REINFORCED CONCRETE
DESIGNER'S HANDBOOK

BY

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SIXTH EDITION

PUBLISHED BY
CONCRETE PUBLICATIONS LIMITED
14 DARTMOUTH STREET, LONDON, S W 1
"CONCRETE SERIES"
BOOKS ON CONCRETE

A list of other "Concrete Series" books on the design and construction of reinforced and prestressed concrete, precast concrete, and cement is given on the page facing page 362.

First Edition (4,000 copies) 1932
Second Edition (7,100 copies) 1939
Third Edition (4,500 copies) 1946
Fourth Edition (8,900 copies) 1948
Fourth Edition (Revised) (10,000 copies) 1951
Fourth Edition (Further Revised) (8,000 copies) 1954
Fifth Edition (20,000 copies) 1957
Sixth Edition 1961
REINFORCED CONCRETE DESIGNER'S HANDBOOK
SIXTH EDITION (1961).

ERRATA.

Page 114, line 17.—British Standard Code should be No. 2001.

,, 163.—\(\beta = 1 + \frac{4.5A}{HL} \); \(P_1(\text{max.}) = k_1wH\).

,, 226.—Rectangular frame, vertical load on beam: \(V_D = aW - 2k'N\).

,, 239.—Second line from bottom: Delete “marked”.

,, 251.—Rectangle: \(I_{XX} = \frac{bd^2}{12} + 2(m - 1)A_{st}(\frac{d}{2} - \frac{d_2}{2})^2\) if \(A_{sc} = A_{st}\).

Trapezium: In formula for \(X\), \(d\) should be \(d_1\).

In formula for \(I_{xx}\), \((d - X)\) should be \((d - X)^2\) and \((X - d_2)\) should be \((X - d_2)^2\).

I-section: In formula for \(X\), \(d\) should be \(d_1\).

,, 321.—Box culverts: \(k = \frac{H(d_h)^3}{L(d_e)}\).

Pressure on walls due to surcharge:

\[ M_A = M_D = -\frac{\rho H^2 k}{12(k + 1)} \]

,, 329.—In formula \(E \cdot H_B\) should be \(H \cdot D_B\).

Also \(M'_{yy} = \frac{8C}{8C}(C' - D')^2\).

May 1962.
PREFACE TO SIXTH EDITION

The arrangement generally and the content of this edition are substantially the same as in previous editions in so far that the descriptive text is in Part I and the tables, with examples applying to the tables, are given in Part II. Notes relating to the tables are now given mainly on the pages facing the tables. Some examples involving the use of several tables and typical details of some common structural parts are given in an Appendix, as are also notes on the effect of various materials on concrete. Specifications and quantities are now included in Part I.

Advantage has been taken of the necessity to reprint the entire book to re-cast and re-number the tables and to re-write and reset the descriptive text in a more convenient form. The result is that without increasing the bulk of the book, the amount of data and the number of tables have been increased. Most of the formulae appearing in Part I of previous editions are now given in the tables. Additional data in the tables include moment-distribution applied to continuous beams and beams with splays, bending-moment diagrams, non-rectangular panels of slabs, portal frames, metric properties of reinforcement bars, load-factor method applied to beams and slabs and to columns subjected to bending, bending in two planes, types of bridges, and formulae abstracted from the original Part I.

The notation throughout has been brought into line with the British Standard Codes where applicable, and effect is given to the recommendations given in B.S. Codes Nos. 114 (buildings) and 2007 (liquid-containers), and other codes and regulations where appropriate.

London, 1961

C. E. R.
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INTRODUCTORY NOTE

A STRUCTURE is an assembly of members each of which is subjected to bending or to direct force (either tensile or compressive) or to a combination of bending and direct force. These primary influences may be accompanied by shearing forces and sometimes by torsion. Secondary effects due to changes in temperature, to shrinking and creep of the concrete, and the possibility of damage resulting from abrasion, vibration, frost, chemical attack, and similar causes may also have to be considered. Design includes the calculation of, or other means of assessing and providing resistance against, the moments, forces, and other effects on the members. An efficiently designed structure is one in which the members are arranged in such a way that the weight, loads and forces are transmitted to the foundations by the cheapest means consistent with the intended use of the structure and the nature of the site. Efficient design means more than providing suitable sizes for the concrete members and the provision of the calculated amount of reinforcement in an economical manner. It implies that the bars can be easily placed, that reinforcement is provided to resist the secondary forces inherent in monolithic construction, and that resistance is provided against all likely causes of damage to the structure. Experience and good judgment may do as much towards the production of safe and economical structures as calculation. Complex mathematics should not be allowed to confuse the sense of good engineering. Where possible, the same degree of accuracy should be maintained throughout the calculations; it is illogical to consider, say, the effective depth of a member to two decimal places if the load is over-estimated by 25 per cent. On the other hand, in estimating loads, costs, and other numerical quantities, the more items that are included at their exact value the smaller is the overall percentage of error due to the inclusion of some items the exact magnitude of which is unknown.

Where the assumed load is not likely to be exceeded and the specified quality of concrete is fairly certain to be obtained, high working stresses can be employed. The more factors allowed for in the calculations the higher may be the working stresses, and vice versa (see pages 57 and 59). If the magnitude of a load, or other factor, is not known precisely it is advisable to study the effects of the probable largest and smallest values of the factor and provide resistance for the most adverse case. It is not always the largest load that produces the greatest stresses in all parts of a structure.

Much design is to-day controlled by regulations or codes, but even within such bounds the designer must exercise judgment in his interpretation of the requirements, endeavouring to grasp the spirit of the requirements rather than to design to the minimum allowed by the letter of a clause. In Great Britain the detail design of reinforced concrete is based largely on the British Standard codes relating to buildings, principally those for "Loading" (Code No. 3, Chapter V), the "Structural Use of Normal Reinforced Concrete in Buildings" (Code No. 114), and "Design and Construction of Reinforced and Prestressed Concrete Structures for the Storage of Water and other Aqueous Liquids" (Code No. 2007). In addition there are such documents as the Memorandum on the design of bridges issued by the Ministry of Transport, and the by-laws of local authorities, such as the London Building By-laws (1952).

The tables given in Part II enable the designer to reduce the amount of arithmetical work. The use of such tables not only makes for speed but also eliminates inaccuracies if the tables are thoroughly understood and their bases and limitations realised. In the succeeding sections of Part I and in the supplementary information given on the pages facing the tables, the basis of the tabulated material is described. Some general information is also tabulated; for example, Table 108 gives fractions of an inch expressed in decimals of a foot; Table 109 is a conversion table for the metric system. Table 110 gives trigonometrical functions of angles to a degree of accuracy sufficient for design purposes; in conjunction with this table, fundamental trigonometrical formulæ and other mathematical formulæ and useful data are given.
Economical Structures.

The cost of a reinforced concrete structure is obviously affected by the prices of concrete, steel, shuttering, and labour. Upon the relation between these prices depend the economical proportions of the quantities of concrete, reinforcement, and shuttering. There are possibly other factors to be taken into account in any particular case, such as the use of available steel shuttering of standard sizes. In Great Britain, economy generally results from the use of simple shuttering even if this requires more concrete compared with a design requiring more complex and more expensive shuttering.

Some of the factors which may have to be considered are whether less concrete of a rich mixture is cheaper than a greater volume of a leaner concrete; whether the cost of higher-priced bars of long lengths will offset the cost of the extra weight used in lapping shorter and cheaper bars; whether, consistent with efficient detailing, a few bars of large diameter can replace a larger number of bars of smaller diameter; whether the extra cost of rapid-hardening cement justifies the saving made by using the shuttering a greater number of times; or whether uniformity in the sizes of members saves in shuttering what it may cost in extra concrete.

There is also the wider aspect of economy, such as whether the anticipated life and use of a proposed structure warrant the use of a higher or lower factor of safety than is usual; whether the extra cost of an expensive type of construction is warranted by the improvement in facilities; or whether the initial cost of a construction of high quality with little or no maintenance cost is more economical than less costly construction combined with the expense of maintenance.

The wording of a contract and the experience of the contractor, the position of the site and the nature of the available materials, and even the method of measuring the quantities, together with numerous other points, all have their effect, consciously or not, on the designer's attitude towards a contract. So many and varied are the factors to be considered that only experience and the study of the trend of design can give any reliable guidance. Attempts to determine the most economical proportions for a given member, based only on inclusive prices of concrete, steel, and shuttering, are often misleading. It is nevertheless possible to lay down certain principles.

For equal weights, combined material and labour costs for reinforcement bars of small diameter are greater than those for large bars, and within wide limits long bars are cheaper than short bars if there is sufficient weight to justify special transport charges and handling facilities.

The lower the cement content the cheaper the concrete but, other factors being equal, the lower is the strength and durability of the concrete. Taking compressive strength and cost into account a concrete rich in cement is more economical than a leaner concrete. In beams and slabs, however, where much of the concrete is in tension and therefore neglected in the calculations, it is less costly to use a lean concrete than a rich one. In columns, where the concrete is in compression, the use of a rich concrete is more economical, since besides the concrete being more efficient, there is a saving in shuttering consequent upon the reduction in size of the column.

The use of steel in compression is always uneconomical when the cost of a single member is being considered, but advantages resulting from decreasing the depth of beams and by reducing the size of columns may offset the extra cost of the individual member. A doubly-reinforced beam that contains the smallest amount of reinforcement is less costly than a beam of the same size containing a greater amount of steel, even although in the latter case the tensile steel is stressed to the maximum permissible value. The most economical doubly-reinforced beam is that in which the compressive stress in the concrete is the maximum permissible stress and the tensile stress in the steel is that which gives the minimum combined weight of tensile and compressive reinforcement. Tee-beams and slabs with compression reinforcement are seldom economical. When the cost of mild steel is high in relation to that of concrete, the most economical slab is that in which the proportion of tensile reinforcement is well below the so-called "economic" proportion. (The "economic" proportion is that at which the maximum permissible stresses in the steel and concrete are simultaneously attained.) Tee-beams are cheaper if the rib is made as deep as practicable; here again
the increase in headroom consequent on reducing the depth may offset the small extra cost of a shallower beam. It is rarely economical to design a tee-beam for the maximum permissible stress in the concrete.

Bent-up bars are more economical than binders for resisting shearing force, and this may be true even if bars have to be inserted specially for this purpose.

Shuttering is obviously cheaper if angles are right-angles, if surfaces are plane, and if there is some repetition of use. Therefore splays and chamfers are omitted unless structurally necessary or essential to durability. Wherever possible architectural features in cast-in situ work should be formed in straight lines. When the cost of shuttering is considered in conjunction with the cost of concrete and steel, the introduction of complications in the shuttering may sometimes lead to more economical construction; for example, large continuous beams may be more economical if they are haunched at the supports. Cylindrical tanks are cheaper than rectangular tanks of the same capacity if many uses are obtained from one set of shuttering. In some cases domed roofs and tank bottoms are more economical than flat beam-and-slab construction, although the unit cost of the shuttering may be doubled for curved work. When shuttering can be used several times without alteration, the employment of steel shuttering should be considered and, because steel is less adaptable than wood, the shape and dimensions of the work may have to be determined to suit. Generally steel shuttering for beam-and-slab or column construction is cheaper than timber shuttering if twenty or more uses can be assured, but for circular work half this number of uses may warrant the use of steel. Timber shuttering for slabs, walls, beams, column sides, etc., can generally be used four times before repair, and six to eight times before the cost of repair equals the cost of new shuttering. Beam-bottom boards can be used at least twice as often.

The use of precast construction usually reduces considerably the amount of shuttering and temporary supports required, and the moulds may be used a greater number of times. Except in some obvious cases, the loss of structural rigidity due to the absence of monolithic construction may offset the economy otherwise resulting from precast construction. To obtain the economical advantage of precasting and the structural advantage of cast-insitu construction, it is convenient to combine both types of construction in some structures.

In many cases the most economical design can be determined only by comparing the approximate costs of different designs. This is particularly true in border-line cases and is practically the only way of determining, say, when a simple cantilevered retaining wall ceases to be more economical than one with counterforts; when a solid-slab bridge is more economical than a slab-and-girder bridge; or when a cylindrical container is cheaper than a rectangular container. Although it is usually more economical in floor construction for the main beams to be of shorter span than the secondary beams, it is sometimes worth while investigating different spacings of the secondary beams, to determine whether a thin slab with more beams is cheaper or not than a thicker slab with fewer beams.

An essential aspect of economical design is an appreciation of the possibilities of materials other than concrete. The judicious incorporation of such materials may lead to substantial economies. Just as there is no structural reason for facing a reinforced concrete bridge with stone, so there is no economic gain in constructing a 4-in. cast-insitu reinforced concrete wall panel if a 4\(\frac{1}{2}\)-in. or 9-in. brick wall is cheaper and will serve the same purpose. Other common cases of the consideration of different materials are the installation of timber or steel bunkers when only a short life is required, the erection of light steel framing for the superstructures of industrial buildings, and the provision of pitched steel roof trusses. Included in such economic comparisons should be such factors as fire-resistance, deterioration, depreciation, insurance, appearance, speed of construction, and structural considerations such as the weight on the foundations, convenience of construction, and scarcity or otherwise of materials.

**Drawings.**

The methods of preparing drawings vary considerably, and in most drawing offices a special practice has been developed to suit the particular class of work done.
The following observations can be taken as a guide when no precedent exists. One of the principal factors is to ensure that on all drawings for any one contract the same conventions are adopted and uniformity of appearance and size should be aimed at, thereby making the drawings easier to read.

In the preliminary stages a general drawing of the whole structure is generally prepared to show the principal arrangement and sizes of beams, columns, slabs, walls, foundations, and other members. Later this, or a similar drawing, is utilised as a key to the working drawings, and should show precisely such particulars as the setting-out of the structure in relation to adjacent buildings or other permanent works, and the level of, say, the ground floor in relation to a datum. All principal dimensions such as the distance between columns and overall and intermediate heights should be indicated, in addition to any clearances, exceptional loads, and other special requirements. The most convenient scale for these general drawings is usually $\frac{1}{3}$ in. to 1 ft. It is often of great assistance if the general drawing can be used as a key to the detailed working drawings by incorporating reference marks for each column, beam, slab panel, or other member.

The working drawings should be large-scale details of the members shown on the general drawing. A suitable scale is $\frac{1}{4}$ in. to 1 ft., although plans of slabs and elevations of walls are often conveniently prepared to a scale of $\frac{1}{2}$ in. to 1 ft., while sections through beams and columns with complicated reinforcement are preferably drawn to a scale of 1 in. to 1 ft. Separate sections, plans, and elevations should be shown for the details of the reinforcement in slabs, beams, columns, frames, and walls, since it is not advisable to show the reinforcement for more than one such member in a single view. An indication should be given, however, of the reinforcement in slabs and columns in relation to the reinforcement in beams or other intersecting reinforcement. Sections through beams and columns showing the detailed arrangement of the bars should be placed as closely as possible to the position where the section is taken.

In reinforced concrete details it is best to indicate the outline of the concrete by a bold line and the reinforcement by a thinner full line representing the centre-line of the bar and, wherever clearness is not otherwise sacrificed, the line representing the bar should be placed in the exact position intended for the centre-line of the bar, proper allowance being made for the amount of cover. Thus the reinforcement as shown on the drawing should represent as nearly as possible the appearance of the reinforcement as fixed on the site, all hooks and bends being drawn to scale.

The dimensions given on the drawing should be arranged so that the primary dimensions connect column and beam centres or other leading setting-out lines, and so that secondary dimensions give the detailed sizes with reference to the main setting-out lines. The dimensions on working drawings should also be given in such a way that the carpenters making the shuttering have as little calculation to do as possible. Thus, generally, the distances between breaks in any surface should be dimensioned. Disjointed dimensions should be avoided by combining as much information as possible in a single line of dimensions.

Marks indicating where cross-sections are taken should be bold and, unless other considerations apply, the sections should be drawn as viewed in the same two directions throughout the drawing; for example, they may be drawn as viewed looking towards the left and as viewed looking from the bottom of the drawing. Consistency in this makes it easier to understand complicated details.

Any notes on general or detailed drawings should be concise and free from superfluity in wording or ambiguity in meaning. Notes which apply to all working drawings can be reasonably given on the general arrangement with a reference to the latter on each of the detail drawings. Although the proportions of the concrete, the cover of concrete over the reinforcement, and similar information are usually given in the specification or bill of quantities, the proportions and covers required in the parts of the work shown on a detail drawing should be described on the latter, as the workmen rarely see the specification. If the bar-bending schedule is not given on a detail drawing, a reference should be made to the page numbers of the bar-bending schedule relating to the details on that drawing.

Notes that apply to one view or detail only should be placed as closely as possible
to the view or detail concerned, and only those notes that apply to the drawing as a whole should be collected together. If a group of notes is lengthy there is a danger that individual notes will be read only cursorily and an important requirement be overlooked.

**Factor of Safety.**

The factor of safety of a reinforced concrete structure is not the ratio of the strength of the materials to the design stress, mainly because of yielding of the reinforcement at a stress less than the tensile strength. The true factor of safety is the ratio of the greatest load that the structure can carry to the design load. The greatest load, which is that causing collapse or so much distortion that the structure fails to fulfil its purpose, cannot be calculated exactly. The ultimate moment of resistance of a rectangular beam or the failure-load of a column can be calculated with reasonable accuracy, but the bending moments and forces acting on an overloaded monolithic reinforced concrete structure are indefinite, as overstressing at one part (say, at the support of a continuous beam) may be relieved by a reserve of strength at another part (say, at midspan). The distribution of bending moments is therefore different from that assumed under the design load. The factor of safety with stresses not exceeding half the yield-point stress of the reinforcement and one-third of the crushing strength of concrete cubes at 28 days is probably not less than three.

A more definite factor of safety can be applied to the lateral stability of a structure. Generally the factor of safety against overturning of a cantilevered retaining wall is about one and a half. For bridges, B.S. No. 153 (Part 3A) requires the stabilising effect to be 1.1 times the overturning effect due to dead loads plus 1.4 times the overturning effect due to other loads.
NOTATION

The symbols adopted in the B.S. Code of Practice No. 114 (1957) are given in the following and are used in the tables and elsewhere in this book. The signification of these and other symbols used are given in the appropriate tables and texts.

- $A_s$: Equivalent area of helical reinforcement (volume of helix per unit length of column).
- $A_{c}$: Cross-sectional area of concrete, excluding any finishing material and reinforcement.
- $A_{k}$: Cross-sectional area of concrete in column core, excluding area of longitudinal reinforcement.
- $A_{sc}$: Cross-sectional area of steel in compression.
- $A_{st}$: Cross-sectional area of steel in tension.
- $A_{sw}$: Cross-sectional area of stirrup.
- $b$: Breadth of a rectangular beam; breadth of flange of a T-beam or L-beam.
- $b_r$: Breadth of the rib of a T-beam or L-beam.
- $D$: Diameter generally.
- $d$: Overall depth.
- $d_e$: Effective depth to the tensile reinforcement in a beam.
- $d_{cm}$: Depth to the compression reinforcement in a beam.
- $d_n$: Depth of concrete in compression in a beam (= depth to neutral plane).
- $d_s$: Depth of slab forming the flange of a T-beam or L-beam.
- $e$: Eccentricity of a load on a column.
- $K_b$: Stiffness of beam.
- $K_{bo}$: Stiffness of beam on one side of a column.
- $K_{bo'}$: Stiffness of beam on the opposite side of a column.
- $K_s$: Stiffness of lower column.
- $K_{u}$: Stiffness of upper column.
- $L$: Length of a column or beam between adequate lateral restraints; in flat slabs, $L$ is the average of $L_1$ and $L_2$.
- $L_1$: Length of panel of flat slab in the direction of span considered.
- $L_2$: Width of panel of flat slab at right-angles to direction of span considered.
- $l$: Effective span of beam or slab; effective height of column.
- $l_s$: Lever arm of the resistance moment.
- $l_s'$: Length of shorter side of slab spanning in two directions.
- $l_g$: Length of longer side of slab spanning in two directions.
- $M$: Bending moment (suffixes as required).
- $M_{zt}$: Bending moment at end of beam framing into a column, assuming fixity at both ends of the beam.
- $M_{zt}$: The maximum difference between the moments at the ends of two beams framing into opposite sides of a column, each calculated on the assumption that the ends of the beams are fixed and that one of the beams is not subjected to live load.
- $M_{zt}$ and $M_{zt}$: Maximum bending moments, for spans $l_s$ and $l_g$ respectively, on strips of unit width in slabs spanning in two directions.
- $m$: Modular ratio.
- $o$: Sum of perimeters of the bars in the tensile reinforcement.
- $P$: Permissible load on a short column subject to load and bending.
- $P_o$: Eccentric load on a column.
- $P_{ob}$: Axial load permissible on a short column.
- $P_{ob'}$: Permissible compressive stress in concrete in bending.
- $P_{ob'}$: Permissible stress in concrete in direct compression.
- $P_{ob'}$: Permissible compressive stress in the reinforcement.
- $Q$: Total shearing force across a section.
- $q$: Shearing stress at a section of a beam or slab.
- $R$: Modulus of rupture.
- $s$: Spacing or pitch of stirrups.
- $u$: Cube crushing strength of concrete.
- $u_{pr} = u$: for preliminary test.
- $u_{tr} = u$: for works test.
- $W$: Total load on beam or slab.
- $W_d$: Dead load.
- $W_g$: Imposed load.
- $w$: Total load per unit area of slab or per unit length of beam.
- $x_s$ and $x_g$: Bending-moment coefficients for the short and long spans respectively of slabs spanning in two directions and freely supported on four sides.
- $\beta_s$ and $\beta_g$: Bending-moment coefficients for the short and long spans respectively of rectangular panels supported on four sides and with provision for torsion at the corners.
# PART I

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SECTION 1

LOADS AND PRESSURES

The loads on a structure are permanent (or dead) loads and transient (or imposed or live) loads. Dead loads include the weights of the structure itself and any permanent fixtures, partitions, finishes, superstructures, and the like. Data for computing dead loads are given in Tables 1 and 2.

Live loads include any external loads imposed upon the structure when it is serving its normal purpose, and include the weight of stored materials, furniture and movable equipment, cranes, vehicles, snow, wind, and people. The accurate assessment of the actual and probable loads is an important factor in the production of economical and efficient structures. Some live loads, such as the pressures and weights due to contained liquids, can be determined exactly; less definite, but capable of being calculated with reasonable accuracy, are the pressures of retained granular materials. Other loads, such as those on floors, roofs, and bridges, are generally specified at safe values. Wind forces are much less definite, and marine forces are among the least determinable.

Imposed Loads.

Floors.—For buildings in most towns the loads imposed on floors, stairs, and roofs are specified in codes or regulations issued by local authorities. The loads given in Table 3 are based on the London By-laws (1952) and the B.S. Code No. 3, Chapter V (1952). For floors and roofs with small intensities of loading the beams are sometimes designed for a smaller imposed load than the slabs, since it is unlikely that the entire floor will be loaded at one time. It is the practice to design short-span slabs and beams for a minimum total imposed load irrespective of the span; the values of the minimum loads are given in Table 3. The limiting span for most slabs is 8 ft., below which length the minimum total imposed loads control the design of the slab. For spans in excess of this limiting value, the normal intensity of imposed loading is the controlling factor. In the case of slabs, the minimum total imposed load is expressed in lb. per foot width of slab, whereas for beams the minimum load is the total load on the beam. In accordance with the London By-laws and B.S. Code, the minimum load on beams applies when the area of floor supported by the beam is not greater than 64 sq. ft. When a beam supports more than 500 sq. ft. of a level floor, it is permissible to reduce the specified imposed load by 5 per cent. for every 500 sq. ft. of floor supported, the maximum reduction being 25 per cent.; this reduction does not apply to floors used for storage. The minimum total imposed loads do not apply to cantilevers. The imposed loads on balconies, corridors, stairs and landings are also given in Table 3.

The loads on floors of warehouses and garages are dealt with on pages 11 and 13. In all cases of floors in buildings it is advisable, and in some localities it is compulsory, to affix a notice indicating the imposed load for which the floor is designed. Floors
of industrial buildings where machinery and plant are installed should be designed not only for the load when the plant is in running order, but for the probable load during erection and the testing of the plant, as in some cases this load may be more severe than the working load. The weights of any machines or similar fixtures should be allowed for if they are likely to cause effects more adverse than the specified minimum imposed load. Any reduction in the specified imposed load due to multiple stories or to floors of large area should not be applied to the gross weight of the machines or fixtures. The approximate weights of some machinery such as conveyors and screening plants are given in Table 2. The effect on the supporting structure of passenger and goods lifts and the forces in colliery pit-head frames of the A-frame type are given on the page facing Table 5. The support of heavy safes requires special consideration, and the floors should be designed not only for the safe in its permanent position but also for the condition when the safe is being moved into position, unless temporary props or other means of relief are provided during installation.

**Structures subject to Vibration.**—For floors subjected to vibration from such causes as dancing, drilling, and gymnastics, the imposed loads specified in Table 3 are adequate to allow for the dynamic effect. For structural members subjected to continuous vibration due to machinery, crushing plant, centrifugal driers, and the like, allowance for dynamic effect can be made by reducing the working stresses by, say, 25 per cent. or by increasing the total dead and live load by this amount; the advantage of the latter method is that ordinary stresses and standard tables and charts are applicable.

**Balustrades and Parapets.**—The balustrades of stairs, landings, and balconies should be designed for a horizontal load acting at the level of the handrail or coping. This load may be 15 lb. per linear foot for light access stairs, 25 lb. per linear foot for stairs in residential buildings, and 50 lb. per linear foot for other cases, including roof parapets. On the parapets of balconies, roofs, bridges, or elsewhere where crowds can assemble a horizontal load of 200 lb. per foot should be allowed. B.S. No. 153 (Part 3A) specifies the load on bridge parapets as 25 lb. to 100 lb. per linear foot depending on conditions of probable loading.

**Roofs.**—The imposed loads on roofs given in Table 3 are additional to all surfacing materials and include snow and other incidental loads but exclude wind pressure. Freshly-fallen snow weighs about 5 lb. per cubic foot, but compact snow may weigh 20 lb. per cubic foot, which should be considered in districts subject to heavy snowfalls. For sloping roofs the snow load decreases with an increase in the slope. According to the B.S. Code the imposed load is zero on roofs sloping at an angle exceeding 75 deg., but a sloping roof with a slope of less than 75 deg. must be designed to support an imposed load of 15 lb. to 20 lb. per square foot or such minimum total imposed load as is given in Table 3.

If a flat roof is used for purposes such as a café, playground, or roof garden, the appropriate imposed load for such a floor should be allowed. The possibility of converting a flat roof to such purposes or for use as a floor in the future should also be anticipated.

**Columns, Walls, and Foundations.**—Columns, walls, and foundations of buildings should be designed for the same imposed loads as the beams of the floors they support. In the case of buildings of more than two stories, and which are not warehouses, garages, or stores and are not factories or workshops the floors of which are designed for less than 100 lb. per square foot, the column and foundation loads
can be reduced thus. Consider the roof as being fully loaded. If two floors are supported, reduce the imposed load on both floors by 10 per cent. If three floors, reduce the imposed load on the three floors by 20 per cent., and so on in 10-per-cent. reductions down to five or more floors for which the imposed load is reduced by not more than 40 per cent. These requirements are in accordance with the London By-laws and the B.S. Code and appropriate factors are given in Table 3. If the unit load on a beam is reduced because of the large area supported the columns or other supporting members can be designed for either this reduced load or for the reduction due to multiple stories.

**Storage Loads.**

**Warehouses.**—Floor slabs, beams, columns, and foundations of buildings of the warehouse class should be designed for a minimum specified imposed load, but the purpose for which each floor of the warehouse is to be used should be considered and, if necessary, heavier loads should be assumed. For example, paper stores and printing works are usually designed for an imposed load of 3 cwt. per square foot. The minimum imposed loads for warehouses for various conditions as given in the B.S. Code and the London By-laws are given in Table 3.

**Containers.**—Wherever possible the actual weights of the materials to be stored in such container structures as tanks, bunkers, and silos should be ascertained. The weights of some of the materials commonly stored in concrete containers are given in Table 4. The horizontal pressures due to such materials are considered on page 19. Where material is floating or submerged in water or other liquid the loads or vertical pressures on the horizontal bottom of the container can be calculated from the expressions and data given in Table 4.

**Live Loads due to Vehicles and Moving Equipment.**

Structures such as bridges, gantries, and buildings used for garages are designed for the loads imposed by vehicles, trains, cranes, and other moving equipment. Reliable data, relating to the weights and other loads, thrusts and other forces, to which suitable allowances for impact should be added, should be obtained from the maker (or elsewhere) of the vehicle or equipment.

**Road Bridges.**—The live load on public road bridges in Britain is specified by the Ministry of Transport, and comprises a uniformly-distributed load combined with a knife-edge load. The intensity of the uniformly-distributed load depends on the "loaded length" (that is the length of the member which must be loaded to produce the greatest stress in that member), and is given in Table 6. Consideration of the "loaded length", and notes on the application of the normal live loads, are given on the page facing the table. Similar loads on reinforced concrete decks of steel bridges are specified in B.S. No. 153, Part 3A; particulars of the normal uniformly-distributed load and alternative wheel-loads for certain slabs are given in Table 6, and notes on their application are on the page facing this table. Reference should be made to B.S. No. 153 for the requirements for stiffening the edges of slabs and for other secondary matters.

On some bridges the effects of an abnormal load must be considered as an alternative to the normal loads; particulars of this load are also given in Table 6, and is the same in B.S. No. 153 as that prescribed by the Ministry of Transport. For the design of slabs this load can be converted to an equivalent distributed load as follows: On slabs spanning in one direction, the equivalent load is the normal uniformly-distributed load
plus the knife-edge load; if the slab spans more than 20 ft. in the direction of the longitudinal axis of the bridge, this equivalent load must be increased by 25 per cent. but need not be greater than the load producing a bending moment of 294,000 in.-lb. per foot width. On slabs spanning in two directions the abnormal load is equivalent to the two-wheel loads specified in Table 6.

If the standard load is excessive for the traffic likely to use the bridge (having regard to possible increases in the future), the load from ordinary and special vehicles using the bridge, including the effect of the occasional passage of steam-rollers, heavy lorries, and abnormally heavy loads should be considered. Axle loads (without impact) and other data for various types of road vehicles are given in Table 6. The actual weights and dimensions vary with different types and manufacturers; notes on weights and dimensions are given on page 150 and include aircraft.

The effect of impact of moving loads is usually allowed for by increasing the static load by an amount varying from 10 per cent. to 75 per cent., depending on the type of vehicle, the nature of the road surface, the type of wheel (whether rubber or steel-tyred), and the speed and frequency of crossing the bridge. An allowance of 25 per cent. on the actual maximum wheel loads is specified in B.S. No. 153, Part 3A for road bridges. A road bridge that is not designed for the maximum loads common in the district should be indicated by a permanent notice stating the maximum loads permitted to use it, and a limitation in speed, and possibly weight, should be enforced on traffic passing under or over a concrete bridge during the first few weeks after completion of the concrete work.

Road bridges may be subjected to forces other than dead load and live loads (including impact); these include wind forces (see page 18) and longitudinal forces due to friction of bearings (Table 92). There is also a longitudinal force due to tractive effort and braking which is assumed to act at the level of the road surface on an area 10 ft. wide and 30 ft. long (or the length of the bridge, if less) so disposed as to have the most adverse effect on the member; this force for bridges designed for the normal loading is \(10 + 0.5(L - 10)\) tons \((L\ ft.\ is\ the\ span\ of\ the\ bridge)\) with a minimum of 10 tons, and 45 tons for bridges of any span to which the abnormal load applies.

Footpaths on road bridges should be designed to carry pedestrians and accidental loading due to vehicles running on the path. A load of 100 lb. per square foot is sufficient to allow for crowds, and an alternative of a single-wheel load of 4 tons, including impact, should be allowed for accidental loading where this can occur. An increase of 50 per cent. in the maximum permissible stresses is sometimes allowed for such accidental loading. If it is probable that the footpath may be converted into a road in the future, the slab and supports should be designed for the same load as the road.

**Railway Bridges.**—The live load for which a main-line railway bridge should be designed is generally specified by the railway authority and may be a standard load such as that in B.S. No. 153, Part 3A. For light railways, sidings, colliery lines, and the like, smaller loads might be assumed and may be a percentage of the standard load, which assumes a number of heavy locomotives to be on the bridge at one time, but for secondary lines the probability of there being only one locomotive and a train of vehicles of the type habitually using the line should be considered in the interest of economy. The weights of some typical rail vehicles are given in Table 6. In addition to dead and live load (including impact), railway bridges have to be designed to resist the effects of lurching and nosing, centrifugal and longitudinal forces, wind pressure, and temperature changes. Data on these effects is given in B.S. No. 153,
LOADS AND PRESSURES

Part 3A, for steel bridges and, in the absence of other information, can be used as a
guide for reinforced concrete bridges, at least for preliminary designs; the data is
summarised on the page facing Table 4.

Railway Sleepers.—The load on a railway sleeper is the load transmitted through
the chair or other rail-seating, and depends on the type of traffic and whether the
sleeper is near or remote from a joint. In B.S. No. 986 (a war-time B.S.) different
design loads are specified for ordinary reinforced concrete sleepers and prestressed
concrete sleepers. The loads specified for the design of reinforced concrete sleepers
are given in Table 5.

Structures supporting Cranes.—Cranes and other hoisting equipment are
commonly supported on columns in factories or similar buildings, or on gantries.
The wheel loads and other particulars for typical overhead travelling cranes are given
in Table 5. It is important that a dimensioned diagram of the actual crane to be
installed is obtained from the makers to ensure that the necessary clearances are
provided and the actual loads taken into account. Allowances for the secondary
effects on the supporting structure due to the operation of overhead cranes are given
on the page facing Table 5.

For jib cranes running on rails on supporting gantries, the load to which the
structure is subjected depends on the disposition of the weights of the crane. The
wheel loads are generally specified by the maker of the crane and should allow for the
static and dynamic effects of lifting, discharging, slewing, travelling, and braking.
The maximum wheel load under practical conditions may occur when the crane is
stationary and hoisting the load at the maximum radius with the line of the jib
diagonally over one wheel.

Garages.—The floors of garages are usually considered in two classes, namely
those for cars and other light vehicles and those for heavier vehicles. Floors in the
lighter class are designed for specified uniformly-distributed imposed loads, or alter-
native minimum loads which depend on whether the floor is to be used as an all-purpose
garage or for parking only. In the design of floors for vehicles in the heavier class,
the bending moments and shearing forces should be computed for the probable weights
and the most adverse disposition of the heaviest vehicles. Actual wheel loads should
be increased by 50 per cent. to allow for dynamic effect. The requirements of the
B.S. Code and the London By-laws are given in Table 5. A load of 1.5 times the
maximum actual wheel load (but not less than 2000 lb.) is assumed to be distributed
over an area 2 ft. 6 in. square. For convenience of design, there are given in Table 5
the equivalent uniformly-distributed loads for the calculation of the bending moments
due to a load of one ton distributed over an area 2 ft. 6 in. square; these equivalent
loads are tabulated for various conditions of continuity for slabs and beams and
should be multiplied by one-and-a-half times the actual load (in tons). With a wheel
directly over a beam and spread on an area 2 ft. 6 in. square, the load is partially
carried by adjacent beams. To allow for this effect, reduction factors are given in
Table 5, the factor being unity for wide beams and for widely-spaced beams; since the
thickness of the slab must also be considered in an accurate computation, the tabulated
factors are approximate.

Dispersal of Concentrated Loads.

A load from a wheel or similar concentrated load bearing on a small but definite
area of the supporting surface (called the contact area) is considered as being further
dispersed over an area dependent upon the combined thicknesses of the road or other surfacing material, filling, concrete slab, and other constructional material. The width of the contact area of the wheel on the slab is equal to the width of the tyre, which may be 1 ft. 6 in. for the heaviest wheels; a minimum of 6 in. is usually assumed. The length of the contact area depends on the type of tyre and the nature of the road surface, and is nearly zero for steel tyres on steel plate or concrete. The maximum contact is probably obtained with an iron wheel on loose metalling or a pneumatic tyre on a tar-macadam surface. A maximum length of 12 in. is reasonable. The dispersal of a concentrated load through the total thickness of the road formation and concrete slab is generally considered as acting at an angle of 45 deg. from the edge of the contact area to the centre of the lower layer of reinforcement, as is shown in the diagrams in Table 6. \[ \text{P} \]

In the case of a pair of wheels, on one axle, on two rails supported on sleepers it can be considered that the load from the wheels in any position is distributed transversely over the length of a sleeper and that two sleepers are effective in distributing the load longitudinally. The dispersal is assumed as 45 deg. through the ballast and deck below the sleepers, as indicated in Table 6. When a rail bears directly on concrete the dispersion may be four to six times the depth of the rail. These rules apply to slowly-moving trains; fast-moving trains may cause a "mounting" surge in front of the train such that the rails, and sleepers, immediately in front of the driving wheels may tend to rise and therefore impose less load in front, but more behind, on the supporting structure.

**Marine Structures.**

The forces acting upon wharves, jetties, dolphins, piers, docks, sea-walls, and similar marine and riverside structures include those due to the wind and waves, blows and pulls from vessels, the loads from cranes, railways, roads, stored goods and other live loads imposed on the deck, and the pressures of earth retained behind the structure.

In a wharf or jetty of solid construction the energy of impact due to blows from vessels berthing is absorbed by the mass of the structure, usually without damage to the structure or vessel if fendering is provided. With open construction, consisting of braced piles or piers supporting the deck in which the mass of the structure is comparatively small, the forces resulting from impact must be considered, and these forces are dependent upon the weight and speed of approach of the vessel, on the amount of fendering, and on the flexibility of the structure. A large vessel has generally a low speed of approach and a small vessel a higher speed of approach. Some examples are a 500-ton trawler berthing at a speed of 1 ft. per second; a 4000-tons vessel at 6 in. per second; a 10,000-tons vessel at 2 in. per second; and a 26,000-tons vessel at 5 in. per second. The kinetic energy of a vessel of 1000-tons displacement moving at a speed of 1 ft. per second and of a vessel of 25,000 tons moving at 2·4 in. per second is in each case about 16 ft.-tons. The kinetic energy of a vessel having a displacement of $W$ tons approaching at a velocity of $V$ ft. per second is $0·016WV^2$ ft.-tons. If the direction of approach is normal to the face of the jetty, the whole of this energy must be absorbed upon impact. More commonly a vessel approaches at an angle of $\theta$ deg. with the face of the jetty and touches first at one point about which the vessel swings. The kinetic energy then to be absorbed is $0·016W[(V \sin \theta)^2 - (\rho\omega)^2]$, where $\rho$ is the radius of gyration (ft.) of the vessel about the point of impact and $\omega$ is the angular
velocity (radians per second) of the vessel about the point of impact. The numerical values of the terms in this expression are difficult, if not impossible, to assess and can vary considerably under different conditions of tide and wind and with different vessels and methods of berthing.

The kinetic energy of approach is absorbed partly by the resistance of the water, but most of it will be absorbed by the fendering, elastic deformation of the structure and the vessel, movement of the ground, and by the energy "lost" upon the impact. The proportion of energy lost upon impact (considered as inelastic impact), if the weight of the structure is $W_s$, does not exceed $\frac{W_s}{W + W_s}$ approximately. It is advantageous to make $W_s$ approximately equal to $W$. The energy absorbed by the deformation of the vessel is difficult to assess, as is also the energy absorbed by the ground. It is sometimes recommended that only about one-half of the total kinetic energy of the vessel be considered as being absorbed by the structure and fendering.

The force to which the structure is subject upon impact is calculated by equating the product of the force and half the elastic horizontal displacement of the structure to the kinetic energy to be absorbed. The horizontal displacement of an ordinary reinforced concrete jetty may be about 1 in., but probable variations from this amount combined with the indeterminable value of the kinetic energy absorbed results in the actual value of the force being also indeterminable. Ordinary timber fenders applied to reinforced concrete jetties cushion the blow, but may not substantially reduce the force on the structure. A spring fender or a suspended fender can, however, absorb a large portion of the kinetic energy and thus reduce considerably the blow on the structure. Timber fenders independent of the jetty are sometimes provided to relieve the structure of all impact forces.

The combined action of wind, waves, currents, and tides on a vessel moored to a jetty is usually transmitted by the vessel pressing directly against the side of the structure or by pulls on mooring ropes secured to bollards. The pulls on bollards, due to the foregoing causes or during berthing, vary with the size of the vessel. A pull of 15 tons acting either horizontally outwards or vertically upwards or downwards is sometimes assumed. A guide to the maximum pull is the breaking strength of the mooring rope, or the power of capstans (when provided) which vary from 1 ton up to more than 20 tons at a large dock.

The effects of wind and waves acting on a marine structure are much reduced if an open construction is adopted and if provision is made for the relief of pressures due to water and air trapped below the deck. The force is not, however, directly related to the proportion of solid vertical face presented to the action of the wind and waves. The magnitude of the pressures imposed is impossible to assess with accuracy, except in the case of sea-walls and similar structures where there is such a depth of water at the face of the wall that breaking waves do not occur. In this case the pressure is merely the hydrostatic pressure which can be evaluated when the highest wave level is known, or assumed, and an allowance is made for wind surge, which, in the Thames estuary for example, may raise the high-tide level 5 ft. above the normal head.

A wave breaking against a sea-wall induces a shock pressure additional to the hydrostatic pressure and reaches its maximum value at about mean water-level and diminishes rapidly below this level and less rapidly above it. The shock pressure may be ten times the hydrostatic pressure and maximum pressures up to 6 tons per square foot are possible with waves 15 ft. to 20 ft. high. The shape of the face of
the wall, the slope of the foreshore, and the depth of the water at the wall affect the maximum pressure and the distribution of pressure. All the possible factors that may affect the stability of a sea-wall cannot be taken into account by calculation, and there is no certainty that the severity of the worst recorded storms may not be exceeded in the future.

Wind Forces.

**Velocity and Pressure of Wind.**—The force due to wind on a structure depends on the velocity of the wind and the shape and size of the exposed members. The velocity depends on the district in which the structure is erected, the height of the structure, and the shelter afforded by buildings or hills in the neighbourhood. In Britain the velocity of gusts may exceed 100 miles per hour but such gusts occur mainly in coastal districts. Gusts of upwards of 80 miles per hour, although infrequent, may be experienced in inland districts. The classification of winds according to the Beaufort scale and the corresponding velocities are given in Table 7. The velocity of wind usually increases with the height above the ground. A relationship between velocity and height specified by the Meteorological Office is given in Table 7; this expression is based on the velocity at a height of 33 ft. (10 metres).

The pressure due to wind varies as the square of the velocity \( V \), and on a flat surface the theoretical pressure is 0.0025 \( V^2 \) lb. per square foot. It is necessary, however, to combine the effect of suction on the leeward side of an exposed flat surface with the positive pressure on the windward side. The resultant pressure can be calculated from the expression given in Table 7, in which are also tabulated the pressures for certain velocities.

The distribution and intensity of the resultant pressures due to wind depend on the shape of the surface upon which the wind impinges. For example, on a vertical cylindrical surface the intensity of pressure varies as shown on the diagram in Table 7. The ratio of height to diameter seriously affects the intensities of the pressures; the greater this ratio, the greater is the pressure. In practice it is usual to allow for the distribution and variation in intensity of the pressure by applying a factor to the normal specified or estimated pressure acting on the projected area of the structure. Such factors are given in Table 7 for cylindrical, octagonal, and square "solid" structures with various ratios of height to width; the corresponding factors for open-frame (unclad) structures and for chimneys and sheeted towers are also given. The factors are in accordance with B.S. Code No. 3 (Chapter V).

The wind pressure to be used in the design of any particular structure should be assessed by consideration of relevant conditions, and especially should be based on local records of velocities.

**Buildings.**—The effect of the wind on buildings is very complex but most building by-laws simply specify a static pressure. For example, local by-laws in Britain sometimes specify that the wind pressure on a building in any horizontal direction shall be assumed to be not less than 15 lb. per square foot on the upper two-thirds of the vertical projected surface of the building with an additional pressure of not less than 10 lb. per square foot on all projections above the general level of the roof. In any particular case it is necessary to determine the requirements of the local authority.

The B.S. Code No. 3 deals with wind forces in more detail. The intensity of external pressure is calculated from the velocity of the wind and the height of the building. The assumed velocity depends on the locality and degree of exposure of
the building. Neglect of the effect of wind on the building as a whole is permitted when, in a building stiffened by walls and floors, the height does not exceed twice the width. The basic pressures recommended in the B.S. Code for buildings of various heights and exposures are given in Table 8; see also the notes relating to Table 8 on the page facing Table 7.

The probable variation and distribution of pressures and suctionso on buildings without permanent openings in the walls are shown in Table 7, but for design purposes, these effects are simplified as follows.

The design of a building to resist wind should take into consideration (i) the stability of the building as a whole and the pressures and forces on the foundations; (ii) the design of the walls; (iii) the design of the roof; and (iv) projections above the roof. For stability and foundations, the primary force is generally the pressure on the vertical faces of the building. The basic intensity of pressure \( p \) (as given in Table 8), is considered to be a pressure of \( 0.5p \) on the windward face and a suction of \( 0.5p \) on the leeward face. These pressures are assumed to act uniformly on the whole height of the vertical face of the building and must be combined with the horizontal components of the pressures and suction on the roof which are assumed to act at right-angles to the slope of the roof. On a building with a flat roof, the windward half of the roof is subjected to a suction of \( p \) and the leeward half to a suction of \( 0.75p \). On a pitched roof the pressures and suction on the windward and leeward slopes depend on the degree of slope and appropriate values are given in Table 8; these pressures and suction apply to the roof as a whole, but for the design of the roof covering and purlins, or other supports, greater local pressures and suction, as also given in Table 8, should be considered. In calculating the stability the total pressure on any projection above the general level of the roof must also be taken into account, the value of \( p \) depending upon the total height of the projection above the ground. In the design of the walls, to the external pressure and suction of \( 0.5p \) an allowance must be added for internal pressure and suction, depending on the area of openings in the walls. In ordinary buildings it is recommended that the alternatives of an internal pressure or an internal suction should be assumed; therefore a wall as a whole should be designed to resist an inward or outward pressure of the amounts given in Table 8, but individual panels of the wall should be designed to resist the greater pressure stated in the table. The walls and roofs of projections above the general roof level should be designed as described for the walls and roof of the main building, the basic intensity of pressure being greater to allow for the greater height. An example of the application of the recommendations in Table 8 is given on the page facing the table.

Curved roofs should be divided into segments as described in Table 3. B.S. Code No. 3 should be referred to for roofs of multiple spans and the effect of wind "drag".

**Chimneys and Towers.**—Since a primary factor in the design of chimneys and similarly exposed isolated structures is the force of the wind, careful consideration of each case is necessary to avoid either under-estimating this force or making an unduly high assessment. Where records of wind velocities in the locality are available an estimate of the probable wind pressures can be made. Due account should be taken of the susceptibility of narrow shafts to the impact of a gust of wind. Some by-laws in Britain specify the intensities of horizontal wind pressure to be used in the design of circular chimney shafts for factories. The total lateral force is the product of the specified pressure and the maximum vertical projected area, and a factor of safety of at least \( 1\frac{1}{2} \) is required against overturning. In some instances specified pressures
are primarily intended for the design of brick chimneys, and in this respect it should be remembered that the margin of safety is greater in reinforced concrete than in brickwork or masonry owing to the ability of reinforced concrete to resist tension, but a reinforced concrete chimney, like a steel chimney, is subject to oscillation under the effect of wind. The pressures recommended in the B.S. Code No. 3 are given in Table 8; these recommendations allow for a variable pressure increasing from a minimum at the bottom to a maximum at the top of the chimney (or tower). A factor, as given in Table 7, to allow for the shape of the structure, can be applied to allow for the relieving effect of curved and polygonal surfaces of chimneys, and of the tanks and the supporting structures of water towers. For cylindrical shafts with fluted surfaces a higher factor than that given in Table 7 should be applied. Local meteorological records should be consulted to determine the probable maximum wind velocity at a height of 40 ft. The chimney, or other structure, can be divided into a number of parts and the average pressure on each part is taken from Table 8.

Chimneys extending above the roof of a building can be designed for the wind forces recommended in the B.S. Code or can be dealt with as described in the foregoing for projections above the general roof level.

**Bridges.**—The forces due to the wind can be neglected on an ordinary reinforced concrete bridge that does not project far above the level of the surrounding land and the width of which is about the same as its height. For high bridges and those erected in exceptionally exposed positions the probable wind forces should be investigated. If other intensities of wind pressure are not specified, and if greater pressures due to exceptional conditions of exposure are not deemed likely, the pressure and rules for calculating the exposed area as given in Table 8 can be adopted.

**Poles and Posts.**

**Transmission-line Poles.**—A transmission-line pole is subject to the weights and pulls of the conductors suspended from it. Due to the pressure of the wind on the conductors, a horizontal force acts transversely to the line at the point at which the conductors are attached. In computing the weight of the conductor and the area exposed to the pressure of the wind, the diameter is considered to be increased by adhering ice, the increase in Britain generally being assumed to be about 3 in., although some regulations require the increase to be 1 in. for high-tension conductors and ½ in. for low-tension conductors. The wind pressure on a high pole must also be taken into account, this being assumed to act at the midpoint of the part of the pole above ground. Formules for the wind forces on the pole and the conductors are given in Table 9 and suitable wind pressures are given in Table 8. Notes on the design of transmission-line poles are given in Table 9.

The wind force on a transmission-line pole is mainly that transferred to the pole by wind pressure on the conductors. In Britain it is usual to assume that a net wind pressure of 8 lb. per square foot (\(P_c\) in the formulæ in Table 9) acts on the projected area of the ice-covered conductor; this pressure includes the allowance for the shape of the conductor. If a pole carries several conductors no allowance should be made for one conductor shielding another, since high-tension conductors are not closely spaced. If 8 lb. per square foot is allowed for the pressure on the conductors of assumed circular section, reasonable values for the pressure on the exposed face of the pole (\(P_p\) in Table 9) are 15 lb. per square foot for a rectangular section (as in
Table 8) and 8 lb. per square foot for a circular section. Greater pressures may be necessary for poles that are perforated.

Lamp Posts.—The load on a lamp post includes, in addition to the weight of the post, the weights of and wind pressure on the bracket, the lantern, and the raising and lowering gear. The weights of the lantern and of the moving part of the raising and lowering gear should be increased by 50 per cent. to allow for dynamic effects. A factor of safety of 2½ on these loads is usually specified. For the calculation of wind pressures on tall lamp posts B.S. No. 1308 specifies a wind velocity of 70 miles per hour for which the corresponding pressures at various heights are given in Table 8.

Active Pressures of Retained and Contained Materials.

The value of the horizontal pressure exerted by a contained material or by earth or other material retained by a wall is uncertain, except when the contained or retained material is a liquid. The formulae, rules and other data in Tables 10 to 14 are given as practical bases for the calculation of such pressures. Reference should also be made to Code No. 2, "Earth-Retaining Structures", of the Institution of Structural Engineers.

Liquids.—At any depth $h$ ft. below the free surface of a liquid, the intensity of pressure $p$ lb. per square foot normal to a surface subject to pressure from the liquid is equal to the intensity of vertical pressure which is given by the simple hydrostatic expression $p = wh$, where $w$ is the unit weight of the liquid in lb. per cubic foot.

Granular Materials.—When the contained material is granular, for example, dry sand, grain, small coal, gravel, or crushed stone, the pressure normal to a retaining surface can be expressed conveniently as a fraction of the equivalent fluid pressure, thus $p = kwh$, where $k$ is a measure of the "fluidity" of the contained or retained material and varies from unity for perfect fluids to zero for materials that stand unrestrained with a vertical face. The value of $k$ depends also on the physical characteristics, water content, angle of repose, angle of internal friction, and the slope of the surface of the material, on the slope of the wall or other retaining surface, on the material of which the wall is made, and on the surcharge on the contained material. The value of $k$ is determined graphically or by calculation, both methods being usually based on the wedge theory or the developments of Rankine or Cain. The total pressure normal to the back of a sloping or vertical wall of height $H$ can be calculated from the formulae in Table 10 for various conditions.

Friction between the wall and the material is usually neglected, resulting in a higher calculated normal pressure which is safe. Friction must be neglected if the material in contact with the wall can become saturated and thereby reduce the friction by an uncertain amount or to zero. Only where dry materials of well-known properties are being stored may this friction be included. Some values of the coefficient of friction $\mu$ are given in Table 14. When friction is neglected (that is $\mu = 0$), the pressure normal to the back of the wall is equal to the total pressure and there is, theoretically, no force acting parallel to the back of the wall.

Generally, in the case of retaining walls and walls of bunkers and other containers, the back face of the wall is vertical (or nearly so) and the substitution of $\beta = 90$ deg. in the general formulae for $k$ gives the simplified formulae in Table 10. Values of $k_1$ (maximum positive slope or surcharge), $k_2$ (level fill) and $k_3$ (maximum negative slope) for various angles of internal friction (in degrees and gradients) are given in Table 11; the value of such angles for various granular materials are given in Tables 11 and 14.
For a wall retaining ordinary earth with level filling $h$, it is often assumed to be 0.3 and, with the average weight of earth as 100 lb. per cubic foot, the corresponding intensity of horizontal pressure is 30 lb. per square foot per foot of height. The formulae assume dry materials. If ground-water occurs in the filling behind the wall, the modified formula given on the page facing Table 10 applies.

The intensity of pressure normal to the slope of an inclined surface is considered on the page facing Table 10 and in Table 11.

**Effect of Surcharge (Granular Materials).**—The effects of various types of surcharge on the ground behind a retaining wall are evaluated in Table 13, comments on which are given on the page facing the table.

**Theoretical and Actual Pressures of Granular Materials.**—In general practice, horizontal pressures due to granular materials can be determined by the purely theoretical formulae of Rankine, Cain, and Coulomb. Many investigators have made experiments to determine what relation actual pressures bear to the theoretical pressures, and it appears that the Rankine formula for a filling with a level surface and neglecting friction between the filling and the back of the wall gives too great a value for the pressure. Thus retaining walls designed on this theory should be on the side of safety. The theory assumes that the angle of internal friction of the material and the surface angle of repose are identical, whereas some investigators find that the internal angle of friction is less than the angle of repose and depends on the consolidation of the material. The ratio between the internal angle of friction and the angle of repose has been found to be approximately between 0.9 to 1. For a filling with a level surface the horizontal pressure given by $p = wh\left(\frac{1}{1 + \sin \theta}\right)$ agrees very closely with the actual pressure if $\theta$ is the angle of internal friction and not the angle of repose. The maximum pressure seems to occur immediately after the filling has been deposited, and the pressure decreases as settling proceeds. The vertical component of the pressure on the back of the wall appears to conform to the theoretical relationship $V = P \tan \mu$. A rise in temperature produces an increase in pressure of about 1 per cent. per 10 deg. F.

The point of application of the resultant thrust on a wall with a filling with a level surface would appear theoretically to be at one-third of the total height for shallow walls, and rises in the course of time and with increased heights of wall. According to some investigators the point of application is at the third-point for fillings the surface of which slopes downwardly away from the wall and rises as the slope increases upward.

Loads imposed on the ground behind the wall and within the plane of rupture increase the pressure on the wall, but generally loads outside the wedge ordinarily considered can be neglected. The increase of pressure due to transient imposed loads remains temporarily after the load is removed. If the filling slopes upwards, theory seems to give pressures almost 30 per cent. in excess of actual pressures.

**Cohesive Soils.**—Cohesive soils include clays, soft clay shales, earth, silts, and peat. The active pressures exerted by such soils vary greatly; due to cohesion, pressures may be less than those due to granular soil, but saturation may cause much greater pressure. The basic formula for the intensity of horizontal pressure at any depth on the back of a vertical wall retaining a cohesive soil is that of Mr. A. L. Bell (derived from a formula by Français). Bell's formula is given in two forms in Table 10. The cohesion factor is the shearing strength of the unloaded clay at the surface.
Some typical values of the angle of internal friction and the cohesion \( C \) for common cohesive soils are given in Table 12, but actual values should be ascertained by test. According to this formula there is no pressure against the wall down to a depth \( \frac{2C}{w\sqrt{k_2}} \) below the surface if the nature of the clay is prevented from changing, but as this condition is unlikely to exist owing to the probability of moisture changes, it is essential that hydrostatic pressure should be assumed to act near the top of the wall. Formulae for the pressure of clays of various types and in various conditions are given in Table 12, together with the properties of these and other cohesive soils. In general, friction between the clay and the back of the wall should be neglected.

**Passive Resistance of Granular and Cohesive Materials.**

The remarks in the previous paragraph relate to the active horizontal pressure exerted by contained and retained materials.

If a horizontal pressure in excess of the active pressure is applied to the vertical face of a retained bulk of material, the passive resistance of the material is brought into action. Up to a limit, determined by the characteristics of the particular material, the passive resistance equals the applied pressure; the maximum intensity that the resistance can attain for a granular material with a level surface is given theoretically by the reciprocal of the active-pressure factor. The passive resistance of earth is taken into account when considering the resistance to sliding of a retaining wall, when dealing with the forces acting on sheet-piles, and when designing earth anchorages, but in these cases consideration must be given to those factors, such as wetness, that may reduce the probable passive resistance. Abnormal dryness may cause clay soils to shrink away from the surface of the structure, thus necessitating a small but most undesirable movement of the structure before the passive resistance can act.

For a dry granular material with level fill the passive resistance is given by the formula in Table 10; expressions for the passive resistance of water-logged ground are given on the page facing the table. It is not easy to assess the passive resistance when the surface of the material is not level, and it is advisable never to assume a resistance exceeding that for a level surface; when the surface slopes downwards the passive resistance should be neglected.

For ordinary saturated clay the passive resistance is given by the formula in Table 10 and the corresponding formulae for clay in other conditions are given in Table 12.

**Horizontal Pressures of Granular Materials in Liquid.**

The effect of saturated soils is considered in preceding paragraphs. The notes on the page facing Table 10 and the numerical values of some of the factors involved for certain materials as given in Table 11 apply to granular materials immersed in or floating in liquids.

**Deep Containers (Silos).**

The foregoing observations relating to the pressures on walls relate only to retaining walls and the walls of shallow containers. In deep containers, termed silos, the arching effect of the contained material considerably relieves the wall of some of the horizontal pressure, so that the principle that the pressure is proportional to the depth of filling is no longer true. The limiting condition that determines whether any particular container shall be considered as "shallow" or "deep" is that if the plane of rupture
strikes the opposite wall before reaching the free surface the container can be treated as "deep"; if otherwise the container must be considered as "shallow". These conditions are shown in the diagrams in Table 14, in which also is given the formula for determining the theoretical minimum depth of a "deep" container. For grain the minimum depth is theoretically equal to about one-and-a-quarter times the breadth of the container, and for cement it is equal to about one-and-a-half times the breadth. During the emptying process the arching action on which the reduction of pressure depends is partially or completely destroyed, and pressures appreciably in excess of the theoretically determined pressures are obtained. It would seem therefore that for absolute security no container that is not at least twice as deep as it is wide should be treated as a deep container.

**Pressures in Silos.**—The value and variation of the pressures in silos is usually computed by either Janssen's or Airy's formula. Janssen's formula gives pressures slightly lower than Airy's formula, and in its general form is given in Table 14. The horizontal pressure \( p_h \) depends on the weight \( w \) per cubic foot of material in the silo, the ratio \( R \) of plan area \( A \) to the perimeter of the silo, the angle of friction \( \mu \) between the contained material and the wall of the silo, the depth \( h \) of material above the plane considered, and a factor involving, among other terms, the ratio \( k \) of the horizontal to the vertical pressure; this ratio equals \( k = \frac{1 - \sin \theta}{1 + \sin \theta} \) for unconfined granular materials, but values for confined materials are given in Table 14, together with other properties of materials commonly stored in silos, and values of \( R \) for silos of common shapes.

The intensity of vertical pressure \( p_v \) on any horizontal plane of the material is \( \frac{p_h}{h} \). Therefore the total pressure on any horizontal plane is \( \frac{p_h A}{h} \), and the load transferred by friction to the walls of the silo is \( (wh - \frac{p_h}{h})R \) per unit length of wall.

For materials such as grain the value of \( k \) is usually about 0.5, and the value of \( \tan \mu \) for concrete walls is about 0.444. On the page facing Table 14 the magnitude of \( \frac{p_h}{w} \) for various values of \( R \) and \( h \), with these values of \( k \) and \( \tan \mu \), are given.

The value of \( k \) equal to 0.5 commonly adopted for reinforced concrete grain silos is on the safe side for the calculation of the horizontal pressure and load carried by the walls, but for the calculation of the vertical pressure on the bottom of the silo, a value of \( k \) equal to \( \frac{1}{3} \) is advisable. For ordinary coarse cement common values are \( w = 90 \text{ lb. per cubic foot}, k = 0.53 \) (assuming an angle of repose of 18 deg.), and \( \tan \mu = 0.577 \). In silos where the cement is pneumatically agitated, the friction between the wall and the cement, on which the reduced pressure depends, does not operate, and the walls of such silos must be designed for "fluid" pressures assuming a weight of, say, 75 lb. per cubic foot.

From the data on the page facing Table 14 it is seen that the pressure on the wall increases very little below a depth of three times the least width of the container. Tests indicate that the maximum pressures occur in silos during filling, and are somewhat greater for a silo filled rapidly than one filled slowly. The pressures \( p_h \) and \( p_v \), which vary with the depth of the filling (see the diagram in Table 14) and attain theoretical maximum values at infinite depth, approach the maximum values of \( p_{\text{max}} = \frac{wR}{\tan \mu} \) and \( p_{v\text{max}} = \frac{wR}{k \tan \mu} \). A practical and safe method is therefore to
design the upper part of the wall of a silo for a pressure that increases according to the formula for a shallow container, $p_h = k_2w\ell h$, until a depth is reached where $p_h = p_{\text{max}}$; below this depth the wall is designed for the pressure $p_{\text{max}}$, as shown in the diagram in Table 14.

Grain will not flow down a slope less steep than 33 deg. to the horizontal; therefore the angle in the valley of a pyramidal bottom of a grain silo should be not less than, say, 30 deg. The corresponding valley angle for clean coal is 45 deg. A method of calculating the valley slope is given on page 342.

Since the weight of barley is about 25 lb. per cubic foot the pressures due to this material are only about half the pressures due to wheat; therefore a silo designed to contain barley must not be filled with other grain. Likewise, any container designed to hold coal should not be filled with stone, and a silo designed to hold static cement may be insufficiently strong to hold cement fluidised by the injection of air. It is important also to ensure that the coefficient of friction between the stored material and the wall is not over-estimated when designing the bottom or under-estimated when designing the wall; some smooth linings installed after the silo has been constructed may invalidate assumptions made in the design.
SECTION 2

BENDING MOMENTS AND FORCES

The bending moments and shearing forces on freely-supported beams are determined readily from the simple statical rules but, for continuous beams and statically-indeterminate frames, one of several methods of analysis should be applied. The most suitable method depends mainly on the type of problem and, to a certain extent, on the appeal a certain method may have to the designer if two or more methods are otherwise equally suitable. The methods given in this section include mathematical methods such as the Theorem of Three Moments and Slope-deflection, and practical methods such as Moment distribution. Many of the tabulated results, the derivation of which are not necessarily given, are obtainable by application of more than one method.

The basic relation between the shearing force, bending moment, slope and deflection caused by a load in a structural member are given in Table 15, in which is also given typical diagrams of bending moments and shearing forces for cantilevers, propped cantilevers, freely-supported beams, and beams fixed or continuous at both supports.

In practice, the deflection of a reinforced concrete beam is restricted to a reasonable amount by limiting the ratio of the span to the depth of the member; the ratios for various types of beams and slabs are given in Table 15. Adoption of these or smaller ratios ensures that in general the efficiency of a structure is not impaired and that cracks are not likely to occur in applied finishes.

Cantilevers and Beams of One Span.

In Table 16 are tabulated the coefficients for maximum shearing forces (that is the reactions of the supports), the coefficients for the maximum bending moments, and the coefficients for the maximum deflections produced by common types of load on cantilevers and freely-supported beams of one span. Similar data are given in Table 17 (general cases) and Table 17a (special cases) for beams rigidly fixed at both ends and beams with one end fixed and one end freely supported (that is propped cantilevers). Notes on the derivation of the factors for propped cantilevers and fixed beams are given on the page facing Table 18.

The bending-moment factors for beams of one span fixed at both supports are the basis of the load-factors used in calculations in some methods of analysing statically-indeterminate structures, and such load-factors (which should not be confused with load-factors used in determining the resistances of members by ultimate-load methods) and notes relating to the methods to which they apply are given in Table 18.

Continuous Beams.

There are several methods of determining the bending moments and shearing forces on beams continuous over two or more spans; some of these methods are dealt with in the following and in the corresponding tables.

Calculated Bending Moments and Shearing Forces.—The bending moments on a beam continuous over two or more spans can be calculated by the Theorem of Three Moments which in its general form for any two contiguous spans is expressed
by the general and special formulae given in Table 19. Notes on the use of the formulae and calculation of the shearing forces are considered on the page facing Table 20; an example is given on the page facing Table 19. The formulae establish the values of the bending moments negative at the supports; the positive bending moment in the span can be obtained graphically or, in the case of uniformly-distributed loads, from the formulae given in Table 20.

It should be noted that the loading producing the greatest negative bending moments at the supports is not necessarily that producing the greatest positive bending moments in the span. The incidence of live load to give the greatest bending moments is illustrated in Table 20 and comments are given on the page facing the table. Some dispositions of live load may produce negative bending moments in adjacent unloaded spans and approximate formulae for calculating such bending moments are given in Table 20.

The moment of inertia of a reinforced concrete beam of uniform depth may vary throughout its length because of variations in the amount of reinforcement. It is common, however, to neglect these variations for beams of uniform depth and for beams having small haunches at the supports. Where the depth of a beam varies considerably, neglect of the variation of moment of inertia when calculating the bending moments leads to results that differ widely from the probable bending moments. Methods of dealing with beams of non-uniform moment of inertia are given in Table 19 and on the page facing Table 20. If the method of moment distribution is used, the coefficients and formulae in Table 33 are applicable.

Another method of determining the bending moments at the supports of beams continuous over equal or unequal spans is given in Table 25; explanatory notes of this method are given on the page facing Table 24 and an example of the use of Table 25 is given on the page facing the table. A graphical method based on "fixed points" is given in Table 24 and described on the page facing the table.

The shearing forces on a span of a continuous beam can be calculated from the basic formulae given in Table 20.

**Coefficients for Bending Moments and Shearing Forces for Equal Spans.**—For beams continuous over a number of equal spans, calculation of the maximum bending moments from basic formulae is unnecessary since the moment and shearing forces can be tabulated. For example, in Table 21 are given the values of the bending-moment coefficients for the middle of each span and at each support for two, three, four, and five or more continuous equal spans carrying identical loads on each span, which is the usual disposition of the dead load on a beam. The coefficients for the maximum bending moments at midspan and support for the most adverse incidence of live loads are also given; the alternative coefficients assuming only two spans to be loaded in the case of the bending moments at the supports are given in brackets. It should be noted that the maximum bending moments do not occur at all sections simultaneously. The types of load considered are a uniformly-distributed load, a single load concentrated at midspan, and equal loads at the two third-points of the span. The bending-moment diagrams for beams continuous over two or more spans are given in Tables 22 and 23. The theoretical bending moments may be adjusted to reduce the peak negative bending moments which can be reduced by 15 per cent. if the numerical value of the reduction is added to the positive bending moments in the two adjacent spans. This adjustment, which is convenient to reduce the inequality between the negative and positive moments, conforms to the recommendations in
B.S. Code No. 114; these recommendations permit an increase or decrease of 15 per cent. with consequent adjustment of the bending moments in the adjacent spans, the amount of the adjustment being different for interior spans and end spans; the adjusted bending-moment coefficients and diagrams are given in Tables 22 and 23. The basis of the diagrams in these tables is described below.

It is generally assumed that an ordinary continuous beam is freely supported on the end supports (unless fixity or other condition of restraint is specifically known), but in most cases the beam is constructed monolithically with the support, thereby producing some restraint; in Table 23 are given coefficients for this case.

The shearing forces produced by a uniformly-distributed load when all spans are loaded and the greatest shearing forces due to any incidence of live load are given in Table 21 for beams continuous over equal spans.

Approximate Bending-moment Coefficients.—The precise determination of the theoretical bending moments on continuous beams may involve much mathematical labour, except in cases which occur often enough to warrant tabulation. Having regard to the general assumptions of unyielding knife-edge supports and uniform moment of inertia, the probability of the theoretical bending moments being greater or less than those actually realised should be considered. The effect of variation of the moment of inertia is given on page 174. The following factors cause a decrease in the negative bending moment at a support: settlement of the support relative to adjacent supports causing an increase in the positive bending moments in the adjacent spans and may be sufficient to convert the bending moment at that support into a positive bending moment; supports of considerable width; support and beam constructed monolithically. The settlement of one or both of the supports on either side of a given support causes an increase in the negative bending moment at the given support and consequently affects the positive bending moments in adjacent spans.

The indeterminate nature of the actual bending moments occurring leads in practice to the adoption of approximate bending-moment coefficients for continuous beams and slabs of about equal spans with uniformly-distributed loads. Such coefficients are given in the lower part of Table 20; notes on the use of the coefficients are given on the page facing the table.

When the bending moments are calculated with the spans assumed to be equal to the distance between the centres of the supports, the critical bending moment in monolithic construction can be considered as that occurring at the edge of the support. When the supports are of considerable width the span can be considered as the clear distance between the supports plus the effective depth of the beam, or an additional span introduced equal to the width of the support minus the effective depth of the beam. The load on this additional span can be considered as the reaction of the support spread uniformly along the part of the beam over the support. When a beam is constructed monolithically with a very wide and massive support the effect of continuity with the span or spans beyond the support may be negligible, in which case the beam should be treated as fixed at the support. Other cases of beams being constructed monolithically with a support of moderate size, such as in ordinary beam and column construction, are considered in Tables 31 and 32, but in these cases the bending moment on the beam at the interior supports does not exceed the calculated bending moment, assuming knife-edge supports.

Bending-moment Diagrams for Equal Spans.—The basis of the bending-moment diagrams in Tables 22 (two and three spans) and 23 (four or more spans) is
as follows. The theoretical bending moments are calculated to obtain the coefficients for the bending moments near the middle of each span and at each support for a uniformly-distributed load, a central load, and loads concentrated at the third-points of each span. The condition of all spans loaded (for example, dead load) and conditions of incidental (or live) load producing the greatest bending moments are considered. As the coefficients are calculated by the Theorem of Three Moments (and are therefore not approximate), the decrease or increase of 15 per cent. of the maximum negative bending moments at the supports, as recommended in the B.S. Code, is permissible. The coefficients are reduced by 15 per cent. to establish the reduced bending moments at the supports, and are increased by 15 per cent. to derive the reduced positive bending moments in the spans. Tables 22 and 23 also give the coefficients for the positive bending moments at the supports and the negative bending moments in the spans which are produced under some conditions of live load; it is not generally necessary to take these small bending moments into account as they are generally insignificant compared with the bending moments due to dead load.

The method of calculating the adjusted coefficients is that the theoretical bending moments are calculated for all spans loaded (dead load), and for each of the four cases of live load that produce maximum bending moments, that is at the middle of an end span (positive), at an inner support (negative), at the middle of the interior span (positive), and at an inner support (positive). For each case, the theoretical bending-moment diagram is adjusted as described in the following. For the diagram of maximum negative bending moments, the theoretical negative bending moments at the supports are reduced by 15 per cent. and the positive bending moments are increased accordingly. For the diagram of maximum positive bending moments in the spans, the theoretical negative bending moments at the supports are increased by 15 per cent. and the positive bending moments are reduced accordingly. The diagrams for live load are combined to give the resultant diagrams for the theoretical bending moments without adjustment and with the greatest degree of adjustment.

**Moment Distribution applied to Continuous Beams.**

The bending moments at the supports of beams continuous over any number of equal or unequal spans, with any type of loading, and with different moments of inertia in each span, can be calculated directly by moment-distribution methods. The general principles of moment-distribution are given in Table 47, but the different sign convention used in Table 26 should be noted. Basic formulae for the bending moments at particular supports in systems of two spans and upwards are given in Table 26. The fundamental formula, which is that for any interior support T in four or more spans, is based on two distributing operations, and the results are reasonably accurate compared with those obtained by more tedious methods. All terms substituted in the tabulated formulae are positive numerical values. If the resultant sign of the support bending moment is negative, tension in the top edge of the beam is indicated. The formulae are applicable to all spans loaded, or to one or more spans only loaded.

Two conditions of end support are allowed for, namely, complete fixity and nominal free support. For any other conditions, such as a beam cantilevered beyond the end support or continuous with an exterior column, the support moments due to the loaded spans are first calculated assuming free support at the end support, and an adjustment is made to the penultimate support bending moment (as indicated in Table 26) for the externally-applied bending moment at the end support. The
externally-applied bending moment is the cantilever bending moment in the case of a beam extending beyond an end support. If the beam is monolithic with an interior column, the externally-applied bending moment can be calculated from Table 32, the bending moment at the end support being the externally-applied bending moment.

The stiffness of a beam freely supported at one support and fixed at the other is less than that of a beam fixed at both supports. In moment-distribution operations it is common to allow for the difference, but in the formula in Table 26 this difference has been neglected, thereby allowing the same expressions to be used for all distribution factors and fixed-end bending moments. Neglect of the theoretically reduced stiffness also makes allowance for the partial restraint which in most cases exists at nominally free supports and for restraint less than complete fixity at supports over which a beam is continuous.

Formulae for special conditions, such as equal spans, uniform moment of inertia, symmetrical inequality of spans, and the like for two, three, four and five or more spans are given in Tables 27, 28, 29 and 30. The effects of bending moments applied at end supports are given in Tables 31 and 32, and the effects of splay adjacent to the supports of beams are given in Table 33. Explanatory notes and the method of using Tables 26 to 33 are given on the pages facing Tables 27, 31, 32 and 33; some examples are given on the pages facing Tables 28, 29 and 30.

An example of the direct application of the principles of moment distribution to the calculation of the bending moments on a continuous beam, with the minimum use of the tables, is given on the page facing Table 30.

Since the bending moments given by the formulae in Tables 26 to 33 are approximate, as are in fact most calculations by moment-distribution except in the case of two continuous spans, the decrease of some peak moments (and the consequent increase of others) is not applicable.

End Restraint.—If, as is common in column-and-beam construction, the end of a continuous beam is subject to a restraining moment due to monolithic construction, the positive and negative bending moments and shearing forces are affected throughout the beam, but principally in the first two spans. The bending moments resulting from such monolithic action can be calculated from the formulae (based on moment distribution) given in Table 32, and the effects of the bending moments can be calculated from the formula in Table 31, in the upper part of which are given the comprehensive, but approximate, formulae in accordance with the method of moment distribution. In the lower part of Table 31 are given more exact expressions for equal spans. Coefficients are given by which the bending moments and shearing forces can be adjusted when a bending moment is applied at one end or at both ends of the beam.

Moving Loads on Continuous Beams.

Bending moments caused by moving loads, such as those due to vehicles traversing a series of continuous spans, are most easily calculated by the aid of influence lines. An influence line is a curve with the span of the beam as a base, the ordinates of the curve at any point being the value of the bending moment produced at a particular section of the beam when unit load acts at the point. The data given in Tables 34 to 37 enable the influence lines for the critical sections of beams continuous over two, three, four, and five or more spans to be drawn. By plotting the position of the load on the beam (drawn to scale), the bending moments at the section being considered
are derived as explained in the example given on the page facing Table 34. The curves in the tables for equal spans are directly applicable to equal spans but the corresponding curves for unequal spans should be plotted from the data tabulated.

The bending moment due to a load at any point is the ordinate of the influence line at that point multiplied by the product of the load and the span, the length of the shortest span being used when the spans are unequal. The influence lines in the tables are drawn for symmetrical inequality of spans. Coefficients for span-ratios not plotted can be interpolated. The symbols on each curve indicate the section of the beam and the ratio of spans to which the curve applies.

**Slabs spanning in One Direction.**

**Uniformly-distributed Load.**—The bending moments on slabs supported on two opposite sides are calculated in the same way as for beams, account being taken of continuity. For slabs carrying uniformly-distributed loads and continuous over nearly equal spans, the coefficients for dead and live load as given in Table 20 for slabs without splays conform to the recommendations of B.S. Code 114. Other coefficients, allowing for the effect of splays on the bending moments, are also tabulated. Spans are considered to be equal if the difference in length of adjacent spans does not exceed 15 per cent. of the shorter span.

If a slab is nominally freely supported at an end support, it is advisable to provide resistance to a probable negative bending moment at a support with which the slab is monolithic. If the slab carries a uniformly-distributed load, the value of the negative bending moment should be assumed to be not less than $\frac{1}{2}wL^2$.

Although a slab may be designed as though spanning in one direction, it should also be reinforced in a direction at right-angles to the span, as described on page 268.

**Concentrated Load.**—When a slab supported on two opposite sides only carries a load concentrated on a part only of the slab, such as a wheel load on the deck of a bridge, there are two principal methods of determining the bending moments. The first method is to assume that a certain width of the slab carries the entire load, and in one such method the contact area of the load is first extended by dispersion through the thickness of the slab as shown in Table 6, giving the dimensions of loaded area as $u$ at right-angles to the span and $v$ parallel to the span $L$. The width of slab carrying the load may be assumed to be $\frac{1}{2}(L + u) + v$. The total concentrated load is then divided by this width to give the load carried on a 1-ft. width of slab for the purpose of calculating the bending moments. The width of slab assumed to carry a concentrated load according to the recommendations of B.S. Code No. 114 is as illustrated in the lower part of Table 43.

Another method is to extend to slabs spanning in one direction the theory of slabs spanning in two directions. For example, the curves given in Table 41 for a slab infinitely long in the direction $L$, can be used to evaluate directly the bending moments in the direction of, and at right-angles to, the span of a slab spanning in one direction and carrying a concentrated load; an example of this application is given on the page facing Table 42.

**Slabs spanning in Two Directions.**

When a slab is supported otherwise than on two opposite sides only, the precise amount and distribution of the load taken by each support, and consequently the
magnitude of the bending moments on the slab, are not easily calculated if assumptions resembling practical conditions are made. Therefore approximate analyses are generally used. The method applicable in any particular case depends on the shape of the panel of slab, the condition of restraint at the supports, and the type of load. The principal bases of the analyses of slabs spanning in two directions are the theory of plates and the yield-line theory.

Distinction must be made between the conditions of free support, fixity, partial restraint, and continuity, and it is essential to establish whether the corners of the panel are free to lift or not. Free support occurs rarely in practice, since in ordinary reinforced concrete beam-and-slab construction, the slab is monolithic with the beams and is thereby partially restrained and is not free to lift at the corners. The condition of being freely supported may occur when the slab is not continuous and the edge bears on a brick wall or on un-encased structural steelwork. If the edge of the slab is built into a substantial brick or masonry wall, or is monolithic with concrete encasing steelwork or with a reinforced concrete beam or wall, partial restraint exists. Restraint is allowed for when computing the bending moments on the slab but the supports must be able to resist the torsional and other effects induced therein; the slab must be reinforced to resist the negative bending moment produced by the restraint. Since a panel of slab freely supported along all edges but with the corners held down is uncommon (because corner restraint is generally due to edge-fixing moments), bending moments for this case are of interest mainly for their value in obtaining coefficients for other cases of fixity along or continuity over one or more edges. A slab can be considered as fixed along an edge if there is no change in the slope of the slab at the support irrespective of the incidence of the load. This condition is assured if the polar moment of inertia of the beam or other support is very large. Continuity over a support generally implies a condition of restraint less rigid than fixity, that is the slope of the slab at the support depends upon the load not only on the panel under consideration but on adjacent panels.

Rectangular Panel with Uniformly-distributed Load.—Empirical formulae and approximate theories have been put forward for calculating the bending moments in the common case of a rectangular panel of slab supported along four edges (and therefore spanning in two directions mutually at right-angles) and carrying a uniformly-distributed load. The bending moments depend on the ratio of the length of the sides of the panel. The "exact" theory of the bending of plates spanning in two directions was established by Lagrange and Navier in the nineteenth century. Pigeaud and others later completed the analysis of panels freely supported along all four edges. Because most theoretical expressions are complex, curves or close mathematical approximations are generally adopted in practice. Westergaard has combined theory with the results of tests and his work is the basis of the bending-moment coefficients which are recommended in B.S. No. 114 and which are represented by the curves in Table 39; these curves apply to freely-supported panels and to panels where there is continuity on one or more sides.

The simplified analysis of Grashof and Rankine can be applied when the corners of a panel are not held down; the bending-moment coefficients are given in Table 38 and the basic formulae are given on the page facing the table. If corner restraint is provided, coefficients based on more exact analyses should be applied; such coefficients for a panel freely supported along four sides are given in Table 38. It has been shown by Marcus that for panels, the corners of which are held down, the midspan bending
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moments obtained by the Grashof-and-Rankine method can be converted to approximately those obtained by more exact theory by multiplying by a simple factor, and this method is applicable not only for conditions of free support along all four edges but for all combinations of fixity on one to four sides with free support along the other edges; the bending moments at the supports are calculated by an extension of the Grashof-and-Rankine method but without the adjusting factors. The Marcus factors for a panel fixed along four edges are given in Table 38 and these and the Grashof-and-Rankine coefficients are substituted in the formula given in the table to obtain the midspan bending moments and the bending moments at the supports.

If the corners of a panel are held down, reinforcement should be provided to resist the tensile stresses due to the torsional strains. The amount and position of the reinforcement required for this purpose, as recommended in B.S. Code No. 114, are given in Table 38. No reinforcement is required at a corner formed by two intersecting supports if the slab is monolithic with the supports.

At a discontinuous edge of a slab monolithic with its support, resistance to negative bending moment must be provided, and the expressions given in the centre of Table 38 give the magnitude, in accordance with the B.S. Code No. 114, of this bending moment, which is resisted by reinforcement at right-angles to the support. The Code also recommends that no reinforcement is required in a narrow strip of slab parallel and adjacent to each support; particulars of this recommendation are also given in Table 38, the coefficients for use in which are taken from Table 39.

Examples of the use of Tables 38 and 39 are given on the page facing Table 39.

If a panel is truly freely supported along one or more edges, the approximate method described on the page facing Table 41 may be applied; this is based on the use of the basic coefficients given in Table 38.

The shearing forces on rectangular panels spanning in two directions and carrying a uniformly-distributed load are considered briefly on the page facing Table 39.

Rectangular Panels with Triangularly-distributed Loads.—In the design of rectangular tanks, storage bunkers and some retaining structures, cases occur of walls spanning in two directions and subjected to triangularly-distributed pressures. The intensity of pressure is uniform at any given level, but vertically the pressure varies from zero near the top to a maximum at the bottom. If there is a support along the top edge of the panels, the probable maximum bending moments on vertical and horizontal strips of unit width are given by the coefficient and expressions in the lower half of Table 40. For fixity along the top, the curves are derived from basic data given by Timoshenko for the exact theory of plates with Poisson’s ratio equal to 0.3. For free support along the top, the curves are interpolated from the basic data. The approximate positions of the maximum bending moments are also indicated. If there is no support along the top of the panel, the bending moment can be determined from the curves in the upper half of Table 40; these curves are based on an analysis by Buchi.

If Poisson’s ratio is assumed to be less than 0.3 the bending moments would be slightly less, but the effect of corner splays, which is to increase slightly the negative bending moments, is ignored. In deriving the curves for positive bending moments it has been assumed that only two-thirds full fixity exists at edges over which the slab is continuous. Further comments on the curves and data and an example are given on the page facing Table 40.
Beams supporting Rectangular Panels.

When designing the beams supporting a panel freely supported along all four edges or with the same degree of fixity along all four edges, it is generally accepted that each of the beams along the shorter edges of the panel carries the load on an area having the shape of a 45-deg. isosceles triangle with a base equal to the length of the shorter side, that is each beam carries a triangularly-distributed load; half the remaining load, that is the load on a trapezium, is carried on each of the beams along the longer edges. In the case of a square panel, each beam carries one-quarter of the total load on the panel, the load on each beam being distributed triangularly. The diagram and expressions in the top left-hand corner of Table 41 give the amount of load carried by each beam. Bending-moment coefficients for beams subjected to triangular and trapezoidal loading are given in Tables 16, 17, and 17A. The formulae for equivalent uniformly-distributed loads given on the page facing Table 41 apply only to the case of the span of the beam being equal to the width or length of the panel.

An alternative method is to divide the load between the beams along the shorter and longer sides in proportion to $K_B$ and $K_L$ (Table 38) respectively. Thus the load transferred to each beam along the shorter edges is $\frac{1}{2}K_{BW}wL_BL_L$, triangularly distributed, and to each beam along the longer edges is $\frac{1}{2}K_{BW}wL_BL_L$, trapezoidally distributed. For square panels the loads on the beams by both methods are identical.

When the panel is fixed or continuous along one, two, or three supports and freely supported on the remaining edges, the subdivision of the load to the various supporting beams can be determined from the diagrams and expressions on the left-hand side of Table 41. Alternatively the loads can be calculated approximately as follows. For the appropriate value of the ratio $K_e$ of the equivalent spans (see page facing Table 41), determine the corresponding values of $K_B$ and $K_L$ from Table 38. Then the load transferred to each beam parallel to the longer equivalent span is $\frac{1}{4}K_{BW}wL_BL_L$ and to each beam parallel to the shorter equivalent span is $\frac{1}{2}K_{BW}wL_BL_L$. Triangular distribution can be assumed in both cases, although this is a little conservative for the load on the beams parallel to the longer actual span. For a span freely supported at one end and fixed at the other, the foregoing loads should be reduced by about 20 per cent. for the beam along the freely-supported edge and the amount of the reduction added to the load on the beam along the fixed or continuous edge.

If the panel is unsupported along one edge or two adjacent edges, the loads on the beams supporting the remaining edges are as given on the right-hand side of Table 41.

Rectangular Panels with Concentrated Load.

The curves in Table 42, based on M. Pigeaud’s theory, can be used directly for calculating the bending moment on a panel freely supported along all four edges with restrained corners and carrying a load uniformly distributed over a defined area symmetrically disposed upon the panel. Wheel loads and similarly highly-concentrated loads are dispersed through the road finish (if any) down to the surface of the slab, or farther down to the reinforcement, as shown in Table 6, to give the dimensions $u$ and $v$ and thence the ratios $\frac{u}{L_B}$ and $\frac{v}{L_L}$, for which the bending moments $m_1$ and $m_2$ for unit load are read off the curves for the appropriate value of the ratio.
of spans \(k\). For a total load of \(W\) on the area \(u\) by \(v\), the bending moments in ft.-lb. per foot width of slab are given by the expressions in Table 42, in which the value of Poisson's ratio is assumed to be 0.15. The positive bending moments calculated from Table 42 for the case of a uniformly-distributed load over the whole panel (that is \(u = v = \text{unity}\)) do not coincide with the bending moments based on the corresponding coefficients \(K'_B\) and \(K'_L\) given in Table 38 unless Poisson's ratio is assumed to be zero, as is commonly recommended.

The curves in Table 42 are drawn for \(k = 1.0, 1.25, \sqrt{2} \approx 1.4\) (approximately), 1.67, 2.0, 2.5, and infinity. For intermediate values of \(k\), the values of \(m_1\) and \(m_2\) can be interpolated from the values above and below the given value of \(k\). The curves for \(k = 1.0\) apply to a square panel.

The curves for \(k = \infty\) apply to a panel of great length \((L_L)\) compared with the short span \((L_B)\) and can be used for determining the transverse (main reinforcement) and longitudinal (distribution reinforcement) bending moments on a long narrow panel supported on the two long edges only. Alternatively the data at the bottom of Table 43 can be applied to this case which is really a special extreme case of a rectangular panel spanning in two directions and subjected to a concentrated load.

When there are two concentrated loads symmetrically disposed or an eccentric load, the resulting bending moments can be calculated from the rules given for the various cases in Table 43. Other conditions of loading, for example, multiple loads the dispersion areas of which overlap, can generally be treated by combinations of the particular cases considered. Case I is an ordinary symmetrically-disposed load. Case VI is the general case for a load in any position, from which the remaining cases are derived by simplification.

The bending moments derived directly from Table 42 are those at midspan of panels freely supported along all four edges but with restraint at the corners. If the panel is fixed or continuous along all four edges, M. Pigeaud recommends that the midspan bending moments are reduced by 20 per cent. The estimation of the bending moment at the support and midspan sections of panels with various sequences of continuity and free support along the edges can be dealt with by applying the following rules, which possibly give conservative results when incorporating Poisson's ratio equal to 0.15; they are applicable to the common conditions of continuity with adjacent panels over one or more supports, and monolithic construction with the supports along the remaining edges. Find \(m_1\) and \(m_2\) from the curves in Table 42 for the appropriate value of \(k = f_1 \frac{L_L}{L_B}\), where \(f_1\) is obtained from Table 43. For similar conditions of support on all four sides, that is Cases (a) and (j), or for a symmetrical sequence as in Case (f), \(f_1 = 1.0\); therefore the actual value of \(\frac{L_L}{L_B}\) is used in these cases. If in Cases (b), (d), and (h) the value of \(f_1 \frac{L_L}{L_B}\) is less than unity, \(L_L\) and \(L_B\) (and consequently \(u\) and \(v\)) should be transposed throughout the calculation of \(m_1\) and \(m_2\). Having found the bending moments in each direction with the adjusted values of \(\frac{L_L}{L_B}\), the bending-moment reduction factors for continuity given in Table 43 are applied to give the bending moments for the purpose of design.

Examples of the use of Tables 42 and 43 are given on the pages facing Tables 42, 43 and 44.
The maximum shearing forces $Q$ per unit length on a panel carrying a concentrated load are given by M. Pigeaud as follows:

$$u > v. - \text{At the centre of length } u, \quad Q = \frac{W}{2u + v}.$$  
$$v > u. - \text{At the centre of length } v, \quad Q = \frac{W}{3v}.$$  

To determine the load on the supporting beams, the rules given for a uniformly-distributed load over the entire panel are sufficiently accurate for a load concentrated at the centre of the panel, but this is not always the critical case for live loads, such as a load imposed by a wheel on a bridge deck, since the maximum load on a beam occurs when the wheel is passing over the beam, in which case the beam carries the whole load.

**Non-rectangular Panels.**

When a panel which is not rectangular is supported along all its edges and is of such proportions that main reinforcement in two directions seems desirable, the bending moments can be determined approximately by the rules given in Table 44, which apply to a trapezoidal panel approximately symmetrical about one axis, to a panel which in plan is an isosceles triangle (or very nearly so), and to panels which are regular polygons or are circular. The case of a triangular panel continuous or partially restrained along three edges occurs in pyramidal hopper bottoms (Table 99); the reinforcement calculated by the expressions for this case should extend over the entire area of the panel, and provision must be made for the negative moments and for the direct tensions which act simultaneously with the bending moments.

If the shape of a panel approximates to a square, the bending moments for a square slab of the same area should be determined. A slab having the shape of a regular polygon with five or more sides can be treated as a circular slab the diameter of which is the mean of the diameters of the inscribed and circumscribed circles; the mean diameters for regular hexagons and octagons are given in Table 44.

For a panel which is circular in plan and is freely supported along the circumference and carries a load concentrated symmetrically about the centre on a circular area, the total bending moment which should be provided for across each of two diameters mutually at right-angles is given by the expression in Table 44. If the panel is restrained around the circumference, the positive bending moment can be reduced to about two-thirds of this value if the load approximates to a "point" load, or to 80 per cent. if the load covers practically the entire panel. The negative bending moment provided for around the edge should be not less than one-third of the "free" positive bending moment. The foregoing are the bases of the corresponding expressions in Table 44. If the degree of restraint is equivalent to complete fixity, the average and maximum positive bending moment can be reduced to two-thirds or less of the "free" positive bending moment if the load covers a considerable area of the entire panel, but the negative bending moment provided for should be not less than two-thirds of the "free" positive bending moment.

If the load covers the entire panel, the bending moments provided for should be not less than those given by the expressions given at the bottom of Table 44. The
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data for the various conditions are based on the following analysis. The total positive bending moment across a diameter of a freely-supported panel of diameter \( D \) ft., carrying a uniformly-distributed load of intensity \( w \) lb. per square foot, is \( \frac{wD^3}{24} \) ft.-lb.

Therefore the average positive bending moment is \( \frac{wD^3}{24} \) ft.-lb. per foot of diameter. The positive bending moment, however, varies from zero at the extremities of the diameter to a maximum at the centre of the panel, where it may be, say, one-and-a-half times the average bending moment. If the panel is partially restrained around the edges, the average and maximum positive bending moments may be reduced to about 80 per cent. of the corresponding “free” positive bending moment, but provision should be made for a negative bending moment around the edge of not less than one-third of the average “free” bending moment. If the panel is completely fixed around the edge, the average and maximum positive bending moments should be not less than one-third of the corresponding “free” positive bending moments, and the negative bending moment provided for around the circumference should be not less than two-thirds of the average “free” positive bending moment.

A circular panel can therefore be designed by one of the following methods.

(i) Design for the maximum positive bending moment at the centre of the panel and reduce the amount of reinforcement or the thickness of the slab towards the circumference; if the panel is not truly freely supported, provide for the negative bending moment acting around the circumference. (ii) Design for the average positive bending moment across a diameter and retain the same thickness of slab and amount of reinforcement throughout the entire panel; if the panel is not truly freely supported around the circumference, provide for the appropriate negative bending moment.

The reinforcement required for the positive bending moments in both the preceding methods must be provided in two directions mutually at right-angles; the reinforcement for the negative bending moment should be provided by radial bars normal to, and equally spaced around, the circumference, or reinforcement equivalent thereto should be provided.

Flat Slabs.

The design of flat slabs, that is beamless slabs or mushroom floors, is based primarily on empirical considerations. The principles which follow and the data summarised in Table 45 and on the page facing the table are in accordance with the empirical method described in B.S. Code No. 114. This type of floor can incorporate drop-panels at the columns or the slab can be of uniform thickness throughout. The tops of the columns may be plain or may be provided with a splayed head having the dimensions indicated in Table 45.

The lengths (or widths) of adjacent panels should not differ by more than 10 per cent. of the greater length (or width). The ratio of the longer to the shorter dimension of a panel which is not square should not exceed 1:4. The length of the drop in any direction should be not less than one-third of the length of the panel in the same direction.

For the purpose of determining the bending moments, the panel is divided into “middle strips” and “column strips” as shown in the diagram on the page facing Table 45, the width of each strip being half the length or width of the panel. If drop-panels of less width than half the panel length or width are provided, the width
of the column strip can be reduced to the width of the drop-panel and the middle strip increased accordingly.

The thicknesses of the slab and the drop-panels must be sufficient to provide resistance to the shearing forces and bending moments, but should be not less than 5 in. nor less than the thickness conforming to the span-thickness ratios on the page facing Table 45.

**Bending Moments.**—For the calculation of the bending moments, the effective spans are \( L_1 - \frac{3}{4}D \) and \( L_2 - \frac{1}{4}D \), where \( L_1 \) and \( L_2 \) are the shorter and longer dimensions respectively of the panel and \( D \) is the diameter of the column or column-head if one is provided. The total bending moments to be provided for at the principal sections of the panel are given in Table 45 and are functions of the effective spans.

Walls and other concentrated loads must be carried on beams, and beams should be provided around openings other than small holes; the Code recommends limiting sizes of openings permissible in the column strips and middle strips.

**Reinforcement.**—It is generally most convenient for the reinforcement to be arranged in bands in two directions, one parallel to each of the spans \( L_1 \) and \( L_2 \). Alternatively the bars should be arranged in two bands parallel to the spans and two diagonal bands, but this method produces congestion of reinforcement in relatively thin slabs.

With two-way reinforcement, 40 per cent. of the bars in the positive-moment reinforcement should remain in the bottom of the slab and extend at least over a length at the middle of the span equal to three-quarters of the span. No reduction of the positive-moment reinforcement should be made within a length of \( 0.5L \) at the middle of the span. No reduction of the negative-moment reinforcement should be made within a distance of \( 0.2L \) from the centre of the support. The negative-moment reinforcement should extend into the adjacent panel for an average distance of \( 0.25L \); if the ends of the bars are staggered the shortest should extend for a distance of \( 0.2L \). The amount of reinforcement in any section should not exceed 1 per cent. of the product of the width of the section and the effective depth.

**Shearing Force.**—The shearing stresses, which should not exceed the values given in Table 57 for conditions when no shearing reinforcement is provided, should be investigated at two sections, namely, around the edge of the drop and around the head of the column. For the latter the critical plane for shearing resistance is at a distance equal to half the total thickness of the slab, or drop, from the edge of the column-head as shown in the diagrams in Table 45.

**Alternative Analysis.**—A less empirical method of analysing flat slabs is described in B.S. Code No. 114, and is applicable to cases not covered by the foregoing rules. The bending moments and shearing forces are calculated by assuming the structure to comprise continuous frames, transversely and longitudinally.

**Framed Structures.**

A structure is statically determinate if the forces and bending moments can be determined by the direct application of the principles of statics. A cantilever, whether a simple bracket or the roof of a grandstand, a freely-supported beam, a truss with pin-joints, a three-hinged arch or frame, are some examples. A statically-indeterminate structure is one in which there is a redundancy of members or supports or both, and which can only be analysed by considering the elastic deformation under load. Restrained beams, continuous beams, portal frames, and other non-triangulated
structures with rigid joints, and two-hinged and fixed-end arches, are examples of statically-indeterminate structures, which can be analysed by the application of one or other of several methods, which include slope-deflection, column analogy, moment distribution, the theorem of least work, and, of more recent development, the displacement method and the ultimate-load method; reference should be made to books dealing fully with each of these methods. Most designers find that one method seems more simple to them than others, and it is therefore generally better for each designer to use the method he prefers; the same results should be obtained irrespective of the method used.

In this section the analysis of primary frames by the methods of slope-deflection and moment-distribution is described. Most analyses of complex rigid frames require an amount of calculation often out of proportion to the real accuracy of the results, and some approximate solutions are therefore given for common cases of building frames and similar structures.

Slope-deflection.—The principles of the slope-deflection method of analysing a restrained member are given in Table 46, and on the page facing the table, in which also are given the basic formulae and the formulae for the bending moments in special cases.

When there is no deflection of one end of the member relative to the other (for example, when supports which are not elastic are assumed), and when the ends of the member are either hinged or fixed, and when the load is symmetrically disposed, the general expressions are simplified and the resulting formulae for the more common cases of restrained members are given in Table 46.

The bending moments on a framed structure are determined by applying the formulae to each member successively. The algebraic sum of the bending moments at any joint equals zero. When it is assumed that there is no deflection (or settlement) \( d \) of one support relative to the other, there are as many formulae for the restraint moments as there are unknowns, and therefore the restraint moments and the slopes at the ends of the members can be evaluated. For symmetrical frames on unyielding foundations and carrying symmetrical vertical loads it is common to neglect the change in the position of the joints due to the small elastic contractions of the members, and the assumption of \( d = 0 \) is reasonably accurate. If the foundations or other supports settle unequally under the load, this assumption is not justified and a value must be assigned to the term \( d \) for the members affected.

If a symmetrical or unsymmetrical frame is subjected to a horizontal force the sway produced involves lateral movement of the joints. It is common in this case to assume that there is no elastic shortening of the member. Sufficient formulae to enable the additional unknowns to be evaluated are obtained by equating the reaction normal to the member, that is the shearing force on the member, to the rate of change of bending moment. Sway cannot be neglected when considering unsymmetrical frames subject to vertical loads, or any frame on which the load is unsymmetrically disposed.

An example of the application of the slope-deflection formulae to a simple problem is given on the page opposite Table 46.

Shearing Forces on Members of a Frame.—The shearing forces on any member forming part of a frame can be determined when the bending moments have been found by considering the rate of change of the bending moment. The uniform shearing force on a member AB due to end restraint only is \( \frac{M_{AB} + M_{BA}}{L_{AB}} \), account
being taken of the signs of the bending moments. Thus if both restraint moments are clockwise, the shearing force is the numerical sum of the moments divided by the length of the member. If one restraint moment acts in a direction contrary to the other, the numerical difference is divided by the length to give the shearing force. For a member with end B hinged, the shearing force due to the restraint moment at A is $\frac{M_{AB}}{L_{AB}}$. The variable shearing forces due to the loads on the member should be algebraically added to the uniform shearing force due to the restraint moments, in a manner similar to that shown for continuous beams in Table 20.

**Portal Frames**—A common type of simple frame used in buildings is the portal frame with either a horizontal top member or two inclined top members meeting at the ridge. In Table 49, general formulae for the bending moments at both ends of the columns, and at the ridge in the case of frames of that type, are given together with expressions for the forces at the bases of the columns. The formulae relate to any vertical or horizontal load and to frames fixed or hinged at the base of the columns. In Table 50 are given the corresponding formulae for special conditions of loading on frames of one bay.

Frames of the foregoing types are statically indeterminate, but a frame with a hinge at the base of each column and one at the ridge, that is a three-hinged frame, can be readily analysed. Formulae for the forces and bending moments are given in Table 48 for three-hinged frames. Approximate expressions are also given for certain modified forms of these frames, such as when the ends of the columns are embedded in the foundations and when a tie-rod is provided at eaves level.

**Method of Moment-distribution.**—The analysis of frames by moment-distribution methods is described, with an example, in Table 47. Similar principles are applied to the solution of continuous beams in Tables 26 et seq., but the different convention of signs adopted in the two cases should be noted.

**Bending of Columns.**

**External Columns.**—Provision should be made for the bending moments produced on the columns due to the rigidity of the joints in monolithic beam-and-column construction of buildings.

The external columns of a building are subjected to a greater bending moment than the internal columns (other conditions being equal), the magnitude of the bending moment depending on the relative stiffness of the column and beam and on the end conditions of the members. The two principal cases for exterior columns are when the beam is supported on the top of the column, as in a top story, and when the beam is fixed to the column at an intermediate point, as in intermediate stories. The second case is shown in the diagrams in Table 46. Since either end of the column or the end of the beam remote from the column can be hinged, fixed, or partially restrained, there are many possible combinations.

For the first case the maximum reverse moment at the junction of the beam and column occurs when the far end of the beam is hinged and the foot of the column is fixed. The minimum reverse moment at the junction occurs when the beam is rigidly fixed at the far end and the column is hinged at the foot. Conditions in practice generally lie between these extremes, and with any condition of fixity of the foot of the column the bending moment at the junction decreases as the degree of fixity at the far end of the beam increases. With any degree of fixity at the far end of the
beam the bending moment at the junction increases very slightly as the degree of
fixity at the foot of the column increases.

The maximum reverse moment on the beam at the junction with the column in
the second case occurs when the beam is hinged at the far end and the column is
perfectly fixed at the top and the bottom as indicated in Table 46. With perfect
fixity at the far end of the beam and hinges at the top and bottom of the column, as
also shown in Table 46, the reverse moment on the beam at the junction is a minimum.
Intermediate cases of fixity follow the following rules: increase in fixity at the end
of the beam decreases the bending moment at the junction; decrease in fixity at either
the top or the bottom of the column decreases the bending moment at the junction,
and vice versa.

Formulæ for the maximum and minimum bending moments are given in Table 46
for a number of frames of a single bay. The bending moment on the beam at the
junction is divided between the upper and lower columns in the ratio of their stiffness
factors $K$ when conditions at the ends of the two columns are identical. When the
end of one column is hinged and the other fixed, the ratio of the bending moments
allocated to each column is in accordance with the expression

$$\frac{\text{Bending moment on hinged portion}}{\text{Bending moment on fixed portion}} = \frac{0.75K}{K} \text{ for hinged portion}.$$

External Columns in Building Frames.—For the framework of ordinary
buildings the bending moments on the external columns can be computed from the
formulæ given in the lower part of Table 46. These expressions, which conform to
those recommended in the B.S. Code, are only approximate as lack of uniformity in
the end conditions of the beams or columns are neglected, but the formulæ are suitable
for use in the design of ordinary buildings.

When there is no upper column (that is when the structure is only one story high
or when the top story is being considered), the stiffness factor $K_D$ is equal to zero.

Corner Columns.—A column at an external corner of a building is generally
subjected to bending moments from beams in two directions at right-angles. These
bending moments can be calculated by considering two frames (also at right-angles)
independently, but practical methods of computing the stresses depend upon the
relative magnitude of the bending moments and the direct load (see page 86).

Internal Columns.—There is less variation in the bending moments due to
continuity between the beams and the internal columns of a building than is the case
with external columns. End-fixing conditions of the various members do not affect
the bending moments so seriously, and the bending moment on a beam at the junction
with a column is less than the bending moment at the support when computed by,
say, the Theorem of Three Moments neglecting fixity with the column. In Table 46
expressions are given for the bending moments in the upper and lower internal columns.
In this case the value of $\frac{A}{L}$, when the spans are equal, should be for the live load only
on one of the adjacent spans of the beam. When the spans are unequal, the greatest
bending moments on the column occur when the value of $M_{es}$ (see Table 46) is greatest,
which is generally when the longer beam is loaded with live and dead load while the
shorter beam is loaded with dead load only.

Approximate Methods.—The methods hitherto described for evaluating the
bending moments in column-and-beam construction with rigid joints involve a fair
amount of calculation, including that of the moments of inertia of the members. In
practice, and especially in the preparation of preliminary schemes, time is not always available to make these calculations, and therefore approximate methods are of value. Designs should be checked by more accurate methods.

For large columns and light beams, the effect on the column of the load on the beam is not great, and in such cases the difference between the permissible compressive stress for direct compression and for bending combined with direct compression (see Table 57) is generally sufficient to enable the preliminary design of the column to be based on the direct load only. Where the effect of the beam on the column is likely to be considerable, and in order to allow a margin for the bending stresses in the column, the column can be designed provisionally for the direct load increased, to allow for the effects of bending, by the amounts shown on the page facing Table 51 for the particular arrangement of beams supported by the column.

**Bending Moments due to Wind.**

In exposed structures such as water towers, bunkers and silos, and the frames of tall narrow buildings, the columns must be designed to resist the effects of wind. When conditions do not warrant a close analysis of the bending moments to which a frame is subjected due to wind or other horizontal forces, the methods described in the following and illustrated in Table 51 are sufficiently accurate.

**Braced Columns.**—For braced columns (of the same cross-section) forming an open tower such as that supporting an elevated water tower, the expressions at (a) in Table 51 give the bending moments and shearing forces on the columns and braces due to the effect of a horizontal force at the head of the columns. The increase or decrease of direct load on the columns is also given.

In general, the bending moment on the column is the shearing force on the column multiplied by half the distance between the braces. If a column is not continuous or is insufficiently braced at one end, as at unconnected foundations, the bending moment has twice this value.

The bending moment on the brace at an external column is the sum of the bending moments on the columns at the intersection with the brace. The shearing force on the brace is equal to the change of bending moment from one end of the brace to the other divided by the length of the brace. These shearing forces and bending moments are additional to those created by the dead weight of the brace and any external loads to which it may be subjected.

The overturning moment on the brace causes an additional direct load on the leeward column and a corresponding relief of load on the windward column, the maximum value of this direct load being approached at the foot of the column and being equal to the overturning moment divided by the distance between the centres of the columns.

The expressions in Table 51 for the effects on the columns and for the bending moments on the braces apply whether the columns are vertical or at a slight inclination. If the columns are inclined, the shearing force on a brace is $2M_B$ divided by the length of the brace being considered.

**Columns supporting Massive Superstructures.**—The case illustrated at (b) in Table 51 is common in bunkers and silos where a superstructure of considerable rigidity is carried on comparatively short columns. If the columns are fixed at the base, the bending moment on a single column is $\frac{Ph}{2N}$, where $N$ is the number of columns if they are all of the same size; the signification of the other symbols is given in Table 51.
BENDING MOMENTS AND FORCES

If the columns are of different sizes, since each column is deflected the same amount, the total shearing force should be divided among the columns in any one line in proportion to their separate moments of inertia. If \( N_1 \) is the number of columns with moment of inertia \( I_1 \), \( N_2 \) the number of columns with moment of inertia \( I_2 \), etc., the total moment of inertia is \( N_1I_1 + N_2I_2 + \ldots = \Sigma I \). On any column having a moment of inertia \( I_n \), the bending moment is \( \frac{P_I I_n}{2\Sigma I} \) as given in diagram (b) in Table 51. Alternatively, the total horizontal shearing force can be divided among the columns in the ratio of their cross-sectional areas (thus giving uniform shearing stress) and with this method the formula for the bending moment on any column with cross-sectional area \( A_n \) is \( \frac{P_A A_n}{2\Sigma A} \), where \( \Sigma A \) is the sum of the cross-sectional areas of all the columns resisting the total shearing force \( P \).

Building Frames.—In the frame of a multiple-story building, the effect of the wind may be small compared with that of other loads, and in this case it is sufficiently accurate to divide the horizontal shearing force on the basis that an external column resists half the shearing force on an internal column. If \( N_T \) is the total number of columns in one frame, in the plane of the lateral force \( P \), the effective number of columns is \( N_T - 1 \) for the purpose of calculating the bending moment on an interior column, the two external columns being equivalent to one internal column; see diagram (c) in Table 51. In a building frame subjected to wind pressure, the pressure on each panel (or story height) \( P_1, P_2, P_3, \) etc., is generally divided into equal shearing forces at the head and base of each story-height of columns. The shearing force at the base of any interior column, \( n \) stories from the top, is \( \frac{\Sigma P + \frac{P_n}{n}}{N_T - 1} \), where

\[ \Sigma P = P_1 + P_2 + P_3 + \ldots + P_{n-1}. \]

The bending moment is the shearing force multiplied by half the story-height.

A bending moment and a corresponding shearing force are caused on the floor beams in the same way as on the braces of an open tower. At an internal column the sum of the bending moments on the two beams meeting at the column is equal to the sum of the bending moments at the base of the upper column and at the head of the lower column.

Earthquake-resistant Structures.

Opinions may differ on whether structures to withstand the disruptive forces of earth tremors and quakes should be designed as rigid or flexible or semi-flexible, but, generally, a rigid construction seems to be favoured. The effect of an earth tremor is equivalent to a horizontal thrust additional to the loads and wind pressures for which the building is commonly designed. There are codes for earthquake-resistant construction in several countries, and recent codes seem more complex than earlier requirements. The most simple consideration is as follows.

The dead and live loads should be increased by 20 per cent. to allow for vertical movement. The magnitude of the horizontal thrust depends on the acceleration of the earth tremor, which may vary from less than 3 ft. per second per second in firm compact ground to over 12 ft. per second per second in alluvial soil and filling. A horizontal thrust equal to about one-tenth of the mass of the building seems to be sufficient for all but major shocks when the building does not exceed 20 ft. in height, and equal to one-eighth of the mass when the building is of greater height. The
horizontal shearing force on the building at any level is one-eighth (or one-tenth) of the total weight of the structure (including live loads) above this level. The analysis of the bending moments and shearing forces on the columns and floor beams is similar to that described for wind pressure on building frames in Table 51.

In order that the structure shall act as a unit, all parts should be effectively bonded together. Panel walls, finishes, and ornaments should be permanently attached to the frame, so that in the event of a shock they will not collapse independently of the main structure. Separate column footings should be connected by ties designed to take a thrust or a pull of one-tenth of the load on the heavier of the two footings connected.

Properties of Members of a Frame.

End Conditions.—Since the results given by the more precise methods of frame analysis vary considerably with different degrees of restraint at the ends of the members, it is essential that the end conditions assumed should be reasonably obtained in the actual construction. Absolute fixity is difficult to attain unless the beam or column is embedded monolithically in a comparatively large mass of concrete. Embedment in a brick or masonry wall represents more nearly the condition of a hinge, and should be considered as such. The ordinary type of separate foundation, designed only for the limiting uniform ground pressure under the direct load on a column, should also be considered as a hinge at the foot of the column. A continuous beam supported on a beam or column is only partly restrained, and where the outer end of an end span is supported on a beam a hinge should be assumed. A column built on a pile-cap supported by two, three, or four piles is not absolutely fixed but a bending moment can be developed if the resulting vertical reaction (upwards and downwards) and the horizontal thrust can be taken on the piles. A column can be considered as fixed if it is built on a thick raft and the necessary restraint can be developed in the raft.

In two-hinged and three-hinged arches, hinged frames, and some types of girder bridges, where the assumption of a hinged joint must be fully realised, it is necessary to form a definite hinge in the construction. This can be done by inserting a steel hinge (or similar), or by forming a hinge within the frame. Some types of hinged joints are illustrated in Table 94.

Moments of Inertia of Reinforced Concrete Members.—The moment of inertia of a reinforced concrete member is theoretically the equivalent moment of inertia of the stressed section about the neutral plane expressed in equivalent units of concrete or of steel. The concrete in tension is neglected (unless the tensile stress is limited as in the design of liquid containers); the reinforcement is considered as being equivalent in concrete units to its area multiplied by the modular ratio (see page 238). Until the cross-section of the member has been determined, or assumed, the calculation of the moment of inertia cannot be made with any precision. Moreover, the moment of inertia of an ordinary beam calculated on this basis changes considerably throughout its length, especially with a continuous or restrained beam in beam-and-slab construction which acts as a tee-beam at midspan but is designed as a rectangular beam towards the ends where reverse bending moments occur. Consideration should be given to whether the probable tensile stresses are sufficiently great to cause cracking, particularly with tee-beams and ell-beams if the flanges are in tension; although the beam may be designed on the assumption that the concrete
BENDING MOMENTS AND FORCES

has cracked and that the reinforcement resists all the tension due to bending, cracking may not take place owing to the comparatively large area of concrete in the flange.

Since early comparisons of moments of inertia are required in the design of frames, the errors due to approximations are of little importance. It is, however, important that the method of assessing the moment of inertia should be the same for all members in a single calculation. It is generally sufficient to compare the moments of inertia of the whole concrete areas alone for members that have somewhat similar percentages of reinforcement. Thus the ratio of the moments of inertia of a rectangular column and a rectangular beam is \( \frac{b_c d_c^3}{b d^3} \); \( b_c \) and \( d_c \) are the breadth and width of the column, and \( b \) and \( d \) are the breadth and depth of the beam.

In Table 64 are given values of the moments of inertia for square, rectangular, octagonal, and some other non-rectangular sections, together with factors and formulae for determining the moment of inertia of tee-beams, for which the breadth of the flange assumed for the purpose of calculating the moment of inertia should not exceed the maximum permissible width given in Table 69. The particulars in Table 64 exclude the effect of the reinforcement, but the data in Tables 65 and 65A for some regular cross-sections take the reinforcement into account.

Alternative methods of assessing the ratio of the moments of inertia of two members are given in the examples on the page opposite Table 51, which show that approximate methods readily give comparative values that are accurate enough not only for trial calculations but for final designs. The two methods described are recommended in the B.S. Code.

Arches.

Arch construction in reinforced concrete occurs mainly in bridges and sometimes in roofs. The principal types of symmetrical concrete arch bridges are shown in Table 93. An arch may be either a three-hinge arch, a two-hinge arch or a fixed-end arch (see the diagrams in Table 52), and may be symmetrical or unsymmetrical, right or skew, or a single arch or one of a series of arches mutually dependent upon each other. The following consideration is restricted to symmetrical and unsymmetrical three-hinge arches and symmetrical two-hinge and fixed-end arches; reference should be made to other publications for information on more complex types. Arch construction may comprise an arch slab (or vault) or a series of parallel arch ribs. The deck of an arch bridge may be supported by columns or transverse walls carried on an arch slab or ribs, in which case the structure may have open spandrels; or the deck may be below the crown of the arch either at the level of the springings (as in a bow-string girder) or at some intermediate level. A bow-string girder is generally considered to be a two-hinge arch with the horizontal component of the thrusts resisted by a tie which generally forms part of the deck. If earth filling is provided to support the deck, an arch slab and spandrel walls are required and the bridge is a closed or solid spandrel structure.

Three-hinge Arch.—An arch with a hinge at each springing and with a hinge at the crown is statically determinate. The thrusts on the abutments, and therefore the bending moments and shearing forces, are not affected by a small movement of one abutment relative to the other, and this type of arch is therefore used when there is a possibility of unequal settlement of the abutments.

For any load in any position the thrust on the abutments can be determined.
from the statical equations of equilibrium. For the general case of an unsymmetrical arch with a load acting vertically, horizontally or at an angle, the expressions for the horizontal and vertical components of the thrusts are given in the lower part of Table 52. For symmetrical arches the formulae for the thrusts given for three-hinge frames in Table 48 are applicable or similar formulae can be obtained from the general expressions in Table 52. The vertical component is the same as the vertical reaction for a freely-supported beam. The bending moment at any section of the arch is the algebraic summation of the moments of the loads and reactions to the thrusts on one side of the section. There is no bending moment at a hinge. The shearing force is likewise the algebraic sum of the reactions and loads, resolved at right-angles to the arch axis at the section considered, and acting on one side of the section. The thrust at any section is the sum of the reactions and loads, resolved parallel to the axis of the arch at the section, and acting on one side of the section.

The extent of the arch that should be loaded with live load to produce the maximum bending moment or shearing force or thrust at a given section is determined by drawing a series of influence lines. A typical influence line for a three-hinge arch and the formulae necessary to construct an influence line for unit load in any position are given in the upper part of Table 52.

Two-hinge Arch.—The hinges of a two-hinge arch are placed at the abutments and thus, as in a three-hinge arch, only thrusts are transmitted to the abutments, there being no bending moment on the arch at the springings. The vertical component of the thrust from a symmetrical two-hinge arch is the same as for a freely-supported beam. Formulae for the thrusts and bending moments are given in Table 52 and notes are given on the page facing the table.

Fixed Arch.—An arch with fixed ends exerts a bending moment on the abutments in addition to the vertical and horizontal thrusts. Like a two-hinge arch, and unlike a three-hinge arch, a fixed-end arch is statically indeterminate, and changes of temperature and shrinking of the concrete affect the stresses. As it is assumed in the general theory that the abutments are incapable of rotation or of translational movement, a fixed-end arch can only be used in such conditions.

Any section of a fixed arch rib or slab is subjected to a bending moment and a thrust, the magnitudes of which have to be determined. The design of an arch is a matter of trial and error since the dimensions and the shape of the arch affect the calculations, but it is possible to select preliminary sizes that reduce the repetition of arithmetical work to a minimum. The suggested method of determining the possible sections at the crown and springing as given in Table 53, and explained on the page facing Table 52, is based on treating the fixed arch as a hinged arch, and estimating the sizes of the cross-sections by reducing greatly the maximum stresses.

The general formulae for thrusts and moments on a symmetrical fixed arch of any profile are given in Table 53 and notes on the application and modification of these formulae are given on pages 230 and 232. The calculations involved in solving the general and modified formulae are tedious, but some labour is saved by preparing the calculations in tabulated form. One such form is that recommended by the Ministry of Transport; this form is particularly suitable for open-spandrel arch bridges because the appropriate formulae, which are those in Table 53, do not assume a constant value of \( s_a \), the ratio of the length of a segment of the arch to the mean moment of inertia of the segment. A similar form is given in Table 55.

For an arch of large span the calculations are further rendered considerably
easier and more accurate by the preparation and use of influence lines for the bending moment and thrust at the crown, the springing, and the first quarter-point. Typical influence lines are given in Table 53, and such diagrams can be constructed by considering the passage over the arch of a single concentrated unit load and applying the formulae for this condition. The effect of the dead load, and of the most adverse disposition of the live load, can be readily calculated therefrom. If the specified live load includes a moving concentrated load, such as the knife-edge load in the Ministry of Transport loading, influence lines are almost essential for the determination of the most adverse position, except in the case of the positive bending moment at the crown for which the most adverse position of the load is at the crown. The method of determining the data to establish the ordinates of the influence lines are given in the form in Table 55.

**Fixed Parabolic Arches.**—Consideration is given in Table 54 and on the facing page to symmetrical fixed arches that can have either open or solid spandrels and can be either arch ribs or arch slabs. The method is based on that of Dr. Strassner as developed by Mr. H. Carpenter, and the principal assumption is that the axis of the arch is made to coincide with the line of pressure due to the dead load. This results in an economical structure and a simple method of calculation. The shape of the axis of the arch is approximately a parabola and this method can therefore only be adopted when the designer is free to select the profile of the arch. The approximately parabolic form of arch may not be the most economical for large spans, although it is almost so, and a profile that produces an arch axis that coincides with the line of thrust for the dead load plus one-half of the live load may be more satisfactory. If the increase in thickness of the arch from the crown to the springing is a parabolic variation, only the bending moments and thrusts at the crown and springing need be investigated. The formulae for the bending moments and forces are given on the page facing Table 54 and include a series of coefficients, values of which are given in Table 54; an example of the application of the method is given on the page facing Table 55. The component forces and moments are as in the following.

The thrusts due to the dead load are relieved somewhat by the effect of the compression causing elastic shortening of the arch. For arches with small ratios of rise to span, or for arches that are thick compared with the span, the stresses due to arch shortening may be excessive. This can be overcome by introducing temporary hinges at the crown and springings, which eliminate all bending stresses due to dead load. The hinges are filled with concrete after arch shortening and much of the shrinking of the concrete have taken place.

There are additional horizontal thrusts due to a rise of temperature or a corresponding counter-thrust due to a fall of temperature. A rise or fall of 30 deg. F. is often used for structures in Britain, but careful consideration should be given to those factors that may necessitate an increase, or may justify a decrease, in the temperature range. The shrinking that takes place when concrete hardens produces counter-thrusts, and can be considered as equivalent to a fall of temperature; with the common sectional method of constructing arches the effect of shrinking may be allowed for by assuming it to be equal to a fall of temperature of 15 deg. F.

The extent of the live load on an arch to produce the maximum stresses in the critical sections can be determined from influence lines, and the following are approximately correct for parabolic arches. The maximum positive bending moment at the crown occurs when the middle third of the arch is loaded; the maximum negative
bending moment at a springing occurs when four-tenths of the span adjacent to the springing is loaded; the maximum positive bending moment at a springing occurs when the whole span is loaded except the length of four-tenths of the span adjacent to the springing. In the expressions in Table 54 the live load is expressed in terms of an equivalent uniformly-distributed load. In Table 6 are given the values of the distributed load specified by the Ministry of Transport, but these loads have to be combined with the prescribed knife-edge load as shown in the example on the page facing Table 54.

When the corresponding normal thrusts and bending moments on a section have been determined, the area of reinforcement and the stresses at the crown and springing are calculated in accordance with the methods described on pages 76 et seq. There only remains to determine the intermediate sections and the profile of the axis of the arch. If the dead load is uniform throughout (or practically so) the axis will be a parabola, but if it is not uniform the axis must be shaped to coincide with the line of pressure for dead load. The latter can be plotted by force and link polygons (as in ordinary graphic statics), the necessary data being the magnitudes of the dead load, the horizontal thrust due to dead load, and the vertical reaction (which equals the dead load on half the span) of the springing. The line of pressure, and therefore the axis of the arch, having been established, and the thicknesses of the arch at the crown and springing determined, the lines of the extrados and intrados can be plotted to give a parabolic variation of thickness between the two extremes.
SECTION 3
MATERIALS AND STRESSES

The properties of reinforcement and of the constituents of concrete are described in regulations, standards and codes of practice. Only those properties which concern the designer directly and working stresses are dealt with in this section; the specification in Section 6 embodies other requirements.

Concrete.

Cement.—Cements suitable for reinforced concrete are ordinary and rapid-hardening Portland cements, Portland-blastfurnace cement, low-heat Portland cement, sulphate-resistant cement, super-sulphate cement, and high-alumina cement. Quick-setting cements are not used in ordinary construction. Calcium chloride is sometimes added to ordinary and rapid-hardening Portland cement to accelerate the initial set, either for concreting in cold weather or to enable the moulds or shuttering to be removed earlier; cements with any such addition do not conform to British Standards. Cements of different types should not be used together.

Ordinary Portland Cement (B.S. No. 12, 1958).—The principal requirements are as follows. The initial setting-time must be not less than 45 minutes and the final setting-time not more than 10 hours. The specific surface area must be not less than 2250 sq. cm. per gramme. The minimum compressive strengths of 1:3 mortar cubes are 2200 lb. per square inch at three days and 3400 lb. at seven days. An alternative test on 4-in. concrete cubes with a cement aggregate ratio of about 1:6 (equivalent to 1:2:4), with aggregate from $\frac{1}{4}$ in. down, a water-cement ratio of 0-6, and a slump of $\frac{1}{2}$ in. to 2 in., is included. The strength of such cubes must be not less than 1200 lb. per square inch at three days and 2000 lb. at seven days. According to the recommendations of B.S. Code No. 114, the crushing strength of 6-in. or 4-in. cubes of 1:2:4 ordinary concrete in preliminary tests should be not less than 2700 lb. per square inch at seven days. It is possible that this strength might not be obtained if cubes tested in accordance with B.S. No. 12 have only the minimum strength of 2000 lb. per square inch, but in this case the concrete would be acceptable so long as the strength at twenty-eight days be not less than 4000 lb. per square inch (see Table 57).

Rapid-Hardening Portland Cement (B.S. No. 12, 1958).—The principal physical difference between ordinary and rapid-hardening Portland cement is the greater fineness of the latter, which must have a specific surface area of not less than 3250 sq. cm. per gramme. The setting-times are the same, but the minimum compressive strengths of mortar cubes are 3000 lb. per square inch at three days and 4000 lb. at seven days. The minimum compressive strengths of concrete cubes are 1700 lb. per square inch at three days and 2500 lb. at seven days. An optional tensile test at one day (300 lb. per square inch) is included. The quicker hardening of this cement may enable shuttering to be removed earlier.

Portland-Blastfurnace Cement (B.S. No. 146, 1958).—The slag content must not exceed 65 per cent. The setting-times and the fineness are the same as for ordinary Portland cement. The minimum compressive strengths of mortar cubes are 1600 lb.
per square inch at three days, 3000 lb. at seven days, and 5000 lb. at twenty-eight days, and of concrete cubes 800 lb., 1600 lb., and 3200 lb. at these ages.

_Sulphate-Resistant Cement._—There is no British Standard for this cement, which is used for concrete liable to chemical attack by sea-water, acid ground-waters, and other medium-sulphate liquids. It is a mixture of blastfurnace slag and Portland cement clinker, and has less free lime. Super-sulphate or high-alumina cements are required when sulphate concentrations are high.

_High-Alumina Cement (B.S. No. 915, 1947)._—This cement has extreme rapid-hardening properties due mainly to the proportion of alumina being up to 40 per cent. compared with about 5 per cent. in Portland cement; a minimum of 32 per cent. of alumina is required. The required fineness is between that of ordinary and rapid-hardening Portland cements. Initial setting must take place between two and six hours, and final setting within two hours after the initial set. The minimum compressive strengths of mortar cubes are 6000 lb. per square inch at one day and 7000 lb. at three days. High-alumina cement is more costly than Portland cement but it is immune from attack by sea-water and many corrosive liquids; because of its high early strength it is also used when saving of time is important.

_Low-Heat Portland Cement (B.S. No. 1370, 1958)._—Low-heat Portland cement generates less heat during setting and hardening than do other cements, and thus reduces the risk of cracks occurring in large masses of concrete due to a reduction of tensile stresses during cooling. The development of strength is slower than that of other Portland cements, but in course of time the strengths may be equal. The minimum compressive strengths of mortar cubes are 1100 lb. per square inch at three days, 2000 lb. at seven days, and 4000 lb. at twenty-eight days. The strength of concrete cubes must be 500 lb. per square inch at three days, 1000 lb. at seven days, and 2000 lb. at twenty-eight days. A high proportion of lime is not compatible with low heat of hydration, and therefore the permissible percentage of lime is less than for other Portland cements. The heat of hydration must not exceed 60 calories per gramme at seven days and 70 calories per gramme at twenty-eight days. The initial setting-time must be not less than one hour and the final setting-time not more than ten hours. The specific surface must be not less than 3200 sq. cm. per gramme.

_Aggregates._—Fine aggregate (sand) and coarse aggregate (stone) must be clean, inert, and hard, non-porous and free from excessive quantities of dust, laminated particles, and splinters. Gravels and crushed hard stone are the commonly used materials for structural concrete. Broken brick is a cheap aggregate for plain concrete, generally of low strength. Clinker, foamed-slag, expanded shale and clay, pellets of pulverised-fuel ash, fire-brick, and pumice, are used as aggregates for lightweight and insulating concrete where great strength is not essential. In general, aggregates for reinforced concrete should comply with B.S. No. 882, but air-cooled blastfurnace slag (B.S. No. 1047), foamed blastfurnace slag (B.S. No. 877), crushed dense clay brick and tile, some proprietary forms of expanded shale or clay, and clean pumice may be suitable for members such as wall panels and floor and roof slabs.

The size and grading of aggregates vary with the nature and source of the material, and the requirements in this respect depend upon the type of structure. For buildings and most reinforced concrete construction the fine aggregate should be graded from \(\frac{1}{8}\) in. down to dust with not more than 3 per cent. passing a B.S. sieve No. 200. The coarse aggregate should be graded from \(\frac{1}{4}\) in. up to \(\frac{1}{2}\) in., and between these limits
the grading should be such as to produce a workable and dense concrete. The largest size of the coarse aggregate should be \( \frac{1}{4} \) in. less than the cover of concrete over the reinforcement (except in slabs) or the space between the bars, and should not exceed a quarter of the smallest dimension of the concrete member. For the ribs and top slab of hollow clay-block slabs, and for shell roofs and similar thin members, the largest size of aggregate is generally \( \frac{3}{8} \) in. In non-reinforced concrete larger aggregate, say, \( 1\frac{1}{4} \) in. to 3 in., is permissible and in concrete in large piers of bridges, massive foundations, or in concrete for filling large cavities or for kentledge, the use of hard stone "plums" is common.

For concrete subject to attrition, such as roads and the floors of garages, factories, and workshops, if a special finish is not applied, an angular aggregate and a coarse sand is preferable. For liquid containers the aggregates should give as dense a concrete as possible.

**Concrete Mixtures.**—The proportions in which the cement, fine aggregate, and coarse aggregate are mixed are usually expressed for convenience as volumetric ratios based on a unit volume of cement, for example, \( 1 : 2 : 4 \), meaning one part by volume of cement, two parts by volume of fine aggregate, and four parts by volume of coarse aggregate. Since it is important that the quantity of cement should be not less than that expected, the cement should be measured by weight. If Portland cement has a nominal weight of 90 lb. per cubic foot, \( 1 : 2 : 4 \) means 90 lb. of Portland cement to 2 cu. ft. of fine aggregate to 4 cu. ft. of coarse aggregate. Since the basis of a batch of concrete is generally a bag containing one hundredweight of cement, this mixture is equivalent to 112 lb. of cement to 2\( \frac{1}{4} \) cu. ft. of fine aggregate to 5 cu. ft. of coarse aggregate, and in specifications and on working drawings the proportions of the concrete should be expressed in these terms.

So long as it is realised that the method of expressing proportions of concrete in nominal volumetric ratios is merely a convenience, its use can greatly facilitate reference to any concrete, and for this purpose may be preferable to expressing the proportions in terms of the volumes of fine and coarse aggregates to a specified weight of cement, which leads to cumbersome expressions, or to giving different proportions distinctive references, such as "Mixture A" or "Grade I". The latter reference is used in the London By-laws, but does not at sight give information regarding the mixture and there is also the difficulty of selecting a suitable reference for proportions that are between, or are variations of, the basic proportions. For these reasons nominal volumetric proportions are adopted in this book, for example, in *Tables 56 and 57*. The latter table relates to the B.S. Code No. 114; in *Table 56* are given the corresponding references in accordance with the London By-laws (1952). The volumes of aggregate required with 1 cwt. of cement when using the more common proportions, which quantities also apply to the B.S. Codes, are given in *Table 56*. Recommendations for special mixtures to give a concrete of specified strength are given in the B.S. Codes, in which the limiting proportions of cement to aggregate are as in *Table 57*.

**Proportions of Cement to Aggregate.**—The proportion of cement to aggregate depends on the strength, impermeability, and durability required. Experience shows that a nominal \( 1 : 2 : 4 \) concrete is suitable for general construction in cost and strength. Leaner concrete is not generally recommended for reinforced concrete unless the mixture has been properly determined by trials with the actual materials it is proposed to use in the work. A nominal \( 1 : 3 : 6 \) concrete is suitable for non-reinforced construction or for concrete temporarily placed and which will be cut away
later. Workable mixtures richer in cement than \(1:2:4\), for example \(1:1\frac{1}{2}:3\) and \(1:1:2\), are stronger but more expensive owing to the higher proportion of cement, and they are not generally economical for beams. They are, however, often economical for heavily-loaded columns or for members subjected to combined bending and direct thrust when the direct thrust predominates. Mixtures richer than \(1:1:2\) contain so large a proportion of cement that, apart from the cost, shrinking upon hardening is excessive. Instead of using a rich mixture it is generally more economical to obtain the necessary compressive strength by careful proportioning of the aggregates and control of the amount of water. Suitable proportions for concrete for various parts and types of structures under ordinary conditions are given in Table 56.

In liquid-containing structures the nominal proportions should be not leaner than \(1:1\frac{1}{2}:3\frac{1}{2}\), that is, 2 cu. ft. of fine aggregate and 4 cu. ft. of coarse aggregate to 1 cwt. of cement. If, however, the thickness of the concrete exceeds 18 in., nominal \(1:2:4\) concrete is permissible. Concrete having the proportions \(1:1\frac{1}{2}:3\frac{1}{2}\) is generally used for precast piles, unprotected roof slabs and for concrete deposited under water, and in other places where a concrete of a better quality than \(1:2:4\) is required. For hollow-block floors and similar narrow ribbed construction and for many precast products \(1:1\frac{1}{2}:3\) concrete is often specified, but with smaller aggregate than is used for ordinary construction. The blinding layer on the bottom of an excavation can consist of concrete having the proportions of 1 part of Portland cement to 8 parts of combined aggregate.

**Proportions of Fine and Coarse Aggregates.**—The ratio between the amounts of fine and coarse aggregate necessarily depends on the grading and other characteristics of the materials in order that the volume of sand shall be sufficient to fill the voids in the coarse aggregate to produce a dense concrete. Until the material for a particular structure has been delivered to the site it is not possible to say what will be the exact grading of the sand or stone. Therefore this information is not always available when the specification is written. Several courses are open to the engineer when specifying the proportions for the concrete. The proportions of a particular sand and a particular stone with the properties of which the engineer is acquainted can be specified. Two or more independent sources of supply should be available within reasonable distance of the site. If the material is specified in this way, the permissible variations of the essential properties should be given. Another method is to specify the proportions of coarse and fine aggregates having definite gradings and leave it to the contractor to supply a material conforming to these requirements. Probably a better method is to specify a provisional ratio of fine to coarse material, the maximum and minimum sizes (with such percentages of intermediate sizes as are necessary), and insert a provision to allow adjustment of the proportions after examination of the actual materials.

Generally the proportion of fine to coarse aggregate should be such that the volume of fine aggregate should be about 5 per cent. in excess of the voids in the coarse material. Since the volume of voids may be up to 45 per cent., a common ratio is one part of fine to two parts of coarse aggregate, as in mixtures such as \(1:2:4\) and \(1:1\frac{1}{2}:3\). Such proportions, however, relate to dry materials. Whereas the water in a damp coarse aggregate does not appreciably affect the volume, the water in a damp fine aggregate may increase the volume by 30 per cent. over the dry (or fully saturated) volume. The proportions specified should therefore apply to dry sand and must be adjusted on the site to allow for bulking due to dampness.

The ratio of \(1:2\) of fine (dry) to coarse aggregate should be altered if tests show
that a denser and more workable concrete can be obtained by using other proportions. Permissible lower and upper limits are generally 1 : 1½ and 1 : 2½ respectively; thus for a nominal 1 : 2 : 4 concrete, the variation of the proportions may result in the equivalent extreme proportions of approximately 1 : 2½ : 3½ and 1 : 1½ : 4½. For liquid-containers, in accordance with B.S. Code No. 2007, the ratio may vary between 1 : 1½ and 1 : 3.

Standards of control to ensure that concrete of uniform quality and strength assumed in design is obtained at the site are described in "Quality of Concrete in the Field" (Inst. Civil Engrs.); the degrees of control and strength categories equivalent to each class of concrete are given in Table 56 and explained briefly on the page facing the table.

Quantity of Water.—The strength and workability of concrete depend to a great extent on the amount of water used in mixing. There is an amount of water for certain proportions of given materials that produces a concrete of greatest strength. A smaller amount of water decreases the strength, and about 10 per cent. less may be insufficient to ensure complete setting of the cement and may produce an unworkable concrete. More than the optimum amount increases workability but decreases strength; an increase of 10 per cent. may reduce the strength by approximately 15 per cent., while an increase of 50 per cent. may reduce the strength by one-half. With an excess of more than 50 per cent. the concrete becomes too wet and liable to separation. The use of an excessive amount of water not only produces low strength but increases shrinking, and decreases density and therefore durability.

Some practical values of the water–cement ratio for structural reinforced concrete are about 0·45 for 1 : 1 : 2 concrete, 0·50 for 1 : 1½ : 3 concrete, and 0·55 to 0·60 for 1 : 2 : 4 concrete.

Concrete placed by efficient mechanical vibrators may generally contain less water than concrete compacted by tamping or rodding, thereby obtaining greater strength. Increased workability can be obtained by incorporating a plasticising agent into the mixture; the consequent reduction in the amount of water required results in a gain in strength.

A practical method of assessing the amount of water required is to make trial mixtures and find the proportion of water which produces a concrete which is just plastic enough to be worked among and around the reinforcement bars. These trial mixtures may also be used to determine the best ratio of fine to coarse aggregate. Several mixtures are made with the amounts of fine and coarse aggregates slightly different in each, but with the total volume of aggregate and weight of cement the same in each mixture. The amount of water is adjusted to give the required workability. The mixture that occupies the least volume, that is the densest, will produce the best concrete. When the best mixture has been determined, the slump may be determined and the slumps of subsequent batches checked. The slump test allows for the porosity and dampness of the aggregates but not for any variation in the grading, size, or shape of the aggregate. A maximum slump for reinforced concrete construction is about 6 in., but a stiffer mixture is often desirable and practicable; a slump of 1 in. may be suitable if the reinforcement is not intricate or congested. For plain concrete in massive foundations, roads and dams, and similar work the concrete may not contain enough water to produce any slump, but sufficient water must be present to hydrate the cement and to enable the concrete to be properly consolidated by vibration or ramming.
Properties of Concrete.

*Weight and Pressure.*—The weight of ordinary concrete is discussed on page 140, and the weights of ordinary reinforced concrete, lightweight concrete and heavy concrete are given in *Table 1.* A weight of 150 lb. per cubic foot is generally adopted in the structural design of reinforced concrete members.

In the design of shuttering a weight of not less than 150 lb. per cubic foot should be allowed for wet concrete. The horizontal pressure exerted by wet concrete is generally assumed to be 140 lb. per square foot of vertical surface per foot of depth placed at one time, but for narrow widths, for drier concretes, and where the reinforcement is intricate, the increase in pressure for each foot of depth is less.

*Lightweight Concrete.*—Concrete having a density less than that of concrete made with gravel or crushed stone is produced by using clinker, foamed-slag, expanded clay and shale, vermiculite, pumice, or other lightweight material. Such concretes have not generally a great strength, especially if they are specially low in weight, and therefore are not in general used alone for structural members; their low densities and high thermal insulation properties make them suitable for partitions and for lining walls and roofs. The use of concrete of medium weight with lightweight aggregates (such as sintered clay or shale) and having sufficient strength for some structural members, with or without reinforcement, is however extending. In some structural members the reinforcement is embedded in ordinary dense concrete and the remainder of the member is made of lightweight concrete.

Other methods of making lightweight concrete include mixing metallic powder or other materials with the concrete; the resulting reactions generate bubbles of gas or entrap air to form cellular concrete. Such concretes are not generally impermeable, and if reinforced to provide lightweight slabs suitable for roofs, wall panels, and the like, the reinforcement should be coated to protect it from corrosion.

"No-fines" concrete is a form of lightweight concrete suitable for cast-insitu, non-reinforced construction. It is generally ordinary gravel concrete with little or no aggregate less than $\frac{3}{4}$ in. in size, and has high thermal insulation properties.

*Air-entrained Concrete.*—Air is entrapped in Portland cement structural concrete with ordinary aggregates by adding resinous or fatty materials during mixing. Generally the amount of air is about 5 per cent. (by volume) and the results are decreases of about 3 per cent. in weight and up to 10 per cent. in strength, an increased resistance to frost and chemical attack and an improvement in workability.

*Compressive Strength.*—With given proportions of aggregates the compressive strength of concrete depends primarily upon age, cement content, and the cement-water ratio, an increase in any of these factors producing an increase in strength. A formula for determining the probable strength of concrete of known mixture is given on the page facing *Table 56.*

Compressive strengths vary from 1500 lb. per square inch for lean concretes to more than 8000 lb. per square inch for special concretes. The rate of increase of strength with age is almost independent of the cement content, and, with ordinary Portland cement concrete, about 60 per cent. of the strength attained in a year is reached at twenty-eight days; 70 per cent. of the strength at twelve months is reached in two months, and about 95 per cent. in six months. Working stresses are generally based on the strength at twenty-eight days. The strength at seven days is about two-thirds of that at twenty-eight days with ordinary Portland cement, and is a good indication of the strength likely to be attained.
The minimum crushing strengths of preliminary laboratory cubes and works cubes, at seven and twenty-eight days for Portland cement concrete of ordinary proportions, and at two days for high-alumina cement concrete, are given in Tables 56 and 57. The strengths of leaner concretes, which are generally used for non-reinforced work, are given in Table 56. The recommendations of B.S. Code No. 114 are given in Table 57. The requirements of the London By-laws for buildings and the recommendations of B.S. Code No. 2007 for liquid-containing structures are given in Table 56. The various grades of concrete included in the tables are the ordinary, lower, and higher qualities and the special grade of concrete described in B.S. Code No. 114, and the two qualities specified in the London By-laws.

Compression tests in Britain are made on 6-in. cubes, which should be made, stored, and tested in accordance with B.S. No. 1181. For cubes made on the site, three should be cast from one batch of concrete. Identification marks should be made on the cubes. Two sets of three cubes each are preferable, and one set should be tested at seven days and the other at twenty-eight days. If only one set of three cubes is made, they should be tested at twenty-eight days. The strengths of the cubes in any set should not vary by more than 15 per cent. of the average, unless the lowest strength exceeds the minimum required. The seven-days’ tests are a guide to the rate of hardening; the strength at this age for Portland cement concrete should be not less than two-thirds of the strength required at twenty-eight days.

In some countries cylinders or prisms are used for compressive tests. For ordinary concrete the compressive strength as measured on 6-in. cylinders is about 85 per cent. of that as measured on 6-in. cubes of ordinary concrete, although the ratio may be two-thirds with high-strength concretes.

*Tensile Strength.*—The direct tensile strength of concrete is considered when calculating resistance to shearing force and in the design of cylindrical liquid-container. The tensile strength does not bear a constant relation to the compressive strength, but is about one-tenth of the compressive strength.

The tensile resistance of concrete in bending is generally neglected in the design of ordinary structural members but is taken into account in the design of slabs and the like in liquid-containing structures. The tensile resistance in bending is measured by the bending moment at failure divided by the section modulus, the result being termed the modulus of rupture. The minimum moduli of structural concretes at three and twenty-eight days as given in B.S. Code No. 114 are given in Table 57. Formulae for estimating the modulus of rupture and the direct tensile strength are given on the page facing Table 56.

*Elastic Properties.*—Notes on the elastic properties such as the modulus of elasticity, modular ratio, and Poisson’s ratio for concrete and reinforced concrete are given on the page facing Table 56.

*Thermal Properties.*—The coefficient of thermal expansion is required in the design of chimneys, tanks containing hot liquids, and exposed or long lengths of construction, and provision must be made to resist the stresses due to changes of temperature or to limit the strains by providing joints. The thermal conductivity of concrete (k on page 107) varies with the density and porosity of the material. Some values of the coefficients of thermal expansion and conductivity are given on the page facing Table 57.

The nature of the aggregate is the principal factor in determining the resistance of concrete to fire, although the type of cement may affect this property to some
extent. The resistance to fire of a reinforced concrete structure is affected considerably by the thickness of cover of concrete over the bars, and for a high degree of resistance covers in excess of those ordinarily specified should be provided, especially for floor and roof slabs and walls; reference should be made to the table on the page facing Table 57, which gives the requirements of the London By-laws and B.S. Code No. 114. Aggregates that have been sintered are superior to other aggregates in their resistance to fire; also of high resistance, but less so than the foregoing, are limestone and artificial aggregates such as broken brick. The aggregate ordinarily used for structural concrete, such as crushed hard stone (excepting hard limestone, but including granite) and flint gravels, are inferior in resistance to fire although such aggregates produce the strongest concrete.

Shrinking.—Unrestrained concrete members exhibit progressive shrinking over a long period while they are hardening. For concrete that can dry completely and where the shrinking is unrestrained, the linear coefficient is approximately 0.00025 at twenty-eight days and 0.00035 at three months, after which period the increase is less rapid until at the end of twelve months it may approach a maximum of 0.0005. In reservoirs and other structures where the concrete does not become completely dry, a maximum value of 0.0002 is reasonable. A concrete rich in cement, or made with finely-ground cement or with a high water content, shrinks more than a lean concrete or one with a low water content.

If a concrete member is restrained so that reduction in length due to shrinking cannot take place, tensile stresses are caused. A coefficient of 0.0002 may correspond to a stress of 500 lb. per square inch when restrained; it is therefore important to reduce or neutralise these stresses by using a strong concrete, by proper curing and by the provision of joints. Shrinking is considered in the design of fixed arches on pages 44, 232 and 234.

Creep.—Creep is the slow deformation, additional to elastic contraction, exhibited by concrete under sustained stress, and proceeds at a decreasing rate over many years however small may be the stress. Characteristic values for creep, expressed in deformation per unit length, for 1:2:4 concrete loaded at 28 days with a sustained stress of 600 lb. per square inch, are 0.0003 at 28 days after loading, and 0.0006 at one year. Thus creep is of the same degree of magnitude as shrinking, and appears to be directly proportional to the stress. The earlier the age of the concrete at which the stress is applied the greater is the creep, which also appears to be affected by the same factors as affect the compressive strength of the concrete; generally the higher the strength the less is the creep. The effect of creep of concrete is not often considered in reinforced concrete design.

Reduction of Bulk upon Mixing.—When the constituents of concrete are mixed with water and tamped into position, a reduction in volume to about two-thirds of the total volume of the dry unmixed materials takes place. The actual amount of reduction depends on the nature, dampness, grading, and proportions of the aggregates, the amount of cement and water, the thoroughness of mixing, and the degree of consolidation. With so many variables it is impossible to assess exactly the amount of each material required to produce a cubic yard of wet concrete when deposited in place. The quantities given in Table 56 are ordinary values for gravel concrete but do not allow for waste. Alternative quantities are given for dry and damp fine aggregates. The quantities of fine and coarse aggregates for other proportions can be obtained from the ratios of the volumes of the materials.
Porosity and Permeability.—The porosity of concrete is the characteristic whereby liquids can penetrate the material by capillary action, and depends on the total volume of the spaces occupied by air or water between the solid matter in the hardened concrete. The more narrow and widely distributed these spaces are the more easily can liquids diffuse in the concrete.

Permeability is the property of the concrete that permits a liquid to pass through the concrete due to a difference in pressure on opposite faces. Permeability depends primarily on the size of the largest voids and on the size of the channels connecting the voids. Impermeability can only be approached by proportioning and grading to make the number and sizes of the voids the least possible, and by thorough consolidation to ensure that the concrete is the densest possible with the given proportions of the materials. Permeability seems to be a less determining factor than porosity when considering the effect on concrete of injurious liquids.

Fatigue.—The effect of repeatedly applied loads, either compressive or tensile, or a frequent reversal of load, is to reduce the strength of concrete; this phenomenon is called "fatigue". If the resultant stress is less than half the strength, as is the case in compression on most concrete members, fatigue is not evident. When a stress exceeding half the strength of the unfatigued concrete is frequently caused, the strength of the concrete is progressively reduced, until it equals the stress due to the applied load, when the concrete fails. The number of repetitions of load to produce failure decreases the more nearly the stress due to the load equals the strength of the unfatigued concrete.

A relatively high frequency of repetition of stress would be ten times and upwards per minute. If intervals of time occur between successive applications of load, the effect of fatigue is delayed. The degree of fatigue differs for direct compression, direct tension, and bending. Since the tensile stress in concrete in bending more nearly approaches the strength than does the compressive stress, it is evident that fatigue due to tensile stress controls fatigue of concrete in bending.

Working Stresses in Concrete.

Compression due to Bending.—The permissible compressive stress \( p_{cb} \) in concrete due to bending is generally assumed to be one-third of the specified minimum crushing strength of cubes at twenty-eight days. Values of \( p_{cb} \) for concrete not leaner than 1:2:4 are given in Tables 56 and 57 and conform to general practice, and in particular to the London By-laws and the B.S. Codes. The London By-laws recognise two grades of concrete. The B.S. Code considers four qualities of Portland cement concrete with aggregates complying with B.S. 882, namely, concrete of normal strength; concrete of lower strength, but not more than 25 per cent. less than ordinary concrete; concrete of higher strength, up to 25 per cent. in excess of ordinary concrete; and special concrete of higher strength. Two qualities of high-alumina cement concrete are considered. If the aggregates do not conform to B.S. 882, the B.S. Code recommends working stresses based directly on the strength of site-made or laboratory-made cubes in accordance with the ratios given in Table 57.

For concrete in which the ratio of fine to coarse aggregate is not 1:2, the permissible stresses are equivalent to those for concrete having the same sum of the proportions of fine and coarse aggregate; thus mixtures of 1:2:3 and 1:4:4 are both equivalent to 1:2:4 as regards working stresses since the sum of the proportions of the fine and coarse aggregates is six in each case. For proportions between
those tabulated, the value of $p_{eb}$ can be computed by proportion between the stresses given for the two nearest mixtures based on the sum of the separate volumes of the fine and coarse aggregate; an example is given on the page facing Table 62.

For liquid-containers made with $1:1\frac{1}{3}:3\frac{1}{3}$ concrete, a maximum compressive stress of 1200 lb. per square inch should be used for slabs and beams subjected to bending. The 28-days' strength of concrete used for such structures should be not less than 5400 lb. per square inch for laboratory-made cubes and 3600 lb. per square inch for cubes made on the works. In thick construction $1:2:4$ concrete is used having a 28-days' strength of 4500 lb. per square inch for laboratory-made cubes and 3000 lb. per square inch for site-made cubes; $p_{eb}$ is 1000 lb. per square inch. These strengths and stresses are in accordance with B.S. Code No. 2007 and are given in Table 56.

**Direct Compression.**—For members in direct compression, such as concentrically-loaded columns, the permissible compressive stress is about 76 per cent. of the permissible compressive stress in bending as is given in Tables 56 and 57. This stress is assumed to occur over the whole cross-sectional area of an ordinary column and on the cross-sectional area of the core of a column with helical binding.

**Combined Bending and Direct Force.**—When a member is subject to bending moment combined with a direct thrust, as in an arch or a column forming part of a frame, or is subjected to a bending moment combined with a direct pull, as in the walls of rectangular bunkers and tanks, the same permissible compressive stress $p_{eb}$ is used for the concrete as if the member were subjected to bending alone. Thus for a column built in $1:2:4$ ordinary concrete (Table 57), the compressive stress should not exceed 760 lb. per square inch for the direct load considered alone, nor exceed 1000 lb. per square inch when the direct load is combined with a bending moment.

**Tension.**—In the design of members subjected to bending the strength of the concrete in tension is commonly neglected, but in certain cases, such as structures containing liquids and in the consideration of shearing resistance, the tensile strength of the concrete is important. For suspension members which are in direct tension and where cracking is not necessarily detrimental, the tensile strength of the concrete can be neglected and the reinforcement then resists the entire load. In a member that must be free from cracks, such as the wall of a cylindrical container of liquids, the tensile stress in $1:1\frac{1}{3}:3\frac{1}{3}$ concrete should not exceed 190 lb. per square inch in accordance with B.S. Code No. 2007; a member in bending should be designed, as described on page 69, so that the tensile stress in the concrete does not exceed 270 lb. per square inch. The corresponding tensile stresses in $1:2:4$ concrete are 175 and 245 lb. per square inch as given in Table 56. Overline railway bridges and similar structures where cracking may permit corrosive fumes to attack the reinforcement should also be designed with a limited tensile stress in the concrete.

**Shearing Stresses.**—The permissible shearing stress $g$ in a beam is about 0.1 of the maximum permissible compressive stress in bending, but if the diagonal tension due to the shearing force is resisted entirely by reinforcement the shearing stress should not exceed $4g$ in accordance with B.S. Code No. 114 for buildings or $2.5g$ in accordance with B.S. Code No. 2007 for liquid-containing structures; a maximum stress of less than $4g$ is advisable in all but primary beams in buildings. Values of the stresses $g$ and $4g$, and $2.5g$, are given in Tables 56 and 57. The relation between $g$ and $p_{eb}$ for concrete of quality other than those tabulated is given in Table 57.

**Bond.**—The permissible bond stress $s_b$ between concrete and plain round bars
is slightly more than the shearing stress. Thus the shearing stress for \(1:2:4\) ordinary concrete is, according to the B.S. Code, 100 lb. per square inch and the average bond stress is 120 lb. per square inch. The local bond stress \(sb\) (see page 62) is about 50 per cent. greater than the average bond stress, that is 180 lb. per square inch. Bond stresses corresponding to other proportions and qualities of concrete are given in Tables 56 and 57. For deformed bars the recommended stresses are 25 per cent. in excess of the stress for plain round bars.

**Bearing on Plain Concrete.**—Plain concrete mixed in leaner proportions than \(1:2:4\) is used for filling under foundations and for massive piers, and thick retaining walls. Suitable working stresses in direct compression are given in Table 56 and these conform generally to the London By-laws. The bearing pressures on plain and reinforced concrete in piers subjected to a concentric load, or to an eccentric load, and permissible local pressures as under bearings are given in Table 104 in accordance with B.S. No. 111.

**Modifications of Permissible Stresses.**—The working stresses given in Tables 56 and 57 and specified in regulations should be related to the method of computing the bending moments and forces to which the member is subjected. These stresses must be reduced if departures are made from the ordinary methods of assessment of the load, the computation of the bending moment, or the calculation of the stresses.

The working stresses should be reduced when any of the following apply.

(a) If tests on works-made cubes made with the available aggregates give at twenty-eight days results lower than three times the permissible stress in bending, the working stresses should be reduced to one-third of the strength at twenty-eight days. The ratios recommended in B.S. Code No. 114 are given in Table 57.

(b) When the structure is constructed by inexperienced men and when competent whole-time supervision is not available.

(c) When the calculations are approximate or when the maximum load is uncertain. In preliminary designs such secondary effects as stresses due to changes in temperature or due to torsion may be omitted and a reduced working stress used. It is necessary, however, when preparing the calculations for the working drawings to include these factors and adopt the ordinary permissible stresses.

(d) Slender columns and piers must be designed for stresses less than the ordinary stress for direct load (see Table 83). Similarly, narrow beams and other members should be designed for lower compressive stresses (see page facing Table 62).

(e) When the structure is subject to vibration or impact the ordinary stresses should be reduced to allow for fatigue unless an allowance for these effects has been made in the calculation of the live load.

(f) If the greatest load can occur before the concrete has obtained a crushing strength equal to three times the compressive stress due to the combined dead and live loads.

(g) If all the load is dead load the working stresses should be reduced, unless means can be taken (for example, by introducing temporary supports) to prevent the full load acting on the structure until three months after completion.

(h) When the work is difficult to construct and when the concrete is deposited under water, in underpinning, or in similar difficult positions.

Under more favourable conditions reasons may exist for increasing the working compressive stresses, although the 28-days' strength divided by three should generally be adopted as a guide. Among such conditions are the following.
(a) When the concrete is specially proportioned and treated with the object of obtaining a high strength.

(b) If the structure, or particular members, will not be loaded for some time after completion of construction, the working stresses may be increased. The amounts of increase, in accordance with the B.S. Code, are given in Table 57.

(c) If the extreme live load occurs infrequently and for a short time, stresses for such a load can be increased at the designer's discretion, but the stresses due to the dead load and the ordinary live load should not exceed those generally permissible.

(d) When the calculated stresses include those due to the bending moments and forces caused by wind pressures on buildings, the stresses can be increased by \( 33\frac{1}{3} \) per cent. in accordance with the London By-laws and 25 per cent. in accordance with the B.S. Code. The stresses without wind should not exceed the ordinary permissible working stresses. This increase should not be made for structures in which the effects of wind are a predominating factor in the design.

In adopting the foregoing reductions or increases much has to be left to the judgment of the designer when considering the conditions of a particular structure and the requirements of any controlling authorities.

Properties of Reinforcement.

Strength.—Reinforcement is most commonly plain round mild steel bars. Less frequently, medium-tensile and high-tensile steels are used; these have higher yield-point stresses than mild steel because, in the case of high-tensile steel (which is obtainable as rolled deformed bars), of the higher carbon content. Medium-tensile steel is a superior type of mild steel. Other high-yield-stress bars are cold-worked bars, such as twisted ribbed bars, twisted square bars, and wire. Cold-drawn mild steel wire and small twisted square bars are generally in the form of oblong-mesh or square-mesh welded or woven fabrics and are used for the reinforcement of slabs and roads. Expanded metal is used for the same purpose. When available, and convenient to use, old rails, disused wire ropes, and light structural steel are occasionally embedded in concrete as reinforcement. The tensile strength, and the yield-point stress or equivalent yield stress, of hot-rolled mild steel, medium-tensile steel, high-tensile steel, hard cold-drawn wire, twisted square bars, and twin-twisted bars are given in Table 58; these data are in accordance with B.S. Nos. 785 and 1144. Twin-twisted bars are not now readily obtainable. The table also gives the equivalent yield stress and tensile strength of twisted ribbed bars, hard-drawn steel wire and expanded metal.

Cross-sectional Area, Perimeter, and Weight.—In Table 59 are given the cross-sectional areas and weights per foot of plain round bars and wire and twisted square bars of common sizes, including wire-gauge sizes. The cross-sectional area of any number up to twenty of plain round bars from \( \frac{3}{8} \) in. to \( 1\frac{1}{4} \) in. diameter is given in Table 60, as well as the cross-sectional areas (per foot) of similar bars at various spacings from 3 in. to 24 in. The weights per foot and per square foot of most of the bars given in Table 60 are given in Table 59. The perimeters of plain round bars and the number of feet in a ton are given in Table 59. The cross-sectional areas and weights of twisted ribbed bars are the same as those of the same size of plain round bar since the nominal size of a ribbed bar is the diameter of the plain round bar having the same weight per foot. The overall diameter of a ribbed bar is 1.1 times the nominal diameter.
In Section 6 some methods of estimating and calculating the weight of mild steel reinforcement are described.

*Table 6* gives data relating to bars of metric sizes, namely, the size of bars in millimetres and the equivalent in inches, the cross-sectional area of a specified number of bars up to ten (in square inches and square centimetres), and the cross-sectional area of bars at specified spacings from 8 cm. (about 3 in.) to 30 cm. (about 12 in.). The data relate to bars from 6 mm. (about ¼ in.) to 25 mm. (1 in.) and in some cases to 30 mm. (about 1½ in.).

**Working Stresses in Reinforcement.**

**Tension.**—For rolled mild steel having a tensile strength of 28 tons to 33 tons per square inch the safe working stress in tension is generally 18,000 lb. per square inch for retaining walls, bridges, industrial and similar structures. For liquid-containers the maximum tensile stress should be 12,000 lb. per square inch when the tensile strength of the concrete is ignored (but see page 69). The permissible tensile stresses for reinforcement provided to resist shearing forces are discussed on page 72. A stress of 20,000 lb. is used for mild steel bars not larger than 1½ in. in buildings designed in accordance with the B.S. Code; for bars larger than 1½ in. the permissible stress is 18,000 lb. per square inch.

Since the yield-point stress of rolled mild steel is about 36,000 lb. per square inch, a working stress of 18,000 lb. per square inch represents a factor of safety of about two on the yield-point stress, and this factor is the true measure of the margin of security. For high-yield-stress steel the maximum working tensile stress permitted is generally 50 per cent. of the yield-point stress of the bar or wire in the form used in the concrete, with a limit of 30,000 lb. per square inch (or 20,000 lb. per square inch in shear reinforcement). The tensile stresses recommended in the B.S. Code and London By-laws are given in *Table 58*.

When deciding the working tensile stress suitable for the reinforcement in a part of a structure, the modifying factors given on page 57 should be considered, but the factors that represent a variation in the strength of the concrete only must be disregarded except where the bond stress is affected.

In liquid-containers in which cracking is to be avoided there is no advantage, so far as the tensile stress is concerned, in using a steel with a high yield-point stress if a working stress of 12,000 lb. per square inch is not exceeded although the tensile stress in mild steel reinforcement near the face of a member not in contact with liquid can, however, be 18,000 lb. or 20,000 lb. per square inch; these stresses are in accordance with B.S. Code No. 2007. The latter stresses might be adopted for a tank lined with an elastic impermeable membrane.

The working tensile stress in the reinforcement in buildings can be increased by one quarter when the increase is due solely to increased bending moments and forces caused by wind pressure. The B.S. Code No. 114 limits the increased stress in reinforcement to 30,000 lb. per square inch. The London By-laws permit an increase of one-third, with a limit of 27,000 lb. per square inch.

**Compression.**—The compressive stress in reinforcement depends on the compressive stress in the surrounding concrete if the modular-ratio theory of the action of reinforced concrete at working loads is applied. Since the strain of the two materials is equal so long as the bond is not destroyed, the stresses are proportional to the elastic moduli. Mild steel has a modulus of elasticity $E_s$ of about 30,000,000 lb. per square
inch, and, if the modulus of elasticity of concrete $E_c$ is assumed to be nominally 2,000,000 lb. per square inch, the compressive stress in the steel $f_{se}$ is fifteen times the compressive stress $f_{cb}$ in the concrete, or generally $f_{se} = mf_{cb}$ if $m$ is the modular ratio $E_s/E_c$. The value of $m$ is considered on the page facing Table 56. It is more convenient to calculate the compressive stress in the steel as additional to that in the concrete; this is $f_{cb}(m - 1)$. When this expression is used the resistance of the concrete can be calculated on the whole cross-sectional area, no deduction being necessary for the area of the bars.

When the moment of resistance of a beam is calculated in accordance with the steel-beam theory (see page 66), the stress in mild steel reinforcement in compression is generally considered to be 18,000 lb. per square inch. For high-yield-stress steel the permissible compressive stress is half the yield stress but not more than 23,000 lb. per square inch in accordance with the B.S. Code No. 114.

The compressive stresses in the main reinforcement in columns, and in other cases where the stress is independent of the stress in the concrete, in accordance with the London By-laws and B.S. Codes are given in Table 58.

**Bond between Concrete and Steel.**

**Length for Bond of Tensile Reinforcement.**—For a bar to resist tensile forces effectively there must be sufficient length of bar beyond any section to develop by bond between the steel and the concrete a force equal to the total tensile force in the bar at that section.

The minimum length for bond, or the minimum length of an overlap, can be expressed as $N$ times the diameter $D$ of the bar; for a plain round bar $N = f_{st} = \frac{f_{st}}{45b}$ where $f_{st}$ is the tensile stress in the bar at the section considered and $s_b$ is the permissible average bond stress. Values of $s_b$ are given in Tables 56 and 57 for various qualities of concrete in buildings according to the London By-laws and the B.S. Code. Values of $N$ for various combinations of common tensile stresses $f_{st}$ and $s_b$ are given in Table 62. The lengths of bond for stresses other than those tabulated are proportional to the tensile stresses, or inversely proportional to the permissible average bond stress. The lowest stress tabulated is 10,000 lb. per square inch and, although the working stress in the bar may be less, it is recommended that in no case should the length of bond provided be less than that required for 10,000 lb. per square inch. The B.S. Code recommends that the minimum length for bond should be $12D$ and the corresponding lengths for bars of various sizes are given in Table 62. If this recommendation is applied rigorously the length for small bars is only a few inches, and it is suggested that a minimum length of 6 in. (or preferably 12 in.) should be adopted, as tabulated. The minimum length of an overlap is $30D$; values are given in Table 62. For the common case of the permissible tensile stress $f_{st}$ being 20,000 lb. per square inch (mild steel) and $s_b = 120$ lb. per square inch (1 : 2 : 4 concrete), $N$ is $41\frac{4}{5}$; the length of bond for bars of various sizes for these stresses is given in Table 62.

If at a particular section of a beam a bar is no longer required to resist tension, it should not be stopped in the tensile zone but carried a short distance beyond the section (and preferably bent up at 45 deg. to extend into the compressive zone). Similar bars in slabs can remain in the tensile zone if continued a distance of $12D$ beyond the section. These requirements conform to the B.S. Code except the recommendations in parenthesis.
Anchorages.—If an anchorage is provided at the end of a bar in tension, the length for bond need not be so great as when no such anchorage is provided. An anchorage may be a semi-circular hook, a 45-deg. hook, a 90-deg. bend, or a mechanical anchorage. To obtain full advantage of the value in bond of an anchorage, the hook or bend must be properly formed. Suitable dimensions for common sizes of bars are given in Table 62, together with the length of bar required to form the anchorage additional to the overall length of the bent bar. The radius of the bend must be not less than 2D. The length for bond must be measured from the point where the bar starts to deviate from the straight, that is, where the bend commences. The bond-resistance value of an anchorage of such dimensions is given in the B.S. Code as equivalent to a length of 4D for each 45 deg. through which the bar is bent; that is 8D for a right-angle bend, 12D for a 45-deg. hook, and 16D for a semi-circular hook. The bond values of each of these types of anchorage are given in Table 62 for bars of various sizes.

According to the B.S. Code No. 114, if an end anchorage is provided, the length required for bond (as represented by ND when N has the values in Table 62) can be reduced by the appropriate bond value (as also given in Table 62) of the anchorage, but the reduced length should be not less than 12D. The bond-lengths required for bars with various forms of anchorages in accordance with B.S. Code No. 114 are given in greater detail in Table 63. For more conservative design, the length given by ND can be considered to be the minimum if a semi-circular hook of value 16D is provided; if other forms of anchorage are provided, the length for bond should be ND plus 16D minus the value of the anchorage as given on the page facing Table 62.

A mechanical anchorage can be either a hook embracing an anchor bar (the internal diameter of the hook being equal to the diameter of the anchor bar) or the end of the bar can be threaded and provided with a plate and nut. The size of the plate should be such that the compression on the concrete at, say, 1000 lb. per square inch of net area of contact (that is the gross area of the plate less the area of the bar) should be equal to the tensile resistance required.

Length of Bond for Compression Reinforcement.—The length of bond required to resist a compressive force in a plain round bar is 80 per cent. of that required for a tensile force of the same magnitude. Therefore the values of N in Table 62, multiplied by 0.8, can be used for compression reinforcement subjected to a stress of the same numerical value. For example, for a bar subjected to a compressive stress of 20,000 lb. per square inch in concrete the permissible average bond stress for which is 120 lb. per square inch, the value of N is 0.8 times 42 approximately, say, 34, and the bond-length is 34D. The same rules apply to the length of overlap of bars forming compression reinforcement. The minimum length of bond for bars in compression is 12D, but the minimum length of an overlap is 24D (as in Table 62). The end of a bar in compression need not be provided with an anchorage.

Bars in Liquid-containers.—For liquid-containers, using 1:1:4:3 1/2 concrete, the length required for bond or for overlap of bars in tension, based on $s_b = 130$ lb. per square inch and $P_{st} = 12,000$ lb. per square inch, as in B.S. Code No. 2007, is about 24D (see Table 62); it is recommended (but it is not required by the Code) that a semi-circular hook should be provided in addition to this length, since it is preferable to provide hooks on all plain bars in direct tension in liquid-containers. Hooks can generally be omitted from reinforcement in planar slabs. Greater lengths must be provided where the permissible tensile stress of 18,000 lb. per square inch is adopted.
For bars in compression, end anchorages are not necessary but a minimum length of 20D should be provided for bond and for overlaps. For liquid-containing structures of 1:2:4 concrete, the lengths should be as for ordinary construction.

**Deformed Bars.**—Twisted-ribbed, twisted-square, and similar bars provide greater mechanical bond resistance than plain round bars. The B.S. Code defines a deformed bar as one having a bond resistance (determined by test) 25 per cent. greater than that of a plain round bar, and for such a bar the bond stresses permissible are 25 per cent. higher than those given in Table 57. Therefore the length provided for bond can be based on a value of \( N \) equal to 0.8 of that given in Table 62; if such a length is provided, no end anchorage is required for a deformed bar. Bond-lengths for deformed bars in tension or compression are given on the page facing Table 63.

**Bond of Beam Reinforcement.**—The local bond stress \( s_b \) due to variation in the tensile stress in reinforcement beams should be investigated by application of the formulae given on pages 246 and 248. The permissible stress \( s_b \) is about 50 per cent. greater than the ordinary bond stress. The permissible local bond stresses in accordance with the B.S. Code No. 114 are given in Table 57.

**Details of Reinforcement.**

**Length and Size of Bars.**—If attention is given to a number of points regarding the length and size of reinforcement bars, fixing the bars is facilitated and the construction is more efficient. As few different sizes of bars as possible should be used, and the largest size of bar consistent with good design should be used, thus reducing the number of bars to be bent and placed. Large bars are cheaper than small bars. The basic price is that of \( \frac{4}{6} \) in. bars, all larger bars being supplied at this rate; smaller bars cost more for each \( \frac{4}{6} \) in. in diameter below \( \frac{4}{6} \) in.

Generally the longest bar economically obtainable should be used, but regard should be paid to the facility with which a long bar can be transported and placed in position. Consideration should also be given to the greatest length that can be handled without being too whippy; these lengths are about 20 ft. for bars of \( \frac{4}{8} \) in. diameter and less, 25 ft. from \( \frac{8}{8} \) in. to \( \frac{4}{8} \) in., 40 ft. for \( \frac{3}{6} \) in., 60 ft. for \( \frac{1}{6} \) in., and 75 ft. for bars over \( 1 \frac{1}{6} \) in. The basic price only applies to bars up to 40 ft. long, and extra for every foot in length over 40 ft. is charged. Bars up to \( \frac{4}{8} \) in. can be obtained in long lengths in coils at ordinary prices and sometimes at lower prices. Over certain lengths it is more economical to lap two bars than to buy long bars, the extra cost of the increase in total length of bar due to overlapping being more than offset by the increased charge for long lengths. Long bars cannot always be avoided in long piles, but bars over 40 ft. require special road or rail vehicles which may result in delay and extra cost.

The total length of each bar should, where possible, be given to a multiple of 3 in. and as many bars as possible should be of one length, thus keeping the number of different lengths of bars as small as practicable.

**Bar-bending Schedule.**—The method of giving bending dimensions and marking the bars should be uniform throughout the bar-bending schedules for any one structure. A system of bending dimensions is illustrated in Table 62 and conforms to B.S. No. 1478 in which bars of other shapes also are given. Most designers have their own form of bending schedule to suit the work in hand. Generally, however, it is necessary to give the following information thereon: Position of the bar in the structure, the diameter and total length of the bar, the number of bars of one type
in a single member, the number of identical members and the total number of bars of one type, the shape and bending dimensions of the bar, and perhaps a reference mark for each bar or bundle of bars.

**Cover of Concrete.**—For the proper protection of the reinforcement and in order to ensure that the thickness of concrete around a bar is adequate to develop the bond resistance, it is necessary to provide sufficient concrete over the bars and sufficient space between adjacent bars. The minimum cover of concrete to a reinforcement bar should be as given on the page facing Table 63 and in no case should it be less than the diameter of the bar.

It should be noted that much of the deterioration of reinforced concrete structures is due to insufficient cover of concrete to the bars; therefore, the designer should not hesitate to increase the minimum covers if it is thought to be desirable to do so for a particular structure.

**Spacing of Bars.**—The minimum clear distance between two bars in any one layer in a beam should be not less than the diameter of the bar, or 1 in., or the largest size of the aggregate plus \( \frac{1}{4} \) in., whichever is the greatest. The minimum clear distance between successive layers of bars in a beam should be \( \frac{1}{2} \) in. and this distance should be maintained by the provision of \( \frac{1}{2} \)-in. diameter spacer bars placed every 3 ft. or 4 ft. throughout the length of the beam wherever two or more layers of reinforcement occur. Where the bars from transverse beams pass between the layers, spacer bars are unnecessary. If the bars in a beam exceed 1 in. in diameter, it is preferable to increase to \( \frac{3}{4} \) in. or 1 in. the clear space between the layers. If concrete is to be compacted by vibration, a space of at least 3 in. should be provided between groups of bars for the insertion of the vibrator if of the internal or poker type.
SECTION 4

RESISTANCE OF STRUCTURAL MEMBERS

Properties of Cross-sections of Members.

The geometrical properties of plane figures, the shapes of which conform to those of the cross-sections of common reinforced concrete members, are given in Table 64. The data include areas, section moduli, moments of inertia, and radii of gyration. In particular are given data for readily calculating the moment of inertia of tee-sections which are also applicable to other flanged sections such as ell-sections and inverted channels. These data are suitable for cases when the amount of reinforcement need not be taken into account as in the case of comparative moments of inertia (see page facing Table 51).

The data in Tables 65A and 65B apply to reinforced concrete members of rectilinear and polygonal cross-sections when the reinforcement is taken into account on the basis of the modular ratio. Two conditions are considered, namely, when the entire section is subjected to stress, and when the concrete in tension in members subjected to bending is not taken into account. The data given for the former condition includes the effective area, the position of the centroid, the moment of inertia, the section modulus, and the radius of gyration. For the condition when a member is subjected to bending and the concrete is ineffective in tension, the data include the position of the neutral plane, the lever-arm, and the moment of resistance. The corresponding general formulae for irregular and regular sections are given on the page facing Table 65B.

Design of Beams: Modular-ratio Method.

There are two fundamental methods of analysing the stresses in, and calculating the resistance of, a reinforced concrete member in bending: the ordinary elastic or modular-ratio method, and the ultimate-load or load-factor method; both methods are included in B.S. Code No. 114. The load-factor method is dealt with on page 70.

The modular-ratio method applies to conditions at working stresses only. The basis of design of a structural member subjected to bending is that the internal resisting couple is equal to the bending moment produced by the external load. In a reinforced concrete member, the equal forces forming the couple are the compressive resistance of the concrete and the tensile resistance of the tensile reinforcement; the arm of the couple, that is the distance between the lines of action of the resultant forces, is called the lever-arm. It is assumed that the strain in any plane of the member is proportional to the distance of the plane from the neutral plane, that the tensile resistance of the concrete is neglected, that the reinforcement resists all the tensile forces (except in certain cases of liquid-containers), and that the concrete and steel are elastic within the range of the permissible working stresses. The formulae for the position of the neutral plane, the lever-arm, the moments of resistance, and the maximum stresses in rectangular beams and flanged beams (that is tee-beams and ell-beams), based on the foregoing principles are given in Table 66. The notation is summarised on the page facing the table. For beams of other regular cross-sections, the expressions for the lever-arm and moments of resistance given in Tables 65A
and 65B are applicable. For a beam of any general or irregular cross-section the method described on the page facing Table 68 can be used.

Formulae (1) and (1a) in Table 66 apply to members of any cross-section subjected to bending and give the neutral-plane factor \( n_1 \) expressed in terms of the permissible and actual stresses respectively; \( n_1 \) is dependent only on the modular ratio \( m \) and the ratio of the maximum stresses in the tensile reinforcement and in the concrete, that is \( p_{st} \) and \( p_{e} \) or \( f_{st} \) and \( f_{e} \) respectively. Values of \( n_1 \) for \( m = 15 \) and for various ratios \( r \) of the maximum stresses are given in Table 68 or can be obtained from the curves in Table 67.

The formulae (14) for the maximum stresses and formulae (15) for the ratio of the maximum stresses (in Table 66) involve \( n_1 \) and \( m \) only, and are therefore applicable to a member of any cross-section.

**Rectangular Beams.**—Formulae (2) and (2a) in Table 66 apply to rectangular beams whether reinforced to resist tension only or to resist tension and compression and give the neutral-plane factor in terms of the proportions of tensile and compression reinforcement, that is \( r_{t} \) and \( r_{c} \) respectively; values of \( n_1 \) for various values of \( r_{t} \), or conversely, are given in Tables 67 and 68 for \( m = 15 \).

The formulae for the lever-arm \( l_{a} \) and the lever-arm factor, \( a_{t} \) in general, \( a_{c} \) for the concrete in compression and \( a_{t} \) for the compression reinforcement, are given by the formulae in the series (5) in Table 66. For values of \( a_{t} = a_{c} \) see Tables 67 and 68.

The moment of resistance of a rectangular beam reinforced in tension only is given by formulae (8) and (8a), depending on whether the resistance to compression or tension \( (M_{re} \) and \( M_{st} \) respectively) determines the strength. If \( n_1 \) is based on the maximum permissible stresses, that is formula (1a), the values of \( M_{re} \) and \( M_{st} \) may differ; if \( n_1 \) is based more properly on formula (2), \( M_{re} \) and \( M_{st} \) will be equal.

The moment of resistance in compression can be expressed conveniently in terms of a factor \( Q_{c} \) such that \( M_{re} = Q_{c} b d_{1}^{3} \). Values of \( b d_{1}^{3} \) for various values of \( b \) and \( d_{1} \) are given in Table 69, and values of \( Q_{c} \) for various stresses with \( m = 15 \) are given in Table 68.

The maximum stresses produced by a given bending moment, with \( n_1 \) based on formula (2), are calculated from formulae (15).

*Tables 70A, 70B and 70C give the moments of resistance and areas of reinforcement for rectangular beams of various depths and of unit width reinforced in tension only for tensile stresses of 18,000 lb., 20,000 lb., and 27,000 lb. per square inch in the reinforcement and compressive stresses of 750 lb., 1000 lb., and 1100 lb. per square inch in the concrete (\( m = 15 \)); Table 70D gives similar data for a tensile stress of 30,000 lb. per square inch and compressive stresses of 750 lb., 1000 lb., and 1250 lb. per square inch.*

When sufficient depth or breadth of beam cannot be obtained to provide enough compressive resistance from the concrete alone compression reinforcement is provided. This extra reinforcement is not generally economical, although some concrete is saved thereby, but in some cases, such as the support sections of continuous beams, the ordinary arrangement of the reinforcement provides compression reinforcement conveniently; the amount of compression reinforcement should not exceed 4 per cent. in accordance with B.S. Code No. II4, and compression reinforcement in excess of this amount should be neglected in the calculation of the resistance of the beam.

If the compressive resistance of the concrete is not neglected, the moment of
resistance of a beam with compression reinforcement is the sum of the moments of resistance of the concrete and compression reinforcement. The moment of resistance of the concrete is calculated as for a beam with tensile reinforcement only, and the additional moment of resistance due to the compressive reinforcement is as given by formula (8b) in Table 66, in which \( n_1 \) is based on formulae (2a). The maximum stresses due to a given bending moment are derived from formulae (14) in which \( I_a \) is based on \( a_1 \) calculated from formula (5a), or approximately from (5b), and \( n_1 \) from formula (2a); note that formula (5b) does not apply if \( r_s \) is small compared with \( r_t \).

The rational limit of application of the formulae for rectangular beams with compression reinforcement is when \( A_{sc} = A_{st} \), and for this condition the moment of resistance if given by

\[
M_r = \left[ \frac{1}{2} \left( n_1 a_c + r_c (m - 1) \left( \frac{n_1 - f_s}{n_1} \right) (1 - f_s) \right) \right] \varphi_{bc} b d_1^2,
\]

and the proportion of tensile reinforcement, which is equal to the proportion of compression reinforcement, is given by

\[
r_t = \frac{0.5 n_1}{r - (m - 1) \left( \frac{n_1 - f_s}{n_1} \right)},
\]

in which \( r \) is the ratio of the permissible stresses \( \left( = \frac{\varphi_{st}}{\varphi_{bc}} \right) \). Values of \( M_r \) and \( r_t \) are given at the foot of Tables 70a, 70b, 70c, and 70d for common permissible stresses and \( m = 15 \). The moments of resistance and areas of reinforcement for rectangular beams of various depths and of unit width reinforced in tension and compression are also given in these tables, from which a beam to resist a specified bending moment can be selected since the moment of resistance for the limits of \( A_{sc} = 0 \) and \( A_{sc} = A_{st} \) are given for stresses of 18,000 lb., 20,000 lb., 27,000 lb., and 30,000 lb. per square inch in the tensile reinforcement and stresses of 750 lb., 1000 lb., and 1100 lb. or 1250 lb. per square inch in the concrete. The compressive resistance of the concrete is not neglected; the stress in the compression reinforcement is therefore \( m \)-times the stress in the surrounding concrete.

To prevent the compression reinforcement from buckling, binders should be provided at a pitch not exceeding twelve times the diameter of the smallest bar in the compression reinforcement. The binders should be so arranged that each bar is effectively tied.

**Balanced Design.**—In the design of a beam it is of course necessary that the permissible stresses in the steel and concrete are not exceeded, but it is also desirable generally for the greatest stresses to be equal to the permissible stresses. When this condition is obtained, the design is considered to be "balanced". There is, for each ratio of permissible stresses, a proportion of tensile and compression (if any) reinforcement which gives balanced design, and expressions for this amount are given in Table 66. The percentage of reinforcement corresponding to the proportion for a given ratio of stresses is sometimes called the "economic percentage", but this may be a misnomer because the relative amounts of steel and concrete in the most economical beam or slab are dependent not only on the permissible stresses but also on the costs of the materials and shuttering.

**Steel-beam Theory.**—If the amount of compression reinforcement required equals or exceeds the amount of tensile reinforcement when using the formulae in
RESISTANCE OF STRUCTURAL MEMBERS

Table 66, a beam may be designed by the "steel-beam theory" in which the compressive resistance of the concrete is neglected and

\[ A_{st} = A_{se} = \frac{M}{(I - f_a)d_kp_{st}}. \]

When this method is adopted the spacing of the binders should not exceed eight times the diameter of the bars forming the compression reinforcement, and \( p_{st} \) should not exceed 18,000 lb. per square inch. The indiscriminate application of the steel-beam theory is not recommended. At first sight it seems that a beam of any size can be designed to resist almost any bending moment irrespective of the compressive stress in the concrete. In fact with a theoretical compressive stress of 18,000 lb. per square inch in the reinforcement, the theoretical compressive stress in the concrete may exceed 1200 lb. per square inch, which for ordinary concrete leaves little margin for accidental overloading, differences between theoretical and actual bending moments and stresses, poor workmanship, and other factors. Partial safeguards against unreasonable use of the steel-beam theory include the provision of sufficient area of concrete to resist the shear forces, the space required for the bars in the top and bottom of the beam, and the reduction in the lever-arm consequent upon large numbers of bars requiring more than one layer of reinforcement in the top and bottom. When \( A_{se} \) approaches equality with \( A_{st} \) it is preferable to design by the load-factor method described on page 70.

Flanged Beams: Tee-beams, Ell-beams and I-beams.—If a beam is constructed monolithically with a slab, the slab forms the compression flange of the beam if the bending moment is such that compression is induced in the top of the beam. If the slab extends an equal distance on either side of the rib, that is the beam is a tee-beam, or if the slab extends on one side only of the rib, as in the case of an ell-beam (or inverted ell), the breadth of slab assumed to form the effective compression flange should not exceed the least of the dimensions given in Table 69.

There are two cases to consider, namely, where the neutral plane is within the thickness of the slab and where the neutral plane is below the slab. In the former case, a flanged beam is dealt with in the same way as a rectangular beam the breadth \( b \) of which is the effective width of the flange. If the neutral plane is below the slab, the small compressive resistance afforded by the concrete between the neutral plane and the underside of the slab is often neglected, and the corresponding formulae in Table 68 apply. Note the approximate expression for the lever-arm in formulae (6b) and (6c); this value is sufficiently accurate for most tee-beams and ell-beams.

It is not common for beams with compression flanges to require compression reinforcement, but if this is unavoidable the same principles apply as for rectangular beams. The theoretical formulae in this case are too complex to be of practical value, although they may be of some use for I-beams, the design of which is dealt with on page 258.

Beams with Concrete effective in Tension.—In the design of liquid-containers, and some other structures, resistance to cracking of the concrete in the tension zone is important; therefore the members concerned are calculated taking the concrete as effective in tension. The corresponding formulae for rectangular and flanged beams are given in Table 69.

Proportions and Details of Beams.—The dimensions of beams are primarily determined from considerations of the moment of resistance and resistance to shearing force, but beams having various ratios of depth to breadth may give the resistances
required. There are in practice several other factors that may affect the relative dimensions.

To limit the deflection, the depth of a rectangular beam should be in accordance with the limiting ratios given in Table 15. A rule for determining a trial section for a rectangular beam or tee-beam is that the total depth in inches should be equal to the clear span in feet. The breadth of a rectangular beam or the breadth of the rib of a tee-beam generally varies from one-third to one times the total depth; for rectangular beams in buildings a reasonable breadth is one-half to two-thirds of the total depth; in industrial structures beams having proportions of breadth to depth of one-half to one-third are often convenient. The lower ratio in each case applies principally to tee-beams. Much, however, depends upon the conditions controlling a structure, especially such factors as clearances below beams and the cross-sectional area required for resistance to shearing. The breadth of beams should also conform with widths of timber as commercially supplied. The cross-sectional dimensions of beams and columns and similar members should be multiples of $\frac{1}{4}$ in. where practicable (see B.S. No. 2539). In buildings the breadth of beams may have to comply with the nominal thicknesses of brick walls, that is $4\frac{1}{2}$ in., 9 in., $13\frac{1}{2}$ in., or 18 in. If the ratio of the span to the breadth of a beam exceeds 30, the permissible compressive stress must be reduced in accordance with the rules given on the page facing Table 62.

The breadth of the rib of a flanged beam is generally determined by the cross-sectional area required for resistance to the shearing force, but consideration must also be given to the accommodation of the reinforcement; a preliminary rule for determining the minimum breadth of rib in inches is $2\frac{1}{2}$ times the number of bars in one layer. When the calculations have been completed the breadth should be checked to ensure that sufficient space is provided for the bars.

Methods of designing beams or determining the stresses in beams either by the use of the tables or by application of the formulæ, are given on the page facing Table 67. Examples of the use of the tables are given on the page facing Tables 68 and 69. Typical details and calculations for freely-supported and continuous beams are given in Appendix II.

**Design of Solid Slabs: Modular-ratio Method.**

A slab is generally calculated for a strip 1 ft. wide; hence a slab is equivalent to a rectangular beam with $b = 12$ in., and the moment of resistance and the bending moment are expressed per foot of width. The formulæ in Table 66 for rectangular beams apply to slabs, but as $b$ has a constant value, the expressions can be modified to facilitate computation; for example, the effective depth and the area of reinforcement required can both be expressed as simple functions of the bending moment as given in Table 71. Notes and examples on the use of Table 71 are given on the page facing the table, which can be used for almost any permissible stresses.

When the tensile stresses are 18,000 lb., 20,000 lb., 27,000 lb., or 30,000 lb. per square inch and the compressive stresses are 750 lb., 1000 lb., or 1250 lb. per square inch, or in the same ratio as pairs of these stresses, suitable slabs can be selected from Table 72 in which are given the moments of resistance of, and area of reinforcement for, slabs 12 in. wide without compression reinforcement; examples are given on the page facing the table.

Common arrangements of, and notes on, the reinforcement in solid slabs are given on pages 266 and 268. It is not usual to provide compression reinforcement in slabs unless a convenient arrangement of reinforcement is obtained thereby, but
when it is necessary to do so the method of calculation is the same as that for rectangular beams. Links or other means to prevent the bars from buckling should be provided at centres not exceeding twelve times the diameter of the bars in compression; otherwise the bars in compression should be neglected when computing the resistance.

Reinforcement for resistance to shearing force is not generally necessary in slabs. Shearing stresses need not be considered unless the span is small.

Although slabs 2 in. thick can be constructed with fine concrete, for ordinary cast-in-situ slabs 3 in. should be the minimum thickness. The minimum thickness of the slab should be not less than the limiting values given in Table 62.

Typical details and calculations for solid slabs are given in Appendix II.

**Slabs in Liquid-containing Structures.**

*Tables 73 and 74 give data for the design of walls and other slabs in a structure containing water or other aqueous liquid in accordance with the recommendations of British Standard Code No. 2007 (1960). There are three principal cases to consider, namely, I. Slabs subjected to bending only; II. Slabs subjected to direct tension only; III. Slabs subjected to bending and direct tension. Table 73 deals with case I and Table 74 with cases II and III. The following conditions are applicable to all cases.*

The tensile stress \( \rho_{st} \) in the reinforcement is not to exceed 12,000 lb. per square inch (except in tensile reinforcement near the face remote from the liquid for slabs not less than 9 in. thick). The nominal volumetric proportions of the concrete are 1:1.6:3.2. The compressive stress in the concrete is not to exceed 1200 lb. per square inch; this requirement is seldom critical. The tensile stress in the concrete is not to exceed 190 lb. per square inch in slabs in direct tension and 270 lb. per square inch in slabs in bending; this requirement applies when designing for resistance to cracking. The cover of concrete over the reinforcement is to be not less than 1\( \frac{1}{4} \) in. or the diameter of the bar if exceeding 1\( \frac{1}{4} \) in.

The basic formulae and conditions from which the expressions and data in *Tables 73 and 74* are derived and examples are given on pages 270, 272 and 274.

**Case I. Slabs subjected to Bending Only.—**Slabs subjected to bending have to be designed to resist cracking and, should cracking occur, they must be reinforced sufficiently and be sufficiently thick to avoid overstressing the reinforcement in tension or the concrete in compression. This basis of design applies to all slabs when the strain at the face in contact with the liquid is tensile, and to slabs less than 9 in. thick whether tensile strain occurs at the face in contact with, or remote from, the liquid. If in a slab not less than 9 in. thick, the tensile strain is at the face remote from the liquid, the method of design is as for an ordinary slab with the stress in the reinforcement not greater than 18,000 lb. per square inch in plain bars or 20,000 lb. per square inch in deformed bars, and the compressive stress in the concrete not greater than 1200 lb. per square inch.

**Case II. Slabs subjected to Direct Tension Only.—**Planar slabs subjected to direct tension only without bending are not common; a curved slab, such as the wall of a cylindrical tank, is the more common case. The two conditions of design are that the tensile stress \( \rho_{st} \) in the concrete should not be so great as to cause cracking, and, should cracking occur, the reinforcement should be able to resist the whole of the tensile force without exceeding the permissible tensile stress. The first of these conditions determines the thickness of the slab. The second condition determines the amount of reinforcement.
Case III. Slabs subjected to Direct Tension combined with Bending.—
The two cases are when the tensile strain is at the face in contact with the liquid, and when this face is subjected to compressive strain and the opposite face to tensile strain.

There are two conditions for the case of tensile strain at the face in contact with the liquid, namely, when tensile stresses only are produced, and when tensile and compressive stresses are produced. When there are tensile stresses only, which condition occurs when \( e \) does not exceed \( \frac{d_1}{d_4} \), the case is like a slab subjected to direct tension only and therefore the tensile stress in the concrete must not exceed the stress permissible in direct tension if no cracking is to occur; the thickness of the slab is determined from this requirement. Should cracking occur, the permissible tensile stress in the reinforcement must not be exceeded; this condition determines the amount of reinforcement.

If there are tensile and compressive stresses, which condition occurs when \( e \) exceeds \( \frac{d_1}{d_4} \), the case is analogous to a slab subjected to bending only, and therefore the limiting stresses for that condition apply. Since the effect of reinforcement in the compression zone of a slab is insignificant, such reinforcement, if any, can be neglected.

When the tensile strain is at the face remote from the liquid, there are two conditions to be considered, namely, when the slab is less than and not less than 9 in. thick. For the former case the design is exactly as in the immediately preceding case, taking into account whether there are tensile stresses only or tensile and compressive stresses.

Design of Beams and Slabs: Load-factor Method.

The load-factor, or ultimate-load, method is based on the behaviour of the member when near failure. The safe resistance under working conditions is a fraction of the ultimate resistance, say, one-third if the crushing strength of the concrete determines the ultimate resistance, or, say, one-half if the yield stress of the reinforcement is decisive. Formulae and factors for these conditions for beams and solid slabs subjected to bending are given in Tables 75 to 79, and are based on the recommendations in B.S. Code No. 114. The notation is given on the page facing Table 75, and the basis of the tables and examples on their use are given on the pages facing Tables 75 to 79. The basic formulae for the moment of resistance and basic design formulae for beams with or without compression reinforcement are given in Table 75, together with the corresponding formulae simplified for cases in which ordinary \( 1:2:4 \) concrete is used and mild steel bars or high-yield-stress bars are provided in beams and solid slabs. Values of some of the factors used in these formulae are given in Table 76. Numerical values of moments of resistance and areas of reinforcement for common conditions for rectangular beams with mild steel reinforcement are given in Table 77A, and with high-yield-stress bars in Table 77B; similar data for solid slabs are given in Table 78, and for flanged beams in Table 79.

Deflection of Beams.

The deflection of reinforced concrete members cannot be calculated with precision, but this is not of serious consequence since in most cases comparative deflections only are required, and the indefinite numerical values offset each other to a large extent. If the amount of the actual maximum deflection of a beam is required to be known,
it can be found approximately from \( \frac{WL^2K}{E_cI_c} \), where \( W \) is the total load on the beam, \( L \) is the span, \( E_c \) is the modulus of elasticity of concrete in compression, \( I_c \) is the equivalent moment of inertia, and \( K \) is the coefficient of deflection depending on the type of load and the conditions at the supports of the beam; values of \( K \) for some common loads are given in Tables 16, 17 and 17A. If all the terms are in inches and pounds, the deflection will be in inches.

An appropriate value for \( E_c \) is 3,750,000 lb. per square inch, corresponding to a modular ratio of 8, but if a more accurate value can be obtained from tests it should be used. The moment of inertia should be expressed in concrete units and should be that at the centre of the span, that is, at the position of the maximum positive bending moment. The moment of inertia in this instance should be computed for the whole area of the concrete within the effective depth, that is, the area of concrete between the neutral plane and the tensile reinforcement should be included as well as the area of the concrete above the neutral plane. The area of the tensile and compression reinforcement should be allowed for by transforming to the equivalent additional area of concrete, that is, the effective modular ratio [say \((8 - 1) = 7\)] multiplied by the area of the reinforcement. The moment of inertia should be taken about the centroid of the transformed area, and is approximately \( \frac{(1 + 4inr)bd^3}{12(I + mnr)} \) for a rectangular beam reinforced in tension only, the proportion of tensile reinforcement being \( r_t \). The corresponding expressions for rectangular beams with compression reinforcement, and for tee-beams, are so complex that it is easier to calculate \( I_c \) with actual numerical values from first principles, or use the data in Tables 65 and 65A or the formulæ on page 352.

Data for limiting the deflection of beams and solid slabs are given in Table 15.

**Resistance to Shearing Force.**

**Shearing Stresses.**—Shearing force produces diagonal tensile stresses in the concrete and, if the stresses exceed the safe tensile stress in the concrete, reinforcement should be provided in the form of either inclined bars or binders, or both, to provide the shearing resistance. The shearing stress \( q \) in a member subjected to a bending moment or a shearing force \( Q \) acting alone is calculated from the formulæ given on the page facing Table 80. If the beam is of uniform cross-section, the shearing force, and therefore the shearing stress, is independent of the bending moment, except in so far that shearing force is the rate of change of bending moment. Values of the safe shearing stresses with or without reinforcement are given in Tables 56 and 57.

The distribution of shearing stress is such that at any point in a beam there exists a horizontal shearing stress equal in intensity to the vertical shearing stress at the point. In a plain rectangular concrete beam of uniform cross-section, the mean intensity of the shearing stress varies parabolically from nothing at the top and bottom edges to a maximum of \( 1.5 \frac{Q}{bd} \) at the mid-depth of the beam. In a reinforced concrete beam in which the concrete in tension below the neutral plane is ignored, there is a parabolic increase from nothing at the top edge to \( \frac{Q}{ld} \) at the neutral plane, and this stress \( q \) is constant from the neutral plane to the centre of the tensile reinforcement. Shearing stresses due to torsion on a rectangular section attain maximum values at the middle of the longer sides.
Without assistance from reinforcement the concrete alone is sometimes considered as capable of resisting the total shearing force when the shearing stress $q$ is not greater than the safe shearing stress. The Ministry of Transport permits this rule to be applied to slabs, but for beams in bridges only one-third of the shearing force may be resisted by the concrete, the remainder being resisted by reinforcement. In industrial structures it is preferable to resist the entire shearing force by reinforcement in the form of inclined bars, and to provide resistance to shearing forces even when the calculations for ordinary loading indicate that the shearing stresses are small; by so doing, some provision is made for abnormal loads to which beams in such structures may be subjected. The B.S. Code recommends that in narrow beams (as defined on page 246), the entire shearing force should be resisted by reinforcement whatever the shearing stress.

If reinforcement is provided to resist the whole of the shearing force the stress $q$ should not exceed four times the safe shearing stress; this is in accordance with the B.S. Code and London By-laws. Generally, however, this extreme value of $q$ should not be used and a limit of $2\frac{1}{2}$ times the safe shearing stress is preferable for secondary beams which may be subjected to greater incidental loads, although the higher limit might be used for main beams in buildings (not warehouses) if the full design load is not likely to occur.

**Reinforcement to resist Shearing Forces.**—The reinforcement provided to resist shearing forces may be in the form of vertical binders or inclined bars. The shearing resistance of such reinforcement is given in Table 81 for tensile stresses of 18,000 lb. and 20,000 lb. per square inch; the resistances at other stresses are proportional. In some cases, such as beams subjected to vibration and impact, the stress in the reinforcement provided to resist shearing forces should be less than 18,000 lb. per square inch, say, 12,000 lb. per square inch, and binders of small diameter closely spaced are preferable. In liquid-containing structures (B.S. Code No. 2007) the stress should not exceed 12,000 lb. per square inch, that is the resistances are two-thirds of those for 18,000 lb. per square inch in Table 81. B.S. Code No. 114 recommends a limiting tensile stress of 20,000 lb. per square inch in shearing reinforcement, comprising mild steel or high-yield-stress bars, in buildings and not more than 18,000 lb. per square inch in mild steel bars greater than 1\frac{1}{2} in. in diameter.

Notes on the spacing, diameter, and shape of binders are given in Table 80 and on the page facing the table.

The principle assumed in evaluating the shearing resistance of inclined bars is that the bars form the tension members of a lattice girder, the inclined compression in the concrete providing the corresponding compression member. The shearing resistance at any section is the sum of the vertical components of all the inclined bars and compression "struts" cut by the section. Notes on the arrangement of inclined bars as affecting the stresses therein are given on pages 288 and 289. The resistances in Table 81 are for bars from $\frac{1}{8}$ in. to $1\frac{1}{2}$ in. in diameter inclined at 45 deg. and 30 deg.

Inclined bars are commonly provided by bending up the main tensile reinforcement, but in so doing inspection must be made to ensure that the bar is not required to assist in providing the moment of resistance beyond the point at which it is bent. The points at which bars can be dispensed with as reinforcement against bending are given in Table 69, which applies to beams having up to eight bars as the principal tensile reinforcement. Although a bar can be bent up at the points indicated, it is
not implied that if it is not bent up it can stop at these points, since it may not have sufficient length of bond from the point of critical stress. This length depends on the rate of change of bending moment, and should be investigated in any particular beam. Examples of the calculation of the shearing resistance at any section of a beam and the design of a member to resist shearing force are given on the page facing Table 81.

**Torsion.**

The design of reinforced concrete members subject to twisting moments is based largely on tests. The data in Table 82 are derived from the results of tests made by Mr. Leslie Turner and Mr. V. C. Davies.

Members subject to twisting moments should not have re-entrant angles. In designing for ordinary shearing force and bending moment, lower stresses than usual should be adopted, thereby leaving a margin for the secondary torsional stresses. The moment of resistance to torsion of a reinforced member can be expressed as \( RD^2/q \), where \( D \) is the equivalent diameter (in.), and \( q \) is the safe shearing stress (lb. per square inch). \( R \) is a section-coefficient the value of which depends upon the percentage of reinforcement and the shape of the member; values are given in Table 82 for the shapes most commonly used. The equivalent diameter \( D \) is the diameter of the largest circle that can be inscribed within the solid section. If the member is subjected to a bending moment or a shearing force that causes the concrete below the neutral plane to crack, the area of the cracked concrete should be ignored when assessing the shape of the cross-section and the value of \( D \). The amount of reinforcement is equal to the sum of the amounts of longitudinal reinforcement and binding inserted solely to resist torsion. Only longitudinal bars near the outer faces of the section can be considered as effective. It is common to provide, for this purpose, equal amounts of binders and longitudinal bars, since lateral binding is of considerable value for resisting torsion especially if arranged diagonally.

The method of designing a member to resist torsion is to design first for the ordinary bending moments and shearing forces (using reduced stresses) and to determine \( D \) for this section. The value of \( q \) given in Table 56 or 57 is then used to determine the section-coefficient from \( R = \frac{T}{D^2q} \), where \( T \) is the applied twisting moment. The amount of reinforcement required for the appropriate shape can be determined from Table 82 and should be divided equally between lateral binders and longitudinal bars. The combination of torsional shearing stress with the shearing stress due to the ordinary shearing force is complex and, failing the adoption of a more accurate analysis, the designer should assure himself that, at any section reinforced for shearing force, the numerical sum of the two stresses does not exceed a reasonable value, say, not more than twice the shearing stress commonly allowed. Consider the numerical example given on the page facing Table 82, in which at the point of contraflexure (approximately at the fifth-point of span) the shearing stress due to the ordinary shearing force is 54 lb. per square inch and the torsional shearing stress does not exceed 100 lb. per square inch; thus the sum of the two shearing stresses does not exceed 154 lb. per square inch which is less than twice 100 lb. per square inch.

**Arcate Beams (Bow Girders).**

Bow girders and beams that are not rectilinear in plan are subjected to twisting moments in addition to the normal bending moments and shearing forces. Beams forming a circular arc in plan may form part of a complete circular system supported
on columns equally spaced and each span equally loaded such as occurs in water towers, silos, and similar cylindrical structures. The equivalent of these conditions also occurs if the circle is incomplete so long as the appropriate negative bending moment can be developed at the end supports. This type of circular beam occurs in structures such as balconies.

The method of designing for twisting moments as given in the foregoing and in Table 82 applies. The maximum twisting moment occurs at the point of contraflexure; therefore the design of the section in torsion can be based on the entire section, assuming no cracking of the concrete due to tensile stresses caused by bending or shearing force. The position of the point of contraflexure expressed in terms of the angular distance from the support is also given in Table 82. The maximum shearing force, equal in value to half the total load on the beam, occurs at the edge of the support. The maximum positive bending moment occurs midway between the supports; it is preferable to design for a positive bending moment in excess of the theoretical value, say, half that of the negative bending moment at the support.

The critical design sections are therefore: (a) Midspan—maximum positive bending moment and zero shearing force and zero twisting moment; (b) Point of contraflexure—zero bending moment, maximum negative twisting moment combined with shearing force; (c) Support—maximum negative bending moment, maximum shearing force, and maximum positive twisting moment.

An analysis of curved beams allows for the torsional rigidity of the section and gives the following expressions for the twisting and bending moment at any point X in a beam carrying a uniformly-distributed load.

\[ M = M_s \cos \theta_X + T_s \sin \theta_X - F_s R \sin \theta_X + w R^2 (1 - \cos \theta_X) \]

\[ T = - M_s \sin \theta_X + T_s \cos \theta_X + F_s R (1 - \cos \theta_X) - w R^2 (1 - \sin \theta_X) \]

where \( M_s, T_s, \) and \( F_s \) are respectively the bending moment, twisting moment, and shearing force at support \( S, \) \( R \) is the radius of the beam, and \( \theta_X \) is the angle defining the position of the point \( X, \) as shown in the diagram on Table 82.

If a curved beam is subjected to a concentrated load \( P \) such that the angle between the radii through the point of application of \( P \) and support \( S \) is \( \alpha, \) the following expressions apply at any point \( X \) if \( \theta_X > \alpha. \)

\[ M = M_s \cos \theta_X + T_s \sin \theta_X - F_s R \sin \theta_X + P R \sin (\theta_X - \alpha) \]

\[ T = - M_s \sin \theta_X + T_s \cos \theta_X + F_s R (1 - \cos \theta_X) - P R [1 - \cos (\theta_X - \alpha)]. \]

If \( \theta_X < \alpha, \) the terms containing \( PR \) equal zero.

For a number of common cases of circular beams, formulæ and coefficients for evaluating \( M_s, T_s, \) and \( F_s \) are given in Table 82, together with the corresponding factors for the bending moment, twisting moment, and shearing force at mid-span and at the point of contraflexure. The general formulæ given in the foregoing are derived from the analyses given by Professor A. J. S. Pippard and Professor J. F. Baker, and the coefficients and expressions in Table 82 are based on the development (as subsequently modified) by Professor W. T. Marshall and Dr. G. G. Meyerhof.

**Safe Loads on Short Columns.**

The imposed loads for which columns in buildings should be designed are the same as those for beams as given in Table 3, except that the minimum load does not apply. The imposed load on the floors supported by the columns may be reduced (see Table 3) when calculating the load on the column in accordance with the scale
given for multiple-story buildings. External columns in buildings, and internal columns under certain conditions, should be designed to resist the bending moments due to the restraint at the ends of beams framing into the columns, and due to wind (see Tables 46 and 51). An approximate method of allowing for the bending moment on a column forming part of a building frame is to design for a concentric load $F$ times the actual load, where $F$ is as given on page 228 for different arrangements of beams framing into the column; so many factors affect the value of $F$ that the tabulated values can be only approximate and the final design must be checked by more accurate calculation.

The working stress for which a column should be designed is the direct compressive stress $p_{cc}$ (Tables 56 and 57) when the column is subjected to direct load only. When the column is subjected to bending in addition to the load, the stresses should be determined in accordance with the methods described in Tables 85 to 88, and the compressive stress in the concrete should not exceed the permissible compressive stress $p_{eb}$ in bending (Tables 56 and 57). Columns subject to bending should, however, be checked to ensure that under the load assumed to be acting alone, the permissible direct compressive stress is not exceeded. The stresses in the foregoing may be exceeded by not more than one-third (London By-laws), or one-quarter (B.S. Code), if the effects of wind are included in the calculation of the load and the bending moment, but the stress $p_{cc}$ or $p_{eb}$ must not be exceeded when these effects are excluded. The modifications to specified working stresses as given on page 57 should be considered when assessing the safe working stress in a column. When the load on the column is ascertained approximately the stress in the concrete should be well below the permissible stress unless it is known for certain that the approximate load has been over-estimated. When the column loads are carefully calculated and the elastic reactions from the beams are taken into account, the calculated stress can be increased to the permissible stress. The foregoing rules apply to "short" columns. The reduction of the safe load on slender columns is discussed below.

Reinforced concrete columns are generally either rectangular in cross-section with separate binders, or circular or octagonal with helical binding. In some multiple-story residential buildings columns of ell-shape or tee-shape are formed at the intersection of reinforced concrete walls. In most reinforced concrete columns the main vertical bars are secured together by means of separate links or binders. Rules for the arrangement of such binding, the limiting amount of main reinforcement and formulae for the safe load on such columns are given in Table 83; safe loads on square columns with separate binders are given in Table 84.

By forming the binding in a column in the form of a continuous helix instead of separate binders, the safe load on the column is increased. The form of the binding must be circular, or nearly so on plan. The conditions given in Table 82 and on the page facing the table conform to the B.S. Code.

Columns with helically-bound cores are either square, octagonal, or circular; generally an octagonal section is the most economical, since the shuttering is less costly than for a circular column and there is less ineffective concrete in the corners than in a square column. The minimum outside size of the column is about 2 in. more than the diameter of the bound core. Although helically-bound columns are not necessarily the most economical form of column construction, the extra cost is mostly offset by the advantages arising from the extra available floor space and reduced dead weight.
So many variants enter into the design of a column that it is not easy readily to
decide which combination gives the most economical member. For a short column
carrying a load exceeding 100 tons, the following may apply.

Other things being equal, the richer the concrete the more economical is the
column. For a square column, the minimum amount of longitudinal reinforcement
produces the cheapest member for a specified quality of concrete. Also, for any
concrete a square column is generally less costly than an octagonal column with helical
binding. Taking eight designs of columns to carry loads from 100 tons to 500 tons,
the order of economy is generally as follows, the most economical design being the
first: 1:1:2 concrete, square column with minimum vertical steel; 1:1:2 concrete,
octagonal column with maximum volume of helical binders and minimum area of
vertical reinforcement; 1:1 1/3:3 concrete, square column with minimum area of vertical
reinforcement; 1:1 1/3:3 concrete, octagonal column with maximum volume of helical
binders and minimum area of vertical reinforcement; 1:2:2 concrete, square column
with minimum area of vertical reinforcement; 1:2:4 concrete, octagonal column
with maximum volume of helical binders and minimum area of vertical reinforcement;
1:2:4 concrete, octagonal column with maximum volume of helical binders and
maximum area of vertical reinforcement; 1:2:4 concrete, square column with
maximum area of vertical reinforcement.

Long Columns.

If the ratio of the effective length of a column to the least radius of gyration
exceeds about 50, the column is a "long" or slender column and the safe load on the
column is less than that on a "short" column. For square and rectangular columns
it is more convenient to calculate the slenderness ratio on the least lateral dimension
(provided that the section has no re-entrant angles) instead of on the radius of gyration.
Formulæ and data relating to both bases of determining the reduction of the con-
centric safe load on "long" columns are given in Table 83 and on the page facing
Table 84, on which page also the effective length compared with the actual length of
the column is considered.

Combined Stresses.

The stresses in structural members such as arches, walls of rectangular containers,
columns subject to eccentric load, and chimneys, are due to the combined effect of
a bending moment and a direct force, which may be either a pull or a thrust. The
method of determining the magnitude and distribution of the stress depends on the
nature of the direct force and the relative magnitudes of the bending moment and
the force. There are three principal cases: (i) When the direct force is a thrust and
the resultant stresses are wholly compressive; (ii) When the direct force is a pull
and the resultant stresses are wholly tensile; (iii) When the direct force is either a pull
or a thrust and both tensile and compressive forces are produced.

The analysis of the stresses for these cases is generally based on a method analogous
to the elastic modular-ratio methods of analysing beams and columns subjected to
concentric loads; this method is accepted by the B.S. Code and is the basis of Tables 85,
86, 87 and 89. The Code also describes a load-factor method of analysis, which is
given in Table 88 and described on page 83.

The permissible stresses in members subjected to bending and compression are
those permitted for bending only. In long columns subjected to bending, the per-
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possible stresses must be reduced by the factor $R_L$ given in Table 83, but no reduction need be made for stresses within one-eighth of the length of the column from either end. In slender beams subjected to axial thrust, the reduction factors should be in accordance with the rules in Table 57 and on page 246.

The effect of a bending moment $M$ and a direct force $N$ acting simultaneously is equivalent to the direct force $N$ acting at a distance $e$ from the centroid of the stressed area where $e = \frac{M}{N}$. The eccentricity $e$ is sometimes measured from the centroid of the concrete section and, except in case (iii) if the eccentricity is small, the error involved by this approximation is small. In some problems the eccentricity of the load about one face of the member is known, and before the stresses can be determined this eccentricity must be converted to that about the centroid of the stressed area (or of the concrete section).

The value of $e$ relative to the dimensions of the member determines into which of the three cases the problem falls. For problems in case (i) the maximum and minimum compressive stresses are calculated by adding and subtracting respectively the stresses due to the direct force alone and to the bending moment alone. The limit of this case is reached when the tensile stress produced by the bending moment alone (assuming that the whole of the concrete and the reinforcement are fully effective) is equal to the compressive stress due to a concentric load $N$. For a rectangular section this limiting condition is reached when $\frac{e}{d}$, $d$ being the total depth of the member, is 0.167 for a section with no reinforcement and up to about 0.3 with large percentages of reinforcement. As a small tensile stress may be permitted in the concrete in some cases, an upper limit for $\frac{e}{d}$ may be about 0.5. If no tensile stress is permitted in the concrete the limiting value of $e$ is $\frac{Z}{A}$, where $A$ is the effective area of the section expressed in concrete units and $Z$ is the modulus of the effective section (also expressed in concrete units) measured about the axis passing through the centroid of the equivalent section. Expressions for the effective area and section modulus of reinforced concrete sections subjected to stress over the entire section are given in Tables 65a and 65b; these expressions take into account the reinforcement; for preliminary approximate calculations it may not always be necessary to allow for the reinforcement, in which case the expressions in Table 64 apply.

When $N$ is a pull and the stresses are entirely tensile, the problem is in case (ii) when $\frac{e}{l_{as}}$ is less than 0.5, where $l_{as}$ is the distance between the centroids of the reinforcement near opposite faces of a reinforced section, the tensile resistance of the concrete being entirely neglected.

When case (i) is applied to a problem in which $N$ is a thrust and an excessive tensile stress is produced in the concrete, or when case (ii) is applied to a problem with $N$ a pull and compressive stresses are produced, the problem is in case (iii). Various methods of calculating the stresses for this case have been devised. Any direct method is complicated since an exact analysis involves the solution of a cubic equation, and rapid computation necessitates an impractically large number of graphs or tables if provision is to be made for all the probable variations of the terms in the equation. In the method given in Tables 85 and 86 the depth to the neutral plane is assumed first; the assumed depth is later checked and adjusted.

For rectangular sections, or sections capable of being reduced to equivalent
rectangles, the notation is as indicated in *Table 86*, and for an irregular section the notation is as shown in *Table 85*. When there is no compression reinforcement the term \( A_{se} \) in the formulae is zero and simplifications consequently follow. An abstract of the methods of determining the stresses in a rectangular member subject to a bending combined with a direct thrust is given in *Table 86*, together with the values of some of the terms involved in the calculation. For values of \( \frac{e}{d} \) exceeding, say, 1.5, an approximate method can be used that gives the stresses with sufficient accuracy.

**Rectangular Section subjected to Bending and Thrust (Modular-ratio Method).**

*When \( e \) does not exceed \( \frac{1}{2}d \).*—In this case, with any amount of reinforcement, compressive stresses only are developed and the maximum and minimum values are given by the formula in *Table 86*. The expression for the section modulus is correct if \( A_{se} \) equals \( A_{st} \) and is approximately correct if \( A_{se} \) differs from \( A_{st} \). For more accurate expressions, see *Table 65A*. The design of a section for this case involves the assumption of trial dimensions and reinforcement. For the special case of \( m = 15 \) and \( A_{se} = A_{st} \) the factors given in *Table 87* can be used, the method being described on the page facing the table.

*When \( e \) is greater than \( \frac{1}{4}d \) and less than \( \frac{1}{2}d \).*—With no reinforcement, tension is developed in one face of the member when \( e \) exceeds \( \frac{1}{4}d \), but as the proportion of reinforcement increases the ratio of \( e \) to \( d \) also increases before tensile stresses are developed. The limiting value of \( \frac{e}{d} \) depends on the amount of \( A_{se} \) and \( A_{st} \) and the relative values of \( d_2 \), \( d_4 \), and \( d \). Cases where \( \frac{e}{d} \) lies between \( \frac{1}{4} \) and \( \frac{1}{2} \) should be first calculated as if \( \frac{e}{d} \) did not exceed \( \frac{1}{4} \), and if no tensile stress is shown to be developed the stresses calculated by this method are the theoretical stresses. Even if a small tensile stress is developed, treatment as in the preceding paragraph is generally justified so long as the tensile stress in the concrete for the worst combination of \( M \) and \( N \) does not exceed about one-tenth of the allowable compressive stress. If the tensile stress exceeds this amount the tensile resistance of the concrete should be ignored, and the stresses calculated as in the next paragraph.

*When \( e \) is greater than \( \frac{1}{4}d_4 \) and less than \( 1 \frac{1}{2}d_4 \).*—This is the general case, when tension in the concrete is ignored, and the method given in *Table 86* is applicable to members with or without compression reinforcement and with any value of \( d_2 \) and any modular ratio.

The first step is to select a trial position for the neutral plane by assuming \( n_1 \), the neutral-plane factor, and then calculating the maximum stresses \( f_{cb} \) and \( f_{st} \) in the concrete and reinforcement respectively from the formulae given in *Table 86*, in which also are given the expressions for the factors and some numerical values of the factors. The term \( \bar{x} \) is the distance from the compressive edge of the section to the centroid of the stressed area. Since \( \bar{x} \) may be very nearly equal to \( \frac{1}{2}d \) it is sufficiently accurate in the first trial calculation to assume this value, but for the second or final calculation, \( \bar{x} \) should be determined from the appropriate expression.

The value of \( n_1 \) obtained by substituting the calculated values of \( f_{cb} \) and \( f_{st} \) in

\[
n_1 = \frac{1}{1 + \frac{f_{st}}{mf_{cb}}}
\]

should coincide with or be very nearly equal to the trial value of \( n_1 \).
If in the first trial there is a difference between the two values of \( n_1 \), the factors \( F, G, H \) and \( J \) should be recalculated with a second trial value of \( n_1 \) and the recalculated values of \( f_{cb} \) and \( f_{st} \) should give a satisfactory value of \( n_1 \). Values of \( n_1 \) for various values of \( \frac{f_{st}}{f_{cb}} \) for different modular ratios and values of \( G \) and \( H \) are given in Table 86, and in Table 68 also values of \( n_1 \) for \( m = 15 \) are given; see also Table 67.

When the member is reinforced in tension only, \( H = 0 \) and the formulæ for the stresses are

\[
f_{cb} = \frac{NF}{Gbd_1} \quad \text{and} \quad f_{st} = \frac{f_{cb}J - N}{A_{st}}.
\]

For the special case of \( m = 15 \) and \( A_{sc} = A_{st} \), the stresses can be obtained approximately from the factors given in Table 87.

A member which does not generally require compression reinforcement can be designed by first assuming a value for \( d_1 \) (and therefore \( d \)) and calculating the breadth required from \( b = \frac{NF}{\phi_{cb}d_1 G} \), in which \( G \) is calculated from the value of \( n_1 \) corresponding to the permissible stresses \( \phi_{cb} \) and \( \phi_{st} \) or taken from Table 86. The area of tensile reinforcement required is given by \( \frac{\phi_{cb}J - N}{\phi_{st}} \). If the value of \( b \) thus obtained is unsuitable, another value of \( d_1 \) may give suitable proportions. (For a slab, \( b = 12 \) in. if \( N \) and \( M \) are expressed in terms of one foot of width.) If suitable proportions cannot be obtained in this way, a convenient section may be found by reducing the stress in the tensile reinforcement, thereby increasing the area of concrete in compression, or by adding compression reinforcement, or by combining both methods.

If reinforcement is added to increase the compressive resistance, or if the member is such that ordinary design or other considerations require the provision of compression reinforcement (for example, columns, piles, the support section of beams, and members subject to reversal of flexure), it is necessary to assume (or determine from other considerations) suitable values of \( b \) as well as \( d_1 \). With these values, and with the ratio of the allowable stresses in the tensile reinforcement and the concrete, the factors \( F, H, J \) and \( G \) can be calculated or read from Table 86. The amount of compression reinforcement required is given by

\[
A_{sc} = \frac{1}{H(1 - f_{cb})} \left( \frac{NF}{\phi_{cb}} - bd_1 G \right).
\]

The area of tensile reinforcement required is given by

\[
A_{st} = \frac{\phi_{cb}(J + HA_{sc}) - N}{\phi_{st}}.
\]

In evaluating \( F \), the value of \( \delta \) may be assumed to be \( \frac{1}{2}d \), but in important members the stresses should be checked using the calculated value of \( \delta \).

If the calculated value of \( A_{sc} \) exceeds \( A_{st} \) both values should be adjusted by reducing the tensile stress or by modifying the dimensions of the concrete.

When \( e \) is greater than \( \frac{1}{4}d_1 \)—When the eccentricity of the thrust is large compared with the dimensions of the member the stresses are primarily determined by the bending moment, the thrust producing only a secondary modification. In this case the stresses should first be calculated for the bending moment acting alone as described on the pages facing Tables 66 and 67. The combined stresses can then be determined approximately by adding a stress \( f_c \) to the maximum compressive stress.
in the concrete, and by deducting $mf_e$ from the tensile stress in the reinforcement
where $f_e$ is given by the formula at the bottom of Table 86.

Examples of the use of Table 86 are given on the page facing the table.

Any Section subjected to Bending and Thrust (Modular-ratio Method).

*Compressive Stresses Only.*—The first step in the determination of the stresses
when the value of $\frac{M}{N}$ is small is to determine the equivalent area $A_e$ and the moment
of inertia $I_X$ of the section about an axis passing through the centroid as given by
the expressions at the top of Table 85; an irregular section is divided into a number
of narrow strips as in the diagram. The maximum and minimum compressive stresses
are obtained from the appropriate formulæ in the table. The limit of this case is
when $f_{eb\text{(min.)}} = 0$. A small negative value of $f_{eb\text{(min.)}}$ may be permissible if this
tensile stress does not exceed, say, one-tenth of the permissible compressive stress.

If the section is symmetrically reinforced and is rectangular (bending about
a diagonal), circular, octagonal, or has any of the symmetrical shapes given in Table 64,
the area of $A$ and the modulus $Z_e$ of the concrete section can be obtained from the
data given in the table. The additional area $A_A$ and the additional modulus $Z_A$ due
to the reinforcement are given by $A_A = (m - 1)\Sigma\delta A_e$ and $Z_A = \frac{2(m - 1)}{D} \Sigma h^2 \delta A_e$, 
where $\delta A_e$ is the area of a bar or group of bars placed at a distance $h$ from the centroid
of the section. Thus $A_e = A_e + A_A$ and $Z = Z_e + Z_A$; the maximum and minimum
compressive stresses in the concrete are given by $\frac{N}{A_e} = \frac{M}{Z}$. The limit for this case is

$\frac{M}{N} = \frac{Z}{A_e}$. For other common sections the expressions for the effective area
and modulus in Tables 65a and 65b can be used.

*Compressive and Tensile Stresses.*—When the stress $f_{eb\text{(min.)}}$ determined in the
preceding paragraph is negative or exceeds the permissible tensile stress, or when
$e$ is so large compared with $D$ that the simultaneous production of compressive and
tensile stresses can be assumed at the outset, the total tension should be resisted by
the reinforcement only. In this case it is necessary to select a trial position of the
neutral plane, either after consideration of the maximum permissible stresses or
otherwise, and to plot the plane on a diagram of the section drawn to scale, as indicated
in the diagram in Table 85. Next find the position of the centre of tension below the
top edge of the section. The next stage is to divide the compression area above the
neutral plane into a number of narrow horizontal strips. The depth $sX$ of each of
these strips need not be the same, as any regularity in the conformation of the section
may suggest convenient subdivisions. When the strips are of equal depth, or when
the section is symmetrical or hollow, simplifications should be readily perceived. For
each strip determine the factors $a$ and $n_a$. The position of the centre of compression
below the top edge can then be found. The distance $x$ of the centroid of the stressed
area below the compressed edge of the section can now be evaluated, and the maximum
compressive and tensile stresses can be calculated from the formulæ in Table 85. The
value of $n_1$ corresponding to these stresses should be compared with the assumed
value, and if necessary a second trial should be made. The values of $p_a$ and $a$ for
individual bars or groups of bars and for individual compression strips are not affected
by the value of $n_1$. An example of the application of this method is given at the
bottom of Table 85.
Bending combined with Tension (Modular-ratio Method).

When \( e \) is less than \( l_{as} - \varepsilon' \).—If the distance between the centroids of the reinforcement on opposite faces of any member is \( l_{as} \), and if \( e \) is measured about the centroid of the combined reinforcement, as shown in the diagram at the top of Table 89, then if \( e \) does not exceed \( l_{as} - \varepsilon' \), the stresses are wholly tensile. The average stresses in the group of bars near the face nearer the line of action of \( N \) and in the group of bars near the face remote from the line of action of \( N \) are given by the formulae for \( f_{st1} \) and \( f_{st2} \) respectively. The maximum stress in a bar depends upon the distance of the farthest bar in any group from the centroid of that group, and is given by the formula for \( f_{st1(max)} \) in the table. The expressions for \( f_{st1} \) and \( f_{st2} \) can be re-arranged to give the areas of reinforcement required for a specified permissible stress.

Simplified formulæ are given in Table 89 for this case for regular sections, such as rectangular sections in which the bars are in two rows only. Further simplification obtains if the area of the bars in each row are equal, as also given in Table 89.

Rectangular Section: When \( e \) is greater than \( l_{as} - \varepsilon' \) and less than \( \frac{1}{4}d_1 \).—This is the general case, and the method of treatment is similar to that given previously for combined bending moment and direct thrust, modifications being introduced to allow for the difference between a direct thrust and a direct pull as given in the lower part of Table 89; the factors \( J \), \( G \), and \( H \) can be obtained from Table 86.

When the section is reinforced in tension only, \( H = 0 \) and the formulæ for the maximum stresses are

\[
f_{cb} = \frac{NL}{bd_1G} \quad \text{and} \quad f_{st} = \frac{f_{cb}J + N}{A_{st}}.
\]

When designing a member to resist a bending moment and a direct tension an approximate method is as follows. If compression reinforcement is not likely to be required, assume \( d_1 \) (and \( d \)) and determine the minimum breadth from

\[
b = \frac{NL}{p_{cb}d_1G},
\]

where \( G \) is calculated (or read from Table 86) from the value of \( n_1 \), corresponding to the permissible stresses. If the value of \( b \) is unsatisfactory, \( d_1 \) should be adjusted or compression reinforcement added. The area of the tension reinforcement required is given by

\[
A_{st} = \frac{p_{cb}J + N}{p_{st}}.
\]

For a singly-reinforced slab subject to a bending moment and a direct tension, such as the wall or bottom of a tank or bunker, a simple approximate procedure is given at the bottom of Table 89. Determine the eccentricity \( e_s \) of the line of action of the direct tension from the centre of the tensile reinforcement. The total tensile reinforcement required is given by

\[
N \left( \frac{e_s}{p_{st}} + 1 \right).
\]

The value of \( d_1 \) (and \( d \)) is that required to resist the bending moment acting alone, and the value of the lever arm \( l_a \) is that corresponding to the permissible stresses.

In designing a member in which compression reinforcement is required, first assume or otherwise determine suitable values of \( b \) and \( d_1 \), and with these values and the allowable maximum stresses determine the area of compression reinforcement from

\[
A_{sc} = \frac{1}{Hl_{as}} \left( \frac{NL}{p_{cb} - bd_1G} \right).
\]

The area of tensile reinforcement is found from

\[
A_{st} = \frac{p_{cb}(J + HA_{sc}) + N}{p_{st}}.
\]

For this method \( L \) can be based on \( \varepsilon = \frac{1}{4}d \), but in important members the stresses should be checked by using the calculated value of \( \varepsilon \).

If it is necessary to reduce the amount of compression reinforcement, this can
often be effected by reducing \( f_t \) and thus increasing \( n_1 \). Generally in problems of bending and direct tension the tension is the deciding factor, and a more economical member can be obtained by decreasing the stress in the concrete.

**Rectangular Section:** When \( e > \frac{1}{14}d_1 \).—The determination of the approximate stresses in this case is similar to that described for the corresponding case for combined bending moment and direct thrust. The stresses are first computed for the bending moment acting alone. The next step is to evaluate \( f_t \) and deduct \( f_c \) from the stress in the concrete and add \( mf_t \) to the tensile stress in the reinforcement to obtain the maximum stresses in the concrete and steel, \( f_t \) being obtained from the expression given for this case near the bottom of Table 89.

To design a member, such as a slab with tensile reinforcement only, the following approximate method is applicable. The depth or thickness \( d \), and the breadth \( b \) in the case of a beam, are determined for the bending moment acting alone. Evaluate the eccentricity \( e_s \) about the tensile reinforcement. The area of tensile reinforcement required is given by substitution in the formula given at the bottom of Table 89 in which \( l_a \) is the lever-arm of the section designed for bending only.

**Any Section:** Compressive and Tensile Stresses.—With the modifications necessary to allow for \( N \) being a pull instead of a thrust, the method given for the corresponding case for combined bending moment and direct thrust can be applied to the determination of the stresses on any section that cannot be treated as rectangular. A trial position for the neutral plane is taken and the part of the section above the neutral plane is divided into a number of narrow horizontal strips as in the diagram in Table 89. The values of \( R, p, \bar{x}, p_c, \bar{x}_a \) and \( a \) are determined and substituted in the formulae for the maximum stresses given in the table. If the value of \( d_n \) corresponding to these stresses is approximately equal to that assumed, the stresses are approximately the maximum stresses produced by the applied bending moment and direct tension. If the difference between the calculated and trial values of \( d_n \) is great, a second trial value must be selected and the summations revised by taking in a greater or smaller number of strips to accord with the different value of \( d_n \).

**Position of the Neutral Plane.**

The accuracy of the results of some of the methods described in the foregoing and the labour entailed in arriving at these results depend upon the accuracy with which the position of the neutral plane is selected. From a consideration of the member and the forces acting upon it, it is possible to assume a value for \( d_n \) very close to that corresponding to the calculated stresses. The maximum stresses for which the section has been designed may indicate a reasonable value of \( d_n \) for the first trial, or consideration can be given to the ratio of stresses for bending only as determined by the proportion of tensile reinforcement. The selected value of \( d_n \) should differ from the value for bending alone in accordance with the following rules. For bending and compression, the selected value of \( d_n \) should be greater than the value for bending alone, the difference increasing as \( \frac{e}{D} \) or \( \frac{e}{d} \) or \( \frac{e}{d_1} \) decreases. For bending and tension, the selected value of \( d_n \) should be less than the value for bending alone, the difference decreasing as \( \frac{e}{D} \) or \( \frac{e}{d} \) or \( \frac{e}{d_1} \) increases. If the difference between the first assumed value, say \( d_{n1} \), and the value corresponding to the calculated stresses, say \( d_{n2} \), is such that it is necessary to select another value, say \( d_{n3} \), intermediate between \( d_{n1} \) and \( d_{n2} \), the following considerations apply. For bending and com-
pression, the value of \( d_{n2} \) should be nearer \( d_{n1} \) than it is to \( d_{n1} \). For bending and tension, the value of \( d_{n2} \) should be nearer \( d_{n1} \) than it is to \( d_{n1} \).

**Columns subjected to Bending: Load-factor Method.**

A load-factor method of designing columns subjected to bending, which is applicable to all similar members, is recommended in B.S. Code No. 114, and formulae and procedures based on this method are given in Table 88 and examples are given on the page facing the table. Since columns subjected to concentric load (no bending) are in general designed by a load-factor method, it is reasonable to apply a similar method to columns subjected to bending instead of changing to the modular-ratio method when bending occurs. The load-factor method in the Code applies directly only to rectangular columns with symmetrical reinforcement. It can be used for checking designs of columns, but it cannot be readily applied to the direct design of columns subjected to bending because many of the factors are related to the dimensions and other properties of the cross-section of the column.

**Notation.**

\( A_c = \) cross-sectional area of concrete (excluding reinforcement). \( A_{sc1} = \) cross-sectional area of reinforcement at each of the two opposite faces normal to the plane of bending, \( A_{sc} = 2A_{sc1} \).

\( b = \) breadth of column.

\( d = \) overall "depth" of column. \( d_1 = " \) effective depth " of column \( (d - d_2) \).

\( d_2 = \) distance from face of concrete to centre of adjacent reinforcement \( (f_d) \).

\( E_s = \) secant modulus of elasticity of the steel at a tensile stress of \( 2\sigma_{st} \) (30 \times 10^8 lb. per sq. in. for mild steel).

\( e = \) eccentricity of applied load \( N \left( = \frac{M}{N} \right) \) if \( M \) is the applied bending moment; this eccentricity and \( e_b \) are measured about the centroid of the gross section.

\( e_b = \) limiting eccentricity of limiting eccentric load \( P_b \).

\( K_I = \left( \frac{P_I}{bd} \right) \), the intensity of the safe load at eccentricity \( e \) for Case I. \( K_{II} = \left( \frac{P_{II}}{bd} \right) \), the intensity of the safe load at eccentricity \( e \) for Case II. \( K_b = \left( \frac{P_b}{bd} \right) \), the intensity of the limiting eccentric load. \( K_0 = \left( \frac{P_0}{bd} \right) \), the intensity of the safe eccentric load.

\( N = \) working load applied at eccentricity \( e \).

\( P = \) maximum safe eccentric load on a short column at eccentricity \( e \); \( = P_I \) when \( N < P_b \); \( = P_{II} \) when \( N > P_b \). \( P_b = \) limiting eccentric load on a short column at limiting eccentricity \( e_b \). \( P_0 = \) maximum safe concentric load on a short column.

\( \sigma_{cc} = \) direct compressive stress permissible in concrete. \( \sigma_{se} = \) compressive stress permissible in reinforcement. \( \sigma_{st} = \) tensile stress permissible in reinforcement.

\( r = \) ratio of cross-sectional area of reinforcement (at the two opposite faces normal to the plane of bending) to gross cross-sectional area of column \( \left( = \frac{2A_{sc1}}{bd} \right) \).

**Limiting Load and Limiting Eccentricity.**—The preliminary stage in the calculation of the safe eccentric load on a column of given size and with specified
reinforcement is the determination of the limiting eccentric load \( P_b \) on the column, and the corresponding limiting eccentricity \( \varepsilon_b \). Then, if the load \( N \) applied at eccentricity \( \varepsilon \) is less than the limiting load \( P_b \), the safe load \( P \) at eccentricity \( \varepsilon \) is determined by the tensile resistance of the column. If the load \( N \) applied at eccentricity \( \varepsilon \) is not less than the limiting load \( P_b \), the safe load \( P \) at eccentricity \( \varepsilon \) is determined by the compressive resistance of the column and is related to the maximum safe concentric load \( P_0 \) on a short column of the same dimensions and reinforcement. \( N \) must not exceed the safe eccentric load \( P \) calculated for the condition obtaining.

The following formulae apply.

\[
P_b = \frac{\rho_{cc}b_d}{100,000 + \frac{60 \rho_{st} \times 10^4}{E_s}}
\]

or

\[
P_b = K_b b_d \text{ in which } K_b = \frac{P_b}{b_d} = \rho_{cc}X(1 - f_1) - (\rho_{st} - \rho_{sc})\frac{r^2}{2} \quad \text{. (A)}
\]

and

\[
X = \frac{0.8\frac{\varepsilon}{\varepsilon_0}}{1 + 600\frac{\rho_{st}}{E_s}}
\]

\[
\varepsilon_b = \frac{1}{P_b} [\rho_{cc}b_d^2 X(1 - 0.8\varepsilon) + A_{sc1p_{sc}(d_1 - d_2)} - \frac{d_1 - d_2}{2}]
\]

or

\[
\frac{\varepsilon_b}{d} = \frac{1}{2} \left[ \frac{(X - 2f_1)K_b}{d} \right] \quad \text{. (B)}
\]

**Case I.—Applied Load less than \( P_b \).**—If the applied load \( N \) acting at eccentricity \( \varepsilon \) is less than \( P_b \), the tensile resistance determines the strength of the member, and the maximum safe load that can be applied at eccentricity \( \varepsilon \) is given by

\[
P = \rho_{cc}b_d \left[ \frac{1}{2} - \frac{\varepsilon}{d} - Y \right] + \sqrt{\left( \frac{1}{2} - \frac{\varepsilon}{d} - Y \right)^2 + \frac{d_1 - d_2}{d} \frac{\rho_{sc}}{\rho_{cc}} + Y \left( \frac{d_1}{d} - Y \right)}
\]

in which \( Y = \frac{r(\rho_{st} - \rho_{sc})}{2\rho_{cc}} \). These formulae can be rewritten as follows.

Maximum safe load (at eccentricity \( \varepsilon \)) = \( P_1 = K_1b_d \) in which \( K_1 \) is the intensity of the maximum safe load (at eccentricity \( \varepsilon \)) and is given by

\[
K_1 = \frac{P_1}{b_d} = \rho_{cc} \left[ U + \sqrt{U^2 + (1 - 2f_1) \frac{\rho_{sc}}{\rho_{cc}} Y} + Y[2(1 - f_1) - Y] \right] \quad \text{. (C)}
\]

where \( U = \frac{1}{2} - \frac{\varepsilon}{d} - Y \). The applied load \( N \) must not exceed \( P_1 \).

**Case II.—Applied Load greater than \( P_b \).**—When the applied load \( N \) is greater than \( P_b \), the compressive resistance determines the strength of the member, and the maximum safe load \( P \) which can be applied at eccentricity \( \varepsilon \) is \( P = \frac{P_0}{1 + \left( \frac{P_0}{P_b} - 1 \right)\frac{\varepsilon}{\varepsilon_b}} \), in which \( P_0 \) is the safe concentric load on a short column, that is

\[
P_0 = \rho_{cc}A_e + \rho_{sc}A_{sc}
\]

These formulae can be rewritten as follows.

Maximum safe concentric load on a short column = \( P_0 = K_0 b_d \), in which \( K_0 \) is the intensity of the maximum safe concentric load on a short column and is given by

\[
K_0 = \frac{P_0}{b_d} = \rho_{cc} + r(\rho_{sc} - \rho_{cc})
\]
Maximum safe load (at eccentricity $e$) = $P_W = K_{II}bd$, in which $K_{II}$ is the intensity of the maximum safe load (at eccentricity $e$) and is given by

$$K_{II} = \frac{P_W}{bd} = \frac{K_b}{1 + (\frac{K_0}{K_b} - 1)\frac{e}{e_b}}.$$  \hspace{1cm} (D)

The applied load $N$ must not exceed $P_W$.

**Special Formulae and Factors.**—Under conditions excluding the effects of wind, the permissible stresses in mild steel bars not exceeding $1\frac{1}{4}$ in. diameter are 20,000 lb. in tension and 18,000 lb. per square inch in compression. For bars exceeding $1\frac{1}{4}$ in. diameter the corresponding stresses are 18,000 lb. and 16,000 lb. per square inch. If the effects of wind are included in the calculations of the loads and bending moments (and therefore the eccentricities) these stresses may be increased by 25 per cent. The elastic modulus for mild steel is 30,000,000 lb. per square inch. The stress coefficients $X$ and $Y$ are non-dimensional. Therefore for mild steel bars not exceeding $1\frac{1}{4}$ in. diameter and excluding the effects of wind, $X = 0.607$ and $Y = \frac{1000\gamma}{P_{cc}}$. If the stresses and coefficients are substituted in the general formulae (A) for $K_b$, (B) for $\frac{e_b}{d}$ and (C) for $\frac{K_1}{P_{cc}}$, the formulae (A), (B) and (C) series (1) to (4) in the upper part of Table 88 are obtained; series (1) and (3) apply when the effects of wind are not included, and series (2) and (4) apply when these effects are included. Similarly series (1) and (2) apply to bars not exceeding $1\frac{1}{4}$ in. diameter, and series (3) and (4) to larger bars. Series (1) apply to the conditions most common for the columns of a building frame, and for these conditions the numerical factors are as given in the lower part of Table 88 and include the preliminary factor $K_1$ and the short-column concentric-load factor $K_0$ for various proportions of mild steel reinforcement with the common range of cover ratios, and for ordinary-quality 1:2:4 concrete. This part of Table 88 also gives values of the safe eccentric load factors $K_1$ for Case I when the applied load is less than the limiting eccentric load $P_b$. The safe eccentric load for Case II, when the applied load is greater than the limiting eccentric load, is calculated readily by direct substitution in formula (D) which is applicable to all conditions of design.

The permissible stress in high-yield-stress bars is half the yield stress, but with limiting stresses of 30,000 lb. per square inch in tension and 23,000 lb. per square inch in compression. If the effects of wind are included the stresses can be increased by 25 per cent. but must not exceed 30,000 lb. per square inch; therefore the maximum permissible stresses when wind is included are 30,000 lb. per square inch in tension and 28,750 lb. per square inch in compression. These stresses apply to bars having a yield stress of not less than 60,000 lb. per square inch. Formulae (A), (B) and (C) series (5) excluding wind, and series (6) including wind, in Table 88, are derived by substituting the foregoing stresses in formulae (A), (B) and (C). The elastic modulus of cold-worked bars depends on the method of manufacture and other conditions; a suitable value should be determined by tests of bars of the type to be used. For preliminary designs a modulus of $15 \times 10^6$ lb. per square inch is reasonable.

**Design Procedure.**—The design procedure in accordance with the load-factor method is as follows. (i) Select a reasonable trial section. (ii) Compare the calculated safe eccentric load with the applied eccentric load. (iii) Make adjustments to the trial section if it is inadequate or too large or contains too much reinforcement. The examples on the page facing Table 88 show the application of this procedure.
Bending Moments about Two Axes.

Some methods of calculating the stresses when a section is subjected to bending moments \( M \) and \( M_1 \) acting about each of the two axes mutually at right-angles simultaneously with a concentric compressive load \( N \) are given in Table 90. The two cases of when the stresses are entirely compressive, and when tensile and compressive stresses are produced, are considered. The method in the former case is accurate, but the method in the latter case is approximate and applies only if \( M \) is much greater than \( M_1 \). If \( M \) and \( M_1 \) are more nearly equal, a semi-graphical method, which is only worth while for important members, can be applied by combining vectorially \( M \) and \( M_1 \) to obtain the resultant moment \( M_R \). Assume a position of the neutral plane, which will be at right-angles to the plane of action of \( M_R \) and proceed as described for the irregular section in Table 85. The foregoing procedures are based on the modular-ratio method.

Combination of Stresses acting in Different Directions.

If three stresses act on a square element of uncracked concrete the principal tensile and compressive stresses, mutually at right-angles, are given by substitution in the general formulae in Table 90; the plane in which the principal tensile stress acts can also be established. The general formulae apply if a tensile stress acts normal to one face of the element, a tensile stress acts normal to an adjacent face, and a shearing stress acts in the plane of the element. If either of the direct stresses are compressive, the sign of the appropriate term in the formula is changed. Formulae are also given for cases in which one or both of the direct stresses are compressive or do not act.
SECTION 5
STRUCTURES AND FOUNDATIONS

The loads and consequent bending moments and forces on the principal types of structural components, and the stresses in, and resistances of, these components are dealt with in the preceding sections. In this section some complete structures, which are mainly assemblies or special cases of such components, and their foundations are considered.

Buildings.

A building may be constructed entirely of reinforced concrete or the roof, floors, walls, stairs and foundations, or one or more of these parts, may be of reinforced concrete in conjunction with a steel frame. Alternatively, the interior and exterior walls may be of cast-insitu reinforced concrete and support the floors and roof, the columns and beams being formed in the thickness of the walls. In the following are given some notes relating to the design of building components.

Floors.—Concrete floors may be of monolithic beam-and-slab (with the slabs spanning in one or two directions), flat-slab, or hollow-slab construction, or may be of precast slabs supported on cast-insitu or precast concrete beams. Some typical details of beams and solid slabs, and hollow-block slabs are given in Appendix II. (B.S. Code No. 114 gives recommendations for the design and construction of floors and flat roofs comprising hollow blocks, ribbed slabs, and precast slabs.)

Openings in Slabs.—The slabs around openings in floors or roofs should be strengthened with extra reinforcement, unless the opening is large compared with the span of the slab (for example, stair-wells or lift-wells) in which case beams should be provided around the opening. For small openings in solid slabs the cross-sectional area of the extra bars placed parallel to the principal reinforcement should be at least equal to the area of principal reinforcement interrupted by the opening. A bar should be placed diagonally across each corner of an opening.

Holes for pipes, ducts, and other services should be formed when the floor is constructed and it should not be permitted to cut such holes afterwards, unless done under the supervision of a competent engineer. It is therefore an advantage to provide, at the time of construction, a number of holes that can be used for electric conduits and small pipes, even when they are not required for the known services. Suitable positions are through floor slabs in the corners of rooms or corridors, and through the ribs of beams immediately below the slab.

Hollow-block Slabs.—If the span of a floor or roof slab exceeds 10 ft. it is often economical to provide a hollow-block slab, which is light in weight and requires less concrete than a solid slab. Such a floor consists of a thin top slab (1\(\frac{1}{2}\) in. to 3\(\frac{1}{2}\) in. thick) overlying concrete ribs. The ribs may be at 6-in. to 36-in. centres and may be from 2\(\frac{1}{2}\) to 5 in. wide. The spaces between the ribs may be left open, but in order to simplify the shuttering they are frequently filled with hollow blocks of burnt clay or lightweight concrete. The combined depth of the rib and slab is determined in the same way as the depth of a solid slab, and the thickness of the top slab is made sufficient
to provide adequate compressive area. The width of the rib is primarily determined by the shearing force. Alternative designs of hollow-block slabs are given in Appendix II. Weights of solid and hollow-block slabs are in Table 1. The principal requirements of B.S. Code No. 114 are that the thickness of the top slab be not less than one-twelfth of the distance between ribs, with a minimum of 2 in. but, if the blocks are assumed to add to the strength of the construction and the clear distance between the ribs does not exceed 18 in., the top slab should be not less than 1 in. thick. The distance between the centres of the ribs should not exceed 3 ft. The width of a rib should be not less than 2½ in. or one-third of its depth; for resistance to shearing, the effective width of the rib is assumed to be the actual width plus the thickness of one wall of the block.

**Stairs.**—Stairs can be designed to span transversely (that is, across the flight) or longitudinally (that is, in the direction of the flight). When spanning transversely supports must be provided on both sides of the flight, either by walls or stringer beams. In this case the waist or thinnest part of the stair construction need be only, say, 2 in. thick, the effective lever-arm for resisting the bending moment being about one-half the maximum thickness from the nose to the soffit measured normal to the soffit. When the slab spans longitudinally the thickness required to resist bending determines the thickness of the waist. The loads for which a flight of stairs should be designed are described on page 9. The bending moments should be calculated from the total weight of the stairs and the total imposed load combined with the horizontal span. The stresses produced by the longitudinal thrust are small and are generally neglected. Unless circumstances otherwise dictate, a suitable shape for a step is 7-in. rise with 10-in. going, which with 1-in. nosing or undercut gives a tread of 11 in.; stairs in industrial buildings may be steeper.

Recommendations for the design of stairs and landings are given in the B.S. Code.

Reference should be made to other publications for the design of helical stairs.

**Flat Roofs.**—A flat reinforced concrete roof is designed similarly to a floor and may be a simple solid slab, or beam-and-slab construction, or a flat slab. In beam-and-slab construction the slab may be a solid cast-insitu slab, a hollow-block slab, or a precast concrete slab. A watertight covering, such as asphalt or bituminous felt, is generally necessary, and with a solid slab some form of thermal insulation may be required. The watertight covering is sometimes omitted from a flat solid slab forming the roof of an industrial building, but in such a case the concrete should be particularly dense, and the slab should be not less than 4 in. thick and should be laid to a slope of at least 3 in. in 10 ft. to expedite the discharge of rainwater. Sodium silicate or tar, well brushed into the surface of the concrete, will improve the watertightness if there are no cracks in the slab.

For ordinary buildings the slab of a flat roof is generally built level and the slope for draining, often about 1 in. in 10 ft., is formed by a mortar topping. The topping is laid directly on the concrete and below the asphalt or other watertight covering, and may form the thermal insulation if it is made of sufficient thickness and of lightweight concrete of low thermal conductivity.

**Sloping Roofs.**—Planar slabs with a continuous steep slope are not common in reinforced concrete, except for mansard roofs, the covering of pitched roofs being generally metal or asbestos-cement sheeting, glass, wood-wool slabs, or other lightweight material. Such coverings and roof glazing require purlins for their support and, although the purlins are frequently of steel, reinforced concrete purlins, which
may be either cast-insitu or more commonly precast, are provided especially if the roof structure is of reinforced concrete.

**Precast Concrete Purlins.**—The size of a precast concrete purlin depends not so much on the stresses due to bending as on the deflection. Excessive deflection, although not necessarily a sign of structural weakness, may lead to defects in the roof covering. The shape of the purlin should be such that lightness is combined with resistance to bending, not only in a vertical plane but also in a direction parallel to the slope of the roof. An ell-shape, which is often used, is efficient in these respects, but a wedge-shape is often less costly to make for small spans. The weight of a precast concrete purlin may be excessive for spans over 15 ft. The dimensions depend on the span and the load, and for purlins spaced at 4 ft. 6 in. centres and carrying ordinary roof sheeting and spanning 15 ft. suitable sizes are 5 in. for the width across the top flange and 8 in. for the overall depth. For a span of 10 ft. the corresponding dimensions are 4½ in. and 6 in. For purlins on sloping roofs, the vertical weights and the wind pressure normal to the slope of the roof should be combined vectorially before computing the bending moment. The stresses should then be calculated with the neutral plane normal to the line of action of the resultant load. A semi-graphical method, as described in Table 85, is most suitable for the calculation of the stresses.

The purlins may be supported on cast-insitu or precast concrete frames or rafters. If the rafters are cast insitu the ends of the purlins can often be embedded in the rafters so as to obtain some fixity, which increases the stiffness of the purlin. If the rafters are of precast concrete, the type of fixing of the purlin is generally such that the purlin should be designed as freely supported.

**Non-planar Roofs.**—Roofs which are not planar, other than the simple pitched roofs considered in the foregoing, may be constructed in the form of a series of planar slabs (prismatic or hipped-plate construction), or as domes or vaults (segmental or cylindrical shells). The elementary analysis of such structural forms is given in Table 91, but reference should be made to other publications for more comprehensive analyses and more complex designs. Notes on the matter given in Table 91 are given on the page facing the table.

In Table 97 expressions are given for the forces in domed slabs such as are used for the bottoms and roofs of cylindrical tanks. In a building a domed roof has generally a much larger ratio of rise to span and, when the dome is part of a spherical surface and has an approximately uniform thickness throughout, the analysis in Table 91 applies. Shallow segmental domes and truncated conical "domes" are also dealt with in Table 91.

Segmental or semi-cylindrical roofs are generally designed as shell structures. A thin curved slab acting as a shell is assumed to offer no resistance to bending and not to deform under distributed loads. Except near the edge and end stiffeners, it is subjected only to direct "membrane" forces, namely, a direct force acting longitudinally in the plane of the slab, a direct force acting tangentially to the curve of the slab, and a shearing force. The membrane forces per unit length of the slab of a segmental shell roof, supported at the ends only, are given by the formulæ in Table 91.

**Panel Walls.**—Panel walls filling in the structural frame and not designed to carry loads (other than ordinary wind pressures) should be not less than 4 in. thick (for constructional reasons), and should be reinforced with not less than ¼-in. mild steel bars spaced at 6-in. centres or the equivalent in bars of other sizes but at not more than 12-in. centres (or an equivalent fabric); this reinforcement should be provided
in one layer in the middle of the wall. Bars \( \frac{3}{8} \) in. or more in diameter should be placed above and at the sides of openings, and \( \frac{1}{2} \) -in. bars 4 ft. long should be placed diagonally across the corners of openings. The slab must be strong enough to resist the bending moments due to spanning between the members of the frame. The connections to the frame must be strong enough to transfer the pressures on the panel to the frame either by bearing, if the panel is set in rebates in the members of the frame, or by the resistance to shearing of reinforcement projecting from the frame into the panel. A bearing is preferable since the panel can be completely free from the frame and therefore not subjected to secondary stresses due to deformation of the frame nor is the connection between the panel and the frame subjected to tensile stresses due to contraction of the panel caused by shrinkage of the concrete or thermal changes. By setting the panel in a chase the connection is also lightproof. The wind pressure which a panel wall should be designed to resist is a pressure or suction of \( 0.8p \) as given in Table 8. If not rigidly connected to the frame the panel of slab should be designated as a slab spanning in two directions without the corners being held down (see Table 38).

**Load-bearing Walls.**—B.S. Codes Nos. 114, 111, and 123—101 give recommendations for load-bearing walls which are summarised in the following. The requirements of the London By-laws are less conservative and should be consulted where applicable.

Walls exposed to the weather should be not less than 6 in. thick, but in other cases not less than 4 in. thick. Party walls should, for acoustical reasons, be cavity walls. A load-bearing reinforced concrete wall should be designed as a column, but the minimum cross-sectional area of the vertical reinforcement need not be less than \( 0.2 \) per cent. of the area of the horizontal cross-sections of the wall, and horizontal reinforcement should be provided, the cross-sectional area of which should be not less than \( 0.2 \) per cent. of the area of the vertical cross-section. If the concrete alone can carry the imposed load, without assistance from the vertical bars, lateral reinforcement tying in the vertical bars is not required, and the minimum amount of vertical reinforcement is \( 0.2 \) per cent. and of horizontal reinforcement \( 0.1 \) per cent. If the

**Reinforcement in Load-Bearing Walls.**

<table>
<thead>
<tr>
<th>Thickness of wall (in.)</th>
<th>Minimum reinforcement (Area)</th>
<th>Applicability</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>o(\cdot)1 per cent.</td>
<td>o(\cdot)2 per cent.</td>
</tr>
<tr>
<td></td>
<td>Total sq. in. On each face</td>
<td>Total sq. in. On each face</td>
</tr>
<tr>
<td>4</td>
<td>0.048  (\frac{1}{8}) in. at 12 in.</td>
<td>0.096  (\frac{1}{8}) in. at 12 in.</td>
</tr>
<tr>
<td>5</td>
<td>0.060  (\frac{1}{8}) in. at 15 in.</td>
<td>0.120  (\frac{1}{8}) in. at 15 in.</td>
</tr>
<tr>
<td>6</td>
<td>0.072  (\frac{1}{8}) in. at 12 in.</td>
<td>0.144  (\frac{1}{8}) in. at 12 in.</td>
</tr>
<tr>
<td>7</td>
<td>0.084  (\frac{1}{8}) in. at 15 in.</td>
<td>0.168  (\frac{1}{8}) in. at 15 in.</td>
</tr>
<tr>
<td>8</td>
<td>0.096  (\frac{1}{8}) in. at 12 in.</td>
<td>0.192  (\frac{1}{8}) in. at 15 in.</td>
</tr>
<tr>
<td>9</td>
<td>0.108  (\frac{1}{8}) in. at 16(\frac{1}{4}) in.</td>
<td>0.216  (\frac{1}{8}) in. at 12 in.</td>
</tr>
</tbody>
</table>

**Notes.** (1)—Area of reinforcement is percentage of cross-sectional area of wall. (2)—Total is cross-sectional area of reinforcement in sq. in. per foot length or height of wall taking into account bars on both faces of wall. (3)—If wall is reinforced by a single layer of bars, the spacings given in the table should be halved.
load on the wall is such that the resistance of the vertical bars is required, it is necessary to provide transverse ties or crimped horizontal bars additional to the horizontal reinforcement; the spacing of the ties should be such that the vertical bars are restrained at points not more than twelve times their diameter apart. The minimum reinforcement in walls of various thicknesses is given in the table on page 90. The minimum amount of reinforcement recommended in B.S. Code No. 114 may be insufficient to resist the effects of temperature and shrinkage, to withstand which the minimum reinforcement recommended in B.S. Code No. 111—201 should be provided; the effect of the method of construction on the shrinkage stresses and the degree of exposure as it affects the probable thermal changes should be considered.

The load-carrying capacity of a reinforced concrete wall depends on the ratio of the effective height $H_e$ of the wall to the effective thickness $D_e$ which is the actual thickness $D$ of a solid wall or, if the wall is stiffened by pilasters, $D_e$ may exceed $D$; if $P_1$ is the ratio of the distance between the centres of the pilasters to the width of the pilasters, and $P_2$ is the ratio of the overall thickness of the pilaster to $D$, empirical values of $D_e$ from which intermediate values can be interpolated are as follows.

<table>
<thead>
<tr>
<th>$P_1$</th>
<th>6</th>
<th>8</th>
<th>10</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_2 = 2$</td>
<td>$1.4D$</td>
<td>$1.3D$</td>
<td>$1.2D$</td>
<td>$1.1D$</td>
</tr>
<tr>
<td>$P_2 = 3$</td>
<td>$2.6D$</td>
<td>$1.7D$</td>
<td>$1.4D$</td>
<td>$1.2D$</td>
</tr>
</tbody>
</table>

If $P_2 = 1$ or is 20 or more, $D = D_e$. The ratio $\frac{H_e}{D_e}$ should not exceed 24. If this ratio exceeds 15, the permissible compressive stress in the concrete (and also in the vertical reinforcement if this is taken into account) should be reduced in accordance with the rules given in Table 83. The effective height $H_e$ of a wall is calculated in the same way as the effective length $L_e$ of a column as described on page 296, but if the wall is stiffened by cross walls or pilasters and the distance between the cross walls or pilasters is less than $H_e$, this distance may be substituted for $H_e$. The safe load calculated for a column may be increased for walls that are long compared with their height, that is if the length $L$ is two or more times the story height $H$, the safe load (or the permissible stresses) may be increased by 20 per cent., and by 10 per cent. if $L = H$; there is no increase if the length is two-thirds or less of the story height. In this consideration $L$ is the overall length or, if there are openings in the wall, the length between adjacent openings.

Walls subjected to Bending.—A reinforced concrete wall subjected to bending should be designed in accordance with the requirements for columns under similar conditions. Where floor beams are monolithic with walls, with or without pilasters, the bending moments on the wall (and at the ends of the beams) can be calculated by the formulae for columns. The bending moment and load from the beam may be concentrated immediately around the junction of the beam and wall (or pilaster), but will be dispersed through the wall. The angle of dispersion in a concrete wall, as specified in B.S. Code No. 111, is 45 deg.; it is reasonable to distribute the bending moments to the same extent.

Bridges.

Types of Bridges.—A bridge may be one of two principal types, namely, an arch bridge or a girder bridge, and either of these types may be statically determinate.
or statically indeterminate. The basic types of bridges are illustrated in Table 93. A bowstring girder is a special type of arch, and a rigid-frame bridge can be considered as a type of girder or arch bridge.

The selection of the type in any particular instance depends principally on the span, the nature of the foundation, and the clearance required. It may be that more than one type is suitable, in which case the economy of one over the others may be the deciding factor. If a bridge is fairly high above the railway, road, or waterway, an arch is generally the most suitable if the ratio of the span to the rise does not greatly exceed ten and if the foundation is able to resist the inclined forces from the arch. If settlement of the foundation is probable, an arch provided with hinges can be used but the ratio of the span to the rise should not greatly exceed five. For other conditions a girder bridge is more suitable; although some girder bridges have the appearance of solid-spandrel arches, they are not arches as they are designed so as not to impose any horizontal thrust on the abutments or piers. In a bowstring-girder bridge, which has the advantages of an arch but does not require the same rigidity in the foundation, the deck is at the level of the springing of the arch and is therefore suitable when the level of the deck is not much above the level of the waterway, road, or railway that is spanned. "Memorandum on Bridge Design and Construction" issued by the Ministry of Transport gives guidance on most aspects of the subject of the general design of bridges and the selection of type.

**Loads.**—The live loads on road and railway bridges are described on pages 11 and 12. Particulars of the Ministry of Transport loading on road bridges, the weights of typical road and rail vehicles, and the loading requirements of B.S. No. 153 (Part 3A) for reinforced concrete decks of steel bridges are given in Table 6. Notes on the foregoing are on the page facing the table.

**Deck.**—The design of the deck of a reinforced concrete bridge is almost independent of the type of the bridge. Some typical cross-sections are given in Table 93. In the simplest case the deck is a reinforced concrete slab spanning between the abutments and bearing freely thereon, as in a freely-supported type of bridge, or built monolithically therewith as in a rigid-frame bridge. This type of deck is suitable only for small spans, say, up to 15 ft. The more common case is for the main arches or girders to support a reinforced concrete slab that spans transversely between these principal members. If the latter are spaced at more than, say, 7 ft., an economical deck is provided by inserting transverse beams and designing the slab to span in two directions. In a solid-spandrel arch bridge with earth filling, the deck may be a concrete slab laid on the filling. A deck common in bridges of large span is one of cellular construction in which the road forms the top slab and a soffit slab forms the bottom of a series of boxes, the sides of which are the longitudinal arch ribs or girders.

The underside of the deck of a bridge over a railway should have a flat soffit, thereby avoiding pockets in which smoke may collect. For such bridges the corrosion of the concrete by the smoke from steam locomotives has to be prevented. Smoke-guards may not entirely protect the structure. A dense concrete, free from cracks through which the fumes can reach and attack the steel reinforcement, is necessary. The cover of concrete should be greater than that provided in buildings, and the tensile stresses in the concrete should be calculated and limited in value, as in liquid-containing structures.

A bridge less than 15 ft. wide is often economical if the deck slab spans transversely between two outer longitudinal girders. These girders may be the parapets of the
bridge, but for major structures the parapets should not be used as principal structural members. If the width of the bridge exceeds 15 ft., an economical design is produced by providing several longitudinal girders or arch ribs spaced at about 7-ft. centres.

Footpaths are sometimes cantilevered off the principal part of the bridge. Water, gas, electrical and other services are generally installed in a duct under the footpaths.

**Clearances.**—The approximate clearances required for a bridge over a railway or road and for subways are given in *Table 92*. Other clearances may be required for railways and roads abroad, and in some instances in Britain. As each railway has its own structure-gauge such requirements are generally specified, as are also the requirements of the navigation authorities for bridges over rivers and canals.

The diagram for a double line of railway in *Table 92* illustrates the minimum desirable clearances required by the Ministry of Transport, but for curved lines these clearances may have to be increased to allow for curvature and superelevation of the line and for the length of the rolling stock. For a new bridge or a bridge being rebuilt, the lateral clearance, after making these allowances, should be not less than 2 ft. 4 in. from the widest part of the broadest vehicle likely to use the line. Except where platforms occur, the clearance of 2 ft. 4 in. must extend for the full height from rail level to the top of the highest carriage-door. The overhead clearance (except for underground railways or for railways having electric traction with overhead equipment) should be 12 in. above the loading gauge, allowance being made for superelevation.

**Parapets.**—The height of the parapets on an underframe railway bridge should be 4 ft. 6 in. above the level of the rail, and should be solid for the whole height. If a check rail is provided in the track the parapet need be solid for a height of only 1 ft. above rail level, a guard rail being provided 4 ft. 6 in. above rail level. The height of the parapets of road bridges or foot-bridges over railways must not be less than 4 ft.

**Vertical Curves.**—If a road bridge is constructed at a slightly higher level than the adjacent land, the approach roads often rise towards the centre of the bridge, where a vertical curve is formed to ease the change of gradient and to provide an adequate view of on-coming traffic. Proportions for such vertical parabolic curves in conformity with British practice are given on *Table 92*. The formula for the sighting distance over the hump of the bridge is based on the level of the eye being 3 ft. 9 in. above the crown of the road.

**Girder Bridges.**—The types of girder bridges shown in *Table 92* are classified on a structural basis. Some of the types shown are statically determinate, the exceptions being the restrained and continuous beams, and the Vierendeel girder. The statically-determinate girders of two or more spans are made up of a series of elements each of which consists of a beam of one span with a cantilever at one or both ends. The junction between two adjacent elements is made by means of a freely-suspended span. To preserve freedom of movement and the condition of statical determinacy it is necessary that such girders should not be rigidly connected to the supporting piers or abutments; the beams should be free to rotate and, at all but one support for each element, the bearing should be free to slide. The formulae in *Table 92* indicate some suitable lengths for the cantilevers in a bridge consisting of a series of equal spans. The bending moments and shearing forces are calculated by determining those for a freely-supported beam, and superimposing those due to the cantilever. For bridges designed for Ministry of Transport loading, the shearing force (lb.) at a point at a distance $aL$ from the centre of the span of a freely-supported girder of span $L$ ft. is

$$0.5w_a LB(a + 0.5)^2 + 2700B(0.5 + a),$$

where $w_a$ is the uniformly-distributed load.
(lb. per square foot) for a loaded span of \((0.5 + a)L\), and \(B\) ft. is the spacing of the longitudinal girders.

The foregoing articulated types of girder bridges are suitable when there is a probability of unequal settlement of the foundations. When unequal settlement of the foundations is not likely, a girder designed as a continuous beam is permissible. This type of girder may have a considerable variation of moment of inertia throughout each span, and it is necessary to adopt the formulae and methods described for this case in Table 19. If the girder is constructed monolithically with the supports, the effect of the fixity at the supports should be allowed for and the structure should be designed as a frame.

An open-type bridge girder may be a lattice girder similar to the truss of a steel bridge and may have a single, double, or treble lattice. These trusses are generally analysed in the same way as a steel truss, assuming pin joints, but the rigidity of the joints in monolithic reinforced concrete construction largely invalidates this method of calculation. The Vierendeel truss, in which the panels are rectangular and the joints rigid, is more appropriate to construction in reinforced concrete. The accurate analysis of such a truss is complex, but a practical approximation is to calculate the shearing forces and consequent bending moments in each member and combine therewith the primary forces due to the general bending action.

**Frame Bridges.**—Bridges incorporating portal frames are statically indeterminate and three principal types are shown in Table 92. The frames, either slabs or ribs, may be hinged at the base or fixed. The simplest form of framed bridge is a rigid frame of one span; for small spans, say, up to 20 ft., the frame may be a slab forming the deck and the abutment wall without beams or counterforts. For larger spans the slab would span horizontally between beams and counterforts which together form the frame. The frame would be analysed as described in Tables 49 and 50.

**Arch Bridges.**—The principal types of arch bridges are illustrated in Table 92 and include arch ribs or arch slabs. In the case of the former type the deck is supported on columns carried on the ribs or, in the case of a bridge with the deck below the level of the crown of the arch, the deck is partly supported on columns and partly suspended from the ribs. In the case of an arch slab, the deck is supported on earth filling deposited on the slab and retained by spandrel walls. Arch ribs or slabs may be three-hinged arches which are statically determinate, or two-hinge or fixed arches which are statically-indeterminate. The design of hinged and fixed arches is dealt with in Tables 52 to 55, and is described on pages 43 et seq.

**Precast Concrete Bridges.**—Where the erection of temporary falsework is prohibited and the working periods are limited, precast concrete can be used with advantage. Some types of bridges incorporating precast construction are given in Table 93. The piers and abutments are constructed first and precast concrete beams (or in some cases precast concrete arch ribs), which are generally I-beams, are erected thereon, being either spaced apart or laid closely together. A cast-in-situ slab is generally laid on the beams. If the beams are spaced apart the slab forms the flange of a tee-beam of which the precast beam forms the rib, the two parts being bonded together by vertical bars projecting from the precast beam. The reinforcement in the precast concrete beam is calculated as the reinforcement required for a tee-beam carrying the total load, but the stresses induced while it is an independent member must be allowed for and must not be excessive. The stresses induced during erection can be kept low by selecting suitable positions for the attachment of the crane slings.
When precast concrete beams are laid together an interlocking device should be provided to distribute the live load over two or more beams to prevent one beam deflecting considerably more than the others. A thin cast-insitu reinforced concrete slab bonded to the top of the precast beams serves this purpose, and the combined construction is a single structural member.

It should be noted that, due to its own weight, the weight of the shuttering, and the weight of the wet concrete in the superimposed slab, the precast beam will be strained before it can act as a beam of the full cross-section of the final structural member. The initial stresses so caused must be included in the calculation of the stresses caused by any further dead load and by the live load, although the additional stresses may be computed for a beam of increased size.

Piers and Abutments.—The piers for girder bridges are generally subjected only to the vertical load due to the total loads on the girders; the abutments of girder bridges have to resist the vertical loads from the girders and the horizontal earth pressure on the back of the abutment. There may also be a horizontal force due to friction on bearings (see Table 91) and, in the case of railway bridges, a horizontal force due to braking, acceleration, etc. (see page 146). Continuity between the girders and the abutments is assumed in rigid-frame bridges, and consequently the foregoing forces on the abutment must be combined with the bending moments and horizontal thrusts due to action as a frame.

The abutments of an arch bridge have to resist the vertical loads and the horizontal thrusts from the arch. Stability is obtained by constructing massive piers in plain concrete or masonry, or by providing tension and compression piles, or by a cellular reinforced concrete box filled with earth. Part of the horizontal thrust on the abutments will be resisted by the active earth pressure on the abutment, but in the case of fixed arches this pressure should be assumed to relieve the thrust from the arch only when complete assurance is possible that this pressure will always be effective. Adequate resistance to sliding should also be assured, and the buoyancy effect of foundations below water should be investigated.

Mid-river piers, if not protected by independent fenders, should be designed to withstand blows from passing vessels or floating débris, and should be provided with cutwaters.

Culverts and Subways.

Pipe Culverts.—For conducting small streams or ditches under embankments, culverts can be constructed with precast reinforced concrete pipes, which must be strong enough to resist the vertical and horizontal pressures from the earth and other superimposed loads. The pipes should be laid on a bed of concrete, and where they pass under a road they should be surrounded with reinforced concrete at least 6 in. thick. The culvert should also be reinforced longitudinally to resist bending due to unequal vertical earth pressure or unequal settlement. Owing to the uncertainty of the magnitude and disposition of pressures on circular pipes embedded in the ground, accurate analysis of the bending moments is impracticable. A basic guide is that the positive bending moments at the top and bottom of a circular pipe of diameter $D$ and the negative bending moments at the ends of a horizontal diameter are $0.0625pD^3$, where $p$ is intensity of downward pressure on the top and of upward pressure on the bottom, assuming the pressures to be uniformly distributed on a horizontal plane.

Loads on Culverts.—The load on a pipe culvert, or on the top slab, includes
the weight of the earth, the imposed load (if any), and the weight of the top slab. When a trench has been excavated in consolidated ground for the construction of the culvert and the depth from the surface of the ground to the roof of the culvert exceeds, say, three times the width of the culvert, it can be assumed that the maximum earth pressure on the culvert is that due to a depth of earth equal to three times the width of the culvert. Although a culvert passing under a newly-filled embankment may be subjected to more than the full weight of the earth above, there is little reliable information concerning the actual load carried, and therefore any reduction in load due to arching of the ground should be made with discretion. If there is no filling and wheels or other concentrated loads can bear directly on the culvert, the load should be considered as carried on a certain length of the culvert. In the case of a box culvert, the length of the culvert supporting the load should be determined by the methods illustrated in Table 6. The concentration is modified if there is any filling above the culvert and, if the depth of filling is \( D^2 \), a concentrated load \( W \) can be considered as spread over an area of \( 4D^2 \). When \( D_1 \) equals or slightly exceeds half the width of the culvert, the concentrated load is equivalent to a uniformly-distributed load of \( \frac{W}{4D_1^2} \) lb. per square foot over a length of culvert equal to \( 2D_1 \). For values of \( D_1 \) less than half the width of the culvert, the bending moments will be between those due to a uniformly-distributed load and a central concentrated load.

The weights of the walls of the culvert can be assumed to produce a uniform pressure on the ground. The weights of the bottom slab and the water in the culvert are carried directly on the ground below the slab and thus do not produce bending moments, although these weights must be taken into account when calculating the maximum pressure on the ground. The horizontal pressure due to the water in the culvert produces an internal triangular load or a trapezoidal load if the surface of the water outside the culvert is above the top, when there will also be an upward pressure on the underside of the top slab. The magnitude and distribution of the horizontal pressure due to the earth against the sides of the culvert can be calculated in accordance with the formulae given in Tables 10, 11 and 12, consideration being given to the possibility of the ground becoming waterlogged with consequent increased pressures and the possibility of flotation.

**Bending Moments on Culverts.**—The maximum bending moments can be calculated by considering the possible incidence of the loads and pressures. Generally there are only two conditions to consider: (a) Culvert empty: full load and surcharge on the top slab, the weight of the walls, and maximum earth pressure on the walls; (b) Culvert full: minimum load on the top slab, minimum earth pressure on the walls, weight of walls, maximum horizontal pressure from water in the culvert, and possible upward pressure on the top slab. In some circumstances these conditions may not produce the maximum positive or negative bending moments at any particular section, and the effect of every probable combination should be considered. The direct thrusts and tensions due to various loads should be combined with the bending moments to determine the maximum stresses.

The bending moments produced in monolithic rectangular culverts are generally determined by considering the four slabs as a continuous beam of four spans with equal bending moments at the end supports. The load can be conveniently divided as follows: (a) A uniformly-distributed load on the top slab and an equal reaction from the ground below the bottom slab; (b) An imposed load on the top slab and an equally-
distributed reaction from the ground below the bottom slab; (c) Upward pressure on the bottom slab due to the weight of the walls; (d) A trianicularly-distributed horizontal pressure on each wall due to the increase in earth pressure in the height of the culvert; (e) A uniformly-distributed horizontal pressure on each wall due to pressure from the earth and any surcharge above the level of the roof of the culvert; (f) The internal horizontal pressure from the contents of the culvert. These loads are indicated in Table 99 and the bending moments at the corners due to the various types of loading can be found from the formulae given in the table. These expressions are applicable when the thicknesses of the top slab and the bottom slab are about equal, but may be equal to or different from the thicknesses of the walls.

For the bending moments due to the outward pressures from the water in the culvert, which are greatest when the culvert is entirely submerged, the formulae for the corresponding inward pressures can be used but with the sign changed.

Subways.—A subway of rectangular cross-section is subjected to external earth pressures similar to those on a culvert and the formulae in Table 99 can be used for the purpose of calculating the bending moments. Internal pressures do not generally have to be considered. The minimum sizes for subways are given in Table 92.

Bearings, Hinges, and Joints.

In the construction of frames and arches, hinges are necessary at points where it is assumed there is no bending moment. Sliding and roller bearings are necessary in some types of bridges to ensure statical determinacy. Some types of bearings and hinges are illustrated in Table 94, and notes on these designs are given on the page facing the table.

Joints in monolithic concrete construction are required to allow free expansion and contraction due to changes of temperature and shrinking in such structures as retaining walls, reservoirs, roads, and long buildings, and to allow unrestrained deformation of the walls of cylindrical containers when it is undesirable to transfer any bending moment or force from the walls to the bottom slab. Some designs of joints for various purposes are illustrated in Table 95 and notes on these designs are given on the page facing Table 94. Joints in road slabs are illustrated in Table 96.

Concrete Roads.

A concrete road may be a concrete slab forming the complete road or may be a slab underlying bituminous macadam, granite setts, asphalt, wooden blocks, or other surfacing. On the site of extensive works it is sometimes convenient to lay concrete roads before constructional work begins, these roads being the bases of permanent roads. A type of concrete road much used for motorways and similar main roads comprises a layer of plain cement-bound granular material (called "dry-lean" concrete), the mixture being about 1:18, with a bituminous surfacing. In this section, the design of all-concrete reinforced concrete roads only is dealt with. For details of the preparation of the foundation (a very important aspect of the construction of a road) and methods of construction, reference should be made to other publications.

The design of concrete roads is based as much on experience as on calculation, since the combined effects of the expansion and contraction of the concrete due to moisture and temperature changes, of the weather, of foundation friction, of spanning over weak places in the foundation, of fatigue, and of carrying the loads imposed by
traffic are difficult to assess. The provision of joints assists in controlling some of these stresses. The notes in the following give the basic principles only.

**Stresses due to Traffic.**—The stresses in a concrete road slab due to vehicles are greatest when a wheel is at the edge or near a corner of the slab, but considerably less when it is remote from an edge or corner; therefore, from the point of view of stresses due to traffic, it is desirable to reduce the number of joints across which loads cannot be transferred and to prevent cracks, thereby reducing the number of effective edges and corners. The empirical formulae derived by Westergaard for the calculation of the stresses are the basis of many subsequent attempts to reconcile the stresses derived theoretically with measured stresses; the formulae (published in 1933) are given in a modified form in Table 96, and have since been modified to apply to aircraft runways. See page 150 and Table 6 for weights of vehicles and aircraft.

**Thickness of Slab.**—For all-concrete roads the slab may be from 5 in. to more than 12 in. thick, depending on the weight of traffic and the type of soil. Some common thicknesses are given in Table 96 for various weights of traffic and types of soil, which are defined approximately in the table. The thicknesses should be increased for particularly adverse conditions, such as for very heavy traffic on dockside roads on very poor soil, for which upwards of 12 in. may be necessary. The concrete should not be leaner than 1:1½:3½ unless special mixtures are “designed” to give a dense strong concrete with a lower cement content. For the wearing surface, rounded aggregates are not recommended and a hard crushed stone should be used. In districts where suitable crushed stone is costly an economical and durable slab can be formed by making the lower part of the slab of 1:2:4 concrete made with uncrushed gravel aggregate, and the upper part, to a depth of about 1½ in., with 1:1½:3 concrete made with crushed stone graded from ½ in. to 3½ in. Exposure to weather and abrasion from traffic subject all-concrete roads to severe conditions, and all reasonable means of attaining a concrete of high quality should be taken.

**Reinforcement.**—When a concrete road is laid on a firm and stable foundation, experience shows that reinforcement is not always necessary, but some engineers take the view that the provision of reinforcement is a precaution that justifies the cost. When mild steel reinforcement is used the amount provided is generally between 6 lb. and 10 lb. per square yard provided in a single layer near the bottom or top of the slab, but for roads subject to heavy traffic reinforcement is provided near the top and bottom to give a total weight of 10 lb. to 20 lb. per square yard. If high-yield-stress steel is used two-thirds of the foregoing weights may be sufficient. Typical weights of mild steel reinforcement, which should preferably be placed near the top of the slab, are given in Table 96 for various types of traffic and soil. The arrangement of the reinforcement depends on the width of the road and the spacing of the transverse joints. If the joints are at distances apart equal to about the width of the slab, the reinforcement should be arranged to give equal strength in both directions, but if the transverse joints are provided at long intervals to form panels of length, say, three or more times their width, nine-tenths of the reinforcement should be parallel to the length of the road; for panels of intermediate proportions, ratios of between 0.5 and 0.9 of the total reinforcement should be placed longitudinally. Additional bars, say, ½ in. diameter, are required in the top at the corners of the panels.

**Joints.**—Although some concrete roads have no transverse joints, the provision of such joints and, in wide roads, the provision of longitudinal joints, may assist in reducing cracking. In Britain the common spacing of expansion and contraction
joints is from 30 ft. to 100 ft. The end of each day’s concreting should coincide with a joint. One form of transverse expansion and contraction joint is illustrated in Table 96; a clear gap of about 1 in. is left between the faces of adjacent panels and the space is almost filled with a resilient material and sealed with a bituminous compound. In the centre of the slab, and at intervals of about 1 ft. to 2 ft. 6 in. across the width of the road, mild steel dowel bars 2 ft. or 3 ft. long and from 1 in. to 3 in. in diameter project horizontally from one panel to the next. One half of each bar is greased or otherwise treated to allow freedom of movement and is fitted with a ferrule; the other half is embedded in the concrete. Dowel bars prevent one panel rising relatively to its neighbour, partially prevent warping and curling, and transfer a part of the load on one panel to the other, thereby reducing the stresses.

Simple contraction joints (or dummy joints) are provided at intervals of about 15 ft. between transverse expansion and contraction joints; such a joint is illustrated in Table 96 and is merely a slot 1 in. wide and 2 in. deep, filled with bituminous compound, formed in the top only (or in some cases in the bottom only) of the slab. The provision of dummy joints enables the slab to crack at intervals without being unsightly, irregular or injurious.

Longitudinal joints are generally provided in roads more than 15 ft. wide so as to divide the road into strips about 10 ft. wide. A longitudinal joint may be a simple butt-joint, but some form of interlock is desirable to avoid one slab rising relatively to the adjacent slab and to enable transfer of load to take place. Thus a dowelled joint is sometimes provided, or a tongued-and-grooved joint as illustrated in Table 96.

The joints shown in Table 96 are typical and are suitable for a reinforced concrete road slab 7 in. thick on a medium or good soil carrying medium traffic. Similar and other designs are given in the publications of the Road Research Board.

Containers.

The weights of materials and the calculation of the horizontal pressure due to dry materials and liquids contained in tanks, reservoirs, bunkers, silos and other containers are given in Tables 4, 10, 11, 12 and 14. This section deals with the design of containers, and with the calculation of the forces and bending moments produced by the pressure of the contained materials. Where containers are required to be watertight, the recommendations given in B.S. Code No. 2007 (1960) for reinforced concrete structures for the storage of water have been adopted. Containers are conveniently classified as tanks containing liquids, and bunkers and silos containing dry materials, each class being sub-divided into cylindrical and rectangular structures. Some typical details of walls of containers are given in Appendix II.

Tanks.

Direct Tension in the Wall of a Cylindrical Tank.—The wall of a cylindrical tank is primarily designed to resist direct tension due to the horizontal pressures of the contained materials, and, if \( p \) lb. per square foot is the pressure at any depth, the direct tension \( T \) in a horizontal ring 1 ft. in depth is \( 0.5pD \) lb., where \( D \) is the internal diameter of the tank (ft.). Sufficient circumferential reinforcement must be provided to resist this tension; appropriate formulae are given in Table 97. For tanks containing liquids, the tensile stress in the circumferential reinforcement should not exceed 12,000 lb. per square inch, but for a container of dry materials a stress of 18,000 lb. per square inch can be allowed. The length of any overlaps in the circumferential reinforcement must be sufficient to enable the maximum tensile stress to be developed,
and for this reinforcement hooks at the ends of the plain round bars should be invariably provided in liquid-containers. For tanks containing liquids the thickness of the wall should be determined in relation to the total tension, as described on page 69 and in Tables 73 and 74, to reduce the risk of cracking. The minimum thickness of the wall is given by the appropriate general expression in Table 97, or the special expressions in Tables 73 and 74. It is sometimes recommended that the thickness of the wall of a tank containing liquid should not be less than 4 in. (or 5 in.) and not less than \(2\frac{1}{4}\) per cent. of the depth of liquid plus 1 in. For cylindrical vessels containing dry materials or for lined tanks containing liquids, it is not so essential to design for no cracking, and the thickness of the wall is determined from practical considerations.

**Bending Moments on the Walls of Cylindrical Tanks.**—In addition to the horizontal tension in the wall of a cylindrical container, bending moments are produced by the restraint at the base of the wall. Unless a joint is made at the foot of the wall, as illustrated at (b) in Table 95, there is some continuity between the wall and the base slab which causes vertical deformation of the wall and reduces the circumferential tension. There are three principal factors, namely, the magnitude of the bending moment at the base of the wall, the point at which the maximum circumferential tension occurs, and the magnitude of the maximum circumferential tension. Coefficients and formulæ for determining these factors are given in Table 97 and are derived from Mr. H. Carpenter’s translation of Dr. Reissner’s analysis. The shape of the wall has some effect on the value of the coefficients, but the difference between the bending moments at the bases of walls of triangular or rectangular vertical section is so small that the common intermediate case of a trapezoidal section can be considered to be the same as a rectangular wall. The small error involved partly offsets the error of assuming perfect fixity at the junction of the wall and the base.

The procedure is first to determine the maximum vertical bending moment and provide a wall having an equal moment of resistance at the bottom. The maximum circumferential tension and the height up the wall at which this occurs are next determined; sufficient area of steel and thickness of concrete must be provided at this height to resist the maximum tension. Above this height the area of reinforcement can be uniformly decreased to a nominal amount, and below it the area of reinforcement can be maintained equal to that required for the maximum circumferential tension, although some reduction towards the bottom may be justified. Some typical details of walls of tanks with and without restraint at the bottom are given in Appendix II.

**Octagonal Tanks.**—If the wall of a tank is in plan, a series of straight sides instead of circular, the shuttering may be less costly but extra reinforcement or increased thickness of concrete or both is necessary to resist the horizontal bending moments which are produced in addition to the direct tension. If the tank is a regular octagon the bending moment at the junction of adjacent sides is \( \frac{\rho L^2}{12} \), where \( L \) is the length of side of the octagon. If the distance across the flats is \( D \), the direct tension in each side is \( \frac{1}{4} \rho D \), and at the centre of each side the bending moment is \( \frac{\rho L^2}{24} \). If the shape of the tank is not a regular octagon, but the lengths of the sides are alternately \( L_1 \) and \( L_2 \), and the corresponding thicknesses are \( D_1 \) and \( D_2 \), the bending moment at the junction of any two sides is

\[
\frac{\rho}{12} \left( L_1^2 + L_2^2 \frac{D_1}{D_2} \right) \left( \frac{D_1}{D_2} \right)^3
\]
Walls of Rectangular Tanks.—The bending moments on, and direct tensions in, the walls of rectangular tanks are calculated in the same manner as described on page 102 for bunkers. For impermeable construction, however, the maximum tensile stress in the reinforcement and, to reduce the risk of cracking, the tensile stress in the concrete should not exceed the values given on page 69; the consequent design formulae are given in Table 74; this data applies to suspended bottoms of tanks as well as to walls.

The walls of large rectangular reservoirs generally span vertically and are monolithically with the roof and floor slab, the floor being generally laid directly on the ground. If the wall is considered as freely supported at the top and bottom, and if \( P \) is the total water pressure on the wall, the force at the top is \( 0.33P \) and at the bottom \( 0.67P \). If the wall is assumed to be freely supported at the top and fixed at the bottom, the forces are \( 0.2P \) and \( 0.8P \) at the top and bottom respectively. As neither of these conditions is likely to be obtained, a practical assumption is that the forces at the top and bottom are \( 0.25P \) and \( 0.75P \) respectively; the positive bending moment at about the midpoint of a wall of height \( H \) ft. and the negative bending moment at the bottom are each \( 0.083PH \). If the walls span vertically and horizontally Table 40 applies. Some typical details are given in Appendix II.

Bottoms of Elevated Tanks.—The type of bottom provided for an elevated cylindrical tank depends on the diameter of the tank and the depth of water. For small tanks a flat beamless slab is satisfactory, but beams are necessary for tanks from 10 ft. up to, say, 25 ft. in diameter. Some designs are indicated on the page facing Table 97; notes on the designs which include bottoms with beams and domed bottoms, and examples, are given on the pages facing Tables 97 and 98.

The weight of liquid in a tank having only one compartment should be considered as a dead load when calculating the bending moments on the slabs and beams of the bottom since all spans will be loaded simultaneously. Tee-beams in the bottom of a tank should be designed so that the tensile stress at the face of the concrete in contact with water, due to negative bending moments, does not exceed the safe tensile stress in the concrete (see page 69); for this purpose the maximum width of slab considered as the flange should be as given in Table 69.

Columns supporting Elevated Tanks.—It is important that there should not be unequal settlement of the foundations of the columns supporting an elevated tank and a raft should be provided if the nature of the ground is such that unequal settlement is likely. In addition to the bending moments and shearing forces due to the pressure of the wind on the tank, as described in Table 8, the wind force causes a thrust on the columns on the leeward side and a tension in the columns on the windward side; the values of the thrusts and tensions can be calculated for a group of columns from the expressions given on the page facing Table 98.

Effects of Temperature.—For a tank containing a hot liquid, the working stresses should be lower than for other tanks, or the probable increase in stress due to the higher temperature should be calculated as described on page 107.

In Britain the effects of temperature due to weather variations are seldom sufficiently great to be considered in the design of the tank, but elsewhere it may be necessary to protect the tank of a water tower from extreme exposure to the sun. External linings of timber, brick, or other material may be provided or the tank should be designed for the effects of the differences of temperature on opposite faces of the wall.

Joints.—Permanent joints are provided in large tanks, reservoirs, and similar
containers to allow for expansion and contraction due to changes of temperature or to shrinkage of the concrete, or to relieve parts of the structure from stresses due to indeterminate restraints that would otherwise be imposed by adjacent parts. Details of joints suitable for reservoirs, swimming pools, and tanks are given in Table 95.

Pipes.—Pipes built into concrete tanks are sometimes made of non-ferrous alloy, since deterioration due to corrosion is much less than for ferrous metals and replacements that may affect the water-tightness of the structure are obviated. Pipes built into the wall of a tank should have an additional intermediate flange cast in such a position that it will be buried in the thickness of the wall and thus form a water-bar.

Underground Tanks.—Underground or submerged tanks are subjected to external pressures due to the surrounding earth or water, which produce direct compression in the walls. The stress produced by this compression in the wall of a cylindrical tank is a maximum when the tank is empty, and is given by the expression in Table 97. Unless conditions are such that the permanence of the external pressure is assured, the relief to the tension provided by the compression should be disregarded in the calculation of the stresses in the tank when full. When empty, the structure should be investigated for flotation if it is submerged in a liquid or is in waterlogged ground.

Reservoirs with earth or other material banked up against the walls should be designed for earth pressure from outside with the tank empty. When the reservoir is full no reduction should be made to the internal pressure by reason of the external pressure, but in cases where the designer considers such reduction justified the amount of the reduction should be considerably less than the theoretical pressure calculated by the formula for active pressures in Tables 10, 11 and 12.

The earth on the roof of a reservoir should be considered as a live load, although it is ultimately a uniformly-distributed load acting on all spans simultaneously. When the earth is being placed in position, conditions may occur whereby some spans are loaded and others are unloaded. Often, however, the designer can ensure that the earth is deposited in such a manner as to keep the bending moments at a minimum.

Bunkers and Silos.

Properties of Contained Materials.—The weights of materials commonly stored in bunkers and silos are given in Table 4, and the pressures of these materials are dealt with in Tables 10 and 11. When calculating the size of a structure of a specified capacity, the weight of the material should not be overrated nor too small a value assumed for the angle of repose. When calculating the weight to be carried on the bottom and the pressures to which the sides will be subjected, the weight should not be under-estimated nor the angle of internal friction over-estimated. Generally two assumptions are therefore necessary in designing a container; examples of these assumptions are given in Table 11.

Walls.—The walls of bunkers and silos are designed to resist bending moments and tensions caused by pressure of the contained material. If the wall spans horizontally, it is designed for the bending moments and direct tension combined as in Table 89. If the wall spans vertically, horizontal reinforcement is provided to resist the direct tension and vertical reinforcement to resist the bending moments. In this case the horizontal bending moments due to continuity at corners should be considered, and it is generally sufficient if as much horizontal reinforcement is provided at any
level at the corners as is required for vertical bending at this level; the amount of reinforcement provided for this purpose, however, need not exceed the amount of vertical reinforcement required at one-third of the height of the wall. The principal bending moment on walls spanning vertically is due to the triaangularly-distributed pressure from the contained material. Bending-moment coefficients for this distribution of load are given in Tables 16 and 17 for conditions of fixity or free-support at one or both ends of the span. The practical assumption described for the walls of rectangular reservoirs should be observed in this connection (see page 101).

For walls spanning horizontally the bending moments and forces depend upon the number and arrangement of the compartments. For structures with several compartments, the intermediate walls act as ties between the outer walls, and in Table 98 expressions are given for the negative bending moments on the outer walls of rectangular bunkers with various arrangements of intermediate walls or ties. The corresponding expressions for the reactions, which are a measure of the direct tensions in the walls, are also given. The positive bending moments can be readily calculated when the negative bending moments at the corners are known. An external wall is subject to maximum stresses when the adjacent compartment is filled, since it is then subjected simultaneously to the maximum bending moment and the maximum direct tension. An internal cross wall is subjected to maximum bending moment when the compartment on one side of it is filled, and to maximum direct tension (but no bending moment) when the compartments on both sides of the wall are filled.

In small bunkers the panels of wall may be of such proportions that they span both horizontally and vertically, in which case Table 40 should be used to calculate the bending moments since the pressure along the horizontal span will be uniformly distributed, while along the vertical span triangular distribution will occur. An example is given on page 320.

In the case of an elevated bunker the whole load is generally transferred to the columns by the walls, and when the span exceeds twice the depth of the wall the wall can be designed as a beam. Owing to the large moment of inertia of the wall (as a beam bending in a vertical plane) compared with that of the columns, the beam can be assumed to be freely supported but the heads of the columns under the corners of the bunker should be designed to resist a bending moment equal to, say, one-third of the maximum positive bending moment on the beam. If the provision of sufficient moment of resistance so requires, a compression head can conveniently be constructed at the top of the wall, but there is generally ample space to accommodate the tensile steel in the base of the wall. When the distance between the columns is less than twice the height of the wall the reinforcement along the base of the wall should be sufficient to resist a direct tension equal to one-quarter of the total load carried by the wall. The total load must include all other loads supported by the wall. These loads may be due to the roof or other superstructure or machines mounted above the bunker and to the weight of the wall.

The effect of wind on large structures should be calculated. In silos the direct compressive force on the leeward walls due to wind pressure is one of the principal forces to be investigated. The stress due to the eccentric compression due to the proportion of the weight of the contents supported by friction on the walls of a silo (see Table 14) must be combined with the stresses produced by wind pressure, and at the base or at the top of the walls there may be additional bending stresses due to continuity with the bottom or the covers or roof over the compartments.
If a wall is thicker at the bottom than at the top it may taper uniformly from bottom to top or the reduction in thickness may be made in steps. The shuttering may be more costly for a tapered wall than for a stepped wall, especially for cylindrical containers. A stepped wall, however, may be subjected to high secondary stresses at the change of thickness where also the day's-work joints generally occur. Stepping on the outside is often objectionable as it provides ledges for the collection of dust. Stepping on the inside may interfere with the free flow of the contents when emptying the container.

The size and shape of a bunker depend on the purpose which it is to serve, and the internal dimensions are therefore generally specified by the owner. Typical calculations are given on page 320 for a design in which the walls span vertically and horizontally. When the walls span horizontally the reinforcement varies from a maximum at the bottom to a nominal amount at the top; the vertical reinforcement need only be sufficient to keep the horizontal bars in place, and generally $\frac{3}{8}$-in. bars at 12-in. to 15-in. centres are satisfactory for this purpose. In the case of tall bunkers each lift of vertical reinforcement should not exceed about 10 ft., although if continuously-moving shuttering is used the vertical bars should be 4 ft. to 6 ft. long.

**Hopper Bottoms.**—The design of sloping hopper bottoms in the form of inverted truncated pyramids consists of finding, for each sloping side, the centre of pressure, the intensity of pressure normal to the slope at this point, and the mean span. The bending moments at the centre and edge of each slope are then calculated. The horizontal direct tension is next computed and combined with the bending moment to determine the amount of horizontal reinforcement required. The direct tension acting in the line of the slope at the centre of pressure and the bending moment at this point are combined to find the reinforcement necessary in the underside of the slab at this point. At the top of the slope the bending moment and the component of the hanging-up force are combined to determine the reinforcement required in the upper face at the top of the slope.

The centre of pressure and the mean span can be found by inscribing on a normal plan of the sloping side a circle touching three of the sides. The diameter of this circle is the mean span and the centre is the centre of pressure. The total intensity of load normal to the slope at this point is the sum of the normal components of the vertical and horizontal pressures at the centre of pressure and the dead weight of the slab. Values for the pressure on an inclined slab are given in Table 11, and expressions for the bending moments and direct tensions along the slope and horizontally are given in Table 99. When using this method it should be remembered that, although the horizontal span of the sloping side is considerably reduced towards the outlet, the amount of reinforcement should not be reduced below that determined for the centre of pressure, since in determining the bending moment based on the mean span adequate transverse support from the reinforcement towards the base is assumed.

The "hanging-up" force in the direction of the slope has a vertical and a horizontal component, the former being resisted by the walls acting as beams. The horizontal component, acting inwards, will produce horizontal bending moments on the beam at the top of the slope, but often this bending moment is neglected as the inward horizontal force is generally resisted by a corresponding outward pressure from the contained material. When there is insufficient head of material above the top of the slope to create the necessary counter-pressure, it is necessary to calculate and to make provision for the horizontal bending moment.
An example of the calculations by this method for a pyramidal hopper bottom is given on the page facing Table 99.

**Chimneys.**

**Dimensions.**—The height and internal sizes of a chimney are usually specified by the engineers responsible for the boiler installation. The reinforced concrete designer has to determine the thickness of, and reinforcement in, the shaft. The two principal forces on a chimney are the wind pressure and the dead weight. At any horizontal section the cantilever bending moment due to the wind is combined with the direct force due to the weight of the chimney and lining above the section considered to find the maximum stresses. Suitable values for wind pressures on circular and other shafts are given in Table 8, and an example showing a graphical method of determining the stresses is given in Table 85. Generally the preliminary determination of the thickness of the wall of the shaft is a matter of trial and error, but the thickness of concrete in inches and the total area of reinforcement required can be found approximately for a circular shaft from the expressions given in the table on the page facing Table 100. These expressions, which were established by Messrs. Taylor, Glenday, and Faber, are for stresses of 400 and 600 lb. per square inch in the concrete and 12,000, 14,000, and 16,000 lb. per square inch in the reinforcement. Low stresses are desirable for the combination of bending moment and weight only, as a margin is left thereby for increases in stresses due to rise of temperature. Having determined the trial dimensions for the shaft, the values of the bending moment and weight can be calculated more closely and the actual stresses obtained either by the graphical method described in Table 85 or by one of the analytical methods given in books on the design of chimneys; the stresses due to thermal effects have to be combined with those due to moment and weight, and the method of so doing is given in other publications. Some notes on thermal stresses are given on page 107.

**Maximum Longitudinal Stresses.**—The maximum stresses on any horizontal plane of a chimney shaft should be investigated for the following conditions. (a) When subjected to direct load only, that is the weight of the concrete shaft and the lining, the maximum compressive stress should not exceed the values given for direct stress in Table 56. For this purpose, the value of the modular ratio used in calculating the effect of the reinforcement can be assumed to be 15. (b) The stresses due to combining the bending moment due to the wind with the maximum direct load should be ascertained either by comparison with the factors given in the table on the page facing Table 100 or by the graphical analysis given in Table 85. To the maximum compressive stress at the inner face of the wall should be added the stress, \( c_T \), due to the change of temperature, and to the maximum tensile stress in the reinforcement on the outer face should be added the stress \( t_T \) due to the change of temperature, thereby giving approximately the maximum stresses; a method of calculating \( t_T \) and \( c_T \) is given in the following (page 107). The maximum compressive stress in the concrete should not exceed the values given in Table 56 for bending.

The various stresses are inter-related, and the addition of, say, the temperature stresses to the combined bending and direct stresses may alter the basis of calculating these stresses by altering the position of the neutral plane, subjecting more of the concrete to tensile stresses in excess of the tensile strength, and thereby causing cracking.

**Transverse Stresses.**—The preceding remarks deal solely with stresses normal
to a horizontal plane. Stresses normal to a vertical plane are also produced both by wind pressure and differences of temperature. In chimney shafts of ordinary dimensions the transverse bending moment resulting from wind pressure is generally negligible, but this is not necessarily so in tanks and cooling towers of large diameter. A uniform pressure of $p$ lb. per square foot produces a maximum bending moment of

$$\pm \frac{pD^2}{12}$$

ft.-lb. on a height of 1 ft. of a cylinder of external diameter $D$ ft. This bending moment causes a compressive stress at the outer face of a wall normal to the line of the action of $p$ and tensile at the inner face. An equal bending moment, but of opposite sign, acts on the wall parallel to the line of action of $p$. If the distribution of wind pressure is as illustrated in Table 7 for cylinders, the bending moment at the end of the diameter normal to the line of action of $p$ may be about three times that given in the foregoing.

A difference in temperature between the two faces of a concrete wall produces a transverse bending moment equal to $\frac{T_{\theta}E_{\theta}I_{\theta}}{d_{\theta}}$. (See pages 107 and 108 for notation.) If the shaft is unlikely to be cracked vertically, the maximum stress in the concrete due to this bending moment is about $0.5T_{\theta}E_{\theta}$, being compressive on the face subjected to the higher temperature and tensile on the opposite face.

**Industrial Structures.**

In addition to the ability of the various members to sustain the forces and moments to which they are subjected, there are other considerations peculiar to each type of industrial structure. Vibration must be allowed for in the substructures for crushing and screening plants. Provision against overstressing a reinforced concrete pit-head frame is obtained by designing for various conditions of working and accidental loading such as described on page 148. Watertightness is essential in slurry basins, coal draining bunkers, settling tanks, and similar hydraulic structures, while airtightness is essential in gas purifiers and in airlock structures in connection with colliery work; the suction in airlocks is generally equivalent to a head of 5 in. to 10 in. of water, that is, 26 lb. to 60 lb. per square foot. The resistance of concrete to corrosion from fumes that are encountered in some industrial processes is one of the properties that recommends the material for industrial construction, but protection of the concrete is needed with other fumes and some liquids (see Appendix I). Provision should be made for expansion in structures in connection with steel-works, coke ovens, gas retorts, and other structures where great heat is experienced. Boiler foundations, especially on clay, should be made sufficiently thick to prevent undue heating and drying out of the subsoil or an insulating layer should be interposed between the foundation and the ground. Firing floors, coke-benches, and rolling-mill floors should be protected from extreme temperatures and abrasion by being covered with steel plates or bricks.

On floors where dust, rubbish, or slime may collect, as in coal washeries, it is advantageous to make a fall in the top surface of 3 in. in 10 ft. to facilitate the cleaning of the floors, but it must be ascertained that such a slope will not be inconvenient to the users of the floor; otherwise suitable channels must be provided to ensure that washing down will be effective.

Structures in mining districts should be designed for the possibility of subsidence of the ground upon which they stand. Thus raft foundations that have at any part equal resistance to negative and positive moments are commonly adopted for small
structures. If isolated foundations are provided for long structures such as gantries, the longitudinal beams should be designed as if freely supported.

**Stresses due to Temperature.**

The following consideration of stresses due to temperature can be applied to chimneys, tanks containing hot liquids, retort foundations, and other structures where there is a difference of temperature between two faces of the concrete.

The first stage is to determine the change of temperature $T$ deg. F. through the concrete. The resistance to the transmission of heat through a wall of different materials, the successive thicknesses of which (in inches) are $d_1$, $d_2$, $d_3$, etc., is given by

$$\frac{1}{k} = \frac{d_1}{k_1} + \frac{d_2}{k_2} + \frac{d_3}{k_3} + \ldots + a_i + a_a + a_o = \sum \frac{d}{k} + a_i + a_a + a_o$$

where $k_1$, $k_2$, $k_3$, etc., are the conductivities of the various materials of which the wall is made, $a_i$ and $a_o$ represent the drop in temperature at the internal and external faces respectively, and $a_a$ is the drop due to a cavity in the wall. Values of the conductivities, in B.th.u. per square foot per hour per degree F., are 6.7 for ordinary concrete, 7.4 for firebrick, and 0.3 for slag wool. The values of $a_o$, $a_i$ and $a_a$ are not readily determinable; $a_a$ for example depends on the degree of exposure and a value of 0.5 deg. F. for the transmission of one B.th.u. per square foot per hour is a reasonable average value, although in sheltered positions facing south the value may be 0.75 and as low as 0.10 for conditions of severe exposure facing north. In a chimney or in a tank containing hot liquid, there may be little difference between the temperature of the flue gas or liquid and the temperature of the face of the concrete or lining in contact with the gas or liquid. Hence $a_i$ may often be neglected or, if some fall of temperature at the internal face is expected, a value of 0.7 may be used. The value of $a_a$ depends on the amount of ventilation of the cavity and may be about unity.

If the temperature of the flue gas or hot liquid is $T_o$ deg. F. and the external air temperature is $T_A$ deg. F., the change of temperature through a concrete wall $d_c$ in. thick is $T = \frac{(T_o - T_A)k_d}{k_c}$ deg. F., where $k_c$ is the conductivity of the concrete. Owing to the numerically indeterminate nature of many of the terms used in the calculation of $T$, extreme accuracy cannot be expected. Therefore only an approximate assessment of the stresses due to a difference in temperature is worth while.

In an uncracked reinforced concrete wall, or in a cracked wall that is entirely in compression, the change in the compressive stress in the concrete due to a temperature difference of $T$ deg. F. is $c_T = \pm 0.5 T e_c E_c$, where $e_c$ is the coefficient of linear expansion of concrete, say, 0.0000055 per degree F., and $E_c$ is the modulus of elasticity of concrete. For $E_c$, a value of 3,000,000 lb. per square inch should be used.

In a cracked wall subject to tension, the concrete being neglected except as a covering for the reinforcement, the change in stress in the reinforcement is $t_T = \pm 0.5 (1 - f) T e_s E_s$. The term $(1 - f)$ is such that $(1 - f)d_e$ is the distance between the centres of the reinforcement on opposite faces of the wall; $e_s$ is the coefficient of linear expansion for steel, say, 0.000006 per degree F., and $E_s$ is the modulus of elasticity of steel, say, 30,000,000 lb. per square inch.

If the wall subjected to temperature strains is already stressed in tension on one face and compression on the other, as may occur in the wall of a tank containing hot liquid, then to the bending moment at any section a bending moment due to
change of temperature equal to \( M_T = \frac{T_{e0}E_cI_e}{d_e} \) should be algebraically added, where \( I_e \) is the moment of inertia of the section expressed in concrete units and ignoring the area of any concrete that may be cracked. In impermeable construction designed to prevent the concrete cracking, the value of \( I_e \) would be based on the whole thickness of concrete together with an allowance for the reinforcement with a modular ratio corresponding to the value assumed for \( E_c \).

It should be noted that the bending moment \( M_T \) due to change of temperature tends to produce compression on the face subject to the higher temperature.

**Protection of Concrete.**

Many structures, such as tanks, chimneys, and the floors of buildings used for certain purposes, have to resist the injurious effects of acids and other chemicals. In Appendix I a list is given which includes non-injurious and corrosive substances and liquids, and for injurious substances some protective treatments are described. Where corrosive liquids are stored in concrete containers the corrosion is generally most marked near the normal level of the liquid, and some liquids, which may not ordinarily affect the concrete, may cause corrosion when the concrete surface is alternately wet and dry.

The life of a reinforced concrete container exceeds that of a steel or timber structure, and can be further prolonged by careful design which reduces the liability to cracking, and thereby increases the resistance of the concrete to deterioration due to abrasion from falling and moving hard material. If attention is given to this matter and a dense concrete is used, lining the wearing faces of the structure with tiles or plates or rails may not be necessary except where coke is stored or where coal or stone is dropped into a bunker from a height. The top surfaces of ties and intermediate walls exposed to falling material should be made in the form of an inverted V and covered by a renewable metal shield. The bottoms of bunkers in which sticky coal or similar material is stored may be lined with glass to facilitate emptying.

**Retaining Walls.**

The weights of, and the methods of calculating the horizontal pressures due to, retained earth are given in *Tables* 10, 11 and 12. This section deals with the design of retaining walls including the calculation of the forces and bending moments produced by the pressure of the retained material. Other recommendations for the design of retaining walls, sheet-piled walls, and the like are given in Code No. 2, "Earth-Retaining Structures", published by the Institution of Structural Engineers.

**Types of Retaining Walls.**—A retaining wall is essentially a vertical cantilever, and when it is constructed in reinforced concrete it can be a cantilevered slab, a wall with counterforts, or a sheet-pile wall. A cantilevered slab is suitable for walls of moderate heights and has a base projecting backwards under the filling, as at (b) in the diagram at the top of *Table* 100, or a base projecting forward as at (a). The former type is generally the more economical. The latter type is only adopted when for reasons relating to buildings or other adjacent property it is not permissible to excavate behind the stem of the wall. If excavation behind the wall is permitted, but to a limited extent only, a wall with a base projecting partly backwards and partly forwards as at (c) can be provided. Any length of base projecting backwards
is advantageous as the weight of earth thereon assists in counterbalancing the over-turning effects.

A wall provided with counterforts is suitable for a greater height than is economical for a simple slab wall. The slab spans horizontally between the vertical counterforts which are arranged as at (d). Where the net height of the wall is great, it is sometimes more economical to adopt the type of wall shown at (e), where the slab spans vertically between horizontal beams which bear against counterforts. By graduating the spacing of the beams the maximum bending moments in each span of the slab can be equal and the slab kept the same thickness throughout. When the shearing stresses allow, the web of the counterfort can be perforated; this saves concrete but complicates the shuttering and reinforcement.

Pressures behind Walls.—The value of the horizontal pressure due to retained earth is often assumed to be \(27h\) lb. per sq. ft. at a depth \(h\) ft. below the level surface of the ground behind the wall. When the ground is compact a lower pressure is sometimes assumed, say, \(22h\), which corresponds to an angle of repose of 40 deg. and a density of 100 lb. per cubic foot. The values \(27h\) and \(22h\) should be increased or reduced when the surface of the ground behind the wall slopes upwards or downwards or when a load is superimposed thereon. The pressure recommended by the Ministry of Transport is \(30h\). In ground that may become accidentally waterlogged, it is often advantageous to design for a nominal factor of safety of 4 against ground pressure, and of \(2\frac{1}{2}\) against the possible water pressure; that is, the equivalent pressure of the water alone after making allowance for the difference in factors of safety is \(40h\).

Cantilevered Retaining Walls.—The factors affecting the design of a cantilevered slab wall are usually considered per foot length of wall when the wall is of uniform height, but when the height of the wall varies a length of, say, 10 ft., or other convenient length, should be treated as a complete unit. For a wall with counterforts the length of a unit is the distance between two adjacent counterforts. The principal factors to be considered are stability against overturning, bearing pressure on the ground, resistance to sliding, and internal resistance to bending moments and shearing forces. Formulae for the bending moments, forces, dimensions, and other factors relating to cantilevered-slab walls are given in Table 100, and typical details of cantilevered-slab walls of types (a) and (b) and of a wall with counterforts, and calculations for these designs, are given in Appendix II.

A factor of safety of not less than \(1\frac{1}{2}\) against overturning should be allowed, rotation being assumed to be about the lowest forward edge of the base. Under the most adverse combination of vertical load and horizontal pressure, the maximum pressure on the ground should not exceed the pressure allowable on the ground upon which the wall is built.

To provide a factor of safety against forward movement of the wall as a whole the minimum total vertical load multiplied by a coefficient of friction should exceed the maximum horizontal pressure by at least 50 per cent. For dry sand, gravel, rock, and other fairly dry soils a coefficient of 0.4 is often used, but for clay, the surface of which may become wet, the frictional resistance to sliding may be zero. In this case the resistance of the earth in front of the wall must provide the necessary resistance to sliding, which can be increased by providing a rib on the underside of the base as shown at (a), (b) and (c) in Table 100. A rib is essential if the depth of earth in front of the wall is shallow. The plane of failure due to shearing in front of the wall is a curve sweeping upwards from the lowest forward edge of the wall. The
resistance of the earth in front of the wall is the passive resistance (see Tables 10 and 12). It is essential that earth be in contact with the front face of the base, otherwise a small, but undesirable, movement of the wall must occur before the passive resistance can operate. In walls of the form shown at (a), where the vertical load is small compared with the horizontal pressure, a rib should be provided either immediately below the wall stem or at the forward edge of the base to increase the resistance to sliding. If the theoretical passive resistance is depended upon to provide the whole of the resistance to sliding, the factor of safety should be at least two.

The foregoing movements of the wall, due to either overturning or sliding, are independent of the general tendency of the bank of a cutting to slip and to carry the retaining wall with it. The strength and stability of the retaining wall have no bearing on such failures, and the precautions necessary to be taken against the wall being carried away are outside the scope of the design of a retaining wall constructed to retain the toe of the bank and is a problem in soils mechanics.

The safe moment of resistance of the stem of the wall should be equal to the bending moment produced by the pressure on the slab. In a cantilevered slab, the critical bending moment may be at the top of the splay at the base of the stem. The base slab should be made the same thickness as the bottom of the wall and equal reinforcement should be provided. The base slab and the stem of the wall should be tapered.

When a single splay only is provided at the base of the stem of a cantilevered-slab wall, the critical bending moment may be at the bottom of the splay instead of at the top, since the increase in effective depth may not cause the moment of resistance to increase as rapidly as the bending moment increases. The effective depth should not be considered to increase more rapidly than is represented by a slope of one in three at each splay.

In walls with counterforts, the slab, which spans horizontally, can also taper from the bottom upwards as the pressure and consequently the bending moments decrease towards the top. Fixity with the base slab near the bottom will produce a certain amount of vertical bending requiring vertical reinforcement near the back face of the slab near the bottom. The horizontal negative and positive bending moments can be assumed to be $\frac{PL^2}{12}$, where $P$ is the intensity of horizontal pressure and $L$ is the distance between the centres of adjacent counterforts. If horizontal beams are provided the slab is designed as a continuous slab spanning vertically, requiring reinforcement near the front face between the beams and near the back face at the beams.

Counterforts are designed as vertical cantilevers, the main tensile reinforcement being in the back sloping face. Owing to the great width at the bottom, reinforcement for resistance to shear is seldom necessary, and when required it is generally most conveniently provided by horizontal binders. Only in the case of very high walls are inclined bars necessary for resistance to shearing.

**Expansion and Contraction.**—Long walls should be provided with expansion joints, suitable designs for which are given in Table 95. To reduce the risk of cracking due to shrinking of the concrete, sectional methods of construction should be specified, and as a further precaution against contraction and temperature cracks appearing on the front face of a wall reinforced near the back face, a mesh of reinforcement, say, $\frac{3}{4}$-in. diameter bars spaced at 12-in. centres horizontally and vertically, should be provided near the front face if the thickness of the wall exceeds 8 in.
Drainage behind Walls.—Sloping the base slab in front of and behind the wall not only economises in concrete but also assists drainage, the provision for which is important, especially for a wall designed for a low pressure. Where the filling behind the wall is gravel or sand, a drain of clean loosely-packed rubble should be provided along the base of the back of the wall, and weep-holes, 3 in. to 6 in. in diameter, provided at intervals of about 10 ft. A weep-hole should be provided in every space between the counterforts, and the top surface of any intermediate horizontal beams should be given a slight slope away from the back of the wall. With backings of clay or other soil of low porosity, hand-packed rubble placed behind the wall for almost the whole height and draining to weep-holes, assists effective drainage of the filling. The filling behind the wall should not be tipped from a height, but should be carefully deposited and consolidated in thin horizontal layers.

Sheet-pile Walls.

When a satisfactory bearing stratum is not encountered at a reasonable depth below the surface in front of the earth to be retained, then a sheet-pile wall is provided. Precast reinforced concrete sheet-piles are driven into the ground sufficiently far to obtain an anchorage for the vertical cantilever and security against sliding and spewing. This type of wall is particularly suitable for waterside works, and in the simplest form the sheet-piles simply cantilever out of the ground, the heads of the piles being generally stripped and bonded into a cast-insitu capping beam.

Designs for typical sheet-piles and for the interlocked type of joint necessary to maintain alignment during driving, are given in the lower part of Table 101, where are also illustrated shapes of the shoes for starting piles and following piles.

If the height of the wall and the pressure on the sheet-piles are such that an excessively thick pile is required, the provision of a tie at the level of the capping beam reduces the maximum bending moment. The tie can be constructed in reinforced concrete or it can be a mild steel bar anchored into the capping beam and wrapped with bituminised hessian to protect it from corrosion. The capping beam must be designed to span between the ties and to transfer the horizontal forces from the top of the sheet-piles to the ties. The end of the tie remote from the wall should be anchored behind the natural slope of the ground, behind one of the lines shown in the upper part of Table 102. The anchorage should be provided by a block of mass concrete, by a concrete wall, by a vertical concrete plate, or by an anchor pile. Although the force in the tie is increased, bending moments on the sheet-piles can be further reduced by placing the tie at some point below the top of the wall, a horizontal beam being provided at the level of the tie. The provision of a tie reduces the depth to which it is necessary to drive the sheet-piles.

Cantilevered Sheet-pile Wall.—The forces on a simple cantilevered sheet-pile wall are indicated in Table 101, where \( P_1 \) is the active pressure due to the filling and surcharge behind the wall and \( P_2 \) and \( P_3 \) are passive pressures producing the necessary restraint moment to resist the overturning effect of \( P_1 \). The shaded diagram illustrates the probable variation of pressure, but the accompanying straight-line diagram is a practical approximation. The sheet-piles tend to rotate about the point \( X \). The maximum bending moment on the sheet-piles occurs at some point \( D \), and the distance \( L \) can be calculated approximately from the factors \( k_1' \) given in the column headed "free" in the tabulation in Table 100 for different angles of repose of the ground in which the pile is embedded. The bending moment on the
sheet-piles is $P_3x$, the value of $P_3$ being conveniently represented by the area of the trapezium ABCD in Table 101; $P_3$ can be determined from Table 11. The distance $x$ indicates the centroid of the area. The embedded length of the sheet-piles must be great enough to enable sufficient passive pressures to be produced, and the factors $k_3'$ (Table 100) enable this length to be calculated approximately.

The foregoing procedure using the factors $k_3'$ and $k_4'$ given in Table 100 is suitable for the preparation of a preliminary design for a simple cantilevered sheet-pile wall. The final design should be checked by the formulae and procedure given in Table 101; the initial formula is derived by equating the forces acting behind the sheet-piles with those acting on the front of the sheet-piles, and by equating to zero the moments of these forces about $E$.

The value of $p_a$, the increase in active pressure per each foot of depth behind the wall, may be different at different depths if various classes of soil are encountered behind the wall and may be affected by waterlogged conditions. No general formula is serviceable under such conditions, and the designer should deal with such problems with caution and adopt safe values for the pressure factors. The two conditions which must be satisfied are that the algebraic sum of the horizontal forces must be zero and that the algebraic sum of the moments of these forces about the bottom of the sheet-piles must also be zero. The available theoretical passive resistance should be in excess of that required by a sufficient margin to allow for over-estimating the passive resistance.

**Sheet-pile Wall with Ties.**—When a tie is provided at the top of the wall the forces acting on the wall are as shown in Table 102; they are similar to those in Table 101 except for the introduction of the horizontal force in the tie. It is not possible to determine the variations of the pressure with any precision, but the diagram shows the probable variation. It is therefore recommended that the following procedure be adopted for preliminary designs. The factor $k_3'$ in the column headed "hinged" in Table 100 gives the minimum value for $h$ to produce sufficient restraint moment. The embedded length $h$ must, however, be not less than the minimum length required to resist forward movement of the toe and not less than the length required to prevent spewing. The wall will be stable if $1.5P_4x' > P_3y'$, where $P_4$ is the total active pressure on the whole depth of the wall as shown and $P_3$ is the total passive pressure in front of the wall. The values of $P_4$ and $P_3$ can be computed from the data in Tables 10 and 11. The factor 1.5 is introduced in order to allow a margin between the theoretically calculated passive resistance and that actually required.

To prevent spewing in front of the wall the embedded length should not be less than $k_3'p_w$, where $p_w$ is the weight per square foot at point $E$ due to the earth and surcharge above this point, $w$ is the weight per cubic foot of the earth in front of wall, and $k_3'$ is the pressure factor taken from Table 10 or 11.

The bending moment on the wall can be calculated by first determining $L$ from the factors given in the column headed "hinged" in Table 100. The sheet-piles can be considered as a propped cantilever of span $L$ built in at $D$ and propped at $A$ and subjected to a trapezoidal load represented by the area ABCD. This load can be divided into a uniformly-distributed load and a triangularly-distributed load, the bending moment coefficients for which are given in Table 16, where is also given the reaction of the prop, which is the force in the tie. Since the security of this type of wall depends on the efficiency of the anchorage, no risk of underrating the force in
the tie should be incurred; it is better to increase the force to be resisted from the value represented by the theoretical reactions to $0.5P_1$. The value of this force should certainly not be less than $P_a - \frac{3}{8}P_6$, which is the part of the outward active pressure not balanced by the reduced passive pressure. The forces in anchor-piles are given in the lower part of Table 102.

Sheet-pile Wall with Tie below Head.—In a wall as in Table 102 it is assumed that the connection between the tie and the head of the wall is equivalent to a hinge, that is, the bending moment at A is zero. If the wall is extended above A, as shown, either by continuing the sheet-piles or by constructing a cast-in-situ wall, a bending moment is introduced at A equal to $P_6\tau$, where $P_6$ is the total active pressure on the extended portion of the wall AF. This bending moment introduces a negative bending moment on the wall at A, but reduces the positive bending moment on the sheet-piles between D and A and also reduces the negative bending moment at D. If the bending moment at A is large enough to produce conditions amounting to complete fixity at A, then the span $L$ can be calculated by the factors $k_1'$ given in the column headed "fixed" in Table 100. The factor $k_2'$ in the same column gives the minimum embedded length $h$, but at the same time $h$ must be sufficient to prevent spewing and forward movement as already described. The equation for stability is given in Table 102. The force in the tie is $P_6 - \frac{3}{8}P_5 + P_6$ or $0.5P_6 + P_6$ whichever is the greater. The bending moment on the wall is calculated from the pressure represented by the area of the trapezium ABCD, considering the beam as fixed at both ends and using the appropriate coefficients given in Table 17. When the bending moment at A is insufficient to provide complete fixity, the bending moments, forces, and values of $L$ and $h$ are intermediate between those for hinged and fixed conditions at A.

A horizontal slab supported on king piles, as sometimes provided at A as shown in the diagram at the bottom of Table 102, has a sheltering effect on the piles since if the slab is carried far enough back it can completely relieve the wall below A from any active pressure due to the earth or surcharge above the level of A.

When a preliminary design has been prepared by the foregoing procedure, using the factors given in Table 100 and the formulae in Table 102, the final design of the sheet-pile wall with anchored ties should be checked by one of the analytical or graphical methods given in text-books on this subject.

Reduced Bending Moments on Flexible Walls.—The pressures behind a flexible retaining wall adjust themselves in such a way that the bending moments on the wall are reduced. Stroyer suggested a formula applicable to reinforced concrete sheet-pile walls with ties. The reduction factors, which are not applicable to simple cantilevered walls, are given in Table 100.

Foundations.

The design of the foundations for a structure comprises three stages, the first of which is to determine from an inspection of the site the nature of the ground and, having selected the stratum upon which to impose the load, to decide the safe bearing pressure. The second stage is to select the type of foundation, and the suitability of one or more types may have to be compared. The third stage is to design the selected foundation to transfer and distribute the loads from the structure to the ground. Reference should be made to Code No. 4, "Foundations," issued by the Institution of Civil Engineers.
Inspection of the Site.—The object of an inspection of the site is to determine the nature of the top stratum and of the strata below in order to detect any weak strata that may impair the load-carrying capacity of the stratum selected for the foundation. Generally the depth to which knowledge of the strata should be obtained should be not less than one and a half times the width of an isolated foundation or the width of a structure with closely-spaced footings.

The nature of the ground can be determined by digging trial holes, sinking bores, or driving piles. A trial hole can be taken down to only a moderate depth, but enables the undisturbed soil to be examined, and the difficulty or otherwise of excavating and the need or otherwise of timbering and pumping to be determined. A bore can be taken very much deeper than a trial hole. A test pile does not indicate the kind of soil it has been driven through, but the driving data combined with local information may give the necessary particulars. A test pile is useful in showing the thickness of the top crust or the depth below poorer soil at which a firm stratum lies. A sufficient number of any of these tests should be made until the engineer is satisfied that he is certain of the nature of the ground under all parts of the foundations.

Reference should be made to British Standard Code of Practice No. 2001

Safe Bearing Pressures on Ground.—The pressures that can be safely imposed on thick strata of soils commonly met with are in some districts the subject of by-laws. Table 104 gives some pressures which are often recommended as a guide, but these pressures must be considered as maxima since there are several factors that may necessitate the use of lower values. Permissible pressures may generally be exceeded by an amount equal to the weight of earth between the foundation level and adjacent ground level but, if this increase is allowed, any earth carried on the foundation must be included in the foundation load. For a soil of uncertain resistance a study of local existing buildings on the same soil may be useful, as may also be the results of a ground bearing test.

Failure of a foundation may be due to consolidation of the ground causing settlement, or rupture of the ground due to failure in resistance to shearing. The shape of the surface along which shearing failure occurs under a strip footing is an almost circular arc extending from one edge of the footing and passing under the footing and continued then as a tangent to the arc to intersect the ground surface at an angle depending on the angle of internal friction of the soil. The average safe resistance of soil therefore depends on the angle of internal friction of the soil, and on the depth of the footing below the ground surface. In a cohesionless soil the resistance to bearing pressure not only increases as the depth increases but is proportional to the width of the footing. In a cohesive soil there is also an increase in resistance to bearing pressure under wide footings, but it is less than in non-cohesive soils. Graphical solutions, such as that attributed to Krey, are sometimes used to find the bearing resistance under a footing of known width and depth. The theoretical formulae, based on Rankine's formula for a cohesionless soil and Mr. Bell's formula for clay, for the maximum bearing pressure on a foundation at a given depth, although giving irrational results in extreme cases, for practical cases give results that are well on the safe side. These formulae are given in Table 104.

Unless they bear on rock, foundations for all but single-story buildings or other light structures should be taken down at least 3 ft. below the ground surface since, apart from the foregoing considerations, it is seldom that undisturbed soil which is sufficiently consolidated is reached at a less depth. In a clay soil a depth of
about 5 ft. is necessary in Britain to ensure protection of the bearing stratum from weathering.

**Eccentric Load.**—When a foundation is subjected to a concentric load, that is when the centre of gravity of the superimposed load coincides with the centroid of the foundation, the bearing pressure on the ground is for practical purposes uniform and its intensity is equal to the total applied load divided by the total area. When the load is eccentrically placed on the base the pressure is not uniformly spread, but varies from a maximum at the side nearer the centre of gravity of the load to a minimum at the opposite side, or to zero at some intermediate point. The variation of pressure between these two extremes depends on the magnitude of the eccentricity and is assumed to be linear. The maximum and minimum pressures are given by the formulæ in *Table 104*. For large eccentricities there may be a part of the foundation under which there is no bearing pressure. Although this state may be satisfactory for transient conditions (such as those due to wind), it is preferable for the foundation to be designed so that there is bearing pressure throughout under ordinary working conditions.

**Blinding Layer.**—For reinforced concrete footings or other construction where there is no mass concrete at the bottom forming an integral part of the foundation, the bottom of the excavation should be covered with a layer of lean concrete in order to provide a clean surface on which to place the reinforcement. The thickness of this layer depends upon the compactness and wetness of the bottom of the excavation, and is generally from 1 in. to 3 in. The safe compressive stress in the concrete should be not less than the maximum bearing pressure on the ground.

**Types of Foundations.**—The most suitable type of foundation depends, primarily, on the depth at which the bearing stratum lies and the safe bearing pressure which determines the area of the foundation. Data relating to common types of separate and combined reinforced concrete foundations, suitable for sites where the bearing stratum is near the surface, are given in *Tables 104 and 105*. Some types of combined bases are also given in *Table 103*. In selecting a type of foundation suitable for a particular purpose the type of structure should be considered. Sometimes it may have to be decided whether the risk of settlement can be taken in preference to providing a more expensive foundation. In the case of silos and fixed-end arches, risk of unequal settlement of the foundations must be avoided, but for gantries and bases for large steel tanks a simple foundation can be provided and probable settlement allowed for in the design of the superstructure. In mining districts, where subsidence is reasonably anticipated, a rigid raft foundation should be provided for small structures in order that the structure may move as a whole; as a raft may not be economical for a large structure, the latter should be designed as a flexible structure or as a series of separate structures each of which, on independent raft foundations, can accommodate itself to movements of the ground without detriment to the structure as a whole.

**Separate Bases.**—The simplest form of foundation for a reinforced concrete column or steel stanchion is the common pyramidal base (*Table 104*). Such bases are suitable for concentric or slightly eccentric loads if the area exceeds about 10 sq. ft. For smaller bases, and for some bases on rock or other ground of high bearing capacity, a rectangular block of plain concrete is probably more economical; the thickness of the block must be sufficient to enable the load to be transferred to the ground under the entire area of the base at an angle of dispersion through the block of not less than 45 deg. to the horizontal.
To reduce the risk of unequal settlement, the sizes of separate bases for the columns of a building founded on a compressible soil should be in proportion to the dead load carried by each column. Bases for the columns of a storage structure should be in proportion to the total load, excluding the effects of wind. In all cases the pressure on the ground under any base due to the combined dead and live load, including wind load and any bending at the base of the column, must not exceed the safe bearing resistance of the ground.

In the design of a separate base the area of a concentrically-loaded base (as in Table 104) is determined by dividing the maximum load on the ground by the safe bearing resistance. The thickness of a reinforced concrete footing of the common pyramidal shape is determined from consideration of resistance to shearing force and bending. The critical shearing stresses are assumed to occur on a plane at a distance equal to the effective depth of the base from the face of the column. This is in accordance with the recommendations of the B.S. Code, as is the requirement that the maximum bending moment at any section shall be the sum of the moments of all the forces on one side of the section. The critical section for the bending moment on a base supporting a reinforced concrete column is at the face of the column, but for a base supporting a steel stanchion it is at the centre of the base. The appropriate formulae are given in Table 104. The moment of resistance of pyramidal bases cannot be determined with precision; the formulae are rational, but conservative.

If the size of the base relative to its thickness is such that the load from the column can be spread by dispersion at 45 deg. over the entire area of the base, no bending moment need be considered and only nominal reinforcement need be provided. If the base cannot be placed centrally under the column, the pressure on the ground is not uniform, but varies as shown in Table 104. The base is then preferably rectangular in plan and the modified formulae for bending resistance are given in Table 104. A special case of an independent base with the equivalent of eccentric loading is a chimney foundation.

A separate base may be subjected to moments and horizontal shearing forces in addition to a vertical concentric load. Such a base should be made equivalent to a concentrically-loaded base by placing the base eccentrically under the column to such an extent that the eccentricity of vertical load offsets the equivalent of the moments and shearing forces. This procedure is impractical if the moments and shearing forces can act either clockwise or anti-clockwise at different times, in which case the base should be provided centrally under the column and designed as an eccentrically-loaded base complying with the two conditions.

**Tied Bases.**—Sometimes, as in the case of the bases under the towers of a trestle or gantry, the bases are in pairs and the moments and shearing forces act in the same sense on each base at the same time. In such conditions the bases can be designed as concentrically-loaded and connected by a tie-beam which relieves them of effects due to eccentricity. Such a pair of tied bases is shown in Table 105, which also gives the formulae for the bending moment and other effects on the tie-beam.

**Balanced Foundations.**—When it is not possible to place an adequate base centrally under a column or other load owing to restrictions of the site, and when under such conditions the eccentricity would result in inadmissible ground pressures, a balanced foundation as shown in Tables 103 and 105 is provided. This case is common in the external columns of buildings on sites in built-up areas.

**Combined Bases.**—If the size of the bases required for adjacent columns is
so large that independent bases would overlap, two or more columns can be provided with a common foundation. Suitable types for two columns are shown in Table 105 for concentrically-loaded bases and for a base that cannot be arranged relative to the columns so as to be concentrically loaded. It may be that, under some conditions of loading on the columns, the load on the combined base may be concentric, but, under other conditions, the load on the same base may be eccentric; alternative conditions must be taken into account. Some notes on combined bases are given on the page facing Table 103.

**Strip Bases.**—When the columns or other supports of a structure are closely spaced in one direction, it is common to provide a continuous base similar to a footing for a wall. Particulars of the design of strip bases are given in Table 105. Some notes on these bases are given, in relation to the diagrams in Table 103, on the page facing the latter table, and an example is given on the page facing Table 105.

**Rafts.**—When the columns or other supports of a structure are closely spaced in both directions, or when the column loads are so high, and the safe ground pressure so low, that a group of independent bases almost or totally covers the space between the columns, a single raft foundation of one of the types shown in (i) to (iv) in Table 103 should be provided. Notes on these designs are given on the page facing the table.

The analysis of a raft foundation supporting a series of symmetrically arranged equal loads is generally based on the assumption of uniformly-distributed pressure on the ground, and the design is similar to an inverted reinforced concrete floor upon which the imposed load is the ground pressure due to the superimposed loads only. Notes on the design of a raft when the columns are not symmetrically disposed are given on the page facing Table 103.

**Basements.**—A basement, a typical cross-section of which is shown at (v) in the lower part of Table 103, is partly a raft, since the weights of the ground floor over the basement, the walls, and other structure above the ground floor, and the weight of the basement itself, are carried on the ground under the floor of the basement. For watertightness it is common to construct the wall and floor of the basement monolithically. In most cases the average ground pressure is low, but owing to the large span the bending moments are high and consequently a thick floor is required if the total load is assumed to be distributed uniformly over the whole area. Since the greater part of the load is transmitted through the walls of the basement, it is more economical to consider the load to be spread on a strip immediately under the walls if by so doing the ground pressure does not exceed the maximum allowable. The bending moment at the edge of the wall due to the cantilever action of this strip determines the thickness of the strip, and the remainder of the floor can generally be less thick.

Where basements are in water-bearing soils the effect of water pressure must be taken into account. The upward water pressure is uniform below the whole area of the basement floor, which must be capable of resisting the pressure less the weight of the slab. The walls must be designed to resist the horizontal pressure of the water-logged ground. It is necessary to prevent the basement from floating. There are two critical stages. When the structure is complete the total weight of the basement and all superimposed dead loading must exceed the maximum upward pressure of the water by a substantial margin. When the basement only is complete, there must also be an excess of downward load. If these conditions are not present, one
of the following steps should be taken. (1) The level of the ground-water near the basement should be controlled by pumping or other measures; (2) Temporary vents should be formed in the floor or at the base of the walls to enable water freely to enter the basement, thereby equalising the external and internal pressures; the vents should be sealed when sufficient dead load from the superstructure is obtained; (3) The basement should be temporarily flooded to a depth such that the weight of water in the basement, together with the dead load, exceed the total upward force on the structure. During the construction of the basement, method (1) is generally the most convenient, but when the basement is complete method (3) is preferable on account of its simplicity. The designer should specify the depth of water required, and a suitable rule for ascertaining this depth in a large basement is to assume 1 ft. for every foot head of ground water less 1 ft. for every 5 in. thickness of concrete in the basement floor above the waterproof layer. The omission of the weights of the basement walls and the ground floor provides a margin of safety.

Foundation Piers.—When a satisfactory stratum is found at 5 ft. to 15 ft. below the natural ground level a suitable foundation can be made by building up piers from the low-level to ground level, and commencing the construction of the columns or other supports on these piers at ground level. The piers are generally square in cross-section, and can be constructed in brick, masonry, or plain or reinforced concrete. The maximum bearing pressure of the construction on the top of the pier depends on the material of the pier. Safe pressures on plain concrete, brickwork, and masonry, in accordance with B.S. Code 111, are given in Table 104; safe pressures on plain concrete in accordance with the London By-laws are given in Table 56.

The economical size of the pier is when the load it carries is great enough to require a base to the pier equal in area to the smallest hole in which men can conveniently work; otherwise unnecessary excavation has to be taken out and refilled. For example, if a man can conveniently work in a hole one yard square at a depth of 12 to 15 ft., the total load would be 36 tons on a stratum capable of sustaining 4 tons per square foot. Generally it is better to provide as few piers as possible and to impose as much of the load as practicable on to each pier, thus making each pier of generous proportions. It may not be necessary to dig a hole larger than is required for the stem of the pier, as the ground at the bottom may be firm enough to be undercut for a widening at the base. In the case of concrete piers this procedure reduces the amount of shuttering.

Reinforced concrete columns can sometimes be economically taken down to moderate depths, but to avoid slender columns it is generally necessary to provide lateral support at ground level.

When piers are impracticable, either by reason of the depth at which a firm bearing stratum occurs or due to the nature of the ground requiring timbering or continuous pumping, piles are adopted.

Wall Footings.—When the load on a strip footing is uniformly distributed throughout its length, as in the general case of a wall footing, the principal bending moments are due to the transverse cantilever action of the projecting portion of the footing. If the wall is of concrete and is built monolithically with the footing, the transverse bending moment at the face of the wall is the critical bending moment. If the wall is of brick or masonry the maximum bending moment occurs under the centre of the wall. Expressions for these bending moments are given in Table 105. When the projection is less than the thickness of the base the transverse bending
moments can be neglected, but in all cases the thickness of the footing should be such that the safe shearing stress is not exceeded.

Whether wall footings are designed for transverse bending or not, if the safe ground pressure is low, longitudinal reinforcement should be inserted to resist possible longitudinal bending moments due to unequal settlement and non-uniformity of the load. One method of providing the amount of longitudinal reinforcement required for unequal settlement is to design the footing to span over a cavity (or area of soft ground) 3 ft. to 5 ft. wide, according to the nature of the ground. The longitudinal bending moment due to non-uniform load is calculated in the same way as for combined footings.

**Foundations for Machines.**—The area of a concrete base for a machine or engine must be sufficient to spread the load on to the ground without exceeding the safe bearing pressure. It is an advantage if the shape of the base is such that the centroid of the bearing area coincides with the centre of gravity of the loads when the machine is working. This reduces the risk of unequal settlement. If vibration from the machine is transmitted to the ground the bearing pressure should be considerably lower than that generally assumed for the class of ground upon which the base bears, especially if the ground is clay or contains a large proportion of clay. It is often essential that the vibration of the machine shall not be transmitted to adjacent structures either directly or through the ground. In such cases a layer of cork or similar insulating material is placed between the concrete base carrying the machine and the ground. Sometimes the base is enclosed in a pit lined with insulating material. When transmission of vibration is particularly undesirable the base may stand on springs, or more elaborate damping devices may be installed. In all cases, however, the base should be separated from surrounding concrete ground floors.

With light machines the bearing pressure on the ground may not be the factor that decides the area of the concrete base, since the area occupied by the machine and its frame may require a base of larger area. The position of the holding-down bolts generally determines the width and length of the base, which should extend 6 in. or more beyond the outer edges of holes left for the bolts.

The depth of the base must be such that the bottom is on a satisfactory bearing stratum and that there is sufficient thickness to accommodate the holding-down bolts. If the machine exerts an uplift on any part of the base, the dimensions of the base must be such that the part subjected to uplift has sufficient weight to resist the uplift with a suitable margin of safety. A single base should be provided under all the supports of one machine, and sudden changes in depth and width of the base should be avoided. This reduces risk of fractures that might result in unequal settlements that might throw the machine out of alignment. If the load from the machine is irregularly distributed on the base, the dimensions of a plain concrete base should be sufficient to resist the bending moments produced therein without overstressing the concrete in tension. If there is any risk of so overstressing the concrete, or if the operation of the machine would be adversely affected by cracking and deformation of the base, reinforcement should be provided to resist all tensile forces.

**Reinforced Concrete Piles.**

**Precast Piles.**—Reinforced concrete piles are either precast or cast insitu. Precast piles have been driven in lengths exceeding 100 ft., although when more than 60 ft. long it is necessary to give special consideration to the design of the pile and
of the lifting and driving plant. Piles less than 15 ft. long are seldom economical. For ordinary work precast piles are generally square or octagonal and are from 8 in. to 18 in. wide. For their support, piles depend either on direct bearing on a firm stratum or on frictional resistance in soft strata, or more often on a combination of both resistances. The safe load on a pile depends on the load that the pile can safely carry as a column and on the load that produces settlement or further penetration of the pile into the ground. So many factors affect the load causing settlement for any particular pile that calculated loads are not very reliable unless associated with loading tests on driven piles. Such tests are often inconvenient and expensive, and frequently an engineer has to rely on computed loads and a large factor of safety.

Formulae for calculating the safe load on a pile are either impact formulae which are applicable to bearing piles, or friction formulae which are applicable to piles that are supported by the frictional resistance of the ground in which they are embedded. The formula of Mr. Hiley includes most of the variants occurring in pile driving such as the weight and the type of the hammer, the fall of the hammer, the penetration per blow, the length of the pile, the type of helmet, the nature of the ground, and the material of which the pile is made. A modified form of this formula is given in Table 106, in which the constant $c$ takes into account the energy absorbed in temporarily compressing the pile, the helmet, and the ground. Since the quake of the ground below the pile shoe is included, it follows that the nature of the ground in which the toe of the pile is embedded affects the value of $c$, and the tabulated values apply to firm gravel; $c$ must be increased if the pile is driven by a long dolly. The dimension $2c$ is a quantity that is measurable on a pile while being driven, since it represents the difference between the permanent penetration for one blow and the greatest instantaneous depression of the pile head as measured at the top of the helmet. The efficiency of the blow depends on the ratio of the weight of the pile (including the weight of the helmet, dolly, cushioning, and the stationary parts of the hammer resting on the pile head) to the weight of the moving parts of the hammer. Values for the efficiency of the blow are given in Table 106, together with values of the effective fall which allow for the freedom or otherwise of the fall of the hammer. The resistance to driving as calculated by Mr. Hiley’s formula is subject to a factor of safety of 1$\frac{1}{4}$ to 3. If a pile is driven into clay or soils in which clay predominates, or into fine saturated sand, the resistance to further penetration may increase after the pile has been at rest for a while. This increase is due to the frictional resistance of the soil settling around the pile, but may be in part offset by a reduction in the bearing resistance, if on clay, which takes place in the course of time. Impact formulae are not therefore very reliable for piles driven into clay, nor for piles that are driven into sand with the assistance of a water-jet.

When piles are driven into soft ground and depend solely upon friction between the sides of the pile and the ground for their support, the safe load can only be estimated approximately by considering the probable frictional resistance offered by the strata through which the pile is driven and the probable bearing resistance of the ground under the toe of the pile. Formulae may be of little assistance in this case; a test load on an isolated pile or on a group of piles is the most satisfactory means of determining the settlement load.

A formula by which an estimate of the safe load on a pile driven entirely into clay can be derived is given in Table 106. An alternative formula is
Safe load (lb.) = \( \frac{1}{F} \left[ \frac{A_5C}{2} + A(7.5C + w_2L_1) \right] \)

The notation is as follows.  \( C \) is the cohesive strength (lb. per sq. ft.) and \( w_2 \) is the density (lb. per cu. ft.) of the clay; \( A \) is the cross-sectional area (sq. ft.) of the pile, the embedded length of which is \( L \) (ft.); \( F \) is a factor of safety (see Table 106).

Precast piles should be designed to withstand the stresses due to lifting, driving, and loading, with appropriate factors of safety. Overstressing the concrete during handling and slinging can be guarded against by arranging the position and number of the points of suspension so that the stresses due to bending moments produced by the weight of the pile are within safe limits. For square piles of \( 1:1\frac{1}{4}:3\frac{1}{2} \) concrete and having the ordinary percentage of longitudinal steel the maximum bending moment due to bending about an axis parallel to one side of the section should not exceed, say, \( 60D^3 \) in.-lb., where \( D \) is the length of the side of the pile in inches, if cracking is to be avoided.

The moment of resistance of a square pile bending about a diagonal is only about two-thirds of that when bending about an axis parallel to one of the sides. For this reason bending about a diagonal should be avoided where possible. If lifting holes are provided there is some assurance that the pile will not be lifted so as to bend about a diagonal. The lifting holes or the points of suspension should be arranged so that the smallest bending moments are experienced during lifting, and the positions for this condition for lifting at one or at two points are given in Table 106.

The greatest compressive stress in a pile is generally that due to the driving and occurs near the head. If driving is severe helical binders should be provided at the top of the pile. Octagonal piles generally have helical binders throughout their length.

In Table 106 is shown the reinforcement in a square precast reinforced concrete pile, in which helical binders at the head of the pile are provided. The arrangement of the lifting holes and spacers is also indicated. For driving into clay, gravel or sand, a pile shoe having an overall taper of about 2 to 1, as shown, is generally satisfactory, but for other types of soil other shapes of shoe are necessary. If the pile has to be driven through soft material to bear on gravel overlying softer ground it is necessary to have a blunter shoe to prevent punching through the thin stratum. For friction piles driven into soft material throughout a shoe is not absolutely necessary, and a blunter end should be formed as shown in Table 106. When driving through soft material to a bearing on soft rock or stiff clay, the form of pile end shown for this case is satisfactory so long as driving ceases as soon as the firm stratum is reached or is only just penetrated. When driving down to hard rock, or where heavy boulders are anticipated, a special shoe or point as shown should be fitted.

Irrespective of the load a pile can carry before settlement occurs, the stresses produced by the load on the pile acting as a column should be considered. For calculation of the reduction of load due to slenderness (see Table 83) the effective length of the pile can be considered as two-thirds of the length embedded in soft soil, or one-third of the length embedded in a fairly firm ground, plus the length of pile projecting above the ground. The end conditions of a pile are generally equivalent to one end fixed and one end hinged.

**Arrangement of Piles.**—In preparing plans of piled foundations attention must be given to the practicability of driving as well as to effectiveness for carrying
loads. In order that each pile in a group shall carry an equal share of the load the centre of gravity of the group should coincide with the centre of the superimposed load. The clear distance between any two piles should generally be not less than 2 ft. 6 in. So far as possible piles should be arranged in straight lines in both directions throughout any one part of a foundation, as this form reduces the amount of movement of the driving frame. The arrangement should also allow for driving to proceed in such a way that any displacement of earth due to consolidation in the piled area shall be free to take place in a direction away from the piles already driven.

**Pile-caps.**—Pile-caps should be designed primarily for punching shear around the heads of the piles and around the column base and for bending moment due to transferring the load from the column to the piles. The thickness of the cap should also be sufficient to provide adequate bond length for the bars projecting from the pile and for the dowel bars for the column. If the thickness is such that the column load can all be transmitted to the piles by dispersion no bending moments need be considered, but generally when two or more piles are placed under one column it is necessary to reinforce the pile-cap for the bending moments produced.

**Loads on Piles in a Group.**—If the centre of gravity of the total load \( W_T \) on a group of \( N \) vertical piles is at the centre of gravity of the piles, each pile will be equally loaded, and will be subjected to a load \( \frac{W_T}{N} \). If the centre of gravity of the load is displaced a distance \( e \) from the centre of gravity of the group of piles, the load on any one pile is \( W_T \left( \frac{1}{N} \pm \frac{e \Sigma d_i}{\Sigma d_i^2} \right) \), where \( \Sigma d^2 \) is the sum of the squares of the distance of each pile measured from an axis passing through the centre of gravity of the group of piles and at right angles to the line joining this centre of gravity to the centre of gravity of the applied load; \( d_i \) is the distance of the pile considered from this axis, and is positive if it is on the same side of the axis as the centre of gravity of the load and negative if it is on the opposite side.

If the structure supported on the group of piles is subject to a bending moment \( M \), which is transmitted to the foundations, the expression given for the load on any pile can be used by substituting \( e = M/W_T \).

The total load that can be carried on a group of piles is not necessarily the safe load calculated for one pile multiplied by the number of piles, as allowance must be made for the overlapping of the zones of stress in the soil supporting the piles. The reduction due to this effect is greater in a group of piles that are supported mainly by friction. For piles supported entirely or almost entirely by bearing the maximum safe load on a group cannot greatly exceed the safe load on the area of the bearing stratum covered by the group.

**Cast-insitu Piles.**—The following advantages are obtained with cast-insitu piles, although all are not applicable to any one system.

The length of each pile conforms to the depth to the bearing stratum and no pile is too long or too short; cutting-off surplus lengths or lengthening insitu is not therefore required. The top of a pile can be at any level below ground, and in some systems at any level above ground. The formation of an enlarged foot giving a greater bearing area is possible with some types of piles. With tube-driven or mandrel-driven piles it is possible to punch through a thin intermediate hard stratum. Boring shows the class of soil through which the pile passes and the nature of the bearing stratum can be observed. A bored pile has little frictional resistance, but greater frictional resistance in soils such as compact gravels is obtained in tube-driven types.
where the tube is withdrawn. Bored piles have no ill-effect on adjacent piles or on
the level of the ground due to consolidation of ground when several piles are driven
in a constricted area. Boring piles is less noisy and is vibrationless; only a small
headroom is required.

Some of the advantages of a precast pile over a cast-insitu pile are that hardening
of the concrete is unaffected by deleterious ground waters; that the pile can be
inspected before being driven into the ground; that the size of the pile is not affected
by water in the ground (this applies also to cast-insitu types with a central core);
and that the pile can be driven into ground that is below water. In neither the
precast pile nor the cast-insitu pile are damage to, or faults in, the pile visible after
it is driven or formed. The designer must consider the conditions of any problem,
and select the pile which complies with the requirements.

Groups of Inclined and Vertical Piles.—Table 107 and the examples on the
pages facing Tables 106, 107 and 108, relate to the loads on piles in a group that project
above the ground, as in a wharf or jetty. For each probable condition of load the
external forces are resolved into horizontal and vertical components \( H \) and \( W \), the
points of application of which are also determined. If the direction of action and
position are opposite to those shown in the diagrams, the signs in the formul\ae\ must
be changed. It is assumed that the piles are surmounted by a rigid pile-cap or super-
structure. The effects on each pile when all the piles are vertical are based on a
simple, but approximate, statical analysis. Since a pile has little resistance to
bending, structures with vertical piles only are not suitable when \( H \) predominates.
The resistance of an inclined pile to horizontal force is considerable. In groups con-
taining inclined piles, the bending moments and shearing forces on the piles are
negligible. The ordinary theoretical analysis, upon which Table 107 is based, assumes
that each pile is hinged at the head and toe. Although this is not an accurate
assumption, the theories based thereon accord fairly well with the behaviour of actual
groups of piles.

Wharves and Jetties.

The loads, pulls, blows, and pressures to which wharves and jetties and similar
waterside and marine structures may be subjected are dealt with on page 144. Such
structures may be a solid wall of plain or reinforced concrete, as are most dock walls
and some quays, in which case the pressures and principles described on pages 19 et seq.
and in Tables 10 to 13 and 100 for retaining walls apply. A quay or similar waterside
wall is more often a sheet-pile wall, which is dealt with in Tables 101 and 102, or it
may be an open-piled structure similar to a jetty. Piled jetties and the piles for such
structures are considered in Tables 106 and 107. If the piles in a group containing
inclined piles are arranged symmetrically, the summations in Table 107 are simplified
thus: \( \Sigma_h = 0; \ G = \Sigma_t \Sigma_b; \ y_0 = - (\Sigma_t \div \Sigma_b); \ \Sigma_3 \) is not required since \( x_0 = 0.5x_n \).
Three designs of the same typical jetty are given on the pages facing Tables 106, 107
and 108.
SECTION 6
SPECIFICATIONS AND QUANTITIES

Contract Documents.

Documents relating to a contract are the Form of Tender, Form of Agreement, Conditions of Contract, Specification, and in most cases Bills of Quantities, a Schedule of Prices, and Contract Drawings. The drawings are later amplified by working drawings and bar-bending schedules. Standard forms of tender and agreement should be used. Since the Conditions of Contract include clauses of a legal nature, it is advisable to use a standard form such as those of the Royal Institute of British Architects for building contracts under the supervision of an architect, of the Institution of Structural Engineers for a contract of purely structural nature, or of the Institution of Civil Engineers for structural work combined with civil engineering. The Specification is generally divided into one part dealing with materials and another with workmanship and manufactured goods. A specification suitable for ordinary reinforced concrete construction follows. It should not be used without making sure that all the requirements apply. Generally some clauses or parts of clauses will have to be omitted or altered and other requirements may have to be inserted.

As British Standards are often revised, the year of publication of the latest Standard should be inserted where required. The engineer should read each British Standard quoted to ensure that his decisions on all optional requirements are expressed in the Specification for the works.

A Specification for Reinforced Concrete Construction.

Materials.—Materials used in the Works shall be new, of the qualities and kinds specified herein and equal to approved samples. Delivery shall be made sufficiently in advance to enable further samples to be taken and tested if required. No materials shall be used until approved, and materials not approved shall be immediately removed from the Works at the Contractor's cost.

Materials shall be transported, handled, and stored on the site or elsewhere in such a manner as to prevent damage, deterioration, or contamination.

Reinforcement.—Mild steel bars shall be plain round hot-rolled steel bars complying with B.S. No. 785. Cold-drawn wire shall be plain round cold-drawn hard mild steel wire complying with B.S. No. 785. Twisted square and twin-twisted bars shall be rolled mild steel bars of suitable sizes which have been twisted when cold and shall comply with B.S. No. 1144. Welded wire fabric, twisted square-bar fabric, or expanded metal shall comply with the appropriate part of B.S. No. 1221. Twisted ribbed bars and other reinforcement for which there is no British Standard shall comply with the Engineer’s requirements.

Reinforcement shall be free from pitting, loose rust, mill scale, paint, oil, grease, adhering earth, ice, or any other material that may impair the bond between the concrete and the reinforcement or that may cause corrosion of the reinforcement or disintegration of the concrete. Adhering limewash or cement grout shall be permitted.

Neither the size nor length of a bar or wire shall be less than the size or length described in the bar schedule or elsewhere, and the length shall be not more than 2 in. in excess of the length so described.

Cement.—The cement shall be ordinary-setting cement of approved manufacture and shall comply with B.S. No. 12 for ordinary and rapid-hardening Portland cement, B.S. No. 146 for Portland-blastfurnace cement, B.S. No. 915 for high-alumina cement,
SPECIFICATIONS AND QUANTITIES

or B.S. No. 1370 for low-heat Portland cement. Compression tests on cement-sand cubes shall be made instead of tensile tests. Sulphate-resistant and other special cements shall comply with the Engineer’s requirements.

All cement shall be fresh when delivered. Cements of different types shall not be mixed one with another. Consignments shall be used in the order of delivery. Admixtures shall be used only if approved.

AGGREGATE.—Materials used as aggregates shall be obtained from a source known to produce aggregates satisfactory for concrete and shall be chemically inert, strong, hard, durable, of limited porosity, and free from adhering coatings, clay lumps, coal and coal residues, and organic or other impurities that may cause corrosion of the reinforcement or may impair the strength or durability of the concrete. Aggregates shall be tested in accordance with the requirements of B.S. No. 812 and the results of such tests shall be as hereinafter specified, the percentages being by weight unless the context indicates otherwise.

FINE AGGREGATE.—Fine aggregate shall be natural sand or sand derived by crushing gravel or stone and shall be free from coagulated lumps. Sand derived from stone unsuitable for coarse aggregate shall not be used as fine aggregate.

The caustic-soda test for organic impurities shall show a colour not deeper than that of the standard solution. The amount of fine particles as ascertained by the laboratory sedimentation test shall not exceed 10 per cent. for crushed stone and 3 per cent. for natural sand or crushed gravel. In the settling test, and after being allowed to settle for three hours, the thickness of the layer of silt deposited on the coarser material shall not exceed 6 per cent. by volume.

The grading of fine aggregate shall be such that not more than 10 per cent. shall exceed \( \frac{1}{8} \) in. in size. For a natural sand or crushed gravel not more than 10 per cent., and for crushed stone not more than 20 per cent., shall pass B.S. sieve No. 100. Between these limits the grading shall conform to the grading for either Zones 1, 2, or 3 (B.S. 882), or Zone 4 if so instructed.

COARSE AGGREGATE.—Coarse aggregate shall be crushed or uncrushed gravel or crushed stone. The pieces shall be angular or, except for concrete surfaces subject to abrasion, rounded in shape and shall have granular or crystalline or smooth (but not glassy), non-powdery surfaces. Friable, flaky, and laminated pieces, mica, and shale shall only be present in such quantities as not to affect adversely the strength and durability of the concrete.

The “aggregate crushing value” shall not exceed 45 per cent. and for concrete surfaces subject to abrasion shall not exceed 30 per cent. The amount of fine particles occurring in a free state or as a loose adherent shall not exceed 1 per cent. when determined by the laboratory sedimentation test. After twenty-four hours’ immersion in water, a previously-dried sample of the coarse aggregate shall not have gained in weight more than 10 per cent. or not more than 5 per cent. if for use in impermeable construction.

The grading of coarse aggregate shall be such that not more than 5 per cent. shall be larger than \( \frac{3}{4} \) in. and not more than 10 per cent. shall be smaller than \( \frac{1}{8} \) in. and not less than 25 per cent. or more than 55 per cent. shall be smaller than \( \frac{1}{8} \) in.

“ALL-IN” AGGREGATE.—All-in aggregate shall comply in all respects except grading with the requirements for fine and coarse aggregates, and for the purposes of all tests except the grading test shall be separated into material smaller than \( \frac{1}{8} \) in. and material \( \frac{1}{8} \) in. and over; these materials shall be considered as fine and coarse aggregates respectively.

The grading of all-in aggregate shall be such that not more than 5 per cent. shall exceed \( 1\frac{1}{4} \) in. and not more than 6 per cent. shall pass B.S. sieve No. 100, and not less than 45 per cent. or more than 75 per cent. shall be smaller than \( \frac{1}{4} \) in. and not less than 25 per cent. or more than 45 per cent. shall be smaller than \( \frac{1}{8} \) in.

WATER.—Water shall be clean and fresh from organic or inorganic matter in solution or suspension in such amounts that may impair the strength or durability of the concrete. Water shall be obtained from a public supply where possible, and shall be taken from any other source only if approved. No sea-water or water from excavations shall be used. Only water of approved quality shall be used for washing out shuttering, curing concrete, and similar purposes.
HOLLOW CLAY BLOCKS.—Hollow clay blocks for use in the construction of slabs shall be accurate in shape, free from twist, and with right-angles at all external corners. The external dimensions shall be as described on the drawings or elsewhere within a tolerance of 5 per cent. under or over. The thickness of material shall nowhere be less than \( \frac{1}{2} \) in. The blocks shall be sound, free from cracks and lime, well and evenly burnt, of uniform density, and with a crushing strength not less than 2,500 lb. per square inch. Tests for shape and strength shall be made in compliance with B.S. No. 1190 and the results of such tests shall be within the limits specified therein. The average weight of any twelve blocks shall not exceed the weight given on the drawings or elsewhere. All faces of blocks intended to receive plaster or rendering shall be scored or roughened in an approved manner to form a key.

FILLING MATERIAL.—Materials for filling shall be uniform in character throughout and free from substances that by decay or otherwise may cause the formation of hollows or cavities or otherwise affect the stability of the filling.

Earth filling shall be selected material obtained from the excavation or other approved source. No soft chalk or clay or earth with a predominating clay content shall be used. Hardcore shall be selected hard clean gravel, broken brick, broken concrete, broken or crushed stone, quarry waste, or similar approved material. Concrete for filling shall have the proportions described.

WORKMANSHIP.—The workmanship shall be of the quality specified, and all persons employed on the Works or elsewhere in connection with the Works shall be competent and skilled in their respective occupations.

EXCAVATION.—The site of the Works shall be cleared and excavated or filled to the levels given on the drawings or elsewhere. Excavations shall be of the width and length necessary for the construction of the foundations or other work below ground. The depths of all excavations shall be decided by the Engineer. The Contractor shall record on a suitable plan, which shall be deposited with the Engineer, the depth of every foundation as constructed.

Any obstacle encountered during the excavation shall be reported to the Engineer and shall be dealt with as then instructed.

Material taken out of the excavation shall be disposed of as instructed.

The Contractor shall supply and maintain and remove any necessary planking and strutting, sheet-piling, or cofferdams, and shall by pumping or other approved means maintain the excavations free from water.

Excavations shall be left open for as short a period as practicable. Immediately before foundations or other work be constructed therein, the sides of the excavation shall be trimmed if necessary, and the bottom shall be cleaned and if in loose or disturbed ground shall be well rammed, and the whole shall be approved.

DEPOSITING FILLING.—Natural hollows or pockets where soft ground has been taken out below the level of foundations or ground slabs or similar shall be filled with hardcore or 1:6 concrete or earth filling as instructed. Earth filling shall be in layers not exceeding 8 in. thick, each layer being well rammed to the same degree of consolidation as the surrounding ground.

Filling behind or at the sides of foundations or other work under ground shall consist entirely of earth surplus from the excavations and shall be deposited as the work proceeds in layers not exceeding 1 ft. thick, each layer being well rammed. Filling behind retaining walls or similar shall consist of approved selected earth deposited gently against the back of the wall in layers not exceeding 3 ft. in depth unless otherwise instructed. The earth shall be prevented from mixing with the rubble backing (if any) behind the wall, and the earth and rubble shall be brought up together.

PREPARATION OF GROUND BELOW PERMANENT CONSTRUCTION.—Plain concrete in foundations or site concrete shall be placed in direct contact with the bottom of the excavation, the concrete being deposited in such a manner as not to be mixed with the earth.

The bottom of excavations for reinforced concrete work shall be covered with a "blinding" layer of 1:6 concrete not less than 2 in. thick, finished to a smooth surface. The required cover of concrete under the reinforcement shall be entirely above the blinding layer.
SPECIFICATIONS AND QUANTITIES

BENDING REINFORCEMENT.—Reinforcement bars shall be bent by machine or other approved means producing a gradual and even motion. Bars shall be bent cold unless the Engineer shall approve bars of over 1 in. in size being bent hot. Bars bent hot shall not be heated beyond cherry-red colour and after bending shall be allowed to cool slowly without quenching. Bars dependent on cold-working for their strength shall always be bent cold.

Bars incorrectly bent shall be used only if the means used for straightening and rebending be such as shall not injure the material. No reinforcement shall be bent when in position in the Works without approval, whether or not it is partially embedded in hardened concrete.

Bends shall comply with the dimensions given in the bending schedule. Dimensions of bent bars and internal dimensions of binders and the like shall not exceed the dimensions given on the bar schedule or elsewhere. Unless described otherwise, bending dimensions shall conform to B.S. 1478.

The internal radii of bends shall be not less than twice the size of the bar unless described to the contrary. The internal radii of the bends at corners of binders or the like shall be half the size of the bar embraced by the binder. For the purpose of this requirement the size of a bar shall mean the diameter of a plain round bar or wire, or one-and-a-half times the nominal dimension of the side of a twisted square bar, or the nominal diameter of a deformed bar of any other type.

FIXING REINFORCEMENT.—Reinforcement shall be accurately fixed and by approved means maintained in the position described on the drawings or elsewhere. Bars intended to be in contact at passing points shall be securely wired together at all such points with No. 16 gauge annealed soft-iron tying wire. Binders and the like shall tightly embrace the bars with which they are intended to be in contact and shall be securely wired or if approved welded thereto.

Immediately before concreting, the reinforcement shall be examined for accuracy of placing and cleanliness and corrected if necessary.

Reinforcement projecting from work being concreted or already concreted shall not be bent out of its correct position for any reason unless approved and shall be protected from deformation or other damage.

The cover of concrete to the reinforcement shall be as described on the drawings and shall be provided and maintained within a tolerance ¼ in. under and over (except where specified as a minimum) by means of distance pieces of cement mortar or other approved material.

The vertical distances required between successive layers of bars in beams or similar members shall be maintained by the provision of mild steel spacer bars inserted at such intervals that the main bars do not perceptibly sag between adjacent spacer bars.

WELDING REINFORCEMENT.—If butt jointing of reinforcement bars by electric-arc welding be approved, the Engineer’s requirements or the regulations of the local or other authority shall be complied with and all operations connected therewith shall be done only by men skilled thereat. These requirements shall not apply to the spot-welding of binders or the like to main bars where approved or to electrically-welded fabric. No welding shall be done in connection with twisted bars or other reinforcement (excepting cold-drawn wire) the strength of which depends on cold-working.

SHUTTERING.—Shuttering for concrete shall be rigidly constructed of approved material and shall be true to the shape and dimensions described on the working drawings. Timber shall be well seasoned, free from loose knots and, except where otherwise approved, wrought on all faces. Faces in contact with concrete shall be free from adhering grout, projecting nails, splits, or other defects. Joints shall be sufficiently tight to prevent the leakage of cement grout and to avoid the formation of fins or other blemishes. Faulty joints shall be caulked. Where described on the working drawings or elsewhere the position and direction of the joints shall be as so described. Unless described on the working drawings or elsewhere to the contrary, 1-in. by 1-in. chamfers shall be formed on the external corners of concrete members. Openings for inspection of the inside of the shuttering and for the escape of water used for washing out shall be formed so that they can be conveniently closed before placing the concrete.
Connections shall be constructed to permit easy removal of the shuttering and shall be either nailed, screwed, bolted, clamped, wired, or otherwise secured so as to be strong enough to retain the correct shape during consolidation of the concrete. Bolt holes in concrete shall be made good after removal of the bolts. Wire ties passing through concrete shall be used only where approved, and the ends of the wires shall be concealed and measures taken to prevent rust stains on the concrete.

Shuttering shall be provided for the top faces of sloping work, and anchored to prevent flotation, where the slope exceeds 1 in 2½.

Shuttering shall be true to line and braced and strutted to prevent deformation under the weight and pressure of the wet concrete, constructional loads, wind, and other forces. The deflection shall not exceed ½ in. Bottoms of beam boxes shall be erected with an upward camber of ½ in. for each 10 ft. of span. If so instructed the designs for the shuttering shall be submitted for approval before construction.

The shuttering for beams and slabs shall be erected so that the shuttering on the sides of the beams and of the soffits of slabs can be removed without disturbing the beam bottoms. Re-propping of beams shall not be done except when, with the approval of the Engineer, props be reinstated in anticipation of loads in excess of the design load. Vertical props shall be supported on wedges, or other measures shall be taken whereby the props can be gently lowered vertically when commencing to remove the shuttering. Props for an upper story shall be placed directly over those in the story immediately below and the lowest prop shall bear upon work sufficiently strong to carry the load.

If the shuttering for a column is erected to the full height of the column, one side shall be left open and shall be built up in sections as placing the concrete proceeds.

Before placing the concrete, bolts and fixings shall be in position, and cores and other devices used for forming openings, holes, pockets, chases, recesses, and other cavities shall be fixed to the shuttering. No holes shall be cut in any concrete unless approved.

An approved mould oil or other material shall be applied to faces of shuttering in contact with wet concrete to prevent adherence of the concrete. Such coatings shall be insoluble in water, non-staining, and not injurious to the concrete, and shall not become flaky or be removed by rain or wash-water. Liquids that retard the setting of concrete shall be used only when approved. Mould oil, retarding liquid, and similar coatings shall be kept from contact with the reinforcement.

Proportions of Concrete.—The aggregate shall be measured by volume (unless instructed to be measured by weight) in an approved container which shall be filled without compacting with the aggregate to a predetermined uniform depth, accurate allowance being made for bulking due to the moisture in the fine aggregate. The cement shall be measured by weight. If stored in bags, one or more complete bags containing 112 lb. of cement shall be used for each batch of concrete. If stored loose the cement shall be measured by weight by approved means.

If the proportions of cement and aggregate are not otherwise described on the drawings or elsewhere, the concrete shall be mixed in the proportions of 1 cwt. of Portland cement to 2½ cu. ft. of sand (measured when dry) and 5 cu. ft. of coarse aggregate. These quantities or others as described shall be altered if instructed and any alteration between the proportions of 1 part of fine aggregate to 1½ parts of coarse aggregate and 1 part of fine aggregate to 2 parts of coarse aggregate shall be made without any alteration in the charge made by the Contractor.

Only sufficient water shall be added to the cement and aggregate during mixing to produce a concrete having sufficient workability to enable it to be well consolidated, to be worked into the corners of the shuttering and around the reinforcement, to give the specified surface finish, and to have the specified strength. When a suitable amount of water has been determined the resulting consistency shall be maintained throughout the corresponding parts of the work, and approved tests shall be conducted from time to time to ensure the maintenance of this consistency.

If difficulty be experienced in placing concrete of the specified proportions and approved consistency between and below the reinforcement bars in the bottom of beams and similar members, the bars shall be embedded in concrete of improved
workability by increasing the amount of cement as approved and by using aggregates of approved smaller maximum size than specified.

MIXING CONCRETE.—The cement and aggregates shall be thoroughly mixed together in the proportions described in a batch-type mechanical mixer, unless otherwise approved. The water shall not be admitted to the drum of the mixer until all the cement and aggregate constituting the batch are in the drum. Mixing shall continue until the concrete is uniform in colour and for not less than two minutes after all the materials and water are in the drum. The entire contents of the drum shall be discharged before the materials for the succeeding batch are fed into the drum. No partly-set or retempered concrete shall be used. Partly-set or excessively wet concrete shall not be used on the Works and shall be immediately removed therewith.

STRENGTH OF CONCRETE.—The compressive strength of the concrete at twenty-eight days shall be not less than 3000 lb. per square inch or such other strength as is described on the drawings or elsewhere, except that if rapid-hardening Portland cement is used the required strength shall be attained in seven days.

The compressive strength shall be ascertained by crushing 6-in. cubes of concrete, the cubes being made on the Works and tested as instructed. The Contractor shall pay all costs incurred in supplying the material for, and in making, maturing, delivering, and testing the cubes and shall be reimbursed for the costs of cubes that attain the required strength.

DISTRIBUTION OF CONCRETE.—The concrete shall be distributed from the mixer to the position of placing in the Works by approved means which do not cause separation or otherwise impair the quality of the concrete.

Mixing and distributing equipment shall be clean before commencing mixing and distribution of the concrete and such equipment shall be kept free from set concrete.

PLACING CONCRETE.—Before proceeding to place the concrete, the shuttering shall be re-aligned if necessary and water and rubbish therein shall be removed by approved means. Immediately prior to placing the concrete, the shuttering shall be well wetted except in frosty weather and inspection openings shall be closed.

The interval between adding the water to the dry materials and the completion of the placing of the concrete shall not exceed twenty-five minutes.

Except where otherwise approved, concrete shall be placed in the shuttering by shovels or other approved implements and shall not be dropped from a height or handled in a manner which will cause separation. Accumulations of set concrete on the reinforcement shall be avoided. Concrete shall be placed directly in its permanent position and shall not be worked along the shuttering to that position.

Each layer of concrete while being placed shall be consolidated by approved methods of ramming, tamping, or mechanical vibration to form a dense material with all surfaces free from honeycombing and tolerably free from water and air holes or other blemishes. Any water accumulating on the surface of newly-placed concrete shall be removed by approved means, and no further concrete shall be placed thereon until such water be removed.

No unset concrete shall be brought into contact with unset concrete containing cement of a different type.

Unless otherwise approved, concrete shall be placed in a single operation to the full thickness of slabs, beams, and similar members, and shall be placed in horizontal layers not exceeding 3 ft. deep in walls, columns, and similar members. Concrete shall be placed continuously until completion of the part of the work between construction joints as specified hereinafter or of a part of approved extent. At the completion of a specified or approved part a construction joint of the form and in the position hereinafter specified shall be made. If stopping of concrete placing be unavoidable elsewhere, a construction joint shall be made where the work is stopped.

Tremies, bottom-opening skips, or other equipment used for placing concrete under water shall be of approved design and shall be used as instructed by the Engineer. No concrete shall be placed if the temperature of the water is below 35 deg. F. Concrete placed between tides shall be shuttered on all faces, the top face being closed immediately after completion of placing and before subsequent submersion. No pumping that may adversely affect the concrete being placed shall be done while placing or within twenty-four hours of placing the concrete.
PLACING CONCRETE IN COLD WEATHER.—No concrete shall be mixed or placed while the temperature is below 34 deg. F. on a rising thermometer or below 36 deg. F. on a falling thermometer. The Contractor shall supply an accurate maximum and minimum thermometer and hang it in an approved position on the Works. Aggregates that have been exposed to frost shall not be used until completely thawed. Concrete shall be maintained by approved means at a temperature of not less than 40 deg. F. during placing and for a period of three days thereafter, or for a period of one day thereafter if rapid-hardening Portland cement or high-alumina cement is used. All concrete placed during cold weather or when a frost is predicted or is likely to occur or occurs contrary to expectation, shall be protected from freezing by approved means.

HOLLOW CLAY BLOCK SLABS.—Hollow clay blocks shall be laid true to line and accurately spaced. Joints between blocks shall be pointed and faces of blocks in contact with concrete shall be given a thick coat of cement slurry immediately before placing the concrete. Broken blocks shall not be used and shall be removed from the Works. Approved stop-ends shall be inserted to close the open ends of each line of blocks. If so described, clay-tile slips of approved thickness shall be provided at the bottom of each concrete rib. Blocks shall be well soaked in water before placing the concrete and slurry.

The concrete in the ribs between the blocks, in the slab over the blocks, and in the solid slab around the edges of the area laid with blocks shall be placed in one operation and shall be mixed in the nominal volumetric proportions of 1:1:3 using coarse aggregate having a maximum size of 1 in.

CONSTRUCTION JOINTS.—Construction joints shall be rebated and of an approved shape and shall be vertical or horizontal, as required, except that in an inclined or curved member the joint shall be at right-angles to the axis of the member.

Construction joints shall be provided in the positions described on the drawings or elsewhere and where not so described shall be in accordance with the following. A joint shall be formed horizontally at the top of a foundation and 3 in. below the lowest soffit of the beams meeting at the head of a column. A joint shall be formed in the rib of a large tee-beam and ell-beam 1 in. below the soffit of the slab. Concrete in a haunch or a splay on a beam or a brace, and in the head of a column where one or more beams meet, shall be placed without a joint at the same time as that in the beam or beams or brace. Concrete in the splay at the junction of a wall and a slab shall be placed without a joint at the same time as that in the slab. Concrete in a beam shall be placed throughout without a joint, but if the provision of a joint is unavoidable the joint shall be vertical and at the middle of a span. A joint in a slab shall be vertical and parallel to the principal reinforcement; where it is unavoidably at right-angles to the principal reinforcement the joint shall be vertical and at the middle of the span.

Before placing new concrete against concrete that has already hardened, the face of the old concrete shall be cleaned and roughened and scum and loose aggregate removed therefrom, and immediately before placing the new concrete the face shall be thoroughly wetted and a coating of neat cement grout applied thereto. The new concrete shall be well rammed against the prepared face before the grout sets.

STRUCTURE JOINTS.—Expansion joints, contraction joints, hinges, or other permanent structure joints shall be provided in the positions and of the form described in the drawings or elsewhere.

PROTECTION OF CONCRETE.—Newly-placed concrete shall be protected by approved means from frost, rain, sun, and drying winds. Exposed faces of concrete shall be kept moist by approved means for seven days after placing or for three days if rapid-hardening Portland cement is used, except if there is a likelihood of curing water or damp coverings freezing, when the period shall be as instructed by the Engineer.

Concrete placed below the ground shall be protected from falling earth during and after placing. Concrete placed in ground containing deleterious substances shall be kept free from contact with such ground and with water draining therefrom during placing and for a period of three days or as otherwise instructed thereafter. The ground-water around a structure below the ground shall be kept to an approved level by pumping, or the Works shall be flooded or other approved means taken to prevent
flotation. Approved means shall be taken to protect immature concrete from damage by debris, ice, excessive loading, vibration, abrasion, deleterious ground-water, mixing with earth or other materials, flotation, and other influences that may impair the strength and durability of the concrete.

Removal of Shuttering.—Shuttering shall be removed by gradual easing without jarring. Before removal of the shuttering the concrete shall be examined and removal shall proceed only in the presence of a competent supervisor and if the concrete has attained sufficient strength to support its own weight and any load likely to be imposed upon it. If the imposition of a load exceeding the design load is anticipated, props shall be provided in an approved manner after removal of the shuttering and before the imposition of the load. The Contractor shall record on the drawings or elsewhere the date upon which the concrete is placed in each part of the work and the date upon which the shuttering is removed therefrom. The assessment of the period elapsing between placing the concrete and removing the shuttering and consequences arising therefrom shall be the Contractor’s entire responsibility.

The shuttering for a part of a structure suspended from concrete placed subsequently to that in or on the shuttering concerned shall not be removed until the supporting concrete has matured, and such shuttering shall be prominently marked as a warning against premature removal.

Finishes.—Honeycombed surfaces shall be made good immediately upon removal of the shuttering, and superficial water and air holes shall be filled in. Unless instructed to the contrary the face of exposed concrete placed against shuttering shall be rubbed down immediately upon removal of the shuttering to remove fins or other irregularities. The face of concrete for which shuttering is not provided, other than slabs, shall be smoothed with a wooden float to give a finish equal to that of the rubbed-down face where shuttering is provided. The top face of a slab which it is not intended to cover with other materials shall be levelled and floated while unset to a smooth finish at the levels or falls shown on the drawings or elsewhere. The floating shall be done so as not to bring an excess of mortar to the surface of the concrete. Indentations in the surface of the concrete shall be formed by approved implements to the depths and patterns described. The top face of a slab intended to be surfaced with mortar, granolithic, or similar material shall be left with a spaded finish.

Concealed concrete faces shall be left as from the shuttering except that honeycombed surfaces shall be made good. Faces of concrete intended to be rendered shall be roughened by approved means to form a key. Faces of concrete that are to have finishes other than those specified shall be prepared in an approved manner as instructed.

Precast Piles.—Shoes for precast piles shall have cast-iron points and wrought-iron or mild steel straps and shall be of the pattern and weight described. Forks for retaining the longitudinal reinforcement in position shall be of cast iron, pressed steel, or other approved material and of approved shape.

Shoes, reinforcement, forks, links, toggle-hole tubes, and other fittings shall be fixed accurately in the position described on the working drawings or elsewhere. Shoes shall be fitted with the point in line with the axis of the pile. One end of the longitudinal bars shall bear on the top of the shoe and the upper ends of such bars shall all be at the same level.

The moulds shall be so made that the pile is not distorted during casting or at any other stage of manufacture. The piles shall be protected from damage from any cause during handling, and shall be slung only from the points described on the working drawings or elsewhere.

The period that shall elapse between making and driving a pile, or between placing the concrete in a pile lengthened in situ and recommencing driving, shall be not less than six weeks if ordinary Portland cement is used and not less than two weeks if rapid-hardening Portland cement is used.

The weight, type, and the height of fall of the driving hammer to be used, and the final set or depth to which each pile shall be driven, shall be as described on the drawings or as instructed. Driving shall proceed without cessation until the pile is driven to the set or depth described. If cessation of driving is unavoidable, or when
the driving of a pile lengthened insitu is recommended, the amount of driving that shall be done before the set is measured shall be as instructed.

During driving, the head of the pile shall be protected by approved means. Piles shall be driven in the sequence described on the working drawings or elsewhere. Sheet piles shall be driven together and shall be keyed together as described on the drawings and elsewhere.

If a pile is driven in an incorrect position, or if a pile intended to be vertical is driven out of plumb, or if an inclined pile is not driven to the correct angle, the Contractor shall take at his own cost such remedial measures as instructed.

Where piles bond with pile-caps or other reinforced concrete work, or if the driven pile be of excessive length, or if a driven pile is to be increased in length, the concrete shall be cut away for a distance below the head of the pile as instructed. The cutting away shall be in an approved manner without damage to the rest of the pile left insitu. Unless instructed to the contrary, the main reinforcement shall not be cut, but shall be embedded into concrete subsequently placed, any bending being done without damage to the concrete in the pile.

Precast Concrete.—The concrete in one precast piece shall be placed in one operation. No piece shall be removed from the mould or erected until sufficiently matured to ensure that no damage shall be done to the piece. A piece shall be suspended or supported only at the points described on the working drawings or elsewhere. A piece that is cracked or otherwise damaged during, before, or after erection shall be removed from the Works and replaced by the Contractor free of charge. Pieces shall be bedded or otherwise fixed in their permanent positions as instructed.

Structure Tests.—The Engineer shall instruct the Contractor to make a loading test on the Works or any part thereof if in the Engineer’s opinion such a test is necessary.

If the test be instructed to be made solely or in part for the reason that the site-made concrete cubes fail to attain the specified strength, the test shall be made at the Contractor’s own cost. If the test be instructed to be made because of one or more circumstances attributable to alleged negligence on the part of the Contractor, the Contractor shall be reimbursed for the cost of the test if the result thereof is satisfactory. If the test be instructed to be made for any reason other than the foregoing, the Contractor shall make the test and shall be reimbursed for all costs relating thereto irrespective of the result of the test.

For the purpose of testing floors, roofs, and similar structures and their supports, the test load shall be equivalent to one-and-a-quarter times the live load for which the Works or part thereof to be tested has been designed. The test load shall not be applied within twenty-eight days of the completion of placing of the concrete in the part of the Works to be tested, and the latter shall be unsupported during the test by the shuttering or other non-permanent support. The test shall be made as instructed.

For a test on a floor, roof, or similar construction, the result shall be deemed to be satisfactory if upon removal of the load the residual deflection does not exceed one-quarter of the maximum deflection after maintaining the load in position for twenty-four hours. If the residual deflection exceeds this amount, the test shall be repeated, and the result shall be deemed to be satisfactory if the residual deflection after removal of the load for the second time does not exceed one-quarter of the maximum deflection occurring during the second test.

If the result of the test is not satisfactory, the Engineer shall instruct that the part of the Works concerned shall be taken down or cut out and reconstructed to comply with this Specification, or that other measures shall be taken to make the Works secure. If in accordance with this Specification the Contractor is liable to conduct the test at his own cost, he shall also at his own cost take down or cut out and reconstruct the defective work or shall execute remedial measures as instructed.
Quantities.

The bills of quantities for a reinforced concrete structure can be divided into six principal divisions (each of which is subdivided into items), such as: Excavation and filling; Plain concrete in foundations and elsewhere; Reinforced concrete; Shuttering; Reinforcement; and Sundries (including asphalt, fixing machinery and pipes, glazing, plumbing, waterproofing, refractory linings, joinery, and other works). The method of measurement and the bills of quantities should be in accordance with a standard method in order to avoid ambiguities. If the contract is for a building under the direction of an architect, the "Standard Method of Measurement of Building Works" should be used. For a civil engineering contract, the "Standard Method of Measurement of Civil Engineering Quantities" issued by the Institution of Civil Engineers should be used. For small reinforced concrete structures, a short bill of quantities, similar to that given later, may be sufficient.

Measurement of Quantities.—The amount of plain and reinforced concrete, shuttering, piling, earthwork, and similar work can generally be measured from the general drawings.

For the measurement of the quantities from the drawings, a system should be followed to ensure accuracy and facilitate checking. The following is recommended for measuring the quantities of concrete and shuttering. In the first column of measurement is entered the number of identical concrete members being measured. In the second column the dimensions of the member in the order of length, breadth and depth are entered. In the third column the volume of concrete represented by the product of the three preceding dimensions multiplied by the number of members is given. The dimensions of the surfaces requiring shuttering are entered opposite the corresponding item for the concrete. The example given later demonstrates the method. The members are measured in order downwards from the top. The quantities for similar members are then abstracted and added to similar items for other parts of the structure before transferring them to the bill of quantities.

Reinforcement.—The total weight can be roughly estimated from the type of structure and volume of concrete. The amount of steel in an ordinary reinforced concrete structure varies from 1 ton in 5 cu. yd. of concrete to 1 ton in 30 cu. yd. The following are approximate ratios of cubic yards of concrete to a ton of reinforcement. Warehouses, girder bridges, rectangular tanks, water towers, silos, circular bunkers, pits, and pit-head gears: 10 to 15; factories, residential buildings, retaining walls, culverts, swimming baths, and open circular tanks: 15 to 20; arch bridges, rectangular bunkers, pyramidal tanks in the ground, and elevated conical tanks: 8 to 10. Exceptional loads and other circumstances may affect the ratios considerably.

If the working drawings or typical details are available, but no bar-bending schedules, a more accurate estimate can be made from consideration of the principal reinforcement. The principal reinforcement is that near the lower edge at midspan in a beam or slab, and the main vertical bars in a column. The area $A$ sq. in. of the principal reinforcement can be computed from Table 59 or 60 and the approximate average weight ($W$ lb. per foot of beam or column, or per square foot of slab) can be calculated from $W = KA$, where $K$ has the following values: freely-supported beams, 3 to 6; continuous beams, 6 to 8 $\frac{1}{4}$; cantilevers, 4 to 6; columns, 5 to 6; continuous slabs spanning in one direction, 5 to 8 $\frac{1}{4}$; continuous slabs spanning in two directions, 4 $\frac{3}{4}$ to 7 $\frac{1}{4}$. For slabs and beams the smaller values apply in cases where there is no
compression reinforcement or no special reinforcement to resist shearing force; the larger values apply where there are equal amounts of compression and tension reinforcement or where a large amount of reinforcement is provided to resist shearing. The smaller values for columns apply to those with independent lateral ties, and the larger for columns with helical binding or where loose splice bars are provided. For slabs spanning in two directions, \( A \) is the sum of the areas of the reinforcement in each direction.

The unit weight multiplied by the length (ft.) of a beam or column or by the area (sq. ft.) of a slab, gives the approximate total weight (lb.) of reinforcement. For members such as important beams and arch ribs, an approximate list of the bars should be made and the total length of the bars of each size multiplied by the weight per foot (Table 59). The weight of reinforcement in walls or other slabs with uniform reinforcement throughout can be found by multiplying the area (sq. yd.) of the wall or slab by the weight per square yard and adding an amount for laps. If bar-bending schedules are available, the weight of reinforcement can be calculated exactly from Table 59. To demonstrate the methods, the weight of reinforcement in the beam shown in the accompanying diagram will be calculated.

The approximate weight is estimated by considering the principal reinforcement, which is four \( \frac{3}{8} \)-in. diameter bars \( (A = 2.41 \text{ sq. in.}) \). Since there is no compression reinforcement at midspan and inclined bars are provided for resistance to shearing force, a suitable value for \( K \) is 6. The average weight of reinforcement per foot of beam is therefore \( 6 \times 2.41 = 14 \) lb. per foot. The length of the span is 18 ft., and therefore the estimated total weight of reinforcement in one beam is \( 18 \times 14 = 252 \) lb.; say, 2\( \frac{1}{2} \) cwt.

A bar-bending schedule giving particulars of each bar is given on the diagram. The total length of bars of each size is calculated and is multiplied by the weight per foot. The system of calculating the weights as given on the diagram avoids confusion and facilitates checking.

The quantities of reinforcement bars are given by weight. Round bars \( \frac{3}{8} \) in. diameter and over are generally given in one item, but the weights of each size of bar less than \( \frac{3}{8} \) in. are given separately. The weights of bent bars need not generally be separated from straight bars unless there is a predominating weight of the latter. Some indication of the length of bars and type of structure should be given, and of the basis of measurement, for example whether tying wire, the rolling margin, or the
like are included in the weight or must be allowed for in the price. Bars over 40 ft. in length and less than 5 ft. should be separately measured. A comprehensive clause describing the reinforcement and enabling a contractor to give a keen price is as follows:

Supply, handle, cut, bend, crank, and fix mild steel bars \( \frac{1}{4} \)-in. diameter and over, but not exceeding \( 1\frac{1}{4} \) in. in diameter, in main reinforcement in beams and columns to all floors and roofs and in foundations, in lengths not less than 5 ft. and not exceeding 40 ft. The price shall include for tying wire. No allowance in the weight paid for shall be made for the rolling margin. The weight of any bar shall be computed from the length given on the drawings or bar-bending schedules, the weight in pounds per foot of bar being calculated as

\[ 2.67 \times \text{the square of the specified diameter of the bar} \times \text{cwt.} \]

Similar items should be given for smaller bars and for longer and shorter bars. When the supply of the reinforcement, or the fixing or bending, or both, is a sub-contract, it should be stated who does the cutting and hoisting and supplies the tying wire.

The "Building Works" method of measuring reinforcement requires bars over 30 ft. long to be given in separate items for each 5 ft. advance in length. Separate items must be given for bent and straight bars, stirrups, helical binding, and bars bent to a large radius. Reinforcement for floors, roofs, walls, beams, and columns must be separate. Bars of each size less than \( \frac{1}{8} \) in. and more than 1 in. must be given separately.

**Estimating Costs.**

In a contracting firm the data and methods for making an estimate of the cost of a reinforced concrete structure are generally available. Otherwise an approximate estimate can be made by pricing each item in the bill of quantities at the average prevailing rates for similar construction. Unless local rates are known, however, this may give misleading results. If a recent priced bill of quantities for previous work in the locality is not at hand, the prices given in technical periodicals might be used to obtain an approximate estimate. A better method is to calculate the probable prices, basing the computation on the cost of labour, materials, and general costs. The wages payable to different classes of workers are given in technical periodicals, and the man-hours required to perform a given piece of work are given in text-books on this subject. The trend of the general productivity of labour should, however, be considered when using published data. The cost of labour can vary considerably. The costs of materials can be obtained from current prices, and the amount of each material required can be determined from the bills of quantities. In *Table 56* are given the quantities of stone, sand, and cement required for a cubic yard of concrete. To the net quantities of aggregate must be added 10 per cent. for tolerances on measurement and for waste, and 5 per cent. to the net quantities of cement for waste and for making grout. If the price does not include delivery, the cost of transport must be added. The general costs, which include overhead charges, profit, and site staff, depend principally upon the contract time and the plant likely to be used. For an approximate estimate for an ordinary reinforced concrete structure, allowance for the general costs can be made by increasing the net cost of materials by, say, 15 per cent. and of labour by, say, 50 per cent. For small structures and for very large works this method is not dependable, and such work should be estimated on a more precise basis.
Beam-and-Slab Construction.—Example for Measurement of Quantities.
## Example of Measuring Quantities.

In the following the quantities are measured for one 20-ft.-square panel of the floor of beam-and-slab construction shown on page 136, the columns being excluded.

### CONCRETE.

<table>
<thead>
<tr>
<th>Slab.</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>20' 0&quot;</td>
<td>20' 0&quot;</td>
<td>4&quot;</td>
</tr>
<tr>
<td>20' 0&quot;</td>
<td>20' 0&quot;</td>
<td>400</td>
</tr>
<tr>
<td><strong>133.3</strong></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Secondary Beams.</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>2/19' 0&quot;</td>
<td>2/2/19' 0&quot;</td>
<td></td>
</tr>
<tr>
<td>1' 4&quot;</td>
<td>1' 4&quot;</td>
<td>101</td>
</tr>
<tr>
<td>8&quot;</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>33.9</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>18' 9&quot;</td>
<td>2/18' 9&quot;</td>
<td></td>
</tr>
<tr>
<td>8&quot;</td>
<td>1' 4&quot;</td>
<td>50</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>16.7</strong></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Main Beams.</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>18' 9&quot;</td>
<td>2/18' 9&quot;</td>
<td></td>
</tr>
<tr>
<td>1' 0&quot;</td>
<td>1' 6&quot;</td>
<td>56</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>28.1</strong></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>2/1/ 2'/ 3&quot;</th>
<th>2/2/1/ 2'/ 3&quot;</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>9&quot;</td>
<td>1' 0&quot;</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>1.7</strong></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| Total          | 213.7 cu. ft. | 609            |

say = 8.0 cu. yard.

### SHUTTERING.

<table>
<thead>
<tr>
<th>Deduct</th>
<th>1' 3&quot;</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1' 3&quot;</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>607 sq. ft.</strong></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

say = 68 sq. yd.

### REINFORCEMENT (see bending schedule on page 136).

<table>
<thead>
<tr>
<th>Diameter</th>
<th>1/8 in.</th>
<th>3/8 in.</th>
<th>1/2 in.</th>
<th>3/4 in.</th>
<th>1 in.</th>
<th>1 1/4 in.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slab (20 ft. sq.)</td>
<td>630</td>
<td>735</td>
<td>756</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Secondary beams (3 No.)</td>
<td>408</td>
<td>108</td>
<td>150</td>
<td>8</td>
<td>86</td>
<td>98</td>
</tr>
<tr>
<td>Main beam: (1 No.)</td>
<td>109</td>
<td>150</td>
<td>20</td>
<td>41</td>
<td>57</td>
<td>69</td>
</tr>
<tr>
<td>Total length</td>
<td>603</td>
<td>141</td>
<td>2049</td>
<td>136</td>
<td>316</td>
<td>233 ft.</td>
</tr>
<tr>
<td>Weight per ft.:</td>
<td>0.167</td>
<td>0.261</td>
<td>0.376</td>
<td>0.668</td>
<td>2.67</td>
<td>4.173 lb. (See Table 59)</td>
</tr>
<tr>
<td>Total weight (net)</td>
<td>105</td>
<td>37</td>
<td>779</td>
<td>91</td>
<td>865</td>
<td>971 lb.</td>
</tr>
<tr>
<td>Gross weight</td>
<td>1</td>
<td>7 1/2</td>
<td>1</td>
<td>16 1/2</td>
<td>cwt.</td>
<td></td>
</tr>
</tbody>
</table>

The foregoing quantities are abstracted and the totals transferred to the bill (see page 138). Price rates can be taken from published data. The basis of the method is unaltered by any fluctuations in the prices.
<table>
<thead>
<tr>
<th>Item No.</th>
<th>Description</th>
<th>Quantities</th>
<th>Rate</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Mixing and placing 1:2:4 concrete in slabs and beams (rough screeding only).</td>
<td>cu. yd. 8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Close-boarded shuttering to soffits of slabs, and sides and soffits of beams and haunches.</td>
<td>sq. yd. 68</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Provide and lay 1-in. granolithic surfacing.</td>
<td>sq. yd. 45</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Supply, cut if necessary, bend, and fix reinforcement bars ½ in. diameter and over, including tying wire.</td>
<td>cwt. 16½</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Ditto ¾ in. diameter.</td>
<td>cwt. 1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>Ditto ¾ in. and ¾ in. diameter.</td>
<td>cwt. 7½</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>Ditto ¼ in. diameter.</td>
<td>cwt. 1</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Total estimated cost for one panel 20 ft. square.
## PART II

### TABLES AND EXAMPLES

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(For complete List of Tables, see pages v and vi.)
DEAD LOADS.

Weight of Concrete.—The primary dead load is usually the weight of the reinforced concrete, which for design purposes can be conveniently assumed to be 144 lb. per cu. ft., that is 1 lb. per lin. ft. for each square inch of cross-sectional area; but the weight of reinforced concrete is rarely less than 150 lb. per cu. ft. and varies with the density of the aggregate and the amount of reinforcement. Some typical weights of plain and reinforced concrete are given in Table 1, together with the weights of solid concrete slabs and hollow clay-block slabs. For members where the dead load predominates, a weight of not less than 150 lb. per cu. ft. should be assumed for ordinary reinforced concrete, this being the minimum weight recommended in the B.S. Code, London By-laws, and by other authorities. The weights of various lightweight concretes are also given in Table 1. Heavy concrete for use as kentledge and nuclear-radiation shielding is made by using aggregates of greater density than ordinary stone, such as barytes, limonite, magnetite, and other iron ores and steel shot or punchings; the weights of such concretes are also given in Table 1.

Other Structural Materials and Finishes.—Dead loads include such permanent weights as those of the finishes and linings on walls, floors, stairs, ceilings, and roofs; asphalt and other applied waterproofing layers; partitions; doors, windows, roof lights, and pavement lights; superstructure of steelwork, masonry, brickwork, or timber; concrete bases for machinery and tanks; fillings of earth, sand, puddled clay, plain concrete, hardcore; cork and other insulating materials; rail-tracks and ballasting; refractory linings; and road surfacing. In Table 1 are given the basic weights of structural materials. In Table 2 are given the weights of glazed, sheeted, and slated roofs, and the average equivalent weights of steel trusses, such weights being useful in estimating the loads imposed on a concrete substructure. Rules for estimating the total weight of structural steelwork based on adding to the sum of the nominal weights of the members an allowance for cleats, connections, rivets, bolts and the like are given in Table 2; extra allowances should be made for stanchion caps, bases, and grillages. The smaller allowances permissible for welded steelwork are also given.

Where concrete lintels support brick walls it is not necessary to consider the lintel as carrying the whole of the wall above it; it is sufficient to allow only for the triangular areas indicated in the diagrams in Table 2.

Partitions.—The weights of partitions should be included in the dead loads of floors and it is convenient to consider such weights as equivalent uniformly-distributed loads \( w_p \). It is usual to consider a minimum load of 20 lb. per sq. ft. of floor area for partitions in offices and buildings of similar use, but this load is only sufficient for timber or glazed partitions. The material of which the partition is constructed and the story-height will determine the weight of the partition, and in the design of floors the actual weight and position of a partition, when known, should be allowed for when calculating shearing forces and bending moments on the slab and beams. Expressions are given in Table 2 for the equivalent uniformly-distributed load if the partition is at right-angles to the direction of the span of the slab and is placed at the middle of the span, or if the partition is parallel to the direction of the span. According to the B.S. Code, the equivalent uniformly-distributed load per sq. ft. of floor for partitions, the positions of which are not known, should be not less than 10 per cent. of the specified minimum imposed load (see Table 3), and in no case less than 20 lb. per sq. ft. for office floors.

In the case of brick or similarly-bodied partitions some relief of loading on the slab occurs due to arching action of the partition if it is continuous over two or more beams, but the presence of doorways or other openings destroys this relieving action.

The uniformly-distributed load on a beam due to partitions can be considered as the proportion of the total weight of the partitions carried by the beam adjusted to allow for non-uniform incidence.
**DEAD LOADS AND WEIGHTS OF MATERIALS.—TABLE 1.**

<table>
<thead>
<tr>
<th>CONCRETE</th>
<th>L.B. PER CUBIC FOOT</th>
<th>OTHER MATERIALS</th>
<th>L.B. PER CUBIC FOOT</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>PLAIN CONCRETE (1:3 OR 1:4)</strong></td>
<td></td>
<td><strong>NATURAL STONE (SOLID)</strong></td>
<td></td>
</tr>
<tr>
<td>GRAVEL OR CRUSHED STONE AGGREGATE</td>
<td>140 to 155</td>
<td>GRANITE, BASALTS, DOLERITES</td>
<td>165</td>
</tr>
<tr>
<td>BROKEN BRICK AGGREGATE <strong>(AV.120)</strong></td>
<td>105 to 135</td>
<td>LIMESTONE, SANDSTONE, YORKSTONE</td>
<td>140</td>
</tr>
<tr>
<td>CRUSHED LIMESTONE AGGREGATE</td>
<td>135 to 150</td>
<td>SLATE</td>
<td>180</td>
</tr>
<tr>
<td><strong>REINFORCED CONCRETE</strong></td>
<td><strong>MINIMUM 144</strong></td>
<td><strong>CRUSHED STONE</strong></td>
<td>60 to 140</td>
</tr>
<tr>
<td>FOR GENERAL DESIGN PURPOSES <strong>(B.S. CODE)</strong></td>
<td><strong>150</strong></td>
<td><strong>GRAVEL, BALLAST, ETC. (LOOSE)</strong></td>
<td>100</td>
</tr>
<tr>
<td>WITH 1% REINFORCEMENT</td>
<td>144 to 154</td>
<td><strong>HARDCORE</strong></td>
<td><strong>AVERAGE 120</strong></td>
</tr>
<tr>
<td>&quot; 2 % &quot; <strong>(AVERAGE 150)</strong></td>
<td>146 to 158</td>
<td><strong>SAND (DRY TO WET)</strong></td>
<td>60 to 120</td>
</tr>
<tr>
<td>&quot; 5 % &quot;</td>
<td>160 to 170</td>
<td><strong>PACKED STONE RUBBLE</strong></td>
<td>140</td>
</tr>
<tr>
<td><strong>LIGHTWEIGHT CONCRETE</strong></td>
<td><strong>STRENGTH</strong></td>
<td><strong>BROKEN BRICK</strong></td>
<td>70 ± 100</td>
</tr>
<tr>
<td><strong>LB./SQ.IN.</strong></td>
<td></td>
<td><strong>ASH FILLING</strong></td>
<td>60 ± 80</td>
</tr>
<tr>
<td>CLINKER <strong>(1:8)</strong></td>
<td>300 to 900</td>
<td><strong>EARTH FILLING (COMPACTED)</strong></td>
<td>100</td>
</tr>
<tr>
<td>PUMICE <strong>(1:6 SEMI-DRY)</strong></td>
<td>200 to 550</td>
<td><strong>PUDDLED CLAY</strong></td>
<td>100 ± 120</td>
</tr>
<tr>
<td>FOAMED BLASTFURNACE SLAG</td>
<td>200</td>
<td><strong>CRUSHED SLAG</strong></td>
<td>50</td>
</tr>
<tr>
<td></td>
<td>800</td>
<td><strong>TIMBER</strong></td>
<td><strong>AVERAGE 50</strong></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>EXPANDED CLAY OR SHALE</td>
<td>800 to 1200</td>
<td><strong>YELLOW PINE</strong></td>
<td>30</td>
</tr>
<tr>
<td><strong>FROZEN SALT (EXPANDED MICA)</strong></td>
<td>2000 to 5000</td>
<td><strong>RED PINE, SPRUCE</strong></td>
<td>30 to 45</td>
</tr>
<tr>
<td><strong>FROZEN SALT (CONSTRUCTIVE)</strong></td>
<td>500</td>
<td><strong>ENGLISH OAK</strong></td>
<td>45 ± 60</td>
</tr>
<tr>
<td><strong>PULVERISED-FUELSASH (FLYASH)</strong></td>
<td>70</td>
<td><strong>LARCH, ELM</strong></td>
<td>35</td>
</tr>
<tr>
<td><strong>FROZEN SALT (CONSTRUCTIVE)</strong></td>
<td>400 to 1000</td>
<td><strong>PITCH PINE</strong></td>
<td>40 ± 60</td>
</tr>
<tr>
<td><strong>NO-FINES GRAVEL AGGREGATE</strong></td>
<td>2000 to 5000</td>
<td><strong>TEAK</strong></td>
<td>40 ± 55</td>
</tr>
<tr>
<td><strong>CELLULAR (AERATED OR GAS)</strong></td>
<td>100 to 120</td>
<td><strong>JARAH</strong></td>
<td>60</td>
</tr>
<tr>
<td></td>
<td>500</td>
<td><strong>GREENHEART</strong></td>
<td>65 ± 75</td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>QUEBRACHO</strong></td>
<td>80</td>
</tr>
<tr>
<td><strong>AIR-ENTRAINED CONCRETE</strong></td>
<td><strong>FOR DESIGN USE SAME WEIGHTS AS PLAIN OR REINFORCED CONCRETE</strong></td>
<td><strong>CORK (SOLID; NOT GRANULAR)</strong></td>
<td>15</td>
</tr>
<tr>
<td><strong>HEAVY CONCRETE</strong></td>
<td></td>
<td><strong>WATER (SEAWATER, G4)</strong></td>
<td>62.4</td>
</tr>
<tr>
<td>BARYTES</td>
<td>200</td>
<td><strong>SNOW (COMPACT 15 to 50) LOOSE</strong></td>
<td>5 ± 12</td>
</tr>
<tr>
<td>LIMONITE, MAGNETITE, AND SIMILAR ORES</td>
<td><strong>AV. 224</strong></td>
<td><strong>METALS (PER INCH THICK)</strong></td>
<td><strong>SQUARE FOOT</strong></td>
</tr>
<tr>
<td>STEEL SHAVINGS OR PUNCHINGS</td>
<td><strong>AV. 33 G</strong></td>
<td><strong>SHEET LEAD</strong></td>
<td>60</td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>COPPER, BRASS, BRONZE, GUNMETAL</strong></td>
<td>45 to 47</td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>CAST OR FORGED IRON</strong></td>
<td>38 to 40</td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>TIN, SPelter, ZINC</strong></td>
<td>37 to 38</td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>ALUMINIUM</strong></td>
<td>14</td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>BITUMEN (PER INCH THICK)</strong></td>
<td>71/2</td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>MACADAM (PER INCH THICK) WATERBOUND</strong></td>
<td>151/2</td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>TAR-MACADAM (</strong>)**</td>
<td>12</td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>DOORS (WOODEN)</strong></td>
<td><strong>AVERAGE</strong></td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>GLASS (PER INCH THICK)</strong></td>
<td>14</td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>WINDOWS (METAL OR WOODEN FRAMES)</strong></td>
<td>5</td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>BRICKWORK (PER INCH THICK)</strong></td>
<td><strong>ENGINEERING, IN P.C. MORTAR</strong></td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>COMMON (IN LIME 3) IN P.C.</strong></td>
<td><strong>10</strong></td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>REFRACTORY</strong></td>
<td>6</td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>HOLLOW-BLOCK PARTITIONS (2 IN. TO 4 IN.)</strong></td>
<td>9 to 15</td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>EARTHENWARE TILES</strong></td>
<td>14</td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>SLATES</strong></td>
<td>8</td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>ASPHALT (1 IN. THICK)</strong></td>
<td>12</td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>FIBRE-BOARD, PAPER BOARD (3/8 IN. THICK)</strong></td>
<td>14</td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>HARDBOARD (1/2 IN. THICK)</strong></td>
<td>1/2</td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>TIMBER BOARDING (1 IN. THICK)</strong></td>
<td>3</td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>RUBBER, PAVING AND SIMILAR (1 IN.)</strong></td>
<td>5</td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>GRANITE SETTS (PER INCH THICK)</strong></td>
<td>14</td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>WOOD-BLOCK, PAVING (PER INCH THICK)</strong></td>
<td>5</td>
</tr>
</tbody>
</table>

See also Table 2.
Examples of the Use of Tables 1, 2 and 3.

Determine the design loads for the following cases.

(a) A 4¼-in. flat roof slab, with an average of 1½ in. of screeding and ¼ in. of asphalt. (Note: Effect of wind on a flat roof is suction.)

<table>
<thead>
<tr>
<th>Load Description</th>
<th>Load (Table 3)</th>
<th>=</th>
<th>lb. per sq. ft.</th>
</tr>
</thead>
<tbody>
<tr>
<td>¼-in. asphalt, 0.5 × 12 (Table 1)</td>
<td>= 6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1¼-in. screeding, 1.5 × 10 (Table 1)</td>
<td>= 15</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4¼-in. slab (Table 1)</td>
<td>= 37</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total load</td>
<td>= 108</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(b) A 4¼-in. floor slab spanning 7 ft. and surfaced with 1-in. boards in residential flats. Since the span is less than 8 ft., the minimum total imposed load will control if the design is to be in accordance with the London By-laws (Class No. 2) or the B.S. Code (Class No. 40); (see Table 3).

Hence the imposed load is 320 lb. ÷ 7 ft. = 46 lb. per sq. ft.

<table>
<thead>
<tr>
<th>Load Description</th>
<th>Load (Table 1)</th>
<th>=</th>
<th>lb. per sq. ft.</th>
</tr>
</thead>
<tbody>
<tr>
<td>4¼-in. slab (Table 1)</td>
<td>= 57</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1-in. boards (Table 1)</td>
<td>= 3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Allow for bearers</td>
<td>= 1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>¼-in. plaster on soffit (Table 1) say</td>
<td>= 5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>= 112</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(c) A 3-in. hollow-block partition 9 ft. high plastered on both faces and supported on a continuous floor slab spanning 10 ft., the partition being placed at midspan and at right-angles to the direction of the span:

Weight of partition, 25 lb. (Table 1) × 9 ft. = 225 lb. per lin. ft. = \( w_p \).

Equivalent distributed load (Table 2), \( w_e = 1.5 \times \frac{225}{10} = 34 \) lb. per sq. ft.

(Minimum, according to the B.S. Code: \( w_e = 0.1w_p = 0.1 \times 225 = 23 \) lb. per sq. ft.)

(d) A secondary beam in an office ground floor; 5-in. slab surfaced with 2-in. granolithic and ¼-in. plaster on underside. Beams 12 in. net depth by 6 in. wide; span 20 ft.; at 10-ft. centres.

Imposed load on 10 ft. width of slab, 10 ft. × 60 lb. (Table 3) = 600 lb. per lin. ft. of beam.

(Check total imposed load = 600 × 20 = 12,000 lb., which exceeds the minimum of 3840 lb.—see Table 3.)

<table>
<thead>
<tr>
<th>Load Description</th>
<th>Load (Table 1)</th>
<th>=</th>
<th>lb. per sq. ft. of slab</th>
</tr>
</thead>
<tbody>
<tr>
<td>5-in. slab (Table 1)</td>
<td>= 63</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Partitions (minimum per Table 2)</td>
<td>= 20</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2-in. granolithic (Table 1)</td>
<td>= 9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>¼-in. plaster (Table 1)</td>
<td>= 5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>= 97</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Deadline load on beam from slab, 10 ft. × 97 lb. = 970 lb. per lin. ft. of beam.

Beam rib, 12 × 6 × 150 ÷ 144 (Table 1) | = 75 |  |

Plaster (hard) on sides of rib,

\[ 2 \times 1 \text{ ft.} \times \frac{1}{2} \times 10 \text{ (Table 1)} = 10 \]  |

1055 |  |

Total dead plus live load = 600 + 1055

= 1655 lb. per lin. ft. of beam.

(e) A 9-in. by 9-in. lintel spanning a clear opening of 7 ft. and supporting a 9-in. brick wall (cement mortar); height of wall exceeds 6 ft.

Height of 60 deg. triangle on 7-ft. base, \( \frac{1}{2} \times 7\sqrt{3} \) (Table 2) = 606 ft.

Weight of wall at 10 lb. per in. thick. (Table 1) = 90 lb. per sq. ft.

Weight supported by lintel, \( \frac{1}{2} \times 606 \times 7 \times 90 \) = 1910 lb.

Weight of lintel, 9 × 9 × 7 × 150 ÷ 144 = 590 lb.

\[ \frac{2500}{2500} \]
**MISCELLANEOUS DEAD LOADS.—TABLE 2.**

<table>
<thead>
<tr>
<th>PARTITIONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>B.S. CODE:</td>
</tr>
<tr>
<td>IF POSITION OF PARTITION IS NOT KNOWN:</td>
</tr>
<tr>
<td>ADDITIONAL DEAD LOAD PER SQ.FT. OF FLOOR ( \pm 0.1 ) (WT. PER LIN. FT. OF PARTITION)</td>
</tr>
<tr>
<td>( \pm 20 ) LB./SQ.FT. FOR OFFICES.</td>
</tr>
<tr>
<td>IF POSITION OF PARTITION IS KNOWN: DESIGN FLOOR TO SUPPORT ACTUAL WEIGHT OF PARTITION.</td>
</tr>
<tr>
<td>EQUIVALENT UNFORMLY-DISTRIBUTED DEAD LOAD ( w_e ) LB./SQ.FT.</td>
</tr>
<tr>
<td>WEIGHT OF PARTITION = ( w_p ) LB./PER LIN. FT.</td>
</tr>
<tr>
<td>PARTITION AT RIGHT-ANGLES TO SPAN OF SLAB:</td>
</tr>
<tr>
<td>SLAB FREELY SUPPORTED: ( w_e = \frac{w_p}{2} )</td>
</tr>
<tr>
<td>SLAB CONTINUOUS OVER BOTH SUPPORTS: ( w_e = \frac{w_p}{1.5} )</td>
</tr>
<tr>
<td>PARTITION PARALLEL TO SPAN OF SLAB:</td>
</tr>
<tr>
<td>( g = d_p + 2 ) (THICKNESS OF SLAB)</td>
</tr>
<tr>
<td>( e = g + 0.06 g \leq g + 3 ) FT. ( w_e = \frac{w_p}{e} )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>LINTELS</th>
</tr>
</thead>
<tbody>
<tr>
<td>LOAD ON LINTELS SUPPORTING BRICK (AND SIMILARLY BONDED) WALLS.</td>
</tr>
<tr>
<td>SHADING DENOTES EXTENT OF WALL CONSIDERED TO BE SUPPORTED BY LINTEL.</td>
</tr>
<tr>
<td>OPENING = ( e )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>ROOFS AND WALLS</th>
</tr>
</thead>
<tbody>
<tr>
<td>CORRUGATED SHEETING (WITH BOLTS, ETC.): GALVANISED IRON OR ASBESTOS-CEMENT</td>
</tr>
<tr>
<td>LIGHT STEEL-.FRAME WALLS WITH 4% BRICK PANELS, WINDOWS, ETC. AVERAGE</td>
</tr>
<tr>
<td>ROOF GLAZING (ASTRAGALS, GLASS, ETC.), STEEL PURLINS AND CONNECTIONS</td>
</tr>
<tr>
<td>SLOPING ROOFS. (FOR WEIGHTS OF TRUSSES—SEE BELOW)</td>
</tr>
<tr>
<td>SLATES OR TILES, BATTENS, STEEL PURLINS, ETC.</td>
</tr>
<tr>
<td>DITTO WITH BOARDING, FELT, ETC.</td>
</tr>
<tr>
<td>CORRUGATED SHEETING (GAL. IRON OR ASBESTOS-CEMENT), STEEL PURLINS, ETC.</td>
</tr>
<tr>
<td>ASBESTOS-CEMENT SHEETING, STEEL PURLINS &amp; TRUSSES, WIND AND LIVE LOAD</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>ROOF STEELWORK</th>
</tr>
</thead>
<tbody>
<tr>
<td>RIVETED STEELWORK = NOMINAL WEIGHT OF STEEL MEMBERS PLUS 10% FOR CLEATS, RIVETS, BOLTS, ETC.</td>
</tr>
<tr>
<td>WELDED STEELWORK = DITTO PLUS 2% TO 5%</td>
</tr>
<tr>
<td>ROLLED SECTIONS: BEAMS = DITTO PLUS 2%</td>
</tr>
<tr>
<td>STANCHIONS = DITTO PLUS 5% (EXTRA FOR CAPS &amp; BASES)</td>
</tr>
<tr>
<td>PLATE-WEB GIRDERS = DITTO PLUS 10% FOR RIVETS, STIFFENERS, ETC.</td>
</tr>
<tr>
<td>CONVEYOR GANTRIES (STEEL FRAMING, CORRUGATED SHEETING, WOODEN FLOOR) 5 TO 6 CWS./LIN. FT.</td>
</tr>
<tr>
<td>STEEL WALL FRAMING TO TAKE SHEETING OR BRICK PANELS 5 TO 7 LB./SQ.FT. OF WALL</td>
</tr>
<tr>
<td>STEEL STAIRS (3 FT. INDUSTRIAL TYPE) 50 LB./PER LINEAR FOOT</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>STRUCTURAL STEELWORK</th>
</tr>
</thead>
<tbody>
<tr>
<td>ROOF</td>
</tr>
<tr>
<td>SPAN OF TRUSS (FT.)</td>
</tr>
<tr>
<td>LB. PER SQUARE FOOT OF PLAN AREA</td>
</tr>
<tr>
<td>TRUSSES</td>
</tr>
<tr>
<td>APPROXIMATE WEIGHT</td>
</tr>
<tr>
<td>OF PLAN AREA</td>
</tr>
</tbody>
</table>

| SMALL ROOF TRUSSES, PURLINS, ETC. (EXCLUDING COVERING) 2½ LB./SQ.FT. OF PLAN AREA |

<table>
<thead>
<tr>
<th>MISCELLANEOUS</th>
</tr>
</thead>
<tbody>
<tr>
<td>RAIL TRACKS (STANDARD GAUGE): RAILS, SLEEPERS, BALLAST, ETC. AVERAGE = 2 CWT. PER SQ. FT.</td>
</tr>
<tr>
<td>RAILS (BULL-HEAD OR FLAT-BOTTOM), CHAIRS, TRANSVERSE SLEEPERS, ETC. = 180 LB./LIN. FT. OF TRACK</td>
</tr>
<tr>
<td>BRIDGE RAILS, LONGITUDINAL SLEEPERS, CONNECTIONS, ETC. = 110</td>
</tr>
<tr>
<td>STEEL-TUBES 2 IN. BORE = 2½ LB./PER LIN. FT.</td>
</tr>
<tr>
<td>¾ IN. GAS PIPE = 1½</td>
</tr>
<tr>
<td>BELT CONVEYORS FOR CEMENT, GRAIN, COAL, CRUSHED STONE, ETC. = ½ TO 2½ CWS./LIN. FT.</td>
</tr>
<tr>
<td>SCREENING PLANT (SHAKER TYPE FOR COAL) INCLUDING SUPPORTING STEELWORK = ½ CWS./SQ.FT.</td>
</tr>
</tbody>
</table>

See also Table 1.
LOADS.

Classification of Floors.—For assessing the imposed (or live) load on floors in buildings, the B.S. Code (No. 3; Chapter V) and the London By-laws classify the floors according to the use to which they are to be put; these classifications and the corresponding minimum imposed loads are given in Table 3. The imposed loads on garage floors are given in Table 5.

Reduction of Imposed Loads on Multiple-story Buildings.—The scales of the reduction of imposed loads on the floors of multiple-story buildings for the design of the columns, walls, and foundations according to B.S. Code (Chapter V, 1952) and the London By-laws (1952) are given in Table 3. No reduction is to be made for factories and workshops designed for 100 lb. per square foot or less, warehouses, garages, and buildings used for storage. For factories and warehouses designed for more than 100 lb. per sq. ft., the reductions given in Table 3 apply, but the total imposed load after reduction is to be not less than 100 lb. per sq. ft. per floor.

Examples.—(a) Calculate the total load on a column supporting a flat roof and ten floors, including the ground floor, of an office building. The total dead weight of the roof is 60 lb. per sq. ft., and of each floor 120 lb. per sq. ft. including partitions. The story-height is 10 ft. and the average size of each column is 18 in. square. Each column supports a panel of floor and roof 20 ft. by 16 ft.

Dead load: Roof

Nine upper floors = 9 × 120 lb. = 1080 lb.

Ground floor = 120 lb.

Total dead load = 1200 lb.

Imposed load: Roof

Nine upper floors = 9 × 0-6 × 50 lb. = 270 lb.

Ground floor = 0-6 × 60 lb. = 36 lb.

Total uniformly-distributed load = 306 lb.

Load on column = 1506 × 20 × 16

Weight of column = (18 × 18 × 150 ÷ 144) × 10 × 10 ft.

= 33,750 lb.

Total load per column (below ground floor) = 544,470 lb.

(b) Assuming that the building in Example (a) is a factory, each floor of which is designed for 150 lb. per sq. ft., calculate the total load on one column.

The normal reduced load for ten floors is 0-6 × 10 × 150 lb., that is 900 lb. per sq. ft., which is less than the specified minimum load of 10 × 100 lb. = 1000 lb. per sq. ft.; the minimum load therefore applies. The total load is therefore

Dead load (as before) = 1200 lb. per sq. ft.

Imposed load: Roof = 30 lb.

Floors = 1000 lb.

Total uniformly-distributed load = 2290 lb.

Load on column = 2290 × 20 × 16 = 732,800 lb.

Weight of column (as before) = 33,750 lb.

Total load on column = 756,550 lb.
## Imposed Loads on Floors, Roofs and Stairs

### Table 3

<table>
<thead>
<tr>
<th>Class No.</th>
<th>B.S. Code CP.3(3) 1952</th>
<th>London By-Laws 1952</th>
<th>Imposed Load (lb, per sq.ft)</th>
<th>Minimum Total Load</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>30</td>
<td>2</td>
<td>5</td>
<td>40, 50, 60, 70, 80, 90, 100, 150, 200</td>
</tr>
<tr>
<td>2</td>
<td>40</td>
<td>3</td>
<td>4</td>
<td>40, 50, 60, 70, 80, 90, 100, 150, 200</td>
</tr>
<tr>
<td>3</td>
<td>50</td>
<td>4</td>
<td>5</td>
<td>40, 50, 60, 70, 80, 90, 100, 150, 200</td>
</tr>
<tr>
<td>4</td>
<td>60</td>
<td>5</td>
<td>6</td>
<td>40, 50, 60, 70, 80, 90, 100, 150, 200</td>
</tr>
<tr>
<td>5</td>
<td>70</td>
<td>6</td>
<td>7</td>
<td>40, 50, 60, 70, 80, 90, 100, 150, 200</td>
</tr>
<tr>
<td>6</td>
<td>80</td>
<td>7</td>
<td>8</td>
<td>40, 50, 60, 70, 80, 90, 100, 150, 200</td>
</tr>
<tr>
<td>7</td>
<td>100</td>
<td>10</td>
<td>10</td>
<td>40, 50, 60, 70, 80, 90, 100, 150, 200</td>
</tr>
</tbody>
</table>

### Use of Floor

- **Residential**
  - Houses: Two Stories (One Occupant)
  - Others (Including Flats)
  - Hospital Wards; Dormitories
  - Hotels: Bedrooms; Private Sitting-Rooms
  - Public Rooms

- **Commercial**
  - Offices: Entrance Floor and Below
  - Other Floors
  - Storage and Filing Rooms
  - Banking Halls
  - Retail Shops
  - Book and Stationery Stores
  - Warehouses (Depending on Type of Goods)
  - Workshops and Factories (Depending on Use)
  - Workrooms: With Central Power-Driven Machines
  - Without Ditti or Storage
  - Power Stations & Machinery Halls: Circulation Space

- **Industrial**
  - School Classrooms
  - Without Fixed Seating; Dance Halls
  - With Fixed Seating; Churches, etc.
  - Restaurants

- **Places of Assembly**
  - Public Pavements over Basements

- **Miscellaneous**
  - Corridors; Same Imposed Load as Floor of Which They Form Part
  - Balconies; Same Imposed Load as Floor to Which They Give Access
  - Minimum Total Load applies if Projection of Canted Levered Balcony is Less Than 8 ft.

### Slope (deg)

- **Flat** Not Steeper than 10 Deg.
  - Without Access 15 lb.
  - With Access 10 lb. (Minimum Total Loads as for Class No. 1 or 20)

- **Not Steeper Than 30 Deg.**
  - Without Access 15 lb.
  - Steeper Than 75° 25 --- 0 lb.
  - Steeper Than 75° Nil

- **Curved**
  - Divide into Four Equal Segments.
  - Imposed and Wind Loads
  - Are Those Applicable to the Slope of the Chord of Each Segment.

- **Not Steeper Than 45 Deg.**
  - Roof Covering (Except Glass) to Be Designed for a Load of 200 lb., Concentrated on 5 in. Square

### Stairs and Landings

- **Class of Floor Supported**
  - No. 30
  - No. 40, 50 or 60

- **Columns, Walls, and Foundations**
  - Number of Floors Supported
  - Proportion of Imposed Load on All Floors Supported
  - Workshops and Factories Designed for More Than 100 lb./sq.ft.
  - Beam Supporting 500 sq.ft. or More of Floor

- **London By-Laws**
  - Total Load (lb) Per ft. Width
  - Total Load (lb) on Beam

### See also
- Table 4 for loads on garage floors;
- Table 2 for loads due to partitions;
- Tables 7 and 8 for wind pressure on roofs.
LIVE LOADS.

Live loads on structures include the weights of stored solid materials and liquids (see Table 4) and the loads imposed by vehicles and moving equipment, the weights of which are given in Tables 5 and 6; notes on the latter tables are given in the following.

Secondary Effects on Railway Bridges.—Weights and other particulars of typical rail vehicles are given in Table 6. To the static forces, such as wheel loads, the following secondary dynamic effects must be added.

Impact on a bridge used by steam locomotives is more severe than on a bridge carrying electric or diesel-operated trains. The effect of impact on the structure is greatly reduced by the provision of ballast, and also depends on rail joints and irregularities in the track and the wheels. Formule suitable for conditions comparable with main lines in Great Britain and elsewhere, and allowances for less-well-laid and maintained tracks, are given in B.S. No. 153 (Part 3A).

Lurching is allowed for on a bridge carrying a main line by assuming that five-eighths of the total axle load may be imposed on any one rail, the remaining three-eighths being carried on the other rail forming the track; these fractions apply to British conditions and can be assumed to be more nearly equal for trains running at low speeds, but more unequal for rolling stock with light springing.

Nosing is allowed for by assuming that a single force of 10 tons acts horizontally at rail level in either direction at right-angles to the track, irrespective of the number of tracks.

The centrifugal effect is considered as a force $F_c$ lb. per lin. ft. acting horizontally at 6 ft. above the level of the rails, where $F_c = \frac{wV^2}{R} \cdot \frac{n}{15}$ in which $w$ lb. per lin. ft. is the live load on one track expressed as an equivalent uniformly-distributed load, $V$ m.p.h. is the speed of the train, $R$ ft. is the mean radius of the track, and $n$ is the number of tracks.

The longitudinal force due to braking and tractive effort is allowed for by introducing at rail level a longitudinal force equal to 10 per cent. of the net live load on one track or 20 per cent. of the total of the net loads on the driving wheels, whichever is greater. For a bridge carrying more than one track the force is calculated assuming two tracks to be occupied, both forces acting in the same direction.

Railway Sleepers.—The various tracks in Table 5 are classified as (i) main lines upon which a frequent service of trains with axle loads up to 23 tons and travelling at speeds exceeding 70 m.p.h. is maintained; (ii) secondary lines upon which the weights and speeds are similar to those for main lines but on which the service is less frequent; (iii) tertiary lines which include lines where speeds not exceeding 45 m.p.h. are reached by trains with axle loads not exceeding 20 tons, and upon which the frequency is less than two-thirds of that on main lines; (iv) refuge sidings, which include heavily-worked sidings, goods loops, and similar lines where the speed of trains does not exceed 30 m.p.h.; (v) sidings which include lightly-worked and storage sidings.

Example of Loading on Deck of Railway Bridge.—To find the maximum total load on a 12-in. concrete slab supporting a single line of standard railway over which can pass at slow speeds any size of locomotive.

Load from ballast, sleepers, chairs, rails, etc. (Table 2) = 224 lb. per sq. ft.
Concrete slab, 12 in. thick (Table 1) = 150

Total dead load = 374

Dispersion of wheel loads (Table 6):
$D$ = say, 12-in. ballast + 10$\frac{1}{4}$-in. effective depth of slab = 1 ft. 10$\frac{1}{4}$ in.
$A = 2D$ + distance over two sleepers (minimum at joints = 3 ft.) = 3 ft. 9 in.
+ 3 ft. = 6 ft. 9 in. (This dimension should not exceed the distance between the axles of the locomotive.)
$B = 2D$ + length of sleeper = 3 ft. 9 in. + 9 ft. = 12 ft. 9 in.

Axle load of 22$\frac{1}{4}$ tons (Table 6), $\frac{22.5 \times 2240}{12.75 \times 6.75} = 586$ lb. per sq. ft.
For infrequent passage and slow speed of such a heavy axle load, allow 50 per cent. for impact and other dynamic effects.
Total load, $374 + (1.5 \times 586) = 1253$ lb. per sq. ft.

(In main-line bridge construction, longitudinal rail-beams might be provided, thereby relieving the slab of loads from the rails and wheels.)
### SOLIDS IN GRANULAR FORM UNLESS STATED OTHERWISE

<table>
<thead>
<tr>
<th>Solids Type</th>
<th>LB. PER CU. FT.</th>
</tr>
</thead>
<tbody>
<tr>
<td>ASHES: GENERAL</td>
<td>40 to 70</td>
</tr>
<tr>
<td>PULVERISED-FUEL</td>
<td></td>
</tr>
<tr>
<td>BRICK: BROKEN, COMMON</td>
<td>70 to 80</td>
</tr>
<tr>
<td>ENGINEERING 80 to 100</td>
<td></td>
</tr>
<tr>
<td>CEMENT: GENERAL</td>
<td>90</td>
</tr>
<tr>
<td>AIR-AGITATED</td>
<td>75</td>
</tr>
<tr>
<td>CLAY: LOOSE IN LUMPS</td>
<td>63</td>
</tr>
<tr>
<td>CLINKER: AVERAGE</td>
<td>60</td>
</tr>
<tr>
<td>COAL: SOLID</td>
<td>80</td>
</tr>
<tr>
<td>LOOSE IN LUMPS</td>
<td>56</td>
</tr>
<tr>
<td>COKE, BREEZE, ETC.</td>
<td>35</td>
</tr>
<tr>
<td>COLLIERY &quot;DIRT&quot; (SHALE, ETC.)</td>
<td>70</td>
</tr>
<tr>
<td>COTTON IN BALES</td>
<td>15 to 40</td>
</tr>
<tr>
<td>EARTH: LOOSE, DRY</td>
<td>90</td>
</tr>
<tr>
<td>COMPACT, DAMP</td>
<td>100</td>
</tr>
<tr>
<td>FLOUR: LOOSE, DRY</td>
<td>40 to 45</td>
</tr>
<tr>
<td>GRAIN: BARLEY</td>
<td>25 to 40</td>
</tr>
<tr>
<td>OATS: 25 to 30</td>
<td></td>
</tr>
<tr>
<td>WHEAT: 40 to 45</td>
<td></td>
</tr>
<tr>
<td>GRAVEL: LOOSE (AVERAGE)</td>
<td>100</td>
</tr>
<tr>
<td>WITH SAND</td>
<td>120</td>
</tr>
<tr>
<td>ALCOHOL</td>
<td>53</td>
</tr>
<tr>
<td>AMMONIA</td>
<td>56</td>
</tr>
<tr>
<td>BEER: G3</td>
<td></td>
</tr>
<tr>
<td>BITUMEN</td>
<td>87</td>
</tr>
<tr>
<td>BENZINE</td>
<td>43</td>
</tr>
<tr>
<td>CLAY SLURRY</td>
<td>76</td>
</tr>
<tr>
<td>CLAY-CHALK SLURRY 100</td>
<td></td>
</tr>
<tr>
<td>LINSEED OIL</td>
<td>56</td>
</tr>
<tr>
<td>MILK</td>
<td>65</td>
</tr>
<tr>
<td>MINERAL OILS (AV.)</td>
<td>58</td>
</tr>
<tr>
<td>NAPHTHA</td>
<td>47</td>
</tr>
<tr>
<td>PARAFIN (KEROSENE)</td>
<td>50</td>
</tr>
<tr>
<td>PETROL (GASOLINE)</td>
<td>44</td>
</tr>
<tr>
<td>PETROLEUM OIL</td>
<td>51</td>
</tr>
<tr>
<td>SEWAGE</td>
<td>G2 to 75</td>
</tr>
<tr>
<td>SULPHURIC ACID (CONC.)</td>
<td>115</td>
</tr>
<tr>
<td>TAR</td>
<td>G4</td>
</tr>
<tr>
<td>TURPENTINE</td>
<td>G2</td>
</tr>
<tr>
<td>WATER, FRESH</td>
<td>G2-4</td>
</tr>
<tr>
<td>SEA-WATER</td>
<td>G4</td>
</tr>
<tr>
<td>WINE</td>
<td>G2</td>
</tr>
</tbody>
</table>

### LIQUIDS AND SEMI-LIQUIDS

<table>
<thead>
<tr>
<th>Solids Type</th>
<th>LB. PER CU. FT.</th>
</tr>
</thead>
<tbody>
<tr>
<td>ALCOHOL</td>
<td>53</td>
</tr>
<tr>
<td>AMMONIA</td>
<td>56</td>
</tr>
<tr>
<td>BEER: G3</td>
<td></td>
</tr>
<tr>
<td>BITUMEN</td>
<td>87</td>
</tr>
<tr>
<td>BENZINE</td>
<td>43</td>
</tr>
<tr>
<td>CLAY SLURRY</td>
<td>76</td>
</tr>
<tr>
<td>CLAY-CHALK SLURRY 100</td>
<td></td>
</tr>
<tr>
<td>LINSEED OIL</td>
<td>56</td>
</tr>
<tr>
<td>MILK</td>
<td>65</td>
</tr>
<tr>
<td>MINERAL OILS (AV.)</td>
<td>58</td>
</tr>
<tr>
<td>NAPHTHA</td>
<td>47</td>
</tr>
<tr>
<td>PARAFIN (KEROSENE)</td>
<td>50</td>
</tr>
<tr>
<td>PETROL (GASOLINE)</td>
<td>44</td>
</tr>
<tr>
<td>PETROLEUM OIL</td>
<td>51</td>
</tr>
<tr>
<td>SEWAGE</td>
<td>G2 to 75</td>
</tr>
<tr>
<td>SULPHURIC ACID (CONC.)</td>
<td>115</td>
</tr>
<tr>
<td>TAR</td>
<td>G4</td>
</tr>
<tr>
<td>TURPENTINE</td>
<td>G2</td>
</tr>
<tr>
<td>WATER, FRESH</td>
<td>G2-4</td>
</tr>
<tr>
<td>SEA-WATER</td>
<td>G4</td>
</tr>
<tr>
<td>WINE</td>
<td>G2</td>
</tr>
</tbody>
</table>

### MATHEMATICAL EQUATIONS

\[
\omega_e = \text{Density of Liquid (lb. per cu. ft.)}
\]

\[
p_v = \text{Pressure on bottom of container (lb. per sq. ft.)}
\]

\[
h = \text{Total depth of liquid (ft.)}
\]

\[
h_m = \text{Thickness of layer of submerged liquid (ft.)}
\]

\[
\omega_m = \text{Weight of solid material (density in lb. per cu. ft.)}
\]

\[
V = \text{Volume of voids in 1 cu. ft. of dry granular material}
\]

\[
\text{Solid materials in liquids}
\]

\[
\omega_e = \frac{\omega_m}{h_m + \omega_e h}
\]

\[
p_v = h_m \left[\frac{1 - V}{1 - V} + \omega_e V\right] + \omega_e (h - h_m)
\]

### Table 4

<table>
<thead>
<tr>
<th>Material</th>
<th>Percentage of Voids ((%))</th>
</tr>
</thead>
<tbody>
<tr>
<td>CRUSHED COAL</td>
<td>77</td>
</tr>
<tr>
<td>CRUSHED STONE SAND</td>
<td>136</td>
</tr>
<tr>
<td>CRUSHED STONE SAND</td>
<td>131</td>
</tr>
<tr>
<td>SOFT-WOODS</td>
<td>30 to 45</td>
</tr>
<tr>
<td>HARD-WOODS</td>
<td>45 to 80</td>
</tr>
<tr>
<td>BASALT, DOLORITE</td>
<td>110 to 140</td>
</tr>
<tr>
<td>LIMESTONE, SANDSTONE</td>
<td>80 to 120</td>
</tr>
<tr>
<td>SHALE</td>
<td>70</td>
</tr>
<tr>
<td>QUARRY WASTE</td>
<td>30</td>
</tr>
<tr>
<td>SUGAR</td>
<td>50</td>
</tr>
<tr>
<td>TEA</td>
<td>28</td>
</tr>
<tr>
<td>TIMBER</td>
<td>40</td>
</tr>
<tr>
<td>SOFT-WOODS</td>
<td>30 to 45</td>
</tr>
<tr>
<td>HARD-WOODS</td>
<td>45 to 80</td>
</tr>
</tbody>
</table>

Other Materials.—Stone, timber, etc., in solid (no voids): see Table 1. Granular materials such as sand, crushed stone and coal, gravel, etc., see Table 11. Cohesive materials such as clay, earth, etc., see Table 12.
Garage Floors: Example.—To find the equivalent uniformly-distributed imposed load at the support section of a beam 12 in. wide, spacing 10 ft., span 10 ft., fully continuous, in a garage floor; the maximum wheel load is 1 1/4 tons. From Table 5, f = 0.95 and w = 330 lb. per ton. With P = 1 1/4 tons, the equivalent uniform load is 1.5 × 330 × 0.95 × 1.5 = 705 lb. per lin. ft.; compare with the normal imposed load of 150 lb. per sq. ft. = 10 × 150 = 1500 lb. per lin. ft. Hence the normal load controls; to the load of 1500 lb. per ft. must be added the dead load of the slab and beams. This combined load, say 2200 lb. per lin. ft., only affects the bending moment, which is about

\[
\frac{2200 \times 10^2 \times 12}{12} = 220,000 \text{ in.-lb.}
\]

The greatest shearing force is 1.5 × 1 1/4 = 2.25 tons plus the shearing force due to the dead load, that is about 8540 lb.

Overhead Travelling Cranes.—To allow for vibration, acceleration, and impact, the maximum static wheel loads, as given in Table 5, of electric overhead travelling cranes should be increased by 25 per cent. Braking or travelling under power produces in the rail-beam a horizontal thrust which is transferred to the supports. The traversing of the crane and load produces a horizontal thrust transversely to the rail-beam. Therefore the additional forces acting on the supporting structure when the crane is moving are (a) a horizontal force acting transversely to the rail and equal to 10 per cent. of the weight of the crane and the load lifted, it being assumed that the force is equally divided between the two rails; (b) a horizontal force acting along each rail and equal to 5 per cent. of the total weight of the crane, crane bridge, and the load lifted (when in the position nearest to the rail). The forces (a) and (b) are not considered to act simultaneously, but the effect of each must be combined with that of the increased maximum vertical wheel loads. The foregoing recommendations are in accordance with B.S. No. 449.

For a crane operated by hand, the vertical wheel loads should be increased by 10 per cent.; for force (a), the proportion of the weight of crane and load can be 5 per cent.

**Loads on Colliery Pit-head Frames.**—The loads to which a pit-head frame is subjected are as follows. (These notes do not apply to the direct vertical winding type of pit-head tower.)

**Dead Loads.**—The dead loads include the weights of (i) the frame and any stairs, housings, lifting beams, etc., attached to it; (ii) winding pulleys, pulley-bearings, pedestals, etc.; (iii) guide and rubbing ropes plus 50 per cent. for vibration.

**Live Loads.**—The live loads are the resultants of the tensions in the ropes passing over the pulleys and (unless described otherwise) are transmitted to the frame through the pulley bearings and may be due to the following conditions. (a) Retarding of descending cage when near the bottom of the shaft; this force is the sum of the net weight of the cage, load, and rope, and should be doubled to allow for deceleration, shock and vibration. (b) Force due to overwinding the cage which is then dropped on to the over-wind platform; this force acts only on the platform (and not at the pulley bearings) and is the sum of the net weights of the cage and attachments and the load in the cage, which sum should be doubled to allow for impact. (c) Force causing rope to break due to cage sticking in shaft or other causes; the force in the rope just before breaking is the tensile strength of the rope. (d) Tension in rope when winding up a loaded cage.

**Combined Loads.**—For a frame carrying one pulley, the conditions to be designed for are the total dead load combined with either live load (a), (b), (c) or (d). Generally condition (c) gives the most adverse effects, but it is permissible in this case to design for stresses, say, double the ordinary permissible working stresses because of the short duration of the maximum force; the procedure would be to design the frame for dead load plus half of force (c) and adopt the ordinary working stresses. If the frame carries two pulleys, the conditions to be investigated are: dead load plus (a) on one rope and (a) on the other (this is the ordinary working condition); dead load plus (a) on one rope and over-wind (b) on the other; dead load plus (a) on one rope and breaking force (c) on other rope (this is generally the worst case; force (c) can be halved as explained for a single-pulley frame).

The weights of the ropes, cages, etc., and the strength of the ropes would be obtained from the colliery pit-head frame from the colliery authorities, and vary too greatly for typical values to be of any use.

**Structures Supporting Lifts.**—The effect of acceleration must be considered in addition to the static loads when calculating the load due to lifts and similar machinery. If a net static load of \( W_B \) is subject to an acceleration of \( a \) feet per second per second the load on the supporting structure is given approximately by \( W_M = W_B (1 + 0.03a) \). The average acceleration of a passenger lift may be about 2 ft. per second per second, but the maximum acceleration will be considerably greater. An equivalent load of 1.25\( W_B \) should be taken as the minimum to allow for dynamic effects. The load for which lift supports, pit-head (Continued on page 150.)
### LIVE LOADS—TABLE 5.

<table>
<thead>
<tr>
<th>Gross Weight of Vehicle</th>
<th>Use of Floor</th>
<th>Member</th>
<th>Ordinary Imposed Load (lb. per sq. ft.)</th>
<th>Alternative Minimum Imposed Load</th>
</tr>
</thead>
<tbody>
<tr>
<td>Not greater than 2½ tons</td>
<td>Parking Only</td>
<td>Slabs and Beams</td>
<td>80 *</td>
<td>Most adverse arrangement of actual wheel loads</td>
</tr>
<tr>
<td></td>
<td>Garage</td>
<td>Slabs</td>
<td>80 *</td>
<td>80 lb./sq.ft. or most adverse arrangement of actual wheel loads, whichever is greater</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Beams</td>
<td></td>
<td>50 # x</td>
</tr>
<tr>
<td>Not greater than 4 tons</td>
<td>Garage</td>
<td>Slabs and Beams</td>
<td>150 *</td>
<td>2000 lb. (minimum) or not less than 1½ x actual wheel load, whichever is greater distributed on area 2 ft. g. in. square</td>
</tr>
</tbody>
</table>

#### Equivalent Uniformly-Distributed Load (w) for B.M. Calculation due to One Ton Distributed over Area 2 ft. g. in. Square

<table>
<thead>
<tr>
<th>Span of Slab</th>
<th>4 ft.</th>
<th>6 ft.</th>
<th>8 ft.</th>
<th>10 ft.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Midspan</td>
<td>310</td>
<td>235</td>
<td>190</td>
<td>140</td>
</tr>
<tr>
<td>Midspan Fixed Support</td>
<td>320</td>
<td>270</td>
<td>230</td>
<td>200</td>
</tr>
<tr>
<td>Midspan Support</td>
<td>300</td>
<td>220</td>
<td>170</td>
<td>140</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Span of Beam</th>
<th>10 ft.</th>
<th>12 ft. g. in.</th>
<th>15 ft.</th>
<th>20 ft.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Midspan</td>
<td>390</td>
<td>320</td>
<td>270</td>
<td>205</td>
</tr>
<tr>
<td>Midspan Fixed Support</td>
<td>500</td>
<td>430</td>
<td>360</td>
<td>280</td>
</tr>
<tr>
<td>Midspan Support</td>
<td>340</td>
<td>280</td>
<td>240</td>
<td>170</td>
</tr>
</tbody>
</table>

#### Reduction Factors

<table>
<thead>
<tr>
<th>Spacing of Beams</th>
<th>5 ft.</th>
<th>10 ft.</th>
<th>15 ft.</th>
<th>Over 15 ft.</th>
</tr>
</thead>
<tbody>
<tr>
<td>f</td>
<td>0.90</td>
<td>0.95</td>
<td>0.975</td>
<td>1.0</td>
</tr>
</tbody>
</table>

#### Overhead Travelling Cranes (Electric)

<table>
<thead>
<tr>
<th>Lifting Capacity (Tons)</th>
<th>Minimum Wheel Base (not less than)</th>
<th>Weight of Crane (Exc. Load) (Tons)</th>
<th>Max. Load (Tons) on Pair of Wheels</th>
<th>Clearances</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>6 ft.</td>
<td>6 to 9</td>
<td>5½</td>
<td>6</td>
</tr>
<tr>
<td>5</td>
<td>8½ x</td>
<td>11½ to 16</td>
<td>11½</td>
<td>11½ 13</td>
</tr>
<tr>
<td>10</td>
<td>14½ to 19</td>
<td>18</td>
<td>20</td>
<td>20 2½ 6½</td>
</tr>
<tr>
<td>20</td>
<td>21 to 26½</td>
<td>31</td>
<td>33</td>
<td>33 35½ 7½</td>
</tr>
<tr>
<td>30</td>
<td>25 to 33</td>
<td>46</td>
<td>48</td>
<td>48 51 8½</td>
</tr>
<tr>
<td>50</td>
<td>43 to 51½</td>
<td>70</td>
<td>72</td>
<td>72 81 10½</td>
</tr>
</tbody>
</table>

#### Railway Sleepers (Reinforced Concrete)

<table>
<thead>
<tr>
<th>Type of Line (See Chart II)</th>
<th>Load (P) from Each Rail SLEEPERS AT JOINTS</th>
<th>Minimum Moment of Resistance at Centre</th>
<th>Distribution of Pressure on Ballast (for purposes of design)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) Main Line</td>
<td>14 Tons</td>
<td>54,500 in.-lb.</td>
<td>P = \frac{1}{2} to \frac{1}{3}</td>
</tr>
<tr>
<td>(2) Secondary Line</td>
<td>12 Tons</td>
<td>45,000</td>
<td>P = \frac{1}{2}</td>
</tr>
<tr>
<td>(3) Tertiary Line</td>
<td>10 Tons</td>
<td>30,000</td>
<td>P = \frac{1}{2}</td>
</tr>
<tr>
<td>(4) Goods Lines</td>
<td>7½ Tons</td>
<td>23,000</td>
<td>P = \frac{1}{2}</td>
</tr>
<tr>
<td>(5) Sidings</td>
<td>5½ Tons</td>
<td>23,000</td>
<td>P = \frac{1}{2}</td>
</tr>
</tbody>
</table>

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frames, and similar structures are designed should be related to the load on the ropes. If the latter is $W_M$ and the ropes have a factor of safety of ten, the design load on the supports should not be less than $24W_M$ to ensure that the structure is as strong as the ropes.

Weights and Dimensions of Road Vehicles.—The data in Table 6 relating to heavy motor vehicles, trailers, public-service vehicles, and road locomotives are abstracted from "The Motor Vehicles (Construction and Use) Regulations 1955" (issued on behalf of the Ministry of Transport). As there are many varieties of vehicle only the maximum loads and dimensions permissible are given. In general the Regulations apply to vehicles registered in or after 1955; vehicles registered before that date may have greater dimensions and weights. Supplementary data are as follows. (Street-cleaning and road-repair vehicles are excluded from Table 6.)

Load imposed by the wheels of a vehicle on a strip of road 2 ft. wide and transverse to the longitudinal axis of the vehicle must not exceed 12 tons.

Tractors.—Overall width must not exceed 7 ft. 6 in. and overhang must not exceed 6 ft.

Motor cars.—Overall length must not exceed 30 ft.; overall width must not exceed 7 ft. 6 in.; overhang must not exceed half the wheel-base. For vehicles not exceeding 20 ft. long, overhang may be 9 in. plus half the wheel-base and not more than $\frac{1}{2}$ of overall length. For rear-tipping lorries, overhang need not conform to the foregoing if not exceeding 3 ft. 9 in.

Trailers.—Total weight of trailers drawn by one locomotive must not exceed 40 tons.

Articulated vehicles.—Overall length must not exceed 35 ft. (except for a vehicle for an indivisible load). Laden weight of trailer must not exceed 20 tons if having not more than four wheels or 24 tons if more than four wheels.

Standard Live Loads for Road Bridges.

Ministry of Transport.—The uniformly-distributed load applicable to the "loaded length" of a bridge or of a structural member forming part of a bridge is selected from Table 6.

The "loaded length" is the length of member which should be considered to be carrying load in order to produce the greatest stresses. On a freely-supported span, the "loaded length" would thus be (a) for bending moment: the entire span; (b) for shearing force at the support: the entire span; and (c) for shearing force at any intermediate section: from the section to the farther support. In arches and continuous spans the "loaded length" can be taken from influence lines.

The live load to be used is in two parts: (1) the uniformly-distributed load which varies with the loaded length; (2) an invariable knife-edge load of 2700 lb. per ft. of width applied at the section where it will, when combined with the uniformly-distributed load, be most effective, that is, on a freely-supported span: (a) for bending moment at midspan: at mid-span; (b) for shearing force at the support: at the support; and (c) for shearing force at any section: at the section.

On slabs, the knife-edge load of 2700 lb. per ft. width is assumed to act parallel to the supporting members (that is, at right-angles to the direction in which the slab spans). On longitudinal girders, stringers, etc., this load is assumed to act transversely to them (that is, parallel to their supports). On transverse beams the load is assumed to act in line with them (that is, 2700 lb. per ft. of beam).

If longitudinal or transverse members are spaced more closely than at 5-ft. centres, the live load allocated to them must be that calculated on a strip 5 ft. wide. With wider spacing the width of this strip will be equal to the spacing of the members.

B.S. No. 153 (Part 3a): Design Loads.—Slabs spanning in One Direction: uniformly-distributed load = $w_L$ lb. per sq. ft. (Table 6); knife-edge load = 2700 lb. per ft. at right-angles to span; transverse reinforcement = sufficient to resist 50 per cent. of live-load bending moment.

Slabs spanning in Two Directions and Cantilevered Slabs (not less than 4-ft. projection): two wheel loads each equal to $W$ (Table 6).

Aircraft Runways.—Aircraft runways and aprons are designed to support a basic load designated as the Equivalent Single Wheel Load (E.S.W.L.), which is equal to the actual wheel load if each undercarriage of the aircraft has only one wheel and, in the case of multiple-wheel undercarriages, is the single wheel load that produces the same stresses in the slab as the group of wheels at the same tyre pressures. The E.S.W.L. varies with the thickness and type of paving, and for each type of aircraft corresponding values are specified by the authorities.

In British practice, a Load Classification Number (L.C.N.) is attributed to each thickness and type of paving and for each aircraft; a given paving can be used by aircraft having the same or lower L.C.N. as the paving. The L.C.N. of an aircraft is equal to the E.S.W.L. (lb.) divided by 1000. The International Civil Aircraft Organisation (I.C.A.O.) has a different load

(Continued on page 152.)
WEIGHTS OF VEHICLES.—TABLE 6.

<table>
<thead>
<tr>
<th>RAILWAY ROLLING STOCK (BRITISH RAILWAYS)</th>
<th>LOGOMOTIVES</th>
<th>TYPE</th>
<th>EXPRESS</th>
<th>MIXED TRAFFIC</th>
<th>SHunting</th>
<th>EXPRESS</th>
<th>NOTE</th>
</tr>
</thead>
<tbody>
<tr>
<td>WEIGHT</td>
<td>G N 1/4 TON 2 TON 2 1/2 TON 3 TON 2 1/4 TON TENDER</td>
<td>13</td>
<td>157</td>
<td>86</td>
<td>5</td>
<td>106</td>
<td></td>
</tr>
<tr>
<td>AXLE LOAD (MAX.) 24 TON 36 TON 40 TON 48 TON 50 TON 60 TON</td>
<td>15</td>
<td>22</td>
<td>17</td>
<td>18</td>
<td>18</td>
<td>2</td>
<td>3-20</td>
</tr>
</tbody>
</table>

DESCRIPTION OF VEHICLE WEIGHTS (TONS)

<table>
<thead>
<tr>
<th>MAX. AXLE LOAD</th>
<th>TOTAL WT.</th>
<th>LADEN</th>
<th>OVER LENGTH</th>
<th>OVERALL LENGTH</th>
<th>GAUGE</th>
<th>WHEEL BASE</th>
</tr>
</thead>
<tbody>
<tr>
<td>OPEN MINERAL WAGONS 24 TON 4 WHEELS 5 TON 8 WHEEL HOPPERED (GAGE)</td>
<td>17</td>
<td>21</td>
<td>34</td>
<td>85</td>
<td>6</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>MISC.</th>
<th>COLLIERY TUBS AND MINE CARS</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>1/2 TO 1/2</td>
</tr>
</tbody>
</table>

ROAD VEHICLES

<table>
<thead>
<tr>
<th>TYPE</th>
<th>NUMBER</th>
</tr>
</thead>
<tbody>
<tr>
<td>STREET TRAM CAR (8 WHEELS)</td>
<td>7 1/4 (DRIVING) 3 1/2 (PONY) 22 1/2</td>
</tr>
</tbody>
</table>

ROLLERS

| STEAM (20 TONS) | 8 (FRONT ROLLER) 12 (DRIVING) 20 |
| DIESSEL 3-WHEEL | 8 |

<table>
<thead>
<tr>
<th>HEAVY MOTOR VEHICLES</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>MAX. 4 WHEELS</td>
<td>16</td>
</tr>
<tr>
<td>&gt;= 4</td>
<td>20</td>
</tr>
<tr>
<td>&gt;= 6</td>
<td>24</td>
</tr>
<tr>
<td>4</td>
<td>12</td>
</tr>
<tr>
<td>4</td>
<td>14</td>
</tr>
<tr>
<td>5</td>
<td>15</td>
</tr>
<tr>
<td>6</td>
<td>18</td>
</tr>
</tbody>
</table>

PUBLIC SERVICE VEHICLES

<table>
<thead>
<tr>
<th>TRAILERS (EXCLUDING VEHICLE FOR INDIVIDUAL LOAD, PART OF ARTICULATED VEHICLE ON LAND IMPLEMENT)</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
</tr>
<tr>
<td>G = 22</td>
</tr>
<tr>
<td>G = 26</td>
</tr>
<tr>
<td>G = 30</td>
</tr>
</tbody>
</table>

ROAD LOCO (M.O.T.)

<table>
<thead>
<tr>
<th>ROAD AREA</th>
</tr>
</thead>
<tbody>
<tr>
<td>11</td>
</tr>
</tbody>
</table>

AIRCRAFT

<table>
<thead>
<tr>
<th>CONTACT AREA = E.S.W.L. X TYRE PRESSURE</th>
</tr>
</thead>
<tbody>
<tr>
<td>LOAD CLASSIFICATION N.P. (L.C.N.)</td>
</tr>
<tr>
<td>I.C.A.O. (1955)</td>
</tr>
<tr>
<td>130</td>
</tr>
<tr>
<td>MINISTRY</td>
</tr>
<tr>
<td>500</td>
</tr>
</tbody>
</table>

NORMAL UNIFORMLY DISTRIBUTED LOAD W L/B. PER SQUARE FOOT

<table>
<thead>
<tr>
<th>STANDARD LOADS</th>
<th>MINISTRY OF TRANSPORT</th>
</tr>
</thead>
<tbody>
<tr>
<td>24 TON 1700 1225</td>
<td>872 644</td>
</tr>
<tr>
<td>220 220 220</td>
<td></td>
</tr>
<tr>
<td>220 212 208 192</td>
<td></td>
</tr>
</tbody>
</table>

WHEEL LOADS (B.S. 153)

<table>
<thead>
<tr>
<th>WHEEL LOADS</th>
<th>DIRECTION OF TRAVEL</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 FT.</td>
<td></td>
</tr>
</tbody>
</table>

SEE NOTES ON PAGE OPPOSITE

<table>
<thead>
<tr>
<th>ABNORMAL LOAD</th>
</tr>
</thead>
<tbody>
<tr>
<td>(M. O. T.)</td>
</tr>
<tr>
<td>B.S. 153</td>
</tr>
</tbody>
</table>

DISPERSION OF WHEEL LOADS ON ROAD BRIDGES, ETC.

<table>
<thead>
<tr>
<th>DISPERSION OF WHEEL LOADS ON RAILS</th>
</tr>
</thead>
<tbody>
<tr>
<td>A = 2A + WIDTH OVER TWO SLEEPERS</td>
</tr>
<tr>
<td>B = 2A + LENGTH OF SLEEPER</td>
</tr>
</tbody>
</table>

FOR TOTAL AXLE LOAD:

<table>
<thead>
<tr>
<th>DISPERSION OF WHEEL LOADS ON RAILS</th>
</tr>
</thead>
<tbody>
<tr>
<td>A = 2D + WIDTH OVER TWO SLEEPERS</td>
</tr>
<tr>
<td>B = 2D + LENGTH OF SLEEPER</td>
</tr>
</tbody>
</table>

151
Aircraft Runways (continued from page 150).

classification, and the equivalent classification and L.C.Ns. are given in Table 6. Design of a pavement depends upon the loading, the contact area (= E.S.W.L. ÷ tyre pressure), and the supporting power of the ground. Values of contact area and the corresponding L.C.N. in accordance with a standard loading curve are given in Table 6. Some examples of contact area (and in brackets the corresponding E.S.W.L.) for various aircraft are as follows.

Dakota, 270 (12,000); Viscount 700, 270 (24,000); Comet 4, 225 (35,000); Constellation, 490 (37,500); Boeing 707, 360 (50,000); and D.C.8, 430 (56,000).


EFFECTS OF WIND.

Wind Pressures on Buildings (Table 8).

Exposures.—Exposure D corresponds to a wind having a velocity of 72 m.p.h. and is applicable to exposed sites near the coast and on inland mountains in Great Britain. Exposure C corresponds to a velocity of 63 m.p.h. and applies to inland sites in open country not exceeding 800 ft. in altitude. Exposure B is for general use and is the condition generally accepted by the London By-laws; it corresponds to a velocity of 54 m.p.h. and applies to inland sites at altitudes not exceeding 500 ft. For sites well protected from the wind by natural features, exposure A (45 m.p.h.) applies.

Pressures.—The basic pressures \( p \) lb. per sq. ft. at any height and for any velocity \( V \) m.p.h. are calculated from \( p = 0.000125V^2p_d \), in which \( p_d \) is the pressure for the same height given for exposure D \( (V = 72 \text{ m.p.h.}) \) in Table 8. The height of the building on which the value of \( p \) is based is the distance from the ground to a point midway between the eaves and the ridge if the roof is sloping, or to the eaves or top of the parapet if the roof is flat. The value of \( p \) for projections above the general level of the roof is based on the total height of the projection above the ground.

General Formula for Wind Pressure. (Table 7).—The following example shows the application of the formula for the velocity and pressure of the wind as given in Table 7.

Example.—If meteorological records for a certain site show that the probable maximum velocity of the wind is 75 m.p.h. at a height of 33 ft. (10 metres), estimate the pressures on a circular chimney erected on this site, and having a height of 200 ft.

For \( V_1 = 75 \text{ m.p.h.}, \ V_H = 17.5 + 49.3 \log H + 16 \).

Allowing a shape factor of 0.77, \( p = 0.0034V_H^2 \times 0.77 = 0.0026V_H^2 \).

Substitution of various values of \( H \) gives

<table>
<thead>
<tr>
<th>( H ) (ft.)</th>
<th>200</th>
<th>150</th>
<th>100</th>
<th>50</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V_H ) (m.p.h.)</td>
<td>107</td>
<td>101</td>
<td>94</td>
<td>82</td>
</tr>
<tr>
<td>( p ) (lb. per sq. ft.)</td>
<td>30</td>
<td>27</td>
<td>24</td>
<td>18</td>
</tr>
</tbody>
</table>

Example of Wind Effects on a Water Tower.—To find the overturning moment due to wind on a water tower comprising a tank of 20 ft. diameter and 12 ft. high supported on an enclosed tower 14 ft. square, the distance from the ground to the underside of the tank being 50 ft. The foundation is 3 ft. below ground. Assume a wind 60 m.p.h. at 40 ft.; therefore from Table 8 the average pressure on the tower \( (h = 25 \text{ ft. average}) \) is about 13 lb. per sq. ft., and 15 lb. per sq. ft. on the tank \( (h = 56 \text{ ft. average}) \).

Total wind pressure on tank = \( 20 \times 12 \times 15 \text{ lb.} \times 0.66 = 2400 \text{ lb.} \)

The factor 0.66 is the reduction due to circular shape \( (Table 7) \); ratio of \( H \) to \( B \) is about 4.

Centre of pressure = \( 3 + 50 + (0.5 \times 12) = 59 \text{ ft.} \) above foundation.

Total wind pressure on tower \( \text{(wind normal to face)} = 14 \times 50 \times 13 \times 1.1 = 10,000 \text{ lb.} \)

Ditto (wind normal to diagonal) = \( 14\sqrt{2} \times 50 \times 13 \times 0.88 = 11,300 \text{ lb.} \)

The factor 0.88 is the reduction due to wind normal to a diagonal \( (Table 7) \).

Centre of pressure = \( 3 + (0.5 \times 50) = 28 \text{ ft.} \) above foundation.

Overturning moment about axis parallel to a face of the tower

\( = (2400 \times 59) + (10,000 \times 28) = 421,000 \text{ ft.-lb.} \)

Overturning moment about a diagonal

\( = (2400 \times 59) + (11,300 \times 28) = 453,000 \text{ ft.-lb.} \)
### Wind Velocities and Pressures

#### Table 7

**Relation of Velocity to Height**

\[ V_H = V_M \cdot (1 + 2.81 \log_{10} (H + 10)) \]

**Relation of Gross Pressure to Velocity**

\[ P = 0.0034 \cdot V^2 \]

**Variation of Pressure on Vertical Cylindrical Surface**

- \( P_1 = 1.0 \cdot P \)
- \( P_2 = 2.0 \cdot P \)
- \( P_3 = 4.0 \cdot P \)
- \( \theta = 33^\circ \)

**Variation of Pressure on Buildings (No Openings in Walls)**

- For pitch of roof < 30°
- For pitch of roof = 30°
- For pitch of roof > 30°

---

**General Effects**

<table>
<thead>
<tr>
<th>No. on Beaufort Scale</th>
<th>Description of Wind</th>
<th>General Effects</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Calm</td>
<td>Smoke rises vertically.</td>
</tr>
<tr>
<td>1</td>
<td>Light Air</td>
<td>Direction shown by smoke.</td>
</tr>
<tr>
<td>2</td>
<td>Light Breeze</td>
<td>Wind vanes not affected.</td>
</tr>
<tr>
<td>3</td>
<td>Gentle Breeze</td>
<td>Leaves and small twigs in motion.</td>
</tr>
<tr>
<td>4</td>
<td>Moderate Breeze</td>
<td>Light flags extended.</td>
</tr>
<tr>
<td>5</td>
<td>Fresh Breeze</td>
<td>Small trees in leaf sway.</td>
</tr>
<tr>
<td>6</td>
<td>Strong Breeze</td>
<td>Crested wavelets on inland water.</td>
</tr>
<tr>
<td>7</td>
<td>Moderate Gale</td>
<td>Large branches in motion; telegraph wires whistle.</td>
</tr>
<tr>
<td>8</td>
<td>Fresh Gale</td>
<td>Whole trees in motion; walking inconvenient.</td>
</tr>
<tr>
<td>9</td>
<td>Strong Gale</td>
<td>Twigs break off; walking impeded.</td>
</tr>
<tr>
<td>10</td>
<td>Whole Gale</td>
<td>Slight structural damage.</td>
</tr>
<tr>
<td>11</td>
<td>Storm</td>
<td>Trees uprooted; structural damage.</td>
</tr>
<tr>
<td>12</td>
<td>Hurricane</td>
<td>Wide-spread damage.</td>
</tr>
<tr>
<td>~</td>
<td>Violent Hurricane</td>
<td></td>
</tr>
</tbody>
</table>

**Ratio of Height to Base**

<table>
<thead>
<tr>
<th>( H/B )</th>
<th>Not Greater Than 4</th>
<th>From 4 to 8</th>
<th>Not Less Than 8</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.6</td>
<td>0.60 (0.66)</td>
<td>0.70 (0.77)</td>
<td></td>
</tr>
<tr>
<td>0.8</td>
<td>0.80 (0.88)</td>
<td>1.0 (1.1)</td>
<td></td>
</tr>
<tr>
<td>0.9</td>
<td>0.90 (0.99)</td>
<td>1.1 (1.2)</td>
<td></td>
</tr>
<tr>
<td>1.0</td>
<td>1.0 (1.1)</td>
<td>1.2 (1.3)</td>
<td></td>
</tr>
<tr>
<td>1.1</td>
<td>1.1 (1.2)</td>
<td>1.3 (1.4)</td>
<td></td>
</tr>
</tbody>
</table>

**Reduction Factors Due to Shape**

- **Circle**
- **Octagon**
- **Square (Wind Normal to Diagonal Face)**

**Open Frames (Unclad Structures)**

- Members of Non-Circular Cross-Section: Ditto
- Members of Circular Cross-Section: Ditto
- Single Isolated Members of Circular Cross-Section: Ditto

**Chimneys and Sheeted Towers (Clad Structures)**

**Reduction Factors**: B.S. Code No. 3
Example.—A building having the cross-sectional dimensions shown in the accompanying diagram is on a site graded as exposure B. In the centre there is a tower 18 ft. wide in a longitudinal direction. The slope of the pitched roof is 3:1. Determine the wind forces for which the various parts of the building should be designed.

Stability and Foundations.

(i) The effective height of main part of building from ground to midpoint between eaves and ridge is 67.5 + (0.5 x 5) = 70 ft. The basic wind pressure (p) for this height (by interpolation) is 11.5 lb. per sq. ft., acting as a positive pressure of 0.5 x 11.5 = 5.75 lb. per sq. ft. on the windward face and a negative pressure (suction) of 5.75 lb. per sq. ft. on the leeward face. The total horizontal force on windward (and on leeward) vertical face is

$$100 \times 67.5 \times 5.75 = 38,613 \text{ lb.}$$

(ii) The pressure normal to windward slope of roof (slope is about 20 deg.) is

$$0.4 \times 11.5 = -4.6 \text{ lb. per sq. ft.}$$

The total pressure normal to windward slope is (100 - 18)75.8 \times (-4.6) = -5970 \text{ lb.}

Horizontal component of ditto $= \frac{1}{\sqrt{10}}(-5970) = -1890 \text{ lb.}$

(iii) The pressure normal to the leeward slope is $-0.45 \times 11.5 = -5.2 \text{ lb. per sq. ft.}$

Horizontal component of pressure normal to leeward slope (by proportion to windward slope) = $-1890 \times \frac{5.2}{4.6} = -2130 \text{ lb.}$

(iv) The height of projection (tower) above ground is 67.5 + 22.5 = 90 ft.

Basic wind pressure for 90 ft. is 12.5 lb. per sq. ft., that is 0.25 lb. per sq. ft. pressure on the windward face and 6.25 lb. per sq. ft. suction on the leeward face. The total horizontal force on windward (and on leeward) vertical face of tower is

$$6.25 \times 22.5 \times 18 = 2525 \text{ lb.}$$

The pressure on the flat roof of the tower has no horizontal component.

Design of Walls.

Assuming ordinary proportion of openings, the walls of the main building as a unit should be designed for a pressure of $\pm 0.7 \times 11.5 = \pm 8 \text{ lb. per sq. ft.}$

Individual wall panels should be designed for $\pm 0.8 \times 11.5 = \pm 9.2 \text{ lb. per sq. ft.}$

Walls of tower should be designed for $\pm 0.7 \times 12.5 = \pm 8.8 \text{ lb. per sq. ft.}$

Panels of walls of tower should be designed for $\pm 0.8 \times 12.5 = \pm 10 \text{ lb. per sq. ft.}$

Design of Roofs.

The windward slope of the pitched roof (about 20 deg.) should be designed for a general suction of $0.4 \times 11.5 = 4.6 \text{ lb. per sq. ft.}$, and the leeward slope for a general suction of $0.45 \times 11.5 = 5.2 \text{ lb. per sq. ft.}$

Each panel of the pitched roof should be designed for the following suction: Windward slope, $0.7 \times 11.5 = 8 \text{ lb. per sq. ft.}$ Leeward slope, $0.75 \times 11.5 = 8.6 \text{ lb. per sq. ft.}$

The windward half of the flat roof of the tower should be designed for a general upward pressure (suction) of $1.0 \times 12.5 = 12.5 \text{ lb. per sq. ft.}$, and the leeward half for a general upward pressure of $0.75 \times 12.5 = 9.4 \text{ lb. per sq. ft.}$

Individual panels of the flat roof should be designed for the following upward pressures (suctions): Windward half, $1.3 \times 12.5 = 16.25 \text{ lb. per sq. ft.}$ Leeward half, $1.05 \times 12.5 = 13.1 \text{ lb. per sq. ft.}$

Design of Roofs.

The windward slope of the pitched roof (about 20 deg.) should be designed for a general suction of $0.4 \times 11.5 = 4.6 \text{ lb. per sq. ft.}$, and the leeward slope for a general suction of $0.45 \times 11.5 = 5.2 \text{ lb. per sq. ft.}$
### Buildings

<table>
<thead>
<tr>
<th>Height (ft.)</th>
<th>A</th>
<th>B*</th>
<th>C</th>
<th>D</th>
<th>Angle of Windward Slope (deg.)</th>
<th>General Pressures</th>
<th>Local Pressures</th>
<th>General (Stability)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>4</td>
<td>6</td>
<td>8</td>
<td>10</td>
<td>0</td>
<td>-1.0 p</td>
<td>-1.3 p</td>
<td>0.6 p</td>
</tr>
<tr>
<td>20</td>
<td>5</td>
<td>7</td>
<td>9</td>
<td>12</td>
<td>10</td>
<td>-0.7 p</td>
<td>-1.1 p</td>
<td>0.8 p</td>
</tr>
<tr>
<td>30</td>
<td>5</td>
<td>8</td>
<td>11</td>
<td>14</td>
<td>20</td>
<td>-0.4 p</td>
<td>-0.7 p</td>
<td>0.7 p</td>
</tr>
<tr>
<td>40</td>
<td>6</td>
<td>9</td>
<td>12</td>
<td>16</td>
<td>30</td>
<td>-0.1 p</td>
<td>-0.4 p</td>
<td>0.8 p</td>
</tr>
<tr>
<td>50</td>
<td>7</td>
<td>10</td>
<td>14</td>
<td>18</td>
<td>40</td>
<td>0.1 p</td>
<td>0.4 p</td>
<td>0.9 p</td>
</tr>
<tr>
<td>60</td>
<td>8</td>
<td>11</td>
<td>15</td>
<td>20</td>
<td>50</td>
<td>0.3 p</td>
<td>0.6 p</td>
<td>1.0 p</td>
</tr>
<tr>
<td>80</td>
<td>9</td>
<td>12</td>
<td>17</td>
<td>22</td>
<td>60</td>
<td>0.5 p</td>
<td>0.8 p</td>
<td>1.1 p</td>
</tr>
<tr>
<td>100</td>
<td>9</td>
<td>13</td>
<td>18</td>
<td>24</td>
<td>70</td>
<td>0.5 p</td>
<td>0.8 p</td>
<td>1.2 p</td>
</tr>
<tr>
<td>120</td>
<td>10</td>
<td>14</td>
<td>19</td>
<td>25</td>
<td>80</td>
<td>0.5 p</td>
<td>0.8 p</td>
<td>1.3 p</td>
</tr>
<tr>
<td>140</td>
<td>11</td>
<td>15</td>
<td>21</td>
<td>27</td>
<td>90</td>
<td>0.5 p</td>
<td>0.8 p</td>
<td>1.4 p</td>
</tr>
<tr>
<td>160</td>
<td>11</td>
<td>16</td>
<td>22</td>
<td>28</td>
<td>100</td>
<td>0.5 p</td>
<td>0.8 p</td>
<td>1.5 p</td>
</tr>
<tr>
<td>180</td>
<td>12</td>
<td>17</td>
<td>23</td>
<td>30</td>
<td>110</td>
<td>0.5 p</td>
<td>0.8 p</td>
<td>1.6 p</td>
</tr>
<tr>
<td>&gt;200</td>
<td>12</td>
<td>17</td>
<td>24</td>
<td>31</td>
<td>120</td>
<td>0.5 p</td>
<td>0.8 p</td>
<td>1.7 p</td>
</tr>
</tbody>
</table>

**P = Basic Pressure (lb. per square foot) on vertical face**

**DATA COMPLIES WITH B.S. CODE No. 3, Chap. III**

*Denotes requirements of London By-Laws.

### Bridges

**Horizontal Pressure = 30 lb. per square foot of exposed area**

**Exposed Areas,**

- For unloaded bridge = \( A \left( 1 + \frac{L}{16d} \right) \)
- Load = Distance between windward and leeward beams.
- \( d = \) Depth or height of beam or parapet.
- \( L = \) Length of bridge.

**A = Net Horizontally-Projected Area of Windward Side of Bridge**

**H = Additional Height of "Area" Subjected to Wind Due to Live Load**

For calculating \( H \) the top of the vehicles, etc., forming the live load is considered to be at a uniform level thus:

- Road Bridge: 8 ft. Above Road
- Railway Bridge: 12 ft. Above Rail
- Footpath: 5 ft. Above Footpath

**Vertical Pressure (for stability calculations) = 5 lb. per sq. ft. of plan area (acting upwards)**

### Chimneys, Towers, etc.

**Horizontal Pressure (lb./sq. ft.) on any part of structure at height \( h \) ft. above ground. Adjustment to be made for shape.**

**Per B.S. Code No. 3, Chap. V, 1958 Revision**

<table>
<thead>
<tr>
<th>Height ( h )</th>
<th>20</th>
<th>40</th>
<th>60</th>
<th>80</th>
<th>100</th>
<th>150</th>
<th>200</th>
<th>300</th>
<th>400</th>
<th>500 or Greater</th>
</tr>
</thead>
<tbody>
<tr>
<td>Velocity of Wind (miles/hr) at Height of 40 ft. (mean velocity during 1 min.)</td>
<td>( \frac{50}{55} )</td>
<td>( \frac{60}{65} )</td>
<td>( \frac{70}{75} )</td>
<td>( \frac{80}{85} )</td>
<td>( \frac{90}{95} )</td>
<td>( \frac{100}{105} )</td>
<td>( \frac{110}{115} )</td>
<td>( \frac{120}{125} )</td>
<td>( \frac{130}{135} )</td>
<td>( \frac{140}{145} )</td>
</tr>
</tbody>
</table>

**Structures more than 100 ft. high to be designed for pressures due to velocities not less than those below the thick line.**

### Poles (B.S.)

**Transmission-Line Poles:** 8 lb. per sq. ft. on projected area of ice-covered conductors on flat windward surface of solid pole 15 ft. or more.

**Lamp Posts: Heights 20 ft. to 40 ft.:** 12 lb. per sq. ft. Subject to reduction for shape. Are to include lantern, etc.
TRANSMISSION-LINE POLES.

If the spans of the conductors on either side of an intermediate pole are unequal the pole is subject to an unbalanced pull in the direction of the line, as is also a pole at the end of a line, and a strut or guy is generally provided at a pole at the end of a line. In Table 9 are given formulae for the forces in the direction of the line on an intermediate pole and on an end pole, with expressions for the forces in a strut or guy in the same plane as the line, and the resultant vertical thrust on the pole. Normally the vertical load on a pole is the sum of the weights of the conductors, of the ice on the conductors, of the brackets, insulators and other fixings, and of the pole itself. The net weight, diameter, and strength of typical conductors are given in Table 9, with an expression for the weight of the conductor and the ice thereon.

A pole at a point where a change in the direction of a transmission line is made is subject to an additional transverse force which is usually resisted by a guy or strut. When the change in direction is small a guy or strut is not always provided, in which case the additional transverse force (see Table 9) has to be taken into account.

If a conductor breaks, an intermediate pole in a line of equal or unequal spans is subject to an unbalanced pull in the direction of the line. If the pole carries several conductors and only one breaks the unbalanced force may be comparatively small, but in a line of only one conductor the unbalanced force on the poles, especially those at either end of the broken span, may have serious effects if the pole has not been designed for this condition. The maximum unbalanced load to which a pole can be subjected when all the conductors in one span break is the pull corresponding to the breaking load of the conductors in the adjacent unbroken span. It is, however, unlikely that all the conductors in one span will break simultaneously, and this extreme case is not usually worthy of consideration except for a pole carrying a single conductor. For small ratios of sag to span the maximum horizontal force that can act on a stiff pole in the direction of the line is about equal to the maximum tension in the conductor, which in the limit equals the tensile strength, some values of which are given in Table 9. If a pole is designed for this extreme load the factor of safety can be less than that for normal working conditions. If a pole carries two or more conductors, the breaking of one causes, in addition to an unbalanced force in the direction of the line, a twisting moment on the pole. The effect of the breaking of one or more conductors can be analysed mathematically having regard to the elastic deformation of the poles, but usually sufficient security against serious overstressing, due to this cause, is obtained by compliance with the regulation of British authorities that the strength of the pole in the direction of the line shall be at least one-quarter of that in the direction normal to the line.

Transmission-line poles are classified in the British Standard according to the minimum ultimate transverse loads they are designed to resist. The working load is the ultimate load divided by a suitable factor of safety. Values of the ultimate loads are given in Table 9.

Light transmission-line poles to carry, say, four low-tension conductors in a vertical plane and spanning 150 ft., and telegraph poles, are usually satisfactory if designed for a working load of 500 lb. applied at the top of the pole.
### TYPICAL CONDUCTORS

<table>
<thead>
<tr>
<th>N° OF STRANDS</th>
<th>COPER</th>
<th>STEEL-CORED ALUMINUM</th>
<th>ALUMINUM</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>7</td>
<td>37</td>
<td>37</td>
</tr>
<tr>
<td>DIA. OF WIRE</td>
<td>0.065</td>
<td>0.081</td>
<td>0.081</td>
</tr>
<tr>
<td>DIA. OF CONDUCTOR (dC)</td>
<td>0.022</td>
<td>0.046</td>
<td>0.061</td>
</tr>
<tr>
<td>STRENGTH (Tu LB)</td>
<td>1550</td>
<td>7240</td>
<td>9920</td>
</tr>
<tr>
<td>WEIGHT PER FT (Wc LB)</td>
<td>0.01</td>
<td>0.05</td>
<td>0.07</td>
</tr>
</tbody>
</table>

### POLES IN STRAIGHT LINE

- **Pole at End of Line**
- **Intermediate Pole**
- **View in Direction of Line**

**Transverse Force Due to Wind on Conductors**

\[
P_\text{w} = \frac{N_p C L_1 (d_c + d_i)}{2.4}
\]

**Horizontal Forces in Direction of Line**

\[
P_\text{h} = \frac{N_w t L_1^2}{8 s_1} + \frac{N_w t L_2^2}{8 s_2} + \frac{N_p C (L_1 + L_2) (d_c + d_i)}{2.4}
\]

**Total Vertical Load**

\[
P = \frac{N_w t L_1^2}{8 s_1} \cos \theta_1 + \frac{N_w t L_2^2}{8 s_2} \cos \theta_2 + \frac{N_p C (d_c + d_i) (L_1 \cos \theta_1 + L_2 \cos \theta_2)}{2.4}
\]

**Notation:**
- \(N\) = number of conductors.
- \(B\) = breadth or diameter of pole.
- \(d_c\) = normal diameter of conductor (in). \(W_c\) = Wt. of pole, attachments, etc.
- \(d_i\) = increase in diameter of conductors due to adhering ice.
- \(W_t\) = weight of conductor (w) and ice (lb. per ft.) = \(w_c + \frac{d_i}{3} (2 d_c + d_i)\).
- \(P_c\) = wind pressure on conductors; \(P_P\) = wind pressure on pole.

If sag is large compared with \(L\), replace \(L_1\) by \(L_1 + \frac{8 g_1}{3 L_1}\) and replace \(L_2\) by \(L_2 + \frac{8 g_2}{3 L_2}\).

### FORCES IN STRUTS AND GUYS

\[
F = \frac{P}{\sqrt{h^2 + L^2}}
\]

For wind pressure on transmission-line poles, see Table 8.
ACTIVE PRESSURES DUE TO RETAINED MATERIALS.

Granular Materials.

Examples of Pressure of Dry Materials.—(a) Find the total horizontal active pressure on the back of a vertical wall 15 ft. high retaining ordinary earth with a level surface subject to an imposed load of 2 cwt. per sq. ft. (= W).

Assume for earth: w = 100 lb. per cu. ft.; \( \theta = 35 \) deg. From Table 11, \( k_2 = 0.271 \).

Pressure at base of wall due to retained earth and surcharge (Table 13):

\[
P_a = k_2 (wH + W) = 0.271[(100 \times 15) + 224] = 467 \text{ lb. per sq. ft.}
\]

Pressure at top of wall due to surcharge: \( P_a = 0.271 \times 224 = 60.6 \text{ lb. per sq. ft.} \)

Total pressure = \( \frac{1}{2}H(467 + 60) = 0.5 \times 15 \times 528 = 3960 \text{ lb.} \)

(b) A vertical wall 20 ft. high retains a heap of dry coal, the top surface of which is sloped downwards from the wall at the natural slope. Find the intensity of active horizontal pressure at the base of wall, neglecting friction on the back of wall.

From Table 11, \( w = 58 \) lb. per cu. ft. and \( \theta = 40 \) deg.;

\( k_3 = 0.16 \);

\[
P_a = k_3 wH = 0.16 \times 58 \times 20 = 186 \text{ lb. per sq. ft.}
\]

Effect of Ground-water.—If ground-water occurs at a depth \( h_w \) below the top of the wall, the intensity of horizontal pressure is \( k_4 wH \) when \( h \) does not exceed \( h_w \); when \( h \) is greater than \( h_w \) the pressure is given by

\[
P_a = k_4 wH + (k_4 w_B + w_w)(h - h_w),
\]

where \( w_B \) is the buoyant density of the soil (about 60 per cent. of the drained density \( w \)), and \( w_w \) is the density of water (62.4 lb. per cu. ft.).

For a dry granular material with level fill, the passive resistance is \( (1 + \sin \theta) \frac{wH}{k_3} \), that is \( \frac{wh}{k_3} \) as given in Table 10; values of \( k_3 \) are given in Table 11. This expression also applies to drained soil above ground-water level; for saturated soil below this level the passive resistance is given by

\[
P_p = \frac{wH}{k_3} + \left( \frac{w_B}{k_3} + w_w \right)(h - h_w).
\]

The symbols have the same significance as in Table 11; \( h_w \) is the depth to ground water.

Active Pressure Normal to Inclined Surfaces.—The intensity of pressure normal to the slope of an inclined surface, such as a hopper bottom loaded with coal, grain, sand, stone, or other granular material, at a depth \( h \) below the level surface of the filling is

\[
wH(h_2 \sin \theta_2 + \cos^2 \theta_2),
\]

where \( \theta_2 \) is the angle between the horizontal and the sloping surface. Values of \( h_2 \) and of \( k_2 \sin \theta_2 + \cos^2 \theta_2 \) for various angles \( \theta \) from 30 deg. to 45 deg. are given in Table 11.

Materials Immersed or Floating in Liquids.

Materials Heavier than the Liquid.—With granular material, of which the specific gravity exceeds that of the liquid in which it is just fully immersed, the intensity of horizontal active pressure on the vertical wall of the container is

\[
P_a = w_L h \left[ 1 + h \left( \frac{w_m}{w_L} - 1 \right) (1 - V) \right],
\]

where \( w_L \) is the weight per cu. ft. of the liquid (lb.), \( w_m \) is the weight per cu. ft. of the solid material (lb.), \( V \) is the ratio of the volume of the voids in a given volume of the dry material, \( h \) is the horizontal pressure factor depending on the slope of the surface of the material and the angle of repose of the dry material (that is, factors \( k_1, k_2, \) or \( k_3 \) in Table 11), and \( h \) is the depth (ft.) from the top of the submerged material to the level at which the pressure is being calculated. If the surface of the liquid is at a distance \( h_3 \) ft. above the top of the submerged material there is an additional pressure of \( w_3 h_3 \). The total intensity of pressure at any depth \( h_3 + h \) below the surface of the liquid is \( w_L(h_0 + hF) \) where

\[
F = \left[ 1 + h \left( \frac{w_m}{w_L} - 1 \right) (1 - V) \right].
\]

If the material is immersed in water, \( w_L \) is 62.4 lb. per cu. ft. and the pressure is 62.4\((h_0 + hF)\) lb. per sq. ft.

(Continued on page 162.)
### Intensity of Pressure Normal to Back of Wall at Depth h ft. \( P = Kw h \sin \beta \)

### Total Pressure Normal to Back of Wall of Height h ft. \( P = \frac{1}{2} Kw h^2 \sin \beta \)

Values of \( K \) for various conditions are given below.

#### Total Pressure on Back of Wall
\[ P_1 = P \sec \mu \]

#### Angle Between Line of Action of \( P_1 \) and Back of Wall
\[ \alpha = (90 - \mu) \]

#### Force Parallel to Back of Wall
\[ F = P \tan \mu = P_1 \sin \mu \]

When friction on back of wall is neglected \((\mu = 0)\):
\[ P_1 = P; \quad F = 0 \quad \text{Angle} = 90^\circ \]

**Notation:**
- \( W \) = Weight of retained material (lb. per cubic foot).
- \( \phi \) = Angle of slope of bank of retained material (deg).
- \( \Theta \) = Angle of internal friction of material retained (deg).
- \( \mu \) = Angle of friction between material and concrete wall (deg).

#### Active Pressures

<table>
<thead>
<tr>
<th>Friction on Wall</th>
<th>Slope of Bank of Retained Material</th>
<th>INCLINED OUTWARDS</th>
<th>INCLINED INWARDS</th>
<th>VERTICAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mu = 0 )</td>
<td>Any Slope ( \phi ) (MAX)</td>
<td>( K_1 = \frac{\sin(\beta - \phi)}{(\beta + 1) \sin \beta} \cdot \frac{1}{\sin \beta} )</td>
<td>( n = \frac{\sin \theta \sin(\Theta - \phi)}{\sin \beta \sin(\beta - \phi)} )</td>
<td>( K = \frac{(\cos \theta)^2}{n + 1} )</td>
</tr>
<tr>
<td>( \phi = 0 )</td>
<td>(LEVEL)</td>
<td>( K_2 = \frac{\sin(\beta - \phi)}{(\beta + \sin \beta) \sin \beta} \cdot \frac{1}{\sin \beta} )</td>
<td>( n = \frac{\sqrt{\sin(\Theta + \mu) \sin(\Theta - \phi)}}{\sin(\Theta + \beta) \sin(\beta - \phi)} )</td>
<td>( K_1 = \cos \theta )</td>
</tr>
<tr>
<td>( \phi = \Theta )</td>
<td>(MIN)</td>
<td>( K_3 = \frac{\sin(\beta - \Theta)}{(\beta + \sin \beta) \sin \beta} \cdot \frac{1}{\sin \beta} )</td>
<td>( n = \frac{2 \sin^2 \theta \cos \Theta}{\sqrt{\sin \beta(\beta + \Theta)}} )</td>
<td>( K_2 = \frac{1 - \sin \theta}{1 + \sin \theta} )</td>
</tr>
</tbody>
</table>

**Dried Granular Materials**

- \( \mu \) = Angle of friction between material and concrete wall (deg).
- \( \Theta \) = Angle of internal friction of material retained (deg).
- \( \phi \) = Angle of slope of bank of retained material (deg).

#### Passive Resistance

**Intensity of Horizontal Passive Resistance Against Vertical Wall at Depth h ft. \((\phi = 0)\)**
\[ P_p = \frac{W h}{K_2} = \frac{(1 + \sin \Theta)}{(1 - \sin \Theta)} \cdot wh \]

**Cohesive Soils**

- \( \phi \) = Angle of internal friction of material retained (deg).
- \( C \) = Cohesion factor = Shearing strength of unloaded soil.

#### Active Pressure

**Intensity of Horizontal Pressure at Depth of h ft. on Vertical Wall \((\phi = 0)\)**
\[ P_2 = \frac{1 - \sin \Theta}{1 + \sin \Theta} \cdot wh - 2C \sqrt{\frac{1 - \sin \Theta}{1 + \sin \Theta}} = \frac{K_2 w h - 2C \sqrt{K_2}}{K_2} \]

**Passive Resistance**

**Intensity of Horizontal Passive Resistance Against Vertical Wall at Depth h ft. \((\phi = 0)\)**
\[ P_p = \frac{w h}{K_2} + \frac{2C}{K_2} \]

Retaining Ordinary Saturated Clay:

- Other notation as for granular materials.
TABLE 11.—PRESSURES DUE TO GRANULAR MATERIALS.

<table>
<thead>
<tr>
<th>Retained Cohesionless Soils Etc.</th>
<th>Angle of Internal Friction $\Theta$</th>
<th>Weight lb. per c.f.t.</th>
<th>Contained Granular Materials</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cravely, Common</td>
<td>35$^\circ$ to 45$^\circ$</td>
<td>110 to 140</td>
<td>Coal: Dry Unwashed</td>
</tr>
<tr>
<td>Shingle Loose</td>
<td>40$^\circ$</td>
<td>115</td>
<td>Coal: Washed</td>
</tr>
<tr>
<td>Sandy Compact</td>
<td>40$^\circ$ to 45$^\circ$</td>
<td>130</td>
<td>Wet (15$%$ moisture)</td>
</tr>
<tr>
<td>Loose</td>
<td>35$^\circ$ to 40$^\circ$</td>
<td>120</td>
<td>Fine</td>
</tr>
<tr>
<td>Sand: Fine DRY</td>
<td>30$^\circ$ to 35$^\circ$</td>
<td>100</td>
<td>Slurry</td>
</tr>
<tr>
<td>Wet</td>
<td>0$^\circ$ to 30$^\circ$</td>
<td>115 to 120</td>
<td>Anthractite</td>
</tr>
<tr>
<td>Well Graded Compact</td>
<td>30$^\circ$ to 40$^\circ$</td>
<td>110 to 120</td>
<td>Coke, Breeze, etc.</td>
</tr>
<tr>
<td>Loose</td>
<td>35$^\circ$ to 40$^\circ$</td>
<td>100</td>
<td>Shale: Broken</td>
</tr>
<tr>
<td>Uniform</td>
<td>35$^\circ$ to 40$^\circ$</td>
<td>100</td>
<td>Colliey Dirt</td>
</tr>
<tr>
<td>Fine or Silty Compact</td>
<td>35$^\circ$ to 40$^\circ$</td>
<td>100</td>
<td>Gravel: Wheat</td>
</tr>
<tr>
<td>Coarse or Medium Compact</td>
<td>35$^\circ$ to 40$^\circ$</td>
<td>100</td>
<td>Barley</td>
</tr>
<tr>
<td>Crushed Rock</td>
<td>30$^\circ$ to 35$^\circ$</td>
<td>100</td>
<td>Coarse</td>
</tr>
<tr>
<td>Granite</td>
<td>30$^\circ$ to 35$^\circ$</td>
<td>100</td>
<td>Air-Agitated</td>
</tr>
<tr>
<td>Basalt, Dolerites</td>
<td>30$^\circ$ to 35$^\circ$</td>
<td>100</td>
<td>Ashes</td>
</tr>
<tr>
<td>Limestone, Sandstone</td>
<td>30$^\circ$ to 35$^\circ$</td>
<td>100</td>
<td>Broken Brick</td>
</tr>
<tr>
<td>Broken Chalk</td>
<td>30$^\circ$ to 35$^\circ$</td>
<td>100</td>
<td></td>
</tr>
</tbody>
</table>

**HORIZONTAL PRESSURE OF GRANULAR MATERIALS SUBMERGED IN WATER**

<table>
<thead>
<tr>
<th>Value of $G_2$ in Slid. Lb. per cu.ft.</th>
<th>Weight when submerged (Lb. per cu.ft.)</th>
<th>Percentage of voids (100)</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>30</td>
<td>35</td>
</tr>
<tr>
<td>40</td>
<td>45</td>
<td>50</td>
</tr>
</tbody>
</table>

**VERTICAL WALLS**

<table>
<thead>
<tr>
<th>Angle of Friction &amp; Contact of Material (Approximate Angle of Repose)</th>
<th>Maximum Positive Surchage</th>
<th>Level Fill</th>
<th>Minimum Negative Surchage</th>
</tr>
</thead>
<tbody>
<tr>
<td>5$^\circ$</td>
<td>0.52</td>
<td>0.84</td>
<td>0.79</td>
</tr>
<tr>
<td>10$^\circ$</td>
<td>0.62</td>
<td>0.70</td>
<td>0.63</td>
</tr>
<tr>
<td>15$^\circ$</td>
<td>0.52</td>
<td>0.68</td>
<td>0.59</td>
</tr>
<tr>
<td>20$^\circ$</td>
<td>0.52</td>
<td>0.54</td>
<td>0.52</td>
</tr>
<tr>
<td>25$^\circ$</td>
<td>0.52</td>
<td>0.53</td>
<td>0.50</td>
</tr>
<tr>
<td>30$^\circ$</td>
<td>0.52</td>
<td>0.50</td>
<td>0.48</td>
</tr>
</tbody>
</table>

**ACTIVE PRESSURES (Friction Between Material and Face of Wall or Surface Neglected)**

<table>
<thead>
<tr>
<th>Friction (Lb. per cu. ft.)</th>
<th>Angle of Internal Friction $\Theta$</th>
<th>$k_1 = \cos^2\Theta$</th>
<th>$k_2 = \frac{1}{1 + \sin\Theta}$</th>
<th>$k_3 = \frac{\sin\Theta}{\sin\Theta}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5$^\circ$</td>
<td>1 in 4</td>
<td>0.99</td>
<td>0.84</td>
<td>0.79</td>
</tr>
<tr>
<td>10$^\circ$</td>
<td>1 in 5</td>
<td>0.97</td>
<td>0.80</td>
<td>0.70</td>
</tr>
<tr>
<td>15$^\circ$</td>
<td>1 in 4</td>
<td>0.94</td>
<td>0.75</td>
<td>0.63</td>
</tr>
<tr>
<td>20$^\circ$</td>
<td>1 in 4</td>
<td>0.93</td>
<td>0.70</td>
<td>0.59</td>
</tr>
<tr>
<td>25$^\circ$</td>
<td>1 in 4</td>
<td>0.92</td>
<td>0.65</td>
<td>0.55</td>
</tr>
<tr>
<td>30$^\circ$</td>
<td>1 in 4</td>
<td>0.90</td>
<td>0.60</td>
<td>0.50</td>
</tr>
<tr>
<td>35$^\circ$</td>
<td>1 in 4</td>
<td>0.88</td>
<td>0.55</td>
<td>0.45</td>
</tr>
<tr>
<td>40$^\circ$</td>
<td>1 in 4</td>
<td>0.86</td>
<td>0.50</td>
<td>0.40</td>
</tr>
<tr>
<td>45$^\circ$</td>
<td>1 in 4</td>
<td>0.84</td>
<td>0.45</td>
<td>0.35</td>
</tr>
<tr>
<td>50$^\circ$</td>
<td>1 in 4</td>
<td>0.82</td>
<td>0.40</td>
<td>0.30</td>
</tr>
<tr>
<td>55$^\circ$</td>
<td>1 in 4</td>
<td>0.80</td>
<td>0.35</td>
<td>0.25</td>
</tr>
<tr>
<td>60$^\circ$</td>
<td>1 in 5</td>
<td>0.78</td>
<td>0.30</td>
<td>0.20</td>
</tr>
</tbody>
</table>

**INCLINED WALLS**

<table>
<thead>
<tr>
<th>Intensity of Pressure Normal to Surface (Lb. per sq. ft.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_n = k_4 w_h + w_i \cos^2 \Theta$</td>
</tr>
</tbody>
</table>

**NOTES ON VERTICAL WALLS**

- Tabulated factors apply only to intensity of active horizontal pressures of granular (cohesionless) material retained or contained by reinforced concrete walls, steel piles, and the like (not heavy gravity walls).

- **Weights (Lb. per cu. ft.)**
  - $\omega = \text{Weight (Lb. per cu. ft.) of retained or contained material} = \omega_a + \omega_b + \omega_c$.

- **$\omega_a$** = Weight of dry soil if no water in ground or of contained material.

- **$\omega_b$** = Weight of moist (drained) soil above level of ground water.

- **$\omega_c$** = Buoyant weight of soil below ground water level ($\approx 0.05 \omega_a$ approx.)
### Pressures Due to Cohesive Soils

**Table 12.**

<table>
<thead>
<tr>
<th>Type of Soil</th>
<th>Saturated Density (lb. per cu. ft.)</th>
<th>Angle of Internal Friction (θ)</th>
<th>Cohesion (lb. per sq. ft.)</th>
<th>Type of Soil</th>
<th>Bulk Density (lb. per cu. ft.)</th>
<th>Angle of Internal Friction (θ)</th>
<th>Cohesion</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clay: Very Stiff Boulder</td>
<td>110 to 140</td>
<td>15°</td>
<td>3580</td>
<td>Clay: Dry Well Damp Drained</td>
<td>110</td>
<td>30°</td>
<td>DETERMINE BY TEST</td>
</tr>
<tr>
<td>Hard Shaley Stiff</td>
<td>110 to 150</td>
<td>6°</td>
<td>750 to 1500</td>
<td>Wet</td>
<td>120</td>
<td>15°</td>
<td>IN ABSENCE OF TEST RESULTS, APPLY FORMULA FOR SATURATED CLAY WITH θ = 0°</td>
</tr>
<tr>
<td>Firm</td>
<td>110 to 120</td>
<td>5°</td>
<td>1120</td>
<td>Gravelly</td>
<td>125</td>
<td>35°</td>
<td></td>
</tr>
<tr>
<td>Moderately Firm</td>
<td>100 to 120</td>
<td>3°</td>
<td>375 to 750</td>
<td>Earth: Top Soil</td>
<td>85</td>
<td>(35°)</td>
<td></td>
</tr>
<tr>
<td>Soft</td>
<td>100 to 120</td>
<td>4°</td>
<td>675</td>
<td>Common</td>
<td>100</td>
<td>(35°)</td>
<td></td>
</tr>
<tr>
<td>Very Soft</td>
<td>100 to 120</td>
<td>3°</td>
<td>450</td>
<td>Moist</td>
<td>90</td>
<td>30°</td>
<td></td>
</tr>
<tr>
<td>Puddle Clay: Soft</td>
<td>100</td>
<td>0°</td>
<td>750 to 1500</td>
<td>Very Wet</td>
<td>100</td>
<td>45° to 50°</td>
<td></td>
</tr>
<tr>
<td>Very Soft</td>
<td>100</td>
<td>&lt; 375</td>
<td>500</td>
<td>Punned</td>
<td>105</td>
<td>17°</td>
<td></td>
</tr>
<tr>
<td>Sandy Clay: Stiff Firm</td>
<td>100</td>
<td>0°</td>
<td>750 to 1500</td>
<td>Peat: Dry</td>
<td>100</td>
<td>65° to 75°</td>
<td></td>
</tr>
<tr>
<td>Silt</td>
<td>120</td>
<td>&lt; 750</td>
<td></td>
<td>Wet</td>
<td>60</td>
<td>15° to 45°</td>
<td></td>
</tr>
</tbody>
</table>

**Formulas for Intensity (lb. per sq. ft.) of Horizontal Pressure at Depth h ft.**

- **Clay (Partially Saturated) and Silt ($θ > 0°$)**
  
  \[ h_o = \frac{2C_s}{w} \left( 1 + \frac{C_w}{C_s} \right) \]

  **Above Ground Water Level:**
  
  \[ h > h_o \Rightarrow p_2 = w_h \]

  **Below Ground Water Level:**
  
  \[ h > h_o \Rightarrow p_2 = k_2 w_h + (k_2 w_b + w_b) (h - h_o) - 2C_s \]

- **Fissured Clay ($θ > 0°$)**
  
  \[ h_o = \frac{2C_s}{w} \left( 1 + \frac{C_w}{C_h} \right) \]

  **Above Ground Water Level:**
  
  \[ h > h_o \Rightarrow p_2 = w_h \]

- **Non-Fissured Clay ($θ = 0°$)**
  
  **Water Level Below Ground Level in Front of Wall**
  
  \[ h_p > h_w \Rightarrow p = \frac{w_h}{k_2} + \frac{2C}{\sqrt{k_2}} \]

  **Water Level Above Ground Level in Front of Wall**
  
  \[ h_p > h_w \Rightarrow p = \frac{w_b}{k_2} (h_p - h_w) + \frac{2C}{\sqrt{k_2}} \]

**Passive Resistance**

**Clay (Partially Saturated) and Silt ($θ > 0°$)**

\[ p = w_h + \frac{2C_h}{\sqrt{k_2}} \]

**Non-Fissured Clay ($θ = 0°$)**

**Active Pressures.**

**Notation.**

\[ w = \text{weight of saturated clay or bulk weight of earth} \]

**Weights (lb. per cu. ft.).**

\[ w_w = \text{weight of water} \]

\[ w_b = \text{buoyant weight} \]

**Cohesion (lb. per sq. ft.).**

\[ C = \text{cohesion at no load on clay} \]

\[ C_s = \text{cohesion of softened clay} \]

\[ C_h = \text{cohesion at depth} \]

\[ C_w = \text{cohesion between clay and wall} \]

\[ k_2 = \frac{1}{\sin \theta} \] (See Table No. 3)
REINFORCED CONCRETE DESIGNER'S HANDBOOK

ACTIVE PRESSURES DUE TO RETAINED MATERIALS

(continued from page 158).

The foregoing expressions apply to materials such as coal or broken stone, the angle of repose of which is not materially affected by submergence. It is advisable to reduce the value of the angle of repose to about 5 deg. below that for the dry material when determining the value of $k$ for the submerged material. In the calculation of $k$ for such material it can be assumed that the angle of repose and angle of internal friction are identical.

For a material such as sand, which has a definite angle of repose when dry but none when saturated, $k$ is unity when submerged.

The values of $F_w$ (lb. per sq. ft.) for unit $h$ for crushed coal, broken stone, and sand, when immersed in water, and with level surface are given in Table 11.

Materials Lighter than the Liquid.—If the specific gravity of the material is less than that of the liquid, the intensity of horizontal pressure, at any depth $h$ below the surface of the liquid in which the material floats, is $w_d h$. Therefore, if stored in water, the horizontal pressure on the walls of the container is equal to the simple hydrostatic pressure of $62.4 h$ lb. per sq. ft.

Cohesive Materials.

Find the intensity of horizontal active pressure at a depth of 25 ft. below the top of wall supporting moderately firm clay assumed to be partially saturated.

From Table 12, $\theta = 5$ deg. and $C = 1120$ lb. per sq. ft.

From Table 11, for $\theta = 5$ deg., $\phi_s = 0.84$; assume $w = 110$ lb. per cu. ft.

Substituting in Bell's formula, $p_s = (0.84 \times 110 \times 25) - [2 \times 1120 \times \sqrt{0.84}] = 260$ lb. per sq. ft.

(This method of calculation should not be adopted unless the values of $\theta$ and $C$ have been well proved by test, and factors that may affect these values in the future have been fully considered.)

SURCHARGE ON GRANULAR MATERIAL.

Non-level Filling.—As is seen from the factors given in Table 11, the slope of the surface of the filling behind the wall has a marked effect on the theoretical pressures. For practical purposes any alteration of the shape of the surface beyond the point B in the diagrams in Table 13, or any additional loading beyond this point, has little or no effect upon the pressure on the wall. The general formula in Table 11 allows for any slope between the limits DB and DB1.

When the surface of the filling is not at a uniform slope, special consideration is required. In the common cases depicted in Table 13 the magnitude of the active horizontal pressure on the wall is between that when the surface is level and when the surface slopes upwards at the maximum angle. On the diagrams are given empirical expressions for the increase of pressure due to these intermediate conditions. The total pressure on the wall is increased above that for a level surface by an amount proportional to the mean increase in the head of the material. These surcharges give no pressure at the top of the wall, and the centre of total pressure is assumed to be one-third of the way up the wall.

Load Imposed on Filling.—When the filling behind a wall is level but liable to self-retaining loads, such as stacked materials, traffic, or buildings, the total imposed load should be converted into an equivalent head of the same material as that retained by the wall, and the pressure intensity on the back of the wall increased uniformly throughout the depth of the wall. In this case there is a positive intensity of pressure at the top of the wall; these conditions are illustrated in Table 11.

Slope Beyond the Natural Angle of Repose.—A type of surcharge not dealt with in the foregoing is shown in Table 13 for the case where the angle of the slope of the surface exceeds the natural angle of repose, as occurs by protecting a bank of earth with turf or stone pitching. For such a case it is suggested that the weight of earth $W$ above the angle of repose, represented by the triangle BCD, should be considered as a load operating at the top of the wall. The magnitude of the resultant horizontal thrust $T$ and increase in pressure would be as shown in Table 13, and each section of the wall should be designed for the extra moment due to $T$.

Concentrated Load.—A single concentrated load imposed on the filling behind a wall can be dealt with approximately by dispersion. With 45 deg. as the angle of dispersion, as indicated in Table 13, the intensity of active horizontal pressure additional to the pressure due to the filling is given by the appropriate formula in Table 13.
**VERTICAL WALL.**

**FRICION ON BACK OF WALL NEGLECTED.**

**GRANULAR MATERIAL: WEIGHT = w LB. PER CUBIC FOOT.**

### Distributed Loads

**a) Uniformly-Distributed Load.**

\[ W = \frac{w}{h} \]

\[ W_s = \text{LB. PER SQ. FT.} \]

\[ P = \frac{k_2 w h}{h} \]

\[ P_s = \frac{k_2 w h}{h^2} \]

\[ P = \frac{1}{2} k_2 w h^2 \]

\[ P_{\text{max}} = k_2 w (h + h_0) \]

\[ \alpha = 45^\circ - k_2 \theta \text{ APPROX.} \]

**b) Line-Load.**

\[ D \]

\[ W_s = \text{LB. PER LIN. FT.} \]

\[ P_s = k_2 w_{\text{lb./ft.}} \]

\[ P = \frac{h}{2} P_{\text{MAX.}} \]

### Concentrated Load

**c) Load Concentrated on Area A x B.**

\[ D \]

\[ W_s = \text{LB.} \]

\[ P = \frac{k_2 w_s}{2d(2d - B + A)} \]

\[ P_s = \frac{k_2 w_s}{(D + B + h)(2h + A)} \]

\[ D = \frac{B}{2} \]

\[ W_s = \text{LB.} \]

**Alternative Method**

**d) Indefinite Extent.**

\[ A = \text{CROSS-SECTIONAL AREA OF "HEAP"} \]

\[ p = k_2 w h \]

\[ P = \frac{H}{2} P_{\text{MAX.}} \]

\[ P_{\text{MAX.}} = k_2 w H \]

\[ \beta = 1 + 4.5 \frac{A}{H} \]

\[ \alpha = 45^\circ + \frac{1}{2} \theta \]

\[ P_{1(\text{MAX.})} = \frac{k_2 w H}{L} \]

\[ P_{3(\text{MAX.})} = \frac{k_2 w H}{L} \]

**e) Limited Extent.**

**f) Artificially-Retained Slope.**

### Heaped Material

\[ \text{For max. slope } + \theta: k = k_1 \]

\[ \text{min. slope } - \theta: k = k_3 \]

\[ T = w \sin \theta \cos \theta \]

\[ P = k_1 w h + h P_{\text{(MAX.)}} \]

\[ P = \frac{k_1 w h}{h} P_{\text{(MAX.)}} \]

For values of \( k_1, k_2 \) and \( k_3 \), see Table II.
Applications of Janssen’s Formula.

Grain Silo.—Consider the special case of grain for which \( \tan \mu = 0.444 \) and \( h = 0.5 \). Substitution in Janssen’s formula (Table 14) gives an expression for the horizontal pressure \( p_h \) at any depth \( h \) in terms of the hydraulic radius \( R \), the depth \( h \), and the density \( w \) of the grain. From this expression values of \( \frac{p_h}{w} \) can be derived for definite values of \( h \) and \( R \) as in the tabulation.

<table>
<thead>
<tr>
<th>( R )</th>
<th>10</th>
<th>7.5</th>
<th>5.0</th>
<th>4.0</th>
<th>3.0</th>
<th>2.5</th>
<th>2.0</th>
<th>1.5</th>
<th>1.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p_h/w )</td>
<td>4/5</td>
<td>4/0</td>
<td>4/0</td>
<td>3/9</td>
<td>3/5</td>
<td>3/3</td>
<td>3/1</td>
<td>2/5</td>
<td>2/2</td>
</tr>
</tbody>
</table>

Increase in lateral pressure is negligible below the depths tabulated.

Example.—Calculate (i) the horizontal pressures at various depths in a 20-ft.-diameter, silo 60 ft. deep containing grain; also (ii) the total load carried on the wall due to the filling and (iii) the intensity of pressure on the bottom. For grain assume \( w = 48 \) lb. per cu. ft., \( h = 0.5 \), and \( \tan \mu = 0.444 \).

(i) Therefore, \( p_h = \frac{p_h}{w} \cdot w = 48 \left( \frac{p_h}{w} \right) \).

The following values of \( \frac{p_h}{w} \) are obtained from the tabulation above for \( R = 0.25 \times 20 = 5.0 \).

At 10 ft. depth: \( p_h = 4.0 \times 48 = 192 \) lb. per sq. ft.

\[ \frac{p_h}{w} = \frac{4.0 \times 48}{48} = 4.0 \]

,, 20 ft. ,, = 6.7 \times 48 = 322 ,, ,, ..

,, 30 ft. ,, = 8.4 \times 48 = 403 ,, ,, ,, ..

,, 40 ft. ,, = 9.4 \times 48 = 452 ,, ,, ,, ,, ..

,, 60 ft. ,, = 10.6 \times 48 = 498 ,, ,, ,, ,, ,, ..

(ii) The load transferred to the wall of the silo by friction is

\[ (w h - \frac{p_h}{h})R = [(48 \times 60) - (2.0 \times 498)]5 \]

= 9420 lb. per ft. of circumference at the bottom of the silo. (This calculation assumes \( h = 0.5 \); if \( h = 0.33 \) for walls the weight per foot carried on the wall will be greater.)

(iii) The intensity of vertical pressure on the bottom of the silo is

\[ \frac{p_h}{h} = \frac{498}{5.0} = 996 \) lb. per sq. ft.

Cement Silo.—Calculate the intensity of horizontal pressure on the wall of a cylindrical cement silo at a depth of 56 ft. from the surface, assuming \( D = 32 \) ft., \( \theta = 18 \) deg., \( w = 90 \) lb. per cu. ft., and \( \mu = 29 \) deg. (Cement not fluidised.)

From Table 14, \( \tan \mu = 0.5543 \).

From Table 11, for \( \theta = 18 \) deg. \( h = k_1 = 0.53 \) approximately.

From Table 14, \( R = 0.25 \times 32 = 8 \) ft. Thus

\[ \frac{k h \tan \mu}{2.303R} = \frac{56 \times 0.5543 \times 0.53}{2.303 \times 8} = 0.842; \]

therefore \( N = 6.95 \) and \( p_h = \frac{90 \times 8}{0.5543(1 - \frac{1}{6.95})} = 1130 \) lb. per sq. ft.
DEEP CONTAINERS (SILOS).—TABLE 14.

\[ D, H = \text{WIDTH AND TOTAL DEPTH OF CONTAINER (FT)} \]
\[ w = \text{WEIGHT OF CONTAINED MATERIAL (LB, PER CU. FT)} \]
\[ p = \text{INTENSITY OF HORIZONTAL PRESSURE AT DEPTH h FT.} = Kp_v \text{ (LB, PER SQ. FT)} \]
\[ P_v = \text{""" VERTICAL """"} = \frac{P_h}{K} \text{ (LB, PER SQ. FT)} \]
\[ K = \frac{P_h}{P_v} \text{ LIMITING VALUE FOR UNCONFINED GRANULAR MATERIALS} \]
\[ = \frac{1 - \sin \theta}{1 + \sin \theta} \text{ FOR CONFINED MATERIALS; APPROXIMATE VALUE FOR GRAIN = 0.55; FOR STATIC CEMENT = 0.55} \]
\[ \theta = \text{ANGLE OF INTERNAL FRICTION OF CONTAINED MATERIAL (DEG)} \]
\[ \mu = \text{ANGLE OF FRICTION BETWEEN CONTAINED MATERIAL AND CONCRETE WALL (DEG)} \]
\[ \alpha = \text{COEFFICIENT DEFINING "DEEP" CONTAINER} = \frac{\tan \alpha}{\tan \theta} + \sqrt{\tan \theta + \frac{\tan^2 \theta}{\tan \theta + \tan \mu}} \]
\[ \alpha = \tan \alpha = \tan (45^\circ + \frac{1}{2} \theta) \]
\[ P_h(\text{max}) = \text{LIMITING VALUE OF HORIZONTAL PRESSURE} = \frac{w R}{\tan \mu} \text{ (PER JANSSEN'S FORMULA)} \]

JANSSEN'S FORMULA:
\[ P_h = \frac{w R}{\tan \mu} \left[ 1 - \frac{1}{N} \right] \]

\[ \alpha x D \]

SHALLOW CONTAINER
(LIMITING CONDITION)

DEEP CONTAINER
(SILO)

<table>
<thead>
<tr>
<th>CONTAINED MATERIALS</th>
<th>WEIGHT LB/CUB FT</th>
<th>FRICTION ON WALL</th>
<th>INTERNAL FRICTION</th>
<th>( \alpha )</th>
<th>( \tan \alpha )</th>
<th>R = 10</th>
<th>5</th>
<th>3</th>
<th>2</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>GRAIN: WHEAT</td>
<td>48</td>
<td>24</td>
<td>0.444</td>
<td>25</td>
<td>0.466</td>
<td>16(1/3)</td>
<td>1030</td>
<td>540</td>
<td>324</td>
<td>216</td>
</tr>
<tr>
<td>BARLEY</td>
<td>25</td>
<td>24</td>
<td>0.444</td>
<td>25</td>
<td>0.466</td>
<td>16(1/3)</td>
<td>560</td>
<td>280</td>
<td>168</td>
<td>112</td>
</tr>
<tr>
<td>SAND, GRAVEL</td>
<td>100</td>
<td>30</td>
<td>0.577</td>
<td>35</td>
<td>0.700</td>
<td>16(1/3)</td>
<td>1740</td>
<td>865</td>
<td>519</td>
<td>346</td>
</tr>
<tr>
<td>CEMENT (static)</td>
<td>90</td>
<td>30</td>
<td>0.577</td>
<td>35</td>
<td>0.700</td>
<td>16(1/3)</td>
<td>1560</td>
<td>780</td>
<td>468</td>
<td>312</td>
</tr>
<tr>
<td>COAL (crushed)</td>
<td>56</td>
<td>35</td>
<td>0.700</td>
<td>40</td>
<td>0.833</td>
<td>21(1/2)</td>
<td>800</td>
<td>400</td>
<td>240</td>
<td>160</td>
</tr>
<tr>
<td>ASHES</td>
<td>60</td>
<td>40</td>
<td>0.839</td>
<td>40</td>
<td>0.833</td>
<td>21(1/2)</td>
<td>715</td>
<td>355</td>
<td>205</td>
<td>145</td>
</tr>
</tbody>
</table>
### Table 15: Bending Moments and Shearing Forces: Basic Data

<table>
<thead>
<tr>
<th>Section Type</th>
<th>Diagram</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cantilever</td>
<td><img src="image" alt="Cantilever Diagram" /></td>
</tr>
<tr>
<td>Simple Beam</td>
<td><img src="image" alt="Simple Beam Diagram" /></td>
</tr>
<tr>
<td>Propped Cantilever</td>
<td><img src="image" alt="Propped Cantilever Diagram" /></td>
</tr>
<tr>
<td>Fixed Beam</td>
<td><img src="image" alt="Fixed Beam Diagram" /></td>
</tr>
<tr>
<td>End Span</td>
<td><img src="image" alt="End Span Diagram" /></td>
</tr>
<tr>
<td>Interior Span</td>
<td><img src="image" alt="Interior Span Diagram" /></td>
</tr>
<tr>
<td>End Conditions and Loading</td>
<td><img src="image" alt="End Conditions and Loading Diagram" /></td>
</tr>
</tbody>
</table>

**Basic Relations**

- **Shearing Force:** \( Q = \sum \text{Loads and Reactions on one side of section} \) = rate of change of \( M \)
- **Bending Moment:** \( M = \sum \text{Moments of loads and reactions on one side of section} \) = rate of change of \( EI\theta \)
- **Slope:** \( \theta = \int \frac{M}{EI} \) = rate of change of \( \Delta \)
- **Deflection (Elastic):** \( \Delta = \int \theta \) = moment of inertia of member at section \( E = \text{Elastic modulus of material} \)

**Typical Diagrams of Bending Moment and Shearing Force**

- **Bending Moments:** \( = \text{Coefficient} \times \text{Total Load} \times \text{Span} \)
- **Shearing Force:** \( = \text{Coefficient} \times \text{Total Load} \)

**Maximum Ratios of Span to Depth**

<table>
<thead>
<tr>
<th>Condition</th>
<th>Beams Spanning in One Direction</th>
<th>Beams Spanning in Two Directions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cantilevered</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>Free</td>
<td>20</td>
<td>30</td>
</tr>
<tr>
<td>Continuous</td>
<td>25</td>
<td>35</td>
</tr>
<tr>
<td></td>
<td></td>
<td>40</td>
</tr>
</tbody>
</table>

*For limiting span-thickness ratios for flat slabs, see page facing Table 45.*

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<table>
<thead>
<tr>
<th>Loading</th>
<th>Total Load W</th>
<th>Bending Moment Coefficient $K_A$</th>
<th>Deflection Coefficient $\Delta_B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>L W</td>
<td>W</td>
<td>-1.0</td>
<td>$\frac{1}{3}$</td>
</tr>
<tr>
<td>A aL W</td>
<td>W</td>
<td>- $a$</td>
<td>$\frac{a^2}{6}(3-a)$</td>
</tr>
<tr>
<td>aL bL W</td>
<td>$\frac{wL}{2}$</td>
<td>$-\frac{1}{2}$</td>
<td>$\frac{1}{6}$</td>
</tr>
<tr>
<td>wL</td>
<td>$-\frac{1}{3}$</td>
<td></td>
<td>$\frac{a^2}{60}(5-a)$</td>
</tr>
<tr>
<td>aL w</td>
<td>$-\frac{2}{3}$</td>
<td></td>
<td>$\frac{1}{60}$</td>
</tr>
<tr>
<td>aL w</td>
<td>$(1-\frac{a}{3})$</td>
<td></td>
<td>$\frac{1}{60}[20-10a+a^3]$</td>
</tr>
<tr>
<td>2/3wL</td>
<td>$-\frac{1}{2}$</td>
<td></td>
<td>$\frac{7}{60}$</td>
</tr>
</tbody>
</table>

**CANTILEVERS**

<table>
<thead>
<tr>
<th>Loading</th>
<th>Total Load W</th>
<th>Reaction Coefficients $r_A$</th>
<th>Coefficient for Max. Pos. B. M. $K_C$</th>
<th>Coefficient for Max. Deflection $\Delta_{MAX}$</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>L W</td>
<td>W</td>
<td>1 - $a$</td>
<td>$\alpha + \alpha(1-a)$</td>
<td>$\frac{a^2}{3}(1-a)^2$</td>
<td>TOTAL LOAD ON BEAM = W</td>
</tr>
<tr>
<td>A aL W</td>
<td>W</td>
<td>$\frac{1}{2}$</td>
<td>$\frac{1}{2}$</td>
<td>$\frac{1}{4}$</td>
<td>SPAN OF BEAM = L</td>
</tr>
<tr>
<td>aL bL W</td>
<td>wL</td>
<td>$1-(a+\frac{b}{2})$</td>
<td>$a + \frac{b}{2}$</td>
<td>$\frac{2}{3}$</td>
<td>REACTION = $rW$</td>
</tr>
<tr>
<td>aL w</td>
<td>$\frac{wL}{2}$</td>
<td>$\frac{1}{2}$</td>
<td>$\frac{1}{2}$</td>
<td>$\frac{1}{8}$</td>
<td>MAX. POS. B.M. = $K_CWL$</td>
</tr>
<tr>
<td>aL w</td>
<td>$\frac{wL}{2}$</td>
<td>$\frac{1}{2}$</td>
<td>$\frac{1}{2}$</td>
<td>$\frac{5}{8}$</td>
<td>MAX. DEFLECTION = $\Delta (\frac{WL^3}{EI})$</td>
</tr>
</tbody>
</table>

**FREELY-SUPPORTED BEAMS**
### Table 17.—Beams Fixed at One or Both Ends. General Formulae

<table>
<thead>
<tr>
<th>Condition</th>
<th>Reaction at B</th>
<th>B.M.S.</th>
<th>Deflection</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reaction at B</td>
<td>$r_B = \frac{1}{2}a^2(3-a)$</td>
<td>$K_A = -\frac{1}{2}a(1-a)(2-a)$</td>
<td>$\Delta_{\text{max}} = \frac{1}{2}(1-a)x^2 \left[ 2x-a(2-a)(3-x) \right]$</td>
</tr>
<tr>
<td><strong>Neg. at Support A</strong></td>
<td></td>
<td>$K_C = -\frac{1}{2}a^2(1-a)(5-a)$</td>
<td><strong>Intermediate values</strong></td>
</tr>
<tr>
<td>(Max. Posy at C) = $K_AWL$</td>
<td></td>
<td></td>
<td><strong>can be interpolated</strong></td>
</tr>
<tr>
<td>Distance of C from B = $cL$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Max. Deflection =</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Note:** Values of coefficients for special cases of loading are given in Table 17A.
**BEAMS FIXED AT ONE OR BOTH ENDS.—TABLE 17A. SPECIAL CASES.**

<table>
<thead>
<tr>
<th>B.M. AT B = 0</th>
<th>LOADING</th>
<th>TOTAL LOAD W</th>
<th>REACTION COEFFICIENT $r_B$</th>
<th>BENDING MOMENT COEFFICIENTS $k_A$, $k_B$, $k_C$, $C$</th>
<th>DEFLECTION COEFFICIENT $\Delta_{\text{max}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>$k_A(\text{at support A})$</td>
<td>$k_C$</td>
<td>$C$</td>
</tr>
<tr>
<td></td>
<td>$aL$ W</td>
<td>$\frac{a^2}{2} (3-a)$</td>
<td>$-\frac{a}{2} (2-a)$</td>
<td>$\frac{a^2}{3} (1-a) x$</td>
<td>$1-a$</td>
</tr>
<tr>
<td></td>
<td>$L$ W</td>
<td>$\frac{5}{16}$</td>
<td>$-\frac{3}{16}$</td>
<td>$+\frac{5}{32}$</td>
<td>$\frac{1}{2}$</td>
</tr>
<tr>
<td></td>
<td>$uL$</td>
<td>$\frac{3}{8}$</td>
<td>$-\frac{1}{8}$</td>
<td>$+\frac{9}{128}$</td>
<td>$\frac{3}{8}$</td>
</tr>
<tr>
<td></td>
<td>$\frac{uL}{2}$</td>
<td>$\frac{11}{32}$</td>
<td>$-\frac{5}{32}$</td>
<td>$+\frac{1}{8}$</td>
<td>$\frac{27}{64}$</td>
</tr>
<tr>
<td></td>
<td>$(1-a)uL$</td>
<td>SEE FORMULA ON FACING PAGE</td>
<td>SEE FORMULA ON FACING PAGE</td>
<td>$\frac{3+2a^2-3a^3}{8}$</td>
<td>SEE FACING PAGE</td>
</tr>
</tbody>
</table>

**BEAMS FIXED AT ONE SUPPORT FREELY-SUPPORTED AT OTHER SUPPORT.**

<table>
<thead>
<tr>
<th>B.M. AT B = 0</th>
<th>LOADING</th>
<th>TOTAL LOAD W</th>
<th>REACTION COEFFICIENT $r_B$</th>
<th>BENDING MOMENT COEFFICIENTS $k_A$, $k_B$, $k_C$, $C$</th>
<th>DEFLECTION COEFFICIENT $\Delta_{\text{max}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>$k_A(\text{at support A})$</td>
<td>$k_B$</td>
<td>$k_C$</td>
</tr>
<tr>
<td></td>
<td>$aL$ W</td>
<td>$\frac{a}{2} wL$</td>
<td>$0.20$</td>
<td>$-0.133$</td>
<td>$+0.067$</td>
</tr>
<tr>
<td></td>
<td>$0.5 wL$</td>
<td>$0.17$</td>
<td>$-0.133$</td>
<td>$+0.057$</td>
<td>$0.47$</td>
</tr>
<tr>
<td></td>
<td>$0.3 wL$</td>
<td>$0.13$</td>
<td>$-0.132$</td>
<td>$+0.053$</td>
<td>$0.49$</td>
</tr>
<tr>
<td></td>
<td>$0.2 wL$</td>
<td>$0.10$</td>
<td>$-0.128$</td>
<td>$+0.047$</td>
<td>$0.59$</td>
</tr>
<tr>
<td></td>
<td>$0.1 wL$</td>
<td>$0.08$</td>
<td>$-0.121$</td>
<td>$+0.041$</td>
<td>$0.57$</td>
</tr>
<tr>
<td></td>
<td>$0.05 wL$</td>
<td>$0.06$</td>
<td>$-0.111$</td>
<td>$+0.032$</td>
<td>$0.62$</td>
</tr>
</tbody>
</table>

**NEGATIVE B.M. AT B = (TABULATED OR CALCULATED COEFFICIENT $k_B$) W L.**

<table>
<thead>
<tr>
<th>B.M. AT B = 0</th>
<th>LOADING</th>
<th>TOTAL LOAD W</th>
<th>REACTION COEFFICIENT $r_B$</th>
<th>BENDING MOMENT COEFFICIENTS $k_A$, $k_B$, $k_C$, $C$</th>
<th>DEFLECTION COEFFICIENT $\Delta_{\text{max}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>$k_A$, $k_B$, $k_C$, $C$</td>
<td>$C$</td>
<td>$C$</td>
</tr>
<tr>
<td></td>
<td>$aL$ W</td>
<td>$\frac{a^2}{2} (5-2a)$</td>
<td>$-a (1-a)^2$</td>
<td>$-a (1-a)^2$</td>
<td>$+2a^2 (1-a)^2$</td>
</tr>
<tr>
<td></td>
<td>$L$ W</td>
<td>$\frac{1}{2}$</td>
<td>$-\frac{1}{8}$</td>
<td>$-\frac{1}{8}$</td>
<td>$+\frac{1}{8}$</td>
</tr>
<tr>
<td></td>
<td>$uL$</td>
<td>$\frac{1}{2}$</td>
<td>$-\frac{1}{12}$</td>
<td>$-\frac{1}{12}$</td>
<td>$+\frac{1}{24}$</td>
</tr>
<tr>
<td></td>
<td>$\frac{uL}{2}$</td>
<td>$\frac{1}{2}$</td>
<td>$-\frac{5}{48}$</td>
<td>$-\frac{5}{48}$</td>
<td>$+\frac{1}{16}$</td>
</tr>
<tr>
<td></td>
<td>$(1-a)uL$</td>
<td>$\frac{1}{2}$</td>
<td>$k_A k_B =$</td>
<td>$-1-2a^2+a^3$</td>
<td>$12 (1-a)$</td>
</tr>
</tbody>
</table>

**BEAMS FIXED AT BOTH SUPPORTS.**

<table>
<thead>
<tr>
<th>B.M. AT B = 0</th>
<th>LOADING</th>
<th>TOTAL LOAD W</th>
<th>REACTION COEFFICIENT $r_B$</th>
<th>BENDING MOMENT COEFFICIENTS $k_A$, $k_B$, $k_C$, $C$</th>
<th>DEFLECTION COEFFICIENT $\Delta_{\text{max}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>$k_A$, $k_B$, $k_C$, $C$</td>
<td>$C$</td>
<td>$C$</td>
</tr>
<tr>
<td></td>
<td>$aL$ W</td>
<td>$\frac{a}{2} wL$</td>
<td>$0.30$</td>
<td>$-0.100$</td>
<td>$-0.067$</td>
</tr>
<tr>
<td></td>
<td>$0.5 wL$</td>
<td>$0.26$</td>
<td>$-0.103$</td>
<td>$-0.062$</td>
<td>$+0.042$</td>
</tr>
<tr>
<td></td>
<td>$0.3 wL$</td>
<td>$0.22$</td>
<td>$-0.105$</td>
<td>$-0.056$</td>
<td>$+0.041$</td>
</tr>
<tr>
<td></td>
<td>$0.1 wL$</td>
<td>$0.18$</td>
<td>$-0.104$</td>
<td>$-0.047$</td>
<td>$+0.040$</td>
</tr>
<tr>
<td></td>
<td>$0.05 wL$</td>
<td>$0.14$</td>
<td>$-0.102$</td>
<td>$-0.038$</td>
<td>$+0.036$</td>
</tr>
<tr>
<td></td>
<td>$0.025 wL$</td>
<td>$0.10$</td>
<td>$-0.096$</td>
<td>$-0.029$</td>
<td>$+0.032$</td>
</tr>
</tbody>
</table>

**Note.**—See also Table 18 for factors for beams fixed at both ends.

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REINFORCED CONCRETE DESIGNER'S HANDBOOK
NOTES ON CANTILEVERS AND BEAMS OF ONE SPAN.

Propped Cantilevers.

A propped cantilever, that is, a beam fixed rigidly at one support and freely supported at the other, can be considered as being subject to two systems of loading: (i) the imposed load; and (ii) the reaction of the prop, the magnitude of which being that which gives the same upward deflection at the end of a cantilever as the downward deflection produced by the imposed load. The coefficients of the maximum deflection given in Table 16 can be used for this purpose if the type of load is not one of those given for the beams fixed at one end and simply supported at the other in Tables 17 and 17a. As comparative deflections only are required the numerical values of I and E are not required. When the reaction of the prop has been calculated, the shearing forces and bending moments can be computed from the basic definitions.

Example.—A beam AB is rigidly fixed at A and freely supported at B. The span AB is 10 ft. and a concentrated load of 10 tons (= W) acts at a point 7 ft. from A. Find the load on the support at B and the bending moment at the support A.

The beam is a propped cantilever and the reaction at B can be found by equating the deflection at B of a cantilever loaded with W, as in the second case in Table 16, to the deflection at B of a cantilever loaded at its extremity (the first case in Table 16). The deflection due to W with a = \( \frac{7}{10} = 0.7 \), is \( \frac{0.7^2(3 - 0.7)10 \times 10^3}{6EI} = \frac{1880}{EI} \). The load at B is the unknown reaction \( R_B \) at this point, and the deflection of a cantilever is

\[
\frac{R_B \times 10^3}{3EI} = \frac{333R_B}{EI}
\]

As one deflection counteracts the other, \( \frac{1880}{EI} = \frac{333R_B}{EI} \), therefore \( R_B = 5.64 \) tons. The bending moment at A is \( (5.64 \times 10) - (10 \times 7) = -13.6 \) ft.-tons, and the shearing force at A is \( 10 - 5.64 = 4.36 \) tons. The shearing force at B is 5.64 tons.

Beams Fixed at Both Ends.

The bending moment on a beam fixed at both ends is derived from the principle that the area of the bending-moment diagram due to the same load imposed on a freely-supported beam of equal span (the "free-moment" diagram) is equal to the area of the restraint-moment diagram; the centroids of the two diagrams are in the same vertical line. The shape of the free-moment diagram depends upon the characteristics of the imposed load, but the restraint-moment diagram is a trapezium. For loads symmetrically disposed on the beam the centroid of the free-moment diagram is above the mid-point of the span, and thus the restraint-moment diagram is a rectangle, giving a restraint bending moment at each support equal to the mean height of the free-moment diagram.

The amount of the shearing force in a beam with one or both ends fixed is calculated from the variation of the bending moment. The shearing force resulting from the restraint moment alone is constant throughout the beam, and equals the difference between the two end moments divided by the span, that is, the rate of change of the restraint moment. This shearing force is algebraically added to the shearing force due to the imposed load considering the beam to be freely supported; that is, the reaction at a support is the sum (or difference) of the restraint-moment shearing force and the free-moment shearing force. For a symmetrically-loaded beam with both ends fixed the restraint moment at each end is the same, and the shearing force due to this moment considered alone is zero; therefore the resultant shearing forces are identical with those for the same beam freely supported, and the reactions both equal half the total load on the beam.
SINGLE-SPAN FIXED-END BEAMS.—TABLE 18.

THE LOAD FACTORS \( C_{AB} \) AND \( C_{BA} \) CAN BE USED THUS:

(i) B.M. AT SUPPORTS OF SINGLE SPAN BEAMS FIXED BOTH ENDS [SEE NOTE (1)]

\[
M_{AB} = -C_{AB}L_{AB} \quad M_{BA} = -C_{BA}L_{AB} \quad \text{WITH SYMMETRICAL LOAD} \quad M_{AB} = M_{BA}
\]

(ii) CONTINUOUS BEAMS — MOMENT DISTRIBUTION METHOD [SEE NOTE (2)]

SPAN-LOAD FACTORS ARE \( F_{AB} = C_{AB}L_{AB} \), \( F_{BA} = C_{BA}L_{AB} \)

WITH SYMMETRICAL LOAD \( F_{AB} = F_{BA} \)

(iii) FRAMED STRUCTURES [SEE NOTE (3)]

LOADING FACTORS \( P \) AND \( Q \) HAVE FOLLOWING VALUES:

\[
P_{AB} = C_{AB}L_{AB} \quad Q_{BA} = C_{BA}L_{AB} \quad \text{WITH SYMMETRICAL LOAD} \quad P_{AB} = Q_{BA} = \frac{A_{AB}}{L_{AB}} = \frac{C_{AB}L_{AB}}{L_{AB}}
\]

(iv) PORTAL FRAMES [SEE NOTE (4)]

\[
N = \text{AREA OF FREE B.M. DIAGRAM} \left( \text{COEFFICIENT TIMES} \frac{A_{AB} + C_{BA}}{L_{AB}} \right) \text{DISTANCE FROM LEFT HAND OR LOWER SUPPORT TO CENTROID OF FREE B.M. DIAG.}
\]

### UNSYMMETRICAL LOADING

<table>
<thead>
<tr>
<th>LOAD</th>
<th>( L_{AB} )</th>
<th>( A )</th>
<th>( B )</th>
<th>( C_{AB} )</th>
<th>( C_{BA} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( aL )</td>
<td>( qL )</td>
<td>( aL )</td>
<td>( W ) (TOTAL)</td>
<td>( \Sigma (1-a)^2 W )</td>
<td>( \Sigma a^2 (1-a) W )</td>
</tr>
</tbody>
</table>

### SYMMETRICAL LOADING

<table>
<thead>
<tr>
<th>LOAD</th>
<th>( L_{AB} )</th>
<th>( A )</th>
<th>( B )</th>
<th>( C_{AB} = C_{BA} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( W ) (TOTAL)</td>
<td>( \Sigma )</td>
<td>( \Sigma )</td>
<td>( \Sigma )</td>
<td>( \Sigma )</td>
</tr>
</tbody>
</table>

### OTHER LOADINGS

CAN GENERALLY BE CONSIDERED BY COMBINING TABULATED CASES, THUS:

- [Diagram]
- [Diagram]
- [Diagram]

**Note 1.**—See also Tables 17 and 17a.

**2.**—Factors for use with Table 26.

**3.**—Factors for use with Table 46.

**4.**—Factors for use with Tables 49 and 50; also for continuous beams (Table 24).
Three-moment Theorem.—Formulas in general and special cases are given in Table 19; the values of the factors $A$ and $x$ for use in these formulas are given in Table 18.

When known factors relating to the load, span, moment of inertia and relative levels of the supports are substituted in the general formula, an equation with three unknown support bending moments is obtained for each pair of spans; that is, for $n$ spans, $n - 1$ equations are obtained containing $n + 1$ unknowns (the moments at $n + 1$ supports). The two excess unknowns represent the bending moments at the end supports and, if these bending moments are known or can be assumed, the bending moments at the intermediate supports can be determined. At a freely-supported end the bending moment is zero. For a perfectly fixed end the support bending moments can be determined if an additional span is considered continuous at the fixed end; this additional span must be identical in all respects with the original end span except that the load should produce symmetry about the original end support with the load on the original end span. The bending moment at the new end support is considered to be equal to that at the original penultimate support, and thus an additional equation is obtained without introducing another unknown.

When the bending moments at the supports have been calculated, the diagram of the support bending moments is combined with the diagram of the "free moments" and the resulting bending moments are obtained.

Example.—Determine by the Theorem of Three Moments the bending moments at the supports of the beam in the diagram, assuming level supports.

From Table 19 the appropriate formula modified for spans AB and BC (span $L_1 = L_{AB}$, and $L_3 = L_{BC}$) is

$$M_A \frac{L_1}{I_1} + 2M_B \frac{L_1}{I_1} + \frac{L_1}{I_1} + M_C \frac{L_3}{I_3} = - \left[ A_1 \frac{x_1}{I_1} + \frac{A_3 (L_3 - x_3)}{I_3} \right] 6 \quad \text{(1)}$$

and for spans BC and CD (span $L_2 = L_{BC}$ and $L_4 = L_{CD}$)

$$M_B \frac{L_2}{I_2} + 2M_C \frac{L_2}{I_2} + \frac{L_2}{I_2} + M_D \frac{L_4}{I_4} = - \left[ A_2 \frac{x_2}{I_2} + \frac{A_4 (L_4 - x_4)}{I_4} \right] 6 \quad \text{(2)}$$

For span AB, $L_1 = \frac{10}{4000}$ = 0.0025 (ft. in.) and $A_1 = \frac{wL_1^3}{24} = \frac{1000 \times 10^3}{24} = 4170$ ft.-lb.

$x_1 = \frac{8}{15}L_1$; thus $\frac{x_1}{L_1} = \frac{8}{15} \times 0.0025 = 0.00133$. Hence $\frac{A_1 x_1}{I_1 L_1} = 4170 \times 0.00133 = 5.55$.

For span BC, $L_2 = \frac{15}{6000} = 0.0025$, and $A_3 = \frac{wL_3^3}{12} = \frac{1500 \times 15^2}{12} = 28,125$ ft.-lb.;

$\frac{x_3}{L_3} = \frac{15}{2L_3} = \frac{L_3 - x_3}{L_3} = 0.5 \times 0.0025 = 0.00125$.

Hence $\frac{A_3 x_3}{I_3 L_3} = 28,125 \times 0.00125 = 35.156$.

For span CD, $L_4 = \frac{12}{5000} = 0.0024$, and $A_4 = \frac{wL_4^3}{24} = \frac{1200 \times 12^2}{24} = 7200$ ft.-lb.; $x_4 = \frac{7L_4}{15}$

thus $\frac{L_4 - x_4}{L_4} = \frac{8}{15} \times 0.0024 = 0.00128$. Hence $\frac{A_4 (L_4 - x_4)}{I_4 L_4} = 7200 \times 0.00128 = 9.21$.

If the beam is simply supported at A and D, $M_A = M_D = 0$, and substituting known values in (1) and (2),

$$2M_B (0.0025 + 0.0025) + 0.0025M_C = - (5.55 + 35.16)6 \quad \text{(3)}$$

Thus

$$0.0025M_B + 0.0025M_C = - 244 \quad \text{(4)}$$

$$0.0025M_B + 0.0098M_C = - 266 \quad \text{(5)}$$

$$0.0025M_B + 0.00392M_C = - 1064 \quad \text{(6)}$$

Multiplying (6) by $0.0100M_B = 4$; $0.0100M_B + 0.00392M_C = - 1064 \quad \text{(7)}$

(Continued on page 174.)
## Continuous Beams: Three-Moment Theorem — Table 19

### General Equation

\[
M_A \frac{L_{AB}}{I_{AB}} + 2 M_B \left( \frac{L_{AB} + L_{BC}}{I_{BC}} \right) + M_C \frac{L_{BC}}{I_{BC}} = -G \left[ \frac{A_{AB} Z_{AB}}{I_{AB}} + \frac{A_{BC} (L_{BC} - Z_{BC})}{I_{BC}} \right]
\]

### Rigid Supports, Uniform Moment of Inertia

\[
M_A \frac{L_{AB}}{I_{AB}} + 2 M_B \left( \frac{L_{AB} + L_{BC}}{I_{BC}} \right) + M_C L_{BC} = -G \left[ \frac{A_{AB} Z_{AB}}{I_{AB}} + \frac{A_{BC} (L_{BC} - Z_{BC})}{I_{BC}} \right]
\]

### Uniformly-Distributed Load (w lb, per foot) on Both Spans

\[
M_A L_{AB} + 2 M_B (L_{AB} + L_{BC}) + M_C L_{BC} = -\frac{w}{4} \left( \frac{3}{L_{AB}} + \frac{3}{L_{BC}} \right)
\]

### Graphical Method

Treat each consecutive pair of spans thus:

1. On the spans drawn to scale, construct the free-moment curves and the moment of inertia curves. Divide the ordinates of the former by those of the latter to enable the curves of free b.m. to be drawn.

2. Find \( A_{AB} = \text{area under curve for span } L_{AB} \) and position of centroid \( G_i \) and \( A_{BC} = \frac{L_{BC}}{L_{AB}} \) and position of centroid \( G_i \) set up \( A_D, A_E, B, C \) and \( G_B \) to a suitable scale to represent any assumed values of the ordinates at \( A, B, C \) respectively. Connect \( DB, AE, BC, \) and \( BF \).

3. Let \( M_A = k_A A_D \), \( M_B = k_B B_E \), \( M_C = k_C C_F \).

4. Divide the ordinates of \( DB \) and \( AE \) by the ordinates of the \( M \) of I curve to give the curves \( AH, HC, \) and \( GB \). Find \( A_V = \text{area under curve } GB \) and position of centroid \( G_V \)

5. \( A_X = \text{area } AH \times \text{AH} = \text{GB} \times \text{GB} \)

6. \( A_Y = \text{area } HC \times \text{HC} = \text{FY} \times \text{FY} \)

7. \( A_U = \text{area } BJ \times BJ = \text{GU} \times GU \)

8. Substitute in

\[
(A_{AB} Z_{AB} + K_A A_V + K_B X_{AB}) = -(A_{BC} Z_{BC} + K_A A_U + K_B Y_{AB})
\]

### Approximate Method

<table>
<thead>
<tr>
<th>APPROXIMATE METHOD</th>
<th>RATIO OF ( I_S : I_C )</th>
<th>BOTH ENDS</th>
<th>MIDSPAN</th>
<th>FIXED</th>
<th>BOTH ENDS</th>
<th>MIDSPAN</th>
<th>FIXED</th>
<th>BOTH ENDS</th>
<th>MIDSPAN</th>
<th>FIXED</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \pm 25 % )</td>
<td>( \pm 50 % )</td>
<td>( \pm 75 % )</td>
<td>( \pm 125 % )</td>
<td>( \pm 150 % )</td>
<td>( \pm 25 % )</td>
<td>( \pm 50 % )</td>
<td>( \pm 75 % )</td>
<td>( \pm 125 % )</td>
<td>( \pm 150 % )</td>
<td>( \pm 25 % )</td>
</tr>
</tbody>
</table>

### Table Representation

| Unknowns are \( k_A, k_B, \) and \( k_C \) and requisite number of equations follows from consecutive pairs of spans and end support conditions. |

---

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CONTINUOUS BEAMS (continued from page 172).

Subtracting (5) from (7): \(0.0367M_C = -820\); hence \(M_C = -22,400 \text{ ft.-lb.}\).
By substituting in (5): \(0.0100M_B = -(244 - 56) = -190\); hence \(M_B = -19,800 \text{ ft.-lb.}\).

**Non-uniform Moment of Inertia.**—When the moment of interia is practically uniform throughout each span of a series of continuous spans, but differs in one span relative to another, the general expressions for the Theorem of Three Moments, the formulae for which are given in Table 19, are applicable as in the example above. When the moment of inertia varies irregularly within the length of each span the semi-graphical method given in Table 19 can be used. The moments of inertia of common reinforced concrete sections are given in Tables 64, 65 and 66.

If the moment of inertia varies in such a way that it can be represented by an equation, the Theorem of Three Moments can be used if for \(M\) is substituted \(\frac{M}{I}\) and if the area of the \(\frac{M}{I}\) diagram is used instead of the area of the free-moment diagram. The solution of the derived simultaneous equations then gives values of the support bending moment \(\frac{M}{I}\), enabling a complete \(\frac{M}{I}\) diagram for the beam to be constructed from which the bending moment at any section is readily obtained by multiplying the appropriate ordinate of the \(\frac{M}{I}\) diagram by the moment of inertia at the section.

When circumstances do not permit the foregoing methods to be used, the bending moments can be calculated on the assumption of uniform moment of inertia, and an approximate adjustment can be made for the effect of the neglected variation. An increase in the moment of inertia near a support causes an increase in the negative bending moment at that support and a consequent decrease in the positive bending moments in the adjacent spans, and vice versa. As a guide in making the adjustment the approximate factors given in Table 19 represent a percentage addition to, or deduction from, the calculated bending moments.

**Incidence of Live Load to Produce Maximum Bending Moments.**—The values of the bending moments at the support and in the span depend upon the incidence of the live load, and for equal spans or with spans approximately equal the dispositions of live load illustrated in Table 20 give the maximum positive bending moment at midspan, and the maximum negative bending moment at a support. The B.S. Code recommends the incidence of live load to be less severe, since when calculating the maximum negative bending moments at any support only the spans immediately on either side of the support under consideration need be loaded. This affects only the coefficients for four or more continuous spans and the reduction is commonly much less than 5 per cent.

**Positive and Negative Bending Moments in the Span.**—When the negative bending moments at the supports of a continuous beam have been determined, the positive bending moments on a loaded span can be determined graphically or, in the case of a uniformly-distributed load, by means of the expressions in Table 20.

Beams and slabs, such as those in bridge decks, where the ratio of live load to dead load is high, should be designed for a possible negative bending moment occurring at midspan. Formulate for the approximate evaluation of this bending moment, which apply if the lengths of adjacent spans do not differ by more than 20 per cent. of the shorter span, are given in Table 20. These formulate make some allowance for the torsional restraint of the supports.

**Shearing Forces.**—The variation of shearing force on a continuous beam is determined by first considering each span as freely supported and algebraically adding the rate of change of restraint moment for the span considered. Formulate for calculating the component and resultant shearing forces are given in Table 20. The shearing force due to the load can be determined from statics. The shearing force due to the restraint moments is constant throughout the span.

**Approximate Bending-moment Coefficients.**—The bending-moment coefficients in Table 20 apply to beams or slabs (spanning in one direction) continuous over three or more spans. The coefficients given for end spans and penultimate supports assume that the beam or slab is nominally freely supported on the end support. The coefficients given for the dead load and live load separately on beams and slabs conform to the recommendations of B.S. Code No. 114; the corresponding coefficients for total load are calculated for various ratios of live to dead load. The coefficients given for slabs only, without spays, are values commonly assumed and apply to the total load on a slab spanning in one direction; they take into account the fact that the slab is partially restrained at the end supports because of monolithic construction. If the slab is provided with spays, of sizes not less than indicated in the diagram, the positive bending moments are decreased and the negative bending moments increased; suitable coefficients for this condition are also given in Table 20.
CONTINUOUS BEAMS: GENERAL DATA—TABLE 20.

INCIDENCE OF LIVE LOAD

TO PRODUCE MAXIMUM POSITIVE B.M. ON SPAN ST.

TO PRODUCE MAXIMUM NEGATIVE B.M. AT SUPPORT S.

B.S. CODE,

LOADS ON SPANS RS AND ST ONLY NEED BE TAKEN INTO ACCOUNT.

UNIFORMLY-DISTRIBUTED LOAD W ON SPAN ST.

MAX. POSITIVE B.M. ON SPAN ST.

\[ M_{\text{max}} = \frac{w l}{2} \left( \frac{M_{ST} - M_{TS}}{w l_{ST}} + \frac{L_{ST}^2}{2} \right) - M_{ST} \text{FT.-LB.} \]

IF W IS IN LB./FT., B.M.'S. MUST BE IN FT.-LB., AND L_{ST} MUST BE IN FT.

\[ X = \frac{L_{ST}}{2} + \frac{M_{ST} - M_{TS}}{w l_{ST}} \text{ (FT.)} \]

NEGATIVE B.M. ON UNLOADED SPAN ST. (BETWEEN TWO LOADED SPANS)

\[ M_{\text{neg}} = (k w_{L} - w_{D}) \frac{L_{ST}}{24} \text{ FT.-LB.} \]

\[ w_{L} = \text{LIVE LOAD LB./FT. FOR BEAMS.} \]

\[ w_{D} = \text{DEAD LOAD LB./SQ. FT. FOR SLABS, FOR BEAMS: } k = \frac{3}{2} \text{ FOR SLABS: } k = \frac{1}{2} \]

DUE TO LOAD.

\[ Q_{T} = \frac{z_{0} w}{L_{ST}} \]

\[ Q_{s} = \frac{L_{ST}}{2} - z_{0} w \]

\[ = \left( w - Q_{T} \right) \]

DUE TO END RERAINT.

\[ Q_{M} = \frac{M_{ST} - M_{TS}}{L_{ST}} \]

RESULTANT SHEARING FORCES.

\[ Q_{S} = Q_{s} + Q_{M} \]

\[ Q_{T} = \left( Q_{T} - Q_{M} \right) \]

NOTE—IF M_{TS} > M_{ST}, Q_{M} IS NEG.

DIFFERENTIALLY DISTRIBUTED LOAD ON EQUAL SPANS

FREE SUPPORT APPLICABLE TO THREE OR MORE SPANS.

A B C D

BEAMS AND SLABS (B.S. CODE)

DEAD LOAD LIVE TOTAL LOAD R=2 LIVE LOAD DEAD LOAD

END SPAN AB ABOUT MIDSAN

+ \frac{1}{12} + \frac{1}{10} + \frac{1}{112} + \frac{1}{10} + \frac{1}{10} + \frac{1}{10}

END SUPPORT A.

- - - - - -

PENULTIMATE SUPPORT B.

- \frac{1}{10} - \frac{5}{8} - \frac{1}{8} - \frac{1}{8} - \frac{1}{8} - \frac{1}{8}

INTERIOR SPAN BC.

+ \frac{24}{12} + \frac{17}{10} + \frac{1}{16} + \frac{1}{16} + \frac{1}{16} + \frac{1}{16}

INTERIOR SUPPORTS C.

- \frac{1}{12} - \frac{5}{8} - \frac{10}{8} - \frac{5}{8} - \frac{5}{8} - \frac{5}{8}

END SUPPORT A.

- - - - - -

SLABS ONLY MONOLITH WITH END SUPPORT A (NOMINALLY FREELY SUPPORTED)

B.M. = (COEFF.)x(TOTAL LOAD)xL

COEFFICIENTS APPLY ALSO TO UNEQUAL SPANS IF INEQUALITY DOES NOT EXCEED 15 PERCENT OF THE SHORTEST SPAN.

BENDING MOMENT COEFFICIENTS

APPROXIMATELY.

END SPAN AB ABOUT MIDSAN

+ \frac{1}{12} + \frac{1}{10} + \frac{1}{112} + \frac{1}{10} + \frac{1}{10} + \frac{1}{10}

END SUPPORT A.

- - - - - -

PENULTIMATE SUPPORT B.

- \frac{1}{10} - \frac{5}{8} - \frac{1}{8} - \frac{1}{8} - \frac{1}{8} - \frac{1}{8}

INTERIOR SPAN BC.

+ \frac{24}{12} + \frac{17}{10} + \frac{1}{16} + \frac{1}{16} + \frac{1}{16} + \frac{1}{16}

INTERIOR SUPPORTS C.

- \frac{1}{12} - \frac{5}{8} - \frac{10}{8} - \frac{5}{8} - \frac{5}{8} - \frac{5}{8}

END SUPPORT A.

- - - - - -

MINIMUM PROPORTIONS OF SPLAYS

d_{2} = \frac{1}{10}

I_{2} = \frac{1}{16}

I_{3} = \frac{1}{24}

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CONTINUOUS BEAMS: EQUAL SPANS.

Note on Table 21.—The coefficients in Table 21 for bending moments at supports due to incidental live load are for alternate spans loaded including the two spans immediately adjacent to the support. The corresponding coefficients given in brackets are for the two spans immediately adjacent to the support only loaded.

Examples of Use of Tables 20 and 21.
(a) Calculate the maximum bending moments at the middles of the end and central spans and at the penultimate and interior supports of a beam continuous over five equal spans of 15 ft. each if the dead load is 600 lb. per ft. and the live load is 1200 lb. per ft.

(i) From Table 21 (using coefficients for all alternate spans being loaded):
Penultimate support: Dead load: \(0.105 \times 600 \times 15^3 = 14,200\) ft.-lb.
Live load: \(0.120 \times 1200 \times 15^3 = 32,300\) ft.-lb.
Total (negative) \(= 46,500\) ft.-lb.

Interior support:
Dead load: \(0.080 \times 600 \times 15^3 = 10,400\) ft.-lb.
Live load: \(0.111 \times 1200 \times 15^3 = 29,900\) ft.-lb.
Total (negative) \(= 40,300\) ft.-lb.

Middle of end span:
Dead load: \(0.078 \times 600 \times 15^3 = 10,250\) ft.-lb.
Live load: \(0.100 \times 1200 \times 15^3 = 27,000\) ft.-lb.
Total (positive) \(= 37,250\) ft.-lb.

Middle of interior span:
Dead load: \(0.046 \times 600 \times 15^3 = 6,200\) ft.-lb.
Live load: \(0.086 \times 1200 \times 15^3 = 23,200\) ft.-lb.
Total (positive) \(= 29,400\) ft.-lb.

(ii) By means of Table 20, using the coefficients recommended in the B.S. Code.
Ratio of live to dead load, \(\frac{1200}{600} = 2\). Total load = 1200 + 600 = 1800 lb. per ft.
Penultimate support: \(- \frac{1}{9.3} \times 1800 \times 15^3 = 43,500\) ft.-lb. (neg.)
Interior support: \(- \frac{1}{9.8} \times 1800 \times 15^3 = 41,300\) ft.-lb. (neg.)
Middle of end span: \(+ \frac{1}{10.6} \times 1800 \times 15^3 = 38,200\) ft.-lb. (pos.)
Middle of central span: \(+ \frac{1}{14.4} \times 1800 \times 15^3 = 28,100\) ft.-lb. (pos.)

(b) Solve, by means of Table 21, Example (a) opposite Table 25.
Bending moment at penultimate support of a beam continuous over four spans:
Dead load: \(0.107 \times 1000 \times 15^3 = 24,100\) ft.-lb.
Live load: \(0.181 \times 10,000 \times 15 = 27,200\) ft.-lb.
Total (negative) \(= 51,300\) ft.-lb.
## CONTINUOUS BEAMS: EQUAL LOADS ON EQUAL SPANS.—TABLE 21.

<table>
<thead>
<tr>
<th>LOAD</th>
<th>ALL SPANS LOADED (E.G. DEAD LOAD)</th>
<th>LIVE LOAD (SEQUENCE OF LOADED SPANS TO GIVE MAX. B. M. OR S. F.)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### BENDING MOMENTS

- **Uniformly-distributed**
  - Uniformly-distributed
  - Concentrated at mid-span
  - Concentrated at three points

### Shearing Force

- **Uniformly-distributed**
  - Concentrated force

### Coefficients for maximum bending moments

- **Coefficients for maximum bending moments**
  - Uniformly-distributed
  - Concentrated at mid-span
  - Concentrated at three points
  - Concentrated force

### Coefficients for maximum shearing force

- **Coefficients for maximum shearing force**
  - Uniformly-distributed
  - Concentrated force

**Bending Moment** = (Coefficient) × (Total Load on One Span) × (Span).

**Shearing Force** = (Coefficient) × (Total Load on One Span).

**B. M. Coefficients Above Line Apply to Negative B. M. at Supports.**
- **Below** = Positive B. M. in Span.
- **S. F. Coefficients Above** = S. F. at Right-Hand Side of Support.
- **Below** = S. F. at Left-Hand Side of Support.

**Coefficients Apply When All Spans Are Equal (or Longest > 15% More than Shortest).**
- Loads on Each Loaded Span Are Equal.
- Moment of Inertia Same Throughout All Spans.

**B. M. Coefficients (Live Load) in Brackets Apply If Two Spans Only Are Loaded.**

---

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# Continuous Beams: Bending Moments Diagrams

## Two & Three Equal Spans
### Uniform Moment of Inertia

<table>
<thead>
<tr>
<th>Two Spans</th>
<th>Three Spans</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Dead Load</strong></td>
<td><strong>Live Load</strong></td>
</tr>
<tr>
<td>(All Spans Loaded)</td>
<td>(All Spans Loaded)</td>
</tr>
<tr>
<td>0.125 (0.106)</td>
<td>0.125 (0.106)</td>
</tr>
<tr>
<td>0.056 (0.041)</td>
<td>0.056 (0.041)</td>
</tr>
<tr>
<td>0.050 (0.040)</td>
<td>0.050 (0.040)</td>
</tr>
<tr>
<td>0.187 (0.159)</td>
<td>0.187 (0.159)</td>
</tr>
<tr>
<td>0.150 (0.123)</td>
<td>0.150 (0.123)</td>
</tr>
<tr>
<td>0.134 (0.114)</td>
<td>0.134 (0.114)</td>
</tr>
<tr>
<td>0.122 (0.103)</td>
<td>0.122 (0.103)</td>
</tr>
<tr>
<td>0.056 (0.040)</td>
<td>0.056 (0.040)</td>
</tr>
<tr>
<td>0.050 (0.040)</td>
<td>0.050 (0.040)</td>
</tr>
<tr>
<td>0.111 (0.091)</td>
<td>0.111 (0.091)</td>
</tr>
<tr>
<td>0.056 (0.040)</td>
<td>0.056 (0.040)</td>
</tr>
<tr>
<td>0.050 (0.040)</td>
<td>0.050 (0.040)</td>
</tr>
<tr>
<td>0.111 (0.091)</td>
<td>0.111 (0.091)</td>
</tr>
</tbody>
</table>

**Equal Load W (Total Load) on Each Loaded Span.**

Bending Moment = (Coefficient) x (W x (Span))

**Coefficients Thus:**
- 0.125 = Theoretical Bending Moments
- 0.106 = Bending Moments with B.M.s at Supports Reduced by 15% (and Reduced Support B.M.s for Live Load)
- 0.101 = Bending Moments with B.M.s at Supports Increased by 15% (Give Adjusted B.M.s for Dead Load)

Diagrams are symmetrical but not drawn to scale.
### Continuous Beams: Bending Moments

#### Four or More Equal Spans

##### Uniform Moment of Inertia

| End Span and Penultimate Span & Support | Other Interior Spans & Supports
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(Based on Four Continuous Spans)</td>
<td>(Based on Central Span of Five Continuous Spans)</td>
</tr>
</tbody>
</table>

#### Dead Load

- **End Span and Penultimate Span & Support**
  - **Dead Load (All Spans Loaded)**
    - $0.107\, (0.091)$
    - $0.077\, (0.084)$
    - $0.170\, (0.183)$
    - $0.072\, (0.086)$
  - **Live Load**
    - $0.116\, (0.099)$
    - $0.099\, (0.097)$
    - $0.174\, (0.148)$
    - $0.072\, (0.105)$

#### Live Load

- **End Span and Penultimate Span & Support**
  - **Dead Load (All Spans Loaded)**
    - $0.053\, (0.061)$
    - $0.053\, (0.061)$
    - $0.053\, (0.061)$
    - $0.053\, (0.061)$
  - **Live Load**
    - $0.036\, (0.042)$
    - $0.036\, (0.042)$
    - $0.036\, (0.042)$
    - $0.036\, (0.042)$

#### Equal Load $W$ (Total Load) on Each Loaded Span

- Bending Moment = (Coefficient) $\times (W) \times (\text{Span})$

#### Coefficients Thus:

- $0.116$ = Theoretical Bending Moments
- $0.099$ = Bending Moments with B.M.S. at Supports Reduced by 15 Per Cent.
- $0.097$ = Bending Moments with B.M.S. at Supports Increased by 15 Per Cent.

#### Diagrams Are Not Drawn to Scale

- Diagrams for Other Interior Spans & Supports Are Symmetrical. (Give Adjusted B.M.S. for Dead Load and Reduced Support B.M.S. for Live Load)

---

**Table 28:** Continuous Beams: Bending-Moment Diagrams—Four or More Equal Spans.
Graphical Determination of Bending Moments by Fixed Points.—A graphical method of determining the bending moments on a continuous beam is given in Table 24. The basis of the method is that there is a point (termed the “fixed point”) adjacent to the left-hand support of any span of a continuous system at which the bending moment is unaffected by any alteration in the bending moment at the right-hand support. A similar point occurs near the right-hand support, the bending moment at this point being unaffected by alteration in the bending moment at the left-hand support. When a beam is rigidly fixed at a support the “fixed point” is one-third of the span from that support; when freely supported the fixed point coincides with the support. For intermediate conditions of fixity the “fixed point” is between these extremes. In two continuous spans, $L_1$ and $L_2$, if the distance from the left-hand (or right-hand) support to the adjacent fixed point is $l_1$, then the distance $l_2$ from the left-hand (or right-hand) support of span $L_2$ to the adjacent fixed point is

$$l_2 = \frac{L_2(L_1 - l_1)}{3(L_1 + L_2)(L_1 - l_1) - L_2}$$

Alternatively $l_2$ can be found from $l_1$ by the graphical construction shown in Table 24. Upon combining the free-moment diagram with the position of the fixed points for a span, as described in Table 24, the resultant negative and positive bending moments throughout the system, due to the load on this span, can be determined. By treating each span separately the envelopes of the maximum possible bending moments throughout the system can be drawn.

Bending Moments on Equal and Unequal Spans.—The factors in Table 25 apply to the calculation of the support moments for beams with uniform moment of inertia and continuous over two, three, or four equal or unequal spans, and carrying almost any type or incidence of live and dead loads. The basis of the method is that the load on any span of a given system can be divided into one or more of the types shown in Table 25. Other types of loading can be allowed for thus: partially-distributed load at one end of span gives coefficients between those for (a) and (c); three or more equal concentrated loads give coefficients between those for (a) and (e).

[Note.—The references to the types of load are: Type a.—Uniformly-distributed load. Type b.—Triangular load with apex at centre. Type c.—Triangular load with apex at left-hand support. Type d.—Central concentrated load. Type e.—Concentrated loads at each of one-third points.]

A given beam system can be divided into a series of identical systems each with only one span loaded with one type of load. Each of these loads produces a bending moment at each support, the total bending moment at any support being the algebraic sum of the moments due to each type of load. When the support moments are known, the mid-span moments can be calculated or determined graphically therefrom.

For equal spans the bending moment at any support due to any one span loaded with any one type of load is given by $M_S = \frac{W}{2}L$, where $M$ is the support moment at the loaded span. The total moment at any support is $EM_S$.

For unequal spans the method is similar to that for equal spans, except for the introduction of the moment-multiplier, $U$, which varies with the support being treated and for the particular span loaded, and depends on the ratio that each span bears to the basic span. Thus for unequal spans $M_S = UFQWL$.

If one type of load extends equally over all spans of a system of equal spans, the beam need not be divided into separately loaded spans, as the values of $Q$ tabulated for this condition apply.

Examples of the use of Table 25 are given on the page facing the table.
CONTINUOUS BEAMS: GRAPHICAL METHOD.—TABLE 24.
UNIFORM MOMENT OF INERTIA.

(a) To determine the positions of the fixed points.

(i) Unequal spans (graphical method)

Draw the beam ABCD... to scale.
Plot the position of the fixed point F1 in left-hand part of span AB.
If the beam is freely supported at A, F1 is at A.
If the beam is fixed at A, \( f1 = 0.333 L_1 \).
Set up verticals through the third points of each span.
Set off \( BE = x_2 = 0.333 (L_2 - L_1) \).
Set off \( CE = x_3 = 0.333 (L_3 - L_2) \).
Set off \( DE = x_4 = 0.333 (L_4 - L_3) \).

(b) To determine the bending moments.

Draw the beam ABCD... to scale and mark the positions of the fixed points F1, F1, F2, F2, etc.
Set up BK and CN at the supports of the loaded span.
\( BK = \frac{3A_2}{L_2} \) and \( CN = \frac{3A_2}{L_2} (L_2 - z_2) \).
For symmetrical load \( BK = CN = \frac{3A_2}{L_2} \).
\( A_2 \) = area of free-moment diagram; \( z_2 \) = distance of centroid of \( A_2 \) from B.

Complete B.M. diagram (shown shaded) by drawing \( PF_1 \), produced, and \( RF_2 \), etc.
Repeat for other loaded spans and combine diagrams to give total b.m.s.

For values of \( \frac{A}{L} (= C_{AB} \text{ for } BK \text{ and } C_{BA} \text{ for } CN) \) and \( x(= z_2L) \), see Table 18.

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EXAMPLES OF THE USE OF TABLE 25.

(a) Find the maximum bending moment at the penultimate support of a beam continuous over four equal spans of 15 ft. carrying a uniformly-distributed load of 1000 lb. per lin.-ft.; a live load of 10,000 lb. can occur at the centre of one or more of the spans simultaneously.

The maximum bending moment occurs when the first, second, and fourth spans are loaded with the concentrated load. Dividing the total load into single-span loads:

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Uniformly distributed</td>
<td>All spans</td>
<td>1:00</td>
</tr>
<tr>
<td>Concentrated load W = 10,000 lb.</td>
<td>1st span</td>
<td>1:50</td>
</tr>
<tr>
<td>Ditto.</td>
<td>2nd span</td>
<td>1:50</td>
</tr>
<tr>
<td>Ditto.</td>
<td>4th span</td>
<td>1:50</td>
</tr>
</tbody>
</table>

\[ \Sigma FQW = -3405 \]

Bending moment = \( L \times \Sigma FQW = -3405 \times 15 = -51,075 \) ft.-lb.

It is seen from the table that these results are the maximum, because if the third span is loaded a positive bending moment is introduced that reduces the total negative bending moment.

(b) Find the bending moment at the centre of the central span of three continuous spans of 10 ft., 15 ft. and 10 ft. respectively. The load on the central span is a uniform dead load of 7000 lb. per lin.-ft. and on each of the end spans the load is triangularly distributed, being 6000 lb. per ft. at the inner supports and zero at the outer supports at which the beam is freely supported. (This type of load occurs in a rectangular tank when the counterforts, tied in at the top, are continuous with the beams of the suspended bottom.)

First determine the bending moment at the inner supports by dividing the load into a series of loads on one span at a time and evaluating the corresponding products \( WL \). Owing to symmetry the bending moments at each of the interior supports are equal.

\[
\begin{align*}
\kappa &= \frac{y}{y + 1} = 0.67 + 1 = 1.67 \\
H &= \frac{5}{(4 \times 1.67 \times 1.67) - 1} = 0.493. \\
U_C &= 3 \times 0.67^4 \times 0.493 = 0.663 \\
U_B &= 0.5 \times 1.67 \times 0.663 = 0.555 \\
\text{For 1st span loaded:} \\
U_C &= 0.493(1.67 + 0.67) = 1.15 \\
\text{For 2nd span loaded:} \\
U_C &= 0.555; U_B = 0.663. \\
\text{For 3rd span loaded:} \\
\text{Owing to symmetry} \\
U_C &= 0.555; U_B = 0.663.
\end{align*}
\]

\[
\begin{align*}
\text{Load factor} \times \text{Support moment coefficient (FQ)} &\text{ } \text{multiplier (U)} \\
\text{Triangularly distributed} \text{ } \text{(apex at left-hand support)} &\text{ } 3rd \text{ } + 0.018 \text{ } U_B = 0.663 + 5370 \text{ ft.-lb.} \\
\text{Uniformly distributed} \text{ } \text{(W)} &\text{ } 7000 \times 15 \text{ } L = 15 \text{ ft.} \\
\text{Triangularly distributed} \text{ } \text{(apex at right-hand support: therefore reverse system)} &\text{ } 1st \text{ } - 0.071 \text{ } U_C \text{ for 3rd span loaded} = 0.555 \\
\text{Net bending moment at inner support} &= -102,870 \text{ ft.-lb.}
\end{align*}
\]

Free bending moment at mid-span = \( \frac{7000 \times 15^2}{8} = 197,000 \) ft.-lb.

Less negative bending moment at mid-span = 102,870

Net positive bending moment at mid-span = 94,130
### Table 25: Uniform Moment of Inertia

<table>
<thead>
<tr>
<th>Type of Load</th>
<th>Load Factor</th>
<th>Max. Free B.M.</th>
</tr>
</thead>
<tbody>
<tr>
<td>F = ( \frac{w_{total}}{L} )</td>
<td>1.00</td>
<td>+125 W/L</td>
</tr>
<tr>
<td>1.25</td>
<td>+167 W/L</td>
<td></td>
</tr>
<tr>
<td>See Note</td>
<td>+128 W/L</td>
<td></td>
</tr>
<tr>
<td>1.50</td>
<td>+250 W/L</td>
<td></td>
</tr>
<tr>
<td>1.33</td>
<td>+167 W/L</td>
<td></td>
</tr>
</tbody>
</table>

#### Notes:
- Divide given beam system into a number of similar systems each having one span loaded with a particular type of load, to find the B.M. at any support due to any one of these loads. Evaluate the following factors for the particular support and type of load:
- Load Factor = \( F \) (unity for distributed load).
- Support Moment Coefficient = \( Q \).
- Moment Multiplier = \( U \) (unity for equal spans).
- B.M. at support = \( FQUxW \times \text{base span} \).

#### Table:

<table>
<thead>
<tr>
<th>No. of Spans</th>
<th>Loaded Span</th>
<th>Equal Spans</th>
<th>Moment Multipliers = ( U )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.</td>
<td></td>
<td></td>
<td>( U_A = \frac{2}{1+k_1} )</td>
</tr>
<tr>
<td>A BASE</td>
<td>k1 B SPAN C</td>
<td>( U_B = \frac{2k_1}{1+k_1} )</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>( U_C = \frac{3k_1^2H}{4} )</td>
<td></td>
</tr>
<tr>
<td>BOTH SPANS LOADED WITH IDENTICAL LOAD.</td>
<td>( U_B = 5yUC )</td>
<td>( x = k_1 + 1 )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( U_C = 3k_1^2H )</td>
<td>( y = k_2 + 1 )</td>
<td></td>
</tr>
<tr>
<td>3.</td>
<td></td>
<td></td>
<td>( U_B = 3k_2^2H )</td>
</tr>
<tr>
<td>A BASE</td>
<td>k1 B SPAN C</td>
<td>( U_C = 5U_B )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>k2 D</td>
<td></td>
<td>( H = \frac{5}{4xy-1} )</td>
</tr>
<tr>
<td>ALL SPANS LOADED WITH IDENTICAL LOAD.</td>
<td>( U_B = \frac{13131}{x}(4 + k_1H) )</td>
<td>( x = k_1 + 1 )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( U_B = zH )</td>
<td>( y = k_1 + k_2 )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( U_C = 2k_1H_1 )</td>
<td>( z = k_2 + k_3 )</td>
<td></td>
</tr>
<tr>
<td>4.</td>
<td></td>
<td></td>
<td>( H_2 = 4.16k_1^2Y(x+1) )</td>
</tr>
<tr>
<td>A BASE</td>
<td>k1 B SPAN C</td>
<td>( U_B = zH )</td>
<td>( H_1 = 14k_1Y )</td>
</tr>
<tr>
<td></td>
<td>k2 H SPAN D</td>
<td>( U_C = 2k_2H_2 )</td>
<td>( H_3 = 4.06k_2^2Y(k_2+1) )</td>
</tr>
<tr>
<td></td>
<td>k3 L</td>
<td></td>
<td>( H_4 = 14k_3Y )</td>
</tr>
<tr>
<td>ALL SPANS LOADED WITH IDENTICAL LOAD.</td>
<td>( U_B = 2k_1H_1 )</td>
<td>( z = k_2 + k_3 )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( U_B = zH_2 )</td>
<td>( u = \frac{13131}{x}(4 + k_1H) )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( U_C = 2k_2H_2 )</td>
<td>( u = 1331H_2(4xy-1) )</td>
<td></td>
</tr>
</tbody>
</table>

For two, three, or four unequal spans loaded simultaneously, determine B.M. for each span loaded separately and add.
MOMENT DISTRIBUTION APPLIED TO CONTINUOUS BEAMS.

The notes which follow explain the basis of the formulæ and other data given in Tables 26 to 30.

The symbols used are given in Table 26. The span-load-factor $F$ is the numerical value of the bending moment produced at the end support of a span $L$ by the load on that span, assuming the beam to be perfectly fixed at both ends. For equal spans, arithmetical work is saved by considering only the load-factor $C, \left(= \frac{F}{L}\right)$, values of which are given in Table 18 for common cases of unsymmetrical and symmetrical loads; note that $F = CL$.

The distribution factor $D$ is derived from the relative stiffness of two adjacent spans meeting at a support; for prismatic beams the stiffness of each span is proportional to the moment of inertia of the beam divided by the span. Consider any two continuous spans ST and TU having span lengths of $L_{ST}$ and $L_{TU}$ respectively. If the moment of inertia of span ST is $I_{ST}$ and of span TU is $I_{TU}$, the stiffness factor of span ST is $\frac{I_{ST}}{L_{ST}}$ and of span TU $\frac{I_{TU}}{L_{TU}}$. If a knife-edge support is assumed at support T, the sum of the distribution factors at this support is unity. The distribution factor of span ST at support T is

$$D_{TS} = \frac{I_{ST}}{I_{TU} L_{ST} + L_{TU} I_{ST}}$$

and the distribution factor of span TU at support T is

$$D_{TU} = \frac{I_{TU}}{I_{ST} L_{TU} + L_{ST} I_{TU}}$$

that is, $D_{TS} = 1 - D_{TU}$.

Since only the ratio of the stiffness factors is involved, it is not necessary for $I$ and $L$ to be in the same units, and it is usually more convenient for $I$ to be in in.\(^4\) units and $L$ to be in feet. Values of $I$ for various sections, but omitting the effect of the reinforcement, are given in Table 64.

At the end support A of an end span AB the distribution factor $D_{AB}$ is zero when the beam is fixed at A and unity when freely supported at A. A cantilever beyond an end support, or a bending moment applied externally at an end support, is considered in Table 31, but for assessing the distribution factors in such cases the end span should be treated as freely supported at the end support.

**Basic Formulae.**—By moment-distribution operations to the extent of two distributions and one carry-over, the bending moment $M_T$ at the support T of the series of continuous spans . . . RS, ST, TU, UV . . . is given by the expression at the bottom of Table 27.

Contrary to the usual signs for moment distribution, the signs in this expression are such that a bending moment producing "concave downwards" deformation is considered as negative and the reverse as positive; as these signs conform to the normally-accepted convention for continuous beams and there is therefore no need to adopt a new conception for negative and positive bending moments. The substitution of the numerical values of the load-factors (or span-load-factors) in the formulæ gives the correct sign to the bending moment at the support.

From the fundamental formula for $M_T$, the basic formulæ for end supports (when fixed), penultimate supports, and any interior supports in systems of two, three, four, five, or more spans, are derived by considering which spans are eliminated and what effect the conditions have on the distribution factors. The basic formulæ given in Table 26 are the foundation of all the expressions in Tables 27 (two spans), 28 (three spans), 29 (four spans), and 30 (five or more spans).

An example of the direct use of the formulæ in Table 26 is given on page 188.
### CONTINUOUS BEAMS: MOMENT-DISTRIBUTION METHOD.—TABLE 26.

<table>
<thead>
<tr>
<th>SUPPORT</th>
<th>APPLICATION</th>
<th>FORMULA FOR B.M. AT SPECIFIED SUPPORT.</th>
<th>REF.</th>
</tr>
</thead>
<tbody>
<tr>
<td>END SUPPORT A</td>
<td>TWO OR MORE SPANS</td>
<td>[-F_{AB} - \frac{D_{BA}}{2} (F_{BA} - F_{BC})]</td>
<td>M_A</td>
</tr>
<tr>
<td>END SUPPORT C</td>
<td>TWO SPANS ONLY</td>
<td>[-F_{CB} - \frac{D_{BC}}{2} (F_{BC} - F_{BA})]</td>
<td>M_C4</td>
</tr>
</tbody>
</table>

### CENTRAL SUPPORT B

<table>
<thead>
<tr>
<th></th>
<th>TWO SPANS ONLY</th>
<th>Formula</th>
<th>REF.</th>
</tr>
</thead>
<tbody>
<tr>
<td>FREE AT A &amp; C</td>
<td>[-\frac{D_{BC}}{2} (2F_{BA} + F_{AB}) - \frac{D_{BA}}{2} (2F_{BC} + F_{CB})]</td>
<td>M_B3</td>
<td></td>
</tr>
<tr>
<td>FIXED AT A</td>
<td>[-D_{BC} F_{BA} - \frac{D_{BA}}{2} (2F_{BC} + F_{CB})]</td>
<td>M_B4</td>
<td></td>
</tr>
<tr>
<td>FREE AT A &amp; C</td>
<td>[-D_{BC} F_{BA} - D_{BA} F_{BC}]</td>
<td>M_B5</td>
<td></td>
</tr>
</tbody>
</table>

### PENULTIMATE SUPPORT B

<table>
<thead>
<tr>
<th></th>
<th>THREE OR, MORE SPANS</th>
<th>Formula</th>
<th>REF.</th>
</tr>
</thead>
<tbody>
<tr>
<td>FREE AT A</td>
<td>[-\frac{D_{BC}}{2} (2F_{CB} + F_{CA}) - \frac{D_{CD}}{2} [2F_{CB} - D_{BC} (F_{BA} - F_{BC})]]</td>
<td>M_B2</td>
<td></td>
</tr>
<tr>
<td>FIXED AT A</td>
<td>[-D_{BC} F_{BA} - \frac{D_{BA}}{2} (2F_{CB} - D_{BC} (F_{BA} - F_{BC})]]</td>
<td>M_B1</td>
<td></td>
</tr>
</tbody>
</table>

### PENULTIMATE SUPPORT C

<table>
<thead>
<tr>
<th></th>
<th>THREE SPANS ONLY</th>
<th>Formula</th>
<th>REF.</th>
</tr>
</thead>
<tbody>
<tr>
<td>FREE AT D</td>
<td>[-\frac{D_{CD}}{2} (2F_{BD} + F_{DE}) - \frac{D_{DE}}{2} [2F_{BD} - D_{CD} (F_{BA} - F_{BC})]]</td>
<td>M_C2</td>
<td></td>
</tr>
<tr>
<td>FIXED AT D</td>
<td>[-D_{CD} F_{BA} - \frac{D_{BA}}{2} (2F_{BC} - D_{CD} (F_{BA} - F_{BC})]]</td>
<td>M_C1</td>
<td></td>
</tr>
</tbody>
</table>

### END SUPPORT D

<table>
<thead>
<tr>
<th></th>
<th>THREE SPANS ONLY</th>
<th>Formula</th>
<th>REF.</th>
</tr>
</thead>
<tbody>
<tr>
<td>FIXED AT D</td>
<td>[-F_{DC} - \frac{D_{CD}}{2} (F_{CD} - F_{CB})]</td>
<td>M_D3</td>
<td></td>
</tr>
</tbody>
</table>

### PENULTIMATE SUPPORT D

<table>
<thead>
<tr>
<th></th>
<th>FOUR SPANS ONLY</th>
<th>Formula</th>
<th>REF.</th>
</tr>
</thead>
<tbody>
<tr>
<td>FREE AT E</td>
<td>[-\frac{D_{DE}}{2} (2F_{DC} + F_{ED}) - \frac{D_{DE}}{2} [2F_{DC} - D_{CD} (F_{BA} - F_{BC})]]</td>
<td>M_D2</td>
<td></td>
</tr>
<tr>
<td>FIXED AT E</td>
<td>[-D_{DC} F_{DE} - \frac{D_{DE}}{2} [2F_{DC} - D_{CD} (F_{BA} - F_{BC})]]</td>
<td>M_D1</td>
<td></td>
</tr>
</tbody>
</table>

### END SUPPORT E

<table>
<thead>
<tr>
<th></th>
<th>FOUR SPANS ONLY</th>
<th>Formula</th>
<th>REF.</th>
</tr>
</thead>
<tbody>
<tr>
<td>FIXED AT E</td>
<td>[-F_{ED} - \frac{D_{DE}}{2} (F_{DE} - F_{DC})]</td>
<td>M_E</td>
<td></td>
</tr>
</tbody>
</table>

### INTERIOR SUPPORT C

<table>
<thead>
<tr>
<th></th>
<th>FOUR OR MORE SPANS</th>
<th>Formula</th>
<th>REF.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>[-\frac{D_{CD}}{2} [2F_{CB} - D_{BC} (F_{BA} - F_{BC})] - \frac{D_{CB}}{2} [2F_{CD} - D_{DC} (F_{DE} - F_{DC})]]</td>
<td>M_C3</td>
<td></td>
</tr>
</tbody>
</table>

### PENULTIMATE SUPPORT B

<table>
<thead>
<tr>
<th></th>
<th>TWO OR MORE SPANS</th>
<th>Formula</th>
<th>REF.</th>
</tr>
</thead>
<tbody>
<tr>
<td>WITH EXTERNALLY</td>
<td>B.M., M_p, D_{BC}</td>
<td></td>
<td>M_B6</td>
</tr>
<tr>
<td>APPLIED NEG. B.M.</td>
<td>A.T.A.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### SUPPORT REFERENCES:—

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### SYMBOLS:—

- L_{AB} = SPAN A-B, etc.
- I_{AB}, etc. = MOMENT OF INERTIA OF SECTION IN SPAN A-B, etc.
- D_{BA}, D_{BC}, etc. = DISTRIBUTION FACTOR AT SUPPORT B FOR SPANS AB AND BC, etc.
- D_{BA} + D_{BC} = 1
- D_{BA} = \frac{1}{1 + I_{AB} - I_{BC}}
- C_{AB} = C_{BA}, WHERE C_{AB}, etc. = LOAD FACTOR (SEE NOTE BELOW)

### GENERAL FORMULA FOR ANY SUPPORT T

<table>
<thead>
<tr>
<th>R</th>
<th>S</th>
<th>T</th>
<th>U</th>
<th>V</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note.—For values of factors C_{AB}, etc., and F_{AB}, etc., see Table 18.
For modified formula for special conditions, see the following tables.
Two spans: Table 27.
Three spans: Table 28.
Four spans: Table 29.
Five and more spans: Table 30.

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MOMENT DISTRIBUTION APPLIED TO CONTINUOUS BEAMS.

**Formulæ for Special Cases.**—In practical design conditions are often such that one or more of the variants in the general case may be a constant resulting in simplification of the formulæ for bending moments at supports. If the load on every span is symmetrically disposed the span-load-factors for each support of any one span are equal. The condition of equal spans has little effect on the formulæ unless accompanied by some other constant factor. Thus the common case of equal spans with uniform section throughout all spans means that the stiffness factors for each span are identical at any interior support; the distribution factors are therefore equal on either side of the support, and each distribution factor is 0.5. Also, since the spans are equal, the load-factor can be used in place of the span-load-factor, and the numerical value of the length of the span is an overall multiplier. The common combination of symmetrical loading, equal spans, and uniform section throughout all spans combines the foregoing simplifications, and the modified expressions for these special cases are tabulated in Tables 27, 28, 29 and 30.

A condition that frequently occurs, for example in bridge girders, is that of symmetrical inequality of spans accompanied by symmetrical inequality of section, in which case there is equality of some of the distribution factors; the modified expressions for this condition in some systems are given in Tables 27 to 29. Symmetrical inequality of spans and section may be accompanied by symmetrical load, with further simplification of the formulæ also as given in the tables.

**Incidental Loading.**—The basic formulæ and the derivatives therefrom for special cases assume that all spans are loaded. Should any span not be loaded, zero must be substituted for the span-load-factors or load-factors for the unloaded span, and from this operation the formulæ for incidental loading are evolved to determine maximum bending moments at supports and in the spans due to various incidences of live loading. The sequence of spans which should be loaded to produce the maximum negative or positive bending moments is indicated in Table 20.

**Condition at End Supports.**—The special formulæ apply to the three conditions at end supports, namely, free support at both ends of the system of spans, free support at one end and fixity at the other, and fixity at both ends. The condition of partial restraint at one or both supports is dealt with in Table 31 and the page facing that table.

**Application of Tables.**—A problem is dealt with in the following stages.

1. Select the table which applies to the given case as regards number of spans and support conditions at the ends.
2. Select the group of formulæ applying to the problem as determined by any condition such as symmetrical loading, equal spans and uniform section, symmetrical inequality of spans, etc.
3. Except in the case of uniform section throughout all spans, determine the moment of inertia of the section in each span and therefrom the stiffness factors for each span. Calculate only those distribution factors required for the group of formulæ applicable.
4. Calculate the span-load-factors (or the load-factors if all spans are equal), using Table 18, for dead and live loads separately. Only those factors required for the group of formulæ which are applicable need be calculated.
5. By direct substitution in the "all-spans-loaded" formulæ determine the bending moments at the supports for dead load only.
6. By direct substitution in the appropriate "incidental-load" formulæ determine the bending moments at supports for live load only.
7. If there is an externally-applied bending moment at one or both end supports, calculate the bending moments at the supports due thereto separately for dead and live load from Table 31.
8. Algebraically add the results from (5), (6) and (7) to give the net bending moments at the supports; combine them with the free bending moments if the bending moments in the spans are required.

An example is given on pages 188, 190 and 192 [Example (b)].
CONTINUOUS BEAMS: MOMENT-DISTRIBUTION METHOD.—TABLE 27.
TWO SPANS.

<table>
<thead>
<tr>
<th>CONDITIONS</th>
<th>SUPPORT</th>
<th>BOTH SPANS LOADED</th>
<th>INCIDENTAL LOADING</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>CASE 2A</strong></td>
<td></td>
<td>MAX. NEG. B. M. AT SUPPORT B</td>
<td>FOR MAX. POS. B. M. IN SPAN AB,</td>
</tr>
<tr>
<td>GENERAL CASE</td>
<td>B</td>
<td>(-\frac{DB}{2}(2F_{BA} + F_{AB}) - \frac{DB}{2}(2F_{BC} + F_{CB}))</td>
<td>(-\frac{DB}{2}(2F_{BA} + F_{AB}))</td>
</tr>
<tr>
<td>SYMMETRICAL LOADING</td>
<td>B</td>
<td>(-\frac{3}{2}(DB_{FA} + DB_{AB} + DB_{BC}))</td>
<td>(-\frac{3}{2}D_{BC}F_{AB})</td>
</tr>
<tr>
<td>EQUAL SPANS AND UNIFORM SECTION</td>
<td>B</td>
<td>(-\frac{3}{4}(C_{BA} + C_{BC}) + C_{AB} + C_{CB})</td>
<td>(-\frac{3}{4}(C_{BA} + C_{AB}))</td>
</tr>
<tr>
<td>EQUAL SPANS UNIFORM SECTION AND SYMMETRICAL LOADING</td>
<td>B</td>
<td>(-\frac{3}{4}C_{AB} + C_{BC})</td>
<td>(-\frac{3}{4}F_{BA})</td>
</tr>
<tr>
<td><strong>CASE 2B</strong></td>
<td></td>
<td>MAX. NEG. B. M. AT SUPPORT B</td>
<td>FOR MAX. POS. B. M. IN SPAN AB,</td>
</tr>
<tr>
<td>GENERAL CASE</td>
<td>A</td>
<td>(-F_{AB} - \frac{DB}{2}(F_{PA} - F_{BC}))</td>
<td>(-F_{AB} - \frac{DB}{2}F_{BA})</td>
</tr>
<tr>
<td>B</td>
<td>(-D_{BC}F_{BC} - \frac{DB}{2}(2F_{BC} + F_{BA}))</td>
<td>(-D_{BC}F_{BA})</td>
<td>(+D_{BC}F_{BC})</td>
</tr>
<tr>
<td>SYMMETRICAL LOADING</td>
<td>A</td>
<td>(-\frac{DB}{2}(F_{BA} + D_{BC}F_{BC}))</td>
<td>(-\frac{DB}{2}(F_{BA} + D_{BC}F_{BC}))</td>
</tr>
<tr>
<td>B</td>
<td>(-D_{BC}F_{BA} - \frac{3}{2}D_{BA}F_{BC})</td>
<td>(-D_{BC}F_{BC})</td>
<td>(+\frac{3}{2}D_{BA}F_{BC})</td>
</tr>
<tr>
<td>EQUAL SPANS AND UNIFORM SECTION</td>
<td>A</td>
<td>(-\frac{3}{4}(4C_{BA} + C_{BC}) - 4(C_{AB} + C_{CB}))</td>
<td>(-\frac{3}{4}(4C_{BA} + C_{AB}))</td>
</tr>
<tr>
<td>B</td>
<td>(-\frac{3}{4}(C_{BA} + C_{BC}))</td>
<td>(-\frac{3}{4}(C_{BA} + C_{BC}))</td>
<td>(+\frac{4}{3}(C_{BC} + C_{CB}))</td>
</tr>
<tr>
<td>EQUAL SPANS UNIFORM SECTION AND SYMMETRICAL LOADING</td>
<td>A</td>
<td>(-\frac{3}{4}F_{AB})</td>
<td>(-\frac{3}{4}F_{AB})</td>
</tr>
<tr>
<td>B</td>
<td>(-\frac{3}{4}F_{Ba})</td>
<td>(-\frac{3}{4}F_{Ba})</td>
<td>(+\frac{4}{3}F_{AB})</td>
</tr>
<tr>
<td><strong>CASE 2C</strong></td>
<td></td>
<td>MAX. NEG. B. M. AT SUPPORTS B &amp; C</td>
<td>MAX. POS. B. M. AT SUPPORT C,</td>
</tr>
<tr>
<td>GENERAL CASE</td>
<td>B</td>
<td>(-D_{BC}F_{BA} - D_{BA}F_{BC})</td>
<td>(-D_{BC}F_{BA})</td>
</tr>
<tr>
<td>C</td>
<td>(-F_{BC} - \frac{DB}{2}(F_{BC} - F_{BA}))</td>
<td>(-D_{BC}F_{BA})</td>
<td>(+D_{BC}F_{BA})</td>
</tr>
<tr>
<td>SYMMETRICAL LOADING</td>
<td>B</td>
<td>(-\frac{DB}{2}(F_{BA} - F_{BC}))</td>
<td>(-D_{BC}F_{BA})</td>
</tr>
<tr>
<td>C</td>
<td>(-\frac{DB}{2}(F_{BC} + F_{BA}))</td>
<td>(-D_{BC}F_{BA})</td>
<td>(+D_{BC}F_{BA})</td>
</tr>
<tr>
<td>EQUAL SPANS AND UNIFORM SECTION</td>
<td>B</td>
<td>(-\frac{3}{4}(C_{BA} + C_{BC}))</td>
<td>(-\frac{3}{4}(C_{BA} + C_{BC}))</td>
</tr>
<tr>
<td>C</td>
<td>(-\frac{3}{4}(C_{BA} + C_{BC}))</td>
<td>(-\frac{3}{4}(C_{BA} + C_{BC}))</td>
<td>(+\frac{4}{3}(C_{BC} + C_{CB}))</td>
</tr>
<tr>
<td>EQUAL SPANS UNIFORM SECTION AND SYMMETRICAL LOADING</td>
<td>B</td>
<td>(-\frac{3}{4}(C_{AB} + C_{BC}))</td>
<td>(-\frac{3}{4}(C_{AB} + C_{BC}))</td>
</tr>
<tr>
<td>C</td>
<td>(-\frac{3}{4}(C_{AB} + C_{BC}))</td>
<td>(-\frac{3}{4}(C_{AB} + C_{BC}))</td>
<td>(+\frac{4}{3}(C_{AB} + C_{CB}))</td>
</tr>
</tbody>
</table>

Note.—For notation, see Table 26.
MOMENT DISTRIBUTION APPLIED TO CONTINUOUS BEAMS (continued).

Examples.—(a) Determine the bending moments at the interior supports of the beam on page 172, by means of the formula in Table 26.

\[ D_{BC} = \frac{1}{1 + \frac{4000 \times 15}{6000 \times 10}} = 0.5; \text{ therefore } D_{BA} = 1 - 0.5 = 0.5. \]

\[ D_{DB} = \frac{1}{1 + \frac{5000 \times 15}{6000 \times 12}} = 0.49; \text{ therefore } D_{CD} = 0.51. \]

From Table 18,

\[ C_{BA} = 0.100 \times 1000 \times 10 \times 0.5 = 500; \quad F_{BA} = 500 \times 10 = 5000 \text{ ft.-lb.} \]
\[ C_{AB} = 0.067 \times 1000 \times 10 \times 0.5 = 333; \quad F_{AB} = 333 \times 10 = 3330 \text{ ft.-lb.} \]
\[ C_{BC} = C_{CD} = 0.083 \times 1500 \times 15 = 1875; \quad F_{BC} = F_{CD} = 1875 \times 15 = 28,125 \text{ ft.-lb.} \]
\[ C_{CD} = 0.100 \times 1200 \times 12 \times 0.5 = 720; \quad F_{CD} = 720 \times 12 = 8640 \text{ ft.-lb.} \]
\[ C_{DC} = 0.067 \times 1200 \times 12 \times 0.5 = 480; \quad F_{DC} = 480 \times 12 = 5760 \text{ ft.-lb.} \]

From Table 26, at interior support B (three spans, free at A):

Bending moment = \(- 0.25[(2 \times 5000) + 3330] - 0.25[(2 \times 28,125) - 0.49(8640 - 28,125)]\)

\[ = -3333 - 16,450 = -19,783 \text{ ft.-lb.} \]

At penultimate support C (three spans only, free at D):

Bending moment = \(- 0.245[(2 \times 8640) + 5760] - 0.255[(2 \times 28,125) - 0.5(5000 - 28,125)]\)

\[ = -5650 - 17,282 = -22,932 \text{ ft.-lb.} \]

(b) Determine the maximum negative bending moment at support B and the maximum positive bending moment in span BC for the beam shown on the Calculation Sheet (on the page facing Table 29) on which are given the spans, loads, and sections from which are calculated the moments of inertia, the distribution factors, and the span-load-factors for the dead and live loads separately, using the data in Tables 18 and 26. The moments of inertia of the sections are calculated from Table 64; for the tee-beam

\[ \frac{b}{d} = \frac{6}{17} = 0.36; \quad \frac{d}{d} = \frac{5}{40} = 0.125; \]

and the factor 0.19 is obtained by extrapolation.

It is unnecessary in this case to draw bending-moment diagrams, since the bending moments for combined dead and live load at each support can be obtained by direct summation of the bending moments calculated from the formula in Table 28, Case 3A.

(i) Maximum negative bending moment at support B.—Substituting the appropriate particulars of dead load from the Calculation Sheet in the formula for support B in Table 28 for conditions of symmetrical load (all spans loaded) and from Table 31 for dead load (general case, three or more spans), the bending moment due to dead load alone is

\[ \left( \frac{3}{2} \times 0.58 \times 140,000 \right) - \frac{0.42}{2} [(2 \times 6 \times 312,500) - (0.6 \times 146,250)] + \frac{0.58 \times 52,500}{2} \]

\[ = 259,000 \text{ in.-lb.} \]

By substitution in the formula for support B from Table 28 for the general case of maximum negative bending moment at supports due to incidental load (in this instance the basic formula \( M_{Bc} \) in Table 26 with \( F_{CD} = 0 \)), the bending moment due to the live load alone is

\[ \frac{0.58}{2} [2 \times 300,000 + 300,000] - \frac{0.42}{2} [(2 \times 115,200) + (0.6 \times 172,800)] = -331,150 \text{ in.-lb.} \]

From Table 31 it is seen that, for the maximum negative bending moment at support B, the effect of the live load on the cantilever should be omitted. Thus the bending moment at support B is \(- (259,000 + 331,150) = -590,150 \text{ in.-lb.} \).

(ii) Maximum positive bending moment in span BC.—Calculate the minimum bending moments at supports B and C and combine the resulting negative bending-moment diagram with the positive freely-supported-span diagram. For dead load alone the negative bending moment at support B is, as already calculated, 259,000 in.-lb. For support C the substitution in the formula for dead load in Tables 28 and 31 gives

\[ \left( \frac{3}{2} \times 0.60 \times 146,250 \right) - \frac{0.40}{2} [(2 \times 58 \times 312,500) - (0.58 \times 140,000)] \]

\[ - \frac{0.40 \times 0.58 \times 52,500}{4} = -280,000 \text{ in.-lb.} \]

(Continued on page 190.)
### Continuous Beams: Moment-Distribution Method

#### Table 28. THREE SPANS.

<table>
<thead>
<tr>
<th>Conditions</th>
<th>Support</th>
<th>All Spans Loaded</th>
<th>Incident Load</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>CASE 3A</strong></td>
<td><strong>B.M. AT SUPPORTS B &amp; C</strong></td>
<td><strong>B. M. AT SUPPORTS B &amp; C</strong></td>
<td><strong>MAX. H.S. B.M. AT SUPPORT, NOTE THESE B.S. DO NOT OCCUR SIMULTANEOUSLY.</strong></td>
</tr>
<tr>
<td>GENERAL</td>
<td>B</td>
<td>BASIC FORMULA $M_{B}$</td>
<td>$M_{B}$ WITH $F_{C} = 0$ + $\frac{D_{AB}}{2}D_{C0}D_{FBA}$ + $D_{B0}D_{FBA}D_{FBA}$</td>
</tr>
<tr>
<td>Symmetrical Loading</td>
<td>C</td>
<td>BASIC FORMULA $MC_{B}$</td>
<td>$MC_{B}$ WITH $F_{C} = 0$ + $\frac{D_{AB}}{2}D_{C0}D_{FBA}$ + $D_{B0}D_{FBA}D_{FBA}$</td>
</tr>
<tr>
<td><strong>CASE 3B</strong></td>
<td><strong>B. M. AT PENULTIMATE SUPPORT B</strong></td>
<td><strong>MAX. H.S. B. M. AT SUPPORT B.</strong></td>
<td><strong>MAX. POS. B. M. AT SUPPORT B.</strong></td>
</tr>
<tr>
<td>General</td>
<td>B</td>
<td>$-\frac{1}{2}F_{C0}D_{B0}D_{FBA}$ + $\frac{1}{2}D_{B0}D_{FBA}D_{FBA}$</td>
<td>$-\frac{1}{2}F_{C0}D_{B0}D_{FBA}$ + $\frac{1}{2}D_{B0}D_{FBA}D_{FBA}$</td>
</tr>
<tr>
<td>Symmetrical Loading</td>
<td>C</td>
<td>$-\frac{1}{2}F_{C0}D_{B0}D_{FBA}$ + $\frac{1}{2}D_{B0}D_{FBA}D_{FBA}$</td>
<td>$-\frac{1}{2}F_{C0}D_{B0}D_{FBA}$ + $\frac{1}{2}D_{B0}D_{FBA}D_{FBA}$</td>
</tr>
<tr>
<td><strong>CASE 3C</strong></td>
<td><strong>B. M. AT SUPPORTS C &amp; D (AND A &amp; B)</strong></td>
<td><strong>MAX. H.S. B. M. AT SUPPORTS C &amp; D (AND A &amp; B).</strong></td>
<td><strong>MAX. POS. B. M. AT SUPPORTS C &amp; D (AND A &amp; B).</strong></td>
</tr>
<tr>
<td>General Case</td>
<td>C</td>
<td>$-\frac{1}{2}D_{B0}D_{FBA}D_{FBA}$</td>
<td>$-\frac{1}{2}D_{B0}D_{FBA}D_{FBA}$</td>
</tr>
<tr>
<td>Symmetrical Loading</td>
<td>D</td>
<td>$-\frac{1}{2}D_{B0}D_{FBA}D_{FBA}$</td>
<td>$-\frac{1}{2}D_{B0}D_{FBA}D_{FBA}$</td>
</tr>
</tbody>
</table>

**Note.**—For notation and basic formulae $M_{B}$ and $M_{C}$, see Table 26. For Case 2B, see Table 27.
MOMENT-DISTRIBUTION APPLIED TO CONTINUOUS BEAMS (continued).

Examples (continued from page 188).

<table>
<thead>
<tr>
<th>SUPPORT REFERENCES</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>LIVE LOADS</td>
<td>4.0 ft.</td>
<td>10.0 ft.</td>
<td>10.0 ft.</td>
<td>15.0 ft.</td>
</tr>
<tr>
<td>DEAD LOADS</td>
<td>CANTILEVER</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SPANS</td>
<td>5.0 ft.</td>
<td>L_AB = 20.0 ft.</td>
<td>L_BC = 25.0 ft.</td>
<td></td>
</tr>
<tr>
<td>SECTIONS</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MомENTS OF INERTIA</td>
<td>I_AB = 12 x 20^3 / 12 = 8,000 in^4</td>
<td>I_BC = 12 x 24^3 / 12 = 13,824 in^4</td>
<td>I_CD = 0.19 x 6 x 17^3 / 12 = 5,570 in^4</td>
<td></td>
</tr>
<tr>
<td>STIFFNESS FACTORS</td>
<td>8,000 / 20 = 400</td>
<td>15,824 / 25 = 633</td>
<td>5,570 / 16 = 342</td>
<td></td>
</tr>
<tr>
<td>DISTRIBUTION FACTORS</td>
<td>D_BA = 1 / [1 + 553 / 400] = 1 - 4.42 / 4.42 = 0.97</td>
<td>D_BC = 1 / [1 + 371 / 555] = 0.58</td>
<td>D_CD = 0.40</td>
<td></td>
</tr>
<tr>
<td>DEAD LOAD SPAN-LOAD FACTORS</td>
<td>F_AB = 350 x 20^2 / 12 = 140,000 in.lbf.</td>
<td>F_BC = 500 x 25^2 / 12 = 312,500 in.lbf.</td>
<td>F_CD = 650 x 15^2 / 12 = 146,250 in.lbf.</td>
<td></td>
</tr>
<tr>
<td>LOAD ON CANTILEVER</td>
<td>M_p = 350 x 5 x 20^2 / 2 = 52,500 in.lbf.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LIVE LOAD SPAN-LOAD FACTORS</td>
<td>F_AB (Symmetrical Loading) = 10,000 x 20 x 12 / 8 = 300,000 in.lbf.</td>
<td>F_BC (Unsymmetrical Loading) = 6 x 4 x 4000 x 25 x 12 = 115,200 in.lbf.</td>
<td>F_CD (Symmetrical Loading) = 800 x 15^2 / 12 = 180,000 in.lbf.</td>
<td></td>
</tr>
<tr>
<td>LOAD ON CANTILEVER</td>
<td>M_p = 2000 x 4.5 x 12 = 108,000 in.lbf.</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

For the live-load bending moments the formula in the column for maximum positive bending moment in span BC in Table 28 are used. From Table 31, the effect of the live load on the cantilever is calculated. Substitution gives

Support B: \(-\frac{0.42}{2}[(2 \times 115,200) + (0.60 \times 172,800)] + \frac{0.58 \times 108,000}{2} = -38,800 \text{ in.-lb.}\)

Support C: \(-\frac{0.40}{2}[(2 \times 172,800) + (0.58 \times 115,200)] - \frac{0.4 \times 0.58 \times 108,000}{4} = -88,800 \text{ in.-lb.}\)

(Continued on page 192.)
## CONTINUOUS BEAMS: MOMENT-DISTRIBUTION METHOD

### Table 29

#### FOUR SPANS

<table>
<thead>
<tr>
<th>Conditions</th>
<th>All Spans Loaded</th>
<th>Incident Loading</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>CASE 4A</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>General Case</strong></td>
<td><strong>B, M, at Supports A, B, C, and D</strong> (and zero)</td>
<td><strong>MAX. POS. B.M. AT SUPPORTS</strong>&lt;br&gt;Note: These B.M. do not occur simultaneously.</td>
</tr>
<tr>
<td><strong>Symmetrical Loading</strong></td>
<td><strong>Basic Formula M&lt;sub&gt;B&lt;/sub&gt;, M&lt;sub&gt;C&lt;/sub&gt;, M&lt;sub&gt;D&lt;/sub&gt;, M&lt;sub&gt;E&lt;/sub&gt;</strong>&lt;br&gt;Note: These B.M. do not occur simultaneously.</td>
<td><strong>Basic Formula M&lt;sub&gt;B&lt;/sub&gt;, M&lt;sub&gt;C&lt;/sub&gt;, M&lt;sub&gt;D&lt;/sub&gt;, M&lt;sub&gt;E&lt;/sub&gt;</strong>&lt;br&gt;Note: These B.M. do not occur simultaneously.</td>
</tr>
<tr>
<td><strong>Equal Spans and Uniform Sections</strong></td>
<td><strong>Basic Formula M&lt;sub&gt;B&lt;/sub&gt;, M&lt;sub&gt;C&lt;/sub&gt;, M&lt;sub&gt;D&lt;/sub&gt;, M&lt;sub&gt;E&lt;/sub&gt;</strong>&lt;br&gt;Note: These B.M. do not occur simultaneously.</td>
<td><strong>Basic Formula M&lt;sub&gt;B&lt;/sub&gt;, M&lt;sub&gt;C&lt;/sub&gt;, M&lt;sub&gt;D&lt;/sub&gt;, M&lt;sub&gt;E&lt;/sub&gt;</strong>&lt;br&gt;Note: These B.M. do not occur simultaneously.</td>
</tr>
<tr>
<td><strong>Symmetrical Inequality of Spans and Sections</strong></td>
<td><strong>Basic Formula M&lt;sub&gt;B&lt;/sub&gt;, M&lt;sub&gt;C&lt;/sub&gt;, M&lt;sub&gt;D&lt;/sub&gt;, M&lt;sub&gt;E&lt;/sub&gt;</strong>&lt;br&gt;Note: These B.M. do not occur simultaneously.</td>
<td><strong>Basic Formula M&lt;sub&gt;B&lt;/sub&gt;, M&lt;sub&gt;C&lt;/sub&gt;, M&lt;sub&gt;D&lt;/sub&gt;, M&lt;sub&gt;E&lt;/sub&gt;</strong>&lt;br&gt;Note: These B.M. do not occur simultaneously.</td>
</tr>
</tbody>
</table>

#### CASE 4B

<table>
<thead>
<tr>
<th>Conditions</th>
<th>All Spans Loaded</th>
<th>Incident Loading</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>B, M, at Supports A, B, C, and D</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>General Case</strong></td>
<td><strong>MAX. NEG. B.M. AT SUPPORTS</strong>&lt;br&gt;Note: These B.M. do not occur simultaneously.</td>
<td><strong>MAX. NEG. B.M. AT SUPPORTS</strong>&lt;br&gt;Note: These B.M. do not occur simultaneously.</td>
</tr>
<tr>
<td><strong>Symmetrical Loading</strong></td>
<td><strong>Basic Formula M&lt;sub&gt;B&lt;/sub&gt;, M&lt;sub&gt;C&lt;/sub&gt;, M&lt;sub&gt;D&lt;/sub&gt;, M&lt;sub&gt;E&lt;/sub&gt;</strong>&lt;br&gt;Note: These B.M. do not occur simultaneously.</td>
<td><strong>Basic Formula M&lt;sub&gt;B&lt;/sub&gt;, M&lt;sub&gt;C&lt;/sub&gt;, M&lt;sub&gt;D&lt;/sub&gt;, M&lt;sub&gt;E&lt;/sub&gt;</strong>&lt;br&gt;Note: These B.M. do not occur simultaneously.</td>
</tr>
<tr>
<td><strong>Equal Spans and Uniform Sections</strong></td>
<td><strong>Basic Formula M&lt;sub&gt;B&lt;/sub&gt;, M&lt;sub&gt;C&lt;/sub&gt;, M&lt;sub&gt;D&lt;/sub&gt;, M&lt;sub&gt;E&lt;/sub&gt;</strong>&lt;br&gt;Note: These B.M. do not occur simultaneously.</td>
<td><strong>Basic Formula M&lt;sub&gt;B&lt;/sub&gt;, M&lt;sub&gt;C&lt;/sub&gt;, M&lt;sub&gt;D&lt;/sub&gt;, M&lt;sub&gt;E&lt;/sub&gt;</strong>&lt;br&gt;Note: These B.M. do not occur simultaneously.</td>
</tr>
<tr>
<td><strong>Symmetrical Inequality of Spans and Sections</strong></td>
<td><strong>Basic Formula M&lt;sub&gt;B&lt;/sub&gt;, M&lt;sub&gt;C&lt;/sub&gt;, M&lt;sub&gt;D&lt;/sub&gt;, M&lt;sub&gt;E&lt;/sub&gt;</strong>&lt;br&gt;Note: These B.M. do not occur simultaneously.</td>
<td><strong>Basic Formula M&lt;sub&gt;B&lt;/sub&gt;, M&lt;sub&gt;C&lt;/sub&gt;, M&lt;sub&gt;D&lt;/sub&gt;, M&lt;sub&gt;E&lt;/sub&gt;</strong>&lt;br&gt;Note: These B.M. do not occur simultaneously.</td>
</tr>
</tbody>
</table>

#### CASE 4C

<table>
<thead>
<tr>
<th>Conditions</th>
<th>All Spans Loaded</th>
<th>Incident Loading</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>B, M, at Supports A and B</strong>&lt;br&gt;(and zero)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>General Case</strong></td>
<td><strong>MAX. NEG. B.M. AT SUPPORTS</strong>&lt;br&gt;Note: These B.M. do not occur simultaneously.</td>
<td><strong>MAX. POS. B.M. AT SUPPORTS</strong>&lt;br&gt;Note: These B.M. do not occur simultaneously.</td>
</tr>
<tr>
<td><strong>Symmetrical Loading</strong></td>
<td><strong>Basic Formula M&lt;sub&gt;B&lt;/sub&gt;, M&lt;sub&gt;C&lt;/sub&gt;, M&lt;sub&gt;D&lt;/sub&gt;, M&lt;sub&gt;E&lt;/sub&gt;</strong>&lt;br&gt;Note: These B.M. do not occur simultaneously.</td>
<td><strong>Basic Formula M&lt;sub&gt;B&lt;/sub&gt;, M&lt;sub&gt;C&lt;/sub&gt;, M&lt;sub&gt;D&lt;/sub&gt;, M&lt;sub&gt;E&lt;/sub&gt;</strong>&lt;br&gt;Note: These B.M. do not occur simultaneously.</td>
</tr>
<tr>
<td><strong>Equal Spans and Uniform Sections</strong></td>
<td><strong>Basic Formula M&lt;sub&gt;B&lt;/sub&gt;, M&lt;sub&gt;C&lt;/sub&gt;, M&lt;sub&gt;D&lt;/sub&gt;, M&lt;sub&gt;E&lt;/sub&gt;</strong>&lt;br&gt;Note: These B.M. do not occur simultaneously.</td>
<td><strong>Basic Formula M&lt;sub&gt;B&lt;/sub&gt;, M&lt;sub&gt;C&lt;/sub&gt;, M&lt;sub&gt;D&lt;/sub&gt;, M&lt;sub&gt;E&lt;/sub&gt;</strong>&lt;br&gt;Note: These B.M. do not occur simultaneously.</td>
</tr>
<tr>
<td><strong>Symmetrical Inequality of Spans and Sections</strong></td>
<td><strong>Basic Formula M&lt;sub&gt;B&lt;/sub&gt;, M&lt;sub&gt;C&lt;/sub&gt;, M&lt;sub&gt;D&lt;/sub&gt;, M&lt;sub&gt;E&lt;/sub&gt;</strong>&lt;br&gt;Note: These B.M. do not occur simultaneously.</td>
<td><strong>Basic Formula M&lt;sub&gt;B&lt;/sub&gt;, M&lt;sub&gt;C&lt;/sub&gt;, M&lt;sub&gt;D&lt;/sub&gt;, M&lt;sub&gt;E&lt;/sub&gt;</strong>&lt;br&gt;Note: These B.M. do not occur simultaneously.</td>
</tr>
</tbody>
</table>

**Note:** For notation and basic formulae, see Table 26.<br>For Case 28, see Table 27.<br>For Cases 3A and 3B, see Table 28.
REINFORCED CONCRETE DESIGNER’S HANDBOOK

MOMENT DISTRIBUTION APPLIED TO CONTINUOUS BEAMS (continued).

Examples (continued from page 190).

For maximum positive bending moment on span BC the support moments are therefore

Support B: \(- (259,000 + 38,800) = -297,800\) in-lb.
Support C: \(- (280,000 + 88,800) = -368,800\) in-lb.

From the loading diagram it is clear that the position of maximum positive bending moment on span BC is at the point of application of the live concentrated load, at which point the free bending moment due to the dead and live loads is

\[
\frac{500 \times 15 \times 10 \times 12}{2} + \frac{4000 \times 15 \times 10 \times 12}{25} = 738,000 \text{ in-lb.}
\]

At this point the negative moment is

\[
- \frac{1}{25} [(297,800 \times 10) + (368,800 \times 15)] = -340,400 \text{ in-lb.}
\]

Therefore the maximum positive bending moment is \(738,000 - 340,400 = 397,600\) in-lb.

(c).—The example in the following shows the method of calculating the bending moments on a continuous beam by direct application of the principles of moment distribution. The tables are used only to the extent of applying the expressions for fixed-end moments (Table 18) and distribution factors (Table 26).
### CASE 5

<table>
<thead>
<tr>
<th>ANY INTERNAL SUPPORT T (EXCEPT PENULTIMATE SUPPORT)</th>
<th>INCIDENTAL LOADING</th>
</tr>
</thead>
<tbody>
<tr>
<td>ALL SPANS LOADED (E.G. DEAD LOAD)</td>
<td>MAX. NEGATIVE B.M.</td>
</tr>
<tr>
<td>GENERAL</td>
<td></td>
</tr>
<tr>
<td>BASIC FORMULA $M_T$</td>
<td>$-\frac{D_T U}{2} \left(2F_{TS} + D_{ST} F_{ST}\right)$</td>
</tr>
<tr>
<td>SYMMETRICAL LOADING</td>
<td></td>
</tr>
<tr>
<td>WITH $F_{TS} = F_{ST}$ AND $F_{UT} = F_{TU}$</td>
<td></td>
</tr>
<tr>
<td>BASIC FORMULA $M_T$</td>
<td>$-\frac{D_T U}{2} \left(2 + D_{ST}\right) F_{TS}$</td>
</tr>
<tr>
<td>EQUAL SPANS AND UNIFORM SECTION</td>
<td></td>
</tr>
<tr>
<td>$-\frac{1}{8} \left[4(S_{ST} + C_{TU}) - C_{SR} + C_{ST} - C_{UV} + C_{TU}\right]$</td>
<td>$-\frac{5L}{8} \left(C_{ST} + C_{TU}\right)$</td>
</tr>
</tbody>
</table>

### ANY INTERNAL SPANN ST

<table>
<thead>
<tr>
<th>SUPPORT</th>
<th>INCIDENTAL LOADING</th>
</tr>
</thead>
<tbody>
<tr>
<td>FOR MAX. POS. B.M. IN SPAN</td>
<td>FOR MAX. NEG. B.M. IN SPAN</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>IF fixed at B.M. at A as CASE 3A.</th>
<th>GENERAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>S</td>
<td>$+\frac{D_T S_D F_{SR}}{2}$ $-\frac{D_T S_D F_{TU}}{2}$</td>
</tr>
<tr>
<td>T</td>
<td>$+\frac{D_T S_D F_{SR}}{2}$ $-\frac{D_T S_D F_{TU}}{2}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>IF fixed at R.</th>
<th>SYMMETRICAL LOADING</th>
</tr>
</thead>
<tbody>
<tr>
<td>S</td>
<td>$+\frac{D_T S_D F_{SR}}{2}$ $-\frac{D_T S_D F_{TU}}{2}$</td>
</tr>
<tr>
<td>T</td>
<td>$+\frac{D_T S_D F_{SR}}{2}$ $-\frac{D_T S_D F_{TU}}{2}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>IF span ST is PENULTIMATE SPAN (Drs = unity if free at R, 0 if fixed at R.)</th>
<th>EQUAL SPANS AND UNIFORM SECTION</th>
</tr>
</thead>
<tbody>
<tr>
<td>S</td>
<td>$-\frac{L}{8} (4 C_{ST} + C_{TS} - C_{RA})$</td>
</tr>
<tr>
<td>T</td>
<td>$-\frac{L}{8} (4 C_{ST} + C_{ST} - C_{UV})$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>IF span ST is PENULTIMATE SPAN (Drs = unity if free at R, 0 if fixed at R.)</th>
<th>EQUAL SPANS AND SYMMETRICAL LOADING</th>
</tr>
</thead>
<tbody>
<tr>
<td>S</td>
<td>$-\frac{L}{8} (5 C_{ST} - C_{QR})$</td>
</tr>
<tr>
<td>T</td>
<td>$-\frac{L}{8} (5 C_{ST} - C_{UV})$</td>
</tr>
</tbody>
</table>

### BASIC FORMULAE

$$M_T = -\frac{D_T U}{2} \left[2F_{TS} - D_{ST} (F_{SR} - F_{ST})\right] - \frac{D_T S_D}{2} \left[2F_{TU} - D_{ST} (F_{UV} - F_{TU})\right]$$

**Note:** For notation, see Table 26.
For Case 2B, see Table 27.
For Cases 3A and 3B, see Table 28.

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CONTINUOUS BEAMS.—END RESTRAINT.

General Case.—If a beam cantilevers beyond an end support, or is monolithic with an external column or other construction, the effect is to produce at the end support a bending moment equal to, and with the same sign as, the cantilever (or equivalent) bending moment. Ignoring any other load on the beam, the bending moments at the penultimate support due to either a negative or positive externally-applied bending moment are given in the upper part of Table 31 for sequence of unequal spans. The factors are based on moment distribution. To the extent of two distributions, an externally-applied bending moment affects only the end and penultimate supports, but with three distributions a small bending moment occurs at the next interior support. The bending moment at the penultimate support is unaffected whether two or three distributions are made. Thus the formulae in Table 31 relate to three distributions for support C and either two or three distributions for support B.

Equal Spans.—The coefficients in the lower part of Table 31 apply to a series of equal spans at one or both end supports of which a bending moment is applied. The effect on the bending moments and shearing forces are taken into account. It must be noted that the units for this part of the table are as follows.

Bending moment.—The units of the bending moment at any support are the same as those in which the externally-applied bending moment is expressed.

Shearing force.—If the externally-applied bending moment is in inch-pound units and the span is in feet, the coefficients are such that the adjustments to the shearing forces are in pounds. If the externally-applied bending moment is in foot-pound units and the span is in feet, the coefficients must be multiplied by 12 to give the shearing forces in pounds.

Example.—If a negative bending moment of 200,000 in.-lb. is applied at one end support of the beam in Example (a) opposite Table 25, find the resultant maximum bending moment at the penultimate support.

Bending moment at penultimate support, $-51,075$ ft.-lb. $= -612,900$ in.-lb.

Bending moment at support B (four equal spans) due to bending moment applied at one end only (Table 31, lower part), $+0.268 \times 200,000$

Resultant net bending moment

\[
\begin{align*}
\text{Bending moment at penultimate support, } & -51,075 \text{ ft.-lb. } = -612,900 \text{ in.-lb.} \\
\text{Bending moment at support B (four equal spans) due to bending moment applied at one end only (Table 31, lower part), } & +0.268 \times 200,000 \\
\text{Resultant net bending moment} & = +53,600 \\
& = -559,300
\end{align*}
\]
# Continuous Beams: Moment-Distribution Method

## Table 31: Bending Moments Applied at End Supports

<table>
<thead>
<tr>
<th>Conditions</th>
<th>Support</th>
<th>Dead Load (or Similar)</th>
<th>Incidental Loading</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>3 or more spans</td>
<td>Two spans</td>
<td>Max. neg. B.M. at support B</td>
</tr>
<tr>
<td></td>
<td>A B C D E</td>
<td>A B C</td>
<td>Max. pos. B.M. in span AB</td>
</tr>
<tr>
<td></td>
<td>2 spans</td>
<td>A B</td>
<td>Max. neg. B.M. in span AB</td>
</tr>
<tr>
<td></td>
<td>A B C</td>
<td>Max. pos. B.M. in span BC</td>
<td></td>
</tr>
<tr>
<td></td>
<td>A B C</td>
<td>Max. pos. B.M. in span BC</td>
<td></td>
</tr>
</tbody>
</table>

### Approximate Method

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
<th>J</th>
<th>K</th>
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</thead>
<tbody>
<tr>
<td>-M&lt;sub&gt;p&lt;/sub&gt;</td>
<td>+Dec&lt;sub&gt;M&lt;sub&gt;p&lt;/sub&gt;&lt;/sub&gt;</td>
<td>-M&lt;sub&gt;p&lt;/sub&gt;</td>
<td>+Dec&lt;sub&gt;M&lt;sub&gt;p&lt;/sub&gt;&lt;/sub&gt;</td>
<td>-M&lt;sub&gt;p&lt;/sub&gt;</td>
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</tr>
<tr>
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<td>C</td>
<td>D</td>
<td>B</td>
<td>A</td>
<td>F</td>
<td>H</td>
<td>J</td>
<td>K</td>
</tr>
<tr>
<td>all support B.M.</td>
<td>as for dead load</td>
<td></td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>

### General Case

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<th>D</th>
<th>E</th>
<th>F</th>
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<th>K</th>
</tr>
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<tbody>
<tr>
<td>-M&lt;sub&gt;p&lt;/sub&gt;</td>
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<tr>
<td>G</td>
<td>E</td>
<td>C</td>
<td>D</td>
<td>B</td>
<td>A</td>
<td>F</td>
<td>H</td>
<td>J</td>
<td>K</td>
</tr>
<tr>
<td>all support B.M.</td>
<td>as for dead load</td>
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### Equal Spans and Uniform Section

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
<th>J</th>
<th>K</th>
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</thead>
<tbody>
<tr>
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<td>D</td>
<td>B</td>
<td>A</td>
<td>F</td>
<td>H</td>
<td>J</td>
<td>K</td>
</tr>
<tr>
<td>as for dead load</td>
<td>as for dead load</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Tabulated Data Are Results of Three Distributions

- M<sub>p</sub> = Numerical Value of Externally-Applied Bending Moment at Support A.
- D<sub>BC</sub>, etc. = Distribution Factor for Span BC, etc., at Support B, etc.
- (See preceding tables for evaluation)

### Bending Moment

<table>
<thead>
<tr>
<th>No. of Spans</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>H</th>
<th>J</th>
<th>K</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
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<td>-</td>
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<td>-104</td>
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### Shearing Force

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### Equal Spans

<table>
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<th>C</th>
<th>D</th>
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<td>J</td>
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<td>T</td>
<td>A</td>
<td>T</td>
<td>B</td>
<td>C</td>
</tr>
</tbody>
</table>

### Key

- Adjustment to B.M. = \((\text{Coefficient}) \times (\text{Applied Bending Moment})\)
- Adjustment to Shearing Force (lb) = \((\text{Coefficient}) \times \left(\frac{\text{Applied B.M. in inch-lb.}}{\text{Span in feet}}\right)\)

---

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In ordinary buildings it is not necessary to consider the interaction of the columns and beams more accurately than by the application of the formulae recommended in the B.S. Code for the bending moments on exterior and interior columns (see Table 46). The formulae in Table 32 are suitable for more complex cases and are based on moment distribution. They are applicable only to vertical loading. (The effect of sway is excluded, so the method requires amplification if large out-of-balance loads act. The effects of wind and other lateral loads are dealt with as in Table 51.) The method is similar to that for continuous beams (on knife-edge supports) in Tables 26 to 31, but the stiffness of the supports is taken into account. The same convention of signs and notation is used. The B.S. Code recommends that the remote ends of members continuous with the span considered be assumed to be fixed, an assumption which simplifies the formulae; therefore, as regards any interior span ST, the ends of the beams at R and U, the ends of the lower columns at O and X, and the ends of the upper columns at P and Y are assumed to be fixed.

Interior Spans.—The stiffness factors and distribution factors are given in Table 32, in which the expressions for the bending moments are the result of two distribution operations and one intermediate carry-over. The bending moments at S and T in any interior span ST are due to vertical loads on interior spans RS, ST, and TU only, since loads on spans QR and UV and beyond do not affect S and T in this limited moment-distribution operation. The formulae for the case of all spans being loaded apply to the dead load on an interior span. To produce the greatest negative bending moments at support S due to live load it is necessary to omit the load from span TU. Similarly, for support T, the live load on span RS is omitted. To produce the maximum positive bending moment on span ST due to live load, it is necessary to load span ST only. The formulae for these conditions of live loading are also given in Table 32.

End Spans.—The formulae for any interior span ST are re-written to apply to an end span AB by substituting A, B, C, etc., for S, T, U, etc.; A is the end support, and there is no span corresponding to RS; the modified stiffness and distribution factors are given in Table 32, together with the formulae for all spans being loaded and also the formulae for the maximum bending moments at the supports due to live load and the formulae for the bending moments at the supports for the incidence of live load giving the maximum positive bending moment in end span AB.

When the bending moments at the supports are known, the positive and negative bending moments in the spans are obtained by combining the diagram of the free bending moments due to the load with the diagram of the corresponding support bending moments.
### Continuous Beams: Moment-Distribution Method — Table 32

#### Continuous Beams

**Support Bending Moments**

<table>
<thead>
<tr>
<th>Conditions</th>
<th>End Span AB</th>
<th>Interior Span ST</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Dead Load</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>All spans loaded</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Maximum bending moment at L.H. support</td>
<td>K = F_{AB} - \frac{D_{BA}}{2}(F_{BC} - F_{BA})(1 - D_{AB})</td>
<td>M_{ST} = - \frac{F_{ST} - D_{ST}}{2}(F_{TU} - F_{TS})(1 - D_{ST}) - D_{ST} F_{SR}</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
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<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Live Load</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Maximum bending moment at R.H. support</td>
<td>M_{AB} = - (F_{AB} + \frac{D_{BA}}{2} F_{BC})(1 - D_{AB})</td>
<td>M_{ST} = - \frac{F_{ST} - D_{ST}}{2}(F_{SR} - F_{ST})(1 - D_{ST}) - D_{ST} F_{SR}</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

#### Beams Monolithic with Supports

<table>
<thead>
<tr>
<th>Conditions</th>
<th>End Span AB</th>
<th>Interior Span ST</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Stiffness Factors (K)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>K_{AB} = \frac{I_{AB}}{L_{AB}}</td>
<td>K_{AG} = \frac{I_{AG}}{L_{AG}}</td>
<td>K_{AH} = \frac{I_{AH}}{L_{AH}}</td>
</tr>
<tr>
<td>K_{BC} = \frac{I_{BC}}{L_{BC}}</td>
<td>K_{BJ} = \frac{I_{BJ}}{L_{BJ}}</td>
<td>K_{BK} = \frac{I_{BK}}{L_{BK}}</td>
</tr>
<tr>
<td>K_{AS} = \frac{I_{AS}}{L_{AS}}</td>
<td>K_{ST} = \frac{I_{ST}}{L_{ST}}</td>
<td>K_{SO} = \frac{I_{SO}}{L_{SO}}</td>
</tr>
<tr>
<td>K_{SP} = \frac{I_{SP}}{L_{SP}}</td>
<td>K_{TU} = \frac{I_{TU}}{L_{TU}}</td>
<td>K_{TY} = \frac{I_{TY}}{L_{TY}}</td>
</tr>
</tbody>
</table>

#### Distribution Factors (D)

| D_{AB} = \frac{K_{AB}}{K_{AB} + K_{AG} + K_{AH}} | D_{BA} = \frac{K_{AB}}{K_{AB} + K_{BC} + K_{BH} + K_{BK}} |

#### Notation

- F_{AB}, etc. = Numerical value of fixed-end B.M. (NEG) at A (ETC) due to load on AB (ETC)
- I_{AB}, etc. = Length of member AB, etc.
- I_{AB}, etc. = Moment of inertia of member AB, etc.
BEAMS WITH SPLAYS AT THE SUPPORTS.

The formulæ in Tables 26 to 32 apply to beams in which the moment of inertia is uniform throughout any span but may differ in each span. If the moment of inertia varies within the span, for example, beams with splays as illustrated in Table 33, the fixed-end moments (the span-load-factor), the carry-over factor, and the stiffness of the beam are affected, each of these items being increased if the moment of inertia is greater at the support than at the middle of the span as in the case a beam with haunches at the supports.

The span-load-factors for supports A and B of a beam AB with non-uniform moment of inertia can be expressed as \( F'_{AB} = G_{AB}C_{AB}L_{AB} \) and \( F'_{BA} = G_{BA}C_{BA}L_{AB} \) respectively. Therefore the coefficient \( G_{AB} \) is \( \frac{F'_{AB}}{C_{AB}L_{AB}} \) and \( G_{BA} \) is \( \frac{F'_{BA}}{C_{BA}L_{AB}} \) and, if the magnitudes of these coefficients are known, \( F'_{AB} \) and \( F'_{BA} \) can be calculated from \( C_{AB} \) and \( C_{BA} \) for a beam of uniform moment of inertia (see Table 18). (For a beam of uniform moment of inertia \( G_{AB} = G_{BA} = 1 \).) For a symmetrical beam with symmetrical load \( G_{AB} = G_{BA} \).

The carry-over factor \( CO_{AB} \) for support A is defined as the moment produced at the fixed end B of a beam AB when unit moment is applied at the freely-supported end A. (If the moment of inertia of the beam is uniform throughout the span, \( CO_{AB} = CO_{BA} = 0.5 \).) For a symmetrical beam \( CO_{AB} = CO_{BA} \).

The stiffness of a beam is the moment which when applied at the freely-supported end A produces unit slope at A if the beam is fixed at B. (For a beam of uniform moment of inertia the stiffness is \( \frac{4EI_{AB}}{L_{AB}} \).)

For a beam with straight splays of the same size at each support \( (K_{A} \text{ and } K_{B} \text{ being factors representing the dimensions of the splays}) \), the values of coefficients \( G_{AB} = G_{BA} \) (for uniformly-distributed load and for a central concentrated load), carry-over factors \( CO_{AB} = CO_{BA} \), and stiffness coefficients \( K_{AB} \) are given in Table 33 for beams with splays of common proportions and symmetrical loads. Intermediate values can be interpolated (plotting is recommended).

The alteration of the span-load-factors, the carry-over factors, and the stiffness due to variation in the moment of inertia affect the basic and derived formulæ for the bending moments at the supports of a beam continuous over two or more spans. The general formulæ for the bending moment at any interior support C (except a penultimate support) are given in Table 33, together with the formulæ for the bending moments at other supports such as the supports at a fixed end and penultimate supports.
### Continuous Beams: Moment-Distribution Method — Table 33

**Beams with Splay**

#### Symmetrical Splay

<table>
<thead>
<tr>
<th>Support</th>
<th>Bending Moments</th>
<th>Application</th>
</tr>
</thead>
<tbody>
<tr>
<td>ANY INTERIOR SUPPORT C (EXCEPT PENULTIMATE)</td>
<td>$-D_{CD} \left[ F_{BC} - CO_{BC} D_{BC} (F_{AB} - F_{BC}) \right] - D_{CB} \left[ F_{CD} - CO_{CD} D_{BC} (F_{DE} - F_{CD}) \right]$</td>
<td>FOUR OR MORE SPANS</td>
</tr>
<tr>
<td>END SUPPORT A (FIXED)</td>
<td>$- F_{AB} - CO_{AB} D_{BA} (F_{AB} - F_{BC})$</td>
<td>TWO OR MORE SPANS</td>
</tr>
<tr>
<td>PENULTIMATE SUPPORT B</td>
<td>FIXED AT A</td>
<td>$-D_{BC} F_{AB} - D_{BA} [F_{BC} - CO_{BC} D_{CB} (F_{CD} - F_{BC})]$</td>
</tr>
<tr>
<td>PENULTIMATE SUPPORT B</td>
<td>FREE AT A</td>
<td>$-D_{BC} F_{AB} (1+CO_{AB}) - D_{BA} F_{BC}$</td>
</tr>
<tr>
<td>MIDDLE SUPPORT C</td>
<td>FIXED AT A AND C</td>
<td>$-D_{BC} F_{AB} - D_{BA} F_{BC}$</td>
</tr>
<tr>
<td>MIDDLE SUPPORT C</td>
<td>FREE AT A AND C</td>
<td>$-D_{BC} F_{AB} (1+CO_{AB}) - D_{BA} F_{BC} (1+CO_{BC})$</td>
</tr>
</tbody>
</table>

For spans AB and BC etc.,

$$D_{BA} = \frac{1}{1 + \frac{K_{AB} I_{AB} L_{BC}}{K_{BC} I_{BC} L_{AB}}} = 1 - D_{bc}$$

$I_{AB}, I_{BC}, etc.$ = Moment of Inertia at Middle of Spans AB, BC, etc.

$K_{AB}, K_{BC}, etc.$ = Stiffness Coefficients of Spans AB and BC, etc.

$L_{AB}, L_{BC}, etc.$ = Length of Spans AB, BC, etc.

$CO_{AB}, CO_{BC}, etc.$ = Carry-Over Factors for Spans AB and BC.

$G_{AB}, G_{BC}, etc.$ = Load Factor at A and B (Symmetrical Load on Span AB) and at B and C (Symmetrical Load on Span BC), etc.

$F_{AB}, F_{BC}, etc.$ = Span-Load Factor for Supports A and B for load on Span AB, and B and C for Span BC, etc.

$= G_{AB} L_{AB} C_{AB} ; G_{BC} L_{BC} C_{BC}; etc.$

$= G_{AB} L_{AB} C_{AB} ; G_{BC} L_{BC} C_{BC}; etc.$

**Symbols**

**Support References:**

For two spans A B C:

For three or more spans A B C D:

<table>
<thead>
<tr>
<th>Splay Length</th>
<th>$k_d = 1.4$</th>
<th>$k_d = 1.6$</th>
<th>$k_d = 2.0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_l$</td>
<td>CO K Stiffness</td>
<td>G</td>
<td>CO K Stiffness</td>
</tr>
<tr>
<td>0.1</td>
<td>0.55</td>
<td>1.21</td>
<td>1.06</td>
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<td>0.2</td>
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<td>1.11</td>
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<tr>
<td>0.3</td>
<td>0.61</td>
<td>1.64</td>
<td>1.13</td>
</tr>
</tbody>
</table>

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INFLUENCE LINES FOR CONTINUOUS BEAMS.

For the determination of the bending moments at any of the critical sections in a system of beams due to a train of loads in any given position, the procedure is as follows:

Draw the beam system to a convenient linear scale.

With the ordinates given in Tables 34, 35, 36 or 37 construct the influence line (for unit load) for the section to be considered, selecting a convenient scale for the bending moment.

Plot on this diagram the train of loads in what appears to be the most adverse position.

Tabulate the value of (ordinate x load) for each load.

Algebraically add the products (ordinate x load) to give the resultant bending moment at the section considered.

Repeat for other positions of loads to ensure that the most adverse position has been assumed.

The following example shows the direct use of the tabulated influence lines for calculating the bending moments on a continuous beam subjected to concentrated loads in specified positions.

Example.—Determine the bending moment at the penultimate (left-hand end) support of a system of four spans (constant moment of inertia, freely supported on end supports) subject to a central load of 10 tons on the first and third spans (counting from left-hand end); the end spans are 20 ft. and the interior spans 30 ft.; hence the span-ratio is 1 : 1 1/4 : 1 1/4 : 1.

With load on 1st span (ordinate c):

\[
\text{Bending moment} = -(0.052 \times 10 \times 2240 \times 20) = 36,800 \text{ (neg.)}
\]

With load on 3rd span (ordinate m):

\[
\text{Bending moment} = + (0.035 \times 10 \times 2240 \times 20) = 15,700 \text{ (pos.)}
\]

Net bending moment at penultimate support = 21,100 (neg.)
CONTINUOUS BEAMS: INFLUENCE LINES.—TABLE 34.
TWO SPANS.

INFLUENCE LINES FOR BENDING MOMENTS
AT MID-SPAN B OF SHORTER SPAN AC
AND MID-SPAN D OF LONGER SPAN CE

INFLUENCE LINES FOR BENDING MOMENTS AT CENTRAL SUPPORT C

<table>
<thead>
<tr>
<th>SECTION</th>
<th>RATIO OF SPANS</th>
<th>ORDINATES</th>
<th>SHORTER SPAN</th>
<th>LONGER SPAN</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>L₁:L₂</td>
<td>a b c d e f g h j k</td>
<td></td>
<td></td>
</tr>
<tr>
<td>SHORTER SPAN prostitution B</td>
<td>1:1</td>
<td>0.063 1.30 2.03 1.21 0.52</td>
<td>0.032 0.046 0.047 0.037 0.037</td>
<td>0.020</td>
</tr>
<tr>
<td></td>
<td>1:1½</td>
<td>0.067 1.37 2.13 1.30 0.58</td>
<td>0.058 0.083 0.084 0.067 0.037</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1:2</td>
<td>0.070 1.42 2.19 1.36 0.62</td>
<td>0.085 1.24 1.25 0.099 0.054</td>
<td></td>
</tr>
<tr>
<td>CENTRAL SUPPORT C</td>
<td>1:1</td>
<td>0.041 0.074 0.094 0.093 0.064</td>
<td>0.064 0.093 0.094 0.074 0.041</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1:1½</td>
<td>0.032 0.059 0.075 0.074 0.051</td>
<td>0.115 0.167 0.165 0.133 0.073</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1:2</td>
<td>0.027 0.049 0.063 0.062 0.042</td>
<td>0.170 0.247 0.250 0.198 0.108</td>
<td></td>
</tr>
<tr>
<td>LONGER SPAN prostitution D</td>
<td>1:1</td>
<td>0.020 0.037 0.047 0.046 0.032</td>
<td>0.052 0.121 0.203 0.130 0.063</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1:1½</td>
<td>0.016 0.030 0.038 0.037 0.025</td>
<td>0.067 0.167 0.231 0.183 0.088</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1:2</td>
<td>0.014 0.025 0.031 0.031 0.021</td>
<td>0.082 0.210 0.375 0.235 0.113</td>
<td></td>
</tr>
</tbody>
</table>

UNEQUAL SPANS.
DATA ENABLES INFLUENCE LINES TO BE DRAWN FOR THE BENDING MOMENTS PRODUCED BY A SINGLE UNIT LOAD MOVING OVER TWO UNEQUAL SPANS.
ORDINATES FOR INTERMEDIATE RATIOS OF SPANS CAN BE INTERPOLATED.

EQUAL SPANS.
INFLUENCE LINES MARKED 1:1:0 CAN BE USED DIRECTLY. (DIAGRAM OF A SUCCESSION OF LOADS MUST BE DRAWN TO THE SAME LINEAR SCALE)
BENDING MOMENT DUE TO LOAD W CONCENTRATED AT ANY POINT.
= (ORDINATE OF APPROPRIATE INFLUENCE LINE) x (SHORTER SPAN L₁) x W.
CONTINUOUS BEAMS: INFLUENCE LINES.—TABLE 35.
THREE SPANS.

INFLUENCE LINES FOR BENDING MOMENTS
AT MID-SPAN B OF END SPAN AC
AND MID-SPAN D OF CENTRAL SPAN CC'

INFLUENCE LINES FOR BENDING MOMENTS
AT INTERIOR SUPPORT C.

<table>
<thead>
<tr>
<th>SECTION</th>
<th>RATIO OF SPANS</th>
<th>ORDINATES</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>L1:L2:L3</td>
<td>END SPAN AC</td>
</tr>
<tr>
<td>MID-SPAN B OF</td>
<td>1:1:1</td>
<td>a b c d e f g h j k l m n</td>
</tr>
<tr>
<td>END SPAN AC</td>
<td>0.062</td>
<td>0.127 0.200 0.117</td>
</tr>
<tr>
<td>1:1:2:1</td>
<td>0.066</td>
<td>0.134 0.209 0.126</td>
</tr>
<tr>
<td>1:2:1:1</td>
<td>0.068</td>
<td>0.139 0.215 0.132</td>
</tr>
<tr>
<td>INTERIOR SUPPORT</td>
<td>1:1:1</td>
<td>0.043 0.078 0.100</td>
</tr>
<tr>
<td>C</td>
<td>0.036</td>
<td>0.065 0.082 0.081</td>
</tr>
<tr>
<td>1:1:2:1</td>
<td>0.030</td>
<td>0.056 0.070 0.069</td>
</tr>
<tr>
<td>MID-SPAN D OF</td>
<td>1:1:1:1</td>
<td>0.016 0.030 0.038</td>
</tr>
<tr>
<td>CENTRAL SPAN CC'</td>
<td>1:1:2:1</td>
<td>0.013 0.023 0.028</td>
</tr>
<tr>
<td>1:2:2:1</td>
<td>0.010</td>
<td>0.019 0.023 0.023</td>
</tr>
</tbody>
</table>

UNEQUAL SPANS.
DATA ENABLES INFLUENCE LINES TO BE DRAWN FOR BENDING MOMENTS PRODUCED BY A SINGLE UNIT LOAD MOVING OVER THREE UNEQUAL SPANS.
ORDINATES FOR INTERMEDIATE RATIO OF SPANS CAN BE INTERPOLATED.

EQUAL SPANS.
INFLUENCE LINES MARKED 1:1:1 CAN BE USED DIRECTLY (DIAGRAM OF A SUCCESION OF LOADS MUST BE DRAWN TO THE SAME LINEAR SCALE.)
BENDING MOMENT DUE TO LOAD W CONCENTRATED AT ANY POINT
   = (ORDINATE OF APPROPRIATE INFLUENCE LINE) x (END SPAN L) x W.
### Table 86—Continuous Beams: Influence Lines

#### Four Spans (Equal or Symmetrical Inequality)

### Influence Lines for Bending Moments

**At Mid-Span B of End Span AB**
**Mid-Span D of Interior Span CE**

**Influence Lines for Bending Moments**

**At Pen-Ultimate Support C.**

**Influence Lines for Bending Moments**

**At Central Support E.**

#### Section

<table>
<thead>
<tr>
<th>Ordinates</th>
<th>Ratio of Spans $\frac{L_1}{L_2} : \frac{L_3}{L_4}$</th>
<th>End Span $AC$</th>
<th>Interior Span $CE$</th>
<th>Interior Span $C'F$</th>
<th>End Span $C'\overline{A}$</th>
</tr>
</thead>
</table>
| **Mid-Span B of End Span AC** | $1 : 1 : 1 : 1$ | $\begin{pmatrix} 0.02 & 0.127 & 0.200 & 0.117 & 0.063 \end{pmatrix}$ | $\begin{pmatrix} 0.028 & 0.035 & 0.037 & 0.027 & 0.013 \end{pmatrix}$ | $\begin{pmatrix} 0.010 & 0.010 & 0.007 & 0.005 & 0.003 \end{pmatrix}$ | | }
| **Pen-Ultimate Support C** | $1 : 1 : 1 : 1$ | $\begin{pmatrix} 0.043 & 0.079 & 0.100 & 0.055 & 0.068 \end{pmatrix}$ | $\begin{pmatrix} 0.057 & 0.078 & 0.074 & 0.053 & 0.025 \end{pmatrix}$ | $\begin{pmatrix} 0.021 & 0.020 & 0.015 & 0.007 & 0.008 \end{pmatrix}$ | | }
| **Mid-Span D of Interior Span CE** | $1 : 1 : 1 : 1$ | $\begin{pmatrix} 0.016 & 0.025 & 0.037 & 0.036 & 0.025 \end{pmatrix}$ | $\begin{pmatrix} 0.061 & 0.039 & 0.175 & 0.064 & 0.040 \end{pmatrix}$ | $\begin{pmatrix} 0.032 & 0.030 & 0.022 & 0.010 & 0.006 \end{pmatrix}$ | | }
| **Central Support E** | $1 : 1 : 1 : 1$ | $\begin{pmatrix} 0.012 & 0.021 & 0.027 & 0.027 & 0.018 \end{pmatrix}$ | $\begin{pmatrix} 0.028 & 0.068 & 0.080 & 0.065 & 0.061 \end{pmatrix}$ | $\begin{pmatrix} 0.085 & 0.080 & 0.008 & 0.001 & 0.026 \end{pmatrix}$ | | }

**Unequal Spans.**
Data enables influence lines to be drawn for bending moments produced by a single unit load moving over four unequal spans.

Ordinates for intermediate ratios of spans can be interpolated.

**Equal Spans.**
Influence lines marked 1:1:1:1:1 can be used directly. (Diagram of a succession of loads must be drawn to same linear scale.)

Bending moment due to load $W$ concentrated at any point is $(\text{ordinate of appropriate influence line}) \times (\text{end span $L_1$}) \times W$. 

---

### Notes
- For bending moments produced by a single unit load moving over four unequal spans, ordinates can be interpolated.
- Influence lines marked 1:1:1:1:1 can be used directly. (Diagram of a succession of loads must be drawn to the same linear scale.)
- Bending moment due to load $W$ concentrated at any point is $(\text{ordinate of appropriate influence line}) \times (\text{end span $L_1$}) \times W$. 

---

### Diagram

- The diagram illustrates the influence lines for bending moments at mid-span B, pen-ultimate support C, and central support E for four spans with equal or symmetrical inequality.
- The influence lines are represented graphically with ordinates and ratios of spans labeled accordingly.
- The table provides specific ordinates for each section, facilitating the calculation of bending moments for given loads and spans.
INFLUENCE LINES FOR BENDING MOMENTS
AT MID-SPAN \{ \text{THIRD-POINT} \} OF INTERIOR SPAN
INTERIOR SUPPORT

INFLUENCE LINES FOR BENDING MOMENTS
AT MID-SPAN \{ \text{THIRD-POINT} \} OF END SPAN
PENULTIMATE SUPPORT

ORDINATES FOR END SPAN AND PENULTIMATE SUPPORT
OF FOUR EQUAL SPANS CAN BE USED FOR FIVE OR MORE
SPANS WITH REASONABLE ACCURACY.

INFLUENCE LINES ABOVE CAN BE USED DIRECTLY (DIAGRAM OF A SUCCESSION OF LOADS MUST BE DRAWN TO THE SAME LINEAR SCALE.)
BENDING MOMENT DUE TO LOAD $W$ CONCENTRATED AT ANY POINT $= (\text{ORDINATE OF APPROPRIATE INFLUENCE LINE}) \times (\text{SPAN}) \times W.$
SLABS SPANNING IN TWO DIRECTIONS.

Notation.—The symbols used in the following and in Tables 38, 39, 41, 42 and 43 are as follows. (The corresponding symbols used in B.S. Code No. 114, where different, are given in brackets.) The symbols used in Tables 40 and 44 are given in the respective tables.

$W = \text{total load (lb.) on the slab and equal to } wL_B^2L_f$ for a completely-loaded panel, and equal to $wuv$ for a partially-loaded panel; $w = \text{uniformly-distributed load (lb. per sq. ft.)}$.

$L_B(l_x)$ and $L_B(l_y) = \text{short and long spans (ft.) respectively, } h = \frac{L_B}{L_L}$.

$M_B$ and $M_L = \text{maximum bending moments at the midspan of the short and long spans respectively; } M_{BA}$ and $M_{BC} = \text{bending moments at supports A and C respectively of the short span; } M_{LD}$ and $M_{LE} = \text{the bending moments at supports D and E respectively of the long span.}$

$K_B$ and $K_L = \text{bending-moment reduction factors for short and long spans respectively, corners not held down; } K_B'$ and $K_L' = \text{corresponding factors with corners held down.}$

$m_B(= \beta_2)$ and $m_L(= \beta_3(\frac{l_y}{l_f})^k) = \text{coefficients for positive bending moments on short and long spans respectively, with corners held down; } m_B'$ and $m_L' = \text{corresponding coefficients for negative bending moments.}$

$m_{BO}(= \alpha_2)$ and $m_{LO}(= \alpha_3(\frac{l_y}{l_f})^k) = \text{coefficients for positive bending moments on short and long spans respectively with corners not held down.}$

Rectangular Panels Freely Supported along All Edges with Uniformly-distributed Load.—For a rectangular panel that is freely supported along all four edges in such a manner that the corners are free to lift, the Grashof and Rankine method is applicable and the bending moment reduction coefficients are $K_B = \frac{h^4}{h^4 + 1}$ and $K_L = 1 - K_B$. The midspan bending moments per foot width $M_B$ and $M_L$ are calculated from the formulae in Table 38. The usual limit of application of this method is when the length of the panel is equal to twice the breadth, that is when $k = 2$. Beyond this limit the slab is considered to span across the short span only, the bending moment per foot width then being $\frac{wL_B^2}{8}$.

For the condition "corners not held down", the bending-moment coefficients in the B.S. Code correspond to $m_{BO}$ and $m_{LO}$ in Table 39.

In cases near the limit of $k = 2$, it is necessary to ensure that the amount of reinforcement in the long direction is not less than the minimum amount of distribution bars required.

For panels that are freely supported along all four edges but with the corners prevented from lifting, the corresponding coefficients $K_B'$ and $K_L'$ in Table 38 conform to a more exact analysis but with Poisson's ratio equal to zero.

The bending moments at midspan based on Dr. Marcus's method are the midspan bending moments calculated by the Grashof and Rankine method multiplied by a factor $C$; for a slab freely supported along all four edges $C = 1 - \frac{5}{6}(\frac{h^2}{h^4 + 1})$, the midspan bending moments per foot width are $M_B = CK_B\frac{wL_B^2}{8}$ and $M_L = \frac{M_B}{h^2}$.

The resultant bending moments by the method of Dr. Marcus and the exact theory (with Poisson's ratio equal to zero) are almost identical. If Poisson's ratio is assumed to be $0.15$, the midspan bending moments per foot of width are $M_B = \frac{wCK_BL_B^2}{8}(1 + \frac{0.15}{h^2})$ and $M_L = \frac{wCK_BL_B^2}{8}(0.15 + \frac{1}{h^2})$. Alternatively the appropriate coefficients can be obtained from the curves in Table 42 for $\frac{u}{L_B} = \frac{v}{L_L} = x$ for a slab completely covered with a load of intensity $w = \frac{W}{L_BL_L}$. The bending-moment coefficients given in the B.S. Code for this case correspond to $m_B$ and $m_L$ in the top left-hand corner in Table 39.

Rectangular Panels Fixed along Four Sides with Uniformly-distributed Load.—If a panel is completely fixed along all four sides, the bending moments are as follows.

Short span: Midspan $M_B = +0.8M_{BA}$; Support $M_{BA} = -K_B'\frac{wL_B^2}{8}$.

Long span: Midspan $M_L = +0.8M_{LD}$; Support $M_{LD} = -K_L'\frac{wL_B^2}{8}$,

where $K_B'$ and $K_L'$ are as in Table 38. (See also B.S.-code method on page 208.)

(Continued on page 208.)
### Table 38: Uniformly-Distributed Load

<table>
<thead>
<tr>
<th>Ratio of Spans</th>
<th>Condition Along Four Edges</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Free Corners Not Held Down</td>
</tr>
<tr>
<td></td>
<td>(K_B) (K_L) (K_B) (K_L)</td>
</tr>
<tr>
<td>1.0</td>
<td>0.50 0.50 0.30 0.30</td>
</tr>
<tr>
<td>1.05</td>
<td>0.55 0.45 0.33 0.27</td>
</tr>
<tr>
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<td>0.59 0.41 0.36 0.24</td>
</tr>
<tr>
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<td>0.64 0.36 0.39 0.22</td>
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<tr>
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<td>0.68 0.33 0.42 0.19</td>
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<tr>
<td>1.25</td>
<td>0.72 0.25 0.45 0.17</td>
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<tr>
<td>1.3</td>
<td>0.74 0.26 0.48 0.15</td>
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<tr>
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<td>0.97 0.03 0.87 0.03</td>
</tr>
<tr>
<td>3.0</td>
<td>0.98 0.02 0.94 0.02</td>
</tr>
</tbody>
</table>

#### Continuity (or Fixity) Along One or More Edges (B.S. Code)

- **Conditions:**
  - Corners Held Down.
  - Torsional Resistance Provided.
  - No reinforcement required in edge strips to resist bending moment parallel to edges of panel.

#### Bending Moments (ft-lb, per foot) in Middle Strips:

- **At Midspan:**
  - Short Span: \(+ m_B w L_B^2\)
  - Long Span: \(+ m_L w L_L^2\)

- **At Discontinuous Edge** (slab monolithic with support):
  - Short Span: \(- m_B w L_B^2 - \frac{3}{2} m_B w L_B^2\)
  - Long Span: \(- m_L w L_L^2 - \frac{3}{2} m_L w L_L^2\)

#### Corner Reinforcement for Torsional Resistance (B.S. Code)

- \(A_B\) \& \(A_L\) = Cross sectional area (per ft.) of reinforcement for positive B.M. at midspan of short and long spans respectively.

#### Note

For values of \(m_B\), \(m_B'\), \(m_L\) and \(m_L'\) (for calculation of bending moments on middle strips) see Table 39.
SLABS SPANNING IN TWO DIRECTIONS (continued from page 206).

Formulate based on Dr. Marcus’s method are given in Table 38; the coefficient
\[ C' = \frac{5}{18} \left( \frac{h^3}{(1 + h^4)} \right) \]
and values are given in Table 38; \( K_L \) and \( K_B \) are the Grashof and
Rankine reduction coefficients given in Table 38.

B.S. Code Method for Rectangular Panels with Uniformly-distributed Load.—
For the conditions common in reinforced concrete construction, that is continuity over the
supports along some edges and discontinuity at the other edges, the method recommended in
the B.S. Code No. 114 is applicable, and the coefficients given by the curves in Table 39
should be substituted in the appropriate formulae for bending moments in the middle strip
shown in Table 38. Provision is made at the discontinuous edge for the possible restraining
demand due to the slab being cast monolithically with reinforced concrete beams supporting
the panel or with concrete encasing steel beams, or due to embedment in a brick or masonry
wall; if the restraint is of such magnitude that full fixity or conditions of continuity exist,
the slab should be considered as continuous, instead of discontinuous, at the edge concerned.
The B.S. Code recommend also other bases of analysis, such as the theory of thin plates
(which gives the factors \( K_B' \) and \( K_L' \) in Table 38) and the yield-line method.

Shearing Force on Rectangular Panel with Uniformly-distributed Load.—The
maximum shearing forces at the edges of a panel spanning in two directions and carrying
a uniformly-distributed load are, according to M. Pigeon, approximately \( 0.333wL_B \) at the
middle of the short edge and \( \frac{wL_B h}{2h + 1} \) at the middle of the long edge. The same values are
applicable to a panel fixed or continuous along all four edges, but for other conditions the
distribution of the shearing force, the stresses due to which are rarely critical, must be adjusted
on the principle that the shearing force is slightly greater at a side where there is continuity
or fixity than at an opposite freely-supported edge.

Examples.

(a) Determine the bending moments on a rectangular slab AEDC freely supported on
all four sides (corners not held down) and subject to an inclusive load of 200 lb. per sq. ft.,
the lengths of the sides being \( L_B = 10 \) ft. and \( A \) and \( C \) being \( L_L = 12 \) ft. 6 in.

Ratio of sides = \( \frac{12.5}{10.0} = 1.25 \).

(i) From Table 38 the coefficients are \( K_B = 0.71 \) and \( K_L = 0.29 \).

Bending moments:

Midspan, short span: \( 0.71 \times \frac{1}{2} \times 200 \times 10^3 \times 12 = 21,300 \text{ in.-lb.} \)

long span: \( 0.29 \times \frac{1}{2} \times 200 \times 12.5^3 \times 12 = 13,600 \text{ in.-lb.} \)

Bending moments at the supports are zero.

(ii) From Table 39 using the coefficients \( m_{BD} \) and \( m_{LO} \) corresponding to B.S. Code for
"corners not held down".

Bending moments.—Midspan, short span = \( 0.09 \times 200 \times 10^3 \times 12 = 21,600 \text{ in.-lb.} \)

long span = \( 0.036 \times 200 \times 12.5^3 \times 12 = 13,400 \text{ in.-lb.} \)

(b) Find the bending moment in the panel in (a) if it is constructed monolithically with
the supports, but discontinuous along all edges (that is, nominally freely supported but with
the corners held down).

(i) From Table 38, using "exact" coefficients \( K_B' = 0.45 \) and \( K_L' = 0.17 \).

Bending moments:

Midspan of short span: \( 0.45 \times \frac{1}{2} \times 200 \times 10^3 \times 12 = 13,500 \text{ in.-lb.} \)

long span: \( 0.17 \times \frac{1}{2} \times 200 \times 12.5^3 \times 12 = 8000 \text{ in.-lb.} \)

(ii) From Table 39, using the B.S. Code coefficients (curves in top left-hand corner).

Bending moments:

in.-lb.

Short span—middle strip, at midspan: \( 0.065 \times 200 \times 10^3 \times 12 = 15,600 \)

at edge: \( -\frac{1}{3} \times 15,600 \)

Long span—middle strip, at midspan: \( 0.032 \times 200 \times 12.5^3 \times 12 = 12,000 \)

at edge: \( -\frac{1}{3} \times 12,000 \)

(c) Determine the bending moments if the panel in (a) is nominally freely supported on
side \( C \) only and is continuous over the other three sides.

Table 39 applies (curves in bottom right-hand corner):

\[ \frac{L_L}{L_B} = \frac{12.5}{10} = 1.25 \]

Bending moments: Short span, midspan: \( 0.043 \times 200 \times 10^3 \times 12 = 10,300 \)

support: \( 0.055 \times 200 \times 10^3 \times 12 = 15,400 \)

Long span, midspan: \( 0.020 \times 200 \times 12.5^3 \times 12 = 7500 \)

support: \( 0.026 \times 200 \times 12.5^3 \times 12 = 9750 \)
SLABS SPANNING IN TWO DIRECTIONS: RECTANGULAR PANELS.—TABLE 39.
UNIFORMLY-DISTRIBUTED LOAD.—B.S. CODE.

BENDING MOMENT COEFFICIENTS $m$

$K = \frac{L_L}{L_B} = \text{RATIO OF SPANS}$

$K = 1.0$ to $2.0$

$1.2$, $1.4$, $1.6$, $1.8$, $2.0$

$E$ DGE AT WHICH THERE IS DISCONTINUITY OR CAST MONOLITHIC WITH SUPPORT.

$E$ DGE AT WHICH THERE IS CONTINUITY.

B.M. COEFFICIENTS $m_B$ AND $m_L$ APPLY TO MAX. POSITIVE AND NEGATIVE B.M.s. RESPECTIVELY ON SHORT SPAN.

Note.—Curves give coefficients to be used in formulae in Table 38. See also page facing Table 38 for corresponding symbols used in B.S. Code.

209
Rectangular Panels with Triangularly-distributed Loads.—The intensity of pressure on the walls of containers is uniform at any given level, but vertically may vary from zero near the top to a maximum \( p \) at the bottom. If there is a support along the top of a rectangular panel spanning in two directions, the curves and expressions in the lower part of Table 40 enable the probable maximum bending moments on vertical and horizontal strips of unit width to be calculated, whether the slab is fixed (Case 3) or freely supported (Case 2) along the top edge. Similar curves are given in the upper part of Table 40 for the condition when there is no support along the top edge (Case 1). In all cases it is assumed that the slab is continuous over the two vertical edges of the panel and is fixed along the bottom edge.

The upper curves for each case indicate the approximate positions of the maximum bending moments. The values given at the ends of the curves are the values to which the coefficients approach as the ratio of spans \( k \) approaches zero, or to which they are asymptotic if \( k \) is infinity. Although the conditions \( k = 0 \) and \( k = \infty \) have no practical significance, the coefficients for these cases apply when the slab spans wholly horizontally or vertically respectively. If \( k \) is less than 0.5 in Cases 2 and 3 or less than 0.5 in Case 1, the bending moments should be calculated on the assumption that the slab spans entirely horizontally. If \( k \) exceeds 2 for Case 1, or 2.5 for Cases 2 and 3, the slab should be assumed to span entirely vertically.

At a nominally freely-supported top edge (Case 2), resistance to negative bending moment equal to two-thirds of the positive bending moment on the vertical span should be provided.

If the slab is assumed to span entirely vertically or entirely horizontally, the amount of reinforcement provided horizontally and vertically respectively, and in other cases at sections where the calculated bending moment is small, should be not less than the minimum amount required in a slab (see page 266).

Since it is common in a container to provide 45-deg. splays at the corners, it should be noted that the critical negative bending moments are not necessarily at the edges of the splays.

A pressure which is distributed trapezoidally can be dealt with by adding the bending moment due to a1 uniformly-distributed load (Table 40) to the bending moment due to a uniformly-distributed load (Table 35 or 36); this applies to the negative bending moment, but only approximately to the positive bending moment.

Example.—Find the maximum bending moments in a wall panel of a rectangular tank that can be considered as freely supported along the top edge and continuous along the bottom edge and along the two vertical sides. The height of the panel is 8 ft. (\( L_v \)) and the horizontal span is 12 ft. (\( L_H \)), the intensity of pressure being 500 lb. per square foot along the bottom edge and decreasing uniformly to zero at the top edge.

Ratio of spans: \( k = \frac{L_H}{L_V} = \frac{12}{8} = 1.5 \). For Case 2, the bending moments are as follows.

At height \( x = 0.45 \) of 0.45 \( \times 8 = 3 \) ft. 7\( \frac{1}{2} \) in., the maximum positive bending moment on the vertical span is \( \frac{500 \times 8^2 \times 0.027 \times 12}{1} = 10,400 \) in.-lb.

At mid-point of the bottom of the panel, the maximum negative bending moment on the vertical span is \( -500 \times 8^2 \times 0.043 \times 12 = 16,500 \) in.-lb.

At mid-span and at a height \( x_1 = 1 \) of \( \frac{1}{2} \times 8 = 4 \) ft., the maximum positive bending moment on the horizontal span is \( \frac{500 \times 12^2 \times 0.004 \times 12}{1} = 3450 \) in.-lb., which is negligible; the minimum amount of reinforcement required in a slab forming part of a liquid-container will provide a greater resistance.

At height \( x = 0.42 \) of 0.42 \( \times 8 = 3 \) ft. 4 in., the maximum negative bending moment at the vertical edges of the horizontal span is \( -500 \times 12^2 \times 0.013 \times 12 = 11,230 \) in.-lb.

The bending moments on the horizontal span have generally to be combined with a direct tension.
SLABS SPANNING IN TWO DIRECTIONS: RECTANGULAR PANELS.—TABLE 40.
TRIANGULARLY-DISTRIBUTED LOAD.

BENDING MOMENT COEFFICIENTS
\[ \frac{B.M}{PL^2} = 0.20 \]

CASE 1.
UNSUPPORTED ALONG TOP EDGE

CURVES FOR THIS CASE ARE APPLICABLE ONLY FOR \( \frac{L_H}{L_V} \leq 0.5 \gg 2.0 \)

CASE 2.
SIMPLY SUPPORTED ALONG TOP EDGE

CASE 3.
FIXED OR CONTINUOUS ALONG TOP EDGE

POSITIONS OF MAXIMUM B.Ms.

\[ P = \text{MAXIMUM INTENSITY OF HYDROSTATIC PRESSURE} \]

MAX. POS. B.M. VERTICAL SPAN

MAX. POS. B.M. HORIZONTAL SPAN

MAX. NEG. B.M. HORIZONTAL SPAN

NOTES

PANELS FIXED OR CONTINUOUS ALONG BOTTOM EDGE AND BOTH VERTICAL SIDES; CONDITION ALONG TOP EDGE AS INDICATED.

FRACTIONS SUCH AS \( \frac{1}{2} \) INDICATE COEFFICIENTS TO WHICH CURVES ARE ASYMMETRIC OR TO WHICH COEFFICIENTS APPROACH AS SPAN RATIO \( \frac{L_H}{L_V} \) APPROACHES ZERO.

VERTICAL SPAN: \( B.M. = (\text{COEFFICIENT})P_L^2 \)

HORIZONTAL SPAN: \( B.M. = (\text{COEFFICIENT})P_L^2 \)

Notes.—Scale on right-hand side is for values of \( x_1, x, \) and \( v \). Ratio of spans \( k = \frac{L_H}{L_V} \).

2II
Rectangular Panel with Free Support along One, Two, or Three Sides with Uniformly-distributed Load.—Bending moments on panels where continuity or fixity occur along one, two, or three edges and free support along the remaining edges, but with corner restraint, can be based on the following approximate method, which gives results reasonably close to those obtained by more accurate methods.

Substitute for \( k = \frac{\text{long span}}{\text{short span}} \) the expression \( k_e = \frac{\text{longer equivalent span}}{\text{shorter equivalent span}} \), where the equivalent span is \( f \) times the actual span. Values of \( f \) for various conditions of supports are approximately as follows: unity for a span freely supported at both ends, \( 0.8 \) for a span continuous at one end and freely supported at the other, and \( 0.67 \) for a span continuous over both supports. The shorter equivalent span may not be the shorter actual span. If the corners of a slab are prevented from lifting and torsional restraint is provided, the coefficients \( K'_B \) and \( K'_L \) (Table 38) for the ratio \( k_e \), instead of \( k \), are combined with the following bending-moment factors.

Slab freely supported along one edge and continuous over the other three supports.—

The bending moment factor is \( \frac{1}{4} \) for the negative bending moment over the three supports where continuity exists, \( \frac{1}{8} \) for the positive midspan bending moment for the span that is continuous at both ends, and \( \frac{1}{6} \) for the positive midspan bending moment for the span that is continuous at one end and free at the other.

Slab freely supported along two opposite edges and continuous over the other two opposite supports.—The bending moment factors are \( \frac{1}{4} \) for the negative bending moments over the two supports where continuity exists and for the positive bending moment at midspan of the span that is freely supported at both ends, and \( \frac{1}{6} \) for the positive bending moment for the span that is continuous at both ends.

Slab continuous over two adjacent supports and freely supported along the other two.—

The bending moment factors are \( \frac{1}{4} \) for the negative bending moments and \( \frac{1}{6} \) for the positive bending moments.

Slab continuous over one support and freely supported along the remaining three edges.—

The bending-moment factors are \( \frac{1}{4} \) for the negative bending moment and for the positive bending moment for the span freely supported at both ends, and \( \frac{1}{6} \) for the positive bending moment for the span that is continuous at one end and freely supported at the other.

Slab continuous over two adjacent supports and freely supported along the remaining edges, but with the corner where the freely supported edges meet not held down.—Combine with the coefficients \( K_B \) and \( K_L \) (Table 38) for the appropriate ratio of \( k_e \), bending moment factors of \( \frac{1}{4} \) for the positive and negative bending moments.

Slab continuous over one support and freely supported along the remaining three edges, but with the corners where the freely-supported edges meet not held down.—The bending moment factors are \( \frac{1}{4} \) for the negative bending moment and for the positive midspan moment of the span continuous at one end and free at the other, and \( \frac{1}{6} \) for the positive bending moment in the span that is free at both ends.

The foregoing method is an alternative method in the B.S. Code; the latter method should be used where applicable.

Beams Supporting Rectangular Panels.—Beams supporting slabs spanning in two directions are subjected to loading from the slab that is distributed approximately triangularly on the beam along each shorter edge (span \( L_B = l_B \)) and trapezoidally on the beam along each longer edge (span \( L_L = l_L \)) as shown in the diagrams in Table 41. For the calculation of the bending moments only on the beams, if the span of a beam is equal to the length (or width) of the panel, and the beam supports one panel only, the equivalent total uniformly-distributed loads on the beams supporting a panel which is freely-supported along all four edges or is subjected to the same degree of restraint along all four edges are as follows.

Short-span beam: \( \frac{wL_B^3}{3} \); long-span beam: \( \frac{wL_B}{2} \left[ 1 - \frac{1}{\left( \frac{L_B}{L_L} \right)^2} \right] \),

where \( w \) is the intensity of total uniformly-distributed load on the slab. If a beam supports two identical panels, one on either side, the foregoing equivalent loads are doubled. If a beam supports more than one panel in the direction of its length, the distribution of the load is in the form of two or more triangles (or trapaziums), and the foregoing formulae do not apply; in such case, however, it is accurate enough to assume that the total load on the beam is uniformly distributed.
### SlabsSpanning in Two Directions: Rectangular Panels

**Loads on Supporting Beams—Table 41.**

<table>
<thead>
<tr>
<th>Panels Supported Along Four Edges</th>
<th>Panels Unsupported Along One (or Two) Edges</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="Diagram 1" /> ( k &gt; 1 )</td>
<td><img src="image2.png" alt="Diagram 2" /> ( W_A = W_C = \frac{1}{2} \omega \varepsilon_x^2 )</td>
</tr>
<tr>
<td>( W_B = W_D = \frac{1}{2} (k-\frac{1}{2}) \omega \varepsilon_x^2 )</td>
<td>( W_A = W_C = 0 )</td>
</tr>
<tr>
<td>( W_B = W_D = \frac{1}{2} (k-\frac{1}{2}) \omega \varepsilon_x^2 )</td>
<td>( W_B = 0 )</td>
</tr>
<tr>
<td>( W_B = W_D = \frac{1}{2} (k-\frac{1}{2}) \omega \varepsilon_x^2 )</td>
<td>( W_B = 0 )</td>
</tr>
<tr>
<td>( W_B = W_D = \frac{1}{2} \omega \varepsilon_x^2 )</td>
<td>( W_B = 0 )</td>
</tr>
<tr>
<td>( W_B = W_D = \frac{1}{2} \omega \varepsilon_x^2 )</td>
<td>( W_B = 0 )</td>
</tr>
<tr>
<td>( W_B = W_D = \frac{1}{2} \omega \varepsilon_x^2 )</td>
<td>( W_B = 0 )</td>
</tr>
</tbody>
</table>

**K = 1**

| ![Diagram 3](image3.png) \( k < 1 \frac{1}{2} \) | ![Diagram 4](image4.png) \( W_A = W_C = \frac{3}{2} \omega \varepsilon_x^2 \) |
| \( W_B = \frac{3}{2} W_D (\text{MIN}) \) | \( W_A = W_B = 0 \) |
| \( W_C = \frac{3}{2} \omega \varepsilon_x^2 (\text{MAX}) \) | \( W_C = \omega \varepsilon_x^2 \) |
| \( W_D = \frac{3}{2} (k-\frac{1}{2}) \omega \varepsilon_x^2 (\text{MAX}) \) | \( W_D = \frac{1}{2} k^2 \omega \varepsilon_x^2 (\text{MAX}) \) |

**K < 1 \frac{1}{2} :**

| ![Diagram 5](image5.png) \( k = 1 \) | ![Diagram 6](image6.png) \( W_A = W_C = \frac{1}{2} \omega \varepsilon_x^2 \) |
| \( W_B = W_D = \frac{1}{2} (k-\frac{1}{2}) \omega \varepsilon_x^2 \) | \( W_A = W_C = \frac{1}{2} \omega \varepsilon_x^2 \) |
| \( W_A = W_C = \frac{1}{2} \omega \varepsilon_x^2 \) | \( W_A = W_C = \frac{1}{2} \omega \varepsilon_x^2 \) |

**K > 1 \frac{1}{2} :**

| ![Diagram 7](image7.png) \( k > 2 \) | ![Diagram 8](image8.png) \( W_A = W_C = \frac{1}{2} \omega \varepsilon_x^2 \) |
| \( W_B = W_D = \frac{1}{2} (k-\frac{1}{2}) \omega \varepsilon_x^2 \) | \( W_A = W_C = \frac{1}{2} \omega \varepsilon_x^2 \) |
| \( W_A = W_C = \frac{1}{2} \omega \varepsilon_x^2 \) | \( W_A = W_C = \frac{1}{2} \omega \varepsilon_x^2 \) |

**K > 2 :**

| ![Diagram 9](image9.png) \( k > 2 \) | ![Diagram 10](image10.png) \( W_A = W_C = \frac{1}{2} \omega \varepsilon_x^2 \) |
| \( W_B = W_D = \frac{1}{2} (k-\frac{1}{2}) \omega \varepsilon_x^2 \) | \( W_A = W_C = \frac{1}{2} \omega \varepsilon_x^2 \) |
| \( W_A = W_C = \frac{1}{2} \omega \varepsilon_x^2 \) | \( W_A = W_C = \frac{1}{2} \omega \varepsilon_x^2 \) |

**Condition of Supports:**

- **— — —** = No Support
- **—— —** = Freely Supported
- **—— —** = Continuity or Fixity

**Loads Marked (MIN.) Apply if Panel is Entirely Freely Supported Along Edge Indicated; If Partially Restrained Load Will Be Slightly Greater Than Given and Load Marked (MAX.) Opposite Edge Will Be Correspondingly Reduced.**
REINFORCED CONCRETE DESIGNER'S HANDBOOK

SLABS SPANNING IN TWO DIRECTIONS (continued).

Square Panels ($k = 1$).—The expression in Table 42 that the bending moment in both directions is $W(m_1 + 0.15m_2)$ applies only if load is over entire panel, or if $u = v$.

Other conditions.—$u$ and $v$ can be in either direction; $m_1$ is the bending moment coefficient in the direction of $u$; $m_2$ is the coefficient in the direction of $v$. Coefficient $m_1$ is based on $u$ and $v$ as selected; for coefficient $m_2$ reverse $u$ and $v$.

Example.—If $\frac{u}{L} = 0.8$ and $\frac{v}{L} = 0.2$, $m_1 = 0.072$; for $\frac{u}{L} = 0.2$ and $\frac{v}{L} = 0.8$, $m_2 = 0.103$.

Bending moments.

On span in direction of $u$: $W[0.072 + (0.15 \times 0.103)] = 0.087W$ ft.-lb. per ft.

On span in direction of $v$: $W[0.103 + (0.15 \times 0.072)] = 0.114W$ ft.-lb. per ft.

Examples of Panels Supporting Concentrated Loads.—The following examples illustrate the use of Tables 42 and 43 for slabs supporting a load which is concentrated uniformly over an area less than the entire area of the panel. Notes on these tables are given on page 32.

\[ \text{(a)} \]

The footpath of a bridge spans 6 ft. between a parapet girder and a main longitudinal girder, and is monolithic with both girders [diagram (a)]. The live load is either 100 lb. per sq. ft. uniformly distributed, or a load of 4 tons from a wheel the contact area of which is 12 in. by 3 in. (With the latter load the stresses may be increased by 50 per cent.; that is at ordinary working stresses the wheel load may be assumed to be about 6000 lb.) These loads comply with the recommendations of the Ministry of Transport.

(i) Assume a 5-in. slab; total uniformly-distributed load = 63 + 100 = 163 lb. per sq. ft. With continuity at both supports, bending moment at midspan and at each support is $1\times163 \times 6.3^3 \times 12 = 6400$ in.-lb. per ft. width.

(ii) Contact area of 12 in. by 3 in. at the wheel can be increased to 20 in. by 11 in. (Table 6); depth to the reinforcement is about 4 in.

The slab spans mainly in one direction; the curves in the lower left-hand corner of Table 42 apply. — $L_B = \frac{u}{77} = 0.143$; $L_B = \frac{v}{77} = 0.26$; $m_1 = 0.22$ and $m_2 = 0.12$.

Free transverse bending moment = $6000[0.22 + (0.15 \times 0.12)]12 + \left(\frac{4}{3} \times 63 \times 6.3^3 \times 12\right) = 17,150 + 3750 = 20,900$ in.-lb. per ft. width.

Allow for continuity (partial fixity) by reducing the free bending moment due to the dead load by one-third, and that due to the live load by 20 per cent.; the transverse bending moment is the same as for simply supported slabs.
SLABS SPANNING IN TWO DIRECTIONS: RECTANGULAR PANELS.—TABLE 42.
CONCENTRIC CONCENTRATED LOADS.

SLABS SIMPLY SUPPORTED ALONG ALL FOUR EDGES.
(WITH CORNER RESTRAINT)
W = TOTAL LOAD (LB) ON AREA
SHORT SPAN B.M. = \( W(m_1 + 0.15m_2) \)
LONG SPAN B.M. = \( W(0.15m_1 + m_2) \)

NOTE: B.M. FORMULAE ARE FOR POISSON'S RATIO = 0.15.
(B.M.s. IN FT²-LB. PER FOOT WIDTH)

\[
\frac{V}{L} = 1.0 \\
0.8 \\
0.6 \\
0.4 \\
0.2 \\
0.0 \\
\frac{L}{L} = 1.0 \\
\frac{U}{L} = 0.2 \quad 0.4 \quad 0.6 \quad 0.8 \quad 1.0 \\
\]

SQUARE PANEL
\( K = 1.0 \\
\frac{U}{L} = \frac{L_B - L}{L} \\
B.M. IN BOTH DIRECTIONS = \( W(m_1 + 0.15m_2) \)

SLABS SUPPORTED ON TWO OPPOSITE SIDES ONLY.
TRANSVERSE B.M. = \( W(m_1 + 0.15m_2) \)
LONGITUDINAL B.M. = \( W(0.15m_1 + m_2) \)

\( K = 0.0 \)
\( m_1 = FULL LINES \)
\( m_2 = DOTTED LINES. \)

Note.—See note regarding square panels on page 214.
Examples of Panels Supporting Concentrated Loads (continued from page 214).

moment = 16,100 in.-lb. [This is greater than the bending moment in case (i); therefore case (ii) controls the design.] With stresses of 1000 lb. and 18,000 lb. per sq. in., the effective depth required is about 3 in., but it is practical to make the footpath slab 5 in. thick, which gives \( d_1 = 4\) in. (3\( \frac{1}{4}\)-in. cover) and \( t_4 = \) about \( 3\frac{1}{4}\) in.

Transverse reinforcement: \( A_{st} = \frac{16,100}{18,000 \times 3.5} = 0.26\) sq. in., say, \( \frac{3}{4}\)-in. bars at 5-in. centres.

Longitudinal bending moment = 6000[(0.15 × 0.22) + (0.12)12] = 11,000 in.-lb.
Allowing 20 per cent. for continuity: net bending moment = about 8800 in.-lb.

Longitudinal reinforcement: \( A_{st} = \frac{8800}{18,000 \times 3.125} = 0.156\) sq. in., say, \( \frac{1}{4}\)-in. bars at 8-in. centres.

(iii) Another method is to calculate the transverse bending moment due to the concentrated load by estimating the width of slab assumed to carry the load from the formula

\[
\frac{1}{2}(L + B) + A = 0.67(77 + 11) + 20 = 79\text{ in.} = 6.55\text{ ft.}
\]

Load carried by 1 ft. of slab is \( \frac{6000}{6.55} = 920\) lb.

Free transverse bending moment = \( \frac{920}{4}(77 - \frac{11}{2}) = 16,400\) in.-lb.; reducing by 20 per cent. for continuity, the net bending moment (after adding 2400 in.-lb. approximately due to the dead load) = 15,500 in.-lb.; this is about the same as calculated from Table 43.

Longitudinal reinforcement in accordance with the requirement of the Ministry of Transport for distribution bars (see page 268): for a span of about 6 ft., 50 per cent. of the transverse reinforcement, that is, \( \frac{2}{3}\)-in. bars at 10-in. centres are sufficient; therefore \( \frac{1}{4}\)-in. bars at 8-in. centres are satisfactory.

(iv) Another method is an adaptation of B.S. Code method. From bottom left-hand corner of Table 43.

\[e = 20 + [2.4 \times \frac{1}{4} \times 77(1 - \frac{1}{4})] \text{ or } = 20 + (0.6 \times 77) = 66\text{ in.} = 5.5\text{ ft.}\]

Free bending moment (symmetrical load) = \( \frac{6000}{4 \times 5.5}(77 - \frac{11}{2}) = 19,200\) in.-lb. per ft.; reducing by 20 per cent. for continuity and adding 2400 in.-lb. for the dead load, maximum positive bending moment = 17,760 in.-lb., which exceeds slightly the bending moments as calculated by the preceding methods.

(b) A panel of the deck slab of a bridge is 12 ft. 6 in. long by 10 ft. wide and is supported on all four sides and is fully continuous over all supports. Calculate the maximum bending moments due to a load of 20 tons symmetrically placed at the centre of the panel, the contact area being 3 in. by 24 in.

Assume a slab 8 in. thick (\( d_1 = 7\) in.) with 2 in. of tarmacadam. Loaded area, allowing for dispersion down to reinforcement, = 21 in. by 42 in. [diagram (b) on page facing Table 42].

\[k = \frac{L_L}{L_B} = \frac{12.5}{10} = 1.25, \quad \frac{21}{10 \times 12} = 0.175; \quad \frac{L_L}{12.5 \times 12} = 0.28. \text{ From Table } 42, m_1 = 0.18, m_2 = 0.13. \text{ Allow a reduction of 20 per cent. for continuity. For the uniformly-distributed dead load of about 120 lb. per sq. ft. (from Table 38) for } k = 1.25, K_B' = 0.45 \text{ and } K_L' = 0.17.

Bending moments (in.-lb. per ft. width).

Midspan of shorter span.—

<table>
<thead>
<tr>
<th>Load condition</th>
<th>Live load: (44,800) [0.18 + (0.15 × 0.13)]</th>
<th>12 × 0.8 = 86,000</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Dead load: 0.45 \times 0.100 \times 120 \times 10^3 \times 12</td>
<td>6,480</td>
</tr>
<tr>
<td>Total</td>
<td>= 92,480</td>
<td></td>
</tr>
</tbody>
</table>

Supports of shorter span.—Live load: As at midspan = 86,000

<table>
<thead>
<tr>
<th>Dead load:</th>
<th>6480 × 0.125 × 0.100 = 8,100</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total</td>
<td>= 94,100</td>
</tr>
</tbody>
</table>

(Continued on page 218.)
SLABS SPANNING IN TWO DIRECTIONS: RECTANGULAR PANELS.—TABLE 48.
ECCENTRIC CONCENTRATED LOADS.

**I**

FIND $m_1$ & $m_2$ FOR:
- \[ \frac{W}{L_B} \text{ AND } \frac{V}{L} \]
- DESIGN B.M.S.:
  - \[ M_B = W(m_1 + 0.15m_2) \]
  - \[ M_L = W(0.15m_1 + m_2) \]

**II**

FIND $m_1$ and $m_2$ FOR:
- \[ (i) \frac{W}{L_B} \text{ AND } \frac{V}{L} \]
- \[ (ii) \frac{W}{L_B} \text{ AND } \frac{V}{L} \]
- \[ (iii) DEDUCT (i) FROM (ii). \]

**III**

FIND $m_1$ & $m_2$ FOR:
- \[ (i) \frac{W}{L_B} \text{ AND } \frac{V}{L} \]
- \[ (ii) \frac{W}{L_B} \text{ AND } \frac{V}{L} \]
- \[ (iii) DEDUCT (i) FROM (ii). \]

**IV**

FIND $m_1$ & $m_2$ FOR:
- \[ (i) \frac{W}{L_B} \text{ AND } \frac{V}{L} \]
- \[ (ii) \frac{W}{L_B} \text{ AND } \frac{V}{L} \]
- \[ (iii) DEDUCT (i) FROM (ii). \]

**V**

FIND $m_1$ & $m_2$ FOR:
- \[ (i) \frac{W}{L_B} \text{ AND } \frac{V}{L} \]
- \[ (ii) \frac{W}{L_B} \text{ AND } \frac{V}{L} \]
- \[ (iii) DEDUCT (i) FROM (ii). \]

**VI**

FIND $m_1$ & $m_2$ FOR:
- \[ (i) \frac{W}{L_B} \text{ AND } \frac{V}{L} \]
- \[ (ii) \frac{W}{L_B} \text{ AND } \frac{V}{L} \]
- \[ (iii) DEDUCT (i) FROM (ii). \]

**CONTINUOUS OVER SUPPORT = \[ \frac{W}{L_B} \] ; NON-CONTINUOUS BUT MONOLITHIC WITH SUPPORT = \[ \frac{W}{L_L} \] \]

**CALCULATE BENDING MOMENTS IN EACH DIRECTION AS IF FREELY SUPPORTED BUT WITH \[ k = \frac{W}{L_L} \] (OR \[ k = \frac{L_B}{L_L} \] IF \[ \frac{1}{2} \]) MULTIPLY BY THE FOLLOWING COEFFICIENTS TO GIVE THE DESIGN BENDING MOMENTS AT THE SPECIFIED SECTIONS.

**MIDSPAN:** INTERIOR SPAN = 0.70; END SPAN = 0.85

**SUPPORTS:** END SUPPORT = 0.25; PENULTIMATE = 0.95

**INTERIOR (EXCEPT PENULTIMATE) = 0.90.**

**ECCENTRIC LOAD (\( \frac{W}{L_B} \))**

**MAX. B.M.** = \[ \frac{W}{L} \left( \frac{L_B}{L} \right) \] PER FT. WIDTH.

**SLABS SPANNING IN ONE DIRECTION (BS.COD).**

**SYM. LOAD (\( \frac{W}{L} \))**

- **MAX. AT MIDSPAN = \[ \frac{W}{4L} \left( L_B - \frac{1}{2} L \right) \] PER FT. WIDTH.
Examples of Panels Supporting Concentrated Loads (continued from page 216).

Midspan of longer span.—

Live load: \(44,800[(0.15 \times 0.18) + (0.13) \times 12 \times 0.8 = 67,600\)

Dead load: \(0.17 \times 0.100 \times 120 \times 12.5^2 \times 12 = 3,830\)

Total = 71,430

Supports of the longer span.—Live load: As at midspan = 67,600

Dead load: \(3830 \times \frac{0.125}{0.100} = 4,790\)

Total = 72,390

(c) Find the bending moments due to the live load in the position shown in diagram (c) (page 214) if the panel in (b) is not continuous over two of its adjacent sides, but is monolithic with the supporting beams on those two sides; continuity exists over the supports on the remaining two sides [Case (f) in Table 43; \(f_1 = 1.0\)]. Adopt the procedure for Case IV, Table 43.

(i) \(u = 2(21 + 39) = 120\) in.; \(v = 42\) in.; \(\frac{u}{L_B} = \frac{120}{120} = 1; \frac{v}{L_L} = \frac{42}{150} = 0.28\); from Table 42, with \(h = 1.25, m_1 = 0.073\) and \(m_2 = 0.063\); these are the factors for a load of \(\frac{2W}{u_1}(u_1 + x)\) on an area of \(2(u_1 + x)\) by \(v\). \(u_1 + x = 21 + 39 = 60\) in.;

\(m_1(u_1 + x) = 0.073 \times 60 = 4.38\);

\(m_2(u_1 + x) = 0.063 \times 60 = 3.78\);

these are the corresponding basic bending moments \(\frac{2W}{u_1}\).

(ii) \(u = 2 \times 39 = 78\) in.; \(v = 42\) in.; \(\frac{u}{L_B} = \frac{78}{120} = 0.65; \frac{v}{L_L} = \frac{42}{150} = 0.28\); from Table 42, \(m_1 = 0.10\) and \(m_2 = 0.088\); these are the factors for a load of \(\frac{2Wx}{u_1}\) on an area \(2x\) by \(v\) symmetrically disposed at the centre of the panel.

\(m_1x = 0.10 \times 39 = 3.9\); \(m_2x = 0.088 \times 39 = 3.43\);

these are the corresponding basic bending moments \(\frac{2W}{u_1}\).

(iii) \(m_1 = 4.38 - 3.9 = 0.48\); \(m_2 = 3.78 - 3.43 = 0.35\); these are the basic bending moments for a total load of \(\frac{2W}{u_1}(u_1 + x) - \frac{2Wx}{u_1} = 2W\) on two symmetrically disposed areas of \(u_1\) by \(v\) (as in Case II); the case under consideration, Case IV, is one half of Case II and thus the immediately preceding bending moments \(m_1\) and \(m_2\) must be multiplied by \(\frac{1}{2} \times \frac{12}{10} = \frac{1}{2}\) to give the basic bending moments for the specified load. \(\frac{W}{u_1} = \frac{44,800}{21} = 2140\) lb. per in.

Free bending moment across short span: \(M_B = \frac{W}{u_1}(m_1 + 0.15m_2)\)

\(= 2140[0.48 + (0.15 \times 0.35)]12 = 13,650\) in.-lb. per ft.

Free bending moment across long span: \(M_L = \frac{W}{u_1}(0.15m_1 + m_2)\)

\(= 2140[(0.15 \times 0.48) + 0.35]12 = 10,800\) in.-lb. per ft.

The maximum bending moments for the live load, allowing for continuity by using the factors given in Table 43, are:

Shorter span.—Midspan: \(+ 0.85 \times 13,650 = + 11,600\) in.-lb.

Inner support: \(- 0.95 \times 13,650 = - 12,950\) in.-lb.

Outer support: \(- 0.25 \times 13,650 = - 3410\) in.-lb.

Longer span.—Midspan: \(+ 0.85 \times 10,800 = + 9180\) in.-lb.

Inner support: \(- 0.95 \times 10,800 = - 10,250\) in.-lb.

Outer support: \(- 0.25 \times 10,800 = - 2700\) in.-lb.

The bending moments due to the uniformly distributed dead load must be added.
### SLABS SPANNING IN TWO DIRECTIONS: NON-RECTANGULAR PANELS — TABLE 44

#### Trapezium

- Calculate B.M.s. as for rectangular panel with:
  \[ k = \frac{e_y}{e_x} \]
  
  IF \( e_y \) IS SMALL COMPARED WITH \( e_x \), APPLY RULES FOR TRIANGULAR PANEL.

#### Isosceles Triangle

- **B.M. (in two directions at centre of circle)**:
  \[ B.M. = \frac{wD_0^2}{16} \]
  
  **Continuous along all sides.**

- **B.M. (in two directions at centre of circle)**:
  \[ B.M. = \frac{wD_0^2}{32} \]
  
  **B.M. (at sides)**

- \( w \): Intensity of uniformly-distributed load
  (or intensity of pressure at centre of circle if pressure varies uniformly.)

#### Regular Polygon

- **Five or more sides.**
  \[ D = \text{Diameter of inscribed circle} = \text{Distance across flats.} \]
  \[ D_0 = \text{Diameter of circumscribed circle} = \text{Distance across corners.} \]

- **For hexagon**
  \[ D_e = \frac{1}{2} (D + D_0) = 1.08D \]
  
  **For octagon**
  \[ D_e = \frac{1}{2} (D + D_0) = 1.04D \]

- **Calculate B.M.s. as for circle of diameter \( D_e \).**

#### Circle (Diameter = \( D \))

<table>
<thead>
<tr>
<th>Condition at Circumference</th>
<th>Positive Bending Moment</th>
<th>Negative Bending Moment</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Average Across Diameter</strong></td>
<td>[ \frac{wD^2}{24} ] per foot of diameter</td>
<td>[ \frac{wD^2}{16} ] per foot at centre</td>
</tr>
<tr>
<td><strong>Alternative Max. at Centre</strong></td>
<td>[ \frac{wD^2}{20} ]</td>
<td>[ \frac{wD^2}{32} ]</td>
</tr>
<tr>
<td><strong>Maximum, ( M_M ), at Centre</strong></td>
<td>[ \frac{wD^2}{32} ]</td>
<td>[ \frac{wD^2}{32} ]</td>
</tr>
</tbody>
</table>

**Notes:**
- Reinforcement to resist positive B.M. to be provided in two directions mutually at right-angles.
- Reinforcement to resist negative B.M. around edge to be radial (or equivalent).
- If max. pos. B.M. is designed for reinf't (or thickness of slab) can be reduced towards edge.

---

For notes on this table, see page 34.
FLAT SLABS.

The notes on flat slabs in the following and the data in Table 45 are in accordance with the recommendations for the empirical method of analysis described in B.S. Code No. 114 (1957).

**Thickness of Slab.**—The slab should be not less than 5 in. thick and the ratio of mean span \( L \) to the thickness should be not greater than the following.

- Internal panels. Without drops . . . 36. With drops . . . 40.
- Edge panels. . . . . 32. . . . . 36.

**Adjustment of Bending Moments.**—If the width of the drop-panel is less than half the panel width, the total bending moment on the middle strip should be increased in proportion to the increased width of the middle strip, and the additional bending moment can be deducted from the total for the corresponding column strips. This adjustment ensures that the total bending moments along the principal axes are unaltered. In edge panels the bending moments should be increased to the values given in Table 45.

**Columns.**—The diameter of the column head \( D \) (Table 45) should not exceed 0.25\( L \). The angle between the slope of the head and the horizontal should be not less than 45 deg. The diameter \( D \) of the head is the diameter 1\( \frac{1}{8} \) in. below the underside of the slab; if the head is not circular, \( D \) is the diameter of the largest circle that can be drawn within the head. Exterior columns should be provided with as much as possible of the head prescribed for interior columns.

Columns should be designed for bending moments equal to 50 per cent. in the case of interior columns, and 90 per cent. in the case of exterior columns, of the negative bending moment required in the column strip. The bending moment should be divided between the upper and lower columns in proportion to their stiffnesses. If the floor panels are cantilevered beyond the outer row of columns, the bending moment on the exterior column can be reduced by the cantilever bending moment due to the dead load.
**Table 45**

<table>
<thead>
<tr>
<th>Total B.M. On Strip of Width $\frac{L_2}{2}$</th>
<th>[ f^2 L_2 \left( L_1 - \frac{2}{3} d \right)^2 ]</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Coefficient ( f )</strong></td>
<td><strong>Interior Panels</strong></td>
</tr>
<tr>
<td></td>
<td>Column Strip</td>
</tr>
<tr>
<td>Without Drops</td>
<td>0.046</td>
</tr>
<tr>
<td>With Drops</td>
<td>0.050</td>
</tr>
</tbody>
</table>

**Note:**
- Other variants are based on these two limiting cases.

**Critical Sections for Shear**

(a) With drop and column head.  
(b) Without drop or column head.

**Bending Moment Diagrams**

**Design with Drop-Panels and Column-Heads**

- **Design without Drop-Panels or Column-Heads**

---

_L. M. Degen_
Bending Moments.—The principles of the slope-deflection method of analysing a restrained member are that the difference in slope between any two points in the length of the member is equal to the area of the \( M \) diagram between these two points; and that the distance of any point on the member from a line drawn tangentially to the elastic curve at any other point, the distance being measured normal to the initial position of the member, is equal to the moment (taken about the first point) of the \( M \) diagram between these two points. In the foregoing \( M \) represents the bending moment, \( E \) the modulus of elasticity of the material, and \( I \) the moment of inertia of the member. The bending moments at the ends of a member subject to the deformation and restraints shown in the diagram at the top of Table 46 are given by the corresponding formulae which are derived from a combination of the basic principles and which are given in the general form and in the special form for members on non-elastic supports.

The symbol \( K \) is the stiffness factor, \( \frac{I}{L} \); the stiffness is proportional to \( EK \), a term in the formula, but as \( E \) is assumed to be constant the term which varies in each member is \( K \). The terms \( P \) and \( Q \) relate to the load on the member. When there is no external load the factors \( P \) and \( Q \) are zero, and when the load is symmetrically disposed on the member \( P = Q = \frac{A}{L} \). Values of \( P \), \( Q \) and \( \frac{A}{L} \) are given in Table 18.

The conventional signs for slope-deflection analyses are: an external restraint moment acting clockwise is positive; a slope is positive if the rotation of the tangent to the elastic line is clockwise; a deflection in the same direction as a positive slope is positive.

Examples of Use of Tables 46 and 49.

(a) Derive the formulae for the bending moments in a column CAD into which is framed a beam AB. The beam is hinged at B and the column is fixed at C and D (see diagram in Table 46). The beam only is loaded. Assume there is no displacement of the joint A.

Selecting the general formulae from Table 46,

\[
M_{AB} = 3EK_{AB}\theta_A - \left( P_{AB} + \frac{Q_{BA}}{2} \right), \quad M_{AC} = 4EK_{AC}\theta_A, \quad M_{AD} = 4EK_{AD}\theta_A.
\]

Therefore \( M_{AB} + M_{AC} + M_{AD} = E\theta_A(3K_{AB} + 4K_{AC} + 4K_{AD}) - \left( P_{AB} + \frac{Q_{BA}}{2} \right) = 0 \).

Thus \( E\theta_A = \frac{P_{AB} + 0.5Q_{BA}}{3K_{AB} + 4K_{AC} + 4K_{AD}} \), and \( M_{AC} = \frac{4K_{AC}(P_{AB} + 0.5Q_{BA})}{3K_{AB} + 4K_{AC} + 4K_{AD}} \).

\( M_{AD} = K_{AD}M_{AC}; \quad M_{AB} = -(M_{AC} + M_{AD}); \quad M_{CA} = 2EK_{AC}\theta_A = \frac{M_{AC}}{2}; \quad M_{DA} = 0.5M_{AD} \).

For symmetrical load, \( P_{AB} + \frac{Q_{BA}}{2} = 1.5\frac{L_{AB}}{L_{AB}}; \quad M_{AC} = \frac{6K_{AC}}{3K_{AB} + 4K_{AC} + 4K_{AD}} \).

(See also numerical example in Table 47.)

(b) Find the bending moments in a rectangular portal frame ABCD subject to a horizontal load \( W = 5000 \) lb. at B (that is, \( a = 1 \), \( b = 0 \), and \( N = 0 \), if \( H = 12 \) ft., \( L = 10 \) ft., and \( R = 1 \), if the frame is fixed at A and D.

From Table 49,

\( H_D = 0.5 \times 5000 = 2500 \) lb.; therefore \( H_A = 5000 - 2500 = 2500 \) lb.

Also \( K_4 = 7 \), and \( K_4 = 4 \); \( C_1 = 0 \); \( M_A = M_D = C_2 = \frac{1}{2 \times 7} = \frac{1}{5000 \times 12 \times 4} = 17,150 \) ft.-lb. (anti-clockwise restraint moment); \( M_B = M_C = \frac{12 \times 2500}{17,150} = 12,850 \) in.-lb. acting at the head of the column of the frame in opposite sense to \( M_B \).
**FRAMED STRUCTURES: BASIC DATA.—TABLE 46.**

### BASIC FORMULAE
(SLOPE DEFORMATION)

![Diagram of B.M. Diagrams and Load Paths](image)

\[
\begin{align*}
M_{AB} &= 2EK_{AB} (2\theta_A + \theta_B - \frac{3a}{L_{AB}}) - P \\
M_{BA} &= 2EK_{AB} (2\theta_B + \theta_A - \frac{3a}{L_{AB}}) + Q
\end{align*}
\]

B.M. DUE TO RESTRAINING MOMENTS

FREE B.M. DIAGRAM FOR LOAD.

**COMBINED B.M. DIAGRAM.**

**Note:** For values of \(P_{AB}\) and \(Q_{AB}\) see Table 18.

---

**THE SUFFIXES**

**AB** relates to joint or supports of any member, \(A\) and \(B\), similarly for other \(AB\) members BC, AC, etc.

- \(L_{AB}\) = LENGTH of MEMBER \(AB\)
- \(I_{AB}\) = MOMENT of INERTIA of CONCRETE UNITS of \(AB\)
- \(K_{AB}\) = STIFFNESS FACTOR of \(AB\)
- \(E\) = YOUNG’S MODULUS FOR CONCRETE
- \(\theta_A\) = SLOPE of DEFORMED MEMBER \(AB\) at \(A\), with no load on \(AB\), \(P_{AB}\) = \(Q_{BA}\) = 0

**SLOPE-DEFORMATION GENERAL FORMULAE NON-ELASTIC SUPPORTS**

<table>
<thead>
<tr>
<th>(M_{AB})</th>
<th>(M_{BA})</th>
<th>(P_{AB})</th>
<th>(Q_{BA})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(2EK_{AB}(\theta_A + \theta_B) - P_{AB})</td>
<td>(4EK_{AB}\theta_A + Q_{BA})</td>
<td>(-P_{AB})</td>
<td>(-Q_{BA})</td>
</tr>
<tr>
<td>(2EK_{AB}\theta_A - P_{AB})</td>
<td>(4EK_{AB}\theta_B + Q_{BA})</td>
<td>(-P_{AB})</td>
<td>(-Q_{BA})</td>
</tr>
</tbody>
</table>

**SLOPE-DEFORMATION ELASTIC SUPPORTS**

<table>
<thead>
<tr>
<th>(M_{AD})</th>
<th>(M_{AB})</th>
<th>(P_{AB})</th>
<th>(Q_{BA})</th>
</tr>
</thead>
</table>
| \(\frac{2K_{AC} + 2K_{AD} - K_{AB}}{K_{AC} + K_{AD} + K_{AB}}\) | \(-\left(\frac{M_{AC} + M_{AD}}{M_{AC}}\right)\) | \(1\) FIXED AT C: \(M_{CA} = 0.5M_{AC}\), etc. | AT HINGES: \(B.M. = 0\)

**B.M. IN EXTERNAL COLUMNS OF BUILDING FRAMES (B.S. CODE)**

\[M_{AC} = \frac{K_{AC}}{K_{AC} + K_{AD} + nK_{AB}} P_{AB}\]

\[M_{AD} = \frac{K_{AD}}{K_{AC} + K_{AD} + nK_{AB}} P_{AB}\]

**INTERIOR COLUMNS**

\[M_{AC} = \frac{K_{AC}}{K_{AC} + K_{AD} + K_{AB}} M_{ES}\]

\[M_{AD} = \frac{K_{AD}}{K_{AC} + K_{AD} + K_{AB}} M_{ES}\]

---

**NOTE:** For values of \(P_{AB}\) and \(Q_{AB}\) see Table 18.
### Table 47.—Framed Structures: Moment-Distribution Method.

**Definition:**
- Consider each member as fixed ended; end moments \((F = CL)\) due to external loads on given member can be derived from the combination of signs for B.M./- restraining moment clockwise = positive.
- If symmetrical load, \(F_{BA} = -F_{AB}\), area of force B.M. diag. = \(A\) or span \(L_{AB}\) if no external load, \(F_{BA} = F_{AB} = 0\).
- Where two or more members meet, sum of B.M. at joint = zero (for equilibrium).
- \(\Sigma F = +F_{BA} - F_{AD} - F_{AC} \text{ etc.} = 0\) since \(\Sigma F\) is unlikely to equal zero, a balancing-moment \(= -\Sigma F\) must be introduced at \(A\).
- The balancing moment is distributed between the members meeting at \(A\) in proportion to their relative stiffnesses \(K = \frac{E}{I}\) by multiplying \(-\Sigma F\) by the distribution-factor \((D)\) for each member:
  \[D_{AC} = \frac{K_{AC}}{K}; \quad D_{AD} = \frac{K_{AD}}{K}; \quad \text{etc.} \quad \Sigma D = 1\]
  - Distributed moment for \(AB = D_{AB} = -\Sigma F\).
- Effect of applying distributed moment at one end of member is to produce a moment of half the magnitude but the same sign at the remote end (framed carry-over). Thus distributed moment \(-\Sigma F D_{AB}\) applied at \(A\) on \(AB\) produces moment \(-O\Sigma F\).
- The carry-over moments at \(A\) from the distributed moments at \(B, C, D, \text{etc.}\) will produce further unbalanced moments at \(A\), which must be distributed and the carry-over process repeated. These successive operations are repeated until the unbalanced moment is negligible. (See example below.)

#### Example
For symmetrical load on \(AB\), value of \(\frac{A}{L} = 24,000\) ft.lbf.
- Ratio of stiffness factors: \(K_{B} = K_{A} = K_{C} = \frac{2}{3} K; \quad K_{D} = \frac{1}{3} K\)
- Formula derived from slope-deflection:
  \[M_{AC} = \frac{4K_{C}L}{4K_{C} + 4K_{D} + 3K} \times 24,000 = 13,714\] ft.lbf.
  \[M_{AD} = \frac{K_{D}L}{K_{C} \times M_{AC}} = \frac{1}{2} (13,714) = 6,857\] ft.lbf.
  \[M_{DA} = \frac{1}{2} M_{AD} = 3,428\] ft.lbf.
  \[M_{AB} = \frac{-M_{AC} - M_{AD} = (13,714 + 6,857)}{2} = -20,571\] ft.lbf.
  \[M_{CA} = \frac{1}{2} M_{AC} = 6,857\] ft.lbf.
- By moment-distribution:
  \[D_{AB} = \frac{K}{K + \frac{3K}{2} + \frac{1K}{2}} = \frac{1}{3}; \quad D_{AC} = \frac{1}{3}; \quad D_{AD} = \frac{1}{3} = \frac{1}{3}(L + \frac{1}{3})\]
- Distribution factors: \(D\)

<table>
<thead>
<tr>
<th>Distribution Factor ((D))</th>
<th>(M_{CA})</th>
<th>(M_{AC})</th>
<th>(M_{AB})</th>
<th>(M_{AD})</th>
<th>(M_{DA})</th>
<th>(M_{BA})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fixed end B.M. ((F))</td>
<td>0</td>
<td>0</td>
<td>-24,000</td>
<td>0</td>
<td>0</td>
<td>+24,000</td>
</tr>
<tr>
<td>1st. Distribution</td>
<td>0</td>
<td>+8,000</td>
<td>+12,000</td>
<td>0</td>
<td>-2,000</td>
<td>+6,000</td>
</tr>
<tr>
<td>1st Carry-over</td>
<td>+4,000</td>
<td>0</td>
<td>-12,000</td>
<td>0</td>
<td>+2,000</td>
<td>0</td>
</tr>
<tr>
<td>2nd. Distribution</td>
<td>0</td>
<td>+4,000</td>
<td>+6,000</td>
<td>+2,000</td>
<td>0</td>
<td>-6,000</td>
</tr>
<tr>
<td>2nd Carry-over</td>
<td>+2,000</td>
<td>0</td>
<td>-3,000</td>
<td>0</td>
<td>+1,000</td>
<td>+3,000</td>
</tr>
<tr>
<td>3rd. Distribution</td>
<td>0</td>
<td>+1,000</td>
<td>+1,500</td>
<td>+500</td>
<td>0</td>
<td>-3,000</td>
</tr>
<tr>
<td>3rd Carry-over</td>
<td>+500</td>
<td>0</td>
<td>-1,500</td>
<td>0</td>
<td>+250</td>
<td>+750</td>
</tr>
<tr>
<td>4th. Distribution</td>
<td>0</td>
<td>+500</td>
<td>+750</td>
<td>+250</td>
<td>0</td>
<td>-750</td>
</tr>
<tr>
<td>Summations</td>
<td>+6,500</td>
<td>+13,500</td>
<td>-20,250</td>
<td>+6,750</td>
<td>+3,250</td>
<td>0</td>
</tr>
</tbody>
</table>

Note: \(M_{AB}\) after 1st distribution = -12,000 ft.lbf.

- 2nd = -18,000
- 3rd = -19,500
- 4th = -20,250
- 5th \(\frac{1}{2}\) shown = -20,457

Note: \(*\) \(F = CL\) can be derived from factors in Table 18.

See Table 46 for formula for slope-deflection method.

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### TABLE 49.—FRAMED STRUCTURES: HINGED AND FIXED PORTALS.
GENERAL CASES.

<table>
<thead>
<tr>
<th>FRAME FORM</th>
<th>LOADING</th>
<th>ENDS</th>
<th>REACTIONS AND BENDING MOMENTS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

---

**FORMULA**

Apply to any type of load (except cases [ ] and [ ] ) and give numerical values of reactions and moments; see loading diagrams for direction of action.

**CENTRE OF GRAVITY**

Of total load \( w \), is at \( ol \). Or \( oh \).

**CENTROID**

Of free B.M. diagram is at \( zl \), or \( Z \). Or \( zl \). From \( l \) or \( Z \) from L.H.S. (vertical loads) or lower support (horizontal loads).

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**Note.**—For values of \( N = \frac{A}{L} \) etc. see Table 18.

For example of use of this table see page facing Table 46.

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**FRAMED STRUCTURES: HINGED AND FIXED PORTALS—TABLE 50.**  
**SPECIAL CASES.**

<table>
<thead>
<tr>
<th>FORM</th>
<th>LOADING</th>
<th>MINGED AT A AND D (ORE) MA = MD = 0 or MA = ME = 0</th>
<th>FIXED AT A AND D (ORE) HA = HD = WHL 4HK1</th>
<th>VA = VD = 1/2 WH</th>
<th>MB = MC = WHL/8K1</th>
<th>MA = MD = WHL/12K1</th>
</tr>
</thead>
<tbody>
<tr>
<td>W = TOTAL LOAD</td>
<td></td>
<td>HA = HD = WHL 4HK1</td>
<td></td>
<td>VD = VD = 1/2 WH</td>
<td>MB = MB = WHL/8K1</td>
<td>MA = MD = WHL/12K1</td>
</tr>
<tr>
<td>IAB = I DC</td>
<td></td>
<td>VA = VA = 1/2 WH</td>
<td></td>
<td>HA = HD = 3WHL 8HK3</td>
<td>MB = MB = 3WHL 8HK3</td>
<td>MA = MD = WHL 8HK1</td>
</tr>
<tr>
<td>R = IBC H / IAB L</td>
<td></td>
<td>HA = HD = 3WHL 8HK3</td>
<td></td>
<td>VD = VD = 1/2 WH</td>
<td>MB = MB = 3WHL 8HK3</td>
<td>MA = MD = WHL 8HK1</td>
</tr>
<tr>
<td>K1 = R + 2</td>
<td></td>
<td>VA = VA = 1/2 WH</td>
<td></td>
<td>HA = HD = 1/2 WH</td>
<td>MB = MB = 1/2 WH</td>
<td></td>
</tr>
<tr>
<td>K2 = 6R + 1</td>
<td></td>
<td>VD (V - VA) = WH</td>
<td></td>
<td>HA = HD = 1/2 WH</td>
<td>MB = MB = 1/2 WH</td>
<td></td>
</tr>
<tr>
<td>K3 = 2R + 3</td>
<td></td>
<td>VD (V - VA) = WH</td>
<td></td>
<td>HA = HD = 1/2 WH</td>
<td>MB = MB = 1/2 WH</td>
<td></td>
</tr>
<tr>
<td>K4 = 3R + 1</td>
<td></td>
<td>IAB = I ED</td>
<td></td>
<td>HA = HD = 1/2 WH</td>
<td>MB = MB = 1/2 WH</td>
<td></td>
</tr>
<tr>
<td>IAB = I ED</td>
<td></td>
<td>IBC = I CD</td>
<td></td>
<td>HA = HD = 1/2 WH</td>
<td>MB = MB = 1/2 WH</td>
<td></td>
</tr>
<tr>
<td>BC = CD = L = (1/2) Y2 + (f)2</td>
<td></td>
<td>HA = HD = 1/2 WH</td>
<td></td>
<td>HA = HD = 1/2 WH</td>
<td>MB = MB = 1/2 WH</td>
<td></td>
</tr>
<tr>
<td>R = IBC H / IAB L</td>
<td></td>
<td>HA = HD = 1/2 WH</td>
<td></td>
<td>HA = HD = 1/2 WH</td>
<td>MB = MB = 1/2 WH</td>
<td></td>
</tr>
</tbody>
</table>

**NOTE:** FORMule give numerical values of reactions and moments; see diagrams for direction of action.

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NOTES ON FRAMED STRUCTURES.

Ratio of Moments of Inertia of Members of Frames.—In the formulae for the bending moments on the members of framed structures, only the relative stiffness factors of adjacent members are required, that is the relative moments of inertia are required. The following examples show the accuracy of various methods of comparison. \( m = 15 \).

![Diagrams of column bending](image)

(i) Find the ratio of the moments of inertia of the column in (b) and the beam in (a).

(ii) Stressed areas only.—Column: Concrete, \( 0.083 \times 15^4 \) in.\(^4 \), Beam: Concrete, \( 0.33 \times 8 \times 8^2 \) in.\(^4 \), Tension steel, \( 1.33 \times 6 \times 5^2 \) in.\(^4 \), Compression steel, \( 1.4 \times 3 \times 4 \times 5^2 \) in.\(^4 \).

\[
\begin{align*}
I_C &= 5540 \\
I_B &= 4700
\end{align*}
\]

(Compare with 1.08.)

(2) Find the ratio of the moments of inertia of the beam in (a) and the column in (c).

(i) Complete concrete areas only.—From Table 64 the moment of inertia of an octagonal section is 0.055D\(^4\). Therefore \( I_C = 0.055 \times 18^4 \) in.\(^4 \).\]

(ii) Stressed areas only: \( I_C \) by calculation similar to (1) \( = 6780 \) in.\(^4 \).

\[
\begin{align*}
I_C &= 6780 \\
I_B &= 4700
\end{align*}
\]

(Compare with 1.49.)

Columns in Buildings: Approximate Method.—For preliminary calculations for the design of columns in a multiple-story building frame, it is often convenient to design for resistance to direct load only and to allow for the effects of bending either (i) by decreasing the maximum stresses, or (ii) by increasing the load to a value which is equivalent to that producing the same stresses as would occur if the actual load and bending moment were combined. The factors in the accompanying table apply to method (ii) and are applicable to most normal building frames. The diagrams indicate arrangements of beams supported by the column.

**Example of Use of Table 51.—**Find the approximate bending moments and forces in the columns of a gantry; the vertical load on each pair of columns is 100 tons, and the horizontal wind load is 5 tons. The height of the columns is 48 ft. braced at top and bottom and with intermediate braces at 12-ft. centres; the distance apart of the columns at the base is 20 ft.; thus \( P = 5 \) tons; \( W = \frac{1}{4} \times 100 = 50 \) tons. \( H = 48.0; h = 12.0; D = 20.0 \).

Bending moment on column at junction with brace, \( \frac{5 \times 120}{20} = 15 \) ft.-tons.

Bending moment in brace at junction with columns, \( 2 \times 15 = 30 \) ft.-tons.

The shearing force on the brace (constant throughout length) = \( \frac{4 \times 48}{20} = 3 \) tons.

The vertical load on the leeward column is \( 50 + \frac{5 \times 48}{20} = 62 \) tons plus the dead load of the column and half of the braces. The vertical load on the windward column is \( 50 - \frac{5 \times 48}{20} = 38 \) tons plus the dead load of the column and half of the braces.
FRAMED STRUCTURES: EFFECTS OF LATERAL LOADS.—TABLE 51.

(A).—OPEN BRACED TOWER

(b).—SUBSTRUCTURE OF SILO (OR SIMILAR STRUCTURE)

ASSUMPTION.—EXTERIOR COLUMN SUBJECTED TO HALF SHEARING FORCE ON INTERIOR COLUMN.

EXAMPLE WITH $N_i = 4$:

$$M_1 = \frac{P_h_1}{12}$$
$$M_2 = \left(\frac{P_1 + \frac{1}{2}P_2}{2}\right) \frac{h_2}{2}$$
$$M_3 = \left(P_1 + P_2 + \frac{1}{2}M_{B_2}\right) \frac{h_3}{2}$$
$$M_{B_3} = \frac{1}{2}M_1$$
$$M_{B_2} = \frac{1}{2} (M_1 + M_2)$$
$$M_{B_3} = \frac{1}{2} (M_2 + M_3)$$

Note.—Explanations of the effects of wind forces (or other lateral loads) on the types of structures illustrated in Table 51 are given on page 40.
Two-hinge Arch.—Referring to the diagram in Table 52, there is no bending moment at the springings of a two-hinge arch. The vertical component of the thrust at the springings is the same as for a freely-supported beam; the horizontal component is \( H \) as given by the formula in Table 52, in which \( M_x \) is the bending moment on a section at a distance \( x \) from the crown considering the arch as a freely-supported beam, that is \( M_x \) as is given by the corresponding expression in Table 52. The summations \( \Sigma M_x S_I \) and \( \Sigma y^2 S_I \) are taken over the whole length of the arch. The formula for \( H \) allows for the elastic contraction of the arch. \( A \) is the average equivalent area of the arch rib or slab; \( s \) is the length of a short segment of the axis of the arch or slab; the ordinates of \( s \) are \( x \) and \( y \) as shown in Table 52, and if \( I \) is the moment of inertia of the arch at \( x, s_I = \frac{S}{I} \). The bending moment at any section is \( M_x = M - Hy \).

The procedure is to divide the axis of the arch into an even number of segments and to calculate the value of \( H \). The calculation can be facilitated by tabulation of the successive steps. The total bending moment need only be determined generally for the crown \( x = 0, y = R \) and the first quarter-point \( x = 0.25L \). The bending moment \( M_x \) at the crown is the bending moment for a freely-supported beam minus \( HR \). For the maximum positive bending moment at the crown the sum of the values of \( M_x \) for all elements of dead load is added to the values of \( M_e \) for only those elements of live load that give positive values of \( M_x \). For the maximum negative bending moment at the crown the sum of the values of \( M_x \) for all elements of dead load is added to the values of \( M_e \) for those elements of live load only that give negative values of \( M_x \). The bending moment at the first quarter-point is the bending moment for a freely-supported beam minus \( HY_0 \), where \( Y_0 \) is the vertical ordinate of the first quarter-point. The bending moment is combined with the normal component of \( H \).

For an arch of large span it is worth while drawing the influence lines \( (W = 1) \) for the bending moments at the crown and at the first quarter-point.

Approximate Determination of Thickness of a Fixed Arch.—Referring to the diagram at the top of Table 53, draw a horizontal line through the crown \( C \), and find \( G \), the point of intersection with the vertical through the centre of gravity of the total load on half the span of the arch. Set off \( GT \) equal to the dead load on the half-span, drawn to a convenient scale; draw a horizontal through \( T \) to intersect GS produced at \( R \). Draw \( RK \) perpendicular to \( GR \), and \( GK \) parallel to the tangent to the arch axis at \( S \). With the same unit of weight used in drawing \( GT \), scale off \( TR \), which equals \( HC \), and \( GK \) which equals \( H_s \).

If \( c \) is the maximum allowable compressive stress in the concrete, \( d \) is the thickness of the arch at the crown, \( d_x \) the thickness of the arch at the springing, and \( b \) the assumed breadth of the arch (12 in. for a slab), then approximately \( d = \frac{1.7HC}{ob} \), and \( d_x = \frac{2H_x}{ob} \).

The method applies only to spans from 40 ft. to 200 ft., and span-rise ratios between 4 and 8. The method does not depend on knowing the profile of the arch (except for solid-spandrel earth-filled arches where the dead load is largely dependent on the shape of the arch), but the span and rise must be known. With \( d_x \) and \( d \) thus approximately determined, the thrusts and bending moments at the crown, springing, and quarter-points can be determined and the stresses on the assumed sections calculated. If this calculation shows the sections to be unsuitable, other dimensions must be assumed and the calculations reworked.

Stresses in a Fixed Arch of Any Profile.—The following method is suitable for determining the stresses in any symmetrical fixed arch if the dimensions and shape of the arch are known, or assumed, and if the shape of the arch must conform to a specified profile. Reference should be made to page 44 for general comments on this method.

On half the arch drawn to scale, as in Table 53, plot the arch axis. Divide the half-arch into a number \( n \) of segments such that each segment has the same ratio \( s_i \) of length \( s \) to mean moment of inertia \( I \) based on the thickness of the arch measured normal to the axis, allowance being made for the reinforcement; \( s_i = \frac{s}{I} \).

The ordinates \( x \) and \( y \) about the axis of the arch at the crown are determined by measurement to the centre of the length of each segment. Calculate the dead load and the live load separately on each segment. Assume the loads act at the centre of the length of each segment. In an open spandrel arch, the dead and live loads are concentrated at the arch at the positions of the columns; these positions should be taken as the centre of the segments; but it may not then be possible to maintain a constant value of \( s_i \), and the value of \( s_i \) for each segment must be calculated; the general formulæ in Table 53 are then applicable.

(Continued on page 232.)
ARCHES: THREE-HINGED AND TWO-HINGED ARCHES.—TABLE 52.

THREE-HINGED ARCH

TWO-HINGED ARCH

FIXED ARCH

TYPES OF ARCHES

THREE-HINGED ARCH

INFLUENCE LINE FOR SECTION AT \( \infty \)

B. M. AT \( \infty \):
\[ M_x = W a \left( \frac{\infty}{L} + \frac{1}{2} \right) \]

HORIZONTAL THRUST:
\[ H = \frac{\sum M_x y y_{s_f}}{(\sum y^2 y_{s_f}) + \frac{1}{L_A}} \]

UNSYMMETRICAL THREE-HINGED ARCHES.

REACTION FORMULE FOR GENERAL CASE (c):

\[ V_A = \sum_a \left[ \frac{1}{1-b} \right] Q \sin \theta + \frac{1}{1-ab} \sum_b \frac{Q \sin \theta}{(y-b)} - \left( \frac{1-b}{1-ab} \right) \sum_e y Q \cos \theta \]

\[ V_B = \frac{1-b}{1-ab} \sum_a \frac{Q \sin \theta}{1-ab} + \frac{1}{1-ab} \sum_b \frac{Q \sin \theta}{(y-b)} \]  

\[ H_A = \frac{l}{(l-a)(l-b)} \left[ \sum_a \frac{Q \sin \theta}{1-ab} + \frac{a}{1-ab} \sum_b \frac{Q \cos \theta}{(y-b)} - \sum_e \frac{Q \cos \theta}{1-ab} \right] \]

\[ H_B = \frac{l}{(l-a)(l-b)} \left[ \sum_a \frac{Q \sin \theta}{1-ab} + \frac{a}{1-ab} \sum_b \frac{Q \cos \theta}{(y-b)} - \sum_e \frac{Q \cos \theta}{1-ab} \right] \]

Note.—See facing page for explanation of symbols in, and notes on, the expressions for two-hinge arches.

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REINFORCED CONCRETE DESIGNER'S HANDBOOK

ARCHES (continued).

Stresses in a Fixed Arch of Any Profile (continued from page 230).

For constant values of $s_I$, the effects at the crown are as follows.

$$H_C = \frac{nEML_Y - EY \cdot EM_L}{2n[\Sigma \Sigma y^2 - (\Sigma y)^2]}; \quad V_C = \frac{\Sigma EM_L}{2\Sigma x^2}; \quad M_C = \frac{EM_L - 2HCy}{2n}.$$  

The summations are taken over one-half of the arch. The term $M_L$ is the moment at the centre of the segment of all loads between the centre of the segment and the crown. Summations are also made for the loads on the other half, for which $V_C$ is negative.

Due to elastic shortening of the arch due to $H_C$:

$$H_C = \frac{H_C \Delta^2}{A}; \quad M_C = \frac{H_C \Delta y}{n},$$

in which $A$ is the cross-sectional area of the segment calculated on the same basis as $I$. Due to a rise (+ $t$) or a fall (- $t$) in temperature:

$$H_C = \frac{t}{2s_I[\Sigma \Sigma y^2 - (\Sigma y)^2]}; \quad M_C = \frac{H_C \Delta y}{n},$$

in which $l$ is the length of the arch axis. Arch shortening due to $H_C$ is neglected. Shrinking of the concrete is equal to a fall of 15 deg. F.

The procedure for applying the foregoing formulae is to calculate $(H_C - H_C)$, $V_C$, and $(M_C - M_C)$ for the dead load. Calculate separately $H_C$ and $M_C$ for a rise and fall of temperature of 30 deg. F. (or any other suitable amount) and separately for shrinking. Calculate $(H_C - H_C)$ and $(M_C - M_C)$ for the live load (reduced to an equivalent uniformly-distributed load) which, for the purpose of finding the maximum bending moment at the crown (and the corresponding horizontal thrust), should be considered as operating on the segments in the middle-third of the arch. (By considering the effect of the live load on one segment more and one less than those in the middle third, the number of segments that should be loaded to give the maximum positive bending moment due to live load at the crown can be determined.) With the live load only on those segments which are unloaded when calculating the maximum positive bending moment, the maximum negative bending moment at the crown due to the live load is obtained. These maximum bending moments are then combined with the bending moments due to dead load and arch shortening, and with the bending moments due to change of temperature and to shrinking, in such a way that absolute maximum and minimum values are obtained. The corresponding thrusts are also calculated and are combined with the appropriate bending moments to determine the stresses at the crown.

The bending moment at the springing due to a load at a point between the springing and the crown of the arch and at a distance $aL$ from the springing (L being the span of the arch) is

$$M_s = (M_C - M_C) + (H_C - H_C)R + 0.5LV_C - WaL,$$

where $R$ is the rise of the arch. For the dead load the values determined for the crown are substituted in this expression, with the term $WaL$ replaced by $2W(0.5L - x)$. To give the maximum negative bending moment at the springing, the live load is considered as acting only on those segments in the part of the arch extending $0.4$ of the span from the support. (As before, the effect of the live load on one more and one less segment should be determined to ensure that the most adverse disposition of the loading has been considered.) By loading only those segments that are unloaded when finding the maximum negative bending moment, the maximum positive bending moment is obtained. These maximum moments are each combined with the bending moments due to dead load, temperature, and shrinking to give the most adverse combination, and the stresses at the springing are obtained by combining the bending moments and normal thrusts.

Thrust normal to the section at the springing: $T_s = (H_C - H_C) \cos \phi + V_S \sin \phi$.

Vertical component of thrust at springing: $V_S = \text{[total load on half arch giving the appropriate value of (H_C - H_C)] - [V_C corresponding to (H_C - H_C)].}$

Shearing force at crown = maximum value of $V_C$ due to any combination of dead and live load.

Shearing force at springing = $(H_C - H_C) \sin \phi + V_S \cos \phi$; the values of $(H_C - H_C)$ and $V_S$ are those due to the incidence of dead plus live load that gives the maximum value for the shearing force, which is generally when the live load extends over the whole arch.

Quarter-point ($xQ = 0.25L$ from the crown).—Bending moment due to a single load $W$ placed between the section and the crown at a distance ($xQ - x'$) from the section: $M = M_C + HCY_Q + V_Cx_Q - W(x_Q - x')$. The procedure is then similar to the analysis of the section at the springing.
Note.—For explanation of the approximate method of determining the size of a fixed arch, see page facing Table 52.
Reference should be made to page 45 for comments on the method of analysing a fixed parabolic arch as given by the data in Table 54. The formulae, basis of the coefficients, and an example are given in the following.

**Dead Load and Elastic Contraction.**—Horizontal thrust due to the dead load alone, 
\[ H = \frac{k_s w d L^2}{R}, \]
where \( w_d \) = dead load per unit length at the crown; \( L \) = span, \( R \) = rise of the arch axis. The coefficient \( k_s \) depends on the dead load at the springing, which varies with the ratio of rise to span and the type of structure, that is whether the arch is open-spandrel or solid-spandrel, or whether the dead load is uniform throughout the span.

Elastic contraction produced by the thrust along the arch axis (assuming rigid abutments). The counter-thrust \( H_D \), while slightly reducing the thrust due to the dead load, renders this thrust eccentric and produces a positive bending moment at the crown and a negative bending moment at the springings. If \( d \) is the thickness of the arch at the crown,
\[ H_D = -k_2 \left( \frac{d}{R} \right)^2 H, \]
in which the coefficient \( k_2 \) depends on the relative thicknesses at the crown and springing.

Due to dead load and arch shortening, the resultant thrusts \( H_C \) and \( H_S \) at the crown and springing respectively, and acting parallel to the arch axis at these points, are
\[ H_C = H - H_D; \quad H_S = \frac{H}{\cos \phi} - H_D \cos \phi, \]
in which \( \phi \) is the angle between the horizontal and the tangent to the arch axis at the springing. Values of \( \cos \phi \) are given in Table 54.

Bending moments due to the eccentricity of \( H_C \) and \( H_S \): \( M_C = k_3 R H_D; \quad M_S = (k_3 - 1) R H_D. \)

**Effect of Change of Temperature.**—Additional horizontal thrust due to a rise in temperature or the corresponding counter-thrust due to a fall in temperature:
\[ H_T = \pm k_4 \left( \frac{d}{R} \right)^2 \frac{dt}{t}, \]
in which \( t \) is the rise or fall in temperature in deg. F. If \( d \) and \( R \) are in feet, \( H_T \) is in lb. per foot width of arch. The values of \( k_4 \) in Table 54 are based on an elastic modulus for concrete \( E_C \) of 2,000,000 lb. per square inch, and a coefficient of linear expansion \( e \) of 0.0000066 per deg. F. If other values, \( E_4 \) and \( e_4 \), are adopted, \( k_4 \) should be multiplied by 0.0076 \( E_4/e_4 \). At the crown the increment or decrease in normal thrust due to change of temperature is \( H_T \) and the bending moment is \(- k_4 R H_T \), accounts being taken of the sign of \( H_T \). The normal thrust at the springing due to change of temperature is \( H_T \cos \phi \), and whether the thrusts due to dead load, etc., are increased or decreased thereby depends upon the sign of \( H_T \). At the springing the bending moment is \((1 - k_5) R H_T \), the sign being the same as that of \( H_T \).

**Shrinking of Concrete.**—Shrinking can be considered to be equivalent to a fall of temperature of 15 deg. F.

**Live Load.**—The intensity of uniformly-distributed load equivalent to the specified live load = \( w \).

Maximum positive bending moment at the crown = \( k_6 L^3 \); horizontal thrust = \( k_6 w L^2 / R \).

Maximum negative bending moment at springing = \( kw L^2 \); horizontal thrust = \( k_6 \frac{w L^2}{R} \);
vertical reaction = \( k_6 w L \).

Maximum positive bending moment at springing = \( k_{10} w L^2 \); horizontal thrust = \( k_{11} L^3 / R \);
vertical reaction = \( k_{11} w L \).

If \( H \) and \( V \) are the corresponding horizontal thrust and vertical reaction, normal thrust at the springing = \( H \cos \phi + V \sqrt{1 - \cos^2 \phi} \).

**Dimensions of Arch.**—The line of pressure, and therefore the arch axis, can now be plotted as described on page 46. The thicknesses of the arch at the crown and springing having been determined, the lines of the extrados and intrados can be plotted to give a parabolic variation of thickness between the two extremes. Thus, the thickness normal to the axis of the arch at any point is given by \( [(d_e - d) + d] \) where \( r \) has the following values: if the ratio of the distance of the point from the springing, measured along the axis of the arch, to half the length of the arch is less than 1, the value of \( r \) is 0.563; if the ratio is 1, \( r \) is 0.250; and if the ratio is 1, \( r \) is 0.063.

An example of the application of Table 54 and the foregoing method is given on the page facing Table 55.
## ARCHES: FACTORS FOR FIXED PARABOLIC ARCHES.—TABLE 54.

<table>
<thead>
<tr>
<th>TYPE</th>
<th>UNIFORM DEADLOAD</th>
<th>OPEN SPANDREL</th>
<th>SOLID SPANDREL</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>RISE:SPAN</strong></td>
<td>0-10 0-15 0-20 0-25</td>
<td>0-10 0-15 0-20 0-25</td>
<td>0-10 0-15 0-20 0-25</td>
</tr>
<tr>
<td><strong>INCLINATION OF AXIS OF ARCH AT SPRINGING COE</strong></td>
<td>0-930 0-848 0-781 0-709</td>
<td>0-918 0-820 0-740 0-650</td>
<td>0-883 0-764 0-665 0-565</td>
</tr>
<tr>
<td><strong>HORIZONTAL THRUST DUE TO DEAD LOAD Values of k₁</strong></td>
<td>-</td>
<td>0-125 0-125 0-125 0-125</td>
<td>0-135 0-140 0-144 0-148</td>
</tr>
<tr>
<td><strong>HORIZONTAL THRUST DUE TO ARCH SHORTENING Values of k₂</strong></td>
<td>1-25</td>
<td>1-10 1-07 1-03 0-99</td>
<td>1-13 1-08 1-03 1-00</td>
</tr>
<tr>
<td><strong>MOMENTS DUE TO ARCH SHORTENING, TEMPERATURE CHANGE AND ECCENTRICITY OF THRUST Values of k₃</strong></td>
<td>1-25</td>
<td>0-284 0-293 0-300 0-307</td>
<td>0-279 0-280 0-281 0-282</td>
</tr>
<tr>
<td><strong>HORIZONTAL FORCE DUE TO TEMPERATURE CHANGE Values of k₄ x 10³</strong></td>
<td>1-25</td>
<td>2-54 2-42 2-32 2-22</td>
<td>2-58 2-46 2-33 2-19</td>
</tr>
<tr>
<td><strong>HORIZONTAL THRUSTS DUE TO LIVE LOAD Values of k₁</strong></td>
<td>0-059 0-059 0-059 0-059</td>
<td>0-062 0-064 0-065 0-066</td>
<td>0-070 0-074 0-077 0-080</td>
</tr>
<tr>
<td><strong>BENDING MOMENTS DUE TO LIVE LOAD Values of k₅ x 10⁴</strong></td>
<td>1-25</td>
<td>48 49 51 52</td>
<td>48 50 52 54</td>
</tr>
<tr>
<td><strong>VERTICAL REACTIONS DUE TO LIVE LOAD Values of k₆</strong></td>
<td>0-358 0-358 0-358 0-358</td>
<td>0-354 0-352 0-350 0-349</td>
<td>0-342 0-337 0-330 0-321</td>
</tr>
</tbody>
</table>

Note.—See facing page for formulae in which coefficients k₁, k₂, etc., should be substituted, as shown in the example on page 236.

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FIXED PARABOLIC ARCHES (continued).

Example of Use of Table 54.—Design of a fixed arch slab for an open-spandrel bridge.

**Span**: 150 ft. measured horizontally between the intersection of the axis of the arch and the abutment.

**Rise**: 22 ft. 6 in., being the rise of the axis of the arch within the 150-ft. span.

**Thickness of arch slab**: 3 ft. 4 1/2 in. at springings, 2 ft. 3 in. at crown.

**Dead load**: 250 lb. per square foot excluding the weight of the arch slab.

**Live load**: 300 lb. per square foot. This is approximately equal to the Ministry of Transport loading; since the live load extends over one-third or four-tenths of the span to give the maximum bending moments, the loaded lengths are 50 ft. and 60 ft.; from Table 6 the equivalent distributed load is 220 lb. per sq. ft. The difference between this and 300 lb. per sq. ft. allows for the single knife-edge load of 2700 lb. per ft. width of slab.

**Temperature range**: ± 15 deg. F. **E_C = 2,000,000 lb. per sq. in.** **\( e = 0.0000066 \)** per deg. F.

**Shrinking**: Equivalent to a fall in temperature of 15 deg. F.

**Geometrical properties**: 
- Thickness at springing \( d_s = \frac{3.375}{2.25} = 1.5 \)
- Thickness at crown \( H = \frac{22.5}{150} = 0.15 \)

**Angle of inclination of arch axis** (from Table 54): \( \cos \phi = 0.82 \).

**Dead load at crown**: 250 lb. per sq. ft. plus weight of 27-in. slab (= 338 lb. per sq. ft. at 150 lb. per cu. ft.; \( w_s = 250 + 338 = 558 \) lb. per sq. ft.). A strip of slab 12 in. wide is considered. The coefficients are taken from Table 54 and substituted in the formula preceding on page 234.

**Horizontal thrusts due to dead load, etc.**

Dead load \( (k_1 = 0.140) \): \( H = 0.140 \times 588 \times \frac{150^2}{22.5} = 82,200 \) lb.

Arch shortening \( (k_2 = 1.39) \): \( H_D = -1.39 \times (\frac{22.5^2}{22.5}) = 1145 \) lb.

Change of temperature \( (k_3 = 3.34 \times 10^4) \):

\[
H_T = \pm 3.34 \times 10^3 \times (\frac{22.5^2}{22.5}) \times 2.25 \times 15 = \pm 1130 \text{ lb.}
\]

**Shrinking**: As for fall in temperature

Crown.—**Maximum positive bending moment.**

Dead load and arch shortening: \( H_C = 82,200 - 1145 \)

\( (k_4 = 0.243) \): \( M_C = 0.243 \times 22.5 \times 1145 \times 12 \)

Fall in temperature: Thrust \( H_T \) as above

\( M_C = 0.243 \times 22.5 \times 1130 \times 12 \)

**Shrinking**: As for fall in temperature

Live load: Thrust \( (k_4 = 0.064) \): \( H = 0.064 \times 300 \times \frac{150^2}{22.5} \)

\( (k_4 = 50 \times 10^4) \); \( M_C = 0.0050 \times 300 \times 150 \times 12 \)

\( + 405,000 \) lb.

Totals: \( + 628,700 \) lb.

Springing.—**Maximum negative bending moment.**

Dead load and arch shortening: \( H_S = 82,200 - 0.820 \)

\( M_S = (1 - 0.243) \times 22.5 \times 1145 \times 12 \)

Fall in temperature : Thrust \( = -1130 \times 0.820 \)

\( M_S = -0.757 \times 22.5 \times 1130 \times 12 \)

**Shrinking**: As for fall in temperature

Live load: Thrust \( (k_4 = 0.038) \): \( H = 0.038 \times 300 \times \frac{150^2}{22.5} \)

\( (k_4 = 0.032) \); \( V = 0.032 \times 300 \times 150 = 15,800 \) lb.

Normal thrust \( = (11,400 \times 0.820) + (15,800 \sqrt{1 - 0.820^2}) \)

\( (k_4 = 0.020) \); \( M_S = 0.020 \times 300 \times 150 \times 12 \)

\( + 1,580,000 \) lb.

Totals: \( -2,276,200 \) lb.

(Continued on page 237.)
### Calculation Chart for Fixed-End Symmetrical Arch

#### Dimensional Properties

<table>
<thead>
<tr>
<th>Segment</th>
<th>x</th>
<th>y</th>
<th>d(x)</th>
<th>s</th>
<th>A</th>
<th>(I_x)</th>
<th>(I_y)</th>
<th>(I_{xy})</th>
<th>(A^2)</th>
<th>(M_x)</th>
<th>(M_y)</th>
<th>(M_{xy})</th>
<th>(M_x^2)</th>
<th>(M_y^2)</th>
<th>(M_{xy}^2)</th>
<th>(C)</th>
<th>(D)</th>
<th>(E)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
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</tr>
<tr>
<td><strong>Total</strong></td>
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</tr>
</tbody>
</table>

#### Summations on Left-Hand Half of Arch

\[ \sum_{i=0}^{n} T_i \sum_{i=1}^{n} t_i \sum_{i=2}^{n} t_i \sum_{i=3}^{n} t_i \sum_{i=4}^{n} t_i \sum_{i=5}^{n} t_i \]

#### Ordinates of Influence Lines

<table>
<thead>
<tr>
<th>Unit Load at A (Crown)</th>
<th>Unit Load at B</th>
<th>Unit Load at C (Crown)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(T_x)</td>
<td>(T_y)</td>
<td>(T_{xy})</td>
</tr>
<tr>
<td>(5)</td>
<td>(6)</td>
<td>(7)</td>
</tr>
<tr>
<td>(8)</td>
<td>(9)</td>
<td>(10)</td>
</tr>
<tr>
<td>(11)</td>
<td>(12)</td>
<td>(13)</td>
</tr>
<tr>
<td>(14)</td>
<td>(15)</td>
<td>(16)</td>
</tr>
<tr>
<td>(17)</td>
<td>(18)</td>
<td>(19)</td>
</tr>
</tbody>
</table>

#### Temperature and Shrinking

\[ H_{ct} = \frac{\Delta T \cdot E \cdot L}{2 \cdot D \cdot T_x} \]

#### Bending Moments at Springs

\[ M_{ba} = M_{c} + \frac{H_{c}}{E} \cdot \frac{L}{2} \cdot \frac{L}{2} \]

#### Bending Moment at Quarter-Point: Repeat Similar to Foregoing with Corresponding Formule

#### Example of Use of Table 54 (continued from page 236)

**Springing:** Maximum positive bending moment.

Bending moment: Thrust

- in. lb: \(234,200\)
- lb: \(+99,260\)

Dead load and arch shortening as before

(Rise of temperature and shrinking neutralize each other.)

Live load: \(k_{10} = 0.089\); \(H = 0.089 \times 300 \times 150^2 = 22.5\)

\(k_{11} = 0.151\); \(V = 0.151 \times 300 \times 150 = 6800 \) lb.

Normal thrust \((26,700 \times 0.820) + 6800 \times 1.820 = 92,400\)

\(k_{10} = 0.023\); \(M_{S} = 0.023 \times 300 \times 150^2 \times 12\)

\(+1,830,000\)

\(+15,595,800\)

\(+123,460\)

The corresponding bending moments and thrusts should be combined to determine the maximum stresses and reinforcement at the crown and springing.
REINFORCED CONCRETE DESIGNER'S HANDBOOK

PROPERTIES OF CONCRETE.

Quality Control.—The standards of control A, B, etc., and the strength categories 1, 2, etc., given under the heading "Grade—I.C.E."
"in Table 56 are from "Quality of Concrete in the Field" (Journal of Inst. Civil Engrs., May 1955).
Strength categories 1, 2, and 3 relate respectively to strengths at twenty-eight days exceeding 4000 lb., 2500 lb., and 1000 lb. per sq. in.; strength of category 4 is below 1000 lb. per sq. in.
Standard A is the most rigid control, requiring tests on materials and trial mixtures, and strict control of all site operations; B requires simple trial mixtures only, batching by weight and reasonable site supervision; C permits proportioning by experience and batching by volume; D applies to concrete with no special properties.

Compressive Strength.—The compressive strengths $u_c$ of concretes made with the same cement and cured and tested under the same conditions are shown by Feret to comply approximately with the expression

$$u_c = a \left( \frac{V_c}{V_c + V_w + V_a} \right)^s$$

in which $V_c$, $V_w$ and $V_a$ are respectively the net volumes of cement, water, and voids in unit volume of mixed concrete; that is if $V_a$ is the net volume of aggregate in unit volume of mixed concrete, $V_c + V_a + V_w = 1$. The term $s$ is a numerical factor determined by tests and depends on the nature of the materials.

Tensile Strength.—The ratio of the direct tensile strength $u_t$ and the compressive strength $u_c$ may vary from 0.05 to over 0.10, the relation being very approximately of the form

$$u_t = b(u_c)^s$$

where $s$ is between 0.5 and unity. A formula, derived by Feret, is

$$u_t = c \sqrt{u_c} - d.$$ 

The parameters $c$ and $d$ are obtained by test and depend on the nature of the cement, the type, grading, and largest size of the aggregate, the amount of water, the conditions of curing, and the methods of testing.

Modulus of Rupture.—For the moduli of rupture of various concretes to be comparable, the test piece must have standard dimensions, say, 4 in. deep by 4 in. wide and 16 in. long. If such a test piece is supported on a span of 12 in. and loaded with a centrally applied load, the modulus of rupture $u_r$ is 28,125 Wlb. per sq. in., where $W$ is the load that causes the test piece to break. Factors that contribute to high compressive strength $u_c$ also cause an increase in the modulus of rupture. The relation of $u_r$ to $u_c$ is given by Feret to be approximately

$$u_r = s \sqrt{u_c} - r,$$

where $s$ and $r$ are parameters affected by the same conditions as affect $c$ and $d$ for the direct tensile strength.

Modulus of Elasticity.—The modulus of elasticity of concrete $E_c$ increases with increase of cement content, age, repetition of stress, and some other factors. Actual values lie between 750 and 1500 times the compressive strength. Thus $E_c$ for a $1:2:4$ concrete may average 3,000,000 lb. per sq. in. and, when combined with a modulus of elasticity for steel $E_s$ of 30,000,000 lb. per sq. in., a modular ratio $\left( \frac{E_s}{E_c} \right)$ of 10 is obtained. The generally accepted arbitrary value for the modular ratio is 15 (corresponding to $E_c$ equal to 2,000,000 lb. per sq. in.) for concretes of all proportions, and is recommended in the B.S. Codes. This ratio compensates in part for the errors involved in the consideration of reinforced concrete as a theoretically elastic substance and for the neglect of the tensile resistance of concrete in bending. It only applies within the range of the working stresses. The modular ratio is not taken into account in the load-factor method of design.

The D.S.I.R. Code of 1934 recommended a variable modular ratio of $\frac{40,000}{u_p}$, where $u_p$ is the compressive strength at twenty-eight days. The Ministry of Transport adopt modular ratios recommended by the D.S.I.R. for the design of bridges.

A modular ratio of 10 is often used when calculating deflections. For calculations involving actual deformations the ratios in the D.S.I.R. Code might also be used.

Poisson's Ratio.—Poisson's ratio, by means of which can be calculated the secondary stress, produced by a primary stress, in a direction normal to that of the primary stress, is about 0.15 for concrete, although in many cases this effect is ignored. The omission of Poisson's ratio makes a difference to the calculated stresses in ordinary slabs spanning in two directions (if in panels that are almost square) and to the analyses of the stresses in shells and in road slabs.
### Table 56

**Concrete: Proportions and Stresses**

<table>
<thead>
<tr>
<th>Nominal Volumetric Proportions</th>
<th>1:8</th>
<th>1:6</th>
<th>1:2:4</th>
<th>1:1(\frac{1}{2} : 3\frac{1}{2} )</th>
<th>1:2:3</th>
<th>1:1:2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aggregate &amp; Quantities per cu. m. Concrete</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Fine Aggregate</td>
<td>10</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Coarse Aggregate</td>
<td>(7\frac{1}{2})</td>
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<td>Finely</td>
<td></td>
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<td>Grading of All-in Aggregate</td>
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</tr>
<tr>
<td>11</td>
<td>&amp; (1(\frac{3}{4})% )</td>
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<tr>
<td>22</td>
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<td>Dependence</td>
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<tr>
<td>Liquid Container</td>
<td>(\frac{1}{2})</td>
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<td></td>
</tr>
<tr>
<td>Bales</td>
<td>4D</td>
<td>4D</td>
<td>2B &amp; 2C</td>
<td>2A</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ord.</td>
<td>A</td>
<td></td>
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<td></td>
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<tr>
<td>Liquid Container</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bales</td>
<td>2B &amp; 2C</td>
<td>2B</td>
<td>2A</td>
<td>2B</td>
<td>2A</td>
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<td>London By-laws</td>
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<tr>
<td>I.C.E.</td>
<td>IV</td>
<td>IV</td>
<td>III</td>
<td>IIIA</td>
<td>HIGH ALUMINA</td>
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<tr>
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<tr>
<td>Crushing Strength, B.D. per cft</td>
<td>-</td>
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<td>-</td>
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<tr>
<td>Without Reinforcement</td>
<td>-</td>
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<tr>
<td>With Reinforcement</td>
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<tr>
<td>Bending</td>
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<tr>
<td>Direct</td>
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<tr>
<td>Without Reinforcement</td>
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<tr>
<td>With Reinforcement</td>
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</tr>
</tbody>
</table>

**Notes**

For rules for properties of mixtures not tabulated see page 246.

Quantities—Tabulated quantities per cu. yd. of mixed concrete are approximate, and vary with grading of aggregate, compaction, etc. Quantities of fine aggregates (in brackets) assume material to be damp and bulked by about 30 per cent.

**Grades.**—Those marked "London By-laws" relate to London Building (Constructural) By-laws 1952. Those marked "I.C.E." relate to recommendations in "Quality of Concrete in the Field" (Journal Inst. Civil Engrs., May 1955); see note on facing page. Those marked "Liquid Containers" relate to B.S. Code No. 2007 (1960), "Design and Construction of Reinforced and Prestressed Concrete Structures for the Storage of Water and other Aqueous Liquids".

Working Stress.—Increase values (London By-laws) by 10 per cent. if vibrated. Working stresses due to effects of wind increased by one-third (London By-laws). Shearing stresses indicated thus * denote maximum (with reinforcement) in design for no cracking. Bond stresses marked ** may be increased by 25 per cent. if deformed bars are used.

B.S. Code No. 114—See Table 57.
## Temperature Coefficients

The coefficient of linear expansion and contraction due to temperature changes increases with an increase in the cement content, depends on the type of aggregate, and varies from 0.00004 to 0.00007 per deg. F. A coefficient of 0.000055 per deg. F., which is about the same as that for mild steel, is commonly used.

## Thermal Conductivity

Average values of the thermal conductivity of concrete \( (k) \), expressed in the number of British thermal units that pass in one hour through 1 in. of concrete 1 sq. ft. in area for each deg. F. difference in temperature between the two faces are: \( 1 : 2 : 4 \) gravel concrete, \( 7:0 \); \( 1 : 2 \) gravel concrete, \( 6:7 \); \( 1 : 2 \) clinker concrete, \( 2:3 \); \( 1 : 2 \) foamed-slag concrete, \( 2:2 \); \( 1 : 2 \) pumice concrete, \( 1:4 \); \( 1 : 2 \) expanded-shale concrete, \( 2:1 \); \( 1 : 8 \) vermiculite concrete, \( 0:6 \).

Recommendations are given in B.S. Code No. 114 relating to the resistance to fire of reinforced concrete construction. This resistance, which depends primarily on the type of aggregate, the thickness of the member, and the cover of concrete over the reinforcement, is expressed by the number of hours of effective resistance as established by tests made in accordance with B.S. No. 476, "Fire Tests on Building Materials and Structures", and values are given in the table below.

The degree of resistance required depends on the size of a building and the use to which the building is to be put, and is specified in the local by-laws. In general, the resistances given in the table conform to those in the London Building By-laws (1952); exceptions and additions are noted.

The Code gives no recommendation regarding the fire-resistance of reinforced concrete stairs, but the London By-laws require at least one hour's resistance which is provided by 2 1/2 in. of solid concrete.

### Least Dimensions to Provide Stated Period of Fire Resistance

<table>
<thead>
<tr>
<th>Member</th>
<th>Period of Resistance (hours)</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1/2</td>
<td>1</td>
</tr>
<tr>
<td><strong>Walls: Thickness</strong></td>
<td>in.</td>
<td>in.</td>
</tr>
<tr>
<td><strong>Solid</strong></td>
<td>3 1/2</td>
<td>4</td>
</tr>
<tr>
<td>Floor Slabs Thickness (minimum)</td>
<td>ft.</td>
<td>in.</td>
</tr>
<tr>
<td><strong>Channel or T-section</strong></td>
<td>( 3 \frac{1}{2} )</td>
<td>4</td>
</tr>
<tr>
<td><strong>Inverted Channel</strong></td>
<td>( \frac{1}{2} )</td>
<td>3</td>
</tr>
<tr>
<td><strong>Box or I-section</strong></td>
<td>2 1/2</td>
<td>3</td>
</tr>
<tr>
<td><strong>Columns: Minimum size</strong></td>
<td>in.</td>
<td>in.</td>
</tr>
<tr>
<td><strong>Mesh within cover</strong></td>
<td>-</td>
<td>5</td>
</tr>
<tr>
<td><strong>Ordinary Construction</strong></td>
<td>( 6 )</td>
<td>8</td>
</tr>
<tr>
<td><strong>Beams: Minimum cover</strong></td>
<td>( 1 \frac{1}{2} )</td>
<td>1</td>
</tr>
<tr>
<td><strong>Stairs: Minimum thickness</strong></td>
<td>( 2 \frac{1}{2} )</td>
<td>(2)</td>
</tr>
</tbody>
</table>

Notes.—(a) Dimensions are for concrete made with aggregates conforming to B.S. No. 882, except that if limestone is used a slightly less thickness is acceptable.

(b) Tabulated data are as given in B.S. Code No. 114 and conform to London By-laws (1952) except that (i) Dimensions marked * are excluded from By-laws; (ii) Dimensions in brackets conform to By-laws but differ from Code.

(c) Dimensions of reinforced concrete members may be reduced if protected by plaster, sprayed asbestos, or the like.
## Table 57: Concrete: Properties and Stresses

**B.S. Code No. 114**

<table>
<thead>
<tr>
<th>Class of Concrete</th>
<th>Nominal Properties</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Quality</td>
</tr>
<tr>
<td></td>
<td>Days</td>
</tr>
<tr>
<td>Lower 1:2:4 (by vol)</td>
<td>7</td>
</tr>
<tr>
<td>28</td>
<td>3000</td>
</tr>
<tr>
<td>Ord. 1:2:4 (by vol)</td>
<td>3</td>
</tr>
<tr>
<td>28</td>
<td>4000</td>
</tr>
<tr>
<td>Higher 1:2:4 (by vol)</td>
<td>7</td>
</tr>
<tr>
<td>28</td>
<td>5000</td>
</tr>
<tr>
<td>Lower 1:2:3 (by vol)</td>
<td>3</td>
</tr>
<tr>
<td>28</td>
<td>3750</td>
</tr>
<tr>
<td>Ord. 1:2:3 (by vol)</td>
<td>7</td>
</tr>
<tr>
<td>28</td>
<td>5000</td>
</tr>
<tr>
<td>Higher 1:2:3 (by vol)</td>
<td>3</td>
</tr>
<tr>
<td>28</td>
<td>4250</td>
</tr>
<tr>
<td>Lower 1:1:2 (by vol)</td>
<td>7</td>
</tr>
<tr>
<td>28</td>
<td>4500</td>
</tr>
<tr>
<td>Ord. 1:1:2 (by vol)</td>
<td>7</td>
</tr>
<tr>
<td>28</td>
<td>6000</td>
</tr>
<tr>
<td>Higher 1:1:2 (by vol)</td>
<td>7</td>
</tr>
<tr>
<td>28</td>
<td>7500</td>
</tr>
<tr>
<td>Ditto But Fine Aggregate Also in Zone 4 (by wt)</td>
<td>1:8 (max) with 3%-ag</td>
</tr>
<tr>
<td>28</td>
<td>Two-thirds of required 28-day strength</td>
</tr>
<tr>
<td>High-Alumina Cement 1:2:4 (by vol)</td>
<td>2</td>
</tr>
<tr>
<td>7</td>
<td>7000</td>
</tr>
<tr>
<td>Aggregate Conforming to B.S. No. 882 Fine Aggregate in Zones 1 to 3 1:1:2 (by vol)</td>
<td>1:8 Max. to 1:5 Min. Fine Aggregate in Zone 4 Also</td>
</tr>
<tr>
<td>7</td>
<td>7000</td>
</tr>
<tr>
<td>General Relation of Stresses</td>
<td>Notation</td>
</tr>
<tr>
<td>28</td>
<td>4$P_{cb}$</td>
</tr>
<tr>
<td>Modifications to Tabulated Stresses</td>
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</tr>
</tbody>
</table>

**Note:**
- For quantities of materials see Table 56.
- For notes on Table 57 see page 246.
### Table 58: Reinforcement: Properties and Stresses

<table>
<thead>
<tr>
<th>Type of Reinforcement</th>
<th>Tensile Properties</th>
<th>Tensile Stress (ψₜ)</th>
<th>Compressive Stress (ψₖ)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Minimum Tensile Strength</td>
<td>Ordinary Conditions</td>
<td>Corrosive Conditions</td>
</tr>
<tr>
<td></td>
<td>Full.</td>
<td>Normal</td>
<td>Including Effects of Wind</td>
</tr>
<tr>
<td>Mild Steel (B.S. No. 783)</td>
<td>1%</td>
<td>62,720</td>
<td>20,000 (18,000)</td>
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<tr>
<td></td>
<td>1%</td>
<td>44,600</td>
<td>22,000 (20,000)</td>
</tr>
<tr>
<td></td>
<td>1%</td>
<td>39,000</td>
<td>19,500 (18,000)</td>
</tr>
<tr>
<td>Hot-rolled Bars (B.S. No. 1144)</td>
<td>1%</td>
<td>51,500</td>
<td>25,750 (24,000)</td>
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<tr>
<td></td>
<td>1%</td>
<td>47,000</td>
<td>23,500 (22,000)</td>
</tr>
</tbody>
</table>

*Not now readily obtainable.* **Per B.S. Code No. 114.

B.S. Code No. 114 (1957).—Tabulated stresses (not in brackets) excluding "Corrosive Conditions". Expanded metal not specifically mentioned in Code.

**LONDON BY-LAWS** (1952).—Stresses given in brackets; otherwise same as stresses in B.S. Code (excluding "Corrosive conditions").

**Corrosive Conditions** include external members, internal members under corrosive conditions, and members against earth (not B.S. Code or London By-laws).

Shearing Reinforcement.—B.S. Code No. 114: Tabulated stresses, but not greater than 20,000 lb. per sq. in. (25,000 lb. with wind). London By-laws: 18,000 lb. per sq. in. in mild steel; otherwise 0.5ψₜ (or 0.6ψₜ with wind), but not greater than 20,000 lb. per sq. in. (26,670 lb. with wind).

**Helical Binding in Columns**.—B.S. Code No. 114: 13,500 lb. per sq. in. for all reinforcement. London By-laws: 13,500 lb. per sq. in. in mild steel; otherwise 0.5ψₜ but not greater than 18,000 lb. per sq. in.

**Liquid-containers**.—See page 59 for stresses in reinforcement in accordance with B.S. Code No. 2007.

---

*Limiting Stresses* (LONDON BY-LAWS 1952):

- CP. 114 (1957): ψₜ = 0.6ψₜ (in mild steel)
- ψₜ = 0.5ψₜ (in other steels)
- ψₜ = 0.4ψₜ (in carbon steel)
- ψₜ = 0.3ψₜ (in stainless steel)

---

*Not now readily obtainable.* **Per B.S. Code No. 114.

---

*242*
## CROSS-SECTIONAL AREA: SQUARE INCH.
WEIGHT: LB. PER FOOT (OR AS STATED)

<table>
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<tr>
<th>DIAM.</th>
<th>3/16&quot;</th>
<th>1/4&quot;</th>
<th>5/32&quot;</th>
<th>7/32&quot;</th>
<th>1/8&quot;</th>
<th>3/32&quot;</th>
<th>7/32&quot;</th>
<th>1/4&quot;</th>
<th>3/8&quot;</th>
<th>7/16&quot;</th>
<th>1/2&quot;</th>
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<tr>
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<td>PERIMETER AREA</td>
<td>0.59</td>
<td>0.73</td>
<td>0.98</td>
<td>1.18</td>
<td>1.57</td>
<td>1.96</td>
<td>2.36</td>
<td>2.75</td>
<td>3.14</td>
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<td>0.028</td>
<td>0.049</td>
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<td>0.601</td>
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<td>0.954</td>
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<td>0.167</td>
<td>0.261</td>
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<td>0.667</td>
<td>1.043</td>
<td>1.502</td>
<td>2.044</td>
<td>2.670</td>
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### TWISTED SQUARE BARS
**SIZE AREA (MIN.)**

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<td>0.045</td>
<td>0.063</td>
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<td>0.250</td>
<td>0.391</td>
<td>0.583</td>
<td>0.766</td>
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</table>

### TWISTED RIBBED BARS
**NOTES**

**PERIMETER OF PLAIN ROUND BARS = 3.1416 D**
**AREAS: PLAIN ROUND BARS = 0.7854 D²**
**WEIGHTS: STEEL = 0.28 LB. PER C.U. IN.**
**3.4 LB. PER FOOT PER SQ. IN.**
**TWISTED SQUARE BARS = D² (MIN.)**
**OF CROSS-SECTION.**
**TWISTED RIBBED BARS = 0.7854 D²**
**PLAIN ROUND BARS = 2.47D² (LB. PER FOOT).**

### WEIGHTS OF PLAIN ROUND BARS (LB.)

<table>
<thead>
<tr>
<th>DIAM.</th>
<th>3/16&quot;</th>
<th>1/4&quot;</th>
<th>5/32&quot;</th>
<th>7/32&quot;</th>
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<th>7/16&quot;</th>
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<tbody>
<tr>
<td></td>
<td>FEET PER TON</td>
<td>23850</td>
<td>13413</td>
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### TOTAL WC. PER FT. OF SPECIFIED NO.

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## REINFORCEMENT: AREAS AND WEIGHTS.—TABLE 59.
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<th>DIAMETER</th>
<th>$\frac{3}{8}$</th>
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<tbody>
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<td>1</td>
<td>0.280</td>
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<td>0.077</td>
<td>0.110</td>
<td>0.150</td>
<td>0.046</td>
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<td>0.554</td>
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</tbody>
</table>

Cross-sectional Areas of Bars at Specified Spacings

Specified Number

N = Number of Bars

$$A_{st} = \frac{1}{4} \pi D^2 N$$

Specified Spacing

S = Spacing (or Pitch) of Bars (in)

$$A_{st} = \frac{12}{S} (0.7854 D^2)$$

$$= \frac{3.425}{S} (\text{sq. in. per foot})$$

A = Cross-sectional Area (sq. in.)

D = Diameter of Bar (in.)

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### Reinforcement: Metric Sizes and Areas—Table 61

<table>
<thead>
<tr>
<th>Diameter of Bar</th>
<th>Number of Bars</th>
</tr>
</thead>
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<td>0.00</td>
</tr>
<tr>
<td>3/4</td>
<td>0.00</td>
</tr>
<tr>
<td>1</td>
<td>0.00</td>
</tr>
</tbody>
</table>

#### Cross-Sectional Area for Specified Number of Bars

<table>
<thead>
<tr>
<th>Square Inches</th>
<th>Square Centimeters</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/8</td>
<td>0.00</td>
</tr>
<tr>
<td>1/4</td>
<td>0.00</td>
</tr>
<tr>
<td>3/8</td>
<td>0.00</td>
</tr>
<tr>
<td>1/2</td>
<td>0.00</td>
</tr>
</tbody>
</table>

#### Spacing of Bars (Cm)

<table>
<thead>
<tr>
<th>Diameter, Mm.</th>
<th>8</th>
<th>10</th>
<th>12</th>
<th>14</th>
<th>16</th>
<th>18</th>
<th>20</th>
<th>22</th>
<th>25</th>
<th>28</th>
<th>30</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>3.53</td>
<td>2.83</td>
<td>2.50</td>
<td>2.02</td>
<td>1.88</td>
<td>1.77</td>
<td>1.57</td>
<td>1.41</td>
<td>1.25</td>
<td>1.18</td>
<td>1.01</td>
</tr>
<tr>
<td>8</td>
<td>2.71</td>
<td>2.03</td>
<td>1.78</td>
<td>1.42</td>
<td>1.28</td>
<td>1.19</td>
<td>1.07</td>
<td>0.95</td>
<td>0.85</td>
<td>0.76</td>
<td>0.64</td>
</tr>
<tr>
<td>10</td>
<td>2.08</td>
<td>1.62</td>
<td>1.43</td>
<td>1.21</td>
<td>1.08</td>
<td>0.97</td>
<td>0.87</td>
<td>0.78</td>
<td>0.69</td>
<td>0.61</td>
<td>0.53</td>
</tr>
<tr>
<td>12</td>
<td>1.65</td>
<td>1.33</td>
<td>1.17</td>
<td>1.00</td>
<td>0.88</td>
<td>0.79</td>
<td>0.71</td>
<td>0.63</td>
<td>0.56</td>
<td>0.50</td>
<td>0.44</td>
</tr>
<tr>
<td>14</td>
<td>1.34</td>
<td>1.10</td>
<td>0.95</td>
<td>0.81</td>
<td>0.71</td>
<td>0.63</td>
<td>0.56</td>
<td>0.50</td>
<td>0.45</td>
<td>0.41</td>
<td>0.37</td>
</tr>
<tr>
<td>16</td>
<td>1.11</td>
<td>0.92</td>
<td>0.80</td>
<td>0.69</td>
<td>0.61</td>
<td>0.55</td>
<td>0.51</td>
<td>0.46</td>
<td>0.42</td>
<td>0.39</td>
<td>0.36</td>
</tr>
<tr>
<td>18</td>
<td>0.92</td>
<td>0.77</td>
<td>0.68</td>
<td>0.59</td>
<td>0.53</td>
<td>0.48</td>
<td>0.45</td>
<td>0.41</td>
<td>0.38</td>
<td>0.35</td>
<td>0.33</td>
</tr>
<tr>
<td>20</td>
<td>0.77</td>
<td>0.66</td>
<td>0.59</td>
<td>0.52</td>
<td>0.47</td>
<td>0.43</td>
<td>0.40</td>
<td>0.37</td>
<td>0.35</td>
<td>0.33</td>
<td>0.31</td>
</tr>
<tr>
<td>22</td>
<td>0.65</td>
<td>0.57</td>
<td>0.51</td>
<td>0.46</td>
<td>0.42</td>
<td>0.39</td>
<td>0.37</td>
<td>0.35</td>
<td>0.33</td>
<td>0.31</td>
<td>0.29</td>
</tr>
<tr>
<td>25</td>
<td>0.56</td>
<td>0.49</td>
<td>0.44</td>
<td>0.40</td>
<td>0.37</td>
<td>0.35</td>
<td>0.33</td>
<td>0.31</td>
<td>0.29</td>
<td>0.27</td>
<td>0.26</td>
</tr>
</tbody>
</table>

#### Notes

- For conversion factors (British units to metric and vice versa) see page 338 and Table 109.
Modification for Proportions not Tabulated.—The permissible compressive stress in bending, on which other permissible stresses depend, for concrete of proportions intermediate between those tabulated in Tables 56 and 57 can be calculated by interpolation, that is

\[ f_{cb} = f_{cb2} + \frac{(V_1 - V)(f_{cb2} - f_{cb1})}{V_2 - V_1}, \]

where \( f_{cb} \) is the compressive stress permissible in concrete for which the sum of the proportions of fine and coarse aggregates is \( V \); \( f_{cb2} \) and \( f_{cb1} \) are the permissible compressive stresses for concretes for which the sum of the proportions of the fine and coarse aggregates are \( V_1 \) and \( V_2 \) respectively, \( V \) being intermediate between \( V_1 \) and \( V_2 \). For example, in \( 1 : 2 \frac{1}{4} : 3 (V_2 = 6) \) and \( 1 : 2 \frac{1}{4} : 4 \) \( f_{cb} = 750 + \frac{(6 - 5)(850 - 750)}{6 - 4\frac{1}{4}} = 817 \text{ lb. per sq. in.} \)

since 750 lb. and 850 lb. per sq. in. are the values of \( f_{cb2} \) and \( f_{cb1} \) for \( 1 : 2 : 4 \) and \( 1 : 2 \frac{1}{4} : 3 \) ordinary concretes respectively. This method also applies to the stresses in Table 57 for the B.S. Code. The other stresses are calculated in a similar manner.

Reduction of Compressive Stresses in Narrow Members.

Narrow Beams.—Compressive-stress reduction factors \( R_B \) for narrow beams in accordance with the B.S. Code can be expressed by

\[ R_B = 1.75 - 0.025 \frac{L}{b} \]

as given in Table 57, where \( L \) = distance between lateral supports, and \( b \) = breadth of the compressive flange. Values of \( R_B \) are as follows.

\[
\begin{array}{ccccccc}
\frac{L}{b} & 30 & 35 & 40 & 45 & 50 & 55 & 60 \\
R_B & 1 & \frac{3}{4} & \frac{3}{2} & 2 & 3 & 4 & 5 \\
\end{array}
\]

The depth of a narrow beam considered effective in resistance to bending should not exceed \( 8b \) and shearing forces should be resisted by reinforcement.

Beams Subjected to Axial Thrust.—If a beam of overall depth \( d \) is subjected to a bending moment \( M \) and an axial thrust \( N \), the compressive stress reduction factor should be as follows (B.S. Code).

(i) If \( N = \frac{M}{2d} \): Reduction factor \( R_L \) as for columns (see page 296 and Table 83).

(ii) If \( N = 0 \): Reduction factor \( R_B \) as for narrow beams (see above).

(iii) For intermediate values of \( N \): Reduction factor calculated by linear interpolation between (i) and (ii).

Bond-lengths with Anchorages.—In accordance with the B.S. Code, if an end anchorage is provided, the bond-length can be reduced by the amounts (expressed in terms of \( D \), the diameter of the bar) given in Table 62 for each type of anchorage. For more conservative design the lengths \( ND \), when \( N \) has the values in Table 62, should be considered as minima if a semi-circular hook of value \( 16D \) is provided; if other forms of anchorage are provided, the length for bond should be \( ND \) plus \( 16D \) minus the value of the anchorage. For example, if the value of \( N \) is \( 37\frac{1}{4} \), the lengths for bond according to the B.S. Code are: \( 37\frac{1}{4}D \) without an end anchorage; \( 37\frac{1}{4}D - 8D = 29\frac{1}{4}D \) with a 90-deg. bend; \( 37\frac{1}{4}D - 12D = 25\frac{1}{4}D \) with a 45-deg. hook; and \( 37\frac{1}{4}D - 16D = 21\frac{1}{4}D \) with a semi-circular hook. For more conservative design the lengths would be: \( 37\frac{1}{4}D + 16D = 53\frac{1}{4}D \) without an end anchorage; \( 53\frac{1}{4}D - 8D = 45\frac{1}{4}D \) with a 90-deg. bend; \( 53\frac{1}{4}D - 12D = 41\frac{1}{4}D \) with a 45-deg. hook; and \( 37\frac{1}{4}D \) with a semi-circular hook.

Liquid-containers.—Stresses of \( p_{th} = 12,000 \text{ lb. per sq. in. and } s_b = 130 \text{ lb. per sq. in.} \) apply to plain round bars in \( 1 \times 6.32 \) concrete; for \( 1 \times 2.4 \) concrete with \( p_{th} = 12,000 \text{ lb. per sq. in. and } s_b = 120 \text{ lb. per sq. in.}, \) \( N = 25 \).

Local Bond Stress.—The local bond stress \( s_{bl} \) due to variation in the tensile stress in reinforcement in beams is given by

\[
(\bar{Q} \pm \frac{M}{d_1} \tan \alpha) \frac{t}{I_{th}}
\]

(Continued on page 248.)
### Minimum Lengths for Bond in Bar (≤12D) Values of N.

<table>
<thead>
<tr>
<th>Permissible Bond Stress S_b (lb. per sq. in.)</th>
<th>90</th>
<th>100</th>
<th>110</th>
<th>120</th>
<th>125</th>
<th>135</th>
<th>150</th>
<th>160</th>
<th>180</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Tensile Stress in Bar f_st</strong></td>
<td>25</td>
<td>30</td>
<td>35</td>
<td>40</td>
<td>45</td>
<td>50</td>
<td>55</td>
<td>60</td>
<td>65</td>
</tr>
<tr>
<td><strong>Load per SQ. IN. N = f_st / 4S_b</strong></td>
<td>43</td>
<td>53</td>
<td>63</td>
<td>73</td>
<td>83</td>
<td>93</td>
<td>103</td>
<td>113</td>
<td>123</td>
</tr>
<tr>
<td>Compression Reinforcement N = S_c / 5S_b  ≤ 12. <strong>Bond Length for Compression Bars = 0.8X(Length of Tension Bars) At Same Stress.</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Diameter of Bar</th>
<th>5/16&quot;</th>
<th>3/8&quot;</th>
<th>7/16&quot;</th>
<th>1/2&quot;</th>
<th>5/8&quot;</th>
<th>7/8&quot;</th>
<th>1&quot;</th>
<th>1 1/8&quot;</th>
<th>1 1/4&quot;</th>
<th>1 1/2&quot;</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minimum Bond Length 12D</td>
<td>N = 20,000 / 18S_b = S_b / 120 = N = 41 S_b</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bond Lengths for Common Case</td>
<td>13&quot;</td>
<td>16&quot;</td>
<td>18&quot;</td>
<td>21&quot;</td>
<td>26&quot;</td>
<td>32&quot;</td>
<td>37&quot;</td>
<td>42&quot;</td>
<td>47&quot;</td>
<td>52&quot;</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Minimum Laps</th>
<th>Tension 30D</th>
<th>9/16&quot;</th>
<th>11/16&quot;</th>
<th>13/16&quot;</th>
<th>15&quot;</th>
<th>19&quot;</th>
<th>22 1/2&quot;</th>
<th>26 1/2&quot;</th>
<th>30&quot;</th>
<th>34&quot;</th>
<th>37 1/2&quot;</th>
<th>45&quot;</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compression 24D</td>
<td>7 1/2&quot;</td>
<td>9&quot;</td>
<td>10 1/2&quot;</td>
<td>12&quot;</td>
<td>15&quot;</td>
<td>18&quot;</td>
<td>21&quot;</td>
<td>24&quot;</td>
<td>27&quot;</td>
<td>30&quot;</td>
<td>34&quot;</td>
<td>37 1/2&quot;</td>
</tr>
</tbody>
</table>

| Liquid Containers 24 D = 12000/Approx | 7 1/2" | 9"  | 10 1/2" | 12"  | 15" | 18" | 21" | 24" | 27" | 30" | 34" | 37 1/2" |

| Compression Bars R = 18,000; S_b = 120; Min. Bond Length = 30 D | 9 1/2" | 11 1/2" | 13 1/2" | 15"  | 19" | 22 1/2" | 26 1/2" | 30"  | 34"  | 37 1/2" | 45"  |

<table>
<thead>
<tr>
<th>Anchorage</th>
<th>H = 9D = 4D = 4D</th>
<th>B = 7D = 4AD = 2AD</th>
<th>B = 5D = 8AD = 2AD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bond Value = 16D = G</td>
<td>3&quot;</td>
<td>3&quot;</td>
<td>3&quot;</td>
</tr>
<tr>
<td>Bond Value = 8D = B</td>
<td>3/2&quot;</td>
<td>3/2&quot;</td>
<td>3/2&quot;</td>
</tr>
</tbody>
</table>

**Binding Dimensions**

- Dimensions A, B, C, E, and R (if > 2D) only to be entered on binding schedule.

---

247
Local Bond Stress (continued from page 246).

where $Q =$ maximum shearing force at the section under consideration (lb.); $l_a =$ lever arm (in.); $o =$ total perimeter of the bars forming the tensile reinforcement at the section (in.);
$M =$ bending moment (in.-lb.) at the section; $d_1 =$ effective depth (in.); and $\alpha =$ angle between the top and bottom edges of the beam at the section. The positive sign applies when the bending moment decreases as $d_1$ increases, as at a haunch at the end of a freely-supported beam. The negative sign applies when the bending moment increases as $d_1$ increases, as at a haunch at an interior support of a continuous beam.

If the beam is of uniform depth, that is, when the top and bottom edges are parallel,

$$s_{b1} = \frac{Q}{l_{b0}}$$

Permissible local bond stresses in accordance with the B.S. Code are given in Table 57; the perimeters of round bars are given in Table 59.

NOTES ON REINFORCEMENT.

Deformed Bars.—Twisted square bars, twisted ribbed bars, and any bars having projections or indentations, have generally a greater bond value than plain round bars. If the bond value of a deformed bar is not less than 25 per cent. more than that of a plain round bar, the bond stress $s_b$ in Table 57 may be increased by 25 per cent. Since many deformed bars are of strength superior to mild steel, and so long as the equivalent yield stress is not less than 60,000 lb. per sq. in., the bond stress $s_b$ is associated with a permissible tensile stress of 30,000 lb. per sq. in. Thus the expression $\frac{p_{bD}}{4s_b}$ for $L_o$ (see Table 63) for high-tensile deformed bars in ordinary 1 : 2 : 4 concrete is $50D$ compared with about $42D$ for plain round bars stressed to 20,000 lb. per sq. in. The greatest permissible compressive stress in high-tensile reinforcement is 23,000 lb. per sq. in.; the corresponding length $L_c$ (Table 63) for high-tensile deformed bars is therefore about $31D$, compared with $30D$ for plain round bars stressed to 18,000 lb. per sq. in. Values of $L_o$ and $L_c$ for high-tensile deformed bars are given below for $s_b = 150$ lb. per sq. in.

<table>
<thead>
<tr>
<th>Diameter (D) of deformed bar (in.)</th>
<th>$\frac{1}{1}$</th>
<th>$\frac{1}{2}$</th>
<th>$\frac{3}{4}$</th>
<th>1</th>
<th>1$\frac{1}{4}$</th>
<th>1$\frac{1}{2}$</th>
<th>1$\frac{3}{4}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minimum bond length (in.) $L_o$ for bars in tension at 30,000 lb. per sq. in. (50D)</td>
<td>25</td>
<td>$31\frac{1}{2}$</td>
<td>$37\frac{1}{2}$</td>
<td>44</td>
<td>50</td>
<td>$56\frac{1}{2}$</td>
<td>$62\frac{1}{2}$</td>
</tr>
<tr>
<td>Minimum bond length (in.) $L_c$ for bars in compression at 23,000 lb. per sq. in. (30$\frac{1}{2}$D)</td>
<td>$15\frac{1}{2}$</td>
<td>$19\frac{1}{2}$</td>
<td>23</td>
<td>27</td>
<td>31</td>
<td>$34\frac{1}{2}$</td>
<td>$38\frac{1}{2}$</td>
</tr>
</tbody>
</table>

Because of the superior continuous bond value of deformed bars it is not common to provide such bars with an anchorage unless the minimum lengths $L_o$ or $L_c$ cannot be provided in a straight length; in this case the bar should be hooked round an anchor bar. If a deformed bar is not circular in section, $D$ in the preceding expressions is the diameter of the circle having the same area as the cross-sectional area of the bar.

Cover of Concrete.—The minimum cover of concrete to a reinforcement bar should be as follows, or not less than the diameter of the bar, whichever is smaller.

Buildings and General Structures.—Slabs and walls: $1\frac{1}{2}$ in. Beams: 1 in. for main bars; $\frac{3}{4}$ in. for binders. Columns: 1 in. for main $\frac{1}{2}$ in. bars, if least dimension does not exceed $\frac{7}{2}$ in.; $1\frac{1}{2}$ in. in larger columns; $\frac{3}{4}$ in. for binders. Cover over ends of bars: 1 in. or not less than twice the diameter of the bar. External members, members in contact with ground, corrosive conditions or the like: $1\frac{1}{4}$ in. (These dimensions are in accordance with the B.S. Code No. 114 and also satisfy the London By-laws except that the By-laws require a minimum cover of 3 in. (except in piles) if the concrete is in contact with earth, unless precautions are taken to protect the concrete, and a minimum of 1 in. of concrete over all reinforcement in factory-made precast concrete products.)

Piles: $1\frac{3}{4}$ in. for main bars; 1 in. for binders.
Marine Structures: $2\frac{1}{4}$ in. for main bars; 2 in. for binders.
Liquid Containers: $1\frac{1}{2}$ in. for all bars; 2 in. in sea-water or other corrosive liquids, but this extra $\frac{1}{2}$ in. should not be taken into account when making the resistance calculations. (B.S. Code No. 2007.)
### REINFORCEMENT: BOND LENGTHS.—TABLE 63.

**B.S. CODE NO. 114.**

<table>
<thead>
<tr>
<th>DIA. OF BAR D</th>
<th>MIN. 12 D</th>
<th>BARS IN TENSION</th>
<th>BARS IN COMPRESSION</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>BOND LENGTH</td>
<td>STRESS IN BAR (LB. PER SQIN.)</td>
<td>MIN. LAP 24D</td>
</tr>
<tr>
<td></td>
<td>10000</td>
<td>12000</td>
<td>16000</td>
</tr>
<tr>
<td>1/4&quot;</td>
<td>L0 5 1/2</td>
<td>L2 2 1/2</td>
<td>L3 3 1/2</td>
</tr>
<tr>
<td></td>
<td>8 1/2</td>
<td>8 1/2</td>
<td>7 1/2</td>
</tr>
<tr>
<td>5/8&quot;</td>
<td>L0 7 1/2</td>
<td>L2 4 1/2</td>
<td>L3 5</td>
</tr>
<tr>
<td></td>
<td>13 1/2</td>
<td>8 1/2</td>
<td>11 1/2</td>
</tr>
<tr>
<td>1/2&quot;</td>
<td>L0 13 1/2</td>
<td>L2 8 1/2</td>
<td>L3 10 1/2</td>
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<td>21 1/2</td>
<td>19 1/2</td>
<td>15 1/2</td>
</tr>
<tr>
<td>3/8&quot;</td>
<td>L0 21 1/2</td>
<td>L2 10 1/2</td>
<td>L3 15 1/2</td>
</tr>
<tr>
<td></td>
<td>25 1/2</td>
<td>23 1/2</td>
<td>21 1/2</td>
</tr>
<tr>
<td>5/16&quot;</td>
<td>L0 29 1/2</td>
<td>L2 13 1/2</td>
<td>L3 18</td>
</tr>
<tr>
<td></td>
<td>34 1/2</td>
<td>32 1/2</td>
<td>29 1/2</td>
</tr>
<tr>
<td>3/16&quot;</td>
<td>L0 37 1/2</td>
<td>L2 15 1/2</td>
<td>L3 20 1/2</td>
</tr>
<tr>
<td></td>
<td>46 1/2</td>
<td>44 1/2</td>
<td>42 1/2</td>
</tr>
<tr>
<td>1/8&quot;</td>
<td>L0 50 1/2</td>
<td>L2 25 1/2</td>
<td>L3 30 1/2</td>
</tr>
<tr>
<td></td>
<td>60 1/2</td>
<td>58 1/2</td>
<td>56 1/2</td>
</tr>
<tr>
<td>1/4&quot;</td>
<td>L0 70 1/2</td>
<td>L2 30 1/2</td>
<td>L3 35 1/2</td>
</tr>
<tr>
<td></td>
<td>80 1/2</td>
<td>78 1/2</td>
<td>76 1/2</td>
</tr>
</tbody>
</table>

**NOTES**

- *BEYOND SECTION WHERE THERE IS NO STRESS, THIS TABLE ONLY APPLIES TO BOND STRESS (20 LB.) BARS IN TENSION.

**MIN. LAP:—**

- TENSILE STRESS OF 14,400 LB. PER SQ. IN. CORRESPONDS TO L1 = L0 - 16D.

- BARS IN COMPRESSION

- TENSILE STRESS OF 14,400 LB. PER SQ. IN. CORRESPONDS TO L3 = L0 - 12D.

**MIN. LAP:—**

- 24D CORRESPONDS TO A COMPRESSIVE STRESS OF 14,400 LB. PER SQ. IN.

---

**Note:** The data in Table 63 is in accordance with B.S. Code No. 114 (1957) for plain round mild steel bars in 1:2:4 concrete; $s_b = 120$ lb. per sq. in.

---

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<table>
<thead>
<tr>
<th>SECTION</th>
<th>AREA</th>
<th>SECTION MODULUS</th>
<th>( \text{2ND MOMENT OF AREA ABOUT X-X)} )</th>
<th>( \text{MOMENT OF INERTIA ABOUT Y-Y) )</th>
<th>( \text{RADIUS OF GYRATION) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>TRAPEZIUM</td>
<td>( \frac{b+d}{2} )</td>
<td>( XX. \frac{BD}{24} )</td>
<td>( XX. \frac{BD^3}{12} )</td>
<td>( YY. \frac{36 BD^3}{12} )</td>
<td>( \sqrt{I_{xx}} )</td>
</tr>
<tr>
<td>REG. HEXAGON</td>
<td>( \frac{8.66 D^2}{3.177 D} )</td>
<td>( XX. \frac{0.12 D^3}{3} )</td>
<td>( XX. \frac{0.06 D^3}{4} )</td>
<td>( YY. \frac{0.104 D^3}{3} )</td>
<td>( \sqrt{I_{xx}} )</td>
</tr>
<tr>
<td>ELLIPSE</td>
<td>( \frac{b+D}{2} )</td>
<td>( YY. \frac{0.098 BD^2}{32} )</td>
<td>( YY. \frac{0.049 BD^2}{D} )</td>
<td>( \frac{I_{xx}}{C+BD} = \frac{0.049 BD^2}{D} )</td>
<td>( \sqrt{I_{xx}} )</td>
</tr>
<tr>
<td>RECTANGLE</td>
<td>( \frac{b+d}{2} )</td>
<td>( XX. \frac{bd^3}{12} )</td>
<td>( XX. \frac{bd^3}{12} )</td>
<td>( \frac{bd^3}{12} )</td>
<td>( \sqrt{I_{xx}} )</td>
</tr>
<tr>
<td>FLANGED BEAM</td>
<td>( b_n )</td>
<td>( \frac{b_n}{d_n} )</td>
<td>( N = d_n - n )</td>
<td>( \frac{I_{xx} + I_{y}y}{N} = \frac{bd^2 + d_n^2 (b-b_n)}{2(b_n^2 + b_n (d-d_n))} )</td>
<td>( \sqrt{I_{xx}} )</td>
</tr>
<tr>
<td>SQUARE</td>
<td>( \frac{D^2}{4} )</td>
<td>( XX. \frac{0.118 D^3}{12} )</td>
<td>( XX. \frac{0.055 D^3}{12} )</td>
<td>( \frac{I_{xx}}{C+BD} = \frac{0.049 BD^2}{D} )</td>
<td>( \sqrt{I_{xx}} )</td>
</tr>
<tr>
<td>REG. OCTAGON</td>
<td>( \frac{0.828 D^2}{4} )</td>
<td>( XX. \frac{0.109 D^3}{12} )</td>
<td>( XX. \frac{0.055 D^3}{12} )</td>
<td>( \frac{I_{xx}}{C+BD} = \frac{0.049 BD^2}{D} )</td>
<td>( \sqrt{I_{xx}} )</td>
</tr>
<tr>
<td>CIRCLE</td>
<td>( \frac{7854 D^2}{32} )</td>
<td>( XX. \frac{0.892 D^3}{32} )</td>
<td>( XX. \frac{0.092 D^3}{32} )</td>
<td>( \frac{I_{xx}}{C+BD} = \frac{0.049 BD^2}{D} )</td>
<td>( \sqrt{I_{xx}} )</td>
</tr>
</tbody>
</table>

**MOMENT OF INERTIA OF TEE SECTION:**

(Also applicable to ell section and inverted channel)

Through centroid

\[ I_{xx} = Cb_1 d^3 \]

Values of \( C \) are tabulated.

<table>
<thead>
<tr>
<th>( \frac{d_s}{d} )</th>
<th>( b/s )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.10</td>
<td>12</td>
</tr>
<tr>
<td>0.15</td>
<td>170</td>
</tr>
<tr>
<td>0.20</td>
<td>180</td>
</tr>
<tr>
<td>0.25</td>
<td>180</td>
</tr>
<tr>
<td>0.30</td>
<td>184</td>
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<tr>
<td>0.35</td>
<td>190</td>
</tr>
<tr>
<td>0.40</td>
<td>197</td>
</tr>
<tr>
<td>0.45</td>
<td>208</td>
</tr>
<tr>
<td>0.50</td>
<td>208</td>
</tr>
</tbody>
</table>
**PROPERTIES OF REINFORCED CONCRETE SECTIONS. — TABLE 65A.**

<table>
<thead>
<tr>
<th>ENTIRE CROSS-SECTION SUBJECTED TO STRESS</th>
<th>BENDING ONLY CONCRETE INEFFECTIVE IN TENSION COMPRESSION ZONE AT TOP (AS DRAWN)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>NOTATION</strong></td>
<td><strong>DISTANCE OF NEUTRAL PLANE BELOW TOP EDGE:</strong> n = c(dn) per B.S. Code</td>
</tr>
<tr>
<td><strong>(ADDITIONAL TO DATA ON DIAGRAMS)</strong></td>
<td>n = c(dn) per B.S. Code</td>
</tr>
<tr>
<td>GEOMETRICAL PROPERTIES ARE EXPRESSED IN</td>
<td>RELATIVED TO MAX. STRESSES: n = c(dn) per B.S. Code</td>
</tr>
<tr>
<td>EQUIVALENT CONCRETE UNITS</td>
<td>LEVER ARM: a = c(εc) per B.S. Code</td>
</tr>
</tbody>
</table>
|                                          | COMPRESSION REINFORCEMENT FACTOR: K = (m-1) Math(h-c)
|                                          | MOMENT OF RESISTANCE: Mrc(Compression) = aCt(εc) unless expressed otherwise |
|                                          | Mrc(Tension) = aεt(εc) otherwise |

**RECTANGULAR**

\[
A_e = b d + (m-1)(A_{st} + A_{se})
\]

\[
X = \frac{1}{A_e} \left( \frac{b^2 d}{2} + (m-1)(A_{st} d + A_{se} d) \right)
\]

\[
I_{xx} = \frac{b^4 d}{12} + \frac{4d}{6}(m-1)(A_{st} d + A_{se} d)^2 - \frac{d^3}{6} A_{st} + \frac{5d^3}{6} A_{se}
\]

**SQUARE**

\[
A_e = d^2 + 4(m-1) A_t
\]

**TRAPEZIUM**

\[
A_e = \frac{(b + b_b)(d + d_b) + (m-1)A_{st} + A_{se}}{b + d}
\]

**TRIANGLE**

\[
A_e = \frac{b d + (m-1)(A_{st} + A_{se})}{b + d}
\]

\[
X = \frac{1}{A_e} \left( \frac{b^2 d}{2} + (m-1)(A_{st} d + A_{se} d) \right)
\]

**I-SECTION**

\[
A_e = b d + b_r d_r + b_r (0 - d_r - d) + (m-1)A_{st} + A_{se}
\]

**TEE SECTION**

\[
A_e = b d + b_r (d_r - d) + (m-1)A_{st}
\]

**ELLIPSE SECTION**

\[
A_e = \frac{b d^3}{12} + 2(m-1)A_{st} + \frac{(d - d_s)^2}{2} A_{st}
\]

\[
A_{se} = A_{st}.
\]
Formule containing summation sign $\Sigma$ apply to irregular sections only (Figs. a and b). Formule containing integration sign $\int$ apply only to regular sections (Fig. c) in which $b_x$ is a mathematical function of $x$.

**Entire section subjected to stress.**

Effective area. — $A_e = \sum_0^D [b_x \cdot 8x + (m - 1)8A_c] = \left[ \int_0^D b_x \cdot dx \right] + (m - 1)(A_{se} + A_4)$.  

Position of centroid. — $X = \frac{1}{A_e} \left\{ \sum_0^D x[b_x \cdot 8x + (m - 1)8A_c] \right\}$  

$Y = \frac{1}{A_e} \left\{ \sum_0^b y[d_x \cdot 8y + (m - 1)8A_c] \right\} = \frac{b}{2}$ (regular section).

Moment of inertia about axes through centroid. — $I_{XX} = \sum_0^D [b_x \cdot 8x + (m - 1)8A_c](X - x)^2$  

$= \left[ \int_0^D b_x(X - x)^2 \cdot dx \right] + (m - 1)\left[A_{se}X - d_x^2 + A_4(d_1 - X)^2\right]$  

$I_{YY} = \sum_0^b [d_y \cdot 8y + (m - 1)8A_c](Y - y)^2$.

Radius of gyration. — $g_X = \sqrt{\frac{I_{XX}}{A_e}}$; $g_Y = \sqrt{\frac{I_{YY}}{A_e}}$.

Modulus of section. — $Z(x = 0) = \frac{I_{XX}}{X}; Z(x = m) = \frac{I_{XX}}{D - X}; Z(y = 0) = \frac{I_{YY}}{Y}; Z(y = b) = \frac{I_{YY}}{b - Y}$.

**Section subjected to bending only.**

$\delta C = (n - x)[b_x \cdot 8x + (m - 1)8A_c]; \delta T = (x - n)8A_t$

Total compression factor. — $C' = \sum_0^n \delta C = \left[ \int_0^n (n - x)b_x dx \right] + (m - 1)(n - d_x)A_{se}$.

Total tension factor. — $T' = \sum_0^n \delta T = (d_1 - n)A_{st}$.

Position of neutral plane. — General: Value of $n = d_n$ satisfying formula $C' - mT' = 0$.

In terms of maximum stresses: $n = \frac{d_n}{m_{fo}} = \frac{d_1}{m_{fo}}$.

(Continued on page 253.)
### Properties of Reinforced Concrete Sections

#### Circular and Octagonal

<table>
<thead>
<tr>
<th>NOTATION (ADDITIONAL TO DATA ON DIAGRAMS)</th>
<th>ENTIRE CROSS-SECTION SUBJECTED TO STRESS</th>
<th>BENDING ONLY CONCRETE INEFFECTIVE INTENSITY, COMPRESSION ZONE AT TOP (AS DRAWN)</th>
</tr>
</thead>
<tbody>
<tr>
<td>GEOMETRICAL PROPERTIES ARE EXPRESSED IN EQUIVALENT CONCRETE UNITS</td>
<td>EFFECTIVE AREA: $A_e = 0.828 D^2 + 8A_t (m-1)$</td>
<td>DISTANCE OF NEUTRAL PLANE BELOW TOP EDGE: $n = \left(\frac{d_2}{D \cdot c_{cb}}\right) \times \frac{d_2}{m \cdot c_{cb}}$</td>
</tr>
<tr>
<td></td>
<td>POSITION OF CENTROID: $X = 0.5D$</td>
<td>RELATED TO MAX. STRESSES: $n = \left(\frac{d_2}{D \cdot c_{cb}}\right) \times \frac{d_2}{m \cdot c_{cb}}$</td>
</tr>
<tr>
<td></td>
<td>MOMENT OF INERTIA ABOUT CENTROIDAL AXIS: $I_{xx} = \frac{1}{12} A_e X^2$</td>
<td>LEVER ARM: $a = \left(\frac{d_2}{D \cdot c_{cb}}\right) \times \frac{d_2}{m \cdot c_{cb}}$</td>
</tr>
<tr>
<td></td>
<td>MODULUS OF SECTION FOR TOP EDGE: $Z_0 = 0.103D^3 + \frac{4A_T R^2}{D}$</td>
<td>UNLESS OTHERWISE EXPRESSED</td>
</tr>
<tr>
<td></td>
<td>FOR BOTTOM EDGE: $Z_o = 0.103D^3 + \frac{4A_T R^2}{D}$</td>
<td>$M_{rc}(compression) = a \frac{f_{cb}}{2} \frac{f_{cb}}{A_T}$</td>
</tr>
<tr>
<td></td>
<td>RADIUS OF GYRATION: $g = \sqrt{\frac{I_{xx}}{A_e}}$</td>
<td>$M_{rt}(tension) = a \frac{f_{cb}}{2} \frac{f_{cb}}{A_T}$</td>
</tr>
</tbody>
</table>

#### Octagon

- $A_e = 0.828 D^2 + 8A_t (m-1)$
- $X = 0.5D$
- $I_{xx} = \frac{1}{12} (D^4 - D_1^4) + (m-1) \frac{R^2}{2}$
- $Z_0 = Z_o = \frac{1}{32} \left( D^2 - D_1^2 \right) + (m-1) \frac{R^2}{2}$

#### Circle

- $A_e = 0.7854D^2 + (m-1) \Sigma A_t$
- $X = 0.5D$
- $I_{xx} = \frac{1}{4} \pi D^4 + (m-1) \frac{R^2}{2} \Sigma A_t$
- $Z_o = Z_0 = \frac{1}{32} \left( D^2 - D_1^2 \right) + (m-1) \frac{R^2}{2} \Sigma A_t$

#### Annulus

- $A_e = 0.7854(D^2 - D_1^2) + (m-1) \Sigma A_t$
- $X = 0.5D$
- $I_{xx} = \frac{1}{4} \pi (D^4 - D_1^4) + (m-1) \frac{R^2}{2} \Sigma A_t$
- $Z_o = Z_0 = \frac{1}{32} \left( (D^2 - D_1^2)^2 + (m-1) \frac{R^2}{2} \Sigma A_t$

(Continued from page 252.)

Lever arm: $l_a = \frac{\sum_{i=1}^{n} x_i \cdot 8T}{T'} - \frac{\sum_{i=1}^{n} c_i \cdot 8C}{C'} = d_1 = \left( \int_{0}^{n} x(n - x) b_{x,d} \cdot dx \right) + (m - 1) (n - d_2) A_{at} d_2$

Moment of resistance: $M_{re} = \frac{p_{el} l_a}{n} C'$, $M_{rl} = \frac{p_{el} l_a m}{n} T'$ = $p_{el} a_{et}$

Notes:
1. For properties of common reinforced concrete sections see Tables 65A and 65B.
2. For properties of sections subjected to stress on entire section, but neglecting reinforcement, omit terms $8A_t$, $A_{at}$, $8A_c$ and $A_{ac}$ from foregoing formulas. Properties of some common sections for this condition are given in Table 64.

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REINFORCED CONCRETE DESIGNER'S HANDBOOK

DESIGN OF BEAMS: MODULAR-RATIO METHOD.

Notation.

Properties of Section.

\(A_{cc}\) = area of compression reinforcement.

\(I_a\) = lever arm = \(a_1d_1\); \(l_{cc}\) (concrete); \(l_{cs}\) (compression reinforcement).

\(A_{st}\) = area of tensile reinforcement.

\(b\) = breadth of a rectangular beam or the breadth of slab of flanged beams or breadth of slab assumed to be effective.

\(b_r\) = breadth of rib of flanged beam.

\(d_1\) = effective depth of beam, that is the distance from the compression edge of the section to the centroid of the tensile reinforcement.

\(d_s\) = thickness of slab of flanged beam; \(s_1 = \frac{d_s}{d_1}\).

\(d_n\) = depth to centroid of compression reinforcement from adjacent edge of beam = \(f_d d_1\)

\(M\) = applied bending moment. \(M_r\) = moment of resistance.

\(d_1\) = depth to neutral plane = \(n_1d_1\).

\(r_t\) = proportion of tensile reinforcement = \(\frac{A_{st}}{bd_1}\); \(r_b\) for balanced design.

\(r_c\) = proportion of compression reinforcement = \(\frac{A_{cc}}{bd_1}\).

\(Q_c\) = factor in moment of resistance formula, \(M_r = Q_c bd_1^2\).

Stresses.

\(f_{cb}\) = maximum permissible compressive stress in concrete.

\(f_{et}\) = actual maximum compressive stress in concrete.

\(f_{st}\) = maximum permissible stress in tensile reinforcement.

\(f_{se}\) = actual stress in tensile reinforcement.

\(f_{se}\) = actual stress in compression reinforcement.

\(m\) = modular ratio = \(\frac{\text{modulus of elasticity of steel}}{\text{modulus of elasticity of concrete}} = \frac{E_s}{E_c}\).

\(r\) = ratio of maximum permissible stresses = \(\frac{\sigma_{et}}{\sigma_{cb}}\).

\(v_1\) = ratio of actual stresses = \(\frac{f_{et}}{f_{cb}}\).

Procedures.

The procedures for the design of rectangular beams and for the determination of stresses in beams make use of the formula in Table 66 and the data in Tables 67, 68, 69 and 70A to 70D. The designs are based on the modular-ratio method with \(m = 15\).

To Design a Rectangular Beam to Resist a Given Bending Moment with Given Stresses.

Method (a).—For the given stresses find \(Q_c\) from Table 68 and find \(bd_1^2\) required from \(\frac{M}{Q_c}\).

Select suitable values of \(b\) and \(d_1\) from consideration of shearing, or from Table 69 to give the required value of \(bd_1^2\). For the ratio of the given stresses find \(a_1\) from Table 68 and the amount of the tensile reinforcement \(A_{st}\) from formula (10) in Table 66 with \(l_a = a_1d_1\).

Method (b).—If the permissible stresses are any of those in Tables 70A to 70D, assume an effective depth \(d_1\) and read from the appropriate table the moment of resistance corresponding to this depth. The applied bending moment divided by this moment of resistance gives the breadth of beam required. If the relative values of \(d_1\) and \(b\) thus derived are unsuitable, select another value of \(d_1\) and repeat. From the same table read the area of reinforcement for the selected value of \(d_1\) and multiply this area by \(b\) to give \(A_{st}\).

Method (c).—If Tables 70A to 70D do not apply, for the permissible stresses read \(r_t\), \(n_1\), \(a_1\) and \(r_1\) from the curves in Table 67, and calculate \(Q_c\) from formula (8) in Table 66. Then \(bd_1^2\) is calculated from \(\frac{M}{Q_c}\) and \(b\) and \(d_1\) are obtained from Table 69. The area of tensile reinforcement is found from \(A_{st} = r bd_1\). Alternatively, the values of \(n_1\), \(a_1\) and \(Q_c\) can be obtained from formula (1a), (5), and (8); then \(bd_1^2 = \frac{M}{Q_c}\); find \(b\) and \(d_1\) from Table 69 and \(A_{st}\) from \(\frac{M}{a_1d_1r_{st}}\).

The values of \(b\) and \(d_1\) should be checked to ensure sufficient resistance to the shearing force.
## Rectangular and Flanged Beams: Formulæ—Table 66.

### Modular Ratio Method

<table>
<thead>
<tr>
<th><strong>Modular Ratio</strong></th>
<th>( m = \frac{E_s}{E_c} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f_{st} )</td>
<td>Actual Stresses in Tensile Reinforcement and Concrete</td>
</tr>
<tr>
<td>( f_{cb} )</td>
<td>Permissible Stresses in Ditto</td>
</tr>
<tr>
<td>( P_{st} )</td>
<td>Compressive Forces in Concrete and Compression Reinforcement</td>
</tr>
</tbody>
</table>

### Rectangular Beam

1. **Position of Neutral Plane**
   - \( \frac{d_n}{n_1} = \frac{1}{f_{st}} \)  
   - \( \frac{n_1}{1 + \frac{P_{st}}{m_{fcb}}} \)

### Flanged Beam (\( d_n > d_s \))

2. **With Tensile Reinforcement Only**
   - \( n_1 = \sqrt{(m_{r_t})^2 + 2m_{r_t} - m_{r_t}} \)

3. **With Compression Reinforcement**
   - \( n_1 = \sqrt{m_{r_t} + r_c} + 2m_{r_t} + 2f_{rc} - m_{r_t} + r_c - r_t \)

### Lever Arm

4. **Concrete Only**
   - \( \ell_a = a_1d_1 \)

5. **Compression Only**
   - \( \ell_{ac} = \frac{a_1d_1}{2n_1} \)

6. **Compression and Tensile Reinforcement**
   - \( \ell_a = \ell_{ac} = d_1 - \frac{d_n}{3}; a_1 = 1 - \frac{3}{2}n_1 \)

### Moments of Resistance

7. **Compression**
   - \( M_{rc} = Q_cbd_1^2 \); \( Q_c = \frac{1}{2}n_1(1 - \frac{3}{2}n_1) f_{cb} \)

8. **Tension**
   - \( M_{rt} = \ell_{ac}d_1f_{st} \)

### Design

9. **To Resist Bending Moment**
   - \( M = M_{rc} \) of Concrete
   - \( M_{st} = M_{rt} \) Required from Compression Reinforcement
   - \( A_s = \frac{M}{Q_{cb}d_1^3}; A_{st} = \frac{M}{Q_{st}d_1^3} \)
   - \( A_{ac} = \frac{M_{rc} + M_{rt}}{Q_{cb}d_1^3} \)

### Proportion of Tensile Reinforcement for Balanced Design

10. **Ratios of Stresses**
    - \( \frac{f_{st}}{f_{cb}} = \frac{n_1}{m_{r_t}} \)

11. **Ratios of Stresses**
    - \( \frac{f_{st}}{f_{cb}} = m \left( \frac{n_1 - 1}{n_1} \right) \)
To Determine the Stresses in a Rectangular Beam Subjected to a Given Bending Moment.

Method (a).—Determine $r_1$ from the given data and from Table 68 find the corresponding values of $a_1$ and $n_1$. Then the maximum stresses are given by formula (14) in Table 66, in which $l_s = a_1d_1$; or $f_{ob} = \frac{f_{ci}}{r_1}$.

Method (b).—Determine from Table 67 the values of $a_1$ and $n_1$ corresponding to the known value of $r_1$ [or calculate $n_1$ from formula (2) and $a_1$ from formula (5) in Table 66]; also find the corresponding value of $r_1$ and calculate the stresses as in Method (a).

To Determine the Moment of Resistance of a Rectangular Beam at Given Maximum Stresses.

Method (a).—Determine $r_1$ and from Table 68 find the values of $a_1$ and $n_1$. The moment of resistance based on the reinforcement is given by formula (8a) or, based on the concrete, by formula (8) in Table 66. If the two results differ, the maximum safe moment of resistance is the smaller of the two calculated moments.

Method (b).—Determine $r_1$ and find the corresponding value of $a_1$ and $n_1$ from Table 67. The moments of resistance can then be calculated in the same way as in Method (a). In Methods (a) and (b) it is not necessary to calculate the moments of resistance for the resistance of the reinforcement and the concrete if the ratio of stresses $r_1$ corresponding to $r_1$ is determined (from Table 68 or 67). If $r_1$ is greater than the ratio of the permissible stresses, the moment of resistance of the reinforcement is the safe moment of resistance, and, if smaller, the moment of resistance of the concrete is the safe value.

To Design a Rectangular Beam with Compression Reinforcement.

Method (a).—With given, or selected, values of $b$ and $d_1$ find $Q_0$ from Table 68 for the specified permissible stresses, and from the same table find the value of $n_1$ for the ratio of these stresses. The required moment of resistance to be provided by the compression reinforcement is $M_g = M - Q_0bd_1^2$. Substitute in formula (10a) in Table 66 to find $A_{se}$, the area of compression reinforcement required. Evaluate the lever arms $l_{as}$ from formula (4b) and $l_{as}$ from formula (5), or approximately from formula (5b), and find the area of tensile reinforcement $A_{st}$ required from formula (10a) in Table 66. If $A_{se}$ is greater than $A_{st}$, Method (c) should be used. If the stresses in Tables 70a to 70d apply, these tables can be used directly if $A_{st} = A_{se}$.

Method (b).—Find $n_1$ and $a_1$ from Table 67 for the given stresses, and calculate $Q_0$. Then proceed as Method (a).

Method (c).—If in Methods (a) or (b) $A_{se}$ exceeds $A_{st}$ the "steel-beam" theory can be applied by substitution in $A_{st} = \frac{M}{l_{as}P_{st}}$ and $A_{se} = \frac{M}{l_{as}P_{se}}$.

To Determine the Stresses in a Rectangular Beam with Compression Reinforcement.

Method (a).—Determine $n_1$ from formula (2a) and $a_1$ from formula (5a) [or approximately from formula (5b)]. Substitute in formula (14), in which $l_s = a_id_1$, to give the maximum stresses.

To Find the Moment of Resistance of a Rectangular Beam with Compression Reinforcement.

Method (a).—Determine $n_1$ and $a_1$ from formula (2a) and (5a), or (5b) for the known values of $r_1$ and $r_2$. Substitute with the permissible stresses in formulae (6b) and (8c) in Table 66; the smaller of the two moments is the safe moment of resistance.

To Design a Flanged Beam for Given Maximum Stresses and $a_1$ Given Bending Moment.

The thickness of the slab $d_s$, the effective depth $d_{ef}$, the breadth of the rib $b$, and the maximum effective width of the flange $b$ are generally known since they are determined from considerations other than the bending moment on the beam. Find $n_1$ for the given stresses from Tables 67 or 68 or from formula (14a). If $n_1d_s$ does not exceed $d_{ef}$, proceed as for a rectangular beam. If $d_s = n_1d_1$ exceeds $d_{ef}$, find $a_1$ from formula (6a) and $A_{st}$ from formula (11). Check the breadth of flange required from formula (11b) in Table 66.

For approximate calculations, $l_s$ is given by formula (6b), $A_{st}$ by formula (11a), and $b$ is checked from formula (11c).
RECTANGULAR BEAMS: DESIGN FACTORS.—TABLE 67.
MODULAR-RATIO METHOD.

LEVER ARM FACTORS: \( a_1 = \frac{L_0}{d_1} \)

PERCENTAGE OF REINFORCEMENT \( m = 15 \)

Neutral Plane Factor \( n_1 = \frac{d_m}{d_1} \)

RATIO OF STRESSES: \( \frac{f}{f_{cb}} = \frac{p_{st}}{p_{cb}} \) or \( \frac{f_{st}}{f_{cb}} \)

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REINFORCED CONCRETE DESIGNER'S HANDBOOK

DESIGN OF BEAMS: MODULAR-RATIO METHOD

Beams with Irregular Cross-section.—Beams of irregular cross-section can be conveniently designed by the following semi-graphical method.

Draw the cross-section to scale as in the diagram and assume a position for the neutral plane. Divide the area above the neutral plane into strips parallel to the neutral plane. For irregular sections, the depth \( h \) of each strip should be equal, but any regularity in the shape of the cross-section may suggest positions for the boundaries of the strips such that each strip is a common geometrical figure. The area \( a_i \) is the area of each bar, or of a number of bars each of which is at the same distance \( p_i \) from the lowest bar or bars. The measurements to be taken from the diagram are (i) \( a_i \) and the corresponding distance \( p_i \); (ii) \( g, h, \) and \( h_0 \) for each of the strips above the neutral plane. The following summations are required: \( S_1 = \Sigma a_i (d_i - d_a - p_i) \); \( S_2 = \Sigma (g h) (d_a - h_0) \); and \( S_4 = \Sigma (g h) (d_a - h_0) h_0 \). The product \( (gh) \) is the area of each strip above the neutral plane and, if there is any reinforcement, say, \( a_o \), in any strip, \( (gh) \) should be increased by \((m - 1)a_o\).

The position of the centre of tension above the lowest bar or bars is \( p_T = \frac{S_4}{S_1} \). The position of the centre of compression below the top edge of the section is \( p_C = \frac{S_2}{S_1} \). The lever-arm \( l_a = d_1 - p_a - p_T \).

Having evaluated numerically the foregoing terms, they are substituted in the following expressions to determine the maximum compressive stress in the concrete and the maximum tensile stress in the steel: \( f_c = \frac{M d_a}{l_a S_2} \) and \( f_t = \frac{f_o (d_1 - d_a)}{S_2} \), where \( M \) is the applied bending moment.

The assumed depth to the neutral axis is checked by substitution in formula (i) in Table 66. If the difference between the assumed and calculated values is greater or less than, say, \( h \) the depth of one strip, another value between the two foregoing values should be assumed. The adjustment to the summations is easily made because the distances \( h_0 \) and \( p_i \) are measured from the outer edges of the section, and the adjustment consists of adding or deducting one or more strips and excluding or including one or more areas \( a_i \) of reinforcement below the neutral plane.

I-Beams.—The formula for the design of a beam of I-section are as follows.

Moment of resistance:

\[ Q' d_4 = M \]

Area of tensile reinforcement:

\[ A_{st} = \frac{(2g h_1 + C_t - C_t) d_4 p_o}{f_t} \]

in which

\[ Q' = Q_o + \left( C_t (1 - f_i) - \frac{C_t}{3} (3 - 2y - n_1) \right) p_o \]

\[ C_t = \frac{1}{4a_t} \left( n_1 - f_i \right) \]

\[ C_t = \left( x - x_1 \right) \left( n_1 - x_1 \right) \]

The method of designing a beam to resist a bending moment \( M \) is first to determine the neutral-plane factor \( n_1 \), from Tables 65, 67 or 68 for the given permissible stresses. The moment of resistance \( Q_o \) is obtained from formulae in Table 66 or from Table 68. Assume, if they are not given, the dimensions \( d_1, d_2, x_b, y_d_1, \) and \( f_d a_1 \). Calculate \( C_t \) from the formula above. The value of \( C_t \) required is

\[ C_t = \left[ M \left( \frac{1}{bd_t^2 p_o} - \frac{Q_o}{p_o} + \frac{C_t}{3} (3 - 2y - n_1) \right) \right] \frac{1}{1 - f_i} \]

The cross-sectional area of compression reinforcement required is: \( A_{cs} = \frac{C_t n_1 b d_1}{14(n_1 - f_i)} \). \( A_{st} \) is calculated from the formula given above. The lever-arm \( l_a \) is \( \frac{M}{A_{st} p_o} \). The maximum shearing stress is \( \frac{M}{x_b l_a} \).
### RECTANGULAR BEAMS: DESIGN FACTORS—TABLE 68.

#### MODULAR-RATIO METHOD.

<table>
<thead>
<tr>
<th>RATIO OF STRESSES $\frac{P_{st}}{P_{cb}}$</th>
<th>NEUTRAL AXIS FACTOR $n_1$</th>
<th>LEVER ARM FACTOR $\alpha_1$</th>
<th>PERCENTAGE OF TENSILE REINFORC. $100r_t$</th>
<th>VALUES OF $Q_c$ ($m = 15$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>MAX. CONC. STRESS $P_{cb}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>MAXIMUM STEEL STRESS $= P_{st}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>12,000</td>
<td>16,000</td>
</tr>
<tr>
<td>5</td>
<td>0.750</td>
<td>0.750</td>
<td>7.50</td>
<td>500</td>
</tr>
<tr>
<td>10</td>
<td>0.600</td>
<td>0.800</td>
<td>3.00</td>
<td>550</td>
</tr>
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<td>0.517</td>
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<td>1.66</td>
<td>700</td>
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<tr>
<td>16</td>
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<td>0.839</td>
<td>1.52</td>
<td>750</td>
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<tr>
<td>17</td>
<td>0.468</td>
<td>0.844</td>
<td>1.38</td>
<td>760</td>
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<tr>
<td>18</td>
<td>0.455</td>
<td>0.848</td>
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<tr>
<td>19</td>
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<tr>
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<td>0.75</td>
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<td>26</td>
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<td>0.881</td>
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<td>1200</td>
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<td>0.56</td>
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<td>0.53</td>
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</tr>
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<td>0.894</td>
<td>0.50</td>
<td>1400</td>
</tr>
<tr>
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<td>0.311</td>
<td>0.896</td>
<td>0.47</td>
<td>1500</td>
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<tr>
<td>34</td>
<td>0.306</td>
<td>0.898</td>
<td>0.45</td>
<td>1650</td>
</tr>
</tbody>
</table>

**FORMULÆ:**

\[ n_1 = \frac{1}{1 + \frac{P_{st}}{mP_{cb}}} \]

\[ \alpha_1 = 1 - \frac{n_1 \times r_t}{2r_t} \]

\[ Q = \frac{M_r}{bd_t^2} = \frac{n_1 \alpha_1 P_{cb}}{2} \]

**FOR ANY MODULAR RATIO ($m$) AND GIVEN VALUES OF $P_{cb}$ AND $P_{st}$—**

**CALCULATE** $r_t = \frac{15P_{st}}{mP_{cb}}$ **AND FOR THIS VALUE OF** $r_t$ **OBTAIN FROM THE TABLE THE CORRESPONDING VALUES OF** $n_1$ **AND** $\alpha_1$. **SUBSTITUTE IN** $Q_c = 0.5n_1\alpha_1 P_{cb}$ **AND** $Q_c = \frac{1}{2}$. **FOR EXAMPLES OF THE USE OF THIS TABLE, SEE PAGE FACING** Table 69.

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REINFORCED CONCRETE DESIGNER’S HANDBOOK

DESIGN OF BEAMS: MODULAR-RATIO METHOD.

Curtailment of Bars.—The positions at which some of the bars in the bottom of a beam can be terminated or bent up can be estimated from the data in Table 69. The positions indicated by the coefficients $k_1$, $k_2$ and $k_3$ are those at which specific bars are no longer required as tensile reinforcement; if they are not bent up, the bar should be continued a distance of not less than twelve times its diameter beyond the position indicated.

Example.—Determine the minimum distance from the supports of the end-span of a continuous beam, carrying a uniformly-distributed load, that two bars can be bent up to assist shearing resistance; the total number of bars of equal diameter in the bottom at mid-span is six; the span of the beam is 20 ft.

From Table 69 for the second bar of six, $k_2 = 0.30$ and $k_3 = 0.18$. Hence the distance from the inner support is $0.30 \times 20 = 6$ ft. and the distance from the outer support is $0.18 \times 20 = 3$ ft. 7 in.

Examples of Use of Tables 68 and 69.—The designs in the following are to be in accordance with the modular-ratio method with $m = 15$.

(a) Design a rectangular beam to resist a bending moment of 500,000 in.-lb. with maximum stresses of 18,000 and 750 lb. per sq. in. From Table 68, $Q_e = 126$. Hence $bd_1^2 = \frac{500,000}{126} = 3968$. From Table 69, if $b = 10$ in., $d_1 = 19$ in.; therefore make the total depth 21 in.; $d_1 = 19\frac{1}{2}$ in., $r_1 = \frac{18,000}{750} = 24$; from Table 68, $a_1 = 0.87$.

Hence $A_{st} = \frac{500,000}{0.87 \times 19\frac{1}{2} \times 18,000} = 1.64$ sq. in.; three $\frac{2}{3}$-in. bars (Table 60).

(b) Calculate the maximum stresses in a rectangular beam 15 in. deep overall (effective depth = 13\frac{3}{8} in.) and 9 in. wide, reinforced in tension with two 1-in. bars, and subjected to a bending moment of 200,000 in.-lb. $A_{st} = 1.57$ (Table 60).

From Table 68: $a_1 = 0.85$ and $r_1 = 18$. $f_{st} = \frac{200,000}{0.85 \times 13\frac{3}{8} \times 1.57} = 11,100$ lb. per sq. in.;

$f_{ob} = \frac{11,100}{18} = 617$ lb. per sq. in.

(c) Calculate the moment of resistance of the beam in (b) if stresses are not to exceed 18,000 lb. and 1000 lb. per sq. in. 100$r_1 = 1.26$, $a_1 = 0.85$, $n_1 = 0.46$ (Table 68).

$M_r$ (tension) = $1.57 \times 18,000 \times 0.85 \times 13\frac{3}{8} = 324,000$ in.-lb.

$M_r$ (concrete) = $9 \times 0.46 \times (0.5 \times 1000) \times 0.85 \times 13\frac{3}{8} = 320,000$ in.-lb.

Hence the compressive stress controls, and safe moment of resistance is 320,000 in.-lb.

(d) Calculate the amount of tensile reinforcement in a tee-beam, the rib of which is 10 in. wide and extends 18 in. below the soffit of a 6-in. slab. The span is 18 ft. and the distance between adjacent beams 8 ft. Maximum stresses not to exceed 18,000 and 750 lb. per sq. in. Bending moment = 1,800,000 in.-lb.

Effective breadth: Available slab = 96 in., or $\frac{1}{2}$ span = 72 in., or $12d + b = (12 \times 6) + 10 = 82$ in.; hence maximum $b = 72$ in.; effective depth, say, 20 in. (Two layers of bars). From Table 68, $d_8$ (for $r_1 = 24$) = $0.39 \times 20 = 8$ in.; the neutral plane is below the slab; $l_a = 20.5 + 3 = 17.5$ in. From formula in Table 66,

check $M_{re} = \frac{750 \times 72 \times 17.5 \times 6}{2 \times 8}[(2 \times 8) - 6] = 3,530,000$ in.-lb., which is ample.

$A_{st} = \frac{1,800,000}{18,000 \times 17.5} = 5.71$ sq. in.; say, six $\frac{1}{4}$-in. bars.

(e) Calculate the moment of inertia of tee-beam of 20 ft. span with rib 8 in. wide and 16 in. deep below a 4-in. slab. $d = 4$ in., $d = 4$ in. + 16 in. = 20 in. $\frac{d_8}{d} = \frac{4}{20} = 0.20$.

$b_r = 8$ in. From Table 69, the effective width of flange $b$ is the smaller of (i) $12d + b_r = (12 \times 4) + 8 = 56$ in., and (ii) $\frac{20}{3} = 6.7$ ft. 8 in. Hence $b = 56$ in.; $\frac{b_r}{b} = \frac{8}{56} = 0.14$.

From Table 64 with $\frac{d_8}{d} = 0.20$ and $\frac{b_r}{b} = 0.14$, $C = 0.175$.

Therefore the moment of inertia is $0.175 \times 8 \times 20^3 = 11,200$ in.\(^4\)
### Table 28. Miscellaneous Data

#### Rectangular and Flanged Beams

<table>
<thead>
<tr>
<th>FORMULAE FOR BEAMS WHEN CONCRETE EFFECTIVE IN TENSION</th>
<th>FLANGED BEAMS</th>
<th>TERMINATING OR BENDING-UP BARS</th>
<th>VALUES OF $bd_1^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>M =</td>
<td>$f_{cd}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$= f_{cd} \times M_{e1}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$= f_{cd} \times M_{e2}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$= f_{cd} \times M_{e3}$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| $Z_o = J_{cd}$ | $f_{cd} = f_{cd}$ | | |
| $= f_{cd} \times Z_{e1}$ | | | |
| $= f_{cd} \times Z_{e2}$ | | | |
| $= f_{cd} \times Z_{e3}$ | | | |

| | | | |
| TOTAL AREA $A_e$ | | | |
| $= A_{e1}$ | | | |
| $= A_{e2}$ | | | |
| $= A_{e3}$ | | | |

| | | | |
| EFFECTIVE AREA $A_e$ | | | |
| $= A_{e1}$ | | | |
| $= A_{e2}$ | | | |
| $= A_{e3}$ | | | |

| | | | |
| MOI OF INERTIA $I_e$ | | | |
| $= I_{e1}$ | | | |
| $= I_{e2}$ | | | |
| $= I_{e3}$ | | | |

| | | | |
| TENSION AREA ONLY WITH COMPRESSION AREA | | | |
| $= A_{e1} - A_{e2}$ | | | |
| $= A_{e2} - A_{e3}$ | | | |
| $= A_{e3} - A_{e1}$ | | | |

| | | | |
| DISTANCE BETWEEN CENTRES OF RESISTANT DIMENSIONS | | | |
| $= d_{e1}$ | | | |
| $= d_{e2}$ | | | |
| $= d_{e3}$ | | | |

| | | | |
| ORDERS OF STANDING-UP OR BENDING-UP BARS | | | |
| | | | |

| | | | |
| PREM-SPANNED TO UNIFORMLY-DISTRIBUTED LOAD ONLY | | | |
| | | | |

| | | | |
| POSITIONS P AT WHICH BARS IN BOTTOM OF BEAMS CAN BE BENT UP OR STOPPED | | | |
| | | | |

| | | | |
| MAXIMUM DISTANCE FROM SUPPORT TO P, $K_1$, MUST BE PROVIDED BEYOND P | | | |
| | | | |

| | | | |
| IF BARS IS NOT BENT UP AT P | | | |
| | | | |

| | | | |
| INCLUDES THE EFFECTIVE DEPTH OF $d_1$ | | | |
| | | | |

| | | | |
| EFFECTIVE DEPTH $d_1$ | | | |
| | | | |

| | | | |
| TOTAL AREA $A_e$ | | | |
| | | | |
REINFORCED CONCÔTÈ DESIGNER’S HANDBOOK

DESIGN OF BEAMS: MODULAR-RATIO METHOD.

Examples of Use of Table 70A or 70B (Mild Steel).

Examples (a), (b) and (c) apply to Table 70A ($\rho_{st} = 18,000$ lb. per sq. in.). The calculations would be similar, but with different numerical values, if $\rho_{st} = 20,000$ lb. per sq. in. (Table 70B). In examples (a) and (b) $\rho_{ob} = 1000$ lb. per sq. in.; $m = 15$.

(a) Design a rectangular beam (tensile reinforcement only) to resist a bending moment of 450,000 in.-lb.

Assume a breadth of 10 in. Moment of resistance required is 45,000 in.-lb. per inch of width, which is given by $d_1 = 16$ in. and $A_{st} = 0.202 \times 10 = 2.02$ sq. in. A beam of 18 in. overall depth and 10 in. wide with three 1-in. bars is satisfactory. Other suitable sections can be determined by assuming other widths. The section must have sufficient bearing resistance and should be comparable in size to the span. If there is a difference between the effective depth of the section selected and that in the table, the reinforcement can be reduced thus: With overall depth 18 in. and 1-in. cover, $d_1 = 16.5$ in.

$Q_c = \frac{450,000}{10 \times 16.5^2} = 166$; for $\rho_{st} = 18,000$ lb., $\rho_{ob} = 900$ lb. per sq. in. (from Table 68). Hence

$r_1 = 20; a_1 = 0.857; A_{st} = \frac{450,000}{18,000 \times 0.857 \times 16.5} = 1.77$ sq. in., that is, three $\frac{3}{4}$-in. bars.

(b) Determine the reinforcement required in rectangular beams of the sizes given if the bending moment is 750,000 in.-lb.

(i) Overall depth 18 in. ($d_1 = 16$ in.); width 10 in.—This beam can resist $49,400 \times 10 = 494,000$ in.-lb. with $A_{st} = 0$, and 130,000 $\times 10 = 1,300,000$ in.-lb. if $A_{st} = A_{st}$. Compression reinforcement is required. The moment of resistance required from the compression reinforcement is 750,000 $- 494,000 = 256,000$ in.-lb. With $r_1 = 18, n_1 = 0.455$ (Table 68).

From the formulæ in Table 66 (assuming $d_2 = 2$ in.) $l_{as} = 16 - 2 = 14$ in.; $d_3 = 0.455 \times 16 = 7.3$ in. $f_{se} = \frac{7.3^2 - 2}{7.3} \times 14 \times 1000 = 10,170$ lb. per sq. in.; $A_{se} = \frac{256,000}{10,170 \times 14} = 1.80$ sq. in., that is, three $\frac{3}{4}$-in. bars in the top. With $1\frac{1}{4}$-in. cover, the actual effective depth is $18 - 1\frac{1}{4} - \frac{3}{4} = 16.2$ in., $\leq 16$ in.

(ii) Overall depth 15 in. ($d_1 = 13$ in.); width 9 in.—The bending moment per inch of width is 750,000 $\div 9 = 83,330$ in.-lb. which, if the concrete is taken into account, requires $A_{se} = A_{st} = 0.416 \times 9 = 3.74$ sq. in., that is three $1\frac{1}{4}$-in. bars in the top and bottom. Actual effective depth $= 15 - 1\frac{3}{4} - \frac{3}{4} = 13.13$ in., $\leq 13$ in. $l_{as} = 13 - 13 - 1\frac{3}{4} - \frac{3}{4} = 11.25$ in. = 0.8641, compared with the assumed 0.94; there is sufficient margin between the assumed and actual effective depths, and between the moment of resistance and the bending moment to compensate for the difference in the assumed and actual lever-arms.

(iii) Overall depth 15 in.; width 8 in.—Bending moment per inch of width is 93,800 in.-lb., $> 85,800$ in.-lb. ($A_{se} = A_{st}$ for $d_1 = 13$ in. and concrete taken into account); apply "steel-beam" theory. Assume two layers of 1-in. bars and 1-in. cover: $l_{as} = 10.5$ in. $A_{st} = A_{se} = \frac{750,000}{18,000 \times 10.5} = 3.97$ sq. in., that is five 1-in. bars top and bottom.

(c) Determine the reinforcement in a beam 22 in. deep ($d_1 = 19$ in.) and 12 in. wide, subjected to a bending moment of 2,750,000 in.-lb. if the stresses are not to exceed 18,000 lb. and 1100 lb. per sq. in.

The bending moment per inch of width is 2,750,000 $\div 12 = 230,000$ in.-lb., which requires the maximum (4 per cent.) of compression reinforcement. $A_{se} = 0.04 \times 19 \times 12 = 9.13$ sq. in.; $A_{st} = 0.792 \times 12 = 9.5$ sq. in., that is, eight $1\frac{1}{4}$-in. bars in the top and bottom. Since the bars must be in two layers, the actual effective depth with $1\frac{1}{4}$-in. cover is 19-25 in., which is not less than 19 in. Also $l_{as} = 16.5$ in. = 0.8751, compared with the assumed ratio of 0.94; since the area of compression reinforcement provided is 9.82 sq. in. compared with a calculated amount not exceeding 9.13 sq. in., there is a margin to allow for the lower stress.

(d) Design a rectangular beam without compression reinforcement to resist a bending moment of 550,000 in.-lb with stresses not exceeding 16,000 lb. and 800 lb. per sq. in. $r_1 = 20$; therefore use Table 70B for $\rho_{st} = 20,000$ lb. and $\rho_{se} = 1000$ lb. per sq. in., and design for a bending moment of 550,000 $\times 20,000 = 687,500$ in.-lb. Assuming a width of 16 in., the moment of resistance required is 57,300 in.-lb. per inch of width, which is given by an effective depth of 18 in. $A_{st} = 0.192 \times 12 = 2.3$ sq. in., that is four $\frac{3}{4}$-in. bars. With 1-in. cover, $d$ should be 20 in. The remarks in example (a) regarding other sections apply.
### Maximum Stress in Tensile Reinforcement = 18,000 lb. per sq. in. \( m = 15 \)

<table>
<thead>
<tr>
<th>EFFECTIVE DEPTH ( d_t ) IN.</th>
<th>( P_{cb} = 750 ) lb. per sq. in.</th>
<th>( P_{cb} = 1000 ) lb. per sq. in.</th>
<th>( P_{cb} = 1100 ) lb. per sq. in.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( A_{sc} = 0 )</td>
<td>( A_{sc} = A_{st} )</td>
<td>( A_{sc} = 0 )</td>
</tr>
<tr>
<td></td>
<td>( M_r )</td>
<td>( A_{st} )</td>
<td>( M_r )</td>
</tr>
<tr>
<td>10</td>
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</tr>
<tr>
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<td>0.156</td>
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<td>0.199</td>
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<td>0.213</td>
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<td>32,300</td>
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<td>0.227</td>
</tr>
<tr>
<td>17</td>
<td>36,400</td>
<td>0.136</td>
<td>0.242</td>
</tr>
<tr>
<td>18</td>
<td>40,800</td>
<td>0.144</td>
<td>0.253</td>
</tr>
<tr>
<td>19</td>
<td>45,500</td>
<td>0.152</td>
<td>0.270</td>
</tr>
<tr>
<td>20</td>
<td>50,400</td>
<td>0.160</td>
<td>0.284</td>
</tr>
<tr>
<td>21</td>
<td>55,600</td>
<td>0.168</td>
<td>0.298</td>
</tr>
<tr>
<td>22</td>
<td>61,000</td>
<td>0.176</td>
<td>0.312</td>
</tr>
<tr>
<td>23</td>
<td>65,400</td>
<td>0.184</td>
<td>0.326</td>
</tr>
<tr>
<td>24</td>
<td>72,500</td>
<td>0.192</td>
<td>0.341</td>
</tr>
<tr>
<td>25</td>
<td>78,800</td>
<td>0.200</td>
<td>0.355</td>
</tr>
<tr>
<td>26</td>
<td>85,200</td>
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</tr>
<tr>
<td>27</td>
<td>91,800</td>
<td>0.216</td>
<td>0.383</td>
</tr>
<tr>
<td>28</td>
<td>98,800</td>
<td>0.224</td>
<td>0.397</td>
</tr>
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<td>29</td>
<td>106,000</td>
<td>0.232</td>
<td>0.412</td>
</tr>
<tr>
<td>30</td>
<td>113,000</td>
<td>0.240</td>
<td>0.426</td>
</tr>
<tr>
<td>31</td>
<td>121,000</td>
<td>0.248</td>
<td>0.440</td>
</tr>
<tr>
<td>32</td>
<td>129,000</td>
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<tr>
<td>36</td>
<td>163,000</td>
<td>0.288</td>
<td>0.511</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>STRESS RATIO ( P_{cb} / P_{st} )</th>
<th>24</th>
<th>18</th>
<th>16.4 * ( A_{sc} = A_{st} ) max.</th>
</tr>
</thead>
<tbody>
<tr>
<td>FACTORS</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( M_r ) = moment of resistance in inch-pounds per one inch width of beam.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( A_{st} ) = area of tensile reinforcement in square inches per inch width of beam.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( A_{sc} ) = area of compressive reinforcement in square inches per inch width of beam.</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\* If any other distance, \( M_r \) and \( A_{sc} \) for \( A_{st} \) \( A_{sc} \) must be adjusted.
### TABLE 70B.—RECTANGULAR BEAMS: RESISTANCE AND REINFORCEMENT. MODULAR-RATIOS METHOD.—\( f_{ct} = 20,000 \) LB. PER SQ. IN.

| MAXIMUM STRESS IN TENSILE REINFORCEMENT = 20,000 LB. PER SQ. IN. | \( m = 15 \) |
|---|---|---|---|---|---|---|---|
| EFFECTIVE DEPTH \( d \) | \( P_{cb} = 750 \) LB. PER SQ. IN. | \( P_{cb} = 1000 \) LB. PER SQ. IN. | \( P_{cb} = 1100 \) LB. PER SQ. IN. |
| \( d_i \) | \( A_{sc} = 0 \) | \( A_{sc} = A_{sb} \) | \( A_{sc} = 0 \) | \( A_{sc} = A_{sb} \) | \( A_{sc} = A_{sb} \) | \( A_{sc} = 0 \) | \( A_{sc} = A_{sb} \) |
| IN | \( M_r \) | \( A_{sb} \) | \( M_r \) | \( A_{sb} \) | \( M_r \) | \( A_{sb} \) | \( M_r \) | \( A_{sb} \) |
| 10 | 12,000 | 0.067 | 1.930 | 0.108 | 18,400 | 0.107 | 40,600 | 0.251 |
| 11 | 14,500 | 0.074 | 2,340 | 0.119 | 22,500 | 0.118 | 49,100 | 0.254 |
| 12 | 17,200 | 0.081 | 2,780 | 0.130 | 26,700 | 0.129 | 56,600 | 0.277 |
| 13 | 20,200 | 0.087 | 3,200 | 0.140 | 31,100 | 0.139 | 68,800 | 0.300 |
| 14 | 23,400 | 0.094 | 3,780 | 0.151 | 36,100 | 0.150 | 79,700 | 0.323 |
| 15 | 26,900 | 0.100 | 4,340 | 0.162 | 41,400 | 0.161 | 91,500 | 0.347 |
| 16 | 30,600 | 0.107 | 4,940 | 0.173 | 46,800 | 0.172 | 104,000 | 0.370 |
| 17 | 34,500 | 0.114 | 5,570 | 0.184 | 53,200 | 0.182 | 117,000 | 0.393 |
| 18 | 38,800 | 0.121 | 6,250 | 0.194 | 59,700 | 0.192 | 132,000 | 0.416 |
| 19 | 43,200 | 0.127 | 6,960 | 0.205 | 66,500 | 0.203 | 147,000 | 0.439 |
| 20 | 47,800 | 0.134 | 7,720 | 0.216 | 73,600 | 0.214 | 162,000 | 0.462 |
| 21 | 52,700 | 0.141 | 8,500 | 0.227 | 81,000 | 0.225 | 179,000 | 0.485 |
| 22 | 58,200 | 0.148 | 9,350 | 0.238 | 89,000 | 0.236 | 197,000 | 0.508 |
| 23 | 63,200 | 0.154 | 10,200 | 0.248 | 97,500 | 0.246 | 215,000 | 0.532 |
| 24 | 68,800 | 0.161 | 11,000 | 0.259 | 106,000 | 0.257 | 234,000 | 0.554 |
| 25 | 74,700 | 0.168 | 12,100 | 0.270 | 115,000 | 0.268 | 254,000 | 0.576 |
| 26 | 80,800 | 0.174 | 13,100 | 0.281 | 125,000 | 0.279 | 275,000 | 0.601 |
| 27 | 87,000 | 0.181 | 14,100 | 0.292 | 134,000 | 0.290 | 296,000 | 0.624 |
| 28 | 93,600 | 0.188 | 15,100 | 0.303 | 144,000 | 0.300 | 318,000 | 0.647 |
| 29 | 101,000 | 0.194 | 16,200 | 0.314 | 155,000 | 0.311 | 342,000 | 0.670 |
| 30 | 108,000 | 0.201 | 17,400 | 0.324 | 166,000 | 0.321 | 365,000 | 0.693 |
| 31 | 115,000 | 0.208 | 18,600 | 0.335 | 177,000 | 0.332 | 390,000 | 0.716 |
| 32 | 122,000 | 0.214 | 19,800 | 0.346 | 188,000 | 0.343 | 416,000 | 0.739 |
| 33 | 130,000 | 0.221 | 21,000 | 0.356 | 200,000 | 0.353 | 443,000 | 0.762 |
| 34 | 138,000 | 0.228 | 22,300 | 0.366 | 213,000 | 0.364 | 470,000 | 0.785 |
| 35 | 146,000 | 0.234 | 23,600 | 0.378 | 225,000 | 0.375 | 498,000 | 0.809 |
| 36 | 155,000 | 0.241 | 25,000 | 0.389 | 239,000 | 0.387 | 526,000 | 0.832 |

| STRESS RATIO | \(2G/\) | 20 | 18.2 |
|.Rad | \(P_{cb}/P_{tcb} | | | |

**M_r** = MOMENT OF RESISTANCE IN INCH-POUNDS PER ONE INCH WIDTH OF BEAM.

**A_{sb}** = AREA OF TENSILE REINFORCEMENT IN SQUARE INCHES PER INCH WIDTH OF BEAM.

**A_{sc}** = AREA OF COMPRESSION REINFORCEMENT IN SQUARE INCHES PER INCH WIDTH OF BEAM.

DISTANCE BETWEEN CENTRES OF TENSILE AND COMPRESSION REINFORCEMENT ASSUMED 0.9 \(d_i\).

IF ANY OTHER DISTANCE, \(M_r\), AND \(A_{sb}\) FOR \(A_{sc}=A_{sc}\) MUST BE ADJUSTED.
## Table 70c: Maximum Stress in Tensile Reinforcement

### Table 70c: Maximum Stress in Tensile Reinforcement

<table>
<thead>
<tr>
<th>Effective Depth d₁</th>
<th>Pₑₑ = 750 LB. Per SQ. IN.</th>
<th>Pₑₑ = 1000 LB. Per SQ. IN.</th>
<th>Pₑₑ = 1100 LB. Per SQ. IN.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Aₛₛ = 0</td>
<td>Aₛₛ = Aₜₜ</td>
<td>Aₛₛ = 0</td>
</tr>
<tr>
<td>Mₑₑ</td>
<td>Mₑₑ</td>
<td>Mₑₑ</td>
<td>Mₑₑ</td>
</tr>
<tr>
<td>10</td>
<td>9,950 0-041 13,900 0-055</td>
<td>15,700 0-066 25,300 0-106</td>
<td>18,200 0-077 31,800 0-134</td>
</tr>
<tr>
<td>11</td>
<td>12,000 0-045 16,800 0-061</td>
<td>19,000 0-073 30,600 0-117</td>
<td>22,000 0-085 38,500 0-147</td>
</tr>
<tr>
<td>12</td>
<td>14,300 0-049 20,100 0-066</td>
<td>22,600 0-079 36,400 0-127</td>
<td>26,200 0-092 45,800 0-161</td>
</tr>
<tr>
<td>13</td>
<td>16,800 0-053 23,700 0-072</td>
<td>26,500 0-086 42,800 0-138</td>
<td>30,700 0-100 53,800 0-174</td>
</tr>
<tr>
<td>14</td>
<td>19,500 0-057 27,300 0-077</td>
<td>30,800 0-092 49,500 0-148</td>
<td>35,700 0-100 62,300 0-188</td>
</tr>
<tr>
<td>15</td>
<td>22,400 0-062 31,300 0-083</td>
<td>35,400 0-099 57,000 0-159</td>
<td>40,900 0-116 71,600 0-201</td>
</tr>
<tr>
<td>16</td>
<td>25,500 0-066 35,600 0-088</td>
<td>40,200 0-106 64,700 0-170</td>
<td>46,600 0-123 81,400 0-214</td>
</tr>
<tr>
<td>17</td>
<td>28,700 0-070 40,100 0-084</td>
<td>45,300 0-112 73,000 0-180</td>
<td>52,500 0-131 92,000 0-228</td>
</tr>
<tr>
<td>18</td>
<td>32,300 0-074 45,100 0-099</td>
<td>50,800 0-119 82,000 0-191</td>
<td>59,000 0-135 103,000 0-241</td>
</tr>
<tr>
<td>19</td>
<td>35,900 0-078 50,200 0-105</td>
<td>56,700 0-125 91,500 0-201</td>
<td>65,800 0-146 114,000 0-255</td>
</tr>
<tr>
<td>20</td>
<td>38,800 0-082 55,600 0-110</td>
<td>62,800 0-132 101,000 0-212</td>
<td>72,800 0-154 127,000 0-268</td>
</tr>
<tr>
<td>21</td>
<td>43,800 0-086 61,300 0-116</td>
<td>69,200 0-139 112,000 0-223</td>
<td>80,300 0-162 140,000 0-281</td>
</tr>
<tr>
<td>22</td>
<td>48,100 0-090 67,300 0-121</td>
<td>76,000 0-145 123,000 0-233</td>
<td>88,000 0-169 154,000 0-295</td>
</tr>
<tr>
<td>23</td>
<td>52,600 0-094 73,500 0-127</td>
<td>83,000 0-152 134,000 0-244</td>
<td>96,200 0-177 168,000 0-308</td>
</tr>
<tr>
<td>24</td>
<td>57,400 0-098 80,000 0-132</td>
<td>90,500 0-158 146,000 0-254</td>
<td>105,000 0-185 183,000 0-322</td>
</tr>
<tr>
<td>25</td>
<td>62,200 0-103 86,900 0-138</td>
<td>98,100 0-165 158,000 0-265</td>
<td>114,000 0-193 198,000 0-335</td>
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<tr>
<td>26</td>
<td>67,300 0-107 94,000 0-143</td>
<td>106,000 0-172 171,000 0-276</td>
<td>125,000 0-200 215,000 0-348</td>
</tr>
<tr>
<td>27</td>
<td>72,500 0-111 101,000 0-149</td>
<td>114,000 0-178 184,000 0-286</td>
<td>133,000 0-206 232,000 0-362</td>
</tr>
<tr>
<td>28</td>
<td>78,000 0-115 109,000 0-154</td>
<td>123,000 0-185 198,000 0-297</td>
<td>143,000 0-216 249,000 0-375</td>
</tr>
<tr>
<td>29</td>
<td>83,700 0-119 117,000 0-159</td>
<td>132,000 0-181 213,000 0-307</td>
<td>153,000 0-223 267,000 0-389</td>
</tr>
<tr>
<td>30</td>
<td>89,600 0-123 125,000 0-165</td>
<td>141,000 0-198 228,000 0-318</td>
<td>164,000 0-231 286,000 0-402</td>
</tr>
<tr>
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<td>95,800 0-127 133,000 0-171</td>
<td>151,000 0-205 243,000 0-329</td>
<td>175,000 0-233 305,000 0-415</td>
</tr>
<tr>
<td>32</td>
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<td>161,000 0-211 259,000 0-339</td>
<td>186,000 0-246 326,000 0-429</td>
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<tr>
<td>33</td>
<td>108,000 0-135 151,000 0-182</td>
<td>171,000 0-218 275,000 0-350</td>
<td>198,000 0-254 346,000 0-442</td>
</tr>
<tr>
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<td>182,000 0-224 292,000 0-360</td>
<td>210,000 0-262 367,000 0-456</td>
</tr>
<tr>
<td>35</td>
<td>122,000 0-144 170,000 0-193</td>
<td>192,000 0-231 310,000 0-371</td>
<td>223,000 0-270 350,000 0-469</td>
</tr>
<tr>
<td>36</td>
<td>129,000 0-148 180,000 0-198</td>
<td>203,000 0-236 328,000 0-381</td>
<td>235,000 0-277 412,000 0-482</td>
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</table>

**Stress Ratios**

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<thead>
<tr>
<th>Mₑₑ</th>
<th>Pₑₑ/ₚₑₑ</th>
<th>Mₑₑ</th>
<th>Pₑₑ/ₚₑₑ</th>
<th>Mₑₑ</th>
<th>Pₑₑ/ₚₑₑ</th>
</tr>
</thead>
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<tr>
<td>36</td>
<td>27</td>
<td>24.5</td>
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</table>

**Note:** Mₑₑ for other combinations of stresses having these ratios are proportional to tensile stress.

- Mₑₑ = Moment of Resistance in Inch-Pounds per One Inch Width of Beam.
- Aₛₛ = Area of Tensile Reinforcement in Square Inches per Inch Width of Beam.
- Aₛₛ = Area of Compressive Reinforcement in Square Inches per Inch Width of Beam.
- Distance between centres of tensile and compressive reinforcement assumed 0.9 d₁.
- If any other distance, Mₑₑ and Aₛₛ for Aₛₛ = Aₛₛ must be adjusted.

---

**265**
Examples of Use of Table 70C or 70D (High-yield-stress Steel).

The following examples apply to Table 70D ($f_{ck} = 30,000$ lb. per sq. in.) for the modular-ratio method with $m = 15$. The calculations would be similar, but with different numerical values, if Table 70C ($f_{ck} = 27,000$ lb. per sq. in.) were used.

(a) A rectangular beam of $1:2:4$ concrete ($f_{ck} = 1000$ lb.) 24 in. deep ($d_1 = 22$ in.) and 12 in. wide is to be reinforced with bars having a yield stress not less than 66,000 lb. per sq. in. ($f_{ys} = 30,000$ lb. and $f_{ty} = 23,000$ lb. per sq. in. from Table 58). Determine the reinforcement required to resist the bending moments in the following examples.

(i) Bending moment $840,000$ in.-lb. ($70,000$ in.-lb. per inch of width). With $A_{st} = 0$, $M_f = 71,600$ in.-lb. per inch of width. Therefore $A_{st} = 0.126 \times 12 = 1.51$ sq. in., say, four $\frac{1}{4}$-in. twisted ribbed bars (Tables 59 and 60) in the bottom.

(ii) Bending moment $1,200,000$ in.-lb ($100,000$ in.-lb. per inch of width). With $A_{st} = A_{sc}$, $M_f = 110,000$ in.-lb. per inch. $A_{st} = A_{sc} = 0.181 \times 12 = 2.17$ sq. in., say, three 1-in. twisted ribbed bars (Tables 59 and 60) in top and bottom. In this design the compressive resistance of the concrete is taken into account.

(iii) Bending moment $1,800,000$ in.-lb. ($150,000$ in.-lb. per inch of width). Since this exceeds $110,000$ in.-lb. the "steel-beam" theory is applied and the resistance of the concrete in compression is neglected. The lever-arm $l_{st}$ is the distance between the centres of the tensile and compression reinforcement, say, $22 - 2 = 20$ in.

\[ A_{st} = \frac{1,800,000}{30,000 \times 20} = 3.0 \text{ sq. in.}; A_{sc} = \frac{1,800,000}{23,000 \times 20} = 3.9 \text{ sq. in.} \]

Using 1-in. twisted ribbed bars, four bars are required in the bottom and five in the top.

**REINFORCEMENT IN SOLID SLABS.**

**Main Bars.**—Various arrangements of the main reinforcement in slabs are illustrated in the diagram. Ordinary floor slabs are generally as the design at (a). When the live load is so large compared with the dead load that reinforcement against negative bending moments is required in the middle of the span, the design at (b) is suitable. Hooks or similar end-anchorages are not generally necessary on bars in slabs, except at the outer supports of end-spans. If the design at (b) is adopted hooks might be provided as the bars may terminate in a tensile part of the slab.

![Diagram of Reinforcement in Slabs]

The spacing of the main bars in a slab should not exceed three times the effective depth of the slab. For slabs spanning in two directions, the spacing of the bars in either direction should not exceed three times the effective depth, and the bars across the shorter span should, at the midspan, be placed under, and at the supports over, the reinforcement at right-angles to this span.

**Minimum Amount of Reinforcement.**—Whatever amount of tensile reinforcement is required by calculation to resist the bending moment, the area of steel should be not less than 0.15 per cent. of the gross area of the concrete.

(Continued on page 268.)

REINFORCEMENT IN SOLID SLABS (continued from page 266).

Distribution (Longitudinal) Bars.—In slabs spanning in one direction bars should be provided at right-angles to the main reinforcement. The B.S. Code recommends that the area of such distribution bars should not be less than 0·15 per cent. of the gross area of the concrete; the spacing of the distribution bars should not be greater than five times the effective depth, although this spacing may be excessive in slabs exceeding 6 in. in thickness. The London By-laws require 10 per cent. of the area of the main reinforcement; in bridge decks the Ministry of Transport requires 40 per cent. for spans of 4 ft., 50 per cent. for 6 ft., 55 per cent. for 8 ft., and 60 per cent. for 10 ft., but in no case need the area exceed 0·5 sq. in. per foot of width. B.S. No. 153 (Part 3A) requires 50 per cent. in reinforced concrete deck slabs for steel bridges. Fabricated meshes do not always have sufficient transverse bars to provide the foregoing amounts and the deficiency has to be made up by an additional sheet or by wiring bars to the main fabric reinforcement.

Where slabs span in one direction there is generally a negative bending moment over main beams parallel to the span. Reinforcement in the top of the slab at right-angles to the main reinforcement is therefore required. The area of the reinforcement provided for this purpose should be 0·3 per cent. of the gross cross-sectional area of the slab. The spacing of these bars should not exceed three times the effective depth, and the bars should extend on each side of the beam an average distance of six times the thickness of the slab. The ends of the bars should be staggered.

DESIGN OF SLABS: MODULAR-RATIO METHOD.

Table 71.—The data in Table 71 are based on the principle that the effective depth of, and reinforcement in, a solid slab required to resist a given bending moment $M$ are direct functions of $\sqrt{M}$. The effective depth can be expressed by $a_1 = k_1 \sqrt{M}$, and the area of reinforcement per foot of width by $A_{st} = k_2 \sqrt{M}$. Values of $k_1$ and $k_2$ are given in Table 71 for various stresses, and for these values $M$ must be in foot-pounds. The method is to find from the table the value of $k_1$ for the permissible stresses and to determine the effective depth from the formula. If this depth is adopted, the value of $k_2$ applicable to the permissible stresses should be used to determine the area of reinforcement per foot of width of the slab. If a depth greater than that determined is adopted find the value of $k_1 = \frac{\text{effective depth adopted}}{\sqrt{M}}$, and find from the table the corresponding value of $k_2$ for the permissible stress in the reinforcement. Multiply this value of $k_2$ by $\sqrt{M}$ to give $A_{st}$. If a depth less than that first calculated is adopted, find the value of $k_1 = \frac{\text{effective depth adopted}}{\sqrt{M}}$, and find from the table the corresponding value of $k_2$ for the permissible stress in the concrete, which should then be multiplied by $\sqrt{M}$ to give the area of reinforcement required.

Example.—Design a slab to resist a bending moment of 2000 ft.-lb. per ft. width with stresses not exceeding 16,000 per sq. in. in the steel and 700 lb. per sq. in. in the concrete.

(i) At maximum concrete and steel stresses:

Effective depth required = 0·091$\sqrt{2000} = 4'07$ in. Total thickness of slab = 5 in.

Area of reinforcement required = 0·0095$\sqrt{2000} = 0'420$ sq. in. per ft.

(ii) Specified thickness of slab, 6 in.—$d_1 = 5'25$ in.; $k_1 = \frac{5'25}{\sqrt{2000}} = 0'118$.

With $f_{st} = 16,000$ lb. per sq. in., the corresponding value of $k_2$ is 0·0071, and the area of reinforcement required is 0·0071$\sqrt{2000} = 0'312$ sq. in. per ft.

(iii) Specified thickness of slab, 4$\frac{1}{2}$ in.—$d_4 = 3'75$ in.; $k_1 = \frac{3'75}{\sqrt{2000}} = 0'084$.

With $f_{st} = 700$ lb. per sq. in., the corresponding value of $k_2$ is 0·014 approximately, and the area of reinforcement required is 0·014$\sqrt{2000} = 0'625$ sq. in. per ft.
### SOLID SLABS: DESIGN FACTORS.—TABLE 71. MODULAR-RATIO METHOD.

<table>
<thead>
<tr>
<th>$P_{cb}$</th>
<th>12,000</th>
<th>14,000</th>
<th>16,000</th>
<th>18,000</th>
<th>20,000</th>
<th>27,000</th>
<th>30,000</th>
</tr>
</thead>
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<tr>
<td>$k_1$</td>
<td>$k_2$</td>
<td>$k_1$</td>
<td>$k_2$</td>
<td>$k_1$</td>
<td>$k_2$</td>
<td>$k_1$</td>
<td>$k_2$</td>
</tr>
<tr>
<td>----------</td>
<td>--------</td>
<td>--------</td>
<td>--------</td>
<td>--------</td>
<td>--------</td>
<td>--------</td>
<td>--------</td>
</tr>
<tr>
<td>100</td>
<td>0.43</td>
<td>0.002</td>
<td>0.46</td>
<td>0.002</td>
<td>0.49</td>
<td>0.002</td>
<td>0.50</td>
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<td>0.10</td>
<td>0.006</td>
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<td>0.005</td>
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<td>0.18</td>
</tr>
</tbody>
</table>

---

**$P_{cb}$** = TENSILE STRESS (LB. PER SQ. IN.) IN REINFORCEMENT.  
**$P_{cb}$** = COMpressive STRESS (LB. PER SQ. IN.) IN CONCRETE.  
**$M'$** = BENDING MOMENT (FOOT-LB.) PER FOOT WIDTH OF SLAB.  
**$K_1$** = $\frac{1}{Q_c}$  
**$K_2$** = $\frac{\alpha_1 d_p}{2}$.  
**$M''$** = BENDING MOMENT (INCH-LB.).  
**$b$** = BREADTH (INCH).  
**$a_1$** = LEVER-ARM FACTOR.  

**FUNCTIONAL DEPTH (INCH):**  
$d_1 = K_1 \sqrt{M'}$.  
**TENSILE REINFORCEMENT (SQ. IN. PER FOOT WIDTH):**  
$A_{sb} = K_2 \sqrt{M'}$.  

**APPLICATION TO RECTANGULAR BEAMS:**  
$d_1 = k_1 \sqrt{M'' \frac{a_1}{b}}$.  
$A_{sb} = \frac{M''}{a_1 d_p}$.  

* For values of lever-arm factor $a_1$, see Table 68.

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REINFORCED CONCRETE DESIGNER'S HANDBOOK

DESIGN OF SLABS: MODULAR-RATIO METHOD.

Table 72.—The data in Table 72 relating to moments of resistance and areas of reinforcement for slabs of various thicknesses from 3 in. to 9 in. are in accordance with the recommendations of B.S. Code No. 114 for the modular-ratio method \( (m = 15) \), using mild steel or high-yield-stress bars and various grades of \( 1:2:4 \) concrete.

Example.—Select a slab to resist a bending moment of 2000 ft.-lb. \( (= 24,000 \text{ inc.-lb}) \) per foot width. Maximum stresses: \( \rho_{st} = 20,000 \text{ lb. per sq. in.} \), and \( \rho_{cb} = 1000 \text{ lb. per sq. in.} \).

The moment of resistance of a \( 4 \text{-in.} \) slab with these stresses is 23,300 inc.-lb. which may be near enough. The area of reinforcement required for this moment is 0.419 sq. in. per ft. width, which is given by \( \frac{1}{2} \text{-in.} \) bars at \( \frac{4}{3} \text{-in.} \) centres (see Table 60) in a \( 4 \text{-in.} \) slab.

If it is imperative that the moment of resistance must not be less than 24,000 inc.-lb., a slab \( \frac{4}{3} \text{-in.} \) thick should be provided \( (M_s = 26,000 \text{ inc.-lb. per ft.}) \). The effective depth with \( \frac{1}{2} \text{-in.} \) bars and \( \frac{1}{4} \text{-in.} \) cover is 3.75 in.; lever-arm \( \text{for} \ r_1 = 20 \text{ from Table 68} \) is 0.857 \( \times \) 3.75 = 3.2 in.

\[
A_{st} = \frac{24,000}{20,000 \times 3.2} = 0.375 \text{ sq. in. per ft.;} \quad \frac{1}{2} \text{-in.} \text{ bars at 6-in. centres are satisfactory.}
\]

SLABS FOR LIQUID-CONTAINING STRUCTURES (B.S. Code No. 2007).

Notation.

\( d, \beta, b, l_s \): Thickness, effective depth, breadth and lever-arm respectively of slab.

\( M'_{re}, M'_{rr} \): Moment of resistance when not cracked and cracked respectively based on the thickness \( (= R_d d^2 \text{ and } R_e d^2 \text{ respectively}) \).

\( Q_e \): Moment-of-resistance factor \( \text{(no compression reinforcement)} \) based on the effective depth.

\( r_0 = \frac{A_{st}}{b d} (\leq 0.003 \text{ with mild steel bars or } 0.0025 \text{ with deformed bars}) \).

Slabs subjected to Bending only. Cases IA and IB, Table 73.

Resistance to Cracking.

\[
M'_{re} = R_d d^2 = \frac{\rho_{st}}{1 - \rho_{cb}} \left[ \frac{d}{m_r} - (1 - n_0)n_0 + r_0(m - 1) \left( \frac{d}{d} - n_0 \right) \right] b d^2,
\]

in which

\[
n_0 = -\frac{1}{r_0 + \frac{d}{d} (m - 1)}.
\]

Resistance when Cracked. \( M'_{rr} = R_e d^2 = \rho_{ef} r_0 \left( 1 - \frac{n_1}{\frac{d}{d}} \right) b d^2 \) in which

\[
n_1 = \sqrt{\left[ \frac{m r_0 \left( \frac{d}{d_1} \right)}{\frac{d}{d_1}} \right] + 2 m r_0 \left( \frac{d}{d_1} \right) - m r_0 \left( \frac{d}{d_1} \right)}.
\]

When the permissible tensile stresses in the concrete \( (\rho_{ct}) \) and reinforcement \( (\rho_{ct}) \) are 270 lb. and 12,000 lb. per sq. in. respectively the expressions are as in Table 73 for \( b = \beta \) in.

The curves give values of \( R_s \) and \( R_e \) for various values of \( \frac{d}{d} \) and \( r_0 \). For each value of \( \frac{d}{d} \) there is a proportion \( r_{eb} \) of reinforcement which gives equal values of \( R_s \) and \( R_e \), say \( R_E \), the adoption of which gives the most economical design. If \( R_E \) is substituted in \( d = \sqrt{\frac{M}{R_E}} \), \( M \) being the bending moment to be resisted, the resulting value of \( d \) is the thickness of slab complying with the two design requirements if \( A_{st} \) in a strip 1 ft. wide is not less than \( 12 r_0 b d \) sq. in. ; corresponding values of \( r_{eb} \) and \( R_E \) are given in Table 73.

The resistance of a slab with the proportion of reinforcement \( r_{eb} \) is obtained by substitution in \( M_s = R_E d^2 \); the resistance and reinforcement of slabs of various thicknesses taking into account probable values of \( \frac{d}{d} \) are given in Table 73.

If in a slab not less than 9 in. thick the tensile strain is at the face remote from the liquid, design as for an ordinary slab with \( \rho_{st} > 18,000 \text{ lb. per sq. in.} \) in plain bars or 20,000 lb. per sq. in. in deformed bars, and the compressive stress in the concrete not greater than 1200 lb.

(Continued on page 272.)
**SOLID SLABS: RESISTANCE AND REINFORCEMENT.—TABLE 72. MODULAR-RATIO METHOD.**

<table>
<thead>
<tr>
<th>MAXIMUM WORKING STRESSES LBS. PER SQ. IN.</th>
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<tbody>
<tr>
<td>m = 15</td>
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<tr>
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</tr>
<tr>
<td><strong>THICKNESS (IN)</strong></td>
</tr>
<tr>
<td>3</td>
</tr>
<tr>
<td>3½</td>
</tr>
<tr>
<td>4</td>
</tr>
<tr>
<td>4½</td>
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<tr>
<td>7½</td>
</tr>
<tr>
<td>8</td>
</tr>
<tr>
<td>9</td>
</tr>
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</table>

<table>
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<tr>
<th>COVER (IN.) (ASSUMED) BAR DIAMETER (ASSUMED) EFFECTIVE DEPTH</th>
</tr>
</thead>
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<tr>
<td>1/2</td>
</tr>
<tr>
<td>1/2</td>
</tr>
<tr>
<td>1/2</td>
</tr>
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<tr>
<td>3/4</td>
</tr>
<tr>
<td>3/4</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>TENSILE</th>
<th>COMPRRESSIVE</th>
<th>( \gamma_t )</th>
<th>( \gamma_e )</th>
<th>BASIC FACTORS</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_{st} )</td>
<td>( P_{cb} )</td>
<td>( M_r )</td>
<td>( A_{st} )</td>
<td>( A_{el} )</td>
</tr>
<tr>
<td>18,000</td>
<td>750</td>
<td>24</td>
<td>450</td>
<td>0.060</td>
</tr>
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</tr>
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<td>144</td>
<td>1590</td>
<td>24000</td>
<td>0.045</td>
</tr>
<tr>
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</tr>
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<td>23000</td>
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<td>6350</td>
<td>9800</td>
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</tr>
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<td>24</td>
<td>13400</td>
<td>19000</td>
<td>0.031</td>
</tr>
</tbody>
</table>

\( M_r \) = MOMENT OF RESISTANCE IN INCH-POUNDS PER FOOT WIDTH OF SLAB REINFORCED IN TENSION ONLY.
\( A_{st} \) = AREA OF TENSILE REINFORCEMENT IN SQUARE INCHES PER FOOT WIDTH OF SLAB.

IF COMBINATION OF ACTUAL COVER AND DIAMETER OF BAR GIVES AN EFFECTIVE DEPTH DIFFERENT FROM THE DEPTH TABULATED, \( M_r \) AND \( A_{st} \) SHOULD BE CALCULATED FROM BASIC FACTORS.

\( M_r \) FOR OTHER COMBINATIONS OF STRESSES HAVING THE SAME RATIOS \( \gamma_t \) AS TABULATED ARE PROPORTIONAL TO THE TENSILE STRESS. \( \gamma_t = \frac{P_{st}}{P_{cb}} \)

For example of use of this table, see page 270.

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SLABS FOR LIQUID-CONTAINING STRUCTURES (continued from page 270).
B.S. CODE No. 2007.

per sq. in. (Case Ia in Table 73); \( d_1 \) must be not less than \( \sqrt{\frac{M}{Q_c}} \) in which \( Q_c = 250 \) for \( p_{st} = 18,000 \) lb., and 239 for \( p_{st} = 20,000 \) lb. per sq. in. If the thickness provided is such that \( d_1 = \sqrt{\frac{M}{Q_c d_1}} \), \( A_{st} = 0.017 bd \) and 0.014bd sq. in. in width \( b \) if \( p_{st} \) is 18,000 lb. and 20,000 lb. per sq. in. respectively. If \( d_1 \) exceeds the minimum depth required, \( A_{st} \) (in width \( b \)) = \( \frac{M}{P_{st} d_1} \), in which \( l_a = 0.83d_1 \), approximately.

The moment of resistance is \( Q_c bd_1^2 \), if \( A_{st} \leq 0.017bd_1 \) (or 0.014bd_1 with deformed bars); moments of resistance and corresponding amounts of reinforcement are given in Table 73 for probable values of \( \frac{d_1}{d} \).

**Examples** (Table 73).—Design walls of tanks for the conditions stated (Case IA).

(a).—Bending moment of 20,000 in.-lb. per foot.
This design can be taken directly from the table headed "Balanced Design," since the moment of resistance of a 6-in. slab is approximately 20,000 in.-lb. The reinforcement required is (also from the table) 0.49 sq. in. per foot, which is provided by \( \frac{8}{8} \)-in. bars at 7\( \frac{3}{4} \)-in. centres with 14-in. cover.

(b).—Bending moment of 24,000 in.-lb. per foot.
The curves in the table can be used directly to obtain a "balanced design." Assume \( \frac{d_1}{d} = 0.7 \). (Compare with values on table headed "Balanced Design"). From curves,

\[
R_e = R_s = R_E = 580 \quad \text{and} \quad r_0 = 0.0067. \quad d = \sqrt{\frac{24,000}{580}} = 6.45 \text{ in.}; \quad \text{that is, a 6\( \frac{1}{2} \)-in. slab.}
\]

\( A_{st} = 0.0067 \times 6\frac{1}{2} \times 12 = 0.522 \) sq. in.; provide \( \frac{8}{8} \)-in. bars at 7-in. centres; 14-in. cover.

(c).—Check that an 8-in. slab reinforced with \( \frac{8}{8} \)-in. bars at 6-in. centres with 14-in cover and subjected to a bending moment of 36,000 in.-lb. per foot conforms to B.S. Code No. 2007.

\[
d_1 = 8 - 1\frac{3}{4} - \frac{8}{8} = 6.19 \text{ in.}; \quad \frac{d_1}{d} = \frac{6.19}{8} = 0.774; \quad \frac{d}{d_1} = \frac{1}{0.774} = 1.29. \quad A_{st} = 0.614 \text{ sq. in.};
\]

\[
r_0 = \frac{0.614}{12 \times 8} = 0.0064. \quad \text{Substitute in formulae in Table A.}
\]

\[
n_0 = \frac{1}{8} + \frac{(14 \times 0.0064 \times 0.774)}{14 \times 0.0064} = 0.523.
\]

\[
R_e = \frac{3240}{1 - 0.523} \left( \frac{1}{8} - (1 - 0.523) \right) \times 0.523 + [14 \times 0.0064 \times (0.774 - 0.523)] = 607.
\]

\[
d \leq \sqrt{\frac{36,000}{607}} = 7.70 \text{ in.}, \quad \text{which does not exceed 8 in.}
\]

\[
r_s = 15 \times 0.0064 \times 1.29 = 0.124; \quad n_1 = \sqrt{(0.124)^2 + (2 \times 0.124)} - 0.124 = 0.39.
\]

\[
a_1 = 1 - \left( \frac{1}{8} \times 0.39 \right) = 0.87; \quad A_{st} \leq \frac{36,000}{12,000 \times 0.87 \times 6.19} = 0.558 \text{ sq. in. per foot.}
\]

which does not exceed 0.614 sq. in. Therefore design is satisfactory.

*Note.*—For designs in Case IB, the procedure is as in the foregoing if \( d < 9 \) in., and as for ordinary designs if \( d \leq 9 \) in.

**Slabs subjected to Direct Tension (Case II, Table 74).**

**Resistance to Cracking**.—Determines the thickness of the slab.

\[
T = p_{ct} \left( d + (m - 1)d_1 \right).
\]

**Resistance when Cracked**.—Determines the amount of reinforcement.

If \( A_{st} = r_0 bd, \quad r_0 = \frac{1}{P_{st} - (m - 1)} \quad \text{and} \quad d = \frac{T}{(1 + (m - 1)r_0)b_p}_{ct}.
\]

The expressions in Table 74 give values of \( r_0 \) and \( d \) for a slab subjected to a tensile force \( T \) lb. per ft. if \( p_{ct} \) and \( P_{st} \) are 190 lb. and 12,000 lb. per sq. in. respectively.

(Continued on page 274.)
SOLID SLABS: LIQUID-CONTAINING STRUCTURES.—TABLE 73.

(B.S. CODE No. 2007.) BENDING ONLY.

IN ACCORDANCE WITH B.S. CODE No. 2007.
CONCRETE: NOMINAL PROPORTIONS 1 1/6: 3:2, m = 15
MINIMUM PROPORTION OF REINFORCEMENT $r_o = \frac{A_{st}}{bd} \leq 0.003$
MINIMUM COVER: 1 1/2 IN.
BENDING MOMENT $= M$ IN.-LB. PERFOOT.

RESISTANCE TO CRACKING. DETERMINES THICKNESS $d$.
$P_{ct} \geq 270$ LB./SQ.IN.
$M_{rc} = R_c d^2$

WHERE $R_c = \frac{3240}{1 - n_0} \left[ \frac{1}{3} - (1 - n_0) n_0 + 14 r_o \left( \frac{d_1}{d} - n_0 \right)^2 \right]$
AND $n_0 = \frac{1 + 14 r_o (d_1/d)}{1 + 14 r_o}$

DESIGN FOR STRENGTH. DETERMINES REINFORCEMENT $A_{st}$.
$P_{st} \geq 12,000$ LB./SQ.IN.
$M_{rs} = R_s d^2$

CASE IA
BENDING ONLY

(TENSILE STRESS AT FACE IN CONTACT WITH LIQUID)

$A_{st} \geq \frac{12,000 a_1 d_i}{R_c}$
WHERE $a_1 = 1 - \frac{1}{3} n_1$
$R_s = 144,000 r_o a_1 d_i d$

BALANCED DESIGN

POINTS Thus indicate
VALUES of
$R_{sc}$ and
$R_{st} = R_e$
FOR BALANCED DESIGN AND
ARE APPLIED IN THE TABLE
BELOW TO SLABS OF THE
THICKNESS STATED.

<table>
<thead>
<tr>
<th>$d$</th>
<th>$d_i$</th>
<th>$R_{sc}$</th>
<th>$M_{sc}$</th>
<th>$A_{st}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>2.25</td>
<td>8,800</td>
<td>2.98</td>
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</tr>
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<td>4/2</td>
<td>2.62</td>
<td>10,300</td>
<td>3.43</td>
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<td>5</td>
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<td>14,200</td>
<td>4.74</td>
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<td>6</td>
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<td>7</td>
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<td>8.87</td>
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<td>9</td>
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<td>10.55</td>
<td>0.95</td>
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<td>61,800</td>
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<td>16.12</td>
<td>1.25</td>
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<td>13.06</td>
<td>144,200</td>
<td>21.12</td>
<td>1.64</td>
</tr>
</tbody>
</table>

CASE IB
BENDING ONLY

(TENSILE STRESS AT FACE REMOTE FROM LIQUID)

(a) $d < 9$ IN. ADOPT CASE IA.

(b) $d < 9$ IN. DESIGN FOR STRENGTH ONLY

$P_{st} = 18,000$ LB./SQ.IN. (PLAIN BARS) OR 20,000 LB./SQ.IN. (DEFORMED BARS)

$P_{cb} = 1200$ LB./SQ.IN.

DESIGN $d_i = \sqrt[3]{\frac{M}{3000}}$ (PLAIN BARS) OR $d_i = \sqrt[3]{\frac{M}{2872}}$ (DEFORMED BARS)

$A_{st} = \frac{M}{15,000 d_i}$ (PLAIN BARS) OR $A_{st} = \frac{M}{16,840 d_i}$ (DEFORMED BARS), $R_o = \frac{A_{st}}{12 d}$

STRESSES GIVEN $d_i$, $d_i$ AND $R_o$, $n_1$ AND $a_1$ AS CASE IA.

$f_{st} = \frac{12 r_o a_1 d_i}{M}$

$\sigma_{cb} = \frac{f_{st}}{15 (1 - n_1)}$

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Slabs Subjected to Direct Tension and Bending (Cases IIIA and IIIB, Table 74).

Tensile Strain at Face in Contact with Liquid. \( e = \frac{M}{T} \).

**Tensile Stresses only.**—\( e > d_1 - \frac{1}{2} d \). 
Resistance to cracking (determines thickness of slab).

\[
\frac{T}{bd} \left[ 1 + \frac{1}{2r_0(m - 1)} \right] + \frac{e}{6} + 4r_0(m - 1) \left( \frac{d_1}{d} - \frac{1}{2} \right)^2 \]

\( \geq \rho_{ct} \) (direct tension).

Resistance when cracked (determines the amount of reinforcement). \( r_0 = \) proportion of reinforcement near each face. \( r_0 \leq \frac{T}{2r_{e1}bd} \left[ 1 + \frac{e}{d_1} \right] \).

Evaluation of these expressions for \( \rho_{ct} = 190 \text{ lb.} \) and \( \rho_{ct} = 12,000 \text{ lb. per sq. in.} \) \( (b = 12 \text{ in.)} \) are given in Table 74.

**Tensile and Compressive Stresses.**—\( e > d_1 - \frac{1}{2} d \). Neglect tension reinforcement.

Resistance to cracking (determines thickness of slab).

\[
\frac{T}{bd} \left[ 1 + \frac{1}{(m - 1)r_0} \right] + \frac{e(1 - n_0)}{d} \left[ \frac{1}{3} - (1 - n_0) \right] \left( \frac{d_1}{d} - n_0 \right)^2 \]

\( \geq \rho_{ct} \) (bending tension).

in which

\[ n_0 = \frac{\frac{1}{3} - (m - 1) \frac{d_1}{d} r_0}{1 + (m - 1) r_0} \].

Resistance when cracked (determines the amount of tensile reinforcement).

\[ r_0 \leq \frac{T}{\rho_{e1}bd} \left( 1 + \frac{e + \frac{1}{2} d - d_1}{r_a} \right) \].

The expressions in Table 74 are for \( \rho_{et} = 270 \text{ lb.} \) and \( \rho_{et} = 12,000 \text{ lb. per sq. in.} \) \( (b = 12 \text{ in.)} \).

**Tensile Strain at Face Remote from Liquid.**

Slab less than 9 in. thick.—Design as for Case IIIA, (a) or (b) as applicable.

Slab not less than 9 in. thick.—If \( d \) and \( d_1 \) are known, or assumed,

\[
A_{et} = \frac{T}{\rho_{et}} \left( 1 + \frac{e_a}{r_a} \right)
\]

in which \( e_a = e + \frac{1}{2} d - d_1 \). \( d_1 = \sqrt{\frac{M}{Q_e d}} \) approximately.

**Examples (Table 74).**—Design walls of tanks for the conditions stated.

(a).—Direct tensile force of 25,000 lb. per foot (no bending).—Case II.

\[
d = \frac{25,000}{2930} = 8.53 \text{ in.; say 81/2-in. slab.}
\]

**b.**—Tensile force of 12,000 lb. per foot, and bending moment of 20,000 in.-lb. per foot. Assume \( d = 8 \text{ in.} \) and \( d_1 = 8 - 2 = 6 \text{ in.}; \) \( d_1 - \frac{1}{2} d = 6 - 4 = 2 \text{ in.} \)

\[
e = \frac{20,000}{12,000} = 1.67 \text{ in.} \]

\[
r' \rho_e = \frac{12,000}{288,000 \times 8} \left( 1 + \frac{1.67}{2} \right) = 0.0095.
\]

\[
f_{et} = \frac{12,000}{12 \times 8} \left[ 1 + \left( 28 \times 0.0095 \right) + \frac{1.67 \times 8}{15} \left( \frac{1}{4} \times 8 \right) + \left( 56 \times 0.0095 \times 2 \right) \right] = 230 \text{ lb. per square inch (approx.) which exceeds 190 lb. per square inch.}
\]

(Continued on page 282.)
SOLID SLABS: LIQUID-CONTAINING STRUCTURES.—TABLE 74.  
(B.S. CODE No. 2007.) TENSION AND BENDING.

IN ACCORDANCE WITH B.S. CODE No. 2007. 
CONCRETE: NOMINAL PROPORTIONS 1:1:6:3.2 
MINIMUM PROPORTION OF REINFORCEMENT: \( r_o = \frac{A_{st}}{b_d} < 0.003 \), \( m = 15 \) 
MINIMUM COVER: 1 \( \frac{1}{2} \) IN. 
DIRECT TENSILE FORCE = T LB. PER FOOT.  BENDING MOMENT = M IN.-LB. PER FT.

### CASE II 
**CONCENTRIC TENSILE FORCE ONLY**

**DESIGN FOR STRENGTH:** 
Determine reinforcement \( A_{st} \). 
\( P_{st} \geq 12,000 \text{LB./SQ.IN.} \) \( A_{st} = \frac{T}{12,000 \text{ SQ.IN./FT.}} \) OR \( r_o = \frac{T}{144,000d} \)

**RESISTANCE TO CRACKING** 
Determine thickness \( d \): 
\( p_{ct} \geq 190 \text{LB./SQ.IN.} \) 
\( A_{st} = 12r_o d \) 
\[ d \geq \frac{T}{2280} - 1.17 A_{st} \]

**BALANCED DESIGN:** 
\( d = \frac{T}{2930} \) IN. 
\( A_{st} = 0.244d \text{ SQ.IN./FT.} \)

### CASE III A 
**TENSILE FORCE COMBINED WITH BENDING** 
(TENSILE STRESS AT FACE IN CONTACT WITH LIQUID) 
\( e = \frac{M}{T} \)

(a) \( e \geq (d_1 - \frac{1}{2}d) \). TENSILE STRESSES ONLY. 
\( P_{ct} \geq 190 \text{LB./SQ.IN.} \) 
\( P_{st} \geq 12,000 \)

Determine \( r_o = \frac{T}{288,000d} \left[ 1 + \frac{e}{d_1 - \frac{1}{2}d} \right] \) 
Provide \( A_{at} \) AT BOTH SIDES \( = \frac{A_{st}}{12r_o d} \)

Substitute in \( f_{cb} = \frac{T}{12d} \left[ \frac{1}{1 + 28r_o + \frac{6d}{d_1 + 56r_o (d_1 - \frac{1}{2}d)^2}} \right] > 190 \)

Adjust \( d \) until expression is satisfied.

(b) \( e > (d_1 - \frac{1}{2}d) \). TENSILE AND COMPRESSION STRESSES. 
\( P_{ct} \geq 270 \text{LB./SQ.IN.} \) 
\( P_{st} \geq 12,000 \)

Determine \( r_o = \frac{T}{144,000d} \left[ 1 + \frac{e + \frac{1}{2}d - d_1}{0.83d_1} \right] \)

Substitute in \( f_{cb} = \frac{T}{12d} \left[ \frac{1}{1 + 14r_o + \frac{\frac{3}{5} - (1 - n_d)r_o + 14r_o (\frac{d_1}{d} - n_d)}{d_1 - \frac{1}{2}d}} \right] > 270 \)

Adjust \( d \) until expression is satisfied.

### CASE III B 
**TENSILE FORCE COMBINED WITH BENDING** 
(TENSILE STRESS AT FACE REMOTE FROM LIQUID) 
\( d \leq 9 \) IN. ADOPT CASE III A.

(b) \( d \leq 9 \) IN. 
\( P_{st} = 18,000 \text{LB./SQ.IN. (20,000 LB./SQ.IN. FOR DEFORMED BARS)} \) 
\( P_{ct} = 1200 \text{LB./SQ.IN. — NOT CRITICAL). DIAGRAM AS CASE III A(b)} \)

Given or Assume \( d \) and \( d_1 \) 
\[ d_1 = \sqrt[3]{\frac{M}{3000 \text{ PLAIN BARS}}} \text{ OR } \frac{M}{2872 \text{ DEFORMED BARS}} \]

\( A_{st} = \frac{T}{18,000 \text{ OR } 20,000 \left[ 1 + \frac{e_s}{0.83d_1} \right]} \text{ SQ.IN./FT.} \)

If \( d_1 \) and \( A_{st} \) given: 
\[ f_{cb} = \frac{T}{A_{st}} \left[ 1 + \frac{e_s}{0.83d_1} \right] \geq 18,000 \text{ OR } 20,000 \]
Notation.

$A_{st}, A_{bt}$, areas of reinforcement in compression and tension respectively; $A_{st}', A_{bt}'$, do. in unit width of a rectangular beam.

$a_1$, lever-arm factor = $l_0/d_1$.

$b$, breadth of a rectangular beam; breadth of flange of a flanged beam; $b_r$, breadth of rib of a flanged beam.

d, overall depth of beam or thickness of slab.

d_n, distance from edge in compression to neutral plane = $n_1d_1$.

d_s, thickness of flange of a flanged beam.

d', $d'_n$, actual effective depth and effective depth required respectively.

d_s, distance of reinforcement in compression below top of beam or of reinforcement in tension above bottom of beam.

$F$, coefficient in formula for $A_{st}$ in rectangular beams with reinforcement in compression.

$K$, coefficient relating to dimensions of flange of a flanged beam = $4b/b_r$.

$l_0$, lever arm of compressive resistance of concrete = $a_1d_1$.

$M$, applied bending moment; $M'$, applied bending moment per unit width.

$M_r$, moment of resistance; $M_r'$, moment of resistance of rectangular beam of unit width.

$m$, modular ratio (= 15).

$n_1$, neutral-plane factor = $d_n/d_1$.

$p_{ab}$, permissible compressive stress in concrete in bending.

$p_{ct}$, $p_{bt}$, permissible or actual compressive and tensile stresses respectively in reinforcement.

$p_{st}' = p_{st} - \frac{q}{m}$.

$q$, shearing force; $q$, shearing stress.

$q_1$, bending-moment factor = $M/bd_1^2$ or $M/bd_1^2$. $q_0$, moment-of-resistance factor (compression); $Q'$, moment-of-resistance factor for rectangular beam of unit width.

$r_{L}$, limiting proportion of reinforcement in tension for balanced design = $A_{st}/bd_1$ or $A_{st}/bd_1$.

$r_t, r_b$, proportion of reinforcement = $A_{st}/bd_1$ and $A_{st}/bd_1$ in tension and compression respectively.

$s'$, ratio of thickness of flange to effective depth = $d_s/d_1$.

$\gamma$, factor in B.S. Code relating to dimensions of flanged beams.

Rectangular Beams with Tensile Reinforcement only.—The moment of resistance of a rectangular beam reinforced only in tension is $(d_1 - 3A_{st}p_{st})A_{st}p_{st}$ if based on the tensile resistance and $\frac{1}{3}p_{ct}bd_1^2$ if based on the compressive resistance. If $Q_0 = \frac{1}{3}p_{ct}$, the two resulting basic formulae are as given in series A for this case in Table 75.

The safe moment of resistance is the smaller of the two moments calculated by these formulae. For a balanced design these moments are identical and the corresponding value of $A_{st}$ is $\frac{2p_{ct}bd_1}{3p_{ct}}$, if this amount is expressed as a ratio of $bd_1$ the limiting proportion $r_{L}$ is $\frac{p_{ct}}{3p_{ct}}$. If the proportion of reinforcement in a beam is less than $r_{L}$, the tensile resistance controls the strength and if greater the compressive resistance controls. Values of $Q_0$ and $r_{L}$ are given in Table 76 for various stresses.

The expression for the moment of resistance if tension controls, can be written as

$$M_r = Q_0bd_1^2,$$

where $Q_0 = r_{sp}t(1 - F_1r_t)$; $F_1 = \frac{3p_{ct}}{4p_{ct}}$.

Values of the factor $F_1$ for various combinations of stresses are given in Table 76.

Example.—Calculate the moment of resistance of a rectangular beam 24 in. deep ($d_1 = 22\frac{1}{2}$ in.) and 12 in. wide ($b = 12$ in.) if reinforced with four 1-in. mild steel bars in tension only ($A_{st} = 3\times14$ sq. in.). The permissible stresses are $p_{ct} = 20,000$ lb. per sq. in., and $p_{ct} = 1000$ lb. per sq. in. $r_t = \frac{3\times14}{12\times22\frac{1}{2}} = 0.0116$.

(Continued on page 278.)
### Rectangular Beams and Slabs: Formulae—Table 75.
#### Load-Factor Method.

<table>
<thead>
<tr>
<th>Column</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Series A</strong></td>
<td>$F = 2 \left( 1 - \frac{Q}{P_{bc}} \right)$</td>
</tr>
<tr>
<td><strong>Series B</strong></td>
<td>$Q_i = b_d$</td>
</tr>
<tr>
<td><strong>Series C</strong></td>
<td>$Q_i = 2.50 b_d^2$</td>
</tr>
<tr>
<td><strong>Series D</strong></td>
<td>$Q_i = 2.50 b_d^2$</td>
</tr>
<tr>
<td><strong>Series E</strong></td>
<td>$Q_i = 2.50 b_d^2$</td>
</tr>
</tbody>
</table>

**Note:** Values of $Q$, $P$, and $F$ are given in Table 76.

---

**Design Formulae**

- **Tensile Reinforcement Only:** $\frac{A_{st}}{f_{yd}} < \frac{r}{L}$
- **Effective Depth Required:** $d_1 = \frac{A_{st}}{f_{yd}}$
- **Effective Depth Provided:** $d_1 = d_1'$

**Moment of Resistance:** $M = P_M b_d d_1$
Example (continued from page 276).

(i) Using Table 75.—Formulæ in series B apply. \( r_L = \frac{1}{60} = 0.0167 \) which exceeds \( r_t \); therefore tension controls and

\[
M_r = 20,000 \times 3.14 \left( 22.5 - \frac{15 \times 3.14}{12} \right) = 1,170,000 \text{ in.-lb.}
\]

(ii) Using Table 76.—\( r_L = 0.017 \) which exceeds \( r_t \); therefore tension controls. \( F_1 = 15 \).

Substituting in formulæ in foregoing,

\[
Q_T = 0.0116 \times 20,000 [1 - (15 \times 0.0116)] = 193;
\]

\[
M_r = 193 \times 12 \times 22.5^2 = 1,170,000 \text{ in.-lb.}
\]

Rectangular Beams with Compression Reinforcement.—If a rectangular beam contains compression reinforcement the moment of resistance is \( 0.25p_{c}b_d d_1^2 + A_{se} p_{se} (d_1 - d_2) \), which can be rewritten in the form given in Table 75 (Series A), which applies if there is sufficient reinforcement in tension to produce at least an equal resistance. The minimum amount of \( A_{st} \) for this purpose is \( r_L b_d d_1 + A_{se} p_{se} \); the first term is the amount required to balance the resistance of the concrete and the second term is the amount required to balance the resistance of the compression reinforcement.

The compressive stress \( p_{se} \) permissible in the compression reinforcement is in general 18,000 lb. per sq. in. in mild steel bars not greater than \( 1\frac{1}{4} \) in. diameter, 16,000 lb. per sq. in. in larger mild steel bars, and 23,000 lb. per sq. in. in high-yield-stress bars; the Code recommends also \( p_{se} \geq 50,000 \left( 1 - \frac{d_2}{d_1} \right) \). If the depth to the neutral plane is assumed to be \( 0.5 d_1 \) (the limiting depth of the compressive zone recommended in the Code) the limiting value of the compressive stress is \( 50,000 \left( 1 - \frac{2 d_2}{d_1^2} \right) \), but this limit may not apply to ordinary designs, since the cover-ratio \( d_2/d_1 \) must exceed \( 0.32 \) (an uncommonly high ratio) for \( p_{se} \) to be less than 18,000 lb. per sq. in.; likewise the cover-ratio for high-yield-stress bars must exceed \( 0.27 \) (also a high ratio) for the permissible stress to be less than 23,000 lb. per sq. in.

The Code recommends that the area of reinforcement in compression shall not exceed \( 4 \) per cent. of the area \( b_d \). With this limiting amount, the moment of resistance is

\[
Q_b d_1^2 + 0.4 b_d p_{se} (d_1 - d_2).
\]

Substituting \( Q_b = 0.25 p_{c}b_d \) and \( d = d_1 + d_2 \),

\[
M_r = \left\{ \frac{p_{c}}{4} + 0.4 p_{se} \left[ 1 - \frac{d_2}{d_1} \right] \right\} b_d d_1^2.
\]

and

\[
A_{se} = 0.04 \left( 1 + \frac{d_2}{d_1} \right) b_d d_1.
\]

Similarly the corresponding area of tensile reinforcement is

\[
A_{st} = \left[ \frac{p_{c}}{3} + 0.04 \left( 1 + \frac{d_2}{d_1} \right) p_{se} \right] b_d d_1 \frac{P_{st}}{P_{se}}.
\]

Example.—Calculate the moment of resistance of a rectangular beam 26 in. deep \( (d_1 = 22\frac{1}{4} \text{ in.}) \) and 12 in. wide \( (b = 12 \text{ in.}) \) reinforced with six \( 1\frac{1}{4} \)-in. mild steel bars in the bottom and two \( 1\)-in. bars in the top. The permissible stresses are \( P_{st} = 20,000 \text{ lb. per sq. in.}, P_{se} = 18,000 \text{ lb. per sq. in.}, \) and \( p_{c} = 1000 \text{ lb. per sq. in.} \). \( A_{st} = 5.96 \text{ sq. in.}, A_{se} = 1.57 \text{ sq. in.} \) \( d_2 = 1\frac{1}{4} \text{ in.} \).

Apply formulæ in series B in Table 75; assuming compression controls,

\[
M_r = 250[12 \times 22.5^2 + (72 \times 1.57)(22.5 - 1.5)] = 2,142,500 \text{ in.-lb.}
\]

Check that there is sufficient tensile reinforcement:

\[
\frac{12 \times 22.5}{60} + (0.9 \times 1.57) = 5.91 \text{ sq. in.} < 5.96 \text{ sq. in.}
\]

(Continued on page 280.)
### Rectangular and Flanged Beams: Design Factors—Table 76.

#### Load-Factor Method.

<table>
<thead>
<tr>
<th>Permissible Compressive Stress $P_{cb}$ (lb. per sq.in.)</th>
<th>Permissible Tensile Stress (t lb. per sq.in.)</th>
<th>16,000</th>
<th>18,000</th>
<th>20,000</th>
<th>25,000</th>
<th>27,000</th>
<th>30,000</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$Q_C$</td>
<td>$r_L$</td>
<td>$F_1$</td>
<td>$r_L$</td>
<td>$F_1$</td>
<td>$r_L$</td>
<td>$F_1$</td>
</tr>
<tr>
<td>750</td>
<td>1.88</td>
<td>0.016</td>
<td>1.6</td>
<td>0.014</td>
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<td>0.015</td>
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<td>0.014</td>
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<td>3.88</td>
<td>0.032</td>
<td>0.0</td>
<td>0.037</td>
<td>0.2</td>
<td>0.035</td>
<td>2.0</td>
</tr>
<tr>
<td>1600</td>
<td>4.00</td>
<td>0.033</td>
<td>0.0</td>
<td>0.040</td>
<td>0.1</td>
<td>0.039</td>
<td>2.0</td>
</tr>
<tr>
<td>1650</td>
<td>4.13</td>
<td>0.034</td>
<td>0.0</td>
<td>0.043</td>
<td>0.0</td>
<td>0.042</td>
<td>2.0</td>
</tr>
<tr>
<td>1700</td>
<td>4.25</td>
<td>0.035</td>
<td>0.0</td>
<td>0.046</td>
<td>0.0</td>
<td>0.046</td>
<td>2.0</td>
</tr>
<tr>
<td>1750</td>
<td>4.38</td>
<td>0.036</td>
<td>0.0</td>
<td>0.050</td>
<td>0.0</td>
<td>0.050</td>
<td>2.0</td>
</tr>
<tr>
<td>1800</td>
<td>4.50</td>
<td>0.037</td>
<td>0.0</td>
<td>0.054</td>
<td>0.0</td>
<td>0.054</td>
<td>2.0</td>
</tr>
<tr>
<td>1850</td>
<td>4.63</td>
<td>0.038</td>
<td>0.0</td>
<td>0.059</td>
<td>0.0</td>
<td>0.059</td>
<td>2.0</td>
</tr>
<tr>
<td>1900</td>
<td>4.75</td>
<td>0.039</td>
<td>0.0</td>
<td>0.065</td>
<td>0.0</td>
<td>0.065</td>
<td>2.0</td>
</tr>
<tr>
<td>1950</td>
<td>4.88</td>
<td>0.040</td>
<td>0.0</td>
<td>0.073</td>
<td>0.0</td>
<td>0.073</td>
<td>2.0</td>
</tr>
<tr>
<td>2000</td>
<td>5.00</td>
<td>0.041</td>
<td>0.0</td>
<td>0.082</td>
<td>0.0</td>
<td>0.082</td>
<td>2.0</td>
</tr>
</tbody>
</table>

**Values of Factor $F = 2 \left[ 1 - \frac{1}{1 - \frac{3Q_1}{P_{cb}}} \right]$**

<table>
<thead>
<tr>
<th>$Q_1$ = $bd_2$, lb. per sq.in.</th>
<th>120</th>
<th>144</th>
<th>150</th>
<th>160</th>
<th>170</th>
<th>180</th>
<th>188</th>
<th>200</th>
<th>220</th>
<th>240</th>
<th>250</th>
<th>260</th>
<th>280</th>
<th>300</th>
<th>313</th>
</tr>
</thead>
<tbody>
<tr>
<td>Permissible Compressive Stress $P_{cb}$ (lb. per sq.in.)</td>
<td>1250</td>
<td>0.037</td>
<td>0.040</td>
<td>0.043</td>
<td>0.046</td>
<td>0.049</td>
<td>0.052</td>
<td>0.056</td>
<td>0.063</td>
<td>0.070</td>
<td>0.074</td>
<td>0.077</td>
<td>0.080</td>
<td>0.085</td>
<td>1.00</td>
</tr>
<tr>
<td>1000</td>
<td>0.040</td>
<td>0.043</td>
<td>0.046</td>
<td>0.049</td>
<td>0.052</td>
<td>0.056</td>
<td>0.060</td>
<td>0.064</td>
<td>0.068</td>
<td>0.073</td>
<td>0.083</td>
<td>0.094</td>
<td>1.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>750</td>
<td>0.056</td>
<td>0.067</td>
<td>0.073</td>
<td>0.080</td>
<td>0.087</td>
<td>0.094</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note.—For basic formulae, see Table 75.

279
Design of Rectangular Beams.—The basic formulae in series A in Table 75 for the moment of resistance can be transposed and simplified to give expressions for determining the size of a beam and the amount of reinforcement to resist a given bending moment. The formula for the effective depth required is given in Table 75; formulae are also given for the amount of reinforcement required for the three cases of when the effective depth provided is equal to, greater than, or less than that required. Compression reinforcement is required in the latter case.

The formulae in series A are further simplified by substituting permissible stresses assuming the concrete to be 1:2:4 ordinary grade and the reinforcement to be mild steel bars or high-yield-stress bars; the formulae in series B and C in Table 75 result. Values of the coefficient $F$ for various values of $Q_1$ and for $\phi_{lb} = 750$ lb., 1000 lb., and 1250 lb. per sq. in. are given in Table 76.

The moments of resistance and areas of reinforcement for rectangular beams of various depths and unit width ($b = 1$ in.) reinforced with mild steel bars not greater than $1\frac{1}{4}$ in. diameter are given in Table 77A for 1:2:4 concrete in which the permissible compressive stresses are 1000 lb. per sq. in. (ordinary grade), 750 lb. per sq. in. (lower grade), and 1250 lb. per sq. in. (higher grade). The data area given for beams without reinforcement in compression, and with the limiting amount of compression reinforcement ($A'_{se} = 0.044d$). The values in Table 77A must be multiplied by the width of the beam to give the actual moment of resistance and the corresponding areas of reinforcement in tension and compression. It is assumed in Table 77A that the distance from the tensile edge of the beam to the centre of the tensile reinforcement is equal to the distance $d_1$ from the compressive edge to the centre of the compression reinforcement; a reasonable value for $d_1$ is assumed for each beam and usually a variation from this value of, say, $\pm 1\frac{1}{4}$ in. has no significant effect on the tabulated data; if $d_1$ is much greater than is assumed, the data should be adjusted to conform to the actual value.

The data in Table 77B is similar to that in Table 77A but relates to rectangular beams reinforced with bars having a yield stress of not less than 60,000 lb. per sq. in.

Examples.—(a) Design a beam, reinforced with mild steel bars, to resist a bending moment of 1,800,000 in.-lb., if the stress in the concrete is not to exceed 1000 lb. per sq. in. Tensile reinforcement only.

(i) By using Table 75; formulae series B apply. (The method in this example applies to any other stresses by using formulae series A.)

Assume $b = 15$ in. $d_1' = \sqrt{\frac{1,800,000}{250 \times 15}} = 21.9$ in. If $d$ is made 24 in., $d_1 = 22\frac{1}{4}$ in. with 1-in. bars and 1-in. cover; therefore $d_1 > d_1'$ and $A_{st} = \frac{bd_1'}{60}$. $Q_1 = \frac{1,800,000}{15} \times 22.5^2 = 237$; from Table 76, $F = 0.93$ approx; $A_{st} = \frac{15 \times 22.5 \times 0.93}{60} = 5.24$ sq. in., say seven 1-in. bars, which can just be accommodated in one layer in a width of 15 in.

If fewer larger bars, say $1\frac{1}{4}$-in. diameter, are provided with $1\frac{1}{4}$-in. cover, $d_1 = 22\frac{1}{4}$ in.

Therefore $d_1 > d_1'$, $Q_1 = 245$, $F = 0.96$ approx, and $A_{st} = \frac{15 \times 22.125 \times 0.96}{60} = 5.32$ sq. in., say, five $1\frac{1}{4}$-in. bars in a beam 24 in. by 15 in.

(ii) By using Table 77A. Assume $b = 15$ in.; therefore $M' = M' = \frac{1,800,000}{15} = 120,000$ in.-lb. per inch; from the table, a beam 24 in. deep ($M' = 122,000$ in.-lb. per inch width) is satisfactory; $A_{st} = 0.369 \times 15 = 5.54$ sq. in., that is, five $1\frac{1}{4}$-in. bars in a beam 24 in. by 15 in. [as in (ii)]. (If high-yield stress bars are provided Table 77B would be used in a similar manner.)

(b) A beam 20 in. deep and 10 in. wide is required to resist a bending moment of 1,200,000 in.-lb. Determine the amount of high-yield-stress reinforcement ($\phi_{st} = 30,000$ lb. per sq. in.) required if higher-grade 1:2:4 concrete is used ($\phi_{lb} = 1250$ lb. per sq. in.).

$d = 20$ in.; $d_1 = \text{say} 18$ in.; $d_4 = \text{say} 2$ in.; $b = 10$ in.

From Table 76, $Q_e = 313 \left( \frac{1250}{4} \right)$; $r_L = 0.014$.

(Continued on page 282.)
## RECTANGULAR BEAMS - MOMENT OF RESISTANCE AND REINFORCEMENT

### LOAD-FACTOR METHOD

<table>
<thead>
<tr>
<th>Mild Steel Bars</th>
<th>$P_{sf} = 20,000$ lb. per sq. in.</th>
<th>$P_{sc} = 18,000$ lb. per sq. in.</th>
</tr>
</thead>
</table>

### LOWER GRADE 1:2:4 CONCRETE

<table>
<thead>
<tr>
<th>$P_{cb} = 750$ lb. per sq. in.</th>
<th></th>
</tr>
</thead>
</table>

### ORDINARY GRADE 1:2:4 CONCRETE

<table>
<thead>
<tr>
<th>$P_{cb} = 1000$ lb. per sq. in.</th>
<th></th>
</tr>
</thead>
</table>

### HIGHER GRADE 1:2:4 CONCRETE

<table>
<thead>
<tr>
<th>$P_{cb} = 1250$ lb. per sq. in.</th>
<th></th>
</tr>
</thead>
</table>

#### Compression Reinforcement

<table>
<thead>
<tr>
<th>Maximum Amount for All Cases</th>
</tr>
</thead>
</table>

#### Tensile Only

<table>
<thead>
<tr>
<th>Overall Depth (Assumed)</th>
<th>Effective Depth (Approx)</th>
<th>$d_1$ in.</th>
<th>$M_r^t$ in lb</th>
<th>$A_{st}$ in sq. in</th>
<th>$Q'_d$ in lb</th>
<th>$A_{st}$ in sq. in</th>
<th>$A_{sc}$ in sq. in</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>14</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>15</td>
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<tr>
<td>16</td>
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<tr>
<td>18</td>
<td></td>
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<tr>
<td>20</td>
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<td></td>
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<tr>
<td>21</td>
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<tr>
<td>22</td>
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</tr>
<tr>
<td>24</td>
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<td>32</td>
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</tr>
<tr>
<td>33</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>34</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td>36</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Notes

- $M_r^t = \text{Moment of Resistance per inch width}$
- $A_{st} = \text{Area of Tensile Reinforcement per inch width}$
- $A_{sc} = \text{Area of Compression Reinforcement per inch width}$
- $Q'_d = 0.25P_{cb} + 720\left(-\frac{d_1}{d_1}\right)$

**Minimum Required to Produce Tabulated $M_r^t$**
Design of Rectangular Beams

Examples (continued from page 280).

From Table 75, \( d' = \sqrt[3]{\frac{1,200,000}{313 \times 10}} = 19.6 \text{ in.}; \) therefore \( d_1 < d_1' \), and compression reinforcement \( (\rho_{se} = 23,000 \text{ lb. per sq. in.}) \) is required. \( Q_1 = \frac{1,200,000}{10 \times 18^2} = 370. \)

\[
A_{se} = \frac{(370 - 313)(10 \times 18)}{23,000(1 - \frac{2}{18})} = 0.5 \text{ sq. in., say, two } \frac{1}{4}\text{-in. bars in the top.}
\]

\[
A_{st} = (0.014 \times 10 \times 18) + \frac{0.5 \times 23,000}{30,000} = 2.88 \text{ sq. in., say, three } 1\frac{1}{4}\text{-in. bars in the bottom. Twisted ribbed bars or similar bars would be used.}
\]

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SLABS FOR LIQUID-CONTAINING STRUCTURES.

B.S. Code No. 2007.

Examples (continued from page 274).

Recalculate with \( d = 9 \text{ in.}, \) and \( d_1 = 7 \text{ in.}; \) \( d_1 - \frac{1}{3}d = 2\frac{1}{3} \text{ in.} \)

\( r_0' = 0.0077; \) \( f_{et} = 195 \text{ lb. per square inch } (\cong 190); \) satisfactory if \( A_{et} \) is slightly greater than \( 12 \times 0.0077 \times 9 = 0.83 \text{ sq. in. per foot}; \) provide say 1-in. bars at 10\frac{1}{4}\text{-in. centres at each face (total = 0.898 sq. in. per foot}).

(c).—Tensile force of 10,000 lb. per foot and bending moment of 48,000 in.-lb. per foot.

Assume \( d = 10 \text{ in. and } d_1 = 8 \text{ in.}; \) \( d_1 = 0.8; \) \( \varepsilon = \frac{48,000}{10,000} = 4.8 \text{ in.} (> 8 - 5 = 3 \text{ in.}; \) therefore Case IIIA b applies.)

\[
r_0 = \frac{10,000}{144,000 \times 10} \left( 1 + \frac{4.8 + 5 - 8}{0.83 \times 8} \right) = 0.0088; \quad 14r_0 = 0.122.
\]

\[
h_0 = \frac{0.5 + (0.122 \times 0.8)}{1 \times 122} = 0.532.
\]

\[
f_{et} = \frac{10,000}{12 \times 10} \left[ \frac{1}{1 \times 122} + \frac{4.8(1 - 0.532)}{[\frac{1}{4} - (1 - 0.532) \times 0.532 + 0.122(0.8 - 0.532)^2]10} \right]
\]

\[
= 275 \text{ lb. per square inch } (\cong 270); \) satisfactory if \( A_{et} \) is slightly greater than \( 12 \times 0.0088 \times 10 = 1.06 \text{ sq. in. per foot}; \) provide 1-in. bars at 8-in. centres \((1 \times 178 \text{ sq. in.}) \) at tensile face.

Note.—For designs in Case IIIB in Table 74, the procedure is as in the foregoing if \( d < 9 \text{ in.}, \) and as for ordinary designs if \( d < 9 \text{ in.} \).
### Rectangular Beams: Resistance and Reinforcement. Table 77b. High-Yield-Stress Bars. Load-Factor Method.

<table>
<thead>
<tr>
<th>$P_{sc}$, kips per sq. in.</th>
<th>30,000</th>
<th>7500</th>
<th>1000</th>
<th>750</th>
<th>100</th>
<th>75</th>
<th>100</th>
<th>75</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{cb}$, kips per sq. in.</td>
<td>23,000</td>
<td>7500</td>
<td>1000</td>
<td>750</td>
<td>100</td>
<td>75</td>
<td>100</td>
<td>75</td>
<td>100</td>
</tr>
<tr>
<td>$Q$, kips per sq. in.</td>
<td>2084</td>
<td>2500</td>
<td>3000</td>
<td>2084</td>
<td>2500</td>
<td>3000</td>
<td>2084</td>
<td>2500</td>
<td>3000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Tensile only tens. &amp; comp.</th>
<th>1875</th>
<th>2500</th>
<th>3000</th>
<th>1875</th>
<th>2500</th>
<th>3000</th>
<th>1875</th>
<th>2500</th>
<th>3000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Effective depth, in.</td>
<td>12</td>
<td>15</td>
<td>18</td>
<td>12</td>
<td>15</td>
<td>18</td>
<td>12</td>
<td>15</td>
<td>18</td>
</tr>
<tr>
<td>Cover, in.</td>
<td>8</td>
<td>10</td>
<td>12</td>
<td>8</td>
<td>10</td>
<td>12</td>
<td>8</td>
<td>10</td>
<td>12</td>
</tr>
<tr>
<td>Moment, in.<em>2</em></td>
<td>2.5</td>
<td>3.0</td>
<td>3.5</td>
<td>2.5</td>
<td>3.0</td>
<td>3.5</td>
<td>2.5</td>
<td>3.0</td>
<td>3.5</td>
</tr>
<tr>
<td>Moment, in.<em>3</em></td>
<td>3.0</td>
<td>3.5</td>
<td>4.0</td>
<td>3.0</td>
<td>3.5</td>
<td>4.0</td>
<td>3.0</td>
<td>3.5</td>
<td>4.0</td>
</tr>
</tbody>
</table>

*Note: $A_{pl}$ is area of tensile reinforcement per inch width.*
REINFORCED CONCRETE DESIGNER'S HANDBOOK

DESIGN OF SOLID SLABS: LOAD-FACTOR METHOD.

Solid Slabs.—Solid slabs can be designed as rectangular beams of 1 ft. width if the applied bending moment is in inch-pounds per foot width of slab and \( A_{sl} \) is the area of reinforcement in square inches per foot width. The basic formula in series A in Table 75 apply directly to solid slabs if \( 12 \) in. is substituted for \( b \). Similarly the formulae in series D for mild steel bars and series E for high-yield-stress bars or wire are obtained from formulae in series B and C respectively by substituting \( b = 12 \) in. No formulae are given for solid slabs with compression reinforcement as such slabs are uncommon; any case of such a slab can be designed directly from the basic formula.

In Table 78 are given the moments of resistance and areas of reinforcement in tension for solid slabs 1 ft. wide and of various thicknesses reinforced with mild steel bars not greater than \( \frac{1}{4} \) in. diameter or with high-yield-stress bars.

Examples.

(a) Design a solid slab to resist a bending moment of 27,000 in.-lb. per ft. width; maximum stresses \( p_{h1} = 18,000 \) lb. and \( p_{ob} = 1000 \) lb. per sq. in.

From Table 76, \( Q_0 = 250 \). From formula in series D in Table 75,

\[
d_1' = \sqrt{\frac{27,000}{3000}} = 3.0 \text{ in.}; \text{ that is 4-in. slab (4-in. cover); } d_1 = 3.25 \text{ in.}\]

From Table 76, \( r_L = 0.019; A_{st} = 0.019 \times 3.0 \times 12 = 0.68 \text{ sq. in.; say, } \frac{1}{10} \text{-in. mild steel bars at } \frac{1}{3} \text{-in. centres.}

(b) Repeat example (a) by using Table 78 if maximum stresses are \( p_{h1} = 20,000 \) lb. and \( p_{ob} = 1000 \) lb. per sq. in.

For \( M_r = 27,000 \) in.-lb., a 4-in. slab is required with \( 0.600 \) sq. in. (say, \( \frac{1}{4} \)-in. mild steel bars at 6-in. centres).

(c) Calculate the moment of resistance of a 4-in. slab reinforced with \( \frac{1}{4} \)-in. bars at \( \frac{1}{4} \)-in. centres. \( p_{h1} \geq 18,000 \) lb. and \( p_{ob} \geq 1000 \) lb. per sq. in.

\[
d_1 = 3.25 \text{ in. } A_{st} = 0.524 \text{ sq. in. } r_t = \frac{0.524}{12 \times 3.25} = 0.0134. \text{ } r_L = 0.019 \text{ (Table 76)}; \text{ therefore } r_t < r_L.

M_r = Q_T b d_1 \text{ (see page facing Table 75). } F_1 \text{ (Table 76) } = 14.

\[Q_T = 0.0134 \times 18,000 \text{ [1 - (14 \times 0.0134)] = 196.}\]

\[M_r = 196 \times 12 \times 3.25 = 24,600 \text{ in.-lb. per ft. width.}\]

(d) Repeat example (b) by using Table 78 if \( p_{h1} \geq 20,000 \) lb. per sq. in. As before, \( d_1 = 3.25 \) in., \( r_t = 0.0134 \) with \( A_{st} = 0.524 \) sq. in. per ft.

From Table 75, \( r_L = \frac{1}{60} = 0.0167 \) which is greater than \( r_t; \) therefore tension controls and, from formula in series D,

\[M_r = 20,000 \times 0.524[3.25 - (1.25 \times 0.524)] = 25,900 \text{ in.-lb. per ft.}\]

DESIGN OF BEAMS: LOAD-FACTOR METHOD.

Flanged Beams.—Flanged beams include tee-beams, ell-beams, I-beams, inverted channels, and the like. The moment of resistance of a flanged beam is \( A_{st} p_{st}(d_1 - 0.5 d_2) \) if based on the tensile resistance, and \( \gamma p_{ob} b d_1^2 \) if based on the compressive resistance, where the coefficient \( \gamma \) is \( b = \frac{b_1}{4} + \frac{1}{3} (1 - \frac{b_2}{b_1}) \left[ \frac{b_2}{d_1^2} - \left( \frac{d_2}{d_1} \right)^2 \right] \). If the beam contains compression reinforcement the moment of resistance is \( \gamma p_{ob} b d_1^2 + A_{st} p_{st}(d_1 - d_2) \) As in the case of rectangular beams, these expressions are simplified to give the formula in Table 79 (corresponding to those for rectangular beams in Table 75).

If \( \frac{b}{b_1} = K, \) \( s' = \frac{d_2}{d_1} \) and \( Q_c = \frac{p_{ob}}{4} \) (as before),

\[K = x + \frac{4s'}{3} \left( \frac{b}{b_1} - x \right) (2 - s') \]

and therefore, with tensile reinforcement only \( M_r = KQ_c b_1 d_1 \). This formula applies when the compressive resistance of the concrete controls. If the tensile resistance of the reinforcement controls, \( M_r = A_{st} p_{st} d_1 (x - 0.5 s') \).

(Continued on page 286.)
| THICKNESS | 3 | 3\frac{1}{2} | 4 | 4\frac{1}{2} | 5 | 5\frac{1}{2} | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 14 | 15 | 16 | 18 |
|-----------|---|-------------|---|-------------|---|-------------|---|---|---|---|---|---|---|---|---|---|
| d_in.     | 2.25 | 2.75 | 3.00 | 3.50 | 4.00 | 4.50 | 4.88 | 5.88 | 6.88 | 7.50 | 8.50 | 9.50 | 10.50 | 12.50 | 13.50 | 14.50 | 16.50 |
| M_f        | 11,400 | 17,000 | 20,300 | 27,600 | 36,000 | 46,600 | 54,800 | 78,100 | 106,800 | 126,700 | 162,600 | 204,000 | 248,000 | 332,000 | 380,000 | 450,000 | 588,000 |
| A_s        | 11,400 | 17,000 | 20,300 | 27,600 | 36,000 | 46,600 | 54,800 | 78,100 | 106,800 | 126,700 | 162,600 | 204,000 | 248,000 | 332,000 | 380,000 | 450,000 | 588,000 |
| M_f        | 15,200 | 22,600 | 27,000 | 36,800 | 48,000 | 60,800 | 71,600 | 104,100 | 142,200 | 168,600 | 216,600 | 271,500 | 331,500 | 442,500 | 519,000 | 600,000 | 783,000 |
| A_s        | 15,200 | 22,600 | 27,000 | 36,800 | 48,000 | 60,800 | 71,600 | 104,100 | 142,200 | 168,600 | 216,600 | 271,500 | 331,500 | 442,500 | 519,000 | 600,000 | 783,000 |
| M_f        | 19,000 | 28,400 | 33,800 | 45,500 | 60,000 | 76,000 | 89,500 | 130,000 | 177,500 | 210,000 | 270,000 | 339,000 | 415,000 | 553,000 | 649,000 | 750,000 | 978,000 |
| A_s        | 19,000 | 28,400 | 33,800 | 45,500 | 60,000 | 76,000 | 89,500 | 130,000 | 177,500 | 210,000 | 270,000 | 339,000 | 415,000 | 553,000 | 649,000 | 750,000 | 978,000 |
| COVER OF CONCRETE = DIA. OF BAR (OR WIRE) AND \( \sqrt{2} \) in. | M_f = MOMENT OF RESISTANCE PER FOOT WIDTH. A_s = AREA OF TENSILE REINFORCEMENT PER FOOT WIDTH. (MINIMUM REQUIRED TO PROVIDE TABULATED M_f)
REINFORCED CONCRETE DESIGNER'S HANDBOOK

DESIGN OF BEAMS: LOAD-FACTOR METHOD

(continued from page facing Table 78).

The limiting proportion \( r_L \) of tensile reinforcement \( \left( \frac{A_{st}}{b_r d_1} \right) \), which determines which condition applies, is given by \( \frac{KQ_e}{\rho_{st}(b - 0.5'')} \).

For a flanged beam with compression reinforcement

\[
M_r = KQ_e b_r d_1^2 + A_{se} \rho_{se}(d_1 - d_2)
\]

which applies if there is sufficient tensile reinforcement to produce at least this resistance; the minimum \( A_{st} \) required for this purpose is \( \frac{KQ_e b_r d_1^2}{\rho_{st}(d_1 - 0.5'd_2)} + \frac{A_{se} \rho_{se}}{\rho_{st}} \).

The basic formulae are given in series \( F \) in Table 79, in which also are given values of \( K \) for various values of the ratio of \( b \) to \( b_r \) and of \( s' \); intermediate values can be interpolated or can be calculated from the formula given at the bottom of Table 79.

The dimensions of a flanged beam are generally determined from considerations other than resistance to bending; resistance to shearing may determine the size of the rib, and the thickness \( d_s \) of the slab may be determined by its span and loading. The dimensions \( d_1, d_s, b \), and \( b_r \) are therefore known, or can be assumed, and the amount of tensile reinforcement required to resist the applied bending moment is calculated from the basic and special formulae for \( A_{st} \) in Table 79. The compressive resistance can be checked by comparing the value of \( K \) corresponding to the known value of \( b/b_r \) and of \( s' \) with the minimum value required, that is \( Q_{1}/Q_{2} \) in which \( Q_1 = \frac{M}{b_r d_1^2} \), if \( K \) is less than this value, compression reinforcement is required or the dimensions of the cross-section of the beam must be increased.

Compression reinforcement may be required in tee-beams and other special beams; if \( d_s < \frac{4}{3} d_1 \), the amounts of compression reinforcement and the corresponding amount of tensile reinforcement required are given by the formulae for \( A_{st} \) and \( A_{se} \) respectively in Table 79.

Examples.—(a) Calculate the moment of resistance of a tee-beam the rib of which is 10 in. wide and 18 in. deep below the soffit of a 6-in. slab. The span is 18 ft. and the distance between adjacent supporting beams is 8 ft. The tensile reinforcement comprises four 1-in. mild steel bars (\( \rho_{st} = 20,000 \text{ lb. per sq. in.} \)) and the concrete is ordinary grade 1:2:4 (\( \rho_{se} = 1000 \text{ lb. per sq. in.} \)).

\[ A_{se} = 0. \]

The special formulae in series \( G \) in Table 79 can be used. \( A_{st} = 3'14 \) sq. in.

Effective breadth: Available slab = 96 in., or \( \frac{1}{3} \) span = 72 in., or \( 12d_s + b = (12 \times 6) + 10 = 82 \text{ in.} \); hence maximum \( b_s = 72 \text{ in.} \); effective depth \( d_2 = (18 + 6 - 1') = 22' \text{ in.} \).

\[
\frac{b}{b_r} = \frac{72}{10} = 7'2; \quad s' = \frac{6}{22'5} = 0'27.
\]

From Table 79, \( K = 4'14 \) approx., or by calculation

\[
K = 1 + \frac{4 \times 0'27(7'2 - 1)(2 - 0'27)}{3} = 4'85. \]

\( Q_e = \frac{1000}{4} = 250. \)

\[
r_L = \frac{250 \times 4'85}{20,000[1 - (\frac{1}{3} \times 0'27)]} = 0'07. \]

\[
\frac{A_{st}}{b_r d_1} = \frac{3'14}{10 \times 22'5} = 0'014, \text{ which is less than } r_L; \therefore \text{tension controls and } M_r = 20,000 \times 3'14(22'5 - 3) = 1,225,000 \text{ in.-lb. (This is the same moment of resistance as calculated by the modular-ratio method.)}
\]

(b) Design the tee-beam in (a) using high-yield-stress steel (\( \rho_{st} = 30,000 \text{ lb. per sq. in.} \)) to resist a bending moment of 3,000,000 in.-lb.

As in (a), \( \frac{b}{b_r} = 7'2 \) but, since a greater amount of tensile reinforcement may be required, \( d_1 \) should be decreased to, say, 22 in., and \( s' = \frac{6}{22} = 0'27 \) as before, and \( K = 4'85 \) and \( Q_e = 250. \)

\[
Q_1 = \frac{3,000,000}{10 \times 22'5} = 620, \quad \frac{Q_1}{\rho_{se}} = \frac{620}{250} = 2'5, \text{ which is less than } K_1; \therefore \text{compression steel is not required. Therefore } A_{st} = \frac{3,000,000}{30,000(22 - 3)} = 5'26 \text{ sq. in., say, seven 1-in. twisted ribbed bars, in two layers (} d_1 = 21'5 \text{ in. approx. compared with 22 in. as assumed).}
## Flanged Beams - Formule for Load-Factor Method

### Basic Formule

<table>
<thead>
<tr>
<th>Moment of Resistance</th>
<th>Tensile Reinforcement Only</th>
<th>Design Formule</th>
</tr>
</thead>
<tbody>
<tr>
<td>( M )</td>
<td>( A_{st} ) ( d_1 )</td>
<td>( Q_c = \frac{P_{cb}}{4} )</td>
</tr>
<tr>
<td>( A_{st} )</td>
<td>( \frac{M}{P_{st} d_1 (1 - s_f^2)} )</td>
<td>( Q_1 = \frac{M}{b_1 q_1^2}; \frac{d_1}{d_s} = s_f )</td>
</tr>
<tr>
<td>( r_L )</td>
<td>( \frac{Q_c K}{P_{st} (1 - s_f^2)} )</td>
<td>( K = 1 + \frac{4 s_f^2 (b + 1)(2 - s_f)}{3} )</td>
</tr>
</tbody>
</table>

### Ordinary Grade 1:2:4 Concrete

- \( P_{cb} = 1000 \text{ lb per sq in} \)
- \( P_{st} = 20,000 \text{ lb per sq in} \)
- \( P_{sc} = 18,000 \text{ lb per sq in} \)
- \( P_{sc} = 30,000 \text{ lb per sq in} \)
- \( P_{sc} = 23,000 \text{ lb per sq in} \)

### Mild Steel Bars

- \( \sigma_{st} = 5000 \) (not greater than \( s_f \))
- \( P_{sc} = 20,000 \text{ lb per sq in} \)
- \( P_{sc} = 18,000 \text{ lb per sq in} \)
- \( P_{sc} = 30,000 \text{ lb per sq in} \)
- \( P_{sc} = 23,000 \text{ lb per sq in} \)

### High-Yield Stress Bars

- \( P_{sc} = 20,000 \text{ lb per sq in} \)
- \( P_{sc} = 18,000 \text{ lb per sq in} \)
- \( P_{sc} = 30,000 \text{ lb per sq in} \)
- \( P_{sc} = 23,000 \text{ lb per sq in} \)

### Formule in Series (G) and (H) Incorporating

- \( P_{sc} = 18,000 \text{ or } 23,000 \text{ lb per sq in} \)
- Apply only if \( \frac{d_1}{d_s} \approx 0.35 \) for mild steel bars
- or \( \approx 0.17 \) for high-yield-stress bars.
RESISTANCE TO SHEARING FORCE.

Shearing Stresses.—The shearing stress \( q \) lb. per sq. in. in a member subject to a bending moment \( M \) (in.-lb.) and a shearing force \( Q \) (lb.) is determined from

\[
q = \frac{M}{b d_a} \tan \alpha,
\]

where \( l_a \) is the lever arm of the member (in.), \( b \) is the breadth of a rectangular beam or the breadth of the rib of a flanged beam (in.), \( d_a \) is the effective depth (in.), and \( \alpha \) is the angle between the top and bottom edges of a beam. The negative sign applies when \( M \) increases as \( d_a \) increases, a condition that occurs at a haunch adjacent to an interior support of a continuous beam. The positive sign applies when \( M \) decreases as \( d_a \) increases, a condition that occurs at a haunch adjacent to a support where a beam is freely supported.

When a beam has parallel edges, as in the common case of a beam of uniform depth,

\[
q = \frac{Q}{b l_a}.
\]

Reinforcement.—The general rule is that if \( q \) exceeds the safe shearing stress, such as given in Tables 56 and 57, the whole of the shearing force should be resisted by reinforcement in the form of binders or inclined bars or both. If \( q \) does not exceed the safe stress, reinforcement to resist shearing is in general not necessary, but the comments on page 72 should be noted.

Binders.—The shearing resistance of vertical binders is given by

\[
\frac{A_{st} p_{st} l_a}{s} = V l_a
\]

in which \( A_{st} = \) cross-sectional area (sq. in.) of the binder taking into account the number of vertical arms, \( p_{st} = \) permissible tensile stress in the binders (lb. per sq. in.), \( l_a = \) lever-arm of the member (in.), and \( s = \) pitch or spacing of the binders (in.). The factor for the resistance to shearing force, taking into account the diameter of the binders, the allowable stress, and the pitch, is \( V = \frac{A_{st} p_{st}}{s} \). Values of \( V \) for 18,000 lb. and 20,000 lb. per sq. in. for various spacings and sizes of binders with two arms are given in Table 8r. The shearing resistance of a system of binders is found by multiplying the appropriate value of \( V \) by the lever-arm of the member.

The spacing of binders to resist a shearing force \( Q \) is given by

\[
s = \frac{A_{st} p_{st} l_a}{Q}
\]

and should not exceed the lever arm; if the calculated value of \( s \) exceeds \( l_a \), the diameter of the binders can be reduced until a suitable pitch is attained. The minimum diameter of bar suitable for binders is \( \frac{1}{8} \) in. for cast-in-situ construction although \( \frac{1}{4} \)-in. steel wire is sometimes used in small precast products; bars over \( \frac{1}{4} \) in. diameter are generally difficult and costly to provide satisfactorily. Although binders may not be required to resist shearing force they are always provided except perhaps in simple lintels, and the maximum spacing should not exceed the effective depth. When compression reinforcement equal in area to the tension steel is provided in a beam and the calculation for resistance moment is based on the "steel-beam" theory, the pitch of the binders should not exceed eight times the diameter of the compression bars, and the diameter of the binders should be calculated accordingly. If the concrete is not ignored in calculating the compressive resistance, the binders should be spaced at distances not exceeding twelve times the diameter of the compression bars.

All binding should be effectively anchored to the top bars in a beam. A number of shapes of binders commonly used are illustrated in Table 8o; the arrangement shown for large tee-beams allows the binders to be open at the top until the reinforcement has been fixed, and, if the top link is provided, is also suitable for heavily-reinforced rectangular beams.

Inclined Bars.—Various arrangements of inclined bars are shown in Table 8o. With a single system of inclined bars arranged as at \( a \) the shearing resistance \( Q \) at any vertical section in the length \( L \) is \( A_{st} p_{st} \sin \theta \) for a single system; when \( \theta = 45 \) deg., \( Q = 0.707 A_{st} p_{st} \), where \( A_{st} = \) cross-sectional area of the inclined bars (sq. in.), \( p_{st} = \) maximum allowable tensile stress (lb. per sq. in.), and \( \theta = \) angle made by the bar with the horizontal.

For a double system, as \( b \), the shearing resistance of any vertical section within the length \( L \) is \( 2 A_{st} p_{st} \sin \theta \) and, when \( \theta = 45 \) deg., \( Q = 1.414 A_{st} p_{st} \). The same resistance can be considered to be provided for the length \( L \), if the shearing stress in the concrete at section AB does not exceed the permissible shearing stress without reinforcement.

(Continued on page 289.)
REINFORCEMENT TO RESIST SHEARING FORCE.—TABLE 80.
ARRANGEMENT OF BINDERS AND BARS.

**BINDERS IN RECTANGULAR BEAM.**
- Ends of binders resist negative B.M. in slab.
- Slab reinforcement

**BINDERS IN ELL-BEAM.**

**BINDERS IN TEE-BEAMS.**
- Slab reinforcement
- Top link optional
- Compression reinforcement in beam
- Double system of binders

**EQUAL STRESS IN STRAIGHT AND INCLINED PARTS.**
- Note relative position of first bar and support
- Single system
- Double system

**REDUCED STRESS IN INCLINED PART FOR COMMON ARRANGEMENT OF BARS.**

**INCLINED BARS.**

**BARS IN DEEP BEAMS.**

**RESISTANCE TO SHEARING FORCE (continued).**

If the inclined bars are spaced in either of the ways shown at (a) and (b), the stress in the inclined part of the bar is equal to that in the straight bottom part of the bar, but if the common arrangement shown at (c) and (d) for bars inclined at 45 deg. is adopted, the stress in the inclined part cannot exceed the proportion of the stress in the straight part stated on the diagram. Inclined bars must be anchored sufficiently beyond the intersection with the neutral plane to enable the stress to be developed. If a bar finishes at the top of the bend as at (e), the allowable stress should not exceed that capable of being developed by the bond on the length e; thus this type of bar is almost valueless except in deep freely-supported beams.

(Continued on page 290)
In Table 81 are tabulated the resistances to shearing force of bars from 3 in. to 2 in. diameter inclined at 45 deg. and 30 deg. for single and double systems at stresses of 20,000 lb. and 18,000 lb. per sq. in. For other stresses, the resistances to shearing force are proportional to the stresses in the inclined bars.

**Design Procedure.**—To calculate the shearing resistance at any section, the value of V for the binders is read from Table 81 and by multiplying the lever-arm the shearing resistance of the binders is obtained. The resistance of the inclined bars is also read from the table. An inspection of the arrangement of the inclined bars will indicate whether a single, double, or a more complex system is provided. The total resistance due to the reinforcement is the sum of the resistance of the binders and the inclined bars. The resistance of the concrete is the product of the lever-arm and the breadth multiplied by the safe shearing stress. The higher of the two resistances (that is, the resistance of the reinforcement or of the concrete) represents the safe resistance of the member.

To design a member, first evaluate \( \frac{Q}{b_{la}} \) and, if less than \( q \) in Table 56 or 57, no reinforcement is required to resist shearing force, but nominal reinforcement should be provided. If it is greater than \( q \) decide from inspection whether the reinforcement shall consist of binders or inclined bars, or both. If binders alone, then \( \frac{Q}{4a} \) gives the value of V required, and a suitable pitch and diameter can be selected from Table 81. If inclined bars form the principal reinforcement there would generally be some binders, if only a nominal amount, and the value of V for these binders can be found from the table. Inclined bars should then be provided to resist a shearing force of \( Q - V_{in} \). If both inclined bars and binders are provided, the procedure is to combine and adjust the values of \( V_{in} \) and \( Q - V_{in} \) to suit the member. This is often done by deciding which bars can be bent up from the bottom of the beam and adding extra inclined bars if necessary; binders are then provided to make up the deficiency of shearing resistance.

In important beams it is advisable to plot the shearing-resistance diagram for the entire beam on the same base and to the same scale as the shearing-force diagram, and be assured that the resistance exceeds the force. For most beams it is generally sufficient to determine the point at which no reinforcement to resist shearing force is required and calculate the reinforcement required at the point of maximum shearing force. Between these two points the reinforcement can generally be allocated by judgment.

**Examples.**—(a) Design the reinforcement for the sections specified to resist a shearing force of 30,000 lb. Allowable stresses in inclined bars and binders, 20,000 lb. per sq. in.; maximum shearing stress in concrete 100 lb. per sq. in. without reinforcement. Lever-arm of all sections, 0.85 \times \text{effective depth}.

(i) Effective depth = 40 in., breadth = 15 in., \( q = \frac{30,000}{0.85 \times 40 \times 15} = 59 \text{ lb. per sq. in.} \)

Therefore no reinforcement to resist shearing is required, but nominal binders throughout and one inclined bar at each end of the beam might be advisable.

(ii) Effective depth = 30 in., breadth = 12 in., \( q = \frac{30,000}{0.85 \times 30 \times 12} = 98 \text{ lb. per sq. in.} \)

This is a border-line case and nominal binders and an inclined bar should be provided as a precaution against accidental overloading, or to offset any other difference between the assumptions in the calculations and actual conditions.

(iii) Effective depth = 20 in., breadth = 10 in., \( q = \frac{30,000}{0.85 \times 20 \times 10} = 178 \text{ lb. per sq. in.} \)

Therefore all the shearing force should be resisted by reinforcement \( L = 0.85 \times 20 = 17 \text{ in.} \). Nominal binding, say, \( \frac{1}{4} \)-in. at 12-in. centres \((V = 256)\) resists \(17 \times 256 = 4350 \text{ lb.}, leaving 25,650 lb. to be resisted by the inclined bars. If \( \frac{1}{4} \)-in. bars are available for bending up, the resistance required is given by one \( \frac{1}{4} \)-in. bar bent up at 45 deg. (double system).

(b) Calculate the resistance to shearing of the section specified in (iii) if reinforced with one \( \frac{1}{4} \)-in. bar bent up at 45 deg. (in double system) and \( \frac{1}{2} \)-in. single binders at 12-in. centres; maximum stress of 20,000 lb. per sq. in. in inclined bars and binders.

Resistence of one \( \frac{1}{4} \)-in. bar at 45 deg. (double system) \( = 22,200 \text{ lb.} \)

... \( \frac{1}{2} \)-in. single binders at 12-in. centres \((V = 368)\) \( = 6,250 \text{ lb.} \)

\( \text{Total} = 28,450 \text{ lb.} \)

The resistance to shearing of the concrete alone \( = 100 \times 17 \times 10 = 17,000 \text{ lb.} \)

Hence the safe shearing resistance is that of the reinforcement and is 28,450 lb.
REINFORCEMENT TO RESIST SHEARING FORCE.—TABLE 81.
RESISTANCE OF BINDERS AND BARS.

<table>
<thead>
<tr>
<th>TENSILE STRESS</th>
<th>18,000 LB. PER SQ. IN.</th>
<th>20,000 LB. PER SQ. IN.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>3/16&quot;</td>
<td>1/4&quot;</td>
</tr>
<tr>
<td>DIA. OF BAR FORMING BINDER</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2&quot;</td>
<td>496</td>
<td>882</td>
</tr>
<tr>
<td>3&quot;</td>
<td>331</td>
<td>587</td>
</tr>
<tr>
<td>4&quot;</td>
<td>248</td>
<td>441</td>
</tr>
<tr>
<td>4 1/2&quot;</td>
<td>221</td>
<td>391</td>
</tr>
<tr>
<td>5&quot;</td>
<td>198</td>
<td>355</td>
</tr>
<tr>
<td>V = Q/A_s evoke alternative values for single binders (two arms) at various spacings (S)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6&quot;</td>
<td>166</td>
<td>294</td>
</tr>
<tr>
<td>7&quot;</td>
<td>142</td>
<td>252</td>
</tr>
<tr>
<td>8&quot;</td>
<td>124</td>
<td>221</td>
</tr>
<tr>
<td>9&quot;</td>
<td>110</td>
<td>186</td>
</tr>
<tr>
<td>10 1/2&quot;</td>
<td>96</td>
<td>168</td>
</tr>
<tr>
<td>V = Q/A_s evoke alternative values for single binders (two arms) at various spacings (S)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>12&quot;</td>
<td>82</td>
<td>147</td>
</tr>
<tr>
<td>14&quot;</td>
<td>72</td>
<td>126</td>
</tr>
<tr>
<td>16&quot;</td>
<td>63</td>
<td>110</td>
</tr>
<tr>
<td>18&quot;</td>
<td>56</td>
<td>88</td>
</tr>
<tr>
<td>24&quot;</td>
<td>41</td>
<td>74</td>
</tr>
</tbody>
</table>

OTHER STRESSES SHEARING RESISTANCE AND V OF BINDERS PROPORTIONAL TO TENSILE STRESS.

MAXIMUM SPACING OF BINDERS

BEAMS WITH COMPRESSION REINFORCEMENTS: S = 12 x DIAM. OF BAR

DITTO STEEL-BEAM THEORY: S = 8 x DIAM. OF BAR

<table>
<thead>
<tr>
<th>TENSILE STRESS</th>
<th>18,000 LB. PER SQ. IN.</th>
<th>20,000 LB. PER SQ. IN.</th>
</tr>
</thead>
<tbody>
<tr>
<td>ANGLE ( \theta )</td>
<td>45 DEG.</td>
<td>30 DEG.</td>
</tr>
<tr>
<td>SYSTEM</td>
<td>SINGLE</td>
<td>DOUBLE</td>
</tr>
<tr>
<td>SHEARING RESISTANCE</td>
<td></td>
<td></td>
</tr>
<tr>
<td>DIA. OF BAR</td>
<td>1/8&quot;</td>
<td>2,490</td>
</tr>
<tr>
<td>1/4&quot;</td>
<td>3,900</td>
<td>7,800</td>
</tr>
<tr>
<td>3/16&quot;</td>
<td>5,630</td>
<td>11,250</td>
</tr>
<tr>
<td>1/2&quot;</td>
<td>7,650</td>
<td>15,300</td>
</tr>
<tr>
<td>Q = A_s P_s \sin \theta</td>
<td></td>
<td></td>
</tr>
<tr>
<td>SYSTEM</td>
<td>SINGLE</td>
<td>DOUBLE</td>
</tr>
<tr>
<td>SHEARING RESISTANCE</td>
<td>DIA. OF BAR</td>
<td>1/8&quot;</td>
</tr>
<tr>
<td>1/4&quot;</td>
<td>3,900</td>
<td>7,800</td>
</tr>
<tr>
<td>3/16&quot;</td>
<td>5,630</td>
<td>11,250</td>
</tr>
<tr>
<td>1/2&quot;</td>
<td>7,650</td>
<td>15,300</td>
</tr>
</tbody>
</table>

MAX. VALUE OF D

| 1/4" | 1,41d_o | 0.71d_o | 2d_o | d_o | 1,41d_o | 0.71d_o | 2d_o | d_o |
| 1/8" | 9,900 | 19,380 | 7,070 | 14,140 | 11,100 | 22,200 | 7,850 | 15,710 |
| 1/4" | 12,650 | 25,290 | 8,950 | 17,890 | 14,050 | 28,100 | 9,940 | 19,880 |
| 1/2" | 15,620 | 31,230 | 11,050 | 22,030 | 17,350 | 34,700 | 12,270 | 24,540 |
| 1" | 18,860 | 37,710 | 13,360 | 26,710 | 20,950 | 41,900 | 14,840 | 29,680 |
| 1 1/2" | 22,410 | 44,820 | 15,910 | 31,810 | 24,900 | 49,800 | 17,670 | 35,340 |

OTHER STRESSES SHEARING RESISTANCE OF INCLINED BARS PROPORTIONAL TO TENSILE STRESS.
REINFORCED CONCRETE DESIGNER’S HANDBOOK

TORSIONAL RESISTANCE AND ARCADE BEAMS (BOW GIRDERs).

Examples.

(a) A bow girder is 18 in. deep and 18 in. wide, has a radius of 12 ft., and subtends an angle of 90 deg. The ends are rigidly fixed and the total load is 45,000 lb. uniformly distributed. Find the maximum negative bending moment, the maximum positive bending moment, and the maximum twisting moment.

$$\theta = 90 \text{ deg.}; \quad R = 12 \text{ ft.}; \quad \text{from Table 82 } G = 1.12;$$

$$w = \frac{45,000}{\pi \times 2 \times 12 \times 0.25} = 2390 \text{ lb. per ft.}; \quad K_w = 2390 \times 12 = 28,700 \text{ lb.}$$

Maximum negative bending moment (at the support):  $$C_1 = 4.5; \quad C_2 = 1.00; \quad C = 0.24.$$  

$$M_{neg.} = \frac{(4.5 \times 1.12) + 0.24}{(4.5 \times 1.12) + 1} \times 28,700 \times 12 = 77,400 \text{ ft.-lb.}$$

Maximum positive bending moment (at midspan):  $$C_1 = 0.32; \quad C_2 = 0.05.$$  

$$M_{pos.} = \frac{(0.32 \times 1.12) + 0.05}{6.04} \times 28,700 \times 12 = 23,200 \text{ ft.-lb.}$$

Maximum twisting moment (at point of contraflexure;  $$\beta = 22 \text{ deg.})$:  $$C_1 = 0.13; \quad C_2 = 0.02.$$  

$$T_{max.} = \frac{(0.13 \times 1.12) + 0.02}{6.04} \times 28,700 \times 12 = 9360 \text{ ft.-lb.}$$

(b) Determine the reinforcement for an 18-in. square beam, circular in plan, radius 12 ft., supported at equal intervals so that the angle subtended at the centre by each arc is 90 deg., the negative bending moment over the support being 89,000 in.-lb., the positive bending moment at mid-span 45,000 in.-lb., and the maximum shearing force at the support 22,500 lb. At the point of contraflexure ($M = 0$) the twisting moment is a maximum and is 136,000 in.-lb. The point of contraflexure is approximately at the fifth-point of the span.

Support section.—With stresses of 18,000 lb. per sq. in. and 1000 lb. per sq. in. and effective depth of 16 in., the moment of resistance (compressive) with $A_{40} = A_{41}$ (but $f_0 > 1000$) is $507 \times 18 \times 16^2 = 2,540,000$ in.-lb. (from Table 70a), which is satisfactory.

With actual effective depth of 16.5 in., $A_{41} = A_{40} = \frac{89,000}{18,000 \times 0.85 \times 16.5} = 3.53$ sq. in., say, four 1/4-in. bars in the top and bottom. These bars can be arranged by providing one 1/4-in. bar in each corner throughout the beam, bars from adjacent spans overlapping both in the top and bottom at the supports to resist negative bending moment.

Shearing stress at support = $\frac{22,500}{18 \times 0.85 \times 16.5} = 89$ lb. per sq. in.; because of the importance of the beam, the entire shearing force is resisted by binders; $V$ required = $\frac{22,500}{0.85 \times 16.5} = 1600$; from Table 81 this is given by 1/4-in. double binders at 4 1/4-in. centres (at 18,000 lb. per sq. in.). The twisting moment at the support section is small and can be ignored.

Midspan section.—Since the bending moment is assumed to be half that at the support, half the support reinforcement is required, that is, two 1/4-in. bars in the top and bottom are sufficient. The shearing force and twisting moment at midspan are zero.

Section at fifth-point of span.—Shearing force = $\frac{0.5 - 0.2}{0.5} \times 22,500 = 13,500$ lb.

Shearing stress = 54 lb. per sq. in., which does not need reinforcement. The bending moment is theoretically zero, but the four bars provided at the support would be continued beyond this section, thus allowing for negative bending moments that may be produced when adjacent spans only are subjected to live load; a margin is also allowed thereby for the longitudinal steel required to resist twisting moment. Twisting moment = 136,000 in.-lb. For a rectangular section with $D_1/D = 1.0$ and $D = 18$ in. (since $M = 0$ the entire cross-section is considered to be uncracked), the section-coefficient varies from 0.30 to 0.39, depending on the percentage of reinforcement. Section-coefficient required: $R = \frac{T}{qD^2}$; with $q = 100$ lb. per sq. in. (for 1:2:4 ordinary concrete, Table 57), $R = \frac{136,000}{100 \times 18^3} < 0.30$. Hence from Table 82, total percentage of torsional reinforcement required is 1.0. This can be divided into two parts: 50 per cent. as longitudinal bars in the corners of the section and 50 per cent. as binding. Assuming that, of the four bars provided at top and bottom, the four corner bars are available for torsional resistance, the percentage is $\frac{3.98 \times 100}{18^2} = 1.23$, which is more than double the amount required. The volume of binding per foot required is $\frac{1.00 \times 18^3 \times 12}{2} = 19.4$ cu. in. The volume of one external 1/4-in. binder = $0.11 \times 4 \times 16 = 7.05$ cu. in. The spacing required = $\frac{12 \times 7.05}{19.4} = 4.35$, say 4-in. centres.
### Torsional Resistance and Arcate Beams — Table 82

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<thead>
<tr>
<th>SHAPE OF SECTION</th>
<th>D_1 + D_2</th>
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<tr>
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<td>OF TORSIONAL</td>
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<td>REINFORCEMENT</td>
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<td>(EQUAL AMOUNTS</td>
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<tr>
<td>OF LONGITUDINAL</td>
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<tr>
<td>BARS AND</td>
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<tr>
<td>BINDERS)</td>
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<tr>
<td>2-25</td>
<td>0-37</td>
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</table>

**Twisting Moment of Resistance**

\[ \frac{R_0 D_1^3}{2} \]

**Section Coefficient as Tabulated**

\[ D = \text{Equivalent Diameter of Uncracked Section} \]

**Safe Shearing Stress**

\[ \sigma = \frac{R_0 D_1^3}{2E} \]

**Applicable to**

1. Beams forming a complete circular system with supports spaced uniformly and with identical load on each span.
2. Beam of single span with ends rigidly fixed.

### Arcate Beams (Circular Arcs)

**Support (Section of Max. Shear Force and Max. Neg. B.M.)**

**Point of Contraflexure (Section of Max. Pos. B.M.)**

**Uniformly-Distributed Load**

\[ (K_w = w R) \]

**Central Conden Load**

\[ (K_w = P) \]

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<th>45°</th>
<th>60°</th>
<th>72°</th>
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<td>0-05</td>
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**G = Torsional Rigidity = NJ**

**FLEXURAL RIGIDITY = EI**

**G is approximately equal for rectangular, T, E, and Ell sections with the same ratio of D : B.**

<table>
<thead>
<tr>
<th>ANGLE θ</th>
<th>10°</th>
<th>15°</th>
<th>18°</th>
<th>22°</th>
<th>30°</th>
<th>52°</th>
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<td>57°</td>
<td>53°</td>
<td>55°</td>
<td>57°</td>
<td>285°/2</td>
<td>57°</td>
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293
Columns with Separate Binders.—The safe concentric load is calculated in general by the load-factor method. If the modular-ratio method is used, the effective area of a column with separate links or binders is the area of the concrete plus \( m \) times the area of the longitudinal reinforcement; for a short rectangular column the safe concentric load is as given by the appropriate formula in Table 83. The value of \( m \) depends upon the concrete, but generally a value of 15 is assumed. The entire area of the concrete is assumed to be effective in carrying the load, but in some cases, as when the cover is considered to have spalled off in a fire in a building with a high fire risk, the safe direct load should be based on the area of the concrete in the core of the column.

The B.S. Code and London By-laws recommend a load-factor method for the design of columns subjected to concentric load. The compressive stress in the reinforcement is independent of the quality of the concrete and is 18,000 lb. per sq. in. in mild steel bars. The safe concentric load is as given by the appropriate formula in Table 83.

The amount of longitudinal reinforcement should be not less than \( 0.8 \) per cent. of the whole concrete area and not more than \( 8 \) per cent. The safe load is calculated in accordance with the B.S. Code and the London By-laws, on square columns with maximum and minimum percentages of longitudinal reinforcement, are given in Table 84; these loads have been calculated with the stresses given in the table, but if other stresses are used the modification to the safe load is not proportional, and the loads should be calculated from the formula.

The pitch of the binders should be not greater than twelve times the diameter of the smallest longitudinal bar or the width of the column, whichever is the smaller. The spacing should not exceed \( 12 \) in. and generally need not be less than \( 6 \) in. For columns in buildings the diameter of the binder should be not less than \( \frac{1}{4} \) in. and not less than one-quarter of the diameter of the largest bar secured by the binder. The largest convenient diameter of binder is \( \frac{1}{4} \) in., but larger diameters can be used, but generally a much smaller bar is sufficient; \( \frac{1}{4} \) in. or \( \frac{3}{8} \) in. are convenient sizes. The arrangement, when the diameter and spacing are known, of separate binders depends on the number of longitudinal bars in the column; a variety of arrangements depending on the number of longitudinal bars is given in Table 83.

Immediately above and below a junction in the longitudinal reinforcement, the binders are sometimes spaced more closely than elsewhere in the column. Such junctions should be made preferably at floor levels or at beam intersections, as in these positions lateral support is obtained and construction is more convenient. Some types of junctions in the longitudinal reinforcement in a column are indicated in Appendix II. The crank in a bar passing from the lower column into the upper column should be not more acute than \( 1 \) in. in a length of \( 1 \) ft. The distance which the bars project into the upper column should be such that the bars overlap the bars in the lower column for a length of not less than 24 times the diameter of the upper bars or for a greater length if such is required to develop by bond the stress in the lower bar. If separate splice bars are provided the area of the splice bars should be not less than the area of the bars in the upper column, and the length of each splice bar above and below the joint should be 24 or more times the diameter of the splice bar. If a column is subjected to bending, the bending moment generally attains its greatest value at a floor level, in which case the calculation of the resistance should take into account the positions of any cranked bars.

Columns with Helical Binding.—The following conditions conform to the recommendations in the B.S. Code.

The area of concrete considered as effective in carrying the load is that in the bound core. The maximum safe axial load which a short column with helical binding can carry is given by

\[
P_0 = P_C + P_T + P_B,
\]

where \( P_C \) is the load carried on the concrete core, \( P_T \) is the load carried on the longitudinal reinforcement, and \( P_B \) is the increase in load due to the helical binding. The values of these components are given in the formulae and data in Table 83.

The area of the longitudinal reinforcement, which should be not less than \( 0.8 \) per cent. or more than \( 8 \) per cent. of the gross area of the concrete. The value of \( P_0 \) must be reduced for a "long" column. The expression 27,000 \( A_b \), for the additional load due to the helical binding, is based on a stress of 13,500 lb. per sq. in. in the binding and, according to the B.S. Code, is the same for all qualities of reinforcement; if high-yield-stress steel is used this stress may be 35 per cent. of the yield stress or not more than 18,000 lb. per sq. in. in accordance with the London By-laws. The stress in the longitudinal reinforcement is as given in Table 58. The spacing of the helical binding should be not less than \( 1 \) in. and not greater than \( 3 \) in. An important limitation is that \( P_C + P_B \) should not exceed \( \frac{1}{4} A u_{oc} \) \( A_{pee} \) is the specified minimum crushing strength of works-test concrete cubes; reducing this to an expression involving the permissible direct compressive stress \( P_C + P_B \) should not be greater than \( \frac{1}{2} \times \frac{3A_{pee}}{0.76} \),

\[
= 1.975A_{pee}.
\]

(Continued on page 296.)
**COLUMNS: GENERAL DATA (CONCENTRIC LOAD).—TABLE 83.**

**SAFE CONCENTRIC LOAD ON SHORT COLUMNS**

\[ P_0 = \text{SAFE CONCENTRIC LOAD ON COLUMN (LB).} \]

**GROSS CROSS-SECTIONAL AREA** \( A = d^2 \times b \) FOR RECTANGULAR COLUMN (SQ. IN.).

**CROSS-SECTIONAL AREA OF MAIN BARS (SQ. IN.).**

\[ A_{bc} = \text{CROSS-SECTIONAL AREA OF MAIN BARS (SQ. IN.).} \]

**VOLUME OF HELICAL BINDING PER INCH OF COLUMN**

\[ V = \frac{4}{3} \pi d_k^2 d_k \]

\[ A_{h} = \frac{V}{d_k} \]

**DIAMETER OF CORE OF COLUMN WITH HELICAL BINDING**

\[ d_k = \text{DIAMETER OF CORE OF COLUMN WITH HELICAL BINDING = MEAN DIA. OF HELIX (IN.).} \]

**DIAMETER OF BAR FORMING HELICAL BINDING (IN.).**

\[ d_k = \text{DIAMETER OF BAR FORMING HELICAL BINDING (IN.).} \]

**PCC**

\[ P_{cc} = \text{PERMISSIBLE DIRECT COMpressive STRESS IN CONCRETE (LB. PER SQ. IN.).} \]

\[ P_{sc} = \text{PERMISSIBLE COMPRESSIVE STRESS IN MAIN BARS (LB. PER SQ. IN.).} \]

**MILD STEEL BARS:**

\[ > \frac{1}{2} \text{ IN. DIA.} = 18,000 \text{ LB./SQ. IN.} \]

\[ > \frac{1}{2} \text{ IN. DIA.} = 15,000 \text{ LB./SQ. IN.} \]

**HIGH-YIELD-STRESS BARS**

\[ = \frac{1}{2} \text{(YIELD-STRESS) AND } > 23,000 \text{ LB./SQ. IN.} \]

**COLUMNS WITH SEPARATE BINDERS (LATERAL TIES)**

**MODULAR-RATIO METHOD.**

\[ P_0 = \left[ A + (m - 1) A_{bc} \right] P_{cc} \]

**LOAD-FACTOR METHOD.**

\[ P_0 = A_{bc} + A_{sc} (P_{cc} - P_{sc}) = A \left( P_{cc} + \frac{r (P_{sc} - P_{cc})}{P_{cc}} \right) \]

**COLUMNS WITH HELICAL BINDING**

**LOAD-FACTOR METHOD (B.S. CODE).**

\[ P_0 = A_{bc} + A_{sc} (P_{cc} - P_{sc}) + 27,000 A_b \]

\[ = 0.7854 \left( \frac{A_{bc}}{P_{cc}} + A_{sc} (P_{cc} - P_{sc}) \right) \times K_2 d_k \]

\[ K_2 = 6,6,600 \frac{d_k}{3} \]

**VALUES OF K_2**

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<th>3</th>
<th>4</th>
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**END CONDITIONS**

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</tr>
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**EFFECTIVE LENGTH L**

**BASED ON LEAST LATERAL DIMENSION DL (RECTANGULAR, ETC., COLUMNS)**

\[ \frac{L}{D_L} > 1.5: R_L = -1.0 \quad \frac{L}{D_L} > 1.6: 33: R_L = 1.5 - \frac{L}{30 D_L} \quad \text{B.S. CODE} \]

**LINEAR INTERPOLATION BETWEEN TABULATED VALUES**

**REDUCTION FACTORS RL**

\[ \frac{L}{D_L} < 33: 57: R_L = 0.55 - \frac{L}{100 D_L} \quad \text{B.S. CODE} \]

**BASED ON LEAST RADIUS OF GRAYRATION \( g \) (GENERAL USE)**

\[ \frac{L}{g} > 50: R_L = 1.0 \quad \frac{L}{g} > 50: 120: R_L = 1.5 - \frac{L}{100 g} \]

**REDUCTION FACTORS RL**

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LONG COLUMNS.—If the ratio of the effective length of a column to the least radius of gyration exceeds about 50, the column is a "long" or slender column and the safe load on the column is less than that on a short column. The reduction factor is \( R_L \) given by the formula in Table 83 in which \( L_E \) is the effective length of the column and \( g \) is the least radius of gyration, \( L_E \) and \( g \) being in the same units. In Table 83 are also given values of \( R_L \) for slenderness ratios up to the generally accepted limit of 150. The permissible load on a long column is \( R_L P_0 \), where \( P_0 \) is the safe concentric load on a short column (that is, a column having a ratio of effective length to least radius of gyration of 50 or less).

For columns with separate binders the least radius of gyration should be calculated on the gross cross-sectional area of the column, and on the core section for columns with helical bindings when the bindings are taken into account in determining the safe load. The radius of gyration may take into account the reinforcement, with a modular ratio of 15. The radii of gyration of some common sections are given in Tables 65 (neglecting reinforcement) and 66A and 66B (taking reinforcement into account).

For square and rectangular columns the slenderness ratio can be based on the least lateral dimension \( D_L \). If the ratio exceeds 15, the reduction coefficient \( R_L \) is

\[
1.5 - \frac{L_E}{30D_L}
\]

up to a ratio of 33 and

\[
0.95 - \frac{L_E}{60D_L}
\]

for ratios from 33 to 57. In Table 83, values of \( R_L \) are given in accordance with these expressions which are based on the B.S. Code for various slenderness ratios up to 57. The London By-laws adopt this basis up to a ratio of 36.

The effective length of a column is the actual length if the end conditions are equivalent to "hinged" at both ends. If a column is "fixed" at both ends, or "fixed" at one end and "free" at the other, or if there are intermediate conditions of restraint, the ratio of effective length to actual length is as given in Table 83 for columns designed in accordance with the B.S. Code or the London By-laws. By "hinged" is meant that the end is properly restrained in position but not in direction. By "fixed" is meant that the end is properly restrained both in position and direction. By "free" is meant that the end of the column is unrestrained in position and direction. By "partially fixed" is meant that the end is fixed in position but imperfectly restrained in direction. The actual length of a column in a building is the distance between floors.

EXAMPLES.—(a) Select from Table 84 square columns of \( 1:2:4 \) ordinary concrete, with separate binders lateral ties, to support a safe concentric load of 400,000 lb.

Suitable columns are 18 in. square with eight \( \frac{1}{2} \)-in. bars; 20 in. square with four \( \frac{3}{4} \)-in. bars or eight \( \frac{1}{2} \)-in. bars; 21 in. square with four \( \frac{3}{4} \)-in. bars or eight \( \frac{3}{4} \)-in. bars.

(b) Calculate the safe direct load on an 18-in. octagonal column of \( 1:1:2 \) concrete, with eight \( \frac{3}{4} \)-in. vertical bars and \( \frac{1}{4} \)-in. helical binding at \( \frac{3}{4} \)-in. centres.

From Table 57, \( P_{k1} = 1,140 \) lb. per sq. in. Diameter of core \( d_k = 18 - 2 = 16 \) in. Area of longitudinal bars (Table 60) = \( 14 \times 14 \) sq. in. \( (K_2 = 3750) \).

\[
P_0 = (0.7854 \times 16^2 \times 1140) + (14 \times 14 \times (18,000 - 1140)) + (3750 \times 16) = 529,000 \text{ lb.}
\]

(assuming a "short" column).

(c) Calculate the safe load on a column 18 in. by 10 in. of \( 1:2:4 \) concrete with six \( \frac{1}{2} \)-in. bars (3-6 sq. in.), if the height of the column extending through several stories is 20 ft. Permissible compressive stresses are 760 lb. per sq. in. and 18,000 lb. per sq. in.

As a "short" column the safe load is \( P_0 = (18 \times 10 \times 760) + (3 \times 6 \times 17,240) = 199,500 \) lb.

Since the least lateral dimension is 10 in., and the effective length is the story-height, that is 20 ft., the slenderness-ratio is given by

\[
\frac{L_E}{D_L} = \frac{20 \times 12}{10} = 24.
\]

The reduction factor \( R_L \) for this ratio is 0.7 (Table 83).

Therefore the safe load is 0.7 \times 199,500 = 139,650 lb.
<table>
<thead>
<tr>
<th>Mixture of Concrete</th>
<th>Size of Square Column</th>
<th>Diameter of Main Reinforcement (Mild Steel) Bars</th>
<th>Four Bars</th>
<th>Eight Bars</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>5/8&quot; 3/4&quot; 7/8&quot; 1&quot; 1 1/8&quot; 1 1/4&quot; 1 3/8&quot; 1 1/2&quot;</td>
<td>1/8&quot; 1/4&quot; 1 1/8&quot; 1 1/4&quot; 1 3/8&quot; 1 1/2&quot;</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>3/4&quot; 7/8&quot; 1&quot; 1 1/8&quot; 1 1/4&quot; 1 3/8&quot; 1 1/2&quot;</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1:2:4</td>
<td>10 in.</td>
<td>97 106 118 130</td>
<td>307 329 355 383</td>
<td>REINFORCEMENT EXCEEDS 8 PERCENT.</td>
</tr>
<tr>
<td></td>
<td>11 in.</td>
<td>113 122 133 146</td>
<td>307 329 355 383</td>
<td>REINFORCEMENT EXCEEDS 8 PERCENT.</td>
</tr>
<tr>
<td></td>
<td>12 in.</td>
<td>131 140 151 164 178</td>
<td>307 329 355 383</td>
<td>REINFORCEMENT EXCEEDS 8 PERCENT.</td>
</tr>
<tr>
<td></td>
<td>14 in.</td>
<td>179 190 205 218 233</td>
<td>LOADS ARE GIVEN</td>
<td>UNITS OF 1000 LB.</td>
</tr>
<tr>
<td></td>
<td>15 in.</td>
<td>213 225 240 256</td>
<td>307 329 355 383</td>
<td>REINFORCEMENT EXCEEDS 8 PERCENT.</td>
</tr>
<tr>
<td></td>
<td>16 in.</td>
<td>236 249 263 279 297</td>
<td>307 329 355 383</td>
<td>REINFORCEMENT EXCEEDS 8 PERCENT.</td>
</tr>
<tr>
<td>1:1:2:3</td>
<td>10 in.</td>
<td>116 125 136 149</td>
<td>368 390 415 444 475</td>
<td>REINFORCEMENT EXCEEDS 8 PERCENT.</td>
</tr>
<tr>
<td></td>
<td>11 in.</td>
<td>136 145 156 169</td>
<td>368 390 415 444 475</td>
<td>REINFORCEMENT EXCEEDS 8 PERCENT.</td>
</tr>
<tr>
<td></td>
<td>12 in.</td>
<td>156 167 178 190 205</td>
<td>368 390 415 444 475</td>
<td>REINFORCEMENT EXCEEDS 8 PERCENT.</td>
</tr>
<tr>
<td></td>
<td>14 in.</td>
<td>184 197 208 220 237</td>
<td>LOADS ARE GIVEN</td>
<td>UNITS OF 1000 LB.</td>
</tr>
<tr>
<td></td>
<td>15 in.</td>
<td>216 227 240 253 270</td>
<td>368 390 415 444 475</td>
<td>REINFORCEMENT EXCEEDS 8 PERCENT.</td>
</tr>
<tr>
<td></td>
<td>16 in.</td>
<td>244 257 270 283 297</td>
<td>368 390 415 444 475</td>
<td>REINFORCEMENT EXCEEDS 8 PERCENT.</td>
</tr>
<tr>
<td>1:1:2:4</td>
<td>10 in.</td>
<td>135 144 155 167</td>
<td>429 450 475 503 535</td>
<td>REINFORCEMENT EXCEEDS 8 PERCENT.</td>
</tr>
<tr>
<td></td>
<td>11 in.</td>
<td>159 168 179 191</td>
<td>429 450 475 503 535</td>
<td>REINFORCEMENT EXCEEDS 8 PERCENT.</td>
</tr>
<tr>
<td></td>
<td>12 in.</td>
<td>185 194 205 217 231</td>
<td>429 450 475 503 535</td>
<td>REINFORCEMENT EXCEEDS 8 PERCENT.</td>
</tr>
<tr>
<td></td>
<td>14 in.</td>
<td>253 264 276 291 306</td>
<td>LOADS ARE GIVEN</td>
<td>UNITS OF 1000 LB.</td>
</tr>
<tr>
<td></td>
<td>15 in.</td>
<td>287 300 314 329 344</td>
<td>429 450 475 503 535</td>
<td>REINFORCEMENT EXCEEDS 8 PERCENT.</td>
</tr>
<tr>
<td></td>
<td>16 in.</td>
<td>332 345 359 375 392</td>
<td>429 450 475 503 535</td>
<td>REINFORCEMENT EXCEEDS 8 PERCENT.</td>
</tr>
<tr>
<td></td>
<td>18 in.</td>
<td>422 436 452 469 486</td>
<td>429 450 475 503 535</td>
<td>REINFORCEMENT EXCEEDS 8 PERCENT.</td>
</tr>
<tr>
<td></td>
<td>20 in.</td>
<td>523 539 556 573 590</td>
<td>429 450 475 503 535</td>
<td>REINFORCEMENT EXCEEDS 8 PERCENT.</td>
</tr>
<tr>
<td></td>
<td>21 in.</td>
<td>569 585 603 622 637</td>
<td>429 450 475 503 535</td>
<td>REINFORCEMENT EXCEEDS 8 PERCENT.</td>
</tr>
<tr>
<td></td>
<td>24 in.</td>
<td>738 763 781 812 845</td>
<td>738 763 781 812 845</td>
<td>REINFORCEMENT EXCEEDS 8 PERCENT.</td>
</tr>
<tr>
<td></td>
<td>27 in.</td>
<td>937 965 997 1101 1126</td>
<td>738 763 781 812 845</td>
<td>REINFORCEMENT EXCEEDS 8 PERCENT.</td>
</tr>
<tr>
<td></td>
<td>30 in.</td>
<td>1160 1192 1226 1256</td>
<td>738 763 781 812 845</td>
<td>REINFORCEMENT EXCEEDS 8 PERCENT.</td>
</tr>
</tbody>
</table>

* This table also applies to concrete Quality A (London By-laws).
### COMBINED BENDING AND THRUST (MODULAR-RATIO METHOD).

**Example.—Annular Member Subjected to Bending and Direct Thrust.**—The method given in Table 85 for the determination of the stresses in a member, the cross-section of which is not rectangular, is applied in the following example to the annular member shown at the bottom of the table.

Eccentricity: \( e = \frac{1,000,000}{50,000} = 20 \text{ in.}; \quad \frac{e}{D} = \frac{20}{36} = 0.556 \). Tensile and compressive stresses are likely to occur. Assume \( d_a = 13 \text{ in.} \) (about \( 0.38d_1 \)).

The properties of the bars below the neutral plane are considered thus:

<table>
<thead>
<tr>
<th>( \delta A )</th>
<th>( \delta A )</th>
<th>( \delta a )</th>
<th>( \beta_a )</th>
<th>( \beta_a - d_a )</th>
<th>( R )</th>
<th>( R\beta_a )</th>
<th>( \delta A \beta_a )</th>
</tr>
</thead>
<tbody>
<tr>
<td>sq. in.</td>
<td>in.</td>
<td>in.</td>
<td>in.</td>
<td>in.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.614</td>
<td>18</td>
<td>5</td>
<td>3.07</td>
<td>55</td>
<td>11.2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.614</td>
<td>24</td>
<td>11</td>
<td>6.75</td>
<td>162</td>
<td>14.7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.614</td>
<td>30</td>
<td>17</td>
<td>10.43</td>
<td>313</td>
<td>18.4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.614</td>
<td>33</td>
<td>20</td>
<td>12.28</td>
<td>405</td>
<td>20.3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.307</td>
<td>34</td>
<td>21</td>
<td>6.45</td>
<td>219</td>
<td>10.5</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\( \Sigma a = 2.76 \)  
\( \Sigma R \beta_a = 1154 \)  
\( \Sigma \delta A \beta_a = 75.1 \)

Centre of the tension: \( \beta_t = \frac{1154}{38.98} = 29.6 \text{ in.} \)

The compressive area above the neutral plane is divided into a series of horizontal strips, the properties of which are tabulated thus:

<table>
<thead>
<tr>
<th>Strip</th>
<th>( b_x )</th>
<th>( \delta_x )</th>
<th>( b_x \delta_x )</th>
<th>( (m - 1)\delta A )</th>
<th>( a )</th>
<th>( x_1 )</th>
<th>( x_m )</th>
<th>( ax_a )</th>
<th>( x_a )</th>
<th>( ax_a x )</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>14</td>
<td>3</td>
<td>42</td>
<td>8.6</td>
<td>51</td>
<td>1.5</td>
<td>11.5</td>
<td>586</td>
<td>76</td>
<td>870</td>
</tr>
<tr>
<td>B</td>
<td>24</td>
<td>3</td>
<td>72</td>
<td>4.3</td>
<td>76</td>
<td>4.5</td>
<td>8.5</td>
<td>645</td>
<td>342</td>
<td>2900</td>
</tr>
<tr>
<td>C</td>
<td>16</td>
<td>3</td>
<td>48</td>
<td>8.6</td>
<td>57</td>
<td>7.5</td>
<td>5.5</td>
<td>314</td>
<td>428</td>
<td>2350</td>
</tr>
<tr>
<td>D</td>
<td>12.5</td>
<td>4</td>
<td>50</td>
<td>8.6</td>
<td>59</td>
<td>11</td>
<td>2.0</td>
<td>118</td>
<td>648</td>
<td>1300</td>
</tr>
</tbody>
</table>

\( \Sigma a = 243 \)  
\( \Sigma ax_a = 1663 \)  
\( \Sigma xa = 1494 \)  
\( \Sigma ax_a x = 7449 \)

Centre of compression: \( \beta_c = \frac{7449}{1663} = 4.47 \text{ in.} \)

If \( m = 15 \), \( x = \frac{(15 \times 75.1) + 1494}{(15 \times 2.76) + 243} = 9.2 \text{ in.} \)

Stresses:  
\( f_{cb} = \frac{50,000 \times 13(20 + 29.6 - 9.2)}{(29.6 - 4.47)1663} = 628 \text{ lb. per sq. in.} \)

\( f_{ct} = \frac{34 - 13 \times [628 \times 1663 - 50,000]}{38.98} = 16,400 \text{ lb. per sq. in.} \)

Check trial position of neutral plane: \( d_a = \frac{34}{1 + \frac{16,400}{15 \times 483}} = 12.35 \text{ in.} \)

compared with the trial value of 13 in.; a second trial is unnecessary.

If a second value were necessary, say, larger than that assumed, the adjustment to the tabulations could be readily made by adding strip E, \( 1\frac{1}{4} \text{ in.} \) deep, the to compressive area and omitting from the reinforcement table any bars now included in the compressive area. For the tensile reinforcement only the values of \( R \) and \( R\beta_a \) are affected. In the compressive area the values of \( x_a \) and consequently \( ax_a \) and \( ax_a x \) are affected. With these modifications the analysis then proceeds as shown for the first trial.
COMBINED BENDING AND THRUST: GENERAL ANALYSIS.—TABLE 85.
MODULAR-RATIO METHOD.

<table>
<thead>
<tr>
<th>Compressive Stresses Only</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equivalent Area of Strip:</td>
</tr>
<tr>
<td>( A_e = \sum a )</td>
</tr>
<tr>
<td>Position of Centroid:</td>
</tr>
<tr>
<td>( \bar{x} = \frac{\sum a x}{A_e} )</td>
</tr>
<tr>
<td>Moment of Inertia About Centroid:</td>
</tr>
<tr>
<td>( I_x = \sum a \left( \frac{x^2}{12} + h \right) )</td>
</tr>
<tr>
<td>Compressive Stresses</td>
</tr>
<tr>
<td>( f_{cb(max)} = \frac{N + M X}{A} \frac{N(\text{THRUST})}{I_x} )</td>
</tr>
<tr>
<td>( f_{cb(min)} = \frac{N}{A} - \frac{M (d - \bar{x})}{I_x} )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Compressive and Tensile Stresses</th>
</tr>
</thead>
<tbody>
<tr>
<td>Assume ( d_n ).</td>
</tr>
<tr>
<td>Centre of Tension:</td>
</tr>
<tr>
<td>( P_t = \frac{\sum R p a}{\Sigma R} )</td>
</tr>
<tr>
<td>If all bars are of same size</td>
</tr>
<tr>
<td>( P_t = \frac{\sum p a (p_a - d_n)}{\sum (p_a - d_n)} )</td>
</tr>
<tr>
<td>Equivalent Area of Strip:</td>
</tr>
<tr>
<td>( a = b e \delta x + (m-1) \delta A_c )</td>
</tr>
<tr>
<td>Centre of Compression:</td>
</tr>
<tr>
<td>( p_c = \frac{\sum x_n x a}{\Sigma x_n a} )</td>
</tr>
<tr>
<td>Position of Centroid of Stressed Area:</td>
</tr>
<tr>
<td>( \bar{a} = \frac{m \sum \delta A_c p_a + \sum a x}{m \Sigma \delta A_t + \Sigma a} )</td>
</tr>
<tr>
<td>Max. Stresses:</td>
</tr>
<tr>
<td>( f_{cb} = \frac{N d_n (e + P_t - \bar{a})}{\Sigma x_n a} )</td>
</tr>
<tr>
<td>( f_{st} = \frac{\sum d_i - d_n \left[ d_n \Sigma x_n a \right]}{\Sigma R} )</td>
</tr>
</tbody>
</table>

**Check Trial Value of \( d_n \).**

**Example**

- \( d_n = 13^\circ \) (1st Trial)
- \( e = \frac{100,000}{50,000} = 2^\circ \)
- \( d_n = 34^\circ \) (2nd Trial)
- B.M. = 1,000,000 in.-lb.
COMBINED BENDING AND THRUST: RECTANGULAR SECTIONS
(MODULAR-RATIO METHOD).

Examples of the Use of Table 86.—A rectangular member is 18 in. deep and 12 in. wide and is reinforced with two 1-in. bars in the top and three 1-in. bars in the bottom. The centres of the bars are \( \frac{1}{2} \) in. from the top and bottom faces respectively. Find the stresses produced in the section by the specified direct thrusts and bending moments.

For all cases \( d = 18 \) in.; \( b = 12 \) in.; \( d_1 = 16.5 \) in.; \( d_2 = 1 \frac{1}{8} \) in.; \( f_1 = \frac{1}{16.5} = 0.091 \);

\( m = 15; \ A_{st} = 2.36 \text{ sq. in.}; \ A_{se} = 1.57 \text{ sq. in.}; \)

\( a) \ M = 200,000 \text{ in.-lb.}, \ N = 100,000 \text{ lb.} \)

\( e = \frac{200,000}{100,000} = 2 \text{ in.}, \) which is less than \( \frac{d_1}{6} (= 3 \text{ in.}) \)

\( A_e = 12 \times 18 = 216 \text{ sq. in.}; \ A_s = 14(2.36 + 1.57) = 55 \text{ sq. in.} \)

\( Z = \frac{2 \times 55}{18} (9 - 1.5)^2 + (0.167 \times 216 \times 18) = 988 \text{ in.}^2 \)

\( f_{eb} = \frac{200,000}{55 + 216} \frac{1}{988} = 571 \text{ lb. (max.) and 165 lb. (min.) per sq. in.} \)

\( b) \ M = 50,000 \text{ in.-lb.}, \ N = 50,000 \text{ lb.} \)

\( e = \frac{200,000}{50,000} = 4 \text{ in.}, \) which is greater than \( \frac{d_1}{6} \) and less than \( \frac{1}{4}d (= 8 \frac{1}{4} \text{ in.}) \)

\( A_e, A_s \) and \( Z \) are the same as in example (a).

\( f_{eb(min.)} = \frac{50,000}{55 + 216} \frac{1}{988} = 184 - 203 = -19 \text{ lb. per sq. in.}, \) which is less than \( \frac{1}{10} \)

of the allowable maximum stress (say, \( \frac{1000}{10} \)) and therefore the maximum compressive stress is

\( 184 + 203 = 387 \text{ lb. per sq. in.} \)

\( c) \ M = 150,000 \text{ in.-lb.}, \ N = 150,000 \text{ lb.} \)

\( e = \frac{300,000}{150,000} = 20 \text{ in.}, \) which is greater than \( \frac{1}{4}d, \) and less than \( \frac{1}{4}d_1 (= 24 \frac{1}{8} \text{ in.}) \).

With \( e = 0.5d \) approx. = 9 in., \( F = \frac{20 - 9}{16.5} + 1 = 1.67 \). Assume \( n_1 = 0.55 \).

\( J = 0.55 \times 12 \times 16.5 \times 0.5 = 55 \). From Table 86, \( G = 0.224 \) and \( H = 11.5 \approx 11.5 \) approx.

Thus \( f_{eb} = \frac{0.224 \times 12 \times 16.5 + (11.5 \times 1.57 \times 0.909)}{15,000 \times 1.67} = 412 \text{ lb. per sq. in.} \)

\( f_{st} = \frac{412[55 + (11.5 \times 1.57)] - 15,000}{2.36} = 6,440 \text{ lb. per sq. in.} \)

The value of \( n_1 \) corresponding to \( f_{st} \) and \( f_{eb} \) is \( \frac{1}{\frac{6440}{412 \times 15}} = 0.49 \), compared with the assumed value 0.55. Re-calculate with an intermediate value of \( n_1 = 0.5 \), for which \( J = 12 \times 16.5 \times 0.5 \times 0.5 = 49.5 \). The accurate value of \( J \) is

\( 16.5[(49.5 \times 0.5) + (15 \times 2.36) + (14 \times 1.57 \times 0.909)] = 6.55 \text{ in.} \)

(It is interesting to compare this value with the assumed approximate value of 0.5d = 9 in.

\( F = \frac{20 - 0.55}{16.5} + 1 = 1.815; \ G = 0.208; \ H = 11.2 \) (or 11.45 by calculation with the exact value \( f_3 = 0.901 \)).

\( f_{eb} = \frac{15,000 \times 1.815}{2.36} = 475 \text{ lb. per sq. in.} \)

\( f_{st} = \frac{475[49.5 + (11.45 \times 1.57)] - 15,000}{2.36} = 7240 \text{ lb. per sq. in.} \)

Corresponding value of \( n_1 = \frac{1}{\frac{7240}{475 \times 15}} = 0.497 \), which is practically the same as the trial value (0.50).

(Continued on page 302.)
### Combined Bending and Thrust: Rectangular Section. — Table 86. Modular-Ratio Method.

#### Eccentricity About Centroid of Stressed Area

\[
e = \frac{B.M.}{N} = \frac{M}{N}
\]

**Method Allows for Any Modular Ratio**

- **Stresses:**
  - \( f_{cb} \text{(MAX)} \) = \( \frac{A_c}{A_{bc}} \) \( \frac{A_c}{A_{bc}} \)
  - \( f_{cb} \text{(MIN)} \) = \( \frac{A_c}{A_{bc}} \) \( \frac{A_c}{A_{bc}} \)

\[
\begin{align*}
  f_{cb} & = \frac{A_c}{A_{bc}} \left( d_2 - d_1 \right) \\
  f_{cb} & = \frac{A_c}{A_{bc}} \left( d_2 - d_1 \right)
\end{align*}
\]

**Modulus of Section:**

\[
Z \text{ (APPROX.)} = \frac{A_c}{d_2} + \frac{2A_s}{d_1} \left( \frac{d_2 - d_1}{2} \right)^2
\]

**Stress:**

\[
\begin{align*}
  f_{cb} & = \frac{A_c}{A_{bc}} \left( d_2 - d_1 \right) \\
  f_{cb} & = \frac{A_c}{A_{bc}} \left( d_2 - d_1 \right)
\end{align*}
\]

**Compute** \( f_{cb} \text{(MIN)} \) **by Method for** \( e < \frac{d_2}{2} \); **if Positive, Method Applies.** IF Negative or Greater than Permissible Tensile Stress IN CONCRETE, **Adopt Method for** \( e > \frac{d_2}{2} \).

**Assume** \( d_n \); **Evaluate** \( n_1 = \frac{d_n}{d_1} \) **and** \( J = \frac{1}{2} bd_n \).

**Determine Position of Centroid of Stressed Area:**

\[
\frac{2J + mA_{bc} + (m-1)A_s}{2} \left( \frac{d_2 - d_1}{2} \right) \text{ APPROX.}
\]

**Calculate F** = \( \frac{e - \frac{Z}{J}}{d_1} \).

**Evaluate** \( C = \frac{n_1}{2} \left( 1 - \frac{m}{n_1} \right) \) **and** \( H = \frac{m}{2} \left( m - 1 \right) \left( n_1 - f_2 \right) \) **or Obtain from Tabulation.**

**Neutral Plane Factor** \( n_1 \):

- 20: 0.043
- 25: 0.144
- 30: 0.155
- 35: 0.155
- 40: 0.165
- 45: 0.185
- 50: 0.185
- 55: 0.205
- 60: 0.205
- 65: 0.205
- 70: 0.215
- 75: 0.225

**Factor G** = \( \frac{n_1}{n_1 (1 - \frac{m}{n_1})} \)

**Ratios of Stresses:**

- **RATIO OF STRESSES**
- **m = 15**
  - \( \frac{f_2}{d_1} \)\( \frac{d_2}{d_1} \):
    - 0.20: 0.02
    - 0.15: 0.03
    - 0.10: 0.04
    - 0.05: 0.06

- **RATIO OF STRESSES**
- **m = 12.5**
  - \( \frac{f_2}{d_1} \)\( \frac{d_2}{d_1} \):
    - 0.20: 0.02
    - 0.15: 0.04
    - 0.10: 0.05
    - 0.05: 0.06

- **RATIO OF STRESSES**
- **m = 10**
  - \( \frac{f_2}{d_1} \)\( \frac{d_2}{d_1} \):
    - 0.20: 0.02
    - 0.15: 0.03
    - 0.10: 0.04
    - 0.05: 0.05

- **RATIO OF STRESSES**
- **m = 8**
  - \( \frac{f_2}{d_1} \)\( \frac{d_2}{d_1} \):
    - 0.20: 0.02
    - 0.15: 0.03
    - 0.10: 0.04
    - 0.05: 0.05

**To Determine Stresses Substitute Evaluated Factors in:**

\[
f_{cb} = \frac{A_c}{A_{bc}} \left( d_2 - d_1 \right) - N
\]

**Check Assumed Value of** \( n_1 \) **from**

\[
\frac{1}{1 + \frac{f_{cb}}{f_{cb}}} = \frac{1}{1 + \frac{f_{cb}}{f_{cb}}}
\]

**Recalculate with Revised Value of** \( n_1 \) **if discrepancy exceeds 5%**

**Calculate** \( f_{cb} \) **and** \( f_{cb} \text{(MAX)} \)** **due to B.M. only, and determine** \( d_n \); **evaluate**

\[
\begin{align*}
f_c & = \frac{N}{b d_n + m A_{bc} + (m-1)A_s} \\
\end{align*}
\]

**Then** \( f_{cb} \text{(MAX)} = f_{cb} + f_c \) **and** \( f_{cb} \text{(MIN)} = f_c - f_m \).
COMBINED BENDING AND THRUST: RECTANGULAR SECTIONS
(MODULAR-RATIO METHOD) (continued from page 300).

Notes on Table 87.—Case I applies to sections subjected to compressive stresses only; Case II is for sections subjected to tensile and compressive stresses. The cross-section of the member is rectangular and there is an equal amount of reinforcement adjacent to each of two opposite faces. The cover ratio is one-tenth of the overall depth.

In a case between Cases I and II, that is, if \( f_{cd(\text{mode})} \) is a small tensile stress, and if tension in the concrete should be ignored, \( f_{cd(\text{max})} \) is greater than the stress obtained for Case I and is about \( \frac{NK_1}{\phi bd} \).

The table applies only if the cover-ratio \( f_b \) is 0·1 for \( A_{st} \) and \( A_{se} \), but the table can be used if the following adjustment is made. If \( f_b > 0·1 \) substitute \( (1·1 - f_b)d \) for the actual value of \( d \). If \( f_b < 0·1 \), the maximum stress in the concrete calculated from the table is slightly in excess of the actual stress.

The table is based on the eccentricity about the centroid of the section. It is accurate for Case I and for large values of \( \varepsilon \), in Case II; for small values of \( \varepsilon \) in Case II the stresses are only approximate; for more exact calculated stresses the case \( \varepsilon > 0·5d \) in Table 86 should be used.

Examples.—Consider a member in which \( d = 20 \text{ in.}, b = 12 \text{ in.}, A_{st} \) and \( A_{se} \) both consist of three 1-in. bars (that is, \( \rho = \frac{100 \times 47\text{I}}{20 \times 12} = 2·0 \) per cent.). With 1½-in. cover, \( d = 18·5 \text{ in.}, d = 2 \text{ in.}; \frac{2}{20} = 0·1; \) therefore Table 87 can be used directly for the determination of the maximum stresses in the following examples, in each of which \( N = 120,000 \text{ lb.} \)

(a) \( M = 240,000 \text{ in.-lb.} \)
\[ \varepsilon = \frac{M}{N} = \frac{240,000}{120,000} = 2 \text{ in.}; \varepsilon_1 = \frac{\varepsilon}{d} = \frac{2}{20} = 0·10. \] This is Case I; for \( \varepsilon_1 = 0·1 \) and \( \rho = 2 \) per cent., \( K_1 = 1·17 \). By substitution
\[ f_{cd(\text{max})} = \frac{1·17 \times 120,000}{12 \times 20} = 585 \text{ lb. per sq. in.} \]

and \( f_{cd(\text{min})} = \frac{2N}{(1 + 0·14\rho)bd} - f_{cd(\text{max})} = \frac{2 \times 120,000}{1·28 \times 12 \times 20} = 585 = 185 \text{ lb. per sq. in.} \)

(b) \( M = 720,000 \text{ in.-lb.} \)
\[ \varepsilon = \frac{720,000}{120,000} = 6 \text{ in.}; \varepsilon_1 = \frac{6}{20} = 0·3. \] This is Case II; for \( \varepsilon_1 = 0·3 \) and \( \rho = 2·0 \) per cent.,
\[ n_0 = 0·80. \] For \( \rho = 2 \) per cent. and \( n_0 = 0·80, K_2 = 0·153. \)
\[ f_{cd(\text{max})} = \frac{720,000}{0·153 \times 12 \times 20^2} = 980 \text{ lb. per sq. in.} \]
\[ f_{st} = 15 \times 980\left(\frac{0·9}{0·8} - 1\right) = 1840 \text{ lb. per sq. in.} \]

COMBINED BENDING AND TENSION.

Example of Use of Table 89.—A rectangular member is 18 in. deep, 12 in. wide, and is reinforced with two 1-in. bars in the top and three 1-in. bars in the bottom. If \( d_1 = 16\frac{1}{2} \text{ in.} \) and \( d_2 = 1\frac{1}{2} \text{ in.}, \) determine the stresses if \( M = 100,000 \text{ in.-lb.} \) and \( N = \) direct pull of 20,000 lb.

Table 89 applies. \( A_{s1} = 2·36 \text{ sq. in.}; A_{s2} = 1·57 \text{ sq. in.} \)
\[ x' = \frac{2·36 \times 15}{3·93} = 9 \text{ in.} \]
\[ \varepsilon = \frac{100,000}{20,000} = 5 \text{ in. which is less than } L_{st} - x' (= 6 \text{ in.}). \]
\[ f_{st1} \text{ (in } A_{s1} = \frac{20,000(5 + 9)}{2·36 \times 15} = 7900 \text{ lb. per sq. in.} \]
\[ f_{st2} \text{ (in } A_{s2} = \frac{20,000(15 - 5 - 9)}{1·57 \times 15} = 850 \text{ lb. per sq. in.} \]
### COMBINED BENDING AND THRUST: SYMMETRICAL SECTIONS.—TABLE 87.
### MODULAR-RATIO METHOD.

#### NOTE:
This table applies when $A_{sc} = A_{st}$ with $m = 15$, and $d_2 = 0.1d$

<table>
<thead>
<tr>
<th>$e_1$</th>
<th>0.5</th>
<th>1.0</th>
<th>1.5</th>
<th>2.0</th>
<th>2.5</th>
<th>3.0</th>
<th>4.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.04</td>
<td>1.14</td>
<td>1.07</td>
<td>1.00</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>0.06</td>
<td>1.25</td>
<td>1.16</td>
<td>1.09</td>
<td>1.01</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>0.08</td>
<td>1.36</td>
<td>1.25</td>
<td>1.17</td>
<td>1.09</td>
<td>1.02</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>0.10</td>
<td>1.47</td>
<td>1.35</td>
<td>1.25</td>
<td>1.17</td>
<td>1.10</td>
<td>1.04</td>
<td>-</td>
</tr>
<tr>
<td>0.12</td>
<td>1.58</td>
<td>1.44</td>
<td>1.33</td>
<td>1.24</td>
<td>1.17</td>
<td>1.10</td>
<td>0.99</td>
</tr>
<tr>
<td>0.14</td>
<td>1.69</td>
<td>1.53</td>
<td>1.42</td>
<td>1.32</td>
<td>1.24</td>
<td>1.17</td>
<td>1.05</td>
</tr>
<tr>
<td>0.16</td>
<td>1.89</td>
<td>1.62</td>
<td>1.51</td>
<td>1.40</td>
<td>1.31</td>
<td>1.24</td>
<td>1.10</td>
</tr>
<tr>
<td>0.18</td>
<td>(1.90)</td>
<td>1.72</td>
<td>1.60</td>
<td>1.48</td>
<td>1.39</td>
<td>1.30</td>
<td>1.17</td>
</tr>
<tr>
<td>0.20</td>
<td>(2.00)</td>
<td>(1.82)</td>
<td>(1.69)</td>
<td>1.57</td>
<td>1.45</td>
<td>1.36</td>
<td>1.22</td>
</tr>
<tr>
<td>0.30</td>
<td>0.70</td>
<td>0.75</td>
<td>0.77</td>
<td>0.80</td>
<td>0.82</td>
<td>0.84</td>
<td>0.86</td>
</tr>
<tr>
<td>0.40</td>
<td>0.54</td>
<td>0.60</td>
<td>0.65</td>
<td>0.67</td>
<td>0.69</td>
<td>0.71</td>
<td>0.74</td>
</tr>
<tr>
<td>0.50</td>
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<tr>
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<td>0.54</td>
<td>0.56</td>
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<tr>
<td>0.70</td>
<td>0.35</td>
<td>0.43</td>
<td>0.47</td>
<td>0.50</td>
<td>0.53</td>
<td>0.55</td>
<td>0.59</td>
</tr>
<tr>
<td>0.80</td>
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<td>0.41</td>
<td>0.45</td>
<td>0.48</td>
<td>0.51</td>
<td>0.52</td>
<td>0.55</td>
</tr>
<tr>
<td>0.90</td>
<td>0.31</td>
<td>0.38</td>
<td>0.43</td>
<td>0.46</td>
<td>0.48</td>
<td>0.50</td>
<td>0.53</td>
</tr>
<tr>
<td>1.0</td>
<td>0.30</td>
<td>0.37</td>
<td>0.41</td>
<td>0.44</td>
<td>0.46</td>
<td>0.48</td>
<td>0.51</td>
</tr>
<tr>
<td>2.0</td>
<td>0.25</td>
<td>0.31</td>
<td>0.35</td>
<td>0.38</td>
<td>0.40</td>
<td>0.42</td>
<td>-</td>
</tr>
<tr>
<td>3.0</td>
<td>0.23</td>
<td>0.29</td>
<td>0.33</td>
<td>0.36</td>
<td>0.40</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>
COLUMNS SUBJECT TO BENDING: LOAD-FACTOR METHOD.

Examples.—A column of 1: 2: 4 ordinary-quality concrete \((\rho_{ce} = 760 \text{ lb. per sq. in.})\) is to be designed for the conditions described in each example. The width \(d\) parallel to the plane of bending is 20 in., and the width \(b\) at right-angles to this plane is 12 in.; the reinforcement comprises three 1-in. mild steel bars near each of the two 12-in. faces; the cover is 1\(\frac{1}{8}\) in., and \(a_s = 2\) in. Therefore, \(bd = 12 \times 20 = 240 \text{ sq. in.}\); \(f_1 = \frac{d}{d} = \frac{2}{20} = 0.1\); \(A_{s1} = 2.36\) sq. in., and \(r = \frac{2A_{s1}}{bd} = 0.02\) (2 per cent.). The effects of wind are not included in the applied load and bending moment; therefore formula series (i) and the factors in the lower part of Table 88 can be applied since the bars are less than \(1\frac{1}{8}\)-in. diameter. \((\rho_{se} = 18,000 \text{ lb. per sq. in.}; \rho_{th} = 20,000 \text{ lb. per sq. in.})\)

From Table 88 for \(100r = 2\) per cent. and \(f_1 = 0.10, K_b = 1103 \text{ lb. per sq. in.}, K_b = 395 \text{ lb. per sq. in.} \text{ and } e_b = 0.621\). Therefore \(P_b = 1103 \times 240 = 264,720 \text{ lb.} \text{ P}_b = 395 \times 240 = 94,800 \text{ lb.} \text{ e}_b = 0.621 \times 20 = 12.4 \text{ in.}\)

(a) Applied load 60,000 lb., which is less than \(P_b\); Case I. Factor \(K_1\) in Table 88 applies.

(i) Bending moment 600,000 in.-lb.; \(\epsilon = \frac{600,000}{60,000} = 10 \text{ in.} < e_b\); \(\epsilon = \frac{10}{20} = 0.5\); therefore \(K_1 = 484 \text{ lb. per sq. in.}; \text{ safe eccentric load} = 484 \times 240 = 116,000 \text{ lb.}, \text{ which is greater than the applied load; therefore the section is satisfactory.}\)

(ii) Bending moment 1,200,000 in.-lb.; \(\epsilon = \frac{1,200,000}{60,000} = 20 \text{ in.} > e_b\); \(\epsilon = \frac{20}{20} = 1.0\); \(K_1 = 238 \text{ lb. per sq. in.}, \text{ and } P_1 = 238 \times 240 = 57,100 \text{ lb.}, \text{ which is less than the applied load and therefore the section is inadequate; therefore either increase the reinforcement to say, six \(1\frac{1}{8}\)-in. bars; or increase } d \text{ to, say, 21 in.; or increase the quality of the concrete to } 1:1:3.\)

(b) Applied load 94,800 lb., which is \(P_1\); therefore the section must be satisfactory (as this is the borderline of Cases I and II) so long as \(\epsilon\) is not greater than \(e_b\).

(i) Bending moment 760,000 in.-lb.; \(\epsilon = \frac{760,000}{94,800} = 8.0 \text{ in.} < e_b\); \(\epsilon = \frac{8.0}{20} = 0.4\); \(K_1 = 555 \text{ lb. per sq. in.}, \text{ which, being greater than } K_b = 395 \text{ lb. per sq. in.}, \text{ shows that the section is satisfactory.}\)

(ii) Bending moment greater than 1,175,000 in.-lb. In this case \(\epsilon = \frac{1,175,000}{94,800} = 12.4 \text{ in.}\) which exceeds \(e_b\); hence \(K_1\) is less than \(K_b\); the section is inadequate.

(c) Applied load 120,000 lb., which is greater than \(P_b\); therefore Case II applies; the appropriate formula is formula (D) for \(K_{II}\) in Table 88. Safe load (at eccentricity \(\epsilon\)) \(P_{II} = K_{II}bd\). The intensity of the applied load is \(\frac{N}{bd} = \frac{120,000}{240} = 500 \text{ lb. per sq. in.}\)

(i) Bending moment 1,200,000 in.-lb.; \(\epsilon = \frac{1,200,000}{120,000} = 10 \text{ in.} < e_b\). From formula \(K_{II} = \frac{\frac{1103}{1} + \left(\frac{10}{395} - 1\right)}{12.4} = 450 \text{ lb. per sq. in.}, \text{ which being less than } \frac{N}{bd} (= 500), \text{ indicates that the section is inadequate although } \epsilon \text{ is less than } e_b\). The greatest bending moment that can be combined with an applied load of 120,000 lb. can be calculated by transposing formula D and substituting \(\frac{N}{bd}\) for \(K_{II}\), thus

\[\epsilon_{limit} = \frac{e_b}{K_b} \times \frac{2.4}{N} \times \frac{1103}{395} - 1 = 8.4 \text{ in.}\]

Max. bending moment with \(N = 120,000 \text{ lb. is } N\epsilon_{limit} = 120,000 \times 8.4 = 1,008,000 \text{ in.-lb.}\)

(ii) Bending moment 1,488,000 in.-lb.; \(\epsilon = \frac{1,488,000}{120,000} = 12.4 \text{ in.} (= e_b)\).

From example (c) it is seen that the section is inadequate to resist this combination of bending moment and load, although \(\epsilon\) is equal to \(e_b\). It is obvious that, for \(\epsilon = e_b\), the greatest load \(P_b\) that can be sustained is 94,800 lb.

(iii) Bending moment greater than 1,488,000 in.-lb. For this case, \(\epsilon\) will exceed \(e_b\) and it is obvious from the foregoing, without further calculation, that the section will be inadequate (that is, \(K_{II} > K_b\)). If \(\epsilon\) exceeds \(e_b\) the applied load must be less than \(P_b\) which is Case I and is dealt with as in example (a).
### Table 88: Load-Factor Method: B.S. Code

#### Combined Bending and Thrust: Rectangular Columns

<table>
<thead>
<tr>
<th>Type of Reinforcement</th>
<th>Series</th>
<th>Permissible Stresses in Reinforcement (lb. per sq. in.)</th>
<th>Limiting Eccentric Load</th>
<th>Limiting Eccentricity $e_b$</th>
<th>Maximum Safe Load at Eccentricity $e$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$P_b = K_b b d$</td>
<td>Formula (A) for $K_b$</td>
<td>$F_b = K_b b d$</td>
<td>Case I Applied Load $&gt; P_b$</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>$20,000$</td>
<td>$18,000$</td>
<td>$0.607 P_{cc} (1-f) - 1000 r$</td>
<td>$0.423 P_{cc} (1-f) + 9000 r (1-2 f)$</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>$25,000$</td>
<td>$22,500$</td>
<td>$0.567 P_{cc} (1-f) - 1250 r$</td>
<td>$0.468 P_{cc} (1-f) + 1125 r (1-2 f)$</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>$18,000$</td>
<td>$18,000$</td>
<td>$0.425 P_{cc} (1-f) - 1000 r$</td>
<td>$0.430 P_{cc} (1-f) + 8000 r (1-2 f)$</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>$22,000$</td>
<td>$20,000$</td>
<td>$0.585 P_{cc} (1-f) - 1250 r$</td>
<td>$0.414 P_{cc} (1-f) + 1000 r (1-2 f)$</td>
</tr>
</tbody>
</table>

#### Mild Steel Bars

<table>
<thead>
<tr>
<th>Size</th>
<th>5</th>
<th>30,000</th>
<th>23,000</th>
<th>$P_{cc} (1-f) - 3500 r$</th>
<th>$V + 11,500 r (1-2 f) + 1250 r (1-3 f)$</th>
<th>$U + \frac{U^2}{12,500} (8-15 f) - 3500 r$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>6</td>
<td>30,000</td>
<td>28,750</td>
<td>$P_{cc} (1-f) - 625 r$</td>
<td>$V + 14,375 r (1-2 f) + 625 r (1-3 f)$</td>
<td>$U + \frac{U^2}{12,500} (24-47 f) - 312 r$</td>
</tr>
</tbody>
</table>

#### Case II Applied Load $< P_b$

$P_{II} = K_{II} b d$  

**FORMULA D:**  

$K_{II} = \frac{K_{II}}{1 + \left(\frac{P_{II}}{P_{II}}\right)^{1/3}}$  

**WHERE $K_{II}$ = FACTOR FOR SAFE CONCENTRATED LOAD ON SHORT COLUMN**

$P_{II} = P_{cc} + r(P_{cc} - P_{II})$

#### All Conditions

$K_b$ and $e_b$ calculated from formula above:

**FORMULA FOR SPECIAL CONDITIONS**

$P_b = 20,000$ lb./sq. in.

**Safe Concentrated Load on Short Column:**

$P_b = K_b b d$

$K_b = 760 + 17,240 r$

**Limiting Eccentric Load:**

$P_b = K_b b d$

$K_b = 460 (1-f) - 1000 r$

**Limiting Eccentricity $e_b$:**

$e_b = \frac{322 (1-f) + 3000 (1-2 f)}{K_b}$

**Max. Safe Load at Eccentricity $e$:**

**Applied Load Less Than $P_b$:**

$P_i = K_i b d$

$K_i = \frac{760 + \frac{U^2 + 2300 (10-15 f)}{r}}{1 - \frac{e}{d} = e/b}$

$U = \frac{1}{2} e - 1.313 r$

---

**Table continued on page 305**
**TABLE 89.—COMBINED BENDING AND TENSION. (MODULAR-RATIO METHOD.)**

<table>
<thead>
<tr>
<th>REGULAR GROUPS OF BARS (CONCRETE, INEFFECIVE IN TENSION)</th>
</tr>
</thead>
<tbody>
<tr>
<td>AVERAGE TENSILE STRESSES IN REINFORCEMENT, ( A_{st} )</td>
</tr>
<tr>
<td>( f_{st1} = \frac{N(e+x)}{\sum A}\times \frac{1}{A_{st}+A_{s2}} )</td>
</tr>
<tr>
<td>IN GROUP A1</td>
</tr>
<tr>
<td>( f_{st2} = \frac{N(e-x)}{\sum A}\times \frac{1}{A_{s2}} )</td>
</tr>
<tr>
<td>MAX. TENSILE STRESSES (IN OUTER BAR OR BARS)</td>
</tr>
<tr>
<td>( f_{st1} = f_{st2} + \frac{(e+\xi)}{\sum A} f_{st} \times f_{st} )</td>
</tr>
<tr>
<td>TOTAL AREA: ( A_{st} )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>IRREGULAR GROUPS OF BARS</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1 ( \neq A_{s2} )</td>
</tr>
<tr>
<td>TENSILE STRESSES</td>
</tr>
<tr>
<td>IN REINFORCEMENT</td>
</tr>
<tr>
<td>( A_{st} )</td>
</tr>
<tr>
<td>( f_{st1} = \frac{N(e+\xi)}{\sum A} \times \frac{1}{A_{s1}} )</td>
</tr>
<tr>
<td>( f_{st2} = \frac{N(e-\xi)}{\sum A} \times \frac{1}{A_{s2}} )</td>
</tr>
<tr>
<td>TOTAL AREA: ( A_{st} )</td>
</tr>
</tbody>
</table>

| ASSUME \( A_{n} = A_{n} \) |
| CENTRE OF TENSION: |
| \( R = (p_1 - p_2) \times A_{n} \) |
| \( P = \sum N \) |
| \( \frac{P_1}{\Sigma N} \) IF ALL \( = \Sigma N \) |
| CENTRE OF COMPRESSION: |
| \( P_c = \frac{N_{cc} \times A_{n}}{\Sigma N_{cc} \times A_{n}} \) |

| POSITION OF CENTROID OF STRESSED AREA: |
| \( A_{n} \) |
| MAXIMUM STRESSES: |
| \( f_{cb} = \frac{N(e+\xi)}{(p_1 - p_2) \sum A_{n} \times N} \) |
| CHECK TRIAL VALUE OF \( A_{n} \) |

<table>
<thead>
<tr>
<th>COMPRESSIVE AND TENSILE STRESSES (CONCRETE, INEFECTIVE IN TENSION)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ASSESS ( A_{cc} )</td>
</tr>
<tr>
<td>( \alpha = (J_{cc} + m A_{cc} + (m-1) A_{cc} \times \sum A_{cc}) )</td>
</tr>
<tr>
<td>EVALUATE ( L = \frac{e + \xi}{2} )</td>
</tr>
<tr>
<td>( G = \frac{M}{I} )</td>
</tr>
<tr>
<td>AND ( H = (m-1) (m-1) )</td>
</tr>
<tr>
<td>DETERMINE STRESSES BY SUBSTITUTING IN N L</td>
</tr>
<tr>
<td>( f_{cb} = \frac{G bd + H A_{cc} (f_{cc})}{A_{cb}} )</td>
</tr>
<tr>
<td>( f_{st} = f_{cb} (J + H A_{cc}) + N )</td>
</tr>
<tr>
<td>CHECK ASSUMED ( A_{n} )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Rectangular Section (Concrete, Ineffective in Tension)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CALCULATE ( f_{cb} ) AND ( f_{st} ) DUE TO B.M. ONLY, AND DETERMINE ( A_{n} = \sum A_{n} ) FOR THESE STRESSES.</td>
</tr>
<tr>
<td>EVALUATE ( f_c = \frac{bd + m A_{cc} + (m-1) A_{cc}}{2} )</td>
</tr>
<tr>
<td>( \frac{N_{cc} \times A_{cc}}{\sum A_{cc} \times A_{cc}} )</td>
</tr>
<tr>
<td>( f_{cb} = f_{cb} + f_{cb} )</td>
</tr>
<tr>
<td>( f_{st} = f_{st} + f_{st} )</td>
</tr>
<tr>
<td>( \text{OR (given } A_{cc}) ) ( f_{cb} = \frac{N_{cc}}{A_{cc}} (1 + \xi \xi) )</td>
</tr>
</tbody>
</table>
| NOTE.—For an example of the use of this table, see page 302.

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### Combined Stresses

**Table 90. Bending in Two Planes. Principal Stresses.**

#### Section Moduli
- \(Z_{xc0}\): About \(xx\), at edge \(A-B = I_y / x\)
- \(Z_{xc}\): " " " C-D = \(I_y / d\)
- \(Z_{yy}\): " " " B-C = \(I_y / d\)
- \(Z_{yb}\): " " " A-D = \(I_y / b\)
- \(M_y = \) B.M. in plane \(yy\)
- \(M_x = \) B.M. in plane \(xx\)
- \(A_e = \) Equivalent area = \(bd + (m-i) A_{sc}\)

#### Stresses
- \(f_{cc} = N / (\text{Concentric Thrust})\); if no thrust, \(f_{cc} = 0\)
  - At A: \(f_{cb} = (f_{cc} + \frac{M_y}{Z_{xc0}}) - \frac{M_x}{Z_{yb}}\)
  - At B (max): \(f_{cb} = (f_{cc} + \frac{M_y}{Z_{xc0}}) + \frac{M_x}{Z_{yo}}\)
  - At D (min): \(f_{cb} = (f_{cc} - \frac{M_y}{Z_{xc0}}) - \frac{M_x}{Z_{yb}}\)

#### Notation as Above (Where Applicable)
- \(Z_n = \frac{d_n b^2}{2b} + \frac{(m-i)(A_{st} + A_{sc}) \delta}{2}\)

#### Additional Notation
- \(A_{sc}\)
- \(d_n\)

#### Stresses (Approx.) When \(M_y > M_x\)
1. Calculate \(f_{st}\), \(f_{cb}\), and \(d_n\) for \(My\) combined with \(N\) or for \(My\) alone if no thrust.
2. Calculate \(\frac{M_x}{Z_{n}} = f_x\)
3. Resultant Stresses:
   - Compression in Concrete at A: \(f_{ca} = f_{cb} - f_x\)
   - Tension (max.) in Bar at D: \(f_{tb} = f_{st} + m f_x\)
   - Compression (max.) in Concrete at B: \(f_{cb} = f_{cb} + f_x\)
   - Tension in Bar at C: \(f_{bc} = f_{st} - m f_x\)

#### Principal Stresses
- \(f_{pt}\): Principal Tensile Stress.
- \(f_{pc}\): Principal Compressive Stress.
- \(\theta\): Inclination of Principal Plane \(PP\).
- \(f_{st}\), \(f_{zt}\): Applied Tensile (Positive) Stresses.
- \(f_{sc}\), \(f_{xc}\): Applied Compressive (Neg.) Stresses.
- \(q\): Applied Shearing Stress

#### Tensile Stresses
- \(f_{pt} > -\frac{1}{2} (f_{zt} + f_{zt}) + \sqrt{\left(\frac{1}{4} (f_{zt} - f_{zt})\right)^2 + q^2}\)
- \(\tan 2\theta = 2q / f_{zt} - f_{zt}\)

#### Compressive Stresses
- \(f_{pc} = \frac{1}{2} (\sqrt{f_{st}^2 + 4q^2} - f_{st})\)
- \(\tan 2\theta = 2q / f_{sc}\)

#### Principal Stresses
- \(f_{pt} = -\frac{1}{2} (\sqrt{f_{sc}^2 + 4q^2} - f_{sc})\)
- \(f_{pc} = \frac{1}{2} (\sqrt{f_{sc}^2 + 4q^2} + f_{sc})\)
- \(\theta = 45\) deg.

### Notes
- \(q\) only: \(f_{pt} = -q\), \(f_{pc} = +q\)

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PRISMATIC STRUCTURES.—To design a simple prismatic roof, or similar structure, comprising a number of planar slabs, the resultant loads $P$ acting perpendicularly to each slab and the unbalanced thrusts $N$ acting in the plane of each slab are determined first, taking into account the thrust of one slab on another. The slabs are then designed to resist the transverse bending moments due to loads $P$ assuming continuity and combination with the thrusts $N$. The longitudinal forces $F$ due to the slabs bending in their own plane under the loads $N$ are, for any two adjacent slabs $AB$ and $BC$, calculated from formula (1) in Table 91, in which $M_{AB}$ and $M_{BC}$ are from formula (1) if the structure is freely supported at the ends of the span $L$. For each pair of slabs, $AB$—$BC$, $BC$—$CD$, etc., there is an equation like (1) containing three unknown forces $F$. If there are $n$ pairs there are $(n - 1)$ equations and $(n + 1)$ unknowns. The conditions at the outer edges (a) and (e) of the end slabs determine the forces $F$ at these edges; for example, if the edges are unsupported, $F_a = F_e = 0$. The simultaneous equations are solved for the remaining unknown forces $F_A, F_B, F_C$, etc. The longitudinal stress at any junction $B$ is calculated from the formula (in Table 91) for $f_B$. Variation of the longitudinal stress from one junction to the next is rectilinear. If $f_B$ is negative, the stress is tensile and the consequent tensile forces should be resisted by reinforcement. Shearing stresses are generally small.

DOMES.—A dome is designed for the total vertical load only, that is, for the weights of the slab and any covering on the slab, the weight of any ceiling or other distributed load suspended from the slab, and the live load. The intensity of total load $= w = e$ the equivalent load per sq. ft. of surface of the dome. Horizontal loads due to the wind and the effect of shrinking and changes in temperature are allowed for by assuming an ample live load, or by inserting more reinforcement than that required for the vertical load alone, or by designing for stresses well below the permissible values, or by a combination of two or all of these methods.

SEGMENTAL DOMES.—Referring to the diagram and formulate in Table 91, the circumferential force acting in a horizontal plane in a unit strip $S$ is $T$, and the corresponding force (the meridional thrust) acting tangentially to the surface of the dome is $N$. At the plane where $\theta = 51$ deg. 48 min., that is, at the plane of rupture, $T = 0$. Above this plane $T$ is compressive and reaches a maximum value of $0.5wR$ at the crown of the dome ($\theta = 0$). Below this plane $T$ is tensile, and equals $0.167wR$ when $\theta = 60$ deg., and $wR$ when $\theta = 90$ deg. The meridional thrust $N$ is $0.5wR$ at the crown, $0.618wR$ at the plane of rupture, and $0.667wR$ when $\theta = 60$ deg., and $wR$ when $\theta = 90$ deg.; that is, $N$ increases from the crown towards the support and has its greatest value at the support.

For a concentrated load $W$ on the crown of the dome $T$ is tensile; and $T$ and the corresponding meridional compressive thrust $N$ are given by the appropriate formulæ in Table 91, the basis of which is that the load is concentrated on so small an area at the crown that it is equivalent to a point load. The theoretical stress at the crown is therefore infinite, but the practical imposibility of obtaining a point load invalidates the application of the formulæ when $\theta$ is nought or very nearly nought. For domes of varying thickness, reference should be made to other publications.

SHALLOW SEGMENTAL DOME.—Approximate analysis only is sufficient in the case of a shallow dome; appropriate formulæ are given in Table 91.

CONICAL DOME.—In a conical dome, the circumferential forces are compressive throughout and at any horizontal plane $s$ from the apex are given by the expression for $T$ in Table 91, the corresponding force in the direction of the slope being $N$. The horizontal outward force per ft. of circumference at the bottom of the slope is $P$ and resistance to this force must be provided by the supports or by a ring beam at the bottom of the slope.

SEGMENTAL SHELLS.—The longitudinal, tangential and shearing membrane forces in an element of a shell roof (defined in position by $s$ and $\theta_s$) are given in Table 91. Note the distribution of dead and live loads on which the formulæ are based. The principal stresses in any part of the shell (due to the membrane stresses only) are computed by the appropriate formula; the reinforcement is arranged to be approximately in line with the principal tensile force and to resist this force. Limitations on the length and width of the shell and the thickness of the slab are as given in the diagram. The effects of a support along the free longitudinal edges and of the end-stiffeners is to modify the forces in the vicinity of the edges and ends; publications dealing with the design of segmental shells should be consulted to determine these effects.
NON-PLANAR ROOFS.—TABLE 91.

<table>
<thead>
<tr>
<th>PRISMATIC STRUCTURE (HIPPED PLATE CONSTRUCTION)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( N_{AB}, \text{etc.} = \text{THrust in Plane of Slab AB, etc.} ) DUE TO COMPONENT OF LOAD AND WEIGHT AND OF THRUST IN SLAB BC, etc.</td>
</tr>
<tr>
<td>( M_{AB} = \frac{N_{AB} L_{AB}}{B}; \quad M_{BC} = \frac{N_{BC} L_{BC}}{B} \text{ etc.} )</td>
</tr>
<tr>
<td>LONGITUDINAL STRESSES (TENSILE IF NEGATIVE) AT JUNCTION B:</td>
</tr>
<tr>
<td>[ \tau_B = \frac{2}{d_{BC}} \left( \frac{3M_{AB}}{L_{AB}} - P_A - 2P_B \right) ]</td>
</tr>
<tr>
<td>3 ( d_{BC} = \left( 2P_B + F_C - \frac{3M_{BC}}{L_{BC}} \right) ) (SIMILARLY FOR JUNCTION C, ETC.)</td>
</tr>
<tr>
<td>FORMULAS FOR STABILISING FORCES, SLABS AB AND BC [FORMULA 2]:</td>
</tr>
<tr>
<td>[ \frac{P_A}{d_{AB} L_{AB}} + \frac{2F_A}{d_{BC} L_{BC}} + \frac{1}{d_{AB} L_{AB}} + \frac{1}{d_{BC} L_{BC}} + \frac{F_C}{d_{BC} L_{BC}} = \frac{3}{2} \left( \frac{M_{AB}}{L_{AB}} + \frac{M_{BC}}{L_{BC}} \right) ]</td>
</tr>
</tbody>
</table>

SEGMENTAL DOME.

\( T = \text{CIRCUMFERENTIAL FORCE (IN HORIZONTAL PLANE) IN UNIT STRIP AT S.} \)

\( N = \text{MERIDIONAL THRUST (ACTING TANGENTIAL) IN UNIT STRIP AT S.} \)

UNIFORMLY-DISTRIBUTED LOAD \( W \) PER UNIT AREA OF SURFACE OF DOME

\[ T = \omega D \sqrt{1 - \cos^2 \Theta} \]

\[ N = \frac{W D \sin^2 \Theta}{2 \pi} \]

FORMULAE NOT APPLICABLE TO SMALL VALUES OF \( \Theta \)

SHALLOW SEGMENTAL DOME (APPROX. METHOD)

\( R = \frac{D^2}{8 \pi} - \frac{D}{2} \) TOTAL LOAD ON SUPPORTS = \( W = 2 \pi R \rho W \)

AT SPRINGING:

\[ N = \frac{W R}{2 \pi} \]

\[ T = \frac{W (R - r)}{2 \pi} \]

TENSILE FORCES IN RING BEAM AT SPRINGING:

\( T_A = \frac{W (R - r)}{2 \pi D} \)

CONICAL DOME.

IN HORIZONTAL PLANE AT \( \infty \):

\[ T = \frac{\omega y \tan \Theta}{2 \cos \Theta} (\text{Compress.}) \]

\[ N = \frac{\omega y}{2 \cos \Theta} \]

AT BOTTOM:

\[ P = \frac{\omega H \tan \Theta}{2 \cos \Theta} \]

PER UNIT LENGTH OF CIRCUMFERENCE

SEGMENTAL SHELLS (MEMBRANE FORCES) (POSITIVE=TENSILE)

TANGENTIAL:

\( F_{y} = \left( \omega_{y} + 1.5 \omega_{x} \cos \Theta_{x} \right) R \cos \Theta_{x} \)

LONGITUDINAL:

\( F_{x} = \left( e \cdot x \right) x R \left[ \omega_{x} \cos \Theta_{x} + 1.5 \omega_{x} \cos^{2} \Theta_{x} - \sin^{2} \Theta_{x} \right] \)

SHEARING:

\[ Q_{x} = \left( \omega_{x} + 1.5 \omega_{x} \cos \Theta_{x} \right) (2x - e) \sin \Theta_{x} \]

AT MID-POINT B OF CROWN (\( \Theta_{x} = 0 \))

\( F_{y} = \left( \omega_{y} + 1.5 \omega_{x} \cos \Theta_{x} \right) R \)

\( F_{x} = \frac{e^{2} \omega_{x}}{4 R} \left( \omega_{y} + 1.5 \omega_{x} \right) \)

PRINCIPAL FORCES (MEMBRANE FORCES ONLY)

\[ F_A = \frac{1}{2} F_{y} + F_{x} \pm \frac{1}{4} (F_{x}^{2} - F_{y}^{2} + Q_{x}^{2})^{1/2} \]

\[ \tan 2 \phi = \frac{2Q_{x} y}{F_{x} - F_{y}} \]
TABLE 92.—BRIDGES: GENERAL DATA.

STRUCTURE GAUGE FOR RAILWAY OVERLINE BRIDGES. (APPROX.)
(BRITISH RAILWAYS: MAIN LINES)

NOTE:
FOR PRIVATE SIDINGS FOR WAGONS ONLY A = 11' "G" RAIL LEVEL

SINGLE LINE

3'-8"
2'-5" RAD IN CORNERS
A = 14'-7/8"

DOUBLE LINE

3'-8"
4'-1" MIN.

RAIL LEVEL

ROAD BRIDGES.

MINIMUM CLEARANCES FOR SUBWAYS.

PEDESTRIANS ONLY.

CYCLISTS ONLY (ONE-WAY TRAFFIC).

CYCLISTS ONLY (TWO-WAY TRAFFIC).

CYCLISTS AND PEDESTRIANS (ONE-WAY TRAFFIC FOR CYCLISTS).

CYCLISTS AND PEDESTRIANS (TWO-WAY TRAFFIC FOR CYCLISTS).

MINIMUM CLEARANCES FORподɰінɡs.

ROAD PROFILES.

CONTACT CURVES.

D = (d + 3.75) / 8

L = LENGTH OF BEARING (IN.)

K = 3.42 FOR 11/2:3 CONCRETE

= 2.640 FOR CAST STEEL (PER WIL. SCOTT)

R = RADIUS OF BEARING (IN.)

PERMISSIBLE STRESSES ON STEEL BEARINGS.

PINS, BENDING = 15/2 TONS/PER SQ.IN.

BEARING = 15 = " = "

SHEARING = 6/2 = " = "

ROLLERS: D = DIA. OF ROLLER (IN.) = 3 3/4

PRESSURE ON STEEL ROLLERS = 0.28D TON PER LIN. IN.

LONGITUDINAL FORCES DUE TO BEARINGS.

FRAC TIONAL RESISTANCE OF ROLLERS = 1 PER CENT.; 5 PER CENT. = 3 ROLLERS

SLIDING PLATES = 25 = (STEEL ON STEEL OR CAST IRON)

EXPANSION OR CONTRACTION DUE TO TEMPERATURE ALLOWED FOR BY CONSIDERING FRICITION ON EXPANSION BEARING UNDER DEAD LOAD AS ADDITIONAL ECCENTRIC THUST OR PULL IN GIRDERS.
<table>
<thead>
<tr>
<th>SLAB</th>
<th>STatically Determinate</th>
<th>FREELY-SUPPORTED SLAB</th>
<th>CAST IN SITU TRANSVERSE SECTIONS</th>
<th>PRECAST</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>SINGLE FREELY-SUPPORTED SPAN</td>
<td>CAST-INSITU OR PRECAST BEAMS CAST-INSITU SLAB</td>
<td>Suspended Span</td>
</tr>
<tr>
<td></td>
<td></td>
<td>DOUBLE - CANTILEVERED CONSTRUCTION</td>
<td>CAST-INSITU CONSTRUCTION</td>
<td>PRECAST BEAMS</td>
</tr>
<tr>
<td></td>
<td></td>
<td>MULTIPLE-SPAN CANTILEVERED CONSTRUCTION WITH PRECAST OR CAST-INSITU SUSPENDED SPAN</td>
<td>TRANSVERSE SECTIONS</td>
<td></td>
</tr>
</tbody>
</table>

**STATICAL DIAGRAMS.**

- **Fixed (Pivoted) Support**
- **Roller or Sliding Bearing**

**FORMULE FOR \( a_1, a_2, \text{etc.} \) APPLY ONLY WHEN EACH SPAN EQUALS L.**

\[
\begin{align*}
\text{LIVE} & = \frac{w_D}{w_L} \\
\text{LIVE} & = \frac{\sqrt{1-2w_D+w_L}}{w_D+w_L} \\
\text{LIVE} & = 0.5 \left(1 - \sqrt{1-4a_1} \right) \\
\text{LIVE} & = \frac{w_D+w_L}{4(2w_D+w_L)}
\end{align*}
\]

**FRAMEs**

- STatically Indeterminate
- Slab
- Slab with Beams and Counterforts
- Cantilevered Beams and Columns

**ARCHES**

- STatically Indeterminate
- Three-Hinged Arch Ribs
- Three-Hinged Arch Slab
- Earth Filling Between Spandrel Walls

- Two-Hinged Arch Ribs
- Two-Hinged Arch Slab
- Bowstring

**TRUSSES**

- STatically Indeterminate
- Lattice Girder
- VIERENDEEL TRUSS

**DECK SIMILAR TO STatically-Determinate Beam-And-Slab**

**ALTERNATIVE PROFILE SHOWN BY BROKEN LINES**

**CLOSED SPANDRELS**

**OPEN SPANDRELS**

**BOWSTRING**

**TRANSVERSE SECTIONS OF ARCH BRIDGES**

3II
Hinges and Bearings.—A hinge is an element that can transmit a thrust and a shearing force, but permits rotation without restraint. If it be essential that hinge action be fully realised, it can be effected by providing a steel hinge, or by forming a hinge monolithic with the member as shown at (a) in Table 94; this hinge is sometimes provided in a frame of a large bunker to isolate the container from the sub-structure or to provide a hinge at the base of the columns of a hinged frame bridge. The hinge-bars (a) resist the entire horizontal shearing force; the area of concrete at D must be sufficient to transfer all the compressive force from the upper to the lower part of the member. The hinge bars should be bound together by binders (d); the main vertical bars (e) should terminate on each side of the slots B and C. It may be advantageous during construction to provide bars extending across the slots, and to cut these bars on completion of the frame. The slots should be filled with bituminous material, lead, or a similar separating layer.

In Table 94 there are illustrated other types of hinges and bearings. (b) A hinge formed by the convex end of a concrete member bearing in a concave recess in the foundation. (c) A hinge suitable for the bearing of a girder where rotation, but not sliding, is required. (d) A bearing for a girder where sliding is required. (e) A mechanical hinge suitable for the base of a large portal frame or the abutment of a large hinged arch rib. (f) A transverse expansion joint in the deck of a bridge, which is suitable if the joint is in the slab or is formed between two transverse beams. (g) A rocker-bearing suitable for girder bridges of spans of over 50 ft.; for spans less than 30 ft. an expansion is not always necessary, and up to 50 ft. a sliding bearing as at (d) is sufficient. (h) A hinge suitable for the crown of a three-hinge arch when a mechanical hinge is not justified. (j) A bearing suitable for the support of a freely-suspended span on a cantilever in an articulated bridge.

The permissible bearing stresses on bearings and the longitudinal forces due to bearings are given in Table 92; the load on bearings formed by two curved surfaces in contact is also given in this table. Permissible bearing pressures on plain and reinforced concrete under bearing plates are given in Table 104.

Expansion and Contraction Joints.—The joints shown in Table 95 are expansion and contraction joints suitable for various structures. At (a) is a vertical joint in the stem of a cantilever retaining wall; the spacing depends on the height of the wall but should not exceed 60 ft. For low walls with thin stems a simple butt-joint is generally sufficient, but unequal deflection or tilting of one part of the wall relative to the next shows at the joint; a keyed joint as at (a) is therefore preferable for walls more than 4 ft. high. The key also reduces the risk of percolation of moisture through the joint. The double chamfer improves the appearance. Where it is necessary to provide for an amount of expansion exceeding the probable amount of shrinkage a space, say, ⅛ in. wide, should be left between the faces of the concrete and filled with resilient material.

At (b) a method of bearing the base of the wall of a cylindrical tank on the bottom is illustrated, which ensures that bending moments are not induced in the wall due to restraint at the bottom. Because of the difficulty of maintaining such joints, monolithic construction and designing the wall to resist bending moment may be preferable.

In the walls of a reservoir a watertight joint must be provided to allow for movements due to shrinking and change of temperature; a suitable joint for a cantilevered wall or a wall that spans vertically is shown at (c).

In slabs laid on the ground (and not forming part of a foundation raft), construction joints should be permanent joints in predetermined positions, such as at the end of a day’s work, at a restricted section, at a change of thickness, or at other positions where cracks are likely to occur. At (d) is shown such a joint that makes a definite break in the slab; this type of joint should be provided at intervals of about 15 ft. in the bottoms of reservoirs or in floors laid directly on the ground. The pad under the joint prevents one panel settling relatively to the other when laid on soft ground, but on firm ground the pad may not be necessary. When the floor is subject to abrasion, as in factories where wheeled containers travel across the joint, the edges should be protected by steel angles as indicated. If the ground is waterlogged, the joint at (f) can be adopted; this joint is also suitable for the floors of reservoirs.

In the case of cantilevered retaining walls and reservoir walls it is not always necessary to extend the joint from the walls into the base slab, but a longitudinal joint parallel to the wall should be provided in the base slab to separate the base of the wall from the remainder of the floor when the area of the floor is considerable.

If permanent joints are provided in buildings it is necessary to carry the joint through the floors, walls [as at (b)], and roof slab [as at (g)]. The joints in the walls should be made at the columns, in which case a double column as at (e) is provided; the space is filled with a joint filler; the copper strip or other type of water-bar must be notched where the binders occur, the ends of the notched pieces being bent horizontally or cut off. At joints through suspended floors and flat roofs it is common for a double beam to be provided. A joint filler and water-bar are required in a joint in a roof, but a joint in a floor should be sealed to prevent rubbish accumulating therein.
SECTION 1.1.

RIB OR WALL OF PORTAL FRAME

BITUMINOUS SEAL

NON-CORROSiVe FORMER

FOUNDATiON

STEEL PLATE WELL GREASED.

CAST STEEL PLUGS.

RESILIENT JOINT FILLER.

M.S. ANCHORS WELDED TO PLATES.

(c) FIXED END BEARING (HINGE)

CAST STEEL SEATINGS

RESILIENT JOINT FILLER

1/4 (APPROX) SHEET LEAD

ANCHOR BOLTS.

(d) FREE END BEARING (SLIDING)

GAP FILLED WITH PLASTIC JOINT FILLER

ANGLES PROTECTING ROAD SURFACE

ROAD SURFACE

PLATE ATTACHED TO A AND SLIDING ON B

EXPANSION GAP IN SLAB OR BETWEEN BEAMS

(f) JOINT IN DECK OF BRIDGE.

3/8 GAP WITH RESILIENT FILLER

3/8 HEAT CEMENT AROUND BARS AT CROSSINGS

(h) HINGE AT CROWN OF ARCH.

(e) MECHANICAL HINGE.

LUGS ENGAGING WITH HOLES IN BEARING PLATE

CAST STEEL PLATE

R.C. ROCKER

SECTION 2.2.

ROCKER BEARING.

1/2 GAP WITH RESILIENT FILLER

SUSPENDED SPAN CANTILEVER

(j) SUSPENSION BEARING.
TABLE 95.—EXPANSION AND CONTRACTION JOINTS.

(a) VERTICAL JOINT IN CANTILEVERED RETAINING WALL.

(b) BASE OF WALL OF CYLINDRICAL TANK.

(c) VERTICAL JOINT IN RESERVOIR WALL.

(d) JOINT IN FLOOR LAID ON THE GROUND.

(e) EXPANSION JOINT AT COLUMN.

(f) JOINT IN FLOOR LAID ON THE GROUND.

(g) ALTERNATIVE DESIGNS FOR JOINTS IN ROOF SLABS.

(h) ALTERNATIVE DESIGNS FOR JOINTS IN EXTERNAL WALLS OF BUILDINGS.

NOTES.—(i) FOR OTHER TYPES OF JOINTS SUITABLE FOR RESERVOIRS, SEE B.S. CODE NO. 2007.

(ii) COPPER-STRIP CAN BE REPLACED BY RUBBER OR SIMILAR WATER-BARS.
### Concrete Roads — Table 96

**Stresses in Road Slabs**

| \( W \) | Wheel Load (lb.) |
| \( d \) | Thickness of Concrete Slab (in.) |
| \( f \) | Maximum Compressive and Tensile Stress (lb. per sq. in.) |
| \( R \) | Radius of Contact Area of Wheel on Slab |
| \( R_1 \) | Equivalent Radius of Contact Area (if \( R > 1.724d, R_1 = R \)) |
| \( E_c \) | Elastic Modulus of Concrete |
| \( \mu \) | Poisson’s Ratio for Concrete (= 0 to 0.3) |
| \( \gamma \) | Radius of Relative Stiffness |
| \( K \) | Coefficient for Soft and Plastic Soils, \( K = 50 \) |
| \( C \) | Coefficient for Firm Soil, \( C = 200 \) |
| \( K_{1} \) | Coefficient for Compact Gravel and Rock, \( K_{1} = 500 \) |

**Maximum Stresses**

- **At Corner:**
  \[ f = \frac{3W}{d^2} \left[ 1 - \left( \frac{R}{R_1} \right)^{\frac{2}{3}} \right] \]

- **At an Edge:**
  \[ f = \frac{0.53(1+0.54\mu)W}{d^2} \left[ \log \left( \frac{E_c d^3}{KR^4} \right) - 0.71 \right] \]

- **Remote from Edge or Corner:**
  \[ f = \frac{W(1+\mu)}{d^2} \left[ 0.275 \log \left( \frac{E_c d^3}{K R^4} \right) - 15 \left( \frac{R}{R_1} \right)^2 \right] \]

### Typical Joints

**Transverse Expansion and Contraction Joint**

- Ferrule (metal or waxed cardboard)
- Dowel treated to prevent bonding with concrete
- Resilient joint filler, perforated to take dowels

**Dowel Bonded with Concrete**

- 3/4" M.S. dowel bars, 2 ft. long

**Typical Details**

<table>
<thead>
<tr>
<th>Type of Soil</th>
<th>Density of Traffic**</th>
<th>Thickness of Slab</th>
<th>Reinforcement</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bad Ground (Loam, Peat, Silt, Sandy Clay, etc.)</td>
<td>Light</td>
<td>7 in.</td>
<td>10 lb. per sq. yd., one layer.</td>
<td>For adverse conditions (e.g., heavy docks, traffic on poor soil)</td>
</tr>
<tr>
<td></td>
<td>Medium</td>
<td>8 in.</td>
<td>Ditto, but in each of two layers.</td>
<td>&lt;= 12&quot; slab.</td>
</tr>
<tr>
<td></td>
<td>Heavy</td>
<td>9 in.</td>
<td>** Traffic: Light = 500 tons per day, Medium = 500 to 5000 x = Heavy = 5000 to 10,000 x =</td>
<td></td>
</tr>
<tr>
<td>Medium Ground (Virgin Clay, compacted fill, except clay fill)</td>
<td>Light</td>
<td>6 in.</td>
<td>8 lb. per sq. yd., one layer.</td>
<td>** Traffic: Light = 500 tons per day, Medium = 500 to 5000 x = Heavy = 5000 to 10,000 x =</td>
</tr>
<tr>
<td></td>
<td>Medium</td>
<td>7 in.</td>
<td>Ditto but in each of two layers.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Heavy</td>
<td>8 in.</td>
<td>Ditto but in each of two layers.</td>
<td></td>
</tr>
<tr>
<td>Good Ground (Gravel, well-graded sand, chalk, rock, etc.)</td>
<td>Light</td>
<td>5 in.</td>
<td>7 lb. per sq. yd., one layer.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Medium</td>
<td>6 in.</td>
<td>Ditto but in each of two layers.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Heavy</td>
<td>7 in.</td>
<td>Ditto but in each of two layers.</td>
<td></td>
</tr>
</tbody>
</table>

*If high-yield-stress steel is used, wt. of reinforcement = \( \frac{2}{3} \) of weight tabulated.*

---

**Typical Joints**

<table>
<thead>
<tr>
<th>Type of Joint</th>
<th>Details</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transverse Dummy Joint</td>
<td></td>
</tr>
<tr>
<td>Longitudinal Joint</td>
<td></td>
</tr>
</tbody>
</table>
Walls of Cylindrical Tanks.

**Example.**—To determine the bending moments and tensions in the wall of a cylindrical water tank 20 ft. in diameter, 30 ft. deep, with walls 10 in. thick at bottom. Thus \( D = 20 \) ft.; 
\[ H = 30 \text{ ft.}; \quad d = 0.83 \text{ ft.}; \quad w = 62.4 \text{ lb. per cu. ft.} = \frac{H}{D} = \frac{30}{20} = 1.5; \quad \frac{H}{d} = \frac{30}{0.83} = 36; \] therefore from Table 97, \( F = 0.003; \quad K_1 = 0.20; \quad K_2 = 0.85. \)

The bending moment (vertical) at \( A \) is \( 0.003 \times 62.4 \times 30^2 = 5050 \) ft.-lb.

Position of point of maximum circumferential tension: \( L = 0.20 \times 30 = 6 \) ft. from bottom.

Maximum circumferential tension \( = 0.5 \times 62.4 \times 30 \times 0.85 = 15,900 \) lb. per ft. height.

**Note.**—If the bottom is suspended, and monolithic with the wall, deformation of the bottom slab may produce in the wall bending moments (vertical) which must be combined with those due to restraint at the base of the wall. Similarly, the bending moments and shearing forces due to this restraint affect the resultant moments and tensions in the bottom.

**Bottoms and Roofs of Cylindrical Elevated Tanks.**

Designs for the bottoms of elevated cylindrical tanks are given in the diagrams above and in Table 97.

**With Beams.**—At (a) each beam spans between opposite columns and carries one-quarter of the load of the bottom of the tank. The remaining half of the load and the weight of the wall and the load from the roof are transferred to the columns through the wall. In the arrangement shown at (b), each length of beam between columns carries the load on the shaded area, and the remainder of the load on the floor of the tank and the weight of the walls and the load from the roof are equally divided between the eight cantilevered lengths of the beams. An alternative to this design is for the columns to be placed almost under the wall, in which case the cantilevers are unnecessary but secondary beams may be required.

**Domed Bottoms.**—For a tank of large diameter a domed bottom and roof of either of the types shown in Table 97 are more economical, and although the shuttering is much more costly the saving in concrete and reinforcement compared with beam-and-slab construction may be considerable. The ring-beams marked \( R \) in the case of a simple domed bottom or roof resist the horizontal component of the thrust from the domes, and the thicknesses of the domes are determined by the magnitude of this thrust. The working compressive stress in the roof dome should be low, say 20 per cent. of the ordinary safe stress, in order to allow ample margin for local increases due to incidental concentrated loads and for irregular distribution of the imposed load. For the bottom dome, where the uniform distribution of the load is more assured, a higher stress can be used, and about \( \frac{1}{4} \) per cent. to 1 per cent. of reinforcement should be provided in each direction. The shearing stress around the periphery of the dome should also be calculated and sufficient thickness of concrete provided to resist the shearing forces. Expressions for the thrust and vertical shearing force around the edge of the dome and the resultant circumferential tension in the ring beams are given in Table 97. Domes can also be analysed by the method described in Table 91.

The bottom comprising a central dome and an outer conical part (called an Intze tank) illustrated at the bottom of Table 97 is economical for the largest tanks. The outward thrust from the top of the conical part is resisted by the ring beams \( S \), and the difference between the inward thrust from the bottom of the conical part and the outward thrust from the domed part is resisted by the ring beam \( A_1 \). Expressions for the forces are given in Table 97. The proportions of the conical and domed parts can be arranged so that the resultant horizontal thrust on \( A_1 \) is zero. Suitable proportions for bottoms of this type are given in Table 97, and the volume for a tank of diameter \( D_0 \) with these proportions is \( 0.585D_0^2 \). The wall of

(Continued on page 318.)
Cylindrical Tanks and Water Towers.—Table 97.

Direct Tension in Wall Due to Internal Pressure (lb. per sq. ft.):

\[ T = \frac{1}{2} p D = \frac{1}{2} w D \]

Minimum Thickness of Wall = \( d \) in.

Direct Compression in Wall of Underground or Submerged Tank:

\[ f_{cc} = \frac{P_k D}{2 \left[ \frac{12 d}{D} + (m-1) A_b \right]} \text{ lb. per sq. in.} \]

Bending Moment Due to Restraint at Base of Wall:

\[ T_H = \text{Max. Circumferential Tension} = \frac{1}{2} w H D K_2 \text{ lb.} \]

Max. B.M. at A = \( F w H^3 \text{ ft.-lb. per ft.} \)

\[ L = \text{Position of Max. Circumferential Tension} = K_1 H \text{ ft.} \]

Values of Factors \( F, K_1, K_2 \) tabulated below.

<table>
<thead>
<tr>
<th>FACTORS</th>
<th>( F )</th>
<th>( K_1 )</th>
<th>( K_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( H + d )</td>
<td>10</td>
<td>20</td>
<td>30</td>
</tr>
<tr>
<td>0.2</td>
<td>0.046</td>
<td>0.028</td>
<td>0.022</td>
</tr>
<tr>
<td>0.3</td>
<td>0.032</td>
<td>0.015</td>
<td>0.014</td>
</tr>
<tr>
<td>0.4</td>
<td>0.024</td>
<td>0.014</td>
<td>0.010</td>
</tr>
<tr>
<td>0.5</td>
<td>0.020</td>
<td>0.010</td>
<td>0.009</td>
</tr>
<tr>
<td>1.0</td>
<td>0.012</td>
<td>0.006</td>
<td>0.005</td>
</tr>
<tr>
<td>2.0</td>
<td>0.006</td>
<td>0.003</td>
<td>0.002</td>
</tr>
<tr>
<td>4.0</td>
<td>0.004</td>
<td>0.002</td>
<td>0.002</td>
</tr>
</tbody>
</table>

Shearing Force: \( F_1 = \frac{W_i}{D} \) (lb. per ft.)

Circumferential Tension in Beam at \( A_1 = \frac{PD}{\cos \theta} \)

\[ T_1 = \text{Thrust at Perimeter of Dome} = F_1 \cos \beta \theta \] (lb. per ft.)

Values of \( F_1, T_1 \) and \( P_1 \) as for simple dome above.

\[ F_2 = \frac{W_2 + W_3}{D} \] (lb. per ft.)

\[ F_3 = \frac{W_3}{D} \] (lb. per ft.)

\[ P_2 = F_2 \cos \beta \] (lb. per ft.)

\[ P_3 = F_3 \cos \beta \] (lb. per ft.)

(Ideal Case: \( P_1 = P_2 \))

Circumferential Tension in Beam at \( A_1 = \frac{D}{2} (P_1 - P_2) \)

Notes.—* For design of slabs forming walls of containers in direct tension, see also Table 74.

** For angles of friction, see Table 11.

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REINFORCED CONCRETE DESIGNER'S HANDBOOK

CYLINDRICAL TANKS AND WATER TOWERS

(continued from page 316).

Domed Bottoms (cont.)

the cylindrical part of the tank should be designed as described previously, account being taken of the vertical bending at the base of the wall and the effect of this bending on the conical part. The floor of a tank must be designed to resist, in addition to the forces and bending moments already described, any direct tension due to the vertical bending of the wall.

Example.—Determine the principal forces in the bottom of a cylindrical tank of the Intze type, if \( D_2 = 40 \) ft.; \( D = 25 \) ft.; \( \phi = 48 \) deg.; \( \theta = 40 \) deg.; \( W_1 = 582,000 \) lb.; \( W_2 = 639,000 \) lb.; and \( W_3 = 296,000 \) lb.

From Table 10: \( \csc \theta = \frac{1}{\sin \theta} = 1.55; \cot \theta = 1.192; \cot \phi = 0.900. \)

From Table 97.

Vertical shearing force along periphery of dome: \( F_1 = \frac{582,000}{3.14 \times 25} = 7400 \text{ lb. per ft.} \)

Thrust at periphery of dome: \( T_1 = 7400 \times 1.55 = 11,450 \text{ lb. per ft.} \)

The values of \( F_1 \) and \( T_1 \) determine the thickness of the dome at the springing.

Outward horizontal thrust from dome on ring beam \( B: P_1 = 7400 \times 1.192 = 8820 \text{ lb. per ft.} \)

Shearing force along inner periphery of conical portion:

\[
F_2 = \frac{639,000 + 296,000}{3.14 \times 25} = 11,900 \text{ lb. per ft.}
\]

Inward thrust on ring beam \( B \) from conical part: \( P_2 = 11,900 \times 0.900 = 10,700 \text{ lb. per ft.} \)

Resultant circumferential force in ring beam \( B = 0.5 \times 25(8820 - 10,700) = 23,500 \text{ lb. (compression).} \)

(If \( P_1 \) exceeds \( P_2 \), the circumferential force is tensile; the ideal case is for \( P_1 \) and \( P_2 \) to be equal, thereby producing zero force in \( B \)—see note below.)

Shearing force along outer periphery of conical part: \( F_3 = \frac{296,000}{3.14 \times 40} = 2350 \text{ lb. per ft.} \)

Outward thrust on ring beam at top of conical part: \( P_3 = 2350 \times 0.900 = 2120 \text{ lb. per ft.} \)

Circumferential tension in beam at top of conical section \( = 0.5 \times 40 \times 2120 = 42,400 \text{ lb.} \)

The vertical wall must be reinforced for circumferential tension due to the horizontal pressure of water: tension \( = 0.5 \times 62.4D_3$, lb. \) The conical part must be reinforced to resist a circumferential tension, and the reinforcement can be either distributed throughout the height of the conical part or concentrated in the ring beams at the top and bottom.

Note.—In Intze tanks of large diameter the width of the ring beam \( B \) may be considerable, in which case the weight of water immediately above the beam should not be considered as contributing to the forces on the dome and conical part. With a wide beam, \( W_3 \) is the weight of the contents over the net area of the dome and \( d \) is the internal diameter of ring beam; \( W_3 \) is the weight of the contents over the net area of the conical part and \( d \) for use with \( W_2 \) is the external diameter of ring beam. If this adjustment is made for a ring beam of reasonable width in the above example \( P_1 \) would more nearly balance \( P_2 \).

Columns Supporting Elevated Tanks.—The thrust and tension produced in columns due to wind forces on a tank supported by the column can be calculated for a group of four columns as follows. If the total wind moment is \( M_W \) ft.-lb and the distance apart of columns is \( D \) ft., when the wind blows normal to the dimension \( D \), the thrust on each column on the leeward side and the tension in each column on the windward side \( = \frac{M_W}{2D} \) lb. When the wind blows normal to a diagonal at the group, the thrust on the leeward corner column and the tension in the windward corner column \( = \frac{M_W}{D} \) lb.

For any other number of columns, the force in any column can be calculated from the equivalent moment of inertia of the group. For example, consider a wind moment of \( M_W \) due to the wind blowing normal to the axis NN of the group of eight columns shown at (c) in the diagram on page 316.

Moment of inertia of the group about NN \( = [2 \times (0.5D)^4] + [4 \times (0.353D)^4] = 1.0D^4. \)

Thrust on the extreme leeward column \( = \frac{M_W \times 0.5D}{1.0D^4} = \frac{M_W}{2D}. \)

The forces on each of the other columns can be determined similarly, substituting the appropriate "arm" for \( 0.5D. \)
### Walls of Rectangular Containers: Table 98

#### Bending Moments and Reactions in Walls Spanning Horizontally and Subjected to Horizontal Pressure of Intensity P.

<table>
<thead>
<tr>
<th>Form of Container</th>
<th>Bending Moments: ( M = \frac{Pd^2}{K} )</th>
<th>Bending Moment Coefficients: ( k )</th>
<th>Reaction Formulæ</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Formulae</td>
<td>Values of ( \frac{d}{y} )</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.5 0.8 1.0 1.2 1.5 1.8 2.0</td>
<td></td>
</tr>
<tr>
<td>M₁</td>
<td>( M_1 = \frac{P(D^3 + B_3)}{12(D+B)} )</td>
<td>16 14.3 12 9.68 6.86 4.92 4</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>M₂</td>
<td>( M_1 = \frac{P(D^3 + B_3)}{12(D+2B)} )</td>
<td>10.1 10.8 12 14.2 22.6 97 60 MINUS</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>M₃</td>
<td>( M_1 = \frac{P(D^3 + 2B^3)}{12(D+2B)} )</td>
<td>19.2 15.4 12 9.15 6.2 4.36 3.53 3.6</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>M₄</td>
<td>( M_1 = \frac{P(3D^3 + 6D^2 B - 3B^3)}{12(3D + 5B)} )</td>
<td>11.2 11.5 12 12.75 14.7 18 22.3</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>M₅</td>
<td>( M_1 = \frac{P(D + B)(2D - B)}{24} )</td>
<td>10.68 11.1 12 13.6 19.2 42.9 4</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>M₆</td>
<td>( M_1 = \frac{P(D^3 + B^3)}{12(D + B)} )</td>
<td>16 14.3 12 9.68 6.86 4.92 4</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>M₇</td>
<td>( M_1 = \frac{P(D + B)(2B - D)}{24} )</td>
<td>22.2 12 7.8 4.8 3.3 2.67 2</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>M₈</td>
<td>( M_1 = \frac{P(6D^3 + 6D^2 B - 3B^3)}{12(6D + 5B)} )</td>
<td>11.5 11.6 12 12.55 13.95 16.4 19.2</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>M₉</td>
<td>( M_1 = \frac{P(D^3 + B^3)}{12(D + 5B)} )</td>
<td>15.4 14.10 12 9.85 7.09 5.12 4.17</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>M₁₀</td>
<td>( M_1 = \frac{P(5B^3 + 9DB^2 - 3D^3)}{12(6D + 5B)} )</td>
<td>22.6 12 7.75 4.75 3.25 2.65 2.65</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>M₁¹</td>
<td>( M_1 = \frac{P(6D^5 + 6D^4 B - 5B^3)}{60(D + B)} )</td>
<td>11.4 11.6 12 12.6 14.11 16.71 20</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>M₁²</td>
<td>( M_2 = \frac{P(D^3 + B^3)}{12(D + B)} )</td>
<td>16 14.3 12 9.68 6.86 4.92 4</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>M₁³</td>
<td>( M_2 = \frac{P(6D^3 + 6D^2 B - D^3)}{60(D + B)} )</td>
<td>80 20 12 8.1 5.12 3.53 2.86 2.86</td>
<td></td>
</tr>
</tbody>
</table>

For example of use of this table, see page 322.
REINFORCED CONCRETE DESIGNER'S HANDBOOK

DESIGN OF BUNKER WITH PYRAMIDAL HOPPER BOTTOM.

**DIMENSIONS OF BUNKER, 15 FT. SQUARE, 12 FT. DEEP TO TOP OF HOPPER SLOPES.**

| CONTENTS:— DRY COAL | W = 50 LB./CU.FT. THETA = 35°. | Kx = 0.27 (PER TABLE 1) |
| CONCRETE | I = 2:1:4 ORDINARY GRADE. | Pcb = 1000 LB./SQ.IN. (PER TABLE 2) |
| MILD STEEL REINFORCEMENT | Fst = 16,000 * * * | COVER = 3/4 IN. GENERALLY. | 
| 1 IN. ON WEARING SURFACES. | CONSIDER 2-IN. NON-STRUCTURAL THICKNESS TO ALLOW FOR WEAR. |
| DESIGN FACTORS (m = 15). | Pfcb = 181; n1 = 0.455; a1 = 0.046; Q = 195 (PER TABLE 2) |

| WALLS |
| VERT. SPAN | d1 = 3 1/4 |
| L = 0.65 X 3 1/4 |
| = 2.6 |
| HORIZ. SPAN | AT MID-SPan | d1 = 3 1/2 |
| e = 3 |
| AT CORNERS | d1 = 3 1/2 |
| e = 0.65 X 3 1/2 |
| = 2.6 |

**RATIO OF EFFECTIVE SPANS**

**B, M, COEFFICIENTS (PER TABLE 2)**

**VERTICAL SPAN,—AT BOTTOM:**

\[ A_{el} = \frac{15,530}{18,000 \times 3.2} = 0.277 \text{ SQ.IN./FT.} \]

**PER TABLE 2**

**HORIZONTAL SPAN,—AT CORNERS:**

\[ A_{el} = \frac{15,530}{18,000 \times 3.2} = 0.277 \text{ SQ.IN./FT.} \]

**PER TABLE 2**

**VERTICAL SPAN,—AT TOP:**

\[ A_{el} = \frac{15,530}{18,000 \times 3.2} = 0.277 \text{ SQ.IN./FT.} \]

**PER TABLE 2**

**HORIZONTAL SPAN,—AT MID-HEIGHT:**

\[ A_{el} = \frac{15,530}{18,000 \times 3.2} = 0.277 \text{ SQ.IN./FT.} \]

**PER TABLE 2**

**HORIZONTAL SPAN,—AT OUTSIDE EDGE OF BUNKER:**

\[ A_{el} = \frac{15,530}{18,000 \times 3.2} = 0.277 \text{ SQ.IN./FT.} \]

**PER TABLE 2**

**HOPPER BOTTOM**

**FORMULAE (BY SCALE OR CALCULATION):**

\[ d = 10 FT. \]

\[ \text{SIN} \theta = \frac{45}{10} = 0.45 \]

\[ \cos \theta = \frac{6.75}{10} = 0.675 \]

**DIAMETER OF INSCRIBED CIRCLE:**

\[ D = 6 FT. \]

\[ D = 6 FT. \]

\[ h = 15 FT. \]

\[ s = 4.5 FT. \]

\[ w = \frac{1875}{500 \times 50} \]

\[ W = \frac{1875}{500 \times 50} \]

\[ W = \frac{1875}{500 \times 50} = 0.0375 \text{ SQ.FT.} \]

**APPROX FOR ALL SECTIONS**

\[ d1 = 3 \frac{1}{4} \]

\[ e = 3 \frac{1}{4} \]

**HORIZONTAL REINFORCEMENT:**

\[ N = 0.8 \times 500 \times 0.75 \times 9.5 = 1775 LB./FT. \]

\[ e = \frac{12,000}{1775} = 6.75 \text{ IN.} \]

\[ e_2 = \frac{4.33}{2} = 2.16 \text{ IN.} \]

**LONGITUDINAL (INCLUDED) REINFORCEMENT**

**AT CENTRE OF SIDE:**

\[ N = 4 \times 0.75 \times 2.5 \times 3.5 = 3300 \text{ LB./FT.} \]

\[ e = \frac{12,000}{3300} = 3.6 \text{ IN.} \]

\[ e_2 = \frac{3.6}{2} = 2.7 \text{ IN.} \]

\[ A_{el} = \frac{1800}{2} = 0.333 \text{ SQ.IN.} \]

**AT TOP OF SLOPE:**

\[ N = 1875 \]

\[ e = \frac{12,000}{1875} = 6.4 \text{ IN.} \]

\[ e_2 = \frac{6.4}{2} = 4.2 \text{ IN.} \]

**BEAM AT TOP OF SLOPE**

\[ W = 240,000 \text{ LB. APPROX.} \]

\[ e = \frac{12,000}{240,000} = 0.05 \text{ IN.} \]

\[ A_{el} = \frac{12,000}{240,000} = 0.0833 \text{ SQ.IN.} \]

**2-\frac{1}{2} BARs**
HOPPER BOTTOMS; BOX CULVERTS—TABLE 99.

**Determination of Horizontal Bars at Midspan and Corners:**

\[
B.M. = 0.375 p_h D^2 \text{ in.-lb. per ft.}
\]

**Direct Tension = 0.5 p_h l \sin \Theta_1 \text{ lb. per ft.}**

**Determination of Longitudinal Bars at Centre of Slope:**

\[
B.M. = 0.375 p_h D^2 \text{ in.-lb. per ft.}
\]

**Direct Tension = \frac{w_1}{2 \sin \Theta (1+\Theta)} \text{ lb. per ft.}**

**Direct Tension = \frac{w}{2(1+\Theta)} \text{ lb. per foot}**

**Vertical Hanging-Up Force at Base of Walls = \frac{w}{2(1+\Theta)} \text{ lb. per ft.}**

---

**Bending Moments at Corners:**

\[
M_A = M_B; \quad M_D = M_C
\]

\[
k = \frac{H}{L} \left( \frac{de}{dh} \right)^3 = \frac{H}{L} \left( \frac{de}{dh} \right)^3
\]

**Loads and Bending Moments are Per Foot Length of Culvert.**

**Concentrated Load on Roof**

\[
W \text{ per ft. length of culvert.}
\]

**B.M. Diagrams**

\[
M_A = -\frac{W L}{12} \left( \frac{2k+4.5}{(k+3)(k+1)} \right)
\]

\[
M_D = -\frac{W L}{24} \left( \frac{k+2}{(k+3)(k+1)} \right)
\]

**Uniformly-Distributed Load on Roof**

\[
W \text{ per ft.}^2
\]

\[
M_A = M_D = -\frac{W L^2}{12(k+1)}
\]

**Weight of Walls**

\[
W \text{ lb. per ft. length of culvert.}
\]

\[
M_A = \frac{W L}{6} \left( \frac{k}{(k+3)(k+1)} \right)
\]

\[
M_D = -\frac{W L}{6} \left( \frac{3+2k}{(k+3)(k+1)} \right)
\]

**Earth Pressure on Walls**

\[
P \text{ lb. per ft.}^2
\]

\[
M_A = -\frac{p H^2}{12} \left( \frac{k}{(k+3)(k+1)} \right)
\]

\[
M_D = -\frac{p H^2}{12} \left( \frac{3k+6k}{(k+3)(k+1)} \right)
\]

**Pressure on Walls Due to Surchage**

\[
P \text{ lb. per ft.}^2
\]

\[
M_A = M_D = -\frac{p H^2}{12} \left( \frac{k}{(k+3)(k+1)} \right)
\]

---

**321**
REINFORCED CONCRETE DESIGNER’S HANDBOOK

RECTANGULAR CONTAINER.

Example.—A single rectangular container, the inside dimensions of which are 12 ft. 9 in. by 10 ft. 6 in., is subject to a uniform horizontal pressure of 200 lb. per sq. ft. at a certain depth. Find the maximum bending moments and direct tensions at this depth, assuming that the walls span horizontally.

Assume the walls are 6 in. thick; effective spans = 13 ft. 3 in. and 11 ft.; hence

\[ \frac{B}{D} = \frac{13 \frac{25}{3}}{11 \cdot 0} = 1.2; \text{ from Table 98, } k = 9.7 \text{ (approx.)}. \]

Bending moment at corners = \(M_1 = \frac{PD^3}{k} = \frac{200 \times 11^2}{9.7} = 2500 \text{ ft.-lb.}\)

Free bending moment on \(D = \frac{200 \times 11^2}{8} = 3025 \text{ "} \)

Deduct bending moment at corner = 2500 "

Positive bending moment at mid-span of \(D = 525 \text{ "} \)

Free bending moment on \(B = \frac{200 \times 13 \frac{25}{3}}{8} = 4390 \text{ "} \)

Deduct bending moment at corner = 2500 "

Positive bending moment at mid-span of \(B = 1890 \text{ "} \)

Direct tension in short side = \(\frac{PB}{2} = 0.5 \times 200 \times 12.75 = 1275 \text{ lb. per ft.}\)

Direct tension in long side = \(\frac{PD}{2} = 0.5 \times 200 \times 10.5 = 1050 \text{ lb. per ft.}\)

The bending moments and direct tensions are combined as described in Table 89.

RETAINING WALLS.

Examples.—See pages 356 to 358 (Appendix II) for examples of designs of retaining walls.

Sheet-pile Wall.

Prepare a preliminary design for a simple cantilevered sheet-pile wall 12 ft. high; angle of internal friction of earth = 35 deg. No surcharge.

\(H = 12 \text{ ft.}; \theta = 35 \text{ deg.}; w = 100 \text{ lb. per cu. ft.} \text{ Hence } k_1' = 1.4; k_2' = 1.0.\)

Embedded length of pile = 11.0 \times 12 = 12 \text{ ft.} \text{ Total length of pile } = 12 + 12 = 24 \text{ ft.} \text{ Span of sheet-pile for calculating bending moment } = L = 11.4 \times 12 = 138.8 \text{ ft.} \text{ Total active pressure on back of wall on height } L(=16.8 \text{ ft.)}) = k_2 \times 100 \times 16.8^2 \times 0.5; k_2 = 0.27 \text{ from Table 11}; \text{ therefore the total pressure } = 3810 \text{ lb.} \text{ The bending moment } = 3810 \times 0.33 \times 16.8 \times 12 = 256,000 \text{ in.-lb. per ft. width, that is, } 21,333 \text{ in.-lb. per inch, which from Table 70A with equal areas of steel in tension and compression and with maximum stresses of 18,000 lb. per sq. in. and 1000 lb. per sq. in. } d_1 = \sqrt{\frac{21,333}{507}} = 6.3 \text{ in.}; \text{ with not less than } 1 \frac{1}{2}-\text{in. cover, the thickness of pile required is about 9 in., } L_3 \text{ is about 6 in., and the reinforcement required in a pile } 15 \text{ in. wide is approximately } \frac{21,333 \times 15}{18,000 \times 6} = 2.97 \text{ sq. in.}, \text{ say a total of six } 1 \frac{1}{2}-\text{in. bars (three bars on each face).}\)

CHIMNEYS.

The factors and formulae given in the table below enable the approximate thickness of the shaft and the amount of reinforcement therein to be determined to a degree of accuracy sufficient for preliminary designs; the stresses in a final design should be checked by more exact methods.

| DIMENSIONS OF CHIMNEY SHAFTS (FOR PRELIMINARY DESIGNS). |
|-----------------|---------------|---------------|---------------|---------------|---------------|
| STRESSES LB. PER SQ. IN. | IN CONCRETE GOO | GOO | GOO | 400 |
| IN STEEL 12 000 | 14 000 | 16 000 | 16 000 |
| FACTORS A 6 408 | 7 551 | 8 685 | 8 600 |
| B 7 776 | 8 990 | 10 250 | 9 698 |
| C 4 592 | 3 409 | 2 867 | 1 376 |
| D 1 372 | 1 410 | 1 418 | 911 |
| R = MEAN RADIUS OF SHAFT (IN.) | TOTAL AREA OF VERTICAL STEEL |
| \(A_{stk} = \frac{M}{AR} - \frac{W}{B} \text{ SQ. IN.} \)|
| THICKNESS OF CONCRETE |
| \(d = \frac{M}{CR^2 + DR} \text{ IN.} \)|
| \(M = BENDING MOMENT (IN.-LB.) \)|
| \(W = TOTAL WEIGHT (LB.) \)|
RETAINING WALLS.—TABLE 100.

CANTILEVERED RETAINING WALLS

Type (b) Surcharge

Pressures & Loads in lb. per ft. length of wall.

W = Resultant of all vertical loads including weights of stem and base of wall, weight of earth on base, and surcharge on ground behind wall.

P = Resultant of all active horizontal pressures on height H.

P = Resultant of all active horizontal pressures acting on stem of wall above A-A.

B.M. at A-A = \( P \) ft.-lb./ft.

(Also check M in with B.M. at bottom of splay)

For stability: \( W \times \frac{F_s}{F_p} \)  \( P_y \).

Resistance: Frictional resistance only = \( \mu W \times \frac{F_s}{F_p} \) (granular)

Sliding: Passive resistance only = Passive resistance of ground in front of wall \( \mu F_p \)

Minimum Factors of Safety:

\( F_s \geq 1.2 \), \( F_p \geq 1.2 \), \( F_p \leq 2 \), \( \mu \) for granular soils \( \geq 0.4 \).

Ground pressure: \( e = \text{eccentricity of } W \) and \( P \) combined

\[ P_{\text{max}} = \frac{W}{L} \left[ 1 + \frac{e}{g} \right] \geq \text{Safe bearing capacity.} \]

Sheet-Pile Retaining Walls

Angle of Internal Friction of Ground

<table>
<thead>
<tr>
<th>Condition</th>
<th>At A</th>
<th>Span of Sheet Piles for Calculation of B.M.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Free</td>
<td>k_1</td>
<td>L = k_1 H</td>
</tr>
<tr>
<td>Hinged</td>
<td>k_2</td>
<td>Minimum Embedded Length of sheeting = h = k_2 H</td>
</tr>
<tr>
<td>Fixed</td>
<td>k_1</td>
<td>For final designs actual conditions should be considered.</td>
</tr>
</tbody>
</table>

* See Table 101 for calculation of bending moments.

** See Tables 11, 12 and 13 for calculation of earth pressures on walls.
TABLE 101.—SIMPLE SHEET-PILE RETAINING WALLS.

\[ P = \text{INCREASE OF PASSIVE RESISTANCE PER FOOT DEPTH,} \]
\[ P_a = \text{ACTIVE PRESSURE} \]

FOR PRELIMINARY DESIGN:
MAX. B.M. ON SHEET-PILE = \( P_i \times \)
\[ P_i = \frac{PL}{2} \left( h_1 + L \right) \]

PROBABLE VARIATION OF PRESSURE

APPROXIMATE DISTRIBUTION OF PRESSURE FOR CHECKING A DESIGN

FORMULAE FOR CHECKING A DESIGN
SUBSTITUTE KNOWN VALUES OF \( P_a, H, h_1 \) AND \( h \) IN
\[ P_a (H+h+h')^3 - P_a h^3 + \left( \frac{P_a h^2 - P_a (H+h+h')^2}{(P_a - P_a)(H+h+2h')} \right)^2 = 0 \]
To give an expression in the form
\[ N_1 - N_2 P_a + \frac{N_3 (P_a - N_4)^2}{P_a - N_5} = 0 \]
in which \( N_1, N_2, N_3, N_4 \) and \( N_5 \) are numerical factors; calculate
\[ a = (N_3 - N_2); \quad b = (N_1 + N_2 N_5 - 2 N_3 N_4); \quad c = (N_4 N_5 - N_1 N_6) \]
SUBSTITUTE IN \( \frac{1}{2a} \left[ b^2 - 4ac \right] - b \)

COMPUTE CALCULATED VALUE OF \( P_a \) WITH PROBABLE VALUE WITH A FACTOR OF SAFETY.

SHOES AND INTERLOCKS

SHOE FOR FOLLOWING SHEET-PILE

324
COMBINED FOUNDATIONS.

Combined Bases (Strip Bases).—When more than one column or load is carried on a single base the centre of gravity of the several loads should, if possible, coincide with the centre of the area of the base, in which case the pressure under the base is uniformly distributed. The base should be symmetrically disposed about the line of the loads and may be rectangular in plan as in Table 105, trapezoidal as in Table 105 or at (i) in the upper part of Table 103, or made up of a series of rectangles as at (ii) in Table 103. In the last case each rectangle should be proportioned so that the load upon it acts at the centre of its area, and the area of each rectangle should be equal to the corresponding applied load divided by a safe bearing pressure, the value adopted for this pressure being the same for all the rectangles.

If it is not practicable to proportion the bases as described the load will be eccentric, and thus the centre of pressure of the upward ground pressure will have the same eccentricity relative to the centre of the area of the base. If the base is so thick that it may be considered to act as a single rigid member, the ground pressure will vary according to the formulae for eccentric loads given in Table 104, and the pressure-distribution diagram is as at (iii) in Table 103. If the base is comparatively thin this distribution may not be realised, and owing to the flexibility of the base the ground pressure may be greater immediately under the loads, giving a pressure-distribution as at (iv) in Table 103.

Balanced Bases.—Referring to the diagrams in the upper right-hand corner of Table 103, a column is supported on the overhanging end C of the beam BC at (i) which is supported on a base at A and subjected to a counterbalance at B. The reaction at A, which depends on the relative values of BC and BA, can be provided by an ordinary reinforced concrete or plain concrete base designed for a concentric load. The counterbalance can mostly be provided by the load from another column as at (ii), in which case the dead load on this column at B should be sufficient to counterbalance the dead and live loads on the column at C, and vice versa, with a sufficient margin of safety. It is often possible to arrange the column B immediately over A. Formulas giving the values of the reactions at A and A1 are given in Table 105, but the notes on the page facing the table should be observed. From the reactions the shearing forces and bending moments on the beam can be calculated.

If no column loads can be conveniently brought into service to counterbalance the column at C an anchorage must be provided at B by other means, such as the construction of a plain concrete counterweight as at (iii) or the provision of tension piles.

If the column to be supported is a corner column loading the foundation eccentrically in two directions, one parallel to each building line, it is sometimes possible to introduce a diagonal balancing beam which is anchored by the adjacent internal column D as at (iv). In other cases, however, the two wall beams meeting at the column can be designed as balancing beams to overcome the double eccentricity. The bending moment due to the cantilever action in the beam EC is equal to \( W_{BE_1} \), where \( W_p \) is the column load, and the upward force on column C is \( \frac{W_{BE_1}}{L_1 - e_1} \). Similarly the bending moment in beam EF is \( W_{BE_2} \) and the upward force on column F is \( \frac{W_{BE_2}}{L_2 - e_2} \).

Rafts.—The load on and the spacing of the columns determine the shearing forces and bending moments which in turn determine the thickness of the raft. If this thickness does not exceed 12 in. a solid slab as at (i) in the lower part of Table 103 is generally the most convenient form. If a slab at ground level is required, it is nearly always necessary to thicken the slab at the edge, as at (ii), to ensure that the edge of the raft is deep enough below the ground to avoid weathering of the ground under the raft. If a greater thickness is required beam-and-slab construction, designed as an inverted floor, as at (iii), is more economical. In cases where the total depth required exceeds 3 ft., or where a level top surface is required, cellular construction as at (iv), consisting of a top and bottom slab with intermediate ribs, is used.

When the columns on a raft are not equally loaded or are not symmetrically arranged, the raft should be designed so that the centroid coincides with the centre of gravity of the loads. As this gives uniformly-distributed pressure on the ground, the area of the raft is equal to the total load (including the weight of the raft) divided by the safe bearing pressure. If this coincidence of centres of gravity is impracticable owing to the extent of the raft being limited on one or more sides, the plan of the raft should be made so that the eccentricity \( e_0 \) (Continued on page 328.)
COMBINED FOUNDATION FOR LINE OF LOADS.

(i) TRAPEZOIDAL.
(ii) RECTANGULAR.
(iii) UNIFORM VARIATION OF PRESSURE.
   (WHEN C.G. AND CENTROID NOT COINCIDENT)
(iv) NON-UNIFORM VARIATION OF PRESSURE.

RAFTS

(i) SOLID RAFT.
(ii) SOLID RAFT WITH THICKENING AT EDGE.
(iii) BEAM-AND-SLAB RAFT.
(iv) CELLULAR RAFT.

BALANCED FOUNDATIONS

(i) A
(ii) B
(iii) C
(iv) E
(v) BASEMENT IN WET GROUND.

WATER
IMPOSED LOAD
WATER PRESSURE

LIMITED EXTENT OF RAFT
THREE LIGHT LOADS
CENTER OF LOADS
CENTER OF RAFT.

(v) PLAN OF ECCENTRICALLY LOADED RAFT.
of the total loading \( W_T \) is a minimum, and this may produce a raft which is not rectangular in plan, as in the example illustrated at vi.

Maximum pressure on the ground (which should not exceed the safe bearing resistance and occurs at \( f = \frac{W_T}{A_R} + \frac{W_T I_R}{I_R} \) where \( A_R \) = total area of raft, and \( I_R \) = moment of inertia about the axis NN which passes through the centroid of the raft and is normal to the line joining the centroid and the centre of gravity of the loads.

Minimum ground pressure (at \( g \)) = \( \frac{W_T}{A_R} - \frac{W_T I_R}{I_R} \).

Intensity of ground pressure along the line NN = \( \frac{W_T}{A_R} \).

When the three pressures have been determined, the pressure at any other point or the mean pressure over any area can be assessed. Having arranged a rational system of beams or ribs dividing the slab into suitable panels, as suggested by the dotted lines at (vi), the panels of slabs and the beams can be designed for the bending moments and shearing forces due to the net upward pressures to which they are subjected.

**SEPARATE BASES (Table 104).**

**Examples.**—(a) Determine the variation in pressure under a plain concrete foundation 10 ft. long by 2 ft. thick, by 8 ft. wide carrying a load of 100 tons placed 1 ft. eccentrically.

Weight of base = \( \frac{8 \times 10 \times 2 \times 140}{2240} = 10 \) tons. Total load = 110 tons.

Moments about the short side of the base = \((10 \times 5) + (100 \times 4) = 450 \) ft.-tons.

Eccentricity = \( \frac{10}{2} - \frac{450}{110} = 0.91 \) ft. Since \( \frac{L}{6} = \frac{10}{6} = 1.67 \) ft. and \( \varepsilon = 0.91 \) ft.,

\( \varepsilon < \frac{L}{6} \). Thus \( k = \left( \frac{1}{10} + \frac{6 \times 0.91}{10} \right) = 1.545 \) or 0.455.

Maximum ground pressure = \( \frac{1.545 \times 110}{10 \times 8} = 2.13 \) tons per sq. ft.

Minimum ground pressure = \( \frac{0.455 \times 110}{10 \times 8} = 0.625 \) ton " "

(b) Determine the ground pressure in (a) if the load is 2 ft. eccentric.

Moments about the short side of the base = \((10 \times 5) + (100 \times 3) = 350 \) ft.-tons.

Eccentricity = \( 5 - \frac{350}{110} = 1.82 \) ft.; \( \varepsilon > \frac{L}{6} \). Thus \( k = \frac{4}{3\left(1 - \frac{2 \times 1.82}{10}\right)} = 2.09. \)

(This condition is not advisable.)

Maximum ground pressure = \( \frac{2.09 \times 110}{10 \times 8} = 2.89 \) tons per sq. ft.

(c) Design a base for a reinforced concrete column 15-in. square carrying a concentric load of 120,000 lb.; ground pressure not to exceed 3000 lb. per sq. ft. Permissible stresses 18,000 lb. and 1000 lb. per sq. in. \( Q_e = 193 \) (Table 68).

Assume \( W_B = 4000 \) lb. \( A \leq \sqrt{\frac{124,000}{3000}} = 6.4 \) ft., say, 6 ft. 6 in. square.

\( \Phi_{net} = \frac{120,000}{6.5^2} = 2840 \) lb. per sq. ft. \( M_{xx} = \frac{2840 \times 6.5}{8} \times (5 - 1.25)^2 = 63,500 \) ft.-lb.

Make \( B = 18 \) in.; \( d \leq \sqrt{\frac{63,500}{193 \times 1.5}} = 14.8 \) in., say, \( D_B = 18 \) in.

If \( D_C = 0.25 \) ft., \( d = 18 - 3 = 15 \) in.; \( A_{et} = \frac{12 \times 63,500}{\frac{3}{4} 	imes 15 \times 18,000} = 3.21 \) sq. in.

Provide twelve 1/2-in. bars (Table 60) in each layer.

If the base is tapered from 18 in. at the column to \( D_v = 6 \) in. at the edge, and \( F = 1.25 + 2(1.5 - 0.25) = 3.75 \) ft.

\( d_1 = 12 \left[ \frac{(D_B - D_v)(A - F)}{A - B} + D_v - D_C \right] = 12 \left[ \frac{10 \times 2.75}{5} + 0.5 - 0.25 \right] = 10 \) in. approx.

Check for shearing resistance: \( Q = \frac{2840}{4 \times 3.75} \times (6.5^2 - 3.75^2) = 5330 \) lb. per ft. approx.

\( d_1 \leq \frac{5330}{10.5 \times 100} = 5.1 \) in., which is less than \( d_1 \) provided (10 in.).
### Foundations: Pressures and Independent Bases

**Table 104.**

<table>
<thead>
<tr>
<th>TYPE OF GROUND</th>
<th>TOTAL PRESSURES (TONS PER SQ. FT.)</th>
<th>CONSTRUCTIONAL MATERIAL</th>
<th>BEARING PRESSURES (TONS PER SQ. FT.)</th>
<th>REDUCTION FOR SLENDER PIERS, ETC.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Silt, alluvial earth, etc.</td>
<td>0 to 7/4</td>
<td>Description</td>
<td>Crushing Strength (lb/sq. in.)</td>
<td>Uniform Pressure Load</td>
</tr>
<tr>
<td>Clay: soft or very soft</td>
<td>3/4 to 7/4</td>
<td>FIRM</td>
<td>1 1/8</td>
<td>1200</td>
</tr>
<tr>
<td>Sandy firm, stiff</td>
<td>1/2 to 7/3</td>
<td>STIFF</td>
<td>1 1/8</td>
<td>1200</td>
</tr>
<tr>
<td>Firm</td>
<td>3/4 to 7/3</td>
<td>STIFF</td>
<td>1 1/8</td>
<td>1200</td>
</tr>
<tr>
<td>Hard shale clay</td>
<td>3 to 7</td>
<td>CONCRETE</td>
<td>1 3/4</td>
<td>1650</td>
</tr>
<tr>
<td>Very stiff (boulder)</td>
<td>3 to 7</td>
<td>CONCRETE</td>
<td>1 3/4</td>
<td>1650</td>
</tr>
<tr>
<td>Sound yellow</td>
<td>3 to 7</td>
<td>CONCRETE</td>
<td>1 3/4</td>
<td>1650</td>
</tr>
<tr>
<td>Blue</td>
<td>4 to 7</td>
<td>CONCRETE</td>
<td>1 3/4</td>
<td>1650</td>
</tr>
<tr>
<td>Sand: uniform loose</td>
<td>2 to 4</td>
<td>COMPACT</td>
<td>2</td>
<td>2000</td>
</tr>
<tr>
<td>Wellgraded loose</td>
<td>2 to 4</td>
<td>COMPACT</td>
<td>2</td>
<td>2000</td>
</tr>
<tr>
<td>Gravel: sandy</td>
<td>2 to 4</td>
<td>COMPACT</td>
<td>2</td>
<td>2000</td>
</tr>
<tr>
<td>Clean loose</td>
<td>2 to 4</td>
<td>COMPACT</td>
<td>2</td>
<td>2000</td>
</tr>
<tr>
<td>Rock: chalk</td>
<td>1 1/2</td>
<td>COMPACT</td>
<td>2</td>
<td>2000</td>
</tr>
<tr>
<td>Soft</td>
<td>3 to 7</td>
<td>MODERATELY HARD</td>
<td>5 to 10</td>
<td></td>
</tr>
<tr>
<td>Hard</td>
<td>12</td>
<td>MODERATELY HARD</td>
<td>5 to 10</td>
<td></td>
</tr>
</tbody>
</table>

**Safe Bearing Pressures (Hamp Shale and Constructional Materials).**

- **Masonry or Brickwork:**
  - Rock filling: depends on consolidation; determine by test.
  - Rock: not more than 15% of crushing strength, sand, gravel, etc., tabulated pressures are for dry materials. Assume 50% if saturated.
  - Earth filling, peat, ashes: may be negligible.

**Maximum ground pressure to prevent spewing.**

- **Cohesive soil:**
  - P = wh/2
  - k = 4A/A
  - K = 3(4A/A)
  - h = 23

- **Values of K and C:** See footnote.

**Intensity of Pressure (lb per sq. ft.)**

\[ P = \frac{kW}{AB} \]

- A = LENGTH OF FOUNDATION (FT)
- B = WIDTH OF FOUNDATION (FT)
- W = TOTAL LOAD ON FOUNDATION INCLUDING WEIGHT OF BASE (LB)

**Distribution of Pressure**

- **Concentric Load:**
  - P = 4A/A
  - E = 3(4A/A)

- **Eccentric Load:**
  - P = 4A/A
  - E = 3(4A/A)

**Notes:** See Tables 11 and 12 for values of K and C.

For notation and units, see Table 105.

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COMBINED BASES.

Strip Base Supporting Several Loads Arranged Eccentrically.

\[ e = \frac{\Sigma Wx}{\Sigma W} - \frac{L}{2} \geq \frac{L}{6}. \]

Calculate \( p_{\text{min}} \), and \( p_{\text{max}} \), from formula in Table 104.

\[ p_x = \left( \frac{L - y}{L} \right) (p_{\text{max}} - p_{\text{min}}) + p_{\text{min}}. \]

\[ M_{XX} = \Sigma Wx - \frac{y^2}{6} (2p_{\text{max}} + p_x) C. \]

Example.—Find the bending moment at the position of load \( W_x \) on a base 50 ft. long and 5 ft. wide, carrying five unequal point loads as shown in the diagram. The loads (in tons) are \( W_1 = 50 \), \( W_2 = 45 \), \( W_3 = 40 \), \( W_4 = 35 \), and \( W_5 = 30 \); the distances (in feet) are \( x_1 = 45 \), \( x_2 = 37 \), \( x_3 = 28 \), \( x_4 = 18 \), and \( x_5 = 5 \). Thus \( \Sigma W = 200 \) and \( \Sigma Wx = (50 \times 45) + (45 \times 37) + (40 \times 28) + (35 \times 18) + (30 \times 5) = 5815. \)

Therefore \( e = \frac{5815}{200} - \frac{50}{2} = 4.075 \) ft., which is less than \( \frac{L}{6} = 8 \frac{1}{3} \) ft. Thus

\[ h = \frac{6 \times 4.075}{50} = 1.489 \text{ or } 0.511. \]

Maximum pressure: \( p_{\text{max}} = \frac{1.489 \times 200}{50 \times 5} = 1.9 \text{ tons per sq. ft.} \)

Minimum pressure: \( p_{\text{min}} = \frac{0.511 \times 200}{50 \times 5} = 0.41 \text{ ton } " \)

At the load \( W_x \), \( y = 50 - 37 = 13 \) ft.

Therefore \( p_x = \frac{(50 - 13)}{50} (1.9 - 0.41) + 0.41 = 0.987 \text{ ton per sq. ft.} \)

Also \( x_1 = 8 \), \( x_3 = 0 \). Therefore \( \Sigma Wx = W_1 x_1 + W_3 x_3 = 400 + 0 = 400 \text{ ft.-tons.} \)

\[ M_{XX} = 400 - \frac{13^2}{6} (2 \times 1.9 + 0.987) 5 = 400 - 470 = -70 \text{ ft.-tons.} \]

Balanced Bases.—The probable variation in loads \( W_1 \) and \( W_2 \) (see diagram in Table 105) must be considered for bases of this type. If \( W_1 \) can vary from \( W_{1(\text{max})} \) to \( W_{1(\text{min})} \), and \( W_2 \) from \( W_{2(\text{max})} \) to \( W_{2(\text{min})} \), the reaction \( R \) will vary from

\[ R_{\text{max}} = \frac{EW_{1(\text{max})}}{L} \text{ to } R_{\text{min}} = \frac{EW_{1(\text{min})}}{L}. \]

Therefore \( R_1 \) and \( R_3 \) may have the following values:

\[ R_{1(\text{max})} = W_{1(\text{max})} + W_{B_1} + R_{\text{max}} + 0.5 W_L \]
\[ R_{1(\text{min})} = W_{1(\text{min})} + W_{B_1} + R_{\text{min}} + 0.5 W_L \]
\[ R_{2(\text{max})} = W_{2(\text{max})} + W_{B_2} - R_{\text{min}} + 0.5 W_L \]
\[ R_{2(\text{min})} = W_{2(\text{min})} - W_{B_2} + R_{\text{min}} + 0.5 W_L \]

Base (1) must therefore be designed for a maximum load of \( R_{1(\text{max})} \) and base (2) for \( R_{2(\text{max})} \); but \( R_{1(\text{min})} \) must always be positive, and should be sufficiently large to ensure a margin of safety in the counterbalance provided by base (2).

Shearing Forces.—The shearing forces on combined bases, rectangular or trapezoidal in plan and carrying two loads, are calculated as for a double-cantilevered beam. For a strip base the shearing force can be calculated by using the basic principle that the shearing force at any section is the algebraic sum of the vertical forces on one side of the section.

Longitudinal Bending Moments.—The longitudinal positive bending moment on a base carrying two loads can be determined graphically from the two negative bending moments under the loads \( W_1 \) and \( W_2 \) and the "free" positive bending moment \( M_{XX} \).

Minimum Depth of Foundation.—The theoretical minimum depth at which a foundation should be placed can be determined from the formula in Table 104 transposed to give

\[ h = \frac{p_{\text{max}}}{w} \frac{k_1^2}{w} \text{ or } \left( p_{\text{max}} - \frac{2C}{\sqrt{k_2(k_2 + 1)}} \right) \frac{k_2^2}{w}. \]
REINFORCED CONCRETE DESIGNER’S HANDBOOK

IMPACT-DRIVEN PILES.

In calculations for the loads on piles a formula can only give comparative values that must be combined with the results of tests and experience when assessing the safe load on a pile.

**Example.**—Estimate the safe load on a pile 40 ft. long, 14 in. square, driven to gravel by a 2-ton single-acting steam hammer dropping 42 in. with 12 blows per inch of final penetration; the weight of the helmet, dolly, and stationary part of hammer is \( \frac{1}{2} \) ton. The dolly is in good condition at the end of the driving. The weight of the pile is

\[
\frac{40 \times 196}{2240} = 3\frac{1}{4} \text{ tons; } P = 3\frac{1}{4} + \frac{1}{2} = 4 \text{ tons; } \frac{P}{w} = \frac{4}{2} = 2.
\]

From Table 106: Effective drop = \( H_1 = 0.90 \times 42 = 38 \) in. For \( \frac{P}{w} = 2, \varepsilon = 0.37 \).

For medium driving, \( c = 0.29 \) for \( L = 40 \) ft.

The settlement load = \[
\frac{2 \times 38 \times 0.37 \times 12}{\frac{1}{1} + (0.29 \times 12)} + 2 + 4 = 75.3 + 6 = 81.3 \text{ tons}.
\]

For this load the driving pressure = \[
\frac{81.3 \times 2240}{14^2} = 930 \text{ lb. per sq. in., which is sufficiently close to the assumed value of 1000 lb. per sq. in. Thus the working load is, say,}
\]

\[
\frac{81.3}{2} - 3\frac{1}{4} = 37\frac{1}{2} \text{ tons.}
\]

DESIGNS FOR PILED JETTIES.

**Vertical Piles Only** [Diagram (a) on page facing Table 107].—Consider a single row of piles; therefore \( N = 1 \) for each line and \( 2N = 4 \). Since the group is symmetrical, \( X = \frac{1}{2} \times 28 = 14 \) ft. From Table 107: \( M = (80 \times 2.5) - (10 \times 15) = +50 \) ft.-tons.

The calculation of the load on the piles can be made by tabulation as follows:

<table>
<thead>
<tr>
<th>Pile No.</th>
<th>( x ) (ft.)</th>
<th>( x^2 ) (ft.(^2))</th>
<th>( \frac{hW}{I} \left( \frac{1}{2N} \right) )</th>
<th>( \frac{hM}{N} \left( \frac{x}{N} \right) )</th>
<th>Axial load ( (hW + hMM) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_1 )</td>
<td>-14</td>
<td>196</td>
<td>( -\frac{14}{435.6} = -0.0321 )</td>
<td>( 0.025 \times 80 ) - ( 0.0321 \times 50 ) = 18.4 tons</td>
<td></td>
</tr>
<tr>
<td>( P_2 )</td>
<td>-4.67</td>
<td>21.8</td>
<td>( -\frac{4.67}{435.6} = -0.0107 )</td>
<td>20 - ( 0.0107 \times 50 ) = 19.5 tons</td>
<td></td>
</tr>
<tr>
<td>( P_3 )</td>
<td>+4.67</td>
<td>21.8</td>
<td>+0.0107</td>
<td>20 + 0.5 = 20.5 tons</td>
<td></td>
</tr>
<tr>
<td>( P_4 )</td>
<td>+14</td>
<td>196</td>
<td>+0.0321</td>
<td>20 + 1.6 = 21.6 tons</td>
<td></td>
</tr>
<tr>
<td>( I = \Sigma N ) ( x^2 ) =</td>
<td>435.6 (ft.(^2))</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The shearing force on each pile is \( \frac{10}{4} = 2.5 \) tons = 5600 lb., and the bending moment on each pile is \( 0.5 \times 2.5 \times 15 = 18.75 \) ft.-tons = 506,000 in.-lb. The maximum load on any pile is 21.6 tons = 48,300 lb., or 56,000 lb. including the pile.

**Vertical and Inclined Piles** [Diagram (b) on page facing Table 107] (tan \( \theta = 0.25 \)). For each pile, \( A \) is assumed to be the same. All piles are driven to the same depth, therefore if \( A \) \( L \) for piles \( P_1 \) and \( P_4 \) is unity, \( A \) \( L \) for \( P_2 \) and \( P_3 \) is \( \frac{4}{\sqrt{1 + 4^2}} = 0.97 \). Since the group is symmetrical, \( \Sigma_2 = 0, \Sigma_4 \) is not required, \( G = \Sigma_1 \Sigma_5, x_0 = \frac{28}{2} = 14 \) ft., and \( y_0 = 0 \).

\( M = 80(16.5 - 14) + 0 = 200 \) ft.-tons.

(Continued on page 334.)
### Hiley Formula (Modified)

<table>
<thead>
<tr>
<th>EFFICIENCY OF BLOW</th>
<th>P/W</th>
<th>0-5</th>
<th>1-0</th>
<th>1-5</th>
<th>2-0</th>
<th>2-5</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>WITH HAMMER AND WOODEN DOLLY</td>
<td>0-69</td>
<td>0-63</td>
<td>0-57</td>
<td>0-50</td>
<td>0-45</td>
<td>0-40</td>
<td>0-35</td>
<td>0-30</td>
<td>0-25</td>
<td>0-21</td>
<td>0-18</td>
</tr>
<tr>
<td>WITH HAMMER AND CAPING IN FROZEN CONDITION</td>
<td>0-67</td>
<td>0-50</td>
<td>0-44</td>
<td>0-37</td>
<td>0-33</td>
<td>0-30</td>
<td>0-25</td>
<td>0-21</td>
<td>0-18</td>
<td>0-15</td>
<td>0-12</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>COMPRESSION FACTOR</th>
<th>C</th>
<th>EASY DRIVING</th>
<th>P&lt;sub&gt;e&lt;/sub&gt; = 500</th>
<th>0-13</th>
<th>0-16</th>
<th>0-17</th>
<th>0-19</th>
<th>0-20</th>
<th>0-20</th>
<th>0-36</th>
<th>0-35</th>
<th>0-44</th>
<th>0-51</th>
</tr>
</thead>
<tbody>
<tr>
<td>MEDIUM DRIVING</td>
<td>P&lt;sub&gt;e&lt;/sub&gt; = 1000</td>
<td>0-24</td>
<td>0-27</td>
<td>0-29</td>
<td>0-33</td>
<td>0-36</td>
<td>0-38</td>
<td>0-40</td>
<td>0-40</td>
<td>0-40</td>
<td>0-40</td>
<td>0-40</td>
<td></td>
</tr>
<tr>
<td>HARD DRIVING</td>
<td>P&lt;sub&gt;e&lt;/sub&gt; = 1500</td>
<td>0-33</td>
<td>0-37</td>
<td>0-40</td>
<td>0-40</td>
<td>0-40</td>
<td>0-40</td>
<td>0-40</td>
<td>0-40</td>
<td>0-40</td>
<td>0-40</td>
<td>0-40</td>
<td></td>
</tr>
<tr>
<td>VERY HARD DRIVING</td>
<td>P&lt;sub&gt;e&lt;/sub&gt; = 2000</td>
<td>0-40</td>
<td>0-40</td>
<td>0-40</td>
<td>0-40</td>
<td>0-40</td>
<td>0-40</td>
<td>0-40</td>
<td>0-40</td>
<td>0-40</td>
<td>0-40</td>
<td>0-40</td>
<td></td>
</tr>
</tbody>
</table>

**Additional Notation:**
- H = Free fall of hammer (in.)
- A<sub>s</sub> = Area (sq. ft.) of sides of pile embedded in ground
- F = Factor of safety (1/2 to 2)

**Points of Lifting:**
- Short piles — Toggle holes near head (for guiding) can generally be used for lifting without producing excessive b.m.s.

**Gas-Pipe Ferrules Forming Toggle Holes**

**Section 1:1**
- Binder: W<sub>1</sub>, 1/4 x 1/8
- Spacers at approx. 4" to 6" intervals
- Helical binding at head of pile only

**Section 2:2**
- G-6/8" bars with c.i. spacers for 1/4" links (ties)
- Helical binding

**Typical Details of Piles and Shapes of Shoes**
- Cast iron shoe with W-1 straps
- Shape suitable for gravels & sands
- Through soft ground to moderately hard ground (no shoe)
- Rock shoe w/ flat W-1

---

**Settlement Load:**
\[ W_M = \frac{w H_1}{1 + c n} + w + P \]

**Working Load:**
\[ \frac{W_M}{b} \text{ tons} \text{ (including weight of pile)} \]

**Effective Drop:**
- For freely-falling drop hammer
- P-9 = Single-acting steam
- P-8 = Winch-operated drop

**Driving Pressure:**
\[ P = \frac{w H_1}{1 + c n} \text{ tons per sq. in.} \]

**B.M. (Max) = 0-043 P.L. tons.**

---

**3" Dia. (W-1) 3/8 flat W-1**

---

**333**
# REINFORCED CONCRETE DESIGNER'S HANDBOOK

## DESIGNS FOR PILED JETTIES (continued from page 332)

![Diagram of pile-supported jetty with calculations]

### Table

<table>
<thead>
<tr>
<th>Pile No.</th>
<th>$\Sigma_1$</th>
<th>$\Sigma_2$</th>
<th>$\Sigma (\text{ft})$</th>
<th>$X (\text{ft})$</th>
<th>$h_p$</th>
<th>$h_W$</th>
<th>$h_H$</th>
<th>$h_M$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_1$</td>
<td>+1</td>
<td>0</td>
<td>0</td>
<td>-14</td>
<td>+196</td>
<td>+1</td>
<td>0</td>
<td>-14</td>
</tr>
<tr>
<td>$P_2$</td>
<td>+0.97</td>
<td>+4/$1+4^4$</td>
<td>+97/$1+4^4$</td>
<td>+14</td>
<td>0</td>
<td>+97</td>
<td>0</td>
<td>-97</td>
</tr>
<tr>
<td></td>
<td>+0.912</td>
<td>$1+4^4$</td>
<td>$1+4^4$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$P_3$</td>
<td>+0.912</td>
<td>+0.957</td>
<td>+14</td>
<td>0</td>
<td>0</td>
<td>+0.941</td>
<td>+0.261</td>
<td>-2.19</td>
</tr>
<tr>
<td>$P_4$</td>
<td>+1</td>
<td>0</td>
<td>+28</td>
<td>+14</td>
<td>+176</td>
<td>+1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Totals</td>
<td>+3.824</td>
<td>+0.114</td>
<td>-</td>
<td>392</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Vertical and Inclined Piles (continuation of example from pages 332 and 334).

Therefore $G = 3.824 \times 0.114 = 0.436$. The axial loads on the piles are

$P_1 = [1(0.261 \times 80) + o - (0.036 \times 200)] = 20.9 - 7.2 = 13.7$ tons.

$P_2 = 0.941(20.9 + (2.19 \times 10) + o) = 40.3$ tons.

$P_3 = 0.941(20.9 - 21.9 + o) = -0.94$ tons (tension) or $+2$ tons including the pile.

$P_4 = 20.9 + o + 7.2 = 28.1$ tons.

The maximum load on any pile is 40.3 tons, say, 43 tons including the pile.

Inclined Piles Only [Diagram (c) on page facing Table 107] (tan $\theta = 0.2$). For each pile $L$ and $A$ are the same; so unity can be substituted for $A^4/L$. For Piles $P_1$ and $P_3$, $B = +5$ and for $P_2$ and $P_4$, $B = -5$. Since the group is symmetrical, $\Sigma_x = 0$, $\Sigma_y$ is not required. $\Sigma_y = 0$, $G = \Sigma_x \Sigma_5$, $x_9 = 9.33$ ft., $y_9 = 0$. $M = 80(11.83 - 9.33) + o = 200$ ft.-tons.

<table>
<thead>
<tr>
<th>Pile No.</th>
<th>$\Sigma_1$</th>
<th>$\Sigma_4$</th>
<th>$x$ (ft.)</th>
<th>$X$ (ft.)</th>
<th>$\Sigma_x = L$</th>
<th>$k_H$</th>
<th>$k_W$</th>
<th>$k_T$</th>
<th>$X/T$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_1$</td>
<td>$-\frac{5}{1+5^2} = +0.96$</td>
<td>$\frac{1}{1+5^2} = +0.0384$</td>
<td>0</td>
<td>$-9.33$</td>
<td>$0.96(-9.33)$</td>
<td>$\frac{5}{\sqrt{1+5^2}}$</td>
<td>$0.956$</td>
<td>$\frac{5}{5\times0.59}$</td>
<td>$-0.33$</td>
</tr>
<tr>
<td>$P_2$</td>
<td>$+0.96$</td>
<td>$+0.0384$</td>
<td>0</td>
<td>$-9.33$</td>
<td>$+8.5$</td>
<td>$+0.0384$</td>
<td>$-0.33$</td>
<td>$334$</td>
<td></td>
</tr>
<tr>
<td>$P_3$</td>
<td>$+0.96$</td>
<td>$+0.0384$</td>
<td>$+1.67$</td>
<td>$+9.33$</td>
<td>$+8.5$</td>
<td>$+0.0384$</td>
<td>$-0.33$</td>
<td>$334$</td>
<td></td>
</tr>
<tr>
<td>$P_4$</td>
<td>$+0.96$</td>
<td>$+0.0384$</td>
<td>$+13.67$</td>
<td>$+9.33$</td>
<td>$+8.5$</td>
<td>$+0.0384$</td>
<td>$-0.33$</td>
<td>$334$</td>
<td></td>
</tr>
<tr>
<td>Totals</td>
<td>$+3.84$</td>
<td>$+0.1536$</td>
<td>-</td>
<td>-</td>
<td>$+3.34$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Therefore $G = 3.84 \times 0.1536 = 0.59$. The axial loads on the piles are:

$P_1 = 0.98[(0.26 \times 80) + (1.305 \times 10) - (0.028 \times 200)] = 0.98(20.8 + 13.1 - 5.6) = 28.3$ tons.

$P_2 = 0.98(20.8 - 13.1 - 5.6) = 21.1$ tons.

$P_3 = 0.98(20.8 - 13.1 - 5.6) = 38.6$ tons.

$P_4 = 0.98(20.8 - 13.1 - 5.6) = 13.1$ tons.

The maximum load on any pile is 38.6 tons, say, 41.8 tons including the pile.

Notes on Designs (a), (b) and (c).—Design (a) comprises vertical piles only, design (b) vertical and inclined piles, and design (c) inclined piles only. In each case the group is symmetrical and is subjected to the same imposed loads. Designs (b) and (c) are special cases of symmetrical groups for which $\Sigma_x = 0$ and therefore $y_9 = 0$; this condition applies only if the inclined piles are in symmetrical pairs, both piles in a pair meeting at the same pile-cap.

The design in (c) requires the smallest pile, but there is little difference between the designs in (b) and (c). Although the maximum load on any pile is least in the design in (a), the bending moment on the pile requires a pile of greater cross-sectional area to provide the necessary resistance to combined thrust and bending. The superiority of design (c) is greater if the horizontal force $H$ is greater, and if $H$ were 20 tons (instead of 10 tons) the maximum loads (excluding the weight of the pile) are 23.4 tons (and a large bending moment of 37.2 ft.-tons) in design (a), 60.6 tons in design (b), and 51.4 tons in design (c). Design (b) is the most suitable when $H$ is small; if $H$ were 1 ton, the maximum loads are 26 tons (and a small bending moment of 1.7 ft.-tons) in design (a), 22 tons in design (b), and 27 tons in design (c). Design (a) is most suitable when there is no horizontal load.
## DECIMAL EQUIVALENTS: TO CONVERT INCHES TO FEET.—TABLE 108.

<table>
<thead>
<tr>
<th>INS.</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
</tr>
</thead>
<tbody>
<tr>
<td>-</td>
<td>-0.333</td>
<td>0.666</td>
<td>1.000</td>
<td>1.333</td>
<td>1.666</td>
<td>2.000</td>
<td>2.500</td>
<td>2.917</td>
<td>3.333</td>
<td>3.666</td>
<td>4.000</td>
<td>4.333</td>
</tr>
<tr>
<td>1/16</td>
<td>0.020</td>
<td>0.040</td>
<td>0.060</td>
<td>0.080</td>
<td>0.100</td>
<td>0.120</td>
<td>0.160</td>
<td>0.190</td>
<td>0.220</td>
<td>0.240</td>
<td>0.300</td>
<td>0.320</td>
</tr>
<tr>
<td>1/32</td>
<td>0.010</td>
<td>0.020</td>
<td>0.030</td>
<td>0.040</td>
<td>0.050</td>
<td>0.060</td>
<td>0.080</td>
<td>0.090</td>
<td>0.110</td>
<td>0.120</td>
<td>0.140</td>
<td>0.150</td>
</tr>
</tbody>
</table>

Note.—For example of the use of this table, see page facing Table 109.
NUMERICAL CONVERSIONS.

Converting Feet and Inches (Table 108).

(a) To express 10 ft. $\frac{5}{8}$ in. as a decimal of a foot:

\begin{align*}
10 \text{ ft.} & = 10 \cdot 00000 \\
5 \frac{1}{8} \text{ in.} & = 0.42969
\end{align*}

\[10.43 \text{ ft.}\]

(b) To convert 0.6732 ft. to inches (nearest $\frac{1}{8}$ in.):

\[0.67187 \text{ ft.} = 8 \frac{5}{8} \text{ in.}\]

Difference = 0.00133 ft. which is slightly more than $\frac{1}{8}$ in.

Therefore 0.6732 ft. = $8 \frac{5}{8}$ in.

Metric Conversion Factors.—1 in. = 2.54 cm. \(\text{1 metre} = 39.3701 \text{ in.}\)

1 pound (av.) = 0.4536 kg. \(\text{1 kg.} = 2.2406 \text{ lb.}\)

1 metric tonne = 0.98421 ton \((= 19.684 \text{ cwt.})\)

To convert the moment of resistance factor \(Q_e\) in inch-pounds to \(Q_M\) in kg.-cm. \((M_r = Q_e bd_1^2)\):

\[Q_M = 0.0703 Q_e\]

\(M_r\) in kilogramme-centimetres = \(Q_M bd_1^2\) (where \(b\) and \(d_1\) are in centimetres).

\(E_0 = 2,000,000 \text{ lb. per sq. in.} = 140.614 \text{ kg. per sq. cm.}\)

\(E_s = 30,000,000 \text{ lb. per sq. in.} = 2,109,210 \text{ kg. per sq. cm.}\)

\(P_b = 18,000 \text{ lb. per sq. in.} = 1266 \text{ kg. per sq. cm.}\)

\(P_s = 1000 \text{ lb. per sq. in.} = 70.31 \text{ kg. per sq. cm.}\)

\(w = 100 \text{ lb. per sq. ft.} = 488.24 \text{ kg. per sq. cm.}\)

\(I = 1000 \text{ in.}^4 = 41,623 \text{ cm.}^4\)

Metric Conversions (Table 109).

(a) To convert 17 ft. 8 $\frac{3}{8}$ in. to metric units:

17 ft. 8 $\frac{3}{8}$ in. = 5.385 metres.

17 ft. 8 $\frac{3}{8}$ in. = 5.393 metres.

(b) To convert 4.067 metres to English units:

4.064 metres = 13 ft. 4 in.

0.003 metre = $\frac{1}{12}$ in. \((\text{At foot of Table 109: } \frac{1}{12} \text{ in.} = 3.18 \text{ mm.})\)

(c) To express $\frac{1}{3}$-in. bars at 8-in. centres in metric units:

8 in. = 0.203 m.

Nearest practical values: 12-mm. bars at 20-cm. centres.

(d) To express 600 lb. per sq. in. in metric units:

\[\frac{600}{42.22} = 42 \text{ kg. per sq. cm. approximately.}\]

(e) To convert 6 lb. per sq. yd. to metric units:

\[6 \frac{6}{2.21} = 2.71 \text{ kg.}\]

1 sq. yd. = $\frac{1}{1.196} = 0.836 \text{ sq. m.}\]

Thus 6 lb. per sq. yd. = $\frac{2.71}{0.836} = 3.24 \text{ kg. per sq. m.}$
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