.5 m.). While chunks of fired wall plaster with impressions of wattling were

¹ Vasić, 1932, p. 10.
quite abundant throughout the deposits, the laminated more or less horizontally laid "floors" (?) composed of a similar substance seemed to be restricted to a layer immediately overlying the pits.\(^1\) Cultural material in situ was fully in view throughout the exposed deposits. Furthermore, the shelf between the bank and the Danube, and to a lesser extent also the horizontal plane of the bank (especially its tilled portion), were fairly strewn with secondarily placed remains. In addition to the Neolithic deposits, which (in the exposed profile) seem to represent a primary settlement, Bronze Age occupation (Yatin phase) is also fully documented, and it is likewise obvious that the site was utilized (perhaps as a military post) during the period of Roman occupation. The Neolithic material includes: the barbotine, fluted, burnished, and hand-smoothed classes of pottery; flint blades, milling stones and querns of sandstone; fired wall plaster and flooring (?); and animal bones (domestic cattle and pig).

5. "Obala-Kusjak," near Prahovo

The village of Prahovo is situated on the bank of the Danube opposite the southern extremity of the Roumanian island of Ostrovul Mare. The site lies about 2 km. upstream from the railroad station of Prahovo and 300 m. downstream from the Kusjak grist mill. Its northern limit coincides with the edge of the gulley which intervenes between the Kusjak grist mill and the bank of the Danube, which at this point has a greater elevation than in any of the previously specified instances of Neolithic sites. The deposits are concentrated within the eroded bank for a distance of about 200 m. The height of the culture bearing level ranges from 2.5 m. to 3.5 m., and the richest exposed deposits are confined to the profile of a prominent depression which forms a triangular pocket in the surface plane of the bank. The deposits consist of large pits, the contours of which are clearly distinguishable

\(^1\) Provisionally, this phenomenon may perhaps be viewed as somewhat of a similar case to "Vinča," where houses with an artificially prepared and fired floor are stratigraphically later than the bothros niveau, which is characterized by simple pits; cf. Vasić, op. cit., and passim.
in the loess foundation, and debris superimposed above these, surmounted, in turn, by recent humus. In 1933 and 1934 the entire profile of the site was exposed by fresh erosion and its limits were precisely traceable in the bank. The artificial deposits were everywhere clearly revealed without a break in their continuous distribution. The sloping beach-like shelf yielded a relatively small quantity of secondarily placed material as compared with other sites. On the surface of the site no cultural evidence was noted; the ground is not under cultivation, serving merely as pasture land. The nature of the deposits and the material finds establish the site as a Neolithic settlement which seems to have remained unoccupied, subsequent to its termination, until the Roman period. The pits and the immediately superimposed layers were found to contain exclusively Neolithic remains. It was only in the highest deposits and within the humus that we noted Roman brick and sherds. The original Neolithic site was probably more or less confined to the depression in the surface of the bank (Pl. VIII: 2) for in the higher elevation above this locus, occupied by recently ploughed fields, we failed to find any cultural remains. This view is further strengthened by the pronounced concentration of the culture bearing deposits within the profile of the depression. The Neolithic material comprises: the barbotine, fluted, burnished, and hand-smoothed classes of pottery; fired wall plaster; flint blades; milling stones of sandstone; slate shoe-last celts.

Our search along the Danube bank downstream from "Obala-Kusjak" met with negative results in so far as Neolithic sites are concerned. In Prahovo proper, near the crossing of the railroad and the highway leading from the boat station to Negotin, we noted Roman deposits in the high bank of the Danube; the distance from this place to "Obala-Kusjak," measured along the immediate river bank, is roughly 1.6 km. I do not mean to imply that this course of the bank is entirely devoid of archaeological deposits; the point I wish to stress is that during our reconnoitering none were obviously exposed. This observation, it seems to me,
has a direct bearing on the following record quoted from Vasić: "Below the site at [the mouth of] Slatinska reka [Slatina brook, which joins the Danube about 15.5 km. northwest of Kusjak] the first nearest prehistoric locality on the bank of the Danube thus far known is Praovo [now called Prahovo]. The site itself lies on the Danube above present day Praovo, not far from the road Negotin-Kusjak. The bank at this place is cut [i.e. eroded] and high." ¹ From this inadequate localization a precise placement of the "prehistoric locality" in question is, of course, not attainable. The road Negotin-Kusjak runs fairly parallel with the Danube for more than 1 km. before Kusjak is reached, and its maximum distance from the river amounts to about 300 m. Beginning with "Obala-Kusjak" the bank of the Danube has an elevation of well over 6 m. (as contrasted with 1.5 m. or less in front of the grist mill called Kusjak), and its height increases to 12 m. immediately farther downstream. This level of the bank continues to and beyond Prahovo; indeed, in several instances it reaches a height of as much as 15 m., especially where sand dunes occur. It seems that Vasić's reference to the high bank, his omission to take into account the position of Kusjak itself, and above all the Late Bronze Age finds which he reported,² have to do not with the site of "Obala-Kusjak" here described, but rather with a separate locality situated, presumably, somewhere farther downstream. According to Vasić's record, as stated in the reference just quoted, the place should be allocated "not far from the road Negotin-Kusjak." Among the finds from this site, as described and illustrated by Vasić,³ there are two sherds which suggest Neolithic provenience.⁴

The site of "Obala-Kusjak" need not necessarily mark the last Neolithic settlement situated on the Danube bank

¹ Vasić, 1910, p. 14. (Free translation.)
² Vasić, 1910, p. 15, and Pl. XI: 92–95, 97 (Pl. XI: 91 a, b, and 96 illustrate Neolithic sherds).
⁴ Vasić, 1910, Pl. XI: 91 a, b, and 96. I should not hesitate to assign both of these sherds to the Neolithic Age as represented at the site of "Obala-Kusjak," or any other Neolithic settlements here described.
between "Obala" at Kostol in the west and the mouth of the Timok in the east. It is necessary to emphasize the observation, however, that beyond "Obala-Kusjak" actual deposits containing evidence *in situ* from which to determine indubitable existence of Neolithic occupation are unsubstantiated at this time. This statement, of course, refers only to the Yugoslavian portion of the lower Danubian bank, for it is well known that sites comparable to the five here detailed appear in the same bank beyond the Timok estuary (in Bulgaria); and like sites have also been ascertained on the Roumanian bank between the Iron Gate and, roughly, the town of Calafatul, as well as on several of the Danubian islands situated within the corresponding portion of the river. (*Vide infra.*)

The five sites just described have a great deal in common with respect to position, nature of deposits, form of dwellings, character of material, size (as revealed in profile), quantity of remains, and manner of erosion. Above all, they are all settlements documenting an essentially uniform type of economy.

*(B) Likely (?) Neolithic Settlements*

Under this heading are here listed localities again located on the bank of the Danube between the Iron Gate and the mouth of the Timok the exposed profiles of which, however, reveal only Late Bronze Age (and sometimes also later) deposits. Yet, in the immediate vicinity of these sites, usually at the base of the bank (on the inclined shelf), but always in a strictly secondary position, Neolithic sherds have been collected. The necessity of segregating these instances is dictated by the ice and water action of the Danube in so far as the two affect the bank, for it is conceivable that dislodged remains might be transported and transplanted. While it must be stressed that during our examination of the sloping shelf extending from the base of the bank to the edge of the river we found only a negligible amount of material attributable to the agencies just mentioned, the contingency
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involved must nevertheless be respected. When the sites themselves yield positive proof of cultural evidence in situ their deposits usually also reveal some clue as to their time placement. That was the case in the five instances thus far detailed, all of which (as far as their profiles disclose) originated as Neolithic settlements, although later occupations are also documented at all these sites. However, there are several instances of finds of Neolithic material retrieved from secondary position at localities now revealing later than Neolithic deposits. Naturally, any prognosis would be necessarily premature; the problematical issue must be dealt with through additional field work. Yet, for the sake of completeness it seems desirable to include the potential data in this treatment. In doing so we must also take into cognizance the sites reported by Vasić,¹ even though his manner of dealing with some of their material fails to be obviously convincing with respect to ceramics earlier than "the Žuto Brdo type." For Vasić described but a sampling of his finds and it is conceivable that his collection may have contained specimens of Neolithic provenience which might not be ordinarily illustrated.² We shall, for convenience, now reverse the order of our previous geographic progression and proceed from the Timok toward the Iron Gate, but still along the Danube.

(a) Radujevac

Radujevac is the first Danubian community on Yugoslav territory above the mouth of the Timok. Practically in its center, opposite the boat station, on the high bank of the Danube, are quite prominent and readily traceable ruins of an extensive Roman castellum. The site is locally called either "Gradić"³ or "Rimski Grad"⁴ and has suffered a great deal

¹Vasić, 1910, pp. 2 ff.
²Vasić, 1910, detailed only the finds illustrated in his plates II–XI by describing them in a sequence corresponding to the numbering of the individual specimens shown on the plates.
³Serbo-Croatian diminutive for fortress. (The diminutive variant is rather difficult to appreciate in view of the extensive nature of the ruins.)
⁴I.e. Roman fortress.
from extensive, and here and there indeed intensive, plundering of a clearly recent date. At this place Vasić found Late Bronze Age material, presumably in secondary position. 1 When I examined the site in 1933 only Roman deposits were revealed in the eroded bank, and very little material, again exclusively Roman, was strewn on the sloping shelf between the base of the bank and the edge of the Danube. The illustrations published by Vasić 2 do not include any Neolithic material from Radujevac. Yet, for reasons previously specified, it seems well to hold this locality in abeyance for further field work in the region.

(b) As has been stated in connection with "Obala-Kusjak," supra, Vasić reported a site "above this village not far from the Negotin-Kusjak road." 3 In his opinion it represented "another example of 'Žuto Brdo' analogies," and as such, of course, was assigned by him to the Iron Age. However, two sherds among his finds from "Praovo" certainly bespeak Neolithic provenience. 4 True, two sherds are meager evidence, but the methodology of Vasić’s report compels us to retain an open mind with respect to possible Neolithic material in any sites "of the 'Žuto Brdo' type" described therein. Since Vasić dealt merely with the material and dismissed the nature of the deposits with a few cursory remarks it is not plausible to do more than this. In any event, great caution must be exercised until actual diagnosis of the several localities is made available through further exploration. Nevertheless, it seems to me that the individual instances of Neolithic material recognizable from the illustrations and description of Vasić, but disregarded by him as such, command attention in the present writing. In this sense Prahovo ("Praovo"), be it on the basis of two sherds, justifies, in my opinion at any rate, its inclusion under localities suggestive of likely (?) Neolithic sites.

1 Vasić, 1910, pp. 15 ff.
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(c) "Ušće Slatinske Reke" 1

The Slatina brook joins the Danube about 4 km. to the north-northeast of the village bearing the same name, at a location situated opposite the southern arm of the bifurcated island of Malo Ostrvo. 2 Vasić recorded "finds of prehistoric objects of the same character" as in the other nine sites examined by him in 1907. 3 None are described or illustrated in his report, the reason being that the author "did not wish to burden the article with a very large number of illustrations." 4 It is to be assumed, then, that the material in question represented Vasić's "usual 'Žuto Brdo' type"; since, as has been stressed, this category embraces Neolithic sherds, I feel that "Ušće Slatinske Reke" must, provisionally at any rate, be mentioned among the likely (?) Neolithic sites.

(d) "Zidinje" (or "Glamija") at Vajuga

The village of Vajuga is located on the Danube at a distance of 6 km. practically due south of Kladovo. The site of "Zidinje," 5 sometimes also called "Glamija," 6 is situated on the bank of the Danube off the eastern edge of the community. Whether or not it is to be identified with Vasić's finds 7 is not possible to ascertain because Vasić referred to them as being collected within the land belonging to Vajuga, without designating any specific location. 8 And he did not

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1 I.e. mouth of the Slatina brook.
2 Malo Ostrvo (i.e. Little Island) is the Serbian name; the island, however, is a Roumanian possession. I am not certain that Ostrovul Mica, a Roumanian equivalent of the Serbian designation, is the recognized official name of this island; my attempts to verify this proved unsuccessful.
5 The name, meaning walls, is derived from the nature of the site which contains ruins of an extensive Roman castellum with the lower portion of its walls and the foundation exposed by "treasure" hunters.
6 The mound-shaped form of the surface features of the site seems to explain this alternate name; this, as has been said, is a corruption of the Slavic term mogila for mound.
7 Vasić, 1910, p. 13 (i.e. "9. Vajuga").
8 Beginning about 300 m. westward of "Zidinje" another site exists; this is known as "Obala" (Vajuga) and is also located on the Danube bank. In the exposed profile visible at the place in 1933 a deposit averaging 0.5 m. in thickness was revealed. However, the material which we obtained from this locality is
mention the Roman ruins. Vasić’s illustrations of his material from Vajuga do not include any Neolithic pieces. Our 1933 finds from “Zidinje” comprise only Late Bronze Age and Roman sherds. However, during our visit to the site in 1934 we collected additional surface pieces among which, aside from the previously noted periods, the Neolithic Age is definitely represented. Yet the deposits, well exposed by fresh erosion on both occasions, did not reveal any evidence in situ older than the Late Bronze Age. The pre-Roman deposits are probably disturbed, in part at least, by the foundation of the castellum, and much additional damage has recently been caused by vandalism. Preserved traces of pre-Roman occupation may perhaps lie buried off the mound proper. A test at the western edge of the ruins, carried inland from the river, might serve to ascertain this presumption, for it is there that Late Bronze Age material has at times been ploughed up. Until a definite conclusion can be drawn from exploration the site may be placed within our group of likely (?) Neolithic localities.

(e) “Obala Southwest” at Korbovo

This site, again, as its name suggests, located on the bank of the Danube, begins at the southern margin of the village and extends on downstream for a distance of about 300 m. After a brief interval at its lower (downstream) terminal similar deposits, in so far as structure, thickness, and remains are concerned, continue farther on toward Vajuga. The bank is considerably lower (maximum height 2.5 m.—Pl. VII: 13) than at the site above the village 2 and the culture bearing deposits are nowhere quite so thick as they are at that locality. The site was identified as a Bronze and Iron Age settlement on the basis of the material found in situ in 1933. 3

wholly either Roman or later, and the site itself is highly suggestive of a settlement contemporary with or subsequent to the existence of the castellum (?) at “Zidinje” (“Glamija”). And it is very doubtful that the material reported from Vajuga by Vasić (op. cit.) may have been collected at “Obala,” Vajuga.

1 Cf. Vasić, 1910, Pls. IX: 78–80, and X: 81–85; these represent the Late Bronze Age.


3 Fewkes, 1934, p. 36, note 23.
In view of the fact, however, that in the collection of surface remains from the sloping shelf extending from the base of the bank to the water edge, several examples of characteristic Neolithic sherds are included, it seems advisable to record this locality in the category now under treatment. It must be especially stressed that I am referring only to that uninterrupted stretch of deposits which lie partially within the village proper and partially immediately downstream from it, designated as “Obala Southwest.” It remains for future investigation to ascertain the relationship of those segments of like deposits located farther down the Danube and individually separated (in 1933 and 1934) by sterile intervals. Our observations enabled us to identify all of these sectors as belonging at least to the Late Bronze Age (and, in certain spots, also to later periods), but nowhere did we locate Neolithic remains. Including the blank spaces, the total span of the culture bearing bank, beginning with “Obala Southwest” and progressing downstream toward Vajuga, measured (in 1933) about 2 km. It seems that several settlements are present unless, of course, the intermittent sterile breaks are merely sporadic blanks in a single occupational area. It is to be remembered that we are dealing only with the exposed face of the bank. Burials are indicated by actual skeletal remains visible in the eroded profile particularly at the downstream margin of the terminal segment of the individual stretches of deposits.1 At “Obala Southwest” itself we noted several coffin-like structures of fired plaster with thin, upright walls, oval in shape (Pl. VII: 10—center foreground), some divided into halves along the long axis, and showing remains of the original roofing likewise of fired plaster. In the deposits most recently washed by the Danube waters we found two of these structures completely exposed, but empty.

1 At the locality known as “Grla,” situated on the Danube between Korbovo and Vajuga, about 4 km. from the first named village, we noted an especially prominent concentration of skeletal burials, either extended or flexed, in well distinguishable grave pits. Although we recorded Roman and later sherds on the adjoining sloping shelf, we found no traces of tile, nor furniture, actually in situ. The site is accessible to excavation with a nominal outlay and should, in my opinion, repay thorough exploration.
Definite remains of other similar instances were also visible in more protected positions, but even in these cases there were no noticeable traces of furniture or burials to give dependable data as to their original purpose. However, we were told by natives that "large human bones" had come to view in a "coffin" freshly disturbed by erosion about the year 1930. In placing this information on record merely as a matter of routine it seems well to recall the burials in "Mulden von schwach gebranten Ton" at Hallstatt.

(f) "Kurvin Grad" at Mala Vrbica

The village of Mala Vrbica lies opposite the Simianu island about 6 km. eastward of Kladovo. "Kurvin Grad" is situated approximately 2 km. upstream from the village, on the bank of the Danube. The ruins of a Roman stronghold, perhaps originally a castellum, identify the locality. These are now largely under water; in a dry season the walls are usually partially exposed, and their contours are distinguishable even when submerged. From this site Vasić recorded certain "finds of Žuto Brdo" analogies, and

1 In 1933 we examined the remains of the structure in question (Pl. VII: 12, 15) and found it to be apparently a half of the original (assumption based on comparison of its measurements with those of other similar remains still more or less well preserved). The portion still resting within the bank had not suffered from erosion although it had been mutilated by curious natives. There were no traces of either skeletal or cultural remains. Although the structure was incomplete and considerably damaged, its former dimensions could be estimated (by comparative calculation in combination with deductions based on extant vestiges) as follows: total length 1.7 m.; thickness of walls 0.05 m. (average); height of chamber near preserved end 0.3 m. (probable maximum at center perhaps as much as 0.4 m.); maximum width 1 m. The shape suggested a fairly regular oval. The plaster was thoroughly fired and was characterized by uniformity in color and texture. There were no signs of ashes or of fire crackling within the contraption, nor, for that matter, in its immediate vicinity. The firing of the plaster impressed me as an intentional process designed to attain sturdy firmness of the walled and vaulted structure. It certainly was not to be interpreted as a remnant of an oven. It would be useless to speculate about the original purpose of the interesting contrivance; the incomplete state of preservation and the recent damage preclude definite deductions. The answer to this puzzling question must be sought by excavation in the undisturbed portion of the deposits.

2 Von Sacken, 1868, pp. 6 ff.
3 The name means fortress of a prostitute.
4 Vasić, 1910, pp. 5 ff., where the locality is described as a settlement.
among the material illustrated by him, two sherds are highly suggestive of Neolithic origins,\(^1\) and I am inclined to believe that two of the several figurines illustrated in the same source are also Neolithic.\(^2\) Although these instances constitute distinctly inadequate documentation I think it proper to include "Kurvin Grad" under the localities suggestive of likely (?) Neolithic sites.

(g) "Pesak" near Kladovo

At this place, situated on the Kladovo-Korbovo road (before reaching the inundation depression, extending inland from the eastern margin of the site of "Obala" at Kostol, \textit{vide supra}), Vasić noted "a locality with 'Žuto Brdo' type of sherds and cinerated human bones."\(^3\) The author illustrated but two sherds, both of which obviously belong to the Late Bronze Age.\(^4\) The locality is occupied by a live dune which seems to have advanced and increased since 1907.\(^5\) It is here included among likely (?) Neolithic sites not only because of necessary caution with respect to Vasić’s interpretation, but also in view of the fact that the definitely Neolithic site of "Obala" at Kostol, lies but a short distance from "Pesak."

(h) "Krvava Bara" at Kladovo

About 0.4 km. south of the fortress of Svetislav (sometimes also called Feth Islam, Fet Izlam, or Fetislav) which lies on the bank of the Danube a little less than a kilometer from the center of Kladovo, brick clay is now being exploited in an area roughly 150 m. long and about 50 m. wide. The place is known as "Krvava Bara."\(^6\) During our reconnaissance in 1933 we examined the open ground and noted several culture pits, more or less mutilated by the clay cutting activities, as

\(^2\) Op. cit., Pl. IV: 18, 19 a, b; cf. note 13, supra.
\(^4\) Op. cit., Pl. III: 10 a, b, and 11 a, b.
\(^5\) The time of Professor Vasić’s survey, \textit{op. cit.;} he specifically mentioned the live dune in his report. The name \textit{Pesak} means sand.
\(^6\) I.e. bloody pool.
well as some remnants of skeletal graves. Although the cultural material recovered there is only of the Roman and later periods, we did note a very few sherds on the surface in the vicinity of the cuts, which, despite their small size and rather atypical character, suggest an earlier, possibly Neolithic, provenience. It is difficult to say whether the few fragments in question originally came from deposits at "Krvava Bara"; but it seems to me desirable to mention this locality as a possible source of Neolithic occupation.¹

Of the eight sites just enumerated six are more or less of the same nature as the definitely Neolithic localities. Their depositional conditions, allocation, and character of debris, as well as extent, show a homogeneous character. The two sites at Kladovo are rather distant from the Danube; "Krvava Bara" seems to have very shallow deposits, and "Pesak" fails to reveal any indications of its depositional nature. With the single exception of "Krvava Bara" the other sites had all been occupied in the Late Bronze Age period; and most, if not all, including "Krvava Bara," also in Roman times. It may be of interest to note, at this point, that Roman ruins are especially numerous along the bank of the Danube between the Iron Gate and the mouth of the Timok (and on into Bulgaria, of course). The work of Kanitz, however amateurish, gives a fair idea of the continuous distribution of Roman military stations in this zone.² The number of those castella and castra which I have visited between Kladovo and Radujevac exceeds a score. And in several instances, as has been stated, prehistoric deposits exist under such ruins. Where the bank is exposed, or the wall foundations disturbed, examination of the remains is readily facilitated. Elsewhere, however, the mounds are fairly intact and only their superficial features give a clue to their Roman origin, while search for earlier relics need not meet with positive results. It seems to me that a specialist in Roman

¹ In the course of our visit to the fortress which, incidentally, is quite rich in interesting Turkish antiquities, we noted Roman sherds, tile, and brick near the water front, but did not collect any positive evidence of any preceding periods. Kanitz, 1892, pp. 42 ff., and his map.
antiquities would find a very rich field in this section of Yugoslovakia and that exploration in sites the occupational history of which outside of Roman times remains obfuscous would benefit him as well as the prehistorian.

(C) Miscellaneous Sites

The Negotin Museum has a small collection of material obtained on various occasions at four localities here listed under (i)–(iv); and the Niš Museum has a sample collection from three additional sites, here listed under (v)–(vii). I have not seen any of these seven places, but have examined, at Negotin and Niš, respectively, the pertinent finds from (i)–(iv), (vi), and (vii). A brief note has been published on (v).¹

(i). “Grad” at Miroč; two shaft-hole axes of (?) copper.
(ii). “Pisk” at Glogovica; several slate (and allied material) celts of the shoe-last type and forms derived therefrom, as well as two perforated axes of quartzitic stone.
(iii). “Gložar” at Negotin; apparently Neolithic sherds.
(iv). “Grad” at Tabakovac; apparently Neolithic sherds.
(v). “Glabarova Glavica” at Klenovac; apparently Neolithic sherds.²
(vi). “Strnjane” at Osmakovo, drainage area of the upper Timok; various Neolithic material (incised, barbotine, fluted, hand smoothed undecorated wares, altars, weights, one celt, one abrading stone) previously described by me.³

¹ Bogdanović and Milenović, 1936.
² Cf. Bogdanović and Milenović, 1936, pp. 13–14; surface finds, reported by amateurs, material in Niš Museum (op. cit., p. 14).
³ Fewkes, 1936, p. 61. I included this site, and “Tumba” at Kalna, vide infra, in the Moravo-Danubian area to which the study just cited was devoted along the lines of the present paper. This was done simply because both localities fall within a geographic locus which embraces parts of both the Morava system and the lower Danubian valley. The two sites are here mentioned more for completeness than by necessity of altering their aforementioned geographical placement. As it happens the material from both is equally characteristic of either periphery of the two drainages (Morava and Timok). It is impossible to draw sharp lines of demarcation against distributional evidence.
(vii). "Tumba" at Kalna, drainage area of the upper Timok; Neolithic fluted, burnished, and hand-smoothed (plain) wares previously reported by me.¹

None of these seven sites have, as far as I am aware, been investigated in detail; it is not possible, therefore, to present any data respecting their deposits. The material was gathered in surface collecting. "Grad," Miroč, "Pisk," Glogovica, "Gložar," Negotin, and "Grad," Tabakovac, contain Roman and perhaps also later ruins. While some of the chance finds must of necessity be viewed with reserve—especially those of (i)–(v)—the abundance and variety of perfectly obvious Neolithic relics at "Strnjane," Osmakovo, and at "Tumba," Kalna, both of which, incidentally, have been examined by the staff of the Niš Museum, warrant their acceptance as Neolithic sites. The two seem to be rather extensive settlements and the material found upon their surface planes had been turned up in the process of cultivation.² They really belong in the category of definitely Neolithic settlements and are included in the group now under discussion for reasons already clarified.

As has been stated, the interior portion of the Yugoslavian possession in the lower Danubian valley is very little known from the standpoint of prehistoric archaeology. A large number of sundry sites exists more or less throughout the territory, each with a specific name, and, in many instances of masonry ruins, also with some local tradition, usually claimed to be a Roman survival. In the latter case finds of coins, glass, slag, bronze objects, "huge pots" (pithoi?), inscribed tiles, etc., are said to have been gathered by natives and at times by outsiders.³ Some such material has been preserved,⁴ but more often it seems to have disappeared without trace. Roman occupation appears to have been

¹ Fewkes, 1936, p. 62.
³ Much is said to have been carried away from the country during the World War while Serbia was under military occupation.
⁴ The Negotin Museum has salvaged several splendid sculptures and a very important collection of Barbaric coins.
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quite general not only along the Danube but deep in the interior as well, and penetrated into the mountainous district evidently in quest of mineral wealth. And it is interesting to note that in some of the mines exploited by the Romans, prehistoric finds are occasionally gathered. How far inland may have reached the various cultural predecessors of the Romans is far from clear at present. It seems advisable, therefore, to record all indications which may lead to a better understanding of this matter in the future.

Description of the Neolithic Material

The homogeneity of the five definitely Neolithic settlements, as revealed in their depositional nature, is further reflected in the relics which they contain. The two marginal settlements ("Strnjane," Osmakovo and "Tumba," Kalna) have yielded precisely the same type of relics as we found in the Danubian loci. Moreover, the material thus far gathered in those places which comprise our category of likely (?) Neolithic sites, as well as certain chance collections, also show comparable characteristics. The total, therefore, may be described collectively. The various classes of ceramics here presented are distinguished along the following qualitative characteristics: surface finish, texture, decoration or embellishment, shape, and color. While it would be possible to arrange a typological seriation this would necessarily be quite premature in view of the complete lack of stratigraphic data definitely established by excavation. Even the significant observation at "Obala-Kusjak," whereby the barbotine class of pottery is indicated as the oldest, must be taken only provisionally despite the seemingly clearly identified case of superimposition of layers. Although the stratigraphic order noted in this particular instance is indeed significant, its precise understanding and interpretation must not be used conclusively without excavating the site and proving its depositional history. We must also point out that the material here specifically treated represents approximately the following proportions: "Obala Northeast," Korbovo, circa
forty-five per cent, "Obala-Kusjak," Prahovo, about the same, and the remaining instances the rest. Nevertheless, even the smallest lots conform to the general character as exemplified by the two large groups. In the classificational arrangement some overlapping cannot very well be avoided, but on the whole the several categories, as they are here stipulated, reflect fairly sharply defined qualitative differences.  

1 This excludes the material from "Strnjane," Osmakovo, and "Tumba," Kalna.

2 It is not a simple matter to classify pottery on a satisfactory rational basis, because the various attributes to be taken into consideration are each highly variable and do not necessarily conform to a coefficient of a stable value. While qualitative aspects provide a dependable means with which to attempt a classificational grouping, they fail to constitute anything like a prescribed rule insofar as their specific application is concerned. In the present work, the individual qualitative characteristics are not used in a constant combination, but rather in conformity with the needs of each individual class. We shall see, then, that surface finish by burnishing (and the manner of surfacing is one of our guiding criteria of distinction) is common to the burnished ware as a class, and also to burnish-decorated pieces, as well as to fluted, incised, and even to barbotine pottery. I should like to explain, at this time, the following technological terms here utilized in connection with pottery description. Fabric refers to the finished, fired product, and is judged by texture (that is to say the measurable size of the granular or platy particles as revealed in the cross section between the outer and inner surfaces of a given sample), inclusions (i.e. the so-called tempering medium), and the nature of firing. It seems to me exceedingly difficult to talk of the original clay, its manipulation in preparation of the paste, and the general treatment preceding construction, because of the altering subsequently occasioned by the firing. Without adequate positive data from which to reconstruct the various steps of the potter's procedure, either theoretical or empirical attempts at an explanation cannot claim serious recognition. Building connotes the manner of constructional procedure wherever extant criteria permit its identification. With respect to what I call the segmental process of building, it should be said that this method comprises, inter alia, true coiling, in which spirally wound fillets of paste are diagnostic, and the circuit (or loop) variant, in which each individual fillet equals the circumference of a given vessel at a given height of its body. Both of these variants are amply documented in the barbotine class here described. The colors of surfaces are given in the terms (but not symbols) of Ridgway, 1912. In order to preclude misunderstanding, and to forestall possible difficulties in translation in the case of the reader who has no access to Ridgway's book, I am including, in foot notes, rough synonyms of the less self-evident color values; obviously, these synonyms are not equivalent to Ridgway's plates. I am led to this step by certain reactions on the part of some of my European colleagues who, without access to Ridgway's book, experienced some difficulty in their attempts to comprehend the color descriptions, after Ridgway, used in Fewkes, 1936, pp. 27 (there note 111) ff. Despite this predicament I am convinced of the general utility of basing color description on matching with a recognized scale. In any event, I am giving only basic colors and their respective ranges.
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(I) Ceramics.

(A) Pottery.

(1) The barbotine class.¹

This, the most constantly-recurring pottery in the sites here described, is perhaps to be understood as the chief domestic ware.² It contains organic (chaff and grasses) and inorganic (small particles of sand grit) inclusions. The basic colors range from vinaceous buff ³ to warm sepia ⁴ on the exterior and from orange cinnamon to fuscous ⁵ on the interior. The characteristics just stated are quite typical of the

¹ The term barbotine is here employed in exactly the same sense as it was applied to the typical material from “Grad,” Starévo; cf. Fewkes, Goldman, Ehrich, 1933, pp. 43 ff. However, its original utilization in Yugoslavia is to be credited to Professor Vasić, who first used it in describing the same class of pottery which he found chiefly in the bothros level of “Vinča”; cf. Vasić, 1932, pp. 82, 90–91, and Pl. XXIX: 132 a, 132 b, and 133. Cf. also Fewkes, 1936, pp. 27 (there note 190) ff. It is interesting to note that in his latest publication on “Vinča,” Vasić, 1936 b, pp. 7 ff., Pl. I, places the barbotine ware at the head of his sundry categories of pottery. However, he still, and more emphatically than ever, insists on his incredible obsession that the site was originally founded at about the break from the seventh to the sixth centuries B.C. The reader cannot help but wonder how and why the excavator of “Vinča” which, in its unusually instructive stratigraphy and in its rich, diversified material, has no equal on the Danube, could have been so grossly misled. Having devoted more than one quarter of a century to the study of “Vinča,” Professor Vasić comes to the amazing conclusion that the settlement was established at a date actually succeeding the period which, as he himself originally stated (1905, pp. 262–263), and subsequently repeated (cf. Fewkes, 1935, pp. 651–652), had witnessed its termination! In his personal attack on my recent paper (Fewkes, 1936, especially pp. 19 ff.), Vasić, 1936 b, pp. iv–v, obviously reveals his failure to comprehend and appreciate my objectives and motivation which I expressly specified (Fewkes, 1936, p. 40, note 237). “The results of the excavations at Vinča,” we are told by Vasić, 1936 b, p. v, “brought forth, successively, more and more proof against the customary ‘scholastic’ theories, and particularly against the dating of Vinča into the Neolithic Age. It was, therefore, only natural that the new data caused an evolution in my dating of the establishment of Vinča. . . .” It is a well known fact that Vasić, 1932, 1936, 1936 a, and 1936 b, has utterly failed to furnish anything like convincing proof of his palpably absurd contentions; cf. Minns, 1933, and 1937. And the reader may recall Vasić’s publication on “Gradac,” 1911, and its review by Menghin, 1913; cf. Fewkes, 1936, pp. 54–55.

² Its character, depositional appearance, and geographic distribution (in lower Danubian Yugoslavia, at any rate) are conducive to a statement that this class of pottery is very likely to be viewed as the chief ware of the primary Neolithic settlers in this region.

³ Pinkish-buff.

⁴ Sepia-brown.

⁵ Dark brownish-gray.
Fig. F, G, and H: Field sketches, approximately four fifths of actual size, of sample material from “Obala-Kusjak,” near Prahovo.
class as a whole, irrespective of surface treatment. However, on the basis of surfacing the following subdivisions are recognizable:

(a) Positive applique.

This category comprises several variants which have a common bond in the manner of their surface finish. The potter added to the outside of the wall a quantity of paste, the constituent quality of which did not differ from that of the body of the plastic vessel, and worked over it to produce either a positive or a negative relief. Consequently, we distinguish the following distinctions in the ware so treated.

(i) Streaking.

This treatment was accomplished either purely manually, and then we speak of finger-stroked streaking, or with the aid of a tool, such as a stick of wood, a spatula (wood, bone), etc; we may call the latter practice tool-treated streaking, always remembering, however, that the paste medium with which the applique was affected was added by hand. Reconstructively —according to inductive tests—this peculiar manner of surfacing may be described as follows. Over a handsmoothed surface the potter applied a thin coating of paste so as to cover the entire area to be subjected to barbotine treatment. Then, by allowing additional similar paste to ooze out between her fingers as she pressed a small quantity held in hand, she superposed, upon the coating, a series of ridge-like formations in positive relief. These she subsequently bonded with the surface coating either by finger or tool stroking (Pl. I: 1–4, 8, 9, 11, 14, 20, 23, Pl. II: 7, 9, 10, 13, Pl. VI: 5, 6, and Fig. G: 1–3). The marks left by the stroking process are as a rule clearly distinguishable megascopically; in
certain border cases microscopic examination must be made to determine the technique.

(ii) Dimpled and/or pinched treatment.

A finger tip, finger nail, or sometimes a tool were used to affect this manner of barbotine surfacing, again upon a previously coated surface. Additional application of paste to the coated area was often, but not always, accomplished (Pl. I: 10, 12, 13, 16, 21, Pl. VI: 1, Fig. G: 4, and Fig. H: 1, 4).

(iii) Lumpy treatment.

This consists of adding, upon a coated surface, a series of dabs of paste, either hand shaped or allowed to ooze out of the hand and to drop on the wall; in either case subsequent adjustment depended on a purely manual manipulation. The lumps are rather irregular in size and shape. (Pl. I: 5, 22, Pl. VI: 19.) At times the arrangement depends on aligned applied warts in combination with incision (Fig. G: 5).

(iv) Applied ridge (or band).

In this category the positive applique is added over a previously coated surface in such manner that the relief stands out as a strip, band, or ridge; the surface plane of such an addition is often embellished by indentations (finger tip, finger nail, tool) or by positive relief (Pl. I: 6, 7, Pl. III: 10, 12, 18, Pl. VI: 2, and Fig. H: 2, 5).

(v) Scattered applique.

When the positive application of the barbotine treatment is done in such manner that a conscious effort at arrangement along the lines of a streaked effect is absent, we speak of scattered applique (Pl. V: 1). The finishing technique is essentially the same as in streaked embellishment.
(vi) Rosette.

Employing the principle of an applied ridge the rosette is hand-modeled into the shape of a disc or oval and then embellished by dimpling (Pl. III: 8, and Fig. A: 2). At times additional appendages radiate from the body of the rosette; however, this particular feature is not a lug irrespective of its size and shape.

(vii) Miscellaneous applique.

This includes such arrangement as the face-like effect shown on Pl. III: 9, and combinations of two or more of the variants previously described.

(viii) Incised and grooved treatment.

Incising is done with a sharp tool, presumably the sharp edge of a cutting implement. Grooving, on the other hand, is dependent upon a blunted tool. In cross section the furrow of such tools is V-shaped (or at least semi-V-shaped) in incising and either semi-circular or semi-oval in grooving. In the barbotine class of ware such embellishment is invariably executed over a paste-coated surface. Another readily-distinguishing criterion which contrasts it with the incised ware of the "Bandkeramik" style is the pronounced crudeness of its technique. As far as I am aware, moreover, the barbotine incised and grooved embellishment runs exclusively along rectilinear patterns. The examples here illustrated (Pl. I: 15, 17-19, and Fig. F: 1, 2) are quite characteristic.

(ix) Plain barbotine ware.

The readily distinguishable qualitative characteristics of the surface coating—the "leading fossil"—of the barbotine ware enables us to segregate its plain, that is to say unembellished, variant. The manner of surfacing ranges from
"rough," i.e. either non-hand-smoothed or brushed (apparently with some vegetable fiber medium), to hand smoothed (identified by finger striations), and even to burnished (with a tool, of course). In any case the surface coating always precedes the surfacing proper.¹

The following illustrations represent plain barbotine ware: Pl. II: 1–6, 8, 11, 12, 14, 15, 19, 20; Pl. V: 4, 22; Pl. VI: 3, 4, 7–14, 16, 23.

With respect to forms of the barbotine ware—speaking of the class as a whole irrespective of surfacing—it is not yet possible to present a conclusive deduction. The profiles here shown (Fig. C: 1–3, 7, 9, 12, 20–23, 25, 27, Fig. E: 6, 9, 13–17, Fig. F: 1, 2, 5–8, 14, 15, 17, 20, Fig. G: 1–4, and Fig. H: 1–10), either rim or bottom fragments, bespeak essentially globular shapes, more or less ablated, sometimes profilated to assume a distinct shoulder, or ovate and even conical shapes. The base is either rounded, slightly curved, or flat; pieces with a flat base often have a short, cup-like pedestal, or a considerably thickened bottom. The latter instances are invariably accompanied by a peculiar feature which seems to be a prominent criterion of the ware as a class. This consists of a composite process wherein the actual bottom of the cup-base or thickened-base ware is made by bonding two disc-like pieces of paste, one applied from within the vessel and the other exteriorly, subsequent to the erection of at least the lower portion of the wall. With a cup base vessel such an operation should have involved a more delicate procedure because of the height of the pedestal which often exceeds 0.05 m. In flat bottom pieces, on the other hand, the manipulation may

¹ At "Grad," Starčevo, a chronological and typological sequence leading from unsmoothed to smoothed and to burnished surfacing is quite obvious. In the region under discussion, however, the material is yet inadequate to warrant a similar deduction despite the suggestive correlates thus far observed.
have been relatively simple. In either case the embrionic, plastic tectomorph had an opening at its body base which was not closed until the composite bottom was constructed in the manner described. Obviously, the traces of the process are most readily revealed in a fresh section of the fabric (Pl. III: 7), and often also superficially by the bonding joints. (A binocular microscope is especially useful in examining the pieces for criteria of constructional methods.)

The surface coating is a diagnostic characteristic of all barbotine pottery; it resembles, especially when quite thin, a slip. But the coating is not qualitatively different from the core of the fabric, although there is an obvious contrast in color which is attributable to the firing. I am unable to state, as this paper goes to the press, whether or not there are any instances of qualitative differences between the coatings and the cores. Until adequate technological studies are completed it is not possible to be conclusive in this matter. The thin-sections of a representative series of samples selected from the barbotine ware from "Grad," Starčevo (vide Horton's Appendix, infra), all of which find duplication amid the material under discussion, uphold the results of my preliminary examination of the material from the lower Danubian sites in Yugoslavia.¹

Extant criteria of the manufacturing technique

¹ Through the courtesy of my colleague Mr. Donald Horton, which I hereby gratefully acknowledge, it becomes possible, on this occasion, to incorporate here the accompanying Appendix dealing with certain microscopic studies of the barbotine samples from "Grad," Starčevo. Macroscopically, the lower Danubian material reveals no substantial differences from the Starčevo series. It is conceivable that in any of the several loci concerned, virtually, if not fully, identical raw materials may have been available. Experiments with sample clays from Starčevo strongly suggest that a combination of loess and top soil formed the basis of the Neolithic potter's paste for the barbotine ware. However, further technological studies as well as inductions are necessary in order to arrive at a conclusive identification of the raw materials employed at the site during its Neolithic existence.
reveal that the barbotine ware was made by segmental building which depended on rolled fillets of paste. The circuit variant seems to have been more in vogue than true coiling (vide note 84, supra), although the latter is amply documented. The separately filled bottoms and the small vessels were produced by hand modeling (direct shaping).

The barbotine class of ware has been ascertained in every one of the definitely Neolithic sites here described (reconnaissance of the American Expeditions and the Negotin Museum), but not, as far as I am aware, in any of the likely (?) Neolithic sites. It also appears at "Strnjane," Osmakovo (Niš Museum—see Miscellaneous Sites, supra).

(2) The incised class.

This, the most sparsely represented class in the sites here considered, falls within the unsatisfactory but well rooted concept of "Bandkeramik"; or perhaps spiral-meandric Danubian. Its decoration embraces well conceived, often complicated designs, executed, as a rule, in fine technique; and it stands out in sharp contrast to the crudely incised barbotine pieces. A further pronounced difference between the two categories lies in qualitative properties. The incised class has a finer fabric (texture), a higher degree of compactness, and is almost exclusively burnished. Its inclusions consist of a greater proportion of mica (which may well be a constituent of the original clay), sand grit (and other minerals yet to be identified by petrographic studies), but rarely chaff. There is no unambiguous proof of a slip. However, in certain instances the mechanical agency of pebble (?) or other tool-burnishing produced a surface "film" which is clearly differentiated from the core of the fabric. Surfaces so affected are indubitably recognizable by accellerated compactness of their texture, the marks left by the tool, and—in
section—by their thickness. However, the color of the fabric is entirely independent of the manner of surfacing.

The incised ware is often excessively carbonized; certain examples bespeak a fuel-smoothed firing process. While a superficial observation suggests a higher degree of heat than in the barbotine class, it is necessary to defer conclusions regarding this aspect until technological studies have been completed.

The basic surface color ranges from ivory yellow to sepia or from smoke gray to deep smoke gray on both surfaces.

The decoration depends on an incising technique executed with sharp tools which may have been made of stone, bone, or wood. The examples thus far noted in our area show both curvilinear and rectilinear motifs. The complete vessel from "Obala Northeast" at Korbovo (Pl. V:15, Fig. B: 3) \(^1\) illustrates both motifs. Its multiple-line spiraliform produces six horizontally placed S-spirals, each defined by a band, and containing either one or two additional lines within the band, roughly paralleling the curvature of the main figure. The individual spirals are mutually interlocked within their bends. This design occupies a greater portion of the spheroid body of the vessel. At the base are four horizontal, more or less parallel lines surmounted by band chevrons at four equidistant places. Above the spirals, upon the shoulder, runs a series of low joined arches consisting of three parallel, unbroken lines. Immediately below the junction of the shoulder and neck are four horizontal, parallel lines, under which, appended to the lower marginal line, hang band chevrons placed opposite to similar (but reversed) figures at the base. The horizontal plane of the squarish

\(^1\) Chance find, purchased from a peasant with reliable information as to its provenience; deposited in Negotin Museum.
platform is decorated by five groups of parallel, straight lines, comprising either quadruple or quintuple arrangement (Fig. B: 3).

In a specimen from “Zidinje” (“Glamija”) at Vajuga (one sherd, not illustrated here) the preserved portion of the design reveals a rectilinear motif arranged in bands and a single line convolution. The human figurines from “Obala Northeast,” Korbovo, see infra, also illustrate the nature of incised decoration.

Band chevrons filled with dimples appear on a sherd reported by Vasić as “from Korbovo.”

The forms of the incised ware are chiefly globular. The intact specimen from “Obala Northeast,” Korbovo, may perhaps be described as a cruse or a bottle shape. It has a peculiar horizontal platform with a circular orifice in its center. The neck is gently profilated, and meets gradually with the shoulder which, in turn, continues to broaden out to produce the belly. The bottom is flat. Otherwise the sherds suggest only spherical and semi-spherical shapes.

No observations are yet available insofar as rims, lug-handles, or handles of this class are concerned. In one rather questionable example from “Obala Northeast,” Korbovo, a cork-shaped lug is fairly well, although not absolutely conclusively, indicated on a damaged sherd. And there are no dependable signs with which to judge the manufacturing technique of the incised ware.

In distribution the incised ware is represented at “Obala Northeast,” Korbovo, “Obala,” Kostol (both definitely Neolithic sites), and at “Zidinje” (“Glamija”), Vajuga (a likely (?) Neolithic site). In the upper Timok drainage it appears at “Strnjane,” Osmakovo (vide supra, Miscellaneous Sites).

(3) Fluted and ribbed ware.

The fabric of this class closely assimilates that of the incised pottery. The basic colors range from drab to olivaceous black on the outside and from pale neutral gray to dark neutral gray on the inside.

The decoration comprises rather shallow, broad fluting executed either with the tip of a finger or with a blunt tool. The ribbing produces positive applique when additional strips of paste are bonded with the surface; otherwise the relief is negative, being due wholly to the fluting, in which the surface plane of the ribbing does not protrude above that of the surrounding area of the wall. In all cases thus far observed the decorative motifs run on a straight line, forming either parallel arrangements or triangular figures (Fig. A: 5, 7, and Pl. IV: 2).

The most common form is a bowl with a differentiated, vertically drawn neck, a gently rounded shoulder, and either an ovate or conical body. In cross section of the shoulder the thickness of the wall within the acute profile ("break") is invariably greater than that of the rest of the body. This is an outstanding characteristic of the fluted and ribbed ware throughout the Danubian area. As yet, however, the angular shoulder profile, so prevalent in the Moravo-Danubian area,\(^1\) has not been noted in the region under discussion. On the basis of the peculiar shapes common to the fluted and ribbed ware (Fig. C: 4–6, 14–17, and Fig. F: 18) it is usually quite simple to recognize sherds of this category even when they are without decoration (Pl. IV: 4, 5, 7–12, 14, 16–18, 23).

The bottom is invariably flat and the rim is either rounded or flattened and thinned or thickened, but only rarely fashioned into a distinct lip.

No lugs have been observed in this category thus

\(^1\) Cf. Fewkes, 1936, pp. 30 ff.
far. There are, however, a few instances of small band handles attached on the shoulder.

There were no dependable traces of manufacturing technique among the pieces which I have examined.

The fluted and ribbed ware has been found at "Obala Northeast," Korbovo, "Obala," Velesnica, and "Obala," Kostol—all on the Yugoslav bank of the lower Danube—and at "Strnjane," Osmakovo, and "Tumba," Kalna, both in the upper Timok drainage (reconnaissance of the Niš Museum).

(4) Burnished ware.

This category includes pottery surfaced by tool-compacting liberally executed more or less over the entire wall of the vessel, predominantly on the outside, but often also on the interior. Although a varying degree of luster is usually present this need not necessarily be due to the technique involved, but rather to a separate agency (polishing). Provisionally three grades of burnishing may be recognized: low, medium, and high. These are distinguished by surface appearance, that is to say by the marks left by the tool, as well as by the degree of compactness, but they are independent of luster which itself can be grouped into at least three categories. The burnishing process consists of rubbing the plastic (preferably, although not absolutely necessarily so, moistened) surface with a hard medium (bone, stone, wood) in such a manner as to compact the paste so affected to a higher degree then can possibly be done merely with hands. It is the pressure, rather than friction, that achieves the desired result. A moist surface is, quite naturally, best conducive to such procedure; and if burnishing follows tectonic operation before air drying has advanced, the potter need not be obliged to wet the surface in order to "smooth" it with a tool.

The fabric of the burnished ware is of fine texture;
mica (constituent?), sand grit, and other minerals are represented among the inclusions. The basic color ranges from pale smoke gray to chaetura black or from drab gray to fuscous on either surface.

In a meretricious sense burnishing may be viewed as having a certain esthetic value. Combined with mottling and luster it certainly produces a striking effect. It is often the manner of surfacing alone that distinguishes the burnished class from other wares of similar shapes and fabric.

The forms are globular jars, conical or ovoid bowls, low casseroles, and cups. All have a flat bottom, and the rims are either rounded, flattened, or slightly drawn out and everted. (Pl. III: 1–7, 15, 17, 19, 20; Pl. IV: 6, 13, 15, 19–21; Pl. V: 8, 13, 14, 16, 17, 19, 20, 23; Pl. VI: 15, 20, 21; Fig. C: 8, 10–13, 18, 19, 24, 26, 28–30; Fig. E: 12; and Fig. F: 3, 4, 9–13, 19, 21, 22.)

Plain lugs, either rounded or cork-shaped, sometimes with a depressed margin or slightly turned upward (Pl. V: 17, Fig. E: 1–5, 7, 8), are common. Handles, roll or band varieties, are appended either wholly on the body or looped from shoulder to rim, and often have a wart-like protuberance surmounting the horizontal plane (Fig. D: 1–5, 8, 9). There are several examples of spout lugs (Pl. IV: 15, 19–21, Fig. D: 11, 12).

The pedestalled pieces have a foot constructed and shaped in the same manner as already described under the barbotine category. The burnished pedestals, however, present a greater variety of form and are often much more slender (Pl. III: 1–7, Fig. F: 11–13, 19, 21, 22).

As far as can be ascertained from extant traces, two techniques of manufacture are manifested: (1) Direct shaping by hand modeling; (2) segmental process depending on a fillet, and comprising both spiral coiling and circuit building.
In distribution the burnished ware has a wide representation. It appears at every one of the five definitely Neolithic sites here treated, as well as at "Tumba," Kalna (upper Timok), and at our likely (?) Neolithic sites of "Zidinje" ("Glamija"), Vajuga, "Obala Southwest," Korbovo, and "Kurvin Grad," Mala Vrbica.

(5) Burnish-decorated ware.

There are only a few sherds of this class among the material I have thus far examined. They are all of the same color ranges as the burnished ware, insofar as the ground is concerned; the decoration proper, however, is predominately blackish mouse gray—thereby contrasting sharply with the rest of the surface. The fabric and inclusions are of the same nature as in the burnished ware.

The decoration consists of lustrous, straight line or arched strips arranged in parallel groups and localized on the belly of the vessel in a vertical order. The ground of the wall is either lightly burnished or simply hand-smoothed so that the contrast between the two treatments is indeed conspicuous. Inasmuch as the difference in color between the decorated and undecorated zones is so pronounced, it appears—on megascopic examination, at any rate—that a coloring medium was employed in connection with the burnish-decorating technique. Until a dependable series of samples is available for the necessary qualitative analysis, no conclusive technological deductions can be made.

There are no restorable shapes, neither bottom nor rim pieces. Likewise, lugs and handles have not yet been noted.

And there are no dependable criteria of the technique of construction, although true coiling is feably suggested in one sherd.

The burnish-decorated pieces here described and
figured (Pl. IV: 3, Fig. A: 1, 3, 4, 6) come from "Obala Northeast," Korbovo—a definitely Neolithic site.

(6) Hand-smoothed ware.

Under this category are included sherds which cannot be placed in any of the previously discussed groups. It is likely that some of the material may represent distinct categories, but at the moment there are no adequate grounds for any specific classificational subdivisions.

The fabrics of the hand-smoothed ware are rather coarser than those of the embellished or decorated classes. The inclusions vary from rough pieces of sundry minerals to fine sand grit, and from chopped straw to chaff. Mica (constituent?) is present in many instances.

The basic surface-color ranges from drab gray to fuscous, or from palid mouse gray to blackish mouse gray.

There is, as a rule, no embellishment, the surfacing being confined to smoothing by plain hand (identified by striations) which may have been incidental to the shaping process. In two instances we have basket impression appearing on the outside plane of bottom pieces (Pl. V: 18, 21).

The shapes include broad bowls of a conical form, large vessels of the pithos type, cylindrical cups, and heavy plates. Flat bottoms predominate in all but the pithos-like pieces where they are oval; the rims are either rounded or flattened. Plain lugs and roll handles are common. In short, the hand-smoothed group represents somewhat of a medley of features, rather than any outstandingly characteristic (save the surfacing) distinctions (Pl. II: 16–18; Pl. III: 11, 13, 14, 16; Pl. IV: 1; Pl. V: 3, 6; Pl. VI: 18, 22; Fig. D: 6, 7, 10; Fig. E: 10, 11; and Fig. F: 16).

The manufacturing technique, the traces of which
are often very well preserved, depended either on direct shaping by modeling or on spirally carried coiling in which rolled fillets were employed, being connected by overlapping.

Distribution: All of the five definitely Neolithic sites; “Strnjane,” Osmakovo, and “Tumba,” Kalna (both upper Timok drainage—material in Niš Museum); “Gložar,” Negotin (material in Negotin Museum), and “Grad,” Tabakovac (material in Negotin Museum).

(B) Figurines.

Among the human figurines we have several head fragments and torsos, all from “Obala Northeast,” Korbovo. The torsos have a flattened body, short, stump-like arms, and a broad neck (Pl. V: 2, Fig. B: 4, Pl. V: 9). The heads are of two varieties. One has a stylized form, flattened on the top and back, and with very little facial modeling (Pl. V: 5, Fig. B: 2), while the other represents a more realistic, though rather crude, product with a rounded face, depressed eyes, and modeled nose and chin (Pl. V: 10).

The decoration of these figurines consists of rectilinear and curvilinear incising executed in the manner of the incised ware. Meandric and spiraliform motifs predominate. The nature of the decorative features (as well as color, fabric, and degree of firing) certainly bespeak close relationship between the figurines and the incised class of ware.

The two incomplete torsos from “Kurvin Grad,” Mala Vrbica, figured by Vasić ¹ suggest Neolithic provenience. The one with short, stumpy arms ² is especially similar in shape to our specimen (Pl. V: 9) illustrating this type.

There is only one zoomorphic figurine from the region

¹ Vasić, 1910, Pl. IV: 18 and 19 a, b; text pp. 6 and 7, where the author considers both specimens as examples of “extreme degeneration of the usual ‘Žuto Brdo’ type of figurine.”
² Ibid., Pl. IV: 18.
under discussion, also from "Obala Northeast," Korbovo. This is a seated specimen with a well modeled head, the face of which is drawn out into a snout, and with outstretched stumpy legs (Pl. V: 7, Fig. B: 1), without decoration.

As far as can be observed from revealed criteria, the figurines appear to have been made by plain hand modeling and out of one lump of paste. Joining of two separate halves, so common in the Moravo-Danubian area, has not been observed thus far. In surface color, texture of the fabric, and nature of firing, the figurines assimilate the incised class of ware.

(C) Altars (or tables).

These specimens are of the tri- or quadri-footed variety with a triangular or rectangular platform which is either planed or somewhat depressed. The decoration consists of incised, rectilinear design. The legs are either triangular or rectangular in horizontal cross section (Pl. V: 11, 12; Fig. A: 8).

In manufacturing technique, nature of firing, fabric, and general appearance the alters (tables) closely approach the figurines; in other words they also fall in the category of incised ware.

(D) Spindle whorls and weights.

Biconical spindle whorls and discoidal weights are represented by a few fragments from "Obala Northeast," Korbovo. These do not allow any deductions except to say that in fabric and shape they conform to the usual Moravo-Danubian specimens of the kind.

(II) Stone artifacts.

(1) The axe-shaped specimen of marble from Vasić's "Korbovo," presumably an amulet, may have been found at the site of "Obala Northeast," Korbovo. That is to say it would readily fall within the Neolithic contexts of that site whereas it would be difficult, at the present, to associate it with "Obala Southwest,"

\^ Vasić, 1910, Pl. V: 77, text p. 12; also Vasić, 1908, Fig. 11.
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Korbovo. The specimen has several very close analogies at "Vinča." ¹

(2) The celts, invariably of slate, shale or allied material, are either of the shoe-last type or trapezoidal in shape (Pl. IV: 22, Fig. E: 18–21). And there are modified forms derived from these two, most characteristic forms. The celts have been noted at every one of the five definitely Neolithic sites, at "Pisk," Glogovica (material in Negotin Museum), and also at "Strnjane," Osmakovo (upper Timok—material at Niš Museum).

(3) Knife blades (some possibly scrapers?) of flint and related hard stone (Fig. F: 23, 24), are common to all of the five definitely Neolithic sites.

(4) Milling stones and querns of limestone and sandstone, roughly rectangular in shape, are common to all definitely Neolithic sites. I speak of those which we actually observed in situ within wholly Neolithic deposits. Otherwise it would be difficult to perceive the relative date of such pieces.

(III) Bone artifacts.

With the single exception of "Obala Northeast," Korbovo, where we found two bone awls in situ in Neolithic deposits, this category of material, although by no means infrequent, cannot be safely assigned a relative date. The same may be said of stray fragments of red deer and roe deer antler pieces with traces of human workmanship which we noted at "Obala Northeast," Korbovo, and at "Obala Kusjak," Prahovo.

(IV) Miscellaneous.

(a) Wall plaster.

Fired fragments of wall plaster, with twig and post impressions, rather coarse in texture and very unevenly fired, have been observed in situ at every one of the five definitely Neolithic sites. The inclusions consist

¹ Cf. Vasić, 1908, pp. 99 ff.; also unpublished specimens in the University Museum and the National Museum, both at Belgrade.
either of minerals or organic admixtures such as chaff and other vegetal material. It should be possible, with a proper technique, to pursue botanical identification of the flora represented therein. The plaster offers direct evidence on the manner of constructing dwellings; daubed wall, over a wattled frame work, supported by heavy posts, seems to have been most prevalent.

The seeming appearance of artificially fired floors suggestive of "Vinča" parallels in this peculiarity is, as already stipulated, not yet to be interpreted conclusively.

(b) Metals.

There are no traces of copper or other metal attributable to the deposits of any of the five definitely Neolithic sites. At "Grad," Miroč, two shaft-hole axes of (?) copper—the celebrated "Hungarian type"—have been found (specimens in Negotin Museum), but nothing seems to be known with respect to their original deposition. Conceivably, such axes may well belong to Neolithic contexts. Additional, similar shaft-hole axes, again of (?) copper, have been collected in the vicinity of Zaječar, i.e. to the south of Negotin (specimens in Zaječar Museum).¹

DISCUSSION

In view of the strictly provisional nature of the observations here presented it is not plausible to proffer any definite conclusions. The evidence of Neolithic economy in the several instances just dealt with does, of course, furnish a useful basis for further investigation. It indicates the signal importance of the Yugoslav portion of the lower Danubian valley in Neolithic and later culture history. While certain deductions can be drawn and some general developments can, to a greater or lesser degree, be perceived, their elaboration

¹ Cf. Fewkes, 1934a, p. 36.—Analyses of any of these axes have not been made; hence the question mark regarding their composition.
at this time would have no more than a tentative value. Strictly objective interpretations are not attainable until actual excavation reveals fully dependable stratigraphic and chronologic factors. It should then become possible to segregate introduced traits from endemic accomplishments. It would be absurd to insist on “feelings,” “views,” or “opinions” in such matters; the existing, inadequate factual orientation would render them premature and subjective. But it does seem permissible to discuss our subject matter from the standpoint of general relationships with developments in adjacent regions. The provisional value of our data must be stressed once more; moreover, it is necessary to stipulate the empirical nature of the broad contentions, especially those regarding the cultural complexity and the type of economy at sites 1–5. To pursue this end, it is convenient to summarize our observations.

The Yugoslavian portion of the lower Danubian valley is adjacent to the southern side of the river between the Iron Gate and the mouth of the Timok. The length of the Danubian bank within these limits totals roughly 100 km. Within this length we now know five rather extensive Neolithic settlements, all with like deposits and relics. Fifteen other localities—a–h of our likely (?) Neolithic sites, i–v of our miscellaneous sites, and “Jelaš” and “Biljevina” near Velesnica—all potential sources of Neolithic remains, require identification of corresponding deposits which may or may not be there. In the southern periphery, on the upper Timok, two Neolithic sites have been recorded.¹ There is, then, an impressive number of sites within a fairly small area. These—taking into consideration all known prehistoric sites in the region—bespeak extensive settlements well separated from one another. Having examined several kilometers of exposed profiles of these sites I am convinced of their permanency in aboriginal times. This is not a feeling but an observation of what appears to be an historical fact.

¹ Records of the Niš Museum, containing reports on archaeological finds submitted by communal offices, indicate that similar sites probably exist farther down the Timok as well; information from Niš Museum, correspondence 1936.
Moreover, the definitely Neolithic sites reveal an interesting phenomenon; their lowest deposits (as brought to view by erosion) are characterized by pits alone. This is true of localities 1–5. And it is in such deposition that the barbotine class of ware occurs most constantly. Presumably, these pits (bothroi) are the remnants of semi-subterranean dwellings.\(^1\) The nature of their original superstructure remains unknown (although by no means unascertainable, provided that proper technique of excavation be applied); it is of interest to note that nowhere in the region under discussion did we find traces of posts in association with pits. With the exception of “Obala,” Kostol, the other definitely Neolithic sites contain remains of dwellings with a packed (sometimes \textit{perhaps} \(^[?]\) fired) floor, apparently without sub-pits.\(^2\) And with these associated post molds have been identified. The two types of dwelling thus distinguished document two stages of architectural history. Huts depending on bothroi undoubtedly belong to the primary Neolithic occupation at all our sites (1–5). Houses\(^3\) with floors, on the other hand, are stratigraphically younger, although still fully Neolithic, in four out of the same five sites. The exception is “Obala,” Kostol; and this is only provisionally so. The most characteristic pottery of such houses is no longer only the barbotine ware—which, alone, seems to be an exclusive class of the bothroi—but predominantly the incised, fluted, burnished, burnish-decorated, and hand-smoothed categories. Figurines and altars (by fabric members of the incised class) are likewise found in houses rather than in pits. These observations \textit{suggest} a relationship between type of dwelling and distinct

\(^1\) Their type has a wide distribution both in space and time.—At “Vinča” bothroi unquestionably antedate houses with floors (cf. Vasić, 1932, pp. 10 ff. and 101 ff., also Fewkes, 1935, pp. 633 ff.); this is equally true—even if not always recognized—of other Danubian sites of a Neolithic date. And at “Vinča” barbotine ware of identical nature with that from “Grad,” Starčevo (Fewkes, Goldman, Ehrich, 1933, pp. 43 ff.), certainly belongs essentially to the bothros niveau; cf. Vasić, 1936b, pp. 7 ff. and Pl. I.

\(^2\) These assimilate, on a smaller scale, the remnants of houses at “Vinča” revealed in vertical profiles; cf. Vasić, 1936, Pls. CXIII and CXIV.

\(^3\) I use the distinction between huts and houses in the same arbitrary sense as in Fewkes, 1935, pp. 656 ff.
wares which is so obviously recognizable at "Vinča." I refer to "Vinča" because that site has been explored on a larger scale than any other of a similar date in the entire Danubian area. This, however, does not imply that similar depositional conditions necessarily exist in other Danubian stations; the question requires specific field investigation.

Sites containing Neolithic material comparable to that found in the Yugoslav portion of the lower Danubian valley exist on the left (Roumanian) bank of the corresponding sector of the Danube. Several are reported from the vicinity of Turnu Severin; an extensive settlement is known at Hinova, and another on the island of Corbului, opposite Korbovo. And similar sites are known on the Bulgarian bank of the Danube (and in the interior) eastward of the Timok on to the Vit. In either series, however, we have no reliable information with respect to the nature of the deposits; nor have any of these sites, as far as sources show, been systematically excavated. Nestor states that the Boian phase, which is said to mark the initial Neolithic development in central Walachia and in central Bulgaria (north of the Balkan range), does not show any traits ascribable to his "Vinča I = Turdaș I—Kultur." At the same time, however, Nestor holds that a Vinča-Boian A relationship is not to be denied. The latter view finds a support in Mikov's observations.

Close parallels to the various classes of ceramics (and other traits) appearing in Neolithic sites in lower Danubian Yugoslavia are found in western Walachia, Siebenbürgen, the

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2 Băreacila, 1924, pp. 280 ff.
3 Nestor, 1932, p. 34.
4 Băreacila, 1924, pp. 283 ff. Cf. also Franz, 1922, pp. 90 ff., for certain finds from Ostrovul Girla (for location see our sketch map); however, the figurine on Franz's Pl. I: 1 a, b, described by him as Neolithic, certainly suggests a Late Bronze Age provenience.
5 Cülingirov, 1911, pp. 147 ff.
6 Nestor, 1932, pp. 34–35.
7 Ibid., p. 34. Cf. also p. 56, therein, respecting a presumable origin of Boian A which the author views as a local specialization rooted in Ariușd (formerly Erős), Vinča I and II, and Bükk material.
8 Cf. Mikov, 1933, pp. 23 ff.
9 Băreacila, 1924, pp. 280 ff., and Nestor, 1932, pp. 33 ff.
10 Schroller, 1933, pp. 6 ff., Nestor, op. cit., and Popescu, 1925, pp. 304 ff.
Banat, the Moravo-Danubian region, and in northwestern Bulgaria. These do not all appear at each site; the notable exception is "Vinča." There, in the extremely rich deposits, we may readily isolate material virtually identical with every one of the several categories here described. The barbotine class of ware has perhaps the widest distribution in the Banat and in the Moravo-Danubian region. Farther west, outside of Yugoslavia, barbotine pottery is especially common in the valley of the upper Tisa. In the Roumanian portion of the lower Danubian valley this ware appears at sites usually designated as of the Boian phase (central Walachia), in certain Moldavian contexts, and, as already stated, more or less throughout Little Walachia. The incised, and the fluted and ribbed wares have a wide distribution not only in the middle Danube valley, but in the lower division as well. The burnish-decorated class finds closest analogies in the Moravo-Danubian region, and the burnished ware is common to the middle and the lower Danube alike. The figurines assimilate those of central Bulgaria and Walachia on the one hand and those of "Vinča" on the other. The stone celts, particularly the shoe-last type, are a common bond of the entire Danubian area.

It appears that the barbotine class is the oldest, i.e. the primary pottery in lower Danubian Yugoslavia. While this

1 Especially at "Grad," Starčevo; cf. Fewkes, Goldman, Ehrich, 1933, pp. 33 ff., and Fewkes, 1936, p. 73, where sources are given in note 486.
2 Fewkes, 1936, pp. 27 ff., and sources cited therein.
3 Mikov, 1933, pp. 22 ff., especially pp. 24 (regarding correlations), 27–32 (dealing with twenty-two caves in northern Bulgaria), 33–50 (listing 123 open sites in northern Bulgaria), and 55–72 (listing 132 habitation mounds in northern Bulgaria). Mikov's work is very exhaustive with respect to local literary sources.
4 Banner, 1932, pp. 32 ff., and 1935, pp. 121 ff.
5 Andrieșescu, 1924, pp. 51 ff. (Sultan of); Dumitrescu, V., 1924, pp. 29 ff., and 1925, pp. 325 ff. (both Gumenița); Stefan, 1925, pp. 138 ff. (Cășcăioarele); Christescu, 1925, pp. 249 ff. (Boian); Dumitrescu, H., 1927–1932, pp. 88 ff. (Bonțești); Dumitrescu, H., 1927–1932a, pp. 150 ff. (Grădiștea Fundeana); Christescu, 1927–1932, pp. 167 ff. (Vădastra).
6 Dumitrescu, H., 1927–1932, pp. 56 ff., and Figs. 20: 9, 21: 4, showing barbotine ware with streaked applique (Ruginosu), and Schmidt, 1932 (Cucuteni), Pl. 24: 1 and 6 (barbotine ware with streaked applique).
7 Bârcăci, loc. cit., and Nestor, loc. cit.
8 Cf. especially Mikov, 1933, pp. 24 ff.
impression is supported by insufficient stratigraphic observation: it is significant that barbotine ware has been found in all of the five sites definitely documenting Neolithic settlements, and that in all cases it has been ascertained in the oldest portion of such deposits. No similar observations are available with respect to the other classes of ceramics. At "Vinča," as well as in a number of related sites, the incised ware apparently belongs even to the oldest deposits (bothroi).¹ The position of the fluted and burnish-decorated wares remains obscure. The burnish-decorated group, however, seems to imply a somewhat later development. At "Vinča" the barbotine ware occurs in the pit level marking the initial occupation of the site. In Roumania, as has been said, it is found in several sites in association with the Boian development. The barbotine ware seems to have a similar relationship in central Bulgaria,² but in the western portion of that country, it appears free of Boian elements.³ At "Grad," Starčevo, this class of ware certainly represents the oldest ceramic group fully antedating the painted pottery which includes elements characteristic of the Sesklo and Dimini painted pieces. There were no "Vinča" imports in the pit level of "Grad," Starčevo, but in its stratigraphically younger niveau examples of "Vinča" fabrics (apparently imports—as yet, however, subjects to technological analysis to prove their provenience), especially fluted sherds, have been found. The total absence at "Grad," Starčevo, of the so-called typically Danubian incised ware furnishes, it would seem, a significant indication of the priority of the barbotine ware at that site.

The somewhat puzzling relationship between the incised and the barbotine wares within the Danubian valley depends largely on additional field work. Somehow, the barbotine ware seems to have escaped adequate attention although its occurrence is rather wide spread; and its generic placement

² Field observations and museum studies, season 1932.
³ Cf., e.g., Popov, 1912/1913, pp. 279 ff., and Figs. 183–185, 189, and 192; these are here selected as unmistakable examples of the barbotine ware illustrated by Popov.
in the Neolithic sphere at large is yet to be elaborated. One important impression stands out with respect to the Danubian distribution of this type of ware; it seems to have survived, at least sporadically—although in an altered style—well into the Bronze Age, while in its earliest appearance it marks (in certain localities, at any rate) the initial phase of the Neolithic Age. It is possible that here we have to deal with a large and long-lived family of pottery, quite individual in its character, and traceable, it seems, to gourd inspiration. In this regard it has a bond in common with the spiral-meandric incised ware, but it embodies gourd imitation more effectively both in shape and in embellishment.

The lower Danubian Neolithic attainments here discussed seem to indicate—as far as tangible evidence permits an interpretation—an extraterritorial primary derivation. There certainly are no obvious local cultural antecedents which might provide a potential foundation for an independent origin of a Neolithic culture. The positive Neolithic evidence before us clearly documents a maturity of cultural accomplishments reflecting the mode of life which readily admits of applying to it the standard—even if inadequate—appellation of the Neolithic Age. Although intraregional differentiation is demonstrated in subsequent developments, the underlying basis, that is to say the primary (or introductory) core, reflects a pronounced uniformity over a large territory. As has been stressed, it is for future exploration to ascertain the criteria with which to segregate, from site to site first and then by correlation, that which is primary as against the subsequent, however inspired, growth.

Collectively considered, the lower Danube valley has a series of signal common bonds: relief, nature of topsoils, hydrography, general means of subsistence (natural resources), economic possibilities, and sundry anthropogeographical factors. Its Neolithic culture history, however, reflects certain diversities not only in material achievement, but also, apparently, in time placement. Despite the impressive number of sites recorded in the Yugoslavian,
Bulgarian, and Roumanian sectors of the lower Danube, and the reconnaissance, sounding, and excavation conducted there, a general synthesis is not yet attainable. For that reason it is not plausible to arrive at a rational appreciation of the seeming complexity of the cultural pattern before us. Yet, at least two geographic zones of Neolithic growth suggest themselves. In the eastern portion of the valley the so-called Boian development seems to be the dominant expression. Characterized by open sites and settlement mounds, and distinctly individual only in its ceramics (but not in its cultural totality), it represents a rather advanced Neolithic phase. In the western portion of the valley, that is to say approximately westward from the rivers Jantra (Bulgaria) and Vede (Roumania), and as far as the Iron Gate, the cultural stamp is more akin to that of the middle Danube valley. There we find several elements in common with "Vinča" and even more strikingly so with "Grad," Starčevo. The sites are predominantly of the open type, although in northern Bulgaria a series of cave stations is on record. Perhaps the most extensive settlements are situated on the Danubian bank, where deposits of 3 m. (or more) in thickness are by no means rare.

Nestor opines that the absence of plastics in Boian A (which are so rich in Gumelniţa A) may indicate that the Boian development is quite old. It is of interest to recall the paucity of figurines at "Grad," Starčevo. And it will be of

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1 Nestor, 1932, pp. 33 ff. (his "Vinča I = Turdaș I-Kultur"); Mikov, 1933, pp. 24 ff; Fewkes, 1936, pp. 73 (note 488) ff.
2 Fewkes, 1934a, pp. 35 ff., and 1936, p. 73.
3 Mikov, 1933, pp. 27–32, where twenty-two caves are listed.
4 Nestor, 1932, p. 56.
5 Cf. Fewkes, Goldman, Ehrich, 1933, p. 48.—In 1934, Mr. L. Nadlački, who served as foreman of the American Expedition at Starčevo in 1932, acquired, by purchase, a ceramic figurine said to have been found by brickmakers at "Grad," Starčevo. From the brief note and sketch published by Nadlački, 1936, it seems that the specimen is painted in "light red stripes on white ground," the design forming irregular, joined diamonds. The specimen has prominently produced breasts, has incisions on the back and shoulders which are suggestive of hair, and is 6.2 cm. high. The head is missing, and the basal portion is said to be damaged. (The statement that heretofore figurines from Starčevo were not known, op. cit., p. 39, is not quite correct; cf. Fewkes, Goldman, Ehrich, 1933, pp. 48–49.)
great importance to determine the true relationship of the primary barbotine ware to the Boian group of pottery, for it is quite possible that the former style forms a foundation of the latter. This, of course, is merely a *provisional thought*.

The Yugoslavian region here treated is most readily accessible via the Danube and its flood plain. The river can be crossed with a nominal effort. Under normal conditions the current is rather slow, landing places are plentiful, and in several instances islands facilitate crossing. The Danube certainly provides a vital natural artery of communication in the open sectors of its valley. However, the Iron Gate, approximately 130 km. long, imposes numerous serious impediments to river or bank passage. It is well to recall the history of modern (steamship) navigation in this portion of the Danube¹ to appreciate the significance of the Iron Gate in relation to antiquity. As far as positive evidence permits deductions, the Iron Gate appears not to have been penetrated either by craft or on foot in pre-Roman times. Insofar as the Neolithic Age is concerned it is significant to note that the distribution of settlements suggests circumvention rather than utilization of the Danube and its banks at least within the most inhospitable stretches of the Iron Gate (such as Pl. VII: 4, 5, 8). This observation seems to be applicable to other prehistoric periods as well. There is no positive proof with which to show that the Iron Gate accommodated cultural traffic before the second century of our era. The remarkable achievement of Trajan's engineers enjoyed but a short-lived success, for the famous road through the Iron Gate (Pl. VII: 7), the backbone of the intricate waterway (used for pulling water craft), fell into disuse and ruin fairly soon after the conquest of Dacia. "Die Stromsnellen," writes Gilsdorf, "lämten die Schifffahrt schon in den ältesten Zeiten. Sie waren die Ursache dafür, dass die Griechen, die die untere Donau befuhren, nicht über die Katarakktenstrecke hinaus stromaufwärts gelangten und den oberen und unteren Teil des Strom für zwei gänzlich verschiedene Wasselaüfe hielten."²

¹ Gonda, 1899.
² Gilsdorf, 1928, p. 18.
Under aboriginal conditions, prior to the construction of the Széchenyi road on the left bank; before the regulation of the navigation channel which now cuts through a series of rapids and uses an ingenious canal (at Sip) to clear the terminal obstacles; and until the construction of the road by Trajan, the Iron Gate apparently discouraged passage both upstream and downstream. The immediate hinterland of the winding gorge consists of a mountainous massif—the Balkan-Carpathian formation—and is heavily forested; there are no transverse gaps or passes to facilitate communication laterally with the Danube. The natural circumstances which originally confronted the Neolithic “explorer-pioneer” cannot, of course, be fully reconstructed and visualized. Yet it is very likely that properly directed research will lead to certain rationalization of the natural conditions then prevailing. It may perhaps be surmized that the adverse conditions which had discouraged the first Roman attempts to build a road on the Moesian side, but which Trajan’s engineers eventually subdued, may well have presented insurmountable difficulties in earlier times. However, strictly objective deductions cannot be made until appropriate field work establishes more dependable data than we have at this time.

Despite the fact that essentially the same type of Bronze Age (i.e. the “Žuto Brdo” development) expression is found in a series of sites immediately above and immediately below the Iron Gate, we have no acceptable proof that contacts between the two regions were maintained via the Danube. Indeed, within the Iron Gate one looks in vain for links with which to support such contacts. The Bronze Age sherds reported from “Orsova” by Wosinsky were, according to Milleker’s information given to Vasić, actually found in the vicinity of Kladovo! Vasić, prompted, as he expressed it, by reasons of an “easier citation of these objects” [sic!] continued to designate them as “finds from Orsova.” As far

1 For the southern bank cf. Vasić, 1910, pp. 5 ff., and his map on Pl. I.
2 Wosinsky, 1905, pp. 69 ff., and Pls. C–CIII.
3 Vasić, 1910, p. 4.
as Neolithic material is concerned, it is only at Dolnea Lupcova, in Roumania (cf. Fig. 1), not far from the western terminal of the Iron Gate, that certain positive data have been recorded. These consist of surface sherds collected and briefly reported by Milleker.\(^1\) There is no information as to the nature of the locality. Otherwise the Iron Gate at large constitutes a lacuna in all archaeological periods antedating its conquest by the Romans. In Neolithic times, judging by extant distributional evidence, the Iron Gate appears to have been circumvented by "detours" on either side of the Danube.\(^2\)

As one observes the exotic configuration of this peculiar gorge one can readily appreciate its negative role in prehistory.

I discussed the matter of natural arteries (river valleys) across the Balkan-Carpathian formation in Yugoslavia, Bulgaria, and Roumania in my paper dealing with the Moravo-Danubian region.\(^3\) To repeat its substance here would be a mere duplication and there are no new significant observations related to the subject to be added at this time. But it seems well to stress the apparent importance of the Struma as an early "route" in contrast to the still grossly "disappointing" valley of the Vardar.\(^4\) Again, our factual knowledge is indeed meager, but it seems that the Struma area is revealing earlier Neolithic remains than have hitherto been noted in the drainage of the Vardar. I am not aware of any dependable analogies with the barbotine class of ware in Greece,\(^5\) and am not dealing with the painted pottery complex; on the other hand the subject of Danubian ceramic traits in Greece is outside of my present concern.\(^6\)

\(^1\) Milleker, 1897, pp. 15 ff.
\(^2\) Fewkes, 1936, p. 71.
\(^3\) Fewkes, 1936, pp. 69 ff.
\(^4\) Cf. Mikov, 1933, p. 44, i.e. "Mursalevo" and "Kadin Most," and pp. 32 ff.
\(^5\) The Peabody Museum of Harvard University has a surface collection of sherds from tumbas near Larisa; this contains separately filled cup bases, a typical characteristic of the barbotine ware. But the manner of surfacing is entirely different. Grundman, 1934, Beilage XII, illustrates a fragment of a marble vessel (see also his reconstruction) which embodies a shape quite commonly appearing in the barbotine class.—With respect to Cretan comparisons cf. Fewkes, 1936, p. 27, note 109.
\(^6\) Cf., however, Grundman, op. cit., and Fewkes, 1936, p. 74 (there sources in notes 493 and 494).
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Provisional as the observations on the Neolithic sites in
the Yugoslavian portion of the lower Danube necessarily are,
they do, nevertheless, help to increase our insight regarding
the culture history of the north-central portion of the Balkan
peninsula. The leads here briefly described call for much
additional field work and study. The chief need of future
work is to adopt a truly objective methodology. It is also
necessary to remember that pottery, though an integral
Neolithic trait, does not in itself make up a culture.

APPENDIX

NOTE ON A MICROSCOPIC STUDY OF A SAMPLE GROUP
OF BARBOTINE SHERDS WITH POSITIVE APPLIQUE
FROM "GRAD," STARČEVO

DONALD HORTON

The sample here described consists of sixteen sherds
which, insofar as typology is concerned, are said to be repre-
sentative of an early phase of the Neolithic manifestations
identified by the American Expedition at Starčevo (informa-
tion by Dr. Fewkes).

On the exterior of each of the sixteen sherds there is
evidence of an applied surface coating, resembling an "ir-
regular slip," which, in some cases, attains a thickness of as
much as ten millimeters. In accordance with a desire speci-
fied by Dr. Fewkes I made it my object to determine, if
possible, the nature of this so-called applique surface coating.
On the basis of my observations I conclude that the applied
surface coating is of essentially the same material as that used
in the construction of the vessels themselves.

1 Mr. Horton is Technical Associate, in charge of the Ceramic Laboratory,
University Museum, Philadelphia; the Laboratory is under the joint auspices
of the University Museum and the Works Progress Administration (Project
No. 2232). This Appendix is an excerpt from Mr. Horton's notes thus far
compiled in the course of his technological research dealing with samples se-
lected from the American share of pottery obtained at the site of "Grad," Star-
čevo, in 1931. The exploratory test excavation accomplished at that time
was sponsored jointly by the Peabody Museum, Harvard University, and the
University Museum, Philadelphia. (Footnote by V. J. Fewkes.)
Although the microscopic study of this material is near completion, a detailed technological analysis is yet to be considered. For the purpose of the study here summarized thin sections of every one of the sixteen sherds were prepared. A fragment of each sherd was fired to approximately 1000° C. for half an hour, and then thin sections were made of the re-fired pieces. The total number of sections prepared and examined was thirty-two. The work was carried out chiefly in the Ceramic Laboratory of the University Museum. Certain of the petrographic determinations were made in the laboratory of the Department of Geology, University of Pennsylvania, through the courtesy of Professor Frederick E. Ehrenfeld.

The following is a description of my observations.

*Microscopic Character.*—All of the sherds have a dense and fine-grained texture. The fabric contains an abundance of silt consisting chiefly of angular particles of quartz and platy particles of sericitic mica. In twelve of the sherds the quartz grains, which are of nearly uniform size, are estimated to have an average diameter on the order of 0.03 mm., and the mica plates have an average length on the same order. The largest quartz grains are approximately 0.1 mm. in diameter, and there are very few of these. Two of the sherds are of even finer texture, and two are slightly coarser. The latter contain a few grains 0.2 mm. in diameter. These few grains of very fine sand are visible to the naked eye, but account for much less than 15 per cent of the volume of the fabric. All sixteen sherds are, therefore, to be described as "sparsely tempered" according to Shepard's (1936, p. 409) scale.

A characteristic feature of the sherds is the presence of thin, elongated cavities, which in cross section are straight, curved, or even hook-shaped. In a few cases, thin, elongated fragments of calcareous material, laminated like shell-fragments, are present, but these account for very few of the cavities observed in thin-section. Examination of the sherds themselves shows that most of the cavities were originally filled wholly or partly with what appears to be more or less
carbonaceous ash of grass blades. These residues of grass are thin and strap-like. They consist of a "skeleton" of amorphous silica, which has a characteristic pattern and an index of refraction of about 1.45 (opal). Where the fabric has not been completely oxidized on firing there is a carbonaceous residue of organic matter adhering to the spodogram. Since these remnants occupy their cavities very loosely they break up on grinding and do not appear in thin-section.

In every case, a strong color-banding is shown both in the sherd and in the thin-section. Margins, which may be very narrow, are buff to light reddish-orange in color; the core is always neutral gray to black. On refiring to 1000° C. the colored margins may be widened and changed to a slightly redder hue, but the cores tend to remain dark. It is evident that the matrix is so dense as not to allow sufficient penetration of oxygen for oxidation of the carbonaceous core even at high temperature. This peculiarity of the material accounts for the circumstance that in many cases the applique surface contrasts in color with the body of the sherd itself.

Examination of thin-sections of the barbotine sherds and of the refired pieces of the same sherds shows definitely that the surface coatings have exactly the same texture and the same inclusions, both mineral grains and grass blades, as the cores themselves. On refiring, the cores and the applique develop the same colors. Except where faint structural differences are evident, the two components of the sherd are undistinguishable under the microscope. There is, therefore,

1 Footnote by V. J. Fewkes: Regarding this point, my colleague Mr. William C. Darrah, Associate Curator of Paleobotany, Harvard Botanical Museum, contributes the following statement: "The elongated thin cavities present in the Starévo sherds are natural 'spodograms' of the stems of certain grasses—probably wheat. It has not yet been possible to identify the genus or species precisely. The stems of many plants, particularly reeds and grasses, contain large amounts of mineral calcite or silica. Upon burning, the mineral ash retains the shape of the stem although all of the carbonaceous material has been volatilized. The name 'spodogram' has been applied to the mineral 'skeletons' of plant parts. In your ceramics the 'spodograms' were produced by the firing of the vessels." (Dated June 7, 1937.)—Mr. Horton and I wish to express our gratitude to Mr. Darrah for his speedy response to our needs in this matter.
convincing evidence that the cores and their surface coatings consist of similar material.

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1-20: Ceramic material from "Obala Kusjak," Prahovo.
Ceramic material (1-21, 23) and celt (22) from "Obala Northeast," Korbovo.
1-17: Ceramic material from "Obala," Kostol; 18-23: Ceramic material from "Glamija-Obala," Ljubičevac.
PLATE VII

1: View of the Danube; taken from the surface of the site of “Vinča.” 2: Golubac (Cuppæ). 3: View of the left bank of the Danube opposite Ljubičevac. 4: The Danube below Veliki Štrobac. 5: The Kazan. 6: Vicinity of Mihajlovac. 7: Portion of the ledge of Trajan’s road in the Kazan. 8: The terminal strait on the Danube (immediately above Sip). 9: Tumulus (Roman and perhaps also earlier) just west of Prahovo. 10, 12, 13, 15: “Obala Southwest,” Korbovo. 11: Southern abutment of Trajan’s Danube bridge; Turnu Severin (Roumania) is seen in the background. 14: The Danube at Prahovo.
PLATE VIII

1: "Obala Northeast," Korbovo; downstream limit of the culture bearing bank. 2: "Obala Kusjak," Prahovo; the depression marking the site is seen in the center and left foreground and the grist mill in the background (center). 3: "Obala Northeast," Korbovo, viewed from the Danube, showing exposed deposits in the bank and the rising surface plane of the terrace. 4: "Kod Ruženke," Mihajlovač; remnants of a Roman road, superimposed over earlier deposits, seen in the exposed bank. (Note the sloping shelf.) 5: View of the Danube taken from the site of "Kula" (a Roman castellum) near the mouth of the Slatinska Reka brook, facing Brza Palanka. 6: View of the flat country near Mihajlovač, taken from the site of "Blato" (a Roman castellum).

(Photographs by the Harvard Expedition to Yugoslavia, season 1933, and by members of the American School of Prehistoric Research, under my direction, in 1934.)
A COMPARATIVE STUDY OF THE ELECTRICAL RESPONSES OF THE EAR

ERNEST GLEN WEVER
Associate Professor of Psychology, Princeton University

AND

CHARLES W. BRAY

(Read April 23, 1937)

ABSTRACT

The electrical responses of the cochlea during sound stimulation were observed in guinea pigs, cats, opossums, and pigeons. Characteristic differences were found in (1) the range of frequencies over which responses occurred, (2) the functional relation between magnitude of response and stimulus intensity, and (3) the variation of sensitivity with frequency. The results form a basis for the comparative study of the auditory mechanism.

The electrical responses that arise in the cochlea during stimulation by sound have been studied in four species of higher vertebrates: guinea pigs, cats, opossums, and pigeons. The results provide a basis for a comparative study of the auditory mechanism in these animals.

The cochlear potentials were picked up by a silver foil electrode in contact with the membrane of the round window, and a grounded electrode on adjacent skin or muscle. After suitable amplification, the responses were observed with a cathode-ray oscillograph. Pure tones were used as stimuli. They were produced by an electrical system consisting of an oscillator, filters, attenuators, and a loudspeaker. This system gave tones of frequencies from 35 cycles upwards, and of any desired intensity within a wide range. A supplementary apparatus was used in experiments on guinea pigs for the production of tones below 35 cycles. Calibrations were carried out so that known intensities of sound could be delivered to the ear of the animals, and so that the potentials
as picked up by the electrodes could be determined in absolute units.\(^1\)

The range of tones for which responses appeared was characteristically different for the four species. Guinea pigs gave responses between 5 and 25,000 cycles.\(^2\) Cats were not tested below 35 cycles, but responded from that frequency up to 30,000 cycles. Opossums gave responses usually between 200 and 25,000 cycles, but sometimes responded to tones as low as 100 cycles.\(^3\) Pigeons gave responses generally from 100 to 10,000 cycles, but occasionally showed a slightly wider range.\(^4\) These ranges were in part a function of the conditions of the experiments, the sound intensities available and the degree of amplification used. However, since these conditions were the same for all animals, the results are adequate for comparative purposes.

Intensity functions were obtained by measuring the electrical response in microvolts (\(\mu\text{V}\)) during stimulation with various tones at different intensities (expressed in terms of sound pressure, in bars). The functions thus obtained were of the same general form for all animals and all tones. Throughout most of the intensity range the magnitude of response was proportional to stimulus intensity raised to a constant power, while at extreme intensities distortion appeared and the responses ceased to bear a simple relation to intensity. The slopes of the curves varied; for the three mammals they were fairly close to unity, but for the pigeon they were in the region of 0.4. These differences are illustrated in Fig. 1, which shows typical results for a tone of 1000 cycles. In this figure, the intensity of stimulation is shown in bars on the abscissa, and the response in microvolts.


is shown on the ordinate. Both scales are logarithmic. In addition to the characteristic forms of the curves, this figure reveals differences in sensitivity, as measured by the magni-

**Fig. 1.** Intensity functions. Each curve shows, for a typical animal of the species indicated, the magnitude of response as a function of intensity of stimulation for a tone of 1000 cycles.

tude of response for a given stimulus intensity. The pigeon and opossum are much below the guinea pig and cat in sensitivity. Individual animals within each species differ somewhat in sensitivity, but the general relations as shown are characteristic.

**Fig. 2.** Equal-response curves. Each curve shows, for a typical animal as indicated, the intensity of sound required at various frequencies to produce a standard response of 10 μV.
The variations of sensitivity are more clearly presented in Fig. 2. Here each curve shows, for a number of frequencies, the intensity of stimulation necessary to produce a standard response of 10 \( \mu \text{V} \). The results are for a typical animal of each species. Considerable differences are observed in both the general level and the form of the curves. Differences in the regions of maximum sensitivity (the least intensity required for the standard response) are particularly striking. The guinea pig is most sensitive in the region of 500–700 cycles, and the cat in the region just above 1000 cycles. The opossum shows poor sensitivity for low tones, and improves fairly uniformly until a maximum is reached around 7000 cycles. The pigeon shows a maximum around 2000 cycles, and falls off rapidly for high tones.

The above results are regarded as reflecting the characteristics of the peripheral mechanism of the ear. However, since further processes in the auditory nerve and central nervous system supervene, a perfect correlation is not expected between the results of these experiments and measurements of hearing made by conditioned reflex and other behavioral methods. But despite this limitation, the results of the electrical studies are in good agreement with what is known from other tests of hearing in these four species.
CORRELATION OF FREQUENCIES OF SEISMIC DISTURBANCES WITH THE HOUR ANGLE OF THE MOON*

HARLAN TRUE STETSON

Cosmic Terrestrial Research, Massachusetts Institute of Technology

ABSTRACT

The possibility that periodic tidal stresses in the earth's crust should be reflected in the frequency with which seismic disturbances occur has been investigated by various workers, usually from the point of view of phases of the moon or with a view to correlation of earthquake frequencies with distances of the epicenter from the sub-lunar point. Results from the former method of treatment have for the most part been conflicting, and the results from the latter method of analysis have been subject to a fortuitous distribution owing to the relatively few occasions when the moon can be near the zenith of the epicenter.

The present investigation, based on some 2,000 major earthquakes, seeks a possible correlation with lunar hour angle alone, which avoids the above-mentioned difficulty. When the seismic disturbances are restricted to major earthquakes recorded over 80° from the epicenter, and confined to the Philippine Island group and the Japanese Archipelago, two maxima are suggested falling approximately 12 hours apart, one corresponding to lunar time 8 hours, and the other to lunar time 20 hours. These occurrences are not far from the time of maximum tidal stress.

More significant are the results of a study of 150 deep-focus earthquakes with origins 100 kilometers or more below the surface. The frequency distribution of these deep-focus earthquakes is compared with a sine curve corresponding to the horizontal tidal force at the epicenters. A least-square solution shows the sum of the squares of the residuals from the most probable sine curve to be \( \frac{3}{4} \) as large as would be the case were there but a chance distribution based on an arithmetic mean of the hourly frequency. Results, therefore, indicate a distinct tendency for major seismic disturbances to follow preferential positions of the moon with maxima occurring near the times when the horizontal component of the tidal force is a maximum.

Various workers from time to time have endeavored to show certain correlations between the occurrences of earthquakes and positions of the moon. Because of considerable indefiniteness and many inconsistencies in results published from such investigations and because of the fact that it is difficult on physical grounds to see how the relatively small tidal forces operating in the Earth's crust could account for the occurrences of earthquakes, seismologists have been

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skeptical concerning the reality of any relation between the moon and the occurrence of major seismographic disturbances. One of the most extensive summaries on this general problem that has come to the author's attention is that of Mr. Leo Cotton, published in the *Bulletin of the Seismographical Society of America*, 12, pp. 49–198. Under the title of "Earthquake Frequency with Special Reference to Tidal Stresses in the Lithosphere," Mr. Cotton gives an extensive survey of the results of many investigations. Near the close of his paper, he appears to show a striking correspondence between earthquake frequencies and the moon's zenith distance. Unfortunately, the increased frequency of earthquakes with increased zenith distance of the moon, while apparently real, is one of statistical rather than physical cause.

Reflection upon the elementary astronomical concepts involved will render it obvious that since the moon rises and sets every lunar day there are always two times in every such interval when its altitude is zero or its zenith distance is 90°. The days, on the other hand, when the moon attains high altitude or a small zenith distance are relatively few, especially for stations in middle latitude. The chance, therefore, for any considerable number of earthquakes coinciding with times of small zenith distance of the moon is meagre. In any investigation of earthquake frequency and astronomical phenomena, it is of primary consequence that astronomical factors be so considered that so far as chance distribution is concerned, there is an equal likelihood of earthquakes happening in any of the intervals selected which may be affected by the astronomical factor considered.

Investigations by Father Rodes 1 which have appeared recently have shown an apparent increase of seismic disturbances when the moon was near perigee compared to times when the moon was near apogee. Furthermore, recent publications by Davidson 2 have appeared to show a connection between the frequencies of earthquake after-shocks and

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2 *Jour. Geol.*, 42, No. 5, July 1934.
the phase of the moon which might be interpreted as a question of the combination of the solar and lunar tidal forces that depend on the relative positions of these two astronomical bodies.

Recent investigations by the writer on apparent changes in the direction of the vertical with positions of the moon have led to the possible conclusion that small changes occur in the geographic coordinates of observing stations that in some way are associated with tides in the earth of somewhat greater magnitude than one would be led to suppose, based on classical ideas alone. Investigations of the Gulf Research Laboratories in Pittsburgh have brought to light through gravimetric measurements a vertical rise and fall in the Earth's crust of the Allegheny region of the order of magnitude of 60 cm. These facts made it appear worth while to investigate again any possible connection between seismic disturbances and preferential positions of the moon. Some 2,000 earthquakes, therefore, have been analyzed from the point of view of a possible relation of the frequency of their occurrence to the moon's position referred to the epicenter at the time the shocks occurred. Curves of vertical and horizontal tidal forces for different latitudes have been drawn for study in comparison with curves of earthquake frequency. To avoid the difficulty of a fortuitous variation with lunar altitude, analysis of the frequencies has been confined to lunar hour angles.

While the lunar-tidal force, both as regards the vertical and horizontal component, is a function of the zenith distance of the moon or the distance of the sub-lunar point from the epicenter, any simple attempt so to correlate earthquake frequency introduces a selectivity in the data that may lead to fortuitous results. The examination of 2,604 seismic disturbances, including both major and minor earthquakes during the years 1918–1924, taken from the British Association catalogue, reveals the following distribution, arranged

by hour angles of the moon, referred to the meridian of the epicenter at the time of the occurrence:

**TABLE I**

<table>
<thead>
<tr>
<th>Number of Earthquakes, Major and Minor, from 1918–1924, Arranged by Lunar Hour Angles. Total in List, 2,604</th>
</tr>
</thead>
<tbody>
<tr>
<td>23–0</td>
</tr>
<tr>
<td>0–1</td>
</tr>
<tr>
<td>1–2</td>
</tr>
<tr>
<td>2–3</td>
</tr>
<tr>
<td>3–4</td>
</tr>
<tr>
<td>4–5</td>
</tr>
</tbody>
</table>

If, from the foregoing list, only the major seismic disturbances are included, such as were reported as having been recorded over 80° from the epicenter, 653 such major disturbances are available for similar analysis. The table (Table II) indicates a maximum around 18 hours and a minimum near 0 for lunar hour angle. A second less pronounced maximum occurs near 6 hours. The maxima correspond to a mean position of the moon for the times approximating the greatest horizontal force at the epicenter.

**TABLE II**

<table>
<thead>
<tr>
<th>Major Earthquakes 1918–1924</th>
</tr>
</thead>
<tbody>
<tr>
<td>0h–2h</td>
</tr>
<tr>
<td>2–4</td>
</tr>
<tr>
<td>4–6</td>
</tr>
<tr>
<td>6–8</td>
</tr>
</tbody>
</table>

An analysis was next made of all of the major earthquakes recorded in the interval 1918–1929, comprising 2,569 entries.

**TABLE III**

<table>
<thead>
<tr>
<th>Major Earthquakes Recorded 80° or More from Epicenter for Years 1918–1929</th>
</tr>
</thead>
<tbody>
<tr>
<td>0h–1h</td>
</tr>
<tr>
<td>1–2</td>
</tr>
<tr>
<td>2–3</td>
</tr>
<tr>
<td>3–4</td>
</tr>
<tr>
<td>4–5</td>
</tr>
<tr>
<td>5–6</td>
</tr>
</tbody>
</table>
Here again maxima are suggested both for lunar hour angle, 7 hours and 18 hours. In plotting the curve (Fig. 1), running means of three have been taken for unbiased smoothing. If

![Graph](image)

**Fig. 1.** Major earthquakes recorded 80° or more from epicenter 1918–1929.

the much-debated lunar-tidal force were to be a factor in the occurrence of earthquakes, one would expect that the frequency would be greatest not necessarily at the time of maximum stress but rather at the time when the stress difference due to the total tidal forces is a maximum. Love has stated that on the assumption of a homogenous incompressible earth, the greatest value of the stress difference on the earth’s surface would occur at places 90° from the sub-lunar point.¹ He emphasizes, however, that the greatest stress difference

found near the surface is only about 3/8 the value at the earth's center. The numerical value he estimates at possibly one pound per square inch. The introduction of corrections for heterogeneity and compressibility would be highly complex and might shift the place of maximum considerably. It should be noted that for earthquake zones of low latitudes the points of greatest stress difference, on the average, would correspond to the lunar hour angles of 6 and 18 hours from the meridian of the epicenter.

Since the earthquakes in the list examined have been widely distributed both as regards latitude and longitude, and undoubtedly represent regions of vastly different crustal structure, perhaps any attempt for finding a relationship between the position of the moon referred to the epicenter and the frequencies of earthquakes taken indiscriminately all over the globe is never likely to yield very significant results. With the material in hand, therefore, it seemed worth while to segregate certain particularly active regions, and analyze similarly the frequencies of occurrence in these localities with respect to the lunar hour angle at the epicenter at the time of the occurrences. Three regions were treated independently. One is that located between the parallels of 0° and 40° north latitude and the meridians of 120° and 140° east longitude. This includes the Philippine Islands and the Southern extremity of the Japanese archipelago. Three hundred and forty-five major disturbances were included in this region during the years 1918–1929 and represent approximately 14 per cent of the total list of about 2,500 entries. The second group selected was that lying between 30° and 60° north latitude and between the meridians 140° and 160° east longitude, comprising the major part of the Japanese archipelago excluded from the first group. The third group selected was that in South American territory comprised between the parallels of 0° and 40° south latitude and the meridians of 60° and 80° west longitude. The results of the distribution of the numbers of earthquakes for each two-hour interval of lunar hour angle is presented in Fig. 2. It will be observed
that in both the first two regions there is a definite trend towards a maximum at about 8 hours. A second maximum for the Philippine territory occurs at 22 hours. The second maximum for the Japanese archipelago occurs at 20 hours.

**Fig. 2.** Number of major earthquakes occurring at hour intervals after moon's passing meridian of epicenter. (a) Philippine Island Group. Lat. 0°–40° N. Long. 120°–140° E. Total 345. (b) Japanese Group. Lat. 30°–60° N. Long. 140°–160° E. Total 233. (c) South American Group. Lat. 0°–40° S. Long. 60°–80° W. Total 147.
The percentage of increase in the frequency at maximum as compared with minimum is more than 100 per cent for these two regions. While a maximum is suggested at 8 hours in the case of the South American region, the frequencies in this case are so much more evenly distributed that one can hardly attach significance to points of maxima and minima. The data for this South American group represent about 6 per cent of the total number of earthquakes in the list.

![Graph showing occurrence of 578 major earthquakes of Philippine and Japanese groups (1918-1929) distributed by hourly intervals following moon's passing meridian of epicenter.](image)

**Fig. 3.** Curve showing occurrence of 578 major earthquakes of Philippine and Japanese groups (1918-1929) distributed by hourly intervals following moon’s passing meridian of epicenter.

In the case of the Japanese archipelago, the number of major earthquake disturbances approximates 9 per cent of the total number of major earthquakes in the list. If the disturbances in the first two groups are combined, we have a list of 564 major disturbances representing approximately 23 per cent of the total list. A graph of these combined regions is shown in Fig. 3—the individual points representing the
total numbers in the hour interval indicated and the circles
representing the moving averages or running means taking
three items at a time. The artifice of running means is a
convenient and unbiased method of drawing a smooth curve,
though considerably reducing the range in amplitude. We
note for the combined results, the peaks of maxima occurred
at 6 hours and 21 hours, the first maximum corresponding to
the time of maximum stress. The second maximum would
appear to precede the time of maximum stress by about three
hours. The position of the sub-lunar point corresponding
would be approximately in the middle of the Pacific Ocean
around longitude 175° west. At the time of the maximum
corresponding to lunar hour angle of 6 hours the sub-lunar
point would lie in the extreme west of the Indian Ocean and
near the east coast of Central Africa.

If we restrict the character of the quakes to those of
deep-focus origin, whose epicenters have been definitely de-
termined to lie from 100–700 km. below the earth’s surface,
an even more significant correlation between lunar hour angle
and frequency appears to result. It is perhaps pertinent to
remark in this connection that the deeper the focus the greater
is the stress difference which is created by the tidal forces.
In the study of these so-called deep-focus earthquakes, a list
of 113 well-determined seismic disturbances with origins
100 km. or more below the surface of the earth served as the
basis for investigation. The list was provided for me by Dr.
J. A. Sharpe of the Massachusetts Institute of Technology,
who utilized the data for another purpose.¹

In Table IV is listed the number of occurrences of these
deep-focus quakes for twenty-four equal intervals correspond-
ing to hourly values in the changing hour angle of the moon
referred to the epicenter at the time of the occurrence of each
deep-focus earthquake. The full line curve in the figure
(Fig. 4) is drawn through points representing the running
means of the numbers of earthquakes for hourly intervals.
The broken line curve represents the east and west component,

¹ "Deep Focus Earthquakes and their Geophysical Significance," by Andrews
### Table IV

**Table Showing Relation of Frequency of Deep-Focus Earthquakes to the Lunar Hour Angle**

<table>
<thead>
<tr>
<th>Hour Angle of Moon</th>
<th>Number of Quakes</th>
<th>Hour Angle of Moon</th>
<th>Number of Quakes</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>7</td>
<td>12</td>
<td>3</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>13</td>
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<td>19</td>
<td>9</td>
</tr>
<tr>
<td>8</td>
<td>3</td>
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<td>2</td>
</tr>
<tr>
<td>9</td>
<td>2</td>
<td>21</td>
<td>2</td>
</tr>
<tr>
<td>10</td>
<td>0</td>
<td>22</td>
<td>4</td>
</tr>
<tr>
<td>11</td>
<td>5</td>
<td>23</td>
<td>2</td>
</tr>
</tbody>
</table>

and the dotted line curve the north and south component of the horizontal lunar tidal force for the corresponding hour angles and declinations of the moon. It will be observed that the curve of the earthquake frequencies shows a much closer correspondence to the curve representing the east and west component of the lunar tidal force than to the curve representing the north and south component. A curve repre-

![Fig. 4](image-url)

*Fig. 4. Curve showing occurrence of deep-focus earthquakes distributed by hourly intervals following moon's passing meridian of epicenter.*
senting the resultant of the north-south and east-west curves would resemble the earthquake frequency curve with striking similarity.

It should be stated that Dr. Sharpe's list shows that the bulk of these deep-focus earthquakes occur in four regions—the Japanese archipelago, the East Indies, the west coast of South America and the Himalayas—the greater number of deep-focus quakes in the list lying in the first three regions mentioned.

Assuming that the frequency of deep-focus earthquakes approximates a sine curve, we may write

\[ m = A \sin (2\theta + \omega) + z, \]

where \( m \) represents the frequency, \( \theta \) the hour angle of the moon from the local meridian of the epicenter, \( \omega \) a phase angle and \( z \) a constant. It is obvious that \( z \) cannot be zero since negative frequencies would be without meaning. The value of \( z \) will be very close to the average frequency per hour throughout the day.

In expanding the equation we may write

\[ m = x \cdot \sin 2\theta + y \cdot \cos 2\theta, \]

where

\[ x = A \cos \omega, \quad y = A \sin \omega, \]

whence

\[ \omega = \tan^{-1} \frac{y}{x}, \quad A = \frac{x}{\cos \omega} = \frac{y}{\sin \omega}. \]

We may solve for the most probable values of \( A, \omega, \) and \( z \) by utilizing 24 normal equations corresponding to the values of \( m \) indicated in the data of Table IV. The coefficients of \( x \) and \( y \) will be given by the sine and cosine of \( 2\theta \) corresponding to intervals of 15°, or one hour in the changing hour angle. The normal equations will take the following forms:

\[
\begin{align*}
[a][x] + [ab][y] + [ac][z] + am &= 0, \\
[ab][x] + [bb][y] + [bc][z] + bm &= 0, \\
[ac][x] + [bc][y] + [cc][z] + cm &= 0,
\end{align*}
\]

where \( a = \sin 2\theta, \ b = \cos 2\theta, \ c = 1, \) in the frequency of the
disturbance per given hour angle, and the brackets represent the sums of the indicated products for the series of 24 equations.

The solution of these equations gives

$$x = 1.83, \ y = -1.47, \ z = 4.7,$$

whence

$$\omega = -39^\circ, \ A = 2.3.$$  

Substituting the numerical values in the equation

$$m = A \sin (2\theta + \omega) + z,$$

we may draw the most probable sine curve to fit the points represented by the data of Table IV.

![Graph showing tidal sine curve computed from least-square solution compared with occurrences of deep-focus earthquakes at hourly intervals following moon’s passing meridian of epicenter.](image)

Fig. 5. Curve showing tidal sine curve computed from least-square solution compared with occurrences of deep-focus earthquakes at hourly intervals following moon’s passing meridian of epicenter.

In Fig. 5 is indicated such a curve together with the points representing the original data (Table IV). If we substitute numerical values into the series of 24 equations written for each of the 24 lunar hour angles we may derive the residuals. If we compare these residuals with those derived from the arithmetical mean of the frequencies, we may take the squares of the residuals in the two cases to represent the relative “fitness” of the sine curve to the observations as compared with the hourly mean for the 24 hour interval. The ratio $[v^2] : [v'^2]$ is 1.33 : 1.77.*

* The expression $[v^2]$ represents the sum of the squares of the residuals derived from the sine curve, and $[v'^2]$ the sum of the squares of the residuals derived from the arithmetical mean.
When the data for the frequency curve are smoothed by taking moving averages of three equally weighted points the improvement in the fitness of the sine curve is remarkable as is indicated by the circles of Fig. 5. If these running means are substituted for the individual points the residuals from the sine curve and from the arithmetical mean are as follows:

<table>
<thead>
<tr>
<th>Hour Angle of Moon</th>
<th>Running Mean of 3</th>
<th>From Equation</th>
<th>From Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>4</td>
<td>0.75</td>
<td>0.56</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
<td>-0.33</td>
<td>0.11</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>-2.53</td>
<td>6.40</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td>-1.49</td>
<td>2.22</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
<td>-0.96</td>
<td>0.92</td>
</tr>
<tr>
<td>5</td>
<td>7</td>
<td>0.16</td>
<td>0.26</td>
</tr>
<tr>
<td>6</td>
<td>6</td>
<td>-0.15</td>
<td>0.23</td>
</tr>
<tr>
<td>7</td>
<td>5</td>
<td>-0.07</td>
<td>0.01</td>
</tr>
<tr>
<td>8</td>
<td>4</td>
<td>0.13</td>
<td>0.02</td>
</tr>
<tr>
<td>9</td>
<td>2</td>
<td>-0.91</td>
<td>0.83</td>
</tr>
<tr>
<td>10</td>
<td>2</td>
<td>-0.44</td>
<td>0.19</td>
</tr>
<tr>
<td>11</td>
<td>3</td>
<td>0.44</td>
<td>0.19</td>
</tr>
<tr>
<td>12</td>
<td>4</td>
<td>0.75</td>
<td>0.56</td>
</tr>
<tr>
<td>13</td>
<td>4</td>
<td>-0.33</td>
<td>0.11</td>
</tr>
<tr>
<td>14</td>
<td>6</td>
<td>0.47</td>
<td>0.22</td>
</tr>
<tr>
<td>15</td>
<td>8</td>
<td>1.51</td>
<td>2.28</td>
</tr>
<tr>
<td>16</td>
<td>8</td>
<td>1.04</td>
<td>1.08</td>
</tr>
<tr>
<td>17</td>
<td>7</td>
<td>0.16</td>
<td>0.03</td>
</tr>
<tr>
<td>18</td>
<td>6</td>
<td>-0.15</td>
<td>0.02</td>
</tr>
<tr>
<td>19</td>
<td>5</td>
<td>-0.07</td>
<td>0.01</td>
</tr>
<tr>
<td>20</td>
<td>4</td>
<td>0.13</td>
<td>0.02</td>
</tr>
<tr>
<td>21</td>
<td>3</td>
<td>-0.91</td>
<td>0.83</td>
</tr>
<tr>
<td>22</td>
<td>3</td>
<td>-0.44</td>
<td>0.19</td>
</tr>
<tr>
<td>23</td>
<td>4</td>
<td>0.44</td>
<td>0.19</td>
</tr>
</tbody>
</table>

\[ \bar{v} = 4.7 \]
\[ v' = 17.48 \]
\[ v'' = 68.96 \]

Considering the relatively small amount of data of deep-focus earthquakes available, it is surprising that the result is as significant as it is. One can say that since the residuals are significantly smaller in the case of the sine curve than in that of the arithmetical mean when the residuals are derived from them, there is some basis for the supposition of a sine curve hypothesis.
In conclusion it may be said that for particular regions of marked seismic activity such as that in the neighborhood of the Japanese archipelago and the Philippine Islands, the number of major seismic disturbances shows a tendency to follow preferential positions of the moon with maxima occurring near the times of maximum tidal stresses. In the case of well-established deep-focus disturbances, the frequency of such phenomena appears to follow more significantly a tidal sine curve. With the rapid accumulation of data relative to such deep-seated disturbances, it may be possible ultimately to gain some knowledge of the building stresses preparatory to the occurrences of earthquakes. A knowledge of the variation in the number of earthquakes accompanying changes in the lunar tidal stresses should help in this direction. One cannot look upon changes in lunar gravitational potential as an adequate cause for earthquakes, but it is not unreasonable to assume that with the building up of stresses to critical values, more major seismic disturbances will occur at times when the added tidal stresses imposed by the moon are near maximum than at times when they are otherwise. One cannot be too cautious, however, in interpreting results and the possibility may have to be entertained that in the apparent correlation noted, some agent other than the moon itself, as for example loading of ocean tides, is a more direct factor in the situation.
THE WAR COMPLEX OF THE PLAINS INDIANS

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ABSTRACT

Warfare was one of the main concerns of the Indian tribes of the Great Plains of North America. A survey of the literature shows that war parties were sporadic but ever-present contingencies. The war path was distinguished from the path of peace by a set of rituals which served also to accent the importance of warrior status to the scale of political and social prestige. The motivations and associations of this warfare link it with the emotional stress of mourning and the practice of scalping is revealed as fundamental to such a linkage.

Popular documents dealing with the warfare of the aboriginals of the New World have fluctuated between the opinion that the red man was a blood-thirsty demon in the throes of an innate savagery and the conviction that he was the noble scion of a dying race which defended its mother-right with heroic courage. Anthropological monographs have dealt rather summarily with warfare, usually being confined to discussions of weapons as elements of material culture and to such of its aspects as fell within the ceremonies of social and religious units. In addition, the warfare of the Plains Indian has come to the attention of anthropologists with an interest in sociology and from this attention has arisen the practice of describing the hostilities of these tribes as an individual competition for social prestige.

The present paper aims to investigate certain aspects of Plains warfare, chosen for their significance in depicting the connections between the war complex and Plains culture as a whole. The problems include: the system of graded war honors and its relation to the life of the individual; the effect of this system upon the fighting situation; the organization of warfare around the war party; the war path
and war dances and the contacts between each of these and society; and lastly, the significance of scalping to the emotional integration of the culture. The final question of prime interest concerns the validity of a description of Plains warfare solely in terms of individual competition.

The paper treats of the typical Plains war complex. An attempt has been made to obviate the danger of over-generalization which accompanies such a treatment by so arranging the footnotes as to indicate tribal distributions.\(^1\)

**The System of Graded War Honors**

The most immediately distinguishing feature of the Plains war complex is that commonly known as the *counting of coup*. This phrase is variously used in the literature to cover several separate, but related, military phenomena which were unfamiliar to the whites who first encountered them, and it has generally been accepted as a blanket term.

The *coup system* depended upon a system of graded war honors.\(^2\) Two or four specifically defined acts of bravery were singled out by each tribe and arranged in a fixed scale; the warrior performing the deed which held first place, received most honor, and so on down the line. Other brave acts might also be recognized and honor accorded to the men performing them; although these were kept distinctly separate, they were not graded among themselves but formed a rather indefinite group below those of the scale.\(^3\) In addition, there seem sometimes to have been a few acts which stood at the pinnacle of bravery which were of such rare occurrence or of such a special nature as not to war-

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\(^1\) In the footnotes the number in parentheses refers to the reference so numbered in the bibliography.

\(^2\) Assiniboin: (13) 560; Blackfoot: (12) VI, 10; Gros Ventre: (47) 191.

\(^3\) Hidatsa: (17) 145; Iowa: (77) 204; Kansa: (1) 752; Mandan: (66) 386.

\(^4\) Crow: (12) IV, 179; (58) 230; (59) 253, 254; Omaha: (24) 331; (30) 437; Ponca: (30) 440; (86); Sarsi: (12) XVIII, 104; Teton: (12) III, 22; Wichita: (23) 7.

\(^5\) Crow: (12) IV, 179; Iowa: (77) 205; Kansa: (1) 752; Mandan: (12) V, 147; Teton: (12) III, 22.
rant a place upon the regular scale. The *coup count* of each man consisted in the number and rank of the deeds which he had performed. The tribes differed in the amount of honor given the same act and the systems of graded war honors varied accordingly.

*Coup*, in a strict sense, was *one among* the deeds for which a man received honor. It consisted in touching, with the hand or with an object held in the hand, some portion of the body of an enemy. Thus, to *count* or *strike coup* was to get in close physical proximity with an enemy. A coup was honored, not because of any result which accrued from the act itself, but because of the danger which had been successfully overcome, and the emphasis naturally fell upon the conditions under which the act was accomplished. Coup might be counted on the body of an enemy fallen in battle by those first to reach the body and count the coup; it might be counted only once, or by two, three, or four warriors; sometimes on only the first enemy slain in a battle or, on the other hand, upon an enemy corpse accidentally discovered, provided that it was within a certain state of preservation. Coup might also be counted on the wounded, or, on an unwounded warrior capable of defense and intent upon his own offensive attack. An accidental

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1 Crow: (58) 231; Teton: (12) III, 22.
2 Assiniboine: (13) 593; Crow: (58) 230; (59) 253; Cheyenne: (38) 303; (39) II, 36; Gros Ventre: (47) 191, 199; Iowa: (77) 205; Kansa: (1) 752; Mandan: (12) V, 147; Ponca: (86); Teton: (12) III, 22; (66) 159.
3 Wichita: (23) 7.
4 Crow: (55) 55; Gros Ventre: (47) 191.
5 Iowa: (77) 205; Omaha: (30) 437; Osage: (25) 116; Ponca: (30) 440; (86).
6 Cheyenne: (39) II, 31.
7 Arapaho: (39) II, 32; Assiniboine: (13) 559; (57) 31; Hidatsa: (17) 145; Mandan: (12) V, 147; Omaha: (24) 329 (note disagreement of authorities, see note 4 above); Santee: (28) xxvii; (72) 220; Teton: (15) 359.
8 Crow: (12) IV, 179; (55) 55; (58) 236; Osage: (85) 36; Teton: (12) III, 23; (15) 359.
9 Arapaho: (46) 23.
10 Blackfoot: (12) VI, 10; Omaha: (30) 437.
11 Blackfoot: (12) VI, 10; Cheyenne: (39) II, 30; Crow: (55) 55; Mandan: (66) 391; Omaha: (30) 437; Ponca: (30) 440; Teton: (12) III, 22, 189.
physical contact during battle was not rated as a coup; true coups on living and active enemies were performed deliberately and before witnesses. As demonstrations of bravery and individual prowess, coups might themselves be graded according to the order in which they were counted or according to the degree of danger involved in their performance, thus forming a separate scale which dovetailed into the war honor scale described above, but did not supersede it. Other acts such as scalping and killing might be ranked in somewhat the same manner but grading of coups was more common.

The term coup is frequently applied to any war deed for which the degree of honor was tribally standardized. In so far as the main element of the coup was the personal daring displayed and since war deeds were graded according to a standard of individual prowess, such usage may be justified. Certainly the situations described often emphasize this factor. Yet, as has many times been pointed out, scalping frequently involved a minimum of risk. And there can be little doubt that the taking of a scalp was one of the deeds on the war honor scale. In view of these facts it seems desirable to limit the use of the term coup to the physical contact described above. The literature some-

1 Blackfoot: (12) VI, 10; Cheyenne: (39) 1, 69, 78, 232, 272; II, 30, 202, 228; Omaha: (24) 330–1; (30) 437; Ponca: (30) 439; Teton: (12) III, 22.
2 Even among the Cheyenne, where interest centered so largely in coup, there is evidence of the existence of a distinct war honor scale which included (1) coup, (2) the capture of a shield, (3) the capture of a gun and (4) the taking of a scalp. (Grinnell, 1910, 303; 1923, Vol. II, 36.) The Omaha and Ponca furnish excellent examples of the way in which graded coups might influence the war honor scale. Thus, a four point scale: (1) to count coup, (2) to kill, (3) to scalp, and (4) to decapitate, or, with the Ponca, to capture horses, becomes a six point scale: (1) to count coup on an unwounded enemy, (2) to count coup on a wounded enemy, (3) to count coup on a dead enemy, (4) to kill, (5) to scalp, and (6) to decapitate, or, as above, to take horses.
3 Assiniboine: (57) 31; Ojibway: (14) 62.
4 Mandan: (66) 386–7; Teton: (15) 359; (66) 159.
5 Arikara: (78) 1, 247; Assiniboine: (57) 31, 68; Blackfoot: (61) 421; Cheyenne: (38) 303; (39) II, 36; Cree: (76) 535; Iowa: (77) 205; Mandan: (12) V, 147; Ojibway: (14) 62; Omaha: (30) 437; Ponca: (30) 440; (86); Sarsi: (12) XVIII, 104; Sioux: (10) 72; (12) III, 22; (28) xxvii; Wichita: (23) 7.
times employs the phrase war honors to indicate all war deeds with standardized degrees of honor, and preference is here given to that term.

War honors might be classified into two groups: coup and the taking of something from the enemy, whether gun, life, scalp or horse. Such a classification would, however, give an entirely false emphasis. As a matter of fact, and quite simply, war honors displayed the military virtues. "Courage," as we know it, is completely insufficient to describe these. Personal fortitude and the protection of women and children, even at great personal risk, were taken as matters of course and did not rank as war honors, whereas "thievery," according to the white man's standard, most emphatically did. Among the military virtues were fearlessness, the capacity to make successful surprise attacks, the power to overcome an antagonist or show superiority to him, and the ability to carry out successful war projects. All of these were connected with the warrior's relationship to the supernatural. But such virtues were not mere abstractions. Each man's military prowess was directly in terms of the deeds he had performed, his coup count was his virtue. After each deed had been accredited it was, with all its accompanying circumstances, kept clearly distinct in the series.

Coup, as has been said, was one among possible war honors. Aside from its emphasis on individual achievement, it had at least two other significant factors. On the open Plains the conditions for long-distance shooting were particularly good. However, the desire to count coup made for a general rush toward a fallen enemy which tended inevitably to decrease the distance between the contending parties and to encourage hand to hand encounters. Coups on living warriors had a similar effect. The other factor, also, tended to keep battles, which were primarily composed

1 (8) 51; (19) II, 208; Blackfoot: (38) 200; (39) II, 33; Cheyenne: (38) 299, 300; Crow: (12) IV, 179; (55) 23-4; Iowa: (77) 204; Omaha: (24) 328; (30) 431, 434, 437; Ponca: (30) 439; Teton: (81) 57; Wichita: (23) 7.
of separate individual engagements, more concentrated than scattered. Although each fighter was occupied with his own opponent, the fact that part of his prestige could be derived through the success of his companions kept him supremely aware of activities over the entire battle ground. Certain war honors seem, indeed, to have specifically recognized this mutual relationship between warriors.\(^1\)

Just as coup emphasized the element of spectacular daring so also did other feats such as the capture of a gun or other weapon\(^2\) and the taking of a shield.\(^3\) The capture of horses is generally classed with these feats. However, it seems evident that unless taken in battle\(^4\) such an act entailed other elements as well. To count as a war honor, horses had to be stolen from the camp of the enemy,\(^5\) had even to be snatched while picketed in his very shadow.\(^6\) There was no dash nor abandon in such an act. The powers exercised were those which might be called "stealthy" and involved an iron control of every movement, complete silence and deception when necessary—the same powers which were indispensable to the successful surprise attack. The suspense of such an act was great. Its success lay in escaping undetected and in evading or outdistancing any pursuit which might be organized when the theft was discovered.

The scout, who likewise exercised the "stealthy" powers, and whose activity contributed to the success of the war venture, was sometimes formally recognized.\(^7\) But the more important of war honors of this nature went to the

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\(^1\) Crow: (12) IV, 179; (58) 231; Teton: (15) 359.

\(^2\) Assiniboin: (13) 560; Blackfoot: (12) VI, 10; (38) 302; Cheyenne: (38) 303; (39) II, 36; Crow: (12) IV, 179; (58) 230; (59) 253; Gros Ventre: (47) 191, 199; Mandan: (12) V, 147; Sarsi: (12) XVIII, 104.

\(^3\) Blackfoot: (38) 302; Cheyenne: (38) 303; (39) II, 36.

\(^4\) Blackfoot: (12) VI, 10.

\(^5\) Assiniboin: (13) 560, 593; (57) 31; Kansa: (1) 752; Mandan: (12) V, 147; Ojibway: (51); Ponca: (30) 440; Sarsi: (12) XVIII, 104; Sioux: (12) III, 22; (66) 152.

\(^6\) Blackfoot: (12) VI, 10; Crow: (12) IV, 179; (58) 230; (59) 253.

\(^7\) Teton: (12) III, 22; (66) 159; Wichita: (23) 3.
leader who had organized and carried through a successful war party.¹

The mortality in Plains fighting was highest when attacks took the enemy unprepared for defense or when offending parties were pursued and overtaken. In such cases the weaker groups were often completely annihilated. The mortality of pitched battles, which were of more frequent occurrence than is generally supposed, was considerably lower. War parties set out with a particular end in view and victory lay in the accomplishment of that purpose and in the safe return of the party.² The leader was responsible for the safety of his men and any loss dampened the sense of triumph. Each man sought his own glory but never to the extent that he lost sight of the ultimate triumphant home-coming of the war party of which he was a member. Unnecessary endangering of lives was consequently avoided. Yet whatever else a battle may be it is primarily a situation in which men kill and are killed. Active participation in such fighting, as evidenced by wounds,³ counted as a war honor. And the act which occurred most frequently on the war honor scale was the killing of a foe.⁴

In his intensive study, one of the few of its kind, of the effect upon the Pawnee of prolonged Sioux hostility, Lesser points out that cholera and "the relentless assaults of the Dakota and Cheyenne" were responsible for a twenty-five per cent reduction of the Pawnee in four years.⁵ His data

¹ Assiniboin: (57) 66; Blackfoot: (12) VI, 10; Crow: (12) IV, 179; (58) 230; (59) 253; Iowa: (77) 203, 204, 205; Teton: (12) III, 22.
² Arapaho: (46) 23; Blackfoot: (1) 458; (12) VI, 10; (37) 253-4; Assiniboin: (13) 409, 555; Cheyenne: (39) II, 40; Crow: (58) 237; Gros Ventre: (6) 318; (47) 192; Kansa: (80) 375; Kiowa: (68) 291; Mandan: (88) 123; Ojibway: (14) 74; Pawnee: (78) II, 722; Sioux: (12) III, 186.
³ Assiniboin: (13) 559; (57) 68, 73; Crow: (58) 232; Gros Ventre: (47) 192; Sioux: (12) III, 22; (74) II, 58.
⁴ Arikara: (78) I, 247; Assiniboin: (13) 560 (see contradiction on page 559); (57) 25, 31, 66, 73; (66) 200; Blackfoot: (61) 421; Gros Ventre: (47) 191, 196, 199; Hidatsa: (17) 145; Iowa: (77) 205; Kansa: (1) 752; Mandan: (12) V, 147; (66) 386-7; Ojibway: (14) 62; Omaha: (24) 331; (30) 437; Osage: (85) 36; Poneca: (30) 440; (86); Sarsi: (12) XVIII, 104; Sioux: (12) III, 22; (15) 359; (66) 150, 152, 159; (72) 220; (74) II, 58; Wichita: (23) 7.
⁵ (52) 15.
are so overwhelmingly against the classical description of the Plains war complex that he feels called upon to add in a footnote: "warfare as carried on by the Sioux in the latter half of the nineteenth century was not the typical pattern of Plains warfare."  

A strange conclusion indeed! The Sioux certainly belong to Plains culture and the great bulk of material upon which any "typical pattern" can be based comes from the latter half of that same century. The Pawnee, under white influence, were relatively defenseless and the pattern of Plains hostility was such that it took immediate advantage of a foe in that condition.

Plains warfare is classically typified as a great game. The prestige of the warrior depended upon his war honors and these were his counters in the game of war, yet, to insist upon such terminology is to attribute a playfulness to Plains warfare which was entirely lacking. The competition for war honors might have been a game among warriors, but the only game played by a warrior and his enemy was the gamble of life and death.

Mutilation of the dead (see page 451) was common, but only scalping and decapitation found their place among the deeds of honor.

Scalping and the capture of horses played a greater part in Plains warfare than the war honor scale would suggest. Indeed, Grinnell has expressed the opinion that only with the coming of the horse did the Blackfoot begin their "systematic sending forth of war parties against neighboring tribes." Both the horse and the gun loomed large among the "indirect Caucasian influences" which rocketed the Plains area to a "cultural intoxication" which is hardly preceded in anthropological annals. With due consideration to their cultural importance, there is, how-

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2 *Iowa: (77) 205; Omaha: (24) 331; (30) 437; Osage: (21) 411; (25) 120.
3 Discussion temporarily postponed. See pp. 452–460.
4 (37) 243.
5 (48) 395.
ever, no conclusive evidence that they revolutionized war procedure. Apparently, their effect was not radically to change the existing war complexes, but to accelerate the momentum of warfare.

The Plains Indian desired horses, yet, however often white men may have found it convenient to name Indian war parties *horse raids*, this desire alone was not sufficient cause for going upon the war path. The shield which the warrior captured was a medicine bundle object and of no utilitarian use to its captor. To take a gun from an enemy was a mark of valor but to pick up one on a battlefield was expediency not bravery. Bodies of white men were more apt to be despoiled than those of Indians, not because corpses were ever systematically plundered, but because only such objects were taken as were of peculiar value or interest. There is no evidence, in Plains material, that robbery ever motivated homicide.

Beyond the maintenance of certain hunting privileges the tribes were not interested in land conquest. With the coming of the horse, intercourse between tribes was greatly facilitated and trade was brisk. It is also important to bear in mind that a great deal of the fluidity of commercial exchange was due to the emphasis upon gift giving. At any rate, there is no reason to believe that warfare was ever an integral part of Plains economy, as it was, for example, among the marauding bands of the Southwest, nor that the prevalence of horse stealing rested upon a purely economic motive.

In view of the somewhat conflicting elements which have been referred to as existent in the acts of the war honor scale, the question arises whether there is not some one common element. The answer is in terms of the underlying cultural interest in self aggrandizement.

Each of the war honors served to demonstrate the superiority of the warrior over his antagonist. To wrest away the weapon with which a man is fighting, to slap an armed enemy in the face and then turn your back on him,
to snatch his prized horses away from under his very nose, these were demonstrations of superiority. Certainly the slayer, under most circumstances, might consider himself superior to the slain. The importance lay not only in doing these things but in doing them without injury to oneself.

The success of the leader was counted by the number of successful expeditions he had led, not in the amount of damage which had been done to the enemy nor in the number of war honors which had been performed under his leadership. Horses and scalps were the proofs of success and, although the first tended to be brought in some numbers, one or two scalps furnished success enough even for a large war party. Groups were successful if they had shown themselves superior to the enemy, however slightly, and had returned safely to tell the tale.

The recounting of war honors, the reënacting by warriors of their martial deeds, was an elaborate boasting. The prestige of each man depended upon his superiority over the enemy and upon his relative standing with other men. This superiority was figured in terms of war honors and it was in this connection that the minute details of the fighting situation gained importance. Men might perform like deeds but never the same deeds; and equally brave men could, nevertheless, differentiate their individualities by such means. No warrior of the Plains was ever as brave as another, for each was brave in his own right, a fact to which his war honors most ably testified.

**The War Party**

Both defensive and aggressive warfare were recognized by the Plains Indian. Defense meant two things: first, the pursuit of small enemy parties which had been successful in driving off horses or in securing a few scalps, and second, actual defense from attack. Neither of these called for any ceremonial organization.

Pursuit parties tended to be composed of those who had suffered at the hands of the pursued, and their relatives, or
of the warrior society which was on police duty at the time.\footnote{Cheyenne: (63) 121.} Unless the anger aroused by such raids was sufficient for the spontaneous formation of a party, no pursuit was attempted.\footnote{Mandan: (34) 57.}

Defense from concerted attack naturally involved the full fighting strength of the camp under whatever men of authority happened to be present. Attacks upon camps of fighting strength equal to that of the attacking force frequently resolved into a lining up of the opposing parties, supremacy being decided through a sort of dueling in which the spectacular powers of a few men were featured. States of siege were also known and although an encampment might be impermanent, it was defended to the death by men, women and children.\footnote{Arikara: (84) 23; Crow: (12) IV, 92; Hidatsa: (65) 61; Mandan: (66) 322; Sioux: (12) V, 105; (14) 113.} Yet when an attack was completely unexpected, allowing no time for preparation, no defense was attempted. In such cases safety consisted in flight and concealment, and depended upon the self-reliance of the individual or of the family group.

It is important to remember that the attitudes of Plains culture were positive rather than negative. All men were warriors of greater or less repute, just as men might, similarly, be hunters or doctors. A man's special powers depended upon the nature of the supernatural powers he had acquired, and, since each man achieved some such powers, each had his role to play. To be a warrior was to have warrior powers. These, like others, were always accompanied by the rites or objects belonging to them. To be a warrior, therefore, meant to assume also these acts or objects. If a man's warrior power was connected with a song of five minutes duration, a hawk wing to be worn in his scalp lock, and a face paint of two colors and an intricate design, he could not manipulate the power unless he sang the song, donned the hawk wing and painted his face. With these he was a warrior, without them he was a desperate man.
faced by danger to himself and to his family. The assumption of warrior powers was an individual necessity for all fighting, whether offensive or aggressive.

The distinction between the war party for defense and that for aggression did not depend upon the mode or manner of fighting. It depended entirely upon which party had assumed the initiative. All the ceremonial care of organization and procedure was lavished upon the war party which began the hostilities. It is this aggressive war party to which the term is more accurately applied.

Such distinctions as were made by various tribes between different types of war party seem to have been based upon the relative amount of ceremonial required by each, rather than on any other considerations. To a certain extent at least the amount of ceremony depended upon the degree to which the tribe as a functioning unit was involved in the expedition; on the other hand, the man-power, or size, of the party might vary indefinitely. Although the average size of the war party might differ for each tribe, it can be said with certainty that the Plains war party was frequently, but not typically, a small one. Whether a war party consisted of one warrior or a man and one or two of his most intimate friends, or of one to four hundred warriors, or even of the whole tribe the purpose and general form of its procedure did not change.

A war party was always organized by a man who wanted to go to war himself and who acted as leader or other important functionary during the entire period of the party's existence. Wherever tribal sanction came into play at all it operated by influence brought to bear in one way or another upon this leader. It was assumed that all boys longed

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1 Assiniboine: (13) 544; Blackfoot: (90) 54; Crow: (55) 138, 185; (59) 254; Osage: (25) 115; Sioux: (15) III, 12-13; (15) 332.
2 Crow: (55) 138; Gros Ventre: (47) 197; Hidatsa: (65) 60; Iowa: (78) I, 171; Omaha: (30) 408; Teton: (12) III, 187.
3 Assiniboine: (6) 42; (66) 440; Crow: (78) II, 669; Ojibway: (14) 68.
4 Assiniboine: (6) 288; (66) 202, 387; (13) 544; (78) III, 1127, 1132; Blackfoot: (78) I, 220, 365; Sioux: (15) 332; (42) 358; (84) 22.
5 Omaha: (24) 319; Teton: (15) 332.
to go to war, an assumption naturally adopted by the boys themselves, and free play was given for the exercise of this longing.

The importance of the central tribal authority depended entirely upon the political set up of each tribe. In general it may be said that the leaders of the group, who had themselves achieved success as warriors, exerted their influence, not to suppress belligerent actions, but to divert them into the war party channel and to allow this fullest expression only at such times as it would not react to the disadvantage of the rest of the group.

Characteristically, a supernatural sanction or guarantee of success was the main distinction between the defensive and the aggressive war party and, likewise, the element of the aggressive party which most concerned the non-participating members of the group. This power (see page 439) was over and above the warrior powers of individuals and gave assurance that the party, as such, would achieve success in the particular venture which was under contemplation. The organization and ceremonial of the war party was largely concerned with obtaining and manipulating this power.

The authority of leadership ceased "with the end of the war"¹ or "with the occasion that called it forth."² This authority might be enforced by disciplinary measures³ but, since it depended upon a power to which participants and non-participants alike looked for success, it was generally expressed as the responsibility of the leader for the welfare of his men.⁴ It was peculiarly the function of the relationship between the members and the leaders of expeditions, which, although they might be organized through enlistment⁵ or invitation⁶ were always felt to be completely

¹ Ecuador: (44) 2.  
² Chipewyan: (31) 159.  
³ Santee: (28) 153; (74) IV, 60.  
⁴ Blackfoot: (61) 420; Crow: (58) 237; Gros Ventre: (6) 317; Omaha: (30) 409; Wichita: (23) 15.  
⁵ Blackfoot: (12) VI, 9; (61) 419; Cree: (89) 337.  
⁶ Assiniboin: (57) 28; Iowa: (77) 201, 202; Omaha: (24) 315; Osage: (25) 115; Ponea: (86); Sioux: (15) 332; (62) 139; Wichita: (23) 7.
voluntary. The more self-evident were the warrior powers of a man the greater were his chances of being accepted as a leader. In societies aristocratically¹ or ceremonially² stratified, only men in certain positions might be considered qualified for martial leadership, but war leadership in North America seems never to have been in the hands of a particular set of permanent officials, and membership in war parties never lost its voluntary character.

There was a positive, but not a one to one, correlation between warrior powers and the powers attendant upon the organization of a war party. Both powers were achieved by the same means. The obvious similarities of the two being offset by the fact that war party powers were operative only for the particular war party in consideration, and accrued to the personalities of the persons who obtained them, not directly, but in the form of war honors. Both these powers were conceived as being individually attained. It is quite possible that in practice sociological factors, such as membership in certain societies because of family prestige, and the inheritance of certain war medicines, influenced the possession of war powers. It is true, however, to a striking degree that war activities may be considered almost without reference to such factors.

Women occasionally accompanied their husbands upon aggressive war parties³ or went upon their own initiative.⁴ As a rule, however, war powers were sought by men. The role of the woman was the encouragement of her male relatives and her main opportunity to demonstrate bravery was in cases of attack.

There is some evidence to show that boys on their first war party were shown special attention.⁵ Male children

¹ Aztec: (3) 129; (70) I, 67; Creek: (82) 427; Natchez: (19) II, 354; (83) 217.
² Southwest.
³ Omaha: (30) 409.
⁴ Arikara: (6) 272; Gros Ventre: (47) 216-221; Hidatsa: (17) 145; Kiowa: (68) 312; Sioux: (11) 326.
⁵ Ojibway: (14) 84; Teton: (12) III, 12; (15) 375.
were accustomed to treat enemy remains as living enemies⁴ (see page 452) and their games often carried through the whole procedure of the war party.⁵ The psychological preparation was such that the beginner was "impelled to reckless deeds of valor."⁶

Information is sadly confused regarding the officials or authoritative persons of a war party. Occasionally in the literature distinctions are drawn between the "leader" and the "partisan" which are useful but insufficient. A tentative terminology based upon function is necessary to clarify the organization of the war party. A large part of the significance of the system outlined below rests in the fact that it is applicable to all of North America east of the Rocky Mountains.

The war party was led upon its journey by a man who may be called its military leader.⁴ This man might have had to obtain the supernatural power necessary for leadership from another⁵ who was the mediator of power or he might get the power himself,⁶ in which case he may be known as the partisan. The mediator of power in aberrant cases accompanied the war party. The partisan who possessed the war party power and led the expedition might select other men as military leaders.⁷ Assistants were often appointed to serve both military leaders and partisans; the assistants of military leaders are commonly known as lieutenants,⁸ while those of partisans may be called kettle-bearers. Kettle-bearers might function in relation to the

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1 Blackfoot: (66) 275; Cheyenne: (63) 263; Omaha: (30) 434.
2 Crow: (55) 23-4; Gros Ventre: (47) 182; Mandan: (20) 319; Santee: (28) 51.
3 Sarsi: (12) XVIII, 104.
4 Blackfoot: (12) VI, 9; Omaha: (30) 409; Sioux: (74) IV, 62.
5 Cree: (89) 337; Crow: (87) 195; (58) 232; Omaha: (30) 416; Sioux: (12) III, 196; (28) 153; (74) II, 184.
6 Arapaho: (11) 41; Assiniboine: (13) 544; (57) 28; Gros Ventre: (6) 293; Kansas: (26) 670; Ojibway: (51); (64) 162; Osage: (25) 115; Teton: (15) 348; Wichita: (23) 7.
7 Kansas: (26) 670; Mandan: (88) 123; Osage: (25) 116; Wichita: (23) 7.
8 Iowa: (77) 201; Ojibway: (14) 91; Wichita: (23) 7.
ceremonial procedure\(^1\) which occurred before setting out upon the journey, or this role might be taken over by a man, also appointed by the partisan, who was to carry the medicine bundle.\(^2\) The distinctive character of the kettle-bearer lay in his connection with cooking arrangements \(^3\) and with the control of certain eating and drinking customs \(^4\) which were common features of war party practice.\(^5\)

With the exception of the position of military leader, which is a role called for by the concept of the war path discussed below, all of these positions were functions of the war party as such and belonged to its ceremonial. None of them have any reference to the mode of fighting. The only position which fulfills any such purpose is that of scout.\(^6\) The *scout* was appointed by the military leader or partisan and served both to protect the party from a surprise attack and to locate an enemy encampment. The actual fighting was entirely in the hands of the warriors and was completely distinct from the ceremonial conditions of either the war party or the war path.

**The War Path**

On the Plains aggressive war activity was encompassed in one blanket concept—the war path. This term occurs early in literature on North America, and is applicable in its full force to this inland area which came into the road of travel relatively late. The war path began when any particular man conceived the idea of getting together any war party and ended with the last dance commemorating

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\(^1\) Kansa: (26) 672; Osage: (25) 116.
\(^2\) Crow: (58) 233; Iowa: (77) 201; Ojibway: (64) 162.
\(^3\) Hidatsa: (69) 320, N.B.; Iowa: (77) 201; Omaha: (30) 409; Osage: (25) 116.
\(^4\) Arapaho: (11) 42; Cheyenne: (39) II, 23–5.
\(^5\) Arapaho: (11) 41; Assiniboin: (57) 12; Cree: (31) 81; Gros Ventre: (47) 191.
\(^6\) (11) 230–1; Assiniboin: (13) 549; Cheyenne: (39) II, 228; Crow: (12) IV, 93, 105; (58) 234; (59) 254; (63) 91; (68) 292; Gros Ventre: (47) 191; Iowa: (77) 203; Omaha: (30) 423; Sioux: (15) 332; 379; (74) IV, 62; Wichita: (23) 3.
the return to the home camp. It was co-extensive with the duration of the war party.

The war path was thought of as a definite road in a number of tribes. The thought was most articulate for the road 1 of the journey itself which was conceived as beginning with the first step 2 taken in leaving the camp, continuing through enemy territory, and ending with the entrance into the home camp. To only a limited degree was this road an actual trail or route 3 through known territory, although a leader would tend to direct his men over ground with which he was familiar. In general, the "road" had a strictly ceremonial significance which influenced the order of march rather than its direction.

"We are now concerned . . . with the preliminaries to setting out on an expedition. In this and particularly in respect to the ceremonial procedures, the available literature is rather deficient." 4 However, there is sufficient evidence to show that the first camp made by the war party after leaving its village was of special ceremonial importance. This camp was not that which would normally be made after the first day's journey, 5 but was a stop specially made for ceremonial purposes a short distance from the home village. 6 The participants in the party might gather at this point, which had been previously designated, or, leaving in a body, halt there for the requisite ceremonies. The war path was co-existent with the war party and the leader's first preparation for his position was a withdrawal from the village. This might be marked by the ceremonial construction of a lodge outside the village 7 and in such cases the ceremonies of the first stop or camp might take place

1 Arikara: (11) 41; Crow: (12) IV, 110; (58) 234; Mandan: (42) 371: (66) 387; Ojibway: (14) 84; Omaha: (24) 322; Teton: (15) 333, 375.
2 Ojibway: (14) 91; Osage: (25) 117.
3 Sioux: (42) 78, 140.
4 Wissler (1) 873.
5 Ojibway: (14) 74.
6 Blackfoot: (12) VI, 9; Kansa: (26) 674; Mandan: (66) 389; Omaha: (30) 416; Osage: (25) 119.
7 Kansa: (26) 675; Ojibway: (64) 161-2; Osage: (25) 118; Sioux: (74) IV, 52.
at this spot. Although the material is not clear, it would seem that even when a stop was made at the outside lodge in preference, or in addition, to a separate location, the concept of the first halt was retained. Such a camp, beyond the boundary of the village, reflected the substitution of the road of war for that of peace.

At the end of each day's journey a night camp was regularly set up. At this camp ceremonies might be performed (see page 444) but they did not contribute directly to the war path as such, except in so far as they showed a tendency to make the road "complete."

If circumstances allowed, a camp was made before an attack upon an enemy village took place. The ceremonies of this camp might occur at the regular nightly camp since attacks were generally at daybreak. Even in such cases a special halt was often called shortly before the attack.

After an attack there is indication that another camp or halt was called for. In raids in which the attacking force was separated, this was often conceived in terms of a rendezvous to which the warriors would return before taking up the homeward march. It is certain that a war party always returned in a body. There is evidence, however, that a ceremonial halt was made either "at the spot where they begin to turn back into their own country" or at a place at which they were safe from enemy pursuit. In the latter case it is not always clear whether the camp referred to was some place on the journey back, or whether it was the last camp before the entrance into the home village. Some tribes certainly made both and there seems little

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1 Winnebago: (71) 157.
2 Assiniboine: (13) 545; (78) III, 1127; Crow: (12) IV, 88; (58) 235; Iowas: (77) 203; Omaha: (24) 327; Sioux: (12) III, 186; (11) 326; (15) 379; (42) 264; (74) IV, 62.
3 (11) 326.
4 Assiniboine: (13) 545.
5 (45) I, 321.
6 (43) 60; (74) II, 57; Assiniboines: (13) 545; Cheyennes: (38) 299; Cree: (1) 535; Crow: (55) 234; (58) 236; Kiowas: (68) 292; Mandans: (66) 387; Ojibways: (14) 110; Omahas: (24) 328, 332; (27) 526-7; Osages: (25) 119; Sioux: (45) 360.
7 Sioux: (8) 52; (28) 153.
reason to doubt that others also of the tribes held such meetings. The material is scanty on both the ceremonial preparation of the scalps and on the determination of the war honors taken by a war party. The former was always, except in parts of the Southeastern Culture area, done before entrance into the home village; the latter seems also to have been determined before the return, what ceremonies occurred later being rather in the nature of an announcement to the tribe at large than any attempt to establish which honors were due each warrior.

There seems to have been no tribe of the Plains area, with the possible exception of the Arikara, in which the returning war party did not camp outside the village before making its entrance. In part, this was in order to prepare for the dances which followed; in part to make that entrance more impressive. But there can be no doubt that the return of a war party was a ceremonial affair which customarily involved the ushering of the successful warriors into the village by all 1 or some part 2 of those who had remained behind.

The actual entrance was an occasion of rejoicing in any tribe from which war parties went out. Although there is some evidence to show that the return of an unsuccessful party need not have been an occasion for tribal comment, 3 successful warriors were always the center of tribal festivities.

The war path was like a piece of string which is doubled so that its ends meet again: the setting out from the home camp, the first halt, the journey, the halt before fighting, the battle, the halt after the battle, the return journey, the halt before entering the home camp, and the entrance itself. The battle, the conflict, marks the point at which the string is bent back upon itself. Nowhere in the literature is there

1 Cree: (1) 535.
2 Assiniboine: (57) 30; Crow: (58) 235; Mandan: (88) 123; Ojibway: (14) 119, 143; (78) III, 1049; Teton: (15) 360.
3 (11) 415; Comanche: (36) II, 317; Cree: (78) II, 521; Gros Ventre: (6) 310.
a record of a war party which did not return after its first
fight. Chance meetings on the return trip were occasional-
ly taken advantage of to add to the victory, but there is
never any mention of an attempt to locate a second encamp-
ment of the enemy after the first had been encountered.
This was true also for the unsuccessful party: a fresh start
must be made from the home village. The length of the
journey varied considerably. Between the time the expedi-
tion was organized and the actual fighting, all ceremony and
every thought was turned to making the outcome of the
combat successful; as soon as the victory was won all atten-
tion turned to the ceremonies which took place when the
party would rejoin the other members of its band. This
becomes clearer as the nature of the ceremonies is investi-
gated.

The Indian is frequently reported as having been suc-
cessful in battle "because of his medicine" 2 without further
description as to its nature, or the report may run some-
thing as follows: "the leader of a war party . . . is careful
to be thought to act under the guidance of the Great Spirit,
of whose secret will he affects to be apprized in dreams, or
by some rites." 3 However inadequate these statements
may be, the differences inherent in them may not be over-
looked.

The great concern of every war party was the success
with which the expedition might be attended. As stated
above, the ceremonies of the war path up to the time of
battle were in large part to prepare for victory.

Frank Russel in speaking of the Pima has suggested
that magic "plays a larger part in the warfare of the
American Indian than is generally known." 4 The South-
west is one of the North American areas where imitative
magic has been most stressed, yet it cannot be denied that
magical control over the outcome of battle was not unknown

1 Crow: (58) 237; Osage: (25) 121; Wichita: (23) 15.
2 (56) 43-4.
3 (74) II, 59.
4 (73) 120.
in other regions. Very generally over the continent were found customs associated in some fashion with sexual intercourse which were considered harmful or beneficial to war activity. Little can be said of these from the available material except that they were often apparently unrelated to other cultural traits of the tribes in which they occurred. Means were sometimes employed by which natural forces, such as mist or dust, were brought to the assistance of an attacking or pursued party, and by which the enemy might be rendered incapable of efficient resistance. Unless the recounting of war deeds is to be considered as imitative magic, such magical techniques can be said to be normally absent from Plains warfare.

Prophecy was the great basis of assurance upon which the Plains Indian depended for success in war. Occasionally this took the form of a true oracle but much more commonly was expressed through the dream or the vision. Reports that success in war was sought by means of penance or self-torture should probably come under this heading, since these were frequent elements of the vision quest. Such dreams or visions might be interpreted as being either "favorable" or "unfavorable" to the success of the war venture. No stigma attached to unfavorable dreams, and either the warrior who had had the dream, or the entire party, immediately turned back from the venture the failure of which had thus been indicated.

The vision influenced the war path even more importantly through the possibility, or necessity, of foreseeing in this fashion the deeds which must transpire if the contemplated expedition was to be carried through. Not only the

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1 Crow: (58) 220, 224, 225. Also Eastern Woodlands, Southeast, and Northwest Coast.
2 Ojibway: (14) 85, 197; Omaha: (24) 321, 327; Santee: (28) 153.
3 Mandan: (66) 361.
4 Ojibway: (64) 161; (51); Assiniboin: (13) 495.
5 Arikara: (53) I, 175; (78) I, 247; Cree: (31) 81; Mandan: (40) 421; (66) 382, 401; Sioux: (78) I, 256; II, 629.
6 Assiniboin: (13) 490; (57) 28, 33; (78) III, 1112; Crow: (12) IV, 85, 105; (58) 232; (59) 8; (87) 185; Gros Ventre: (6) 317; Ojibway: (14) 86; (51); Sioux: (12) III, 159; (28) 153; (72) 225; (74) IV, 62.
chance of success or failure was predicted, but all the attendant details of the war path, such as the whereabouts of the enemy, the weather, the hunting conditions, the number of the enemy and the nature of the victory, might have to be indicated before the war party left the village, or during the journey itself. Such details were widely discussed and upon their acceptance often depended the fate of the war party, certainly its personnel.

The power to have prophetic visions might depend upon the ownership of a war bundle, i.e. a medicine bundle devoted to war. Or the war bundle employed might be tribal, kept under the guardianship of some society or kin group of the tribe. The character of such visions did not differ from those obtained solely on individual initiative: their prophetic nature was explicit.

One of the elements of success with which every member of the war party was naturally concerned was his own safety. To insure such safety he depended upon his skill as a warrior and upon certain articles from his own, or another’s medicine bundle, or upon specific "war medicines," or charms. Such charms played an important role as protection against bullets, but were by no means limited to this.

Wissler writes that "among the general features of the war trait-complex is the use of a war bundle. With the Central Algonkin we have specific bundles for this, but . . . the Pawnee took particular objects from bundles having general functions. It is not clear, but the Iowa may have done likewise, and the Ponca, Osage, Omaha, Oto and Kansa had war bundles. Among the Blackfoot there were certain bundles whose chief function was war. The use of individual war charms, or small bundles was highly developed among the Blackfoot, the Gros Ventre, and the Plains-Ojib-

1 Crow: (58) 232 ff.
2 Iowa: (1) 715; Kansa: (26) 671; Omaha: (30) 404, 405, 431; Osage: (25) 120; Ponca: (30) 439.
3 Assiniboin: (13) 553; (66) 257; Cheyenne: (39) II, 223; (63) 63; Crow: (58) 232, 234, 236; Gros Ventre: (47) 192; Mandan: (66) 388; Ojibway: (14) 98, 106; Sioux: (42) 430; (15) 350.
4 (1) 873-4.
way." It has been shown above that the function of these war bundles might differ considerably; those of the Southern Sioux and the Crow and Central Algonkin were mainly adjuncts to vision, whereas the Blackfoot charm was in large part protective. Both types of "bundle" were, however, used for prophetic validation. Indeed, in the whole medicine bundle area, in which the power of the bundle was so closely tied to the strength and personality of the individual, it is difficult to conceive of any war party being unmindful of its bundles, and their relation to the war path as such seems also to have been close.

War Dances

In 1819 Franklin found the Cree performing apparently similar dances "before going to war" and "on meeting with strange tribes." In either case, the dance involved certain definite components. Since so many of the social activities of the Plains contained a martial element, these components are present in a number of dances which would not ordinarily be classed as war dances. However, it is possible to show that these components are a part of war activity and connected with this rather than with any other behavior complex.

The most common military or war element used in non-military dances was that sometimes known as counting coup, called in this paper the recounting of war honors. In other than war dances, the war honors recounted were always those of the narrator and, however realistic, functioned to give him prestige and to differentiate his personality from that of others.

The recounting of war honors illustrates one of the main components of the war dance, the reënactment of the fighting situation. A returning war party rushed upon its home village in much the same manner as it would rush an enemy camp even though news had previously been sent ahead

\[^{1} (31) 81.\]
\[^{2} \text{Cheyenne: (38) 307; Crow: (59) 254; Gros Ventre: (6) 82; (47) 191; Kiowa: (68) 291; Teton: (15) 361.}\]
as to the details of the victory. An interesting expression of this tendency is found among the women in the scalp dance, who carried men's weapons or imitated the dress of men. The female relatives of a warrior seem in this way to have gained a certain right of participation in his victory and might, at future scalp dances, recount his war deeds. In other regions mimic battles were sometimes waged after the return of a war party and upon the Plains sham battles between visiting tribes were not unusual. But there the interest centered around the deeds of individual warriors and found full expression in the recounting of war honors, during which the most minute and graphic details of an engagement were reënacted.

Whether the interest in realistic reproduction of the fighting situation can be said to have influenced the locale of the war dance, it is not possible to say. It is true, however, that war dances commonly took place in the open.

In 1832 George Catlin wrote: "no part of the human race could present a more picturesque and thrilling appearance on horseback than a party of Crows, rigged out in all their plumes and trappings—galloping about and yelping, in what they call a war parade... an amusement of which they are excessively fond, and great preparations are invariably made for these occasional shows." This "tournament" not only imitated war behavior but also incorporated a second component of the war dance—the parade or procession. The war path ritual commonly in-

\[1\] Assiniboine: (13) 556; (57) 30; Blackfoot: (78) II, 580; Hidatsa: (66) 397; Omaha: (25) 330; Teton: (33) 136.
\[2\] (33) 43; Blackfoot: (66) 255; (78) II, 580; Crow: (59) 256; Hidatsa: (66) 442-3; Osage: (26) 127-8; Ute: (16) 157.

\[3\] Whenever women themselves had performed brave actions as warriors they could, of course, recount war honors. The statement above, although nowhere given in the literature, seems a reasonable conclusion from the observation that women did share the honor of their male relatives, and recounted their honors in the scalp dances, yet these prerogatives were at other times strictly reserved for participants in the victory.

\[4\] Plateau and Southwest.

\[5\] Blackfoot-Sioux: (61) 277; Cheyenne-Mandan: (42) 359; Crow and Shoshoni-Mandan and Hidatsa: (87) 182-3.

\[6\] (20) 111.
cluded leaving the camp and traveling in a column in which a certain order of march was observed and a featured portion of the war dance was a sort of procession,\(^1\) which might make a circuit of the entire camp.\(^2\)

War dances were matters of group concern. There are a few plates representing scalp dances which show scattered groupings of uninterested persons, but all the descriptions indicate general group participation. Dances preparatory to the formation of a war party were commonly carried into several bands, or even tribes,\(^3\) and the scalp was sometimes shared by villages which had not taken an active part in the victory.\(^4\) The preparation for both the war dances and the feasts which might accompany them must have been elaborate. Yet upon the question of their organization the literature maintains a complete silence. It is certain that the war dances were not under the control of any of the so-called dancing societies. Welcoming dances seem to have been such natural expressions of individual responsiveness to a given situation as to need no organization other than the suggestion of the chiefs. This is true also of the dances which had to do with particular war parties. Although there seems little doubt that the instigation of and the preparation for such dances lay with the leaders and members of the party and their relatives, this did not serve in any way as a restriction upon the responsive participation of the group.

A war dance was held in order to obtain the volunteers of whom the war party was composed. This dance was directly instigated by the prospective leader of the expedition. It might be simple in character,\(^5\) such as the offer and

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\(^1\) Arapaho: (75) 23; Blackfoot: (1) 458; Comanche: (36) II, 315; Cree: (1) 535; Crow: (58) 237; (59) 255; (60) 365; Hidatsa: (66) 442–3; Kansa: (1) 752.

\(^2\) Assiniboine: (57) 30; Crow: (59) 254; Kansa: (26) 678; Kiowa: (68) 291; Osage: (9) 105; Ute: (16) 152.

\(^3\) Assiniboine: (41) 304; Cree: (42) 159, 194; (89) 337; Ojibway: (14) 87, 126; Sioux: (12) III, 12; (28) 152; (43) 147.

\(^4\) Assiniboine: (1) 558; Crow: (13) 491; Gros Ventre: (6) 310; Ojibway: (14) 126.

\(^5\) (19) II, 349; (43) 336; (74) II, 59–60; Cree: (89) 338; Ojibway: (14) 86, 124; Sioux: (62) 139; (15) 332.
acceptance of a pipe or a march in which the volunteers took part, or the dance might be elaborate. In either case the enlistment of the warrior was sealed by his performance of the prescribed act.

Perhaps the most important of the war dances were those which followed upon the return of the successful war party. As an old Sioux warrior put it: "the honor was in coming home victorious, and the demonstration was reserved to see whether it would be needed when we returned." The period over which such demonstrations continued might vary from a day and a night to a whole season. They began with the ceremonial entrance of the party into the village and continued for an indefinite number of days or nights.

Although no details are known as to the organization of these dances, two factors undoubtedly contributed: the desire on the part of the relatives of the leaders and members of the party to add to the prestige of the newly crowned victors, and the willingness of the rest of the tribe, especially those persons to whom scalps had been presented (see page 454), to honor them. Lowie reports for the Assiniboine that when a scalp dance was held "the victors' wives cooked food for the dancers" and although the element of rendering honor to the victors was always prominent, it seems probable that the members of the war parties bore the brunt of the material outlay necessary for the victory dances.

The dances themselves seem to have been largely under the supervision of certain unidentified old men and women. The role of women was so marked as to make its description

1 Assiniboine: (13) 544.
2 (15) 333.
3 Crow: (59) 255.
4 Mandan: (66) 388.
5 (57) 30.
6 Assiniboine: (13) 557; Blackfoot: (38) 303; (1) 458; Cree: (1) 535; Crow: (12) IV, 110; (55) 132; (58) 234; (60) 364, 387; (1) 492.
7 Assiniboine: (13) 556; (19) II, 210; (57) 30; Blackfoot: (20) 102; (38) 303; (78) II, 580; (1) 458; Cheyenne: (39) II, 142; Crow: (12) IV, 110; (59) 236; (60) 364; Hidatsa: (60) 364; Mandan: (66) 357; Ojibway: (14) 146; Sioux: (28) xx.
practically synonomous with descriptions of the scalp dance.\textsuperscript{1} The dances might include the female relatives of warriors, all the warriors with war honors to relate as well as the newly returned victors, and the old men and women in whose memories were stored accounts of the deeds of warriors long dead. Few restrictions seem to have been placed upon active participation in the post-warpath dances.

Inevitably, the victory dances expressed the group’s feeling of triumph and superiority. They were occasions of “wild rejoicings,”\textsuperscript{2} festivities “to which all came and enjoyed themselves,”\textsuperscript{3} celebrations of “uninhibited triumph” in which all was “individualism, all exultation and triumph.”\textsuperscript{4} Probably, however, they were not truly orgastic. The warriors recounted their war honors with every appearance of abandon, heightening the drama of the fighting situation to the best of their abilities. Yet these were examples, not of abandon, but of a very remarkable control, for each account was carefully rehearsed, the songs were learned and the record was an expression of studied dramatic effect.

Although in the heat of battle the warriors allowed themselves every excess in mutilating the bodies of their

\textsuperscript{1} (11) 141; (19) II, 201; (32) 112–3; (33) 43; (43) 80, 334; Arapaho: (22) 34; (40) 325; (75) 23; Assiniboine: (13) 556–8; (57) 30; Blackfoot: (1) 458; (12) VI, 10; (38) 303; (66) 255; (78) II, 580; Cheyenne: (38) 304, 307; (39) II, 37; (40) 281; (63) 256; Comanche: (36) II, 316; Cree: (31) 107; (1) 535; Crow: (12) IV, 110; (58) 234, 237; (59) 254; (60) 364; (78) I, 399; Gros Ventre: (6) 83, 319; (12) V, 106; (47) 191, 209; (66) 325; Hidatsa: (17) 155; (66) 397, 442–3, 449; Iowa: (1) 150; (20) 150; Kansa: (1) 757; (26) 678; (80) 375; Kiowa: (68) 291; (1) 851; Mandan: (20) 319; (66) 357, 388; (88) 123; Ojibway: (1) 492; (12) III, 285; (14) 113, 118, 121, 124, 126, 145; Omaha: (10) 432; (24) 328, 330 354; Osage: (25) 127; Pawnee: (18) 65; (54) 208, 211; Ponea: (1) 791; Sarsi: (12) XVIII, 104; Sioux: (20) 308–9; (74) II, 12; (78) I, 256; (78) II, 629; Sauvie: (28) 157; (72) 226; Teton: (1) 80; (15) 261; (34) 39; (40) 572; (53) 136; (81) 57; Ute: (16) 146, 156; (35) 245, 248; Wichita: (23) 7; (12) XIX, 43.

\textsuperscript{2} (74) II, 57.

\textsuperscript{3} (43) 334.

\textsuperscript{4} (5) 116–7.
enemies and severed body members were even carried home for the women and young boys to gloat over and "enjoy," such action was an expression of fighting fervor, attained by the men only in the midst of battle, and by the women in retaliation for loved ones lost at the hands of the enemy. Occasionally severed body parts were carried in the post-warpath dances. But most commonly such abandon occurred immediately upon the return of the party and was not part of the dance situation, and the dances, which were largely social in character, were exultant rather than sadistic or grim.

It is important to note that competition for individual honor was not the only feature of the post-warpath dances: interest centered in the superiority of the returned party in particular and of the group in general over the enemy, and the recounted war honors emphasized the part each had played in demonstrating it. The aggrandizement of the entire group was sought through declarations of superiority and taunting insistence upon the inferiority of the foe.

Scalping

It is not easy to fit scalping into the system of graded war honors, nor to see the scalp merely as a trophy in the war dances, as either of these have been described. For this reason the tendency has been to subordinate the role which scalping played as an actual part of the Plains war complex, in favor of the more distinctive custom of coup counting. In reality only two Plains tribes did so subordinate it: the Cheyenne and the Crow. Plains war parties set out with either or both of two avowed purposes: to capture horses, and to take scalps. Practically these were not

1 Assiniboin: (13) 491, 551; Cheyenne: (7) 483; (63) 22, 29, 263; (68) 387; Comanche: (36) II, 317; Cree: (13) 492; Crow: (13) 491; (59) 261; Gros Ventre: (47) 211; Hidatsa: (66) 404; Ojibway: (1) 492; (28) 151; Omaha: (24) 332; Sioux: (28) 156; (42) 260.

2 Assiniboin: (13) 492; Gros Ventre: (47) 191; Mandan: (66) 387; (88) 123; Omaha: (24) 313; (30) 434; Sioux: (11) 326; (28) xxix.

3 Blackfoot: (1) 458; (12) VI, 83; (37) 254; (38) 303; Gros Ventre: (6) 146.
necessarily divorced, and war honors could be achieved equally in either quest, yet they were disparate. The capture of horses has already been discussed in its relation to the system of graded war honors. Attention must now turn to the taking of scalps. The following paragraphs deal with the function of the scalp in what G. A. Dorsey has called the "mourning-war" situation.¹

Revenge is frequently said to have been among the motives for the formation of war parties. Certainly one of the surest ways of arousing the fighting spirit was to remind young men of the wrongs which their tribe had suffered at the hands of an enemy. There was, however, no state of continued blood feud existent in any Plains tribe. The whole machinery for long-enduring and systematic revenge was completely absent. War parties were loose aggregates of individuals only temporarily united and they cut across all the religious, political and social groupings which normally function to join men for cooperative effort. Men might be urged by relatives and leaders to go to war, but only one thing served to bind the members of a party together: the desire to fight and to return home successful. Nevertheless, if the material is examined closely certain connections between death and war emerge. War parties were commonly instigated by persons who had lost a spouse, a child or a close relative,² yet the impulse was the same whether the loved one had been killed by the enemy or not. The causal agent of death seems to have been relatively unimportant. The attitude toward death was realistic, sorcery and witchcraft were not connected with it. Yet, "saddened and angered by sorrow, it is a common thing for the men, at loss of friend or kin to announce the fact that their heart is bad, and start at once on the war-path."³

¹ (21).
² Blackfoot: (12) VI, 83; Crow: (12) IV, 83, 105; (58) 233; (78) I, 398; Gros Ventre: (6) 290; (47) 192; Kansa: (10) 54; (26) 670; Osage: (9) 165; (25) 115; (79) 168; Oto: (67) 189; Sioux: (12) III, 187; (72) 212; (74) IV, 60.
³ (11) 263.
Leaders or members of war parties might have been mourners. The scalps taken might have been given to mourners of the village upon the return of the war party to its home camp, or might simply have been placed upon the graves of the recently deceased, and "dancing the scalp" put an end to the period of mourning. This common interest in the death situation seems also to have played its part not only in cementing the members of the war party together for the duration of the war path but also in maintaining a close contact between each war party and the rest of the people.

The emphasis upon dignity, generosity and equanimity which the mature Plains Indian insisted upon in his outlook upon life broke down completely when he faced the crisis of death. His grief took the most extreme forms of unkemptness, of mutilation, of blood letting by piercing or gashing his arms, legs or head. Not only were these agonies of grief permitted by the society, they were expected. At such times individuals were recognized as being unstable and were carefully guarded to prevent suicide. It was in this state of mind that the mourner turned to war.

Mourning seems to have been compounded of the emotions of grief, anger and shame. It followed upon the loss of a near relative but was not limited to such occasions. A man might go to war when he was in mourning either "for dead kindred or for stolen property." A man who "felt aggrieved" might turn to warfare. The key to the situation seems to lie in the simple fact that war was the occasion

1 Arikara: (6) 272; Assiniboine: (13) 483; Blackfoot: (78) II, 508; Gros Ventre: (47) 181; Kansa: (25) 670, 678; Omaha: (30) 408; Osage: (21) 404.
2 Arapaho: (75) 23; Assiniboine: (13) 556; Crow: (58) 234; (59) 401; Hidatsa: (17) 145; Osage: (21) 404; Oto: (10) 62; Sioux: (28) xx; Ute: (16) 152; Wichita: (23) 15.
3 Assiniboine: (10) 44; Sioux: (10) 23; (28) 105, 131; (2) 86.
4 Assiniboine: (13) 556; Blackfoot: (12) VI, 83; Crow: (58) 233, 237; (60) 227; (78) I, 399; Kansa: (10) 54; Osage: (21) 411; Sioux: (28) 101, 226; Wichita: (23) 15.
5 Upper Missouri (13) 573.
6 (25) 121.
7 (74) IV, 60.
par excellence for reinstating one's pride. The ideal gamut of excited emotion was that which began with such mourning, ran through self-violence to violence upon an enemy, and terminated in social recognition of success in war.

It is important to recognize that these situations, mourning and war, were the only situations in which violence was culturally recognized. Murder was discountenanced, quarreling and bickering were ideally tabu, and nothing could be less violent than the matter-of-fact way in which sacrificial self-mutilation was undertaken. Yet the Plains Indians were decidedly not a calm people and the effect of this cultural concept was to direct all violent emotional expression into warfare and its accompanying ceremonials.

The writer recognizes that to bandy with the emotions of an almost dead culture in the above fashion is to tread on controversial ground. The conclusion she wishes to draw in relation to scalping, however, is rather simple: a tie existed between mourning for a dead relative and going to war which cannot be satisfactorily explained as revenge and, in some way or other, the Plains Indian thought of this tie in terms of the scalp.

Although this material is from Siouan village tribes, the connection of ideas which it expresses has been shown to be general in the Plains. Its significance lies in the fact that, despite its ceremonial uniqueness, its conceptual content is not unique. Because of this significance the material is here given in some detail.

On July 27, 1827, six Osage landed at Havre. They were the remnant of twenty-five who had started down the Missouri in the beginning of the same year. Evidently, the two Frenchmen who accompanied them had planned the same sort of mission as those later taken over by the great showman, George Catlin. Their arrival at Havre was heralded by an article probably dwelling upon the spectacular side of the life of these "savages" for it aroused Paul
Vissier to a refutation. The same year, in Paris, he published an article dealing with the circumspection they used in carrying on that most glaring of their supposed atrocities—war. Vissier admits to having drawn freely upon the "Journal des Voyages" but it is probable that he interviewed the four visiting Osage warriors. He shows himself, at any rate, as a rather exceptional ethnologist.

In 1884 J. Owen Dorsey published a full account of the war parties of the Osage. At this time the war party was recent enough to have a living reality for the people. The material of George A. Dorsey obtained in April 1902 indicates that the war party was a matter of memory but the importance of the rite as such remained in full force. Speck's material published in 1907-8 says of the same ceremony that the Osage themselves recognize it as "their most characteristic religious observance." The recent information of Francis la Flesche serves mainly to fill out the ritual form of J. O. Dorsey's very full account and to lend the whole a cosmic symbolism which is largely lacking in earlier reports.

The change in tone of the various accounts is noteworthy. In part it is to be explained by the differences in the accounts themselves. Yet to a marked degree the change is explicable in terms of cultural change over a period of years. The Vissier and J. O. Dorsey descriptions are clearly methods by which a war party was gotten under way. Vissier carries the party through to a gory conclusion; Dorsey brings it home in triumph. In both the situation is that of actual warfare. The ceremony of 1902 was a mockery of war but it held all of its former significance in relation to the situation of death; it was a ceremony to bring the period of mourning to a close. Speck's ceremony forms a "ransom" for the human soul which cannot pass

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1 (85).
2 (25).
3 (21).
4 (79).
5 (49) and (50).
into the spirit land until a scalp has been placed at the grave. Saucy-Calf gave his ritual knowledge to la Flesche that its cosmic significance might not be lost forever. In any attempt to arrive at a concept, however static, of the ceremony itself not one of these aspects may be overlooked. By a happy combination of opportune time and capable reporting, the account of 1884 caught them all. Three types of Osage war party were distinguished by J. O. Dorsey—an annual, tribal expedition, a sacred bag party which might occur at any season of the year, and the horse stealing raid in which the "customs are similar to the others." The customs pertaining to each of these "are incomplete, being obtained from a member of the principal peace gens. Moreover, there are customs peculiar to each gens, which are not familiar to members of other gens."

"When the warriors were about to organize for war they chose a man to act as representative, not only of the warriors but of all the people." 1 After a period of "vigil" this mourner selected a man from the opposite side of the camp circle to act as military leader of the expedition and to "mourn with him." "Ce choix étant un grand honneur, est toujours accepté." 2 It is clear that the partisan was "a very near relative of the decedent." 3 The great difference between the tribal and sacred bag parties was that in the former the entire tribe was represented in the persons of the two mourners, whereas in the latter the mourner or partisan represented only his own gens. Although each party was composed of standard bearers and warriors from both sides of the camp circle, one of them assumed the responsibility of allaying the mourning of the tribe, the other functioned only for the gens of the deceased. The ceremony of the tribal party included all of the warriors of the tribe, although all of these need not go upon the war path, and the whole group was included in certain encircling processions not found in the ritual of the smaller party. Both

1 (50) 610.
2 (85) 33.
3 (21) 404.
parties entailed the selection of a kettle bearer, of standard bearers and of lieutenants. These officers, who were drawn from both sides of the camp circle, assumed completely reciprocal ceremonial roles. Each mourner was instructed by a "teacher," the mediator of power, who controlled the ritual of the sacred packs of war. The ceremonies of the four days preceding the departure of the party included the reciprocal war path rituals, the medicine bundle rituals, and the mourning ceremonies. The horse stealing expedition differed from the above only by the multiplication of the possible number of partisans, each of whom had assumed individual responsibility as "a mourner for dead kindred, or for stolen property." 1

When the party met the enemy, at least one of these was killed ceremonially, his head severed (an act which entailed special honor) and the body arranged in a certain fashion which included the use of the standards, prepared beforehand, borne by special officers and now left upon the field of battle. In May of 1863 a band of Confederate officers, who had set out to incite the Indians to war against the Union, was slain by a group of loyal Osage: "the heads, besides being scalped, had been, according to the Osage custom, severed from the bodies . . . some being found at a considerable distance" from the bodies. 2 In the massacre of the Kiowa in 1833 the Osage also beheaded their enemies, making such an impression upon the Kiowa survivors that they declared that the Osage "never scalped." 3 The scalp, likewise, was ceremonially taken and prepared 4 by the partisan to whom belonged all the scalps taken by the members of the party. The partisan had, of course, no claim upon the war honors of the members, which were

1 (25) 121.
2 (4) 65.
3 (78) 260. In his review (American Anthropologist, IX, 1907) of G. Friederici's "Skalpieren und ähnliche Kriegsgebrauche" Mooney reiterated his statement that in 1833 the Osage did not scalp. "Aber gerade über die Skalpgebrauche der Osages haben wir ausserst gute und eingehende Berichte" (32, 23) and we must conclude, as did Friederici, that in general the Osage certainly took scalps.
4 (25) 120; (27) 526-7.
established by a trial over the sacred bags of war, after the return to the home camp.

The scalp dance was not a necessary part of the dances which followed upon the return of the victorious warriors. But the scalp itself was retained to allay the grief of the gens, and placed in the medicine bundle by the mourner of the sacred bag war party, or, distributed by the partisan of the tribal expedition to those of the tribe who were in mourning, the portions being placed upon the graves. The ceremonies which accompanied this were not tribal and served to bring to an end the individual’s period of mourning.

J. O. Dorsey also recorded a description of the “Mourning and War Customs of the Kansas” which corresponds very closely to the account of the Osage given twenty-five years later. In both, the war ceremony which had lost all of its former military reality was retained as necessary in dealing with the crisis of death. “When a member of a family dies, a warrior of the band to which the family belongs is chosen to make propitiation . . . takes a body of warriors . . . and goes out on a war expedition against some hostile tribe. If he is successful in taking scalps or stealing horses he returns, and the widow can put aside her mourning and is at liberty to marry again. If a woman dies, the husband selects the one to make propitiation; the father if a child dies.” The Kansa emphasis upon this motivation for war went so far as to rob the familiar coup of some of its meaning; the chief war deeds were “killing, or killing and scalping a foe,” stealing horses was “creditable, but counting coup was of no importance, although it was looked upon as a brave act.” In 1902 the Osage party had killed a deer; a “wolf, bird, or other creature” could also have served; the Kansa in 1883 killed several prairie

1 (26).
2 (21).
3 (10) 54.
4 (1) 752.
5 (79) 170.
chickens. But in each case a war party bearing a scalp was a necessary ending to the period of mourning.

The connection between the death of an enemy and the death of a tribal member, and the role of the scalp as intermediate between the two, cannot be avoided. Although it is beyond the scope of this paper adequately to prove such a thesis, the writer believes that scalping was one aspect of an underlying New World complex, that in North America, east of the Rockies, at least a relationship can be established between this complex and what may be called for want of a better name a cult of the dead, and finally, that as such it was closely knit into the war complex and into the emotional life of the Plains Indian.

CONCLUSIONS

Certain conclusions as to the organization and significance of the Plains war complex may be drawn from the preceding discussions:

(1) The system of graded war honors depended upon the inclusion of martial prowess in the Plains ideal of manhood and of social virtue. This system gave peculiar play to individualism and to a culturally accepted form of self aggrandizement.

(2) The accomplishment of just those feats which, in recounting, enhanced the prestige and differentiated the personality of the individual, served, in actual performance, to knit these individuals into a functioning unit.

(3) The deeds chosen for social recognition may be classified according as they represented: (a) fearlessness, (b) the ability to make surprise attacks, (c) the powers of leadership and organization and (d) the taking of a scalp. It is probable that these indicate a standard of the martial values.

(4) Defense was included in warfare only in so far as it contained elements of aggression.

(5) The aggressive fighting unit was the war party, both
the leadership and membership of which was temporary and voluntary.

(6) The war path embodied certain ceremonies which emphasized the cohesion of the fighting unit and which sharply defined the duration of such unity.

(7) War dances marked the organization of the war party and its dissolution, and served to give publicity to both.

(8) There was a definite connection between mourning and war. The emotions of grief, anger and shame which the former excited were allayed by social recognition of success in the latter.

(9) Individuals who could not themselves go upon the war path were dependent upon the success of others to allay the emotions of mourning. The scalp was the symbol around which such emotions centered and to receive a scalp was to cease to mourn. There was a consequent compulsion and social obligation on the part of warriors to obtain scalps. This obligation did not negate the voluntary character of the war party.

(10) Within the mourning-war behaviors were channelled all of the violent and socially disruptive reactions.

(11) Notwithstanding its elements of individual competition, the warfare of the Plains was one of the strongly socializing institutions which the culture recognized.

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THE SINKING OF DECOMPOSING PLANKTON IN SEA WATER AND ITS RELATIONSHIP TO OXYGEN CONSUMPTION AND PHOSPHORUS LIBERATION

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Abstract

The present investigation represents an attempt to obtain certain fundamental theoretical data on physical and chemical processes accompanying decomposition of marine organic matter which are needed, in particular, for investigation of the origin of the minimum oxygen concentration in the ocean basins. Five separate experiments (in duplicate), ranging up to 363 hours duration, were carried out under controlled laboratory conditions. For each experiment weighed amounts of fresh zooplankton from Woods Hole vicinity were permitted to decompose in a light-proof apparatus; and rates of oxygen consumption, phosphorus liberation, and settling velocities of the organic debris estimated at stated intervals.

The analytical results indicate a divergence of the two chemical processes in so far as laboratory conditions are concerned. The rate of oxygen consumption from 0 to 280 hours after death decreased approximately linearly with time. On the other hand, phosphorus liberation showed no apparent continuous regularity, but there was a rapid liberation of phosphate for approximately 24 hours after death of the organisms with a rapid decline thereafter. Settling velocities of the decomposing organic debris in sea water (density, $\sigma_t$: 20.66-21.70) decreased from about 0.37 to 0.03 cm/sec between 2 and 363 hours after death of the organisms. These data are subjected to special treatment on the assumption of the applicability of Stokes' Law (for motion of a sphere through a viscous medium), and the mean value of the density and variation of the "equivalent radius" estimated. The "equivalent radius" of the organic particles appears to decrease linearly with time.

Results of the analysis of settling velocities were applied to an oceanic area (represented by "Atlantis" station 1170) not with the intention of illustrating actual conditions in nature, but rather to show how results of controlled laboratory experiments may be combined with a natural oceanographic set up to obtain basic information on laws regulating decomposition in the sea. Calculations were made, on the basis of derived equations, to illustrate relationship between the reduction of "equivalent radii" with depth and the relationship between time and depth for organic particles (of various initial sizes) as they sink through water of known density stratification. It appears reasonable to assume that the results indicate, in a general way, the mechanism underlying certain of the biological activities which alter the distribution of non-conservative properties (such as oxygen) in the sea.

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I. Introduction

This paper is a report on a series of laboratory experiments carried out for the purpose of obtaining elementary data on the relation between the sinking of decomposing marine zooplankton in sea water and its consumption of oxygen and liberation of phosphorus. The experiments, carried out under controlled laboratory conditions, were planned as an approach to the quantitative measurement of decomposition activity in the sea, and, while the artificial conditions of the laboratory disturb the equilibria of the natural reactions and limit the practicability of the application of the final results to the sea, an even greater hindrance in investigations of this kind is in the selection of the kind of material for study.

The interpretation of organic decomposition studies is difficult particularly in view of the limited knowledge of the fundamental structure on which all problems of marine organic decomposition rest, namely, the relative significance of its various size classes in nature. Hence, until this and similar questions of practical oceanography are investigated the application of results of physical and chemical laboratory experiments can do little but suggest, in a general way, the laws underlying the biochemical and physiological mechanisms of the sea.

It is the plan of this paper to present first the results of the laboratory experiments and then to apply these data to a specified part of the North Atlantic ocean. No claim is made that this oceanic application gives a correct picture of what happens in nature, but rather indicates how a type of organic matter might behave in the water if it should act in nature as in the laboratory.

II. Methods

The animal plankton (chiefly copepod) used for analyses was collected with a one meter diameter stramin net at depths of 10 to 30 meters in the vicinity of Woods Hole. Weighed amounts of wet plankton were placed in amber glass bottles in a light proof apparatus where a continuous flow of sea water
was maintained over the decomposing material. Samples both of water and plankton were removed at various intervals (up to 280 hours after death) for estimation of oxygen consumption and phosphorus liberation on the former, and of free fall velocities on the latter. An approximately constant oxygen pressure was maintained in the bottles at temperatures between 22° and 25° throughout the experiment.

Oxygen and phosphate were estimated by standard methods of oceanographic procedure (Seiwell, 1934 and 1935), and, for estimation of the free fall velocities, small quantities of the decomposing organic matter were allowed to settle in a Nessler tube through a measured distance (25 centimeters) of sea water of known density.

III. Experimental Results

(a) Oxygen Consumption

Amounts of plankton ranging from 0.4 to 2.4 grams (wet weight) were used for each of the experiments. The results, expressed as grams of oxygen consumed \(- \frac{dO_2}{dt}\) per hour per gram of wet plankton, which are plotted in Fig. 1 against

![Graph](https://via.placeholder.com/150)

**Fig. 1.** Time variation of oxygen consumption based on data from five controlled laboratory experiments. The straight line trend has been fitted by the method of least squares.

age of the dead plankton (referring from the time the plankton was put in the apparatus at the beginning of each experiment) suggest a linear reduction with time in the consumption rate. The best straight line fitting these data (for 0 to 280 hours), by the method of least squares, has the equation:

\[
- \frac{dO_2}{dt} = 10^{-3} \left[ 0.2154 - 0.000551 \ t \right]
\]
<table>
<thead>
<tr>
<th>Experiment No. 1</th>
<th>Experiment No. 2</th>
<th>Experiment No. 3</th>
<th>Experiment No. 4</th>
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</thead>
<tbody>
<tr>
<td>Age dead plankton</td>
<td>g. P per g. per hour</td>
<td>g. P per g. per hour</td>
<td>g. P per g. per hour</td>
</tr>
<tr>
<td>200' 00'</td>
<td>10.000×10⁻⁶</td>
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<td>negative</td>
</tr>
<tr>
<td>400' 00'</td>
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<td>negative</td>
</tr>
<tr>
<td>500' 00'</td>
<td>3.600×10⁻⁶</td>
<td>negative</td>
<td>negative</td>
</tr>
<tr>
<td>600' 00'</td>
<td>3.000×10⁻⁶</td>
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<tr>
<td>700' 00'</td>
<td>3.000×10⁻⁶</td>
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<td>900' 00'</td>
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<tr>
<td>1000' 00'</td>
<td>3.000×10⁻⁶</td>
<td>negative</td>
<td>negative</td>
</tr>
</tbody>
</table>

TABLE 1
where \( t \) (hours) is the age of the dead plankton, and, \[-\frac{dO_2}{dt}\]
the oxygen consumption expressed as grams of oxygen per hour per gram of (wet) plankton at temperatures of 22° to 25°. In these experiments fresh sea water continually flowed over the decomposing material and the results would appear to be but little influenced by the decomposition of other suspended or dissolved organic material present in the sea water itself such as, for instance, has been reported by Waksman and Renn (1936).

(b) Phosphate Liberation

The determinations (given in table 1) were made simultaneously with those for oxygen consumption.

The data in table 1 show little consistency but there is a rapid (and apparently the maximum) liberation of phosphate phosphorus in the vicinity of 24 hours after death of the plankton; still later a rapid drop in rate of phosphate liberation occurs and it appears frequently that the rate of liberation is exceeded by the rate of consumption of phosphate (both from sea water and the decomposing plankton) by the growth of organisms associated with the decomposition. A similar liberation of phosphate has been observed by Cooper (1935); from his results we judge that about one third to one fourth of the amount of phosphate added as zooplankton to sea water was set free during the first twelve hours, while an amount equivalent to that added as zooplankton was liberated in about 6 days.

(c) Free Fall Velocity of Decomposing Plankton

The measurements (table 2) were made in sea water with a temperature range of 22° to 25°, a salinity range of 30.97 0/00 to 31.69 0/00, and a density (\( \sigma_i \)) range of 20.66 to 21.70. The actual density values for each measurement were calculated from temperature and salinity of the water. Each value in table 2 is the mean of ten to fifteen individual measurements.

In the treatment of this data we have assumed that the
<table>
<thead>
<tr>
<th>Experiment No. 1</th>
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<th>Experiment No. 3</th>
<th>Experiment No. 4</th>
<th>Experiment No. 5</th>
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<tbody>
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<td>Age dead plankton</td>
<td>Age dead plankton</td>
<td>Age dead plankton</td>
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<td>Age dead plankton</td>
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<tr>
<td>cm./sec.</td>
<td>velocity cm./sec.</td>
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<td>velocity cm./sec.</td>
<td>velocity cm./sec.</td>
</tr>
<tr>
<td>2h</td>
<td>0.27</td>
<td>0.25</td>
<td>0.27</td>
<td>0.27</td>
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<tr>
<td>1h</td>
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</tr>
<tr>
<td>2h</td>
<td>0.03</td>
<td>0.03</td>
<td>0.03</td>
<td>0.03</td>
</tr>
</tbody>
</table>

**TABLE 2**
rate of sinking of the particles of organic matter followed Stokes formula for the rate of settling \((v)\) of small particles of radius, \(r\), and density, \(\rho_0\), under the influence of gravity, \(g\), through a liquid (or gaseous) dispersing medium of density, \(\rho\), and viscosity, \(\mu\), thus:\(^1\)

\[
v = \frac{2g}{9\mu} r^2 (\rho_0 - \rho)
\]

The formula stipulates that the particle should be spherical and since this condition is obviously not fulfilled in the case of organic particles we have used an "equivalent radius" such as has been introduced by Oden (Gripenberg, 1934). Since the size of the organic particle becomes smaller as decomposition proceeds the "equivalent radius" will become smaller with the increased state of the decomposition. Hence, at the outset an effort was made to determine \(r\) as a function of time by using the following procedure.

In the Stokes equation (2) the two unknown quantities were the density of the sinking particles, \(\rho_0\), and their radii, \(r\) (the viscosity, \(\mu\), and density, \(\rho\), of the dispersing medium are calculated from its temperature and salinity).\(^2\) To determine \(\rho_0\) the assumption was made that the density was the same for all particles studied and remained constant throughout the duration of the experiments (this is not strictly correct but appears necessary if the data are to be reduced). Then, by taking various pairs of free fall measurements obtained for the same age of the decomposing plankton, but from different experiments where the densities of the dispersing media were different, a number of pairs of simultaneous equations were obtained and solved for \(\rho_0\). (This method of attack also makes use of the assumption that equal "equivalent radii" characterize equal ages of the various samples of decomposing plankton.) Finally, from the various individual values of \(\rho_0\) a determination of the arithmetic mean gave, what was as-

\(^1\) For the material studied, it was observed that for any particle sinking through the water a practically constant velocity appeared to persist from the beginning.

\(^2\) Viscosity of sea water was calculated for data given by Krümmel (1907), and density from Knudsen (1901).
sumed to be, the most likely value for the density of the particles. Thus, we obtained from the data under consideration:

\[ \rho_{0,\text{mean}} = 1.02351 \]

as the most likely value of \( \rho_0 \) to be used for calculation of the time variation of the equivalent radius, \( r \). The mean density of the media in which the plankton decomposed was found to be 1.02109; thus, giving a density difference, between organic matter and the external media, of 0.00242 (a value which seems reasonable).

After insertion of the proper values in Stokes formula (2) the change of \( r \) with time was calculated (from data in table 2); the results are plotted in figure 2. The points show a linear decrease of \( r \) with time and the best straight line fit (by the method of least squares) has an equation of the form:

\[ r = r_0 - \beta t \]

where

\[ r_0 = 7.968 \times 10^{-2} \text{ cm.} \quad \text{and} \quad \beta = 5.356 \times 10^{-8} \text{ cm./sec.} \]

The validity of the linear decline of \( r \) with time may be verified on the assumption that the rate of decomposition is
probably proportional to the area of the particles, and if $M$ be the mass of a particle and $r$ its radius, then:

$$\frac{dM}{dt} = -kr^2$$

or

$$\frac{dr}{dt} = -\beta$$

and consequently:

$$r = r_0 - \beta t$$

The determination of $r$ may be useful as an index for characterization of certain decomposition studies of this type. The value of $\beta$ is calculated from results obtained at temperatures of $22^\circ$ to $25^\circ$.

IV. Application of Experimental Results to the Ocean

Application of the foregoing experimental results to the sea is made, not with any intention of illustrating actual conditions in nature, but of showing how results of controlled laboratory experiments may be combined with an observed oceanographic set up in nature for the purpose of obtaining information on the laws regulating decomposition in the sea.

In the concluding part of this paper we calculated for an oceanic area, as represented by "Atlantis" station 1170 (14° 47'N, 40° 58'W), the change with depth of "equivalent radii" of particles of various initial sizes as they sink from the surface layer into the depths; and the time required for these particles to sink. The data are based on an assumed constant density of the particles corresponding to the known density of the water at a great depth (e.g. 1950 meters); the effect of eddying motion of the water is disregarded.

(a) Distribution of Density at "Atlantis" Station 1170

The vertical distribution of density (0–1950 meters) at "Atlantis" station 1170 (14° 47'N., 40° 58' W.) is illustrated by Fig. 3. Between depths of 1950 and 50 meters (taking the
$z$ axis as positive upwards) the density distribution may be approximated by the hyperbola:

$$
\sigma_z = 27.81 + \frac{z}{4.5889z - 92.4629 \times 10^4}
$$

the distance $z$ is measured in centimeters upwards from 1950 meters. The values of $\sigma_z$ determined by this equation for various values of $z$ are indicated along the curve in Fig. 3.
(b) Change in "Equivalent Radii" of Particles Sinking Through the Water Column at Station 1170

We considered the sinking through the water column of organic particles originating in the upper 50 meter layer; the density of the particles corresponded to that of the water at a depth of 1950 meters \( z = 0; \sigma_t = 27.81, \rho_0 = 1.02781 \) and remained constant during the sinking process.

The vertical distribution of density between \( z = 0 \) (1950 meters) and \( z = 19.00 \times 10^4 \) cms. (50 meters) at station 1170 is:

\[
\sigma_t = 27.81 + \frac{z}{4.5889z - 92.4629 \times 10^4}
\]

or

\[
\rho = \rho_0 + \frac{z}{az + b},
\]

where

\[
\rho = \sigma_t \times 10^{-3} + 1.00000,
\]
\[
\rho_0 = 1.02781,
\]
\[
a = 4.5889 \times 10^3,
\]
\[
b = -92.4629 \times 10^7.
\]

From Stokes' law we have:

\[
v = -\frac{dz}{dt} = \frac{2g \mu}{9\mu} r^2 (\rho_0 - \rho)
\]

where \( \rho_0 \) is the density of the sinking particle taken to equal 1.02781, and after substitution we have:

\[
\frac{dz}{dt} = \frac{2g}{9\mu} r^2 \left( \frac{z}{az + b} \right)
\]

also we may write:

\[
\frac{dz}{dt} = \frac{dz}{dr} \cdot \frac{dr}{dt}
\]

and since from equation (3)

\[
r = r_0 - \beta t
\]

or

\[
\frac{dr}{dt} = -\beta
\]
and
\[ \frac{dz}{dt} = -\frac{dz}{dr} \beta \]

Substituting in the above equation we finally obtain:

\[ \frac{dz}{dr} = \frac{k}{\beta} r^2 \left( \frac{z}{az + b} \right) \]

where
\[ k = -\frac{2g}{9\mu} \]

Integrating equation 5 gives the change of \( r \) with \( z \) or:

\[ az + b \ln z = \frac{kr^3}{3\beta} + C \]

where \( C \) is the constant of integration to be evaluated. We now let \( h \) equal the distance above \( z = 0 \) at which depth the particles of initial radius \( r_0 \) begin to sink into the water. Hence, the conditions imposed on the above equation are when
\[ z = h, \quad r = r_0 \]
or
\[ C = ah + b \ln h - \frac{kr_0^3}{3\beta} \]

and after substituting for \( C \) we obtain as the final relation between \( r \) and \( z \):

\[ r^3 = r_0^3 + \frac{3\beta}{k} \left[ a(z - h) + b \ln \frac{z}{h} \right] \]

The following constants were determined for the water column between \( z = 0 \) and \( z = 19.00 \times 10^4 \) cm. (1950 to 50 meters) at station 1170. \( T_{\text{mean}} = 7.45^\circ; S_{0/00\text{mean}} = 35.11; \mu_{\text{mean}} = 0.0151 \) dyne-sec/cm.²; from the experimental data \( \beta = 5.356 \times 10^{-8} \) cm./sec., and finally \( a = 4.5889 \times 10^3, b = -92.4629 \times 10^7 \).

Since the effect of pressure on viscosity of the water is not known we have used a value of \( \mu \) determined from the mean temperature and salinity of the water between 50 and 1950
meters (any further refinement appears to be superfluous). Furthermore, the value of $\beta$ is assumed to be independent of temperature, this is probably not correct but in the light of our present knowledge is the best that can be done. We, thus, calculated the values:

$$ k = -\frac{2g}{\gamma \mu} = -1.4432 \times 10^4 $$

$$ \frac{\beta}{k} = -3.7112 \times 10^{-12} $$

**Fig. 4.** Relation between the “equivalent radii” $r$ and depth $z$ for organic particles of four initial radii, $r_0$, when sinking through a water column represented by “Atlantis” oceanographic station 1170.

The relation between $r$ and $z$ is calculated for three values of initial radii ($r_0$) at 50 meters depth; these initial values are:
\[ r_0 = 0.0885 \text{ cm.} \]
\[ r_0 = 0.177 \text{ cm.} \]
\[ r_0 = 0.354 \text{ cm.} \]
\[ r_0 = 0.708 \text{ cm.} \]

and the results are graphically illustrated by Fig. 4. The value, \( r_0 = 0.354 \), is sufficient for the particle radius to theoretically become 0 at a distance of 10 meters (\( z = 10^3 \) cm.) above the level of its identical density. The curves illustrate that, if the particle is sufficiently large, it will approach asymptotically the level of equivalent density, whereas if the particle is completely broken down before it reaches this level (because of its small initial size) it will approach some level higher up in the water column in a similar manner.

(c) Time Required for Particles to Sink Through the Water at Station 1170

This calculation is based on the data of the previous section and the same assumptions and restrictions apply as formerly indicated. Thus, from equation (3) we have:

\[ r = r_0 - \beta t \]

or

\[ t_z = \frac{r_0 - r_z}{\beta} \]

where \( \beta \) is taken as \( 5.356 \times 10^{-8} \) cm./sec. \( (1.928 \times 10^{-4} \) cm./hour),\(^1\) and from the value of \( r \) at any level \( z \) (Fig. 4) we obtain the time, under specified conditions, for organic particles having a density of 1.02781 and various initial equivalent diameters, as formerly, to sink from the fifty meter level into the depths. The results, plotted in Fig. 5, show that particles of larger equivalent radii sink faster and correspondingly remain for a longer time close to the level of similar density.

(d) Phosphate Liberation and Oxygen Consumption of Organic Particles Sinking Through a Water Column

Laboratory studies on decomposing organic material have shown that at the end of 24 hours after death there is a much

\(^1\) A second determination of \( \beta \) recently made on decomposing zooplankton at Bermuda (0–250 hours) gave a value \( (3.888 \times 10^{-8}) \), close to that found for Woods Hole plankton.
greater decline in the rate of phosphate liberation than in the rate of oxygen consumption; and, if this be a true comparison of the relative activities of the two processes in nature, com-

Fig. 5. Time required for organic particles of various initial radii, \( r_0 \), to sink into the water column represented by "Atlantis" oceanographic station 1170.

bination of these results with those of the time required for particles to sink through a water column suggests that relatively large quantities of phosphate may be liberated in the sea within the layer of photosynthesis. However, such a suggestion is merely tentative.

Since the consumption of oxygen by organic matter (as well as other reactions of decomposition) is, among other things, a function of the size of the body (page 473), the consumption equation obtained for the particular material studied cannot be generalized. However, a study of the time required for par-
particles of various sizes to sink in a water column of variable
density shows that unless these particles are of a relatively
high density and of a relatively large "equivalent radius"
much of the oxygen consumption will be confined to the upper
part of the water column. From the data presented here we
may generalize to the extent that since particles (having a
density not greater than that of the bottom water) sinking
through water of variable density will approach some level
asymptotically, and if organic debris in the sea should be
sufficiently homogeneous (as to density and "equivalent
radius"), a stratum of maximum oxygen consumption may be
conceived of as occurring in the sea. The position of such a
stratum will depend on the density of the water as well as that
of the organic debris, and its thickness on the homogeneity of
the debris.

Conclusions

Application of the foregoing results to the ocean cannot be
expected to show definitely what happens in nature. All sorts
of possibilities exist, since not only is the relative importance of
the size classes of organic matter in the sea unknown, but also
the application of the results of controlled laboratory experi-
ments to natural conditions can hardly be expected to illus-
trate with any degree of exactness a natural phenomenon.

It appears likely, however, that our results indicate, in a
general way, the mechanism underlying certain of the bio-
logical activities in the ocean, in so far as these activities tend
to alter the distribution of the non-conservative properties of
the sea. One thing which appears noteworthy is that, since
more biological activity apparently occurs in the upper levels
of the ocean, it probably is the result either of a low density
(similar to that of the upper levels) of the great mass of organic
material, or else, in the case of a higher density, the great mass
of organic matter has very small equivalent radii. However,
in either case the physical conditions of organic matter in the
sea need investigation to see if the equivalent radii and the
density of the great mass of organic debris are sufficiently
homogeneous so that biological activity may, in certain levels
of the water column, offset the effects of dynamic processes and control the distribution of the non-conservative substances in the sea, or if, on the other hand, organic debris is so heterogeneous that the distribution of non-conservative substances is controlled largely by the horizontal and vertical circulation.

A somewhat similar question surrounds the origin of the minimum oxygen concentration and the minimum oxygen layer in the Atlantic. A recent study (Seiwell, 1937) has revealed that the minimum oxygen concentration in the western North Atlantic basin tends to occur at a density value (regardless of its depth) of \( \sigma_t = 27.232 \pm 0.008 \) which may suggest that this density value is somewhere near that of the great mass of organic matter. On the other hand, certain dynamic factors attached to such a situation prevent any rigid conclusion in favor of the biological argument as controlling the minimum oxygen concentration.

**LITERATURE**


NEW AUTOMATIC CALORIMETER FOR THE CONTINUED MEASUREMENT OF HEAT PRODUCTION OVER LONG PERIODS OF TIME*

W. M. BARROWS, JR., AND J. R. MURLIN

Department of Vital Economics, University of Rochester, Rochester, N. Y.

ABSTRACT

The principle upon which this apparatus operates is as follows. In addition to the heat generated within the calorimeter by the experimental animal, heat is supplied in the form of electrical energy, which is easily measured. By means of a device analogous to the self-balancing Wheatstone bridge, the rate of total heat supply is maintained constant by automatic control of the electric heat. The animal heat is then simply the difference between the total (constant) heat supply and the electrical heat supply.

The calorimeter chamber, well insulated thermally, contains an electric heater and a cooling coil which carries a steady stream of water under constant hydraulic pressure. Entering and leaving the chamber the water passes over electrical resistance thermometers, which form the two variable arms of a Wheatstone bridge, the galvanometer of which reflects a beam of light onto a photoelectric cell. The resulting photoelectric current, after amplification, operates a relay of the double-throw type. The latter controls a reversible motor, which moves a worm drive rheostat connected in series with the heater, a direct-current ammeter and a source of direct current. The null point of the galvanometer is so chosen that the reflected light beam falls half on, half off the sensitive surface of the photoelectric cell. If now the heat supply is for any reason (such as a drop in the D.C. line voltage) altered, the change will appear as a decrease or increase in temperature of the outgoing water. The bridge becomes unbalanced, the galvanometer is deflected swinging the light beam further off (say) the photoelectric cell. A decrease or increase in photo-electric current results, the relay is closed right or left and the motor will turn in such a direction as to decrease or increase the series resistance (increase or decrease the heating current). Thus the change plus or minus which started the mechanism will be counteracted and the heat supply brought back to its former value.

Temperature of the chamber wall and heat loss through the wall are automatically recorded. Heat of evaporation is measured by collecting the water from the ventilating current of air (previously dried) as it leaves the chamber. The outgoing air also is sampled continuously, and automatically for measurement of oxygen absorbed and carbon dioxide produced.

With groups of 12 to 16 rats experiments of 3 to 16 hours duration have shown good agreement between heat as calculated and as measured.

The apparatus described is the result of an attempt to produce a respiration calorimeter for small animals which shall

*Supported by a grant from the Penrose Fund of The American Philosophical Society.
be as nearly as possible automatic in its operation and thus require a minimum of attention during an experiment of long duration. This condition is satisfactorily fulfilled by the small-animal calorimeter now in use in this laboratory.

The principle upon which this apparatus operates is as follows:

In addition to the heat generated within the calorimeter by the experimental animal, heat is supplied in the form of electrical energy, which is easily measured. By means of a device analogous to the self-balancing Wheatstone bridge, the rate of total heat supply is maintained constant by automatic control of the electric heat. The animal heat\(^1\) is then simply the difference, total (constant) heat supply minus electrical heat supply.

![Schematic diagram of automatic calorimeter.](image)

Fig. 1. Schematic diagram of automatic calorimeter.

The mechanism whereby this control of the total heat is exercised is illustrated schematically in Fig. 1. In the upper right-hand corner is represented a conventional, double-walled calorimeter chamber, well insulated thermally, containing an electric heater and a cooling coil which carries a steady stream

\(^1\) Save that part represented by evaporation of water from the animal's body.
of water under constant hydraulic pressure. Upon entering and again upon leaving, the water passes over electrical resistance thermometers. These thermometers form the two variable "arms" of a Wheatstone bridge, the galvanometer of which reflects the beam of light from a fixed lamp onto a photoelectric cell. The resulting photoelectric current, after amplification by a direct current amplifier, operates a relay. This relay is of the double-throw type, being closed on low current, open on intermediate current and closed on the other side on high current. The relay controls a reversible motor, starting, stopping or reversing it, depending upon the magnitude of the photoelectric current. The motor drives a worm-drive rheostat, which is connected in series with the heater, a direct-current recording ammeter, and a source of direct current.

To illustrate the operation of the apparatus, suppose the empty calorimeter to be closed, the heating current to be adjusted to a value where it produces within the calorimeter an amount of heat certainly greater than that expected from the experimental animal, and the cooling water to be started flowing. After allowing sufficient time to reach thermal equilibrium, the bridge current is turned on and the "ratio arms" of the bridge (by means of a variable resistor not shown in Fig. 1) adjusted for zero galvanometer deflection. This null point is so chosen that when no galvanometer current flows, the reflected light beam will fall half on, half off the sensitive surface of the photoelectric cell. Now by adjusting the fixed grid potential ("bias"), the plate current is regulated so that it holds the relay in its intermediate, or open position, and the 110 volt supply to the motor is turned on. Providing true equilibrium has been reached, that is, that all thermal and electrical conditions remain unchanged, the apparatus will remain in this state indefinitely, the motor stationary and the electric heat supply constant.

Now suppose that for any reason whatever (let us say a drop in the D.C. line voltage) the heat supply is diminished. The change will appear immediately as a decrease in tempera-
ture of the outgoing water thermometer. The bridge will become unbalanced, the galvanometer will be deflected, swinging the light beam farther off (say) the photoelectric cell, the decrease in photoelectric current will decrease the grid potential (making it more negative) thus decreasing the plate current, the relay will close in its low current position and the motor will turn in such a direction as to decrease the series resistance (increase the heating current). Thus the change which started the mechanism will be counteracted and the heat supply brought back to its former value.

Due to the inherent time lag between the change in heat supply and the appearance of its effect at the outgoing water thermometer, and to the time required by the motor to move the rheostat slider sufficiently (through a reduction gear not shown in Fig. 1), the control device will “overshoot its mark” and overcorrect the change, provided the change was large compared to the sensitivity of the apparatus and was comparatively sudden. Then the whole process is repeated in the reverse direction and so on and the heat supply oscillates about its equilibrium value until the oscillations have become smoothed out to a level within the limits of sensitivity. With the values shown in Fig. 6 for the electrical constants, the limits are about ± 0.3 kg-calories per hour in total heat supply, or about ± 1 per cent of that usually supplied. In this respect, the device may be compared with an analytical balance, which swings to and fro about its null position, rather than remaining “stuck” in a position of static, and perhaps unstable, equilibrium. The electric heat supply, which is now total heat supply, may be read from the ammeter record.

Without disturbing the electrical adjustments, the calorimeter is now opened and the experimental animal introduced. The added source of heat will immediately throw the control device into operation and reduce the electric heat supply proportionately. After thermal equilibrium, disturbed by opening the calorimeter, has been reattained, the electric heat supply will level off at such a value as to reestablish the predetermined constant total, or will follow changes in the animal’s
Fig. 2. General view of calorimeter, with door open and animal cage in position. The vertical pipe in the foreground is the air intake from out-doors. The wooden box, lower right, contains the constant-temperature bath for pre-heating air and water. The table, right background, bears the air-sampling equipment and the shelf, upper right, the electrical accessory apparatus.
Fig. 3. Close-up of calorimeter, showing details of construction. The water thermometers, $T_1$, $T_2$, are housed in brass tubes entering the calorimeter at its lower right front corner. $B$ indicates the box containing the constant-temperature bath. Note the electrical heating ribbons in the extreme lower right of the photograph. These are used in room-temperature control (p. 594).
heat so as to maintain this constant total. The rate of heat production by the animal is then measured by subtracting that now being recorded from the known total, correcting for losses of heat by evaporation and by other means presently to be discussed.

An alternative method of establishing the total heat supply, obviating the expenditure of time in running the preliminary "electric period" described above, is that of calibrating the ratio arms of the bridge directly in terms of water temperature difference and measuring, during the animal experiment, the rate of flow of water. In this way the total heat removed (equal to the total supply) is measured directly and the electric heat subtracted from it to give animal heat. This method is the one now used with this calorimeter.

The complete control mechanism is shown in detail, photographically and diagrammatically, in figures 4, 5, 6, 7 and 8. The sensitivity controls (battery rheostat and galvanometer series resistance) are necessary to prevent the light beam from entirely traversing the photoelectric cell and going off the wrong side, which would of course result in the electric heat being changed in the wrong direction. The shunt on the primary relay adapts the actuating coil of the relay to the plate current of the amplifier tube, which would otherwise be much too great for it. (With the values shown the relay closes on the low side at 2 milliamperes plate current and on the high side at 4 ma., being fully open at 3 ma. plate current, or about 0.25 ma. through the relay itself.) The secondary relay, consisting actually of two double-pole Ward-Leonard relays connected as a reversing switch with an open or neutral position, is necessary since the points of the primary relay will not handle the current necessary to operate the motor. An open or neutral position is required in order that the heat-control rheostat may be stopped at any of various positions it may assume in maintaining thermal equilibrium, or in other words, since the balance of the bridge depends not upon the electric heat alone, but upon the total heat remaining constant.
Fig. 4. The control box (left) and the recording ammeter (right).
Fig. 5. Interior of control box. L, lamp; P, photoelectric cell; A, amplifier tube; G, galvanometer; B, B, radio "B" batteries; R, primary relay. The small box, housing the lamp and photoelectric cell, has its own cover which, when closed, allows light to enter only through the window before the photoelectric cell. Normally the entire control box is also closed by means of its cover. Power for the lamp, tube filament and Wheatstone bridge is derived from an external storage battery; the transformers on the floor of the box no longer are used.
Fig. 6. Detailed wiring diagram of control box. The switches, rheostats, meters, etc., are shown in approximately the same relative positions which they occupy on the front panel of the box (Fig. 4).
Fig. 7. The heat-control mechanism. M, motor; R, secondary relays; S, motor-speed control resistances; T, rectifying transformer for actuating relays; G, elements of reduction gear; Rh, heat-control rheostat.
Fig. 8. Diagram of heat-control mechanism. The elements are shown in the inverse (end for end) relative positions which they occupy in Fig. 7.
The recording ammeter (Fig. 4) is a Cambridge and Paul recording galvanometer with appropriate shunt and series resistances so that its full scale deflection corresponds to 1 ampere heating current. It is to be noted that when the current is measured in this manner, the resistance of the heater must be known and must be constant, that is, it must have negligible temperature coefficient of resistance, because the voltage drop across the heater itself is not measured; therefore, the heat developed is calculated from the product of the resistance and the square of the current. For this reason, the heat cannot be read directly from the ammeter record, but each ordinate must be squared and multiplied by the heater resistance and by an appropriate constant proportionality factor (see p. 507).

As may be seen in Fig. 4, the control box, containing the essential elements of the automatic heat regulation system, is a unit separate from the other elements of the apparatus, being connected therewith by electric cables and plugs. Therefore the control box may be used with different calorimeters interchangeably, which depend for heat measurement upon a pair of differential resistance thermometers, regardless of size or capacity, and with any system of secondary relays and heating control capable of being operated by the double-throw primary relay. In fact, the primary relay itself, which is mounted and connected on the outside of the control box (Fig. 4 and R, Fig. 5), could be dispensed with in favor of a current-control device operated directly by the amplifier plate current. Numerous such devices suggest themselves but their cost would probably be greater than that of the simple relay-motor system now in use.

**The Calorimeter Proper**

The calorimeter itself is illustrated in Figs. 2 and 3. It consists of an outer chamber of no. 16 gauge, galvanized iron, measuring $120 \times 94 \times 80$ cm., including the door, which comprises $10$ cm of the length and is supported on hinges along the left side of the open end of the chamber (invisible in the photograph), and an inner chamber of 24 ounce cold-rolled
sheet copper, measuring $100 \times 75 \times 60$ cm., which slides into the outer box and is spaced 10 cm. from it all around by means of guide rails riveted onto the walls of the outer box ($S$, Fig. 3).

The open end of the inner chamber is provided with a flange in the form of a rectangular frame of $2.5 \times 2.5 \times 0.3$ cm. brass angle, welded together at the corners to provide a smooth face and riveted, then soldered, to the copper walls of the chamber. A cover is provided in the form of a rectangular frame of the same size as the face of the flanges to which is cemented a pane of 6 mm thick plate glass. A gasket of sheet rubber is cemented to the metal face of the cover plate and 7.8 mm. holes are drilled through the cover-plate frame, gasket and flange, at intervals of about 20 cm. so that the cover-plate, with the face of the gasket greased with petroleum jelly, may be bolted in place by means of 6 mm. eye-bolts the nuts of which are soldered in place on the back of the flange, thereby hermetically closing the inner chamber. All seams in the copper chamber are securely soldered, inside and out, and pipe fittings, traversing the walls for the purpose of conducting the cooling water and ventilating air, are provided with flanges, soldered to the metal wall. The binding posts for connecting the electric heater consist of two no. 8–32 machine screws passing through the centers of two 9 mm. holes in the copper wall, held in this position by two strips of 3.1 mm. thick bakelite through which they pass, inside and outside the wall, the strips being bolted to the wall through another pair of holes just large enough to accommodate the securing screws. In assembly, the strips, screws and an appropriate area of the wall were covered thoroughly with Koenig cement (one part rosin, two parts beeswax, by weight) and the strips bolted tightly together while hot. After cooling, the assembly was painted thoroughly on all exposed parts with more of the hot wax, to insure airtightness.

The space between chambers is filled with loosely packed "Dry Zero" ($I$, Fig. 3), a vegetable fibre insulating material resembling "Kapok" and the outer chamber is completely
covered, on the outside, by a 1/2-inch thickness of "insulite" board, an insulating, vegetable-fiber wall board.

On the floor of the inner chamber are placed two wooden rails (5 × 10 cm. pine, impregnated with paraffin), standing on edge (10 cm. dimension vertical) and extending longitudinally from front to back of the chamber. These are so spaced as to be vertically above the metal beams between walls and hence to be supported thereby. On the upper surfaces of these rails are fastened lengths of brass channel, 2.5 cm. × 0.9 cm. deep, which serve as tracks, upon which the animal cage may be rolled in and out of the calorimeter on wheels fastened to the cage. Slots and holes are cut through the wooden rails to accommodate the cooling tubes and heater wires (see Fig. 3 and pp. 498–499).

The cage itself, shown in position in the calorimeter in Fig. 2, is made of 1.25 cm. mesh no. 16 gauge galvanized wire screen, reinforced at the top edges with 9 mm. iron wire, at the vertical edges with no. 16 gauge sheet metal angles and along the bottom edges with 2.5 × 2.5 × 0.3 cm. angle irons. Two pairs of solid metal wheels are borne on cross pieces fastened to the angle irons. The cage measures 85 × 45 × 50 cm. deep and will accommodate a dog weighing up to about 18 kg. or any smaller animal. It is provided with a false floor, supported by hinges at one end and by springs at the other, so that movements of the animal may be recorded by means of a pneumograph, on a kymograph drum outside the calorimeter.

For experiments on much smaller animals, such as rats, the cage is replaced by a rack, rolling on wheels which fit the tracks in the calorimeter. Individual cages containing the small animals are placed in the rack, which is then rolled into the calorimeter. By means of this device, the calorimeter has been used to measure the simultaneous heat production of groups of 16 rats and could easily accommodate many more (up to a total of 32).

The entire calorimeter is supported, with its lower surface about 35 cm. from the floor, upon a wooden stand (Fig. 2). A wooden trestle is provided, which bears brass-channel tracks
similar to those in the calorimeter. When this is placed against the open end of the calorimeter, the tracks join those of the calorimeter and form a continuation thereof. It is by means of this device that the cage is rolled into or out of the calorimeter, the trestle and cage forming a portable unit in the latter case.

**Cooling System**

The operation of the calorimeter depends upon the removal of heat at a steady rate by a stream of water. This is accomplished as follows: Distilled water is pumped from a storage tank in an adjacent room to a smaller tank near the ceiling, from which it may overflow and be conducted back to the main tank. Thus the small tank is kept filled to a constant level, or constant hydraulic head. An outlet pipe from this tank conducts the water through the wall into the calorimeter room. It then traverses a coil of tubing immersed in a constant-temperature bath (maintained by means of a toluene-mercury thermoregulator) and immediately enters the calorimeter. At the inlet to the calorimeter, the water flows longitudinally through a section of 2.5 cm. brass tubing, some 20 cm. long, which contains, coaxially, the sensitive element of a Leeds and Northrup electrical resistance thermometer \((T\), Fig. 3) of approximately 100 ohms resistance. Upon issuing from this tube, the water enters directly the cooling coil inside the inner chamber of the calorimeter. This coil, of 8 mm. copper tubing, is rectangular in cross-section and coaxial with the long dimension of the chamber (Fig. 3). The turns of the coil are spaced 10 cm. apart and are supported within the chamber by means of four wooden bars of 45° right-triangular cross-section, to which the tubing is fastened and which occupy the dihedral angles formed at the horizontal edges of the oblong chamber. In this way the coil is spaced 2.5 cm. (to center of tubing) from the copper walls, except of course at the corners, which are necessarily rounded. A few turns are added, in the form of a spiral, at the back (closed end) of the chamber. Additional rigidity is incidentally given the coil by soldering to it, at intervals, 10 cm. wide strips of thin (28
gauge) sheet metal, of a length equal to that of the coil, in a
direction parallel to the axis and across the back (Fig. 3).
The entire coil and the inside faces of the metal strips are
painted black, in order to facilitate absorption of radiation.
Care was taken in the assembly of the calorimeter not to dull
the naturally polished appearance of the copper wall, in order
that it might reflect back as much as possible of the radiation
incident upon it.

After traversing the entire coil, which includes some 17.5
meters of tubing, the water is led back to a point near the inlet
tube, where it leaves the calorimeter, passing over a resistance
thermometer exactly similar to the first one. From this point
it is conducted back through the wall of the room and is dis-
charged, through a flow meter, into the main storage tank.
 Provision is made for cooling the water slightly before it re-
traces its path through the system.

The functions of the cooling coil and resistance thermom-
eters have been discussed above. The remainder of the
hydraulic system is the same apparatus as used with the
human calorimeter in this laboratory, and has been described
in detail elsewhere (Murlin and Burton, 1935).

**The Electric Heater**

The electric heater consists of a coil of 24-gauge “chrom-
el-A” wire (a nickel-chromium alloy manufactured by the
Hoskins Mfg. Co., Detroit, and similar in properties to
“nichrome” alloy) which shows negligible variation of re-
sistance with temperature in the temperature range used.
This coil is exactly symmetrical to and coaxial with the cooling
coil, being spaced and supported by means of 2 cm. wooden
dowels, impregnated with paraffin wax and fastened by brack-
etts to the wooden supports of the cooling coil (Fig. 3). The
electrical connection of this heater with the other apparatus
has been described above.

**The Ventilating System**

The calorimeter may be ventilated by either of the con-
ventional methods commonly used in respiration calorimetry,
open-circuit or closed-circuit. Since, however, long periods with a minimum of attention by the operator are desirable, the following, modified open-circuit method has been adopted: Air is drawn from outdoors, through a drying train consisting of a sulfuric acid absorbing bottle (lower right, Fig. 2) (and a bottle of "dehydrite" to remove moisture), through a coil of tubing immersed in the constant temperature bath described above, and into the calorimeter through a port in the top, near the front of the chamber. Here the oxygen is partially consumed by the experimental animal and carbon dioxide and water vapor added. Air is withdrawn from a port in the back near the bottom (Fig. 7), passes through the air pump, and is forced through another water-absorbing bottle. This bottle is weighed periodically, its gain in weight representing the amount of water vapor evolved within the calorimeter. From here the air passes through a gas-flow meter, which measures the total volume of flow, and after traversing several decimeters of rubber tubing is exhausted into the room.

Inserted in the rubber exhaust tube, near the flow-meter, is the horizontal cross arm of a glass T-tube (Fig. 9). The vertical arm of this T-tube is connected with a mercury-filled gas-sampling tube, which is supported in a vertical position on a framework provided for the purpose. The reservoir tube of the sampling device is affixed to a wooden slide, which is free to move vertically in a set of guides. The slide is suspended from a cord which passes over 2 pulleys and thence to a small drum on the axis of a kymograph, about which it is wound for several turns and then secured. When the clockwork of the kymograph is started, the drum turns in such a direction as to unwind the cord and lower the slide bearing the reservoir tube. Mercury flows into the reservoir from the sampling tube and draws in air from the ventilating system to replace it. By adjusting the size of the drum and the rate of the kymograph, the sampling device may be made to draw one full sample (about 100 cc.) during any predetermined period of time. Two of these sampling devices, operated by a single clockwork, are used in practice, in order to furnish duplicate
A NEW AUTOMATIC CALORIMETER

Fig. 9. The air-sampling apparatus.
samples for comparison. The entire assembly is illustrated in Fig. 9. Since the rate of sampling is uniform, the contents of the tubes will be samples of the average composition of the air exhausted from the calorimeter during the period. The samples are then analyzed by the Haldane method and the results of the analysis multiplied by the total volume of flow as determined from the flow-meter. Comparison of this product with the composition of an equal volume of outdoor air gives the quantities of oxygen consumed and carbon dioxide evolved by the animal during the period. Knowing these, we may then calculate the “respiratory quotient,” which gives some insight into the nature of the substances oxidized, and the amount of heat which should be released in this oxidation. This heat, calculated on a chemical basis from the products of respiration and called “indirect heat,” is then checked against that measured physically by the calorimeter, called “direct heat.”

**Evaporation**

Since the vaporization of water inside the calorimeter involves the absorption of heat (heat of vaporization) and the water is withdrawn from the calorimeter in the form of vapor, the ventilating system constitutes an avenue of escape of heat which is not measured by the water thermometers. Consequently, the amount of water caught in the absorber in the exhaust side of the system, and the heat of vaporization represented thereby, must be measured and the heat added to that removed by the cooling water as a contribution to the total heat evolved. However, since evaporation in the calorimeter takes place at a substantially constant rate, the assumption of a constant rate of removal of heat will not be badly in error and this avenue of escape will not seriously affect the operation of the control device.

Similarly, the heat removed by absorption into the air itself must be taken into account. This is done by placing thermometers (ordinary mercury thermometers reading to 0.1°C. are sufficiently precise) in the ventilating system at the entrance and exit, respectively, of the calorimeter. The
temperature difference shown, multiplied by the volume of flow, by the density of the air and by its specific heat, is a measure of the amount of heat removed via this route. This amount is, in practice, almost negligibly small and is quite constant, for reasons about to be described.

Heat Loss to the Room

Finally, there is one more avenue of escape of heat, which is considerable and must be taken into account. It is practically impossible so to construct a calorimeter that no heat escapes through its insulation into the room, without an elaborate system of measurement and manual control of wall temperatures, e.g. the Russell Sage calorimeter (Riche and Soderstrom, 1915). Hence, it is the practice in this laboratory (cf. Murlin and Burton, loc. cit.) to impede this heat flow as much as possible by means of insulation, then to measure it and take it into account in the calculations. The measurement is accomplished in exactly the same manner as in the human calorimeter of Murlin and Burton, namely by measuring the temperature difference between inner and outer chamber walls (temperature gradient across the insulation) and assuming linear gradient proportional to heat flow (constant insulation resistance).

For this purpose the walls of the calorimeter are furnished with electrical resistance thermometers. Each of these consists of a single length of cotton-covered nickel wire (no. 40 gauge Driver Harris grade A nickel), laid to and fro in the form of a grid, held snugly against the metal wall and painted lightly but thoroughly with hot Koenig cement. These wires are thus fastened against five faces (all except the open end) of each chamber, before assembly, that of the inner chamber on its exterior faces and that of the outer chamber on its interior faces. They are connected as the variable arms of a Wheatstone bridge circuit, housed in the box directly under the recording ammeter (Fig. 4). The recorder itself is the galvanometer. It is so constructed that it records each of two different quantities on alternate minutes. These are, in the present
application, wall temperature differences (gradient) and amperes heating current, each recorded every two minutes. It is unnecessary to calibrate the wall thermometers directly in terms of temperature; the galvanometer readings may be correlated directly to heat loss through the wall, by calibration experiments in which a known amount of heat is introduced to the calorimeter and heat losses by all avenues except that through the wall are directly measured.

Now suppose the calorimeter to be placed in an environment of constant temperature. If the heat evolved within it remains constant, and the rate of removal by cooling water, ventilating air and evaporation is constant, then the loss through the wall, hence the temperature gradient, hence the interior temperature, must necessarily be constant. Therefore, the partition of heat between the various avenues of escape remains uniform, and the operation of the control device, which depends upon one element of this partition, will be steady and reliable.

**Room Temperature Control**

For the reason just stated, the calorimeter is installed in a constant temperature room. The temperature control is effected as follows: Around the walls of the room, about 36 cm. from the ceiling and 10 cm. from the wall, passes a single turn 3.1 cm. pipe, which conducts a stream of cold brine, pumped to the room by the refrigerating plant in the building.1 (Actually the brine is diverted from the cooling coil of the human calorimeter room; cf. Murlin and Burton, *loc. cit*.). Each of the long walls of the room (it measures 4.1 × 2.5 m.) is provided with a false wall of 1.25 cm. thick insulite board, 10 cm. away from the permanent (brick) wall extending vertically from a point 15 cm. above the floor to a point 15 cm. below the ceiling, with a flare at the top to clear the brine pipe and its drain gutters, (not shown) all supported on studs bolted to the brick wall. Ten cm. farther (20 cm. altogether) from the brick wall and in a plane parallel to it is a strip of

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1 Part of this pipe, frost-covered, is visible in Fig. 2.
6 mm. thick "alco" (mineral fibre) board, extending upward a distance of 36 cm. from the floor itself, hence overlapping the insulite panel 15 cm. in a vertical direction and 10 cm. away from it. In the space between the insulite and alco boards, in the region of overlap, seven strands (actually a single strip doubled back and forth 7 times) of Hoskins "chromel C" electric heating ribbon, 6 mm. wide and 0.25 mm. thick (no. 30 B & S gauge) are strung parallel to each other, with their edges in horizontal planes, their widths vertical and lengths parallel to the walls. These strands are kept 1.25 cm. apart by insulating spacers at 1.2 m. intervals throughout their length and are held taut by helical springs, of the screen-door type, to which they are connected by cords passing over pulleys and which compensate for expansion and contraction of the ribbons. The ribbons are connected, in series electrically, to a 110-volt A.C. main. The combined resistance of those on the two sides of the room is approximately 22 ohms, so that some 5 amperes flow continuously in the ribbons, producing about 550 watts in the form of heat.

The room air near the ceiling, cooled by contact with the brine pipe, falls behind the insulite curtain and is forced upward over the hot ribbons as it issues into the room, whence it rises and repeats the circuit. The two air currents from opposite sides of the room cause sufficient turbulence, and the convection currents produced by the alternate cooling and heating cause sufficient circulation, that fanning or other stirring of the air is rendered unnecessary.

A thermo-regulator of the ether-and-ether-vapor-over-mercury type, depending for its action on the change with temperature of vapor pressure, which supports a column of mercury in a Torricelli vacuum, rather than directly on change of volume with pressure, is placed as nearly as possible in the geometric center of the room and is connected through appropriate relays to the heaters, in such a way that when the thermo-regulator circuit is closed by falling temperature, the power relay closes and shunts out part of the heater resistance. When this occurs, the heating current, varying inversely with
resistance, increases, thereby increasing the heat supply and raising the room temperature. When the temperature becomes too high, the converse series of events occurs, reducing the heat supply. This type of thermo-regulator is of quite high sensitivity, depending chiefly on the thermal capacity of the glass bulb containing the ether-vapor. Changes of temperature of the order of a few thousandths of a degree C. at the thermostat bulb are sufficient to operate it. In the present application, the temperature at the calorimeter is maintained constant within less than ± 0.1° C., this in spite of the intermittent presence in the room of one or more persons and the occasional momentary opening of the doors of the room. Note that some heat is being supplied at all times, tending to counteract the cooling by the refrigerant, hence convection is taking place at all times, regardless of the action of the thermostat.

**Sample Data-sheet**

*Illustrating Method of Calculating Results*

Subject: 16 rats (fed diet high in sucrose) of average weight 206 g.

Date: April 9, 1937.

Period I Began 1:02 p.m. Ended 4:02 p.m.

Duration of Period: 3 hours

Setting of ratio arm (Kohlraush bridge) = 5.040

Calibration equation of bridge:

\[ \Delta T = 10.40 (P - 5) + 0.539^\circ C. \]

where \( \Delta T \) = water temperature difference,

\( P \) = setting of slide wire in arbitrary units,

whence: Water temperature difference = 0.955°

Rate of flow of water (measured by water meter) = 28.60 liters per hour

Heat removed by water = \( 27.31 \) Cal. per hour

Air temperature difference (measured by mercury thermometer) = 0.68° C.
Rate of flow of air (by air flow meter) = 512.4 liters per hour
Correction factor to reduce to standard conditions from 25.2°
C. and 743 mm. = 0.863
Heat removed by air = $F \times k \times p \times C_p$
where $F =$ rate of flow,
$k =$ factor to reduce to standard conditions,
$p =$ density of air at standard conditions,
$C_p =$ specific heat at standard conditions.
Heat removed by air = $512.4 \times .863 \times 1.0293 \times 0.237$
= 0.08 cal./hr.
(The degree of accuracy used here in $F$ and $k$ is unnecessary,
but will be used later.)
Average reading of wall temperature record = 17.3 scale
divisions.
Heat loss through wall (read from calibration curve)
= 5.83 cal./hr.
Water evaporated in calorimeter (collected in absorber and
weighed) = 29.405 grams in 3 hours or 9.802 grams
per hour.
Latent heat removed by evaporation
= $9.802 \times 0.583 =$ 5.71 cal. per hr.
Total Heat Removed (sum of items italicized above)
= 38.93 cal. per hr.
(Item referred to on p. 495.) Average
electric heat supply (obtained by
squaring each ordinate of ammeter
record, multiplying by heater re-
sistance and by factor of 0.8600 to
reduce watts to Calories per hour)
= 20.23 cal. per hr.
Animal heat (direct) (Total heat minus
electric supply)
= 18.70 cal. per hr.
Calculation of Indirect Heat:
Oxygen loss from air (by Haldane anal-
ysis) = 0.842 per cent.
Oxygen consumed by animals = 512.4
$\times 0.863$ (from air flow) $\times 0.00842$
= 3.723 liters per hour
Carbon dioxide increase in air = 0.921 per cent.
Carbon dioxide produced = 512.4 \times 0.863 
\times 0.00921 = 4.073 \text{ liters per hour}

Respiratory Quotient = \frac{\text{CO}_2 \text{ produced}}{\text{O}_2 \text{ consumed}} 
= 1.09.

Excess of \text{CO}_2 \text{ production over } \text{O}_2 \text{ consumption} = 4.073 - 3.723 = 0.350 \text{ liters per hour}

\text{Indirect Heat} (\text{By Zuntz & Schumburg table, as corrected by Lusk, 1924}) 
= 3.723 \times 5.047 + 0.350 \times 1.09 
= 18.79 + 0.38 
= 19.17 \text{ cal. per hr.}

Difference: Indirect Heat minus direct heat = 0.47 \text{ cal per hr.} = 2.51\% 
(of direct heat).

\text{Experimental Results on Rats}

As evidence that the calorimeter is capable of measuring direct heat with a satisfactory degree of accuracy, the results of a few experiments on groups of rats are given briefly below. The rats were being pair-fed on glucose and sucrose diets containing every other constituent in like amount except the 68 per cent of sugar. The problem which was under investigation has not yet been completed; it will be reported elsewhere in due time.

\text{Series I (1936).}

At the time the experiments were run the two groups of 12 rats each in Series I weighed on the average 179.3 and 189.6 grams respectively. In all, twenty-four 3 hr. experiments were done, 12 on each group. Table 1 compares the direct and

\text{It is a coincidence that the heat value of a liter of } \text{CO}_2 \text{ above the amount required to give a non protein R.Q. of 1.0, should have the same numerical value as the total R.Q. given six lines above. For the origin of this heat value, 1.09 kg. cal. per liter, see G. Lusk's Science of Nutrition, Philadelphia, 1928, p. 397.}
TABLE 1

Comparison of Indirect and Direct Hourly Heat Production of Groups of 12 Rats in 24 Experiments

<table>
<thead>
<tr>
<th></th>
<th>Indirect</th>
<th>Direct</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean Cal.</td>
<td>12.39 ± 0.18</td>
<td>12.77 ± 0.21</td>
</tr>
<tr>
<td>St. Dev.</td>
<td>1.28 ± 0.13</td>
<td>1.55 ± 0.15</td>
</tr>
<tr>
<td>Coef. Var.</td>
<td>10.3%</td>
<td>12.2%</td>
</tr>
</tbody>
</table>

Av. Dif. bet. D. & Ind. 3.07%

Indirect heat in these 24 experiments. The mean indirect heat is 12.39 kg. calories per hour with a probable error of ± 0.18 cal. The mean direct heat is 12.77 kg. cal. per hour with a probable error of ± 0.21 cal. The mean difference is 0.38 cal. or 3.07 per cent of the indirect. The standard deviations and coefficients of variation amongst the 24 experiments are self-explanatory. The closed-circuit system of ventilation was used.

Table 2 compares the heat productions taken singly as the mean between the indirect and direct methods for each experiment, and taken together as the mean for twelve experiments on each group. The average rat of the sucrose group showed a slightly higher metabolism per hour than the average rat of the glucose group. This difference is not quite proportional to the difference in weight but is quite exactly proportional to the difference in surface area as expressed by Diack’s (1930) formula.

TABLE 2

Average Hourly Heat Production per Unit Weight and Surface of 2 Groups of 12 Rats Each

<table>
<thead>
<tr>
<th></th>
<th>Glucose</th>
<th>Sucrose</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean wt.</td>
<td>179.3 g.</td>
<td>189.6 g.</td>
</tr>
<tr>
<td>Mean surf.</td>
<td>0.0238 sq. mm.</td>
<td>0.0245 sq. mm.</td>
</tr>
<tr>
<td>Mean heat prod. rat/hr.</td>
<td>1.016 cal.</td>
<td>1.064 cal.</td>
</tr>
<tr>
<td>Mean heat prod./100 g. wt.</td>
<td>0.566</td>
<td>0.561</td>
</tr>
<tr>
<td>Mean heat prod./sq. m. surf.</td>
<td>43.3 ± 1.05</td>
<td>43.4 ± 1.29</td>
</tr>
<tr>
<td>St. deviation</td>
<td>3.64 ± 0.50</td>
<td>4.47 ± 0.62</td>
</tr>
<tr>
<td>Coef. variability</td>
<td>8.4%</td>
<td>10.3%</td>
</tr>
</tbody>
</table>

There was no control of activity of the animals other than that imposed by the rather small individual cages in which the animals were confined, not only while in the calorimeter but throughout the feeding experiment. The rats in this series of
experiments were in the post absorptive state for the six or seven hours during which they were in the calorimeter. The apparatus was not yet in its final form when the first series was conducted.

*Series II (1937)*

A second paired-feeding experiment was conducted, and heat productions were measured with the apparatus in its final form as described in this paper. There were 16 rats in each group in this series, one fed on a 68 per cent glucose diet, the other on 68 per cent sucrose. The calorie intake of the two groups was identical, until the calorimeter experiments were begun. During the course of the experiments the food intakes were deliberately changed in order to induce changes in weight. Food and water were placed in the small individual cages, so that the metabolism recorded includes the effects of activity incident to feeding and the specific dynamic action of the food—in short, it was the total normal energy metabolism which was desired. On four of the 12 experimental days the measurements were made in only two 3 hr. periods as in the first series; but on the remaining 8 days these two short periods were followed by one long 16 hr. period lasting from around 5:30 P.M. to 9:30 A.M. The calorimeter ran itself without any attention—in fact without any person in the laboratory throughout the night. The open-circuit system of ventilation was used.

**Table 3**

<table>
<thead>
<tr>
<th>Direct</th>
<th>Indirect</th>
<th>Difference of Averages</th>
<th>Average of Individual differences</th>
</tr>
</thead>
<tbody>
<tr>
<td>cal./hr</td>
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In table 3 is shown the close agreement between the mean direct and indirect heat measurements obtained with the ap-
paratus in its completed form. The reader should note that the percentage difference between the averages is quite another value from the average of the individual percentage differences. Claims for accuracy, which in reality are misleading, have sometimes been made on the former basis. The real test of the closeness of agreement which may be expected in any experiment is the average of the percentage differences. The value found in these experiments is slightly better than the average found in this laboratory with the human calorimeter; namely 5.8 per cent, where much greater quantities of heat were involved (Murlin, Burton and Barrows, 1936). The apparatus therefore is capable in long periods of giving satisfactory results.

**Summary**

1. An automatic calorimeter is described capable of measuring both directly and indirectly at the same time, the heat production of groups of 12 to 30 rats or of a dog weighing 10 to 20 kg.

2. The apparatus operates on the principle of successive differential calorimetry. In addition to the heat generated within the calorimeter by the experimental animal or animals, heat is supplied in the form of electrical energy, which is accurately measured. By means of a device analogous to a self-balancing Wheatstone bridge the rate of total heat supply is maintained constant, first, with the calorimeter empty, and then with the animal or animals contained. The animal heat is the difference between this total constant heat and the electrical heat required when the animal is contained.

3. Complete description is given of the controlling and recording mechanisms.

4. The chamber may be ventilated by either a closed-circuit or an open-circuit method. It has been operated successfully in both ways giving close agreement between direct and indirect measurements. The open-circuit, however, has given somewhat better results, samples of air being analyzed by an improved Haldane apparatus.

5. The room in which the apparatus stands is thermostatically controlled by a new method, which is described.
6. Sample data sheets with an illustration of the calculations involved in an experiment are given.

7. Two series of experiments on groups of paired-fed rats are presented as evidence of the close agreement between direct and indirect heats which may be obtained.

LITERATURE CITED


MELANOPHORE RESPONSES AND BLOOD SUPPLY  
(VASOMOTOR CHANGES)

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(Read by title, Nov. 27, 1937)

ABSTRACT

It has long been known that in many animals, such as chameleons among lizards and catfishes and killifishes among the bony fishes, the changeable color cells in the skins of these animals are under the control of nerves. It has not been ascertained whether these nerves cause the color cells to expand or contract by direct action as motor nerves activate muscles or whether the chromatophore nerves act on blood-vessels which in turn control the color cells by an increased or decreased output of particular substances, thus resulting in an indirect influence for the nerves. This question was tested by cutting off the blood supply from a given area of skin of such fishes as the dogfish, the killifish, and the catfish and then exciting the nerves to the color cells to ascertain whether these cells could still react. Such was found to be true. For half an hour after the blood had been excluded from the skin but while the tissues were still alive the nerves when excited could induce either contractions or expansions of the color cells. This also occurred even in areas of skin from which the quiescent blood had been fully washed out by irrigation of the blood-vessels with Ringer's solution. It is therefore concluded that nerves to color cells act directly on these cells and not indirectly through the blood supply.

1. INTRODUCTION

For at least a decade and a half students of chromatophores and particularly of vertebrate melanophores have intimated in one way or another that in experiments on color changes too little attention has been paid to the possible effect of associated interferences with the blood supply. Thus Hogben in his volume on "The Pigmentary effector System" (1924, p. 36) states in discussing this situation in lizards that "there exists, at present, no well-authenticated experiments demonstrating positive effects of nerve reaction or stimulation in Reptiles or Amphibia, conducted in such a way as to exclude the possible influence of concomitant vasomotor changes."

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Lundstrom and Bard (1932, p. 8) in commenting on the part played by adrenalin in blanching Mustelus remark that "it may well have been that" through this substance a "vasoconstriction of cutaneous vessels led to an asphyxiation and consequent contraction of the melanophores." Young (1933, p. 580) concludes a brief discussion of this instance with the remark that the paleness of the dogfish under adrenalin is probably "a secondary effect due to vascular change," and Wykes (1936, p. 463) finds "considerable support" for the assumption that in elasmobranchs adrenalin affects the color indirectly "as a result of its vaso-constriction action." None of these statements is accompanied by any evidence experimental or otherwise that tends to show their correctness. They are in short unsupported opinions on a matter which appears to call for some comment and which from the frequency with which such comments have been made requires an answer. The present paper will be devoted to a consideration from an experimental standpoint of the relation of the vascular system to the melanophore reactions of three fishes, *Mustelus canis*, *Ameiurus nebulosus*, and *Fundulus heteroclitus*. This work was done at the Marine Biological Laboratory at Woods Hole, to the Director and Assistants of which I am under obligation for aid in many ways.

2. Mustelus

The dark phase in the smooth dogfish *Mustelus* is due to the action on its melanophores of a secretion from the intermedite lobe of the pituitary gland (Lundstrom and Bard, 1932) and the pale phase to the action of a lipohumor from the concentrating nerve-fibers on these pigment cells (Parker, 1935). Consequently the only phase in this fish which is significant for the present discussion is the pale one and the most favorable part of the fish in which to study this phase is the pectoral fin. Some ten minutes after a cut transverse to the rays of the pectoral fin of a dark dogfish has been made in a position to interfere least with the blood supply, a pale band appears reaching from the cut directly to the margin of the fin.
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This band results from a concentration of the pigment in the melanophores of the region concerned.

It was now attempted to cut off the flow of blood in the pectoral fin of this fish and to ascertain whether or not the formation of a pale band would occur after this step had been taken. In an active adult dogfish dark in color the subclavian artery was exposed and tied off and two cuts were made in the fin so as to intercept the chief veins in this organ. An inspection of the middle of the edge of the fin, under the microscope, showed no movement of blood in the vessels in which previous to the operation a copious flow had been seen. On making a transverse cut to incite the nerves in such a bloodless fin the pale band appeared as usual in some ten minutes or so (Fig. 1). But an inspection of the lateral edges of the fin showed more or less collateral circulation, as in fact Wykes (1936) had reported for other elasmobranchs similarly treated, and since the body of the fin in Mustelus was too thick for microscopic examination this method was abandoned as not fully reliable.

A second method was tried in which a stout cord was bound round the base of the pectoral fin directly next the trunk of the fish and then tightened by being twisted as a surgical tourniquet is made tight. This method proved entirely effective as a means of checking completely the flow of blood in the fin. About an hour and a half after the fin had been tied off it began to blanch slightly. This change in color could be seen particularly when the corded fin was compared with the opposite, normal fin. After about three hours the corded fin was very much paler than its mate. At this stage the cord was removed from the fin and in less than half an hour the fin had become as dark as the rest of the fish and indistinguishable in appearance from its opposite. An inspection of its edge under the microscope showed a fully returned circulation. In a second test the cord was kept on the fin for some six days. In this test the fin blanched as the first one did, then became mottled and discolored and finally after about five days its tissues showed necrotic changes.
The cord was then removed but the fish died the day following. Whether death resulted from the condition of the fin or from some other cause was not determined. In several instances fins bound in the way described were cut from the fish just distal to the ligature. In none of these was there any bleeding from the central stump as was regularly so when unbound fins were cut off. These preliminary trials showed that this method was reliable so far as the stoppage of the circulation of blood was concerned, but that it was safe to regard the responding tissues as normally active for only an hour or so after the ligature had been applied.

When cuts such as would have incited pale bands in a normal Mustelus were made immediately after the flow of blood had been checked by ligating the fin as already described, pale bands were regularly formed (Fig. 2), but they were not so pronounced as the ordinary pale bands in normal fishes and they could be incited only during the first quarter of an hour or so after the circulation of the blood had been cut off. They were, however, invariably and unquestionably present under the circumstances. The fact that these reactions are to be seen for only a short time after the flow of blood in the fin has been checked makes clear why Wykes failed to get conclusive evidence on this particular question. After having tested the effect of obstructing the blood supply in a number of elasmobranchs she concluded (1936, p. 465) "where the circulation is impeded melanophore changes are temporarily eliminated." Had she tested the elasmobranchs on which she worked within a quarter of an hour after the occlusion of the circulation instead of some three to four hours thereafter, she might have reached different results, though it is to be borne in mind that the elasmobranchs on which she worked probably do not possess concentrating nerves.

In a third method the young of Mustelus, the pups, were employed instead of the adults. These pups were some 30 centimeters long and remarkable for the activity of their color changes. Dark individuals were opened ventrally so as to expose the heart, and the ventral aorta was firmly ligated.
An inspection of the edge of the pectoral fins under the microscope showed before this operation a good flow of blood. After the aorta had been ligated, no circulation could be seen, as might have been expected. On ligating the ventral aorta appropriate cuts were made in the pectoral fins and pale bands were invariably formed soon thereafter (Figs. 3 and 4). These bands, though not so pronounced as those in normal pups, were more striking than those in the ligated fins of the adults. The records from the pups as well as those from the adults show without doubt that the blanching of *Mustelus* on nerve cutting in the absence of an active blood supply is still possible though this capacity persists for only a part of an hour after the blood has been cut off. These tests are sufficiently conclusive to show that no vasomotor or other vascular changes are necessary links in the chain of events that intervene between the action of the concentrating nerve endings and the responding of the melanophores. It is concluded that the melanophores in *Mustelus* are directly stimulated by their concentrating nerves.

3. *Ameiurus* and *Fundulus*

There is good reason to believe that the melanophores of *Fundulus* (Mills, 1932; Abramowitz, 1936) and of *Ameiurus* (Parker, 1934) are doubly innervated in that they possess one set of nerve-fibers for the concentration of their pigment and another for its dispersion. In *Ameiurus* the action of the dispersing fibers is supplemented by a pituitary secretion and that of the concentrating fibers possibly by a concentrating one (Abramowitz, 1936). In *Fundulus* the control of the melanophores seems to be purely nervous except that the action of the dispersing fibers may possibly be aided by a weak pituitary hormone (Kleinholz, 1935; Abramowitz, 1937). In discussing the condition in these two fishes attention will be directed to the nervous control of their melanophores and the particular question to be kept in mind is whether the nerves act directly on the melanophores or indirectly through vascular changes.

If the electrodes of an induction apparatus are applied to
the medulla of a dark *Ameiurus* or a dark *Fundulus*, the whole fish quickly blanches. If such electrodes are brought to bear on a single ray in the tail of one of these fishes in the dark condition this ray from the region of application to the edge of the fin will blanch. These responses have long been known to be due to the stimulation of the concentrating nerve-fibers of the parts concerned. If the autonomic trunk in the region of the internal ear of a *Fundulus* or of an *Ameiurus* is cut, the corresponding side of the head from the region of the cut to the anterior end will darken by the dispersion of the pigment in the melanophores of this region. If a ray and its contained nerve in the tail of either of these fishes is cut, the ray will darken by melanophore expansion and become evident as a band extending from the cut to the edge of the tail. These responses, which were originally believed to be due to paralysis of the concentrating nerve-fibers, are now known to be the result of an excessive stimulation of the dispersing nerve-fibers. The question to be determined in this connection is the extent to which these two sets of nervous reactions, the concentration and the dispersion of melanophore pigment, are dependent upon vascular change.

The pale phases in *Ameiurus* and in *Fundulus* are easily excited locally by a faradic stimulation of a nerve trunk. Does such stimulation induce through the nerve a contraction of the blood vessels with a resultant anoxemia whereby the melanophore pigment is induced to concentrate? So far as our present knowledge goes such a relation is possible and it is to instances of this kind that Hogben (1924) has rightly directed attention.

I have attempted to approach this question in the following way. Six catfishes which had been blanched by three days' retention in a white-walled illuminated vessel were cut each in two spots in the tail so that all the fishes showed on their caudal fins two well defined separated dark bands. Several hours after these bands had been formed each fish was subjected to the following test. By a single cut of the scissors through the most posterior right and left gill-slits completely
from side to side the ventral aorta of the fish was severed and the circulation of blood thus stopped. The fish was then laid on its side, the tail spread out, and the points of a pair of electrodes were immediately applied across one of the dark bands close to the root of the tail. In from three to five minutes, during which time the stimulating current was running, the band began to blanch with the result that in a minute or so later the band had disappeared in that its area had become as pale as the rest of the fish was. The remaining fishes were treated in the same way except that in a few cases the electrodes were first applied to the cut ray proximal to where it was severed with the result that the band failed to blanch. When, after this preliminary test, the electrodes were next placed on the band distal to the cut, the band blanched as in the previous instance. During all this procedure, the severed band on the tail remained unchanged showing that the general condition of the fish in this respect was unaltered. The tail of one fish after the ventral aorta had been severed was examined under the microscope to ascertain the condition of the circulation. As was to have been expected there was no flow of blood in the vessels. Exactly similar results were obtained from tests carried out on the tails of a group of Fundulus. It, therefore, appears certain that melanophores can concentrate their pigment under nerve stimulation in regions devoid of a flow of blood. Tests of this kind repeated on catfishes at intervals of time after the cutting of the ventral aorta showed that melanophore contractions could be induced as described for some 25 minutes following the elimination of the blood supply. After half an hour it was generally impossible to excite the contraction of the melanophore pigment electrically, and in an hour or two what may have been left of the control band and all of the band especially tested gradually disappeared by blanching. This late response I believe to be the first sign of true anoxemia which characterizes Ameiurus and Fundulus after death. In my opinion it has nothing to do with the earlier blanching of the caudal bands by electric stimulation.
As a final test of the relation of blood to the dispersing reactions of melanophores I tried the following experiment on large catfishes. An *Ameiurus* was restrained ventral side up and its pericardial cavity opened. Into its ventral aorta a canula was tied through which the whole circulatory system of the fish could be perfused with Ringer’s solution. The Ringer’s solution was introduced under a pressure of some seventy centimeters of water and after seven to eight minutes of flow it escaped free of red color from the opened auricle of the fish. An inspection under the microscope of the tail of the Ameiurus showed the blood-vessels of this organ to be free of blood corpuscles. Presumably this amount of irrigation freed the circulatory system in this fish of its blood. At this point in the operation a ray in the tail was cut in the usual way and about ten minutes thereafter a dark caudal band appeared in appropriate position. This test was carried out on four catfishes in all and invariably with the same result. It seems clear, therefore, that in *Ameiurus* the absence of standing blood in the blood-vessels offers no hindrance to the action of the dispersing nerves on their associated melanophores.

These observations confirm in all essential respects Smith’s studies (1931) on *Phoxinus* in which he showed that after the loss of the heart and the consequent cessation of the blood flow, the melanophores in this minnow could be made to contract by electric stimulation. Here in fact the tests were repeated several times on the same fish. The period over which such tests were possible was set down by Smith at about half an hour. In a similar way Abramowitz (1936) showed that when the autonomic strands in the posterior part of a catfish that had been severed from the rest of the fish and consequently was without circulating blood are stimulated electrically, the innervated melanophores contract and the tail blanches. Both these sets of observations are confirmed in general by my own work just recorded.

The dark phase of *Ameiurus* and of *Fundulus* may be induced by the severance of dispersing nerves. When these
are cut in a pale fish, a dark area or a dark band results, due to the dispersion of melanophore pigment. Such responses were first described in fishes by Pouchet (1876). As already stated they were regarded by the earlier workers as the result of the paralysis of concentrating nerve-fibers, but they have since been shown to be due to the over-excitation of dispersing fibers in consequence of the mechanical stimulation due to cutting.

In both *Fundulus* and *Ameiurus* two sets of nerves are favorable for cutting, the ophthalmic nerves in the orbits and the radial nerves in any of the rays in the tail. By a small incision through the skin dorsal to the eye-ball the orbit of either of these fishes may be opened and the ophthalmic nerve easily reached as it crosses on the dorsal bony roof of that cavity. It is not difficult to sever this nerve without injuring the accompanying blood-vessel. When such a cut is made the appropriate half of the head of the fish darkens from the cut anteriorly to the snout. This condition has been noted recently by such workers as Smith (1931) and especially Abramowitz (1936). Abramowitz observed further in *Ameiurus* that if only the blood-vessel was severed, the nerve having been left intact, no color change occurred. It is highly improbable that this vessel supplies exactly the region on the head that darkens when the nerve alone is cut or that collateral circulation may not replace a part of the true vascular field after the vessel has been cut.

To test still further the significance of the blood supply in this respect in both *Ameiurus* and in *Fundulus* I excised the hearts of these fishes and then immediately cut on one side of each fish the ophthalmic nerve. In all such instances the appropriate side of the head now without circulation darkened as it did when the nerve was cut and the blood was flowing normally.

What is true of the ophthalmic region in these two fishes is also true of their caudal rays. When a bony ray with its contained nerve in a heartless *Fundulus* or *Ameiurus* is cut, the ray immediately darkens even though there is no circula-
tion of blood in the fin. These reactions like those of the ophthalmic darkening of these fishes appears then not to be immediately dependent upon any vascular factor, but to take place through the direct action of the dispersing nerve-fibers on the melanophores.

Such a conclusion is consistent with an interesting and somewhat remarkable condition in the tail of Fundulus first pointed out, I believe, by Abramowitz. When the posterior autonomic chain on one side of a Fundulus is severed, the posterior quadrant of the fish on the appropriate side darkens. This darkening extends over the tail and is limited to the corresponding face of this relatively thin organ as can be seen by inspecting the melanophores of its two sides. It is indeed remarkable that the change does not spread to both sides. The blood-vessels of the tail freely anastomose; in fact there is only one general vascular supply to this organ. The chromatophoral nerves on the other hand are apparently bilateral in their distribution in that the nerve-fibers from the right side of the body remain on that side of the tail and those from the left on their side. The excitation of appropriate nerves thus induces unilateral responses. Although the peculiar condition herein described is not a proof of melanophore independence of the vascular system, the understanding of this condition is made possible where such independence is assumed.

What part the blood system could play as a mediating mechanism between nerves and melanophores in the darkening of such fishes as Ameiurus and Fundulus is not easy to state. Vasodilation might bring more of a dispersing hormone, such as intermedin, into the lymph surrounding the melanophores and thus induce expansion, but the reactions we are considering are nervous and not those due to pituitary secretions. It is in fact difficult to ascribe a dispersing nervous activity in these reactions to vasomotor changes. Vasoconstriction on the other hand might produce tissue anoxemia and hence concentration of melanophore pigment (blanching). But it has already been shown that nervous blanching can take place
long before tissue anoxemia sets in so that it is unnecessary to assume any such vascular condition as a part of the normal blanching. So far as Ameirus and Fundulus are concerned the pale and the dark nervous responses certainly take place under appropriate stimulation in individuals in which the circulation of blood has ceased and in which consequently vasomotor effects are quite impossible. This condition favors distinctly the view that the dispersing and the concentrating nerve-fibers act directly on the melanophores and not through vascular intervention.

4. Discussion

The conclusion to be drawn from these studies is that in the three fishes investigated, Mustelus, Ameirus, and Fundulus the melanophore nerves, both concentrating and dispersing, act directly on the melanophores through their neurohumors and not indirectly by exciting vasomotor changes. The vascular supply is of significance in these color changes only in so far as it provides a favorable environment in which these changes may take place. In an hour or so after the blood is cut off from a given area of skin the melanophores begin to lose their ability to respond probably from anoxemia, but the blood supply plays no special part in the details of pigment concentration or dispersion. Not only do the melanophore nerves act in this way but such neurohumors as intermedin appear to do the same, for this substance seems to act directly on the melanophores, in that it is difficult to conceive of any vasomotor change that might induce dispersion of pigment. Whether adrenalin acts in the same way or not is uncertain for its well known vasoconstrictor effect may induce anoxemia of the tissues which might be followed by the concentration of melanophore pigment usually ascribed to this hormone. However adrenalin will contract melanophores at concentrations so weak as to produce no vascular effects whatever and it is more than probable that this hormone also influences melanophores directly and not through vasomotor channels. It is therefore probable that the suspicions of such workers as Hogben, Lundstrom and Bard,
Young, and Wykes, that vasomotor influences play a necessary part in melanophore responses must be considered without weight until at least positive evidence in favor of this view has been brought forward.

The improbability that vasomotor changes are concerned with the ordinary color responses of animals has been often pointed out. When a cut is made in the caudal fin of any such fish as one of those discussed in this paper, blood-vessels are cut as well as nerves and a band of redness is traceable from the cut toward the edge of the fin. This band is made up chiefly of veins congested with immobile blood. After a mélanophore band has been formed, a close comparison between its limits and that of the reddened area will usually show lack of agreement. This can be well seen especially in Mustelus. Here the pale band when formed in a darkish fish has sharply defined rectilinear edges that follow very accurately the radial nerves whereas the reddish area overspreads these limits and has no well defined contours. Moreover if the smaller and more superficial blood-vessels including the capillaries are examined under the microscope they will be found in most parts of the pale band except immediately

DESCRIPTION OF FIGURES

All figures are taken from the pectoral fins of the smooth dogfish Mustelus canis. I am under obligation to Dr. F. M. Carpenter for the preparation of the photographs.

Fig. 1. Dorsal view of the right pectoral fin of an adult dogfish. The subelavian artery of this fin had been ligated and its vein had been cut in two places as shown by the incisions near its anterior and posterior edges. After the blood was in larger part thus excluded from the fin the more nearly central cut was made whereupon a pale band was formed extending from this cut toward the edge of the fin.

Fig. 2. Dorsal view of a left fin of an adult dogfish. The blood supply to this fin had been cut off by a general ligature at the base of the fin. On making an exciting cut near the middle of the fin, a faint, pale band developed from the cut toward the edge of the fin.

Fig. 3. Dorsal view of a right fin from a pup the blood circulation of which had been brought to a standstill by ligating its ventral aorta. After this operation, the exciting cut near the base of the fin was made whereupon a pale band developed.

Fig. 4. The same fin as that shown in Fig. 3 but viewed from the ventral side. The pale band is more evident on this side than on the dorsal side.
around the wound to exhibit a flow of blood hardly different from that seen in normal skin. It is these small vessels rather than the larger ones that are next the melanophores and it is these vessels through which a complete collateral circulation is restored. This lack of agreement between the pale area of nervous distribution and the red area of blood disturbance as well as the almost immediate establishment of a collateral blood supply in the experimental area has led many workers to discredit the suspicion that vascular disturbances are concerned with the details of color changes. All these conditions are still more emphasized in the oph-thalmic region where areas of blood supply and of nerve distribution disagree strikingly. Moreover, as Abramowitz (1936) has shown in *Ameiurus*, the ophthalmic artery can be cut, leaving this nerve intact and yet without the least interference with the color changes. These conditions, some of which were pointed out years ago, seem to have escaped the attention of those who have raised the question of vasomotor factors in color change.

No general conditions such as those which have been described in the preceding paragraph are known, so far as I am aware, in support of the vasomotor hypothesis, and since these conditions as well as the experimental results given in this paper are unfavorable to this view, I conclude that the details of melanophore changes are not dependent upon vasomotor factors; in other words, that vasomotor activities do not form an essential part of the connection between nerve and melanophore.

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THE HISTORIOGRAPHY OF IDEAS

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(Read April 24, 1937)

ABSTRACT

The branch of historiography which is concerned with the history of ideas is now divided among more than a dozen nominally distinct specialties. These divisions, though natural and necessary, do not correspond to real discontinuities in the phenomena to be studied; identical 'ideas' pass over from one of these conventionally distinguished provinces of the history of thought to others, and their roles as factors in the historic process can often be understood only by observing their operation in diverse fields. Specialists, consequently, have become increasingly aware of the necessity of going beyond the bounds of their own specialties in order to discover the causes or to interpret the significance of facts lying within those bounds. This is well illustrated in recent work in English literature, especially in Miltonic studies. An individual scholar cannot, however, hope to become a competent specialist in many fields of intellectual history. The only solution, therefore, lies in a greater practice of organized cooperation, by which the special knowledge of investigators in different parts of the whole subject can be focussed for the illumination of specific problems which arise in one or another of these divisions. Only by means of such cooperation will it be possible to carry out satisfactorily certain large and desirable projects, which American scholarship might well undertake. Among these may be mentioned an adequate volume of historical and interpretative annotations and essays on Paradise Lost; and a history of the development of the idea of 'evolution,' in its most comprehensive sense, before Darwin.

UNLIKE most of the communications laid before the Society, this paper is not a contribution to knowledge. It belongs to that inferior class of writings which are concerned with questions of program and method in research, rather than with results. It is, nevertheless, desirable that such questions should from time to time be discussed, especially at the meetings of learned academies; and the present situation in certain branches of historical inquiry seems to make opportune some consideration of the matters with which I propose to deal.

I ought to begin by explaining what I mean, in the title, by the word 'ideas,' but that would demand a long preamble, and
I have attempted the explanation at some length elsewhere;¹ for both reasons I shall dispense with a preliminary definition, hoping that the meaning of the term, for the present purpose, will become fairly evident from its context in what follows.

Historical study having to do, more or less, with ideas and their rôle in human affairs is now actively pursued in our universities and by non-academic scholars under at least twelve different labels:

1. The history of philosophy.
2. The history of science.
3. Folklore and some parts of ethnography.
4. Some parts of the history of language, especially semantics.
5. The history of religious beliefs and theological doctrines.
6. Literary history, as it is commonly presented, namely, the history of the literatures of particular nations or in particular languages—in so far as the literary historians interest themselves, as some do in but small degree, in the thought-content of literature.
7. What is unfortunately called “comparative literature,” which is apparently, by its most competent investigators, understood to be the study of international intellectual relations, of the transfer of tendencies of thought and taste, and of literary fashions, from one country to another, with especial attention to the modifications or metamorphoses which these undergo when transplanted into a new milieu.
8. The history of the arts other than literature, and of changes of taste in these arts.
9. Economic history and the history of economic theory, which, though they are not the same thing, are so closely related that they may here, for brevity, be grouped together.
10. The history of education.
11. Political and social history, and
12. The historical part of sociology, in so far as specialists in these subjects take account, as they now increasingly do, of

¹ The Great Chain of Being (1936), pp. 7–20.
intellectual or quasi-intellectual processes, of "ruling ideas" or "climates of opinion," either as causal factors in, or as consequences or "rationalizations" of, the political institutions, laws, mores, or social conditions prevalent in a given period—the subject sometimes designated as Wissensssoziologie. The enumeration might be extended and further subdivided; but these twelve appear to be the principal recognized divisions of the general field.

These subjects have usually in the past been studied in relative, though scarcely ever in complete, isolation. They are assigned in universities to separate departments, between which there frequently has not, I suspect, been much consultation concerning the interrelations of their respective provinces. Those who investigate them have their separate journals and their special learned societies, and, for the most part, do not and, indeed, cannot give much time to reading the journals or attending the meetings of their brethren in other fields—unless they have the good fortune of membership in some non-specialized society such as this. This division of the general domain of intellectual history has, of course, been inevitable and highly useful. Increasing specialization, and with it the development and refinement of distinctive techniques of inquiry, is obviously a necessary condition of progress in all branches of knowledge, and not least in the historical disciplines. Nevertheless the divisions—in so far as these several disciplines are concerned with the historiography of ideas—are artificial, though not, in general, arbitrary; that is to say, they correspond to no lines of absolute cleavage in the historical phenomena under investigation. They are in part temporarily convenient isolations of certain objects from their contexts, to facilitate more minute scrutiny; and in part they are fortuitous, results of accidents in the history of educational institutions or of the idiosyncratic limitations of the intellectual interests of influential scholars. And in the present phase of the development of, at least, several of these nominally distinct disciplines the lines of division are breaking down. They are breaking down because
questions originally raised within the traditional limits of one or another of these subjects prove incapable of adequate and accurate answer without going beyond those limits. Ideas are commodities which enter into interstate commerce. One notable example of the growing recognition of this has been the emergence, out of the study of separate national literatures, of the study of comparative literature. But the observation of what happens to ideas when they cross national or linguistic boundary lines is but a small part of the process of which I am speaking, even in the specific case of the history of literature. This may be illustrated by recent tendencies in the study of English literature. Scholars who primarily set out to be specialists in that field, and even in a limited part of it, have found themselves compelled to confess, not only how little they know of English literature who only English literature know—that has long been obvious—but also how little they know of English literature who only literature know. A scholar, for example, decides to attempt a special study of Milton, or, narrowing his subject of investigation still further, of Paradise Lost. It is, of course, possible to treat that work from an exclusively aesthetic point of view, as "pure literature," without raising any historical questions about it—though, if I may thus parenthetically dogmatize, half even of the aesthetic values of the poem will thereby be lost. In any case Paradise Lost is, inter alia, an extremely interesting phenomenon in the history of the activities of the human mind; and it is, in part, as such that most scholars in English literature now approach it. Now Paradise Lost is not merely, as the schoolboy noted with surprise, full of familiar quotations; it is also full of ideas, which, if only as a means to the understanding of what Milton meant, and of the movement of his mind as he composed, need to be seen in their historical perspectives. Scarcely one of them is original with him, though many of them receive a special twist or coloring, or enter into novel combinations, in consequence of personal characteristics of his. Even to recognize what is distinctive of either his style or his thought as distinctive, it is necessary
to have both an extensive and a fairly intimate acquaintance with manifestations of the same ideas elsewhere, especially among his contemporaries and among those of his predecessors with whom he is known, or can be fairly presumed, to have been acquainted. It is as impossible to appreciate the characteristic qualities of a poet's mind and art, when he is expressing a general idea, without knowing the idea and also other expressions of it, as it is to appreciate the art of a painting of the Madonna and Child without knowing the opening chapters of the Gospels of Matthew and Luke and without having seen any other paintings of the same subject. But the history of the ideas in Milton in great part does not, by the conventional classification of 'subjects,' lie in the field of English literature; it belongs to the history of philosophy, of theology, of religious poetry in other languages, of science, of aesthetic doctrines, and of taste.

For example, in the Eighth Book, it will be remembered, Adam and the Archangel Raphael engage—somewhat oddly—in a long discussion of the theories of seventeenth-century astronomy. Even for the exegesis of Milton's text—for the mere identification of the hypotheses referred to, which are by the poet sometimes rather loosely expressed—it is necessary to be extensively acquainted with the doctrines and reasonings of the astronomers from Copernicus's time to Milton's concerning the arrangement and motions of the celestial bodies; and this is the more plainly necessary if any competent judgment is to be formed as to Milton's knowledge of and attitude towards the new science of his age. The student of Paradise Lost, therefore, is forced by the nature of the historical inquiry in which he is engaged to turn to a part of the history of science. And if he is a cautious and critical scholar, he will not be content to get up a little information on the subject from Dreyer, Duhem, or other general survey, scarcely even to review the more recent monograph-literature on the history of early modern astronomy—especially as this will often not give him what he needs for his special purpose. He will feel constrained to study the relevant astronomical texts
themselves, and to attempt to make himself really at home in the theories of the period; and he may be enabled, in consequence, to make fresh contributions to the history of that science, of interest to those who know not Milton and are indifferent to the reputed astronomical opinions of the Archangel Raphael.

I am not describing a hypothetical case; I am describing what has actually been happening in a single part of recent Miltonic study \(^1\) directed upon a passage of some two hundred lines in one Book. If the meaning and the background of the ideas in the whole of *Paradise Lost* are dealt with in a similar manner, the student will find a wide range of other conceptions the history of which, once more, is not a part of what has commonly been considered the province of the historian of English literature, but lies within the domains of the specialists in many other branches of learning. When, for one brief example, Milton’s Adam quotes Aristotle (without acknowledgment) to his Creator, observing that while the deity is self-sufficient and “best by himself accompanied,” he (Adam) needs a human companion even in so agreeable a place as Eden, it is desirable that the careful student of the poem, as a historical phenomenon, should know this fact. For, in the first place, without a knowledge of it, it is not impossible for the reader to miss much of the point of Milton’s lines. In the second place, the identification of self-sufficiency with the supreme good—which, however, Adam is here made to declare, is a good for God but not for man—is one of the most influential and widely-ramifying ideas in Western thought;\(^2\) and in this larger historical vista Milton’s expression of the idea gains a great enrichment of interest—an increase, so to say, of voluminosity. In the third place, Milton’s particular way of employing the Aristotelian theorem, on the one hand illuminates his conception of God and on the other

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\(^1\) As examples among American scholars in this field I may mention the work of Dr. Marjorie Nicolson, Dr. F. R. Johnson and Dr. Grant McColley.

\(^2\) *P.L.*, VIII, 415 f., 427 f.

\(^3\) See the writer’s *The Great Chain of Being* (1936), 161, and 42 f., 48 f. 62, 83, 159, 300, 351.
hand amounts to a fairly explicit denial of the proposition—which had been assumed as axiomatic in most orthodox Christian theology—that man’s chief good is the *imitatio dei.* And finally, a recognition of the Aristotelian source of Adam’s theology lends to the passage, I can’t but think, an agreeable touch of humor—not, I admit, probably intended by the poet. But all this appears not to be generally known to the Milton-commentators. They have, no doubt, usually been too little acquainted with Aristotle, and with the history of philosophy in general, and the Aristotelian specialists have been too little concerned with Milton, for either to establish the connection. Similar examples might be multiplied by the hundred, all illustrating the general fact that the quest of a historical understanding even of single passages often drives the student into fields which at first seem remote enough from his original topic of investigation. The more you press in towards the heart of a narrowly bounded historical problem, the more likely you are to encounter in the problem itself a pressure which drives you outward beyond those bounds.

If, instead of literary history, we had taken as a starting-point any one of several other fields of historical inquiry, we should have encountered similar, and in many cases more important, illustrations of the necessity of this sort of correlation; and we should, if I am not mistaken, have found among the keener-minded specialists in those fields a growing sense of that necessity. It is perhaps not too much to say that, in the history of historiography itself, we have now reached a juncture at which the indispensability of a closer and wider *liaison*—or, to better the metaphor, of a great deal more cross-fertilization—between primarily distinct disciplines, is much more apparent and more urgent than it has ever been before. It would be wholly false to say that the phase of increasing minute specialization in these studies is over—though in some of them, I suspect, the period of diminishing returns from the customary methods of cultivation has been reached; it would not, I believe, be false to say that increasing specialization has actually “passed over,” like a category in the Hegelian
logic, into its own apparent opposite, and now manifests itself as a demand for more historical synthesis—for the establishment of concrete and fruitful interconnections at a large number of specifiable points. And if this is so, we are confronted with a difficult situation pertinent to what may be called the general strategy of historical inquiry, and in some degree also to the organization of advanced instruction in universities, which demands practical consideration.

The nature of the difficulties is, I suppose, evident; explicit consideration of it may perhaps suggest some alleviations, if not a complete remedy. The divisions of the total domain having to do in any degree with the rôle of ideas in history exist; and it is neither possible nor desirable to abolish them in favor of any vague "universal history." Yet it is now plain that the scholar who wishes to understand sufficiently the material within almost any one of these divisions must take account of material lying, according to the conventional boundary-lines, in other—often in several other—divisions. But no man, obviously, can be a competent original investigator in many provinces even of history. Yet the specialist often—and, I am disposed to think, usually—cannot get what he needs even from the more substantial general treatises or manuals in the subjects which he finds his own overlapping. One reason, though not the only one, why he cannot is that the authors of those works, having preoccupations different from his, may have left out precisely the portions of their subjects which are most pertinent to his. It would be possible to cite, if time permitted, specific instances, in which the initial specialized interest of investigators in one province has produced a kind of blindness to aspects of the historical material with which they deal that are of great significance in relation to other parts of intellectual history. That it is easy, in observing any object—including historical sources—to overlook a good deal of what is there and is important, unless you know what to look for, is a truism sufficiently illustrated by the classic anecdote of the student who, being required to describe a fish-skeleton placed
before him, faithfully enumerated all the features of the object except the most conspicuous—bilateral symmetry. Learned historians of literature, philosophy, religion, science, or social or political movements, sometimes fall into comparable omissions, simply because, knowing only their own subjects, they do not know all that is to be looked for in those subjects.

But it is time to pass on to the question how the difficulties of this situation can be diminished. Upon this question I submit three observations.

1. The first will perhaps be the most repugnant, and may seem only the expression of a professional bias, of the tendency of a specialist to fancy his own subject to be of peculiar interest and importance. However that may be, I think one of the desiderata in the juncture I have described is a more general recognition of the fact that in the history of philosophy is to be found the common seed-plot, the locus of initial manifestation in writing, of the greater number of the more fundamental and pervasive ideas, and especially of the controlling preconceptions, which manifest themselves in other regions of intellectual history. To offer proof of this here, for those, if there be such, who doubt it, is manifestly impossible. But if it is a fact, it has two practical implications: first, that in the preparation of scholars for competent investigation in most other historical fields, a sound training both in the history of philosophical ideas and—what is not less important—in the methods of philosophical analysis—of taking idea-complexes apart—is especially needful; and second, that the history of philosophy needs to be studied with more attention to the repercussions of philosophic ideas outside the great technical systems, and to be presented in a manner rather different from the usual one, which will make it more digestible and nutritious for non-philosophers. In explanation and justification of this last thesis I might, but I shall not here, expatiate at length; but what I mean by it may in part be gathered from what I shall say under the next head.

2. The history of individual ideas as such—or the ideas entertained by men on individual questions which have seemed
to them significant—is in great part still to be adequately investigated and the results to be written. On this I have gone into print elsewhere \(^1\) and shall on this occasion speak only summarily. There are, I have suggested, many ‘unit-ideas’—types of categories, thoughts concerning particular aspects of common experience, implicit or explicit presuppositions, sacred formulas and catchwords, specific philosophic theorems, or the larger hypotheses, generalizations or methodological assumptions of various sciences—which have long life-histories of their own, are to be found at work in the most various regions of the history of human thinking and feeling, and upon which the intellectual and affective ‘reactions’ of men—individuals and masses—have been highly diverse. There is here another distinct realm of historiography, which needs to be added to the dozen mentioned at the outset, partly because it is concerned with a class of historical phenomena of extraordinary interest in themselves, which the others do not wholly cover, and partly (which is the point that I here wish to make) because their progress depends greatly upon it—as its progress, not less truly, depends upon theirs. Until these units are first discriminated, until each of them which has played any large rôle in history is separately pursued through all the regions into which it has entered and in which it has exercised influence, any manifestation of it in a single region of intellectual history, or in an individual writer or writing, will, as a rule, be imperfectly understood—and will sometimes go unrecognized altogether. "There are few things in the world more interesting," Professor Lowes has remarked, "than the disclosure of facts which throw into fresh perspective a mass of other historic facts." \(^2\) Through the sort of study of which I am now speaking, the study of the (so far as possible)

\(^1\) In the introductory lecture of *The Great Chain of Being*, the rest of the volume being an attempt to give, so far as the author's resources and the limitations of a single course of lecture permitted, an illustration of such a study of a single idea, in its interactions with others. Professor George Boas and the writer have attempted a similar study, by a somewhat different method, in *Primitivism and Related Ideas in Antiquity*, (1935).

\(^2\) In his "Teaching and the Spirit of Research," *The American Scholar*, 1933.
total life-history of individual ideas, in which the many parts that any one of them plays upon the historic scene, the differing facets which it exhibits, its interplay, conflicts and alliances with other ideas, and the diverse human reactions to it, are traced out with adequate and critical documentation, with analytical discrimination, and, finally, with imagination—through this, I am persuaded, are to be disclosed many facts which will throw into fresh perspective, and thereby invest with heightened interest and greater intelligibility, facts in other branches of intellectual history which, lacking such perspective, sometimes appear dull, unrelated, and more or less incomprehensible.

I do not mean to imply that this form of the historiography of ideas is as yet non-existent; some excellent examples of it, or at least approximations to it, have long been in our libraries, and numerous scholars in different quarters are now contributing to it. But if it is not in its infancy, it is still, I think, barely in its adolescence; and its methods, its requirements, its aims, and its interest, are less generally understood than could be desired. Its program, you will observe, is one of both isolation and synthesis—the provisional isolation of an idea for separate study, but the bringing together, for that study, of material from all the historical provinces into which the idea has penetrated.

3. Finally, from all that has been so far said, one conclusion seems to me to emerge almost too plainly to require statement. It is that in almost all of the branches of historiography which deal with the history of men's thoughts or opinions, and the affective attitudes and behavior associated with these, there is imperative need of more definite, responsible, organized collaboration between specialists in these several branches than has hitherto been customary—collaboration too, in some cases, between historians and specialists in non-historical disciplines, notably the natural sciences. Trustworthy historical synthesis is not a one-man job. If the pieces that are to be put together—even for the understanding of one part of one subject—are to be sound pieces,
they must be provided, or at least be critically inspected, by those having special training and up-to-date technical knowledge in the fields to which the pieces primarily belong. And by coöperation I do not mean the sort of thing exemplified by the *Cambridge Modern History* and *History of English Literature*, admirable and useful as those great works are. What I have in mind is not simply the parcelling-out of the subdivisions of a large subject among specialists in those subdivisions; it is the convergence upon each of them of all the special knowledge from all of these subdivisions which is genuinely pertinent to it. The indispensability of such coöperation is especially evident if we consider possible large enterprises of scholarship which now wait to be undertaken. I will briefly mention only two such enterprises, of different types and scales of magnitude, and in quite dissimilar provinces, either of which might well be sponsored by a great learned society or a great university. The first is a book of annotations on *Paradise Lost* and of studies on special historical and literary aspects of that poem. Such books were fairly frequent in the eighteenth century, though usually very badly done. I am unable to discover, through some bibliographical search and inquiries of English scholars, that there exists any modern work of this character, bringing together all the knowledge needed for placing that great English classic in its historical relations, and for the adequate illumination of the ideas which it contains. And the reason, no doubt, is that such a work cannot now decently be done by one man, unaided; it needs, as what I have earlier said implies, the coöperation not only of a number of specialists in English, but also of a classical scholar, a patristic scholar, a medievalist, a philosopher, a student of rabbinical and other Jewish literature, a theologian versed in early Protestant divinity, special students both of French and Italian literature of the sixteenth and seventeenth centuries, and a historian of science especially at home in early modern astronomy. I do not, once more, mean to imply that no studies in these fields, pertinent to Milton, as yet exist. Much valuable work on
them has been and is being done, chiefly by English scholars who have found themselves compelled, in the manner already mentioned, to deviate into provinces not primarily their own.\(^1\) The task to which I am referring would consist in part in bringing together in a connected and synoptic way the results of these previous studies now scattered through many scores of monographs, books, and articles. But those studies are, as a rule, sound and dependable precisely in the degree in which the English specialists have been able to obtain collaboration and criticism from specialists of thorough competence in the other provinces in question; some of them would be of more value if considerably more such coöperation could have been had by their authors; and in any case, much further research is needed in some of these collateral fields, as well as a better correlation and cross-illumination between them. If such a piece of work could be coöperatively carried out, there could be focussed upon Milton's text a mass of facts which would, we may be confident, throw many parts of it into fresh perspectives of great and of diverse interest.

To turn to the history of (primarily) scientific conceptions: there as yet exists, so far as I know, no historically and philosophically respectable account of the total development of the idea of evolution before Darwin—using the term in its broader sense to include not only the theory of the transformation of species, but also developmental conceptions in astronomy, geology, anthropology, social philosophy, cosmology and theology, and the influence of all of these in other provinces of thought.\(^2\) Historically, the various phases of

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\(^1\) Four further recent examples of this may be given: Mr. Harris F. Fletcher's *Milton's Rabbinical Studies* (1930); Miss Kathleen E. Hartwell's *Milton and Lactantius* (1929), both of which convincingly demonstrate the relevance and interest, for the student of Milton, of such excursions into other branches of learning; the coöperative enterprise of a group of scholars at the University of North Carolina, under the leadership of Professor U. T. Holmes, who, in order to contribute to the definitive clarification of one important question concerning the background and sources of *Paradise Lost*, and Milton's way of using his sources, have undertaken a carefully documented biography of Du Bartas and a critical edition of *La Semaine*; and the work of Professor G. C. Coffin on the same subject (*Milton's Use of Du Bartas*, 1934).

\(^2\) Substantially the same opinion is expressed by Professor P. T. Sorokin in his monumental *Social and Cultural Dynamics*, II, 371 (published since this paper was
the progress of what may be called the genetic way of thinking—which has been a long, complex, and extremely gradual process—are closely related. The reason why there exists no adequate history of the process as a whole is, in part, that much of the *grundlegend* detailed study of sources still remains to be done; but the task, in any case, can scarcely be executed properly by any one scholar. For it requires a competent acquaintance with many special fields—not only with the several natural sciences mentioned, and with the history of particular subdivisions of these—in biology, for example, of taxonomy, comparative anatomy, paleontology, embryology and genetics—but also with an extremely wide range of the philosophical, theological and general literature of the seventeenth, eighteenth, and early nineteenth centuries, and with the history of ideas about history and its general movement. A part of the story, for example, can be verified only through a thorough study of the works of Leibniz; and another part demands an intimate acquaintance with the literature, the metaphysics, and even the aesthetic theories and fashions, of the German Romantic period. And all of these need to be illuminated from the special history of certain relevant individual ideas—for instance, of the principle of continuity and of the concept or pseudo-concept of 'species.' The thing can be done, and it could be wished that American scholarship would undertake it. But unless it is undertaken in the manner suggested, through large-scale, planned, and articulated coöperation, it is likely to be much more imperfectly done than even the present state of knowledge in the numerous special fields pertinent to it would make possible.

It may be that the kind and measure of coöperation needed, at these and at many other points in the historiography of ideas, is unattainable. There are undeniably great obstacles to it, both in the organization and traditions of most of our universities, and in human nature. Natural scientists have, presented), with especial reference to the medieval part of the story: “The real history of the idea of progress . . . is not written yet. Works like J. B. Bury’s *The Idea of Progress*, or Delvaille’s work, . . . only most superficially touch the problem.”
if I am not mistaken, realized earlier than historians the necessity for planned team-work, and have thus far developed it more skillfully and on a much greater scale. But in a number of historical disciplines encouraging beginnings of it are now discernible; Professor Nitze’s paper has presented an important example of this. The difficulty of realizing it, however, is still so considerable that I have thought it perhaps not wholly useless to take this occasion to emphasize its necessity in the present phase of that large business which is the endeavor to investigate the history, and thereby, it may be hoped, to understand better the nature, of the workings of the human mind.
INTERNATIONAL PEACE *

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I have been devoting myself since the World War to the problems of international peace. I sometimes think of myself in that regard as an academic war casualty; but I have been carrying on in the hope that this problem can be reduced to something like usable terms. If you will permit me, in a few minutes—because I shall not keep you long—I should like to summarize the main points that underlie my work of these last years.

First of all I should say that as a historian I have received almost no help from historians in the study of war in history. War as a technique has its record, but war as an instrument of policy is a subject that has not been properly studied by those who analyze the processes by which civilization has moved along. In the protest of this post-war generation there has been a natural and perhaps inevitable tendency to turn against war as an institution, deny its legitimacy in the past, present, and future. This is utterly unsound from the historical point of view, for war has served two kinds of purposes. If it has been the instrument of anarchy and despotism, it also has been the one instrument by which protest or defence could be made successful against anarchy and despotism. It is still today the one action of police against anarchy and despotism menacing the world in certain quarters.

The complications that confront one when one deals with the problems of international peace have never been squarely faced by the social scientists and historians. I have tried to furnish some of the material upon which what I would regard as sounder conclusions might be drawn. Let me say that as I look over the rôle that war has played I am conscious of the

* Dinner address, April 24, 1937.
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fact that the society of the past, in which it was a prime agent both for destruction and for progress, has been a society based upon a repetitive system of life and society. Men had to do the same thing over and over again, generation after generation, age after age, in a limited environment created by hand labor. The iron mould of custom was therefore broken now and again by war. But the clue to peace in the modern world is science—science which has brought us that change which makes life no longer a mere prisoner of routine and repetition, science which takes the world's work out of the hand and moves it over through the brain to the materials which we have at our disposal. So in this revolutionary moment in the long processes of the centuries we are now faced with the fact that the technique of war is no longer in the same setting in relation to the technique of peace as it was in the days of Julius Caesar.

 Trying to boil down the elements or phenomena of the World War to understandable size to make it usable, it is pretty well summarized in this simple aphorism, that war was no longer a pertinent instrument of policy, because it was no longer controllable, directable, among highly developed industrialized countries. The total military history of the World War could be narrowed down to this, that it was a war of blockade. The battles on the front, the movement of troops under the direction of general staffs, provided little chance for either side, but the exhaustion of materials meant disaster for that nation that could not get out of the iron ring. England at the start began with the idea it could bring war back to the old techniques, and it hung out the sign, "Business as usual." But Rathenau over in Berlin came to the War Ministry with the suggestion that he be placed in control of raw materials, or at least that someone be, and the War Material Department of the War Ministry then saw the beginning of mobilization of the industrial world for war. In the course of the war every other nation followed that lead, and at the end the exhausted nation lost; but it lost only after the totalitarian effort of nation against nation. Where was the direction of the plans
of the general staff, and what was the direction of the victory? The two do not coincide.

The problem of war has changed as a procedure, and I venture to maintain that most of its mechanism, valid in a different day, is no longer valid. And if it is no longer valid, and war is no longer, as Clausewitz called it, the continuation of policy, then there can be no doubt in the mind of any civilized man that it is a crime when waged between civilized nations. I am ready to admit that war between high-grade societies and low-grade societies may be directed strategically, as well as between low-grade societies and in these cases we have to have other controls and other devices for achieving our ends—those of peace. But I am speaking now of the wars in which the United States conceivably might become involved.

If it is in the nature of war that we must, therefore, turn to other techniques, we cannot turn to them by an Eighteenth Amendment, simply saying war must not be; because war is not like crime or vice, a thing that has been wholly wrong and has always been recognized as wholly wrong. There may be those of us here who think it wholly wrong, but the nation as a whole will not share that view. When the questions of national prestige, national honor and the like are rung in their ears, old history, habits and attitudes come back to us from the historic past and control our actions in the time of crisis. So it is necessary that we have something which will function before the crisis, something that can be counted upon with relative confidence to function with pertinence to the end before us. And that is the kind of thinking that we as a people have steadily refused to do. Instead, we mobilize our emotions in support of moral attitudes and then imagine that our actions in the crisis will follow the moral line instead of what we think of then as the immoral. But, we cannot be counted upon to have stable moral attitudes until we have had a longer experience in the arts and procedures of peace itself.

And, now, where have these been experimented upon? Primarily in the League of Nations, of course. Do not be
alarmed, I am not going to try at this late hour to bring us back into the League, although I should be glad to do so if I could. But as an experiment, as a laboratory, there has been something there of which we should learn. War has to be met primarily by two devices, first, by some device that will give security for law and order in the world, calculable prosperity, and the like. The problem of security is one that, naturally, we as Americans have never thought through because we have it naturally, from nature itself. Security that is artificial, that is political, lies beyond the immediate necessities of our thinking. But the League of Nations in the years following the War made security the key to its policy; I mean the nations in the League. And let me say that Germany and Austria and Hungary were just as anxious in those years for the status quo, as over against a possible breaking of the barriers of war upon Europe, as was France or England. It was a period when status quo and peace were more important than anything else to all the nations concerned.

But such policies will never keep back war for more than a period. You have got to have machinery for pacific means as well, not simply for the settlement of disputes, but to keep pace with change, with the changing situations that are bound to arise with great rapidity in a scientific world. Provision must be made for international adjustment to changing conditions. Now, the League of Nations has failed so far to find adequate machinery for peaceful change, but its Committee on Intellectual Cooperation has organized a study conference in which the problems connected with peaceful change are taken up. I know of no other way to approach the problems than by research. We have to lay the foundations of a new problem. We have been at it now for the last two years. Let me pause to say that the name, Committee of Intellectual Cooperation, is a translation from the French, and "intellectual" is not an insult in French. We have been working for two whole years on raw materials, colonies, and the technique that is pertinent to each and every one of the questions likely to come up in the immediate future. I don't know whether we
have made progress; none of us know; I wish we did. All we know is that we have probed into fallacies in popular thinking not only here but in Europe, and that our discussions and the results of our research will be placed before students for further analysis not only in the universities and colleges but generally through our publications. All that we can do is to ask that our generation address itself to consider such measure of advance as we have made, and that then if we are wrong, they right it; and if we are right, they build upon it. That has always been the method of the progress of knowledge.

In this connection I should like to add a word of praise for the conference that has just been held in Washington by the International Labor Organization, on the textile problem, one of the most difficult international problems in the world today. The International Labor Organization has in that, and in the Maritime Conference of last October, discovered a technique for solving some of the most serious international difficulties. It has drawn together representatives of labor, capital, and government in the specific industries concerned in an international conference. And as I sat there in Geneva last October and heard the seamen and the ship-owners discuss with the Board of Trade and our Commerce Department the problems that lay before them when they put three shifts on the ships, to get an eight-hour day, and as I saw the interplay of capital and labor, and heard the debates by those who knew what they were talking about because their lives were spent in it, I had a very different impression from listening to certain political gatherings in certain capitals by those who are representatives of great peoples making laws about things of which they know little.

We have, therefore, a specific technique that is being worked out without any front-page news for the newspapers, but reaching into the heart of the controversies in the field of economics; and I hope that that same device will soon be taken up in finance, in trade, commerce, and the whole economic sphere.
And then I should say that I look forward to some kind of a league of nations that would have nations in it devoting themselves to economic problems, without necessarily becoming members of a diplomatic corps or a diplomatic conference, and members of a diplomatic conference that need not become members of the other,—autonomous houses directing their energies upon the specific solution of specific difficulties. That is the only way we can approach the fundamental cause of war, which cause is not economics but is prestige. We must take the problems out of the sphere of emotion. We must bring them to their realities in their own terms, and then there is little left for that kind of hot political debate which is never on the subject matter itself, but is what you or I think about each other, instead of what we both think about it. That is what war is fought over. That is what they debated in those fateful days in the Austro-Hungarian Cabinet when they decided to go to war. It was prestige. That was what made Germany come to Austria's aid—a world war for prestige.

Colonies and raw materials are not one problem in the world today, although they are given out as one by one great nation. They are entirely separate and soluble entirely on different terms. They must be approached in those terms. Such analysis is the exercise of intelligence. There is a place in the world for intelligence. The day is coming when science will pass from engineering to statecraft. It has just begun to tear down the old local frontiers of our loyalties to ancient things and is now creating truer loyalties to ourselves and our State by making them part and parcel of a great world problem.

I close by suggesting to you that a body such as this cherish the kind of tradition and attitude which is necessary if the world should have permanent peace.
THE LAW OF ANOMALOUS NUMBERS

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(Introduced by Irving Langmuir)

(Read April 22, 1937)

ABSTRACT

It has been observed that the first pages of a table of common logarithms show more wear than do the last pages, indicating that more used numbers begin with the digit 1 than with the digit 9. A compilation of some 20,000 first digits taken from widely divergent sources shows that there is a logarithmic distribution of first digits when the numbers are composed of four or more digits. An analysis of the numbers from different sources shows that the numbers taken from unrelated subjects, such as a group of newspaper items, show a much better agreement with a logarithmic distribution than do numbers from mathematical tabulations or other formal data. There is here the peculiar fact that numbers that individually are without relationship are, when considered in large groups, in good agreement with a distribution law—hence the name "Anomalous Numbers."

A further analysis of the data shows a strong tendency for bodies of numerical data to fall into geometric series. If the series is made up of numbers containing three or more digits the first digits form a logarithmic series. If the numbers contain only single digits the geometric relation still holds but the simple logarithmic relation no longer applies.

An equation is given showing the frequencies of first digits in the different orders of numbers 1 to 10, 10 to 100, etc.

The equation also gives the frequency of digits in the second, third \ldots place of a multi-digit number, and it is shown that the same law applies to reciprocals.

There are many instances showing that the geometric series, or the logarithmic law, has long been recognized as a common phenomenon in factual literature and in the ordinary affairs of life. The wire gauge and drill gauge of the mechanic, the magnitude scale of the astronomer and the sensory response curves of the psychologist are all particular examples of a relationship that seems to extend to all human affairs. The Law of Anomalous Numbers is thus a general probability law of widespread application.

PART I: STATISTICAL DERIVATION OF THE LAW

It has been observed that the pages of a much used table of common logarithms show evidences of a selective use of the natural numbers. The pages containing the logarithms of the low numbers 1 and 2 are apt to be more stained and frayed by use than those of the higher numbers 8 and 9. Of

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course, no one could be expected to be greatly interested in the condition of a table of logarithms, but the matter may be considered more worthy of study when we recall that the table is used in the building up of our scientific, engineering, and general factual literature. There may be, in the relative cleanliness of the pages of a logarithm table, data on how we think and how we react when dealing with things that can be described by means of numbers.

*Methods and Terms*

Before presenting the data collected while investigating the possible existence of a distribution law that applies to numerical data in general, and to random data in particular, it may be well to define a few terms and outline the method of attack.

First, a distinction is made between a digit, which is one of the nine natural numbers $1, 2, 3, \ldots 9$, and a number, which is composed of one or more digits, and which may contain a $0$ as a digit in any position after the first. The method of study consists of selecting any tabulation of data that is not too restricted in numerical range, or conditioned in some way too sharply, and making a count of the number of times the natural numbers $1, 2, 3, \ldots 9$ occur as first digits. If a decimal point or zero occurs before the first natural number it is ignored, for no attention is to be paid to magnitude other than that indicated by the first digit.

*The Law of Large Numbers*

An effort was made to collect data from as many fields as possible and to include a variety of widely different types. The types range from purely random numbers that have no relation other than appearing within the covers of the same magazine, to formal mathematical tabulations that admit of no variation from fixed laws. Between these limits one will recognize various degrees of randomness, and in general the title of each line of data in Table I will suggest the nature of the source. In every group the count was continuous from the beginning to the end, or in the case of long tabulations, to a sufficient number of observations to insure a fair average.
The numbers counted in each group is given in the last column of Table I.

**TABLE I**

Percentage of Times the Natural Numbers 1 to 9 are Used as First Digits in Numbers, as Determined by 20,229 Observations

<table>
<thead>
<tr>
<th>Group</th>
<th>Title</th>
<th>First Digit</th>
<th>Count</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Rivers, Area</td>
<td>1  2  3  4  5  6  7  8  9</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>Population</td>
<td>1  2  3  4  5  6  7  8  9</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>Constants</td>
<td>1  2  3  4  5  6  7  8  9</td>
<td></td>
</tr>
<tr>
<td>D</td>
<td>Newspapers</td>
<td>1  2  3  4  5  6  7  8  9</td>
<td></td>
</tr>
<tr>
<td>E</td>
<td>Spec. Heat</td>
<td>1  2  3  4  5  6  7  8  9</td>
<td></td>
</tr>
<tr>
<td>F</td>
<td>Pressure</td>
<td>1  2  3  4  5  6  7  8  9</td>
<td></td>
</tr>
<tr>
<td>G</td>
<td>H.P. Lost</td>
<td>1  2  3  4  5  6  7  8  9</td>
<td></td>
</tr>
<tr>
<td>H</td>
<td>Mol. Wgt.</td>
<td>1  2  3  4  5  6  7  8  9</td>
<td></td>
</tr>
<tr>
<td>I</td>
<td>Drainage</td>
<td>1  2  3  4  5  6  7  8  9</td>
<td></td>
</tr>
<tr>
<td>J</td>
<td>Atomic Wgt.</td>
<td>1  2  3  4  5  6  7  8  9</td>
<td></td>
</tr>
<tr>
<td>K</td>
<td>$n^{-1}$, $\sqrt{n}$, ...</td>
<td>1  2  3  4  5  6  7  8  9</td>
<td></td>
</tr>
<tr>
<td>L</td>
<td>Design</td>
<td>1  2  3  4  5  6  7  8  9</td>
<td></td>
</tr>
<tr>
<td>M</td>
<td>Digest</td>
<td>1  2  3  4  5  6  7  8  9</td>
<td></td>
</tr>
<tr>
<td>N</td>
<td>Cost Data</td>
<td>1  2  3  4  5  6  7  8  9</td>
<td></td>
</tr>
<tr>
<td>O</td>
<td>X-Ray Volts</td>
<td>1  2  3  4  5  6  7  8  9</td>
<td></td>
</tr>
<tr>
<td>P</td>
<td>Am. League</td>
<td>1  2  3  4  5  6  7  8  9</td>
<td></td>
</tr>
<tr>
<td>Q</td>
<td>Black Body</td>
<td>1  2  3  4  5  6  7  8  9</td>
<td></td>
</tr>
<tr>
<td>R</td>
<td>Addresses</td>
<td>1  2  3  4  5  6  7  8  9</td>
<td></td>
</tr>
<tr>
<td>S</td>
<td>$n^2$, $n^2 \cdots n!$</td>
<td>1  2  3  4  5  6  7  8  9</td>
<td></td>
</tr>
<tr>
<td>T</td>
<td>Death Rate</td>
<td>1  2  3  4  5  6  7  8  9</td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>Average</strong></td>
<td>1  2  3  4  5  6  7  8  9</td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>Probable Error</strong></td>
<td>+0.8</td>
<td>±0.4</td>
</tr>
</tbody>
</table>

At the foot of each column of Table I the average percentage is given for each first digit, and also the probable error of the average. These averages can be better studied if the decimal point is moved two places to the left, making the sum of all the averages unity. The frequency of first 1's is then seen to be 0.306, which is about equal to the common logarithm of 2. The frequency of first 2's is 0.185, which is slightly greater than the logarithm of 3/2. The difference here, log 3 - log 2, is called the logarithmic integral. These resemblances persist throughout, and finally there is 0.047 to be compared with log 10/9, or 0.046.
The frequency of first digits thus follows closely the logarithmic relation

$$F_a = \log \left( \frac{a + 1}{a} \right),$$

(1)

where $F_a$ is the frequency of the digit $a$ in the first place of used numbers.

**TABLE II**

**Observed and Computed Frequencies**

<table>
<thead>
<tr>
<th>Natural Number</th>
<th>Number Interval</th>
<th>Observed Frequency</th>
<th>Logarithm Interval</th>
<th>Observed - Computed</th>
<th>Prob. Error of Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1 to 2</td>
<td>0.306</td>
<td>0.301</td>
<td>+0.005</td>
<td>±0.008</td>
</tr>
<tr>
<td>2</td>
<td>2 to 3</td>
<td>0.185</td>
<td>0.176</td>
<td>+0.009</td>
<td>±0.004</td>
</tr>
<tr>
<td>3</td>
<td>3 to 4</td>
<td>0.124</td>
<td>0.125</td>
<td>-0.001</td>
<td>±0.004</td>
</tr>
<tr>
<td>4</td>
<td>4 to 5</td>
<td>0.094</td>
<td>0.097</td>
<td>-0.003</td>
<td>±0.003</td>
</tr>
<tr>
<td>5</td>
<td>5 to 6</td>
<td>0.080</td>
<td>0.079</td>
<td>+0.001</td>
<td>±0.002</td>
</tr>
<tr>
<td>6</td>
<td>6 to 7</td>
<td>0.064</td>
<td>0.067</td>
<td>-0.003</td>
<td>±0.002</td>
</tr>
<tr>
<td>7</td>
<td>7 to 8</td>
<td>0.051</td>
<td>0.058</td>
<td>-0.007</td>
<td>±0.002</td>
</tr>
<tr>
<td>8</td>
<td>8 to 9</td>
<td>0.049</td>
<td>0.051</td>
<td>-0.002</td>
<td>±0.002</td>
</tr>
<tr>
<td>9</td>
<td>9 to 10</td>
<td>0.047</td>
<td>0.046</td>
<td>+0.001</td>
<td>±0.003</td>
</tr>
</tbody>
</table>

There is a qualification to be noted immediately, for Table I was compiled from numbers composed in general of four, five and six digits. It will be shown later that Eq. (1) is a distribution law for large numbers, and there is a more general equation that applies when considering numbers of one, two ... significant digits.

If we may assume the accuracy of Eq. (1), we then have a probability law of the most general nature, for it is a probability derived from "events" through the medium of their descriptive numbers; it is not a law of numbers in themselves. The range of subjects studied and tabulated was as wide as time and energy permitted; and as no definite exceptions have ever been observed among true variables, the logarithmic law for large numbers evidently goes deeper among the roots of primal causes than our number system unaided can explain.

*Frequency of Digits in the qth Position*

The second-place digits are ten in number, for here we must take 0 into account. Also, in considering the frequency
THE LAW OF ANOMALOUS NUMBERS

$F_b$ of a second-place digit $b$ we must take into account the digit $a$ that preceded it. The logarithmic interval between two digits is now to be divided into ten parts corresponding to the ten digits 0, 1, 2, \ldots 9. Let $a$ be the first digit of a number and $b$ be the second digit; then using the customary meaning of position and order in our decimal system a two-digit number is written $ab$, and the next greater number is written $ab + 1$.

The logarithmic interval between $ab$ and $ab + 1$ is $\log (ab + 1) - \log ab$, while the interval covered by the ten possible second-place digits is $\log (a + 1) - \log a$. Therefore the frequency $F_b$ of a second-place digit $b$ following a first-place digit $a$ is

$$F_b = \log \left( \frac{ab + 1}{ab} \right) / \log \frac{a + 1}{a}.$$  \hspace{1cm} (2)

As an example, the probability $F_b$ of a 0 following a first-place 5 in a random number is the quotient

$$F_b = \log \frac{51}{50} / \log \frac{6}{5}.$$  

It follows that the probability for a digit in the $q$th position is

$$F_b = \frac{\log \frac{abc \cdots \ p \ (q + 1)}{abc \cdots \ pq}}{\log \frac{abc \cdots \ o \ (p + 1)}{abc \cdots \ p}}.$$  \hspace{1cm} (3)

Here the frequency of $q$ depends upon all the digits that precede it, but when all possible combinations of these digits are taken into account $F_q$ approaches equality for all the digits 0, 1, 2, \ldots 9, or

$$F_q \simeq 0.1.$$  \hspace{1cm} (4)

As a result of this approach to uniformity in the $q$th place the distribution of digits in all places in an extensive tabulation of multi-digit numbers will be also nearly uniform.
TABLE III

Frequency of Digits in First and Second Places

<table>
<thead>
<tr>
<th>Digit</th>
<th>First Place</th>
<th>Second Place</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.000</td>
<td>0.120</td>
</tr>
<tr>
<td>1</td>
<td>0.301</td>
<td>0.114</td>
</tr>
<tr>
<td>2</td>
<td>0.176</td>
<td>0.108</td>
</tr>
<tr>
<td>3</td>
<td>0.125</td>
<td>0.104</td>
</tr>
<tr>
<td>4</td>
<td>0.097</td>
<td>0.100</td>
</tr>
<tr>
<td>5</td>
<td>0.079</td>
<td>0.097</td>
</tr>
<tr>
<td>6</td>
<td>0.067</td>
<td>0.093</td>
</tr>
<tr>
<td>7</td>
<td>0.058</td>
<td>0.090</td>
</tr>
<tr>
<td>8</td>
<td>0.051</td>
<td>0.088</td>
</tr>
<tr>
<td>9</td>
<td>0.046</td>
<td>0.085</td>
</tr>
</tbody>
</table>

Reciprocals

Some tabulations of engineering and scientific data are given in reciprocal form, such as candles per watt, and watts per candle. If one form of tabulation follows a logarithmic distribution, then the reciprocal tabulation will also have the same distribution. A little consideration will show that this must follow for dividing unity by a given set of numbers by means of logarithms leads to identical logarithms with merely a negative sign prefixed.

The Law of Anomalous Numbers

A study of the items of Table I shows a distinct tendency for those of a random nature to agree better with the logarithmic law than those of a formal or mathematical nature. The best agreement was found in the arabic numbers (not spelled out) of consecutive front page news items of a newspaper. Dates were barred as not being variable, and the omission of spelled-out numbers restricted the counted digits to numbers 10 and over. The first 342 street addresses given in the current American Men of Science (Item R, Table IV) gave excellent agreement, and a complete count (except for dates and page numbers) of an issue of the Readers’ Digest was also in agreement.

On the other hand, the greatest variations from the logarithmic relation were found in the first digits of mathe-
mathematical tables from engineering handbooks, and in tabulations of such closely knit data as Molecular Weights, Specific Heats, Physical Constants and Atomic Weights.

TABLE IV

<table>
<thead>
<tr>
<th>Order</th>
<th>Item</th>
<th>Nature</th>
<th>Difference</th>
<th>Order</th>
<th>Item</th>
<th>Nature</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>D</td>
<td>Newspaper Items</td>
<td>2.8</td>
<td>11</td>
<td>N</td>
<td>Cost Data, Concrete</td>
<td>12.4</td>
</tr>
<tr>
<td>2</td>
<td>F</td>
<td>Pressure Lost, Air Flow</td>
<td>3.2</td>
<td>12</td>
<td>S</td>
<td>Design Data Generators</td>
<td>13.8</td>
</tr>
<tr>
<td>3</td>
<td>G</td>
<td>H.P. Lost in Air Flow</td>
<td>4.8</td>
<td>13</td>
<td>L</td>
<td>Population, U. S. A.</td>
<td>16.6</td>
</tr>
<tr>
<td>4</td>
<td>R</td>
<td>Street Addresses, A.M.S.</td>
<td>5.4</td>
<td>14</td>
<td>B</td>
<td>Drainage Rate of Rivers</td>
<td>16.6</td>
</tr>
<tr>
<td>5</td>
<td>P</td>
<td>Am. League, 1936</td>
<td>6.6</td>
<td>15</td>
<td>I</td>
<td>Molecular Wgts.</td>
<td>21.6</td>
</tr>
<tr>
<td>6</td>
<td>Q</td>
<td>Black Body Radiation</td>
<td>7.2</td>
<td>16</td>
<td>K</td>
<td>Specific Heats</td>
<td>22.8</td>
</tr>
<tr>
<td>7</td>
<td>O</td>
<td>X-Ray Voltage</td>
<td>7.4</td>
<td>17</td>
<td>H</td>
<td>Physical Constants</td>
<td>23.2</td>
</tr>
<tr>
<td>8</td>
<td>M</td>
<td>Readers' Digest</td>
<td>8.4</td>
<td>18</td>
<td>E</td>
<td>Atomic Weights</td>
<td>24.2</td>
</tr>
<tr>
<td>9</td>
<td>A</td>
<td>Area Rivers</td>
<td>9.8</td>
<td>19</td>
<td>C</td>
<td>34.9</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>T</td>
<td>Death Rates</td>
<td>11.2</td>
<td>20</td>
<td>J</td>
<td>35.4</td>
<td></td>
</tr>
</tbody>
</table>

These facts lead to the conclusion that the logarithmic law applies particularly to those outlaw numbers that are without known relationship rather than to those that individually follow an orderly course; and therefore the logarithmic relation is essentially a Law of Anomalous Numbers.

PART II: GEOMETRIC BASIS OF THE LAW

The data so far considered have been composed entirely of used numbers; that is, numbers as they are used in everyday affairs. There must be some underlying causes that distort what we call the "natural" number system into a logarithmic distribution, and perhaps we can best get at these causes by first examining briefly the frequency of the natural numbers themselves when arranged in the infinite arithmetic series 1, 2, 3, ... n, where n is as large as any number encountered in use.

Let us assume that each individual number in the natural number system up to n is used exactly as often as every other individual number. Starting with 1, and counting up to
10,000, for example, 1 would have been used 1,112 times, or 11.12 per cent of all uses. If the count is extended to 19,999 there are 9,999 1's added, and first 1's occur in 55.55 per cent of the 19,999 numbers. When number 20,000 is reached there is a temporary stopping of the addition of first 1's and 90,000 of the other digits are added to the series before

\[
\text{FREQUENCY OF FIRST PLACE DIGITS} \quad \text{OBSERVED}
\]

\[
\log \frac{a+1}{a}
\]

\[
\text{FREQUENCY}
\]

\[
0.30
0.20
0.10
0.0
1 2 3 4 5 6 7 8 9
a
\]

Fig. 1. Comparison of observed and computed frequencies for multi-digit numbers.

1's are again brought into the series at 100,000. At this point the percentage of 1's is again reduced to 11.112 per cent as illustrated in curve A of Fig. 2. This curve is \( F_n \) and \( \log n \) plotted to a semi-logarithmic scale. If the equations for \( A \) are written for the three discontinuous but connected sections 10,000–20,000, 20,000–99,999 and 99,999–100,000 the area under the curve will be very closely 0.30103, where the entire area of the frame of coordinates has an area 1. But an integration by the methods of the calculus is merely a quick way of adding up an infinite number of \( \text{equally} \) spaced ordinates to the curve and from this addition finding the average height of
the ordinates and hence the area under the curve. But if we are satisfied with a result somewhat short of the perfection of the integral calculus we may take a finite number of equally spaced ordinates and by plain arithmetic come to practically the same answer. By definition each point of $A$ represents

![Graph](image)

**Fig. 2.** Linear frequencies of the natural number system between 10,000 and 100,000.

the frequency of first 1’s from 1 up to that point, and an integration (by calculus or arithmetic) under curve $A$ gives the average frequency of first 1’s up to 100,000. The finite number corresponding to equally spaced ordinates now represents a geometric series of numbers from 10,000 to 100,000, and it is substantially this series of numbers, in this and other orders of
the natural number scale that lead to the numerical frequencies already presented.

Curve $B$ of Fig. 2 is for 9 as a first digit. The frequency of 9's decreases in the number range from 10,000 to 89,999 and then increases as 9's are added from 90,000 to 99,999, and an integration under curve $B$ leads to a good numerical approximation to the logarithmic interval $\log 10 - \log 9$, as called for by the previous statistical study.

*Geometric and Logarithmic Series*

The close relationship of a geometric series and a logarithmic series is easily seen and hardly needs formal demonstration. The uniformly spaced ordinates of Fig. 2 form a geometric series of numbers for these numbers have a constant factor between adjacent terms, and this constant factor is determined in size by the constant logarithmic increment.

*Semi-Log Curves*

A geometric series of numbers plotted to a semi-logarithmic scale gives a straight line. In the original tabulation of observed numbers the line of data marked "R" is designated simply as "street addresses." These are the street addresses of the first 342 people mentioned in the current *American Men of Science*. The randomness of such a list is hardly to be disputed, and it should therefore be useful for illustrative purposes.

In Fig. 3 these addresses are first indicated by the height of the lines at the base of the diagram. The height of a line, measured on the scale at the left, indicates the number of addresses at, or near, that street number. Thus there were five addresses at No. 29 on various streets. In order to make the trend clearer, the heights of these lines were summed, beginning at the left and proceeding across to the right. It was found that four straight lines could be drawn among these summation points with fair fidelity of trend, and these four lines represent four geometric series, each with a different factor between terms. Each line will give the observed fre-
quency over the numerical range it covers, and hence satisfies the logarithmic relationship.

*The Natural Numbers and Nature's Numbers*

In natural events and in events of which man considers himself an originator there are plenty of examples of geometric or logarithmic progressions. We are so accustomed to labeling things 1, 2, 3, 4, \ldots and then saying they are in natural order that the idea of 1, 2, 4, 8, \ldots being a more natural arrangement is not easily accepted. Yet it is in this latter manner that a surprisingly large number of phenomena occur, and the evidence for this is available to everyone.

First, let us consider the physiological and psychological reaction to external stimuli.

The growth of the sensation of brightness with increasing illumination is a logarithmic function, as illustrated by Fechner's Law. The growth of sensation is slow at first while the rods of the retina are alone responsive, and a straight line on semi-logarithmic paper (the stimulus being on the logarithmic scale) can represent the intensity-brightness function in this region. When the cones come into action there is a sharp change in the rate of growth, and another straight line represents our working range of vision. When over-excitation and fatigue set in, a third line is needed; and thus three geometric series could be used to state the relation between illumination and the sensation of brightness. If the literature contained sufficient numerical references, the brightness function should give an extremely close approximation of the logarithmic law of distribution.

The sense of loudness follows the same rules, as does the sense of weight; and perhaps the same laws operate to make the sense of elapsed time seem so different at ages ten and fifty.

Our music scales are irregular geometric series that repeat rigidly every octave.

In the field of medicine, the response of the body to medicine or radiation is often logarithmic, as are the killing curves under toxins and radiation.
In the mechanical arts, where standard sizes have arisen from years of practical experience, the final results are often geometric series, as witness our standards of wire diameters and drill sizes, and the issued lists of "preferred numbers."

The astronomer lists stars on a geometric brightness scale that multiplies by 100 every five steps and the illuminating engineer adopts the same type of series in choosing the wattage of incandescent lamps.

In the field of experimental atomic physics, where the results represent what occurs among groups of the building units of nature, and where the unit itself is known only by mass action, the test data are statistical averages. The action of a single atom or electron is a random and unpredictable event; and a statistical average of a group of such events would show a statistical relationship to the results and laws here presented. That this is so is evidenced by the frequent use made of semi-log paper in plotting the test data, and the test points often fall on one or more straight lines. The analogy is complete, and one is tempted to think that the 1, 2, 3, \ldots scale is not the natural scale; but that, invoking the base \( e \) of the natural logarithms, *Nature* counts

\[
e^0, \ e^z, \ e^{2z}, \ e^{3z}, \ldots
\]

and builds and functions accordingly.

**Part III. Digital Orders of Numbers**

The natural number system is an array of numbers in simple arithmetic series, but on top of this we have imposed an idea taken from a geometric series. Numbers composed of many digits are ordinarily separated into groups of three digits by interposing commas, and here we unknowingly give evidence of the use of these numbers on a geometric scale.

For convenience of description the natural numbers 1 to 10 are called the first digital order numbers, those from 10 to 100 the second digital order, etc. It will be noted that 10 is both the last number of the first order and the first number of the second order, and when an integration is carried out, as will
be done later, 10 appears as both an upper and a lower limit, and it is thus used in this case as a boundary line rather than a unit zone in the natural number system.

In Fig. 4 the curves show the frequency with which the natural numbers occur in the Natural Number System, beginning at the left edge, where 1 is the only number, its frequency is 1; that is, until a second number is added 1 is the entire number system. When 2 is reached the frequency is 0.50 for 1

![Graph showing linear frequencies of the natural numbers 1 to 1000.](image)

**Fig. 4.** Linear frequencies of the natural numbers in the first three orders.

and 0.50 for 2. At 5, for example, the frequency for each of the first 5 digits is 0.20, and the equal division continues until 9 is reached. At 10, the digit 1 has appeared twice and has a frequency of 0.20 against 0.10 for each of the other eight digits that have appeared but once.

It will be observed that the curve rising from 9 on the scale of abscissae is for only the digit 1, while the curve continuing downward from 9 is for the digits 2 to 9 inclusive. At 19 the frequency curve for 2 rises to join the curve for 1 at 29 and 1
and 2 have a common curve until 99 is reached and a third first 1 is about to be added to the series. At any ordinate the curves therefore tell the frequency of the total number of natural numbers up to that point.

Fig. 5. Continuous and discontinuous functions in the neighborhood of the digit 9.

The curves are drawn as if we were dealing with continuous functions in place of a discontinuous number system. The justification for using a continuous form is that the things we use the number system to represent are nearly always perfectly continuous functions, and the number, say 9, given to any phenomenon will be used in some degree for all the infinite sizes of phenomena between 8 and 10 when we confine ourselves to single digit numbers.
An enlarged sketch of the linear frequency curves at the junction of the first and second orders is given in Fig. 5. The lines $h-b$ and $b-j$ are the computed ratios of 1 in this region, while the lines $8-b$ for the ratio of 9 begins at 8, for as soon as size 8 is passed there is a possibility of our using a 9, while for

![Graph showing frequency of single digits 1 to 9](image)

**Fig. 6.** Theoretical and observed frequencies of single digits.

size $8\frac{1}{2}$ the chances are about equal for calling it either 8 or 9. The summation of area under the curve $8-b-c$ is taken as the probability of using a 9 for phenomena in this region. This is about equivalent to knowing accurately the size of all phenomena in this region and deciding to call everything between 8.5
and 9.5 by the number 9. Once 9 is passed the curve for 1, 9--j, begins to rise in anticipation of the phenomena between 9 and 10 that will be called 10.

It has been noted that for high orders of numbers the areas under the curves of Fig. 2 are proportional to the frequency of use of the first digit. The same demonstration will now be made with the aid of the calculus in regions that are markedly discontinuous.

Selecting the third digital order, Fig. 4, the area under the 1-curve can be written

\[ A_{1'''} = \int_{100}^{199} y_1 \, dx + \int_{199}^{999} y_2 \, dx + \int_{999}^{1000} y_3 \, dx, \]  

(5)

where the ordinates of the first rising section of the curve are

\[ y_1 = \frac{a - 88}{a}. \]

(6)

The descending section of the curve has ordinates

\[ y_2 = \frac{111}{a} \]

(7)

and the last rising section between 999 and 1,000 has ordinates

\[ y_3 = \frac{a - 888}{a}. \]

(8)

The curves are plotted to semi-logarithmic coördinates and

\[ x = \log a, \]

\[ dx = da/a. \]

(9)

(10)

The integrals after making these substitutions give the value

\[ A_{1'''} = \log_e \frac{1990}{999} + \frac{8}{1000}. \]

A similar operation yields for the 1-curve in the second digital order

\[ A_{1''} = \log_e \frac{190}{99} + \frac{\frac{8}{100}}{} \]
and in the first order

\[ A_1' = \log_e \frac{10}{9} + \frac{8}{10}. \]

From the symmetry running through these solutions and from the solutions for the eight other first digits, we can write the general equation for the Law of Anomalous Numbers

\[
F_{1'} = \left\lfloor \log_e \frac{10(2 \cdot 10^{r-1} - 1)}{10^r - 1} + \frac{8}{10^r} \right\rfloor \frac{1}{N} \\
F_{a'} = \left\lfloor \log_e \frac{(a + 1)10^{r-1} - 1}{a10^{r-1} - 1} - \frac{1}{10^r} \right\rfloor \frac{1}{N}
\]

where \( N = \log_e 10 \) is the factor to convert the expressions from the natural logarithm system, base \( e \), to the common logarithm system, base 10.

If high orders of \( r \) are considered, as was unwittingly done in the original statistical work, these expressions simplify by dropping the terms \(-1\) in both numerator and denominator, and the numerical terms having \( 10^r \) in the denominator become negligible. Hence the general equations become

\[
F_{1'} = \log_{10} \frac{2}{1},
\]

\[
F_{a'} = \log \frac{a + 1}{a},
\]

but these two expressions no longer have a difference in form, and they may be merged into

\[
F_{a'} = \log_{10} \frac{a + 1}{a},
\]

which was the relationship originally observed for multi-digit numbers.

In Table V numerical values are given for the theoretical frequencies of used numbers for the first, second, third and limiting digital orders.
The frequencies of the single digits 1 to 9 vary enough from the frequencies of the limiting order to allow a statistical test if a source of digits used singly can be found. The footnotes so commonly used in technical literature are an excellent source, consisting of units that are indicated by numbers, letters or symbols.

The procedure of collecting data for the first-order numbers was to make a cursory examination of a volume to see if it contained as many as 10 footnotes to a page, for obviously no test of the range 1 to 9 could be made if the maximum number fell short of the full range. The numbers here recorded in Table VI are the number of footnotes observed on consecutive pages, beginning on page 1 and continuing to the end of the book, or until it seemed that a fair sample of the book had been obtained. The books used were the Standard Handbook for Electrical Engineers, Smithsonian Physical Tables, Handbuch der Physik and Glazebrook’s Dictionary of Applied Physics.

In Table VI the observed percentages of single digits 1 to 9 are given along with the number of pages used in each volume and the number of footnotes observed. The frequency for 1 is seen to be 43.2 per cent as against the theoretical frequency of 39.3 per cent, and for the digit 9 the observations agree with theory with \( F_9' = 0.8 \) per cent.
FRANK BENFORD

In general the agreement with theory is as good as the computed probable errors of the observation.

TABLE VI
COUNT OF FOOTNOTES

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<th>Volume</th>
<th>Pages Used</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>Total Count</th>
</tr>
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<tr>
<td>1. S. H. E. E.</td>
<td>All</td>
<td>55.1</td>
<td>22.7</td>
<td>12.3</td>
<td>5.0</td>
<td>2.4</td>
<td>1.7</td>
<td>0.3</td>
<td>0.3</td>
<td>0.3</td>
<td>1586</td>
</tr>
<tr>
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<td>All</td>
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<td>22.1</td>
<td>6.6</td>
<td>6.1</td>
<td>5.0</td>
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<td>0.0</td>
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</tr>
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<td>360</td>
<td>52.8</td>
<td>23.6</td>
<td>8.5</td>
<td>5.5</td>
<td>4.0</td>
<td>3.2</td>
<td>0.8</td>
<td>0.0</td>
<td>1.6</td>
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</tr>
<tr>
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<td>37.2</td>
<td>25.7</td>
<td>12.1</td>
<td>9.5</td>
<td>4.8</td>
<td>5.2</td>
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<td>1.8</td>
<td>1.0</td>
<td>1.4</td>
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</tr>
<tr>
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<td>19.5</td>
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<td>6.1</td>
<td>5.8</td>
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<td>27.5</td>
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<td>5.9</td>
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<td>56.8</td>
<td>23.2</td>
<td>6.7</td>
<td>7.6</td>
<td>2.4</td>
<td>1.4</td>
<td>0.5</td>
<td>1.4</td>
<td>0.0</td>
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<td>9. Glazebrook</td>
<td>All</td>
<td>49.5</td>
<td>22.3</td>
<td>13.7</td>
<td>6.9</td>
<td>2.3</td>
<td>1.5</td>
<td>1.5</td>
<td>1.5</td>
<td>0.8</td>
<td>394</td>
</tr>
<tr>
<td>10. Glazebrook</td>
<td>All</td>
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<td>25.2</td>
<td>13.4</td>
<td>9.1</td>
<td>4.7</td>
<td>3.2</td>
<td>1.7</td>
<td>0.5</td>
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Observed Ave. 43.2 23.6 11.8 8.3 4.9 3.9 1.9 1.6 0.8 2968
Predicted Ave. 43.2 25.7 13.3 8.1 5.3 3.6 2.4 1.5 0.8
Difference +3.0 -2.1 -1.5 +0.2 -0.4 +0.3 -0.5 +0.1 0.0
Probable Error ±3.0 ±0.6 ±0.7 ±0.5 ±0.6 ±0.5 ±0.4 ±0.4 ±0.4

**Summation of Frequencies**

One of the conditions that must be met by these expressions for the frequencies of the integers is that, in any one order, the sum of the frequencies must equal unity; that is, the sum of their probabilities must equal certainty.

Selecting the first-order digits, Eq. 11, and remembering the logarithmic rule that the sum of the logarithms of a group of numbers is equal to the logarithm of their combined products, we have the probability $P'$

$$P' = \log_{10} \frac{10 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7 \cdot 8 \cdot 9}{9 \cdot 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7 \cdot 8} + \left| \frac{8}{10} - \frac{1}{10} - \frac{1}{10} - \frac{1}{10} - \frac{1}{10} - \frac{1}{10} - \frac{1}{10} - \frac{1}{10} - \frac{1}{10} \right| \frac{1}{N},$$

which reduces to

$$P' = \log_{10} 10 + 0 = 1.$$ 

In a similar manner from the complete set of equations
indicated by Eq. 11 we have

\[ P'' = \log_{10} \frac{190\cdot9-8-9\cdot9-9\cdot9-5-9\cdot6-9\cdot7-9\cdot8-9\cdot9}{99\cdot1-9\cdot8-9\cdot8-9\cdot9-9\cdot8-9\cdot7-9\cdot6-9\cdot5-9\cdot4-9\cdot3-9\cdot2-9\cdot1-100} \]

\[ + \left[ \frac{8}{100} - \frac{1}{100} - \frac{1}{100} - \frac{1}{100} - \frac{1}{100} - \frac{1}{100} - \frac{1}{100} - \frac{1}{100} - \frac{1}{100} \right] \frac{1}{N} \]

\[ = \log_{10} 10 + 0 \]

\[ = 1 \]

and similar proof can be worked out for the other orders.

**Summary of Part III**

Single digits, regardless of their relation to the decimal point and also regardless of preceding or following zeros, have a specific natural frequency that varies sharply from the logarithmic ratios. The second digital order, which is composed of two adjacent significant digits, has a specific frequency approximating the logarithmic frequency; and for three or more associated digits the variation from the latter frequency would be extremely difficult to find statistically.

The basic operation

\[ F = \int \frac{da}{a} \]

or

\[ F = \sum \frac{\Delta a}{a} \]

in converting from the linear frequency of the natural numbers to the logarithmic frequency of natural phenomena and human events can be interpreted as meaning that, on the average, these things proceed on a logarithmic or geometric scale. Another way of interpreting this relation is to say that small things are more numerous than large things, and there is a tendency for the step between sizes to be equal to a fixed fraction of the last preceding phenomenon or event. There is
no necessity or implication of limits at either the upper or the lower regions of the series.

If the view is accepted that phenomena fall into geometric series, then it follows that the observed logarithmic relationship is not a result of the particular numerical system, with its base, 10, that we have elected to use. Any other base, such as 8, or 12, or 20, to select some of the numbers that have been suggested at various times, would lead to similar relationships; for the logarithmic scales of the new numerical system would be covered by equally spaced steps by the march of natural events. As has been pointed out before, the theory of anomalous numbers is really the theory of phenomena and events, and the numbers but play the poor part of lifeless symbols for living things.
THE MULTIPLE SCATTERING OF ELECTRONS

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Cornell University, Ithaca, New York

ABSTRACT

The multiple scattering of electrons is calculated in terms of the well-known cross section for single scattering (§2). The transmission of a collimated beam through a plane parallel plate is treated, neglecting energy loss; the transmitted intensity is found to be inversely proportional to the thickness of the plate for large thicknesses (§3). Under certain conditions (nearly random distribution of velocities) the motion of the electrons can be treated as a diffusion problem (§4) and their energy loss taken into consideration; several applications are given. The absorption of fast electrons due to energy loss and scattering is discussed quantitatively (§5). These considerations are applied to the determination of γ-ray energies from the absorption of their secondary electrons and very good agreement with experimental data is obtained.

§1. INTRODUCTION

The cross section of the elementary scattering of fast electrons by atoms has been known for a considerable time. However, for the problem of penetration of electrons through foils which is of great practical importance for the evaluation of experiments, multiple scattering must be considered since the thickness of the foils used in practice is very large compared to the mean free path of the electrons. It is the purpose of this paper to develop a theory of the multiple scattering expressing it in terms of the well known cross section for single scattering.

In §2, we shall set up the Boltzmann equation which governs the development in time of a given initial electron distribution in space and direction of motion. The cross section of single scattering enters this Boltzmann equation only insofar as it determines the transport mean free path. The Boltzmann equation is applied in §3 to the penetration of electrons through a plane parallel plate. The transmission

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of the plate and the angular distribution of the emerging electrons will be calculated. In §4, the problem of diffusion of the electrons is considered. These considerations are applicable when there is no greatly preferred direction of motion, i.e. (1) when the electrons are emitted in all directions from a source (emergence of electrons from a radioactive target), or (2) when an initially collimated beam of electrons has lost most of its forward motion due to scattering. In the diffusion theory, the energy loss of the electrons can easily be taken into account. In §5, the penetration of electrons through foils is treated including the energy loss. This has been done only approximately but the approximation should be rather good. Theoretical absorption curves are obtained for electrons of given energy. From these, it is possible to derive the absorption curves for the electrons produced by γ-rays of given energy. This absorption method has been used recently by experimenters for determining the energy of γ-rays produced in nuclear reactions. In one case where the γ-ray energy is known from other sources (Th C'' γ-ray, 2.6 MV) the observed absorption agrees practically exactly with the theoretical value. For higher energy, the theoretical curve should give better results than the curve used by the experimenters which was based on an extrapolation of empirical data.

§2. THE BOLTZMANN EQUATION

We shall consider the number of electrons at a given position \( \mathbf{r} \) and with a given direction of motion \( \mathbf{u} \), \( f(\mathbf{r}, \mathbf{u}, t) \) \( \sin \phi d\phi d\varphi \), where \( \phi \) and \( \varphi \) give the direction of the unit vector \( \mathbf{u} \) and \( t \) is the time. The density \( f \) changes with time because of convection and because of scattering. In the usual manner, we find the Boltzmann equation

\[
\frac{\partial f}{\partial s} = \frac{1}{v} \frac{\partial f}{\partial t} = - \mathbf{u} \cdot \nabla f + \int \sin \alpha \, d\alpha \, d\beta \, N\sigma(\alpha, v) \times [f(\mathbf{r}, \mathbf{u}', t) - f(\mathbf{r}, \mathbf{u}, t)]. \tag{1}
\]

Here \( v \) is the velocity, and \( ds = v \, dt \) an element of the path of
the particle. The gradient is to be taken in ordinary (\( r \))
space. \( N \) is the number of scattering atoms per unit volume,
\( \sigma \) the scattering cross section for deflection of electrons of
velocity \( v \) through the angle \( \alpha \). \( \alpha \) is the angle between the
unit vectors \( u \) and \( u' \), and \( \beta \) is the azimuth of \( u' \). The first
term in (1) represents the convection, the first term in the
square bracket the scattering into the solid angle at \( u \) and
the last term the scattering out of this solid angle.

The scattering cross section \( \sigma \) is known to be (cf. below)
small for large scattering angles. Therefore the important
directions \( u' \) will be very close to \( u \); we write \( u' = u + w \)
where \( |w| = 2 \sin \alpha/2 \ll 1 \). Then we can make a Taylor
expansion of \( f(u') \) in powers of \( w \) (always on the surface of
the unit sphere), viz.:

\[
f(u') - f(u) = w_x \frac{\partial f}{\partial u_x} + w_y \frac{\partial f}{\partial u_y} + \frac{1}{2} w_x^2 \frac{\partial^2 f}{\partial u_x^2} \\
+ w_xw_y \frac{\partial^2 f}{\partial u_x \partial u_y} + \frac{1}{2} w_y^2 \frac{\partial^2 f}{\partial u_y^2}. \quad (2)
\]

Now \( w_x = w \cos \beta, \; w_y = w \sin \beta \); therefore upon averaging
over the azimuth \( \beta \) all terms vanish except those with \( w_x^2 \)
and \( w_y^2 \), and (1) becomes

\[
\frac{\partial f}{\partial s} = -u \cdot \text{grad}_r f + \Delta_u f/\lambda, \quad (3)
\]

where \( \lambda \) is the "transport mean free path"

\[
1/\lambda = \pi N \int \sin \alpha \, d\alpha \sigma(\alpha, v)(1 - \cos \alpha) \quad (4)
\]

and \( \Delta_u \) is the Laplacian operator in the angular coordinates,

\[
\Delta_u = \frac{1}{\sin \vartheta} \frac{\partial}{\partial \vartheta} \left( \sin \vartheta \frac{\partial}{\partial \vartheta} \right) + \frac{1}{\sin^2 \vartheta} \frac{\partial^2}{\partial \varphi^2}. \quad (5)
\]

The differential cross section \( \sigma \) for single scattering by an
atom of nuclear charge Ze is given by

\[ \sigma(\alpha) \sin \alpha \, d\alpha = e^4 Z^2 \frac{W^2}{(cp)^4} \left( \frac{1 - F}{1 - \cos \alpha} \right)^2 \sin \alpha \, d\alpha. \]  

(6)

Here \( W \) is the energy of the electron including \( mc^2 \) and \( p \) its momentum. The formula is valid for any energy greater than the binding energy of the \( K \)-electrons in the atom, and approximately valid for energies considerably below this limit. \( F \) is the atomic form factor known from X-ray scattering, divided by \( Z \); it represents the contribution of the atomic electrons to the elastic scattering. \( F \) may be calculated using the Thomas-Fermi model of the atom; it is a function of

\[ \xi = 2(ap/h)Z^{-1/3} \sin \alpha/2, \]  

(7)

where \( a \) is the Bohr radius. \( F \) is unity for \( \xi = 0 \), about 1/2 for \( \xi = 1 \) and falls rapidly to zero for larger \( \xi \). Because of the denominator \( (1 - \cos \alpha)^2 \), the cross section becomes very small for large angles, a fact which we have used in the derivation of (3).

For the transport mean free path we obtain

\[ \frac{1}{\lambda} = \frac{\pi N e^4 Z^2 W^2/(W^2 - m^2 c^4)^2}{ \int_0^{\pi} \sin \alpha \, d\alpha (1 - \cos \alpha) \times \left( \frac{1 - F}{1 - \cos \alpha} \right)^2}. \]  

(8)

Since \( F \) is near unity for \( \xi < 1 \) and near zero for \( \xi > 1 \), we replace the integral by

\[ \int_{\xi = 1}^{\xi = \pi} \sin \alpha \, d\alpha/(1 - \cos \alpha) \]

\[ = 2 \log 2ap/hZ^{1/3}, \]  

(9)

\[ \frac{1}{\lambda} = 2\pi N e^4 Z^2 W^2/(W^2 - m^2 c^4)^2 \log 2ap/hZ^{1/3}. \]  

(10)

---

Bethe, *Handbuch der Physik*, 24/1, p. 491. It will not be necessary to consider inelastic scattering because this is negligible except for very small angles. Inelastic scattering must, of course, be considered for the energy loss.

A table for \( F \) is found, e.g., in *Ann. der Phys.*, 5, 325 (1930).

\( e \) begins to decrease rapidly when \( \xi \) becomes larger than unity. For \( p = mc \) and \( Z = 30 \), \( \xi = 1 \) corresponds to \( \alpha = 1.3^\circ \).
Table I gives the values of $\lambda$, in g./cm.$^2$, for various substances and kinetic energies $E$.

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<tr>
<th>Scattering Material</th>
<th>0.1</th>
<th>0.2</th>
<th>0.5</th>
<th>1.0</th>
<th>2.0</th>
<th>3.0</th>
<th>8.5</th>
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<tr>
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<td>1.87</td>
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</tbody>
</table>

§3. Penetration through a Plane Parallel Plate

We consider a plane parallel plate of thickness $d$; $x$ (going from 0 to $d$) is the coördinate perpendicular to the surface. An electron beam is incident normally on the plate with uniform intensity (1 electron per second per cm.$^2$ over the surface $x = 0$). We are interested in the number of electrons emerging from the surface $x = d$ (transmission) and their angular distribution, and also in the angular distribution of the electrons scattered back through the surface $x = 0$.

The electron distribution in the plate will depend on $x$ and $\theta$. We consider the stationary case $\partial f / \partial s = 0$ and have

$$\frac{1}{\lambda} \frac{\partial}{\partial \mu} (1 - \mu^2) \frac{\partial f}{\partial \mu} - \mu \frac{\partial f}{\partial x} = 0$$

(11)

with $\mu = \cos \theta$. This equation may be separated putting

$$f = g(x)\psi(\mu),$$

(12)

$$\frac{d}{d\mu} (1 - \mu^2) \frac{d\psi}{d\mu} + k \mu \psi = 0,$$

(13)

$$\frac{dg}{dx} + \frac{k}{\lambda} g = 0,$$

(14)

$k$ being the separation parameter. Eq. (14) has the solution

$$g_k = ce^{-kx/\lambda} \quad \text{(for } k \neq 0).$$

(15)

*This equation was already derived by Bothe, *Zeits. f. Phys.*, 54, 161 (1929) but not correctly treated due to application of improper boundary conditions.
(13) is similar to the differential equation for the spherical harmonics, the only difference being the factor $\mu$ in the coefficient of $\psi$. The boundary conditions are the same as for the spherical harmonics, viz.: $\psi$ must be finite everywhere between $\mu = -1$ and $\mu = +1$. The solutions of (13) for $k \neq 0$ can be obtained either by numerical integration or (not very practical) by expansion in spherical harmonics or, for sufficiently large $k$, by the Debye (WKB) method. This latter method gives for the eigenvalues

$$k_{n\pm} = \pm 6.88 (n + \frac{1}{2})^2. \quad (16)$$

The first eigenvalue $k_1$, is 15.4 from (16) whereas a numerical integration of (13) gives $k_1 = 14.476$. Even better agreement will be obtained for the higher eigenvalues. The WKB solution shows that $\psi$ decreases rapidly (exponentially) for negative $k\mu$. Thus for positive $k$ most of the electrons move in a forward direction ($0 < \theta < \pi/2$), for negative $k$, in a backward direction.

Since the eigenvalues $k$ calculated above are all very large, the corresponding solutions $f = e^{-kz/\lambda}\psi_k$ die out at a very small depth in the foil ($x \approx \lambda/15$). If these were the only solutions of (11), the electron absorption would be extremely strong. However, there is in addition a singular solution

$$f = \alpha + \beta(\mu - 2x/\lambda), \quad (17)$$

where $\alpha$ and $\beta$ are arbitrary constants. This solution is the only important one for the actual penetration of the electrons through the foil, the exponential solutions are necessary only to satisfy the boundary conditions at the surfaces of the foil.

An approximate solution may be obtained by neglecting the exponential solutions altogether. The boundary conditions are that no particles go towards smaller $x$ at the exit surface $x = d$, and that at $x = 0$ we have a collimated incident beam of unit flux. With our approximate solution, we can

---

7 In addition to these solutions, there are solutions $e^{-k(d-x)/\lambda}\psi_{-k}(\mu)$ which are only important near the back surface $x = d$. 

of course not satisfy these conditions accurately but only in the mean. At \( x = d \), we require that the total current to the left is zero, viz.:

\[
\int_{-1}^{0} \mu f(d, \mu) \, d\mu = 0. \tag{18}
\]

At \( x = 0 \), we set the total current to the right equal to unity, viz.:

\[
\int_{0}^{1} \mu f(0, \mu) \, d\mu = 1. \tag{19}
\]

Inserting (17), we obtain the values of \( \alpha \) and \( \beta \). The transmitted intensity is then given by

\[
J = \int_{0}^{1} \mu f(d, \mu) \, d\mu = \frac{2}{3} \beta = \frac{1}{(3d/2\lambda) + 1}. \tag{20}
\]

The transmission is unity for zero thickness and inversely proportional to the thickness for large \( d \). Therefore the absorption is not nearly as strong as was believed previously (exponential absorption, cf. e.g. reference 5).

To obtain an accurate solution, the exponential eigenfunctions must be considered in the boundary conditions at both surfaces. The result for the transmission is

\[
J = \frac{0.862}{d/\lambda + 0.719}, \tag{21}
\]

provided \( d/\lambda \) is large enough to make the exponentials negligible, i.e. \( d > 0.2\lambda \). It is seen that the transmission is somewhat greater than that given by (20) (about 30 per cent for large thicknesses).

The angular distribution of the emerging electrons has been calculated; it is

\[
f(d, \mu) \sim 0.717 + \cos \vartheta - 0.181 \psi_1(\mu) - 0.082 \psi_2(\mu) - \cdots. \tag{22}
\]

\( \psi_n \) is the solutions of (13) corresponding to the eigenvalue \( k_n \). Except near \( \mu = 0 \), the distribution is almost proportional to \( c_1 + c_2 \cos \vartheta \).
§4. Diffusion

After a large number of collisions, the motion of the electrons will be almost random in direction. In this limit, the motion may be treated as a diffusion problem. We introduce the total density of electrons

$$F(r, s) = \int d u f(r, u, s)$$  \hspace{1cm} (23)

and the current density

$$J(r, s) = \int u f(r, u, s) \, d u.$$  \hspace{1cm} (24)

Then, with the assumption of small anisotropy, we may write

$$f(r, u, s) = (1/4 \pi) (F + 3 S \cdot u).$$  \hspace{1cm} (25)

We integrate the Boltzmann equation (3) over all directions of $u$ and obtain the continuity equation

$$\partial F/\partial s = - \text{div } J,$$  \hspace{1cm} (26)

since $\int d u \Delta_u f = 0$. Multiplying (3) by $u$ and integrating over all directions yields

$$\partial J/\partial s = - 1/3 \text{ grad } F + 2 J/\lambda.$$  \hspace{1cm} (27)

The process may be considered a diffusion only if $F$ and $J$ do not change appreciably over one mean free path $\lambda$. Therefore $\partial J/\partial s \ll 2 J/\lambda$, and combining (26) and (27) we obtain in sufficient approximation

$$\partial F/\partial s = - (\lambda/6) \Delta F,$$  \hspace{1cm} (28)

where $\Delta$ is the ordinary Laplacian operator.

$F$ depends on the coordinates $x$, $y$, $z$ and on the total length $s$ of the path of the electrons from their time of emission. This length of path may be considered as uniquely connected with the energy loss suffered by the electrons, except for the rather small straggling of the energy loss.$^8$ Thus the

---

$^8$ The straggling is small only as long as the energy loss by radiation is negligible. Therefore this procedure is only valid for energies up to $800/Z$ MV in a material of atomic number $Z$. However, for higher energies the scattering is negligible.
distribution in space of electrons of a given energy is given by the function \( F \) for the corresponding value of \( s \). With a given initial energy \( E_0 \), there will be a certain total length of path \( s_0 \) required for stopping the electrons completely. After being stopped, the electrons are, of course, absorbed by recombination with the atoms of the material. \( F(r, s_0) \) gives the distribution of electrons in space when they are absorbed, and

\[
\int (ds/v)F(r, s) \text{ gives the total density of electrons of all energies at the point } r.
\]

Equation (28) will be simplified by introducing the new variable

\[
\tau(s) = \frac{1}{6} \int_0^s \lambda \, ds. \tag{29}
\]

Since \( s \) is a function of the electron energy \( E \), and \( \lambda \) is as well (cf. 10), \( \tau \) will also depend on \( E \) (and the initial energy \( E_0 \)). If \( dE/ds \) is the energy loss per unit path,

\[
\tau(E) = \frac{1}{6} \int_{E_0}^E \lambda(E) \, dE/|dE/ds|. \tag{30}
\]

Explicit formulas for \( \tau(E, E_0) \) can easily be derived.

With (29), (28) becomes

\[
\frac{\partial F}{\partial \tau} = -\Delta F, \tag{31}
\]

which is the ordinary diffusion equation. Its solution depends on the boundary conditions and the initial condition. Some cases of particular interest are:

(1) **Point source of unit intensity in homogeneous medium.**—

If \( r \) is the distance from the source

\[
F = (4\pi\tau)^{-3/2} e^{-r^2/4\tau}. \tag{32}
\]

The mean square distance from the source at the moment of absorption is

\[
r^2_{\lambda\nu} = 6\tau_0 = \int_{E_0}^\infty \lambda(E) \, dE/|dE/ds|. \tag{33}
\]

The "Unwegfaktor" \( U \) may be defined as the actual length of
path of an electron, $s_0 = \int_0^{2\theta_0} dE/|dE/ds|$, to the average distance from the source at the moment of absorption, $r_{A\nu}$ or $\sqrt{r_{A\nu}^2}$. $U$ is a measure of the importance of scattering.

(2) Point source of unit intensity in a medium bounded by the plane $x = 0$.—Position of source $x = d, y = z = 0$.

$$F = (4\pi\tau)^{-3/2}e^{-(y^2+z^2)/4\tau}(e^{-(x-d)^2/4\tau} - e^{-(x+d)^2/4\tau}).$$  \hspace{1cm} (34)

Number of particles of energy between $E$ and $E + dE$ emerging from the surface is

$$I dE = \frac{d}{(4\pi\tau)^{1/2}} e^{-d^2/4\tau} \frac{1}{\tau} \frac{d\tau}{dE} dE.$$  \hspace{1cm} (35)

(3) Uniform distribution of sources over the interior of a medium bounded by the plane $x = 0$.—If $q$ is the number of electrons produced per cm.$^3$ and sec., the total number of electrons emerging from the surface per cm.$^2$ and sec. is

$$n = 2\sqrt{\tau_0/\pi} q = \left[ \frac{2}{3\pi} r_{A\nu}^2 \right]^{1/2} q,$$

where $r_{A\nu}^2$ is given in (33). Thus, if radioactive atoms are distributed uniformly in a target (e.g. neutron produced radioactivity), the number of emerging electrons is proportional to the "effective target thickness" $(2/3\pi)^{1/2}(r_{A\nu}^2)^{1/2}$. Table II

**Table II**

<table>
<thead>
<tr>
<th>Scattering Material</th>
<th>$E$</th>
<th>0.1</th>
<th>0.2</th>
<th>0.5</th>
<th>1.0</th>
<th>2.0</th>
<th>5.0</th>
<th>10.0</th>
<th>20.0 mc²</th>
</tr>
</thead>
<tbody>
<tr>
<td>Al</td>
<td></td>
<td>0.0063</td>
<td>0.0155</td>
<td>0.050</td>
<td>0.124</td>
<td>0.304</td>
<td>0.990</td>
<td>2.43</td>
<td>5.95</td>
</tr>
<tr>
<td>Cu</td>
<td></td>
<td>0.0047</td>
<td>0.0116</td>
<td>0.034</td>
<td>0.063</td>
<td>0.227</td>
<td>0.725</td>
<td>1.76</td>
<td>4.35</td>
</tr>
<tr>
<td>Pb</td>
<td></td>
<td>0.0031</td>
<td>0.0084</td>
<td>0.027</td>
<td>0.065</td>
<td>0.160</td>
<td>0.505</td>
<td>1.23</td>
<td>3.00</td>
</tr>
</tbody>
</table>

gives these thicknesses for various elements and initial electron energies. The energy distribution is given by

$$I dE = \frac{n(E)}{2[\tau_0 \tau(E)]^{1/2}} \frac{d\tau}{dE} dE.$$  \hspace{1cm} (37)

Other cases can easily be treated.
§5. Absorption of Fast Electrons

A beam of fast electrons will in the beginning of its path suffer energy loss but very little scattering and thus move in almost a straight line. With decreasing energy, scattering will become more important until finally the stage of diffusion is reached where the direction of motion becomes almost random. Both the limiting cases of straight motion and diffusion (§4) are easy to treat whereas the intermediate region is not. We therefore approximate the problem by assuming a direct transition from straight motion into diffusion. As the transition point we take the point at which the average cosine of the angle between the direction of motion and the primary beam becomes $1/e$. The average of $\cos \vartheta$ after traversal of a path $s$ can easily be shown to be

$$(\cos \vartheta)_{A\nu} = \exp \left[ -2 \int_0^s ds/\lambda \right].$$

The energy $E_d$ at which the diffusion starts is therefore given by

$$\int_{E_d}^{E_0} \frac{dE}{\lambda |dE/ds|} = \frac{1}{2}.$$  \hspace{1cm} (39)

The average penetration of the electrons in the "straight" part of the path is

$$x_{A\nu} = \int_0^{x_{s'}} [\cos \vartheta(s')]_{A\nu} ds'.$$  \hspace{1cm} (40)

We assume that the source of diffusing electrons is a plane distribution at a distance $x_{A\nu}$ from the starting point. Then the current of electrons emerging from a plane at a distance $x_{A\nu} + d$ from the starting point is obtained by integrating (35) over $\tau$ from 0 to $\tau_0$

$$J(x_{A\nu} + d) = 1 - \Phi(d/2\tau_0^{1/2}),$$  \hspace{1cm} (41)

where $\Phi$ is the error integral.

We apply these formulas to the absorption of electrons produced by $\gamma$-rays (cf. reference 1). The $\gamma$-rays pass through a thick Al foil in which they produce electron pairs and Comp-
ton electrons. These electrons, if emitted in the forward direction, go through two Geiger counters separated by an absorbing foil of Al. The number of coincidences of the counters is measured as a function of the thickness of the absorbing foil, due corrections being made for the absorption in the walls of the counters. In order to obtain the theoretical absorption curve, we must consider that electrons are produced throughout the first foil. If the thickness $x$ of the absorbing foil is less than $x_{AV}$ and if the electrons have all the same energy, the intensity is approximately proportional to $1 - x/x_{AV}$. In reality, the electrons have various energies and an average must be taken over the known energy distribution of Compton and pair electrons. The theoretical absorption curve thus obtained is in good agreement with the experimental curve for monochromatic $\gamma$-rays (Th C'', cf. Fleischmann, loc. cit.). In the general case, the absorber thickness at which the intensity is reduced to one-half is taken as a measure of the $\gamma$-ray energy. Figure 1 gives the calculated
half-value thickness as a function of the $\gamma$-ray energy. Three experimental points are also given ($\text{Th C}', H + n \rightarrow D, \text{Th C''}$); the agreement is excellent, especially considering that there is no adjustable constant in the theory. The assumption of a linear relation between $\gamma$-ray energy and half value thickness which was used by Fleischmann gives reasonably good results (energy 20 per cent too high at $30 \, mc^2$); much better is the relation proposed by Bothe and used by Mitchell and Langer, viz.

$$d_{1/2} = 0.065(h\nu)^2/(h\nu + mc^2)mc^2,$$  \hfill (42)

the constant being chosen to fit the Th C'' measurement.

We have also calculated the quarter value thickness as a function of $h\nu$. A comparison of the quarter and the half value thickness gives an indication as to whether the $\gamma$-rays are monochromatic or not. If $d_{1/4}/d_{1/2}$ has about the theoretical value, the $\gamma$-rays will be monochromatic or nearly so whereas larger values of this ratio indicate the presence of several components with comparable intensity. The $\gamma$-rays emitted in the capture of slow neutrons by nuclei have electron absorption curves which show that they deviate markedly from being monochromatic as is to be expected from the theory of that process. Similar calculations for the absorption of radioactive $\beta$-particles are in progress.
EXPERIMENTS CONCERNING THE SECRETORY ACTIVITY OF THE LIVER

RUDOLF HÖBER
Visiting Professor of Physiology, University of Pennsylvania

AND
ELINOR MOORE

ABSTRACT

Earlier experiments have shown that the frog liver when perfused with Ringer’s solution containing natural or artificial pigments is able to secrete these substances in concentrations several hundred times those in the perfusing solutions. This secretory activity of the surviving organ can be stimulated by certain organic substances and inhibited by others. In the group of compounds exhibiting an inhibitory effect have been found those which possess a strong affinity for water, are surface inactive, and produce dehydration and condensation of hydrophilic colloids. Examples of such substances are sugars, polyhydric alcohols, amino acids, and bivalent aliphatic acids and oxaacids. The members of the group promoting secretion are surface active and disclose organophilic and hydrophobic properties on one side of their polar molecules and hydrophilic qualities on the other side. To this group belong the bile acids, higher fatty acids, carbamates and saponin. The same division into groups has been met with in experiments upon the influence of a fairly large number of organic substances on the dispersity of lecithin sols and in experiments on the production of injury potentials in muscle and nerve. From these results it is concluded that the secretory activity is in some way dependent upon the alteration of the condition of the cell colloids in the direction of either a higher or a lower degree of dispersion.

SECRETORY activity designates not only synthesis by special living cells of specific chemical compounds like hormones, enzymes, bile acid or milk sugar, but also collection, transport and concentration as in the removal from the blood of substances like urea and uric acid by the kidney and of pigments by the liver. We have found that such concentration can be accomplished by the frog liver even when separated from the body of the animal and when the blood is replaced by a simple saline solution. When a very

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small amount of either normal bile pigment or of any technical dyestuff is added to the saline solution, the pigment reappears in the biliary secretion in a concentration several hundred times that of the entering solution. Such transformation of a dilute into a stronger solution requires work on the part of the living cell just as the concentration, the compression of air requires an expenditure of energy. Furthermore, during suspension of the vital activity of the liver caused by administration of an anaesthetic the capacity for accumulation disappears and the unchanged solution leaks through. After removal of the anaesthetic the liver regains its concentrating power. The mechanism of this type of activity, which is exhibited by the kidney in the transportation of urea, of uric acid or of certain dyestuffs from the blood into the urine, by the gills of marine fishes in the excretion of salt from their bodies into the salty sea, by the skin of the frog and by plant roots in the accumulation of salt from the highly diluted source in the surrounding fresh water or in the soil, is an unsolved physiological problem. We have been trying to attack this general problem in experiments on the liver.

It is well known, mainly from clinical observations, that the secretory activity of the liver can be stimulated by several substances, namely, the normally present salts of bile acids, as well as those of oleic and salicylic acids. Substances like these have been called "choleretics," which are distinct from the so called "cholagogues," i.e. substances causing discharge of the filled gall bladder. In order to get better information concerning this choleretic, secretory activity, we have undertaken a study of the ability of a large number of organic substances to promote or to inhibit the above mentioned dyestuff secretion of the liver.

Before giving a résumé of this part of our investigation, we wish to turn the attention to an unexpected situation. We have not been able to find substances other than the dyestuffs to be concentrated by the liver. Some carbohydrates and inorganic ions pass through the livers of
frogs and of rabbits unconcentrated as they would through a filter, while simultaneously injected dyes show the characteristic strong accumulation. This behavior is illustrated e.g. by a series of experiments on frogs published recently by Dr. Haywood and one of us.

| Table 1 |
| Dyestuff Concentration Compared with that of Inulin |

<table>
<thead>
<tr>
<th>Dyestuff Concentration Factor</th>
<th>p.e. in perfusion fluid</th>
<th>p.e. in secretion</th>
</tr>
</thead>
<tbody>
<tr>
<td>70</td>
<td>0.069</td>
<td>0.049</td>
</tr>
<tr>
<td>45</td>
<td>0.069</td>
<td>0.068</td>
</tr>
<tr>
<td>60</td>
<td>0.040</td>
<td>0.037</td>
</tr>
<tr>
<td>30</td>
<td>0.040</td>
<td>0.041</td>
</tr>
<tr>
<td>40</td>
<td>0.064</td>
<td>0.053</td>
</tr>
</tbody>
</table>

Table 1 gives the results of five experiments in which the liver was perfused by inulin and dyestuff. Whereas the dyestuff reappears in concentrations 30 to 70 times higher than that in the perfusion fluid, the inulin passes unconcentrated.

The fairly great number of organic substances tested for their influence upon secretion can be divided into two groups, one comparatively harmless to the liver, but distinctly decreasing its ability of dyestuff accumulation, the other promoting in small, but poisonous in greater concentrations. In the actual experimental procedure of testing for these effects the liver is perfused with faintly stained isotonic salt solution until a secretion dark from concentrated dyestuff appears in the cannula inserted in the bile duct. 1/8 to 1/10 of the salt is then replaced by the isosmotic amount of, for example, the sodium salt of an organic acid belonging to the first group of substances characterised as harmless to the liver. The behavior is illustrated by the Table 2, Exp. 1 to 5.
### TABLE 2
**Anticholeretic and Choleretic Effects**

<table>
<thead>
<tr>
<th>Exp.</th>
<th>Sodium Salts</th>
<th>Molarity</th>
<th>Dyestuff Concentration Factor</th>
<th>Amount of Secretion γ/Hour</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Before</td>
<td>During</td>
</tr>
<tr>
<td>1.</td>
<td>Acetate</td>
<td>0.028</td>
<td>40</td>
<td>0</td>
</tr>
<tr>
<td>2.</td>
<td>Glycolate</td>
<td>0.014</td>
<td>600</td>
<td>200</td>
</tr>
<tr>
<td>3.</td>
<td>Glutarate</td>
<td>0.028</td>
<td>730</td>
<td>6</td>
</tr>
<tr>
<td>4.</td>
<td>Malate</td>
<td>0.028</td>
<td>260</td>
<td>30</td>
</tr>
<tr>
<td>5.</td>
<td>Succinate</td>
<td>0.028</td>
<td>690</td>
<td>6</td>
</tr>
<tr>
<td>6a.</td>
<td>Taurocholate</td>
<td>30 × 10⁻⁴</td>
<td>1000</td>
<td>3200</td>
</tr>
<tr>
<td>6b.</td>
<td></td>
<td>170 × 10⁻⁴</td>
<td>1150</td>
<td>640</td>
</tr>
<tr>
<td>7.</td>
<td>Olate</td>
<td>5 × 10⁻⁴</td>
<td>760</td>
<td>900</td>
</tr>
<tr>
<td>8.</td>
<td>Salicylate</td>
<td>7 × 10⁻⁴</td>
<td>150</td>
<td>280</td>
</tr>
</tbody>
</table>

The amount of secretion as well as the dyestuff concentration falls off. But the effect is reversible. After switching over to the original perfusing solution, the former secretory power reappears. On the other hand, supplying the liver with a substance belonging to the second group mentioned produces an increase in dyestuff concentration as well as in volume of secretion (Exp. 6 to 8), but only when the concentration is kept below a certain value. Otherwise irreversible decrease of dyestuff concentration results (Exp. 6 b).

Before enumerating more fully the organic compounds, which appear to belong either to one group or to the other, we wish to characterise these two groups not only according to their physiological effects, but also in terms of their physico-chemical properties, which seem to be correlated with their physiological behavior. The group of organic compounds disclosing an inhibitory effect on the liver activity comprises compounds with a strong affinity for water. For this reason, they are surface inactive and they produce dehydration and condensation of hydrophilic colloids like those in the cell surface and interior by competing with them for water. Such substances are the sugars, the polyhydric alcohols, amino acids, the sodium salts of bi-
valent aliphatic acids and oxyacids. In contrast, the secretory power of the liver seems to be increased by compounds which show, more or less, the opposite type of behavior. These substances, though water-soluble and hydrophilic, are likewise organophilic and hydrophobic. Their molecules have polar configurations, one part showing affinity for organic substances, the other part for water. Therefore, these substances are surface active, i.e. they are adsorbed at the interface between an organic structure and its environment. Such an interface would be located, for example, between the organic colloids forming the cell body and water. When the affinity for water is sufficiently strong, the colloidal structure is loosened instead of being condensed and tightened as in the former case. Substances belonging to this type are the salts of bile acids, of higher fatty acids, of salicylic acid, or the carbamates or saponin.

Briefly, our experiments on the liver suggest as a working hypothesis that the secretory activity of the liver is in some way dependent upon the colloidal condition of the active structures, which can be altered in one or in the other direction by the two groups of substances bringing about either functional improvement or functional impairment, either a choleretic or an anticholeretic effect.

Of necessity, this is a rather vague statement, since, at the present stage of our investigation, we do not feel in a position to be more definite. In order to test further this suggestion that colloidal processes may underlie the physiological effect of the organic compounds, it seemed worthwhile to extend the investigation to other physico-chemical and physiological systems, since approximately the same colloidal material is present in all cellular structures. As an example of a purely physical system, a model experiment was designed using emulsions of lecithin, a constituent of all cell stromata. The influence of the same two groups of organic compounds on the degree of dispersion of these sols was investigated. Turning to physiological material,
parallel studies were conducted upon the electric manifestations of nerve and muscle. Since the injury and action potentials are generally believed to arise from alterations of the colloidal membranes, structures essential in these physiological reactions, we have studied the ability of our compounds to produce muscle and nerve injury potentials. The following summary of the results (in Tables 3 and 4) shows to what extent the different processes supposed to correspond to each other really appear to be comparable.

### Table 3

**Substances with Increasing (+) Effects**

<table>
<thead>
<tr>
<th>Substance</th>
<th>Dyestuff Secretion</th>
<th>Injury Potential</th>
<th>Dispersity of Leisthin</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Liver</td>
<td>Muscle</td>
<td>Nerve</td>
</tr>
<tr>
<td>Glycocholate</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>Taurocholate</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>Oleate</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>Caprylate</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>Heptylate</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>Nonylate</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>Salicylate</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>Codeine</td>
<td>+</td>
<td>+</td>
<td>-</td>
</tr>
<tr>
<td>Cocaine</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>Novocaine</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>Veratrine</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>Butylalcohol</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>Amylalcohol</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>Heptylalcohol</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>Propylurethane</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>Butylurethane</td>
<td>+</td>
<td>+</td>
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<td>Saponine</td>
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Obviously, the results are in fairly good agreement with the concept that we are dealing in the liver experiments with alterations of the colloidal state of the structures concerned, since the augmenting as well as the inhibiting effects of the compounds in question are paralleled by cor-
responding effects on the lecithin suspension and since the muscle and nerve potentials, which frequently have been considered as due to an increase of membrane permeability to ions, are likewise brought about by the corresponding group of chemicals.

Therefore, summing up, we come to the conclusion that the characteristic ability of the liver to concentrate dye-stuffs in its secretion to an extraordinary degree, could be regarded as based upon the adsorption on special carriers inside the cells and that entrance and transport are either increased or decreased accordingly as the condition of the cell colloids is altered either in the direction of a higher or a lower dispersion.

Citations

THE NEUROMOTOR SYSTEM OF ANOPLOPHRYA LUMBRICI, A DEGENERATE CILIATE, FROM THE EARTHWORM OCTOLASIUM CYANEUM *

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ABSTRACT

1. The neuromotor system of Anoplophrya lumbrici (Schrank), a simple astomatus ciliate from the intestine of the lumbricoid earthworm Octolasmum cyaneum (Savigny) consists of 60 longitudinal rows of cilia, 20 on the ventral surface, 30 on the dorsal, and 5 more closely-set ones on each lateral margin. Ciliary rootlets below the basal granules of the ciliary rows connect with 60 primary longitudinal fibrils all of which join the anterior sutural fibril which crosses the anterior end of the ventral surface, and at the relict cytoplasmic depression passes internally and joins the motorium. From the angles of the motorium two other fibrils enter the endoplasm. A secondary set of 60 more superficial longitudinal fibrils lies between the primary ones and is joined to them by cross commissures. The motorium lies in the region where the ancestral pharynx may have been located.

2. The cilia beat in regular synchronized metachronic waves, propelling the organism with relatively great speed.

3. The anterior sutural fibril and primary longitudinal fibrils are interpreted as functioning in transmission of impulses initiating ciliary movements in the lines of cilia, while the secondary longitudinal fibrils and cross commissures provide for coordination which results in the orderly progression of waves of ciliary action.

4. In an early stage of binary fission the ciliary lines are crowded inward at both lateral margins at the future plane of division; a process of multiplication of the basal granules occurs on both sides of this plane, and an ectoplasmic wall of demarcation appears which later grows inward forming the plane of separation between the two daughters.

5. The neuromotor system of Anoplophrya may have arisen from that of the simple stomatous ciliates by degeneration of all of the fibrillar complex of the pharyngeal region except the connection of the motorium with the sutural fibril.

* Grateful acknowledgment is made to the Penrose Fund of the American Philosophical Society for a grant-in-aid. Acknowledgment is made also to the Works Progress Administration for assistance rendered through Project No. 6079–5797.

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Introduction

Since the time when Sharp (1914), under the direction of the senior author, demonstrated the presence of a coordinating fibrillar system in *Diplodinium*, a series of investigations has shown that such a structurally integrated neuromotor system is characteristic of a number of diverse groups of ciliated Protozoa, and presumably of all Ciliata. Most of these investigations have been concerned with the more complex genera. It is, therefore, a matter of considerable interest to determine whether the simpler representatives of the ciliates also have a neuromotor system, and, if so, what relation it bears to that of the more highly differentiated and diversely specialized genera. This paper deals with a holotrich of simple type the simplification of which has resulted from a parasitic mode of life.

The parasitic mode of life tends to the simplification of the structural differentiation of any persisting organ systems by reduction in numbers of repeated elements and restriction of structural modifications among parts of series of organs. It may even go so far as to eliminate wholly an organ system or an unused part of it, or to retain it in some stage of differentiation or reduction. In conformity to these processes of change induced by the parasitic mode of life the astomatous ciliates, some of which, such as *Anoplophrya*, live in the fluids of the digestive tract of earthworms and absorb food from the medium in which they are immersed, exhibit the loss of the pharyngeal organelles consisting, in other holotrichs, of specially differentiated cilia and even membranelles the function of which in the ancestors of the astomatous ciliates was to pick up organisms such as bacteria and pass them into and down the pharynx into the endoplasm, often shaping them into food balls at the inner end of this circumscribed route. Cilia serving these functions, their structural differentiation, and their specialized groupings have been described by Lund (1933) in *Paramecium*, and Rosenberg (1937) in *Nyctotherus*.
Since Anoplophrya has neither mouth nor pharynx it is a matter of interest to find out how far its parasitic mode of life has reduced the pharyngeal apparatus and the motorium, found near the pharynx, and from which fibrils are sent out to the pharyngeal apparatus and to other parts of the ciliary mechanism.

As material for attacking this problem a relatively simple astomatous ciliate, Anoplophrya lumbrici (Schrank), has been chosen by us. However, the simple structure of this ciliate is undoubtedly to be regarded as secondary, since this species appears to have been derived from an ancestral holotrichous form with a cytostome and pharynx. This makes the problem doubly interesting, for if this hypothesis as to ancestry is correct it is to be expected that some vestige of a more complex ancestral organization of the neuromotor system may be carried over in some stage or stages of degeneration.

The only previous investigation of the neuromotor system of astomatous ciliates is that of Bush (1934) on Haptophrya michiganensis Woodhead, parasitic in the intestinal tract of the four-toed salamander. This ciliate has a well-developed sucker functioning for attachment, and a complex neuromotor system with parts homologous to those described in stomatous ciliates, including a motorium, fibrillar ring and connecting fibrils. Cépède (1910) has shown that the astomatous ciliates are not monophyletic in origin. Thus, the simple Anoplophryidae are not closely related to the more complex Haptophryidae, although both are astomatous ciliates.

Sarcely any work has been done on the neuromotor system of the Anoplophryidae except that of Tchang (1931) who gave a brief description of the superficial fibrillar system of Anoplophrya brasili Leger and Duboscq. Cheissin (1930) figured a silver impregnation preparation of Mesnilella rostrata Rossolimo of the related family Hoplophryidae. No other work on the fibrillar system of the Astomata is known to us.
Because of the relatively simple structure of the astomatous ciliates their systematics has been in a state of confusion. The latest extended work on this subject is that of Heidenreich (1935). This author places in synonymy with *A. lumbrici* (Schrank) several later described species of *Anoplophrya*, including *A. alluri* Cépède, *A. complanata* Rossolimo, *A. marylandensis* Conklin, and *A. sp.* Eksemparskaja. He considered the commonly accepted name *A. striata* Dujardin as a synonym of the earlier *A. lumbrici* (Schrank), which name will be used in this study.

**Methods**

*Anoplophrya lumbrici* (Schrank) was found in abundance in *Octoplasium cyaneum* (Savigny), a species of earthworm collected from the banks of Strawberry Creek in Berkeley, California. The earthworm was identified by Luther C. Altman, of Seattle, Washington, to whom we wish to express our appreciation. *Anoplophrya lumbrici* has not been previously reported from this host, although it occurs in several other genera of the Lumbricidae.

The ciliates occur in the anterior part of the intestine in considerable numbers. Seventeen of 21 or 81 per cent of the worms of this species examined were infected with *Anoplophrya lumbrici*, both immature and mature worms being infected, as a rule. The very small worms, below two inches in length, were usually free from infection.

In studying the living ciliates a small amount of physiological saline was added to the digestive fluid containing them. In preparations within vaseline rings they live for several hours. Neutral red was used as a vital stain in studying the morphology of the living organisms.

For permanent preparations smears of the intestinal contents were made on cover slips, and fixed in Schaudinn’s, Bouin’s, Flemming’s, Champy’s, or Susa’s fluid. Heidenhain’s iron-hæmatoxylin was used as a stain in either aqueous or alcoholic solution. Klein’s (1926) silver method and a modification of Gelei’s wet silver method
(Hammond, 1937) were used for silver impregnation preparations. Material fixed in Schaudinn's or in Flemming's fluid was stained with aqueous iron-hæmatoxylin for examination.

**General Morphology**

*Anoplophrya lumbrici* is elongate-ovoidal in shape and flattened dorso-ventrally (Pl. 1, Fig. 5). One hundred specimens averaged 54.1 μ by 22.3 μ with a range from 76.0 by 28.5 to 39.9 by 16.1. The body is widest in life towards the anterior end, but in fixed specimens nearer the middle. The anterior end is more narrowly rounded than the posterior one. Although the organism is relatively simple in structure it has differentiated dorsal and ventral surfaces, and right and left sides. The ventral surface is slightly concave and has fewer rows of cilia than the convex dorsal surface. The anterior end is a trifle wider than the posterior and distinguished by the presence of the suture of the ciliary lines (Pl. 1, Fig. 6; Pl. 2, Fig. 1). The three contractile vacuoles are on the left side of the body and the micronucleus on the right. The cilia are fine, very numerous and arranged in closely-set longitudinal rows. They are relatively long and nearly uniform throughout, with an average length of 11.6 μ, nearly half the transdiameter of the body.

In fixed material the ectoplasm consists of a relatively thick peripheral homogeneous layer lighter in appearance than the endoplasm and separated from it by a distinct boundary (Pl. 1, Fig. 1). There is a slightly denser or more deeply staining region near the posterior end of the suture line at the region where the cytostome and pharynx occur in stomatous ciliates. In life a slight depression in this region may be detected. The endoplasm varies in appearance according to the fixation used and the condition of the organism. Thus it ranges from a homogeneous condition relatively free from visible inclusions to a reticular or alveolar structure with numerous formed bodies in it.
The macronucleus is rod-shaped, lies in the sagittal plane, and extends through three-fourths of the total length equidistant from the two ends (Pl. 1, Fig. 5). In fixed material it has an irregular angular contour and is surrounded by a clear zone. Since these features do not appear in living organisms they must be an artifact of the process of fixation. The micronucleus is a relatively small, ovoidal, deeply staining structure located midway of the length, but displaced toward the right side near the margin.

The contractile vacuoles are located on the left side of the body near the margin. They are usually three in number, although just prior to division they increase to from four to six, and each daughter schizont has two or three after fission. In physiological saline solution the contractile vacuoles do not discharge their contents but gradually increase in size until the death of the organism. Each vacuole has a short, clearly defined canal leading to the excretory pore on the left dorsal surface (Pl. 1, Fig. 3). In wet silver preparations these pores can be plainly distinguished. The pores of the different vacuoles are all situated in one parasagittal plane between the same two rows of cilia, or in the two adjacent intervals.

Movements

*Anoplophrya lumbrici* lives in the viscous fluid content of the intestine of the earthworm and to make its way through this medium a powerful system of locomotor organs is required. The length and abundance of cilia are factors meeting this requirement. In physiological saline the organism moves very rapidly, but naturally moves more slowly in the denser medium of the undiluted digestive fluid.

The direction of progression is usually straight forward in line with the major axis, either with or without rotation. The rotation may be either clockwise or anti-clockwise. A peculiar mode of locomotion occurs occasionally, at which times the organism changes direction and advances for a
short distance in a sidewise fashion, possibly a relict or modified reaction. For study of the mechanism of movement the organisms were retarded by sodium amytyl and by the preparation of smears several hours before use so that the movement was not so rapid as in freshly prepared mounts. The organisms are much more sensitive to sodium amytyl than are free-living forms such as Spirostomum teres and Euplotes patella.

Fauré-Fremiet (1908) in studying A. striata observed that the cilia move in metachronic waves comparable to those of the cilia of epithelial cells. This was confirmed by Cépède (1910) in his studies of A. alluri. He found in organisms slowed down prior to death that definite ciliary waves could be seen progressing from the posterior to the anterior end of the body. Eksemplarskaja (1931) also noticed this wave-like motion of the cilia. Because of the length of the cilia this shows very beautifully in the species we have examined.

**Neuromotor System**

This system consists of the cilia with their basal granules, the superficial fibrillar system consisting of the primary and secondary longitudinal fibrils and their cross commissures, the sutural fibril, and the deeper lying motorium with pharyngeal and internal projections, indicative of fibrillar organs. The rows of cilia are about 60 in number arranged longitudinally over the entire surface of the body. The number was determined by a count of rows on specimens prepared by the wet silver method, by which means the basal granules of the cilia are made clearly visible along all the ciliary lines. The rows of cilia are parallel, converging towards the ends of the body. They fall into four regions as to spacing, namely the dorsal, ventral, and two lateral regions. The dorsal region has 30 equidistant rows and the ventral 20 in about the same space. In the two lateral regions there are 5 more closely set rows (Pl. 2, Figs. 2, 3 and 6) along each lateral angle. These are
on the margins of the slightly concave ventral surface and are adaptations for attachment to or gliding over the substrate formed by the intestinal cells of the host. A comparable adaptive differentiation of ciliary lines on dorsal and ventral surfaces occurs in Parachaetina myae (see Kofoiid and Bush, 1936), a parasitic ciliate from the gills of the clam, in which the cilia of the ventral surface are much shorter but the lines more closely spaced than on the dorsal region.

It was impossible to determine whether or not the number of rows is uniformly constant in all individuals because of the difficulty of counting them on the margins of the body. It would be expected that, since at binary fission each daughter is supplied with the same number of rows as the parent, the number of rows would be uniform in all individuals of the species. In confirmation of this expectation, six individuals which were more favorable for counting showed exactly 60 rows of cilia. However, on some other less favorable specimens there was a variation of one to three rows in our count which may have been due to the above-mentioned difficulty in counting all of the rows.

Conklin (1930) in investigating Anopllophrya marylandensis which Heidenreich (1935) later assigned to the species lumbrici, counted 31 to 45 rows of cilia in five specimens examined. This is a lower number and a much greater range of variation than we obtained with the ciliates with which we were working. In haematoxylin preparations the rows are not so clearly differentiated as in silver preparations, a condition which might lead to more seeming differences in number. There are also the possibilities of specific differences and that Heidenreich was in error in including Conklin's species in A. lumbrici.

The sutural line near the anterior margin of the ventral surface of the body is the region at which all of the 60 rows of cilia originate. Each one of these rows, with the exception of the five closely-set ones on each lateral margin, continues without interruption around the posterior end of the
body, turning anteriorly to end again at the suture. The five lateral rows on each side pass posteriorly from the anteriorly located sutural line and end near the posterior end without turning anteriorly or acquiring any other terminal connections. To the right of the median line the suture makes a rather abrupt sigmoid curve posteriorly and then continues to the right margin of the body, resulting in a small elongated comma-shaped area of the ventral surface being covered by lines continued over from the dorsal surface (Pl. 1, Fig. 6). Near the right end of this area, namely, the right anterior ventral surface of the body, occurs, a shallow depression, visible clearly only in living organisms. This depression was described by Heidenreich (1935c). In our studies it does not appear so deep nor so extensive as described by him. He regarded it as an aid in attachment. We interpret it morphologically as the remnant of the ancestral cytostome but have no evidence that it serves as a sucker for attachment. The ciliary lines on the ventral surface are bent slightly toward the right in the left anterior region of the body, and this is correlated with the presence of the depression in this area. The relations of the anterior sutural fiber, the overlapping of the dorsal fibrils onto the ventral surface, the dextral curvature of the anterior ends of the ventral longitudinal fibrils, the denser cytoplasm, and the superficial depression all combine to support the view that this is the region where the cytostome of the ancestral holotrichous ciliate was located.

Superficial Fibrillar System

The surface of the organism is transversed by alternate longitudinal ridges and furrows. The longitudinal rows of cilia coincide in position with the furrows, from the bottoms of which the cilia arise. The basal granules of the cilia are located within the layer of ectoplasm. From the basal granules ciliary rootlets extend inward to join the subjacent primary longitudinal fibril situated just periphe-
rally to the boundary between ectoplasm and endoplasm (Pl. 1, Fig. 1). Cépède (1910) saw the basal granules and ciliary rootlets of *Anoplophrya alluri*, but showed them as passing inward to the endoplasm with no further connection. The fibril connecting the basal granules in the same longitudinal row will here be designated the primary longitudinal fibril. This shows indistinctly in the silver preparations but is especially well differentiated in the Flemming-haematoxylin preparations, in which it appears as a distinct line beneath the basal granules. In these preparations the ciliary rootlets connecting the basal granules to the primary longitudinal fibril are also clearly seen. In silver impregnations the superficial structural elements of the organisms are well shown. In these methods, particularly in the drying method of Klein, the pattern of the surface contour of the organism often influences the results obtained (Lund, 1933). The contour and contour lines must therefore be taken into consideration in interpreting these preparations. The wet silver preparations show a more regular pattern of superficial structure than do the dry silver ones because the distortion from drying is eliminated.

With the silver impregnation methods the primary longitudinal fibrils are only indistinctly differentiated, but the secondary longitudinal fibrils are more clearly shown. These run between each two adjacent primary ciliary rows with their primary fibrils. Cross commissures connect each secondary fibril with the primary fibrils on either side of it (Pl. 1, Fig. 4; Pl. 2, Figs. 2–6). They are somewhat irregular in occurrence. On the right side of each secondary fibril there are cross commissures connecting it with the primary fibril at intervals of from 2 to 4 basal granules. On the left side these commissures are only about half as frequent. The secondary fibril does not run down midway between ciliary rows, but slightly to the left of the middle. It is significant that the convex ridge which occupies this
area is also slightly to one side, and thus coincides in location with the secondary fibril. The secondary fibril and cross commissures are therefore related to the surface contour and for this reason they are shown to better advantage with the silver impregnation methods than are the primary fibrils. The superficial fibrillar system is visible also in specimens stained by the alcoholic hæmatoxylin method.

The primary fibrils, because of their position and relation to the cilia, undoubtedly function in the coordination of the sequences of ciliary movement within each of the longitudinal rows. The cross commissures and secondary fibrils taken together serve to connect the adjacent rows of cilia through the primary fibrils and thus are morphologically suited for the function of transverse coördination of the ciliary movements along the ciliary lines in successive waves. The secondary fibrils are related to the pattern of surface contour inasmuch as they follow the ridges between rows of cilia, but their primary relationship is with the fibrillar system through the cross commissures.

The anterior sutural fiber crosses the anterior end of the body on the ventral surface in a flat sigmoid curve, which curve originates on the right margin at the region of the degenerate cytostomal depression. The fiber terminates on the left margin where it is joined by the group of 5 crowded lateral longitudinal fibrils. All of the primary longitudinal fibrils take their origin from this fibril. Those of the ventral surface pass immediately posteriorly. Those of the dorsal surface arise from the sutural fibril on the ventral surface, pass anteriorly to the anterior margin where they turn over its edge to continue posteriorly (Pl. 2, Fig. 1). Connection of the superficial fibrillar system with the motorium is established by a continuation of the sutural fibril transversely through the endoplasm to the deeply placed motorium. Since this fibril lies beneath the surface of the organism it does not show in the silver preparations. Also, because of its deep-lying position it shows well only in
favorably differentiated hæmatoxylin preparations. This anterior sutural fibril connects with all of the primary longitudinal fibrils, and thus is interpreted as having the function of coördination of ciliary movements of the organism.

The motorium is an elusive structure detected only in specimens stained in iron hæmatoxylin and decolorized with care. It cannot be demonstrated by any silver method and its absence or questionable presence in some hæmatoxylin preparations may be due only to artifact. There is a possibility of varying degrees of degeneration but it does not seem possible to establish this. When demonstrable it is a small deeply staining subquadrangular body, placed transversely, embedded in the endoplasm at the level of the cytostomal depression and of the right end of the sigmoid curve of the anterior sutural fiber. It lies deep in the endoplasm at the right and above the anterior end of the macronucleus. This is precisely the region where the ancestral pharyngeal wall would have been located and the motorium would then lie at some point on this wall, presumably near the inner end of the pharyngeal cavity.

From the angles of this deeply staining structure three distinct fibrils can be traced. The subtransverse one from the right lower angle runs to the right margin in the region of the cytostomal depression (which is obliterated in fixation) where it is continued as the anterior sutural fibril. No fiber has been demonstrated emerging from the upper right angle. The two angles of the left end of the motorium each give rise to fine fibers which are quickly lost in the endoplasm. These two structures are homologous to the "postpharyngeal bundle" and other unnamed small fibrils entering the endoplasm from the motorium in Nyctotherus hyle and the anterior sutural fibril to some one or several of the pharyngeal fibers of the complex pharyngeal system of that species (Rosenberg, 1937).

As is to be expected, the neuromotor system of Ano-
plophrya is much simpler than that of the more complex Protozoa. It consists only of the motorium, the anterior sutural fibril to which are attached the primary longitudinal fibrils running along the ciliary rows and serving to link the cilia in an integrated system, and two endoplasmic fibrils. The complex pharyngeal fibrillar system, in structural relationship with the pharynx, which we might expect the stomatous ancestor to have had, has degenerated along with the cytostome, leaving as a distinct structure only the anterior fibril and the motorium to function as central coördinating elements. The motorium persists in the endoplasm at what may have been its location in the ancestral holotrichous ciliate.

The simplification of structure as a result of the parasitic mode of life has progressed further in Anoplophrya than is the case with Haptophrya, which has a relatively complex neuromotor system (Bush, 1934) associated with a sucker for attachment. Another possible explanation of the difference in complexity is that the free-living ancestral form from which Anoplophrya is derived was simpler in structure than the ancestor of Haptophrya, and neither Anoplophrya nor its ancestor evolved a sucker as an organ of attachment.

The presence of a simple neuromotor system in Anoplophrya is correlated with its lack of diversity in modes of progression. Such forms with complex neuromotor systems as Euplotes have a series of diversified and uniformly adhered to types of movements. On the other hand, Anoplophrya with its simple neuromotor system, as would be expected, has no such consistently followed and varied patterns of movement.

**Binary Fission**

During division the macronucleus is halved between the two daughters without undergoing any demonstrated elaborate process of reorganization. Heidenreich (1935b) has
described in astomatous ciliates such a process which is not necessarily connected with binary fission. This process involves a series of divisions of the micronucleus and a replacement of the old macronucleus by one of the products of these divisions. According to the description of Heidenreich the products of the last micronuclear division fuse again to form the definite micronucleus. The whole process is termed parthenogenesis by him.

Although it was not possible in the present study to follow out completely, the process of reorganization in *A. lumbrici*, several stages showing, respectively, the early stage of the new macronucleus, its growth to the definite size, and the breakdown of the old macronucleus, were seen. This portion of the process of reorganization observed by us conforms to the description of Heidenreich.

This process in *Anoplophrya* and related forms is peculiar in that it is not of necessity linked with binary fission, encystment, or conjugation as is the case with most of the other ciliates which have been thoroughly investigated (Hammond and Kofoid, 1937). However, this process of reorganization may be considered to be a basic phase of the life cycle in itself, somewhat comparable to endomixis. Since there is apparently no reorganization of the macronucleus during binary fission the organism must undergo a gradual senescence until the resorption or “death” of the old macronucleus and its replacement by a new one during the process of reorganization (Kofoid, 1923, 1935). The micronucleus undergoes a form of mitosis, the details of which were not worked out in this study.

In the process of binary fission provision is made for division and consequent doubling in number of the neuromotor organelles. The first evidence of the series of changes by which this is accomplished is a rearrangement of the fine ciliary lines at the lateral margins of the body. At the site of the future plane of division, the transverse break occurs across the middle members of these two
groups of closely placed lateral ciliary lines and the adjacent lines then bend in toward the clear area thus developed (Pl. 1, Fig. 4). The effect is similar to what would occur if a pinching-in had taken place at this region. This change is the beginning of the rearrangement of ciliary lines to form a posterior end for the anterior daughter and an anterior one for the posterior daughter. The transverse break in ciliary lines then develops around the entire circumference of the organism at the site of the future plane of division. This results in the formation of a transverse furrow which marks the place at which the daughters will be separated.

The nature of this line of demarcation is of considerable interest. In hematoxylin preparations it appears as a distinct and continuous deeply staining transverse line. This line was noticed by Cépède (1910) during binary fission in *A. alluri*, by Rossolimo and Perzewa (1929) in *Protoradiophrya* and by Eksemplarskaja (1931) in *Anoplophrya* sp. In wet silver method preparations a furrow in the ectoplasm is visible running transversely through the plane at which the ciliary lines are broken. This break in continuity of the ciliary lines is made more apparent because the free ends of the ciliary lines anterior to the break are displaced toward one side. It is possible that the line represents a condensation of the ectoplasm, a condition due to local contraction.

In the later stages this ectoplasmic depression grows inward into the endoplasm as a wall, forming an incomplete partition between the two daughters. In stages just prior to separation of the daughters this extends inward almost to the macronucleus.

This formation of an ectoplasmic plane of demarcation and its ingression as a partitioning wall resembles the localized cortical ingrowth in division of amphibian eggs (Schechtman, 1937). The ectoplasm of the ciliate might be compared to the cortex of the Metazoan egg. Although
details differ, there is in both cases growth from the surface inward along the plane of division. Schechtman assigns this growth to the category of sol-gel phenomena.

In the process of binary fission in ciliates two new sets of locomotor systems are organized with the inclusion of the continuously functioning parental system on either side of the plane of fission. The cilia are increased in number before, during, and after binary fission so that the two new individuals resemble the parent in respect to this system of organelles, although both the transmitted ancestral parts and the newly organized parts have certain differences in relations in the two schizonts in polarity (as a whole), in morphology, and in function. In *Anoplodphrya* the process of increase in number of cilia occurs chiefly, if not entirely, during the process of binary fission. In wet silver preparations it is seen that, during the first stages of this process, the ciliary lines on either side of the future plane of division become noticeably thicker due to increase in fibrillar substance. Slightly later the basal granules of the cilia in this region show a zigzag arrangement indicating a process of multiplication and a pressure of expansion of the fibrillar structures. It is of interest that the process of multiplication of cilia is localized in the middle region of the body as in *Paramecium* (Gelei, 1934) and in the sensory bristles of *Euplotes* (Hammond, 1937).

The questions as to what extent, if any, dedifferentiation occurs in the ancestral neuromotor system and if so how the motorium and sutural line behave during binary fission remain to be solved.

It is a matter of great interest that the motorium, the fibril from it which integrates the entire ciliary system including the peripheral fibrillar net, and the endoplasmic fibrils persist after the cytostome, pharynx and its specialized fibrillar ciliary structures have entirely disappeared. This persistence is indicative of the fundamental integrative function and morphological significance of the neuromotor system and especially of its motorium.
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**EXPLANATION OF PLATES**

**PLATE I**

All figures of *Anoplophyra lumbrici* (Schrank) drawn with camera lucida.

Fig. 1. Diagrammatic sketch of structure of the cortical layer of cytoplasm. The outer lighter layer represents ectoplasm, in the middle of which are embedded the basal granules of the cilia. From the basal granules ciliary rootlets extend inward to the primary longitudinal fibril lying just peripheral to the boundary between ectoplasm and endoplasm. × 2460.

Fig. 2. From wet silver method preparation of an individual in the early stage of binary fission showing the rearrangement of the cilia in the five lateral rows at the level of the future plane of division. × 1140.

Fig. 3. Outline showing contractile vacuoles with their excretory canals leading to the pores on the dorsal surface of the organism. × 820.

Fig. 4. A portion of fibrillar system showing rows of basal granules, primary longitudinal fibrils connecting them, secondary longitudinal fibrils running between the rows of the basal granules, and the cross commissures. × 820.

Fig. 5. Dorsal view of a specimen fixed in Champy’s fluid and stained with iron-haematoxylin, showing contractile vacuoles, micronucleus, and macronucleus. × 820.

Fig. 6. Ventral view of anterior end of a specimen fixed in Champy’s fluid and stained with iron-haematoxylin. Anterior sutural fibril and primary longitudinal fibrils of the dorsal and ventral surfaces which connect with it are shown, the former only as they come over upon the ventral surface. × 1640.

Fig. 7. Ventral view of anterior end from a specimen fixed in Schaudinn’s fluid and stained with iron-haematoxylin. Just anterior and laterally to the macronucleus is the motorium with a fibril connecting it at the right with the anterior sutural fibril. There are also two fibrils given off from the left end of the motorium which end blindly in the endoplasm. × 1640.
Plate II

Photomicrographs of *Anoplophrya lumbrici* (Schrank), taken by Mr. J. E. Gullberg.

Fig. 1. Photomicrograph of dry silver method preparation showing suture at anterior end. $\times 550$.

Fig. 2. Photomicrograph of dry silver method preparation showing basal granules of the cilia, primary and secondary longitudinal fibril, and cross commissures. $\times 825$.

Fig. 3. Photomicrograph of dry silver method preparation showing basal granules of cilia, primary fibrils and lateral region of five crowded ciliary rows. $\times 825$.

Fig. 4. Same as fig. 2, at a higher magnification. $\times 1262$.

Fig. 5. Photomicrograph of dorsal surface in wet silver method preparation. Note the greater regularity of pattern of the fibrils and rows of basal granules. $\times 825$.

Fig. 6. Photomicrograph of ventral surface of specimen in wet silver method preparation. Note five closely-placed lines of cilia at either side of the 20 ventral rows of basal granules of cilia. $\times 825$. 
THE BLOOD VESSELS OF THE BRAIN SUBSTANCE IN SOME AMPHIBIANS

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(Introduced by Henry H. Donaldson)

ABSTRACT

In *Necturus*, *Cryptobranchus*, *Triturus*, and *Plethodon*, the central nervous tissue is vascularized entirely by slender loops which rise independently from the meningeal reticulum. These may branch by secondary looping but the capillaries never ramify or anastomose.

In *Ambystoma*, similar loops are associated with a very simple capillary network, some being independent of the latter, others connected with its meshes. The network is supplied by peripheral but not central arteries. *Ambystoma* is thus intermediate in this respect between the other Caudata studied and the frog.

Loops occur only in the central nervous organs.

The frog has no capillary loops and the brain and spinal cord are vascularized entirely by a spongy reticulum more complicated than that in *Ambystoma* and similar to that in other tissues. The network in the medulla oblongata and the cord is supplied only by central arteries, in the rest of the brain basal arteries preponderate but peripheral arteries also occur.

Caliber of the capillaries is large, largest in *Necturus*. It is usually larger in one limb of a loop than in the other but the loops do not differ significantly from the network in *Ambystoma* in this respect.

Evidence as to phylogeny is inconclusive but seems to favour primitiveness of the capillary loops.

In the year 1882 Schöbl published a description of the blood vessels of the brain and spinal cord of *Salamandra maculosa*, in which he reported that neither arteries nor veins enter the substance of the central nervous system, but that such vessels branch only upon the surface, in the pia mater, and send into the interior of the organs only simple, unbranched capillary loops. The same condition he found also in *Triton*, *Proteus*, *Ambystoma*, and *Mehobranchus*.

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Sterzi ('04) studied the vessels of the spinal cord, particularly in *Triton* and *Salamandra*, and confirmed fully the above statements of Schöbl. He states that the arteries of the cord break into capillaries at the surface and these suddenly turn inwards, mostly in a radial direction. Most of them reach the gray matter, then suddenly turn back without change of caliber and return to the surface close beside the arterial limb of the loop. Sometimes the limbs are twisted together spirally. Most of these loops enter from the dorsal half of the surface of the cord. The arteries and veins on the dorsal and lateral surfaces are connected only by such loops but those on the ventral aspect frequently are connected directly on the surface.

In contrast to the agreement between the two authors just quoted was the finding of Roofe ('35) in *Ambystoma tigrinum*, where he states that "The curious simple capillary loops and arterio-venous anastomoses described by Schöbl do not exist in our specimens, and his account was probably based on poorly preserved material." Herrick ('35) writes, "As described by Roofe, the arteries of the brain break up into a capillary net which pervades the nervous tissue and the choroid plexuses."

In view of the diametrically opposed nature of these statements and of the consideration that such divergent observations could hardly be attributed to differences in technique, an examination of injected brains of a number of Caudata was undertaken. As no detailed account of the intracerebral vessels of Salientia was found in the literature, a comparative study of the brain of the frog was also included.

**Material and Methods**

The species to be studied were selected so as to represent five families and four suborders of tailed amphibians besides one species of frog.

Preparations satisfactory for the purpose were obtained from the following animals, which were adults in all cases.
Order Caudata. Suborder Proteida. Family Proteidæ.
*Necturus maculosus* Raf. (2 specimens).
Suborder Cryptobranchoidea. Family Cryptobranchidæ.
*Cryptobranchus alleganiensis* (Daudin) (7 specimens).
Suborder Salamandroidea. Family Salamandridæ.
*Triturus viridescens* (Raf.) (4 specimens).
Family Plethodontidae.
*Plethodon cinereus* (Green) (2 specimens).
Suborder Ambystomoidea. Family Ambystomidæ.
*Ambystoma maculatum* Shaw (3 specimens).
*Ambystoma jeffersonianum* (Green) (3 specimens).
*Ambystoma tigrinum* (Green) (10 specimens).

Order Salientia. Family Ranidæ.
*Rana pipiens* Schreber (10 specimens).

*Ambystoma tigrinum* is represented by four specimens from Manitoba and six obtained from a Chicago dealer. The latter were received in two lots which differed markedly in external appearance. For comparison, two brains of the lamprey (*Petromyzon marinus*), several of the turtle, and various mammalian specimens were available.

The animals were killed with illuminating gas, amyl nitrite, or chloroform, or usually with combinations of these and were at once opened and injected with carmine gelatin from the heart, the aorta and brachial arteries having first been clamped or tied off. Immediate injection was
found in this laboratory to give better results than delayed injection. The pressure was gradually increased to a maximum varying in different cases from 150 to 300 mm. of mercury, 200 or 250 mm. being most frequently used.

The neck was ligated and the brain was partly exposed immediately after injection, being then at once immersed in 10 per cent formalin. Later, it was dissected out and was eventually imbedded in celloidin and cut into serial transverse sections. In the case of the frog, one series was cut coronally and one sagittally. In most series each eleventh section was either 150 μ or, more usually, 200 μ thick, while the intervening sections were 20 μ thick. These were mounted as complete series and lightly stained with picric acid.

**Observations**

*Necturus*

The internal vessels of the central nervous system of *Necturus* conform with Schöbl's description of those of *Salamandra*. They have the form of simple, elongated loops mostly disposed in a roughly radial manner (Figs. 1, 9, 10). Many, but not all, reach the central gray and penetrate it to varying depths. Some turn back soon after entering the gray matter, while others reach right to the ependyma, and very occasional vessels seem actually to pass among the cells of the latter layer.

The arrangement is essentially the same in both the spinal cord and the brain.

The two limbs of a loop always lie close together and, as observed by Sterzi, the loops are usually twisted spirally to some extent, not infrequently to that of a complete turn or even two. In these spirals, one limb usually remains relatively straight while the other is twisted round it.

The course of a loop within the tissue is not usually straight, but rather is curved, bent, or somewhat undulating. Occasional loops have even a hooked shape. As they follow these irregular courses, the distances between them
vary considerably, but they are in general scattered through the tissue in a manner approaching uniformity (apart from definite regional differences in frequency). Never do two loops approach each other so closely that an anastomosis might be overlooked on account of faulty ob-

Fig. 1. Freehand drawings of capillary loops in the brain of Necturus selected to show various degrees of twisting and branching.

ervation or defective injection. Never does a single vessel appear cut off in a section without its companion, except in the occasional cases where the extreme tip of a loop or a tangential slice of one limb is isolated. There is not a single trace of anastomosis within the substance of the central nervous organs.

Not infrequently loops are branched, as shown in figure 9. This branching is produced by the formation of a
secondary loop from one limb of a primary one. The simple twisted loops often show a rather sharp turn of one limb at a particular point and it seems probable that branching has been accomplished by an extension of such a sharp turn to form a short loop, which may then become more or less elongated. Examples of what appear to be various stages of such a process, as illustrated in figure 1, may readily be picked out. In no case was there observed a branching of the capillary composing a loop.

Not only are the capillaries within the central nervous organs all disposed in the unconnected loops just described, but that arrangement appears to be confined strictly to these organs. The capillaries in the roots of the olfactory, trigeminal, and facial nerves which appear in the same series of sections run singly, branch, and anastomose to form large-meshed, three-dimensional networks (Fig. 9). No vessels were found in the substance of the optic nerves. The neural portion of the hypophysis has a network of wide vessels which, however, is only the superficial reticulum infolded between the convolutions of the infundibular wall. The epithelial portion of the hypophysis has wide capillary vessels disposed in a dense spongy network, while the wall of the infundibulum and the most caudal part of the hypothalamus lack vessels within their substance. The rest of the hypothalamus has loops identical with those elsewhere in the brain tissue. The nodus vasculosus and the chorioid plexuses have highly developed reticula. In none of these networks were there detected any traces of loops or of anything which could be recognized as transitional between the two types of arrangement.

As quoted above, Sterzi reports that, in Salamandra and Triton, at the end of a loop the capillary turns back towards the surface without change of caliber. In Necturus, while there is not regularly a sudden change of caliber at the point mentioned, inspection indicates that the two limbs of a loop are frequently of unequal diameter. To check this, the diameter of both limbs of twenty-five loops
was measured in each of two regions in each specimen available, with the results indicated in table 1. In the material used, the medial longitudinal bundle is not distinct and as the vessels in it are not numerous no attempt was made to be sure of measuring only capillaries within this fasciculus. The data recorded under this heading in tables 1 to 5 apply to capillaries in the medial longitudinal bundle or in immediately adjacent white matter.

**TABLE 1**

**Average Diameter in µ of Capillaries in Each Limb of Twenty-Five Loops in Necturus, not Corrected for Shrinkage**

<table>
<thead>
<tr>
<th></th>
<th>Total length in cm.</th>
<th>Injection pressure in mm. of Hg</th>
<th>Primordium hippocampi</th>
<th>Medial longitudinal bundle</th>
<th>All capillaries measured</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Thicker limb</td>
<td>Thinner limb</td>
<td>Average of both</td>
</tr>
<tr>
<td>Necturus 1</td>
<td>24</td>
<td>200</td>
<td>13.8</td>
<td>10.4</td>
<td>12.1</td>
</tr>
<tr>
<td>Necturus 2</td>
<td>27</td>
<td>200</td>
<td>14.1</td>
<td>10.6</td>
<td>12.4</td>
</tr>
<tr>
<td><strong>Average</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td><strong>12.2</strong></td>
</tr>
</tbody>
</table>

From this table it will be observed, further, that the caliber of the capillaries in *Necturus* is notably large, a fact which may be clearly seen in the accompanying photographs (Figs. 9, 10).

**Cryptobranchus**

In *Cryptobranchus*, the vessels of the substance of the central nervous system are essentially the same as in *Necturus*. As in the latter, they consist entirely of non-anastomosing loops, and although the loops may branch once, twice, or even oftener, this branching is produced only by the formation of secondary loops (Fig. 2), the component capillaries never ramifying. Also, as in *Necturus*, such loops appear to be confined strictly to the tissues of the brain and spinal cord, the contrast between them and the capillary net in the attached nerve roots being striking.
In both animals, each loop lies in a single perivascular space, which surrounds the loop as a whole, not being divided to enclose the limbs separately. This would be a natural outcome of the method of development observed by Sterzi ('04) in the salamander, in which the loop as such grows inward from the meningeal net, its limbs becoming gradually more closely approximated, though unpublished studies announced by Wislocki at a recent meeting show that Sterzi's view needs confirmation.

The loops in Cryptobranchus are longer and more slender than those in Necturus (Figs. 11, 12) and the limbs of each tend to be even more closely apposed, though they may spread apart, particularly towards the end of the loop. They also seem to be less regularly twisted in the spiral manner described above. One limb frequently lies directly in front of the other, so that the loop appears as a single line in a transverse section. This is particularly common in the more ventral parts of the brain stem, and in the peripheral parts of the loops. In Necturus also one limb seems most frequently to enter the brain substance just in front of the other.

Branched loops are very much in the minority (as in Necturus also) but are sufficiently numerous for one or several to appear in each thick section. While counts were not made, the impression is received that the proportion of branched and unbranched loops is about the same in all parts of the brain and in the upper portion of the spinal cord.

The apparently greater slenderness of the loops in Cryptobranchus as compared with Necturus is partly due to smaller caliber of the capillaries themselves. Measure-
ments of diameter of the capillaries were made as in *Necturus* and are recorded in table 2.

**TABLE 2**

**AVERAGE DIAMETER IN μ OF CAPILLARIES IN EACH LIMB OF TWENTY-FIVE LOOPS IN *CRYPTOBANCHUS*, NOT CORRECTED FOR SHRINKAGE**

<table>
<thead>
<tr>
<th></th>
<th>Total length in cm.</th>
<th>Injection pressure in mm. of Hg</th>
<th>Primordium hippocampi</th>
<th>Medial longitudinal bundle</th>
<th>All capillaries measured</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Thicker limb</td>
<td>Thinner limb</td>
<td>Average of both</td>
</tr>
<tr>
<td>Cryptobranchus 2</td>
<td>♀ 43</td>
<td>200</td>
<td>7.3</td>
<td>5.6</td>
<td>6.4</td>
</tr>
<tr>
<td>Cryptobranchus 4</td>
<td>♂ 38</td>
<td>140</td>
<td>11.0</td>
<td>9.5</td>
<td>10.3</td>
</tr>
<tr>
<td>Cryptobranchus 15</td>
<td>♀ 39</td>
<td>300</td>
<td>10.0</td>
<td>8.0</td>
<td>9.1</td>
</tr>
<tr>
<td><strong>Average</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

It may be seen that the animals were considerably larger than the specimens of *Necturus* used, yet the average diameters of the capillaries are markedly smaller. Thus the difference in caliber of the capillaries can not be linked up with that in the size of the animals as can be done roughly in comparing the diameters of the vessels in small and large mammals (Luna, ’24; Craigie, ’38a).

Since the pressure used in injection was not the same in all cases, the pressure employed in preparing these particular specimens is indicated in the tables. It is evident that the difference in caliber of the capillaries in *Necturus* and in *Cryptobranchus* is not related to injection pressure, nor can the smaller diameters in *Cryptobranchus* 2 as compared with *Cryptobranchus* 4 be a direct result of this factor.

**Triturus**

The tissues of the central nervous system of the newt, like those of the two species just dealt with, are vascularized entirely by capillary loops which rise independently from a meningeal network and never anastomose. Their character and relations are identical with those in the pre-
vious cases. Branched loops are a little less frequent but are common, the branching being again entirely a secondary looping and no capillary ever being seen to ramify within the substance of the brain. Nerve roots and epithelial portions of the hypophysis have capillary networks.

Fig. 3. Drawing of a slightly oblique section 200 μ thick of the medulla oblongata of Triturus. The meningeal net on the ventral surface is shown in the lower part of the picture, with the loops running from it independently into the brain substance.

In slightly oblique, thick sections of these small brains (Fig. 3), the relation of the capillary loops to the superficial network is particularly clear. As described by Sterzi, the wide capillaries in the meninx, which follow an irregular course, suddenly turn at right angles directly into the brain substance, being somewhat constricted at the turning point. (This constriction is probably an artifact.) The other limb of the loop emerges close to the point of entry of the first one and, like it, turns at a right angle, expands to a larger diameter, and continues along its more or less sinuous path through the meninx.

The two limbs of the loop seem often to be rather less closely apposed than in Cryptobranchus but the loops are
not so coarse as in *Necturus*. The caliber of the vessels, though somewhat less than in *Cryptobranchus*, is relatively wide in spite of the small size of the brain in which they lie. Measurements similar to those reported for the previous species are presented in table 3.

**TABLE 3**

**AVERAGE DIAMETER IN µ OF CAPILLARIES IN EACH LIMB OF TWENTY-FIVE LOOPS IN *Triturus*, NOT CORRECTED FOR SHRINKAGE**

<table>
<thead>
<tr>
<th></th>
<th>Total length in cm.</th>
<th>Injection pressure in mm. of Hg</th>
<th>Primordium hippocampi</th>
<th>Medial longitudinal bundle</th>
<th>All capillaries measured</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><em>Triturus</em> 2</td>
<td>Г 8</td>
<td>120</td>
<td>7.4</td>
<td>6.0</td>
<td>6.7</td>
</tr>
<tr>
<td><em>Triturus</em> 3</td>
<td>Г 8</td>
<td>230</td>
<td>9.9</td>
<td>7.0</td>
<td>8.4</td>
</tr>
<tr>
<td><em>Triturus</em> 5</td>
<td>Г</td>
<td>150</td>
<td>9.5</td>
<td>7.5</td>
<td>8.5</td>
</tr>
</tbody>
</table>

**Average**

7.9

7.3

7.6

**Plethodon**

The lungless and entirely terrestrial *Plethodon* is probably the most highly specialized of the genera of Caudata studied (Noble, ’31). Nevertheless, the above description of the disposition and relations of the capillaries in *Triturus* is fully applicable to those of *Plethodon* also. Nowhere in the central nervous system is there any trace of branching of individual vessels or of anastomosis. One of the specimens was left surrounded by the related skeletal parts and muscles and the sections show clearly that the tissues of the central nervous system itself are the only ones in which the independent capillary loops occur. Even the sensory root ganglia have reticula. It is notable that in the midbrain and forebrain most of the loops do not run far through the gray matter but either fail to reach it or just enter its outer parts with their tips.

Measurements of diameter, as in the previous cases, are recorded in table 4 and show the average diameter of
the capillaries to be somewhat less than in the three preceding genera.

**TABLE 4**

**Average Diameter in μ of Capillaries in Twenty-five Loops in *Plethodon*, not Corrected for Shrinkage**

<table>
<thead>
<tr>
<th></th>
<th>Total length in cm.</th>
<th>Injection pressure in mm. of Hg</th>
<th>Primordium hippocampi</th>
<th>Medial longitudinal bundle</th>
<th>All capillaries measured</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Thicker limb</td>
<td>Thinner limb</td>
<td>Average of both</td>
</tr>
<tr>
<td><em>Plethodon</em></td>
<td>4</td>
<td>7.9</td>
<td>300</td>
<td>5.3</td>
<td>4.7</td>
</tr>
<tr>
<td><em>Plethodon</em></td>
<td>5</td>
<td>7.9</td>
<td>250</td>
<td>7.1</td>
<td>6.3</td>
</tr>
<tr>
<td><strong>Average</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>5.9</td>
</tr>
</tbody>
</table>

**Ambystoma**

In the material derived from two species of this genus, *A. maculatum* and *A. jeffersonianum*, no differences were detected. Particular stress was laid upon the study of *A. tigrinum*, since it was upon this species that the accounts of Roofe and of Herrick were based. As noted above, it was represented by three different strains which seemed to be alike in respect of cerebral vascularization, and to differ little from the other two species examined. The differences between *Ambystoma* and the animals of the four genera described in the preceding pages, however, were very remarkable.

In brief, all specimens of *Ambystoma* possess in the substance of the brain and in the uppermost part of the spinal cord (the only part thereof examined) a true, three-dimensional capillary network, with meshes of large size, as described by the authors cited. Contrary, however, to the statement of Roofe, capillary loops exactly like those in *Necturus*, *Cryptobranchus*, and *Triturus* are also present.

In the upper end of the spinal cord, simple capillary loops, arising independently from a superficial, meningeal plexus, exactly as in the previous genera, lie among the
large meshes of the internal net. Thus some vessels run singly, branch and anastomose, while others run in closely associated pairs, form a simple loop, do not branch, and do not anastomose.

Fig. 4. Two loops from *Ambystoma maculatum*, the left one showing the peripheral parts of the limbs widely separated, the right one a single capillary branch which anastomosed with the reticular vessels.

In the medulla oblongata, the loops are fewer in most regions, but in the cerebellum, in some portions of the tectum (Fig. 13), and in the pallium they are numerous. They occur in all parts of the brain, but no part seems to be vascularized by them alone (Figs. 13, 14, 17, 18). The large-meshed network is also present throughout. The simple loops are less numerous in *A. tigrinum* than in the other two species, being rather infrequent in the spinal cord and medulla oblongata of that animal.

Very rarely, simple branching of the loops was seen and in every case this was of the type already described, the branches being formed by secondary looping, never by simultaneous ramification of the two limbs such as has been described and illustrated for certain lizards by Schöbl ('78) and by Sterzi ('04) and for the brain of the opossum by Wislocki and Campbell ('37).
All the loops, however, are not independent of the capillary network and a few examples of inter-relation are illustrated in figures 4, 5, 14, and 17. These connections are of quite varied character. In some cases, one limb of a loop gives off a single branch capillary which anas-

tomoses with an adjacent component of the net. Again, an otherwise typical loop may arise from the side of a mesh of the internal net instead of from the meningeal vessels. Two capillaries entering the brain substance singly have been observed to come together some distance inside and form a typical narrow loop from that point onwards.

The internal capillary reticulum appears to be fed and
drained through its connections with the meningeal net all over the surface. There is no sign of central arteries or other main vessels connecting the deep and the superficial networks, and no vessels of larger than capillary size were observed within the central nervous tissue. Thus a strictly centrifugal course for the blood such as Sterzi demonstrated in the Salientia is not even suggested in the slightest degree.

![Diagram](image)

**Fig. 6. Ambystoma tigrinum.** Three single arches, each with a slender loop connected, and one loop giving off a single branch capillary to the reticulum.

The relatively simple network within the nervous tissue is not of the spongy, three-dimensional character shown by the fully-developed capillary bed of the central nervous system in higher vertebrates. Rather, it is strikingly suggestive of the conditions in the brains of chick embryos of five to six days incubation as described and illustrated by Williams ('37). The vessels entering the nerve substance run perpendicularly from the surface for a varying distance and then bend or branch to form a more or less arch-like anastomotic connection with one or more similar
vessels (Figs. 7, 8). Branching may occur successively at different depths, but more frequently the perpendicular vessel ends with its first branching. Thus the vessels are seen in sections as a series of interconnected arches, with occasional anastomoses between the stems thereof. The

![Diagram](image)

**Fig. 7.** *Ambystoma tigrinum.* Capillary network in the left primordium hippocampi, showing it to be composed largely of simple anastomosing arches. No loops appear in this section.

**Fig. 8.** *Ambystoma tigrinum.* Capillaries in a section of the right ventral thalamus and preoptic nucleus. This shows in the ventral thalamus one of the most complicated pieces of reticulum observed. Two simple loops also appear.

system as a whole forms a net roughly parallel with the surface, with numerous vessels passing between it and the meningeal network. This internal net, however, is not all in one plane for, besides the secondary anastomoses mentioned, some of the arches are in the white matter, some are near the ventricle (occasionally even penetrating between the cells of the ependyma), and others occur at all levels between these. The majority lie in or near the more peripheral part of the gray matter.
VESSELS OF BRAIN SUBSTANCE IN AMPHIBIA

The internal network does not appear to be continuous throughout the substance of the brain and cord, as it does in higher vertebrates. There occur groups of two, three, or more arches which seem to give off no branches except to each other, thus forming little independent units. Single arches, apparently quite independent, have also been observed (Fig. 6). In the upper part of the spinal cord, anastomoses across the median plane are very rare, so that the internal networks of the two sides are practically unconnected. In the brain, however, such connections are numerous and no separation of the two sides is indicated.

To try to determine whether the different arrangement of the capillaries found in *Ambystoma* is associated with any significant difference in the caliber of the vessels, the diameters of the two limbs of twenty-five loops and also the diameters of twenty-five capillaries which formed part of the reticulum were measured in the primordium hippocampi in two specimens of each of *Ambystoma maculatum*

**TABLE 5**

**AVERAGE DIAMETER IN μ OF CAPILLARIES IN EACH LIMB OF TWENTY-FIVE LOOPS AND OF SINGLE ANASTOMOSING CAPILLARIES IN *Ambystoma*, NOT CORRECTED FOR SHRINKAGE**

<table>
<thead>
<tr>
<th></th>
<th>Total length in cm.</th>
<th>Injection pressure in mm. of Hg</th>
<th>Primordium hippocampi</th>
<th>Medial longitudinal bundle</th>
<th>All capillaries measured</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><em>A. maculatum</em> 2</td>
<td>15 200</td>
<td>7.7 6.2 6.4 6.8</td>
<td>6.6 6.7</td>
<td></td>
<td></td>
</tr>
<tr>
<td><em>A. maculatum</em> 3</td>
<td>17 200</td>
<td>8.1 6.4 6.6 7.7</td>
<td>6.0 6.6</td>
<td></td>
<td></td>
</tr>
<tr>
<td><em>A. tigrinum</em> 5</td>
<td>14.7 200</td>
<td>6.1 4.6 5.6 5.4</td>
<td>5.6 5.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td><em>A. tigrinum</em> 13</td>
<td>17.7 200</td>
<td>7.6 5.5 6.0 6.4</td>
<td>6.1 6.3</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Average</strong></td>
<td></td>
<td>7.4 5.7 6.2</td>
<td>6.1 6.3</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

and *A. tigrinum*. In the medial longitudinal bundle capillary loops, while they occur, are too few to be treated separately, so only vessels running singly and presumably
entering into mesh formation were considered, fifty being measured in each case. The results, which are recorded in table 5, show that there is no significant difference between the caliber of the capillaries forming a network and those disposed as independent loops in *Ambystoma*. The caliber of the capillaries of both types is about the same as that of those composing the loops in *Plethodon* and is smaller than in any of the other Caudata studied.

*Rana pipsiens*

In the frog there is no trace of slender capillary loops such as have been observed in the Caudata and described in the preceding pages. Vascularization of the tissues of the central nervous system is accomplished entirely by a three dimensional capillary network which is considerably more complicated than that found in *Ambystoma* and differs significantly in its connections.

Sterzi ('04) reported that the spinal cords of *Rana esculenta*, *R. fusca*, and *Bufo vulgaris* are supplied with blood through central arteries which reach the gray matter and bifurcate to form two longitudinal channels, one on each side, a little ventral to the central canal. Branches from these give rise to capillaries of about 7 µ diameter which form a continuous network. In the white matter, the meshes and the capillaries composing them both increase in size towards the periphery, where the capillaries empty into superficial veins. The anterior spinal artery gives off

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**Fig. 9. Necturus maculosus.** Photomicrograph of transverse section of medulla oblongata, 150 µ thick. × 50. In the roots of nerves VII and VIII at the right of the figure may be seen the capillaries forming a reticulum, in contrast with those within the brain.

**Fig. 10. Necturus.** Transverse section 100 µ thick through the caudal part of tectum. × 40.

**Fig. 11. Cryptobranchus alleganiensis.** Transverse section through left half of medulla oblongata 200 µ thick. × 50. While the looped formation is less distinct in this photograph and the next than in those of *Necturus*, the more slender proportions of the loops and the absence of anastomosis are evident.

**Fig. 12. Cryptobranchus.** Transverse section through medial wall of right cerebral hemisphere, 200 µ thick. × 50.
only rare and small lateral branches and these do not send any vessels into the substance of the cord. Venous tribu-
taries, however, leave the cord all over its surface. Thus
the flow of blood through the substance of the spinal cord
is entirely centrifugal, passing from the central arteries to
the superficial veins.

The vessels within the Salientian brain do not appear
to have been described, but Socha ('30) in an account of
the superficial arteries and veins of *Rana temporaria* notes
that the anterior and middle cerebral arteries give off
branches into the substance of the prosencephalon, where
they break into capillaries at some distance from the sur-
face. He observes that the brain substance is very rich in
vessels.

The cerebral arteries of *Rana pipiens* correspond closely with
those of European frogs as described by Gaupp ('96), Hofmann
('00), and Socha ('30). The internal carotid artery enters the
eranial cavity (A. carotis cerebralis) and divides into anterior and
posterior rami. The former ramus passes forward on the ventro-
lateral aspect of the diencephalon and divides into a ramus hemi-
sphærii ventralis, which continues forward on the ventral surface
of the cerebral hemisphere, and a ramus hemisphærii medialis,
which passes dorsomedial to the nodus vasculosus, gives a branch
to the chorioid plexus, and then proceeds forward on the medial
surface of the hemisphere. The posterior ramus of the cerebral
carotid artery unites with its fellow of the opposite side dorsal to
the hypophysis to form the basilar artery. Also, a little farther

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**Fig. 13. Ambystoma maculatum.** Transverse section of brain passing through caudal part of tectum, thickness 100 μ. × 60. Distinct loops appear in the tectum while the capillaries in the tegmentum are mainly single and branch to form a net, of which the meshes are too large to appear in a section of this thickness.

**Fig. 14. Ambystoma tigrinum.** Transverse section through right cerebral hemisphere, thickness 150 μ. × 50. In the primordium hippocampi two long loops are clearly visible, the more ventral one being crossed by part of a mesh and a loop arising from it.

**Fig. 15. Petromyzon marinus.** Transverse section of left half of medulla oblongata, thickness 200 μ. × 50.

**Fig. 16. Petromyzon.** One unusually much-branched capillary loop in a section 200 μ thick from the upper end of the spinal cord. × 200. The central canal appears in the upper right of the picture and the ventral surface of the cord is at the bottom.
forward, a wide transverse channel runs between them, dorsal to
the infundibulum. This is not named by the authors quoted but it
corresponds with the posterior communicating ramus of Roofe’s
description of Ambystoma. In front of either end of the anastomo-
tic channel just mentioned, each posterior ramus gives off a large
superior mesencephalic artery (Gaupp) which runs to the dorsal
surface of the isthmus. From the basilar artery spring two pairs
of large vessels, the more anterior being the arteria auditiva, which
passes to the auditory organs and apparently does not contribute
to the blood supply of the brain. The more posterior branches are
the Rr. communicantes cum A. vertebralis of Gaupp. Few if any
small lateral branches to the surface of the medulla oblongata are
present. Soeha’s middle cerebral artery to the anterior surface
of the optic lobe was not recognized but a large vein occurs in this
situation, the vena diencephali posterior of Gaupp and of Hofmann
(‘01).

The tissue of the more caudal part of the medulla ob-
longata is vascularized in the same way as the spinal cord,
as described by Sterzi. The basilar artery gives off a
series of central arteries which run vertically into the brain
close to the median plane and branch in the deeper part of
the tissue (Figs. 19, 21). The chain of longitudinal anastomo-
moses in the spinal cord can be seen in coronal and in sagittal
sections to continue forward through the oblongata. Each artery branches to one side of the brain only, though
capillary anastomoses occur freely across the median
plane. The branches and capillaries stream out towards
the surface and enter the veins all over the latter, so that
the strictly centrifugal course of the blood appears to be
preserved in the oblongata until a level near the isthmus
is reached.

The posterior ramus of the cerebral carotid artery
gives off at intervals along its course branches which pene-

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**Fig. 17. Ambystoma tigrinum.** Transverse section of left ventral thal-
amus and preoptic nucleus, thickness 200 μ. × 100. Independent capillary
loops are seen in the ventral thalamus and in the preoptic area a short loop
appears originating from a mesh of the reticulum.

**Fig. 18. Ambystoma tigrinum.** Transverse section of right thalamus a
little farther forward than Fig. 18, thickness 200 μ. × 100. Numerous capil-
lar loops and parts of a few meshes are visible.
trate directly into the brain substance, and as it approaches its fellow, short superficial branches also occur, which ramify on the ventral surface and apparently turn into the brain substance in some cases. From the transverse anastomotic channel described above a pair of relatively large arteries run directly dorsad within the brain tissue and branch to supply much of the midbrain and the more caudal part of the diencephalon. Some smaller superficial branches also occur.

The superior mesencephalic artery gives off a large branch which enters the ventral margin of the optic lobe and ramifies within its deeper portion. The artery then continues, to end in numerous branches on the posterior surface of the optic lobe, into which many of them penetrate, supplying mainly the torus semicircularis, where the capillary reticulum is notably rich (Fig. 21). The superior mesencephalic artery also provides a few smaller rami to the anterior part of the cerebellum. The cerebellum is supplied mainly, however, by central arteries sweeping round the ventricle from the oblongata below it (Fig. 19).

The anterior ramus of the cerebral carotid artery near its origin gives off one or more deep branches which run medially in the plane between the thalamus and the hypothalamus ramifying to both. It then proceeds rostrad to near the caudal end of the hemisphere, where it gives rise to small branches which penetrate the diencephalon. Immediately thereafter it divides into the ventral and medial hemispheric arteries.

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**Fig. 19.** *Rana pipiens.* Transverse section of medulla oblongata and caudal part of cerebellum, thickness 200 µ. × 40. Two central arteries appear, passing to right and left sides respectively, the latter sending branches into the cerebellum.

**Fig. 20.** *Rana pipiens.* Transverse section of left cerebral hemisphere, thickness 200 µ. × 40. A branch from the medial hemispheric artery is seen entering the primordium hippocampi and dividing into dorsal and ventral rami, of which the former sweeps round to enter the capillary reticulum of the dorsolateral area. On the way it contributes to the supply of the dorsomedial and dorsal areas. The ventral ramus branches in the deep parts of the dorsomedial and ventromedial regions.
The ventral hemispherical artery sends a large internal branch into the anterior end of the diencephalon, where it divides to supply both that region and the caudal part of the hemisphere (Fig. 22). A little farther forward another branch passes into the ventral surface of the hemisphere to ramify in its caudoventral portions. The ventral hemispherical artery gives off occasional small branches to the ventral surface and one or two larger ones which penetrate directly into the anterior half of the hemisphere. It ends by ramifying over the surface of the olfactory bulb.

The medial hemispherical artery in the earlier part of its course sends branches over the lateral surface of the diencephalon and the caudal surface of the hemisphere, and from these arterioles penetrate the nervous substance. After passing the nodus vasculosus, the artery breaks into a few large vessels running forward on the medial surface of the hemisphere, gives one or two rather large branches into its substance (Figs. 20, 22), and ramifies on the anteromedial surface and within the region of fusion of the two hemispheres.

Venous tributaries emerge from all surfaces of the brain (Figs. 19–22).

From the foregoing it will be evident that the arrangement of the blood vessels in the brain of the frog corresponds fundamentally with that in the spinal cord, the blood flowing mainly in a centrifugal manner, but that, except in the medulla oblongata, the centrifugal course is not preserved in the strict way in which it appears to be in the cord.

The medulla oblongata is probably supplied entirely by central arteries. The cerebellum is fed mainly by such
vessels but also receives small tributaries through its anterior surface. The midbrain, besides receiving basal arteries as described above, has a considerable number of peripheral vessels derived from the superior mesencephalic artery and entering the more caudal part of the tectum. The diencephalon has rather extensive arterial ramifications (from the medial hemispherical artery) over its lateral surface, from which small vessels penetrate it. The hemispheres are fed mainly by basal arteries from the two hemispherical arteries but the latter also give rise to some superficial branches which eventually penetrate the medial or ventral walls. Thus, while the flow in the hemispheres is not entirely centrifugal, most of it is, and much is from the medial and ventral walls to the dorsal and lateral ones.

In the medulla oblongata, capillaries in considerable number run to the floor of the ventricle and there turn to course and ramify parallel to the ventricular surface. In this way there is formed a paraventricular capillary network of some richness, which lies partly just under the ependyma, but mainly among the cells of the latter. This group of capillaries, while continuous everywhere with the spongy capillary reticulum of the oblongata, is sufficiently richer than that in the immediately adjacent gray matter to suggest the character of a partly differentiated two-dimensional net. In contrast, the lining of the central canal of the spinal cord lacks any such vascular supply, as do also the ventricular surface of the cerebellum and most of that of the dorsal (cochlear) part of the acoustic area. Capillaries penetrate the ependyma also in the more anterior parts of the brain, and these are sufficiently numerous to form a superficial net over parts of the walls of the third

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Fig. 22. *Rana pipiens*. Coronal section of forebrain. × 40. At each side is seen a basal artery derived from the ventral hemispherical and bifurcating to the anterior part of the diencephalon and the posterior part of the hemisphere, where it branches in the gray matter of both medial and lateral walls. A fairly large branch enters each medial wall from the medial hemispherical artery. The arteries supply a continuous capillary reticulum, from which small venous tributaries leave all surfaces.
ventricle. Nowhere else, however, is the ependymal net so rich as in the floor of the fourth ventricle.

A quantitative study of the vascular supply in the brain of the frog is to be reported in a separate paper (Craigie, '38b) and it is intended also to extend such investigations to representative Caudata but for comparison with the diameters of the capillaries in Caudata the values in table 6 are presented here.

**Table 6**

**Average Diameters in μ of 50 Capillaries in the Frog (Rana pipiens), Not Corrected for Shrinkage**

<table>
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<tr>
<th></th>
<th>Body length in cm.</th>
<th>Injection pressure in mm. of Hg</th>
<th>Primordium hippocampi</th>
<th>Medial longitudinal bundle</th>
<th>Average of capillaries in five areas</th>
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<tr>
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<tr>
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<td>♀</td>
<td>6.5</td>
<td>50</td>
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<td>3.3</td>
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<td>5.1</td>
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<tr>
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<td>♀</td>
<td>6.2</td>
<td>175</td>
<td>5.9</td>
<td>7.1</td>
</tr>
<tr>
<td>Frog 124</td>
<td>♀</td>
<td>7.0</td>
<td>200</td>
<td>5.7</td>
<td>5.7</td>
</tr>
</tbody>
</table>

The figures in the table show that the average diameter of the capillaries in the frog is comparable with that in *Plethodon* and in *Ambystoma* and is a little less than in the other tailed amphibians studied. Whether or not the differences are mathematically significant, their importance is increased by the fact that the frog is a considerably larger animal than *Ambystoma, Triturus*, or *Plethodon*, which would lead to the expectation that its capillaries would be wider than those of these animals. Thus it would appear that in the amphibians the presence of a fully developed capillary net is associated with a relatively small caliber in the capillaries.

**Discussion**

The observations reported above confirm for all the animals examined except *Ambystoma* the descriptions of
Schöbl and Sterzi and are in agreement with their conclusion that the disposition of the vessels within the central nervous system is radically different in Caudata and Salientia. The observations are new, however, in the finding of an intermediate condition in the genus Ambystoma.

Wislocki and Campbell ('37) recently reported the discovery that the central nervous organs of the opossum are vascularized by non-anastomosing capillary loops essentially similar to those in tailed amphibians but differing from the latter in branching dichotomously, often many times. Further, this branching in the opossum sometimes involves parallel ramification of both limbs of a loop, instead of being accomplished only by secondary looping as in amphibians. In this it resembles the conditions in most lizards found by Schöbl ('78) and by Sterzi ('04).

Sterzi points out that the simple loops constitute a much less efficient arrangement of capillaries than a network and expresses the belief that they are the more primitive disposition. Wislocki and Campbell, on the other hand, maintain with justice that this view is open to question and that "the facts at hand could be interpreted as indicating that the condition of terminal loops is a form of specialization of the capillary bed which has arisen independently in several widely separated groups of vertebrates."

It seems highly probable that the primitive condition of the vertebrate nervous system was one in which the blood supply was entirely superficial (Craigie, '38a) and it is unlikely that the two types of internal capillary pattern originated from this independently and have both been carried through the vertebrate series. It seems more probable that one condition must have been derived from the other, but as to which is the more primitive available evidence is inconclusive. Embryological studies of Ambystoma may throw some light upon the question and such studies of certain lizards in which also Schöbl and Sterzi found intermediate conditions are also desirable.
Meanwhile, however, it may be pointed out that in the lamprey, as in *Amphioxus*, the spinal cord has only superficial vessels, presumably the most primitive state, while the brain of the lamprey has numerous independent, non-anastomosing loops. The latter (Figs. 15, 16) are exactly like those in tailed amphibians except in that they are composed of much more slender capillaries. They are usually unbranched but occasionally branch in the same way as do those of the Caudata. Figure 16 shows a loop branched to an unusual degree.

Sterzi ('07) described the conditions in both the lamprey and the hag-fish (*Myxine*) and states that the latter has a network of anastomosing capillaries in both spinal cord and brain. Towarnicki ('35) confirmed this. Thus both types of disposition occur even in the Cyclostomes, but since the brain of *Myxine* is so highly modified and since the vascularization of the spinal cord of the lamprey is more primitive than that of *Myxine* in being purely superficial (except at its upper end), the probability seems greater that the condition in the brain of the lamprey (*i.e.* the simple loops) is the more primitive.

If this be assumed to be the case, then it is possible that the looped arrangement in simple or more complicated form may have been transmitted through the stem fishes to the early Stegocephalia and from them to modern Caudata, to the ancestral reptiles, and even to the most primitive mammals. In that case a reticular condition must have arisen independently in Myxinoidea, fishes, Salientia, reptiles, and perhaps mammals. The alternative interpretation is that reversion from a reticulum to loops has occurred independently in tailed amphibians, lizards, and marsupials.

If the reticular is the more primitive condition, then loops must have been produced by parallel specialization in each of these groups and in the lampreys. It is difficult to see that such a modification can be of adaptive value or to
imagine any factor which could act to produce it independently in such diverse places in the phylogenetic series unless it is a reversion to a more primitive condition. Thus it appears to the writer that, so far as it goes, the evidence favours primitiveness of the simple loops.

**Summary**

The arrangement and caliber of the vessels in the substance of the central nervous system have been studied in sections of injected material from seven species of tailed amphibians and one species of frog. The tailed amphibians belong to five families and four suborders.

In *Necturus, Cryptobranchus, Triturus,* and *Plethodon* the central nervous tissue is vascularized entirely by slender loops which arise independently from the meningeal reticulum. The loops may branch by secondary looping, but the component capillaries never branch and anastomosis between them never occurs.

In three species of *Ambystoma* there occur scattered loops of the same character as those in the four genera just mentioned, but there is in addition a very simple capillary network. Some of the loops are independent of this, others rise from its meshes or send off single branches to anastomose with it. Both afferent and efferent vessels of small size connect the network with the meningeal reticulum at points all over the surface.

The loops in these Caudata occur only in the central nervous organs. Peripheral nerve roots and ganglia, the chorioid plexuses, the hypophysis, and the muscles have no trace of them, but are supplied entirely by reticula of anastomosing capillaries.

Capillary diameter differs markedly in the different genera but not in the loops and the reticulum of *Ambystoma*.

In the frog there are no loops like those of the Caudata even in the substance of the brain or spinal cord. These
are vascularized by a spongy reticulum which is more highly developed than that in *Ambystoma*. In the spinal cord and medulla oblongata this reticulum receives blood only through central arteries, so that the flow is entirely centrifugal. In the rest of the brain basal arteries preponderate, but there are also peripheral arteries playing a minor rôle. The hemisphere is supplied only through its ventral and medial walls, while efferent vessels leave all surfaces.

Evidence as to the phylogenetic relationship of the looped and the reticular dispositions of the capillaries is inconclusive, but seems on the whole to favour the view that the loops are the more primitive arrangement.

**Literature Cited**


EVAPORATION AND RAINFALL STUDIES IN THE NORTHWEST MINNESOTA LAKE REGION

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ABSTRACT

The evaporation and rainfall studies made in the Northwestern Minnesota Lake Region during the summer of 1935 were continued with essentially the same equipment during the spring and summer of 1936. The rainfall deviations noted in the preliminary report were duplicated to a large extent during the summer of 1936, the heavier precipitation in the vicinity of Evansville being especially noticeable. Tabulated data are given to show the wide variations of thunderstorm rainfall in stations only a few miles apart. This is in agreement with the studies of local storms made by Thornthwaite in Oklahoma.

The large additional moisture content in the air on the lee-side of lakes was again noted and additional information obtained regarding the rate of evaporation by observation of several lakes. It is shown that the heat absorption resulting from one-half inch of evaporation per day is considerably more than the total incident solar radiation on the corresponding surface; therefore, it is evident that the presence of the lake will have a decided cooling effect in addition to adding to the moisture content of the atmosphere. Radiation falling on stubble and dry soil, on the other hand, is mostly reflected and thus raises the temperature of the surrounding air. This cooling effect of exposed water surface is one of the important contributions of the lakes and ponds which the author contends should be greatly increased in number.

The investigation was made possible by additional support from the Penrose Fund of the American Philosophical Society.

A progress report of investigations on rainfall distribution in the Northwest Minnesota Lake Region for the summer of 1935 called attention to the desirability of continuing the studies in order to obtain additional data. The equipment of the preceding year being still available, a small additional grant from the Penrose Fund of the American Philosophical Society made it possible to continue the work during the summer of 1936.

* This investigation was supported by a grant from the Penrose Fund of the American Philosophical Society.

The rain gauges having been left with the observers who coöperated during the summer of 1935, as shown by the map, Fig. 1, rainfall records were begun on May 1, 1936. The recording instruments were brought from the writer’s laboratory at Lincoln, Nebraska, on July 14, which proved

one of the hottest days of the entire summer. Camp on West Battle Lake, Minnesota, was reached at about 1:00 a.m. on the 15th, and by noon of that day the microbarographs were lined up on a table for adjustment, the thermometer and hygrograph were in position, and by noon of the following day the recording rain gauge and recording anemometer at Base Station were installed ready for service. Barographs, thermographs, and hygrographs were taken to the recording stations at Pelican Rapids, Fergus Falls and Elbow Lake on the seventeenth, and the work was carried on continuously under direct supervision until August 24. The rain gauges were left in service until October 1, but recording stations with the exception of one set in charge of Prof. Lind of the high school at Fergus Falls, were discontinued on August 24.

Some changes in personnel were made necessary because of illness, death, or removal of persons who had assisted during the preceding summer. The loss of Mr. C. E. Kissenger, the U.S. Weather Bureau observer at Fergus Falls, whose advice and assistance were so valuable during the first summer, was keenly felt. His successor, Mr. H. C. Featherstone, gave every possible assistance. The death of Mr. Miller, the efficient observer at Deer Creek, and that of Mr. Haarsted, who kept very complete records at his farm between Fergus Falls and Elbow Lake, removed two other experienced men. The records at Deer Creek were kept by Mr. Max Nastansky. Mrs. Haarsted and her daughter efficiently continued the work of that station. The continued illness of Mr. Buerkle at New York Mills left the records for that station very incomplete. Mr. C. H. Weiby, who had charge of the station near Amor, was succeeded by Mr. A. E. Lindquist. The other observers continued in the same friendly and accurate manner as in the preceding summer.
Fig. 2. Rain gauge and anemometer in position.
Method of Attack

The procedure followed was similar to that of 1935. Each of the observers in charge of a rain gauge was provided with a book in which records were entered after each shower. During the time when the investigation was under personal supervision, the records were collected once each week from most of the stations. Postcard forms were provided for stations off the regular routes between recording stations. In case an observer was missed on the regular circuit, he was asked to report by postcard, the blank form provided having a space for each day of the week to make certain that no showers, however small, were overlooked. During the remainder of the time semi-monthly postcard reports were made from all stations. The recording instruments were serviced and checked for accuracy each Monday. The instruments at Base Station were checked daily, the recording rain gauge and anemometer using a twenty-four hour chart. These instruments were set away from the banks of the lake on the open sand where there was excellent exposure for rainfall, and for wind from every direction except north, in which case the trees provided considerable shelter, Fig. 2.

An attempt was again made to obtain pictures of lightning discharges, but none of unusual interest were taken. In order to get an unobstructed view of approaching storms it was necessary to go out on the open beach on the north side along the lake front. This produced the two-fold hazard of rain on the cameras and of possible injury by lightning to an observer standing on the sand a hundred yards from the nearest timber. As a safeguard in this connection, a large umbrella, such as is used on harvesters and other farm machinery, was fastened to a post set deep in the sand. Copper wires attached to the ribs of the umbrella extended along the post which reached down below the water level. The umbrella gave shelter from the rain excepting under conditions of very high winds, and the cop-
per wires should have carried to ground the major portion of any direct lightning discharge in the vicinity.

**Rainfall Distribution**

Table I gives a summary of the rainfall at the various points under observation, including the U. S. Weather Bureau stations at Fergus Falls, Alexandria, Wadena, and Detroit Lakes. The records for May were incomplete in a number of instances and have been omitted. Only those days are included in the table on which there was rainfall at one or more stations.

The large variations in precipitation, even at points separated by only a few miles, should be noted. In the previous report attention was called to the fact that Evansville had more rainfall than most of the other stations. One of the interesting phenomena of the 160 mile circuit covered each Monday was the fine corn and alfalfa between Evansville and Lake Christine. The nearest Weather Bureau stations are Fergus Falls and Alexandria. Reference to the table shows rainfall of 0.13 inches on June 30 at Fergus Falls, 0.81 at Alexandria, and 1.42 at Evansville. This same local shower gave a precipitation of 1.56 inches at the Haarsted farm eight miles southeast of Fergus Falls. During July the total for Fergus Falls was 0.05 inches, while Evansville was again favored with 1.48 inches on July 19. During August, Fergus Falls once more shows the lowest rainfall, while Wadena and Alexandria, both in the eastern part of the lake section, had much more precipitation.

The thunderstorms on August 20 were of the line-squall type with large variations in intensity at points only a few miles apart, as shown by the microbarograph records, Fig. 3. The total for two showers at Base Station was 0.53 inches, while eight miles north at Ottertail, the gauge showed 1.64 inches, and Henning, ten miles east of Base Station, received 1.15 inches. South of Henning at Vining
# TABLE I

## Rainfall Report, 1936

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<th></th>
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<th></th>
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M = report missing.
on East Battle Lake there was evidence of a much heavier rain, estimated at 2 inches. At Parkers Prairie it had dwindled to 0.95 and at Alexandria to 0.04 inches. Fergus Falls showed 0.67 inches, while the Haarsted farm eight miles away showed 2.5 times as much. Other similar variations will be evident from an examination of the table. Thus, in direct line with a report by C. W. Thornthwaite of the U. S. Soil Conservation Service,¹ it is evident that a rainfall report for the area under observation based on official records for Fergus Falls for June, July, and August, 1936, would give a very imperfect and incorrect estimate of the true precipitation.

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**Drought Conditions**

The summer of 1936 was one of most severe drought for all of Minnesota. It is interesting in this connection to study the barographic record for the week August 3–10, as shown in Fig. 4. It will be noted that the barometric pressure was practically constant during that week. There was a moderate high on Tuesday and a moderate low on Friday but very little air movement during the first part of the week. The total wind movement for the 66 hours ending at 6 A.M. on August 6 was only 105 miles. A mod-

erate low pressure area to the northwest on the 7th and 8th resulted in high winds from the south. The wind velocity reached 25 miles per hour at 10 a.m. on the 7th and continued high during the afternoon, but without precipitation. On the 9th it varied from complete calm to seven or eight miles per hour.

The graphs in the lower part of Fig. 4 show the humidity records for three of the stations during this same period. The thin, uniform line is Pelican Rapids, the heavy line, Fergus Falls, and the broken line was the record for Base Station. Pelican Rapids and Fergus Falls, being on the west side of the lake area where winds from the southwest should bring dry air from the Dakotas, it would be expected that their relative humidity should be lower than at Base Station where the instrument was located on the north shore of a large lake. This is well illustrated in the record. It will be noted that the relative humidity increased rapidly each night at Base Station, while the increase at the other stations was much less marked. During the windy conditions of Friday and Saturday the curves more nearly coincide due to the fact that the moisture added to the air by the lake at Base Station was carried away by the wind. The variations in time for the humidity peak at the three stations are largely a matter of exposure. The Fergus Falls Kiosk is on the roof of a hotel adjacent to the Ottertail River. Here the increase in humidity was quite marked in calm weather, but the curves were in almost complete coincidence during the high winds of Thursday afternoon and Friday.

Table III gives a selected set of observations showing the relative and the absolute humidity on dry, windy days at selected points. Maximum temperature differences of more than eight degrees were found between points two or three miles from a lake and those near the leeward side of the same lake. Corresponding differences in relative humidity were noted. The variations in absolute humidity, which are more important, ranged as high as 38 per cent.
It is thus seen that the lakes were adding quite appreciable amounts of moisture to the air, the moisture content of which after blowing over the parched plains of the Dakotas, was unusually low. The effect of this higher humidity in the formation of local thunderstorms was pointed out on page 755 of the previous report, where it was shown that on the basis of the Neuhoff diagram, air of approximately 20 per cent higher humidity should reach the saturation point, expanding adiabatically, at a level half a mile lower than the less humid air. In this connection it would be well to emphasize the fact that the advocates of lakes and ponds as a means of increasing rainfall under drought conditions do not contend that the construction of such water reservoirs will of itself result in thunderstorms. As Dr. C. F. Brooks has pointed out,\(^1\) thunderstorm conditions prevail with some regularity during the driest seasons over the Mississippi Valley. Light showers result from these local storms, and as Dr. Brooks said, “Give them more vapor

---

and they will rain." The shortage of rainfall in Minnesota in recent summers has been cited as proof that the large amount of water there exposed has no appreciable influence on rainfall. It should be borne in mind that the winds coming in from the Dakotas were extremely dry; that they had already dried up many of the smaller Dakota and Minnesota lakes and that they continue to take a large toll every hot day from the lakes which remain. It is too much to expect that a few hundred square miles of scattered lakes should be able to make up the large deficit in moisture content of the air which prevailed to the westward.

**Lake Levels**

Twin Lakes northwest of Amor are equipped with a level gauge under the supervision of the U. S. Geological Survey. There is a similar gauge at the west end of Rush Lake. Since Twin Lakes have neither inlet nor outlet, the gauge readings there are a good index to the extent of water losses in this area. These losses totaled 5.16 inches from May 22 to July 2, 6.48 inches during July, 1.44 inches in August, and 3.6 inches in September. The rainfall noted in Table I accounts for the smaller net loss in August. West Battle Lake may also be considered an isolated lake at the present time, as it has evaporated far below the level of the stream which normally discharges water from its west end. It lost about 7 inches during the winter of 1935–36. A gauge installed near Base Station showed a drop of 3.50 inches from July 23 to August 4 and 2.50 inches from August 4 to August 19. The Geological Survey gauge at Rush Lake showed a total loss of 10.32 inches during the month of July. The water was then below the lower index of the gauge and was not immediately reset. A gauge on the east shore of Ottertail Lake at Bay View resort showed a drop of 6.38 inches from July 22 to August 13 and an additional loss of 0.50 from August 13–24, this latter including the heavy rain of August 20. The Ottertail
River runs through Rush Lake and Ottertail Lake and on toward the southwest, supplying power to a hydraulic plant at Fergus Falls and furnishing water for several towns down stream. The large drop in Rush Lake was due primarily to lowering the dam at the outlet of the lake. The dam at the outlet of Ottertail Lake was also lowered somewhat, but it did not lose as much proportionately. Rush Lake, which is surrounded by marshes, was lowered several inches at the request of farmers who wanted to obtain additional hay on the wet ground bordering the lake. The effect of such a policy from the standpoint of moisture conservation needs little comment here except to quote from page 11 of President Roosevelt's Great Plains Drought Area Committee in its report of August, 1936, "In a land of little rain it is imperative that water should never be allowed needlessly to go to waste."

The cooling effect resulting from the evaporation of water is worthy of consideration. Mr. C. D. Reed, Senior Meteorologist of the U. S. Weather Bureau at Des Moines, has made extensive studies of drought conditions in Iowa during the summer of 1936 with special reference to causes leading to failure of the corn crop. He contends that Iowa had sufficient rain to produce a satisfactory crop, had the temperatures not reached such great extremes. His position in this respect may not be wholly tenable in view of the oasis near Evansville, Minnesota, above referred to, resulting from a few local showers in that immediate area where the temperatures must have been essentially the same as elsewhere. There is also abundant evidence to show that corn yields of 30 to 70 bushels per acre were obtained in 1936 by means of pump irrigation in the Platte Valley around Kearney and Lexington, Nebraska, and that record yields of watermelons and alfalfa were obtained by means of irrigation at Orleans, Nebraska, although surrounding crops which did not have irrigation were completely burned up.

It is interesting to compute the amount of heat required
to evaporate half an inch of water per day. That this estimate is not excessive is shown by the fact that the daily average for Lincoln, Nebraska, for July, 1936 was 0.545 inches and for August, 1936, it was 0.417 inches. The maxima were 0.855 inches on July 24 and 0.678 on August 17, as reported by the U. S. Weather Bureau.

The solar radiation constant, according to Abbot, is 1.92 calories per minute on each square centimeter of the earth normally exposed. This amounts to 1.065 B.T.U. per second, or more than one horse-power, per square yard. In a twelve-hour day the total would be 46,000 B.T.U., but much of this heat is absorbed by the atmosphere and does not really reach the earth's surface due to atmospheric absorption. Milankowitch in the Köpper-Geiger Handbuch der Klimatologie gives a total of 487 gm. cal. per day at latitude 40° for August 8 with the sun at a longitude of 135° and an estimated transmission coefficient of 0.7. This would be the equivalent of 508 gm. cal. per day at latitude 46° where this investigation was carried out. Weather Bureau records show a mean of 523 gm. cal. per day at Lincoln, Nebraska, and 442 gm. cal. per day at Madison, Wisconsin, for August, 1936. There is considerable haziness in Minnesota, especially under drought conditions, so that a transmission coefficient of 0.7 would not be far from correct.

On the basis of 508 gm. cal. per square centimeter per day, the average heat received at the surface would equal 17,000 B.T.U. per square yard. 578.3 calories are required to evaporate one gram of water at 90° F. This is equivalent to 24,380 B.T.U. to evaporate a one-half inch layer of water per square yard, or 43 per cent more than the average incident radiation. The radiation corresponding to the maximum of 0.855 inches evaporation mentioned above was 630 calories, requiring 41,600 B.T.U. with only 21,000 B.T.U. available, or a deficiency of nearly 50 per cent.

1 Smithsonian Tables, p. 235.
This available heat should be reduced by 6 to 8 per cent for reflection from a water surface when evaporation from lakes and ponds is under consideration. It is thus seen that a pond losing water at the above rate will not only contribute its moisture to the atmosphere but will absorb much more than the average solar radiation incident upon it in the process. In other words, the heat absorbed in evaporation will be equal to about one and one-half times that incident on its own surface, while dry earth or stubble re-radiates most of the heat which falls upon it.

A similar computation shows that 21 per cent of the incident radiation will be required to evaporate 9 inches of water during a summer of 5 months. This would be 50 per cent of an 18 inch annual rainfall. Mr. Adolf F. Meyer has shown that the runoff and moisture evaporated from the soil may easily exceed the 9 inches estimated in this connection. Since the ground water level remains practically constant, the disposition of the rainfall at any given locality must be divided between runoff, evaporation, and transpiration. If half of the moisture that falls is put back into the air through vegetation, the other half is either lost by drainage or evaporated locally. In an ideal situation under semi-arid conditions no water would be permitted to escape from the region. Therefore, the greater the storage, either in the soil or in basins, the greater the cooling effect must be.

**Conclusions**

(1) The irregular rainfall distribution in the area studied has demonstrated the value of the investigation of precipitation and humidity variations in small areas because of their bearing on the larger problems of moisture conservation. The data at hand is not sufficient to be conclusive, but there is evidence of a direct relationship between the 40 mile chain of lakes beginning with Lake Lizzy and

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¹ *Bull. American Meteorological Society, 17, 4, April, 1936, p. 103.*
extending through Lake Lyda, Star Lake, Ottertail Lake, the two Battle Lakes, and Lake Christine and the larger rainfall recorded to the southeastward around Evansville and Elbow Lake in two successive seasons. Further studies where similar conditions exist may bring out corresponding situations which have escaped unnoticed in regular Weather Bureau reports due to the distance between stations.

(2) It has been shown that higher humidity and lower temperatures, both of which are conducive to increased rainfall from local thunderstorms, obtain on the leeward side of lakes and ponds.

(3) The cooling effect of the evaporation from exposed water surfaces and its probable relation to crops in the vicinity have been pointed out, together with the desirability of increasing the total water exposure over all areas where the rainfall is too low for successful agricultural pursuits. It is also suggested, that with the approach of a wet cycle, the outlet levels of all natural water basins should be raised in order to conserve and retain the maximum water storage in anticipation of future droughts.

Acknowledgments

As stated in the beginning of this report, some of the equipment used in this investigation was obtained through loans from the U. S. Weather Bureau, the University of Nebraska, and from instruments purchased by grants from the Hodgkins Fund of the Smithsonian Institution and from the Bache Fund of the National Academy of Sciences, all of these instruments having been used in previous investigations. The splendid assistance of the U. S. Weather Bureau observers and the sixteen others who were temporarily drafted for this service in their respective communities is gratefully acknowledged. Valuable suggestions on certain technical aspects of the problem were offered by
Dr. C. F. Brooks of the Blue Hill Observatory at Milton, Massachusetts.

The continuation of the project was made possible by an additional grant from the Penrose Fund of the American Philosophical Society to cover necessary operating expenses. For this continued assistance the writer expresses his sincere appreciation.
THE PANAMA CANAL TOLLS PROBLEM AND ITS SOLUTION

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(Read February 19, 1938)

On the closing day of the regular session of Congress, the 21st of August, 1937, final action was taken upon legislation giving the President definite authority to fix, within limits designed by law, the tolls to be paid for the use of the Panama Canal, and to prescribe the rules to be applied in measuring vessels to determine the tonnage upon which the tolls shall be levied. This action settled and disposed of a question that has been before Congress for nearly a quarter of a century.

In 1912, two years before the completion and opening of the Panama Canal, Congress passed an act to govern the use and operation of the Canal, and, by that Act, Congress intended to vest in the President the power to fix, within prescribed limits, the charges levied on transiting vessels. The law provided that the tolls might be levied upon the gross or net registered tonnage of vessels or upon some other basis; but the statute stipulated that “If the tolls [upon ships of commerce] shall not be based upon net registered tonnage, they shall not exceed the equivalent of $1.25 per net registered ton . . . nor be less than 75 cents per net registered ton.”

For commercial vessels, a ton is 100 cubic feet of closed-in space. The tonnage of war ships is their weight or displacement in tons of 2240 pounds. The gross tonnage of commercial vessels when accurately determined is their entire closed-in space in cubic feet divided by 100; while their net tonnage, when correctly calculated, is their earn-
ing capacity in units of 100 cubic feet—their gross tonnage less the tonnage of the spaces used for the operation of the vessels. When a commercial vessel is registered under the national flag, its gross and net tonnages are determined by the national vessel measurement rules and its gross and net registered tonnages are stated in the vessel's certificate of government registration.

Acting upon the authority given him by the Panama Canal Act of August 24, 1912, President Taft, in November of that year, issued a proclamation prescribing a canal toll on laden commercial vessels of $1.20 per net ton—each 100 cubic feet—of earning capacity. For such vessels in ballast the toll was 72 cents per net ton. For war ships the toll was made 50 cents per ton (2240 pounds) of weight or displacement at the time of passing through the Canal. The Secretary of War, under whose authority the Canal is operated, was directed to prepare rules for measuring vessels to determine the tonnage upon which the prescribed rates of toll were to be levied. The Panama Canal vessel measurement rules were promulgated by President Wilson by a proclamation issued in November 1913.

A few months after the Canal was opened—which was in August 1914—the power of the President over Canal charges was largely nullified by an interpretation that the Attorney General gave to the tolls section of the Panama Canal Act of 1912. He held that, inasmuch as the Panama Canal vessel measurement rules had not been prescribed when the Act of 1912 was adopted, Congress, in fixing the maximum toll at $1.25 per net registered ton, must have referred to tonnage determined by the United States rules by which vessels are measured for registration under the American flag. The President has no authority over formulating and interpreting the national vessel measurement rules which are based upon statutes and are framed and applied by the Director of the Bureau of Marine Inspection and Navigation in the Department of Commerce. The determination of the maximum tolls for the Panama Canal
was taken from the President and shifted to the Department of Commerce.

When the Canal began operations in 1914 the tonnage resulting from measuring vessels by the National registration rules was but little less than the tonnage determined by the Panama Canal rules; but, beginning in 1915, the National measurement rules, in order to make them correspond to the British rules, have from time to time been so modified as greatly to reduce the net registered tonnage of American vessels. In 1936, the aggregate net registered tonnage of vessels transiting the Panama Canal was only 69.5 per cent of their tonnage by the Canal rules. Thus a maximum toll of $1.25 per net registered ton was much less than a toll of $1.20 per net ton Panama Canal measurement. On some types of ships, a large share of the spaces used for passengers and cargo were, under the National measurement rules, not included in the tonnage upon which tolls were levied.

When early in 1915 the intention of Congress to vest in the President authority over Panama Canal charges and fiscal policy was defeated by the opinion of the Attorney General, it was expected that Congress would promptly correct the situation by appropriate legislation; but those who benefited from the successive lowering of Canal charges by changes made in the United States measurement rules by the Bureau of Marine Inspection and Navigation successfully opposed action by Congress. Although 26 bills were introduced between 1915 and 1936, and four passed the House, no action was taken by the Senate. Finally, in 1936, Congress authorized the President to appoint a special committee of three men to investigate Panama Canal tolls and vessel measurement rules and to recommend such changes as the committee found to be needed. The Committee on Panama Canal Tolls and Vessel Measurement Rules devoted seven months to its task, and in February 1937 recommended legislation giving the President sole authority to fix the Panama tolls within statutory
limits, and to prescribe the rules to determine the tonnage upon which charges shall be levied. The Chairman of the Committee was the author of this statement, presumably because it was he who had prepared the schedule of tolls prescribed in 1912 and the Panama Canal vessel measurement rules promulgated in 1913. The Committee recommended that Congress should authorize tolls on laden commercial vessels to be fixed within a maximum of $1.00 per net ton and a minimum of 75 cents per net ton. These recommendations were embodied in the Act passed August 21 and approved by the President August 24.

The Committee recommended several changes in the Panama Canal Measurement Rules of 1913, modifications made desirable by changes in the design of vessels and the substitution of fuel oil for coal. Tolls of 90 cents per net ton for laden commercial vessels and 72 cents per net ton for such vessels in ballast were also recommended. The President adopted the Committee’s recommendations as to rates of toll and as to changes in the Panama measurement rules. The amended rules and new schedule of tolls were made effective March 1, 1938, six months notice of the changes being required by statute. This very satisfactory disposition of a long-standing controversy between the users of the Canal and the Canal administration is most gratifying to the Canal Authorities and to the Committee whose recommendations have been embodied in legislation and executive decrees.
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