Design of Prestressed Concrete Structures

Asian Students Edition

Design of Prestressed Concrete Structures

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To engineers who, rather than

blindly following the codes of practice,

seek to apply the laws of nature

Preface

In the United States prestressed-concrete construction has now developed to such a stage that a general understanding of its principles and methods of design is urgently demanded by engineers. This book is intended to fill such a need by presenting a systematic and comprehensive coverage of the field so that the average intelligent engineer can learn to design with ease and confidence. The design of prestressed-concrete structures is not nearly so complicated as most engineers presume. Basic knowledge regarding the behavior of such structures is already available. It is only necessary to express the information in simple forms so that popular understanding may result and wider application may follow.

This book deals essentially with *design*. However, a designer must necessarily be acquainted with the materials and methods for prestressing as well as with economic factors and other special problems. To supply such information, a substantial portion of the book is devoted to these related subjects. Thus engineers interested in the general application of prestressed concrete also may find this book a handy reference. Only the features of design peculiar to *prestressed concrete* are presented, it being assumed that the readers will possess a working knowledge of strength of materials, reinforced concrete, and elementary structural analysis and design. Emphasis is placed on the design of *structures*, although the basic principles are equally applicable to non-structural uses of prestressed concrete. Fundamentals of design are stressed rather than details, which can be readily obtained in so many other publications.

Examples in the book are generally short. Each example is intended to clarify one particular method or formula brought out in the text. It is believed that by singling out a specific point for each example the attention of the reader is focused on it and understanding can be obtained more easily. Having mastered sufficient individual concepts, the reader may then refer to the complete example of design worked out in Chapter 8. These complete designs illustrate the sequence and

procedure followed for conventional structures, while the design of special structures is left to the ingenuity of the engineer who has mastered the design of simpler ones.

Since it is not possible to treat in detail every phase of prestressed-concrete design, references are listed at the end of each chapter to suggest further studies on certain problems. These references are by no means inclusive, nor are they intended to give proper acknowledgment. Whenever an English version of a publication is available, the original is not mentioned. Readers desiring more extensive literature are referred to a *Bibliography on Prestressed Concrete* published by the American Concrete Institute in 1954.

Rapid progress is still being made in prestressed concrete. Although basic principles and fundamentals of design have been stabilized, new materials, techniques, and knowledge are being constantly developed. Every effort has been made to bring the discussion in the book up to date, but it would be appreciated if the readers kindly call the attention of the author to any later developments so that they may be included in future editions.

Throughout the book, elastic and ultimate designs are presented side by side, not only for the case of flexure, but also for investigations of shear, bond, and direct loads. Such a dual approach is deemed necessary from the points of view of economy and of safety. Methods for the design of continuous beams and slabs, including the location and concordancy of cables, are presented in simple terms of moment diagrams and moment distribution. It is hoped that such simple design methods will help to advance the development of prestressed concrete in this country and elsewhere.

T. Y. Lin

Berkeley, California
July, 1955

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Introduction

1-1 Résumé of Development of Prestressed Concrete

The basic principle of prestressing was applied to construction perhaps centuries ago, when ropes or metal bands were wound around wooden staves to form barrels (Fig. 1-1-1). When the bands were

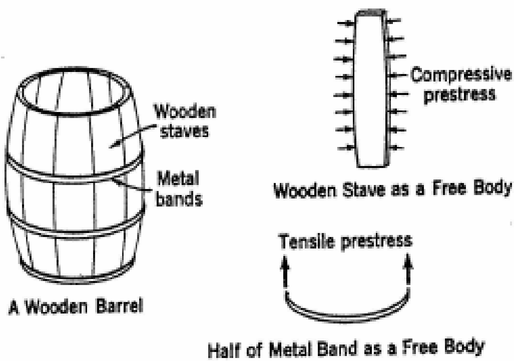

Fig. 1-1-1. Principle of prestressing applied to barrel construction.

tightened, they were under tensile prestress which in turn created compressive prestress between the staves and thus enabled them to resist hoop tension produced by internal liquid pressure. In other words, the bands and the staves were both prestressed before they were subjected to any service loads.

The same principle, however, was not applied to concrete until about 1886, when P. H. Jackson, an engineer of San Francisco, California, obtained patents for tightening steel tie rods in artificial stones and concrete arches to serve as floor slabs. Around 1888, C. E. W. Doehring of Germany independently secured a patent for concrete reinforced with metal that had tensile stress applied to it before the slab was loaded. These applications were based on the conception that concrete, though strong in compression, was quite weak in tension,

and prestressing the steel against the concrete would put the concrete under compressive stress which could be utilized to counterbalance any tensile force due to dead or live loads.

These first patented methods were not successful because the low prestress then produced in the steel was soon lost as a result of the

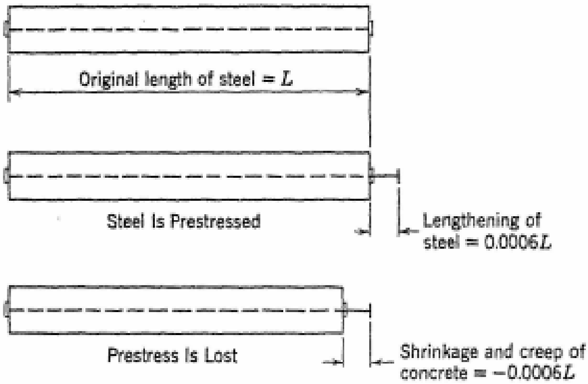

Fig. 1-1-2. Prestressing concrete with ordinary structural steel.

shrinkage and creep of concrete. Consider an ordinary structural steel bar prestressed to a working stress of 18,000 psi (Fig. 1-1-2). If the modulus of elasticity of steel is 30,000,000 psi, the unit lengthening of the bar is given by

$$\begin{aligned}\delta &= \frac{f}{E} \\ &= \frac{18{,}000}{30{,}000{,}000} \\ &= 0.0006\end{aligned}$$

Since eventual shrinkage and creep often induce comparable amounts of shortening in concrete, this initial unit lengthening of steel could be entirely lost in the course of time, not to mention possible creep in steel itself. At best, only a small portion of the prestress could be retained, and the method cannot compete economically with conventional reinforcement of concrete.

In 1908, C. R. Steiner of the United States suggested the possibility of retightening the reinforcing rods after some shrinkage and creep of concrete had taken place, in order to recover some of the losses. In 1925, R. E. Dill of Nebraska tried high-strength steel bars coated to prevent bond with concrete. After the concrete had set, the steel rods

were tensioned and anchored to the concrete by means of nuts. But these methods were not applied to any appreciable extent, chiefly for economic reasons.

Modern development of prestressed concrete is credited to E. Freyssinet of France, who in 1928 started using high-strength steel wires for prestressing. Such wires, with an ultimate strength as high as 250,000

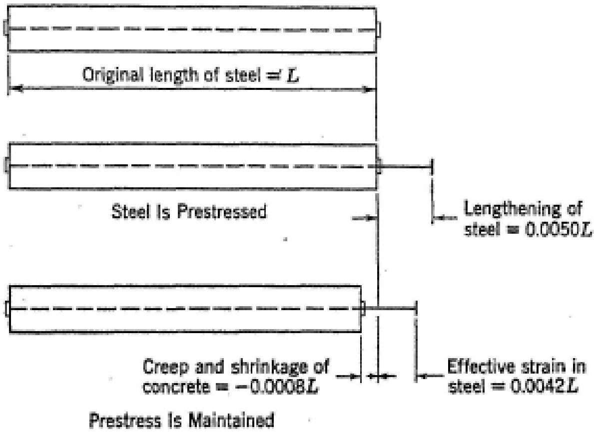

Fig. 1-1-3. Prestressing concrete with high-tensile steel.

psi and a yield point of around 180,000 psi, are prestressed to about 150,000 psi, creating a unit strain of (Fig. 1-1-3)

$$\begin{aligned}\delta &= \frac{f}{E} \\ &= \frac{150{,}000}{30{,}000{,}000} \\ &= 0.0050\end{aligned}$$

Assuming a total loss of 0.0008 due to shrinkage and creep of concrete and other causes, a net strain of $0.0050 - 0.0008 = 0.0042$ would still be left in the wires, which is equivalent to a stress of

$$\begin{aligned}f &= E\delta \\ &= 30{,}000{,}000 \times 0.0042 \\ &= 126{,}000 \text{ psi}\end{aligned}$$

Although Freyssinet also tried the scheme of pre-tensioning where the steel was bonded to the concrete without end anchorage, practical application of this method was first made by E. Hoyer of Germany.

The Hoyer system consists of stretching wires between two buttresses several hundred feet apart, putting shutters between the units, placing the concrete, and cutting the wires after the concrete has hardened. This method enables several units to be cast between two buttresses.

Wide application of prestressed concrete was not possible until reliable and economical methods of tensioning and of end anchorage were devised. In 1939, Freyssinet developed conical wedges for end anchorages and designed double-acting jacks which tensioned the wires

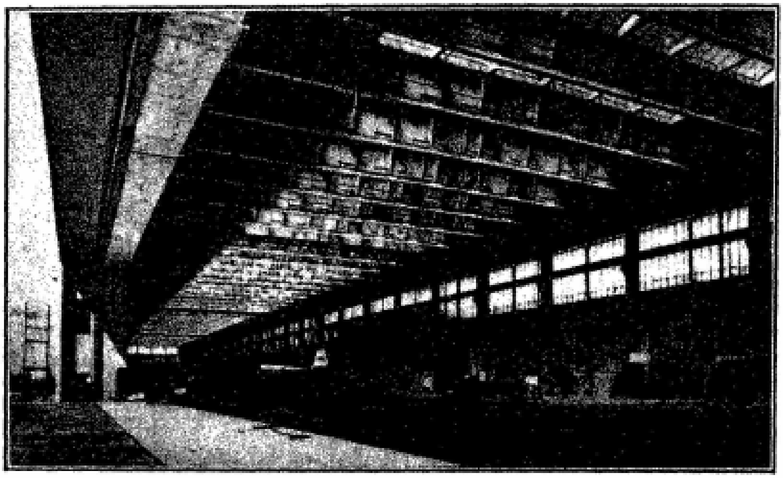

Fig. 1-1-4. 110-ft prestressed-concrete girders carry roofs of London airport.

and then thrust the male cones into the female cones for anchoring them (Fig. 3-4-1, p. 67). In 1940, Professor G. Magnel of Belgium developed the Magnel system, wherein two wires were stretched at a time and anchored with a simple metal wedge at each end (Fig. 3-4-2, p. 68). About that time, prestressed concrete began to acquire importance, though it did not actually come to the fore until about 1945. Perhaps the shortage of steel in Europe during the war had given it some impetus, since much less steel is needed for prestressed than for reinforced concrete. But it must also be realized that time was needed to prove and improve the serviceability, economy, and safety of prestressed concrete as well as to acquaint engineers and builders with a new method of design and construction.

Although France and Belgium led the development of prestressed concrete, England, Germany, Switzerland, and Holland quickly followed their lead[1-1] (Figs. 1-1-4 through 1-1-7). A survey made in

Germany showed that, of about 500 bridges built of concrete during the years 1949–1953, 350 were prestressed and only 150 were reinforced.

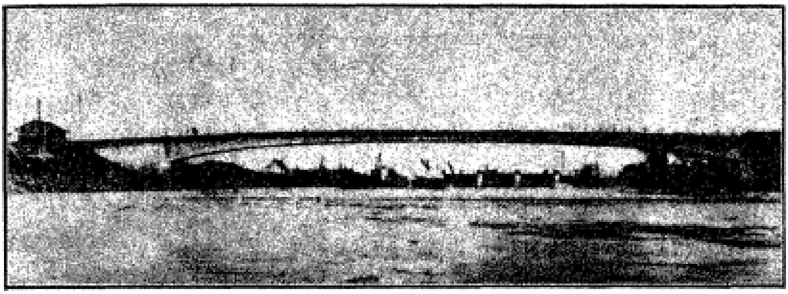

Fig. 1-1-5. Prestressed-concrete bridge of 312-ft span crosses the Neckar Canal near Heilbronn, Germany. Girder depth 5 ft 3 in. at midspan.

Prestressed concrete in the United States followed a different course of development. Instead of linear prestressing, a name given to prestressed concrete beams and slabs, circular prestressing especially as

Fig. 1-1-6. A prestressed-concrete bridge over the Drac, France; 149-ft, 179-ft, and 149-ft spans. (Technical Society for Utilization of Prestressing.)

applied to storage tanks took the lead (Fig. 1-1-8). This was performed almost entirely by the Preload Company, which developed special wire winding machines, and which, from 1935 to 1953, built

no less than 700 prestressed-concrete tanks throughout this country and other parts of the world.

Linear prestressing did not start on any appreciable scale in this country until 1949, when construction of the famed Philadelphia

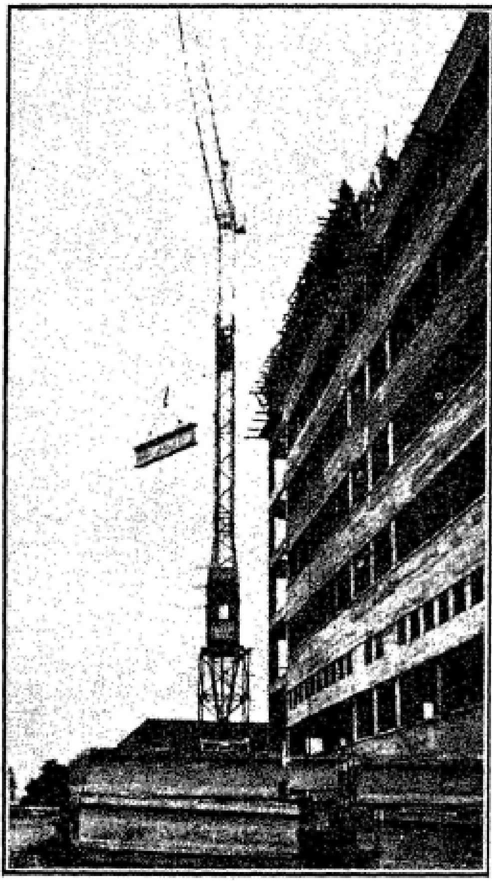

Fig. 1-1-7. Prestressed-block beams provide clear space for 10-story hospital at Renaix, Belgium. (Dr. F. G. Riessauw.)

Walnut Lane Bridge was started (Fig. 1-1-9). The first prestressed-concrete bridge in this country, however, was completed October, 1950, in Madison County, Tennessee, though it was a much smaller structure than the Philadelphia Bridge.

By the middle of 1951, it was estimated that about 175 bridges and

50 buildings incorporating prestressed concrete had been constructed in Europe.[1-2] At that time, no more than 10 such structures could be found in this country. A year later, a survey conducted by the

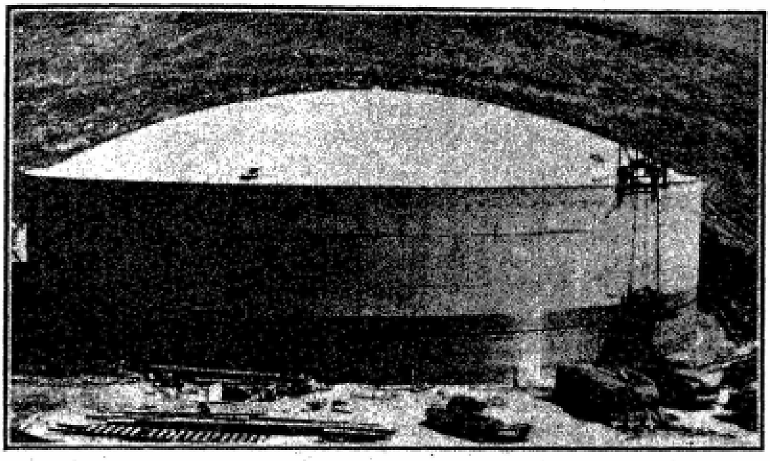

Fig. 1-1-8. Prestressed-concrete tank of 3,000,000-gal capacity, Walnut Creek, California. (East Bay Municipal Utility District, Oakland.)

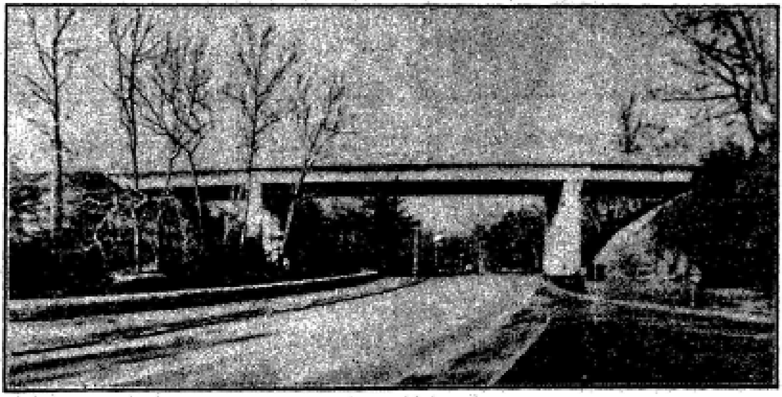

Fig. 1-1-9. Philadelphia Walnut Lane Bridge, 160-ft span, using the Magnel system, the first major prestressed structure in the United States. (The Preload Co.)

Portland Cement Association showed at least 100 such structures either completed or under construction, of which two-thirds were bridges and one-third buildings.[1-3] From then on, it was impossible

to keep track of all prestressed structures in this country, but the number for 1953 has been estimated as 300 to 500. In the state of Pennsylvania alone,[1-4] there were 75 bridges of prestressed concrete in 1953. In 1954, 34 plants were counted in this country turning out prestressed-concrete units on a mass-production basis.[1-5] At the same time, many prestressed-concrete lift slabs were built into all types of buildings (Fig. 1-1-10).

The United States has no monopoly on this type of construction in the New World. A survey of prestressed concrete in Latin American

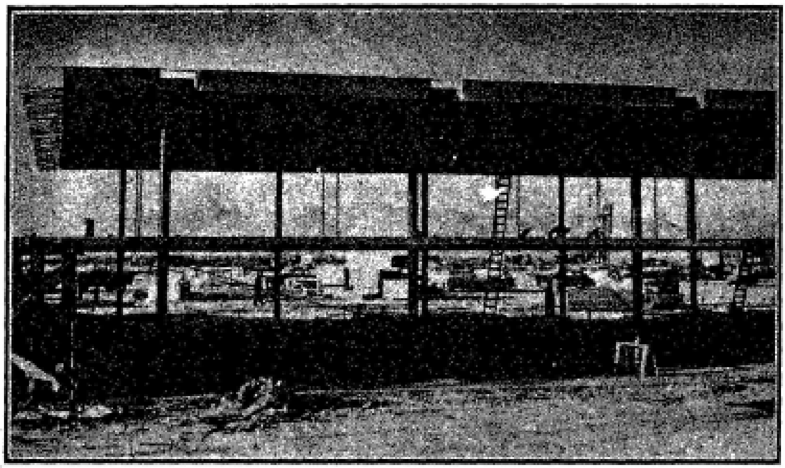

Fig. 1-1-10. Prestressed-concrete lift slabs, Las Vegas, Nevada. (Designed by author. Roof slab 4½ in., floor slab 6 in. thick, both supported by inverted roof beams spanning 52 ft. *Eng. News-Rec.*, April 21, 1955, p. 51.)

countries in 1952 indicated that its use was rapidly growing.[1-6] In Brazil, long-span prestressed-concrete bridges have been built, including continuous ones.[1-7] Argentina took the lead in factory production of prestressed items. Long-span arch bridges embodying prestressed concrete have been constructed in Venezuela.[1-8] Several continuous bridges, up to 300-ft-span length, were completed in Cuba, 1953–1955.

Outside the field of tanks, bridges, and buildings, prestressed concrete has been occasionally applied to dams, by anchoring prestressed steel bars to the foundation, or by jacking the dam against it.[1-9] Piles, posts, and pipes all have been constructed of prestressed concrete. In certain structures, it is possible to prestress the concrete without

using prestressing tendons. For example, the Freyssinet method of arch compensation introduces compensating stresses in the arch rib by a system of hydraulic jacks inserted in the arch. Such stresses are intended to neutralize the effects of shrinkage, rib shortening, and temperature drop in the arch. The Plougastel Bridge near Brest, with 3 spans of 612 ft each, is an example of such application.[1-10]

The basic principle of prestressing is not limited to structures in concrete; it has been applied to steel construction as well. When two plates are joined together by hot-driven rivets or high-tensile bolts, the connectors are highly prestressed in tension and the plates in compression, thus enabling the plates to carry tensile loads between them. The Sciotoville bridge, of 720-ft spans, had its members prestressed in bending during erection in order to neutralize the secondary stresses due to live and dead loads.[1-11] More recently, a continuous truss prestressed with high-tensile wires was built into the airplane hangars at Brussels, Belgium, and two similar ones were tested at the University of Ghent.[1-12]

Whether prestressing is applied to steel or concrete, its ultimate purpose is twofold: first, to induce desirable strains and stresses in the structure; second, to counterbalance undesirable strains and stresses. In prestressed concrete, the steel is pre-elongated so as to avoid excessive lengthening under service load, while the concrete is precompressed so as to prevent cracks under tensile stress. Thus an ideal combination of the two materials is achieved. The basic desirability of prestressed concrete is almost self-evident, but its widespread application will eventually depend on the development of new methods of design and execution which will enhance its economy relative to more conventional types of structures.

1-2 General Principles of Prestressed Concrete

One of the best definitions of prestressed concrete is given by the ACI Committee on Prestressed Concrete:

> Prestressed concrete: Concrete in which there have been introduced internal stresses of such magnitude and distribution that the stresses resulting from given external loadings are counteracted to a desired degree. In reinforced-concrete members the prestress is commonly introduced by tensioning the steel reinforcement.

It might be added that prestressed concrete, in the broader sense of the term, might also include cases where the stresses resulting from internal strains are counteracted to a certain degree, such as in arch compensation. This book, however, will deal essentially with prestressed-concrete structures as defined by the ACI Committee, i.e.,

concrete whose stresses resulting from external loadings are counterbalanced by prestressing reinforcements placed in the structure. This is at present by far the most common form of prestressed concrete.

The basic behavior of this form of prestressed concrete can be explained from three points of view. It is highly important that a designer visualize at least the first two viewpoints in order to proportion and design prestressed concrete structures with intelligence and understanding. These will be explained as follows:

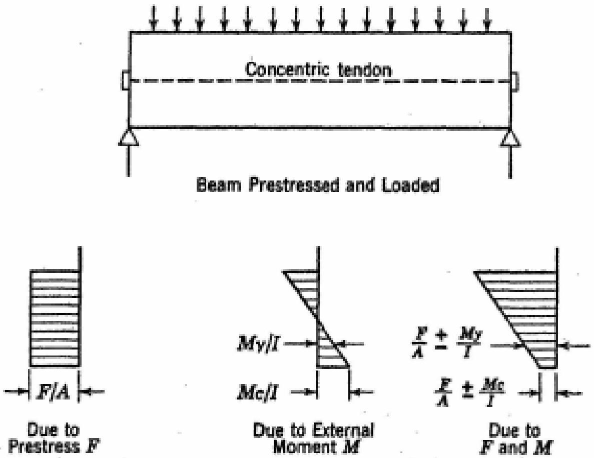

Fig. 1-2-1. Stress distribution across a concentrically prestressed-concrete section.

1. The first point of view is to consider prestressed concrete as essentially a *concrete* structure with the tendons* supplying the prestress to the concrete. This is the most common point of view among engineers. From this standpoint, concrete is visualized as being subject to two systems of forces: prestress and external load, with the tensile stresses due to the external load counteracted by the compressive stresses due to the prestress. Similarly, the cracking of concrete due to load is prevented or delayed by the precompression produced by the tendons. So long as there are no cracks, the stresses, strains, and deflections of the concrete due to the two systems of forces can be considered separately and superimposed if necessary.

In its simplest form, let us consider a simple rectangular beam prestressed by a tendon through its centroidal axis, Fig. 1-2-1, and

* See Appendix A for definition of tendon.

loaded by external loads. Owing to the prestress F, a uniform stress of

$$f = \frac{F}{A} \tag{1-2-1}$$

will be produced across the section with an area A. If M is the external moment at a section due to the load on and the weight of the beam, then the stress at any point across that section due to M is

$$f = \frac{My}{I} \tag{1-2-2}$$

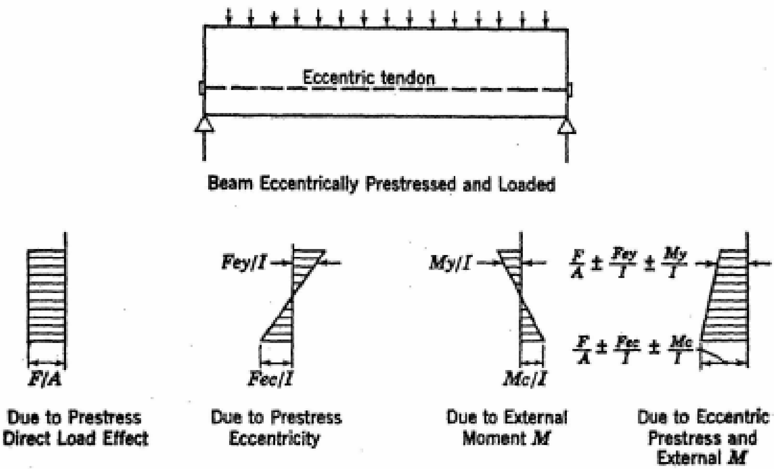

Fig. 1-2-2. Stress distribution across an eccentrically prestressed-concrete section.

where y is the distance from the centroidal axis and I is the moment of inertia of the section. Thus the resulting stress distribution is given by

$$f = \frac{F}{A} \pm \frac{My}{I} \tag{1-2-3}$$

as shown in Fig. 1-2-1.

The solution is slightly more complicated when the tendon is placed eccentrically with respect to the centroid of the concrete section, Fig. 1-2-2. Owing to an eccentric prestress, the concrete is subject to a moment as well as a direct load. The moment is Fe, and the stresses due to this moment are

$$f = \frac{Fey}{I} \tag{1-2-4}$$

Thus, the resulting stress distribution is given by

$$f = \frac{F}{A} \pm \frac{Fey}{I} \pm \frac{My}{I} \tag{1-2-5}$$

as shown in the figure.

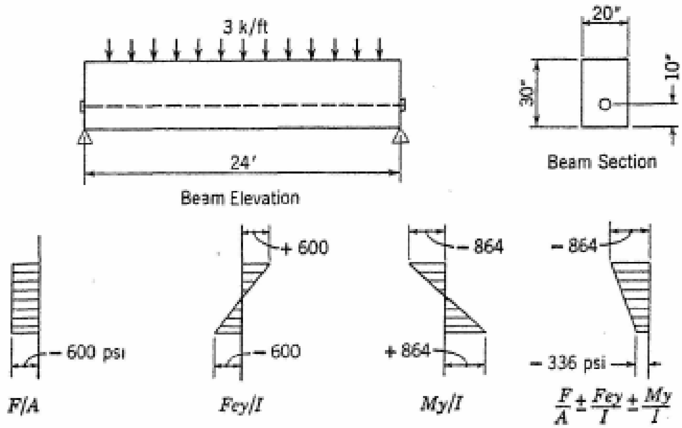

Fig. 1-2-3. Example 1-2-1.

Example 1-2-1

A prestressed-concrete rectangular beam 20 in. by 30 in. has a simple span of 24 ft and is loaded by a uniform load of 3 k/ft including its own weight, Fig. 1-2-3. The prestressing tendon is located at the lower third point and produces an effective prestress of 360 kips. Compute fiber stresses in the concrete at the midspan section.

Solution. Using formula 1-2-5, we have $F = 360$ k, $A = 20 \times 30 = 600$ in.^{2} (neglecting any hole due to the tendon), $e = 5$ in., $I = bd^3/12 = 20 \times 30^3/12 = 45{,}000$ in.^{4}; $y = 15$ in. for extreme fibers.

$$M = 3 \times 24^2/8 = 216 \text{ k ft.}$$

Hence

$$\begin{aligned} f &= \frac{F}{A} \pm \frac{Fey}{I} \pm \frac{My}{I} \\ &= \frac{-360{,}000}{600} \pm \frac{360{,}000 \times 5 \times 15}{45{,}000} \pm \frac{216 \times 12{,}000 \times 15}{45{,}000} \\ &= -600 \pm 600 \pm 864 \\ &= -600 + 600 - 864 = -864 \text{ psi for top fiber} \\ &= -600 - 600 + 864 = -336 \text{ psi for bottom fiber} \end{aligned}$$

The resulting stress distribution is shown in Fig. 1-2-3.

This method of approach becomes a little more complicated when the axis of the member or the tendons are bent or curved. Then it will be more convenient to take the concrete as a free body, cut loose from the tendons. Consider a beam whose tendon is bent at midspan,

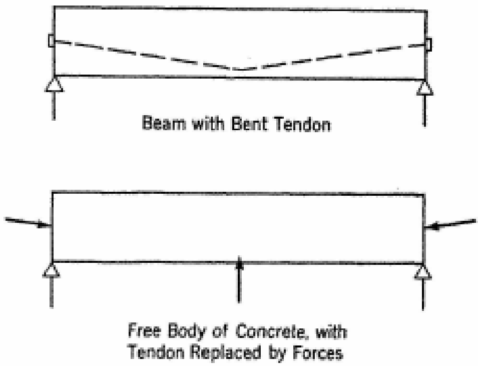

Fig. 1-2-4. Prestressed beam with bent tendon.

Fig. 1-2-4. To simplify the discussion, let us further assume that we have a sudden sharp bend, that there is no frictional loss along the tendon, and that the deviation produced by the bend is small compared to the length of the beam. Now taking the concrete as a free

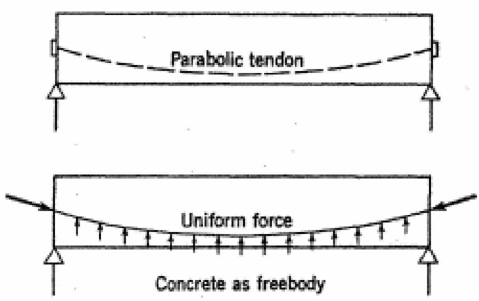

Fig. 1-2-5. Prestressed beam with parabolic tendon.

body, we have a vertical upward force applied at the midspan in addition to the prestress applied at the ends of the beam. Thus, the tendon supplies not only a direct prestress but also an upward force at midspan which can help to balance the external loads, very effectively at times.

Now, this is an important concept. A tendon can be laid out so

as to supply the most desirable system of forces to the concrete. These forces may include: a direct prestress and a moment applied at each end, plus vertical forces acting along the span. It is also evident that, if the tendon has a parabolic shape, a uniform upward force would be supplied instead of a concentrated force, Fig. 1-2-5. The following example illustrates this method.

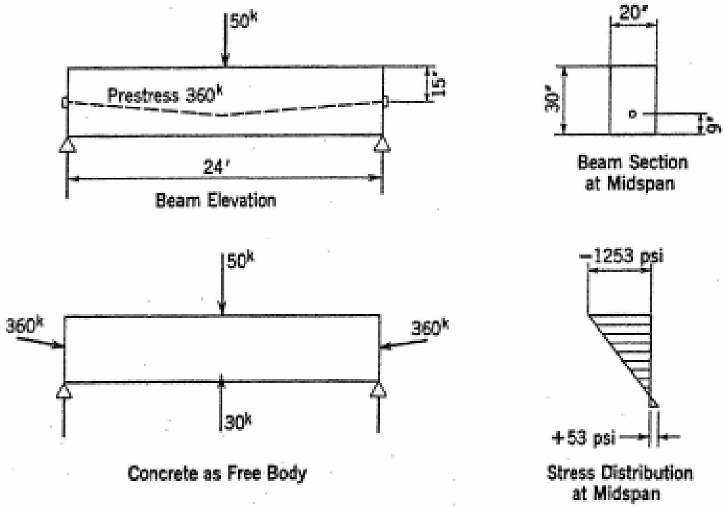

Fig. 1-2-6. Example 1-2-2.

Example 1-2-2

A concrete beam with the same span, cross section, and prestress as in example 1-2-1 is prestressed with a tendon bent as shown, Fig. 1-2-6. It carries a concentrated load of 50,000 lb at the midspan and its own uniform load of 600 plf. Compute the extreme fiber stresses at midspan.

Solution. The tendon can be replaced by an inclined force of 360 kips at each end and an upward force at the center equal to

$$2 \times \frac{6}{12 \times 12} \times 360 = 30 \text{ k}$$

The horizontal component of the 360 kips at the ends can be taken as 360 kips and will produce a uniform stress of 600 psi in the beam. The vertical components at the ends are applied directly over the supports and hence have no effect on the moments in the beam. The upward force of 30 kips will balance part of the load of 50 kips, leaving a net downward force of 20 kips at the center. Thus the moment at midspan due to the concentrated load is

$$M = \frac{PL}{4} = \frac{20{,}000 \times 24}{4} \times 12 = 1{,}440{,}000 \text{ in.-lb}$$

and that due to the weight of the beam is

$$M = \frac{wL^2}{8} = \frac{600 \times 24^2}{8} \times 12 = 518,400$$

$$\text{Total } M = 1,958,400 \text{ in.-lb}$$

for which

$$f = \frac{Mc}{I} = \frac{1,958,400 \times 15}{45,000} = \pm 653 \text{ psi}$$

The resulting stresses are

$$-600 \pm 653 = +53 \text{ psi bottom fiber}$$
$$= -1253 \text{ psi top fiber}$$

2. A second point of view is to consider steel and concrete acting together, with steel taking tension and concrete taking compression so that the two materials form a resisting couple against the external moment, Fig. 1-2-7. This is often an easy concept for engineers

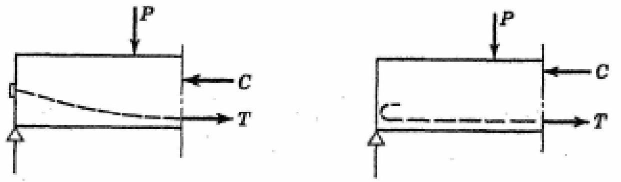

Fig. 1-2-7. Internal resisting moment in prestressed- and reinforced-concrete beams.

familiar with reinforced concrete where the steel supplies a tensile force and the concrete supplies a compressive force, the two forces forming a couple with a lever arm between them. Few engineers realize, however, that similar behavior exists in prestressed concrete.

In prestressed concrete, high-tensile steel is used which will have to be elongated a great deal before its strength is fully utilized. If the high-tensile steel is simply buried in the concrete, as in ordinary concrete reinforcement, the surrounding concrete will have to crack very seriously before the full strength of the steel is developed, Fig. 1-2-8. Hence it is necessary to prestretch the steel with respect to the concrete. By prestretching and anchoring the steel against the concrete, we produce desirable stresses and strains in both materials: compressive stresses and strains in concrete, and tensile stresses and strains in steel. This combined action permits the safe and economical utilization of the two materials which cannot be achieved by simply

burying steel in the concrete as is done for ordinary reinforced concrete. There were isolated instances where medium-strength steel was used as simple reinforcement without prestressing while the steel was specially corrugated for bond, in order to distribute the cracks. This process avoids the expenses for prestretching and anchoring but cannot be applied to high-tensile steel and does not have the desirable effects of precompressing the concrete and of reducing the deflections.

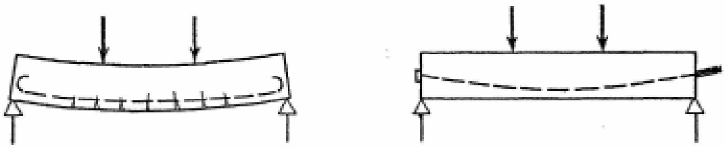

Fig. 1-2-8. Concrete beam using high-tensile steel.

From this point of view, prestressed concrete is no longer a strange type of design. It is rather an extension of the applications of reinforced concrete to include steels of higher strength. From this point of view, prestressed concrete cannot perform miracles beyond the capacity of the strength of its materials. Although much ingenuity can be exercised in the proper and economic design of prestressed-concrete structures, there is absolutely no magic method to avoid the eventual necessity of carrying an external moment by an internal couple. And that internal resisting couple must be supplied by the steel in tension and the concrete in compression, whether it be prestressed or reinforced concrete.

Once the engineer sees this viewpoint, he understands the basic similarity between prestressed and reinforced concrete. Then much of the complexity of prestressing disappears, and the design of prestressed concrete can be intelligently accomplished and not performed by groping in the dark among a lot of complicated and confusing formulas.

The following example illustrates a simple application of the above principle in the analysis of prestressed-concrete beams; more extensive treatment will be presented in Chapter 6.

Example 1-2-3

Solve the problem stated in example 1-2-2 by applying the principle of the internal resisting couple.

Solution. Take one half of the beam as a free body, thus exposing the internal

couple, Fig. 1-2-9. Total external moment at the section is

$$\begin{aligned} M &= \frac{PL}{4} + \frac{wL^2}{8} \\ &= \frac{50 \times 24}{4} + \frac{0.6 \times 24^2}{8} \\ &= 300 + 43.2 \\ &= 343.2 \text{ k ft} \end{aligned}$$

The internal couple is furnished by the forces $C = T = 360$ k, which must act with a lever arm of

$$\frac{343.2}{360} \times 12 = 11.44 \text{ in.}$$

Since T acts at 9 in. from the bottom, C must be acting at 20.44 in. from it. Thus the center of the compressive force C is located.

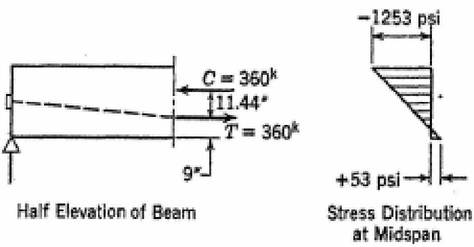

Fig. 1-2-9. Example 1-2-3.

So far we have been dealing only with statics, the validity of which is not subject to any question. Now, if desired, the stress distribution in the concrete can be obtained by the usual elastic theory, since the center of the compressive force is already known. For $C = 360{,}000$ lb acting with an eccentricity of 5.44 in.,

$$\begin{aligned} f &= \frac{P}{A} \pm \frac{Mc}{I} \\ &= \frac{-360{,}000}{600} \pm \frac{360{,}000 \times 5.44 \times 15}{45{,}000} \\ &= -600 \pm 653 \\ &= +53 \text{ for bottom fiber} \\ &= -1253 \text{ for top fiber} \end{aligned}$$

(The same answers as in the previous example.)

3. The third point of view is to conceive of a prestressed-concrete beam as essentially a steel member similar to a suspension bridge where the wires form the main load-carrying body. Imagine the

tendons as the cables of a suspension bridge self-anchored against the concrete and stiffened by the concrete. In other words, compare the beam to a suspension bridge, with the concrete serving as both the stiffening truss and the self-anchor. Such a concept will not be utilized in the design methods presented in this book, because the approach is useful only when the spans become excessive and the tendons curve and bend appreciably. Hence it will not be expounded further.

1-3 Classification and Types

Prestressed-concrete structures can be classified in a number of ways, depending upon their features of design and construction. These will be discussed as follows.

1 · Externally or Internally Prestressed. Although this book is devoted to the design of prestressed-concrete structures internally prestressed, presumably with high-tensile steel, it must be mentioned that it is sometimes possible to prestress a concrete structure by adjusting its external reactions. The method of arch compensation was mentioned previously, where a concrete arch was prestressed by jacking against its abutments.

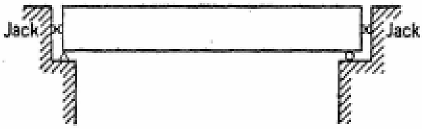

Fig. 1-3-1. Prestressing a simple concrete beam by jacking against abutments.

Theoretically, a simple concrete beam can also be externally prestressed by jacking at the proper places to produce compression in the bottom fibers and tension in the top fibers, Fig. 1-3-1, thus even dispensing with steel reinforcement in the beam. Such an ideal arrangement, however, cannot be easily accomplished in practice, because,

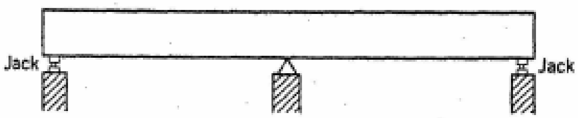

Fig. 1-3-2. Prestressing a continuous beam by jacking its reactions.

even if abutments favorable for such a layout are obtainable, shrinkage and creep in concrete may completely offset the artificial strains unless they can be re-adjusted. Besides, such a site would probably be better suited for an arch bridge.

For a statically indeterminate structure, like a continuous beam, it is possible to adjust the level of the supports, by inserting jacks, for example, so as to produce the most desirable reactions, Fig. 1-3-2. This

is sometimes practical, though it must be kept in mind that shrinkage and creep in concrete will modify the effects of such prestress so that they must be taken into account or else the prestress must be adjusted from time to time.

2 · Linear or Circular Prestressing. Circular prestressing is a term applied to prestressed circular structures, such as round tanks, silos, and pipes, where the prestressing tendons are wound around in circles. This topic is discussed in Chapter 13. As distinguished from circular prestressing, the term linear prestressing is often employed to include all other structures such as beams and slabs. The prestressing tendons in linearly prestressed structures are not necessarily straight; they can be either bent or curved, but they do not go round and round in circles as in circular prestressing.

3 · Pre-Tensioning and Post-Tensioning. The term pre-tensioning is used to describe any method of prestressing in which the tendons are tensioned before the concrete is placed. It is evident that the tendons must be temporarily anchored against some abutments or stressing beds when tensioned and the prestress transferred to the concrete after it has set. This procedure is employed in precasting plants or laboratories where permanent beds are provided for such tensioning; it is also applied in the field where abutments can be economically constructed. In contrast to pre-tensioning, post-tensioning is a method of prestressing in which the tendon is tensioned after the concrete has hardened. Thus the prestressing is almost always performed against the hardened concrete, and the tendons are anchored against it immediately after prestressing. This method can be applied to members either precast or cast in place.

4 · End-Anchored or Non-End-Anchored Tendons. When post-tensioned, the tendons are anchored at their ends by means of mechanical devices to transmit the prestress to the concrete. Such a member is termed end-anchored. Occasionally, though rarely, a post-tensioned member may have its tendons held by grout with no mechanical end anchorage. In pre-tensioning, the tendons generally have their prestress transmitted to the concrete simply by their bond action near the ends. The effectiveness of such stress transmission is limited to wires and strands of small size. More recently, anchorages have been developed for pre-tensioning so as to permit the use of tendons of larger diameter. Different types of end anchorages will be discussed in Chapter 3.

5 · Bonded or Unbonded Tendons. Bonded tendons denote those bonded throughout their length to the surrounding concrete. Non-end-anchored tendons are necessarily bonded ones; end-anchored ten-

dons may be either bonded or unbonded to the concrete. In general, the bonding of post-tensioned tendons is accomplished by subsequent grouting; if unbonded, protection of the tendons from corrosion must be provided by galvanizing, greasing, or some other means. Sometimes, bonded tendons may be purposely unbonded along certain portions of their length.

6 · Precast, Cast-in-Place, Composite Construction. Precasting involves the placing of concrete away from its final position, the members being cast either in a permanent plant or somewhere near the site of the structure, and eventually erected at the final location. Precasting permits better control in mass production and is often economical. Cast-in-place concrete requires more form and falsework per unit of product but saves the cost of transportation and erection, and it is a necessity for large and heavy members. In between these two methods of construction, there are tilt-up wall panels and lift slabs which are constructed at places near or within the structure and then erected to their final position; no transportation is involved for these. Oftentimes, it is economical to precast part of a member, erect it, and then cast the remaining portion in place. This procedure is called composite construction. The precast elements in a structure of composite construction can be more easily joined together than those in a totally precast structure. By composite construction, it is possible to save much of the form and falsework required for total cast-in-place construction. However, the suitability of each type must be studied with respect to the particular conditions of a given structure.

7 · Partial or Full Prestressing. A further distinction between the types of prestressing is sometimes made depending upon the degree of prestressing to which a concrete member is subject. When a member is designed so that under the working load there are no tensile stresses in it, then the concrete is said to be fully prestressed. If some tensile stresses will be produced in the member under working load, then it is termed partially prestressed. For partial prestressing, additional mild-steel bars are frequently provided to reinforce the portion under tension. In practice, it is often difficult to classify a structure as being partially or fully prestressed since much will depend upon the magnitude of the working load used in design. For example, highway bridges in this country are always designed for full prestressing, though actually they are subject to tensile stresses during the passage of heavy vehicles. On the other hand, roof beams designed for partial prestressing may never be subject to tensile stresses since the assumed live loads may never act on them.

1-4 Stages of Loading

One of the considerations peculiar to prestressed concrete is the plurality of stages of loading to which a member or structure is often subjected. Some of these stages of loading occur also in non-prestressed structures, but others exist only because of prestressing. For a cast-in-place structure, prestressed concrete has to be designed for at least two stages: the initial stage during prestressing, and the final stage under external loadings. For precast members, a third stage, that of handling and transportation, has to be investigated. During each of these three stages, there are again different periods when the member or structure may be under different loading conditions. These will be analyzed below:

1 · Initial Stage. The member or structure is under prestress but not subjected to any superimposed external loads. This can be further subdivided into the following periods, some of which may not be important and hence may be neglected in certain designs:

A · *Before Prestressing.* Before the concrete is prestressed, it is quite weak in carrying load; hence the yielding of its supports must be prevented. Provision must be made for the shrinkage of concrete if it might occur. This is often significant because any shrinkage cracks will destroy the capacity of the concrete to carry tensile stresses.

B · *During Prestressing.* This is a critical test for the strength of the tendons. Oftentimes, the maximum stress to which the tendons will be subject throughout their life occurs at that period. It occasionally happens that an individual wire may be broken during prestressing, owing to defects in its manufacture. But this is seldom significant, since there are often many wires in a member. If a bar is broken in a member with only a few bars, it should be properly replaced. For concrete, the prestressing operations impose a severe test on the bearing strength at the anchorages. Since the concrete is not aged at this period while the prestress is at its maximum, crushing of the concrete at the anchorages is possible if its quality is inferior. Again, unsymmetrical and concentrated prestress from the tendons may produce overstresses in the concrete. Hence the order of prestressing the various tendons must often be studied beforehand.

C · *At Transfer of Prestress.* For pre-tensioned members, the transfer of prestress is accomplished in one operation and within a short period. For post-tensioned members, the transfer is often gradual, the prestress in the tendons being transferred to the concrete one by one. In both cases, there is no external load on the member except its own

weight. Thus the initial prestress, with little loss as yet taking place, imposes a serious condition on the concrete and often controls the design of the member. For economic reasons the design of a prestressed member often takes into account the weight of the member itself in holding down the cambering effect of prestressing. This is done on the assumption of a given condition of support for the member. If that condition is not realized in practice, failure of the member might result. For example, the weight of a simply supported prestressed girder is expected to exert a maximum positive moment at midspan which counteracts the negative moment due to prestressing. If the girder is cast and prestressed on soft ground without suitable pedestals at the ends, the expected positive moment may be absent and the prestressing may produce excessive tensile stresses on top fibers of the girder, resulting in its failure.

D · *Decentering and Retensioning.* If a member is cast and prestressed in place, it generally becomes self-supporting during or after prestressing. Thus the falsework can be removed after prestressing, and no new condition of loading is imposed upon the structure. Some concrete structures are retensioned, i.e., prestressed in two or more stages. Then the stresses at various stages of tensioning must be studied.

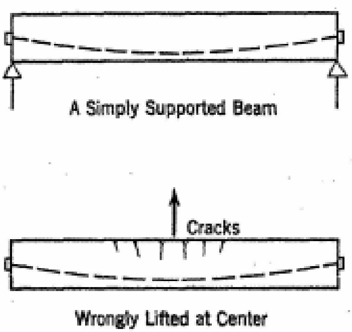

Fig. 1-4-1. Failure of beam due to careless handling.

2 · Intermediate Stage. This is the stage during transportation and erection. It occurs only for precast members when they are transported to the site and erected in position. It is highly important to ensure that the members are properly supported and handled at all times. For example, a simple beam designed to be supported at the ends will easily break if lifted at midspan, Fig. 1-4-1. Figure 1-4-2 shows a correct way to lift a prestressed simple beam.

Not only during the erection of the member itself, but also when adding the superimposed dead loads, such as roofing or flooring, attention must be paid to the conditions of support and loading. This is especially true for a cantilever layout, when partial loading may result in more serious bending than a full loading, Fig. 1-4-3.

3 · Final Stage. This is the stage when the actual working loads come on the structure. As for other types of construction, the designer must consider various combinations of live loads on different portions

of the structure with lateral loads such as wind and earthquake forces, and with strain loads such as those produced by settlement of supports and temperature effects. For prestressed-concrete structures, espe-

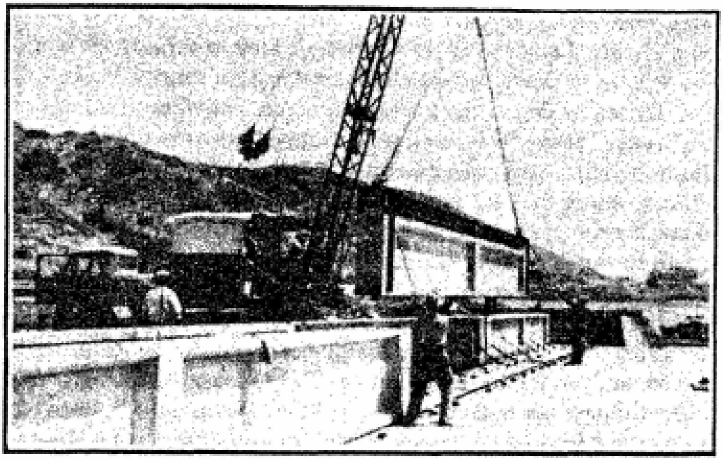

Fig. 1-4-2. Lifting a 40-ft pretensioned beam from casting bed. (Southwest Structural Concrete Corp., San Diego, California.)

cially those of unconventional types, it is often necessary to investigate their cracking and ultimate loads, in addition to the working load. These will be discussed as follows:

A · *Working Load.* To design for the working load is a check on excessive stresses and strains. It is not necessarily a guarantee of suffi-

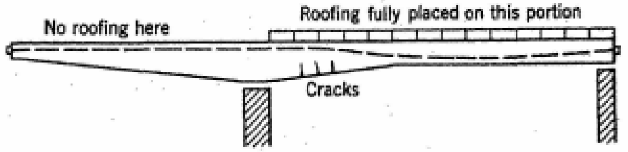

Fig. 1-4-3. Cracking of beam due to wrong sequence in adding superimposed load.

cient strength to carry overloads. However, an engineer familiar with the strength of prestressed-concrete structures may often design conventional types and proportions solely on the basic of working-load computations.

B · *Cracking Load.* Cracking in a prestressed-concrete member signifies a sudden change in the bond and shearing stresses. It is sometimes a measure of the fatigue strength. For certain structures, such

as tanks and pipes, the commencement of cracks presents a critical situation. For structures subject to corrosive influences, for unbonded tendons where cracks are more objectionable, or for structures where cracking may result in excessive deflections, an investigation of the cracking load seems important.

C · *Ultimate Load.* Structures designed on the basis of working stresses may not always possess a sufficient margin for overloads. This is true, for example, of prestressed-concrete members under direct tensile loads. Since it is desirable that a structure does possess a certain minimum overload capacity, it is often necessary to determine its ultimate strength. In general, the ultimate strength of a structure is defined by the maximum load it can carry before collapsing. However, before this load is reached, permanent yielding of some parts of the structure may already have developed. Although any strength beyond the point of permanent yielding may serve as additional guarantee against total collapse, some engineers consider such strength as not usable and prefer to design on the basis of usable strength rather than the ultimate strength. However, ultimate strength is more easily computed and is more commonly accepted as a criterion for design.

In addition to the above normal loading conditions, some structures may be subject to repeated loads of appreciable magnitude which might result in fatigue failures. Some structures may be under heavy loads of long duration, resulting in excessive deformations due to creep, while others may be under such light external loads that the camber produced by prestressing may become too pronounced as time goes on. Still others may be subject to undesirable vibrations under dynamic loads. These are special conditions which the engineer must consider for his individual case.

The above discussion outlines the relatively new and complex problems encountered in the design of prestressed-concrete as compared with reinforced-concrete structures. It is unfortunate that the design of prestressed concrete is more complicated, but the difficulty is by no means excessive. The new problems must be understood and solved. Ignorance of the situation might result in tragic failures such as are experienced by careless practitioners in almost any new field of endeavor.

With some experience in design, many of the loading stages mentioned above are automatically eliminated from consideration by inspection. Calculations will actually have to be made for only one or two controlling conditions. Besides, as will be shown in later chapters, calculations can be greatly simplified if the correct methods of approach and analysis are chosen. It is the observation of the

author that an engineer who belittles the complications of prestressed-concrete design will encounter problems beyond his expectations, while the majority of engineers will find it not as difficult as they may imagine.

1-5 Prestressed vs. Reinforced Concrete

As it is assumed that readers are already acquainted with reinforced concrete, it will be interesting to compare prestressed concrete with it. The most outstanding difference between the two is the employment of materials of higher strength for prestressed concrete. In order to utilize the full strength of the high-tensile steel, it is necessary to resort to prestressing to prestretch it. Prestressing the steel and anchoring it against the concrete produces desirable strains and stresses which serve to reduce or eliminate cracks in concrete. Thus the entire section of the concrete becomes effective in prestressed concrete, whereas only the portion of section above the neutral axis is supposed to act in the case of reinforced concrete.

The use of curved tendons will help to carry some of the shear in a member. In addition, precompression in the concrete tends to reduce the diagonal tension. Thus it is possible to use a smaller section in prestressed concrete to carry the same amount of external shear in a beam.

High-strength concrete, which cannot be economically utilized in reinforced-concrete construction, is found to be desirable and even necessary with prestressed concrete. In reinforced concrete, using concrete of high strength will result in a smaller section calling for more reinforcement and will end with a more costly design. In prestressed concrete, high-strength concrete is required to match with high-strength steel in order to yield economical proportions. Stronger concrete is also necessary to resist high stresses at the anchorages and to give strength to the thinner sections so frequently employed for prestressed concrete.

Each material or method of construction has its own field of application. When welding was first developed in the 1930's, some engineers were overenthusiastic and believed that it would replace riveting altogether, which it has not done even yet. Prestressed concrete is likely to have a similar course of development. Not for a long time will it be used in as great quantity as reinforced concrete. But a new type of construction, basically sound in its strength and economy, is likely to have a rapid rate of growth and to be adaptable to new and unprecedented situations and requirements.

The advantages and disadvantages of prestressed concrete as com-

pared with reinforced concrete will now be discussed in respect to their serviceability, safety, and economy.

1 · Serviceability. Prestressed-concrete design is more suitable for structures of long spans and those carrying heavy loads, principally because of the higher strengths of materials employed. Prestressed structures are more slender and hence more adaptable to artistic treatment. They yield more clearance where it is needed. They do not crack under working loads, and whatever cracks may be developed under overloads will be closed up as soon as the load is removed, unless the load is excessive. Under dead load, the deflection is reduced, owing to the cambering effect of prestress. This becomes an important consideration for structures such as long cantilevers. Under live load, the deflection is also smaller because of the effectiveness of the entire uncracked concrete section, which has a moment of inertia 2 to 3 times that of the cracked section. Prestressed elements are more adaptable to precasting because of the lighter weight.

So far as serviceability is concerned, the only shortcoming of prestressed concrete is its lack of weight. Although seldom encountered in practice, there are situations where weight and mass are desired instead of strength. For these, plain or reinforced concrete could serve just as well and at lower cost.

2 · Safety. It is difficult to say that one type of structure is safer than another. The safety of a structure depends more upon its design and construction than upon its type. However, certain inherent safety features in prestressed concrete may be mentioned. There is partial testing of both the steel and the concrete during prestressing operations. For many structures, during prestressing, both the steel and the concrete are subjected to the highest stresses that will exist in them during their life of service. Hence, if the materials can stand prestressing, they are likely to possess sufficient strength for the service loads.

When properly designed by the present conventional methods, prestressed-concrete structures have overload capacities similar to and perhaps slightly higher than those of reinforced concrete. For the usual designs, they deflect appreciably before ultimate failure, thus giving ample warning before impending collapse. The ability to resist shock and impact loads and repeated working loads has been shown to be as good in prestressed as in reinforced concrete. The resistance to corrosion is better than that of reinforced concrete for the same amount of cover, owing to the non-existence of cracks. If cracks should occur, corrosion can be more serious in prestressed concrete. Regarding fire resistance, high-tensile steel is more sensitive to high temperatures, but, for the same amount of minimum cover, prestressed tendons can have

a greater average cover because of the spread and curvature of the individual tendons. These problems are discussed in Chapter 16.

Prestressed-concrete members do require more care in design, construction, and erection than those of ordinary concrete, because of the higher strength, smaller section, and sometimes delicate design features involved. Although prestressed-concrete construction has been practiced only since the late 1940's, it is possible to conclude from experience that the life of such structures can be as long as if not longer than that of reinforced concrete.

3 · Economics. From an economic point of view, it is at once evident that smaller quantities of materials, both steel and concrete, are required to carry the same loads, since the materials are of higher strength. There is also a definite saving in stirrups, since shear in prestressed concrete is reduced by the inclination of the tendons, and the diagonal tension is further minimized by the presence of prestress. The reduced weight of the member will help in economizing the sections; the smaller dead load and depth of members will result in saving materials from other portions of the structure. In precast members, a reduction of weight saves handling and transportation costs.

In spite of the above economies possible with prestressed concrete, its use cannot be advocated for all conditions. First of all, the stronger materials will have a higher unit cost. More auxiliary materials are required for prestressing, such as end anchorages, conduits, and grouts. More formwork is also needed, since non-rectangular shapes are often necessary for prestressed concrete. More labor is required to place 1 lb of steel in prestressed concrete, especially when the amount of work involved is small. More attention to design is involved, and more supervision is necessary; the amount of additional work will depend upon the experience of the engineer and the construction crew, but it will not be serious if the same typical design is repeated many times.

From the above discussion, it can be concluded that prestressed-concrete design is more likely to be economical when the same unit is repeated many times or when heavy loads and long spans are encountered. It should also find suitable application when combined with precasting or semi-precasting such as composite or lift-slab construction. Each structure must be considered individually. The availability of good designers, of experienced crews, of pre-tensioning factories, and of competitive bidding often helps to tip the balance in favor of prestressed concrete.

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Chapter

2

Materials

2-1 Concrete, Strength Requirements

Stronger concrete is required for prestressed than for reinforced work. Present practice in this country calls for 28-day cylinder strength of 4000 to 5000 psi for prestressed concrete, while the corresponding value for reinforced concrete is around 2500 psi. The usual cube strength specified for prestressed concrete in Europe is about 450 kg/cm^2, based on 10-, 15-, or 20-cm cubes at 28 days. If cube strength is taken as 1.25 times the cylinder strength, this would correspond to

$$450 \times 14.2/1.25 = 5100 \text{ psi cylinder strength}$$

Although the above are the usual values, strengths differing from these are also occasionally specified.

Higher strength is necessary in prestressed concrete for several reasons. First, in order to minimize their cost, commercial anchorages for prestressing steel are always designed on the basis of high-strength concrete. Hence weaker concrete either will require special anchorages or may fail under the application of prestress. Such failures may take place in bearing or in bond between steel and concrete, or in tension near the anchorages. Next, concrete of high compressive strength offers high resistance in tension and shear, as well as in bond and bearing, and is desirable for prestressed-concrete structures whose different portions are under higher stresses than ordinary reinforced concrete. Another factor is that high-strength concrete is less liable to the shrinkage cracks which sometimes occur in low-strength concrete before the application of prestress. It also has a higher modulus of elasticity and smaller creep strain, resulting in smaller loss of prestress in the steel.

Experience has shown that 4000- to 5000-psi strength will generally work out to be the most economical mix for prestressed concrete. Although the strength of concrete to be specified for each job must be

considered individually, there are some evident reasons why the economical mix usually falls within a certain range. Concrete strength of 4000 to 5000 psi can be obtained without excessive labor or cement. The cost of 5000-psi concrete averages about 15% higher than that of 2500-psi concrete, while it has 100% higher strength, which can be well utilized and is often seriously needed in prestressed structures. To obtain strength much greater than 5000 psi, on the other hand, not only will cost more but also will call for careful design and control of the mixing, curing, and placing of concrete which cannot be easily achieved in the field.

To attain a strength of 5000 psi, it is necessary to use a water-cement ratio of not much more than 0.45 by weight. In order to facilitate placing, a slump of 2 to 4 in. would be needed, unless more than ordinary vibration is to be applied. To obtain 3-in. slump with water-cement ratio of 0.45 would require about 8 bags of cement per cu yd of concrete. If careful vibration is possible, concrete with ½-in. or zero slump can be employed, and 7 bags of cement per cu yd may be quite sufficient. Since excessive cement tends to increase shrinkage, a lower cement factor is desirable. To this end, good vibration is advised whenever possible, and proper admixtures to increase the workability can sometimes be advantageously employed.

Not only should high-strength concrete be specified for prestressed work, but, when called for, such strength should be more closely attained in the field than for reinforced concrete. It will be shown later (in Chapter 14) that the factor of safety against ultimate compressive failure in concrete is around 2.5, which is certainly a sufficient factor but is not as excessive as in reinforced concrete, where the factor is more nearly 3.5. For this, as well as for other reasons mentioned previously, care should be exercised to produce as good concrete as called for in the design. However, it is also evident that, with a factor of safety of 2.5, it would be no cause for alarm if the concrete in the structure should possess a strength 10 or 20% below the required value. In fact, many engineers believe that, if the concrete is not crushed under the application of prestress, it should be able to stand any subsequent loadings, since the strength of concrete usually increases with age and since excessive overloads are very rare except for some structures.

The above discussion is not intended to encourage careless concreting in the field. Indeed, more parts of a prestressed-concrete member are subjected to high stresses than of a reinforced one. Consider a simple prestressed beam, for example. While the top fibers are highly compressed under heavy external loads, the bottom fibers are

under high compression at the transfer of prestress. While the midspan sections resist the heaviest bending moments, the end sections carry and distribute the prestressing force. Hence, in a prestressed member, it is often more important to secure uniformity of strength, whereas in reinforced concrete the critical sections are relatively limited. It would be foolish to tear down a structure just because its concrete did not test up to the specified strength, but the engineer should use reasonable precautions to obtain good and strong concrete.

It is general practice to specify a lower strength of concrete at transfer than its 28-day strength. This is desirable in order to permit early transfer of prestress to the concrete. At transfer, the concrete is not subject to external overloads, strength is necessary only to guard against anchorage failure and excessive creep, hence a smaller factor of safety is considered sufficient. For example, in pre-tensioning work, a strength of 4000 psi at transfer is often sufficient for a specified 28-day strength of 5000 psi.

Direct tensile strength in concrete is a highly variable item, generally ranging from $0.06f_c'$ to $0.10f_c'$, and may be zero if cracks have developed as the result of shrinkage or other reasons. Modulus of rupture in concrete is known to be higher than its direct tensile strength, varying from about $0.15f_c'$ for 3000-psi concrete to $0.10f_c'$ for 6000-psi concrete. Direct shearing strength, not often used in design, ranges from $0.50f_c'$ to $0.70f_c'$. Beam shear produces the principal tensile stress, whose limiting value is commonly gaged on the basis of direct tensile strength in concrete.

2-2 Concrete, Strain Characteristics

In prestressed concrete, it is important to know the strains produced as well as the stresses. This is necessary to estimate the loss of prestress in steel and to provide for other effects of concrete shortening. For the purpose of discussion, such strains can be classified into four types: elastic strains, lateral strains, creep strains, and shrinkage strains.

1 · Elastic Strains. The term elastic strains is perhaps a little ambiguous, since the stress-strain curve for concrete is seldom a straight line even at normal levels of stress, Fig. 2-2-1. Neither are the strains entirely recoverable. But, eliminating the creep strains from consideration, the lower portion of the instantaneous stress-strain curve, being relatively straight, may be conveniently called elastic. It is then possible to obtain values for the modulus of elasticity of concrete. The modulus varies with several factors,[2-1, 2-2] notably the strength of concrete, the age of concrete, the properties of aggregates and cement,

and the definition of modulus of elasticity itself, whether tangent, initial, or secant modulus. Furthermore, the modulus may vary with the speed of load application and with the type of specimen, whether a cylinder or a beam. Hence it is almost impossible to predict with any accuracy the value of the modulus for a given concrete.

As an average value, for concrete at 28 days old, and for compressive stress up to about $0.40f_c'$, the secant modulus has been approximated by the following empirical formulas:

A. The ACI Code for Reinforced Concrete specifies the following empirical formula:

$$E_c = 1000f_c' \tag{2-2-1}$$

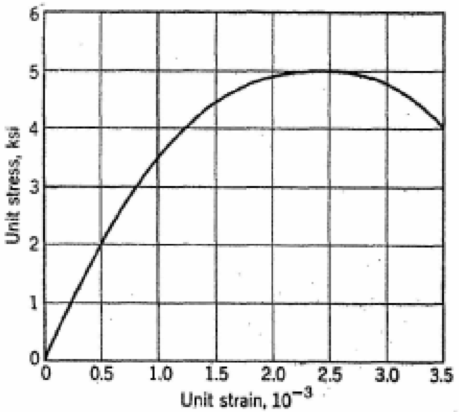

Fig. 2-2-1. Typical stress-strain curve for 5000-psi concrete.

which is a simple approximation but apparently close enough only for f_c' around 3000 psi, the usual strength for reinforced concrete. For concrete of higher strength, such as employed for prestressed construction, this formula seems to yield values of E_c somewhat too high.

B. Empirical formula proposed by Jensen:

$$E_c = \frac{6 \times 10^6}{1 + (2000/f_c')} \tag{2-2-2}$$

which gives more correct values for f_c' around 5000 psi.

C. Empirical formula proposed by Hognestad:

$$E_c = 1{,}800{,}000 + 460f_c' \tag{2-2-3}$$

which gives results similar to the last one.

D. Instead of a formula, German specifications for prestressed concrete give the following set of values:

Cube Strength kg/cm^2	Corresponding Cylinder Strength, psi	Modulus of Elasticity, psi
300	3400	4,800,000
450	5100	5,700,000
600	6800	6,200,000

Plotting the above four proposals in Fig. 2-2-2, we can see that those of Jensen and Hognestad come quite close together but the ACI and the German values are relatively high. It is believed that the German values were intended to represent modulus used for computing in-

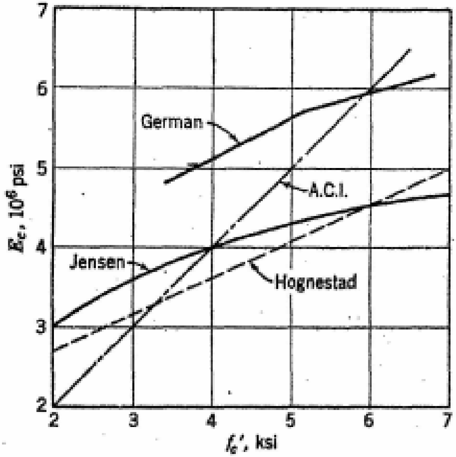

Fig. 2-2-2. Empirical formulas for E_c.

stantaneous beam deflections while the others were based on measured strains from cylinder specimens. Authorities differ on the relation between the two kinds of moduli. Some tests indicate the agreement of these two values; others tend to show that the modulus for beams is higher than that for cylinders. Not too much work has been done for the modulus of elasticity of concrete in tension, but it is generally assumed that, before cracking, the average modulus over a length of several inches is the same as in compression, although the local modulus in tension is known to vary greatly.

2 · Lateral Strains. Lateral strains are computed by Poisson's ratio.[2-2] Owing to Poisson's ratio effect, the loss of prestress is slightly decreased in biaxial prestressing. Poisson's ratio varies from 0.15 to 0.22 for concrete, averaging about 0.17.

3 · Creep Strains. Creep of concrete is defined as its time-dependent deformation resulting from the presence of stress. A great deal of work has been done in this country on the creep or plastic flow of concrete.[2-3, 2-4] However, most of it was for concrete under lower stress, such as exists in reinforced concrete. For concrete under higher stress, data are lacking. But the mechanics of creep is fairly well understood, and it is agreed that many factors affect its value. The important ones are the magnitude of stress, the duration of stress, the age of concrete at application of stress, the quantity of mixing water, the strength of concrete, and the properties of aggregates and cement.

In Europe, the term coefficient of creep C_c is employed to indicate the ratio of the eventual strain δ_t after a lengthy period of constant stress to the instantaneous strain δ_i immediately obtained upon the application of stress,[2-6] thus,

$$C_c = \frac{\delta_t}{\delta_i}$$

This coefficient varies widely as reported from different tests, essentially because of the difficulty of separating shrinkage from creep. For purposes of design, it is considered safe to take C_c as around 3.0. For post-tensioned members, where the prestress is applied late, the coefficient could be a little less; for pre-tensioned members, where the prestress is applied at an early age, the coefficient could be a little more.

For a creep coefficient of 3.0, the amount of creep strain is 2.0 times the instantaneous elastic strain, Fig. 2-2-3. Of this 2.0, it can be roughly estimated that about ¼ takes place within the first 2 weeks after application of prestress, another ¼ within 2 to 3 months, another ¼ within a year or two, and the last ¼ in the course of many years. German specifications state that creep ceases at the end of 4 years.

There is good reason to believe that, for smaller members, creep as well as shrinkage takes place faster than for larger members. Upon the removal of stress, part of the creep can be recovered in the course of time. Again, owing to the difficulty of separating shrinkage from creep, the amount and speed of such recovery have not been accurately measured.

4 · Shrinkage Strains. As distinguished from creep, shrinkage in concrete is its contraction due to drying and chemical changes, dependent on time and on moisture conditions but not on stresses. At least a portion of the shrinkage resulting from drying of the concrete is recoverable upon the restoration of the lost water. The amount of shrinkage strain also varies with many factors,[2-5] and it may range

from 0.0000 to 0.0010. At one extreme, if the concrete is stored under water or very wet conditions, the shrinkage may be zero. There may even be expansion for some types of aggregates and cements. At the other extreme, for a combination of certain cements and aggregates, and with the concrete stored under very dry conditions, as much as 0.0010 can be expected.

Other things being equal, shrinkage of concrete is almost directly proportional to the amount of water employed in the mix. Hence if minimum shrinkage is desired, the water-cement ratio and the proportion of cement paste should be kept to a minimum. Thus aggregates of larger size well graded for minimum void will need a smaller amount of cement paste and shrinkage will be smaller.

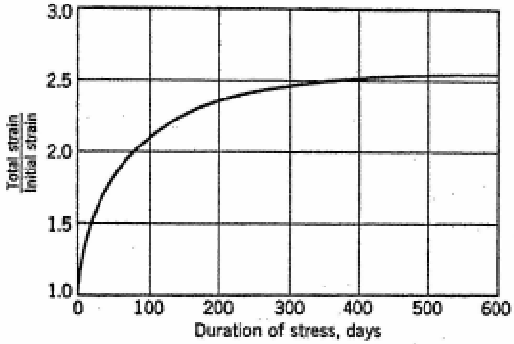

Fig. 2-2-3. Creep in concrete. Typical variation of strain with time.

The quality of the aggregates is also an important consideration. Harder and denser aggregates of low absorption and high modulus of elasticity will exhibit smaller shrinkage. Concrete containing hard limestone is believed to have smaller shrinkage than that containing granite, basalt, and sandstone of equal grade, approximately in that order. The chemical composition of cement also affects the amount of shrinkage. For example, shrinkage is relatively small for cements high in tricalcium silicate and low in the alkalies expressed as the oxides of sodium and potassium.[2-7]

The amount of shrinkage varies widely, depending upon the individual conditions. For the purpose of design, an average value of shrinkage strain would be about 0.0002 to 0.0004 for the usual concrete mixtures employed in prestressed construction. The rate of shrinkage depends chiefly upon the weather conditions. Actual structures exposed to weather show measurable seasonal changes in the shrinkage

of concrete, swelling during rainy seasons and shrinking during dry ones. If the concrete is left dry, there is reason to believe that most of the shrinkage would take place during the first 2 or 3 months. If it is always wet, there may be no shrinkage at all.

2-3 Concrete, Special Manufacturing Techniques

Most of the techniques for manufacturing good concrete, whether for plain or reinforced work, can be applied to prestressed concrete. However, they must be investigated for a few factors peculiar to prestressed concrete. First, they must not decrease the high strength required; next, they must not appreciably increase the shrinkage and creep; they must not produce adverse effects, such as inducing corrosion in the high-tensile wires.

Compacting the concrete by vibration or some other means is usually desirable and necessary. In order to produce high-strength concrete without using an excessive amount of mortar, a low water-cement ratio and a low-slump concrete must be chosen. Such concrete cannot be well placed without compaction. There are only a few isolated applications in which concrete of high slump is employed and compaction may be dispensed with. But it will be found preferable to use at least a small amount of compaction for corners and around reinforcements and anchorages.

Good curing of concrete is most important. Too early drying of concrete may result in shrinkage cracks before the application of prestress. Besides, only by careful curing can the specified high strength be attained in concrete. In order to hasten the hardening process, steam curing is often resorted to in the precasting factory; it can also be employed in the field where the amount of work involved justifies the installation. When field work of casting must be carried out in cold weather, steam can profitably be used to raise the temperature of the ingredients and the placed concrete in order that high strength may be attained within a reasonable time.

Where early hardening of the concrete is desirable, as in pre-tensioning work, high-early-strength cement is used. Admixtures to accelerate the strength should be employed with caution. For example, calcium chloride, the most commonly used accelerator, even applied in normal amounts will increase shrinkage. There is also some evidence that it may cause corrosion which could be serious for the prestressing wires. When accelerators are used, care must also be taken not to have the initial set take place too soon.

Wetting admixtures to improve the workability of concrete may be found to be profitable, since they may permit easy placing of high-

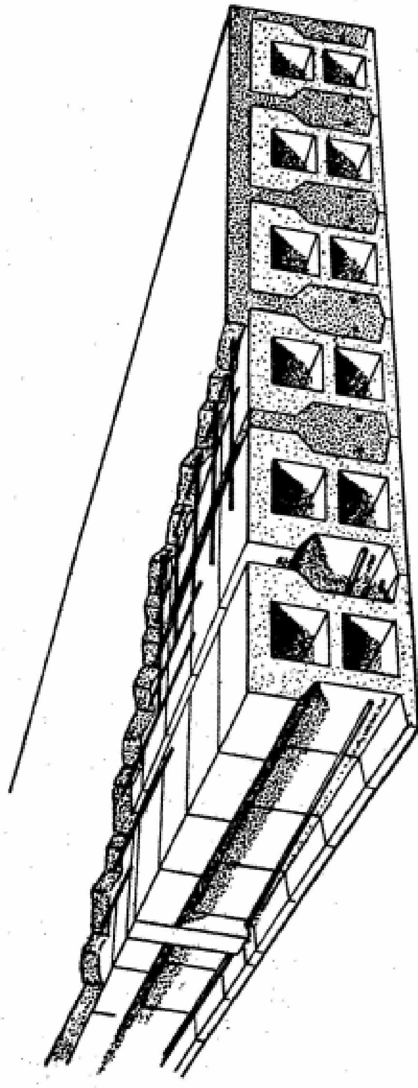

Fig. 2-3-1. Heavy bridge deck using prestressed-concrete blocks.

strength concrete without too high a cement content. Some of these admixtures tend to increase the shrinkage and may offset the advantage of saving cement. Each must be judged on its own merits, in conjunction with the nature of the aggregates and cement. Air entrainment of 3 to 5% improves workability and reduces bleeding. When well-recognized air-entraining agents are employed, there is no evidence of increased shrinkage or creep. Hence proper application of air entrainment is considered beneficial for prestressed concrete.

Light-weight concrete has not yet found wide application in prestressed work. But, when dead weight of the structure becomes an important consideration, light-weight aggregates may be economically employed. The main difficulty lies in the attainment of high strength consistent with high modulus of elasticity plus low shrinkage and creep. It is difficult to produce light-weight concrete which will satisfy all these requirements, but some sacrifice may be justified where dead load of the structure becomes a primary factor in the design.

Concrete blocks have been frequently manufactured for prestressed beams.[2-8] Breaking up a beam into blocks reduces the individual weight and facilitates casting and handling. These blocks can be mass-produced in a plant where rigid inspection and control can be effected. Forms for the blocks can also be reused many times, resulting in substantial economy. A typical construction using precast blocks is shown in Fig. 2-3-1.

For the joints between the blocks, two methods are employed. One is to grind the concrete surface for perfect bearing. This is usually a costly procedure. Another is to put mortars or grouts in between. By using high-early-strength cement for the grouts and applying a slight tension to the tendons to tighten the joints, a good bearing can be obtained. Full prestress is to be applied only after a few days.

Clay tiles of 10,000-psi strength have been prestressed to form beams and columns.[2-9] The use of ceramics of high strength, although not likely to be economical for massive structures, may prove desirable for light structures such as airplanes. Where resistance to high temperatures or to acid attacks is required, prestressed ceramics may prove to be an ideal material for certain structures.

2-4 Steel, Manufacture

High-tensile steel is almost the universal material for producing prestress and supplying the tensile force in prestressed concrete. Such steel can take any of three forms: wires, strands, or bars. The most widely used at present are the wires, which are grouped, in parallel,

into cables. Strands are fabricated in the factory by twisting wires together, thus decreasing the number of units to be handled in the tensioning operations. Since 1954, strands have been adopted in the majority of pre-tensioning plants in the United States. Steel bars of high strength have also been developed and successfully applied to prestressed concrete, resulting in considerable economy at times.

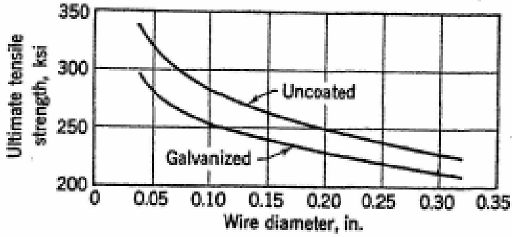

Fig. 2-4-1. Typical variation of wire strength with diameter.

The so-called prestressing wires now in the market are mostly high-tensile wires obtained by cold-drawing high-tensile steel bars through a series of dies. The process of cold drawing tends to realign the crystals, and the strength of the wires is increased by each drawing so that, the smaller the diameter of the wires, the higher their ultimate unit strength. The ductility of wires, however, is somewhat decreased as a result of cold drawing. A curve giving the general variation of strength with diameter is shown in Fig. 2-4-1. It must be borne in mind, though, that the actual strength will necessarily vary with the composition and manufacture of the wire as well as with its diameter.

The "as-drawn" wire, although possessing a high ultimate strength, has a relatively low proportional limit, for example, about 60,000 to 80,000 psi, above which the stress-strain curve flattens at an increasing rate. This is objectionable, since the deformation characteristics are relatively uncertain and the amount of elongation during prestress cannot be easily determined. Hence various methods commercially known as the "stress-relieving" process have been used to increase the proportional limit of the "as-drawn" wire. Two common methods of stress-relieving are as follows.

1 · Time-Stress Treatment. This treatment consists of stretching the wire to a stress level higher than that to be used in the final application. This increases the proportional limit to about 60 or 70% of the ultimate strength while the ultimate strength itself remains about the same. After this process of stretching, the wire will still have slight

creep at an eventual stress of 50% of the ultimate, but, when stressed up to 70%, the creep will not be much more than 5%.

2 · **Time-Temperature Treatment.** This consists of heating the wire to 750° to 800°F for a period of 30 to 40 seconds. The heating is accomplished by drawing the wire through a molten lead bath, or through a hot-air tunnel such as a ceramic tube with heat applied on the outside. This treatment will have an effect on the proportional limit and ultimate strength of the wire similar to that of the previous process. But this "time-temperature treated" wire has practically no creep when subjected to 50% of the ultimate strength. At 60% of the ultimate strength it shows slightly more creep than the "stretched" wire, and at 70% and above the creep becomes excessive.

In order to prevent corrosion in unbonded prestressed concrete, wires are sometimes galvanized. When galvanized, the tensile strength is slightly reduced. But its other characteristics are similar to those of the time-temperature treated wire. It has practically no creep when used within 55% of its ultimate strength, but its tendency to creep at stresses above 55% cannot be well controlled.

At present, the American Society for Testing Materials has no specification directly applicable to prestressing wires. The nearest one would be ASTM specification A227-41 for hard-drawn spring wire. Hence it is left to the engineer to specify the physical properties and sometimes the chemical composition desired. The chemical composition of prestressing wire may vary with the manufacturer, but a sample analysis is about as follows:

Carbon	0.60–0.85%
Manganese	0.70–1.00%
Phosphorus	0.050% max
Sulfur	0.055% max

Some manufacturers may use a certain amount of silicon in the steel.

In the United States, there are available two kinds of high-tensile wire strands: one for pre-tensioning and another for post-tensioning. Pre-tensioning strands are made of 7 or 2 small uncoated wires as drawn. The strands are then drawn through a lead bath for stress-relieving and also to improve their bond characteristics.

For post-tensioning and unbonded work, strands consisting of 7 to 61 galvanized wires are produced. These strands are machine fabricated and stress-relieved to increase their proportional limit and to minimize creep. When the strands are to be bonded to the concrete, the wires should preferably be ungalvanized.

High-strength bars up to 150,000 psi or more are made by cold-working special alloy steels. By alloying high-carbon steel with proper agents such as silicon and manganese, high strength is obtained. Then the proportional limit is raised by cold working. The chemical contents of these bars again may differ. A sample composition of high-strength steel bars is:

Carbon	0.6%
Silicon	2.0–2.5%
Manganese	0.7–1.0%
Phosphorus	0.2%
Sulfur	0.2%

In order to get better bond between steel and concrete, especially in pre-tensioning, where bond is critical at the ends, corrugated or waved wires are employed in Europe. Various forms of surface indentation afford direct mechanical keys with the surrounding concrete. It is assumed that the corrugations now commercially used will not alter the stress-strain properties of the wires, although some question has been raised as to their fatigue strength in comparison with the straight ones. Some pre-tensioning factories pass their wires through a small machine, forming permanent waves which are believed to increase their bond resistance.

2-5 Steel, Physical Properties

The ultimate strength of steel wires, strands, or bars varies with their manufacture, so that it is frequently necessary to obtain sample tests for each lot of products. However, the general range of values is listed in the table.

	Ultimate Strength, psi
Wires of varying diameters and make	200,000–330,000
Strands of 7 uncoated small wires	230,000–270,000
Strands of 19 or more galvanized wires	200,000–220,000
Strands of 19 or more uncoated wires	220,000–240,000
Bars	140,000–170,000

While the ultimate strength of high-strength steel can be easily determined by testing, its elastic or proportional limit, or its yield point, cannot be so simply ascertained. First, there is no yield point for high-strength steel as there is for ordinary low-carbon steel. Second, the gradual curving of the stress-strain curve makes it difficult

to fix a point for the proportional limit. Consequently, different methods for defining the yield point of high-tensile steel have been adopted:

1. The point where the stress-strain curve deviates from a straight line. As mentioned, this is often difficult to fix and hence seldom adopted.

2. At 0.2% set, i.e., where the inelastic or permanent deformation is 0.2%. This is obtained by drawing a line parallel to the initial tangent modulus at a horizontal distance of 0.2% from it, Fig. 2-5-1.

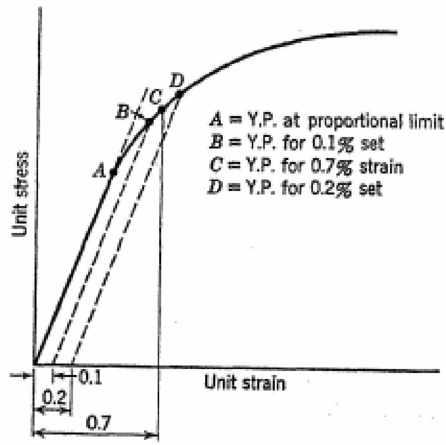

Fig. 2-5-1. Methods of determining yield point for high-tensile steel.

If the lower portion of the stress-strain curve is relatively straight, this can be rather accurately done. At present, this is probably the most popular method.

3. At 0.1% set. This is similar to the above method except that 0.1% is taken instead of 0.2%. It actually gives a value nearer the proportional limit than method 2, but it is not as often employed.

4. At 0.7% total strain. This may be a correct measure of the yield point for steel with proportional limit around 150,000 psi, but it does not afford a good comparison for steels of varying strengths and proportional limits. It has been used in limited instances only.

Yield point and proportional limit must be obtained by testing the particular steel. But as a rough approximation the following tabulation gives the usual values for high-tensile steels expressed in terms of the respective ultimate strength.

Approximate Yield Point and Proportional Limit
(in terms of ultimate strength f_s')

	Yield Point at 0.2% Set	Proportional Limit
Wires as drawn	$0.75f_s'$	$0.35f_s'$
prestretched	0.85	0.55
time-temperature treated	0.87	0.70
galvanized	0.85	0.55
Strands, pre-tensioning		
as drawn	0.85	0.35
stress-relieved	0.90	0.75
Strands, post-tensioning		
prestretched	0.85	0.55
Bars	0.90	0.60

Typical stress-strain curves for a high-tensile wire and a high-tensile bar are given in Fig. 2-5-2. When these curves are available, the

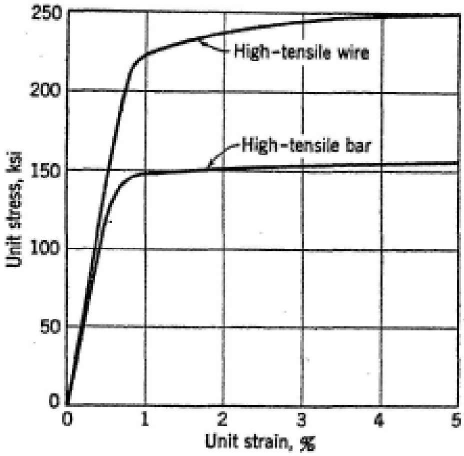

Fig. 2-5-2. Typical stress-strain curves for high-tensile steel.

modulus of elasticity for the steel can be accurately computed. Approximate average values for the secant modulus at the proportional limit are listed as shown.

	Secant Modulus at Proportional Limit, psi
Wires	27,000,000–30,000,000
Strands for pre-tensioning and stress-relieved	27,000,000–29,000,000
Strands for post-tensioning and prestretched	24,000,000–26,000,000
Bars	25,000,000–28,000,000

In order to avoid brittle failures in the prestressed concrete, a certain amount of ductility in the steel is desirable. This is measured by the amount of elongation in a certain gage length, generally 10-in. gage in this country. The average ultimate elongation is about 4% for wires and 5% for bars. For evident reasons, it cannot be easily measured for strands; only the wires making up the strands are measured for ductility.

2-6 Steel, Creep Characteristics

One of the important characteristics required of prestressing steel is minimum creep under maximum stress. Creep in steel is the loss of its stress when it is prestressed and maintained at a constant strain for a period of time. It can also be measured by the amount of lengthening when maintained under a constant stress for a period of time. The two methods give about the same results when the creep is not excessive, but the constant-strain method is more often employed as a basis for measurement, because of its similarity to the actual conditions in prestressed concrete. Creep varies with steel of different compositions and treatments; hence exact values can be determined only by test for each individual case if previous data are not available.

Approximate creep characteristics, however, are known for most of the prestressing steels now in the market. Speaking in general, the percentage of creep increases with increasing stress, and when a steel is under low stress, the creep is negligible. To summarize what is already stated in section 2-4, the creep characteristics of different steels are as follows. Compared to stress-relieved wires, the "as-drawn" wires have somewhat higher creep. Prestretched wires will have about 2 to 3% creep when subject to $0.50f_s'$, but when stressed to $0.70f_s'$, the creep will still be no more than 5%. Time-temperature treated wires have practically no creep when subject to $0.50f_s'$. At $0.60f_s'$, it has slightly more creep than "prestretched" wires, and at 0.70 to $0.80f_s'$ the creep becomes excessive. Galvanized wires have about the same creep characteristics as the time-temperature treated wires, and should preferably not be subjected to any stress above $0.60f_s'$ without carefully considering the effect of creep.

Creep in stress-relieved strands has been determined by W. O. Everling of the United States Steel Corporation, Cleveland, Ohio, 1953–1955. In general, their characteristics are similar to those of stress-relieved wires. For high-tensile bars, some limited tests seemed to show that, for stress up to about $0.55f_s'$, creep is not more than 5%.

While creep in steel is a function of time, there is evidence to show that under the ordinary working stress for high-tensile steel,[2-6] creep takes place mostly during the first few days. Under constant strain, creep ceases entirely after about 2 weeks. If the steel is stressed to a few per cent above its initial prestress and that overstress is maintained for a few minutes, the eventual creep can be greatly lessened, and it practically stops in about 3 days.

2-7 Steel, Some Practical Notes

Information about the available size and length of various prestressing steels can be obtained from the respective manufacturers (see Appendix B). For the purpose of design, however, it will be convenient to know in advance what sizes and lengths are available so that designs can be made accordingly. In continental Europe, smooth wires 2 and 3 (sometimes 2.5) mm in diameter and corrugated wires of 4 and 5 mm are most frequently employed in pre-tensioning work. Small wires possess higher unit strength and furnish better bond, which is often vital in pre-tensioning. In order to save labor and anchorage costs, larger wires are preferred for post-tensioning. For the common systems, such as the Freyssinet and the Magnel, the anchorages are manufactured for the 5- and 7-mm wires. Such wires still possess high strength and there are fewer units to handle. Hence they have become popular and almost monopolize the post-tensioning field.

In England, wires are based on the British Imperial Gauge, No. 2 of which has a diameter of 0.276 in., exactly 7 mm, while No. 6 has a diameter of 0.192 in., which is very close to 5 mm. Hence gage Nos. 2 and 6 are sometimes called for in post-tensioning, while the exact equivalents of 7 and 5 mm (0.276 and 0.196 in., respectively) are also frequently employed. In Germany, corrugated wires with flattened cross section are widely used for post-tensioning. These wires have areas equal to 5- or 7-mm round ones.

In this country, wires are manufactured according to the U.S. Steel Wire Gage, No. 2 of which has a diameter of 0.2625 in. and No. 6 has a diameter of 0.1920 in. Neither of these is the exact equivalent of the millimeter counterparts. Hence, when the European types of anchorages are adopted, 0.276-in. and 0.196-in. wires are often specified. For post-tensioning systems developed in the United States, ¼-in. wires have been most commonly incorporated. Smaller wires, with diameters of 3 mm (0.118 in.), are common for pre-tensioning without mechanical end anchorages. Where mechanical end anchorages

are provided, ⅜-in. wires have been successfully employed for pre-tensioning.

Seven-wire strands for pre-tensioning have nominal diameters of ¼, 5/16, ⅜, 7/16, and ½ in.; while post-tensioning strands with more wires per strand range from ½ in. to 1 11/16 in. in diameter, with steel area up to 1.73 sq in.

High-tensile bars are available from ½ to 1⅛ in. in diameter, with length up to 80 ft. Because of difficulty in shipping, the length may have to be further limited. But sleeve couplers are available to splice the bars to any desired length. These couplers have tapered threads in order to develop very nearly the full strength of the bars. They have outside diameters about twice that of the bar and a length about 4 times its diameter.

Wires are supplied in drums. They are cut to length and assembled either at the plant or in the field. Drum diameters are made as big as practicable, at least 5 to 6 ft, so that wires may be wound around them with the least permanent set. However, most wires when unwound from the drums do have a slight permanent set and require some straightening. Some wires also need a certain amount of degreasing and cleaning before placement, in order to ensure good bond with concrete. Loose rust or scale should be removed, but a firmly adherent rust film may be advantageous in improving the bond.

As fabricated, wire strands are several thousand feet long. When post-tensioning anchorages are required, the strands are cut and fitted in the factory. When unwinding strands, care must be taken not to pull them. Strands can best be laid out by rolling them along the path. Pulling them might result in kinking and permanent twisting of the strands, which would be very difficult to undo.

2-8 Glass Fibers

The possibility of using glass fibers for prestressing has been under investigation for some years[2-11] (Fig. 2-8-1). Glass fibers are now manufactured in three forms: parallel cords, twisted strands, and parallel fibers embedded in a plastic. Although that material has not yet been commercially applied in prestressed construction, it does possess certain qualities that are superior to those of high-tensile steel so that experiments to determine its practicality are considered worth while. Ultimate strength as high as 5,000,000 psi has been reported for individual silica fibers 0.00012 in. in diameter. When made into cords or strands, an ultimate strength of 150,000 to 400,000 psi could be expected. The specific gravity of glass fibers is about the same as that of concrete.

Another advantage of glass fibers is the low modulus of elasticity,

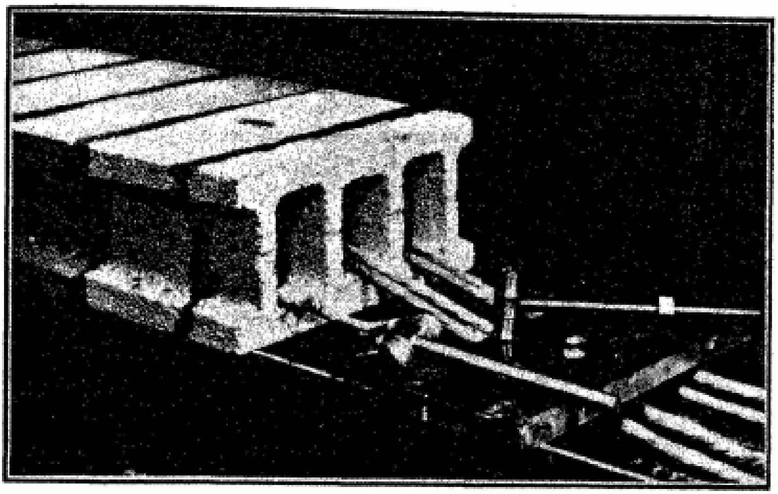

Fig. 2-8-1. Glass fiber strands lapped over the end holding devices in a test beam. (Owens-Corning Fiberglas Corp., Newark, Ohio.)

which ranges from 6,000,000 to 10,000,000 psi. With their high stress and low modulus, the percentage of loss of prestress would be quite small. Other advantages claimed for this material are high resistance to acids and alkalies and the ability to withstand high temperature. However, some major problems must be solved before it can be applied in practice:

1. Methods of fabricating cords from the glass fibers to obtain an even distribution of stress so as to increase the ratio of the strength of cords to the strength of individual fibers.

2. The determination of the chemical stability of the glass fibers, such as their reaction to the surrounding concrete, under both wet and dry conditions.

3. The minimizing of static fatigue in glass fibers, since it is known that the duration of loading has a pronounced effect on their strength.

4. The design of suitable end anchorages, since the brittle material is liable to fail in the grip under the effect of combined stresses.

Practically all the above problems are being studied at one institution or another. If they can be solved, there still remains a last hurdle: the economics of the application of the material.

2-9 Auxiliary Materials

Among the special auxiliary materials required for prestressed concrete are those for the provision of proper conduits for the tendons. For pre-tensioning, no such conduits are necessary. For post-tension-

ing, there are two types of conduits, one for bonded, another for unbonded prestressing.

When the tendons are to be bonded, generally by grouting, the conduits can be made of aluminum, steel, tin, or other metal sheathing or tubes. For small cables, corrugated sheet-metal pipes are often employed. For example, the Freyssinet system employs metal hose with outside diameters varying from 1⅛ to 1⅝ in. for 8 to 18 wires, with thickness of hose about $\frac{1}{16}$ in.

It is also possible to form the duct by withdrawing steel tubing or rod before the concrete hardens. More frequently, the duct is formed by withdrawing extractable rubber cores buried in the concrete. Several hours after the completion of concreting, these cores can be withdrawn without much effort, because the lateral shrinkage of the rubber under a pull helps to tear the rubber away from the surrounding concrete. In order that the rubber cores may remain straight during concreting, they are stiffened internally by inserting steel pipes or rods into axial holes provided in the rubber. To maintain the cores in position during concreting, transverse steel rods are placed under and over them at 3- to 4-ft intervals. For the Magnel system, for example, rubber cores approximately 2⅛ by 2 in. and 2⅛ by 3 in. to accommodate 0.196-in. wires, are obtainable; 2½ in. by 2½ in. and 2½ in. by 3½ in. cores are used for 0.276-in. wires. The combinations of these sizes make possible various numbers of wires in a cable. Sometimes, rubber tubes inflated from their normal diameter can be substituted for the above rubber cores. These tubes can then be deflated and withdrawn.

When the tendons are to be unbonded, plastic or heavy paper sheathing are frequently used, and the tendons are properly greased to facilitate tensioning and to prevent corrosion, Fig. 2-9-1. Such sheathing should be wound with wires or tapes at frequent intervals. Plastic tubes should be properly overlapped and taped along the seams, so as to seal them against any leakage of mortars, which might bind the tendons to the tubes. When papers are spirally wrapped around the tendons, care should be exercised in wrapping so as to avoid jamming of the papers when the tendons are tensioned. Direct wrapping using heavy papers of about 4-ft length is advised. It is desirable to wind the paper twice around the tendon to prevent any leakage of mortar.

For bonding the tendons to the concrete after tensioning (in the case of post-tensioning), cement grout is injected, which also serves to protect the steel against corrosion. Entry for the grout into the cableway is provided by means of holes in the anchorage heads and

cones, or pipes buried in the concrete members. The injection can be applied at one end of the member until it is forced out of the other end. For longer members, it can be applied at both ends until forced out of a center vent. Either ordinary portland cement or high-early-strength cement may be used for the grout. Coarse sand is

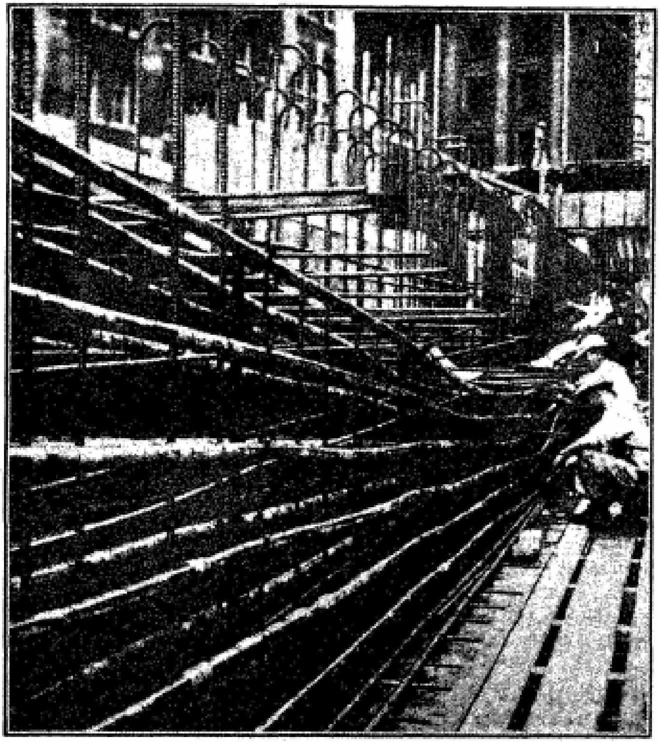

Fig. 2-9-1. 1½-in. Roebling strands wrapped with plastic tubing for heavy prestressed girders.[2-12] (Ellison and King, Consulting Engineers, San Francisco, California.)

preferred for bond and strength, but sufficient fineness is necessary considering the limited space through which the grout has to pass. To ensure good bond for small conduits, grouting under pressure is desirable; however, care should be taken to ensure that the bursting effect of the pressure on the walls of the cable enclosure can be safely resisted. Machines for mixing and injecting the grouts are commercially available.

Where larger space between the wires is obtained, such as in a Magnel cable, a 1 : 1 cement-sand mix is often used with water-cement ratio of about 0.5 by volume and a pressure of a few psi may be sufficient for short cables. Where the space is limited, as in a Freyssinet or Strescon cable, neat cement paste with about the same water-cement ratio should be employed. When it is desired to save cement on a big job, fine sand of $\frac{1}{64}$-in. grain size can be added. The water : cement : sand proportion should be about 1.0 : 1.3 : 0.7 by volume. Grouting pressure generally ranges from 80 to 100 psi. After the grout has discharged from the far end, that end is plugged and the pressure is again applied at the injecting end to compact the grout. It is also good practice to wash the cables with water before grouting is started, the excess water being removed with compressed air.

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Chapter 3 Prestressing Systems—End Anchorages

3-1 Introduction

On account of the existence of different systems and their patents for tensioning and anchoring the tendons, the situation appears a little confusing to a beginner in the design and application of prestressed concrete. Actually, present practice in this country does not require of the designer a thorough knowledge of the details of all systems or even of the system that he intends to use for his particular job. In order to encourage competitive bidding, the engineer often specifies only the amount of effective prestressing force required so that the bid is open to all systems of prestressing. However, he should have a general knowledge of the methods available and keep it in mind while dimensioning his members so that the tendons of several systems can be well accommodated. Sometimes he must compute the actual size and number of tendons for his members in order to obtain feasible arrangements and an accurate estimate of the quantities of materials. A knowledge of the details of end anchorages and the tensioning jacks is required to design the ends of his members so as to be ready to house the anchorages and to receive the jacks.

In the United States alone, there are already more than one hundred patents and patents pending on various systems of prestressing.[3-1] Many of these patents have never been commercially or economically applied, but many others are still being developed. It would now take the job of a specialized lawyer in addition to a specialized engineer to look into the matter, or to apply for and obtain a new patent. The situation is further complicated by the development of similar methods in other countries having reciprocal patent arrangements with ours, although, in general, any patents obtained abroad must be at least registered in this country before being effective here. These problems, however, are only for the inventors to worry about. The practical engineer, who simply wants to design some structures of prestressed concrete, is free to specify and design for any

system without studying the intrigues of patent rights. In fact, the owner of the structure would not have to pay any direct royalty to the patent holder. The royalty is indirectly included in the bid price for the supply of prestressing steel and anchorages, which sometimes also includes the furnishing of equipment for prestressing and some technical supervision for jacking. Owing to the keen competition already existing in this country in the field of prestressing, the matter of patent royalty is not a serious cost item for the owner. In order to bid on a structure embodying prestressed concrete, the general contractor usually approaches the various, prestressing concerns for sub-bids on the prestressing portions. Thus the standard method of bidding applicable to reinforcing steel and other trades is being extended to prestressing work.

In some other countries, the conditions are different. In Germany, for example, there is a tendency not to accept any general contractor for a job employing prestressed concrete unless he himself has devised a system of prestressing. As a result, each contractor is forced to devise some system of his own, and he tends to charge an excessive royalty for his method if his competitors want to use it. In France and Belgium, for historic and other reasons, one or two prestressing systems are much more widely applied than the others; hence the direction of growth tends toward the development of these particular systems rather than a multiple approach.

The basic principles of prestressing cannot be patented, but the details of its application can. There are some patents on the methods of application, such as special designs for prestressed pavements or pipes, using processes of construction different from the ordinary. Fortunately, these patents are based more on the construction procedures than on the design features and seldom affect the designing work of the engineer. Furthermore, it is not good policy for an engineer to try to hold a monopoly on his design. Hence engineers are seldom obstructed from using any design in prestressed concrete.

The so-called "prestressing system" comprises essentially a method of stressing the steel combined with a method of anchoring it to the concrete, including perhaps some other details of operation.[3-2] Hence most of the patents on prestressed concrete are based on either or both of the following two operational details: (1) the methods of applying the prestress; (2) the details of end anchorages. In addition to these, sometimes the size and number of wires also form part of the patented process, although most patents contain a variety of combinations of these. At the present, no less than two to three dozen prestressing systems are known to the author. Linear-prestress-

TABLE 3-1-1

Prestressing Systems

Type	Classification	Description		Name of System	Country
Pre-tensioning	Methods of stressing	Against buttresses or stressing beds		Hoyer	Germany
		Against central steel tube		Shorer Chalos	U.S. France
	Methods of anchoring	During pre-stressing	Wires	Various wedges	
			Strands	Strandvise	U.S.
		For transfer of prestress	Bond, for small wires	For most systems	
			Corrugated clips, for big wires	Dorland	U.S.
Post-tensioning	Methods of stressing	Steel against concrete		For most systems	
		Concrete against concrete		Leonhardt	Germany
				Billner	U.S.
		Expanding cement		Lossier	France
		Electrical prestressing		Billner	U.S.
		Bending steel beams		Preflex	Belgium
	Methods of anchoring	Wires, by frictional grips		Freyssinet	France
				Magnel	Belgium
				Morandi	Italy
				Holzmann	Germany
				Preload	U.S.
		Wires, by bearing		B.B.R.V.	Switzerland
				Strescon or Prescon	U.S.
				Texas P.I.	U.S.
		Wires, by loops and combination of methods		Billner	U.S.
				Monierbau	Germany
				Huttenwerk Rheinhausen	Germany
				Leoba	Germany
				Leonhardt	Germany
		Strands, by bearing		Roebling	U.S.
				Wayss and Freytag	Germany
		Bars, by bearing		Lee-McCall	England
				Stressteel	U.S.
				Finsterwalder	Germany
				Dywidag	Germany
				Karig	Germany
				Polensky and Zollner	Germany
				Wets	Belgium
				Bakker	Holland

ing systems are grouped and classified in Table 3-1-1. Systems for circular prestressing will be discussed in Chapter 13.

Since it is almost impossible and perhaps also unnecessary to discuss all these systems, only the more prominent and common ones will be described in this chapter. The eventual success of each system will depend upon a number of conditions, the chief criterion being its economy and convenience. It is difficult to predict whether new and better systems, or major improvements of existing ones, may not be developed in the future, but it does seem that most of the practical ideas have been embodied, in one form or another, in the systems mentioned.

3-2 Pretensioning Systems and End Anchorages

A simple way of stressing a pre-tensioned member is to pull the tendons between two bulkheads anchored against the ends of a stressing bed. After the concrete hardens, the tendons are cut loose from the bulkheads and the prestress is transferred to the concrete. Such stressing beds are often used in a laboratory and sometimes in a prestressing factory. For this set-up, both the bulkheads and the bed must be designed to resist the prestress and its eccentricity.

For mass production of pre-tensioned members, the Hoyer system is generally used. It consists of stretching the wires between two bulkheads some distance apart, say several hundred feet, Fig. 3-2-1. The bulkheads can be independently anchored to the ground, or they can be connected by a long stressing bed. Such a bed is costly, but it can serve two additional purposes if properly designed. First, intermediate bulkheads can be inserted in the bed so that shorter wires can be tensioned. Then the bed can be designed to carry vertical loads, thus permitting the prestressing of bent tendons.

With this Hoyer process, several members can be produced along one line, by providing shuttering between the members and concreting them separately, Fig. 3-2-2. When the concrete has set sufficiently to carry the prestress, the wires are freed from the bulkheads and the prestress is transferred to the members through bond between steel and concrete or through special pre-tensioning anchorages at the ends of members. This long-line production method is economical and is used in almost all pre-tensioning factories. Since the amount of lengthening is appreciable, it is convenient to use hydraulic jacks of long ram travel and to pull the wires by an electric motor. In the United States, these equipments are made by companies such as the Rogers Hydraulic Incorporated of Minneapolis, Minnesota.

Devices for gripping the pre-tensioning wires to the bulkheads are

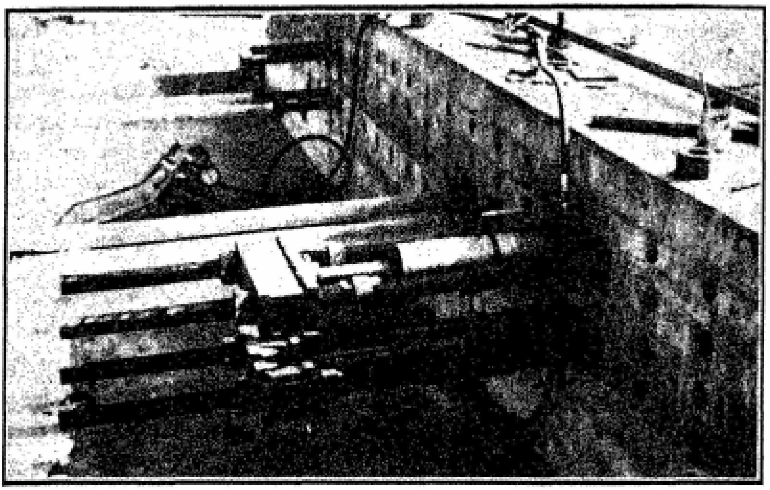


Fig. 3-2-1. Hydraulic jacks pre-tension the wires against heavy bulkheads. (Concrete Engineering Co., Tacoma, Wash.)

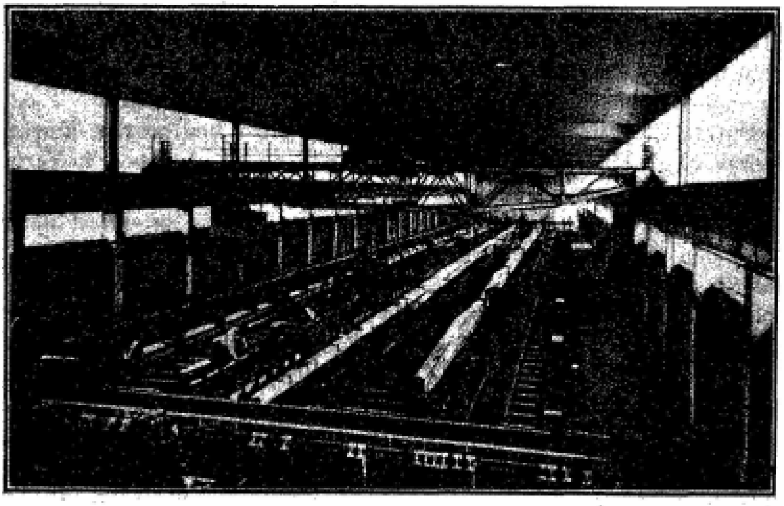


Fig. 3-2-2. Pre-tensioning plant, using Hoyer system. (Concrete Development Co., Ltd., London.)

usually made on the wedge and friction principles.[3-3] A typical split cone wedge is shown in Fig. 3-2-3*a*, made from a tapered conical pin. The pin, drilled axially and tapped, is cut in half longitudinally to

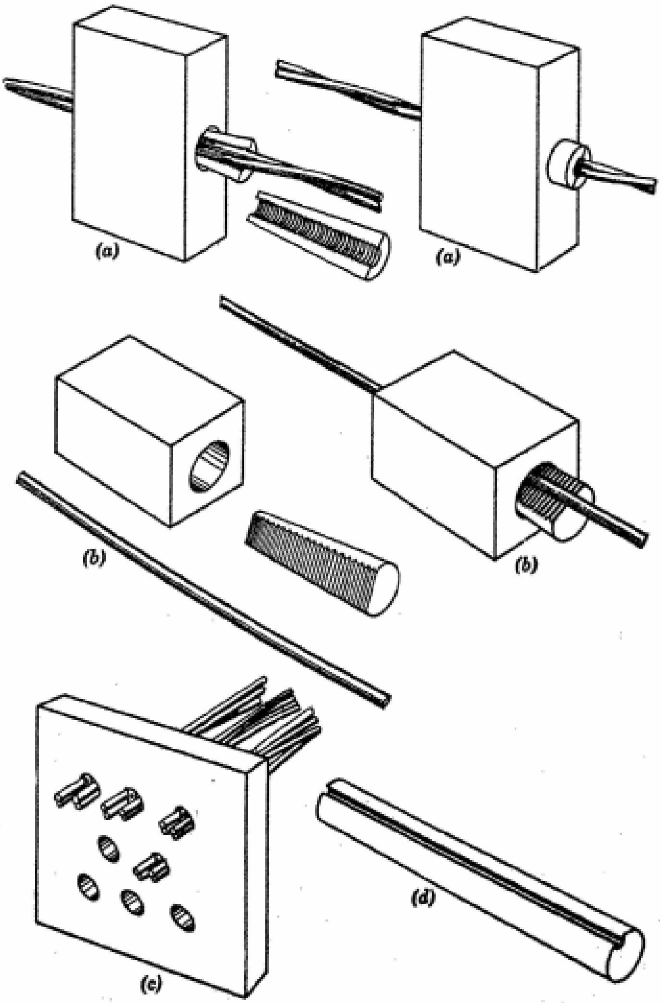


Fig. 3-2-3. Some typical wire gripping devices.

form a pair of wedges. These grips can be used for single wires as well as for twisted wire strands. Another grip is shown in Fig. 3-2-3*b*, made from a conical pin on which a flat surface has been machined

and serrated. The pin fits into a conical hole in a block and holds the wire between the serrated face and the block. In addition, there are quick release grips, which are more complicated and more costly, but effect a great saving in time. If the wires are to be held in tension only for short periods, these quick-release grips may be found to be more economical. Quick-release grips for holding strands have also been manufactured in this country, for example, the Strandvises by the Reliable Electric Company of Chicago.

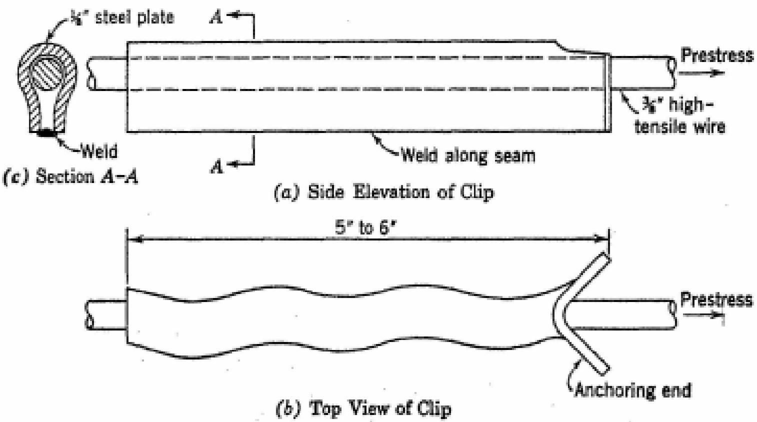


Fig. 3-2-4. Dorland clip anchorages for pre-tensioning. The clip is placed straight on the wire or strand and then crimped in a die with high pressure; then the seam is welded.

The dependence on bond to transmit prestress between steel and concrete necessitates the use of small wires to ensure good anchorages. It is also necessary to effect a gradual transfer of prestress from the bulkheads to the member in order not to destroy the end bond between steel and concrete. Wires greater than about ⅛ in. are used only if they are waved along their length or if they are corrugated. In any case, a minimum length of transfer is required to develop the bond. Should there be insufficient length of transfer, e.g., when cracks occur near the end of a beam, the bond may be broken and the wires may slip. A more reliable method is to add mechanical end anchorage to the pre-tensioned wires. One method developed in San Diego, California, is the Dorland anchorage, which can be gripped on to the wires or strands at any point, thus supplying positive mechanical anchorage in addition to the bond, Fig. 3-2-4. The clips are gripped to the tendon under high pressure, and the edges of the clips are then

welded together at several points. It should be noted that such anchorage makes possible the use of bigger tendons and sometimes permits an earlier transfer of prestress, resulting in considerable economy in the laying and stressing operations.

The Shorer system[3-4] involves an ingenious feature, doing away with stressing beds and bulkheads. In their stead, a central tube of high-strength steel carries the prestress from the surrounding wires, and the entire assembly is placed in position and concreted. After the concrete has set and attained a certain strength, the tube is removed and the prestress is transferred to the concrete by bond. The hole left by the tube is then filled with grout. This method has not found wide application in this country. In France, the method is credited to Chalos and is known as the Chalos system.

3-3 Post-Tensioning, Tensioning Methods

The methods of tensioning can be classified under four groups: (1) mechanical prestressing by means of jacks; (2) electrical prestressing by application of heat; (3) chemical prestressing by means of expanding cement; (4) miscellaneous.

1 · Mechanical Prestressing. In both pre-tensioning and post-tensioning, the most common method for stressing the tendons is jacking. In post-tensioning, jacks are used to pull the steel against the hardened concrete; in pre-tensioning, to pull it against some bulkheads. Hydraulic jacks are often used, because of their high capacity and the relatively small force required to apply the pressure. Occasionally, screwjacks are used when the force to be exerted does not exceed about 5 tons. Levers may be found convenient only when very small wires are to be tensioned individually.

When hydraulic jacks are employed, one or two rams are worked by one pump unit with a control valve in the pipe circuit, Fig. 3-3-1. The capacity of jacks varies greatly, from about 3 tons up to 100 tons or more. A strand of 1½-in. nominal diameter may require an initial tensioning of 90 tons; it may then be desirable to have two jacks of 60-ton rated capacity for the tensioning, for an ample margin of safety, Fig. 3-3-2. For some prestressing systems, jacks are specially designed and rated to perform the job of tensioning certain particular cables containing a given number and size of wires. Most systems either sell or rent their jacks. For some systems, any jack of sufficient capacity can be employed, provided suitable grip for the tendon is available. Care must be taken to see that the jack can be properly mounted on the end bearing plates, and that there is enough room at the tensioning ends to accommodate the jacks.

It is not possible to compile all the data necessary for designing each system of prestressing. New systems are being developed and existing ones are being modified from time to time. Engineers interested in a particular system should obtain the company's pamphlets

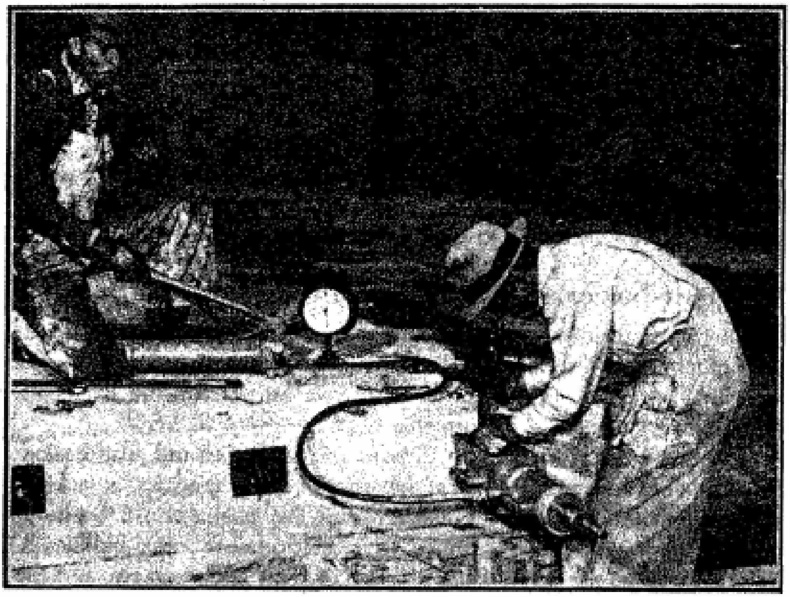


Fig. 3-3-1. Pumping a jack to stress a six-wire cable for the Prescon system. Note pressure indicated by jack gage.

or consult its representatives for particular details. In order to facilitate such consultation, the addresses of some prestressing systems in this country are listed in Appendix B.

Systems of jacking vary from pulling one or two wires up to several hundred wires at a time. The Clifford-Gilbert system in England employs a small screwjack weighing about 20 lb which pulls one wire at a time and can be easily handled. In the Magnel system, tensioning is carried out by a hydraulic jack which pulls two wires at a time by means of a temporary wire grip, Fig. 3-3-3. The jacks are designed for both 0.196-in. and 0.276-in. wires. The frame for the jack is made big enough so that several pairs of wires may be tensioned with the jack remaining in the same working position.

The Freyssinet double-acting jack pulls up to 18 wires at a time, Fig. 3-3-4. The wires are wedged around the jack casing and are stretched by the main ram which reacts against the embedded anchor-

age. When the required tension is reached, an inner piston pushes the plug into the anchorage to secure the wires; the pressure on the main ram and that on the inner piston are then released gradually, and the jack is removed.

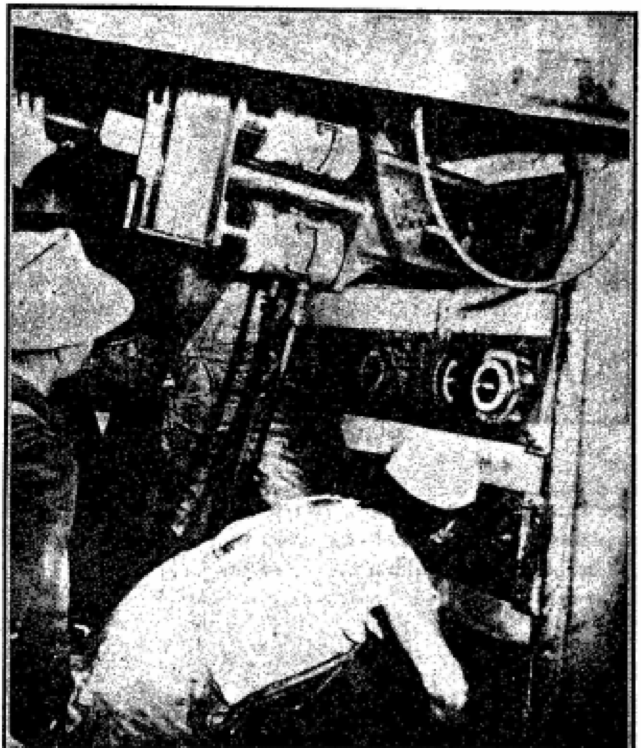


Fig. 3-3-2. Two rams, each of 60-ton capacity, apply a prestress of 85 tons on a 1½-in. Roebling cable.

Jacks for the Roebling system are fitted with threaded bars to be screwed into the strand fitting for tensioning. Jacks for the Lee-McCall or Stressteel system are provided with an adjustable connector to accommodate various bar diameters.

Some hints regarding the practice of jacking may be helpful for designers as well as for supervising engineers. In order to minimize creep in steel and also to reduce frictional loss of prestress, tendons are often jacked a few per cent above their specified initial prestress. Overjacking is also necessary to compensate for slippage and take-up

in the anchorage at the release of jacking pressure. When tendons are long or appreciably curved, jacking should be done from two ends. During the process of jacking, anchorage screw nuts and wedges

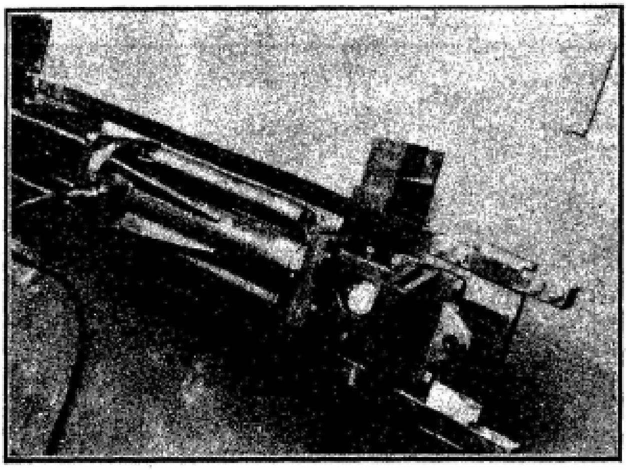


Fig. 3-3-3. The Magnel jack.

should be run all the way home and seated moderately tight against the end plates. This may help to avoid serious damages in the event of a wire breakage or a sudden failure of the jacks.

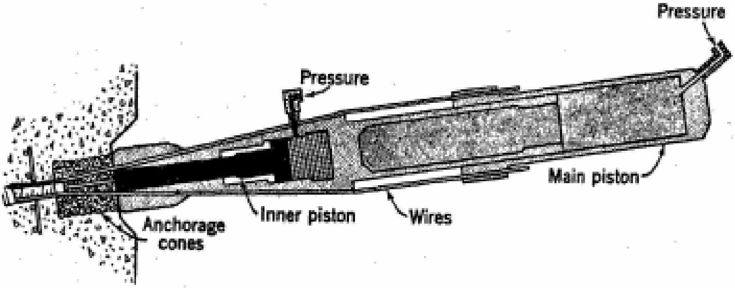


Fig. 3-3-4. The Freyssinet double-acting jack.

Pressure gages for jacks are calibrated either to read the pressure on the piston or to read directly the amount of tension applied to the tendon. It is usual practice to measure the elongation of steel so that the magnitude of prestress can be computed from the modulus of elasticity and checked against the gage indications.

When several tendons in a member are to be tensioned in succession, care should be taken to pull them in the proper order so that no serious eccentric loading will result during the process. If necessary, some tendons may have to be tensioned in two steps so as to reduce the eccentric loading on the member during tensioning.

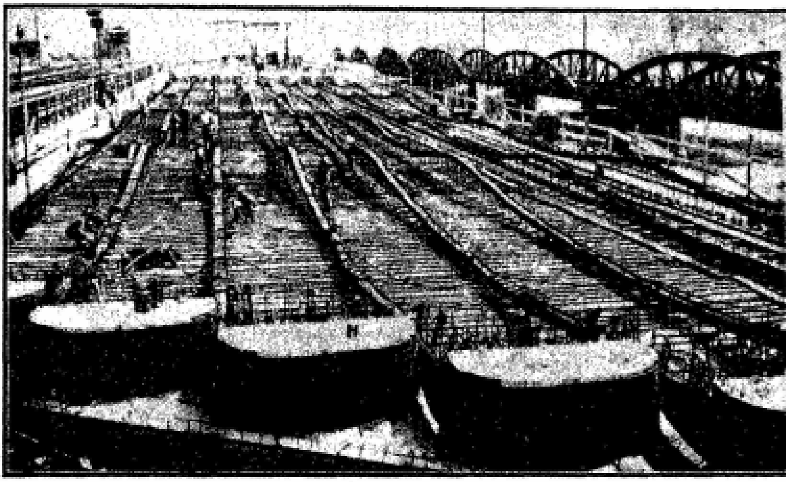


Fig. 3-3-5. Continuous cable and end jacking blocks for the Leonhardt system.

Instead of being attached to the steel, the jacks are sometimes inserted between two portions of concrete to force them apart, one against the other. Notably, this procedure is used in two systems: the Leonhardt system of Germany, and the Billner system of the United States. In the Leonhardt system,[3-5] one reinforced-concrete anchor block is poured at each end of a structural portion, and prestressing cables are wound around the blocks to be stressed all at once by hydraulic jacks inserted between the blocks and the main body of the structure, Fig. 3-3-5. The advantage of this method lies in the reduction of the number of stressing operations. But naturally much heavier jacks are required. In order to reduce the cost of the heavy jacks, built-in ones of reinforced concrete are made on the job. They are eventually left in the structure.

The reinforced-concrete jack of the Leonhardt system is made of several metal cylinders serving as pistons, Fig. 3-3-6. The ends of the cylinders are filled with concrete and fixed horizontally to the vertical surface of the anchorage block. Over each piston is placed an outer metal sleeve surrounded by a coil of heavy reinforcing steel, which

forms the cylinder. These jacks are interconnected by tubes and actuated by water pressure which forces the end blocks away from the major portion of the structure, thus prestressing the entire structure in one operation. When the desired elongation of the cables has been attained, cement grout is pumped in to fill the jacks and the gap left by the jacking is filled with concrete. Elongation of the wires around the loop is facilitated by special wax lubrication between the

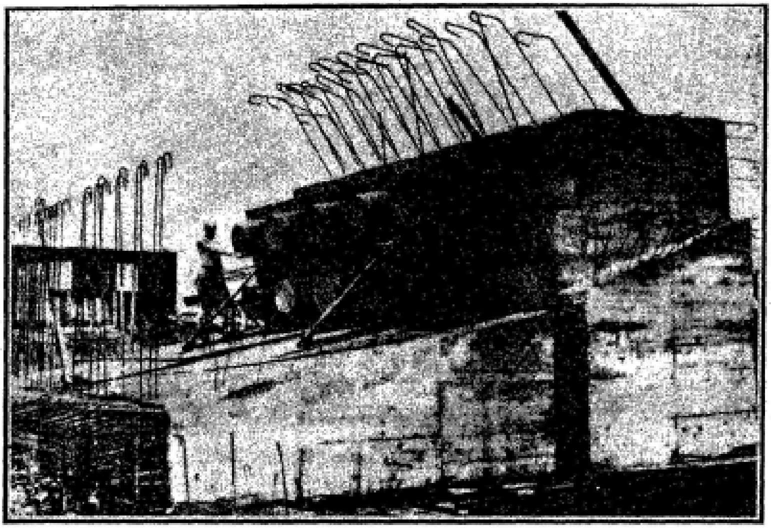


Fig. 3-3-6. Arrangement of reinforced-concrete jacks for the Leonhardt system.

wires and the contact surface. The lubricant melts under pressure and offers very little friction during tensioning.

In contrast to the Leonhardt system, which is specifically designed for large structures, the Billner system[3-6] is better suited to small ones. In the Billner system, the member is cast in two portions, split at the midspan. Jacks separated by a comblike partition are inserted between the two portions. Concrete is cast on both sides of the partition, forming two separate units. The prestressing wires pass through the slots provided in the partition plate and are not bonded to the concrete except at the ends. The economy of this method lies in the saving of the more expensive end anchorages otherwise required for post-tensioning work: i.e., anchorages that can be gripped by a jack, prestressed, and anchored. In this system, jacking is done near the midspan and between the concrete; hence the end anchorages only have to perform the task of anchoring, which is simply achieved by looping the tendons around the concrete.

2 · Electrical Prestressing. The electrical method of prestressing[3-7] dispenses with the use of jacks altogether. The steel is lengthened by heating with electricity. This electrical process is a post-tensioning method where the concrete is allowed to harden fully before the application of prestress. It employs smooth reinforcing bars coated with thermoplastic material such as sulfur or low-melting alloys and buried in the concrete like ordinary reinforcing bars but with protruding threaded ends. After the concrete has set, an electric current of low voltage but high amperage is passed through the bars. When the steel bars heat and elongate, the nuts on the protruding ends are tightened against heavy washers. When the bars cool, the prestress is developed and the bond is restored by the resolidification of the coating.

This method, as originally developed, was intended for steel bars stretched to about 28,000 psi, which requires a temperature of about 250°F. Owing to the high percentage of loss of prestress for steel with such a low prestress, and to other expenses involved in the process, this method has been found to be uneconomical in competition with prestressing using high-tensile steel. It has not been applied to high-tensile steel because a much higher temperature would be required for its prestressing. Such a high temperature could involve a number of complications, including damage to some physical properties of high-tensile steel.

3 · Chemical Prestressing. The use of self-expanding cement[3-8] for stressing steel has not yet been found economically feasible. One purely practical difficulty is the trouble encountered when a structure expands by itself in all directions. Hence the method cannot be easily applied to structures cast in place. At present, the technique of manufacturing expanding cement has developed to such a degree that the amount of expansion can be well controlled. For example, a strain of 0.0030 to 0.0110 can be obtained for neat cement paste, which would mean a strain of about 0.0010 to 0.0040 for concrete using 9 sacks of cement per cubic yard. The high value of 0.0040 would indicate a stress in the steel of about 120,000 psi, but important plastic effects take place when the concrete is under high stress, tending to reduce the total amount of expansion. Thus a stress of no more than 40,000 to 50,000 psi can be obtained with a percentage of wire reinforcement of about 0.8%. Therefore it cannot compete with the ordinary method of mechanical prestressing. In addition, the sensitiveness of such expanding cements to sulfates, sea water, or atmospheric moisture presents a problem yet unsolved.

It must be mentioned, however, that expanding cement has been successfully applied for many interesting projects, especially in France.

These generally involved prestressing of the member itself without attempting to develop the stress in the reinforcing steel. When a concrete block of expanding cement is cast as the keystone for a concrete arch, it serves as a jack, producing the desired arch compensation to balance rib shrinkage and shortening. When used for underpinning buildings, it tends to lift the structure without jacking. It can be used for pressure grouting or for producing concrete pavements and slabs with no shrinkage joints. Thus, it must be concluded that, at the present time, expanding cement can be applied for the purpose of prestressing in special cases, but the method is not yet competitive with mechanical procedures for prestressing high-tensile steel.

4 · **Miscellaneous.** Still another method of prestressing, not belonging to any of the above groups, was developed and applied in Belgium; it is known as the "Preflex" method.[3-9] The procedure consists in loading a high-tensile steel beam in the factory with a load equal to that anticipated in use. While the beam bends considerably under this load, its tensile flange is clothed with concrete of high compressive strength. After the concrete hardens, the load on the beam is removed, and the concrete is compressed as the beam regains a measure of its original shape. Then the beam is transported to the site to form a part of the structure, generally with the top flange and the web then also encased in concrete. Thus a composite section is obtained combining the strength of high-tensile steel with the rigidity of concrete.

3-4 Post-Tensioning Anchorages for Wires by Wedge Action

There are essentially three principles by which steel wires are anchored to concrete:

1. *By the principle of wedge action producing a frictional grip on the wires.*
2. *By direct bearing from rivet or bolt heads formed at the end of the wires.*
3. *By looping the wires around the concrete.*

Several dependable systems have been developed based on the principles of wedge action and of direct bearing. Little can be said about the relative advantages of these two principles, the superiority of each system depending upon the method of application rather than upon the principle itself. The last method, looping the wires around the concrete, has not been widely applied, although it also has its advantages.

Two popular prestressing systems anchor their wires by wedge action: the Freyssinet system and the Magnel system. The Freyssinet

system makes use of concrete cylinders and cones reinforced with steel wires, Fig. 3-4-1. Each anchorage unit consists of a cylinder with a conical interior through which the wires pass, and against the walls of which the wires are wedged by a conical plug lined longitudinally with grooves to receive them. The cylinder is buried flush with the face of the concrete and serves to transmit the reaction of the jack as well as the prestress of the wires to the concrete (see Fig. 3-3-4).

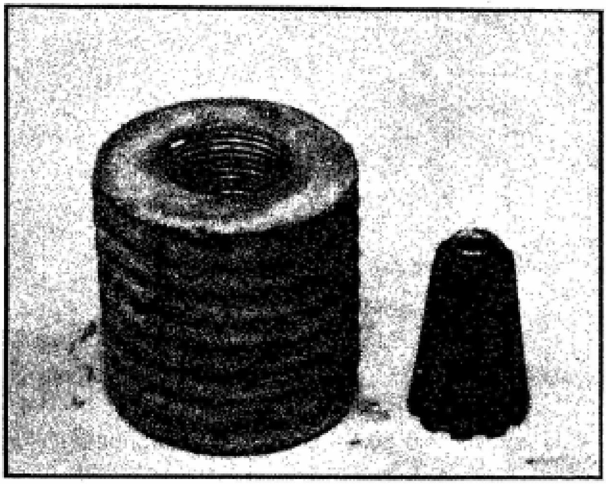


Fig. 3-4-1. A set of Freyssinet cones for end anchorage.

After the completion of prestressing, grout is injected through a hole at the center of the conical plug.

The Freyssinet cones are made for wires of 5-mm (0.196-in.) diameter, with the number of wires ranging from 2 through 8, 10, 12 and up to 18 per cable, 12 and 18 wires being most common. The outside dimensions of a 12-wire anchorage are about 3¾ in. for diameter and 4 in. for length; of an 18-wire anchorage, about 4¾ in. for diameter and 4¾ in. for length. Cones are made also for 7-mm (0.276-in.) wires, with 12 wires per cable. For exact dimensions of various units see Appendix B and also consult catalogues of the Freyssinet Company.

The Magnel system, also known as the Magnel-Blaton system, uses rectangular sandwich plates of steel which have tapered notches to receive the wedges. Wires of 0 196 or 0.276 in. are gripped between the grooves of the wedges and the sandwich plate, Fig. 3-4-2. The most commonly used plates are those for 8 wires, but plates for 2, 4,

or 6 wires are also available. Each cable is formed of 1 to 8 of these plates laid one against the other, which react against a cast-steel distributing plate interposed between them and the concrete. The entire anchorage assembly is usually laid after the setting of the concrete,

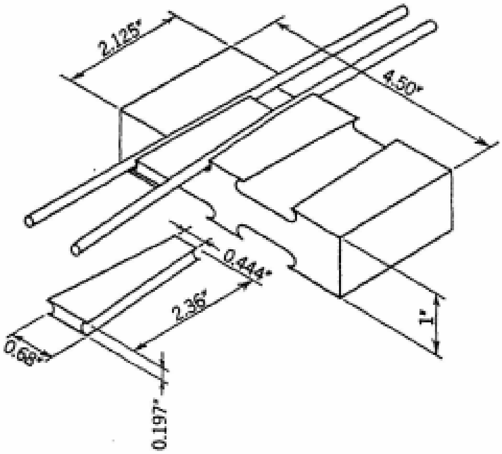


Fig. 3-4-2. A Magnel sandwich plate with wedges (for 0.196-in. wires).

with the steel distribution plates cemented to the concrete at proper angles. These plates may be cast into the member at the proper place during concreting if desired.

The number of wires for the Magnel cable varies from 2 to 64 per cable for both sizes of wires. At intervals of several feet the wires are spaced by grilles made of thin, mild-steel strips. In order to resist the transverse component of curved tendons, the grilles are sometimes stiffened with blocks placed along the center of the grilles. The cable can be encased in a light metal sheath which is concreted into the structure. More often it is drawn into preformed ducts made by means of extractable rubber cores. In some jobs, the wires are placed outside of the concrete, thus avoiding the use of rubber cores or metal sheathing; then the cable is wrapped with cement mortar for protection.

A typical Magnel cable of twenty-four 0.196-in. wires consists of 6 layers with 4 wires per layer. The outside dimensions of the grilles for such a cable are about 2 in. by 2½ in. The end anchorages consist of 3 sandwich plates bearing on a distribution plate about 7 in. by 10 in., with a maximum thickness of about 1½ in.

The Preload Company of the United States developed an anchorage

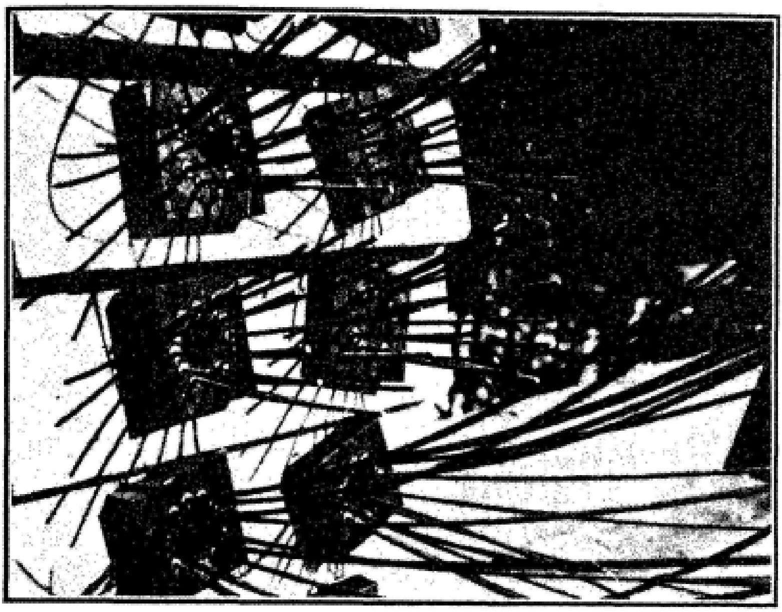
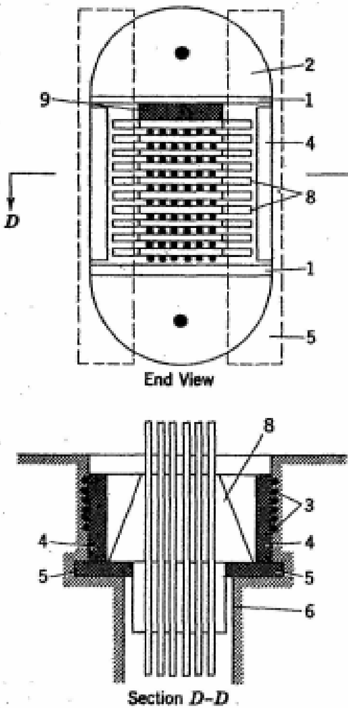


Fig. 3-4-3. The Preload anchorage.

in which the wires are gripped by split cones wedged in the holes of a heavy distribution plate, Fig. 3-4-3, similar to pre-tensioning bulkheads. Holzmann of Germany developed another system, in which the ends of the prestressing wires are wrapped around with spiral wires holding the prestressing wires in hoop tension. By this system, only one set of wedges is required to hold many wires, Fig. 3-4-4. In Italy, the Morandi system makes use of cables of 16 wires, with every pair of wires anchored individually by a small conical wedge. These latter methods have not been applied in the United States.

3-5 Post-Tensioning Anchorages for Wires by Direct Bearing

Two prominent systems employing cold-formed rivet heads for direct bearing at the ends of stressing wires are now used throughout the United States. Both systems have special head-forming machines for the purpose. One of these is the Strescon or the Prescon system, developed by the Prestressed Concrete Corporation of Kansas City, Missouri. By this method, rivet heads are cold-formed at the proper place for high-tensile wires of ¼-in. diameter. Static tests on these heads have shown that the full strength of the wire can be developed.



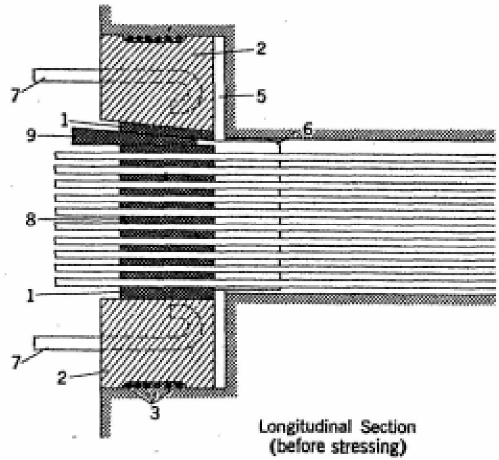


Fig. 3-4-4. The Holzmann anchorage. Numbers on the drawing refer to:

1. Pressure distributing plate.
2. Precast concrete ends.
3. Spiral wire.
4. Stressing plate.
5. Support plate.
6. Pipe piece.
7. Construction bolt.
8. Filler plates.
9. Flat wedge.

If the wires are grouted, there being practically no change in stress at the ends, no danger of fatigue failure is expected. Even if the wires are not grouted, since extreme variations of stress do not exist at the ends the possibility of failure under repeated loads is not believed to be serious.

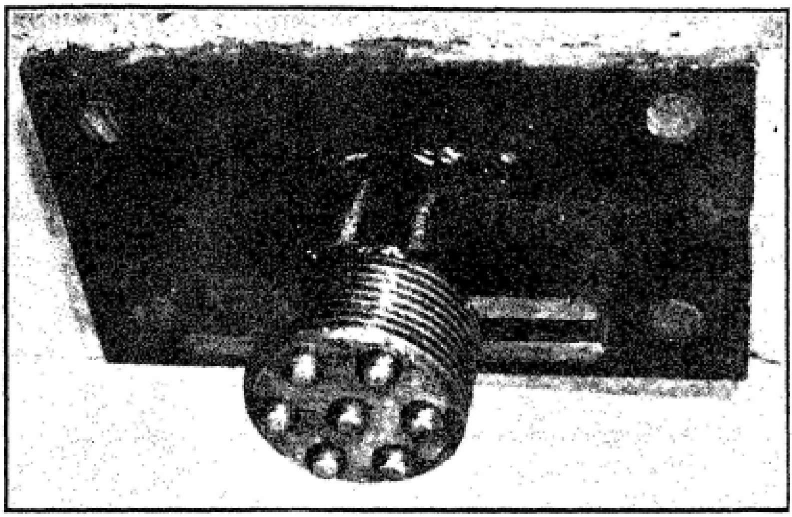


Fig. 3-5-1. End anchorage, Strescon system.

The Prescon system uses cables of 2 to 16 wires arranged in parallel. The wires are threaded through a stressing washer at each end, Fig. 3-5-1, before having their heads formed. A hole is provided in the stressing washer to permit grouting. The stressing jack has a special stressing collar which is screwed over the stressing washer and pumped to give the required elongation. A slight excess elongation will enable the shims to be inserted more easily. Then the jack is relieved to transmit the pressure to the shims. The height of the shims must be calculated for each particular case, depending upon the length and modulus of elasticity of the wire, the amount of prestress, and the frictional force along the cable. After completion of the prestressing operations, the entire end anchorage is enclosed with concrete for protection against corrosion and fire. In order to minimize handling of individual wires in the field, they are made into cables in the plant and shipped to the site ready for installation.

If the cables are used for bonded work, metal hose is required. In order to permit the passage of grout, the inside diameter of the hose

is at least ¼ in. greater than required to house the wires. For unbonded work, grease is applied to the wires, which are then wrapped with heavy papers into a cable. For a 6-wire unit, the cable has a diameter of ¾ in., and the stressing washer has a diameter of about 2 in. and a thickness of ¾ in. and bears against the steel shims which rest on a 5 in. by 4½ in. steel plate ½ in. thick. For the non-stressed ends, the wire head bears directly on the steel bearing plate without the shims. Both the stressing washer and the bearing plate are made of high-strength steel, such as plow steel.

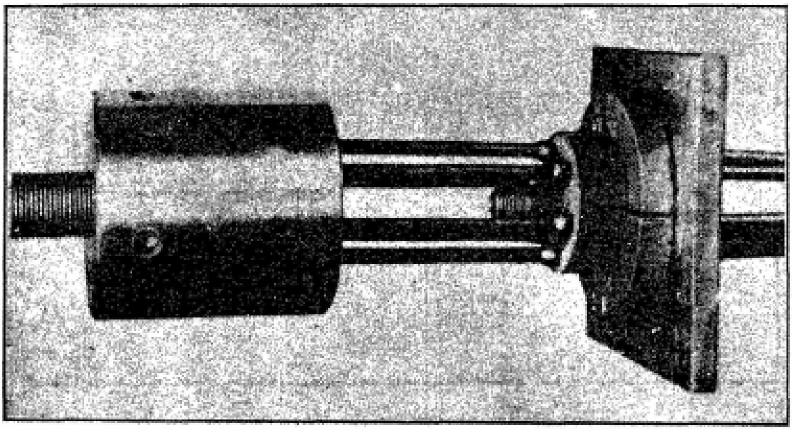


Fig. 3-5-2. Complete P.I. assembly. Left to right: pulling rod, stressing adapter, stressing assembly (contained within adapter), grouting nipple, anchorage assembly, split holding rings, bearing plates.

The Texas P.I. (Texas Prestressing Incorporated) system differs from the Prescon system in that two rivet heads instead of one are formed on the wire at the stressing end, Fig. 3-5-2. The first head at the end of the wire is used for pulling and the second one for anchoring. After tensioning, the extruding portion is cut off up to the second rivet head. By employing two heads, long shims and thick concrete coverage at the ends sometimes required for the Strescon system are eliminated. Since it is necessary to pull the second rivet head through the bearing plate, its anchorage to the plate requires special hardware such as shown in the figure. The usual wire size is also ¼ in. for the Texas P.I. system, and the number of wires per cable varies from 4 to 12. A typical bearing plate for a 6-wire unit is 5 in. by 5 in. by ½ in. thick, with an accompanying split holding ring 3½ in. in diameter by ½ in. thick.

In Switzerland, a system of anchorage using cold-formed rivet heads similar to the Prescon system was developed. It was known as the B.B.R.V. system and has been applied both in that country and abroad.

In certain systems the wires are connected to a short rod of high-tensile steel which can be anchored by nuts and washers. In Germany, three such systems were developed, differing in the method of connecting the wires to the end rod. In the Leoba system, a short

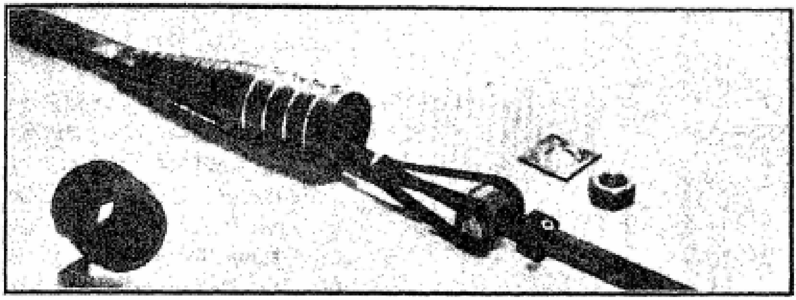


Fig. 3-5-3. End anchorage, Leoba system.

tee is formed at one end of the stub, around which the wires are looped, Fig. 3-5-3. The Monierbau system spreads the wires in a steel cone and anchors them with zinc or lead, as in the case of Roebling anchorage for strands, described later in the chapter. The Huettenwerk Rheinhausen system uses a cylinder for connection. For the Leonhardt and the Billner systems, the wires loop around the concrete and bear directly on it (see Fig. 3-3-5).

3-6 Post-Tensioning Anchorages for Bars

A suitable end anchorage for high-tensile steel bars in prestressed concrete was developed by Donovan Lee of England where it is known as the Lee-McCall system. In this country, it is known as the Stressteel system of the Republic Steel Corporation. The ends of the bars are threaded and anchored with nuts on washers and bearing plates. The essential point is the proper threading of the ends to take a special nut capable of developing as nearly as possible the full strength of the bar. By using tapered threads, about 98% of the bar strength is developed, Fig. 3-6-1.

Only a short length of the bar is threaded at the untensioned end, sufficient to receive the nut resting on a washer. For the jacking end, a long threaded end is required; the total length of thread is such that, after tensioning to the full value, the nut will be turned to the very

bottom of the taper so as to develop the full strength of the bar. If, owing to non-uniformity of material or of construction, the bar has to be lengthened more than the calculated amount in order to obtain the desired prestress, split washer shims may be inserted between the nut and the regular washer. Overtensioning will be necessary if because of friction or for other reasons the bar cannot be lengthened to the predicted amount under the desired prestress.

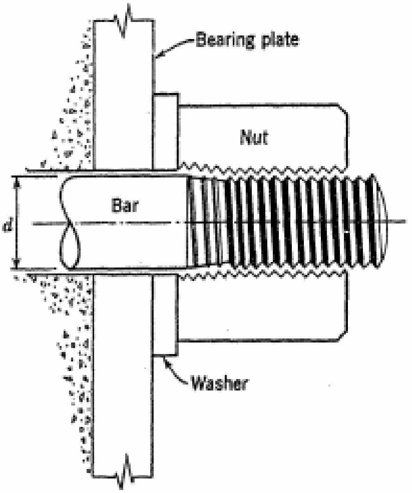


Fig. 3-6-1. End anchorage for the Lee-McCall or Stressteel system.

During jacking, an adapter from the jack screws into the threaded end of the bar to apply the pull. Since the jacking force for each bar is never more than 60 or 70% of its ultimate strength, the net section at the root of thread is not critical during jacking. However, as mentioned previously, it is a good safety measure to keep the nut near the washer at all times during the process of jacking. After completion of the prestressing operations, the protruding threaded ends can be either cut off or buried in concrete together with the anchorage plates.

The bars can be either bonded or unbonded to the concrete. If unbonded, they can be encased in flexible metal tubing or coated with grease and wrapped with heavy paper. They are then placed and supported in the forms before concreting. If bonded, the bars can be placed either before or after the pouring of concrete. For prepouring placement, flexible metal tubes with inside diameter about ¼ in. greater than the bar size are used to facilitate grouting. For post-pouring placement, hole-forming cores such as inflated rubber or rubber with stiffening bars can be employed.

The hexagonal nuts for the bars have a short diameter equal to about twice the bar diameter and a thickness about 1.6 times the bar diameter. The standard and split washers are made of 3⁄16-in. and 14-gage metal. The anchorage plates differ in size and can accommodate 1 to 3 bars per plate. The plates have a thickness of ⅝ to 1½ in. and an area per bar equal to about $(5d)^2$, where d is the bar diameter. If sleeve couplers are used to splice the bars, it is necessary to

provide enough space near the couplers to permit movement during the tensioning process.

In other countries, similar methods of anchorage for high-tensile bars have been devised. They differ in detail from the Lee-McCall or Stressteel system, but thread and nuts bearing against washers are employed in all methods. In Germany alone, four such methods have been developed, namely, the Dywidag, Finsterwalder, Karig, and Polensky and Zollner systems; in Belgium, the Wets system; in Holland, the Bakker system. These will not be described here.

3-7 Post-Tensioning Anchorages for Strands

For pre-tensioning, wire strands can be gripped by strand vises mentioned in section 3-2. For post-tensioning, the only commercial strand anchorages in this country are those of the Roebling system. The Roebling system uses anchorages similar to those long employed for rope suspenders of suspension bridges. The ends of the wires of a strand are spread into a bushing and buried with zinc poured in a conical funnel of cast-steel tube, Fig. 3-7-1. The outer end of the tube is threaded both inside and outside. During prestressing, a threaded rod on the jack is attached to the inside of the tube in order to pull the cable. After the cable is pulled to its desired elongation and stress, the nut threaded to the outside of the tube is turned tight to bear against the bearing plate, which in turn rests on the concrete. A small length of pipe is embedded in the concrete to house the fittings before prestressing and also to transmit, through bond, part of the prestress from the bearing plate to the concrete. The same anchorage is used on the unjacked end with the nut bearing on the plate previous to tensioning.

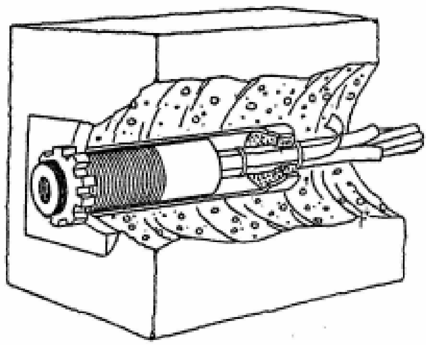


Fig. 3-7-1. End anchorage for the Roebling system.

A second type of the Roebling anchorage consists of a long threaded stud at the prestressing end, to which are attached both the jack adapter and the anchoring nut. The nut is turned tight after the cable is jacked to its desired prestress. For both types, the amount of lengthening must be figured out beforehand to make sure that the threaded portion is long enough for the anchorage. If the length of

the threaded portion is insufficient, special split washers must be added for bearing. In other cases, the entire threaded portion is hidden within the pipe before jacking. Then the pipe must be large enough to house the jack adapter for tensioning.

3-8 Comparison of Systems

It is very difficult to compare the advantages of various systems of prestressing. Speaking in general, established systems that have been proved by tests and service can all be considered safe ones. That does not preclude the possibility that newer and perhaps better systems may be developed. Any new system, however, should be subjected to adequate tests before it can be safely adopted in practice.

Owing to the manner in which prestressed concrete has developed, any "prestressing system" generally embodies several essential features, some of which are also adopted by other systems in one form or another. For example, the methods of providing the conduits, the size, number, and arrangement of wires, and the basic principles of jacking and of anchoring are common to many methods. The essential difference between the systems, then, lies usually in the following three features: the material for producing the prestress, the details of jacking process, and the method of anchoring.

First of all, there is the choice between pre-tensioning and post-tensioning. When a pre-tensioning plant is accessible, and the precast member can be conveniently transported, pre-tensioning will generally be found to be the cheaper, because of the saving in end anchorages, in conduits, and in grouting, and because of the centralization of the production process. If a plant is too far away, the cost of transportation may be excessive. If a plant has to be established just for one job, the costs may be prohibitive unless the job is big enough to justify such an establishment. Long and heavy members can best be poured in place or cast in blocks to be post-tensioned at the site, and pre-tensioning may not be economical.

In the United States, pre-tensioning using small wires has proved to be relatively uneconomical and has given way to the adoption of wire strands. Seven-wire strands up to ⅜-in. diameter have been successfully employed. Also, pre-tensioning with anchorages has been found to be desirable. Anchored pre-tensioning allows much larger tendons to be used and hence reduces the expense of handling and installing. Such anchorage serves only to anchor the tendons and does not have to provide for the jacking grips, and hence is cheaper than the post-tensioning anchorages.

One important shortcoming in pre-tensioning is the fact that its

application has been limited to the employment of straight wires tensioned between two bulkheads. Hence advantage cannot be taken of the curving and bending of cables so beneficial to many beam layouts. In modern pre-tensioning plants, however, provisions are made so that pre-tensioning wires can be bent at almost any point. Heavy girders using bent-up wires anchored to the beds at the points of bending were built for the New Northam Bridge, Southampton, England, in 1954.

For post-tensioning, the members can be either precast or cast in place. There is a further choice between bonded and unbonded reinforcing. Most present-day systems permit the use of either the bonded or the unbonded type. Certain systems yield a slightly better bond than others, depending upon the passage provided for grouting and the bonding perimeter afforded per unit of prestressing force. For other systems, the tendons can be more easily greased and wrapped for unbonded reinforcing. When competition is keen between systems, the choice of the bonded or the unbonded type may decide the economy of one system as against the others.

Another important decision is the choice of proper materials for prestressing, whether wires, strands, or bars. Wires possess higher unit strength than the others; strands and bars mean fewer units for handling. The strength of strands is close to that of wires, but the strands cost more per pound. Bars possess the least strength; they cost more than the wires; but they are easier to handle and cheaper to anchor.

Anchorages for strands are more costly, but the percentage cost of anchorages decreases with the length of tendons. Bars require splices for longer length, whereas strands and wires can be supplied without splice for almost any length of tendon. These are some of the inherent advantages and disadvantages of each material. Once the choice of materials is made, the choice of prestressing systems is further narrowed down. In the United States, only one or two systems are available for strands or bars, though several systems are employed for wires. Since the choice of materials almost automatically dictates the choice of prestressing systems and tends to eliminate competition, it is common practice not to commit the design to any one material. This is often achieved by specifying the amount of effective prestress instead of the material and area of the tendons.

The final decision is often an economic one, i.e., which system will work out the cheapest. There are some basic advantages to each system. For example, when fewer wires are stretched per operation, smaller jacks will be needed; they are easier to handle but take more

time for the total tensioning. Systems where the jacking is done all at once demand jacks of much greater capacity, which are naturally more costly and more cumbersome to move.

For any particular structure at a given time and location, one system will come out to be the most economical. This is usually the result of the surrounding conditions as much as the inherent advantages of that system. The availability of service from the system's representatives, the accessibility of materials and equipments, the acquaintance of the designing engineer with a particular system, and the desirability and ability of the system to get that job often form the deciding factors. Most surviving popular systems possess a number of merits of their own, but the economy of each system will vary with each job.

Finally, an engineer designing for any particular system should refer to the pamphlets issued by the respective companies for details so that his structure may be designed accordingly. It is also possible that the representatives or licensees of a system will be able to furnish special advice as to how the structure can be suitably designed. The engineer can learn much from such advice, although he should always depend upon his own judgment for the final decision. In order that engineers in this country may conveniently get in touch with the various system representatives, a list of the head offices is given in Appendix B.

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Chapter 4 Loss of Prestress: Friction

4-1 Elastic Shortening of Concrete

Let us first consider pre-tensioned concrete. As the prestress is transferred to the concrete, the member shortens and the prestressed steel shortens with it. Hence there is a loss of prestress in the steel. Considering only the shortening of concrete due to the direct load effect of prestressing (the effect of bending of concrete will be considered in section 4-5), it is given by

$$\text{Unit shortening } \delta = \frac{f_c}{E_c}$$

$$= \frac{F_0}{A_c E_c}$$

where F_0 is the final prestress after the shortening has taken place. Loss of prestress in steel is

$$\Delta f_s = E_s \delta = \frac{E_s F_0}{A_c E_c} = \frac{n F_0}{A_c} \tag{4-1-1}$$

The value of F_0, being the prestress after transfer, may not be known exactly. But exactness is not necessary in the estimation of F_0, because the loss due to this shortening is only a few per cent of the total prestress, hence an error of a few per cent in estimating the loss will have no practical significance. It must be further remembered that the value of E_c cannot be accurately predicted either. However, since the value of F_i is usually known, a theoretical solution can be obtained by the elastic theory. Using the transformed-section method, with $A_t = A_c + nA_s$, we have

$$\delta = \frac{F_i}{A_c E_c + A_s E_s}$$

$$\Delta f_s = E_s\delta = \frac{E_s F_i}{A_c E_c + A_s E_s}$$

$$= \frac{nF_i}{A_c + nA_s}$$

$$= \frac{nF_i}{A_t} \qquad (4\text{-}1\text{-}2)$$

Example 4-1-1

A straight pre-tensioned concrete member 40 ft long, with a cross section of 15 in. by 15 in., is concentrically prestressed with 1.2 sq in. of steel wires which are anchored to the bulkheads with a stress of 150,000 psi (Fig. 4-1-1). If

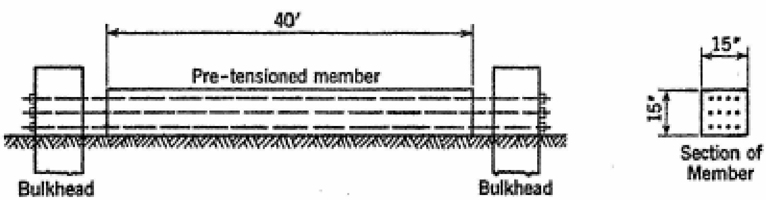


Fig. 4-1-1. Example 4-1-1.

$E_c = 5{,}000{,}000$ psi and $E_s = 30{,}000{,}000$ psi, compute the loss of prestress due to the elastic shortening of concrete at the transfer of prestress.

Solution. Using formula 4-1-1 and the initial prestress of $150{,}000 \times 1.2 = 180{,}000$ lb, we have

$$\Delta f_s = \frac{nF_o}{A_c} = \frac{6 \times 180{,}000}{225} = 4800 \text{ psi}$$

indicating a loss of $4800/150{,}000 = 3.2\%$.

Note that, theoretically, an A_c of $225 - 1.2 = 223.8$ sq in. could have been used, but most of the time the gross concrete area can be employed with little error.

If a more exact solution is desired, formula 4-1-2 yields

$$\Delta f_s = \frac{nF_i}{A_c + nA_s} = \frac{6 \times 180{,}000}{223.8 + 6 \times 1.2} = 4660 \text{ psi}$$

indicating a loss of $4660/150{,}000 = 3.1\%$, only slightly different from the above approximate solution.

For post-tensioning, the problem is different. If we have only a single tendon in a post-tensioned member, the concrete shortens as that tendon is jacked against the concrete. Since the force in the cable is measured after the elastic shortening of the concrete has

taken place, no loss in prestress due to that shortening need be accounted for.

If we have more than one tendon and the tendons are stressed in succession, then the prestress is gradually applied to the concrete, the shortening of concrete increases as each cable is tightened against it, and the loss of prestress due to elastic shortening differs in the tendons. The tendon that is first tensioned would suffer the maximum amount of loss due to the shortening of concrete by the subsequent application of prestress from all the other tendons. The tendon that is tensioned last will not suffer any loss due to the elastic concrete shortening, since all that shortening will have already taken place when the prestress in the last tendon is being measured. The computation of such losses can be made quite complicated. But, for all practical purposes, it is accurate enough to determine the loss for the first cable and use half of that value for the average loss of all the cables. This is shown in example 4-1-2.

Example 4-1-2

Consider the same member as in example 4-1-1, but post-tensioned instead of pre-tensioned. Assume that the 1.2 sq in. of steel is made up of 4 tendons with 0.3 sq in. per tendon. The tendons are tensioned one after another to the stress of 150,000 psi. Compute the loss of prestress due to the elastic shortening of concrete.

Solution. The loss of prestress in the first tendon will be due to the shortening of concrete as caused by the prestress in the other 3 tendons. Although the prestress differs in the 3 tendons, it will be close enough to assume a value of 150,000 psi for them all. Hence the force causing the shortening is

$$3 \times 0.3 \times 150{,}000 = 135{,}000 \text{ lb}$$

The loss of prestress is given by formula 4-1-1,

$$f_s = \frac{nF_0}{A_c} = \frac{6 \times 135{,}000}{225} = 3600 \text{ psi}$$

Note that it is again unnecessary to use the more exact formula 4-1-2; neither is it necessary to use the net concrete area for A_c, although theoretically it would be more accurate.

Similarly, the loss in the second tendon is 2400 psi, in the third tendon 1200 psi, and the last tendon has no loss. The average loss for the 4 tendons will be

$$\frac{3600 + 2400 + 1200}{4} = 1800 \text{ psi}$$

indicating an average loss of prestress of 1800/150,000 = 1.2%, which can also be obtained by using one-half of the loss of the first cable,

$$3600/2 = 1800 \text{ psi}$$

The above method of computation assumes that the tendons are stretched in succession and that each is stressed to the same value as

indicated by a manometer or a dynamometer. It is entirely possible to jack the tendons to different initial prestresses, taking into account the respective amount of loss, so that all the tendons would end up with the same prestress after deducting their losses. Considering the above example, if the first cable should be tensioned to a stress of 153,600 psi, the second to 152,400, the third to 151,200, and the last to 150,000, then, at the completion of the prestressing process, all the tendons would be stressed to 150,000 psi. Such a procedure, although theoretically desirable, is seldom carried out because of the additional complications involved in the field. When there are many tendons and the elastic shortening of concrete is appreciable, it is sometimes desirable to divide the tendons into three or four groups; each group will be given a different amount of overtensioning according to its order in the jacking sequence.

In actual practice, either of the following two methods is used:

1. Stress all tendons to the specified initial prestress (e.g., to 150,000 psi in example 4-1-2), and allow for the average loss in the design (e.g., 1800 psi in example 4-1-2).

2. Stress all tendons to a value above the specified initial prestress by the magnitude of the average loss (e.g., to 150,000 + 1800 = 151,800 psi in example 4-1-2). Then, when designing, the loss due to the elastic shortening of concrete is not to be considered again.

If the loss due to this source is not significant, the first method is followed. If the steel can stand some overtensioning, and if a high effective prestress is desired, the second procedure can be adopted.

The above discussion refers to the case when the prestress in the tendons is measured by manometer or dynamometer and only approximately checked by elongation measurements. At other times, prestress is measured by the amount of elongation, the gages being used merely as a check. The preference for one or the other depends on many factors: the personal practice of the engineer, the accuracy of the different instruments available, the constancy of the modulus of elasticity of steel, the amount of friction in the tendons, as well as the system of prestressing employed. In the previous discussion, when the gages are used as the final means of measuring, it is evident that if the tendons are tensioned to the same initial stress they will eventually possess different effective prestresses, and hence different effective elongations, although their elongations may not vary by more than 2 or 3% and may not be easily detectable. In other words, the tendons are supposedly stretched to the same initial elongation, but, as the concrete shortens gradually, the tendons that are stretched earlier lose more of their tensile strains and therefore lose more of their tensile stresses.

If, on the other hand, the eventual elongations are used as the measure of stress, the problem would be different. Consider the Texas P.I. system, for example, where a second rivet head is preformed at a distance inside the pulling head equal to the computed elongation, the elongation of each tendon relative to the concrete is kept constant at all times, and therefore any loss of prestress due to the shortening of concrete would be uniform for all tendons irrespective of their order of tensioning. This means that the wires will have different initial prestresses, that the initial elongation of the tendons will be different, but that the eventual elongations will be the same for all. This is also true for the Prescon system, if all inserted wedges at the ends of anchorages are made of the same length. For a system like the Prescon, if the tendons are stressed to the same initial prestress, they will have varying effective prestress, and the wedges will have to vary in length. The difference in length, however, is seldom more than 2 or 3% and is hardly noticeable.

4-2 Creep and Shrinkage in Concrete

The amounts and nature of shrinkage and creep having been discussed in section 2-2, their effect on the loss of prestress will now be considered. First, let us consider creep. Since the amount of creep is often 1 to 2 times the elastic shortening, it is obviously an important item. Furthermore, although the loss due to elastic shortening can be counterbalanced for post-tensioned members, the loss due to creep cannot be easily compensated for. It is not possible to overtension the wires excessively in order to allow for such loss, because that would mean very high initial stresses in the steel which might increase its creep or approach its yield point. If the steel is unbonded or not yet bonded to the concrete, it is sometimes possible to retension the steel after some of the losses have taken place. But this might be expensive and undesirable.

It is known that the failure of early efforts at prestressing was attributed largely to the lack of knowledge concerning creep in concrete. In fact, it is still one of the main sources of loss and a serious one if the prestress in the steel is low and the compression in the concrete is high.

Assuming a prestress in the concrete of 1000 psi, for $E_c = 5{,}000{,}000$ psi, $E_s = 30{,}000{,}000$ psi, and creep coefficient $C_c = 2.5$, the loss of prestress in the steel due to creep in concrete is

$$\Delta f_s = (C_c - 1)\frac{f_c E_s}{E_c} = (C_c - 1)nf_c = 1.5 \times 6 \times 1000$$
$$= 9000 \text{ psi}$$

For an initial prestress of 150,000 psi in the steel, this is a loss of 9000/150,000 = 6%.

So far as creep due to prestress is concerned, the amount of prestress producing the creep can be assumed to be constant in computing the losses. Actually the prestress decreases as creep takes place. But neither can we predict creep coefficient with any great accuracy, and there is no point in splitting hairs in a practical design.

A more complicated problem is to determine the deflection of beams due to eccentric prestressing. Here not only the deflection of beams due to prestress is affected by creep, but also the deflection due to external load is similarly affected. Hence the eventual deflection of a prestressed beam will depend upon the duration of the external load which often cannot be well predicted. It must be remembered here that the same holds true for the deflection of ordinary reinforced-concrete beams. And the magnitude may or may not be significant, depending upon the specific conditions.

Since the age of concrete at transfer affects the amount of creep, it is generally true that the pre-tensioned members will have more loss than post-tensioned ones. This is because transfer of prestress usually takes place earlier in pre-tensioned members.

Shrinkage of concrete, as discussed in section 2-2, varies widely. For ordinary prestressed concrete, an average value of shrinkage strain of 0.0003 appears to be about right. The corresponding loss of prestress in steel with a value of E_s = 30,000,000 psi is given by

$$\Delta f_s = 0.0003 \times 30{,}000{,}000 = 9000 \text{ psi}$$

For an initial steel stress of 150,000, this means a loss of another 6%.

To express the amount of loss of prestress in the form of formulas, we can write, for both creep and shrinkage,

$$\Delta f_s = (C_c - 1)nf_c + \delta_s E_s \cdots \tag{4-2-3}$$

The amount of shrinkage varies greatly with the proximity of the concrete to water and the time of application of prestress. Prestress was applied to a certain concrete tank after most of the shrinkage had taken place, and it was found that, when the tank was later filled with water, the restoration of water content to the concrete resulted in considerable expansion which balanced all the creep. On the other hand, if the concrete is prestressed early, before shrinkage has taken place, and then subjected to a very dry atmosphere, the loss of prestress due to shrinkage could be excessive.

The British First Report on Prestressed Concrete, for example, recommends a total shrinkage of 0.0003 for pre-tensioning. For transfer

at 2 to 3 weeks, a shrinkage of 0.0002 is considered sufficient. For creep in concrete, a strain of 0.0000004 for each psi of stress is recommended for pre-tensioning, and 0.0000003 for post-tensioning applied at about 2 to 3 weeks of age.

For pre-tensioning, the amount of shrinkage is independent of the age at transfer. The total shrinkage starting from the setting of concrete must be considered. The amount of creep, however, would be less if the transfer took place later.

German Specifications DIN 4227, Spannbeton, 1953, specify the following:

	Creep Coefficient C_c	Shrinkage Strain
1. Under water	1.5–2.0	0
2. In very moist air	2.5–3.0	0.0001
3. In ordinary atmosphere	3.0–4.0	0.0002
4. In dry air	4.0–5.0	0.0003

Some of these creep coefficients appear to be unusually high and cannot be confirmed by results obtained in the United States.

Since the coefficient of expansion of steel is nearly the same as that for concrete, there is practically no loss of prestress due to temperature drop if the two materials are subject to the same changes. However, because of the heat of setting of the cement, the eventual temperature drop in concrete after the dissipation of heat can result in loss of prestress. German specifications mentioned that, if the prestress is introduced before the concrete has hardened, the equivalent fall in temperature should be assumed to be 45°F; if the prestress is introduced after the concrete has hardened, the equivalent temperature fall may range from 10° to 40°F, depending upon the hardness of concrete at the time of prestressing.

Except for very massive structures, the heat of setting is almost entirely dissipated within the first week after placing. Hence no such loss needs to be considered in post-tensioning work where the prestress is seldom applied before the concrete is 1 or 2 weeks old. Furthermore, if, during prestressing, the steel has the same temperature as the concrete, there will be no loss due to a temperature drop. For pre-tensioning work, steel is tensioned at room temperature and the concrete sets at a higher temperature; hence there will be a loss due to the temperature drop in the concrete. Assuming a drop of 20°F, for a coefficient of expansion of concrete of 0.000006, the shrinkage strain amounts to nearly

$$0.000006 \times 20 = 0.0001$$

This means a loss of prestress in the steel of

$$30{,}000{,}000 \times 0.0001 = 3000 \text{ psi}$$

which is about 2% for an initial prestress of 150,000 psi.

4-3 Creep in Steel

The amount of creep in steel varies with the kinds of steel, and with the level and duration of stress as described in section 2-6. The

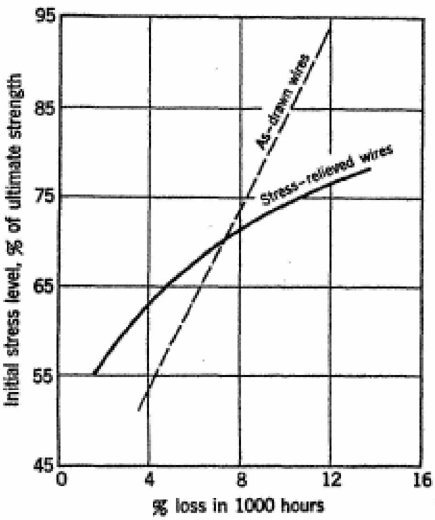


Fig. 4-3-1. Creep of high-tensile 0.196-in. wires from various stress levels. (From "Prestressing Wires—Stress-Relaxation and Stress-Corrosion up to Date," by G. T. Spare, *Wire and Wire Products*, December, 1954.)

effect of such creep on loss of prestress together with its control will now be discussed.

For most kinds of steel now in the market, stressed to the usual allowable values, the percentage of creep varies from 1 to 5%, and an average of 3% should be a fair guess. It is best for the engineer to know the creep characteristics of his steel and to take at least the ordinary precautions to minimize creep. If no precautions are taken, creep may occasionally exceed 10%; Fig. 4-3-1.

Besides choosing the steel with the best creep characteristics, there are several means for diminishing or balancing the loss due to its creep. One well-known method is to overtension the steel and keep it there for a few minutes. Almost any high-tensile steel would

respond to such treatment, and its creep may easily be cut in half if it is overtensioned by 5 to 10% and held there for 2 to 3 minutes. Since overtensioning is often required to balance friction and other losses, this practice is frequently followed, killing several birds with one stone.

Theoretically, it is possible to stress the steel to a certain level and hold it there for a few days until most of the creep has taken place. In practice, retensioning or aftertensioning is sometimes employed, i.e., retightening the steel after most of its creep has taken place. But the expense and inconvenience involved in such operations can be justified only under special circumstances.

Wire strands are usually stress-relieved in the factory before being delivered. Their creep characteristics are believed to be similar to those of individual wires subjected to the same treatment.

The question has sometimes been raised as to the possibility of excessive creep under repeated loads. But, so far as available evidence from fatigue tests show, no such creep need be feared for the ordinary range and duration of stress to which the wire is subject. There has also been some question as to the difference in creep between bonded and unbonded steel. Although no experimental data are available on this, it appears that creep in steel is dependent on its own properties, and not on its bond with concrete.

Some authors have attempted to define a "creep limit" for steel. Professor Campus suggested two possible limits.[4-1] The absolute or theoretical creep limit for a steel is defined as the highest stress that can be applied to it and maintained for a few days without producing any noticeable creep. Since this limit is difficult to determine, a conventional creep limit is proposed and defined as the stress that will not produce more than 1% creep in the steel. Tests have also been performed to determine these creep limits for certain steels.

4-4 Loss Due to Anchorage Take-Up

For most systems of post-tensioning, when a tendon is tensioned to its full value, the jack is released and the prestress is transferred to the anchorage. The anchorage fixtures that are subject to stresses at this transfer will tend to deform, thus allowing the tendon to slacken slightly. Friction wedges employed to hold the wires will slip a little distance before the wires can be firmly gripped. The amount of slippage depends upon the type of wedge and the stress in the wires, an average value being around 0.1 in. For direct bearing anchorages, the heads and nuts are subject to a slight deformation at the release of the jack. An average value for such deformations

may be only about 0.03 in. If long shims are required to hold the elongated wires in place, there will be a deformation in the shims at transfer of prestress. As an example, a shim 1 ft long may deform 0.01 in.

When tensioning heavy strands, the high force in the end fixtures of the strands may produce some slippage in the wires. Depending upon the type of anchorage and the size of the strands, this deformation may be as much as 0.2 in. In such cases, it would be best to retighten all the strands after such losses have taken place if accurate prestress is to be attained.

A general formula for computing the loss of prestress due to anchorage deformation Δ_a is

$$\Delta f_s = \frac{\Delta_a E_s}{L} \tag{4-4-1}$$

Since the loss of prestress is caused by a definite total amount of shortening, the percentage of loss is higher for short wires than for long ones. Hence it is quite difficult to tension short wires accurately, especially for systems of prestressing whose anchorage losses are relatively large. For example, the total elongation for a 10-ft tendon at 150,000 psi is about

$$\frac{150{,}000 \times 10 \times 12}{30{,}000{,}000} = 0.6 \text{ in.}$$

and a loss of 0.1 in. would be a loss of 17%. On the other hand, for a wire of 100 ft, a loss of only 1.7% would be caused by such slippage, and it can be easily allowed for in the design or counterbalanced by slight overtensioning.

4-5 Loss Due to Bending of Member

Loss of prestress due to a uniform shortening of the member under axial stress was discussed in sections 4-1 and 4-2. When a member bends, further changes in the prestress may occur: there may be either a loss or a gain in prestress, depending upon the direction of bending and the location of the tendon. If there are several tendons and they are placed at different levels, the change of prestress in them will differ. Then it will be convenient to consider only the centroid of all the tendons (the c.g.s. line) to get an average value of the change in prestress.

This change in prestress will depend on the type of prestressing: whether pre- or post-tensioned, whether bonded or unbonded. Before the tendon is bonded to the concrete, bending of the member will

affect the prestress in the tendon. Neglecting frictional effects, any strain in the tendon will be stretched out along its entire length, and the prestress in the tendon will be uniformly modified. After the tendon is bonded to the concrete, any further bending of the beam will only affect the stress in the tendon locally but will not change its "prestress."

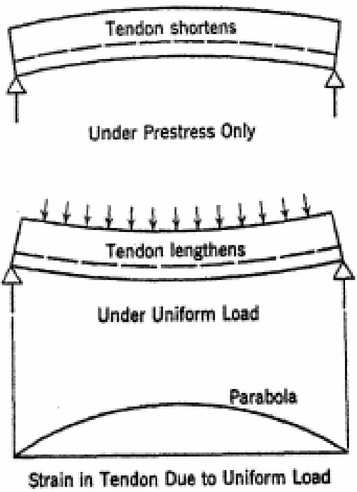


Fig. 4-5-1. Variation of strain in a bonded tendon.

Consider a simple beam, where the tendon is bonded to the concrete either by pre-tensioning or by grouting after post-tensioning, Fig. 4-5-1. Before any load is applied to the beam, it possesses a camber, as shown. Then an external load is applied and the beam deflects downward. The external load produces bending moment in the beam. Bending in the beam changes the unit stresses, hence the unit strains, in the tendon. The stress in the tendon near the mid-span changes quite a bit, but that at the end does not change at all since there is no change in bending moment at the ends. If the "prestress" from the steel on the concrete is considered to be force applied at the ends, the change in "stress" along the length is not considered as a change in "prestress." After the tendon is bonded to the concrete, the steel and concrete form one section, and any change in stress due to bending of the section is easily computed by the transformed section method. Hence, it is convenient to say that "prestress" does not change as the result of bending of a beam after the bonding of steel to concrete, although the stress in the tendon does change.

The same is true of pre-tensioned members bending under prestress and their own weight. Again referring to Fig. 4-5-1, after the transfer of prestress, the beam bends upward and the wires shorten because of that bending. For the same reason as discussed above, that shortening of the wires due to bending is not considered a loss of prestress, just as the eventual lengthening under load is not considered a gain in prestress. In both cases, the prestress considered is the prestress at the ends of the members, which does not change under bending.

For post-tensioned bonded beams before grouting, the bending of

the member will affect the prestress in the steel. Referring to Fig. 4-5-2, suppose that the cables are tensioned one by one and the beam cambers upwards gradually as more cables are tensioned. Then the cables that are tensioned first will lose some of their prestress due to this bending, in addition to the elastic shortening of concrete due to axial precompression. In general, these losses will be small and can be neglected. But when the camber is appreciable, it may be desirable to retension the cables after completing the first round of tensioning or to allow for such losses in the design. Since it is the curvature of the beam at the moment of grouting that determines the length of the tendons, the effect of creep in concrete will exaggerate the curvature and should be taken into account when allowing for such changes in prestress.

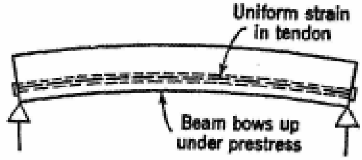


Fig. 4-5-2. Post-tensioned member before grouting.

For post-tensioned unbonded members, there is not only a loss of prestress due to bending caused by prestressing but there is also a gain in prestress when the member is eventually loaded. If the tendons are permitted to slide freely within the concrete they will lengthen and shorten along their entire length as the beam bends. If a tendon does not remain at a constant distance from the c.g.c. line, the computation of the change in length will be quite complicated. Fortunately, the loss or gain due to this source is ordinarily not more than 2 or 3% and for all practical purposes can be approximately estimated and allowed for.

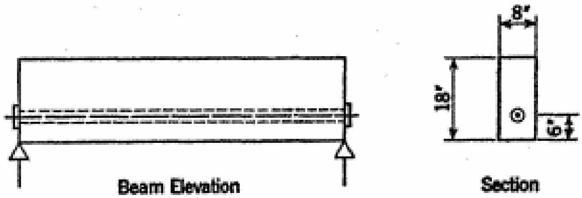


Fig. 4-5-3. Example 4-5-1.

Example 4-5-1

A concrete beam 8 in. by 18 in. deep is prestressed with an unbonded tendon through the lower third point, Fig. 4-5-3, with a total initial prestress of 144,000 lb. Compute the loss of prestress in the tendon due to the bowing up of the beam under prestress, neglecting the weight of the beam itself. $E_s = 30{,}000{,}000$, $E_c = 4{,}000{,}000$ psi. Beam is simply supported.

Solution. Owing to the eccentric prestress, the beam is under a uniform bending moment of

$$144{,}000 \times 3 \text{ in.} = 432{,}000 \text{ in.-lb}$$

The concrete fiber stress at the level of the cable due to this bending is

$$f = \frac{My}{I} = \frac{432{,}000 \times 3}{8 \times 18^3/12} = 333 \text{ psi compression}$$

(Note that stress due to the axial prestress of 144,000 lb is not included here; also, the gross area of the concrete is used for simplicity.)

Unit compressive strain along the level of the tendon is therefore

$$333/4{,}000{,}000 = 0.000083$$

Corresponding loss of prestress in steel is

$$0.000083 \times 30{,}000{,}000 = 2500 \text{ psi}$$

However, if the beam is left under the action of prestress alone, the creep of concrete will tend to increase the camber and will result in further loss of prestress. On the other hand, if the prestress in the tendon is measured after the bowing of the beam has taken place, this loss due to bending of beam need not be considered.

4-6 Frictional Loss, Practical Considerations

Valuable and extensive research work has been carried out to determine the frictional loss of prestress in prestressed concrete,[4-2] so that now it is possible to estimate such losses within the practical requirement of accuracy. First of all, it is known that there is some friction in the jacking and anchoring system so that the stress existing in the tendon is less than that indicated by the pressure gage. This is especially true for some systems whose wires change direction at the anchorage. This friction in the jacking and anchoring system is generally small though not insignificant. It can be determined for each case, if desired, and an overtension can be applied to the jack so that the calculated prestress will exist in the tendon. It must be remembered, though, that the amount of overtensioning is limited by the yield point, the creep limit, and the strength of the wires.

More serious frictional loss occurs between the tendon and its surrounding material, whether concrete or sheathing, and whether lubricated or not. This frictional loss can be conveniently considered in two parts: the length effect and the curvature effect. The length effect is the amount of friction that would be encountered if the tendon is a straight one, i.e., one that is not purposely bent or curved. Since in practice the duct for the tendon cannot be perfectly straight, some friction will exist between the tendon and its surrounding material even though the tendon is meant to be straight. This is some-

times described as the wobbling effect of the duct and is dependent upon the length and stress of the tendon, the coefficient of friction between the contact materials, and the workmanship and method used in aligning and obtaining the duct. Values for these losses are given in Table 4-6-1.

The loss of prestress due to curvature effect results from friction and intended curvature of the tendons in addition to the wobble of the duct. This loss is again dependent on the coefficient of friction between the contact materials and the pressure exerted by the tendon on the concrete. The coefficient of friction, in turn, depends on the smoothness and nature of the surfaces in contact, the amount and nature of lubricants, and sometimes the length of contact. The coefficient has been determined for various conditions as listed in Tables 4-6-1 and 4-6-2. The pressure between the tendon and concrete is dependent on the stress in the tendon and the total change in angle.

For multiple wires arranged in one duct, there are other sources of friction around curves. If the wires in one duct are tensioned in succession, those tensioned later may be subject to excessive friction because the radial component of the tension in the outer wires will tend to press against the inner wires. It would be desirable to tension the inner wires first, but in a tendon with reversed curves there is little choice to make. If wire separators are employed in the duct they may be disarranged, as the units are tensioned successively, and thus produce additional friction. These are individual points which may sometimes be significant, depending upon the existing conditions.

The Cement and Concrete Association of England has conducted extensive experiments in an effort to determine the coefficient of friction and the wobble effect for computing the frictional loss in various systems.[4-2] The coefficient of friction between the tendon and the duct is denoted by μ; the wobble effect of the duct is expressed by K, giving the loss of prestress per foot of length. It is evident that these coefficients, μ and K, will depend upon a number of factors: the type of steel used, whether wires, strands, or rods; the kind of surface, whether indented or corrugated, whether rusted or cleaned or galvanized. The amount of vibration used in placing the concrete will affect the straightness of the ducts; so will the overall size of the duct and its excess over the enclosed steel, and the spacing of the supports for the duct-forming material. Table 4-6-1 gives the average values for a few common cases. Individual values vary greatly, and the readers are referred to the original publication if more accurate information is desired. As an example, for Freyssinet

TABLE 4-6-1

Frictional Coefficients μ and Wobble Effect K

	Freyssinet System		Magnel System		Lee-McCall System	
	μ	K/ft	μ	K/ft	μ	K/ft
Duct formed by withdrawing steel tubing or rods	0.55	0	0.30	0.0010	0.55	0.0005
Rubber core unstiffened	0.55	0.0020	0.30	0.0005	0.55	0.0010
Rubber core stiffened internally	0.55	0.0005	0.30	0.0005	0.55	0.0005
Metal sheathing	0.35	0.0010	0.30	0.0005	0.30	0.0005

cables in light metal sheathing, the K value can be as high as 0.0050 if heavy vibration is applied to the concrete and light-gage metal is used for sheathing without adequate supports.

Additional frictional coefficients have been determined by Dr. Leonhardt for different stressing wires and strands on various underlays.[4-3] He also mentioned many factors that cannot be predetermined, such as thin sheet metal casings which may be worn through so that the wires slide against the concrete, or the lateral binding of wires in curved sections, or the uneven movement of the separators due to the elongation of wires. Some of his values are tabulated in Table 4-6-2, but they are intended only as a guide for the normal conditions.

TABLE 4-6-2

Frictional Coefficients μ for Stressing Wires on Various Underlays (Dr. F. Leonhardt)

Type of Wire	Underlay	μ
Drawn wires, 0.196-in. diameter	Smoothly finished concrete	0.29–0.31
	Roughly finished concrete	0.35–0.44
	New black sheet metal	0.16–0.22
Two 0.079-in. wire strands	Smoothly finished concrete	0.38–0.40
	Roughly finished concrete	0.40–0.46
	New black sheet metal	0.19–0.22
Seven 0.099-in. wire strands	Black sheet metal	0.20–0.25
	Paraffin, under pressure of 30 psi	0.10
	Paraffin, under pressure of 700 psi	0.02–0.025

The coefficient of friction depends a great deal on the care exercised in construction. For unbonded reinforcement, lubricants can be used to advantage. Cables well greased and carefully wrapped in plastic tubes will have little friction, but if mortar leaks through openings in

the tube the cables may be tightly stuck. For bonded reinforcement, where lubricants may occasionally be employed, they must be applied very carefully in order not to destroy the eventual bond to be effected by grouting. The use of soluble oils has also been suggested

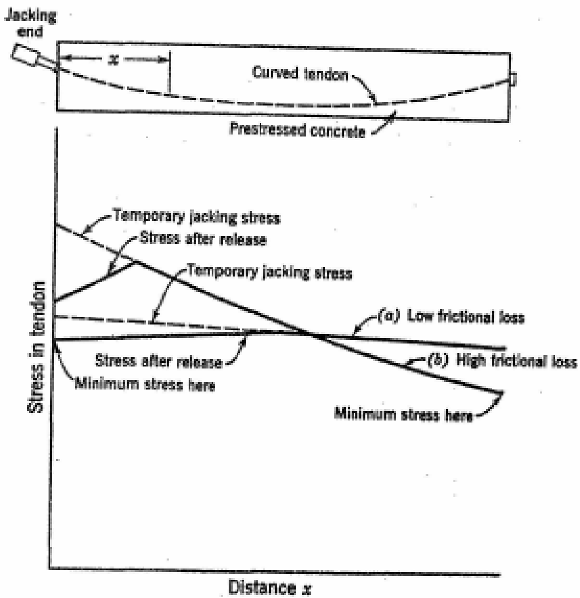


Fig. 4-6-1. Variation of stress in tendon due to frictional force.

to reduce the friction while tensioning, and the lubricant is to be flushed off with water afterwards.

There are several methods of overcoming the frictional loss in tendons. One method is to overtension them. When friction is not excessive, the amount of overtension is usually made to equal the maximum frictional loss. The amount of wire lengthening corresponding to that overtension and the estimated friction can also be computed to serve as a check. This amount of overtension required for overcoming friction is not cumulative over that required for overcoming anchorage losses or for minimizing creep in steel. It is sufficient to take the greatest of the three required values and overtension for that amount. This is because in all three cases the overtensioning consists of an overstretching and a subsequent release-back. It must be noted that, if most of the friction exists near the jacking end, overtensioning to balance that friction will not produce any overstretching

of the main portion of the tendon and hence will not serve to minimize creep to any extent.

The effect of overtensioning with a subsequent release-back is to put the frictional difference in the reverse direction. Thus, after releasing, the variation of stress along the tendon takes some shape as in Fig. 4-6-1. When the frictional loss is a high percentage of the prestress, it cannot be totally overcome by overtensioning (curve *b*, Fig. 4-6-1), since the maximum amount of tensioning is limited by the strength or the yield point of the tendon. The portion of the loss that has not been overcome must then be allowed for in the design.

Jacking from both ends, of course, is another means for reducing frictional loss. It involves more work in the field but is often resorted to when the tendons are long or when the angles of bending are large.

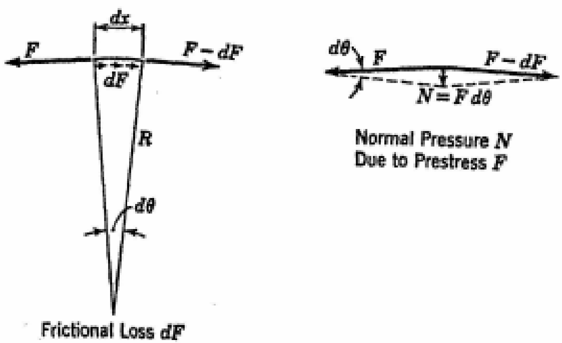


Fig. 4-7-1. Frictional loss along length dx.

4-7 Frictional Loss, Theoretical Considerations

The basic theory of frictional loss of a cable around a curve is well known in physics. In its simple form, it can be derived as follows. Consider an infinitesimal length dx of a prestressing tendon whose centroid follows the arc of a circle of radius R, Fig. 4-7-1, then the change in angle of the tendon as it goes around that length dx is

$$d\theta = \frac{dx}{R}$$

For this infinitesimal length dx, the stress in the tendon may be considered constant and equal to F; then the normal component of pressure produced by the stress F bending around an angle $d\theta$ is given by

$$N = F d\theta = \frac{F\,dx}{R}$$

The amount of frictional loss dF around the length dx is given by the pressure times a coefficient of friction μ, thus,

$$dF = -\mu N$$
$$= \frac{-\mu F\, dx}{R} = -\mu F\, d\theta$$

Transposing, we have

$$\frac{dF}{F} = -\mu\, d\theta$$

Integrating this on both sides, we have

$$\log_e F = -\mu\theta$$

Using the limits F_1 and F_2, we have the conventional friction formula

$$F_2 = F_1 e^{-\mu\theta} = F_1 e^{-\mu L/R} \tag{4-7-1}$$

since $\theta = L/R$ for a section of constant R.

For tendons with a succession of curves of varying radii, it is necessary to apply this formula to the different sections in order to obtain the total loss.

The above formula can also be applied to compute frictional loss due to wobble or length effect. Substituting the loss KL for $\mu\theta$ in formula 4-7-1, we have

$$\log_e F = -KL \qquad F_2 = F_1 e^{-KL} \tag{4-7-2}$$

If it is intended to combine the length and curvature effect, we can simply write

$$\log_e F = -\mu\theta - KL$$

For limits F_1 and F_2,

$$F_2 = F_1 e^{-\mu\theta - KL} \tag{4-7-3}$$

Or, in terms of unit stresses,

$$f_2 = f_1 e^{-\mu\theta - KL} \tag{4-7-3a}$$

The above formulas are theoretically correct ones which take into account the decrease in tension and hence the decrease in the pressure as the tendon bends around the curve and gradually loses its stress due to friction. A chart is given in Fig. 4-7-2 for the solution of equation 4-7-3a. If, however, the total difference between the tension in the tendon at the start and that at the end of the curve is not excessive (say not more than 15 or 20%), an approximate formula using the initial tension for the entire curve will be close enough.

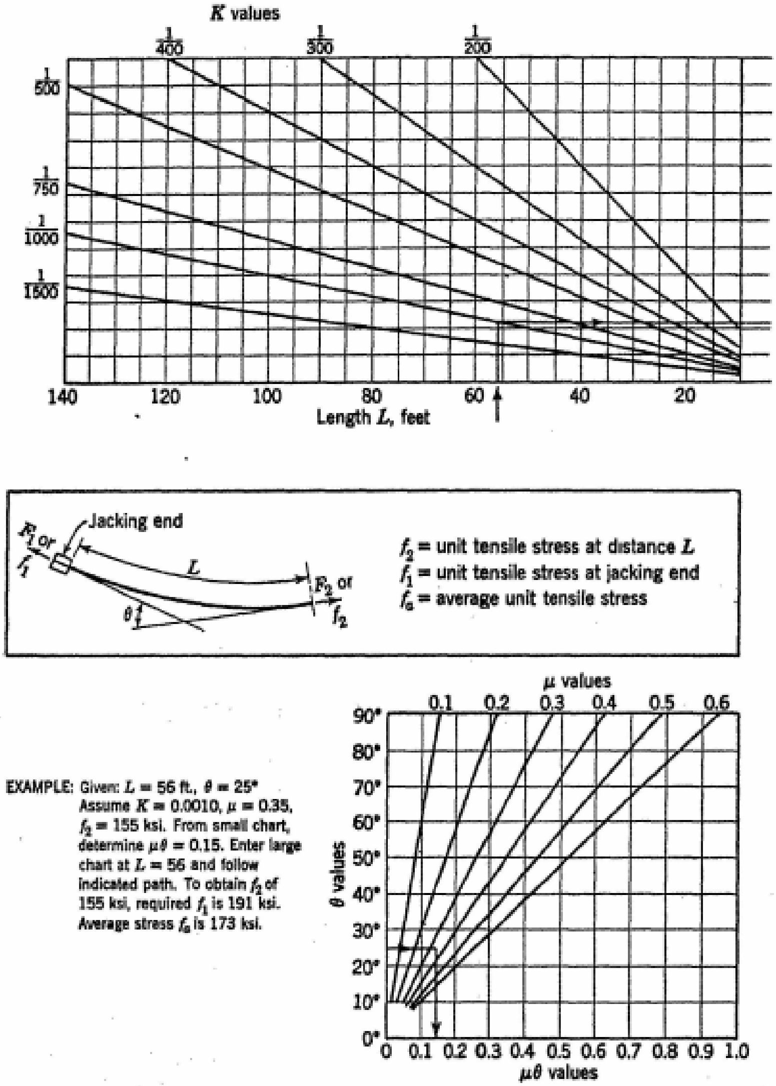
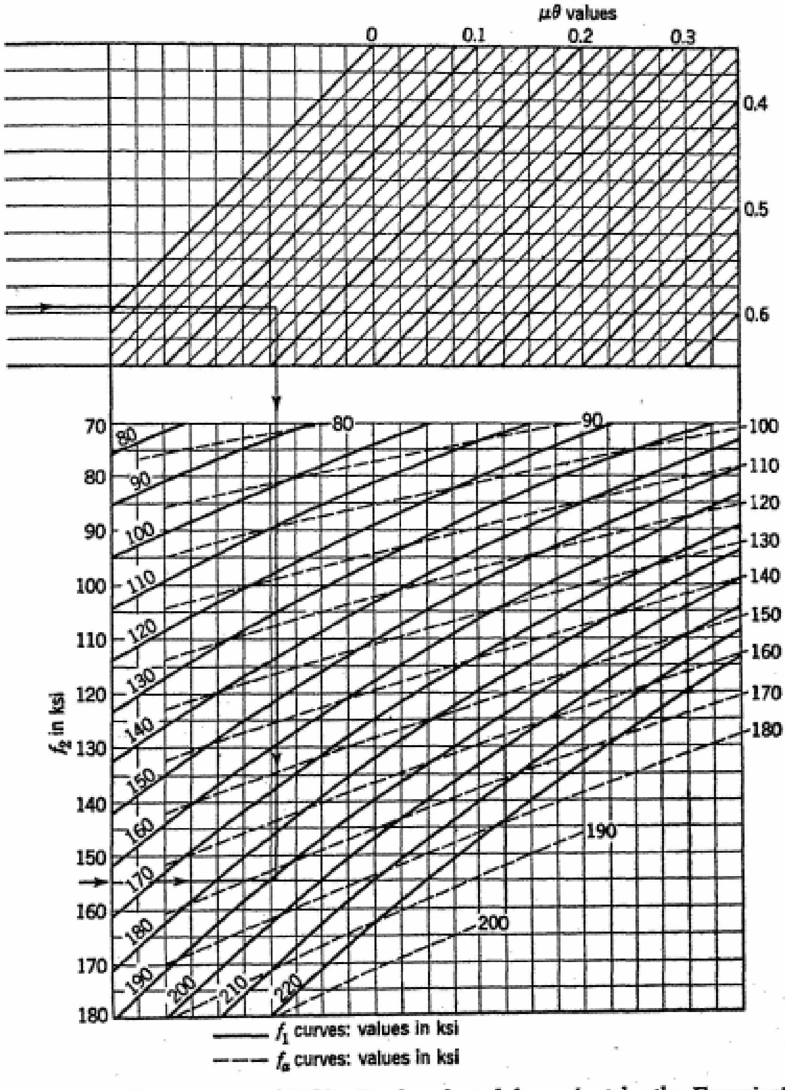


Fig. 4-7-2. Chart for solution of equation 4-7-3g (from *Criteria for Prestressed* Company,



Concrete Bridges, Bureau of Public Roads, adapted from chart by the Freyssinet New York).

On this assumption, a simpler formula can be derived in place of the above exponential form. If the normal pressure is assumed to be constant, the total frictional loss around a curve with angle θ and length L is, Fig. 4-7-3,

$$F_2 - F_1 = -\mu F_1\theta = -\frac{\mu F_1 L}{R} \tag{4-7-4}$$

For length or wobble effect, we can again substitute KL for $\mu\theta$, thus,

$$F_2 - F_1 = -KLF_1 \tag{4-7-5}$$

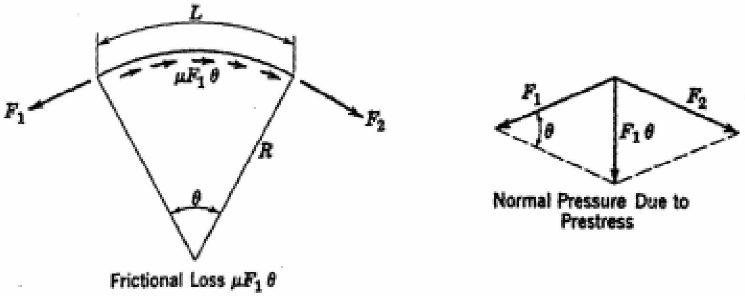


Fig. 4-7-3. Approximate frictional loss along circular curve.

To compute the total loss due to both curvature and length effect, the above two formulas can be combined, giving

$$F_2 - F_1 = -\mu F_1\theta - KLF_1$$

Transposing terms, we have

$$\frac{F_2 - F_1}{F_1} = -KL \quad \mu\theta = -\left(K + \frac{\mu}{R}\right)L \tag{4-7-6}$$

The loss of prestress for the entire length of a tendon can be considered from section to section, with each section consisting of either a straight line or a simple circular curve. The reduced stress at the end of a segment can be used to compute the frictional loss for the next segment, etc.

Since, for practically all prestressed-concrete members, the depth is small compared with the length, the projected length of tendon measured along the axis of the member can be used when computing frictional losses. Similarly, the angular change θ is given by the transverse deviation of the tendon divided by its projected length, both referred to the axis of the member.

Example 4-7-1

A prestressed-concrete beam is continuous over two spans, Fig. 4-7-4, and its curved tendon is to be tensioned from both ends. Compute the percentage loss of prestress due to friction, from one end to the center of the beam (A to E). The coefficient of friction between the cable and the duct is taken as 0.4, and the average "wobble" or length effect is represented by $K = 0.0008$ per ft.

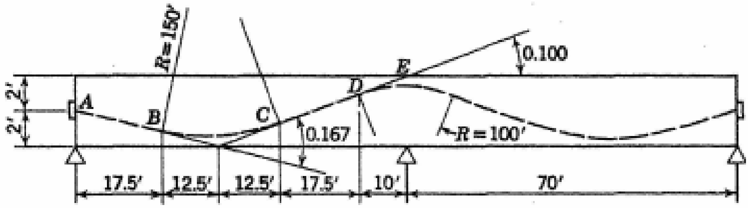


Fig. 4-7-4. Example 4-7-1.

Solution 1. A simple approximate solution will first be presented. Using formula 4-7-6,

$$\frac{F_2 - F_1}{F_1} = -KL - \mu\theta$$

$$= -0.0008 \times 70 - 0.4(0.167 + 0.100)$$

$$= -0.056 - 0.107$$

$$= -0.163$$

Solution 2. The above solution does not take into account the gradual reduction of stress from A toward E. A more exact solution would be to divide the tendon into 4 portions from A to E, and consider each portion after the loss has been deducted from the preceding portions. Thus, for stress at $A = F_1$,

AB, length effect: $KL = 0.0008 \times 17.5 = 0.014$

Stress at $B = 1 - 0.014 = 0.986F_1$

BC, length effect: $KL = 0.0008 \times 25 = 0.020$

Curvature effect: $\mu\theta = 0.4 \times 0.167 = 0.067$

Total: $0.020 + 0.067 = 0.087$

Using the reduced stress at B of 0.986, the loss is $0.087 \times 0.986 = 0.086$.

Stress at $C = 0.986 - 0.086 = 0.900F_1$

CD, length effect: $KL = 0.0008 \times 17.5 = 0.014$

Using the reduced stress of 0.900 at C, the loss is $0.014 \times 0.900 = 0.013$.

Stress at $D = 0.900 - 0.013 = 0.887F_1$

DE, length effect: $KL = 0.0008 \times 10 = 0.008$

Curvature effect: $\mu\theta = 0.4 \times 0.100 = 0.040$

$$\text{Total: } 0.008 + 0.040 = 0.048$$

$$\text{Loss} = 0.048 \times 0.887 = 0.043$$

$$\text{Stress at } E = 0.887 - 0.043 = 0.844F_1$$

$$\text{Total loss from } A \text{ to } E = 1 - 0.844 = 0.156 = 15.6\%$$

The above computation can be tabulated in order to simplify the work. It can be further noticed that this second method yields a loss only slightly less than the first approximate method.

Solution 3. A still more exact solution is to use the conventional friction formula 4-7-3, which takes into account not only the variation of stress from segment to segment but also that from point to point all along the cable. The solution is tabulated as shown.

Segment	L	KL	θ	$\mu\theta$	$KL + \mu\theta$	$e^{-KL-\mu\theta}$	Stress at End of Segment
AB	17.5	0.014	0	0	0.014	0.986	$0.986F_1$
BC	25	0.020	0.167	0.067	0.087	0.916	$0.903F_1$
CD	17.5	0.014	0	0	0.014	0.986	$0.890F_1$
DE	10	0.008	0.100	0.040	0.048	0.953	$0.848F_1$

The total frictional loss from A to E is given as

$$1 - 0.848 = 0.152 = 15.2\%$$

4-8 Total Amount of Losses

Initial prestress in steel minus the losses is known as the *effective* or the design *prestress*. The total amount of losses to be assumed in design will depend upon the basis on which the initial prestress is measured. First, there is the *temporary maximum jacking stress* to which a tendon may be subject for the purpose of minimizing creep in steel or for balancing frictional losses. Then there is a slight release from that maximum stress back to the normal *jacking stress*.

As soon as the prestress is transferred to the concrete, anchorage loss will take place. The *jacking stress* minus the anchorage loss will be the stress at anchorage after release and is frequently called the *initial prestress*. For post-tensioning, losses due to elastic shortening will gradually take place, if there are other tendons yet to be tensioned. This elastic shortening of concrete may be considered in two parts: that due to direct axial shortening and that due to elastic bending, as discussed in sections 4-1 and 4-5. For pre-tensioning, the entire amount of loss due to elastic shortening will occur at the transfer of prestress.

Depending upon the definition of the term *initial prestress*, the amount of losses to be deducted will differ. If the jacking stress minus the anchorage loss is taken as the initial prestress, as described

in the previous paragraph, then the losses to be deducted will include the elastic shortening and creep and shrinkage in concrete plus the creep in steel. This seems to be the most common practice. If the jacking stress itself is taken as the initial prestress, then anchorage losses must be deducted as well. If the stress after the elastic shortening of concrete is taken as the initial prestress, then the shrinkage and creep in concrete and the creep in steel will be the only losses. For points away from the jacking end, the effect of friction must be considered in addition. Frictional force along the tendon may either increase or decrease the stress, as discussed in section 4-6.

The magnitude of losses can be expressed in four ways:

1. In unit strains. This is most convenient for losses such as creep, shrinkage, and elastic shortenings of concrete.

2. In total strains. This is more conveniont for the anchorage losses.

3. In unit stresses. All losses when expressed in strains can be transformed into unit stresses in steel, if the modulus of elasticity of steel is known.

4. In percentage of prestress. Losses due to creep in steel and friction can be most easily expressed in this way. Other losses expressed in unit stresses can be easily transformed into percentages of the initial prestress. This often conveys a better picture of the significance of the losses.

It is difficult to generalize the amount of loss of prestress, because it is dependent upon so many factors: the properties of concrete and steel, curing and moisture conditions, magnitude and time of application of prestress, and the process of prestressing. For average steel and concrete properties, cured under average air conditions, the tabulated percentages may be taken as representative of the average losses.

	Pre-Tensioning, %	Post-Tensioning, %
Elastic shortening and bending of concrete	3	1
Creep of concrete	6	5
Shrinkage of concrete	7	6
Creep in steel	2	3
Total loss	18	15

The table assumes that proper overtensioning has been applied to reduce creep in steel and to overcome friction and anchorage losses. Any frictional loss not overcome must be considered in addition. The common allowance for loss of prestress is 15% for post-tensioning and

about 18% for pre-tensioning. Such allowance is seen to be not too far from the probable values. It must be borne in mind, however, that, when conditions deviate from the average, different allowances should be made accordingly. That is why an understanding and analysis of the sources of loss are of prime importance to the designing engineer. Some empirical formulas for estimating loss of prestress are given in Appendix D and may be used as a general guide.

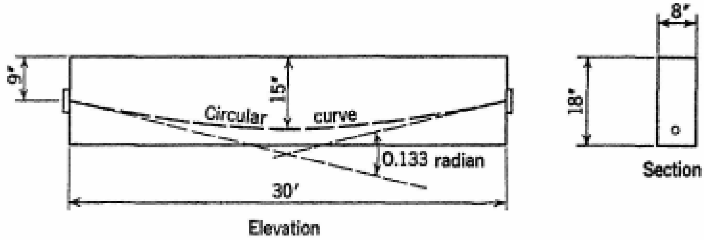


Fig. 4-8-1. Example 4-8-1.

Example 4-8-1

A post-tensioned concrete beam, Fig. 4-8-1, with a cable of 24 parallel wires (total steel area = 1.20 sq in.) is tensioned with 2 wires at a time. The jacking stress is to be measured by jack gage pressure. The wires are to be stressed from one end to a value of f_1 to overcome frictional loss, then released to a value of f_2 so that immediately after anchoring an initial prestress of 120,000 psi would be obtained. Compute f_1 and f_2. Then compute the final design stress in the steel after all losses have taken place. Assume the following:

1. Coefficient of friction $\mu = 0.6$ between steel and concrete, K for length effect = 0.0010.

2. Deformation of anchorage and slippage of wires estimated at 0.05 in. per end. $E_s = 30{,}000{,}000$ psi.

3. Elastic shortening of concrete to be computed for $E_c = 4{,}000{,}000$ psi. Neglect shortening of steel due to bending of beam.

4. Creep coefficient of concrete = 2.2.

5. Shrinkage of concrete = 0.0002.

6. Creep of steel = 3% of initial steel stress.

Solution. 1. The percentage loss of prestress due to friction is given by

$$\text{Length effect} = KL = 30 \times 0.0010 = 0.03$$

$$\text{Curvature effect} = \mu\theta = 0.6 \times .133 = 0.08$$

$$\text{Total } 0.03 + 0.08 = 0.11$$

Hence it is necessary to tension the steel to

$$f_1 = 120{,}000/(1 - 0.11) = 135{,}000 \text{ psi}$$

at one end in order to overcome the friction and obtain a stress of 120,000 psi at the unjacked end. Note that, should the frictional loss be more than 20 or 30%, it may be desirable to apply formula 4-7-3.

2. The anchorage slippage of 0.05 in. occurs only on one end since the unjacked end would have its slippage taking place before the release of jack. The loss in unit strain is given by

$$0.05/360 = 0.00014$$

and the loss in steel stress is

$$0.00014 \times 30{,}000{,}000 = 4200 \text{ psi}$$

Hence, by tensioning the steel to $f_1 = 135{,}000$ psi and then releasing it to $f_2 = 124{,}200$ psi for anchoring, the minimum stress after anchoring will be 120,000 psi. This minimum stress will occur at both ends of the beam.

3. Since the wires are tensioned two by two, the first pair will lose some stress due to the elastic shortening of concrete under the action of the subsequent 11 pairs, and the amount will be approximately

$$\frac{\frac{11}{12} \times 120{,}000 \times 1.20 \times 30{,}000{,}000}{4{,}000{,}000 \times 18 \times 8} = 6800 \text{ psi}$$

The average loss for all the wires will be $6800/2 = 3400$ psi.

4. The total stress in the steel is very nearly 144,000 lb, for which the elastic shortening of concrete is

$$\frac{144{,}000}{4{,}000{,}000 \times 18 \times 8} = 0.00025$$

Creep of concrete $= (2.2 - 1)0.00025 = 0.00030$, which corresponds to a stress of

$$0.00030 \times 30{,}000{,}000 = 9000 \text{ psi in the steel}$$

5. For shrinkage of 0.0002, the loss in the steel stress is

$$0.0002 \times 30{,}000{,}000 = 6000 \text{ psi}$$

6. Creep of steel at 3% of 120,000 = 3600 psi

The total loss can now be summarized:

Elastic shortening	3,400
Creep of concrete	9,000
Shrinkage of concrete	6,000
Creep in steel	3,600
Total	22,000 psi

Eventual loss of prestress = 22,000/120,000 = 18%.

4-9 Elongation of Tendons

It is often necessary to compute the elongation of a tendon caused by prestressing. When fabricating the anchorage parts in the Roebling and Stressteel systems, the expected amount of elongation must be known approximately. For the Prescon and Texas P.I. systems, it must be known rather accurately. For all systems, the measured elongation is compared to the expected value, thus serving as a check on the accuracy of the gage readings or on the magnitude

of frictional loss along the length of the tendon. The computation of such elongation is discussed in two parts as follows.

1 · Neglecting Frictional Loss along Tendon. If a tendon has uniform stress f_s along its entire length L, the amount of elongation is given by

$$\Delta_s = \delta_s L = f_s L/E_s = FL/E_s A_s \qquad (4\text{-}9\text{-}1)$$

For prestress exceeding the proportional limit of the tendon, this formula may not be applicable, and it may be necessary to refer to the stress-strain diagram for the corresponding value of δ_s.

Before any tendon is tensioned, there almost always exists in it a certain amount of slack. For systems requiring shim plates, such as the Prescon system, that slack must be allowed for when computing the length of the shims. In addition, it may be desirable to allow for the shrinkage and elastic shortening of concrete at the time of tensioning. Hence the length of the shims must equal the elastic elongation of the tendon plus the slack in the tendon plus the shortening of concrete at transfer. Conversely, the elastic elongation of the tendon must be computed from the apparent elongation by deducting the initial slack and the shortening of concrete.

It is not easy to determine the slack in a tendon accurately, hence the usual practice is to give the tendon some initial tension f_{s1} and measure the elongation Δ_s thereafter. Then, neglecting any shortening of the concrete, the total elastic elongation of the tendon can be computed by

$$\text{Elastic elongation} = \frac{f_s}{f_s - f_{s1}} \Delta_s \qquad (4\text{-}9\text{-}2)$$

Example 4-9-1

A Prescon cable, 60 ft long, Fig. 4-9-1, is to be tensioned from one end to an initial prestress of 150,000 psi immediately after transfer. Assume that there is no

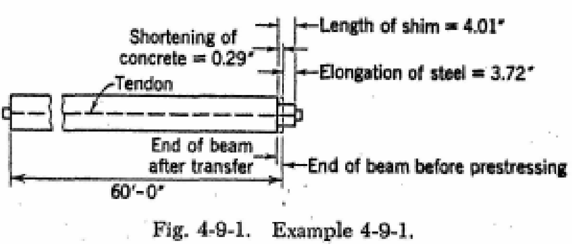


Fig. 4-9-1. Example 4-9-1.

slack in the cable, that the shrinkage of concrete is 0.0002 at time of transfer, and that the average compression in concrete is 800 psi along the length of the tendon. $E_c = 3{,}800{,}000$ psi; $E_s = 29{,}000{,}000$ psi. Compute the length of shims required, neglecting any elastic shortening of the shims and any friction along the tendon.

Solution. From equation 4-9-1, the elastic elongation of steel is

$$\Delta_s = f_s L / E_s = 150{,}000 \times 60 \times 12/29{,}000{,}000 = 3.72 \text{ in.}$$

Shortening of concrete due to shrinkage is

$$0.0002 \times 60 \times 12 = 0.14 \text{ in.}$$

Elastic shortening of concrete is

$$800 \times 60 \times 12/3{,}800{,}000 = 0.15 \text{ in.}$$

Length of shims required is

$$3.72 + 0.14 + 0.15 = 4.01 \text{ in.}$$

If shims of 4.01 in. are inserted in the anchorage, there should remain an initial prestress of 150,000 psi in the steel immediately after transfer.

Example 4-9-2

Eighteen 0.196-in. wires in a Freyssinet cable, 80 ft long, are tensioned initially to a total stress of 3000 lb. What additional elongation of the wires as measured therefrom is required to obtain an initial prestress of 160,000 psi? $E_s = 28{,}000{,}000$ psi. Assume no shortening of concrete during the tensioning process and neglect friction.

Solution.

$$A_s = 18 \times 0.03 = 0.54 \text{ sq in.}$$

$$f_{s1} = 3000/0.54 = 5500 \text{ psi}$$

Total elastic elongation of tendon from 0 to 160,000 psi is

$$f_s L/E_s = 160{,}000 \times 80 \times 12/28{,}000{,}000 = 5.48 \text{ in.}$$

From equation 4-9-2,

$$5.48 = \frac{f_s}{f_s - f_{s1}} \Delta_s$$

$$= \frac{160{,}000}{160{,}000 - 5500} \Delta_s$$

$$\Delta_s = 5.28 \text{ in.}$$

Thus, with zero reading taken at a total stress of 3000 lb, an elongation of 5.28 in. must be obtained for a prestress of 160,000 psi.

2 · Considering Frictional Loss along Tendon. It was shown in section 4-7 that, for a curved tendon with a constant radius R, the stress at any point away from the jacking end is

$$F_2 = F_1 e^{-(\mu\theta + KL)}$$

The average stress F_a for the entire length of curve with stress varying from F_1 to F_2 can be shown to be

$$F_a = F_2 \frac{e^{\mu\theta + KL} - 1}{\mu\theta + KL} \qquad (4\text{-}9\text{-}3)$$

This equation is solved graphically in Fig. 4-7-2, where the dotted lines give the values of $f_a = F_a/A_s$.

The total lengthening for length L is given by

$$\Delta_s = \frac{F_a L}{E_s A_s} = \frac{F_2 L}{E_s A_s} \frac{e^{\mu\theta + KL} - 1}{\mu\theta + KL} \tag{4-9-4}$$

If only an approximate solution is desired, the medium value of F_1 and F_2 can be used in computing the elongation, thus

$$\Delta_s = \frac{F_1 + F_2}{2} \frac{L}{E_s A_s} \tag{4-9-5}$$

Example 4-9-3

A tendon 80 ft long is tensioned along a circular curve with $R = 102$ ft, Fig. 4-9-2. For a unit stress of 180,000 psi applied at the jacking end, a total elonga-

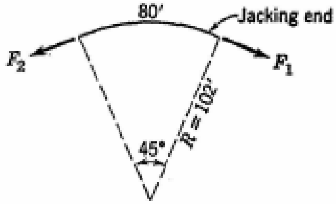


Fig. 4-9-2. Example 4-9-3.

tion of 4.80 in. is obtained. $E_s = 30{,}000{,}000$ psi. Compute the stress f_2 at the far end of the tendon.

Solution. 1. Approximate solution. Average stress in the tendon is given by

$$f_a = \Delta_s E_s / L = 4.80 \times 30{,}000{,}000 / (80 \times 12)$$
$$= 150{,}000 \text{ psi}$$

Since the maximum stress is 180,000 psi, the minimum stress f_2 must be 120,000 psi, assuming uniform decrease in the stress.

2. Exact solution. Using Fig. 4-7-2, enter the diagram with $f_1 = 180$ ksi and $f_a = 150$ ksi. At the intersection of these two curves, read $f_2 = 125$ ksi.

References

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4-2 E. H. Cooley, *Friction in Post-Tensioned Prestressing Systems*, and *Estimation of Friction in Prestressed Concrete*, Cement and Concrete Assn., London, 1953.

4-3 F. Leonhardt, "Continuous Prestressed Concrete Beams," *J. Am. Conc. Inst.*, March, 1953 (*Proc.*, Vol. 49), p. 617.

Chapter 5 Analysis of Sections for Flexure

5-1 Introduction and Sign Conventions

Differentiation can be made between the *analysis* and *design* of prestressed beam sections, as is done for reinforced-concrete beams. By *analysis* is meant the determination of stresses in the steel and concrete when the form and size of a section are already given or assumed. This is obviously a simpler operation than the *design* of the section, which involves the choice of a suitable section out of many possible shapes and dimensions. In actual practice, it is often necessary to first perform the process of design when assuming a section, and then to analyze that assumed section. But, for the purpose of study, it is easier to learn first the methods of analysis and then those of design. This reversal of order is desirable in the study of both prestressed and reinforced concrete.

This chapter will be devoted to the first part, the *analysis;* the next chapter will deal with *design*. The discussion is limited to the analysis of sections for flexure, meaning members under bending, such as beams and slabs. Only the effect of moment is considered here; that of shear and bond is treated in Chapter 7.

A rather controversial point in the analysis of prestressed-concrete beams is the choice of a proper system of sign conventions. Many authors have used positive sign (+) for compressive stresses and negative sign (—) for tensile stresses, basing their convention on the idea that prestressed-concrete beams are normally under compression and hence the plus sign should be employed to denote that state of stress. The author prefers to maintain the common sign convention as used for the design of other structures, i.e., minus for compressive and plus for tensile stresses. The arguments in favor of this common convention are as follows:

1. This system is consistent with that used in the design of other types of construction such as steel or reinforced concrete. For a

structure using a combination of several materials, it will be less confusing to use the same system for all parts of the structure.

2. All engineers are used to this common system. Prestressed concrete is only one of several methods of construction. It is more natural to follow than to reverse the practice.

3. In prestressed concrete, if the minus sign is used for denoting tensile stresses in concrete, the choice of a proper sign for the tensile stresses in steel becomes a difficult problem.

4. Tensile stresses are now quite often permitted in prestressed concrete; they are also investigated when determining its cracking strength. Hence there is less justification to reserve the plus sign for the supposedly more predominant compressive stress.

The author is quite convinced that the common system of plus for tension is also the logical system for prestressed concrete, and it is not too late to start in the right direction. It is also interesting to note that in Germany this common standard has been established for prestressed concrete. Throughout this treatise, plus will stand for tension and minus for compression, whether we are talking of stresses in steel or concrete, prestressed or reinforced. When the sense of the stress is self-evident, signs will be omitted.

5-2 Stresses in Concrete Due to Prestress

Some of the basic principles of stress computation for prestressed concrete have already been mentioned in section 1-2. They will be discussed in greater detail here. First of all, let us consider the effect of prestress. According to present practice, stresses in concrete due to prestress are always computed by the elastic theory. Consider the prestress F existing at the time under discussion, whether it be the initial or the final value. If F is applied at the centroid of the concrete section, and if the section under consideration is sufficiently far from the point of application of the prestress, then, by St. Venant's principle, the unit stress in concrete is uniform across that section and is given by the usual formula

$$f = \frac{F}{A}$$

where A is the area of that concrete section.

For a pre-tensioned member, when the prestress in the steel is transferred from the bulkheads to the concrete, Fig. 5-2-1, the force that was resisted by the bulkheads is now transferred to both the steel and the concrete in the member. The release of the resistance from the bulkheads is equivalent to the application of an opposite force F_i to the

member. Using the transformed section method, and with A_c = net sectional area of concrete, the compressive stress produced in the concrete is

$$f_c = \frac{F_i}{A_c + nA_s} \tag{5-2-1}$$

while that induced in the steel is

$$f_s = nf_c = \frac{nF_i}{A_c + nA_s} \tag{5-2-2}$$

which represents the immediate reduction of the prestress in the steel as a result of the transfer.

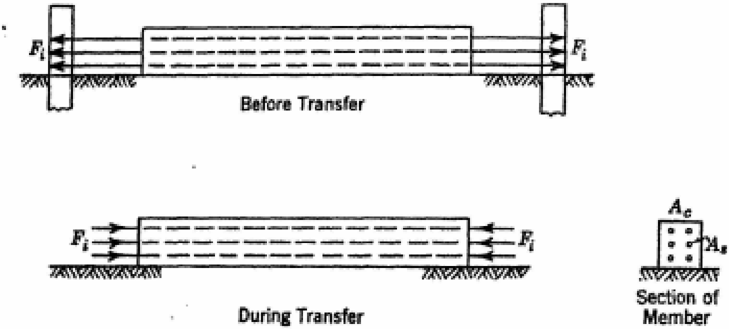

Fig. 5-2-1. Transfer of concentric prestress in a pre-tensioned member.

Although this method of computation is correct according to the elastic theory, the usual practice is not to follow such a procedure, but rather to consider the prestress in the steel being reduced by a loss resulting from elastic shortening of concrete. As discussed in the previous chapter, that loss is given by

$$f_s = n\frac{F_i}{A_c} \quad \text{or} \quad n\frac{F_i}{A_g} \tag{5-2-3}$$

which differs a little from formula 5-2-2 but is close enough for all practical purposes, since the total amount of reduction is only about 2 or 3% and the value of n cannot be accurately known anyway.

After the transfer of prestress, further losses will occur owing to the creep and shrinkage in concrete. Theoretically, all such losses should be calculated on the basis of a transformed section, taking into consideration the area of steel. But, again, that is seldom done, the practice being simply to allow for the losses by an approximate per-

centage. In other words, the simple formula $f = F/A$ is always used, with the value of F estimated for the given condition, and the gross area of concrete used for A. For a post-tensioned member, the same reasoning holds true. Suppose that there are several tendons in the member prestressed in succession. Every tendon that is tensioned becomes part of the section. The effect of tensioning any subsequent tendon on the stresses in the previously tensioned ones should be calculated on the basis of a transformed section. Theoretically, there will be a different transformed section after the tensioning of every tendon. However, such refinements are not justified, and the usual procedure is simply to use the formula $f = F/A$, with F based on the initial prestress in the steel.

Example 5-2-1

A pre-tensioned member, similar to that shown in Fig. 5-2-1, has a section of 8 in. by 12 in. It is concentrically prestressed with 0.8 sq in. of high-tensile steel wire, which is anchored to the bulkheads at a unit stress of 150,000 psi. Assuming that $n = 6$, compute the stresses in the concrete and steel immediately after transfer.

Solution 1. An exact theoretical solution. Using the elastic theory, we have

$$f_c = \frac{F_i}{A_c + nA_s} = \frac{F_i}{A_g + (n-1)A_s}$$

$$= \frac{0.8 \times 150{,}000}{12 \times 8 + 5 \times 0.8} = 1200 \text{ psi}$$

$$nf_c = 6 \times 1200 = 7200 \text{ psi}$$

Stress in steel after transfer = 150,000 − 7200 = 142,800 psi.

Solution 2. An approximate solution. The loss of prestress in steel due to elastic shortening of concrete is estimated by

$$= n\frac{F_i}{A_g}$$

$$= 6\frac{120{,}000}{8 \times 12} = 7500 \text{ psi}$$

Stress in steel after loss = 150,000 − 7500 = 142,500 psi. Stress in concrete is

$$f_c = \frac{142{,}500 \times 0.8}{96} = 1190 \text{ psi}$$

Note that, in this second solution, the approximations introduced are: (1) using gross area of concrete instead of net area, (2) using the initial stress in steel instead of the reduced stress. But the answers are verv nearly the same for both solutions. The second method is more convenient and is usually followed.

Next, suppose that the prestress F is applied to the concrete section

with an eccentricity e, Fig. 5-2-2; then it is possible to resolve the prestress into two components: a concentric load F through the cen-

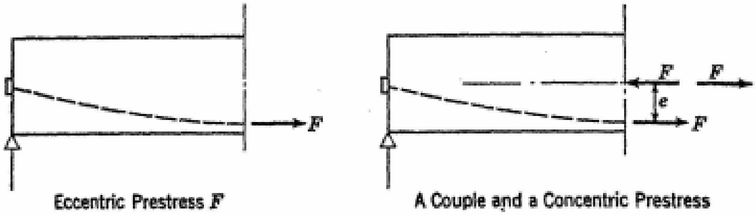

Fig. 5-2-2. Eccentric prestress on a section.

troid, and a moment Fe. By the usual elastic theory, the fiber stress at any point due to moment Fe is given by the formula

$$f = \frac{My}{I} = \frac{Fey}{I} \tag{5-2-4}$$

Then the resultant fiber stress due to the eccentric prestress is given by

$$f = \frac{F}{A} \pm \frac{Fey}{I} \tag{5-2-5}$$

The question now again arises as to what section should be considered when computing the values of e and I, whether the gross or

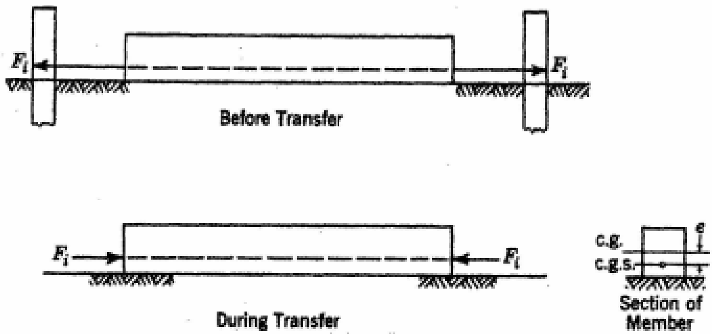

Fig. 5-2-3. Transfer of eccentric prestress in a pre-tensioned member.

the net concrete section or the transformed section, and what prestress F to be used in the formula, the initial or the reduced value. Consider a pre-tensioned member, Fig. 5-2-3. The steel has already been bonded to the concrete; the release of the force from the bulkhead is equivalent to the application of an eccentric force to the composite

member; hence the force should be the total F_i and I the moment of inertia of the transformed section, e being measured from the centroidal axis of that transformed section. However, in practice, this procedure is seldom followed. Instead, the gross or net concrete section is considered, and either the initial or the reduced prestress is applied. The error is negligible in most cases.

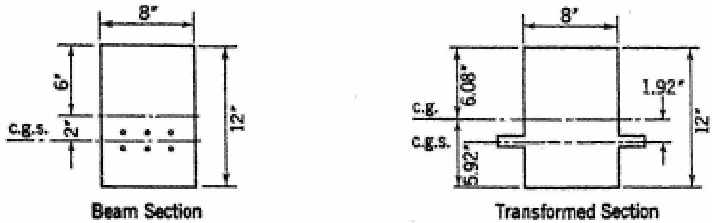

Fig. 5-2-4. Example 5-2-2.

Example 5-2-2

A pre-tensioned member similar to that shown in Fig. 5-2-3 has a section of 8 in. by 12 in. deep. It is eccentrically prestressed with 0.8 sq in. of high-tensile steel wire which is anchored to the bulkheads at a unit stress of 150,000 psi. The c.g.s. is 4 in. above the bottom fiber. Assuming that $n = 6$, compute the stresses in the concrete immediately after transfer.

Solution 1. An exact theoretical solution. Using the elastic theory, the centroid of the transformed section and its moment of inertia are obtained as follows. Referring to Fig. 5-2-4, for $(n-1)A_s = 5 \times 0.8 = 4$ sq in.,

$$y_0 = \frac{4 \times 2}{96 + 4} = 0.08 \text{ in.}$$

$$\begin{aligned} I_t &= \frac{8 \times 12^3}{12} + 96 \times 0.08^2 + 4 \times 1.92^2 \\ &= 1152 + 0.6 + 14.7 \\ &= 1167.3 \text{ in.}^4 \end{aligned}$$

$$\begin{aligned} \text{Top fiber stress} &= \frac{F_i}{A_t} + \frac{F_i e y}{I_t} \\ &= \frac{-120{,}000}{100} + \frac{120{,}000 \times 1.92 \times 6.08}{1167.3} \\ &= -1200 + 1200 \\ &= 0 \end{aligned}$$

$$\begin{aligned} \text{Bottom fiber stress} &= \frac{-120{,}000}{100} - \frac{120{,}000 \times 1.92 \times 5.92}{1167.3} \\ &= -1200 - 1170 \\ &= -2370 \text{ psi} \end{aligned}$$

Solution 2. An approximate solution. The loss of prestress can be approximately computed, as in example 5-2-1, to be 7500 psi in the steel. Hence the reduced prestress is 142,500 psi or 114,000 lb. Extreme fiber stresses in the concrete can be computed to be

$$
\begin{aligned}
f_c &= \frac{F}{A} \pm \frac{Fey}{I} \\
&= \frac{-114{,}000}{96} \pm \frac{114{,}000 \times 2 \times 6}{(8 \times 12^3)/12} \\
&= -1187 \pm 1187 \\
&= 0 \text{ in the top fiber} \\
&= 2374 \text{ psi in the bottom fiber}
\end{aligned}
$$

The approximations here introduced are: using an approximate value of reduced prestress, and using the gross area of concrete. This second solution, although approximate, is more often used because of its simplicity.

Now consider a pre-tensioned curved member as in Fig. 5-2-5. If the transfer of prestress is considered as a force F_i applied at each end,

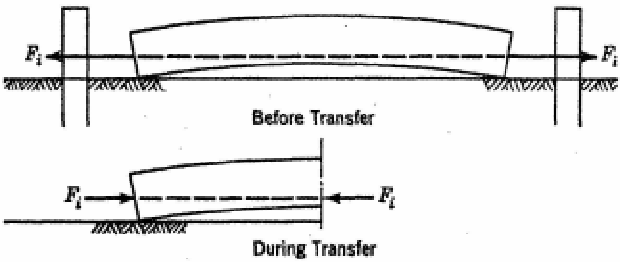

Fig. 5-2-5. Transfer of prestress in a curved pre-tensioned member.

the eccentricity and the moment of inertia will vary for each section. If the exact method of elastic analysis is preferred, different I's and e's will have to be computed for different sections. If an approximate method is permitted, a constant I based on the gross concrete area would suffice for all sections, and the eccentricity can be readily measured from the mid-depth of the section.

For a post-tensioned member before being bonded, the prestress F to be used in the stress computations is again initial prestress minus the estimated losses. For the value of I, either the net or the gross concrete section is used, although, theoretically, the net section is the correct one. After the steel is bonded to the concrete, any loss that takes place actually happens to the section as a whole. However, for the sake of simplicity, a rigorous analysis based on the transformed

section is seldom made. Instead, the reduced prestress is estimated and the stresses in the concrete are computed for that reduced prestress acting on the net concrete section (gross concrete section may sometimes be conveniently used). Stresses produced by external loads, however, are often computed on the basis of the transformed section if accuracy is desired; otherwise, gross section is used for the computation.

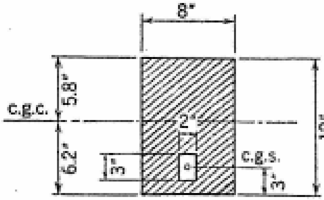

Fig. 5-2-6. Example 5-2-3.

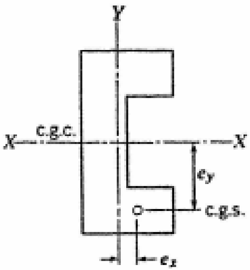

Fig. 5-2-7. Eccentricity of prestress in two directions.

Example 5-2-3

A post-tensioned beam has a midspan cross section as shown, Fig. 5-2-6. It is prestressed with 0.8 sq in. of steel to an initial stress of 150,000 psi. Immediately after transfer the stress is reduced by 5% owing to anchorage and other losses. Compute the stresses in the concrete at transfer.

Solution 1. Using net section of concrete. The centroid and I of the net concrete section are computed as follows:

$$A_c = 96 - 6 = 90 \text{ sq in.}$$

$$y_0 = \frac{6 \times 3}{96 - 6} = 0.2 \text{ in.}$$

$$I = \frac{8 \times 12^3}{12} + 96 \times 0.2^2 - \frac{2 \times 3^3}{12} - 6 \times 3.2^2$$

$$= 1152 + 3.8 - 4.5 - 61.5$$

$$= 1090$$

Total prestress in steel = 150,000 × 0.8 × 95% = 114,000 lb

$$f_c = \frac{-114{,}000}{90} \pm \frac{114{,}000 \times 3.2 \times 5.8}{1090}$$

$$= -1270 + 1940 = +670 \text{ psi for top fiber}$$

$$f_c = -1270 - 2070 = -3340 \text{ psi for bottom fiber}$$

Solution 2. Using gross section of concrete. An approximate solution using

the gross concrete section would give results not so close in this case:

$$f_c = \frac{-114{,}000}{96} \pm \frac{114{,}000 \times 3 \times 6}{(8 \times 12^3)/12}$$
$$= -1187 \pm 1783$$
$$= +596 \text{ psi for top fiber}$$
$$= -2970 \text{ psi for bottom fiber}$$

If the eccentricity does not occur along one of the principal axes of the section, it is necessary to further resolve the moment into two component moments along the two principal axes, Fig. 5-2-7; then the stress at any point is given by

$$f = \frac{F}{A} \pm \frac{Fe_x x}{I_x} \pm \frac{Fe_y y}{I_y}$$

Since concrete is not a really elastic material, the above elastic theory is not exact. But, within working loads, it is considered an accepted form of computation. When the stresses are excessively high, the elastic theory may no longer be nearly correct.

The above method further assumes that the concrete section has not cracked. If it has, the cracked portion has to be computed or estimated, and computations made accordingly. The computation for cracked section in concrete is always complicated. Fortunately, such a condition is seldom met with in actual design of prestressed concrete. In general, any high-tensile stresses produced by prestress are counterbalanced by compressive stresses due to the weight of the member itself, so that in reality there do not exist any cracks under prestress and the beam's own weight. Hence the entire concrete section can be considered as effective, even though, at certain stages of the computation, high-tensile stresses may appear on paper.

During post-tensioning operations, concrete may be subjected to abnormal stresses. Suppose that there is one tendon at each corner of a square concrete section. When all four tendons are tensioned, the entire concrete section will be under uniform compression. But when only one tendon is fully tensioned, there will exist high tensile stress as well as high compressive stress in the concrete. If two jacks are available, it may be desirable to tension two diagonally opposite tendons at the same time. Sometimes it may be necessary to tension the tendons in steps, i.e., to tension them only partially and to retension them after others have been tensioned. Computation for stresses during tensioning is also made on the elastic theory. It is believed that the elastic theory is sufficiently accurate up to the moment of cracking, although it can hardly be used to predict the

ultimate strength under prestress if it should be desired to determine such strength.

5-3 Stresses in Concrete Due to Loads

Stresses in concrete produced by external bending moment, whether due to the beam's own weight or to any externally applied loads, are computed by the usual elastic theory:

$$f = \frac{My}{I} \tag{5-3-1}$$

For a pre-tensioned beam, steel is always bonded to the concrete before any external moment is applied. Hence the section resisting external moment is the combined section. In other words, the values of y and I should be computed on the basis of a transformed section, considering both steel and concrete. For approximation, however, either the gross or the net section of concrete alone can be used in the calculations; the magnitude of error so involved can be estimated and should not be serious except in special cases.

When the beam is post-tensioned and bonded, for any load applied after the bonding has taken place, the transformed section should be used as for pre-tensioned beams. However, if the load, the weight of the beam itself, is applied before bonding takes place, it acts on the net concrete section, which should hence be the basis for stress computation. For post-tensioned unbonded beams, the net concrete section is the proper one for all stress computations. It should be borne in mind, though, that, when the beam is unbonded, any bending of the beam may change the overall prestress in the steel, the effect of which can be separately computed or estimated as discussed in section 4-5.

Oftentimes, only the resulting stresses in concrete due to both prestress and loads are desired, instead of their separate values. They are given by the following formula, a combination of (5-3-1) and (5-2-5):

$$\begin{aligned} f &= \frac{F}{A} \pm \frac{Fey}{I} \pm \frac{My}{I} \\ &= \frac{F}{A}\left(1 \pm \frac{ey}{r^2}\right) \pm \frac{My}{I} \\ &= \frac{F}{A} \pm (Fe \pm M)\frac{y}{I} \end{aligned} \tag{5-3-2}$$

Any of these three forms may be used, whichever happens to be the most convenient. But, to be strictly correct, the section used in com-

puting y and I must correspond to the actual section at the application of the force. It quite frequently happens that the prestress F acts on the net concrete section, while the external loads act on the transformed section. Judgment should be exercised in deciding whether refinement is necessary or whether approximation is permissible for each particular case.

When prestress eccentricity and external moments exist along two principal axes, the general elastic formula can be used:

$$f = \frac{F}{A} \pm \frac{Fe_x x}{I_x} \pm \frac{Fe_y y}{I_y} \pm \frac{M_x x}{I_x} \pm \frac{M_y y}{I_y}$$

$$= \frac{F}{A} \pm (Fe_x \pm M_x)\frac{x}{I_x} \pm (Fe_y \pm M_y)\frac{y}{I_y} \qquad (5\text{-}3\text{-}3)$$

Care must be exercised to use the correct I and y for each case, if theoretically exact values are desired.

Example 5-3-1

A post-tensioned bonded concrete beam, Fig. 5-3-1, has a prestress of 350 kips in the steel immediately after prestressing, which eventually reduces to 300 kips. The beam carries two live loads of 10 kips each in addition to its own weight

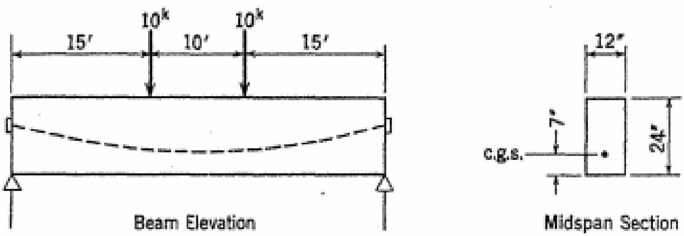

Fig. 5-3-1. Example 5-3-1.

of 300 plf. Compute the extreme fiber stresses at midspan, (*a*) under the initial condition with full prestress and no live load, and (*b*) under the final condition, after the losses have taken place, and with full live load.

Solution. To be theoretically exact, the net concrete section should be used up to the time of grouting, after which the transformed section should be considered. This is not deemed necessary, and an approximate but sufficiently exact solution is given below, using the gross section of concrete at all times, i.e.,

$$I = 12 \times 24^3/12 = 13{,}800 \text{ in.}^4$$

1. *Initial condition.* Dead-load moment at midspan, assuming that the beam is simply supported after prestressing:

$$DLM = \frac{wL^2}{8} = \frac{300 \times 40^2}{8} = 60{,}000 \text{ ft-lb}$$

$$f = \frac{F}{A} \pm \frac{Fey}{I} \pm \frac{My}{I}$$

$$= \frac{-350{,}000}{288} \pm \frac{350{,}000 \times 5 \times 12}{13{,}800} \pm \frac{60{,}000 \times 12 \times 12}{13{,}800}$$

$$= -1215 + 1520 - 625 = -320 \text{ psi, top fiber}$$

$$= -1215 - 1520 + 625 = -2110 \text{ psi, bottom fiber}$$

2. *Final condition.* Live-load moment at midspan = 150,000 ft-lb; hence total external moment = 210,000 ft-lb, while the prestress is reduced to 300,000 lb; hence,

$$f = \frac{-300{,}000}{288} \pm \frac{300{,}000 \times 5 \times 12}{13{,}800} \pm \frac{210{,}000 \times 12 \times 12}{13{,}800}$$

$$= -1040 + 1300 - 2190 = -1930 \text{ psi, top fiber}$$

$$= -1040 - 1300 + 2190 = -150 \text{ psi, bottom fiber}$$

The above describes the conventional method of stress analysis for prestressed concrete, but it will be recalled that in section 1-2 a second

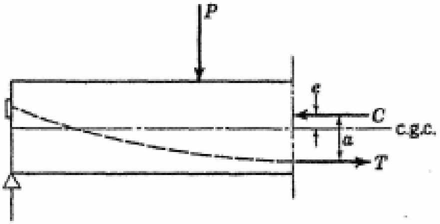

Fig. 5-3-2. Internal resisting couple C-T with arm a.

method of approach is described in which the center of pressure C in the concrete is set at distance a from the center of prestress T in the steel such that

$$Ta = Ca = M \tag{5-3-4}$$

By this method, the stresses in concrete are not treated as being produced by prestress and external moments separately, but are determined by the magnitude and location of the center of pressure C, Fig. 5-3-2. Since most beams do not carry axial load, C equals T, and is located at a distance from T

$$a = M/T$$

Since the value of T is the value of F in a prestressed beam it is quite accurately known. Thus the computation of a for a given moment M is simply a matter of statics. Once the center of pressure C is located for a concrete section, the distribution of stresses can be

determined either by the elastic theory or by the plastic theory. Generally the elastic theory is followed, in which case we have, since

$$C = T = F, \qquad f = \frac{C}{A} \pm \frac{Cey}{I} = \frac{F}{A} \pm \frac{Fey}{I} \tag{5-3-5}$$

where e is the eccentricity of C, not of F.

Following this approach, a prestressed beam is considered similar to a reinforced-concrete beam with the steel supplying the tensile force T, and the concrete supplying the compressive force C. C and T together form a couple resisting the external moment. Hence the value of A and I to be used in the above formula should be the net section of the concrete, and not the composite section. If a beam has conduits grouted for bond, the stress in the grout is actually different from that in the adjacent concrete, and an exact theoretical solution would be quite involved. Under such conditions it is advisable to use the gross section of concrete for all computations for the sake of simplicity. Only when investigating the stresses before grouting should the net concrete section be used.

It can be noted that formula 5-3-5 is only a different form of formula 5-3-2, with e measured to C, thus combining the effect of M with the eccentricity of F. Although the formulas are in fact identical, the approaches are different. By following this second approach, all the inaccuracies are thrown into the estimation of the effective prestress in steel, which can generally be estimated within 5%. After that, the location of C is a simple problem in statics, and the distribution of C across the section can be easily computed or visualized. This method of approach will be further explained in the next chapter on the design of beam sections.

Example 5-3-2

For the same problem as in example 5-3-1, compute the concrete stresses under the final loading conditions by locating the center of pressure C for the concrete section.

Solution. Referring to Fig. 5-3-3, a is computed by

$$a = (210 \times 12)/300 = 8.4 \text{ in.}$$

Hence e for C is $8.4 - 5 = 3.4$ in. Since $C = F = 300{,}000$ lb,

$$\begin{aligned} f &= \frac{C}{A} \pm \frac{Cey}{I} \\ &= \frac{-300{,}000}{288} \pm \frac{300{,}000 \times 3.4 \times 12}{13{,}800} \\ &= -1040 - 890 = -1930 \text{ psi, top fiber} \\ &= -1040 + 890 = -150 \text{ psi, bottom fiber} \end{aligned}$$

Also, by inspection, since the center of pressure is near the third point, the stress distribution should be nearly triangular as is shown. By comparing this solution with that for example 5-3-1, the directness and simplicity of this method seem to be evident.

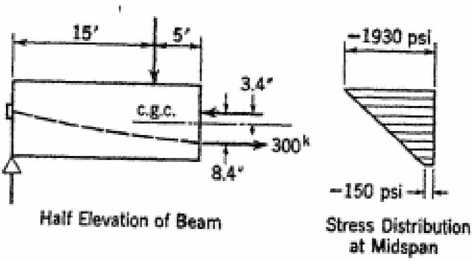

Fig. 5-3-3. Example 5-3-2.

5-4 Stresses in Steel Due to Loads

In prestressed concrete, prestress in the steel is measured during tensioning operations, then the losses are computed or estimated as described in Chapter 4. When dead and live loads are applied to the member, minor changes in stress will be induced in the steel. In a reinforced-concrete beam, steel stresses are assumed to be directly proportional to the external bending moment. When there is no moment, there is no stress. When the moment increases, the steel stresses increase in direct proportion. This is not true for a prestressed-concrete beam, whose resistance to external moment is furnished by a lengthening of the lever arm between the resisting forces C and T which remain relatively unchanged in magnitude.

In order to get a clear understanding of the behavior of a prestressed-concrete beam, it will be interesting to study first the variation of steel stress as the load increases. For the midspan section of a simple beam, the variation of steel stress with load on the beam is shown in Fig. 5-4-1. Along the X-axis is plotted the external load, and along the Y-axis is plotted the stress in the steel. As prestress is applied to the steel, the stress in the steel changes from A to B, where B is at the level of f_0, which is the initial prestress in the steel after losses due to anchorage and elastic shortening have taken place.

Immediately after transfer, no load will yet be carried by the beam if it is supported on its falsework and if it is not cambered upward by the prestress. As the falsework is removed, the beam carries its own weight and deflects downward slightly, thus changing the stress in the steel, increasing it from B to C. When the dead weight of the beam is relatively light, then it can be bowed upward during the course of

the transfer of prestress. The beam may actually begin to carry load when the average prestress in the steel is somewhere at B'. There may be a sudden breakaway of the beam's soffit from the falsework so that the weight of the beam is at once transferred to be carried by the beam itself, or the weight may be transferred gradually, depending

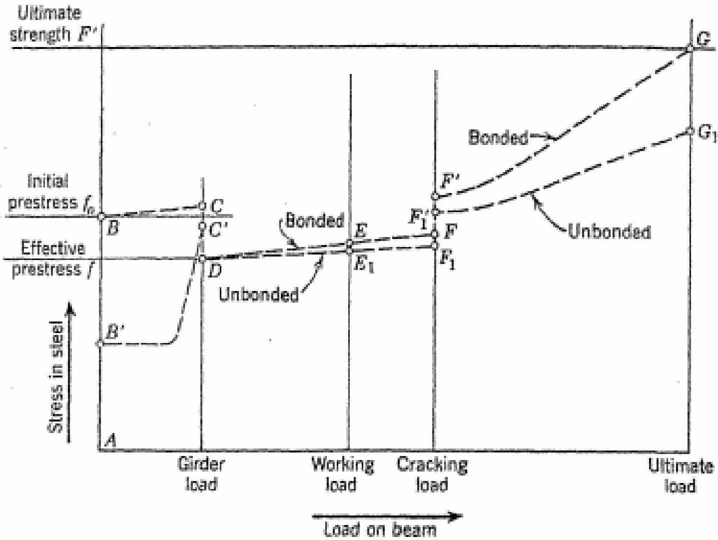

Fig. 5-4-1. Variation of steel stress with load.

upon the actual conditions of support. But, in any event, the stress in steel will increase from B' up to point C'. The stress at C' is slightly lower than f_0 by virtue of the loss of prestress in the steel as caused by the upward bending of the beam. Consider now that the losses of prestress take place so that the stress in the steel drops from C or C' to some point D, representing the effective prestress f for the beam. Actually, the losses will not take place all at once but will continue for some length of time. But, for convenience in discussion, let us assume that all the losses take place before the application of superimposed dead and live loads.

Now let us add live load on the beam until the full design working load is on it. The beam will bend and deflect downward, and stress in the steel will increase. For a bonded beam, such increase can be simply computed by the usual elastic theory,

$$f_s = n \frac{My}{I} = nf_c$$

where I and y correspond to the transformed section and n is the modular ratio of steel to concrete. Since the maximum change in concrete stresses at the level of steel is not more than about 2000 psi in most cases, the corresponding change of stress in steel is limited to $2000n$, or 12,000 psi for a value of $n = 6$. This stage is represented by the line DE in Fig. 5-4-1. It is significant to note that, in prestressed concrete, the variation in steel stress for working loads is limited to a range of about 12,000 psi even though the prestress is as high as 120,000 psi.

If the beam is overloaded, beyond its working load, and up to the point of cracking, the increase in steel stress still follows the same elastic theory. Hence the line DE is prolonged to point F. This would represent a tensile stress in the concrete around 600 psi, indicating an increase in steel stress of about $6 \times 600 = 3600$ psi from E to F.

When the section cracks, there is a sudden increase of stress in the steel, from F to F' for the bonded beam. After cracking, the stress in the steel will increase faster with the load. As the load is further increased, the section will gradually approach its ultimate strength, the lever arm for the internal C-T couple cannot be increased any more, and increase in load is accompanied by a proportional increase in steel stress. This continues up to the point of failure. From the results of various tests, it is known that the stress in the steel approaches very nearly its ultimate strength at the rupture of the beam provided compression failure does not start in the concrete and failure of the beam is not produced by shear or bond. Hence the stress curve can be approximately drawn as from F' to G.

The computation of steel stress beyond cracking and up to the ultimate load is a complicated problem which cannot yet be solved until more test data are available. But it must be pointed out that between the two points, F' and G, there is one point when the steel ceases to be elastic, elastic in the sense that no appreciable permanent set is caused by the external load. This point is considered by some engineers to be the limit to which a structure, such as a bridge or a building, should ever be subjected. If it can be conveniently determined, it may be a more significant criterion for design than the cracking or ultimate load used at present.

If the beam is unbonded, the stress in the steel will be different from the bonded beam. Assuming that the same effective prestress is obtained before the addition of any external load, we can discuss the stress in an unbonded tendon as follows: Starting from point D, when load is added to the beam, the beam bends while the steel slips with

respect to the concrete. Owing to this slip, the usual method of a composite steel and concrete section no longer applies. Before cracking of the concrete, stress in the concrete due to any external moment M is given by

$$f = \frac{My}{I}$$

where I and y refer to those for the net concrete section. But it must be remembered that the prestress changes as load is applied, Fig. 5-4-2. Hence the question becomes more complicated.

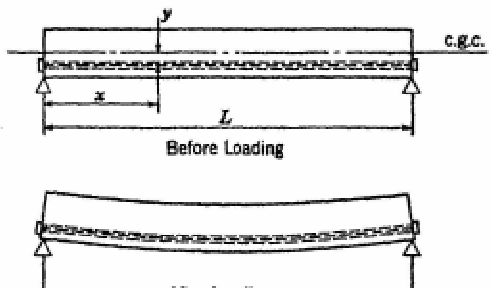

Fig. 5-4-2. Change of cable length in an unbonded beam.

At the section of maximum moment, the stress in an unbonded tendon will increase more slowly than that in a bonded tendon. This is because any strain in an unbonded tendon will be distributed throughout its entire length. Hence, as the load is increased to the working or the cracking load, the steel stress will increase from D to E_1, F_1, and F_1', below E, F, and F', respectively. To compute the average strain for the cable, it is necessary to determine the total lengthening of the tendon due to moments in the beam. This can be done by integrating the strain along the entire length. Let M be the moment at any point of an unbonded beam; the unit strain in concrete at any point is given by

$$\delta = \frac{f}{E} = \frac{My}{E_c I}$$

The total strain along the cable is then

$$\Delta = \int \delta\, dx = \int \frac{My}{E_c I}\, dx$$

The average strain is

$$\frac{\Delta}{L} = \int \frac{My}{LE_cI}\,dx$$

The average stress is

$$f_s = E_s\frac{\Delta}{L} = \int \frac{MyE_s}{LE_cI}\,dx = \frac{n}{L}\int \frac{My}{I}\,dx \qquad (5\text{-}4\text{-}1)$$

If y and I are constants and M is an integrable form of x, the solution of this integral is simple. Otherwise, it will be easier to use a graphical or an approximate integration.

After cracks have developed in the unbonded beam, stress in the steel increases more rapidly with the load, but again it does not increase as fast as that at the maximum moment section in a similar bonded beam. In an unbonded beam, it is generally not possible to develop the ultimate strength of the steel at the rupture of the beam. Thus the stress curve is shown going up from F_1' to G_1, with G_1 below G by an appreciable amount. It is evident that the ultimate load for an unbonded beam is less than that for a corresponding bonded one, although there may be very little difference between the cracking loads for the two beams. Most engineers believe that there is a tendency for the unbonded beams to develop large cracks before rupture. These large cracks tend to concentrate strains at some localized sections in the concrete, thus resulting in lower ultimate strength than bonded beams. However, opinions and observations differ in this respect. It is further known that the strength of unbonded beams may be greatly increased by the addition of non-prestressed bonded reinforcements, which will be discussed later.

Example 5-4-1

A post-tensioned simple beam on a span of 40 ft is shown in Fig. 5-4-3. It carries a superimposed load of 750 plf in addition to its own weight of 300 plf. The initial prestress in the steel is 138,000 psi, reducing to 120,000 psi after deducting all losses and assuming no bending of the beam. The parabolic cable has an area of 2.5 sq in., $n = 6$. Compute the stress in the steel at midspan, assuming: (1) the steel is bonded by grouting; (2) the steel is unbonded.

Solution 1. Moment at midspan due to dead and live loads is

$$\frac{wL^2}{8} = \frac{(300 + 750)40^2}{8}$$

$$= +210{,}000 \text{ ft-lb}$$

Moment at midspan due to prestress is

$$2.5 \times 120{,}000 \times 5/12 = -125{,}000 \text{ ft-lb}$$

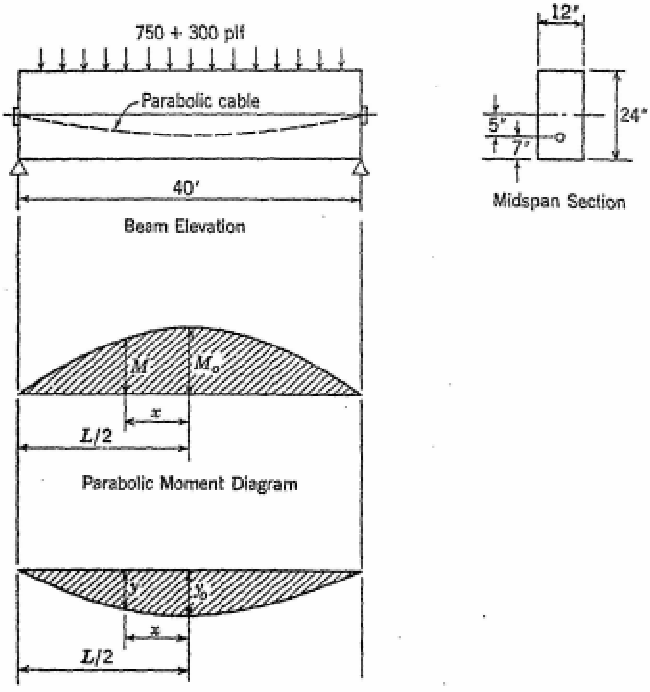

Fig. 5-4-3. Example 5-4-1.

Net moment at midspan is 210,000 − 125,000 = 85,000 ft lb. Stress in concrete at the level of steel due to bending, using I of gross concrete section, is

$$= \frac{My}{I} = \frac{85{,}000 \times 12 \times 5}{13{,}800} = 370 \text{ psi}$$

Stress in steel is thus increased by

$$f_s = nf_c = 6 \times 370 = 2220 \text{ psi}$$

Resultant stress in steel = 122,220 psi at midspan.

Solution 2. If the cable is unbonded, the average strain or stress must be obtained for the whole length of cable as given by formula 5-4-1,

$$f_s = \frac{n}{L}\int \frac{My}{I}\,dx$$

Using y_0 and M_0 for those at midspan, and measuring x from the midspan, we can express y and M in terms of x, thus,

$$M = M_0\left[1 - \left(\frac{x}{L/2}\right)^2\right]$$

$$y = y_0\left[1 - \left(\frac{x}{L/2}\right)^2\right]$$

$$f_s = \frac{n}{LI}\int_{-L/2}^{+L/2} M_{0}y_0\left[1 - \left(\frac{x}{L/2}\right)^2\right]^2 dx$$

$$= \frac{nM_0y_0}{LI}\left[x - \frac{2}{3}\,\frac{x^3}{(L/2)^2} + \frac{x^5}{5(L/2)^4}\right]_{-L/2}^{+L/2}$$

$$= \frac{8nM_0y_0}{15I}$$

which is 8/15 of the stress for midspan of the bonded beam, or 8/15 (2220) = 1180 psi.

Resultant stress in steel is 120,000 + 1180 = 121,180 psi throughout the entire cable. In this calculation, the I of the gross concrete section is used and the effect of the increase in the steel stress on the concrete stresses is also neglected. But these are errors of the second order. Since the change in steel stress is relatively small, exact computations are seldom required in an actual design problem.

5-5 Cracking Moment

The moment producing first hair cracks in a prestressed concrete beam is computed by the elastic theory, assuming that cracking starts when the tensile stress in the extreme fiber of concrete reaches its modulus of rupture. Questions have been raised as to the correctness of this method. First, some engineers believed that concrete under prestress became a complex substance whose behavior could not be predicted by the elastic theory with any accuracy.[5-1] Then it was further questioned whether the usual bending test for modulus of rupture could give values to represent the tensile strength of concrete in a prestressed beam. However, most available test data seem to indicate that the elastic theory is sufficiently accurate up to the point of cracking, and the method is currently used.

Attention must be paid to the fact that the modulus of rupture is only a measure of the beginning of hair cracks which are often invisible to the naked eye. A tensile stress higher than the modulus is necessary to produce visible cracks. On the other hand, if the concrete has been previously cracked by overloading, shrinkage, or other causes, cracks may reappear at the slightest tensile stress. If the beam is made of concrete blocks, the cracking strength will depend on the tensile strength of the joining material.

Referring to formula 5-3-2, if f' is the modulus of rupture, it is seen that, when

$$-\frac{F}{A} - \frac{Fec}{I} + \frac{Mc}{I} = f'$$

cracks are supposed to start. Transposing terms, we have the value of cracking moment given by

$$M = Fe + \frac{FI}{Ac} + \frac{f'I}{c} \tag{5-5-1}$$

where $f'I/c$ gives the resisting moment due to modulus of rupture of concrete, Fe the resisting moment due to the eccentricity of prestress, and FI/Ac that due to the direct compression of the prestress.

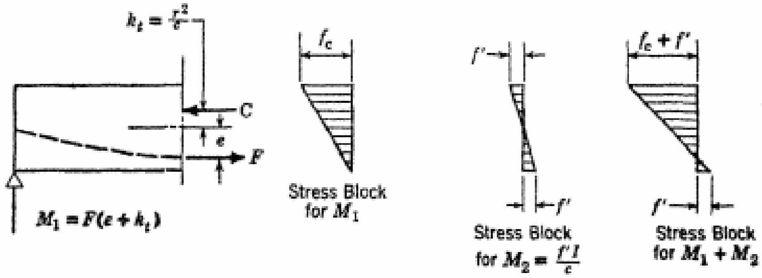

Fig. 5-5-1. Cracking moment.

Formula 5-5-1 can be derived from another point of view. When the center of pressure in the concrete is at the top kern point, there will be zero stress in the bottom fiber. The resisting moment is given by the prestress F times its lever arm measured to the top kern point, Fig. 5-5-1, thus,

$$M_1 = F\left(e + \frac{r^2}{c}\right)$$

Additional moment resisted by the concrete up to its modulus of rupture is $M_2 = f_tI/c$. Hence the total moment at cracking is given by

$$M = M_1 + M_2 = F\left(e + \frac{r^2}{c}\right) + \frac{f'I}{c} \tag{5-5-2}$$

which is identical with formula 5-5-1.

In order to be theoretically correct when applying the above two formulas, care must be exercised in choosing the proper section for the computation of I, r, e, and c. For computing the term $f'I/c$, the transformed section should be used for bonded beams, while the net concrete section should be used for unbonded beams (proper modification being made for the value of prestress due to bending of the beam as explained in section 4-5). For the term $F\left(e + \frac{r^2}{c}\right)$, either

the gross or the net section should be considered, depending upon the computation of the effective prestress F. For a practical problem, these refinements are often unnecessary, and it will be easier to use one section for all the computations. In order to simplify the computations, the gross section of the concrete is most often used. If the area of holes is an important portion of the gross area, then net area may be used. If the percentage of steel is high the transformed area may be preferred. The engineer must use his own discretion in choosing a simple method of solution consistent with the degree of accuracy required for his particular problem.

Example 5-5-1

For the problem given in Example 5-4-1, compute the total dead and live uniform load that can be carried by the beam, (1) for zero tensile stress in the bottom fibers, (2) for cracking in the bottom fibers, with tensile stress = the modulus of rupture at 600 psi.

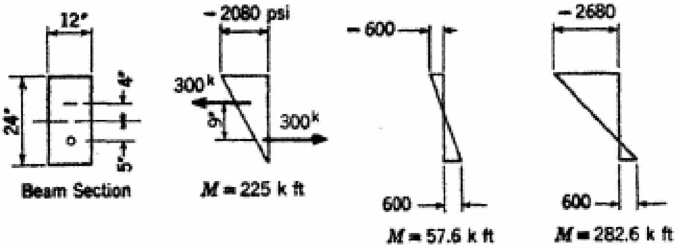

Fig. 5-5-2. Example 5-5-1.

Solution 1. Considering the critical midspan section and using the gross concrete section for all computations, k_t is readily computed to be at 4 in. above the mid-depth, Fig. 5-5-2. To obtain zero stress in the bottom fibers, the center of pressure must be located at the top kern point. Hence the resisting moment is given by the prestress multiplied by the lever arm, thus

$$F(e + k_t) = 300(5 + 4)/12 = 225 \text{ k ft}$$

Solution 2. Additional moment carried by the section up to beginning of cracks is

$$\frac{f'I}{c} = \frac{600 \times 13{,}800}{12}$$

$$= 690{,}000 \text{ in.-lb}$$

$$= 57.6 \text{ k ft}$$

Total moment at cracking is 225 + 57.6 = 283 k ft, which can also be obtained directly by applying formula 5-5-1 or 5-5-2.

5-6 Ultimate Moment

Exact analysis for the ultimate strength of a prestressed-concrete section under flexure is a complicated theoretical problem, because both steel and concrete are generally stressed beyond their elastic range. However, for the purpose of practical design, where an accuracy of 5–10% is considered sufficient, relatively simple procedures can be developed. With the immense amount of data available on this subject, it is an easy matter to predict such strength within the required degree of accuracy.

Many tests have been run, and many papers written, on the ultimate flexural strength of prestressed-concrete-beam sections. Worthy of special mention are the group of papers on this thesis[5-2] presented before the first International Congress on Prestressed Concrete, held in London, October, 1953, and the laboratory investigations carried on by the University of Illinois.[5-3] The first group of papers represent the present status of the subject in Europe. On the surface, widely different formulas are proposed by various authors, based on full-size and specimen tests as well as on theoretical studies. In actuality, these formulas give values which do not differ by more than a few per cent from one another. At the University of Illinois, where 27 beams were tested in one series, the results also showed excellent agreement between theory and experiments, so that it is now generally agreed that such ultimate strength can be predicted with sufficient accuracy.

The method for determining ultimate flexural strength presented herewith is based on the results of the above-mentioned tests as well as many others. All these tests, however, are limited to the following conditions:

1. The failure is primarily a flexural failure, with no shear, bond, or anchorage failure which might decrease the strength of the section.
2. The beams are bonded. Unbonded beams possess different ultimate strength and are discussed later.
3. The beams are statically determinate. Although the discussions apply equally well to individual sections of continuous beams, the ultimate strength of continuous beams as a whole is explained by the plastic hinge theory to be discussed in Chapter 10.
4. The load considered is the ultimate load obtained as the result of a short static test. Impact, fatigue, or long-time loadings are not considered.

Of the methods proposed for determining the ultimate flexural strength of prestressed-concrete sections, some are purely empirical and others highly theoretical. The empirical methods are generally simple but are limited only to the conditions which were encountered

in the tests. The theoretical ones are aimed for research studies and hence unnecessarily complicated for the designer. For the purpose of design, a rational approach is presented in the following, consistent with test results, but neglecting refinements so that reasonably correct values can be obtained with the minimum amount of effort. The method is based on the simple principle of a resisting couple in a prestressed beam, as that in any other beam. At the ultimate load, the couple is made of two forces, T' and C', acting with a lever arm a'. The steel supplies the tensile force T', and the concrete the compressive force C'.

Before going any further with the method, let us first study the modes of failure of prestressed-beam sections. The failure of a section may start either in the steel or in the concrete, and may end up in one or the other. The most general case is that of an under-reinforced section, where the failure starts with the excessive elongation of steel and ends with the crushing of concrete. This type of failure occurs in both prestressed- and reinforced-concrete beams, when they are under-reinforced. Only in some rare instances may fracture of steel occur in such beams; that happens, for example, when the compressive flange is restrained and possesses a higher actual strength. A relatively uncommon mode of failure is that of an over-reinforced section, where the concrete is crushed before the steel is stressed into the plastic range. Hence there is only elastic but no plastic deflection before rupture, and a brittle mode of failure is obtained. This is similar to an over-reinforced non-prestressed-concrete beam. Another unusual mode of failure is that of a too lightly reinforced section, where failure may occur by the breaking of the steel immediately following the cracking of concrete. This happens when the tensile force in the concrete is suddenly transferred to the steel whose area is too small to carry that tension.

There is no sharp line of demarcation between the percentage of reinforcement for an over-reinforced beam and that for an under-reinforced one. The transition from one type to another takes place gradually as the percentage of steel is varied. For the materials presently used in prestressed work, the normal reinforcement ranges between 0.3% and 0.8%. Such ratios of reinforcement almost always end in plastic failure and can be termed as under-reinforced ratios. A limiting ratio for "balanced" design is given in Appendix D, applicable, however, only to rectangular sections. If the ratio is over 1%, sudden crushing of concrete without substantial elongation of steel will be likely to take place. If it is less than about 0.15%, breaking of the wires following cracking of concrete may occur.

A proper definition of the percentage of steel p cannot be easily

given for prestressed sections, because of their irregular shapes. For certain purposes, the ratio p is A_s/A_c, where A_c refers to the total area of concrete. For ultimate strength it is not the total concrete area but the concrete area in the compressive flange that matters; hence p will be more indicative of the relative strength of concrete and steel if it is expressed in terms of A_s/bd, where b is the width or average width of the compressive flange and d the effective depth. Similarly, for investigating the minimum percentage of steel to prevent sudden fracture at cracking, the width of the tensile flange is the proper value for b.

1 · Under-Reinforced Bonded Beam. For under-reinforced bonded beams, the steel is almost always stressed to its ultimate strength at the point of rupture. In fact, there are some test data which seem to show that the steel was stressed even beyond its ultimate strength. Though this does not seem to be possible, it might perhaps be explained by the fact that the group strength of wires forced to fail together at one section of a beam might be higher than the tested strength of the specimens, since, during specimen tests, only the strength of the weakest link is recorded. Other engineers believe that the tensile resistance in the concrete contributes toward the ultimate strength,[5-4] although such participation is not likely to be significant. For the purpose of practical design, it will be sufficiently accurate to assume that the steel is stressed to the ultimate strength at the rupture of under-reinforced beams.

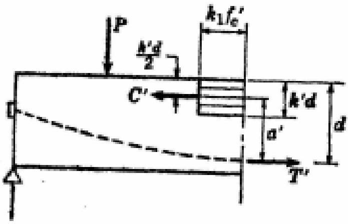

Fig. 5-6-1. Ultimate moment capacity of a section.

If it is assumed that the steel is stressed to its ultimate strength at the rupture of the beam, the computation of the ultimate resisting moment is a relatively simple matter and can be carried out as follows. Referring to Fig. 5-6-1, the ultimate compressive force in the concrete C' equals the ultimate tensile force in the steel T', thus,

$$C' = T' = A_s f_s' \tag{5-6-1}$$

Let a' be the lever arm between the forces C' and T'; then the ultimate resisting moment is given by

$$M' = T'a' = A_s f_s' a' \tag{5-6-2}$$

To determine the lever arm a', it is only necessary to locate the center of pressure C'. There are many plastic theories for the distribution of

compressive stress in concrete at failure,[5-5] assuming the stress block to take the shape of a rectangle, trapezoid, parabola, etc. Although the actual stress distribution is a very interesting problem for research, for the purpose of design, any of these methods would be sufficiently accurate, because they would yield nearly the same lever arm a', seldom differing by more than 5%.

Choosing the simplest stress block, a rectangle, for the ultimate compression in concrete, the depth to the ultimate neutral axis $k'd$ is computed by

$$C' = k_1 f_c' k' bd$$

where $k_1 f_c'$, is the average compressive stress in concrete at rupture. Hence,

$$k'd = \frac{C'}{k_1 f_c' b} = \frac{A_s f_s'}{k_1 f_c' b} \tag{5-6-3}$$

$$k' = \frac{A_s f_s'}{k_1 f_c' bd} \tag{5-6-4}$$

These formulas apply if the compressive flange has a uniform width b at failure.

Locating C' at the center of the rectangular stress block we have the lever arm

$$a' = d - k'd/2$$

$$= d\left(1 - \frac{k'}{2}\right) \tag{5-6-5}$$

Hence, the ultimate resisting moment is

$$M' = A_s f_s' d\left(1 - \frac{k'}{2}\right) \tag{5-6-6}$$

Now the determination of the value of k_1 deserves some comments. According to Whitney's plastic theory of reinforced-concrete beams, k_1 should be 0.85, based on cylinder strength. According to some authors in Europe, k_1 should be 0.60 to 0.70 based on the cube strength; since cube strength is 25% higher than cylinder strength, this would give approximately 0.75 to 0.88 for k_1 based on the cylinder strength. The important thing for the designer to see is the fact that variation of the value of k_1 does not appreciably affect the lever arm a'. Hence it is considered accurate enough to adopt some approximate value, such as 0.85 for k_1. Since the center of pressure C' is actually located slightly above the middle of $k'd$, we are on the safe

side when assuming a rectangular stress block. Using 0.85 for k_1, formula 5-6-4 can be written as

$$k' = \frac{A_s f_s'}{0.85 f_c' bd} \tag{5-6-7}$$

In order to illustrate the computation of ultimate strength in beam sections by the above method, two examples will be given below. Example 5-6-1 deals with a rectangular section, while example 5-6-2 has a T section.

Example 5-6-1

A rectangular section 12 in. by 24 in. deep is prestressed with 1.5 sq in. of steel wires to an initial stress of 150,000 psi. The c.g.s. of the wires is 4 in.

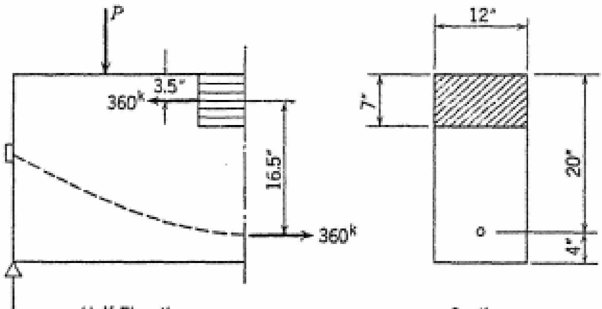

Fig. 5-6-2. Example 5-6-1.

above the bottom fiber of the beam, Fig. 5-6-2; $f_s' = 240{,}000$ psi; $f_c' = 5000$ psi. Estimate the ultimate resisting moment of the section.

Solution. Assuming that the wires will be stressed to their ultimate strength, then the total T' at rupture is

$$1.5 \times 240{,}000 = 360{,}000 \text{ lb}$$

Assuming that the average stress in concrete is $0.85 f_c' = 4250$ psi, the depth to neutral axis $k'd$ is, from formula 5-6-7,

$$k'd = \frac{360{,}000}{4250 \times 12} = 7.0 \text{ in.}$$

The center of pressure C is located at $7/2 = 3.5$ in. from the top. Hence the tensile force T in the wires has a lever arm of $20 - 3.5 = 16.5$ in., and the ultimate resisting moment is, from formula 5-6-2,

$$360{,}000 \times 16.5 = 5{,}940{,}000 \text{ in.-lb}$$

For a more exact solution, see example 5-6-3.

Example 5-6-2

A T-section is shown in Fig. 5-6-3, with 1.5 sq in. of wires prestressed to 150,000 psi; $f_s' = 240{,}000$ psi; $f_c' = 5000$ psi. Estimate the ultimate resisting moment.

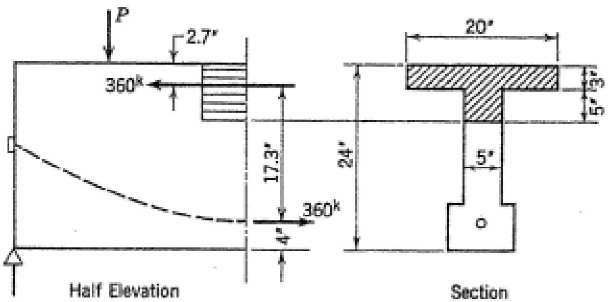

Fig. 5-6-3. Example 5-6-2.

Solution. This being a T-section, formulas 5-6-4 and 5-6-7 do not directly apply. But similar procedure can be followed. Assuming the average stress in concrete to be $0.85f_c' = 4250$ psi, and the steel stressed to its ultimate strength of $1.5 \times 240{,}000 = 360{,}000$ lb, the total compressive area required for concrete is

$$360{,}000/4250 = 85 \text{ sq in.}$$

The flange supplies an area of 60 sq in., leaving an area of 25 sq in. to be supplied by the web. Thus the neutral axis is located $25/5 = 5$ in. below the flange. If only an approximate solution is desired, the effect of compression in the web can be neglected and the center of pressure can be located at mid-depth of the flange, 1.5 in. below the top. Then the lever arm for the steel is 18.5 in., and the ultimate resisting moment is

$$360{,}000 \times 18.5 = 6{,}660{,}000 \text{ in.-lb}$$

If a more exact solution is desired, still based on a rectangular stress block, the center of pressure C' can be located at the centroid of the compressive area, thus:

$$\frac{20 \times 3 \times 1.5 + 5 \times 5 \times 5.5}{20 \times 3 + 5 \times 5} = 2.7 \text{ in.}$$

$$a' = 20 - 2.7 \text{ in.} = 17.3 \text{ in.}$$

$$M' = 360{,}000 \times 17.3 = 6{,}220{,}000 \text{ in.-lb}$$

2 · Over-Reinforced Bonded Beams. The above method assumes that the ultimate strength of the steel can be developed at the rupture of the beam. But when a section is over-reinforced, the neutral axis at rupture will be low, and compressive failure will take place in the concrete before the ultimate strength in the steel is developed. In a case like this, to determine the stress of steel at rupture of the beam

it is necessary to study the strain relations in the section and relate them to the stress-strain diagram of steel. This is done as follows.

The maximum strain of concrete at failure is believed to vary between 0.003 and 0.004. Tests at the University of Illinois gave an average value of 0.0034 for the strain of the top fiber of concrete at failure. Assuming that plane section remains plane at rupture (which was also found to be approximately correct by the University of Illinois tests), we can obtain the strain of steel at rupture of beam to be

$$e_{s2} = 0.0034\,\frac{d - k'd}{k'd} = 0.0034\,\frac{1 - k'}{k'}$$

That strain in the steel is in addition to the prestressed strain in the steel e_{s1} at the time when concrete strain is zero on the top fiber, Fig. 5-6-4. The total strain e_s is given by $e_{s2} + e_{s1}$. From the stress-

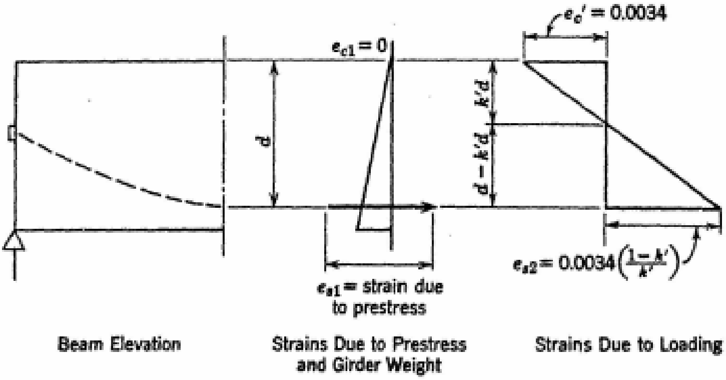

Fig. 5-6-4. Strains in steel and concrete at rupture.

strain diagram of steel, the corresponding stress f_s can be obtained. If that stress f_s is near the ultimate value f_s', the section is not over-reinforced and the previous method using the ultimate strength of steel is accurate enough. If f_s is appreciably lower than f_s', the solution has to be modified. To obtain the actual value of f_s at rupture, a method of trial and error can be followed, repeating the above process, until the assumed and computed values of f_s agree within limits. This procedure will be illustrated in the following example.

Example 5-6-3

For the same section as in example 5-6-1 compute the ultimate resisting moment. Assume that the high-tensile wires have a stress-strain diagram as shown in Fig. 2-5-2. Use the trial-and-error method.

Solution. Corresponding to the first trial in example 5-6-1, assuming a stress of 240,000 psi in steel, the neutral axis at rupture was located at 7 in. from the top. If the maximum concrete strain is assumed to be 0.0034, the strain in the steel can be obtained from the simple relation shown in Fig. 5-6-5.

$$\epsilon_{s2} = 0.0034 \times \frac{13}{7} = 0.0063$$

If the effective prestress in the steel is assumed to be 126,000 psi, for $E_s =$ 30,000,000 psi, this indicates a strain of 0.0042. Thus the total strain at failure

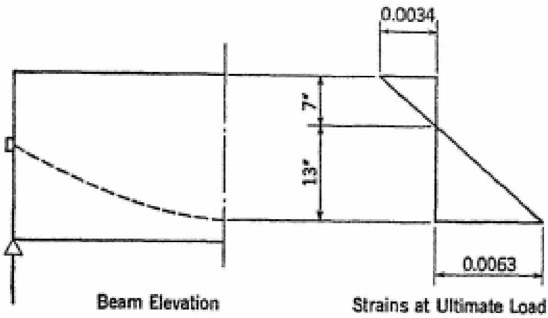

Fig. 5-6-5. Example 5-6-3.

is 0.0063 + 0.0042 = 0.0105, which corresponds to a stress of 224,000 psi, not 240,000 psi as previously assumed.

Next assuming a stress of 224,000 psi in the steel,

$$T' = 1.5 \times 224{,}000 = 336{,}000 \text{ lb}$$

$$k'd = \frac{336{,}000}{4250 \times 12} = 6.6 \text{ in.}$$

$$\epsilon_{s2} = 0.0034 \frac{13.4}{6.6} = 0.0070$$

$$\epsilon = 0.0070 + 0.0042 = 0.0112$$

which corresponds to a stress of 225,000 psi. This is close enough to the assumed value. Hence, the ultimate moment is

$$1.5 \times 225{,}000 \times (20 - 6.6/2) = 5{,}630{,}000 \text{ in.-lb}$$

which is about 5% lower than the more approximate value obtained in example 5-6-1.

In the above example, it is seen that about 95% of the ultimate strength of steel is developed at rupture. Appreciable elongation of steel and consequently considerable deflection and cracking of the beam would have taken place before rupture. This is not considered as an over-reinforced beam although the ultimate deflection would

be much greater if the percentage of steel were lowered. A beam would be called over-reinforced when the value of k' was greater than about 0.5. Such a condition can be readily detected by equating the concrete strength above the mid-depth to about 85% of the ultimate strength of the steel reinforcement. For a rectangular section, we have

$$0.85f_s'A_s = 0.5 \times 0.85f_c'bd$$

$$p = A_s/bd = 0.5f_c'/f_s'$$

For $f_c' = 5000$, $f_s' = 250{,}000$ psi, we have $p = 1\%$ as the limiting value.

It must again be stated that there is no abrupt change from an under-reinforced beam to an over-reinforced one. The change in mode of failure is a gradual one as the percentages of steel are increased. It must be further added that the above formulas should be modified if some of the tendons are positioned far away from the c.g.s. of the section, since the stress in such tendons will be quite different at the rupture of the section.

3 · Unbonded Beams. An accurate calculation for the ultimate strength of unbonded beams is more difficult than for that of bonded ones, because the stress in the steel at rupture of the beam cannot be closely computed. Also there have not been sufficient data on the ultimate strength of unbonded beams to establish definitely a reliable method of computation. It is agreed, however, that unbonded beams are weaker than the corresponding bonded ones in their ultimate strength, the difference being placed at 10–30%.

Explanations have been offered for the lower strength of unbonded beams. First, since the tendon is free to slip, the strain at the critical section is lessened and the stress is uniform along the entire length. Hence the stress in the tendon is increased only slowly so that, when the crushing strain has been reached in the concrete, stress in the steel is often far below its ultimate strength. When there are no cracks in the beam, stress in steel can be computed as in solution 2, example 5-4-1. As soon as part of the beam cracks or is stretched into the plastic range, the stress cannot be conveniently calculated. For the purpose of design, however, it may be possible to estimate the stress in the steel at the rupture of the beam and to compute the corresponding lever arm so as to approximate the ultimate resisting moment. Until further test data are available, such estimation may often err by 10–15%. Fortunately, unbonded beams are not often used where ultimate strength is an important consideration, and they

are generally designed for the working loads by the elastic theory rather than for the ultimate load.

Another reason for the lower ultimate strength of unbonded beams is the appearance of a few large cracks in the concrete instead of many small ones well distributed. Such wide cracks tend to concentrate the strains in the concrete at these sections, thus resulting in premature failure. However, authorities differ in this belief, and definite conclusions cannot be reached until more extensive tests are performed.

Some tests tend to prove that the ultimate strength of unbonded beams can be materially increased by the employment of a limited amount of non-prestressed steel. Such increase has been attributed to the resistance of the non-prestressed steel itself as well as to its effect in distributing the cracks in the concrete. This will be discussed in Chapter 11.

5-7 Composite Sections

In prestressed-concrete construction, it is often advantageous to precast part of a section (either by pre-tensioning or by post-tensioning), lift it to position, and cast the remainder of the section in place.

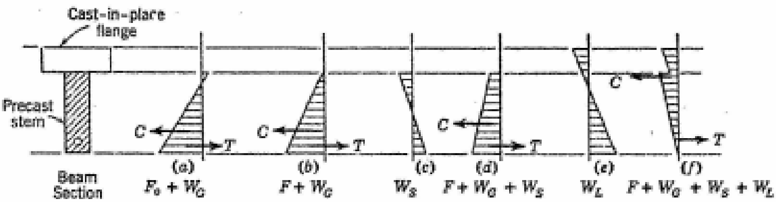

Fig. 5-7-1. Stress distributions for a composite section.

The precast and cast-in-place portions thus act together (with proper keys if necessary) and form a composite section. Members of composite sections laid side by side may be eventually connected together by transverse prestressing, while such members laid end to end may be further prestressed longitudinally in order to attain continuity. These points will be discussed in later chapters. We shall first describe here the basic method of analysis commonly employed for such composite sections.

Figure 5-7-1 shows a composite section at the midspan of a simply supported beam, whose lower stem is precast and lifted into position with the top slab cast in place resting directly on the stem. If no

temporary intermediate support is furnished, the weight of both the slab and the stem will be carried by the stem acting alone. After the slab concrete has hardened, the composite section will carry any live or dead load that may be added on to it.

In the same figure, stress distributions are shown for various stages of loading. These are discussed as follows:

(*a*) Owing to the initial prestress and the weight of the stem, there will be heavy compression in the lower fibers and possibly some small tension in the top fibers. The tensile force T in the steel and the compressive force C in the concrete form a resisting couple with a small lever arm between them.

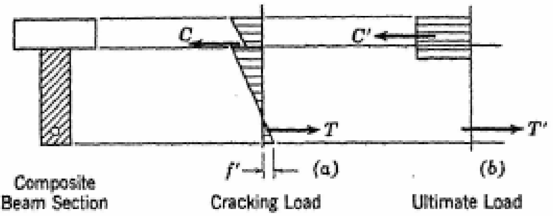

Fig. 5-7-2. Stress distributions for cracking and ultimate loads.

(*b*) After losses have taken place in the prestress, the effective prestress together with the weight of the stem will result in a slightly lower compression in the bottom fibers and some small tension or compression in the top fibers. The C–T couple will act with a slightly greater lever arm.

(*c*) Owing to the addition of the slab, its weight produces additional moment and stresses as shown.

(*d*) Owing to the effective prestress plus the weight of the stem and slab, we can add (*b*) to (*c*), and a somewhat smaller compression is found to exist at the bottom fibers and some compression at the top fibers. The lever arm for the C–T couple further increases.

(*e*) Stresses resulting from live load moment are shown, the moment being resisted by the composite section.

(*f*) Adding (*d*) to (*e*), we have stress block as in (*f*), with slight tension or compression in the bottom fibers, but with high compressive stresses in the top fibers of the stem and the slab. The couple T and C now acts with an appreciable lever arm.

The above shows the stress distribution under working load conditions. For overloads, the stress distributions are shown in Fig. 5-7-2. For the load producing first cracks, it is assumed that the lower fibers reach a tensile stress equal to the modulus of rupture.

This is obtained when the live-load stresses shown in Fig. 5-7-1 (e) are big enough to result in a stress distribution as shown in Fig. 5-7-2 (a), computed by the elastic theory.

Under the ultimate moment, however, the elastic theory is no longer nearly correct. As an approximation, the ultimate resisting moment is best represented by a tensile force T almost equal to the ultimate strength of the steel acting with a compressive force C supplied by the concrete. If failure in bond and shear is prevented, the ultimate strength of a composite section can be estimated by a method similar to that previously described for a simple prestressed section. It must be emphasized, however, that a composite section may fail in horizontal shear between the precast and the cast-in-place portions, unless proper keys or connectors are provided.

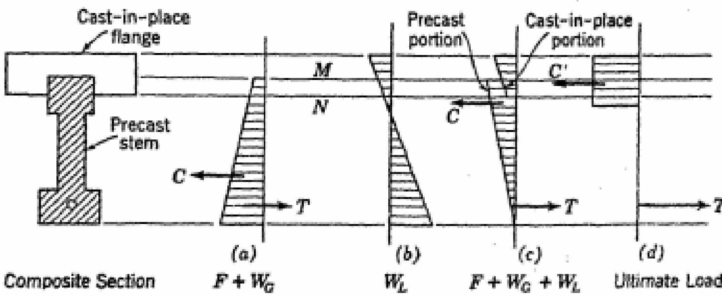

Fig. 5-7-3. Stress distribution for a special composite section.

The above describes a simple case of composite action; there are many possible variations. First, the precast portion may be supported on falsework while the cast-in-place slab is being poured or placed, the falsework being removed only after the hardening of the slab concrete. This will permit the entire composite section to resist the moment produced by the weight of the slab. It is also possible to prop up the falsework so that the stem will carry practically no moment by itself. Then the moments due to the weight of the stem will also be carried by the composite section. Since the composite section has a greater section modulus than the stem alone, the resulting stresses will be more favorable. The desirability of such methods depends upon the cost of falsework for the particular structure.

Another variation happens when the cast-in-place slab overlaps with the precast portion as shown in Fig. 5-7-3. Here, the stresses in the concrete between levels M and N will follow two different variations, as shown in (c), one for the precast and another for the cast-in-

place portion. At the ultimate range, however, they will all be stressed to the maximum and the difference will be hardly noticeable. Then the section can be analyzed as if it were a simple one, Fig. 5-7-3 (*d*).

If the precast portion is only a small part of the whole section, it may be prestressed for direct tension only, or with a slight eccentricity of prestress. One method used in England (known as the Udall system), Fig. 5-7-4, employs both prestressed and non-prestressed wires in the groove of precast blocks, with the major top portion cast in place so as to be well bonded to the wires. For such

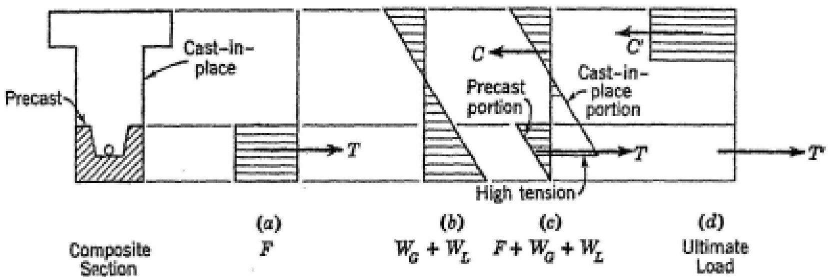

Fig. 5-7-4. Stress distribution for a special (Udall) composite section.

a construction, high tension may exist in the bottom fibers of the cast-in-place portion, (*c*), resulting in cracks under working load. But the ultimate strength in flexure is not affected by the tensile stresses, (*d*).

In other instances, the section is prestressed in two stages. Only part of the tendons are prestressed first in order to hold the stem together. The remaining tendons are prestressed after the slab has been cast and has hardened. If the process of retensioning is not too costly, this may result in an economical design. The stress distribution must be studied for the various stages, but the allowable stresses need not be the same as for an ordinary simple section. In certain instances, considerable tension may be permitted. The problem of different shrinkage of portions of a composite section also needs study. Since the portions are cast at different times, shrinkage and creep of one portion might induce stresses in another. Such stresses may or may not be serious, depending upon the conditions.

Example 5-7-1

The midspan section of a composite beam is shown in Fig. 5-7-5. The precast stem 12 in. by 36 in. deep is post-tensioned with an initial force of 550 kips, Fig. 5-7-5*a*. The effective prestress after losses is taken as 480 kips. Moment

due to the weight of that precast section is 200 k ft at midspan. After it is erected in place, the top slab of 6 in. by 36 in. wide is to be cast in place producing a moment of 100 k ft. After the slab concrete has hardened, the composite section is to carry a maximum live load moment of 550 k ft. Compute stresses in the section at various stages. $A_s = 3.7$ sq in. $f_s' = 240{,}000$ psi. $f_c' = 5000$ psi. Estimate the ultimate moment.

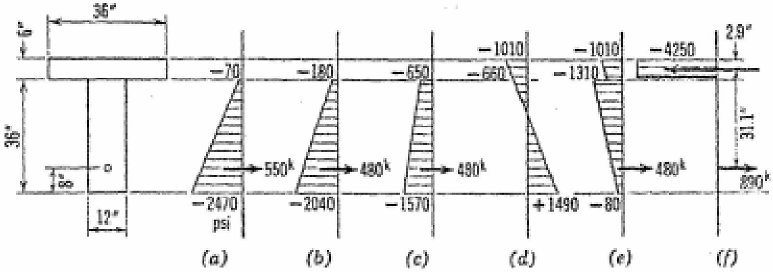

Fig. 5-7-5. Example 5-7-1.

Solution. C.g.c. of the composite section is located at 25 in from the bottom fiber. The area and moment of inertia of the rectangular and the composite sections are computed and listed below:

	Rectangular Section	Composite Section
Area, sq in.	432	648
I, in.^{4}	46,600	111,000

(*a*) Immediately after prestressing, the stresses in the rectangular section will be

$$f = \frac{F}{A} \pm \frac{(M - Fe)c}{I}$$

$$= \frac{-550{,}000}{432} \pm \frac{(200{,}000 \times 12 - 550{,}000 \times 10)18}{46{,}600}$$

$$= -1270 \pm 1200$$

$$= -70 \text{ psi top fiber}$$

$$= -2470 \text{ psi bottom fiber}$$

(*b*) After loss of prestress, the stresses will be

$$f = \frac{-480{,}000}{432} \pm \frac{(200{,}000 \times 12 - 480{,}000 \times 10)18}{46{,}600}$$

$$= -1110 \pm 930$$

$$= -180 \text{ psi top fiber}$$

$$= -2040 \text{ psi bottom fiber}$$

(*c*) After pouring of top slab, the stresses will be

$$= \frac{-480,000}{432} \pm \frac{(300,000 \times 12 - 480,000 \times 10)18}{46,600}$$

$$= -1110 \pm 460$$

$$= -650 \text{ psi top fiber}$$

$$= -1570 \text{ psi bottom fiber}$$

(*d*) The live load acts on the composite section, producing stresses,

$$f = \frac{-550,000 \times 12 \times 17}{111,000} = -1010 \text{ psi top fiber of composite section}$$

$$f = \frac{550,000 \times 12 \times 25}{111,000} = +1490 \text{ psi bottom fiber}$$

By proportioning, the stress at top fiber of the rectangular portion is found to be −660 psi due to this live load.

(*e*) The combined stresses due to prestress and dead and live loads are given in Fig. 5-7-5 (*e*), which yields −80 psi for bottom fiber and −1310 psi for top fiber of the rectangular section.

(*f*) The ultimate moment capacity of the section can be estimated as follows. Assume that ultimate strength of steel is developed; then total tensile force is

$$3.7 \times 240,000 = 890,000 \text{ lb}$$

Area of compression concrete, for an average stress of $0.85f_c' = 4250$ psi, is

$$\frac{890,000}{4250} = 210 \text{ sq in.}$$

or a width of $210/36 = 5.8$ in. The center of compressive force is about $5.8/2 = 2.9$ in. from top; hence the lever arm for the resisting moment is $42 - 2.9 - 8 = 31.1$ in., and the ultimate moment capacity is

$$890,000 \times 31.1/12,000 = 2310 \text{ k ft}$$

The total applied dead and live load moment is only 850 k ft, indicating a factor of safety of $2310/850 = 2.7$.

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Chapter

6 Design of Sections for Flexure

6-1 Preliminary Design

Preliminary design of prestressed-concrete sections for flexure can be performed by a very simple procedure, based on a knowledge of the internal C–T couple acting in the section. In practice the depth h of the section is either given, known, or assumed, as is the total moment

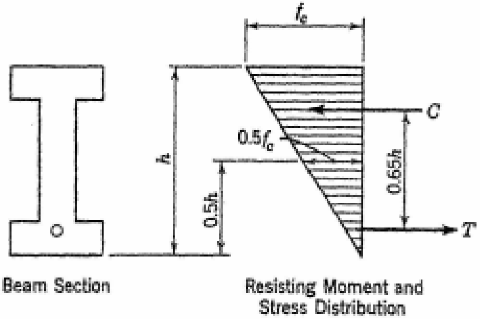


Fig. 6-1-1. Preliminary design of a beam section.

M_T on the section. Under the working load, the lever arm for the internal couple averages about 65% of the overall height h. Hence the required effective prestress F can be computed from the formula

$$F = T = \frac{M_T}{0.65h} \tag{6-1-1}$$

assuming the lever arm to be $0.65h$, Fig. 6-1-1. If the effective unit prestress is f_s for the steel, then the area of steel required is

$$A_s = \frac{F}{f_s} = \frac{M_T}{0.65hf_s} \tag{6-1-2}$$

The total prestress $A_s f_s$ is also the force C on the section. This force will produce an average unit stress on the concrete of

$$\frac{C}{A_c} = \frac{T}{A_c} = \frac{A_s f_s}{A_c}$$

For preliminary design, this average stress can be assumed to be about 50% of the maximum allowable stress f_c, under the working load. Hence,

$$\frac{A_s f_s}{A_c} = 0.50 f_c$$

$$A_c = \frac{A_s f_s}{0.50 f_c} \tag{6-1-3}$$

Note that, in the above procedure, the only approximations made are the coefficients of 0.65 and 0.50. These coefficients vary widely, depending upon the shape of the section. However, with some experience and knowledge, they can be closely approximated for each particular section, and the preliminary design can be made rather accurately.

The above procedure is based on the design for working load, with little or no tension in the concrete. Preliminary designs can also be made on the basis of ultimate strength theories with proper load factors. Such an alternative procedure will be discussed in section 6-8.

Example 6-1-1

Make a preliminary design for a section of a prestressed-concrete beam to resist a total moment of 320 k ft. The overall depth of the section is given as 36 in. The effective prestress for steel is 125,000 psi, and allowable stress for concrete under working load is −1600 psi.

Solution. From formulas 6-1-1, 6-1-2, and 6-1-3,

$$F = T = M_T/0.65h$$
$$= (320 \times 12)/(0.65 \times 36) = 164 \text{ k}$$
$$A_s = F/f_s = 164/125 = 1.31 \text{ sq in.}$$
$$A_c = 164/(0.5 \times 1.60) = 205 \text{ sq in.}$$

Now a preliminary section can be sketched with a total concrete area of about 205 sq in., a height of 36 in., and a steel area of 1.31 sq. in. Such a section is shown on Fig. 6-1-2. A T-section is chosen here because it is an economical shape when M_G/M_T ratio is large.

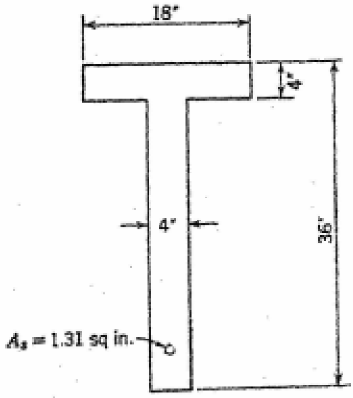


Fig. 6-1-2. Example 6-1-1.

In estimating the depth of the section, an approximate rule is to use 70% of the corresponding depth for conventional reinforced-concrete construction. Some other empirical rules are also available. For example, the thickness of prestressed slabs may vary from $L/35$ for heavy loads to $L/55$ for light loads. The depth of beams of the usual proportions can be approximated by the following formula:

$$h = k\sqrt{M}$$

where h = depth of beam in inches.
M = maximum bending moment in k ft.
k = a coefficient varying from 1.5 to 2.0.

It is needless to add that such empirical rules apply only under the average conditions and should be used merely as a preliminary guide.

A more accurate preliminary design can be made if the girder moment M_G is known in addition to the total moment M_T. When M_G is much greater than 20 to 30% of M_T, the initial condition under M_G generally will not control the design, and the preliminary design needs be made only for M_T. When M_G is small relative to M_T, then the c.g.s. cannot be located too far outside the kern point, and the design is controlled by $M_L = M_T - M_G$. In this case, the resisting lever arm for M_L is given approximately by $k_t + k_b$, which averages about $0.50h$. Hence the total effective prestress required is

$$F = \frac{M_L}{0.50h} \tag{6-1-4}$$

When M_G/M_T is small, this formula should be used instead of equation 6-1-1. Formulas 6-1-2 and 6-1-3 are still applicable.

Example 6-1-2

Make a preliminary design for the beam section in example 6-1-1, with $M_T = 320$ k ft, $M_G = 40$ k ft, $h = 36$ in., $f_s = 125{,}000$ psi, and $f_s = -1600$ psi.

Solution. Since M_G is only 12% of M_T, it is not likely that the c.g.s. can be located much outside the kern. Hence it will be more nearly correct to apply formula 6-1-4. Thus,

$$M_L = M_T - M_G = 320 - 40$$
$$= 280 \text{ k ft}$$
$$F = M_L/0.50h = 280 \times 12/(0.50 \times 36)$$
$$= 187 \text{ k}$$

Applying formulas 6-1-2 and 6-1-3, we have

$$A_s = F/f_s = 187/125$$
$$= 1.50 \text{ sq in.}$$

$$A_c = A_s f_s / 0.50 f_c = 187/(0.50 \times 1.60)$$
$$= 234 \text{ sq in.}$$

Now a preliminary section can be sketched with a total concrete area of about 234 sq in., a height of 36 in., and a steel area of 1.50 sq in., as shown in Fig. 6-1-3. An I-section is chosen because it is a suitable form when the M_G/M_T ratio is small.

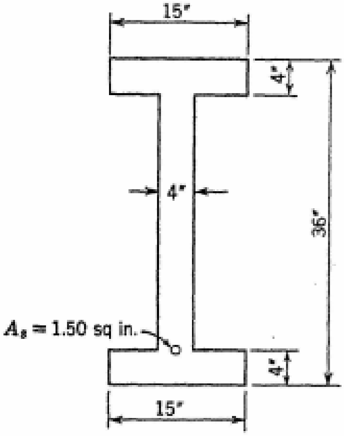


Fig. 6-1-3. Example 6-1-2.

When it is not known whether M_T or M_L should govern the design, one convenient way is to apply both equations 6-1-1 and 6-1-4, and use the greater of the two values of F. For example, if $M_G = 80$ k ft in example 6-1-1, we have, from equation 6-1-1,

$$F = M_T/0.65h$$
$$= (320 \times 12)/(0.65 \times 36)$$
$$= 164 \text{ k}$$

From equation 6-1-4 we have

$$F = M_L/0.50h$$
$$= [(320 - 80)12]/(0.50 \times 36)$$
$$= 160 \text{ k}$$

$F = 164$ k controls the design.

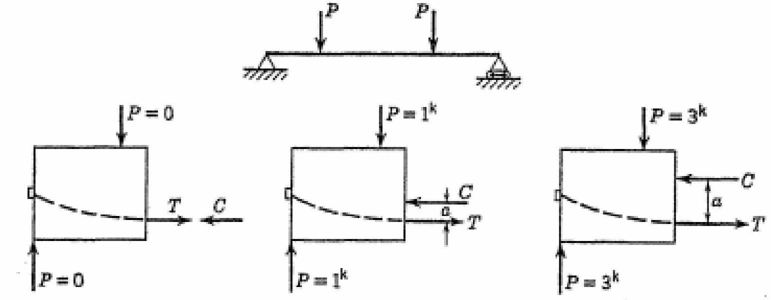
6-2 Elastic Design, General Concepts

There is a prevailing impression that the design of prestressed-concrete sections is much more complicated than that of reinforced ones. This is not true if the procedure recommended in this chapter is followed. However, the design of a section is based on a knowledge of its analysis. Hence readers must be familiar with the methods of analysis discussed in the previous chapter before they can master the methods of design.

The method of elastic design presented in section 6-1 is based on the fact that the section is governed by two controlling values of external bending moment: the total moment M_T, which controls the stresses under the action of the working loads; and the girder load moment M_G, which determines the location of the c.g.s. and the stresses at transfer.

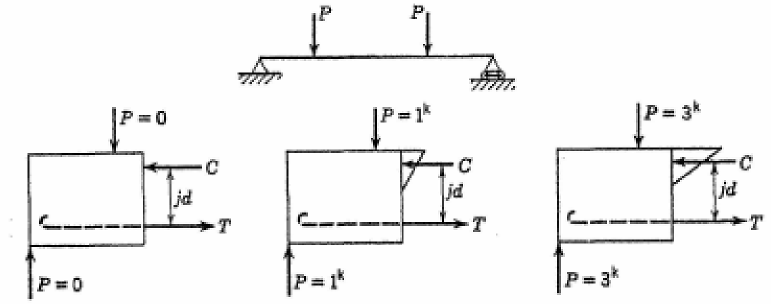
It is desirable to reiterate here the basic concept of a resisting couple in a prestressed-concrete-beam section. From the law of statics, the internal resisting moment in a prestressed beam, as in a reinforced-concrete beam, must equal the external moment. That internal moment can be represented by a couple, C–T, for either the prestressed- or the reinforced-concrete-beam section, Figs. 6-2-1 and 6-2-2. T is

the centroid of the prestress or tensile force in the steel; and C is the center of pressure or the center of compression on the concrete.



(*a*) External Moment = 0, $a = 0$ (*b*) Small External Moment, a is small. (*c*) Large External Moment, a is large.

Fig. 6-2-1. Variable a in a prestressed-concrete beam.



(*a*) External Moment = 0, $C = T = 0$. (*b*) Small External Moment, small C and T. (*c*) Large External Moment, large C and T.

Fig. 6-2-2. Constant jd in a reinforced-concrete beam.

There is, however, an essential difference between the behavior of a prestressed- and of a reinforced-concrete-beam section. The difference is explained as follows:

1. In a reinforced-concrete-beam section, as the external bending moment increases, the magnitude of the forces C and T is assumed to increase in direct proportion while the lever arm jd between the two forces remains unchanged, Fig. 6-2-2.

2. In a prestressed-concrete-beam section under working load, as the external bending moment increases, the magnitude of C and T remains practically constant while the lever arm a lengthens almost proportionately, Fig. 6-2-1.

Since the location of T remains fixed, we get a variable location of C in a prestressed section as the bending moment changes. For a given moment M, C can be easily located, since

$$Ca = Ta = M \tag{6-2-1}$$

$$a = M/C = M/T \tag{6-2-1a}$$

Thus, when $M = 0$, $a = 0$, and C must coincide with T, Fig. 6-2-1 (a). When M is small, a is also small, Fig. 6-2-1 (b). When M is large, a is also large, Fig. 6-2-1 (c).

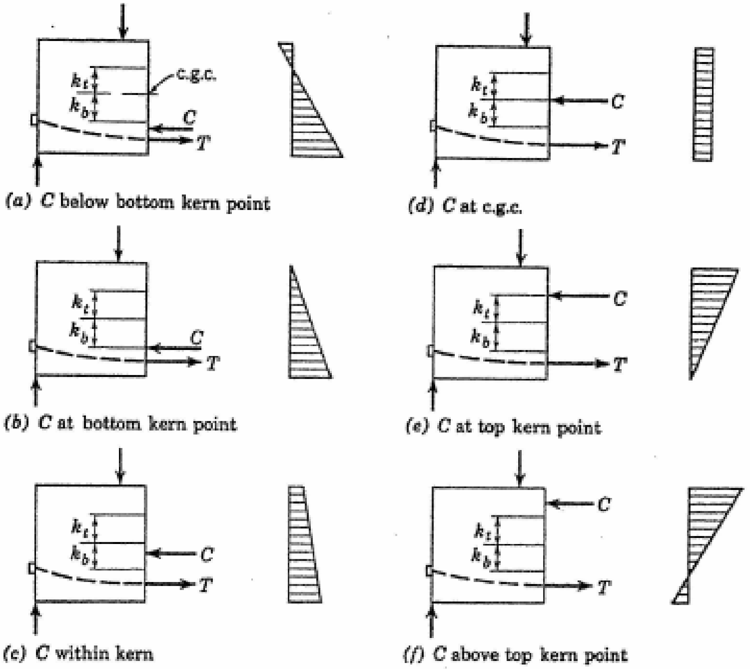


Fig. 6-2-3. Stress distribution in concrete by the elastic theory.

In a prestressed-concrete beam, the amount of initial prestress F_0 is measured and is rather accurately known. At the time of transfer of prestress, $T = F_0$. After all losses have taken place, $T = F$. Although the value of T does change as the beam bends under loading, the change is small within the working range and can be neglected in design.

Once the magnitude of T is known, the value of a can be computed from equation 6-2-1a for any value of M. The location of C can thus

be determined. With the position and magnitude of C known, stress distribution across the concrete section can be obtained either by the elastic or the plastic theory, although only the elastic theory is usually followed.

It will be well to mention some of the simple relations between stress distribution and the location of C, according to the elastic theory, Fig. 6-2-3. If C coincides with the top or bottom kern point, stress distribution will be triangular, with zero stress at bottom or top fiber, respectively. If C falls within the kern, the entire section will be under compression; if outside the kern, some tension will exist. If C coincides with c.g.c., stress will be uniform over the entire concrete section. (See Appendix A for k_t and k_b, defining the kern.)

In the actual design of prestressed-concrete sections, similar to any other type of section, a certain amount of trial and error is inevitable. There is the general layout of the structure which must be chosen as a start but which may be modified as the process of design develops. There is the dead weight of the member which influences the design but which must be assumed before embarking on the moment calculations. There is the approximate shape of the concrete section, governed by both practical and theoretical considerations, which must be assumed for the trial. Because of these variables, it has been found that the best procedure is one of trial and error, guided by known relations which enable the final results to be obtained without excessive work.

6-3 Elastic Design, No Tension in Concrete

In this section will be discussed the final flexural design of sections based on the elastic theory and allowing no tension in the concrete both at transfer and under working load. Two cases will be considered, one for small and one for large ratios of M_G/M_T.

Case I · Small Ratios of M_G/M_T. For the section obtained from the preliminary design, the values of M_G, k_t, k_b, A_c are computed. When the ratio of M_G/M_T is small, c.g.s. is located outside the kern just as much as the M_G will allow. Since no tension is permitted in the concrete, c.g.s. will be located below the kern by the amount of Fig. 6-3-1 (b)

$$e - k_b = M_G/F_0 \tag{6-3-1}$$

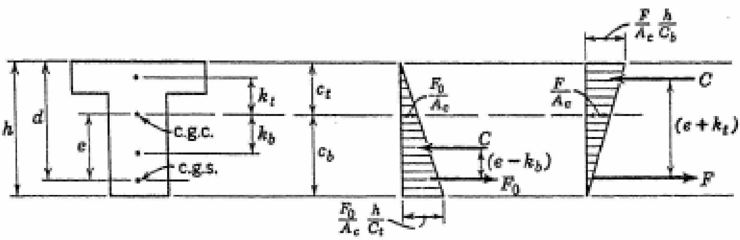
If c.g.s. is so located, C will be exactly at the bottom kern point for the given M_G, and the stresses at the top and bottom fibers will be

$$f_t = 0$$

$$f_b = \frac{F_0}{A_c}\frac{h}{c_t} \tag{6-3-2}$$

$$A_c = \frac{F_0 h}{f_b c_t} \tag{6-3-2a}$$

If c.g.s. is located farther up, C will fall within the kern; then the top fibers will be under some compression, and the bottom fibers will be



(a) Section Properties (b) Just after Transfer C at bottom kern point (c) Under Working Load C at top kern point

Fig. 6-3-1. Stress distribution, no tension in concrete, case 1.

stressed less than given by equation 6-3-2. If c.g.s. is located farther below, C will fall outside the kern; then there will be some tension in the top fibers, and the bottom fibers will be stressed higher than given by equation 6-3-2.

With c.g.s. located as above, the available lever arm for the resisting moment is given by $e + k_t$, and the effective prestress F is given by

$$F = \frac{M_T}{e + k_t} \tag{6-3-3}$$

Under the action of this effective prestress F and the total moment M_T, C will be located at the top kern point, and the top and bottom fiber stresses are, Fig. 6-3-1 (c),

$$f_t = \frac{F}{A_c}\frac{h}{c_b} \tag{6-3-4}$$

$$A_c = \frac{Fh}{f_t c_b} \tag{6-3-4a}$$

$$f_b = 0$$

If F is smaller than the value given by equation 6-3-3, there will be tension in the bottom fibers and the compressive stress in the top fibers will be greater than that given by equation 6-3-4; if F is greater, there will be some residual compression in the bottom fibers and the compressive stress in the top fibers will be less than that indicated by equation 6-3-4.

If f_b or f_t exceeds the allowable value, it will be necessary to increase the area of concrete A_c, or to decrease the ratio of h/c_t or h/c_b respectively. If f_b and f_t are both less than the respective allowable values, A_c can be decreased accordingly. Slight changes in the dimensions of the section may not affect the k_t, k_b, and other values. But if major changes are made, it may be desirable to go over the procedure once more to obtain a new location for the c.g.s. and compute new values for F and check over the required A_c.

To summarize the procedure of design, we have:

Step 1. From the preliminary design section, locate c.g.s. by

$$e - k_b = M_G/F_0$$

Step 2. With the above location of c.g.s., compute the effective prestress F (and then the initial prestress F_0) by

$$F = \frac{M_T}{e + k_t}$$

Step 3. Compute the required A_c by

$$A_c = F_0 h/f_b c_t$$

and

$$A_c = Fh/f_t c_b$$

Step 4. Revise the preliminary section to meet the above requirements for F and A_c. Repeat steps 1 through 4 if necessary.

From the above discussion, the following observations regarding the properties of a section can be made:

1. $e + k_t$ is a measure of the total moment-resisting capacity of the beam section. Hence, the greater this value, the more desirable is the section.

2. $e - k_b$ locates the c.g.s. for the section, and is determined by the value of M_G. Thus, within certain limits, the amount of M_G does not seriously affect the capacity of the section for carrying M_L.

3. h/c_b is the ratio of the maximum top fiber stress to the average stress on the section under working load. Thus, the smaller this ratio, the lower will be the maximum top fiber stress.

4. h/c_t is the ratio of the maximum bottom fiber stress to the average stress on the section at transfer. Hence, the smaller this ratio, the lower will be the maximum bottom fiber stress.

To facilitate design computations, properties of different sections are listed in Appendix C, Tables 6-2-1 through 6-2-6. Values of A_c, I, k_t, k_b, c_t, c_b, etc., are given in these tables. Properties for a rectangular section are included in Table 6-2-1 under the headings $b'/b = t/h =$

1, i.e., section 1-q By the use of the above formulas and the tables, it is possible to develop formulas which will give directly the required section modulus for a given shape. But for a practical design, it is generally preferable to follow a method of trial and error as outlined above, because dimensioning and other practical considerations do not often permit keeping to an assumed shape of section.

Example 6-3-1

For the preliminary section obtained in example 6-1-2, make a final design, allowing $f_b = 1.80$ ksi, $f_0 = 150$ ksi. Other given values were: $M_T = 320$ k ft; $M_G = 40$ k ft; $f_t = -1.60$ ksi; $f_s = 125$ ksi; $F = 187$ k. And the preliminary section is the same as in Fig. 6-1-3.

Solution. For the trial preliminary section, compute the properties as follows:

$$A_c = 2 \times 4 \times 15 + 4 \times 28 = 232 \text{ sq in.}$$

$$I = \frac{15 \times 36^3}{12} - \frac{11 \times 28^3}{12}$$

$$= 58{,}200 - 20{,}100$$

$$= 38{,}100 \text{ in.}^4$$

$$r^2 = 38{,}100/232$$

$$= 164 \text{ in.}^2$$

$$k_t = k_b = 164/18 = 9.1 \text{ in.}$$

Step 1. For an assumed

$$F = 187 \text{ k}$$

$$F_0 = \frac{150}{125} 187 = 225 \text{ k}$$

c.g.s. should be located at $e - k_b$ below the bottom kern, where

$$e - k_b = \frac{M_G}{F_0} = \frac{40 \times 12}{225} = 2.1 \text{ in.}$$

$$e = 9.1 + 2.1 = 11.2 \text{ in.}$$

Step 2. Effective prestress required is recomputed as

$$F = \frac{M_T}{e + k_t} = \frac{320 \times 12}{11.2 + 9.1}$$

$$= 189 \text{ k}$$

$$F_0 = \frac{150}{125} 189 = 227 \text{ k}$$

Step 3. A_c required is

$$A_c = \frac{F_0 h}{f_b c_t}$$

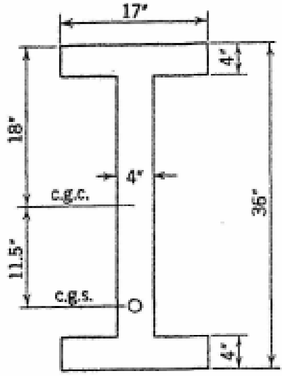


Fig. 6-3-2. Example 6-3-1.

$$= \frac{227 \times 36}{1.80 \times 18}$$

$= 252$ sq in. controlling

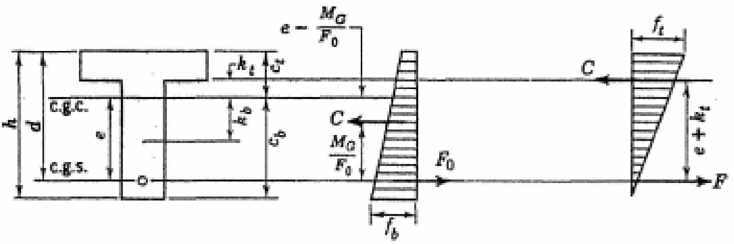
$$A_c = \frac{Fh}{f_t c_b}$$

$$= \frac{189 \times 36}{1.60 \times 18}$$

$= 236$ sq in.

Step 4. Try a new section as shown in Fig. 6-3-2, with $A_c = 248$ sq in. For this new section, $I = 42{,}200$ in.^{4}; $k_t = k_b = 9.4$ in.; $e - k_b = 2.1$ in.; $F = 320 \times 12/(11.5 + 9.4) = 184$ k; $F_0 = 221$ k; A_c required for bottom fiber $= 246$ sq in., for top fiber $= 230$ sq in. Hence the section seems to be quite satisfactory. And no further revision is needed.

Case 2 · Large Ratios of M_G/M_T. When the ratio of M_G/M_T is large, the value of $e - k_b$ computed from equation 6-3-1 may place c.g.s. outside of the practical limit, e.g., below the section of the beam.



(*a*) Section Properties (*b*) Just after Transfer, *C* above bottom kern point (*c*) Under Working Load, *C* at top kern point

Fig. 6-3-3. Stress distribution, no tension in concrete, case 2.

Then it is necessary to place the c.g.s. only as low as practicable and design accordingly.

For such a condition, the bottom fiber stress is seldom critical. Under the initial condition, just after transfer, the bottom fiber stress is shown in Fig. 6-3-3 (b) and is given by the formula

$$f_b = \frac{F_0}{A_c} + \frac{(F_0 e - M_G)c_b}{I}$$

$$= \frac{F_0}{A_c}\left(1 + \frac{e - (M_G/F_0)}{k_t}\right)$$

from which the required area A_c can be computed as

$$A_c = \frac{F_0}{f_b}\left(1 + \frac{e - (M_G/F_0)}{k_t}\right) \qquad (6\text{-}3\text{-}5)$$

The top fiber is always under some compression and does not control the design under this condition.

Under the working load, the stress distribution is the same as in Case 1, and is pictured in Fig. 6-3-3 (c). The design is practically the same as in Case 1 except that equation 6-3-5 should be used in place of equation 6-3-2a. For convenience, the procedure will be outlined as follows:

Step 1. From the preliminary section, compute the theoretical location for c.g.s. by

$$e - k_b = M_G/F_0$$

If it is feasible to locate c.g.s. as indicated by this equation, follow procedure for Case 1. If not, locate c.g.s. at the practical lower limit and proceed as follows.

Step 2. Compute F (and then F_0) by

$$F = \frac{M_T}{e + k_t}$$

Step 3. Compute the required area by equation 6-3-4a and equation 6-3-5.

$$A_c = Fh/f_t c_b$$

$$A_c = \frac{F_0}{f_b}\left(1 + \frac{e - (M_G/F_0)}{k_t}\right)$$

Step 4. Use the greater of the two A_c's and the new value of F, and revise the preliminary section. Repeat steps 1 through 4 if necessary.

Example 6-3-2

Make final design for the preliminary section obtained in example 6-1-1, M_G = 210 k ft, allowing $f_b = -1.80$ ksi, $f_0 = 150$ ksi. Other values given were M_T = 320 k ft; $h = 36$ in.; $f_s = 125$ ksi; $f_t = -1.60$ ksi. The preliminary section is shown in Fig. 6-3-4, with $A_c = 200$ sq in., $c_t = 13.5$ in., $c_b = 22.5$ in., $I = 26{,}000$ in.^{4}, $k_t = 5.8$ in., $k_b = 9.6$ in., $F = 164$ k, $F_0 = 164(150/125) = 197$ k.

Solution. Step 1. Theoretical lowest location for c.g.s. is given by

$$\begin{aligned} e - k_b &= M_G/F_0 \\ &= (210 \times 12)/197 \\ &= 12.8 \text{ in.} \end{aligned}$$

indicating 12.8 in. below the bottom kern, or 0.1 in. above the bottom fiber, which is obviously impossible. Suppose that for practical reasons the c.g.s. has to be kept 3 in. above the bottom fiber to provide sufficient concrete protection. This problem then belongs to Case 2, and we proceed as below.

Step 2. The effective prestress required is, corresponding to a lever arm of

$$22.5 - 3 + 5.8 = 25.3 \text{ in.,}$$

$$F = (320 \times 12)/25.3 = 152 \text{ k}$$

$$F_0 = 152(150/125) = 182 \text{ k}$$

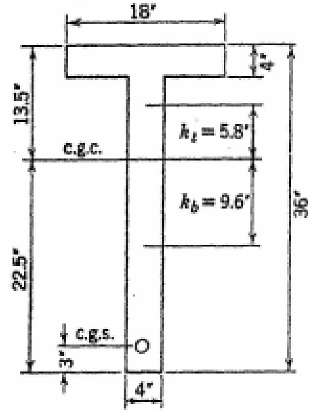


Fig. 6-3-4. Example 6-3-2. Trial section.

Step 3. Compute the area required by

$$A_c = \frac{Fh}{f_t c_b}$$

$$= \frac{152 \times 36}{1.60 \times 22.5}$$

$$= 152 \text{ sq in.}$$

$$A_c = \frac{F_0}{f_b}\left(1 + \frac{e - (M_G/F_0)}{k_t}\right)$$

$$= \frac{182}{1.80}\left(1 + \frac{19.5 - 210 \times 12/182}{5.8}\right)$$

$$= 199 \text{ sq in.}$$

which indicates that the trial preliminary section with $A_c = 200$ sq in. is just about right for the stress in the bottom fibers, but much more than enough as far as the top fibers are concerned. In other words, if practical conditions permit, it may be desirable to reduce the concrete area in the top flange and to add concrete area to the bottom flange, to obtain a more economical section. The reader may try this out to see whether a better section is obtainable for this example.

6-4 Elastic Design, Remarks on Allowing Tension

The above section discusses the design of prestressed-concrete sections allowing no tensile stresses. This requirement may sometimes be an extravagance that cannot be justified. When compared to reinforced concrete, where high tensile stresses and cracks are always present under working load, it seems only logical that at least some tensile stresses should be permitted in prestressed concrete. On the other hand, there are several reasons for limiting the tensile stresses in prestressed concrete. These are:

1. The existence of tensile stress in prestressed concrete may indicate an insufficient factor of safety against ultimate failure. When high tensile stress exists in prestressed concrete, the working lever arm a for the resisting couple is a large ratio of h, Fig. 6-4-1, so that no substantial increase in the lever arm can take place in case of overloads.

Thus the margin of safety is not as high as when no tension is permitted.

2. The existence of tensile stress may indicate an insufficient factor of safety against cracking and may easily result in cracking if the concrete has been previously cracked. Although cracking may not be significant under static load, it could be an important criterion when a member is subject to repeated loads. Cracking also signifies a

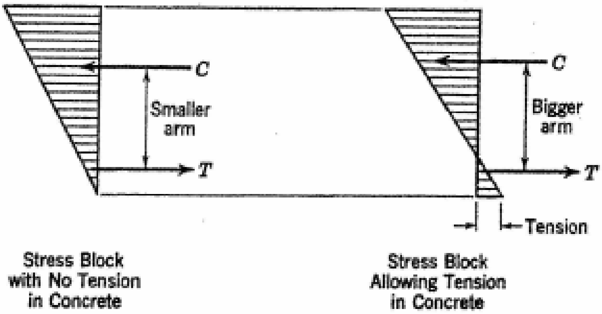


Fig. 6-4-1. Bigger arm for steel when allowing tension in concrete.

change in the nature of bond and shearing stresses. Furthermore, it is sometimes believed that the small wires in prestressed concrete are more susceptible to corrosion in the event of permanent cracks, although opening of cracks under passing loads is seldom held as contributing to corrosion to any significant extent.

3. If tension in the concrete constitutes a major portion of the total tensile force in the internal resisting couple, the cracking of concrete may result in total collapse of the beam without advance warning. Hence it is often believed that, although tension can be permitted in concrete, it should not be considered in computing the internal resisting couple.

4. The original idea of prestressing concrete was to produce a new material out of concrete, by putting it permanently under compression. Any excessive tensile stress might result in cracks and hence violate this basic idea of prestressed concrete.

Since none of the above reasons are absolutely correct, tensile stresses can apparently be permitted in prestressed concrete when the conditions warrant. This is especially true if mild steel can be used to reinforce the tensile portions. When tensile stresses are permitted under working loads, the term "partial prestressing" is often employed, indicating that the concrete is only partially compressed by the prestress. It is the author's opinion that there is really no basic difference

between partial and full prestress. The only difference is that, in partial prestressing, there exists a certain amount of tension in concrete under working loads. Since most structures are subject to occasional overloads, tensile stresses will actually exist in both partial and full prestressing, at one time or another. Hence there is no basic difference between them.

It is often argued that the allowing of tension in concrete is a dangerous procedure, since the concrete might have cracked previously and could not take any tension. This is true if the tensile

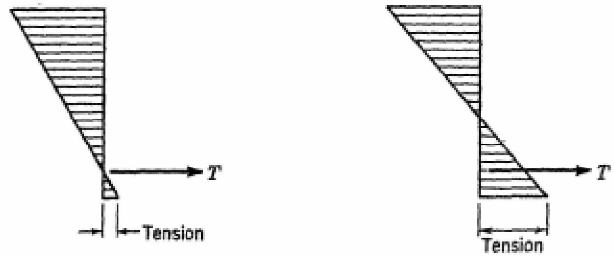


Fig. 6-4-2. Relative significance of tension in concrete.

force in concrete is a significant portion of the tensile force in the steel, in which event it will be necessary to neglect tensile force furnished by the concrete, Fig. 6-4-2. On the other hand, if the tensile force in the concrete is only a small proportion of that in steel, the calculations will not be very different whether it is neglected or included.

In the following two sections, two methods of design will be presented, both allowing tension in the concrete. For the first method, tensile force in concrete will be neglected in the computation, assuming all concrete under tension to be cracked. This is believed to be a safe method of design, provided that the tensile stress allowed in concrete is not excessive. The second method of design takes into account the tensile force in concrete, assuming uncracked sections. This gives a better representation of the actual stress distribution before cracking, but it is not conservative enough unless the portion of the tensile force taken by concrete is small or the cracking of concrete is prevented at all times. It is, however, a convenient method of design and gives practically the same results as the first method when the tensile force in concrete is relatively small.

6-5 Elastic Design, Allowing but Neglecting Tension

Case 1 · Small Ratios of M_G/M_T. Designing by this method will yield steel areas and sometimes concrete areas somewhat less than when tension is not allowed. It is usually best first to make a design by the method in section 6-3, allowing no tension. This will give a fairly close section to start with. Using this as a trial section, the portion under compression or the uncracked portion at transfer can be assumed as in Fig. 6-5-1 (*a*), with the allowable tensile stress on

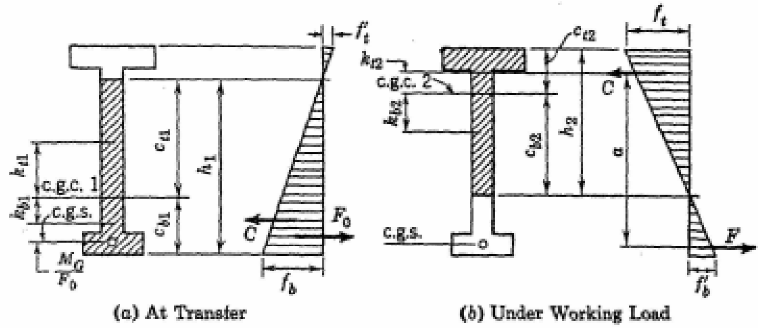


Fig. 6-5-1. Allowing but neglecting tension in concrete.

top fiber f_t' and the allowable compressive stress on bottom fiber f_b shown thereon. For this uncracked portion with depth h_1 compute the section properties, A_{c1}, I_1, c_{t1}, c_{b1}, k_{t1}, k_{b1}. The c.g.s. can now be located below the bottom kern of the uncracked portion by the amount of M_G/F_0.

Under the working load, if some tension f_b' is permitted in the bottom fibers, the uncracked portion with depth h_2 can be assumed as shown in Fig. 6-5-1 (*b*). The properties of this second uncracked portion can be computed as A_{c2}, I_2, c_{t2}, c_{b2}, k_{t2}, k_{b2}. The center of compression is located at the top kern, and the total lever arm for the internal resisting moment is a as shown in the figure. Thus the required value of effective prestress is computed as

$$F = M_T/a \tag{6-5-1}$$

The bottom fiber stress under the initial condition at transfer is given by

$$f_b = \frac{F_0 h_1}{A_{c1} c_{t1}} \tag{6-5-2}$$

For an allowable stress of f_b, the required area A_{c1} is given by

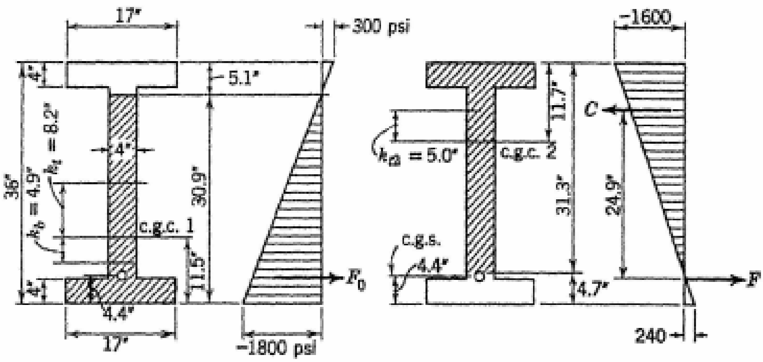
$$A_{c1} = \frac{F_0 h_1}{f_b c_{t1}} \tag{6-5-2a}$$

The top fiber stress under working load is given by

$$f_t = \frac{F h_2}{A_{c2} c_{b2}} \tag{6-5-3}$$

and similarly, for an allowable stress of f_t, the required A_{c2} is

$$A_{c2} = \frac{F h_2}{f_t c_{b2}} \tag{6-5-3a}$$



(a) At Transfer (b) Under Working Load

Fig. 6-5-2. Example 6-5-1.

From the above four equations, checking can be done either for the stresses produced or for the areas to be furnished.

Design by this method is illustrated in the following example.

Example 6-5-1

Redesign the section in example 6-3-1, allowing tension in concrete: $f_t' = 0.30$ ksi, $f_b' = 0.24$ ksi. Other values given for the trial section were: $M_T = 320$ k ft, $M_G = 40$ k ft, $f_t = -1.60$ ksi, $f_b = -1.80$ ksi, $F = 184$ k, $F_o = 221$ k.

Solution. *Step* 1. Assuming the bottom fiber stress at transfer to be $f_b = -1.80$ ksi and the top fiber stress to be 0.30 ksi, the uncracked portion is shown in Fig. 6-5-2 (*a*). Properties of the uncracked portion are:

$$A_{c1} = 4 \times 17 + 26.9 \times 4 = 68 + 108 = 176 \text{ sq in.}$$

$$c_{b1} = \frac{68 \times 2 + 108 \times 17.5}{176} = 11.5 \text{ in.}$$

$$I_1 = 68\left(\frac{4^2}{12} + 9.5^2\right) + 108\left(\frac{26.9^2}{12} + 6^2\right)$$

$$= 6200 + 10{,}400 = 16{,}600 \text{ in.}^4$$

$$r_1^2 = 16{,}600/176 = 94.3 \text{ in.}^2$$

$$k_{t1} = 94.3/11.5 = 8.2 \text{ in.}$$

$$k_{b1} = 94.3/19.4 = 4.9 \text{ in.}$$

$$M_G/F_0 = 40 \times 12/221 = 2.2 \text{ in.}$$

Hence c.g.s. can be located at $11.5 - 4.9 - 2.2 = 4.4$ in. above bottom fiber.

Step 2. Under working conditions, for a stress at top fiber of $f_t = -1.60$ ksi and at bottom fiber of $f_b' = 0.24$ ksi, the uncracked portion is shown in Fig. 6-5-2 (*b*). Properties of the uncracked portion are:

$$A_{c2} = 4 \times 17 + 27.3 \times 4 = 68 + 109 = 177 \text{ sq in.}$$

$$c_{t2} = \frac{68 \times 2 + 109 \times 17.7}{177} = 11.7 \text{ in.}$$

$$I_2 = 68\left(\frac{4^2}{12} + 9.7^2\right) + 109\left(\frac{27.3^2}{12} + 6.0^2\right)$$

$$= 6470 + 10{,}700 = 17{,}200 \text{ in.}^4$$

$$r_2^2 = 17{,}200/177 = 97.0 \text{ in.}^2$$

$$k_{t2} = 97.0/19.6 = 5.0 \text{ in.}$$

$$\text{Total lever } a = 36 - 4.4 - 11.7 + 5.0 = 24.9 \text{ in.}$$

$$F = M_T/a$$

$$= 320 \times 12/24.9$$

$$= 154 \text{ k}$$

$$F_0 = 154 \times 150/125 = 185 \text{ k}$$

Step 3. Area A_{c1} required will be

$$A_{c1} = \frac{185 \times 30.9}{1.80 \times 19.4} = 164 \text{ sq in.}$$

A_{c1} furnished was 176 sq in., indicating a slight excess. Area A_{c2} required will be

$$A_{c2} = \frac{154 \times 31.3}{1.60 \times 19.6} = 154 \text{ sq in.}$$

A_{c2} furnished was 177 sq in., again indicating an excess. Hence a somewhat smaller section can be tried, and the above procedure repeated if desired.

Case 2 · Large Ratios of M_G/M_T. In this case, the c.g.s. has to be located at the lowest practicable point. At transfer, the stress distribution will be as shown in Fig. 6-3-3 (*b*), and there will be no critical tensile stresses at the top fibers. The area of concrete required

will be given by formula 6-3-5,

$$A_c = \frac{F_0}{f_b}\left(1 + \frac{e - (M_G/F_0)}{k_t}\right)$$

Example 6-5-2

Revise the design for the section in example 6-3-2 allowing tension in concrete, $f_t' = 0.30$ ksi and $f_b' = 0.24$ ksi. Other values given were: $M_T = 320$ k ft; $M_G = 210$ k ft; $F = 152$ k; $F_0 = 182$ k; $A_c = 200$ sq in.; $c_t = 13.5$ in.; $c_b = 22.5$ in.; $I = 26{,}000$ in.^{4}; $k_t = 5.8$ in.; $k_b = 9.6$ in. (Fig. 6-5-3).

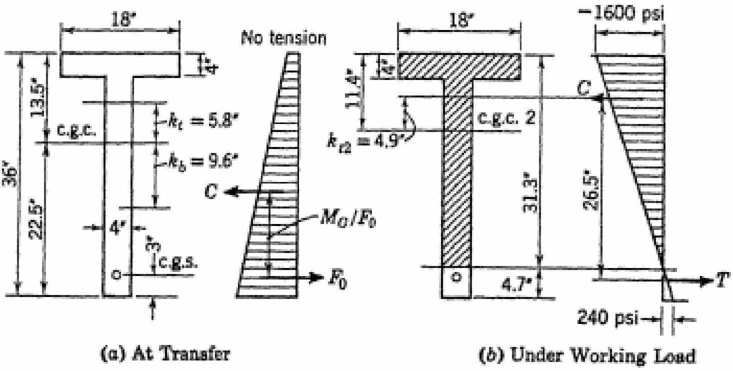


Fig. 6-5-3. Example 6-5-2.

Solution. Step 1. Proceeding as in example 6-3-2, the theoretical lowest position for c.g.s. without producing tension in top fiber is given by

$$\begin{aligned} e - k_b &= M_G/F_0 \\ &= (210 \times 12)/182 \\ &= 13.8 \text{ in.} \end{aligned}$$

indicating 13.8 in. below the bottom kern, or 0.9 in. below the bottom fiber, which is not feasible. Suppose that the c.g.s. is located at 3 in. above bottom fiber; then no tension will be in top fiber.

Step 2. Under the working load, assuming the top fiber stress to be -1.60 ksi and the bottom fiber stress to be 0.24 ksi, the uncracked portion has a depth of 31.3 in., as shown in Fig. 6-5-3 (*b*). Properties of the uncracked portion are:

$$A_{c2} = 4 \times 18 + 4 \times 27.3 = 72 + 109 = 181 \text{ sq in.}$$

$$c_{t2} = \frac{72 \times 2 + 109 \times 17.6}{181} = 11.4 \text{ in.}$$

$$I_2 = 72\left(\frac{4^2}{12} + 9.4^2\right) + 109\left(\frac{27.3^2}{12} + 6.2^2\right)$$

$$= 6480 + 11{,}100 = 17{,}600$$

$$k_{t2} = 17{,}600/(181 \times 19.9) = 4.9 \text{ in.}$$

Total lever arm a for bending is

$$a = 36 - 3 - 11.4 + 4.9 = 26.5 \text{ in.}$$

$$F = (320 \times 12)/26.5 = 145 \text{ k}, \; F_0 = 174 \text{ k}$$

Step 3. As governed by top fiber in compression,

$$A_{c2} = \frac{145 \times 36}{1.60 \times 19.9} = 164 \text{ sq in.}$$

As governed by the bottom fiber, using the entire section for $F_0 = 174$ k,

$$A_c = \frac{174}{1.80}\left(1 + \frac{19.5 - (210 \times 12/174)}{5.8}\right)$$

$$= 180 \text{ sq in.}$$

The above calculation shows that the top flange area can be reduced while the bottom flange area can be slightly increased, in order to obtain a more balanced section. Compare this with the solution for example 6-3-2.

6-6 Elastic Design, Allowing and Considering Tension

This method should be used only with caution. It may not always be a safe procedure when tension in concrete constitutes a major part of the total tensile force in resisting bending. It is, however, a convenient method and yields results comparable to those of the method in section 6-5 when the tensile force in concrete considered is only a small portion of the total tension. The method will be explained as follows.

Case 1 · Small Ratios of M_G/M_T. If tensile stress f_t' is permitted in the top fibers, the center of compression C can be located below the bottom kern by the amount of

$$e_1 = f_t'I/F_0c_t = f_t'Ak_b/F_0 \tag{6-6-1}$$

For a given moment M_G, the c.g.s. can be further located below C by the amount of

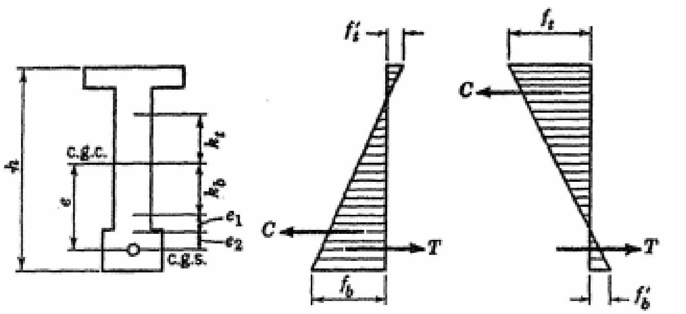
$$e_2 = M_G/F_0 \tag{6-6-2}$$

Hence the maximum total amount that the c.g.s. can be located below the kern is given by

$$e_1 + e_2 = \frac{M_G + f_t'Ak_b}{F_0} \tag{6-6-3}$$

The c.g.s. having been located at some value e below c.g.c., the lever arm a under working load is known. For an allowable tension in the bottom fiber, the moment carried by the concrete is

$$f_b'I/c_b = f_b'Ak_t$$



(a) Properties of Section (b) Stress Distribution at Transfer (c) Stress Distribution under Working Load

Fig. 6-6-1. Allowing and considering tension in concrete.

The net moment $M_T - f_b'Ak_t$ is to be carried by the prestress F with a lever arm acting up to the top kern point; hence the total arm is, Fig. 6-6-1,

$$a = k_t + e \tag{6-6-4}$$

and the prestress F required is

$$F = \frac{M_T - f_b'Ak_t}{a} \tag{6-6-5}$$

The bottom fiber stress at transfer is given by

$$f_b = \frac{F_0h}{A_cc_t} + f_t'\frac{c_b}{c_t} \tag{6-6-6}$$

from which we have

$$A_c = \frac{F_0h}{f_bc_t - f_t'c_b} \tag{6-6-6a}$$

Similarly, the top fiber stress under working load is given by

$$f_t = \frac{Fh}{A_cc_b} + f_b'\frac{c_t}{c_b} \tag{6-6-7}$$

from which

$$A_c = \frac{Fh}{f_tc_b - f_b'c_t} \tag{6-6-7a}$$

Example 6-6-1

Redesign the beam section in example 6-3-1, allowing and considering tension in concrete. $f_t' = 0.30$ ksi, $f_b' = 0.24$ ksi. Other given values were: $M_T = 320$ k ft; $M_G = 40$ k ft; $f_t = -1.60$ ksi; $f_b = -1.80$ ksi; $F = 184$ k; $F_0 = 221$ k.

Solution. *Step* 1. From example 6-3-1, we have $k_t = k_b = 9.4$ in.; $A_c = 248$ sq in. Using equation 6-6-3, we have

$$e_1 + e_2 = \frac{40 \times 12 + 0.3 \times 248 \times 9.4}{221} = 5.3 \text{ in.}$$

Hence c.g.s. can be located 5.3 in. below the bottom kern, or 3.3 in. above the bottom fiber, Fig. 6-6-2.

Step 2. The net moment to be carried by the prestress is

$$M_T - f_b'Ak_t = 320 \times 12 - 0.240 \times 248 \times 9.4$$
$$= 3840 - 560 = 3280 \text{ k in.}$$

For a resisting lever arm of $9.4 + 9.4 + 5.3 = 24.1$ in., the prestress required is

$$F = 3280/24.1 = 136 \text{ k}$$
$$F_0 = 136 \times 150/125 = 163 \text{ k}$$

Step. 3. To limit the bottom fibers to -1.80 ksi, we need

$$A_c = \frac{163 \times 36}{1.80 \times 18 - 0.30 \times 18}$$
$$= 218 \text{ sq in.}$$

To keep the top fibers to -1.60 ksi, we need

$$A_c = \frac{136 \times 36}{1.60 \times 18 - 0.24 \times 18}$$
$$= 200 \text{ sq in.}$$

which indicates that the trial section can be appreciably reduced and a new section tried over again.

Case 2 · Large Ratios of M_G/M_T. When M_G/M_T is large, C will be within the kern at transfer, and the allowing of tension on top fiber will have no effect on the design. The c.g.s. has to be located within practical limits. Otherwise, the design is made as for Case 1. This is illustrated below.

Example 6-6-2

Revise the design for the section in example 6-3-2 allowing and considering tension in concrete. Other values given were: $M_T = 320$ k ft; $M_G = 210$ k ft; $F = 152$ k; $F_0 = 182$ k; $A_c = 200$; $c_t = 13.5$ in.; $c_b = 22.5$ in.; $k_t = 5.8$ in.; $k_b = 9.6$ in. (Fig. 6-6-3).

Solution. *Step* 1. Referring to example 6-5-2, since the possible theoretical location for c.g.s. is 13.8 in. below the bottom kern (0.9 in. below bottom fiber) without producing tension in top fiber, whereas the practical location of c.g.s. has to be 3 in. above bottom fiber, no tension will exist in top fiber.

Step 2. Net amount to be carried by prestress is

$$M_T - f_b'Ak_t = 320 \times 12 - 0.240 \times 200 \times 5.8$$
$$= 3840 - 280 = 3560 \text{ k in.}$$

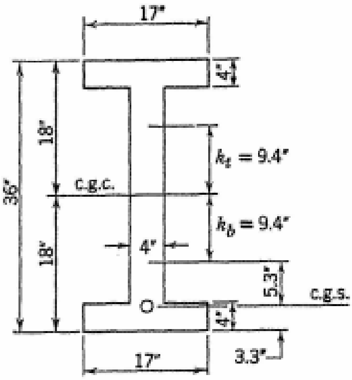


Fig. 6-6-2. Example 6-6-1.

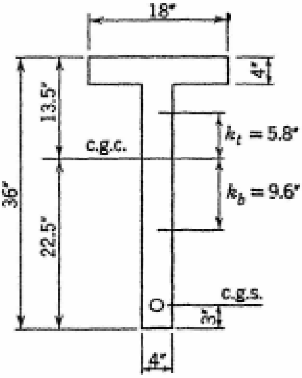


Fig. 6-6-3. Example 6-6-2.

The resisting lever arm is

$$36 - 3 - 13.5 + 5.8 = 25.3 \text{ in.}$$

The required prestress is

$$F = 3560/25.3 = 141 \text{ k}$$

$$F_0 = 141(150/125) = 169 \text{ k}$$

To keep the bottom fiber stress within limits, we can apply equation 6-3-5,

$$A_c = \frac{F_0}{f_b}\left(1 + \frac{e - (M_G/F_0)}{k_t}\right)$$

$$= \frac{169}{1.80}\left(1 + \frac{19.5 - (210 \times 12/169)}{5.8}\right)$$

$$= 168 \text{ sq in.}$$

To keep the top fiber stress within limit, we have, from equation 6-6-7*a*,

$$A_c = \frac{141 \times 36}{1.60 \times 22.5 - 0.24 \times 13.5}$$

$$= 155 \text{ sq in.}$$

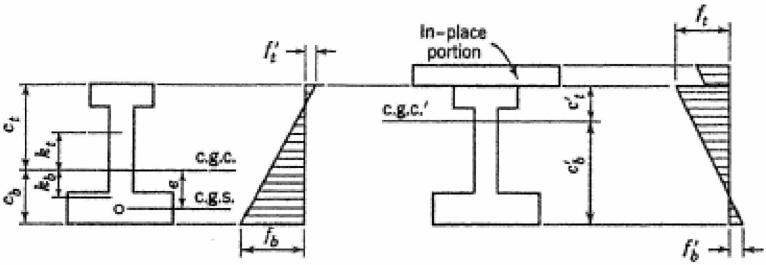
The area furnished is 200 sq in., which can be reduced if desired.

6-7 Elastic Design, Composite Sections

As described previously, a composite section consists of a precast prestressed portion to be combined with another cast-in-place portion which usually forms part or all of the top flange of the beam. The design of composite sections is more complicated than that of simple ones because there are many possible combinations in the make-up of a composite section. Only a very common case will be treated

here, leaving the possible variations to the designer after he has mastered the principles here presented.

In the case considered here the precast portion forms the lower flange and the web while part or the whole of the top flange is cast in place. Tension is usually permitted in the top flange at transfer and often also in the bottom flange under working load. Hence, formulas will be derived to include tensile stresses. These can be easily simplified when tensile stresses are not permitted. For such composite sections compressive stress in the cast-in-place portion is seldom critical and hence will be checked only at the end of the design. When the cast-in-place portion becomes the major part of the web, or when falseworks are employed to support the precast portion during casting, the method presented here has to be modified accordingly.



(a) Precast Portion, Stress Distribution at Transfer

(b) Composite Section, Stress Distribution under Working Load

Fig. 6-7-1. Elastic design of composite sections.

The procedure of design here presented follows closely the basic approach previously adopted for non-composite sections. It is essentially a trial-and-error process, simplified by a systematic and fast converging procedure and assisted by the use of some simple relations and formulas. One additional concept introduced for composite action is the reduction of moments on the composite section to equivalent moments on the precast portion. This is accomplished by the ratio of the section moduli of the two sections. Steps in the design and the formulas employed will now be explained:

Step 1. Location of c.g.s. For a given trial section, the c.g.s. must be so located that the precast portion will not be overstressed and yet will possess the optimum capacity in resisting the applied external moments. Thus, the c.g.s. must be situated as low as possible but not lower than given by the following value of eccentricity, Fig. 6-7-1 (a),

$$e = k_b + e_1 + e_2$$

where

$$e_1 = \frac{f_t'I}{c_tF_0}$$

$$e_2 = \frac{M_G}{F_0}$$

where

f_t' = allowable tension stress on top fiber of precast portion at transfer.
I = moment of inertia of precast portion.
c_t = distance to top fiber from c.g.c. of precast portion.

Step 2. Compute the equivalent moment on the precast portion. For any moment M_C acting on the composite section, it will produce stresses on the precast portion as follows, Fig. 6-7-1 (*b*),

$$f_t = \frac{M_Cc_t'}{I'}$$

$$f_b = \frac{M_Cc_b'}{I'}$$

where $I' = I$ of composite section, c_t' and c_b' = distance to extreme fibers of the precast portion measured from c.g.c.′ of the composite section. With A_c, k_t', k_b' referring to the precast portion, let

$$m_t = \frac{I/c_t}{I'/c_t'}$$

$$= \frac{Ak_b}{A_ck_b'}$$

and

$$m_b = \frac{I/c_b}{I'/c_b'}$$

$$= \frac{Ak_t}{A_ck_t'}$$

we have

$$f_t = \frac{m_tM_Cc_t}{I}$$

$$f_b = \frac{m_bM_Cc_b}{I}$$

which indicate that M_C can be modified by the coefficients m_t and m_b

so that it can be reduced to equivalent moments for computation based on the precast-portion properties.

Step 3. Compute the amount of prestress required for the moments as follows. If M_P = the total moment acting on the precast portion, and f_b' = allowable tensile stress at the bottom fiber, we have

$$\frac{F}{A_c}\left(-1 - \frac{e}{k_t}\right) + \frac{M_P}{A_c k_t} + \frac{m_b M_C}{A_c k_t} = f_b'$$

$$F = \frac{M_P + m_b M_C - f_b' k_t A_c}{e + k_t} \tag{6-7-1}$$

or

$$F = \frac{M_P + m_b M_C}{e + k_t}$$

if

$$f_b' = 0$$

from which compute the required initial prestress F_0. Revise the location of c.g.s. by this new value of F_0 if necessary.

Step 4. In order to limit the bottom fiber stress to the allowable value at transfer, we have

$$f_b = \frac{F_0}{A_c} + \frac{(F_0 e - M_G)}{A_c k_t}$$

from which

$$A_c = \frac{1}{f_b}\left[F_0 + \frac{F_0 e - M_G}{k_t}\right] \tag{6-7-2}$$

In order to limit the top fibers of the precast portion to within allowable compressive stress f_t under working load, we have

$$f_t = \frac{F}{A_c} + \frac{M_P + m_t M_C - Fe}{A_c k_b}$$

$$A_c = \frac{1}{f_t}\left[F + \frac{M_P + m_t M_C - Fe}{k_b}\right] \tag{6-7-3}$$

The greater of the two formulas will control the A_c required for the precast portion. The top fiber of the cast-in-place top flange can be computed by the formula $f = Mc/I$, using the applicable values.

Example 6-7-1

The top flange of a composite section is given as a slab 4 in. thick and 60 in. wide cast in place. Design a precast section with a total depth of 36 in. (including the slab thickness) to carry the following moments: M_T = 320 k ft, M_G =

40 k ft, $M_r = 100$ k ft, $M_G = 220$ k ft. Allowable stresses are: $f_t = -1.30$ ksi, $f_b = -1.80$ ksi, $f_t' = 0.30$ ksi, $f_b' = 0.16$ ksi. Initial prestress = 150 ksi, effective prestress = 125 ksi.

To assume the section, make a preliminary design, assuming a lever arm of $0.65h$ for the prestressing force in resisting the total moment; we have

$$F = \frac{M_T}{0.65h} = \frac{320 \times 12}{0.65 \times 36} = 164 \text{ k}$$

$F_0 = 164 \times 150/125 = 197$ k. For an inverted T-section, the concrete area required can be approximated by

$$A_c = 1.5\frac{F_0}{f_b} = 1.5\frac{197}{1.8} = 164 \text{ sq in.}$$

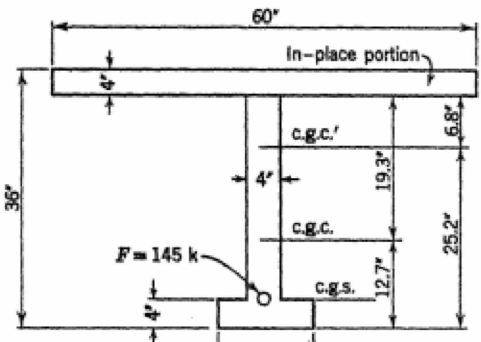


Fig. 6-7-2. Example 6-7-1.

From this preliminary section, a sketch of a trial section is shown, Fig. 6-7-2, and the design proceeds as follows.

For the precast portion, the section properties are:

$$\begin{array}{rl} 4 \times 14 = 56 \times 2 = & 112 \\ 28 \times 4 = \underline{112} \times 18 = & \underline{2016} \\ A_c = 168 & 2128 \div 168 = 12.7 \text{ in.} = c_b \end{array}$$

$$\begin{array}{rl} 56(4^2/12 + 10.7^2) = & 6{,}500 \\ 112(28^2/12 + 5.3^2) = & \underline{10{,}450} \\ & 16{,}950 \div 168 = 101 = r^2 \end{array}$$

$$k_t = 101/12.7 = 8.0 \text{ in.}$$

$$k_b = 101/19.3 = 5.2 \text{ in.}$$

For the composite section, the properties are:

$$
\begin{array}{llll}
4 \times 60 = 240 \times 2 & & = & 480 \\
\underline{168} \times 23.3 & & = & \underline{3920} \\
408 & & & 4400 \div 408 = 10.8 \text{ in.}
\end{array}
$$

$$
\begin{array}{lcr}
240(4^2/12 + 8.8^2) & = & 18{,}800 \\
168(12.5^2) & = & 26{,}200 \\
I \text{ of precast portion} & = & \underline{16{,}950} \\
 & & 62{,}000
\end{array}
$$

$$m_t = \frac{I/c_t}{I'/c_t'} = \frac{16{,}950/19.3}{62{,}000/6.8} = 0.10$$

$$m_b = \frac{I/c_b}{I'/c_b'} = \frac{16{,}950/12.7}{62{,}000/25.2} = 0.54$$

Step 1. Location of c.g.s.

$$e_1 = \frac{f_t'I}{c_tF_0}$$

$$= \frac{0.30 \times 16{,}950}{19.3 \times 197} = 1.3 \text{ in.}$$

$$e_2 = M_G/F_0$$

$$= (40 \times 12)/197 = 2.4 \text{ in.}$$

$$k_b = 5.2 \text{ in.}$$

$$e = 1.3 + 2.4 + 5.2 = 8.9 \text{ in.}$$

Thus c.g.s. can be located at 12.7 − 8.9 = 3.8 in. above bottom fiber.

Step 2. As computed above,

$$m_t = 0.10$$

$$m_b = 0.54$$

Step 3. Compute the required F,

$$F = \frac{M_P + m_bM_C - f_b'k_tA_c}{e + k_t}$$

$$= \frac{(100 + 0.54 \times 220)12 - 0.16 \times 8.0 \times 168}{8.9 + 8.0}$$

$$= \frac{2430}{16.9}$$

$$= 144 \text{ k}$$

$$F_0 = 144 \times 150/125 = 173 \text{ k}$$

For F_0 = 173 k instead of 197 k, revise e_1 and e_2 as follows;

$$e_1 = 1.3 \times 197/173 = 1.5 \text{ in.}$$

$$e_2 = 2.3 \times 197/173 = 2.7 \text{ in.}$$

$$e = 5.2 + 1.5 + 2.7 = 9.4 \text{ in.}$$

which indicates that c.g.s. can be located at $12.7 - 9.4 = 3.3$ in. above bottom fiber. With new $e + k_t = 9.4 + 8.0 = 17.4$, F can be revised to be $144 \times 16.9/17.4 = 140$ k. $F_0 = 140 \times 150/125 = 168$ k.

Step 4. To keep bottom fiber within allowable stress f_b,

$$A_c = \frac{1}{f_b}\left(F_0 + \frac{F_0 e - M_G}{k_t}\right)$$

$$= \frac{1}{1.80}\left(168 + \frac{168 \times 9.4 - 40 \times 12}{8.0}\right)$$

$$= 170 \text{ sq in.}$$

To keep top fiber within allowable f_t,

$$A_c = \frac{1}{f_t}\left(F + \frac{M_P + m_t M_C - Fe}{k_b}\right)$$

$$= \frac{1}{1.60}\left(140 + \frac{(100 + 0.10 \times 220)12 - 140 \times 9.4}{5.2}\right)$$

$$= 106 \text{ sq in.}$$

The top fiber is not controlling in this case, and the A_c required for bottom fiber stress is 170 sq in., which is very close to the A_c of 168 sq in. furnished by the trial section. The design is considered satisfactory.

6-8 Ultimate Design

Only the ultimate design for simple sections will be discussed here. Basically, the procedure is also applicable to the ultimate design of composite sections, the details of which will be left to the reader, however.

1 · Preliminary Design. The amount of mathematics involved in the design of prestressed-concrete sections is less in ultimate design than in elastic design, since the ultimate flexural strength of sections can be expressed by simple semiempirical formulas. For preliminary design, it can be assumed that the ultimate strength of bonded prestressed sections is given by the ultimate strength of steel acting with a lever arm of about $0.80h$. Hence the area of steel required is computed as

$$A_s = \frac{M_T \times m}{0.80h \times f_s'} \tag{6-8-1}$$

where m = factor of safety or the load factor.

Assuming that the concrete on the compressive side is stressed to $0.85f_c'$, then the required ultimate concrete area under compression is

$$A_c' = \frac{M_T \times m}{0.80h \times 0.85f_c'} \tag{6-8-2}$$

The web area and the concrete area on the tension side are designed

to provide the shear resistance and the encasement of steel respectively. In addition, concrete on the tension side has to stand the prestress at transfer. For a preliminary design, these areas are obtained by comparison with previous designs rather than by making any involved calculations.

The chief difficulty in ultimate design lies in the proper choice of the factor of safety or the load factor, which will be discussed in detail in Chapter 14. For the present it will be assumed that a load factor of 2 will be sufficient for steel and one of 2.5 for concrete. The application of the method is illustrated in the following example.

Example 6-8-1

Make a preliminary design for a prestressed-concrete section 36 in. high to carry a total dead and live load moment of 320 k ft, using steel with an ultimate strength of 220 ksi and concrete with $f_c' = 4$ ksi. Use ultimate design, and assume a bonded beam.

Solution. Using a load factor of 2 for steel, we have, from equation 6-8-1,

$$A_s = \frac{320 \times 12 \times 2}{0.80 \times 36 \times 220} = 1.21 \text{ sq in.}$$

Using a load factor of 2.5 for concrete, from equation 6-8-2,

$$A_c' = \frac{320 \times 12 \times 2.5}{0.80 \times 36 \times 0.85 \times 4} = 98 \text{ sq in.}$$

Thus a preliminary section can be sketched as in Fig. 6-8-1, providing an ultimate area of 98 sq in. under compression, assuming the ultimate neutral axis to be 10 in. below the top fiber. Note that the exact location of the ultimate neutral axis cannot and need not be obtained for a preliminary design but can be assumed to be about 30% of the effective depth of section.

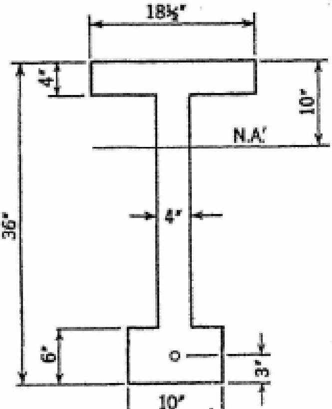


Fig. 6-8-1. Examples 6-8-1 and 6-8-2.

2 · Final Design. Although the above illustrates a preliminary design based on ultimate strength, a final design is more complicated in that the following factors must be considered.

1. Proper and accurate load factors must be chosen for steel and concrete, related to the design load and possible overloads for the particular structure.

2. Compressive stresses at transfer must be investigated for the tensile flange, generally by the elastic theory. In addition, the tensile flange should be capable of housing the steel properly.

3. The approximate location of the ultimate neutral axis may not be easily determined for certain sections.

4. Design of the web will depend on shear and other factors.

5. The effective lever arm for the internal resisting couple may have to be more accurately computed.

6. Checks for excessive deflection and overstresses may have to be performed.

7. The ultimate flexural strength for unbonded sections is not so well known.

For the above reasons, a final design entirely based on the ultimate theory cannot be easily made at the present time. However, as far as the flexural strength is concerned, a final design can be made for bonded sections based on ultimate strength. This is illustrated in the following example.

Example 6-8-2

Make a final design for the beam in example 6-8-1, based on its ultimate strength.

Solution. A trial-and-error procedure is considered convenient for the purpose. Using the preliminary section obtained in example 6-8-1 as the first trial section, Fig. 6-8-1, we can proceed as follows:

With the ultimate axis 10 in. below top fiber, the centroid of the ultimate compressive force is located by

$$\frac{74 \times 2 + 24 \times 7}{74 + 24} = 3.2 \text{ in.}$$

or 3.2 in. from top fiber. With the c.g.s. located 3 in. above bottom fiber, the ultimate lever arm for the resisting moment is

$$36 - 3.2 - 3 = 29.8 \text{ in.}$$

Now the area of steel required may be recomputed as

$$A_s = \frac{320 \times 12 \times 2}{29.8 \times 220} = 1.17 \text{ sq in.}$$

which is very near to the preliminary value of 1.21 sq in., and no further trial is necessary. Design of the top flange may be done as in example 6-8-1. Our present knowledge does not permit the design of the bottom flange by the ultimate theory, and it has to be checked by the elastic theory; the tensile stress also should be checked in the top flange at transfer. The web, of course, has to be checked for shear, which will be discussed in Chapter 7.

3 · Ultimate vs. Elastic Design. At the present time, both the elastic and the ultimate designs are used for prestressed concrete, the majority of designers still following the elastic theory. It is difficult to state

exact preference for one or the other. Each has its advantages and shortcomings. But, whichever method is used for design, the other one must often be applied for checking. For example, when the elastic theory is used in design, it is the practice to check for the ultimate strength of the section in order to find out whether it has sufficient reserve strength to carry overloads. When the ultimate design is used, the elastic theory must be applied to determine whether the section is overstressed under certain conditions of loading and whether the deflections are excessive. Overstressing is objectionable because it may result in undesirable cracks and creep and fatigue effects. When the design is of conventional types and proportions, such checking becomes unnecessary, because it is then generally known that designing by one method will yield safe results when checked by the other. This is, in fact, the reason why such checking is not required of reinforced-concrete structures designed by the usual codes. When we delve into new types and proportions, it is possible that elastic design alone might not yield a sufficiently safe structure under overloads, while the ultimate design by itself might give no guarantee against excessive overstress under working conditions. At this stage of our knowledge regarding prestressed-concrete design, it is deemed desirable to learn and apply both the elastic and the ultimate methods, especially for structures of unusual proportions.

An understanding of both theories of design is also essential in forming judgment when designing structures. Sometimes, design based on one method will yield different proportions from those based on the other. In order to illustrate the point, let us compare a symmetrical I-section and its circumscribing rectangular section, example 6-8-3. Based on elastic design, allowing no tension in the concrete, the I-section will carry greater moment than the rectangular section; the rectangular section, however, has a higher ultimate strength. If strength is a more important consideration, the design can be based on ultimate strength. If tensile stress, cracking, creep, or deflection is a critical limit, elastic design should be followed. If both strength and stress are controlling criteria for a structure, we are forced to apply both methods in order to ensure safety, even at the sacrifice of some economy.

Example 6-8-3

An I-section and another circumscribing rectangular section are both prestressed with 0.9 sq in. of steel, Fig. 6-8-2. $f_c' = 5$ ksi, $f_s = 125$ ksi, $f_s' = 250$ ksi. Compute (*a*) the resisting moment capacity of each section by the elastic theory, allowing no tension in concrete, (*b*) the ultimate moment capacity of each section.

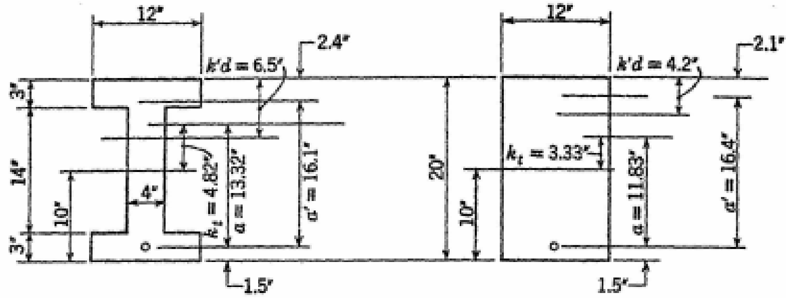


Fig. 6-8-2. Example 6-8-3.

Solution. (*a*) For no tension in concrete, using formula 6-3-3, we have

	I-Section	Rectangular Section
Area, sq in.	128	240
I, in.^{4}	6170	8000
k_t, in.	4.82	3.33
Lever arm between c.g.s. and k_t, in.	13.32	11.83
Effective prestress, kips	112.5	112.5
Resisting moment, k ft	125	111

(*b*) For the ultimate moment capacity, following the method in section 5-6, we have, assuming $k_1 f_c' = 4.5$ ksi,

	I-Section	Rectangular Section
$k'd$, in.	6.5	4.2
Ultimate distance of centroid of compression force from top fiber, in.	2.4	2.1
Ultimate lever arm a' for resisting couple, in.	16.1	16.4
Ultimate tension in steel, k	225	225
Ultimate resisting moment k ft	302	307

The above example illustrates that, when designed by the elastic theory, the I-section can carry greater moment; when designed by ultimate strength, the rectangular section carries the greater moment.

There is another case where the application of the elastic and the ultimate designs yields radically different results. Consider two sections of exactly the same steel and concrete dimensions, but one with bonded steel and the other unbonded. By the elastic design, both sections will carry the same moment; but by the ultimate design, the

unbonded section will carry much less moment. Which method should be used in design will depend upon the particular conditions of the structure. When overloads are likely, and ultimate strength is an important consideration, the bonded section must be given preference; if only working loads are concerned, the bonded and unbonded sections can be considered equally strong.

6-9 Shapes of Concrete Sections

Having studied both the elastic and ultimate designs, we are now ready to discuss the selection of the best shapes for prestressed-concrete-beam sections. The simplest form is the rectangular shape possessed by all solid slabs and used for some short-span beams. As far as formwork is concerned, the rectangular section is the most economical. But the kern distances are small, and the available lever arm for the steel is limited. Concrete near the centroidal axis and on the tension side is not effective in resisting moment, especially at the ultimate stage.

Hence other shapes are frequently used for prestressed concrete, Fig. 6-9-1:

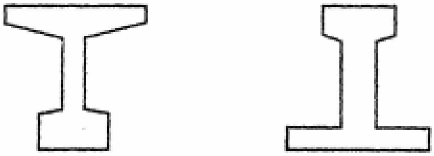
1. The symmetrical I-section.
2. The unsymmetrical I-section.
3. The T-section.
4. The inverted T-section.
5. The box section.

The suitability of these shapes will depend upon the particular requirements. The I-section has its concrete concentrated near the extreme fibers where it can most effectively furnish the compressive force, both at transfer of prestress and under working and ultimate loads. The more the concrete is concentrated near the extreme fibers, the greater will be the kern distances, and the greater will be the lever arm furnished for the internal resisting couple. However, this principle of concentrating the concrete in the extreme fibers cannot be carried too far, because the width and thickness of the flanges are governed by practical considerations, and the web must have a minimum thickness to carry the shear, to avoid buckling, and to permit proper placement of concrete.

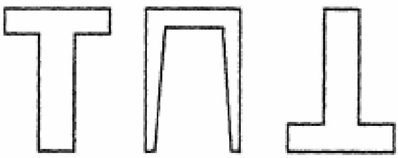
If the M_G/M_T ratio is sufficiently large, there is little danger of overstressing the flanges at transfer and concrete in the tension flange can be accordingly diminished. This will result in an unsymmetrical I-section which when carried to the fullest extent becomes a T-section. A T-section, similar to that for reinforced beams, is often most economical, since the concrete is concentrated at the compression flange

where it is most effective in resisting the external moment. It may not be properly used, however, where the M_G/M_T ratio is small because then the center of pressure at transfer may lie outside the

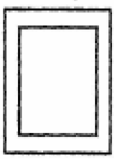
(a) Rectangular Section (b) Symmetrical I Section



(c) Unsymmetrical I Sections



(d) T and Inverted T Sections



(e) Box Section

Fig. 6-9-1. Shapes of concrete sections.

kern. Then tensile stresses may result in the compression flange and high compressive stresses in the tension flange.

The unsymmetrical I-section with a bigger flange on the tensile side, like a rail section, is not an economical one in carrying ultimate moment, since there is relatively little concrete on the compression flange. However, there is a great deal of material to resist the initial prestress. It can be economically used for certain composite sections, where the tension flange is precast and the compression flange is poured in place. This section requires very little girder moment

to bring the center of pressure within the kern and hence is suitable when the M_G/M_T ratio is small. When carried to the extreme, this section becomes an inverted T-section.

The box section has the same properties as the I-section in resisting moment. In fact, their section properties are identical and both are listed in Table 6-2-6 of Appendix C. The adoption of one or the other will depend upon the practical requirements of each structure.

The above discussion can be summarized as follows. For economy in steel and concrete it is best to put the concrete near the extreme fibers of the compression flange. When the M_G/M_T ratio is small, more concrete near the tension flange may be necessary. When the M_G/M_T ratio is large, there is little danger of overstressing at transfer, and concrete in the tension flange is required only to house the tendons properly.

In choosing the shapes, prime importance must be given to the simplicity of formwork. When the formwork is to be used only once, it may constitute the major cost of the beam, so that any irregular shapes for the purpose of saving concrete or steel may not be in the interest of overall economy. On the other hand, when the forms can be reused repeatedly, more complicated shapes may be justified.

For plants producing precast elements, it is often economical to construct forms that can be easily modified to suit different spans and depths. For example, by filling up the stems for the section in Fig. 6-9-2 (*g*), several depths can be obtained. Again, by omitting the center portions of a tapered beam or decreasing the distance between the side forms, one set of forms can be made to fit many shorter spans.

Sections must be further designed to enable proper placement of concrete around the tendons and the corners. This is especially true when proper vibration cannot be ensured The use of fillets at corners is often desirable. It is also common practice to taper the interior side of the flanges, Fig. 6-9-1 (*c*). Such tapering will permit easier stripping of the formwork and easier placement of concrete.

Examples of some actual beam sections are shown in Fig. 6-9-2. (*a*) shows a precast and post-tensioned girder of 160-ft span whose M_G is quite heavy and a T-section is employed. (*b*) shows a cast-in-place section of T-shape, whose own weight on a span of 62 ft was not sufficient to hold down the eccentricity due to prestress. Here the shape was required for architectural reasons, and it was necessary to put additional dead load on the girder previous to tensioning the tendons. (*c*) shows a symmetrical I-section for a pre-tensioned beam. The I-shape was adopted in order to reduce the stresses both at transfer and under working load. (*d*) shows a post-tensioned roof girder

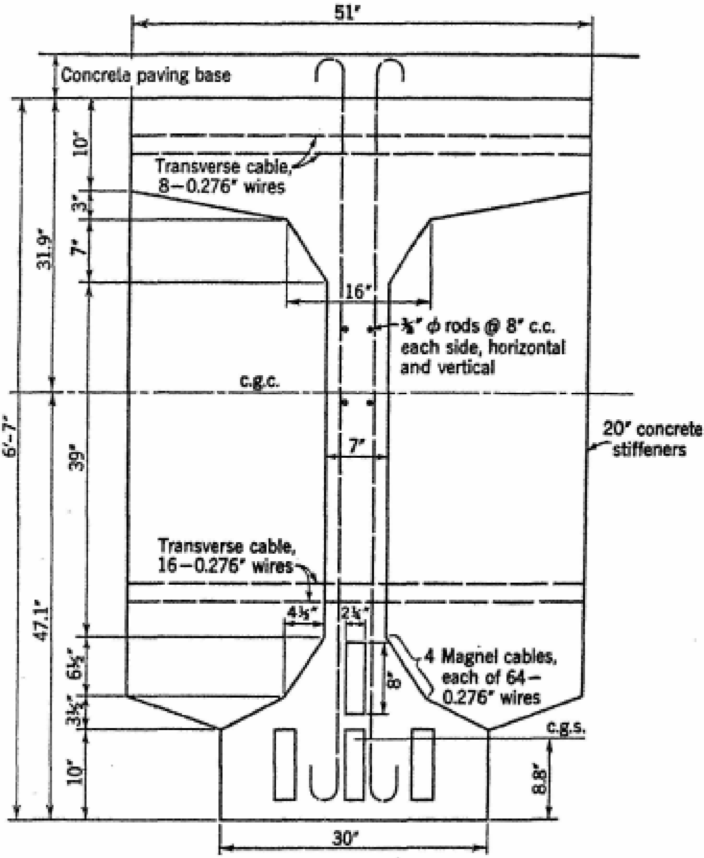


Fig. 6-9-2. Examples of actual beam sections. (*a*) Precast post-tensioned girder, Walnut Lane Bridge, Philadelphia. Magnel system. (Span = 160 ft.)

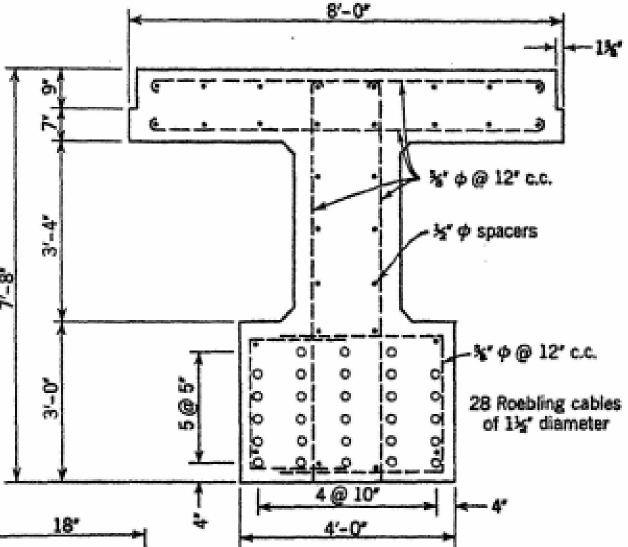


Fig. 6-9-2 (*b*). Cast-in-place post-tensioned girder, Lick Garage, San Francisco. Roebling system. (Span = 62 ft.)

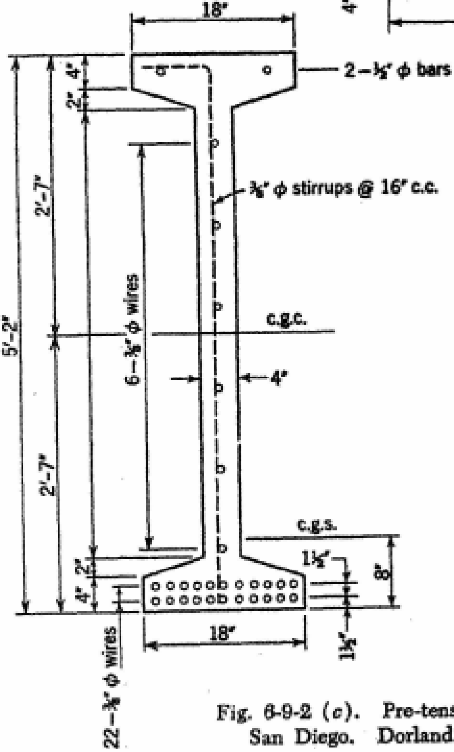


Fig. 6-9-2 (*c*). Pre-tensioned girder, B Street Warehouse, San Diego. Dorland anchorages. (Span = 60 ft.)

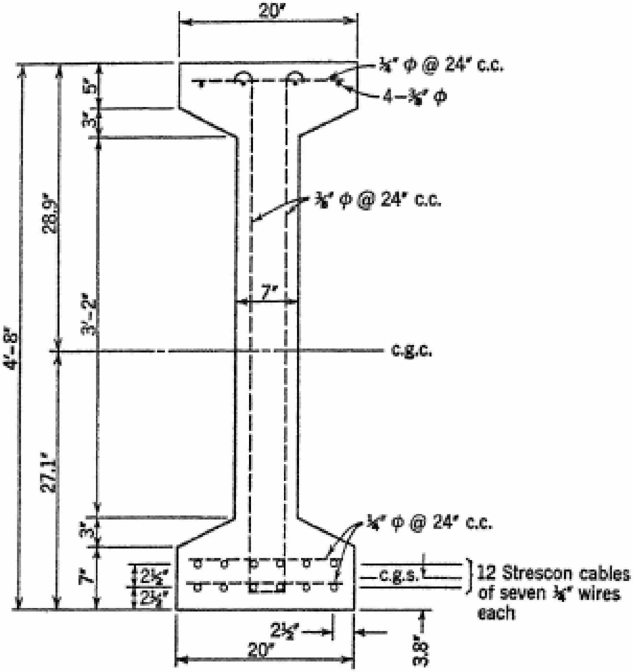


Fig. 6-9-2(d). Post-tensioned precast girders, warehouse, Long Beach, California. Strescon system. (Span = 74 ft.)

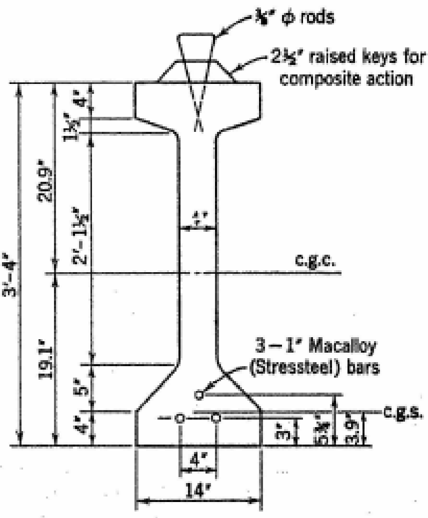


Fig. 6-9-2 (e). Pre-tensioned precast girders, Tampa Bay Bridge, Florida. Stressteel or Lee McCall system. (Span = 49 ft.)

of approximately symmetrical I-shape, the bottom flange being made a little thicker to facilitate housing of tendons and placement of concrete. (*e*) and (*f*) show inverted T- or unsymmetrical I-sections with larger flange at bottom. They were used for composite sections,

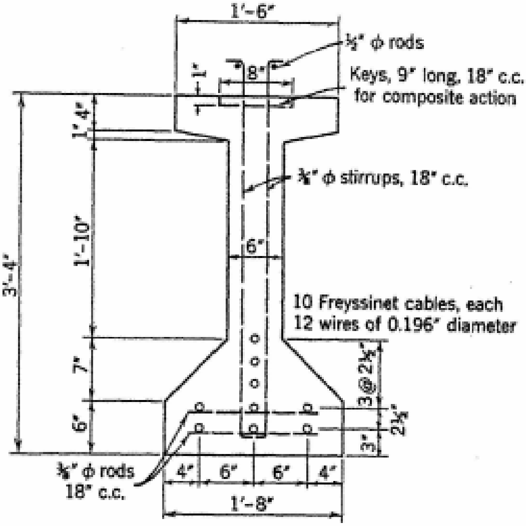


Fig. 6-9-2(*f*). Post-tensioned precast girders, Ten Mile Creek Bridge, Oregon. Freyssinet system. (Span = 59 ft.)

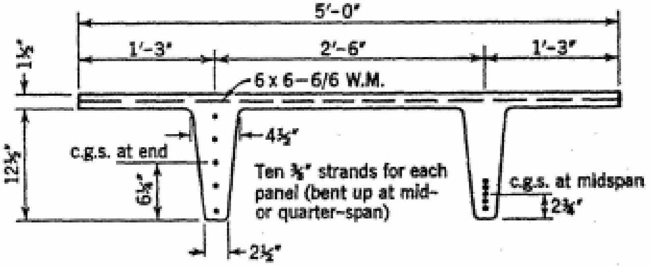


Fig. 6-9-2(*g*). Pre-tensioned precast roof panels using bent wires. (Span = 50 ft.)

with slabs to be cast on top to form the top flange. (*g*) shows a double-T or π section used for roof panels. When concrete topping is added, it can also serve as floor panels. Figure 6-9-3 shows a pre-tensioned channel section for roofs with top only 1¼ in. thick.

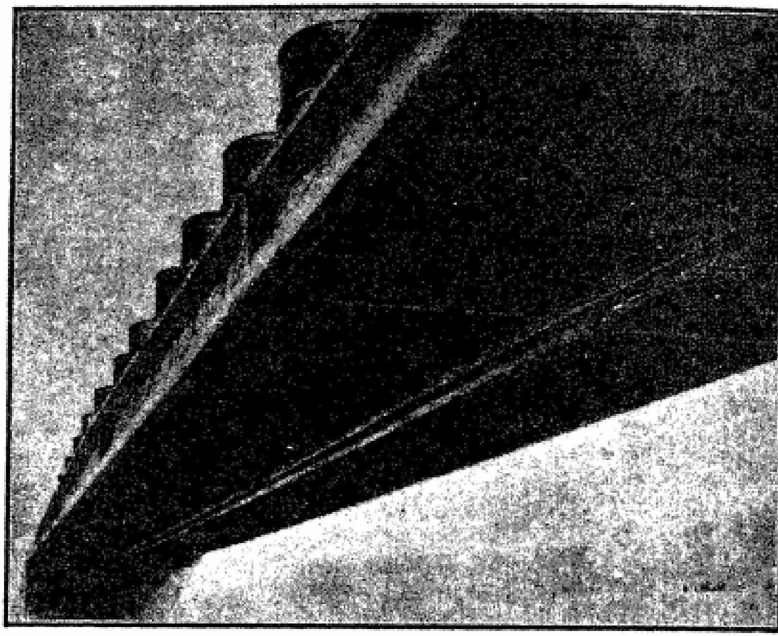


Fig. 6-9-3. A 60-ft pre-tensioned roof panel being tested for overloads of long duration. (Southwest Structural Concrete Corp., San Diego, California.)

It spans 60 ft and has been loaded with twice the design live load since the early part of 1953.

6-10 Arrangement of Steel

The arrangement of steel is governed by an important principle: in order to obtain the maximum lever arm for the internal resisting moment, it must be placed as near the tensile edge as possible. This is the same for prestressed as for reinforced sections. But, for prestressed concrete, one more condition must be considered: the initial condition at the transfer of prestress. If the c.g.s. is very near the tensile edge, and if there is no significant girder moment to bring the center of pressure near or within the kern, Fig. 6-10-1, the tension flange may be overcompressed at transfer while the compression flange may be under high tensile stress. Hence, this brings up a special condition in prestressed concrete: a heavy moment is desirable at transfer so that the steel can be placed as near the edge as possible. However, it must be noted that no economy is achieved by adding unnecessary dead weight to the structure in order to enable a bigger

lever arm for the steel, because whatever additional moment capacity was thus obtained would be used in carrying the additional dead load, although some additional reserve capacity is obtained for the ultimate range. Loads that will eventually have to be carried by the beam

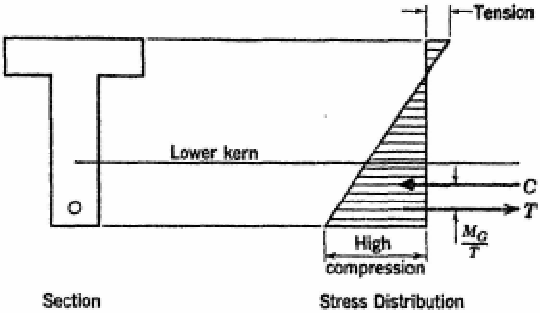


Fig. 6-10-1. Girder moment insufficient to bring C within kern.

can be more economically put on the structure before transfer rather than after, because moments produced by such loads will permit the placement of steel nearer the tensile edge.

Another method sometimes used in order to permit placement of steel near the tensile edge is to prestress the structure in two or more

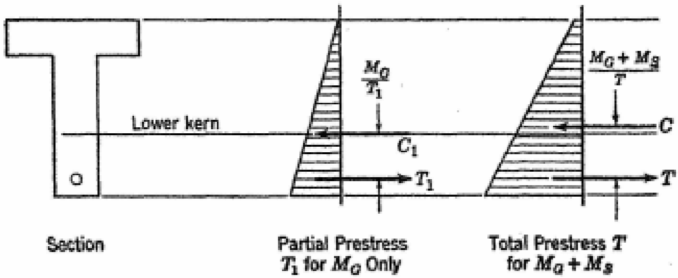


Fig. 6-10-2. Prestressing in two stages to keep C within kern.

stages; it is known as retensioning. At the first stage, when the moment on the beam is small, only a portion of the prestress will be applied; the total prestress will be applied only when additional dead load is placed on the beam producing heavier moment on the section, Fig. 6-10-2. Thus the center of pressure can be kept within the kern at all times, and excessive tension in the compression flange, as well as high compression in the tension flange, can be avoided.

For certain sections, the tendons are placed in the compression flange as well as in the tension flange, Fig. 6-10-3. Generally speaking, this is not an economical arrangement, because it will move the c.g.s. nearer to the c.g.c. and thereby decrease the resisting lever arm. At the ultimate range, tendons in the compressive flange will neutralize some of its compressive capacity, whereas only those in the tension flange are effective in resisting moment. However, under certain circumstances it may be necessary to put tendons in both flanges in spite of the resulting disadvantages. These conditions are:

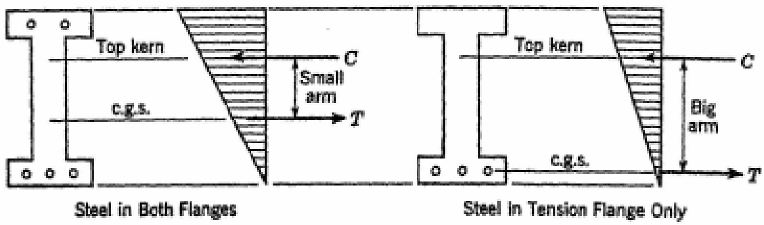


Fig. 6-10-3. Prestressing steel in both flanges reduces lever arm for resisting moment.

1. When the member is to be subject to loads producing both $+M$ and $-M$ in the section.

2. When the member might be subject to unexpected moments of opposite sign, during its handling process.

3. When the M_G/M_T ratio is small and the tendons cannot be suitably grouped near the kern point. Then the tendons will be placed in both the tension and the compression flanges with the resulting c.g.s. lying near the kern.

No definite rules have yet been established for the spacing of tendons or the minimum concrete protection required for them. The minimum spacing of tendons is governed by several factors. First, the clear spacing between tendons must be sufficient to permit easy passage of concrete. Here we may apply the general rule for reinforced concrete, which limits the clear spacing to a minimum of 1⅓ times the size of the maximum aggregates. This requirement may be reduced for prestressed concrete when good vibration can be ensured. Second, to properly develop the bond between steel and concrete, we may again apply the rule for reinforcing bars: the clear distance between bars should be at least the diameter of the bars for special anchorage and 1½ times the diameter for ordinary anchorage, with a minimum of 1 in. These limitations may not be necessary for small wires and strands used in prestressed work, especially if concrete is

to be well vibrated. For pre-tensioned tendons without mechanical anchorage, a clear spacing of 2 to 3 diameters is often specified for the ends, while closer spacing is permitted for the intermediate portions.

Two additional requirements must be considered for prestressed concrete regarding the minimum clear distance between wires or tendons. First, the wires in one tendon must be sufficiently far apart so as to permit easy passage of grout. While this will depend upon the pressure used for grouting and the fluidity of the grout, a general value of ¼ in. is often considered sufficient, although this cannot always be maintained for all systems. Next, when the tendons are sharply curved, they will exert radial thrusts on the concrete along the bends; hence some substantial thickness of concrete between the tendons may be required to resist such thrusts.

The minimum concrete protection for tendons is controlled by requirements for fire resistance, which will be discussed in Chapter 16. For reinforced concrete, the usual values are ¾ in. for slabs and 1½ in. for beams. Minimum distance between reinforcing bars and the side forms is often limited to 1⅓ times the size of the maximum aggregates; again this limit may be diminished if good vibration can be ensured. When the tendons are placed outside of the concrete to be eventually covered with mortar, then only the problem of fire protection need be considered.

To help dimension a beam section, the sizes of some tendons and their conduits are listed in Appendix B. Typical spacing of tendons for several systems is shown in Fig. 6-9-2.

References

6-1 T. Y. Lin and A. C. Scordelis, "Selection and Design of Prestressed Concrete Beam Sections," *J. Am. Conc. Inst.*, November, 1953 (*Proc.*, Vol. 49), pp. 209-224.

Chapter

7

Shear, Bond, Bearing

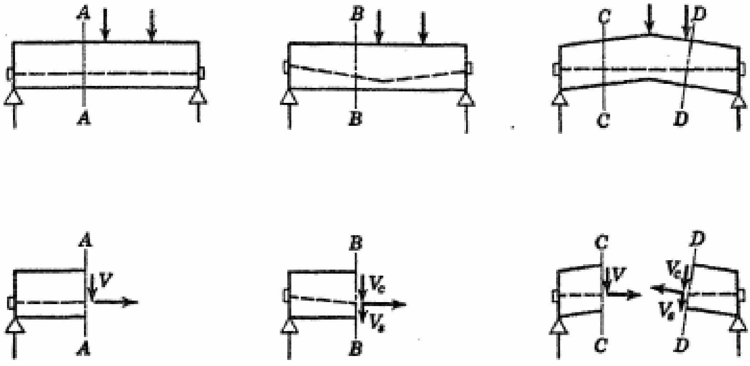
7-1 Shear, General Considerations

The strength of prestressed-concrete beams in flexure is quite definitely known, but their strength in resisting shear cannot be predicted with any precision. In fact, the same situation exists for reinforced-concrete beams, with the exception that more tests have been conducted to investigate their shearing strength so that some empirical methods of design have been derived.[7-1] Even though these empirical methods differ appreciably in different countries and are known to be inaccurate, they have been used for years without serious consequences. For prestressed-concrete beams, no extensive tests are available at the moment. In Europe, most prestressed-concrete beams that were tested to failure were of such proportions that failure simply did not start in shear. In the United States, one series of such tests has been conducted,[7-2] but it is too early to arrive at any general conclusions. Hence no accurate method for estimating the shear strength of prestressed beams can yet be derived.

Fortunately, this rather dim view of the present status of knowledge concerning shear does not stop us from designing prestressed-concrete beams. This is because many prestressed-concrete beams have been built and designed on the basis of some assumed theories for shear which seldom, if ever, ended in disastrous results. Such commendable performance of structures so designed does not necessarily prove the safety of the present method of design. In fact, when we get into proportions and loadings beyond the present scope of practice, serious adverse effects might occur at any moment owing to the lack of rationality in our method of design. Nevertheless, our experience does indicate that structures of normal proportions can be designed at least safely, if not economically, by the conventional method of design. The degree of safety, however, is open to question. Some structures may be too safe in shear, others not so safe,

even though all have functioned safely under the usual service loads.

A general picture of the shear in a prestressed-concrete beam will now be presented. Consider three beams carrying transverse loads as shown in Fig. 7-1-1. Beam (*a*) is prestressed by a straight tendon. Taking an arbitrary section *A–A*, the shear *V* at that section is carried



(*a*) Beam with Straight Tendon

(*b*) Beam with Inclined Tendon

(*c*) Beam with Inclined Axis but Straight Tendon

Fig. 7-1-1. Shear carried by concrete and tendons.

entirely by the concrete, none by the tendon which is stressed in a direction perpendicular to the shear. Beam (*b*) is prestressed with an inclined tendon. Section *B–B* shows that the transverse component of the tendon carries part of the shear, leaving only a portion to be carried by the concrete, thus,

$$V_c = V - V_s$$

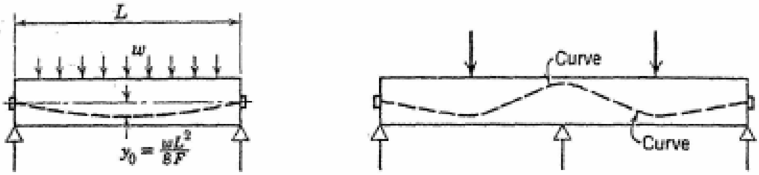
This may be compared to reinforced-concrete beams with bent-up bars where the inclined portion of the steel carries some of the shear. It must be noted, though, that, even if the tendon is inclined to the axis of the beam, it does not carry any vertical shear. This is illustrated by section *C–C* in beam (*c*). Whenever the tendon is not perpendicular to the direction of shear, then it does assist in carrying the shear, e.g., section *D–D*. It is interesting to note that, in some rare instances, the transverse component of the prestress increases the shear in concrete.

In prestressed concrete, it is sometimes possible to design a beam with no shear in the concrete under a given condition of loading. Take Fig. 7-1-2, for example; if the simple beam carrying a uniform

load is prestressed by a parabolic cable with a sag equal to

$$y_0 = \frac{wL^2}{8F}$$

where F is the prestress in the cable, then the transverse component of the cable equals the shear at any point, and there is no shear to be carried by the concrete. For beams carrying concentrated loads, or for continuous beams over the intermediate supports, Fig. 7-1-2 (*b*), the problem is more complicated since the tendon cannot be bent sharply to conform with the theoretically sudden change in shear at



(*a*) Beam with No Shear in Concrete

(*b*) Continuous Beam with Concentrated Loads

Fig. 7-1-2. Varying inclination of tendons to carry shear.

the point of concentration. It may be remarked, however, that the actual change in shear under a concentrated load is not likely to be as abrupt, although its form of variation is little known.

The amount of shear acting on the concrete having been determined, the next step is to compute the shear resistance of the concrete. It is generally believed that prestressed beams, similar to reinforced ones, practically never fail under direct shear or punching shear. They fail as a result of tensile stresses produced by shear, known as diagonal tension in reinforced concrete and as principal tension in prestressed concrete. Before cracking, prestressed concrete can be considered as made up of a homogeneous material; the computation of principal tensile stresses can thus be made by the usual method in strength of materials for the state of stress in a homogeneous body.

Although the principal tensile stresses can be computed, the strength of concrete in resisting such stresses is not definitely known. It must be remembered that for concrete there are many theories of failure, of which the maximum tensile stress theory is only one. After the cracking of concrete, and with the addition of web reinforcement, the problem becomes even more complicated. Tests on both reinforced- and prestressed-concrete beams have seemed to indicate that, when shear failure occurs at a section, not only the shear but also the

moment at that section has effect on its ultimate strength.[7-2–7-4] Thus the problem of shear strength is complicated indeed, if a clear understanding is desired. For the purpose of design, empirical methods are employed. They are relatively simple and are presented in the following sections.

7-2 Shear, Conventional Design

Conventional design for shear in prestressed-concrete beams is based on the computation of the principal tensile stress in the beam and the limitation of that stress to a certain specified value. The first part of this method, the computation of the principal tension, can be performed graphically by Mohr's circle of stress, which is a correct procedure so long as the concrete has not cracked. The second part of this method, limiting the principal tension to a definite value, is known to be an inaccurate approach, because there is evidence to show that the resistance of concrete to such principal tension is not a consistent value but varies with the magnitude of the axial compression.[7-5] It seems, however, that, when the axial compression is not too high, say less than about $0.50f_c'$, the resistance of concrete to its principal tensile stress is relatively consistent. Hence, the computation of principal tensile stress can be regarded as a proper criterion for the stress conditions within the working range, though it may not give a correct measure of safety when considering overloads or when the concrete has cracked.

The conventional method of computing principal tensile stress in a prestressed-concrete-beam section is based on the elastic theory and on the classical method for determining the state of stress at a point as explained in any treatise on mechanics of materials. The method can be outlined as follows:

1. From the total external shear V across the section, deduct the shear V_s carried by the tendon to obtain the shear V_c carried by the concrete, thus,

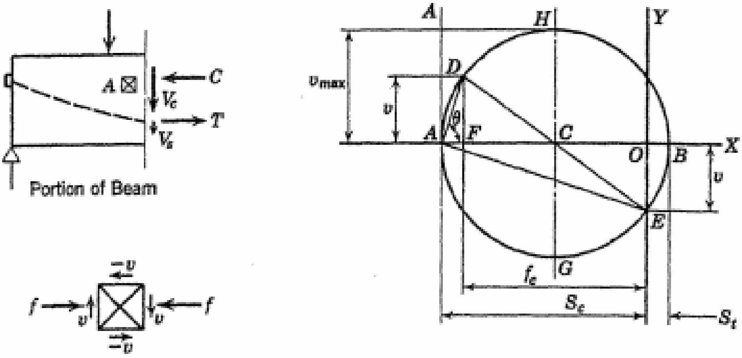
$$V_c = V - V_s \tag{7-2-1}$$

Note again that occasionally, though rarely, $V_c = V + V_s$; this happens when the cable inclination is such that it adds to the shear on the concrete.

2. Compute the distribution of V_c across the concrete section by the usual formula, Fig. 7-2-1,

$$v = V_cQ/Ib$$

where v = shearing unit stress at any given level.



Element A from Above

Mohr's Circle of Stress for Element A

Fig. 7-2-1. State of stress in concrete.

Q = statical moment of the cross-sectional area above (or below) that level about the centroidal axis.

b = width of section at that level.

3. Compute the fiber stress distribution for that section due to external moment M, the prestress F, and its eccentricity e by the formula

$$f_c = \frac{F}{A} \pm \frac{Fec}{I} \pm \frac{Mc}{I}$$

4. The maximum principal tensile stress S_t corresponding to the above v and f_c is then given by the formula

$$S_t = \sqrt{v^2 + (f_c/2)^2} - (f_c/2) \qquad (7\text{-}2\text{-}2)$$

Graphically it can be solved by Mohr's circle of stress* as shown in Fig. 7-2-1. One advantage of this graphical method lies in the indication of the plane of principal tension, as shown in Fig. 7-2-1 and listed in the table. (Note: AA = plane perpendicular to AB.)

Plane	Shearing Stress	Fiber Stress
AD = vertical plane	v	f_c
AE = horizontal plane	$-v$	0
AB = principal tensile plane	0	S_t
AA = principal compressive plane	0	S_c

* Mohr's circle is constructed as follows:

Choose a pair of rectangular axes X–Y with origin at O. Measure OE equal to v. Measure OF equal to f_c and FD equal to v. Draw a circle with DE as diameter. Then OB is the principal tension, OA the principal compression, and $CG = CH$ = principal shear.

It can be seen from the table that the angle between the principal tensile plane AB and the vertical plane AD is greater than 45°. Also note that the principal compressive stress, although somewhat greater than the compressive fiber stress, is seldom considered in design. It is considered sufficient to limit the compressive fiber stress to an allowable value. Similarly, no account is taken of the maximum shearing stresses which occur on planes at 45° to the principal planes, since it is the tension, rather than shear, that produces ultimate failure.

It should be noted further that the greatest principal tensile stress does not necessarily occur at the centroidal axis, where the maximum vertical shearing stress exists. At some point, where f_c is diminished, equation 7-2-2 will often yield a higher principal tension even though v is not a maximum. For I-sections, the junction of the web with the tensile flange is often a critical point for computing the greatest principal tension. This is illustrated in the following example.

Example 7-2-1

A prestressed-concrete beam section under the action of a given moment has a fiber stress distribution as shown in Fig. 7-2-2. The total vertical shear in the concrete at the section is 520 kips. Compute and compare the principal tensile stresses at the centroidal axis N–N and at the junction of the web with the lower flange M–M.

Solution. I of the section about its centroidal axis is computed as 3,820,000 in.^{4} Other values are listed separately for the two levels M–M and N–N as tabulated.

	M–M	N–N
Q, in.^{3}	$36 \times 48 \times 32.6$ $= 56{,}200$	$14.6 \times 24 \times 7.3$ $+ 56{,}200 = 58{,}800$
$v = \dfrac{V_c Q}{Ib}$, psi	$\dfrac{520{,}000 \times 56{,}200}{3{,}820{,}000 \times 24}$ $= 319$	$\dfrac{520{,}000 \times 58{,}800}{3{,}820{,}000 \times 24}$ $= 335$
f_c, psi	720	1012
$S_t = \sqrt{v^2 + \left(\dfrac{f_c}{2}\right)^2} - \dfrac{f_c}{2}$	$\sqrt{319^2 + \left(\dfrac{720}{2}\right)^2} - \dfrac{720}{2}$ $= 121$	$\sqrt{334^2 + \left(\dfrac{1012}{2}\right)^2} - \dfrac{1012}{2}$ $= 100$

Instead of using equation 7-2-2 as above, S_t can be obtained directly from Fig. 7-2-3, using the curves plotted therein, or it can be measured graphically by constructing Mohr's circles as in Fig. 7-2-1. The same answers will be obtained by any of these methods

In this example, the greatest principal tension occurs at M–M rather than the centroidal axis N–N where v is a maximum. As in

most prestressed sections, the principal tension is much smaller than the vertical shearing stress.

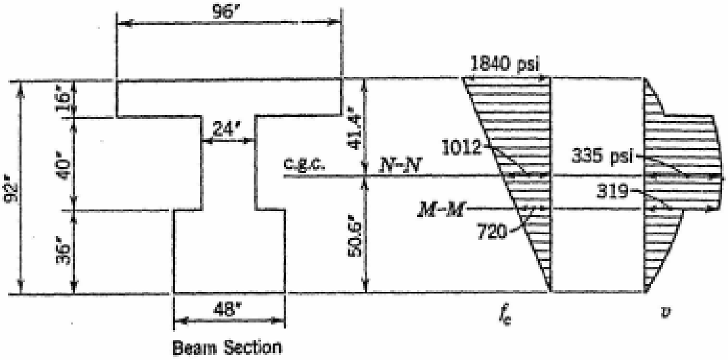


Fig. 7-2-2. Example 7-2-1.

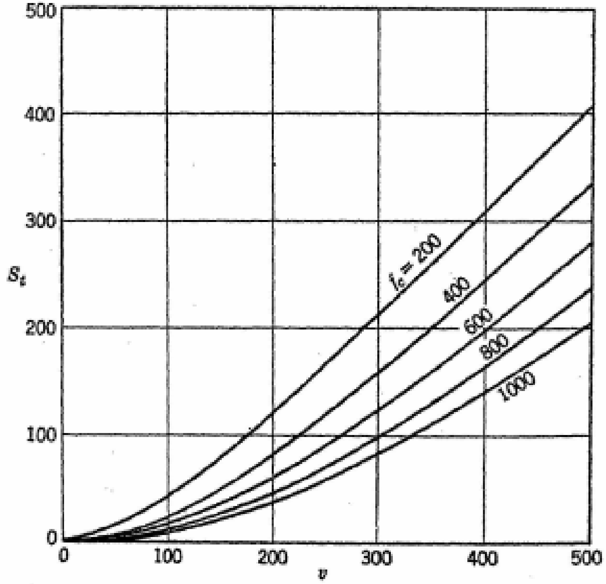


Fig. 7-2-3. Graph for principal tension.

It is usual in design to limit the principal tensile stress to about $0.02f_c'$ or $0.03f_c'$ (see Chapter 14 for further discussion on the allowable stresses). A value of $0.04f_c'$ is occasionally permitted when at least some nominal stirrups are provided. Since the resistance of concrete to principal tensile stress is about $0.08f_c'$ or more, the above

allowable stress appears to be sufficiently conservative at first thought. But one important point must be noted. When the beam is overloaded, the unit vertical shearing stress v increases while the compressive fiber stress f_c near the tensile flange decreases. Furthermore, with the increase in V, V_s changes but little; hence V_c tends to increase fast. Thus the principal tensile stress S_t increases far more rapidly than the increase in external load. Roughly speaking, S_t varies more nearly as the square of v rather than as its first power, so that, if v is doubled, S_t is not doubled but nearly quadrupled. Such a relationship can be seen either from Fig. 7-2-3 or from equation 7-2-2. Thus, if overloads are possible, designing for shear by the conventional method may not be considered safe and some ultimate design may be a more rational approach. The effect of overloading on the increase in principal tension is illustrated in example 7-2-2.

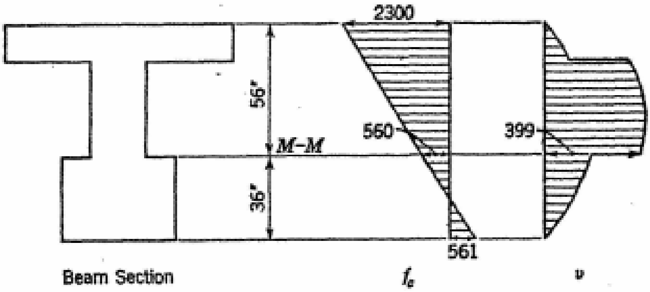


Fig. 7-2-4. Example 7-2-2.

Example 7-2-2

For the beam section in example 7-2-1, suppose that the external load is increased by 25% so that the fiber stress distribution is shown in Fig. 7-2-4 and the total vertical shear in the concrete is 520 × 1.25 = 650 kips. Compute the principal tensile stress at *M–M*.

Solution. The compressive fiber stress at *M–M* is computed to be 560 psi, assuming that cracks have not occurred at the bottom fiber for the tensile stress of 561 psi. The unit vertical shearing stress is

$$v = \frac{V_c Q}{Ib} = \frac{650{,}000 \times 56{,}200}{3{,}820{,}000 \times 24} = 399 \text{ psi}$$

Hence the principal tensile stress at *M–M* is

$$S_t = \sqrt{v^2 + \left(\frac{f_c}{2}\right)^2} - \frac{f_c}{2}$$

$$= \sqrt{399^2 + \left(\frac{560}{2}\right)^2} - \frac{560}{2} = 208 \text{ psi}$$

By comparing this value with $S_t = 121$ psi in example 7-2-1, it is seen that, for this particular point, an increase of 72% in principal tension has taken place corresponding to a 25% increase in loading. Note that, in this example, V_s is assumed to be zero. If V_s is not zero, the percentage increase in the value of S_t will be still higher.

7-3 Shear, Ultimate Strength

As discussed in the previous section, the conventional method for "analyzing" the principal tension, based on the state of stress in a homogeneous material, is a rational method of analysis as long as the concrete has not cracked. However, when applied for "designing," the members so proportioned will possess different factors of safety, since slight increases in loads may produce appreciable and varying increases in the principal tension while the resistance of concrete to principal tension may also change with the magnitude of the com-

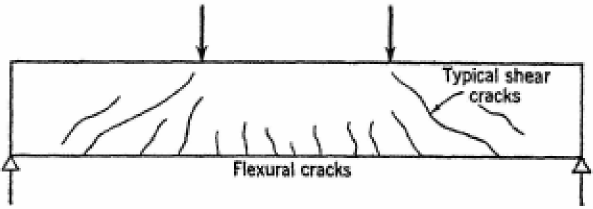


Fig. 7-3-1. Shear failure in prestressed beams.

pressive fiber stress. Furthermore, after the cracking of concrete, whether produced by flexural or principal tension, the method of analysis is no longer applicable. Hence it is evident that our present approach for shear design is not a satisfactory one, especially if the member is to be subjected to overloads.

In order to facilitate more logical and accurate design of structures, particularly those of unusual proportions, it is necessary to have a clear understanding of the shear strength of beams so that design can be based on their strength under overloads rather than on their stresses under the working loads. Unfortunately, few data are available at the present concerning such strength, only one series having been conducted in this country.[7-2] The results of these tests, though not conclusive, will be freely incorporated in the following discussions.

In prestressed-concrete beams, cracks may be produced by either flexural or principal tension, Fig. 7-3-1. For certain beams, generally those with low percentages of reinforcement and with high moment-to-shear ratios, the flexure cracks will develop faster than the principal tension cracks, the steel will be highly stressed in the region

of high bending moment, and final failure will occur by crushing of concrete above the flexural cracks. When the beams are over-reinforced but still subject to high moment rather than high shear, failure may occur by crushing of concrete while the steel is still in the elastic range. When the shear is heavy, the principal tension cracks will develop faster than the flexural cracks; the presence of principal tension cracks will tend to reduce the compressive depth of concrete, and the beam will fail at a load lower than its capacity under pure flexure. Flexure cracks are not necessarily objectionable, unless they combine with and develop into principal tension cracks. Existing by themselves, flexure cracks do not indicate any imminent failure of the beam unless it is highly over-reinforced. Principal tension cracks, on the other hand, often result in sudden failure and should be avoided if possible.

Methods for predicting the shear strength of prestressed beams are not yet available, but, qualitatively speaking, shear strength varies with several factors, as follows:

1. It increases with the strength of concrete f_c'.
2. It increases with the percentage of steel.
3. It increases with the effective prestress in the steel.
4. It increases with the shear carried by the prestressed steel.
5. It increases with the amount of web reinforcement.

It is interesting to note that the conventional method of shear design, unsatisfactory as it is, does take care of these factors in an indirect manner. Hence, lacking a more reliable solution to the problem, we are somewhat justified in following the conventional method as explained in the previous section.

Tests seem to indicate that the so-called shear failure of beams is actually a combination of moment and shear failure, starting in principal tension cracks and ending in the crushing of concrete under bending. This is especially evident when high moment and high shear exist at the same section, as under some concentrated loads of simple beams or over the intermediate supports of continuous beams. It must be admitted, however, that we do not have at this moment sufficient understanding of the cause of shear failure in both reinforced and prestressed beams. Even if we did, it would take some time before we could formulate our knowledge so that it could be simply and efficiently applied in design.

7-4 Shear, Web Reinforcement

Theoretically speaking, if the principal tension in concrete is kept within the allowable limit, no web reinforcement is required. But,

in practice, nominal stirrups are used whether or not they are required by computations: ⅜-in. mild-steel bars are most commonly employed for these nominal stirrups, although other sizes are also used. Stirrup spacing usually ranges from 12 to 24 in. Single stirrups are used for thin webs; double ones, for thick webs.

If the principal tension exceeds the allowable, it will then be necessary to compute the amount of web reinforcement. Since we do not possess a logical method of shear design without web reinforcement, it is obviously more difficult to obtain a satisfactory method when web reinforcement is added. However, in the field of prestressed concrete, as in many other branches of engineering, practice often precedes theory; and the lack of a rational theory does not necessarily stop engineers from attempting new designs, guided by their own experience and intuition plus some method of computation, right or wrong.

To understand the behavior of prestressed-concrete beams with web reinforcement would require extensive tests well conducted and interpreted. To formulate such findings into rules for design requires further study and judgment. Since engineers are often confronted with urgent problems which they have to solve before such research results are available, they have arbitrarily developed semiempirical methods of design, generally yielding safe, though perhaps not economical, results. Two such methods are currently used: the elastic design, and the ultimate design, based on some simple relations between the internal and external forces and on knowledge of reinforced concrete, applied with a proper factor of safety or of ignorance, whichever we prefer to call it.

Even for reinforced concrete, the design for shear differs in different countries. In this country, it is assumed that concrete takes a certain part of the shear, leaving the remainder to be carried by the stirrups. This method is based on the fact that the shear resistance of concrete beams does actually increase with f_c', which can be interpreted to mean that concrete does help to carry the shear to some extent. In Germany, it is assumed that the web reinforcement takes all the shear, with no shear taken by the concrete. This is based on the fact that, under heavy shear, concrete would have cracked and would be ineffective in carrying the load. It is not our purpose here to compare the relative merits of these methods. Attention is simply called to the divergency of methods even in reinforced-concrete design, although both methods have been extensively applied and apparently have yielded safe results.

There is little reason to believe that either of these methods for

reinforced concrete can be directly applied to prestressed concrete, since the methods are more empirical than rational and hold good only within the range of conditions encountered in the tests, which did not include prestressed concrete. However, completely lacking data for the behavior of web reinforcement in prestressed concrete, engineers have been forced to transpose the reasonings for reinforced-concrete design and apply them to prestressed concrete in a perhaps more cautious manner.

Of the two methods, elastic design is based on the computation of principal tension under working loads as in section 7-2. If the computed tension exceeds the allowable value, then web reinforcement is designed to carry the total amount of the tensile stresses, assuming concrete to carry no principal tension. This method is parallel to the German method for reinforced concrete. Since the strength of concrete does affect the shear strength of beams to some extent, this approach is certainly not entirely correct. But some conservatism is desirable considering our ignorance of the situation and the fact that the principal tension increases rapidly with the increase in load.

The application of this elastic design can be outlined as follows:

1. For each section of the beam, compute the maximum principal tension S_t as in section 7-2, and the direction of the principal tensile plane, making an angle θ with the vertical plane, Fig. 7-4-1.

2. For vertical stirrups with spacing s, the force to be taken by each stirrup should be

$$\frac{S_t b s}{\sin \theta}$$

where b is the width of the beam at the point.

3. Since the force supplied by each stirrup with area A_v at working stress f_v is $A_v f_v$, we can equate the resisting and working forces, obtaining

$$A_v f_v = \frac{S_t b s}{\sin \theta} \qquad \text{and} \qquad s = \frac{A_v f_v \sin \theta}{S_t b} \tag{7-4-1}$$

Let us now examine the validity of this method, together with its assumptions and approximations. First, the method is based on the elastic theory applied to working load conditions; hence the principal tension computed is a reliable value. Next, the method makes the conservative assumptions that the concrete carries no tension and that the steel is worked to its allowable stress. Then, for overloads, the principal tension increases rapidly, and, when concrete cracks under flexure, the method of analysis is no longer correct. Hence the method

is a conservative one under working load but does not necessarily ensure a sufficient factor of safety against overloads. One of the inconsistencies in this elastic design is the fact that the web steel evidently cannot be stressed to its design value unless the concrete has

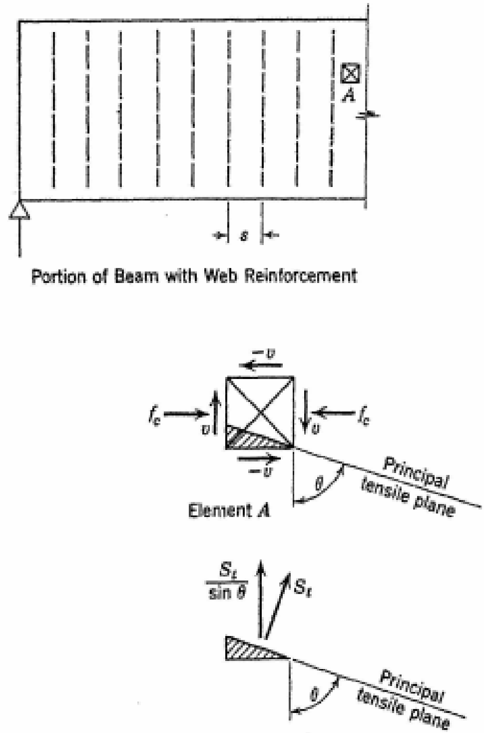


Fig. 7-4-1. Elastic design for web reinforcement.

cracked, while any cracks in the concrete would completely invalidate the assumption of elastic behavior.

To ensure safety under overloads, ultimate design must be adopted. Unfortunately, for web reinforcement, ultimate design cannot be easily formulated because no test data are available. An approximate solution is sometimes made by assuming the direction of the potential shear crack. When the fibers are under flexural compression, the principal tensile plane makes an angle with the beam axis smaller than 45°. When the fibers are not under compression, the principal tensile plane makes a 45° angle with the beam axis. As an approxi-

mation, it can be assumed that the potential cracks lie at 45° with the horizontal. This yields a conservative design if the angle is actually less than 45°. Note that if high moment exists simultaneously with high shear the average inclination of the crack can be greater than 45°. Assuming 45°, a free body can be taken as in Fig. 7-4-2, and a method developed as follows:

1. Compute the total ultimate shear V' at the section, using a proper load factor, say about 2 (i.e., assuming the ultimate load to be twice

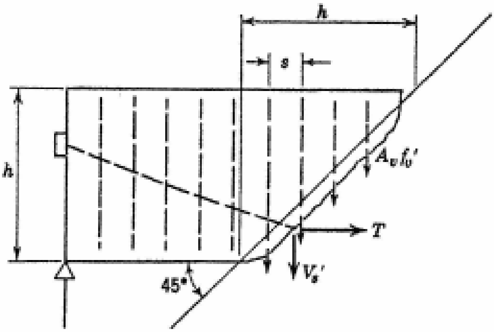


Fig. 7-4-2. Ultimate design for web reinforcement.

the working load). Estimate the ultimate shear V_s' carried by the tendons. The ultimate shear V_c' to be carried by the concrete is, then,

$$V_c' = V' - V_s'$$

2. Assume the shear to be carried entirely by the stirrups across a 45° potential crack; the total number of stirrups is then h/s, where h is the overall depth of the section.

3. Assume the stirrups to be stressed to their yield point f_v'; then the total resistance of the intercepted stirrups is $(h/s)A_v f_v'$. Equating this to the total shearing force in the concrete, we have

$$V_c' = \frac{h}{s} A_v f_v'$$

$$s = \frac{h A_v f_v'}{V_c'} \tag{7-4-2}$$

which gives the spacing of stirrups based on the ultimate design.

Except for the assumption of 45° for the direction of crack and the neglecting of shear in the concrete, this method appears to be logical.

Note that both assumptions err on the safe side except when high moment exists at the same section. The chief difficulty lies in the proper choice of a load factor. Furthermore, the state of stress of such a section under working loads should be checked after the design has been made on the basis of ultimate load. Pending the development of more reliable methods of design, procedures such as this one may have to be adopted even though they can hardly be recommended.

Example 7-4-1

A prestressed-concrete beam has a rectangular section as shown in Fig. 7-4-3 and is subjected to a shear of 170 kips under working loads. The effective prestress in the tendons totals 300 kips and is inclined at an angle of arc sin $\alpha = 1/6$.

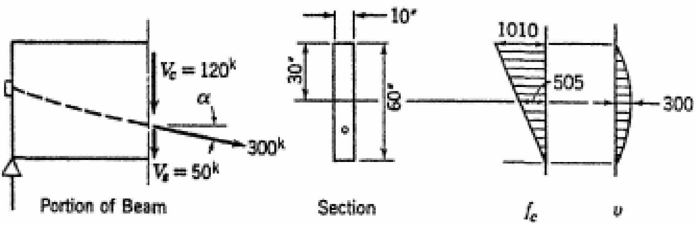


Fig. 7-4-3. Elastic design in example 7-4-1.

The fiber stress distribution under working load is 1010 psi at top fibers and 0 psi at bottom fibers. Allowable principal tension is 100 psi; ½-in. U-stirrups are to be used ($A_v = 0.40$ sq in.), $f_v = 20{,}000$ psi, $f_c' = 40{,}000$ psi. (*a*) By elastic design for the working-load conditions, determine the stirrup spacing, assuming all tension in the concrete to be carried by the stirrups; (*b*) by ultimate design method with a load factor of 2, compute the stirrups spacing, assuming that the ultimate stress in the longitudinal steel is 1.8 times its effective prestress.

Solution. (*a*) Under working load, shear carried by the tendons is

$$V_s = 300/6 = 50 \text{ k}$$

$$V_c = V - V_s$$

$$= 170 - 50 = 120 \text{ k}$$

As an approximation, let us consider the state of stress at the centroidal axis,

$$v = \frac{3}{2}\frac{V}{A}$$

$$= \frac{3}{2}\left(\frac{120{,}000}{10 \times 60}\right)$$

$$= 300 \text{ psi}$$

From Fig. 7-4-3, $f_c = 505$ psi, $f_c/2 = 252$ psi. Hence,

$$S_t = \sqrt{300^2 + 252^2} - 252$$
$$= 140 \text{ psi}$$

which exceeds the allowable of 100 psi. The inclination of the principal tension plane with the vertical plane is found to be 70°, using Mohr's circle of stress.

Applying equation 7-4-1, we have

$$s = \frac{A_v f_v \sin\theta}{S_t b}$$
$$= \frac{0.40 \times 20{,}000 \times \sin 70°}{140 \times 10}$$
$$= 5.2 \text{ in.}$$

(*b*) For ultimate design with a load factor of 2, $V' = 2 \times 170 = 340$ kips; $V_c' = 1.8 \times 50 = 90$ kips; $V_c' = 340 - 90 = 250$ kips. Hence,

$$s = \frac{hA_v f_v'}{V_c'}$$
$$= \frac{60 \times 0.40 \times 40{,}000}{250{,}000}$$
$$= 3.8 \text{ in.}$$

7-5 Flexural Bond at Intermediate Points

For post-tensioned concrete, bond is supplied by grouting. For pre-tensioned concrete, bond is secured directly when placing the concrete. When a bonded beam is subject to shear, bond stresses are produced. In order to design against bond failure, it is necessary to determine two things: first, the amount of bond stress existing between steel and concrete; second, the bond resistance between the two materials. Pre-tensioning, where the end anchorage of the tendons is secured solely by bond, is discussed in the next section. Bond at intermediate points along the length of beam, whether pre-tensioned or post-tensioned, will now be discussed.

To determine the bond stress existing between concrete and the tendons, two stages have to be considered: before and after cracking of concrete. Before the cracking of concrete, bond stress can be calculated similar to that for compressive steel in reinforced concrete. Consider a prestressed beam loaded externally, Fig. 7-5-1. To determine the bond stress, taking an elementary segment dx between sections A and B as a free body, we have

$$V\,dx = dM = M_B - M_A$$

Since $M_B = f_B I_t/y$ and $M_A = f_A I_t/y$, we have

$$V\,dx = \frac{f_B I_t}{y} - \frac{f_A I_t}{y}$$

which can be written as

$$V\,dx = \frac{I_t}{nA_s y}\,(nA_s f_B - nA_s f_A)$$

Since $nA_s f_B - nA_s f_A = F_B - F_A$, we have

$$V\,dx = \frac{I_t}{nA_s y}\,(F_B - F_A)$$

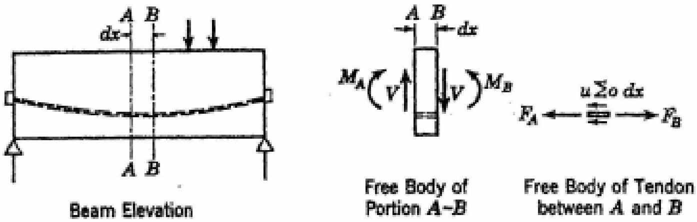


Fig. 7-5-1. Bond stress before cracking.

Taking the wires as a free body, we have

$$F_B - F_A = u\Sigma o\,dx$$

hence,

$$V\,dx = \frac{I_t}{nA_s y}\,(u\Sigma o\,dx)$$

Transposing,

$$u = \frac{nA_s yV}{\Sigma o I_t} \tag{7-5-1}$$

For round wires, $A_s/\Sigma o = D/4$, we have

$$u = \frac{VynD}{4I_t} \tag{7-5-2}$$

When wires are encased in metallic hoses, bond stress must be calculated for two contact areas: first, between wires and the grout; then between hoses and the concrete. For the latter computation, equation 7-5-1 and not 7-5-2 should be applied; A_s is the area of the encased wires, and Σo stands for the perimeter of the hoses. The same

principle holds when considering a group of wires and computing the bond between the group and the surrounding grout or concrete.

Example 7-5-1

A prestressed-concrete rectangular beam is post-tensioned and then grouted. The steel consists of three tendons, each made up of twelve ¼-in. wires (A = 0.05 sq in. per wire) encased in a thin metallic hose 1.25 in. in diameter. $E_c = 4 \times 10^6$ psi; $E_s = 28 \times 10^6$ psi. The beam spans 30 ft and carries a con-

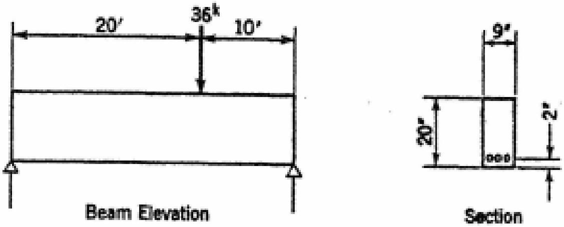


Fig. 7-5-2. Example 7-5-1.

centrated load as shown, Fig. 7-5-2. Compute the unit bond stress, (a) between each wire and the grout, (b) between the hose and the concrete.

Solution. Since the beam is grouted after being post-tensioned, no bond stress is produced by the weight of the beam. The maximum shear V to be used for computing bond is that due to live load only, thus,

$$V = 24{,}000 \text{ lb}$$

Instead of using the transformed section including the steel, it will be close enough to use the gross concrete section; hence,

$$I_t = I = \frac{9 \times 20^3}{12} = 6000 \text{ in.}^4$$

and y can be measured from the mid-depth of the section,

$$y = 8 \text{ in.}$$

(a) For bond stress between each wire and the grout, apply equation 7-5-2,

$$u = \frac{VynD}{4I_t}$$

Since $n = 28/4 = 7$, and $D = 0.25$, we have

$$u = \frac{24{,}000 \times 8 \times 7 \times 0.25}{4 \times 6000}$$
$$= 14 \text{ psi}$$

(b) For bond stress between the hoses and the concrete, we can compute either on the basis of one hose or of all three hoses, obtaining the same results. Considering one hose, we have $A_s = 12 \times 0.05 = 0.60$ sq in., and $\Sigma o = 1.25\pi$

$= 3.92$ in. Applying equation 7-5-1,

$$u = \frac{nA_s yV}{\Sigma o I_t}$$
$$= \frac{7 \times 0.60 \times 8 \times 24{,}000}{3.92 \times 6000}$$
$$= 34 \text{ psi}$$

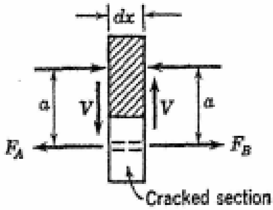


Fig. 7-5-3. Bond stress after cracking.

After the cracking of concrete, the problem is more complicated. First of all, it is known that bond stresses change suddenly at the cracks owing to the abrupt transfer of tension from concrete to steel at such points.[7-6] Thus there exists a varying bond stress near the cracks which cannot be easily determined. However, if it is assumed that the bond stress is uniform along the length, then a formula similar to that for tensile bars in reinforced-concrete beams can be applied.

Referring to Fig. 7-5-3, again taking the element dx as a free body, we have

$$V\,dx = (F_B - F_A)a$$
$$= (u\Sigma o\,dx)a$$

Hence,

$$u = \frac{V}{a\Sigma o} \tag{7-5-3}$$

In the ultimate range, the value of a can be approximated by $\frac{7}{8}d$ as for reinforced-concrete beams. Equation 7-5-3 gives a much higher bond stress than equation 7-5-1 or 7-5-2. This indicates that the bond stress increases suddenly when the section changes into a cracked one.

Example 7-5-2

For the same beam as in example 7-5-1, compute the bond stress between the hoses and concrete, if the load is doubled and cracks have occurred on the tensile side.

$$\text{Shear } V = 2 \times 24{,}000 = 48{,}000 \text{ lb}$$

Assuming $a = \frac{7}{8}d = \frac{7}{8}18 = 15.7$ in., we have, from equation 7-5-3, considering all three hoses,

$$u = \frac{V}{a\Sigma o}$$
$$= \frac{48{,}000}{15.7 \times 1.25 \times 3.14 \times 3}$$
$$= 260 \text{ psi}$$

indicating a very high value of bond stress. Comparing this with example 7-5-1, it is seen that, while the shear is only doubled, the unit bond stress is increased from 34 to 260 psi. This again illustrates the inadequacy of the elastic theory and the importance of investigating the ultimate strength of prestressed elements.

The existing bond stress having been determined, the next step is to find out the resisting unit bond strength between steel and concrete. This, obviously, depends on many factors, foremost of which are the surface of steel and the strength of the adjoining concrete or grout. While many tests have been run for bond resistance of reinforcing bars, only a few data are available concerning the bond strength between prestressed steel and concrete.[7-13] Referring to ordinary reinforced concrete, it is seen that the bond strength varies from $0.08f_c'$ for plain bars to $0.30f_c'$ for new deformed bars.[7-7] For prestressing steel, ordinary plain wires have bond strength comparable to that of plain reinforcing bars, while corrugated wires (and perhaps waved wires) will have greater bond resistance. It is also known that twisted wire strands have higher bond than straight wires.

Besides the above bond stresses produced by shear, there are bond stresses produced by flexure alone, even in a region of zero shear. Consider, for example, a beam loaded at the third points of the span. The middle third of the beam has no shear but is under heavy bending moment. When that section begins to crack, stress in the steel right at the cracks necessarily differs from that away from the cracks. Thus there exists a rather high and varying bond stress adjacent to any cracks produced by flexure, whether there is shear or not. Such local bond stress is often high enough to result in failure of bond between steel and concrete near the cracks. However, this local bond failure may not be significant as far as the overall safety of the beam is concerned.

7-6 Prestress Transfer Bond in Pre-Tensioned Concrete

1 · Nature of Bond and Length of Transfer. When tendons are pretensioned, their stress is often transferred to the concrete solely by bond between the two materials. Thus there is a length of transfer at each end of the tendons to perform the function of anchorage, when mechanical end anchorages are not provided. The condition of bond stress existing at these ends is radically different from that along the intermediate length of a beam. At intermediate points, the bond stress is produced by the external shear or by the existence of cracks. Where there are no cracks and no shear, the bond stress is zero. At anchorage, bond stress exists immediately after transfer. The stress in the tendons varies from zero at the exposed end to a

full prestress at some distance inside the concrete. That distance is known as the length of transfer; and such bond stress is termed as prestress transfer bond.[7-13]

The nature of prestress transfer bond is entirely different from the flexural bond stress produced by shear or cracks. At intermediate points along a beam, the bond stress is resisted by adhesion between steel and concrete, aided by mechanical resistance provided by corrugations in the steel when deformed bars are used. At end anchorages, the pre-tensioned tendons almost always slip and sink into the concrete

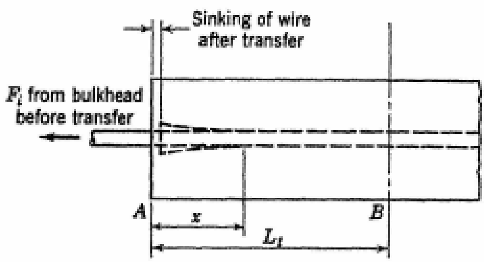


Fig. 7-6-1. Prestress transfer at end of pre-tensioned beam.

at the moment of transfer. This slippage destroys most of the adhesion for the length of transfer and part of the mechanical resistance of the corrugations, leaving the bond stress to be carried largely by friction between steel and concrete.

Immediately after transfer, at end A, Fig. 7-6-1, the wire will have zero stress and its diameter will be restored to the unstressed diameter. At B, the inner end of the length of transfer, the wire will have almost full prestress, and, owing to Poisson's ratio effect, its diameter will be smaller than the unstressed diameter. Thus along the length of transfer, there is an expansion of the wire diameter which produces radial pressure against the surrounding concrete. Frictional force resulting from this pressure serves to transmit the stress between steel and concrete. In other words, a sort of wedging action takes place within that length of transfer.

On the supposition of lateral expansion, Hoyer[7-14] has derived an equation giving the length of transfer, L_t, as

$$L_t = \frac{d}{2\mu}(1 + m_c)\left(\frac{n}{m_s} - \frac{f_i}{E_c}\right)\frac{f_e}{2f_i - f_e} \tag{7-6-1}$$

where m_c = Poisson's ratio for concrete.

m_s = Poisson's ratio for steel.

$n = E_s/E_c$.
E_c = modulus of elasticity for concrete.
f_i = initial prestress in steel.
f_e = effective prestress in steel.
μ = coefficient of friction between steel and concrete.
d = diameter of wire.

If we assume that $m_c = 0.1$, $m_s = 0.3$, $n = 6$, $E_c = 5{,}000{,}000$ psi, $f_i = 150{,}000$ psi, and $f_e = 125{,}000$ psi, we will reduce equation 7-6-1 to

$$L_t = 8d/\mu$$

It can be observed, therefore, that the length of transfer varies directly as the diameter of wire and inversely as the coefficient of friction, assuming this wedging action to be the only bonding force.

More recently, a similar elastic analysis has been made by Janney,[7-18] also based on the elastic theory of a thick-walled cylinder, considering only the frictional bonding phenomenon and neglecting the adhesion or mechanical bond due to corrugation. This analysis leads to the following formula, which gives the ratio of steel stress f_s/f_e at a distance x from the free end,

$$\log \frac{f_e - f_s}{f_e} = \frac{-4\mu m_s x}{d[1 + (1 + m_c)n]} \qquad (7\text{-}6\text{-}2)$$

In order to show the variation of stress along the length of transfer, equation 7-6-2 is plotted in Fig. 7-6-2, giving f_s/f_e for wires of diameter 0.1, 0.2, and 0.3 in., with a coefficient of friction $\mu = 0.3$. Values of m_c, m_s, and n are again assumed to be 0.1, 0.3, and 6, respectively. It can be observed from equation 7-6-2 that the distance x required to develop a given ratio of f_s/f_e varies directly with the diameter of the wire and inversely with the coefficient of friction. Hence, for wires of other diameters and other coefficients of friction, the distance x for a given f_s/f_e can be proportionately obtained from these curves. The results given by this equation are necessarily approximate because only the frictional force is considered and because concrete is not elastic but plastic.

Tests made by the Portland Cement Association to measure the variation of steel stress along the length of transfer seem to verify the above theory in a qualitative manner. Some of the essential findings are listed below:

1. The length of transfer is about 15 to 20 in. for wires of 0.100- to 0.276-in. diameter. However, it is only moderately greater as the wire diameter increases.

2. Only slightly shorter length of transfer is obtained for concrete of 6500-psi strength as against that of 4500-psi strength.

3. Rusted wires develop the full prestress at a more rapid rate than clean wires; lubricated wires require appreciably longer length of transfer. Lubricated wire of 0.276-in. diameter, pretensioned to 120,000 psi, has a length of transfer of about 36 in., while 0.162-in. diameter rusted wire pre-tensioned to the same stress has a length of transfer as short as 12 in.

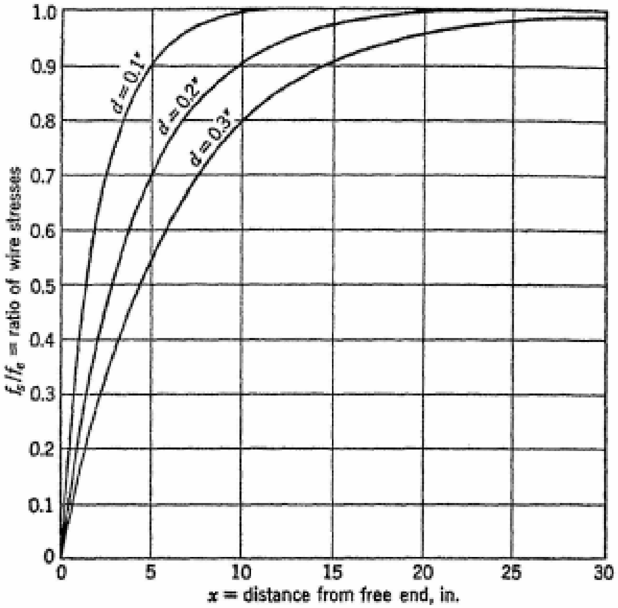


Fig. 7-6-2. Theoretical variation of wire stress along length of transfer. (From equation 7-6-2 with $\mu = 0.3$, $n = 6$, $m_c = 0.1$, $m_s = 0.3$.)

According to tests by Guyon,[7-8] at the moment of transfer, wires of 0.10-in. diameter sank into the concrete an average amount of about 0.16 in. This amount of sinking seems to be much larger than usual, although it was partly explained by the fact that the concrete used in these tests was of relatively lower strength.

Limited measurements by the author on the sinking of 7-wire strands of ⅜-in. nominal diameter indicated values from 0.04 to 0.10 in. These strands were stressed to 175 ksi initially and probably had a

stress of 160 ksi immediately after transfer. The compressive strength of concrete was 5000 psi.

It must be noted here that bond at end anchorage cannot be measured by the usual pull-out tests as applied to reinforced-concrete specimens. For such tests, the steel is cast in the concrete without any prestress. So, when the steel is pulled, its diameter shrinks, producing a tendency to tear itself away from the surrounding concrete. For the prestress transfer in pre-tensioned steel, the wire diameter expands, thus producing wedge action and increasing the frictional force between steel and concrete.

2 · Design for Prestress Transfer by Bond. Although the general nature of bond at end anchorage is now known, the ultimate bond strength for all types and sizes of tendons with different surface conditions has not yet been carefully determined. European practice limits the size of smooth plain wires to 2, 2.5, and 3 mm (0.08 in., 0.10 in., and 0.13 in. respectively), while corrugated wires of 4 and 5 mm (0.16 and 0.20 in.) are also employed for pre-tensioning. Wires up to ⅜-in. diameter have been successfully used in this country when mechanical anchorages, such as Dorland anchorages, have been provided. It is believed that, for the same nominal diameter, strands have greater resistance in bond than wires, and thus possess shorter length of transfer. Although only scattered tests are yet available, strands up to ⅜ in. have been successfully employed for pre-tensioning in this country. The design of beams to avoid end bond failure hinges upon several factors. Load tests on beams have indicated that failure in end bond does not occur until the beam has cracked under flexure. Pre-cracking flexure increases the bond stress only slightly and seldom produces failure in bond. At cracking, however, the bond stress increases suddenly. If the bond stress exceeds the ultimate value, local slipping may occur. Such slipping releases the bond stress in that portion but increases the bond stress in the adjacent region. Thus a wave motion of bond stress may take place, progressing toward the end of the beam. A superposition of this flexural bond with the prestress transfer bond near the end of the beam may often result in complete bond failure of the beam. Failure can be prevented only when the distance from the outermost crack to the end of the beam is sufficient to develop the bond. Hence it is believed that bond failure in pre-tensioned beams will depend to a large degree upon the extent and magnitude of moment along the length of the beam, as much as upon the shear which produces the bond stress.

In actual structures, it must be further noted that the amount of

overhang beyond the edge of supports will be likely to affect the bond strength of the beam. If there is sufficient overhang, the resistance to flexural bond can be considerable. If the overhang is insufficient, the bond stress produced by shearing and cracking will be additive to that produced by transfer and the results may be detrimental. Figure 7-6-3 shows cracks in a pre-tensioned beam tested with overhang on one side only, resulting in bond failure at one end which has practically no overhang.[7-10] After the failure, the same beam was tested with sufficient overhang on both sides, and the full flexural strength of the beam was developed.

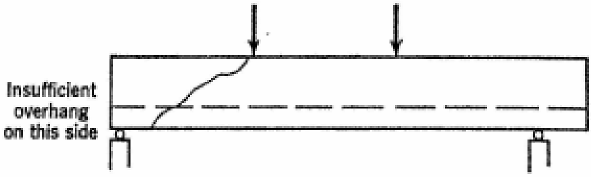


Fig. 7-6-3. Effect of overhang on bond failure.

Only a few data are available on the fatigue strength at the ends of unanchored pre-tensioned wires and strands. Some tests showed that, with overhang of 35 in. for wires of 0.2-in. diameter, an indefinite number of repeated loads could be carried without bond failure even when some of the wires were purposely oiled to reduce their bond resistance. Other tests on strands indicated their ability to stand repeated loads, but the details of the test, such as the amount of overhang, are not definitely known.

One precaution must be mentioned regarding the shear resistance of the end portion of beams with unanchored pre-tensioned tendons. For that end portion within the length of transfer where full prestress is not obtained in the concrete, design for shear should be made accordingly. In other words, near the end of the beam, we have more nearly a plain or reinforced-concrete region rather than a prestressed one.

7-7 Bearing at Anchorage

For tendons with end anchorages, where the prestress is transferred to the concrete by direct bearing, various designs may be used for transmitting the prestress: steel plates, steel blocks, or reinforced-concrete ones.

The design of an anchorage consists of two parts: determining the

bearing area required for concrete, and designing for the strength and detail of the anchorage itself. Stress analysis for any anchorage is a very complicated problem, because not only the elasticity but also the plasticity of concrete enters into the picture. As a result, anchorages are designed by experience, tests, and usage rather than by theory. Since anchorages are generally supplied by the prestressing companies which have their own standards for different tendons, the engineer does not have to design for them. Anchorages that have been successfully adopted are usually considered reliable, and no theoretical check on their stresses is necessary. For a new type of anchorage, the most reliable check is to run a test to determine its ultimate strength. A proper safety factor can then be applied to obtain the allowable load.

Sometimes it is necessary to design or to check the bearing areas for end anchorage, as governed by the allowable bearing in concrete. Since the cost of anchorage increases greatly if the allowable bearing stress is low, it has been the practice to use as high a bearing stress as is consistent with safety, much higher than permitted in reinforced concrete. This is true for practically all systems of prestressing. Besides reasons of economy, such high bearing stress can be justified on the following grounds:

1. The highest bearing stress that will ever exist at the anchorage occurs at transfer. As loss of prestress takes place, the bearing stress gradually diminishes.

2. The strength of concrete increases with time. Hence, if failure does not take place immediately at transfer, there is little possibility that it will happen later.

3. For bonded tendons with anchorages at the end of members, externally applied load will not increase the force on the anchorage. For unbonded tendons, the force on the anchorage will increase with load; but the increase is limited, hence a high factor of safety is not required.

The allowable bearing stress depends upon several factors, such as the amount of reinforcement at the anchorage, the ratio of bearing to total area, and the method of stress computation. A value commonly allowed is $0.60f_c'$, assuming uniform bearing over the entire contact area. If the anchorage is rigid, the variation of pressure will be small over the contact area; if only a thin plate is used, high bearing pressure may exist near the tendons. If the anchorage simply bears on the end of concrete without being buried in it, the prestress is transferred entirely through bearing. If the anchorage is buried in the concrete, then part of the prestress may be transferred through bond along the

sides of the anchorage. Take the Freyssinet cone, for example; it is believed that about a third of the prestress is transmitted through the sides.

Because of the strict economy followed in the design of end anchorages, it has not been unusual that when the concrete is poor it actually crushes under the application of the prestress. Hence it is important that concrete for prestressed work should be of high quality and should be carefully placed around the anchorages.

Besides supplying strength and rigidity, anchorage must be detailed to suit the dimensions of the jack and the ends of the beam. When both anchorages and jacks are supplied by the prestressing company, the designer will not have to worry about such details. If the thickness of the bearing plate has to be designed, a procedure similar to that used for designing column bearing plates may be followed. This consists of designing the critical section for bending at an allowable stress. Here, again, the allowable stress can be somewhat higher than ordinary, since there is no danger of overload or fatigue effect. The design for a Roebling anchorage is shown in the following example.

Example 7-7-1

Compute the thickness required for the bearing plate to carry a Roebling cable 1½ in. in diameter, with an initial tension of 170 kips. Size of the plate is chosen as 10 in. square, being limited by jacking requirements. Compute the bearing pressure on concrete. $f_c' = 5000$ psi. Other details of the anchorage are shown in Fig. 7-7-1.

Solution. Allowable load carried by bond may be calculated as follows:

Area of contact $= 6.625\pi \times 16 = 333$ sq in.

Allowable bond stress assumed at $= 0.04f_c' = 200$ psi.

Permissible load by bond $= 200 \times 333 = 67$ kips.

Load to be carried by bearing $= 170 - 67 = 103$ kips.

Bearing area $= 100 - 28$ (area of pipe) $= 72$ sq in.

Bearing stress $= 103{,}000/72 = 1430$ psi.

Assume allowable stress to be $0.60f_c' = 3000$ psi, the stress of 1430 psi is evidently low. But it is not possible to reduce the bearing area because of jacking requirements in this case.

To compute the thickness of the plate, consider critical section Y–Y. Assuming uniform distribution of pressure, the lever arm of force from concrete on plate $= 2.8$ in., the lever arm of force from screw nut on plate $= 2.0$ in. The total force for half of the plate $= 103/2 = 51.5$ kips.

Total bending moment

$$M = 51{,}500 \times (2.8 - 2.0)$$

$$= 41{,}200 \text{ in.-lb}$$

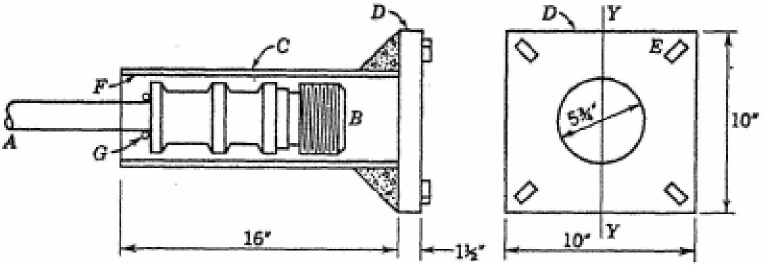


Fig. 7-7-1. Example 7-7-1.

A = 1½" Roebling strand.
B = end fitting for above strand, in position before prestressing.
C = 6" standard pipe 16" long with 11/16" holes for rods G to pass; holes to be sealed with mastic. Outside diameter = 6.625".
D = 1½" x 10" x 10" steel plate, with center hole slightly greater than inside of pipe.
E = ½" x ½" x 1" centering lugs for jack, welded to 2" plate.
F = paper packing.
G = keeper rods ⅝".

Section modulus of Y–Y, for net width $b = 4.25$ in.,

$$\begin{aligned} Z &= bd^2/6 \\ &= 4.25d^2/6 \\ &= 0.708d^2 \end{aligned}$$

Allowing a high stress of 30,000 psi in the steel plate, we have

$$f = M/Z$$

$$30{,}000 = 41{,}200/0.708d^2$$

$$d = 1.4 \text{ in.}$$

Use a 1½-in. plate.

7-8 Transverse Tension at End Block

The portion of a prestressed member surrounding the anchorages of the tendons is often termed the end block. Throughout the length of the end block, prestress is transferred from more or less concentrated areas and distributed through the entire beam section. The theoretical length of the end block is the distance through which this change takes place and is sometimes called the lead length. It is known from theoretical and experimental investigations that this lead is not more than the height of the beam and often is much smaller.

Referring to Fig. 7-8-1, the prestress at section A–A, whether hori-

zontal or inclined, is applied as concentrated or somewhat distributed loads. At section B–B, the end of the lead length, the resistance from the beam consists of linearly distributed fiber stresses and corresponding shearing stresses as calculated by the usual beam theory. For the portion between sections A–A and B–B, the stress distribution is

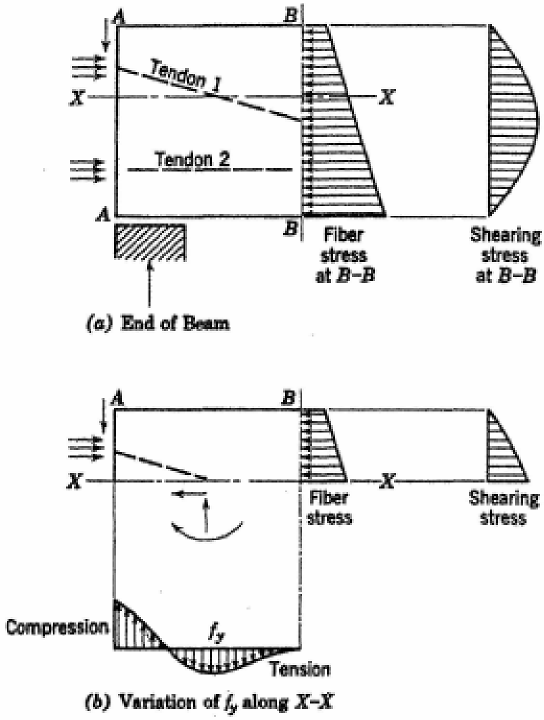
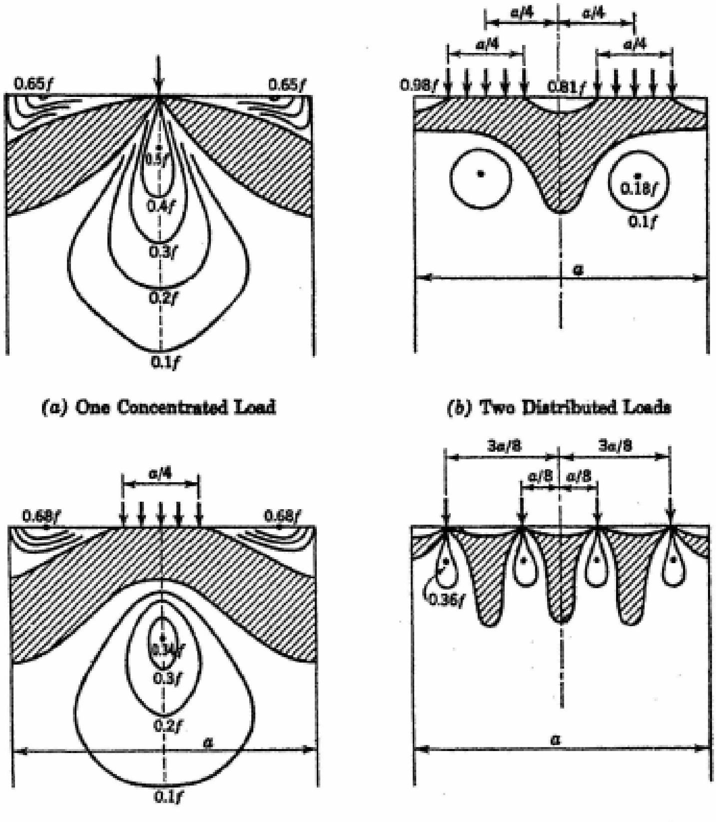


Fig. 7-8-1. Stresses at end block.

rather complicated. If we cut a longitudinal section X–X, and take a free body as in Fig. 7-8-1 (b), there will exist moment, shear, and a direct load on that section. These components of forces can be simply computed from statics, but their distribution along X–X cannot be easily determined. It is not possible to apply the usual beam theory assuming a plane section remaining plane, because that theory is far from being correct when applied to a short block like A–A–B–B. It can only be solved by the advanced theory of elasticity, which is complicated even for the simplest conditions of loading.

In order to simplify the solution, an assumption is made that the load is uniformly distributed across the width of the beam; thus,

instead of a load concentrated at one point, we can assume a knife-edge load extending the entire width of the beam. Then the problem is reduced from a three- to a two-dimensional problem. On this assumption, and on the theory of elasticity, stress distributions within



(a) One Concentrated Load

(b) Two Distributed Loads

(c) One Distributed Load

(d) Four Concentrated Loads

Fig. 7-8-2. Isobars for transverse tension in end block (in terms of average compression f). Shaded areas represent compressive zones. From Guyon's *Prestressed Concrete*.

the end block have been solved, and tables and graphs are available for certain conditions of loading.[7-12] For these graphs, Fig. 7-8-2, it is convenient to express the stresses in terms of the average direct compression f, where

$$f = F/A$$

F = total axial prestress at end of beam, and A = cross-sectional area of beam.

In general, along any longitudinal section, such as X–X, in Fig. 7-8-1, the shearing stress is small and does not cause any trouble; only the transverse tensile stress f_y can be serious. Hence we are interested only in the variation of f_y.

Graphs in Fig. 7-8-2 are for rectangular sections and are intended to indicate the general nature of the tensile stresses in end blocks. Lines of equal f_y, also termed "isobars," are shown in the graphs. From these "isobars," it can be observed that there are two general areas of tension. One area in the center of the section is termed the "bursting zone." It has a maximum tension along the line of the load and at some distance from it. Another area is on the sides of the load close to the end surface, termed the "spalling zone." This zone is subject to high tensile stresses but only over a small area.

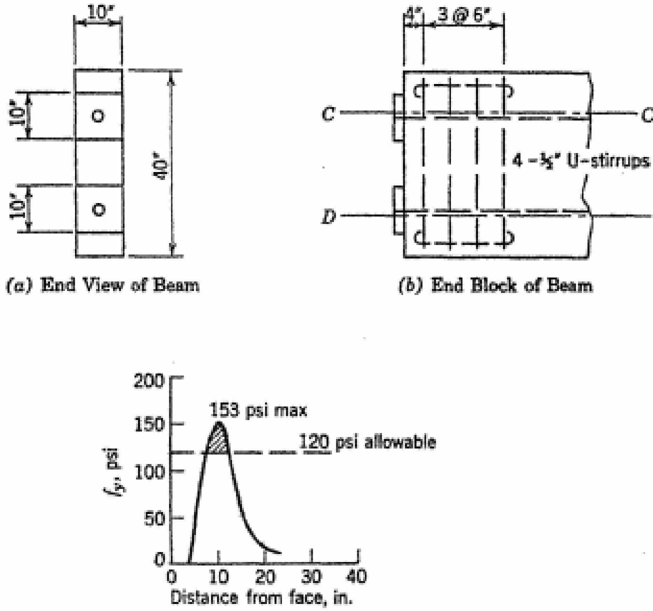
Additional graphs are available in Guyon's *Prestressed Concrete*, to which readers are referred. Even though these graphs are theoretically correct and some of them have been confirmed by photoelasticity, their application to design is another problem. First, concrete is not a perfectly elastic material and will act plastically especially when part of it is overstressed. Second, what should be the allowable tension in the concrete? Third, if the allowable tension is exceeded, how shall we design the reinforcing steel? Fourth, the pattern of forces applied at the end is often more complicated than can be handled by the theory of elasticity. Hence, though theory is needed in analysis, judgment must be exercised in design.

Guyon recommends that the allowable tensile stress be set at about a tenth of that for compression, i.e., about $0.04f_c'$. Wherever the tension exceeds that value, steel reinforcement should be designed to take the entire amount of tension on the basis of the usual allowable stress in steel.

In the spalling zone, the tensile stresses are very high and will generally exceed the allowable value. However, these stresses act on only a small area, and the total tensile force is therefore small. For most cases, it has been found sufficient to provide steel for a total transverse tension of $0.03F$. This steel is placed as close to the end as possible. Either wire mesh or steel bars may be used.

To carry the tension in the bursting zone, either stirrups or spiral steel may be used. For local reinforcement under the anchorage, ¼-in. spirals at 2-in. pitch or ⅜-in. spirals at 1½-in. pitch are sometimes adopted. For overall reinforcement, stirrups can be efficiently employed. A design of these stirrups is illustrated in example 7-8-1.

It is fortunate that under ordinary conditions the number of stirrups required to resist the transverse tension is not excessive. Hence nominal amounts of reinforcement will suffice. However, longitudinal cracks have been produced by transverse tension when reinforcement was not provided or was insufficient.



(a) End View of Beam

(b) End Block of Beam

(c) Variation of Transverse Tensile Stress along C–C or D–D

Fig. 7-8-3. Example 7-8-1.

Example 7-8-1

The end of a prestressed beam is rectangular in section, and is acted on by two prestressing tendons anchored as shown, Fig. 7-8-3. The initial prestress is 170 kips per tendon. $f_c' = 4000$ psi. Design the reinforcement for the end block, allowing a maximum of 120 psi for the tension in the concrete.

Solution. Tensile stresses for the bursting zone can be obtained from Fig. 7-8-2 (*b*). Critical tensile stresses exist through sections *C–C* and *D–D* of Fig. 7-8-3 (*b*), and their variation is plotted in Fig. 7-8-3 (*c*). The greatest tensile stress is given as $0.18f$.

$$0.18f = 0.18\frac{2 \times 170{,}000}{10 \times 40}$$

$$= 0.18 \times 850$$

$$= 153 \text{ psi}$$

Suppose that reinforcement is required for the portion whose tension exceeds 120 psi; then the shaded portion of about 5 in. would require reinforcement. Assuming an average tension of 140 psi for the 5-in. length, the total tensile force to be resisted by steel is

$$140 \times 5 \times 10 = 7000 \text{ lb}$$

For an allowable stress of 20,000 psi in the steel, the area of steel required is

$$A_s = \frac{7000}{20,000} = 0.35 \text{ sq in.}$$

Four ½-in. U-stirrups will be provided as shown, giving a total area of 1.57 sq in. Note that computation such as this simply serves as a guide. Judgment must be exercised in design. Since the tension is not excessive, liberal provision of steel is possible without much additional cost.

If concrete around the anchorages is thin, it is desirable to add some spiral steel such as ¼-in. wires at 2-in. pitch.

For the spalling zone, stresses as high as $0.98f = 0.98 \times 850 = 830$ psi exist. The total force, however, is small and can be approximated by an average of 400 psi over a length of 2 in., which amounts to

$$400 \times 2 \times 10 = 8000 \text{ lb}$$

Using the value of $0.03F$ as suggested, we would get

$$0.03 \times 340,000 = 10,200 \text{ lb}$$

showing not too bad an agreement in this problem. Steel required to resist 8000 lb is

$$8000/20,000 = 0.40 \text{ sq in.}$$

which is adequately provided by the ½-in. stirrups.

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Chapter 8 Beam Deflections and Layouts

8-1 Beam Deflections

Before cracking, the deflections of prestressed-concrete beams can be predicted with greater precision than that of reinforced-concrete beams. Under working loads, prestressed-concrete beams do not crack; reinforced ones do. Since prestressed concrete is a homogeneous elastic body which obeys quite closely the ordinary laws of flexure and shear, the deflections can be computed by methods available in elementary strength of materials.

As usually encountered for any concrete member, two difficulties still stand in the way when we wish to get an accurate prediction of the deflections. First, it is difficult to determine the value of E_c within an accuracy of 10% or even 20%. Tests on sample cylinders may not give the correct value of E_c, because E_c for beams may differ from that for cylinders. Besides, the value of E_c varies for different stress levels and changes with the age of concrete. The second difficulty lies in estimating the effect of creep on deflections. The value of the creep coefficient as well as the duration and magnitude of the applied load cannot always be known in advance. However, for practical purposes, an accuracy of 10% or 20% is often sufficient, and that can be attained if all factors are carefully considered.

Deflections of prestressed beams differ from those of ordinary reinforced beams in the effect of prestress. While controlled deflections due to prestress can be advantageously utilized to produce desired cambers and to offset deflections due to loadings, there are also known cases where deflections due to prestress have caused serious troubles. Deflections due to prestress can be computed by two methods. The first method is to take the concrete as a free body, separated from the tendons, which are replaced by a system of forces acting on the concrete, Fig. 8-1-1. This would necessitate the computation of proper components of forces at the end anchorages plus transverse or radial forces at every bend of the tendons.

This method is applicable to both simple and continuous beams. For the sake of simplicity, the following assumptions are usually made:

1. The gross section of concrete can often be used in computing the moment of inertia, although the net concrete section would be a more correct value.

2. The prestress producing deflection is somewhere between the initial and the final effective value. It is considered sufficiently accurate to assume a reasonable value for the purpose of computation.

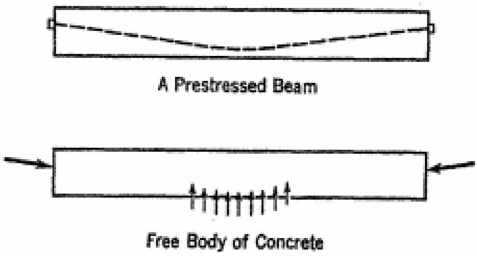


Fig. 8-1-1. Computation for deflection due to prestress.

3. The component of the prestress along the beam axis is assumed constant unless the inclination of the tendons becomes excessive. The component transverse to the beam is computed by the prestress times the tangent of the angle of bending unless the angle becomes unusually large.

4. Where the tendons bend suddenly, the transverse components may be assumed to be concentrated; where they form a flat curve, the transverse load may be assumed to be uniformly distributed along the bend.

5. All computations may be based on the c.g.s. line, the tendons being treated as a whole instead of individually.

6. Shearing deflections are small for ordinary proportions of prestressed beams and can be neglected.

The second method of computation is based on the same assumptions as above, but, without calculating the forces from the tendons on the concrete, a moment diagram produced by the tendons is directly drawn from the c.g.s. profile. For statically determinate beams the moment diagram is similar to the eccentricity profile of the c.g.s. line; hence it is only necessary to plot the eccentricity profile to another scale to obtain the moment diagram. Then the computation of deflections from the moment diagram is performed by any method given in elementary strength of materials. This procedure

is often simpler than the first, since it does away with the computation of forces from the tendons. But when applied to statically indeterminate beams it has to be modified because of moments produced by the redundant reactions as a result of prestressing, which will be explained in Chapter 10.

Acting simultaneously with the prestress is the weight of the beam itself, which will produce deflections depending on the conditions of support. Such deflections can again be computed by the usual elastic theory. The resultant deflections of the beam at transfer are obtained by summing algebraically the deflections due to prestress and those due to beam weight.

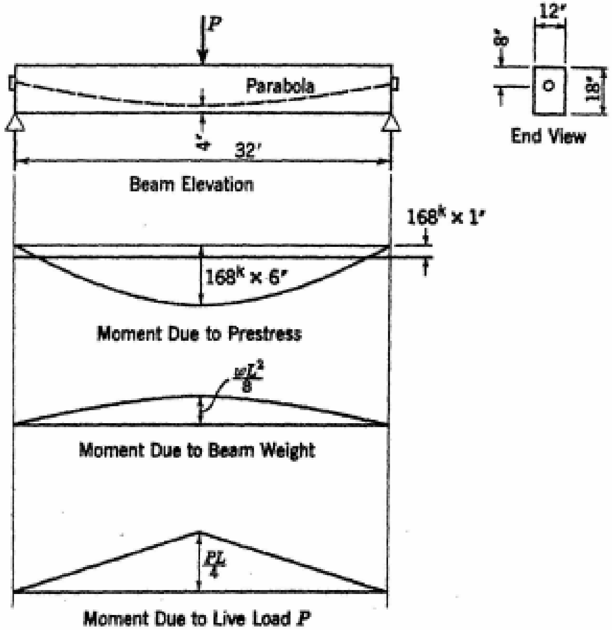


Fig. 8-1-2. Examples 8-1-1 and 8-1-2.

Example 8-1-1

A concrete beam of 32-ft simple span, Fig. 8-1-2, is post-tensioned with 1.2 sq in. of high-tensile steel to an initial prestress of 140 ksi immediately after prestressing. Compute the initial deflection at midspan due to prestress and the beam's own weight, assuming $E_c = 4{,}000{,}000$ psi. Estimate the deflection after 3 months, assuming a creep coefficient of $C_c = 1.8$ and an effective prestress of 120 ksi at that time.

Solution. Using the first method, take the concrete as a free body and replace

the tendon with forces acting on the concrete. The parabolic tendon with 6-in. midordinate is replaced by a uniform load acting along the beam with intensity

$$w = \frac{8F6}{L^2} = \frac{8 \times 140{,}000 \times 1.2 \times 6}{32^2 \times 12} = 655 \text{ plf}$$

In addition, there will be two eccentric loads acting at the ends of the beam, each producing a moment of $140{,}000 \times 1.2 \times 1/12 = 14{,}000$ ft-lb.

Since the weight of the beam is 225 plf, the net uniform load on concrete is $655 - 225 = 430$ plf, which produces an upward deflection at midspan given by the usual deflection formula

$$\begin{aligned}\Delta &= \frac{5wL^4}{384EI} \\ &= \frac{5 \times 430 \times 32^4 \times 12^3}{384 \times 4{,}000{,}000 \times (12 \times 18^3)/12} \\ &= 0.434 \text{ in.}\end{aligned}$$

The end moments produce a downward deflection given by the formula

$$\begin{aligned}\Delta &= \frac{ML^2}{8EI} \\ &= \frac{140 \times 1.2 \times 1 \times 32^2 \times 12^2}{8 \times 4{,}000{,}000 \times (12 \times 18^3)/12} \\ &= 0.133 \text{ in.}\end{aligned}$$

Thus the net deflection due to prestress and beam weight is

$$0.434 - 0.133 = 0.301 \text{ in. upward}$$

If we follow the second method, it will not be necessary to compute the forces between the tendon and the concrete. Instead, the moment diagram is drawn from the eccentricity curve of the tendon, and the deflection computed therefrom. For convenience in computation, the moment diagram can be divided into two parts, a parabola and a rectangle (Fig. 8-1-2). By area-moment principles or any similar method, the upward deflection due to prestress can be computed to be

$$\begin{aligned}\Delta &= \frac{5F6L^2}{48EI} - \frac{ML^2}{8EI} \\ &= \frac{5 \times 140 \times 1.2 \times 6 \times 32^2 \times 12^2}{48 \times 4{,}000{,}000 \times (12 \times 18^3)/12} - \frac{140 \times 1.2 \times 1 \times 32^2 \times 12^2}{8 \times 4{,}000{,}000 \times (12 \times 18^3)/12} \\ &= 0.661 - 0.133 \\ &= 0.528 \text{ in.}\end{aligned}$$

Downward deflection due to beam weight of 225 plf is given by

$$\Delta = \frac{5WL^4}{384EI} = \frac{5 \times 225 \times 32^4 \times 12^3}{384 \times 4{,}000{,}000 \times (12 \times 18^3)/12} = 0.227 \text{ in.}$$

The resultant deflection is $0.528 - 0.227 = 0.301$ in. upward, the same answer as by the first method.

While the above gives the initial deflection, the eventual deflection should be modified by two factors: first, the loss of prestress, which tends to decrease the deflection; and second the creep effect, which tends to increase the deflection. Since the prestress is reduced from 140 to 120 ksi, the deflection due to prestress can be modified by the factor 120/140. Then, for the creep effect, the net deflection should be increased by the coefficient 1.8. Thus, if the beam is not subject to external loads the eventual deflection after 3 months can be estimated as

$$\left(0.528 \times \frac{120}{140} - 0.227\right) 1.8 = 0.407 \text{ in. upward}$$

The calculation for deflections due to external loads is similar to that for non-prestressed beams. So long as the concrete has not cracked, the beam can be treated as a homogeneous body and the usual elastic theory applied to it for deflection computations.

If the beam is bonded at the time of application of the load, the transformed section including steel should be used in computing the moment of inertia. If it is unbonded, to be theoretically correct, the net section of the concrete should be used and the effect of the change in prestress in the tendons under loading should be taken into account. For practical purposes, however, it will be close enough to consider the gross section of concrete in the computations and to neglect the change in prestress. This will simplify the procedure a great deal and will yield practically the same results. It must always be remembered that the greatest difficulties in arriving at correct deflections are the proper choice of a value for E_c and an accurate allowance for creep effect.

When the beam is loaded beyond its working load (or near its working load, for some cases), tensile stresses will exist in the beam. So long as the beam has not cracked, the elastic theory can still be applied for the computation of deflections. Although the tensile modulus of elasticity may be different from the compressive, the difference is not significant enough to alter the nature of deflection, since, at that stage, tension exists only in a small portion of the beam.

When cracks begin to occur in the beam, the nature of deflection will start to change. Even at the beginning of cracks, when they are still hair cracks hardly visible to the unaided eye, the effective section in resisting moment will be the cracked section instead of the entire concrete section. As the cracks extend deeper and deeper, the moment of inertia of the section will become smaller and smaller until eventually the cracked section may have a moment of inertia about one-half or one-third that of the uncracked section. Besides, the concrete will be under higher average stress and therefore will possess

a lower average value of E_c. Hence the deflection of the section will increase much faster than before cracking. It must be noticed, however, that only the part of the beam subjected to higher moment has cracked, while the remaining portion under lower moments may still remain intact. Thus the deflection of the beam will increase faster as more cracks develop. This is shown graphically in Fig. 8-1-3.

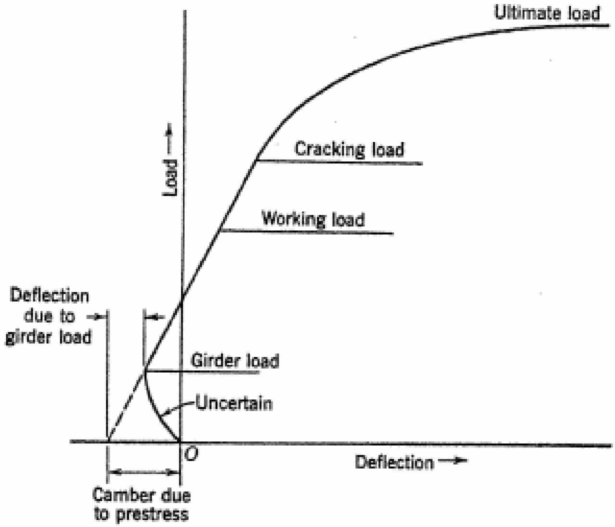


Fig. 8-1-3. Load deflection curve of a prestressed beam.

Upon the removal of the applied load, the beam will return to its original position even though cracks have already developed, provided that the prestress in the steel has not suffered any losses due to the overload. There will, in general, be some residual deflection left in the beam, depending upon the degree and duration of loading. Such residual deflection is often attributed to the plasticity of concrete and can amount to a few per cent of the total deflection upon the first application of the load, but will be hardly noticeable for the second and third similar applications. If the loading is sustained for some time, residual deflections will be produced as a result of creep but can be recovered in the course of time.

When cracks have developed to an appreciable degree, portions of the steel near and across the cracks may be stressed beyond the elastic or creep limit. In such cases, there will be loss of prestress upon the removal of the load. The amount of loss naturally depends

upon the degree of overload; if the amount of permanent deformation equals or exceeds the prestressed strain, the prestress can be entirely lost. Then, upon reloading, that section of the beam will behave like an unprestressed one reinforced with high-tensile steel. Cracks will appear much earlier, even though the ultimate load of rupture may not be decreased.

The above description of the deflections of beams applies to both bonded and unbonded beams. It is believed that unbonded beams are almost as strong as the bonded ones in so far as the elastic limit of steel is concerned. Bonded beams, however, can carry higher load before the eventual crushing of concrete.

After concrete has cracked, the cracks will reappear as soon as tensile stresses again exist in that portion. The tensile stress does not have to approach the modulus of rupture for the cracks to reappear. Hence, between the working load and the cracking load, the beam will deflect slightly more after it has been previously cracked.

Example 8-1-2

For the beam in example 8-1-1, compute the center deflection due to a 10-kip concentrated load applied at midspan, when the beam is 3 months old after prestressing.

Solution. If the beam is bonded, the moment of inertia for the section should be computed on the basis of the transformed section including steel, but it can be approximated by using the gross concrete section. Also note that the modulus of elasticity E_c may be greater at the time of application of load than at transfer, but will be assumed to be 4,000,000 psi for simplicity. Using the usual formula for deflection, we have

$$\Delta = \frac{PL^3}{48EI}$$

$$= \frac{10{,}000 \times 32^3 \times 12^3}{48 \times 4{,}000{,}000 \times (12 \times 18^3)/12}$$

$$= 0.505 \text{ in.}$$

which is the instantaneous downward deflection due to a load of 10 kips. Since the deflection before the application of load was 0.407 in. upward, the resultant deflection is 0.505 − 0.407 = 0.098 in. downward. If the load is kept on for a time, the creep effect due to that load must be considered. Also, if the load is heavy enough to produce cracking, then the elastic theory for computing deflection can be used only as guidance for an approximation.

8-2 Simple Beam Layout

The layout of a simple prestressed-concrete beam is controlled by two critical sections: the maximum moment and the end sections.

After these sections are designed, intermediate ones can often be determined by inspection but should be separately investigated when necessary. The maximum moment section is controlled by two loading stages, the initial stage at transfer with minimum moment M_G acting on the beam and the working-load stage with maximum design moment M_T. The end sections are controlled by the area required for shear resistance, bearing plates, anchorage spacings, and jacking clearances. All intermediate sections are designed by one or more of the above requirements, depending upon their respective distances from the above controlling sections. A common arrangement is to employ some shape, such as I or T, for the maximum moment section and to round it out into a simple rectangular shape near the ends. The design for individual sections having been explained in Chapters 5, 6, and 7, the general longitudinal layout of simple beams will now be discussed.

The layout of a beam can be adjusted by varying both the concrete and the steel. The section of concrete can be varied as to its height, width, shape, and the curvature of its soffit or extrados. The steel can be varied occasionally in its area but mostly in its position relative to the centroidal axis of concrete. By adjusting these variables, many combinations of layout are possible to suit different loading conditions. This is quite different from the design of reinforced-concrete beams, where the usual layout is either a uniform rectangular section or a uniform T-section and the position of steel is always as near the bottom fibers as is possible.

Consider first the pre-tensioned beams, Fig. 8-2-1. Here straight cables are preferred, since they can be more easily tensioned between two abutments. Let us start with a straight cable in a straight beam of uniform section, (*a*). This is simple as far as form and workmanship are concerned. But such a section cannot often be economically designed, because of the conflicting requirements of the midspan and end sections. At the maximum moment section generally occurring at midspan, it is best to place the cable as near the bottom as possible in order to provide the maximum lever arm for the internal resisting moment. When the M_G at midspan is appreciable, it is possible to place the c.g.s. much below the kern without producing tension in the top fibers at transfer. The end section, however, presents an entirely different set of requirements. Since there is no external moment at the end, it is best to arrange the tendons so that the c.g.s. will coincide with the c.g.c. at the end section, so as to obtain a uniform stress distribution. In any case, it is necessary to place the c.g.s. within the kern if tensile stresses are not permitted at the ends.

It is not possible to meet the conflicting requirements of both the midspan and the end sections by a layout such as (*a*). For example, if the c.g.s. is located all along the lower kern point, which is the lowest point permitted by the end section, a satisfactory lever arm is not yet attained for the internal resisting moment at midspan. If the c.g.s. is located below the kern, a bigger lever arm is obtained for resisting the moment at midspan, but stress distribution will be more unfavorable at the ends. Besides, too much camber may result from such a layout, since the entire length of the beam is subjected to negative bending due to prestress. In spite of these objections, this simple arrangement is often used, especially for short spans.

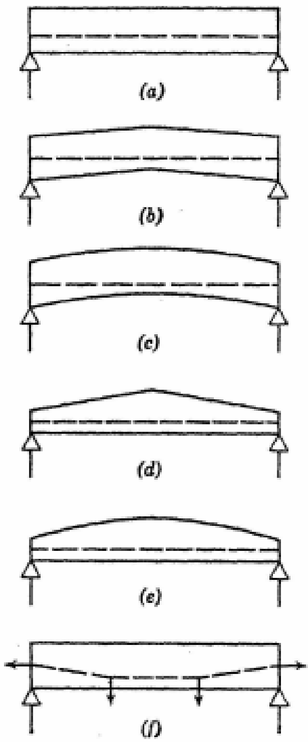


Fig. 8-2-1. Layouts for pre-tensioned beams.

For a uniform concrete section and a straight cable, it is possible to get a more desirable layout than (*a*) by simply varying the soffit of the beam, as in Fig. 8-2-1 (*b*) and (*c*); (*b*) has a bent soffit, while (*c*) has a curved one. For both layouts, the c.g.s. at midspan can be depressed as low as desired, while that at the ends can be kept near the c.g.c. If the soffit can be varied at will, it is possible to obtain a curvature that will best fit the given loading condition, e.g., a parabolic soffit will suit a uniform loading. While these two layouts are efficient in resisting moment and favorable in stress distribution, they possess two disadvantages. First, the formwork is more complicated than in (*a*). Second, the curved or bent soffit is often impractical in a structure, for architectural or functional reasons.

When it is possible to vary the extrados of concrete, a layout like Fig. 8-2-1 (*d*) or (*e*) can be advantageously employed. These will give a favorable height at midspan, where it is most needed, and yet yield a concentric or nearly concentric prestress at end sections. Since the depth is reduced for the end sections, they must be checked for shear resistance. For (*d*), it should also be noted that the critical section may not be at midspan but rather at some point away from it

where the depth has decreased appreciably while the external moment is still near the maximum. Beam (*d*), however, is simpler in formwork than (*e*), which has a curved extrados.

Modern pre-tensioning plants have buried anchors along the stressing beds so that the tendons for a pre-tensioned beam can be bent, Fig. 8-2-1 (*f*). It may be economical to do so, if the beam has to be of straight and uniform section, and if the M_G is heavy enough to warrant such additional expense of bending. Means must be provided to reduce the frictional loss of prestress produced by the bending of the tendons.

It is evident from the above discussion that many different layouts are possible with prestressed beams. Only some basic forms are described here, the variations and combinations being left to the discretion of the designer. The correct layout for each structure will depend upon the local conditions and the practical requirements as well as upon theoretical considerations.

Most of the layouts for pre-tensioned beams can be used for post-tensioned ones as well. But, for post-tensioned ones, Fig. 8-2-2, it is not necessary to keep the tendons straight, since slightly bent or curved tendons can be as easily tensioned as straight ones. Thus, for a beam of straight and uniform section, the tendons are very often curved as in Fig. 8-2-2 (*a*). Curving the tendons will permit favorable positions of c.g.s. to be obtained at both the end and midspan sections, and other points as well.

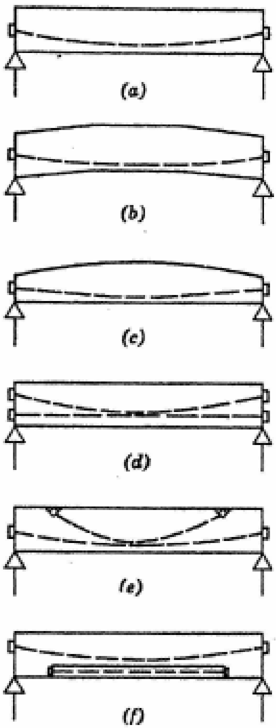


Fig. 8-2-2. Layouts for post-tensioned beams.

A combination of curved or bent tendons with curved or bent soffits is frequently used, Fig. 8-2-2 (*b*), when straight soffits are not required. This will permit a smaller curvature in the tendons, thus reducing the friction. Curved or bent cables are also combined with beams of variable depth, as in (*c*). Combinations of straight and curved tendons are sometimes found convenient, as in (*d*).

Variable steel area along the length of a beam is occasionally preferred. This calls for special design of the beam and involves details which may offset its economy in weight of steel. In Fig. 8-2-2 (*e*),

some cables are bent upward and anchored at top flanges. In (*f*), some cables are stopped part way in the bottom flange. These arrangements will save some steel but may not be justified unless the saving is considerable as for very long spans carrying heavy loads.

8-3 Cable Profiles

It is stated in the previous section that the layout of simple beams is controlled by the maximum moment and end sections so that, after these two sections are designed, other sections can often be determined by inspection. It sometimes happens, however, that intermediate points along the beam may also be critical, and in many instances it would be desirable to determine the permissible and desirable profile for the tendons. To do this, a limiting zone for the location of c.g.s. is first obtained, then the tendons are arranged so that their centroid will lie within that zone.

The method described here is intended for simple beams, but it also serves as an introduction to the solution of more complicated layouts, such as cantilever and continuous spans, where cable location cannot be easily determined by inspection. The method is a graphical one, giving the limiting zone within which the c.g.s. must pass in order that no tensile stresses will be produced. Compressive stresses in concrete are not checked by this method. It is assumed that the layout of the concrete sections and the area of prestressing steel have already been determined. Only the profile of the c.g.s. is to be located.

Referring to Fig. 8-3-1, having determined the layout of concrete sections, we proceed to compute their kern points, thus yielding two kern lines, one top and one bottom, (*c*). Note that for variable sections, these kern lines would be curved, although for convenience they are shown straight in the figure.

For a beam loaded as shown in (*a*), the minimum and maximum moment diagrams for the girder load and for the total working load respectively are marked as M_G and M_T in (*b*). In order that, under the working load, the center of pressure, the C-line, will not fall above the top kern line, it is evident that the c.g.s. must be located below the top kern at least a distance

$$a_1 = M_T/F \qquad (8\text{-}3\text{-}1)$$

If the c.g.s. falls above that upper limit at any point, then the C-line corresponding to moment M_T and prestress F will fall above the top kern, resulting in tension in the bottom fiber.

Similarly, in order that the C-line will not fall below the bottom

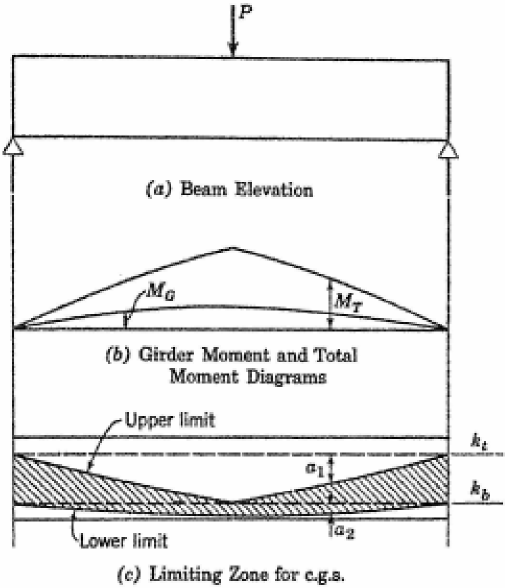


Fig. 8-3-1. Location of limiting zone for c.g.s.

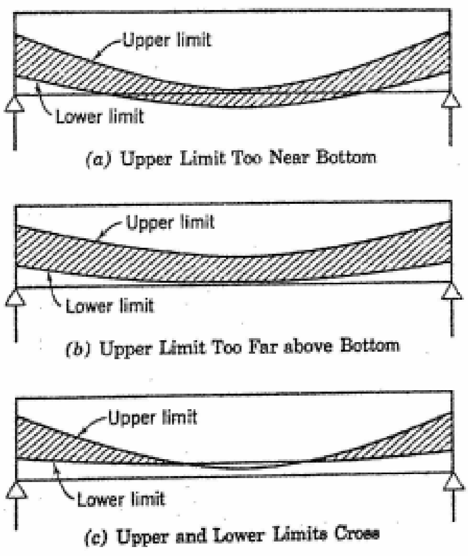


Fig. 8-3-2. Undesirable positions for c.g.s. zone limits.

kern line, the c.g.s. line must not be positioned below the bottom kern by a distance greater than

$$a_2 = M_G/F_0 \tag{8-3-2}$$

which gives the lower limit for the location of c.g.s. If the c.g.s. is positioned above that lower limit, it is seen that the C-line will be above the bottom kern and there will be no tension in the top fiber under the girder load and initial prestress F_0.

Thus, it becomes clear that the limiting zone for c.g.s. is given by the shaded area in Fig. 8-3-1 (c), in order that no tension will exist both under the girder load and under the working load. The individual tendons, however, may be placed in any position so long as the c.g.s. of all the cables remains within the limiting zone.

The position and width of the limiting zone are often an indication of the adequacy and economy of design, Fig. 8-3-2. If some portion of the upper limit falls outside or too near the bottom fiber, in (a), either the prestress F or the depth of beam at that portion should be increased. On the other hand, if it falls too far above the bottom fiber, in (b), either the prestress or the beam depth can be reduced. If the lower limit crosses the upper limit, in (c), it means that no zone is available for the location of c.g.s., and either the prestress F or the beam depth must be increased or the girder moment must be increased to depress the lower limit if that can be done.

The application of the above graphical method is illustrated in example 8-3-1.

Example 8-3-1

Preliminary design for a 50-ft pre-tensioned beam gives a layout with tapered top flange and symmetrical I-sections as shown in Fig. 8-3-3. Ten steel wires of ⅜-in. diameter with anchorages are used for prestressing. $f_0 = 130$ ksi, $f_s = 110$ ksi, $f_c' = 5000$ psi. Determine the position for the c.g.s. line. Live and superimposed dead load on the beam totals 450 plf, in addition to the weight of the beam itself. $M_G = 58$ k ft and $M_T = 194$ k ft at midspan.

Solution. To get an accurate graphical solution, 4 or 5 points should be calculated for half of the span, but only calculations for the midspan section will be shown here. Note that the sections near the end are of rectangular shape; hence there is a sudden jump in the kern lines at the junction. Also, theoretically, there is no external moment for the portions directly over the supports.

First, locate the kern lines. Values for the midspan section are as follows:

I of section $= 28{,}200$ in.^{4}

A of section $= 204$ in.^{2}

r^2 of section $= 140$ in.^{2}

k_t and k_b $= 140/17 = 8.2$ in.

With $F = 10 \times 0.11 \times 110 = 121$ kips, minimum resisting arm required for M_T is, from equation 8-3-1,

$$M_T/F = (194 \times 12)/121 = 19.2 \text{ in.}$$

which is measured down from the top kern line and located as shown.

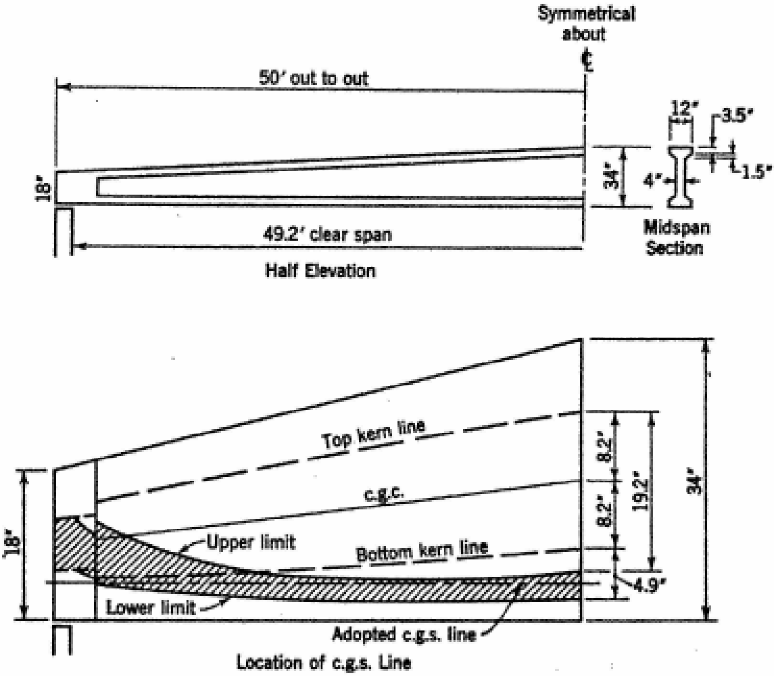


Fig. 8-3-3. Example 8-3-1.

For initial prestress $F_0 = 10 \times 0.11 \times 130 = 143$ kips, the lever arm corresponding to $M_G = 58$ k ft is, from equation 8-3-2,

$$M_G/F_0 = (58 \times 12)/143 = 4.9 \text{ in.}$$

These limiting points are calculated for several other sections, and the limiting zone is indicated by the shaded area. Straight tendons are preferred for pretensioning, while it is impossible to get a straight c.g.s. line within this shaded area. The best recourse in this design is perhaps to permit some tension near the supports and to reinforce the ends with some mild steel. Then it is possible to adopt a c.g.s. line as shown which will result in no tension in the bottom fibers under working loads but will have some tension in the top fibers near the supports. If such tension is to be avoided, it will be necessary to use a greater prestressing force, thus raising the upper limit of the zone and enabling a c.g.s. line to be located at about 6 in. from the bottom. Deflection of this beam at transfer should be computed to see whether the camber is excessive, but it will not be illustrated in this example.

The location of the c.g.s. line as described above is based on the elastic theory, allowing no tensile stress both at transfer and under the working loads. If some tension is permitted, then it is possible to place the c.g.s. line slightly outside the previous limiting zone. Referring to Fig. 8-3-4, for an allowable tensile stress of f_t' in the top fibers at transfer, we have

$$
\begin{aligned}
f_t' &= \frac{Mc_t}{I} \\
&= \frac{F_0 e_b c_t}{I} \qquad (8\text{-}3\text{-}3)
\end{aligned}
$$

where e_b = the amount c.g.s. may fall below the lower limit. For an allowable tensile stress of f_b' in the bottom fibers under the working load, we have

$$f_b' = \frac{Fe_t c_b}{I} \qquad (8\text{-}3\text{-}4)$$

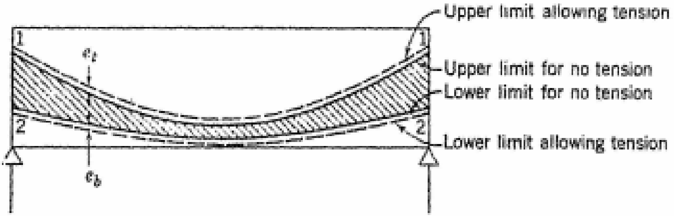


Fig. 8-3-4. Limiting zone for c.g.s. allowing tension in concrete.

where e_t = the amount c.g.s. may rise above the upper limit. From equations 8-3-3 and 8-3-4, we can write

$$e_b = \frac{f_t' I}{F_0 c_t} = \frac{f_t' A k_b}{F_0} \qquad (8\text{-}3\text{-}5)$$

and

$$e_t = \frac{f_b' I}{F c_b} = \frac{f_b' A k_t}{F} \qquad (8\text{-}3\text{-}6)$$

Hence, the limiting zones for no tensile stresses can be extended to lines 1-1 and 2-2 if some tensile stresses are permitted, Fig. 8-3-4.

The above graphical method can also be applied when there are changes in the cross-sectional area of steel. It is only necessary to use the corresponding value of prestress existing at the particular point when computing the position of the limiting zone. Thus, at points of change in the steel area, there will be sudden jumps in the

limiting lines. If the prestress is applied in two stages, two lower limits should be computed, each based on its own prestressing force. However, if too many complications are involved, the graphical method may not be efficient.

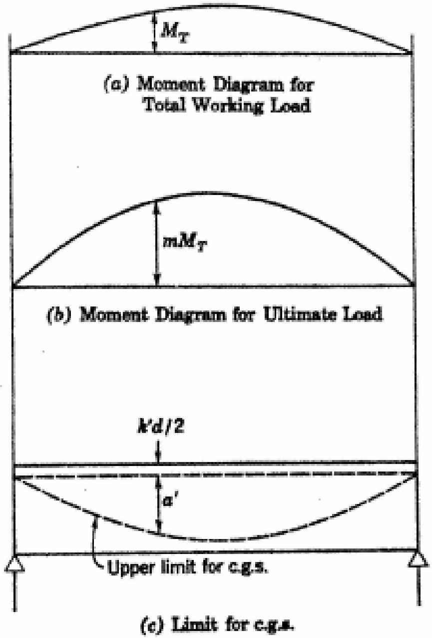


Fig. 8-3-5. Location of c.g.s. by ultimate design.

If ultimate-strength design is to be used, the c.g.s. line can also be located by a graphical method, Fig. 8-3-5. But, since ultimate design applies only to the maximum loading stage, the lower limit for the c.g.s. still has to be determined by the elastic theory or some other method. The upper limit, however, can be obtained by the ultimate theory as follows. If M_T is the total moment, and m the load factor, then the ultimate moment is mM_T, which is to be resisted by the ultimate strength of the steel (in the case of bonded reinforcement) with a level arm,

$$a' = \frac{mM_T}{A_s f_s'}$$

The line of pressure at ultimate load is located at $k'd/2$ below top

fiber, where $k'd$ is obtained by

$$k'd = \frac{A_s f_s'}{k_1 f_c' b}$$

if a uniform width b is obtained for the top flange at the ultimate load.

8-4 Cantilever Beam Layout

Because of the balancing and reduction of moments, cantilever beams can be economically utilized in prestressed-concrete structures, especially for certain favorable span ratios and for long and heavy beams. The basic theories and methods for the design of cantilever beams are the same as those for simple beams. But the work of designing is more complicated, because of several factors which must be more carefully considered. These are:

1. Certain portions of a cantilever are subjected to both positive and negative moments, depending on the position of live loads.

2. To obtain most severe loading conditions, partial loading of the spans must sometimes be considered.

3. In a cantilever, moments produced by loads on a certain portion are often counterbalanced by loads on other portions. Hence the moments are sensitive to changes in external load. Because of this, the sequence of the application of superimposed loads on the beam must be carefully considered and executed.

4. If the beam is precast, care must be exercised during erection and transportation of the beam. At all times the supporting conditions assumed in design must be realized for the beam. Even slight changes in the position of supports may affect the moments seriously.

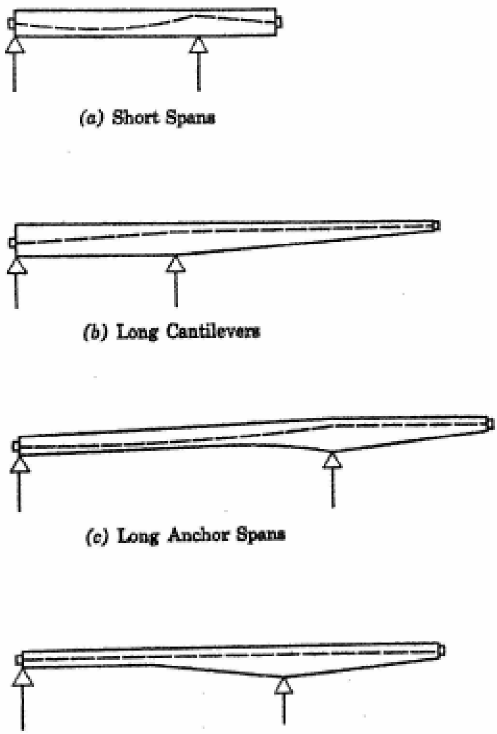
5. Cantilever beams are more sensitive to temperature changes which might result in excessive deflections.

6. The ultimate capacity of cantilever beams may be relatively low if heavy partial loading is a possibility. The coexistence of high moment and shear at certain critical sections may also tend to reduce the ultimate strength in a cantilever.

In spite of the complications, cantilever beams are often used, because of their economy and their adaptability to certain structures. In fact, the above-mentioned complications should not be held against the use of cantilever beams. They only indicate that greater care must be exercised in their design and construction.

Two general layouts are possible for cantilevers: the single and the double cantilevers. Some typical layouts for the single cantilevers are shown in Fig. 8-4-1. (*a*) shows the layout for a short

span with a short cantilever, where a straight and uniform section may be the most economical. In such a design, it is only necessary to vary the c.g.s. profile so that it will conform with the requirements of the moment diagrams. When the cantilever span becomes longer,



(a) Short Spans

(b) Long Cantilevers

(c) Long Anchor Spans

(d) Straight Tendons

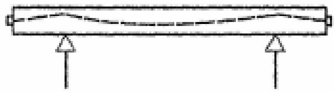
Fig. 8-4-1. Typical layouts for single cantilevers.

it is advisable to taper the beam as in (b). If the anchor span is short compared to the cantilever, it may be entirely subjected to negative moments, and the c.g.s. may have to be located above the c.g.c. at all points.

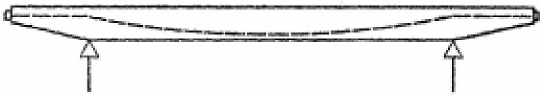
For longer anchor spans, it may be desirable to haunch them as in (c) and (d). Then the c.g.s. profile can be properly curved as in (c) or may remain practically straight as in (d) where conditions permit.

For short double cantilevers, a straight and uniform section can be adopted as shown in Fig. 8-4-2 (a). When the cantilevers are

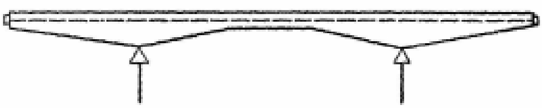
long, they may be tapered as in (*b*). If the anchor span is long, it may be haunched as in (*c*). If the anchor span is short compared with the cantilevers, the c.g.s. line may lie near the top of the beam at all points, as in (*d*).



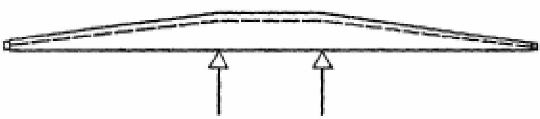
(*a*) Short Spans



(*b*) Tapered Cantilevers



(*c*) Straight Tendons



(*d*) Long Cantilevers

Fig. 8-4-2. Typical layouts for double cantilevers

Cable location for cantilevers can be obtained graphically as for simple beams, except that more thought should be given to the possibilities of partial live loads and the reversal of moments. Figure 8-4-3 (*a*) shows a cantilever beam. Assume that the beam is under the action of its own weight and the action of uniform live load on any portion. Moment due to dead weight of the beam is pictured in (*b*). Moment due to live load on the anchor span is shown in (*c*); that due to live load on the cantilever is shown in (*d*). For convenience in discussion, maximum moment will signify the greatest positive or the smallest negative moment, while minimum moment

will mean the smallest positive or the greatest negative moments. So, for this beam, the maximum moment will be given by $(b) + (c)$, and the minimum moments by $(b) + (d)$. Both are plotted in (e).

In order to obtain the limiting zone for the c.g.s. line, first plot the top and bottom kern lines for the beam, k_t and k_b lines in (f). If no

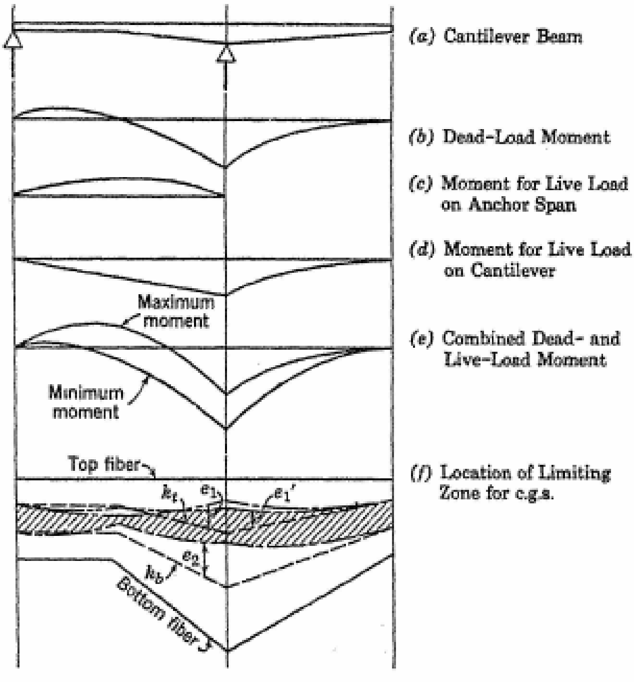


Fig. 8-4-3. Graphical method for locating c.g.s.

tension is permitted in the concrete, one limiting line is obtained by plotting from each kern line the permissible eccentricity e, with

$$e = M/F$$

Note that e may be plotted from either the k_t or the k_b line, whichever gives the more critical limit. But e due to $+M$ is always plotted downward, since it tends to shift the required c.g.s. line downward. By similar reasoning, e due to $-M$ is always plotted upward. In general the upper limit for the zone is plotted from the k_t line with a distance

$$e_1 = M_{max}/F$$

The lower limit for the zone is plotted from the k_b line with a distance

$$e_2 = M_{\min}/F$$

Consideration should also be given to the action of dead load alone, since in this case we may have the initial prestress which is greater than the effective prestress and may impose a more critical situation. With the dead load acting alone, another limit is obtained by plotting from the k_t line a distance

$$e_1' = M_G/F_0$$

again plotting the $+M$ downward and the $-M$ upward. In this figure it is not necessary to plot e_1' from the k_b line, because evidently it will not be controlling. When plotted from the k_t line, it is seen that, for certain portions of the beam, e_1' will be controlling rather than e_1. The resulting limiting zone is shaded as in (*f*).

For long cantilevers carrying heavy loads, it is sometimes economical to cut off some of the prestressing wires at intermediate points. The number and location of cut-offs can also be established by a graphical method, which is the reverse of the above procedure and will be illustrated in example 8-4-1.

Example 8-4-1

Compute the variation of steel area required along the 140-ft length of the cantilever roof girder having a layout as shown in Fig. 8-4-4. Given the following data:

1. Concrete: $f_c' = 5000$ psi, allowable $f_c = 2250$ psi for working load, and 2500 psi under initial prestress, allowable tension in concrete = 0 under working load.

2. Steel: $f_s = 240{,}000$ psi, initial prestress = 150,000 psi, final effective prestress = 125,000 psi.

3. Live load and superimposed dead load = 1.60 kips per linear foot of girder, producing moment at support

$$wL^2/2 = 1.60 \times 140^2/2 = 15{,}700 \text{ k ft}$$

4. Moments due to weight of girder: For the trial layout, the girder load moments are computed for various points on the cantilever, with a maximum of 15,200 k ft at the support.

Solution. After some preliminary investigation, it is found that, owing to the relatively heavy girder load moment, there will exist compressive stresses along most of the bottom flange. Hence it is not necessary to check for any tensile stress in the bottom flange except near the cantilevering end. For the given layout, the c.g.s. line is computed and plotted in Fig. 8-4-5. The resisting lever arm a_1 available for the internal resisting couple is measured from the c.g.s. line to the k_b line at each point. Corresponding to a_1, the minimum amount of prestress required is

$$F = M_T/a_1$$

and the steel area required is

$$A_s = F/125 = M_T/125a_1$$

Similarly, the maximum steel permitted without producing tension in the bottom fiber is

$$A_s = M_G/150a_2$$

where a_2 is the distance between the c.g.s. and the k_t lines.

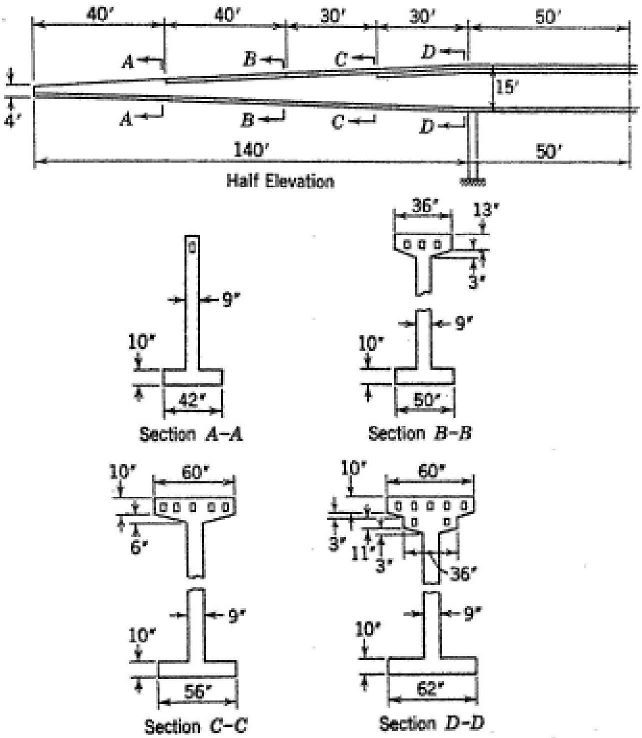


Fig. 8-4-4. Example 8-4-1. Girder sections.

First, the M_T and M_G moment diagrams are drawn. Next the distances a_1 and a_2 between the c.g.s. line and the k_b and k_t lines are measured. Then the minimum and maximum A_s lines are computed by the above formulas. Several points may be necessary for an accurate determination of these curves, but only some sample computations will be illustrated here. At the support, the total moment is

$$M_T = 15{,}200 + 15{,}700 = 30{,}900 \text{ k ft}$$

The c.g.s. is located 9 in. from the top, and the k_b is located 42.6 in. above the

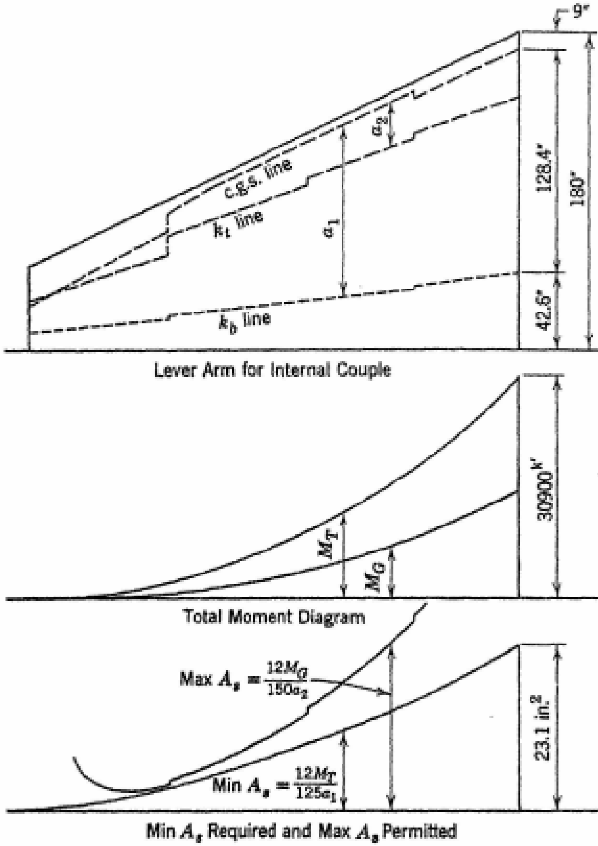


Fig. 8-4-5. Example 8-4-1. Computation for A_s.

bottom fibers; hence the available lever arm a_1 is

$$a_1 = 180 - 42.6 - 9 = 128.4 \text{ in.}$$

The minimum area of steel required at the support is, therefore,

$$\min A_s = \frac{M_T}{125a_1} = \frac{30{,}900 \times 12}{125 \times 128.4} = 23.1 \text{ sq in.}$$

At 35 ft from the cantilevering end,

$$M_G = 600 \text{ k ft}$$

a_2 is measured to be 10.5 in. The maximum steel area permitted at this point is

$$\max A_s = \frac{M_G}{150a_2} = \frac{600 \times 12}{150 \times 10.5} = 4.57 \text{ sq in.}$$

Similar computations are made for other points, and the curves are drawn as

shown. It will be seen that the maximum A_s curve is actually required only for a short portion of the beam near the cantilevering end. Keeping as close as possible to the minimum A_s curve, but without crossing the maximum A_s curve, the adopted steel area may be tailored and cut off as desired.

The checking of compressive stresses in concrete and other design features will not be discussed here. Note that all moments are obviously negative in this solution, hence no particular attention has been paid to the signs of the moments.

One advantage of such a graphical solution is that it gives a visual presentation. The variations of the lever arm a_1 and a_2 and of the A_s curves both follow certain simple laws so that necessary modifications to suit changes in design can be easily made, either by shifting the c.g.s. location or by varying the steel areas.

8-5 Design of a Post-Tensioned Bridge Girder

The following example illustrates the complete design of a precast post-tensioned girder for a highway bridge.

Example 8-5-1

Precast girders of a highway bridge are to be post-tensioned, grouted, then lifted to the bridge site to be connected together by concrete poured in place. The two-lane bridge is to carry H20-S16-44 loading, and the girders are spaced 6 ft on centers. Overall length of girder is 96 ft, with 95 ft between centers of supports. Following the 1953 AASHO Specifications for Highway Bridges and the 1954 Criteria for Design of Prestressed Concrete Bridges by the Bureau of Public Roads (Appendix D), design an interior girder as follows:

(*a*) Design the midspan section, indicating the required amount of prestressing steel.

(*b*) Design the end section, showing the mild-steel stirrups.

(*c*) Design a longitudinal layout of the girder showing the profile for c.g.s. and the intermediate and end diaphragms.

(*d*) Investigate the factor of safety of the girder at cracking and ultimate strengths.

(*e*) Compute the deflection of the girder at transfer and under the working load.

(*f*) Detail the midspan and the end sections using the Freyssinet system. Compute the loss of prestress and the initial prestress required at the jack.

Strength of concrete is to be 4500 psi at 28 days and 4000 psi at transfer. According to Appendix D, the allowable stresses for concrete at transfer are:

$$\text{Compression in extreme fiber} = 0.55f_{ci}' = 0.55 \times 4000 = 2200 \text{ psi}$$

$$\text{Tension in extreme fiber} = 0.05f_{ci}' = 0.05 \times 4000 = 200 \text{ psi}$$

and the allowable stresses under the working load are:

$$\text{Compression in extreme fiber} = 0.4f_c' = 0.4 \times 4500 = 1800 \text{ psi}$$

$$\text{Tension in extreme fiber} = 0$$

The high-tensile steel used is to have a minimum ultimate tensile strength of 240,000 psi and a minimum yield point of 200,000 psi at 0.2% plastic set. The steel stress at transfer will be 160,000 psi, and the effective prestress at 85% of 160,000 psi = 136.000 psi [these values will be checked in part (*f*) of this problem]. $E_s = 28{,}000{,}000$ psi. $E_c = 4{,}000{,}000$ psi. Use intermediate-grade reinforcing bars for the mild-steel reinforcement.

Solution. (*a*) From Appendix A of the AASHO Specifications for Highway Bridges, the maximum moment for one lane of H20-S16-44 loading on a span of 95 ft is found to be 1433 k ft. The proportion of lane load carried by an interior stringer can be approximated by the AASHO formula $S/10$ for interior stringers (Art. 3.3.1 of AASHO Specifications). Note that this formula is considered conservative for spans as long as 95 ft. Since spacing $S = 6$ ft, the moment for each girder is

$$LLM = 1433 \times 6/10 = 860 \text{ k ft}$$

Impact on highway bridges is given by the formula

$$\begin{aligned} I &= 50/(L + 125) \\ &= 50/(95 + 125) \\ &= 0.227 \end{aligned}$$

$$\text{Impact moment } IM = 860 \times 0.227 = 195 \text{ k ft}$$

$$LLM + IM = 860 + 195 = 1055 \text{ k ft}$$

Note the above value of maximum moment actually does not occur at midspan, but for all practical purposes it can be assumed to occur there.

After some preliminary design and trials, a section is assumed as in Fig. 8-5-1, from which

Bituminous paving 2 in. at 150 pcf = 25 psf × 6 ft	= 150 plf
In place concrete slab and diaphragms	= 133 plf
Total added *DL*	= 283 plf

Added DL moment $= (283 \times 95^2)/8 = 319$ k ft
Weight of girder and diaphragms = 940 plf
Girder load moment $M_G = (940 \times 95^2)/8 = 1060$ k ft
Total moment $M_T = 1055 + 319 + 1060 = 2434$ k ft

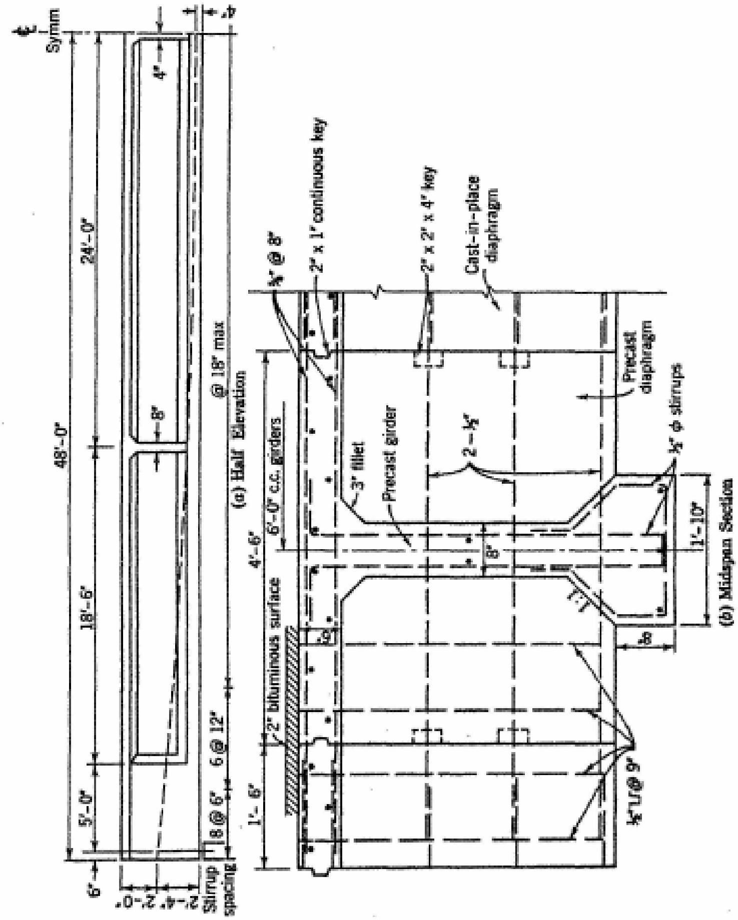


Fig. 8-5-1. Example 8-5-1. Girder layout.

The c.g.c. of the section, using the gross area, is as follows:

$$
\begin{aligned}
6 \times 54 &= 324 \times 3 &&= 972\\
3 \times 3 &= 9 \times 7 &&= 63\\
38 \times 8 &= 304 \times 25 &&= 7600\\
7 \times 7 &= 49 \times 41.67 &&= 2040\\
22 \times 8 &= \underline{176} \times 48 &&= \underline{8450}\\
& 862 && 19{,}125 \div 862 = 22.5 = c_t\\
& && 52 - 22.5 = 29.5 = c_b
\end{aligned}
$$

The moment of inertia of the concrete section about the c.g.c. is

$$
\begin{aligned}
324(6^2/12 + 19.5^2) &= 124{,}000\\
9(3^2/18 + 15.5^2) &= 2{,}200\\
304(38^2/12 + 2.5^2) &= 38{,}600\\
49(7^2/18 + 19.17^2) &= 18{,}000\\
176(8^2/12 + 25.5^2) &= \underline{115{,}000}\\
& 297{,}800 \div 862 = 345 = r^2
\end{aligned}
$$

$$k_t = r^2/c_b = 345/29.5 = 11.7 \text{ in.}$$

$$k_b = r^2/c_t = 345/22.5 = 15.4 \text{ in.}$$

Owing to the relatively large ratio of M_G/M_T, it is evident that the c.g.s. can be located as low as practicable without producing any tension in the top fiber. Assuming that the c.g.s. is located 4 in. above the bottom fiber, the total arm for the internal resisting moment is

$$a = 11.7 + 29.5 - 4 = 37.2 \text{ in.}$$

The total effective prestress required is

$$F = M_T/a = (2434 \times 12)/37.2 = 786 \text{ k}$$

For a loss of prestress at 15%, the initial prestress required will be

$$F_0 = F/0.85 = 786/0.85 = 925 \text{ k}$$

To limit the top fibers to a maximum stress of 1.8 ksi, we must have

$$A_c = \frac{Fh}{f_t c_b} = \frac{786 \times 52}{1.8 \times 29.5} = 770 \text{ sq in.}$$

To limit the bottom fibers to a maximum of 2.2 ksi, we must have,

$$
\begin{aligned}
A_c &= \frac{F_0}{f_b}\left(1 + \frac{e - (M_G/F_0)}{k_t}\right)\\
&= \frac{925}{2.2}\left(1 + \frac{25.5 - 1060 \times 12/925}{11.7}\right)\\
&= 841 \text{ sq in.}
\end{aligned}
$$

The actual gross area furnished is 862 sq in., which seems to be just about sufficient for the required area of 841 sq in. The top fibers will not be stressed to the allowable value, but the width and thickness of the top flange are governed by the slab requirements, which will not be discussed here. Also note that the in-place concrete will further reduce the stress in the top fibers, and may be included in the computation if desired. Hence the section is considered satisfactory and is adopted without further changes. Note that it generally takes two or three trials to arrive at this adopted section rather than just one trial as illustrated here.

To supply the effective prestress of 786 k at an allowable stress of 136 ksi, steel area required will be

$$786/136 = 5.79 \text{ sq in.}$$

(*b*) Shearing stresses can be checked for two sections, one at the support and another 5 ft from the support where the web is 8 in. thick. At the support, the web is 22 in. thick; shear is evidently not controlling. The shear at 5 ft from support is

LL shear, from AASHO Specifications	= 61.3 k/lane
61.3 × 6/10	= 36.8 k/girder
Impact shear = 0.227 × 36.8	= 8.4 k
Bituminous paving and in-place concrete 0.283 × 42.5	= 12.0 k
Girder own wt 0.940 × 42.5	= 40.0 k
Total shear	97.2 k

Shear V_s carried by the tendons at end of span, assuming a parabolic rise of $h = 2$ ft on a length of $L = 96$ ft, is given by

$$\begin{aligned} V_s &= 4Fh/L \\ &= (4 \times 786 \times 2)/96 \\ &= 65.5 \text{ k} \end{aligned}$$

At 5.5 ft from end of girder,

$$\begin{aligned} V_s &= (42.5/48)65.5 \\ &= 58.0 \text{ k} \end{aligned}$$

Hence V_c by concrete is

$$97.2 - 58.0 = 39.2 \text{ k}$$

Maximum shearing stress in concrete occurs at c.g.c. and is given by

$$v = V_cQ/Ib$$

Since Q, the statical moment of the area above the c.g.c. about it, is

$324 \times 19.5 + 9 \times 15.5 + 8 \times 16.5^2/2 = 7550,$

$$v = \frac{39{,}200 \times 7550}{297{,}800 \times 8} = 124 \text{ psi}$$

The compressive fiber stress at c.g.c. is given by F/A_c

$$\begin{aligned} f_c &= F/A_c \\ &= 786{,}000/862 \\ &= 912 \text{ psi} \end{aligned}$$

The principal tensile stress is

$$\begin{aligned} S_t &= \sqrt{v^2 + (f_c/2)^2} - f_c/2 \\ &= \sqrt{124^2 + 456^2} - 456 \\ &= 20 \text{ psi} \end{aligned}$$

The moment is relatively small at this section of maximum shear; hence the fiber stress is nearly uniform throughout the depth of the section, and the maximum principal tensile stress at the c.g.c. represents the greatest tensile stress. This maximum principal tension does not exceed the allowable value of $0.03f_c' = 0.03 \times 4500 = 135$ psi. Hence no stirrups are required under the working load. To investigate the ultimate strength for shear, from Appendix D, we have ultimate shear

$$\begin{aligned} &V' \text{ for } D + 3(L + I) = 52.0 + 3(45.2) &&= 188 \text{ k} \\ &V' \text{ for } 2(D + L + I) = 2 \times 97.2 &&= 194 \text{ k} \end{aligned}$$

Design for $V' = 194$ k, and assuming $V_s' = V_s = 58.0$ k,

$$V_c' = 194 - 58.0 = 136 \text{ k}$$

$$v' = \frac{136{,}000 \times 7550}{297{,}800 \times 8} = 431 \text{ psi}$$

Since f_c remains practically the same as under working load, we have

$$\begin{aligned} S_t' &= \sqrt{431^2 + 460^2} - 460 \\ &= 170 \text{ psi} \end{aligned}$$

This principal tension is much higher than under working load. But the allowable ultimate principal tension is $0.08f_c' = 0.08 \times 4500 =$ 360 psi. Hence theoretically no stirrups are required under the ultimate load. However, Appendix D also recommends that some nominal stirrups be used irrespective of whether computations show that they are needed. The recommended maximum stirrup spacing is to be

not more than three-fourths the depth of the beam, or $52 \times \frac{3}{4} = 39$ in. The sum of the cross-sectional areas of the legs of stirrups should be not less than 0.08% of the cross-sectional area of the beam for the maximum spacing, which means not less than $0.0008 \times 862 = 0.69$ sq in. per 39 in. of beam, or $0.69/(39/12) = 0.21$ sq in. per ft. Use $\frac{1}{2}$-in. U-stirrups at 18 in. spacing, giving an area of $2 \times 0.20/1.5 = 0.27$ sq in. per ft. This is more than enough but, as a matter of good proportioning, will be used throughout the entire beam.

For the end section, stirrups are required to distribute the anchorage stresses. Since the anchorages are to be fairly uniformly distributed, the computed tensile stresses in the anchorage zone will be low and analysis is not required. Nominal stirrups, however, are provided as shown on Fig. 8-5-1.

(*c*) A half elevation of the girder is shown in Fig. 8-5-1. The midspan section is adopted for the entire girder except the 5 ft near the ends where a uniform web thickness equal to the bottom flange width of 22 in. is used in order to accommodate the end anchorages, to permit the curving up of some tendons, and to distribute the prestress. Three intermediate diaphragms are placed along the length of the span. Sometimes transverse prestressing is employed to bind the girders together. But, for this design, transverse dowels are provided in these diaphragms to be joined together by in-place concrete. In either case, the theoretical calculation for the steel in these diaphragms can be a complicated problem. But the amount of steel is not excessive for these girders; some nominal reinforcement is employed as shown.

The most common location of c.g.s. for a simple beam is a parabola with c.g.s. near the c.g.c. at the ends. Such a profile will give ample moment resistance along the entire beam. If the c.g.s. is above the c.g.c. at the ends, the tendons will carry greater shear but lose some of the reserve moment resistance. If the c.g.s. is below the c.g.c. at the ends, the tendons will carry less shear, but the positive prestressing moment at the ends will tend to decrease the principal tension. Also note that the c.g.c.'s for the midspan and the end sections actually differ slightly. For this design, the c.g.s. will be placed a little below the c.g.c. of both the end and the midspan sections.

(*d*) The cracking moment is computed as follows: The resisting moment up to zero stress in the bottom fiber is given by

$$Fa = 786 \times 37.2/12 = 2434 \text{ k ft}$$

Assuming the modulus of rupture at bottom fiber to be $0.14f_c' = 0.14 \times 4500 = 630$ psi, the additional resisting moment from zero

stress to 630 psi is

$$M = fI/c_b = 630 \times 297{,}800/29.5 \times 12{,}000$$
$$= 530 \text{ k ft}$$

Total resisting moment at cracking is

$$2434 + 530 = 2964 \text{ k ft}$$

Overall factor of safety against cracking is

$$2964/2434 = 1.22$$

Factor of safety for live load and impact is

$$\frac{2964 - (1060 + 319)}{1055} = 1.50$$

which indicates that the girder will begin to crack only when the live load plus impact is increased by as much as 50%.

The method for computing ultimate moment given in Appendix D is not applicable to T-sections. Hence the ultimate resisting moment will be computed by the procedure in section 5-7, as follows:

Assuming the ultimate strength of steel to be developed, the ultimate tensile force is

$$f_s'A_s = 250 \times 5.79 = 1450 \text{ k}$$

Assuming an average stress in concrete of $0.85f_c' = 0.85 \times 4500 = 3.82$ ksi, the total concrete area under compression is

$$1450/3.82 = 380 \text{ sq in.}$$

Neglecting the in-place concrete, the ultimate neutral axis can be located at about 9 in. below the top fiber. Thus the distance from the ultimate neutral axis to the c.g.s. is about 39 in., indicating that, at rupture of the girder, the steel is stressed to very near its ultimate strength. The centroid of the 380 sq in. of concrete is located at

$$(324 \times 3 + 9 \times 7 + 47 \times 8.9)/380 = 3.8 \text{ in.}$$

below top fiber, and the ultimate lever arm is

$$52 - 4 - 3.8 = 44.2 \text{ in.}$$

Hence the ultimate moment is

$$1450 \times 44.2/12 = 5330 \text{ k ft}$$

indicating an ultimate factor of safety of

$$5330/2434 = 2.19$$

for total live and dead load, or

$$\frac{5290 - (319 + 1060)}{1055} = 3.70$$

for live load and impact alone. The factors of safety required by the Criteria are 2 and 3, respectively. Hence the ultimate strength is

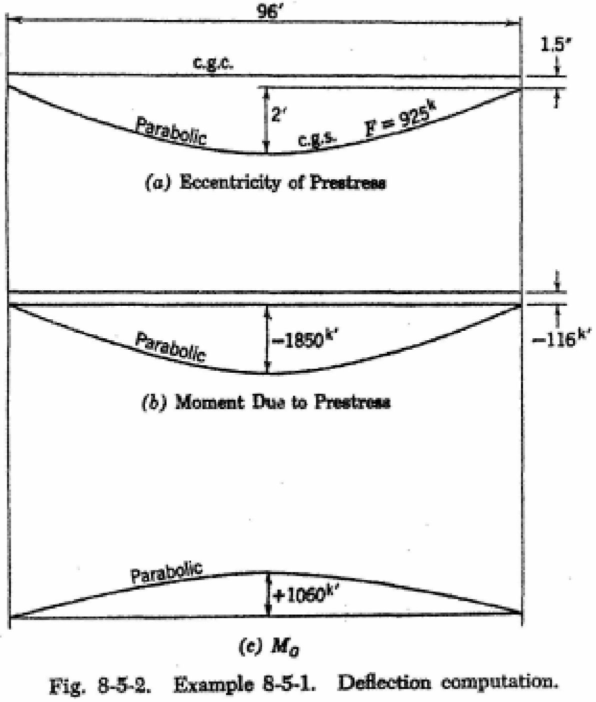


Fig. 8-5-2. Example 8-5-1. Deflection computation.

considered sufficient. Owing to the relatively shallow depth of the girder compared to its span, ultimate failure in shear is unlikely and need not be investigated.

(*e*) Referring to Fig. 8-5-2, it is seen that the deflection due to the initial prestress of 925 k can be computed as due to a uniform moment of $M_1 = 116$ k ft for the whole length of the beam plus a parabolic moment with $M_2 = 1850$ k ft at midspan. Downward deflection due to M_G is given by a parabolic moment with 1060 k ft at midspan. Thus the instantaneous upward deflection due to prestress is given by

$$M_1L^2/8EI + 5M_2L^2/48EI$$

$$= \left(\frac{116}{8} + \frac{5 \times 1850}{48}\right)\frac{96^2 \times 12^2 \times 12{,}000}{4{,}000{,}000 \times 297{,}800}$$

$$= 2.78 \text{ in.}$$

To simplify the numerical work, gross I for the concrete is used for all computations. Owing to the loss of prestress, this deflection will reduce to

$$0.85 \times 2.78 = 2.36 \text{ in.}$$

Downward deflection due to M_G is

$$\frac{5M_GL^2}{48EI} = \frac{5 \times 1060 \times 96^2 \times 12^2 \times 12{,}000}{48 \times 4{,}000{,}000 \times 297{,}800}$$

$$= 1.47 \text{ in.}$$

Hence the immediate upward deflection of the girder at transfer will be

$$2.78 - 1.47 = 1.31 \text{ in.}$$

The added dead load will produce a downward deflection of

$$(319/1060) \times 1.47 = 0.44 \text{ in.}$$

Not considering the composite action of in-place concrete, the eventual upward deflection will be decreased by the effect of loss of prestress, but increased by the creep effect of concrete. Assuming creep coefficient of 2 for the period considered, we have the resultant upward deflection after loss of prestress

$$(2.36 - 1.47 - 0.44)2 = 0.90 \text{ in.}$$

The instantaneous downward deflection due to the design live load and impact, assuming parabolic moment diagram, is

$$(1055/1060)1.47 = 1.46 \text{ in.}$$

(*f*) The above design will be applicable to most prestressing systems now used in this country, although minor modifications may be desirable for certain cases. For purpose of illustration, detailed arrangement of the tendons will be shown for the Freyssinet system, as in Fig. 8-5-3. Using cables of eighteen 0.196-in. wires, 11 tendons are required (see Appendix B). The midspan and end sections are drawn showing the arrangement of tendons to give the required locations of c.g.s. Curving of the tendons in both horizontal and vertical planes is necessary to conform with the required location of c.g.s. It must be noted that some deviation from the required parabola is

permissible, because it will not affect the strength of the girder. A recommended order for tensioning the cables is indicated as shown.

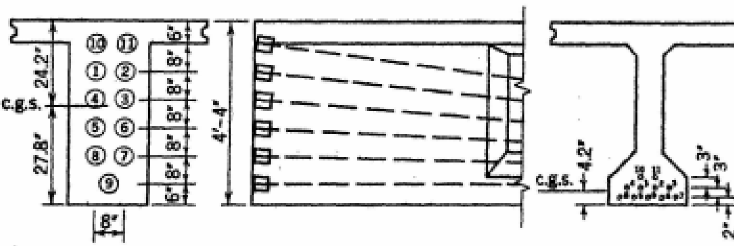
The jacking stress and the actual loss of prestress can now be computed as follows. Using the formula in Appendix D for loss of prestress,

$$3000 + 11f_{cs} + 0.04f_{si}$$

we have, for $f_{cs} = 925{,}000/862 = 1070$, $f_{si} = 160{,}000$,

$$3000 + 11 \times 1070 + 0.04 \times 160{,}000 = 21{,}200 \text{ psi}$$

indicating a loss of 13.3%, which is less than the assumed value of 15%.



(a) End Section Number indicates order of tensioning (b) End Elevation (c) Midspan Section

Fig. 8-5-3. Example 8-5-1. Cable location for Freyssinet system

Loss for the anchorage slip in Freyssinet cones may be assumed to average 0.2 in., which, if averaged throughout the entire length of 96 ft, indicates a loss of prestress equal to

$$[0.2/(96 \times 12)] \times 28{,}000{,}000 = 4900 \text{ psi}$$

To estimate the frictional loss, let us assume a coefficient of friction = 0.35 for Freyssinet cables in metal sheathing and a $K = 0.0010$ per ft for wobble effect. The average change in direction for the cables is given by $8 \times 2/96 = 0.167$ radian, for a parabolic rise of 2 ft in a span of 96 ft. Hence the maximum frictional loss, if tensioned from one end, can be computed as

$$\begin{aligned} u\theta + KL &= 0.35 \times 0.167 \times 0.0010 \times 96 \\ &= 0.058 + 0.096 \\ &= 15.4\% \end{aligned}$$

This loss will be reduced to half of 15.4% = 7.7% if tensioned from both ends; 7.7% of 160,000 = 12,300 psi. Hence, if the tendons are

overtensioned by an average of 12,300 psi at the anchorages, the anchorage loss of 4900 psi will automatically be balanced. Using the Freyssinet jack, there is an additional loss at the jack of about 8000 psi. Hence the maximum initial stress at the jack should be

$$160{,}000 + 12{,}300 + 8000 = 180{,}300 \text{ psi}$$

According to Appendix D, this temporary jacking stress should not exceed $0.80f_s' = 0.80 \times 240{,}000 = 192{,}000$ psi. The effective prestress should not exceed $0.60f_s' = 0.6 \times 240{,}000 = 144{,}000$ psi, or $0.80f_y' = 0.8 \times 200{,}000 = 160{,}000$ psi. Hence the stress in the steel is considered safe under all conditions.

8-6 Design of Pre-Tensioned Roof Beam

The following example illustrates the design of a tapered roof beam, precast and pre-tensioned. The design was used in a warehouse, San Diego, California, Fig. 8-6-1. Since a roof beam is not likely to be subjected to any overload, checking for cracking and ultimate strength is not necessary. However, one of the beams was tested to failure and showed a factor of safety of 1.3 (girder + D + L) against cracking and a factor of 2.2 (girder + D + L) at rupture.

Example 8-6-1

Design a precast pre-tensioned roof beam to span a clear distance of 64 ft with an additional length of 1 ft over each support. The beam carries a superimposed dead and live load of 520 plf, in addition to its own weight. The top chord of the beam is to be tapered so that the depth of beam is 40 in. at midspan and 16 in. at the ends, the bottom of the beam to remain straight. Use concrete with $f_c' =$ 4500 psi at 28 days and $f_{ci}' = 4000$ psi at transfer. Steel for pre-tensioning will have an ultimate strength of 200,000 psi. Further assume that ⅜-in. wires with Dorland anchorages are to be used. Allowable stresses are as follows:

	At Transfer	Under Working Load
Concrete:		
Compressive fiber	$f_b = 0.60f_{ci}' = 2400$	$f_t = 0.45f_c' = 2025$ psi
Tensile fiber	$f_t' = 0.06f_c' = 240$	$f_b' = 0$
Principal tension without web steel		$s_t = 0.03f_c' = 135$ psi
Steel:		
Initial prestress	$f_i = 130{,}000$ psi	
Prestress just after transfer	$f_o = 120{,}000$ psi	
Effective prestress		$f_e = 110{,}000$ psi

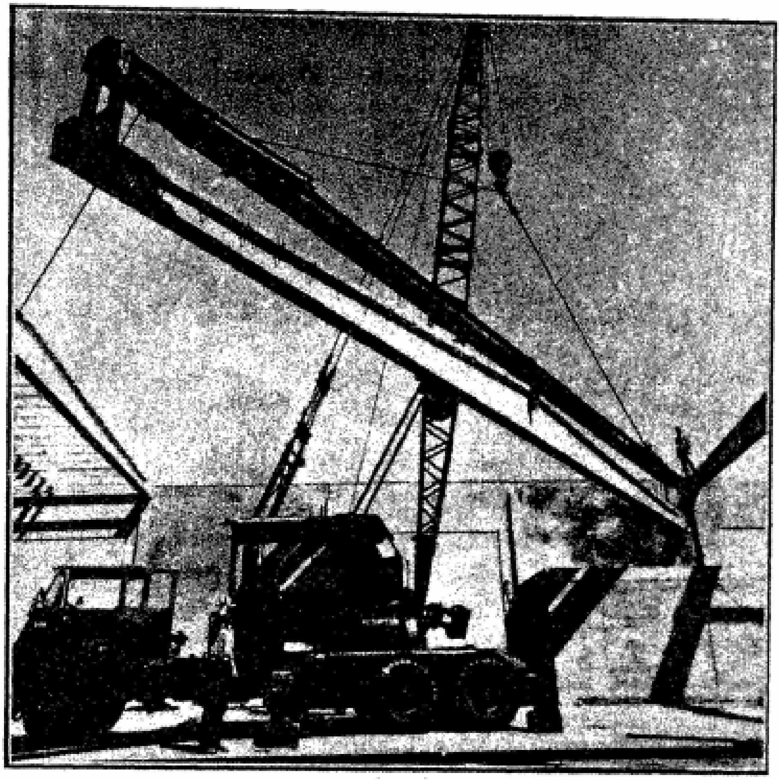


Fig. 8-6-1. Erecting a 66-ft pre-tensioned roof beam. (Beam anchored with Dorland anchorages.)

(*a*) Make a preliminary design for the beam.

(*b*) Design the beam for flexure, and locate the profile for the c.g.s.

(*c*) Design the beam for shear, and provide necessary web reinforcement.

Solution. (*a*) Preliminary Design. For this tapered beam carrying a uniform load, the controlling section for flexure is not at midspan but is nearly at the third point. For $w_S = 520$ plf,

$$M_S = \frac{w_S L^2}{8} = \frac{520 \times 65^2}{8} = 274 \text{ k ft}$$

The weight of the beam is estimated by an empirical formula

$$A_c = \frac{5M}{hf_c} = \frac{5 \times 274 \times 12}{40 \times 2.025} = 203 \text{ in.}^2$$

Hence, w_G = about 220 plf. The total load on the beam is 520 + 220 = 740 plf. Since the ratio of M_G/M_T is not too high, critical stresses may be found both at transfer and under the working load; hence an I-shape is considered economical. The moments at midspan and third points are, for span of 65 ft,

		Midspan	Third Points
$M_S = 520 \times 65^2/8$	=	274 k ft $\times 8/9$ =	244
$M_G = 220 \times 65^2/8$	=	116 k ft $\times 8/9$ =	103
M_T	=	390 k ft	347 k ft

The depth of beam at the third points is nearly 32 in. Using formulas 6-1-1 and 6-1-4, we have

$$F = \frac{M_T}{0.65h} = \frac{347 \times 12}{0.65 \times 32} = 200 \text{ k}$$

and

$$F = \frac{M_L}{0.50h} = \frac{244 \times 12}{0.50 \times 32} = 183 \text{ k}$$

Corresponding to $F = 200$ k and $f_c = 2025$ psi, we have, from formula 6-1-3,

$$\begin{aligned} A_c &= F/0.50f_c \\ &= 200/(0.50 \times 2.025) = 198 \text{ sq in.} \end{aligned}$$

A trial midspan section is sketched as in Fig. 8-6-2 with $A_c = 208$ sq in. at the third point. In order to maintain a uniform flange section throughout the entire length of the beam, a trial layout for the beam is now sketched as shown. The end 3 ft of the beam is made rectangular in section in order to distribute the stress at anchorage and to provide sufficient area for the end shear.

For a section 3 ft from the end of beam, the shear is

$$V_T = 740 \times 30 = 22{,}200 \text{ lb}$$

Neglecting the shear taken by steel, the maximum unit shear stress in concrete can be approximated by

$$\begin{aligned} v &= 1.2 \frac{V}{A_{\text{web}}} \\ &= 1.2 \frac{22{,}200.}{4 \times 18.2} = 366 \text{ psi} \end{aligned}$$

for a web 4 in. thick and a depth of beam = 18.2 in. Corresponding to a compressive fiber stress of about 600 psi, this would indicate a principal tension of

$$\sqrt{366^2 + 300^2} - 300 = 173 \text{ psi}$$

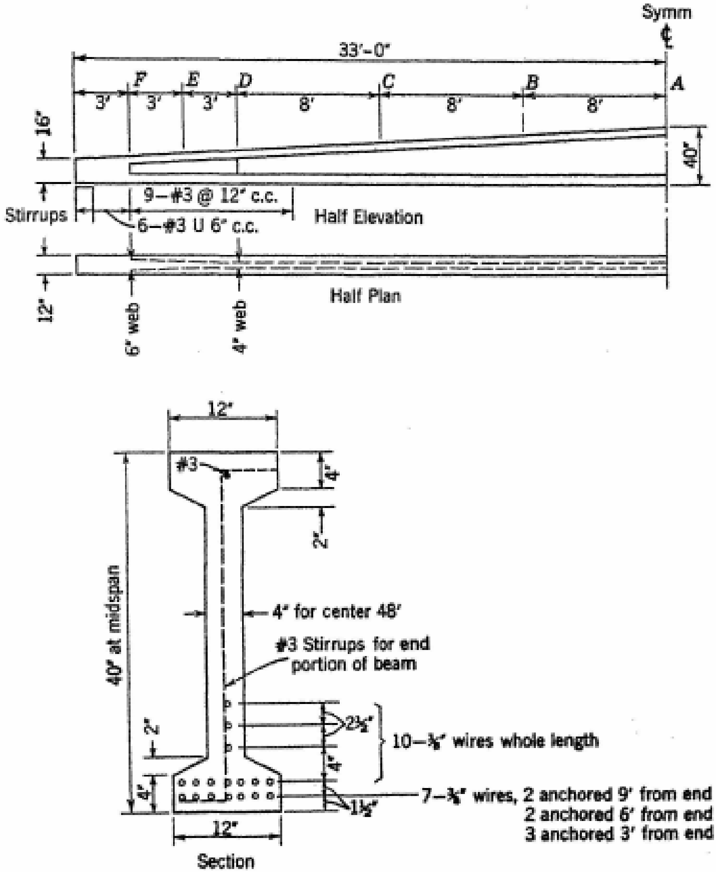


Fig. 8-6-2. Example 8-6-1.

which is somewhat above the allowable value of 135 psi. Hence it seems desirable to thicken the web near the support. A gradual increase from 4 in. to 6 in. within a distance of 6 ft is thus adopted as shown. This also increases the A_c near the ends so as to keep the flexural stress within limits.

(*b*) Design for Flexure. To provide for effective prestress of about 200 k, seventeen ⅜-in. wires will be used, furnishing total effective prestress

$$F = 17 \times 0.11 \times 110 = 206 \text{ k}$$

Although the initial prestress in the steel is 130 ksi, immediately after

transfer, the creep in steel and the elastic shortening of concrete would have already taken place, and the steel stress at that time can be considered to be 120 ksi. Hence,

$$F_0 = 206 \times 120/110 = 225 \text{ k}$$

Now, again considering the section at 3 ft from end of beam, the A_c is 169 sq in., and the maximum compression can be as high as

$$(225/169) \times 2 = 2660 \text{ psi} = 2.66 \text{ ksi}$$

which is somewhat too high. It would be desirable to cut off some of the wires before they reach the ends. After some trial, it is decided to cut off 3 wires at F, 2 at E, and another 2 at D, Fig. 8-6-2.

In order to determine the location of the c.g.s., a procedure similar to that of example 8-3-1 is followed. Since both A_c and A_s vary along the beam, it will be convenient to tabulate the computations as shown.

Section	F		E		D		C	B	A
x, ft from center of support	2.5		5.5		8.5		16.5	24.5	32.5
h, in.	18.2		20.3		22.5		28.4	34.2	40.0
Web thickness, in.	12	6	5		4		4	4	4
A_c, in.^{2}	218	169	171		170		195	217	240
I, in.^{4}		5700	7700		10,000		18,800	30,500	46,000
r^2, in.^{2}		34	45		60		96	141	192
$k_t = k_b$, in.	3.0	3.8	4.4		5.4		6.9	8.2	9.6
No. of $\frac{5}{16}''$ wires	10	13	13	15	15	17	17	17	17
F_0 at 120 ksi	132	172	172	198	198	225	225	225	225
F at 110 ksi	121	158	158	182	182	206	206	206	206
M_T, k ft	59		121		179		294	365	390
M_G, k ft	17		36		53		87	109	116
M_T/F, in.	5.9	4.5	9.2	8.0	11.8	10.4	17.1	21.4	22.7
M_G/F_0, in.	1.5	1.2	2.5	2.2	3.2	2.8	4.6	5.8	6.2
$f_t'A_ck_b/F_0$, in. ($f_t' = 0.24$ ksi)	1.2	0.9	1.1	1.0	1.1	1.0	1.4	1.9	2.5
e_b, in.	1.7	0.9	2.2	1.4	2.3	1.4	2.1	2.2	1.8
$f_b = \frac{F_0}{A_c}\left(2 - \frac{e_b}{k_t}\right) + 0.24$, ksi	1.11	2.03	1.74	2.19	2.08	2.54	2.29	2.04	1.94
e_t, in.	1.5	3.9	1.1	2.7	0.9	2.5	0.4	0.4	3.4
$f_t = \frac{F}{A_c}\left(2 - \frac{e_t}{k_b}\right)$, ksi	.83	.91	1.62	1.48	1.96	1.86	2.07	1.85	1.42

Values from the table are plotted in Fig. 8-6-3, which shows a half profile of the beam. First, the kern points are plotted from the c.g.c.; then, from the respective kern lines, the values of M_T/F and $M_G/F_0 + f_t'A_ck_b/F_0$ are plotted to obtain the limiting zone for c.g.s. If the c.g.s. is located within this zone, there will be no tension in the bottom

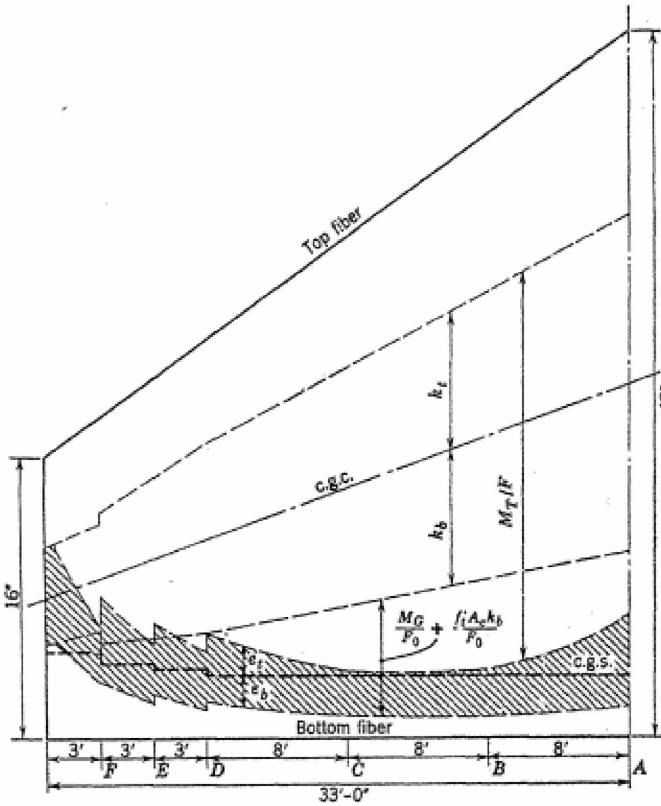


Fig. 8-6-3. Example 8-6-1. Location of c.g.s.

fiber under working load and the tension in the top fiber will be no greater than 240 psi at transfer.

The actual location of c.g.s. is given by the heavy dotted line in Fig. 8-6-3, with the wires placed and cut off as indicated in Fig. 8-6-2. For this profile of c.g.s., the bottom compressive stress at transfer is

$$f_b = \frac{F_0}{A_c}\left(\frac{h}{c_t} - \frac{e_b}{k_t}\right) + 0.24$$

and the top fiber compressive stress under working load is

$$f_t = \frac{F}{A_c}\left(\frac{h}{c_b} - \frac{e_t}{k_b}\right)$$

where e_b and e_t are the distances of c.g.s. within the limiting zone measured from the bottom and top limits, respectively. These values of f_b and f_t are computed and listed in the table. It will be observed that the greatest compressive stress occurs at D at transfer; its magnitude of 2.54 ksi slightly exceeds that of the allowable of 2.40 ksi but is not considered serious. The greatest compressive stress under working load is 2.07 ksi, very close to the allowable of 2.03 ksi, and is considered satisfactory.

The percentage of reinforcement at D is

$$(17 \times 0.11)/170 = 1.1\%$$

which indicates that the beam is somewhat over-reinforced and compression failure in concrete may occur without too much elongation in the wires.

Actual test showed a live load deflection of 7½ in. at rupture.

(*c*) Design for Shear. After a little investigation, it is seen that the critical section for shear is at 3 ft from the end of beam, where we have a 6-in. web and a shear of

$$30 \times 740 = 22{,}200 \text{ lb}$$

which is also very nearly the shear perpendicular to the c.g.c. line. The prestressing steel makes an angle of 1/33 with the c.g.c. line and hence will carry some of the shear. The 10 wires at this section with a prestress of 121 kips carry a shear of

$$121{,}000/33 = 4000 \text{ lb}$$

leaving $V_c = 22{,}200 - 4000 = 18{,}200$ lb to be carried by the concrete. The maximum shearing stress at the c.g.c. is

$$\begin{aligned} v &= \frac{V_c Q}{Ib} = \frac{18{,}200(48 \times 7.1 + 6 \times 4.4 + 30.6 \times 2.55)}{5700 \times 6} \\ &= 240 \text{ psi} \end{aligned}$$

The compressive stress at this point is 121,000/169 = 716 psi. Hence the principal tension is

$$\sqrt{240^2 + 358^2} - 358 = 72 \text{ psi}$$

Although this is well within the allowable of 135 psi, it is desirable to provide some web steel to increase the shear resistance of the beam.

For the end 3 ft, $\frac{3}{8}$-in. U-stirrups at 6-in. spacing are used to distribute the load, Fig. 8-6-2. For the next 9 ft, single stirrups at 12-in. spacing are employed. No stirrups are used for the remainder of the beam.

Chapter 9 Partial Prestress and Non-Prestressed Reinforcements

9-1 Partial Prestress and Beam Behavior

When prestressed concrete was introduced in the 1930's, the philosophy of design was to create a new material by putting concrete under compression so that there would never be any tension in it, at least not under working loads. In the later 1940's, observations on those earlier structures indicated that oftentimes extra strength existed in them. Therefore some engineers believed that a certain amount of tensile stresses could be permitted in design. In contrast to the earlier criterion of no tensile stress, which may be called "full prestressing," this later method of design allowing some tension is often termed "partial prestressing." As discussed in section 6-4, there is no basic difference between the two, because, while a structure may be designed for no tension under working loads, it will be subjected to tension under overloads. Besides, overloading is very common for certain structures, such as highway bridges in this country. The difference is rather a matter of degree; tensile stresses will be higher and occur more frequently for the same structure if designed for partial prestressing rather than full prestressing.

In order to provide additional safety for partially prestressed concrete, non-prestressed reinforcements are often added to give higher ultimate strength to the beam, and to help carry the tensile stresses in the concrete. For these beams, some of the reinforcements are prestressed and others not. This situation also lends itself to the use of the term "partial prestress," so that, sometimes, "partial prestress" may mean either or both of the following two conditions, although more frequently it is employed to denote the first condition only:

1. Tensile stresses are permitted in the concrete under working loads. Design on that basis is described in sections 6-4 through 6-6.

2. Non-prestressed reinforcements are employed in the member. This will be described in the following sections of this chapter.

In order to understand the design of partially prestressed beams, it is necessary to study the behavior of such beams with varying amount of reinforcement and subjected to varying amount of prestress. The difference in behavior of over-reinforced and under-reinforced beams is seen by comparing curves (*a*) and (*b*) of Fig. 9-1-1. The difference in behavior of over-prestressed and under-prestressed beams is seen by comparing curves (*a*), (*b*), (*c*), and (*d*) in Fig. 9-1-2.

When a section is over-reinforced, Fig. 9-1-1, it will fail by compression in concrete before the steel is stressed beyond its elastic limit. Thus, the ultimate deformation of the steel and the ultimate

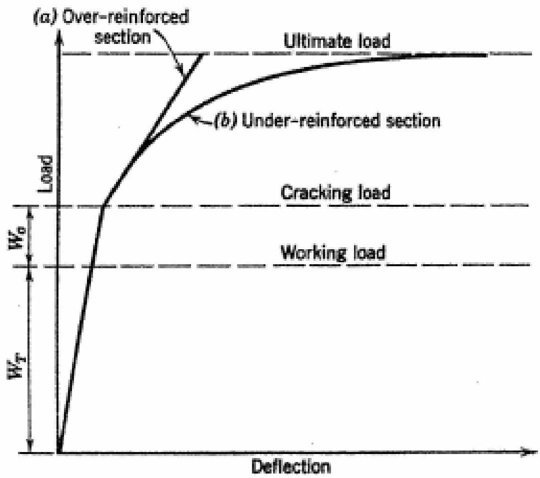


Fig. 9-1-1. Load-deflection curves, over- and under-reinforced sections.

deflection of the beam are rather small, and the failure is brittle. When seriously over-reinforced, even if the steel is not prestressed, the deflection of the beam before rupture will still be limited. When a section is under-reinforced, its deflection increases very appreciably before failure, thus giving ample warning of impending collapse. Failure starts in the excessive elongation of steel and ends in the gradual crushing of concrete on the compressive side.

In order to avoid sudden and brittle failures, and also for general economy in design, most beams are under-reinforced. When an under-reinforced section is designed for full prestressing, allowing no tension in concrete under the working loads, the load-deflection relation is given by curve (*b*) of Fig. 9-1-2. Before cracking, the section will carry an additional load W_0 above the working load W_T, the

magnitude of that additional load being

$$W_0 = \frac{f'I}{c_b}$$

where f' is the modulus of rupture and c_b the distance from c.g.c. to the tensile extreme fibers.

If the same under-reinforced section with the same amount of steel is given somewhat smaller prestress so that cracking is reached just at the working load, the tensile stress being equal to the modulus of

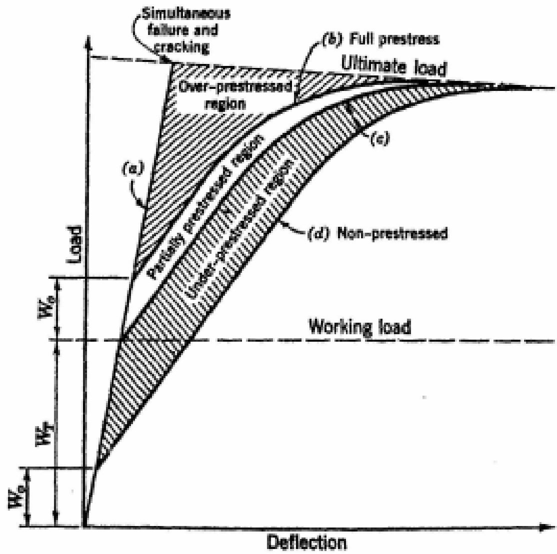


Fig. 9-1-2. Load-deflection curves for varying degrees of prestress (for under-reinforced sections of bonded beams).

rupture under working load, the load-deflection relation will be given by curve (*c*), where the deflection corresponding to the cracked section starts at the working load. If the beam is not prestressed at all, but still reinforced with the same amount of steel, provided that the steel is bonded to the concrete, the beam will behave as in curve (*d*). It will start cracking as soon as load W_0 is reached, although its ultimate strength may not be greatly reduced.

If the beam is over-prestressed, it will crack only after the load exceeds $W_T + W_0$, and its load-deflection curve will fall between curves (*a*) and (*b*), Fig. 9-1-2. In the extreme case, when the beam is very much under-reinforced but highly over-prestressed, cracking

and failure may take place simultaneously so that brittle failure occurs with sudden rupture in the steel. In principle, a partially prestressed beam may have a load-deflection curve lying anywhere between curves (*b*) and (*d*), depending upon the amount of prestress. But, in practice, seldom is cracking permitted under working load; hence the actual load-deflection curve will usually fall between curves (*b*) and (*c*), and hardly ever below curve (*c*).

The desired amount of prestress will depend upon the type of service to which the structure is to be subjected. For structures in which the possibility of cracking under working loads must be avoided and where overload might occur rather frequently, full prestress yielding load-deflection curve (*b*) is preferred. For structures which are seldom overloaded, such as certain types of buildings, partial prestress between curves (*b*) and (*c*) may be permitted. Some prestressing steel is saved by designing for partial prestress, but, if the same ultimate strength is desired, at least the same amount of total reinforcement must be used.

Where greater resilience is desired, over-prestressing is objectionable. The area under the load-deflection curve is a measure of the ability of a beam to stand impact load and to absorb shocks. Hence the conventional design yielding load-deflection curve (*b*) supplies a reasonable amount of resilience, while partial prestressing may increase this capacity, at the sacrifice of earlier appearance of cracks.

Partial prestress may be obtained by any of the following measures:

1. By using less steel for prestressing; this will save steel, but will also decrease the ultimate strength, which is almost directly proportional to the amount of steel.

2. By using the same amount of high-tensile steel, but leaving some non-prestressed; this will save some tensioning and anchorage, and will increase resilience at the sacrifice of earlier cracking and slightly smaller ultimate strength.

3. By using the same amount of steel, but tensioning them to a lower level; the effects of this are similar to those of method 2, but no end anchorages are saved. Hence the method is seldom used.

4. By using less prestressed steel and adding some mild steel for reinforcing; this will give the desired ultimate strength and will result in greater resilience at the expense of earlier cracking.

The engineer must use his own judgment as to which method is desirable for his particular structure.

The advantages and disadvantages of partial prestress as compared to full prestress may be now summarized:

Advantages:

1. Saving in the amount of prestressing steel.
2. Saving in the work of tensioning and of end anchorages.
3. Greater resilience in the structure.
4. Economical utilization of mild steel.

Disadvantages:

1. Earlier appearance of cracks.
2. Greater deflection under overloads.
3. Higher principal tensile stress under working loads.
4. Slight decrease in ultimate flexural strength for the same amount of steel.

9-2 Uses of Non-Prestressed Reinforcements

One of the more recent developments in prestressed concrete is the use of non-prestressed flange reinforcements. These reinforcements can be made up of high-tensile wires, wire strands, bars, or merely ordinary mild-steel bars. When used in conjunction with prestressed steel, they form an effective combination, one supplementing the other. The prestressed steel supplies the rigidity and the major part of the strength, while the non-prestressed steel distributes the cracks, increases the ultimate strength, reinforces those portions not readily reached by prestressed steel, and provides additional safety for unexpected conditions of loading. With proper design, both economy and safety can be attained in many cases. However, it must be admitted that not too many experiments are yet available regarding the exact behavior of such designs, although the general nature is known and many structures have been built utilizing such combinations.

Non-prestressed reinforcements can be placed at various positions of a prestressed beam to serve different purposes and to help carry the loading at different stages. Oftentimes, one position of these reinforcements can serve to strengthen the beam in several ways. This will become evident upon examining the following functions performed by them:

1. To provide strength immediately after transfer of prestress:

A. When the compressive flange may be under some tension at transfer, non-prestressed steel will help to reinforce that flange against any possible fracture, Fig. 9-2-1 (a). This design is often desirable when the beam's own weight is small compared to its live load. The use of such non-prestressed steel will permit the placing of the prestressing steel nearer to the extreme tensile fibers, thus gaining a bigger lever arm for the resisting moment.

B. When straight tendons are used for straight beams, top flange at the ends of the beam may be subjected to tensile stresses. Non-prestressed reinforcements can be placed therein for reinforcement, Fig. 9-2-1 (*b*).

C. When high compressive stresses are produced in the tensile flange by high prestressing, steel bars may be added to reinforce that flange, Fig. 9-2-1 (*c*). Such bars will also tend to minimize creep in the concrete. On the other hand, when these bars are subjected to high compressive stresses, especially when considering the effect of creep and shrinkage, lateral expansion of the bars due to Poisson's ratio effect may have a tendency to split the surrounding concrete. German specifications call for a minimum cover of about 3 times the bar's diameter to prevent such splitting. The use of proper stirrups may also be helpful.

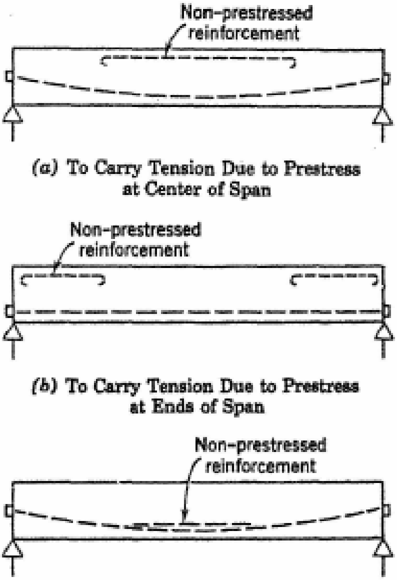


Fig. 9-2-1. Non-prestressed reinforcements to strengthen beam just after transfer of prestress.

2. To reinforce certain portions of precast beams so as to be able to carry special or unexpected loads during handling and erection, Fig. 9-2-2. This may either permit easier handling of the beams or may prevent serious rupture in case of careless handling.

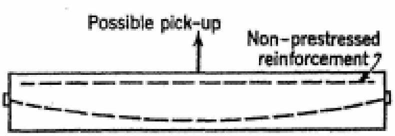


Fig. 9-2-2. Non-prestressed reinforcement to strengthen precast beam during handling and erection.

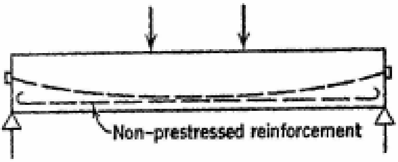
3. To reinforce the beam under working loads:

A. Either high-tensile or ordinary steel can be placed side by side with prestressed steel, Fig. 9-2-3 (*a*). This will help to distribute cracks when they occur and also to increase the ultimate strength, especially when the prestressed steel is not bonded to the concrete. By preventing the formation of concentrated big cracks, it is possible that the shear resistance of the section may also be increased. Non-prestressed steel can often be economically em-

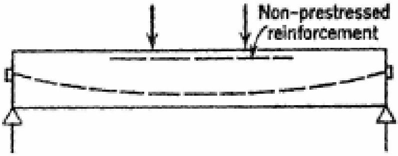
ployed because it has to be placed only over certain critical portions, while the prestressed steel generally has to extend the whole length of the beam.

B. Ordinary steel bars can be added to the compressive flange to reinforce it against high compression, Fig. 9-2-3 (*b*). This is generally uneconomical but may be required under certain conditions.

The use of non-prestressed reinforcements is certainly not limited to simple spans. For cantilevers and continuous spans, where peak moments exist it is often economical to reinforce such portions with non-prestressed steel, Fig. 9-2-4. Here, again, the use of some short length of ordinary steel may save some long prestressed steel, and economy is thereby achieved.



(*a*) To Distribute Cracks and Increase Ultimate Strength



(*b*) To Reinforce Compression in Concrete

Fig. 9-2-3. Non-prestressed reinforcements to reinforce beams under working and ultimate loads.

It must be remembered, however, that, when prestressed and non-prestressed reinforcements are combined in a structure, the cooperation of the two should be carefully investigated. Most of the time,

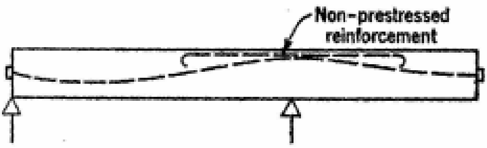


Fig. 9-2-4. Non-prestressed reinforcement to reinforce moment peaks in cantilevers.

the non-prestressed steel will not be acting effectively until the cracks have formed. Its effect on the start of hair cracks and on the elastic deflection of the beam will be small. But after cracking occurs, such steel will distribute the cracks and prevent the formation of big ones which may sometimes be detrimental in producing diagonal tension cracks and compression failures. The ultimate strength of beams under both static and repeated loads can be materially increased by proper employment of non-prestressed steel.

9-3 Non-Prestressed Reinforcement—Elastic Stresses

It is difficult, if not impossible, to design non-prestressed reinforcement by the elastic theory, because, within the elastic range, the tensile stresses in the reinforcements are very small and the reinforcements are consequently ineffective, although in the ultimate range

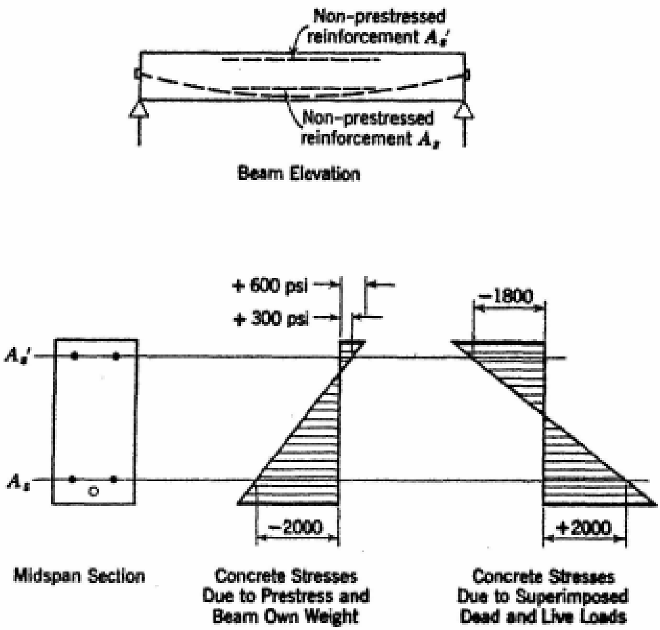


Fig. 9-3-1. Concrete stresses at levels of non-prestressed reinforcements.

they are usually stressed to the yield point and function effectively. However, a study of the elastic stresses is significant in helping to understand the behavior of such beams and to design them properly. As discussed in section 9-2, non-prestressed steel can be placed on either or both sides of the beam: the tension side which is counter-compressed by prestressing, and the compression side which could be under tension before the application of external loads. Let us investigate the stresses in both sides together as in Fig. 9-3-1.

Assume that there is no shrinkage of concrete; then there is no stress in the non-prestressed steel until prestress is transferred. At the transfer of prestress, the non-prestressed steel will have strains corresponding to the adjacent concrete, and stresses can be computed by

the elastic theory with the usual formula

$$f_s = n\left[\frac{(M_G + F_0e)y}{I_t} + \frac{F}{A}\right]$$

Owing to creep in concrete, stresses in the steel will be modified by the creep coefficient so that they will increase from f_s to $C_c f_s$.

The above method can be assumed to apply equally to the tensile and to the compressive stresses in the steel. Thus both the tensile and the compressive stresses in the steel will be increased by the effect of creep in concrete, and both can be modified by the coefficient of creep applicable for the given duration of time.

Next, let us consider the effect of shrinkage in concrete, due to which compressive stresses will be produced in steel to the amount of

$$f_s = \delta E_s$$

where δ is the unit shrinkage strain in the concrete. Thus the resulting stresses in the steel before the application of external loads is given by the formula

$$f_s = C_c n\left(\frac{(M_G + F_0e)y}{I} + \frac{F}{A}\right) + \delta E_s \tag{9-3-1}$$

using proper signs for each of the items.

The approximate magnitudes of these stresses can be shown as below. Assuming that the concrete fiber at the level of the non-prestressed reinforcement A_s, Fig. 9-3-1, is stressed to -2000 psi, for a value of $n = 6$ and $C_c = 1.5$, the compressive stress in steel will be

$$-2000 \times 6 \times 1.5 = -18{,}000 \text{ psi}$$

Add to this the effect of a shrinkage strain of -0.0002, which will induce a compressive stress in the steel of

$$-0.0002 \times 30{,}000{,}000 = -6000 \text{ psi}$$

Hence the total stress in the steel A_s on the tension flange may be around $-18{,}000 - 6000 = -24{,}000$ psi. Thus the steel will be stressed appreciably in compression.

For the steel A_s' on the compression flange the stresses cannot be very high. Assuming a tensile stress of 600 psi in the extreme top fibers of concrete, there may be only about 300 psi tension at the level of steel. For the same creep and shrinkage as above the resulting

stress in the steel A_s' will be

$$(1.5 \times 300 \times 6) - 6000 = -3300 \text{ psi}$$

Hence this non-prestressed steel in the compression flange, which is intended to carry tension in that flange under girder loads, will probably be under compression instead. In other words, within the elastic range, such reinforcements may not serve their intended purpose at all.

Now, when the external load is applied on the beam, steel A_s' on the compression flange will be further compressed, while the steel A_s on the tensile side will be decompressed. These stresses can again be computed by the elastic theory, the effect of shrinkage and creep being taken into account if necessary. To get an idea of the magnitude of the elastic stresses produced by loading, assume a compression of about 1800 psi in the concrete fiber near the A_s', and a decompression of about 2000 psi in the concrete fiber at the A_s; we have, Fig. 9-3-1,

$$f_s = -3300 - 1800 \times 6 = -14{,}100 \text{ psi in } A_s'$$

$$f_s = -24{,}000 + 2000 \times 6 = -12{,}000 \text{ psi in } A_s$$

which indicates that the tensile steel A_s, which is intended to carry the tension under working loads, may actually still be under compression instead. Hence it is impossible to design such non-prestressed steel for working loads. They will not serve the intended function within the elastic range. They are provided to increase the ultimate strength of the beam and to minimize its deflection after cracking. For members whose serviceability is impaired by cracking, non-prestressed reinforcements cannot suitably be employed.

9-4 Non-Prestressed Reinforcements, Ultimate Strength

It is shown in the previous section that non-prestressed reinforcements, when used in conjunction with prestressed ones, do not function effectively in carrying any tension within the elastic working range. Similar to bars in ordinary reinforced concrete beams, they act efficiently in tension only after the concrete has cracked. Before the cracking of concrete, their tensile stresses, if any, are limited. Since almost all prestressed beams are designed for no cracks within the working loads, the non-prestressed reinforcements are useless under such conditions. The interesting phenomenon is that, though they do not serve within the working range, they are often as effective as the prestressed ones near the ultimate load. Thus, if the ultimate strength is of prime importance rather than the elastic strength, non-prestressed reinforcements can be profitably employed.

Figure 9-4-1 shows, for various reinforcements, the variation of stresses with strains produced by external loads. Consider first a prestressed wire, with effective prestress of 125 ksi, elastic limit of 180 ksi, and ultimate strength of 250 ksi. As the load on the beam increases, the strain and hence the stress increases as shown in curve (*a*). Next consider a non-prestressed wire of the same qualities embedded at the same level. Its stress-strain variation is given by

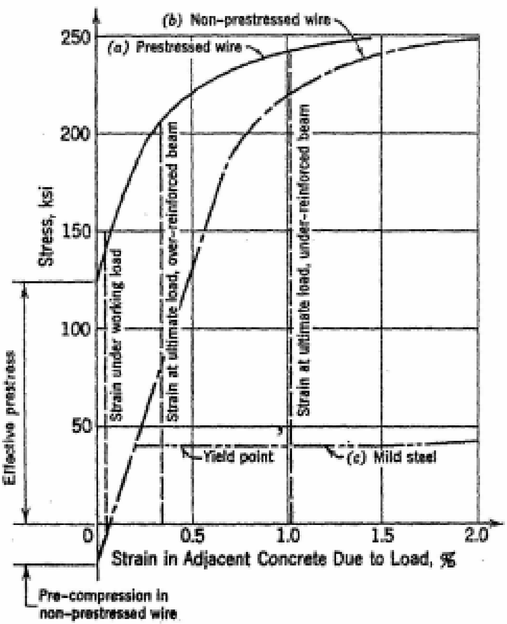


Fig. 9-4-1. Stress-strain diagrams of prestressed and non-prestressed steel.

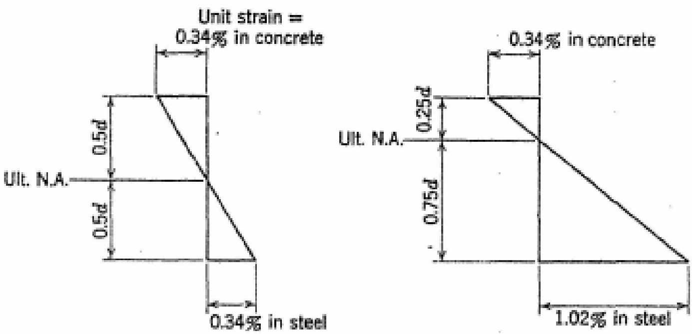
curve (*b*). The wire will actually be precompressed during the transfer of prestress, so that, before any external load is applied to the beam, the wire will be under compression. If the amount of precompression is 20 ksi, the total difference in stress between this wire and the prestressed one is of the order of 145 ksi.

Under working load, producing a strain in the steel of about 0.05%, stress in the prestressed wire will be increased to 140 ksi, while that in the non-prestressed one will be changed to 5 ksi compression. Hence the non-prestressed wire is still not functioning at all.

Now, going into the ultimate range, let us refer to Fig. 9-4-2, which shows the conditions of strain in a prestressed bonded beam section at

the ultimate load. In (*a*), we see that, for an average over-reinforced beam, the strain in steel at failure is about 0.34%. Figure 9-4-1 shows that, at this strain, the stress in the prestressed wire is about 207 ksi while that in the non-prestressed one is only 80 ksi. This means that the non-prestressed wire is picking up some stress but still has been worked only to about a third of its capacity, even at the ultimate load.

Figure 9-4-2 (*b*) shows the strain relations of an average under-reinforced beam, with a strain in the steel amounting to about 1.02% at the ultimate load. Corresponding to this strain, it is seen from



(*a*) An Over-Reinforced Beam (*b*) An Under-Reinforced Beam

Fig. 9-4-2. Strain relations at ultimate load for a bonded beam.

Fig. 9-4-1 that the prestressed wire will be stressed to about 243 ksi and the non-prestressed one to 222 ksi, the two values being quite close. This means that the non-prestressed wire is now almost as effective as the prestressed one. It can thus be concluded that, for an under-reinforced beam, the non-prestressed wires will be quite efficient in resisting the ultimate load, although under ordinary working loads it is hardly functioning at all.

The stress in a non-prestressed ordinary mild-steel bar can be studied by referring to curve (*c*), Fig. 9-4-1. Under ordinary working loads, the bar may be under some compression similar to the non-prestressed wire. But at the ultimate load, it will be stressed to its yield point for either an over-reinforced or an under-reinforced beam. In the latter case, it is possible that the bar may sometimes be stressed even beyond its yield point.

The above discussion has been confirmed by many tests, such as mentioned in the references for this chapter. Although the detailed behavior of non-prestressed reinforcements may still need experi-

mental investigation before they can be definitely formulated, enough data are on hand to permit designs made within the usual range of accuracy desired in practice.

Having described the general behavior of non-prestressed steel in a prestressed beam, we can now proceed to the design of such beams on the basis of ultimate strength. It must again be remembered that ultimate strength is only one measure of the safety of a structure. High stresses and local strains, which may be detrimental if repeated often enough, and deflections and cracks, which may impair the serviceability of the structure long before the ultimate strength is reached, should be carefully studied in each case before a design can be adopted.

It is most difficult to formulate a proper basis for the design of non-prestressed reinforcements in the compression flange, where they are needed to strengthen the beam during handling. First of all, there is no way to tell exactly how the beam is to be handled unless careful supervision is given. Next, the effect of prestress on the ultimate strength in such a case is not exactly known. Figure 9-4-3 (a) shows one half of a beam which is being lifted at the midspan point. With half of the beam as a free body and taking moments about point A, the c.g.s. of the prestressed steel, we can write, for conditions at failure,

Moment of tension in non-prestressed steel about A
= Moment of weight of member about A

assuming that the ultimate center of compression in concrete coincides with the c.g.s. The factor of safety to be used in such a computation and the actual factor of safety possessed have not been experimentally or analytically determined, although it is agreed that such reinforcements will add to the safety of the member.

On the other hand, the ultimate design of non-prestressed steel in the tension flange can be definitely formulated. Referring to Fig. 9-4-3 (b), the total tension in the steel, both prestressed and non-prestressed, can be estimated. Corresponding to that total tension, the depth of compression in the concrete can be figured,

$$k'd = \frac{T' + T_1'}{k_1 f_c' b}$$

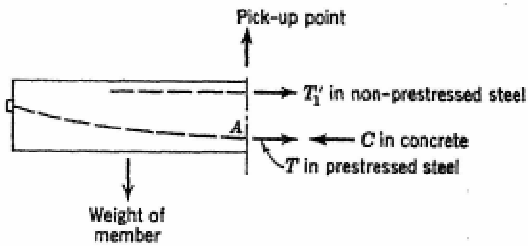
With the neutral axis thus located for the ultimate load, the ultimate tension in the steels can be obtained from diagrams and curves such as Figs. 9-4-1 and 9-4-2. Then $k'd$ can be revised, if necessary. The lever arms for the tensile forces, a' and a_1', are easily obtained and

the ultimate moment computed,

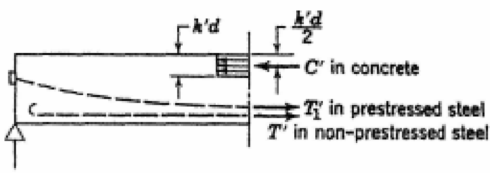
$$M' = T'a' + T_1'a_1'$$

Then the allowable moment can be obtained from M' by using a proper factor of safety.

This procedure will be illustrated in the following example.



(a) Non-Prestressed Steel in Compression Flange, an Arbitrary Assumption for Design



(b) Non-Prestressed Steel in Tension Flange

Fig. 9-4-3. Ultimate design of non-prestressed steel.

Example 9-4-1

A prestressed concrete beam has a T section as shown, Fig. 9-4-4. It is prestressed with high tensile wires (A_s = 1.06 sq in.) and additionally reinforced with non-prestressed wires (A_s = 0.47 sq in.) and non-prestressed mild steel bars (A_s = 1.32 sq in.). The c.g.s. of each type of steel is shown in the figure. f_c' = 5000 psi; for wires, f_s' = 250 ksi, f_s = 125 ksi; for mild steel bars, f_y = 40 ksi. Estimate the ultimate flexural strength of the section, assuming no failure in shear or bond.

Solution. Assuming that at rupture the prestressed wires will be stressed to 250 ksi, the non-prestressed wires stressed to 230 ksi, and the mild steel to 40 ksi, then the total tensile force at rupture will be

$$250 \times 1.06 = 265 \text{ kips}$$
$$230 \times 0.47 = 108 \text{ kips}$$
$$40 \times 1.32 = \underline{53 \text{ kips}}$$
$$\text{Total} = 426 \text{ kips}$$

Assuming the average stress in concrete to be $0.85f_c' = 4250$ psi, the ultimate depth of compression will be

$$k'd = \frac{426{,}000}{30 \times 4250}$$

$$= 3.4 \text{ in.}$$

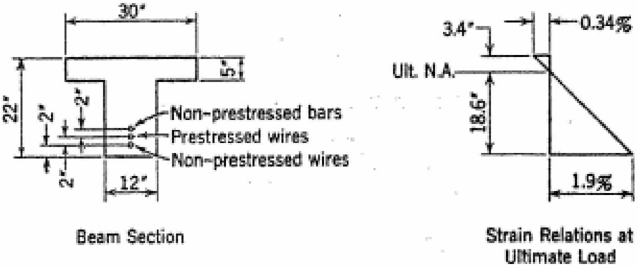


Fig. 9-4-4. Example 9-4-1.

For a concrete ultimate strain of 0.0034 or 0.34%, the ultimate strain in steel can be computed by a simple diagram as in the figure, thus,

$$(0.34/3.4) \times 18.6 = 1.9\%$$

corresponding to which the stresses in both the prestressed and the non-prestressed wires can be taken as 250 ksi (see Fig. 9-4-1).

Further revision of $k'd$ is deemed unnecessary; hence the resisting moments of the various steels, taken to the mid-depth of $k'd$, can be listed as below:

$$\begin{aligned} 250 \times 1.06 \times 16.3 &= 4320 \text{ k in.} \\ 250 \times 0.47 \times 18.3 &= 2150 \text{ k in.} \\ 40 \times 1.32 \times 14.3 &= \underline{750 \text{ k in.}} \\ \text{Total} \qquad &= 7220 \text{ k in.} = 600 \text{ k ft.} \end{aligned}$$

which is considered a rather close estimate of the ultimate resisting moment of the section. The design moment should be determined by applying a proper factor of safety to the ultimate moment. In addition, the stresses in the concrete and the amount of deflection under the working load should be investigated.

References

9-1 *First Report on Prestressed Concrete*, Institution of Structural Engineers, London, 1951 (see Appendix II).

9-2 P. W. Abeles, "How Much Prestress," *Eng. News-Rec.*, July 5, 1951, pp. 32-33; also see reader's comment on p. 46, April 13, 1950, and on p. 10, Jan. 15, 1951.
9-3 P. W. Abeles, *Partially Prestressed Concrete Construction Built in the Eastern Region of British Railway*, 1948-1952.
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Chapter
10
Continuous Beams

10-1 Continuity, Pros and Cons

A simple comparison between the strength of a simply supported and a continuous beam will demonstrate the basic economy inherent in continuous construction of prestressed concrete. Consider a simple prestressed beam loaded with a uniformly distributed load of intensity w, Fig. 10-1-1 (a). The total load w' that can be ultimately carried by the beam is determined by the ultimate moment capacity of the midspan section. If the ultimate tension developed in the tendon is T', acting with a lever arm a', then the ultimate resisting moment at midspan is $T'a'$. With one half of the span as a free body, Fig. 10-1-1 (b), and taking moment about the left support, we have

$$\frac{w'L^2}{8} = T'a'$$

$$w' = \frac{8T'a'}{L^2} \tag{10-1-1}$$

The moment diagram produced by the load w' is shown in Fig. 10-1-1 (c). It should be noted that the ultimate load w' carried by the beam is controlled by the capacity of the midspan section and cannot be increased by any change in the end eccentricities of the c.g.s.

Now consider the intermediate span of a continuous beam, Fig. 10-1-2 (a), with the same section, same span, and same prestressing steel as the simple beam of Fig. 10-1-1 (a). Again with one half of the span as a free body, Fig. 10-1-2 (b), and taking moment about the left support, we have

$$\frac{w_c'L^2}{8} = 2T'a'$$

$$w_c' = \frac{16T'a'}{L^2} \tag{10-1-2}$$

noting that there are two resisting moments, one at midspan and another over the support. Hence the load-carrying capacity is definitely affected by the position of c.g.s. over the intermediate support. The moment diagram produced by the load w_c' is now plotted in Fig. 10-1-2 (c).

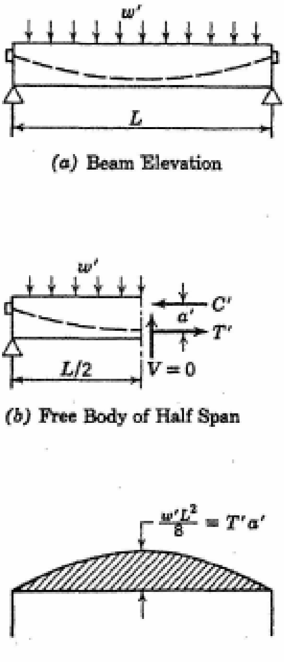


Fig. 10-1-1. Load-carrying capacity of a simple beam.

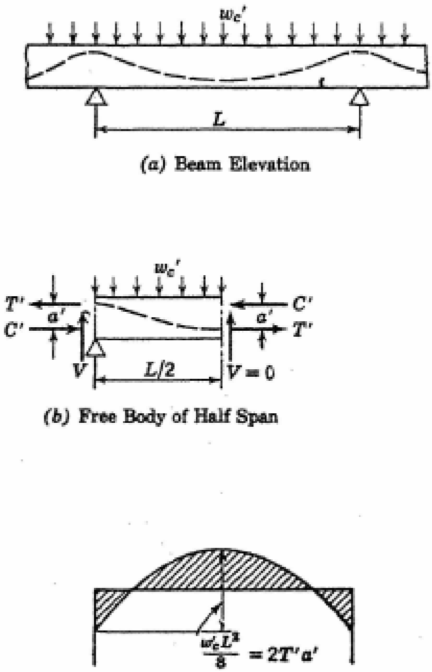


Fig. 10-1-2. Load-carrying capacity of a continuous beam.

Comparing Fig. 10-1-1 (c) with Fig. 10-1-2 (c), or equation 10-1-1 with equation 10-1-2, it is readily seen that $w_c' = 2w'$. This means that twice the load on the simple span can be carried by the continuous span for the same amount of concrete and steel. This represents a very significant basic economy that should be realized by engineers designing prestressed concrete structures. Because of this strength inherent in continuous construction, it is possible to employ smaller concrete sections for the same load and span, thus reducing the dead weight of the structure and attaining all the resulting economies.

Although it is generally conceded that continuity is economical in reinforced concrete, it is seldom known that, from certain points of view, even greater economy can be attained in prestressed construction. In reinforced concrete, the negative steel often laps with the positive steel bars, and both sets of bars are extended for additional anchorage, thus canceling some of the economy of continuity. In prestressed concrete, the same cable for the $+M$ is bent over to the other side to resist the $-M$, with no loss of overlapping. In addition, continuity in prestressed concrete saves end anchorages otherwise required over the intermediate supports, thus resulting in further economy and convenience.

The above discussion refers to the ultimate capacity of continuous beams, but the same general principles hold true within the elastic range. For both the elastic and the plastic ranges, there is a resisting couple at each intermediate section of the beam. For both ranges, with one half of the beam a free body, there are two resisting moments in a continuous beam, but only one in a simple beam. Within the elastic range, however, the positive and negative moments acting on the beam may not be equal in magnitude. Then one of the moments will control the design, and the resisting capacity of the continuous may not be as high as twice that of the simple beam.

Economical design of continuous prestressed beams can be achieved in several ways. Owing to the variation of moment along the beam, the concrete section and the amount of steel are often varied accordingly. The peaks of the negative moments can be reinforced with non-prestressed steel, thus reducing the amount of prestressing steel. Advantage can be taken of the redundant reactions to obtain favorable lines of pressure in the concrete, which will be discussed in sections 10-4 and 10-5. Designs can be based on the ultimate strength of such beams, applying the principles of limit design. Some of these, however, are more delicate problems which should be handled with care.

It is perhaps unnecessary to add that, as is true with other continuous structures, the deflections will be less than simple spans. Hence, for continuous spans, smaller depth is sufficient not only for strength but also for rigidity.

Like any type of construction, there are advantages and also shortcomings, which, under certain conditions, could outweigh the advantages. The choice of a particular type of design must be made after considering all the factors involved in the job. Disadvantages inherent in continuous prestressed concrete beams can be enumerated as follows:

1. Frictional loss in continuous tendons. This can be serious if there are many reversed curves, if the curves possess large deflection angles, or if the tendons are excessively long. Such loss can be minimized by using relatively straight cables in undulating or haunched beams. The usual methods of overtensioning, of stressing from both ends, can also be used to reduce frictional losses, as discussed in Chapter 4.

2. Shortening of long continuous beams under prestress. This may produce excessive lateral force and moments in the supporting columns, if they are rigidly connected to the beams during prestressing. Provisions are usually made to permit rocking of the columns.

3. Secondary stresses. Secondary stresses due to prestressing, creep and shrinkage effects, temperature changes, and settlements of supports could be serious for continuous structures unless they are controlled or allowed for in the design. One interesting point in continuous prestressed structures is that these secondary stresses can oftentimes be utilized to good advantage so that they will add to the economy of the structure.

4. Concurrence of maximum moment and shear over supports. It is believed that the concurrence of maximum moment and shear at the same section may decrease the ultimate capacity of a beam. This happens over the supports of most continuous beams. Hence care must be taken to reinforce such points properly for both shear and moment if high ultimate strength is desired. The elastic strength, however, is not affected by such concurrence.

5. Reversal of moments. If live loads are much heavier than dead load, and if partial loadings on the spans are considered, continuous beams can be subjected to serious reversal of moments which often cannot be economically designed with prestressed concrete. This is particularly true for short continuous beams of equal spans but is not so serious when long spans alternate with short ones or when some optimum ratios of the span lengths are adopted.

6. Moment peaks. Peaks of maximum negative moments may sometimes control the number of tendons required for the entire length of the beam. These peaks, however, can be strengthened by employing deeper sections or by adding prestressed and non-prestressed reinforcements over the portions where they are needed.

7. Difficulty in achieving continuity for precast elements. It is easy and natural to obtain continuity for cast-in-place construction, but continuity for precast elements cannot always be achieved without special effort. On account of difficulties in handling precast con-

tinuous beams, they are often precast as simple elements, to be made continuous after they are erected in place.

8. Difficulty in designing. It is more difficult to design continuous than simple structures. But, with the development of simpler and standardized methods, the design of continuous prestressed concrete beams can be made into a more or less routine procedure applying basic principles for continuous structures familiar to most engineers. These methods will be presented in the following sections.

10-2 Layouts for Continuous Beams

Several methods for providing continuity in prestressed concrete construction have been applied in practice.[10-1] These methods permit various layouts to be adopted, some of which are shown in Figs. 10-2-2 and 10-2-3 and will be described below. There are other methods and layouts that are perhaps less frequently used. Still other arrangements are being developed. But it is considered sufficient to present the more common methods, leaving the variability as well as the desirability of each to the judgment of the designer, who should of course take into account the particular conditions surrounding each structure when selecting his layout.

Continuous beams may be divided into two classes: fully continuous beams and partially continuous beams. For full continuity, all the tendons are prestressed in place and are generally continuous from one end to the other, although some can be anchored at intermediate points if found desirable. The concrete may be either poured in place or made of blocks assembled on falsework. The tendons may be encased in the concrete during pouring, threaded through preformed holes, or placed outside the webs, Fig. 10-2-1. They may be either bonded or unbonded, depending upon the requirements of the structure. Some of the typical layouts for full continuity are shown in Fig. 10-2-2:

(*a*) In (*a*) is shown a straight beam with curved tendons, which follows in general the tensile side of the beam. This layout is often used for slabs or short-span beams, where simple formwork is more important than the saving of steel and concrete. The main objections here are the heavy frictional loss and the difficulty of threading the tendons through when they are continuous over several spans.

(*b*) For longer spans and heavier loads, it will be more economical to haunch or curve the beams, as in (*b*). This will not only save concrete and steel but also permit the use of straight tendons, likewise positioned on the tensile side of the beam. However, it is often diffi-

cult to get the optimum eccentricities all along the beam if the tendons are to remain entirely straight.

(*c*) The best layout is often obtained with a compromise of the above two arrangements, using curved beams and slightly curved tendons at the same time, as in (*c*). This would permit optimum

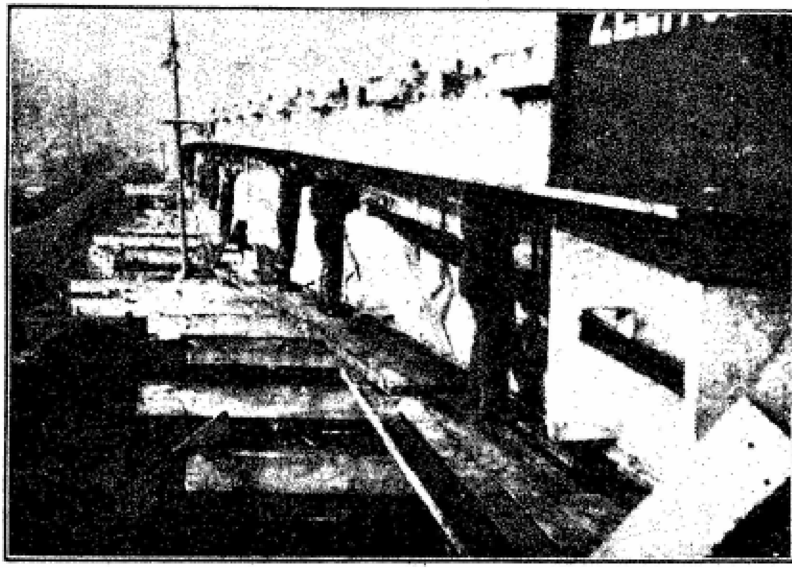


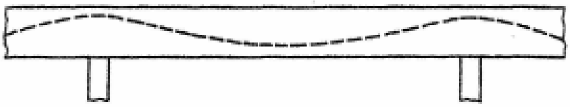
Fig. 10-2-1. Threading an exterior cable through transverse stiffeners of a continuous prestressed bridge in Belgium. (Span 85′—170′—85′, depth 4′ 11″ at center and 8′ 10″ over support.) (Dr. Van de Pitte.)

depth of beam as well as ideal position of steel at all points, while avoiding excessive frictional loss.

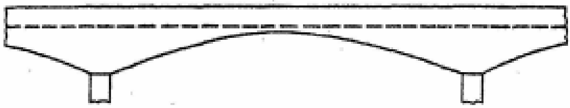
(*d*) Cables protruding at intermediate points, as in (*d*), offer a possibility of varying prestressing force along the beam. The arrangement here shown has no reversed curves in the tendons, so that heavier cables and rods can be more easily threaded through and stressed with less frictional loss.

For partial continuity, each span is first precast as a simple beam, a sufficient amount of prestressed steel being used for handling and erection. Concrete blocks can also be used if desired. Generally, no falsework is required for erection. After the simple elements are erected in place, additional elements, sometimes non-prestressed, but

often prestressed longitudinally or transversely, are inserted to provide continuity over the supports. These are termed partially continuous beams, Fig. 10-2-3.

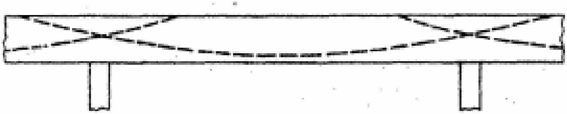


(a) Curved Tendons in Straight Beams



(b) Straight Tendons in Curved Beams

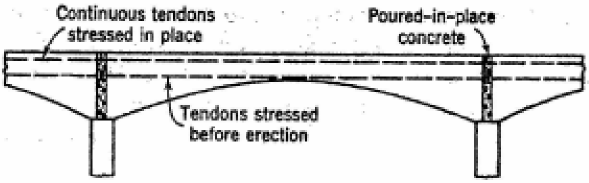
(c) Curved Tendons in Haunched or Curved Beams



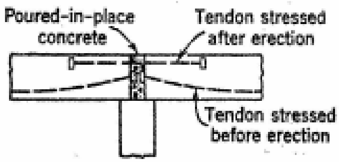
(d) Overlapping Tendons

Fig. 10-2-2. Layouts for fully continuous beams.

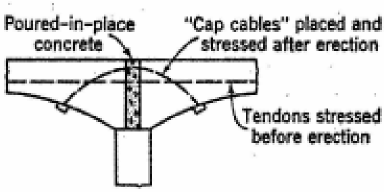
(*a*) In (*a*) are shown continuous prestressed cables placed in conduits or grooves left in the structure. After erection, concrete is poured between the beams over the supports. When the concrete hardens, the continuous cables can be stressed to provide continuity. The construction is relatively simple, but economy in steel cannot be easily attained since the same cable area is provided throughout the entire length, whether needed or not.



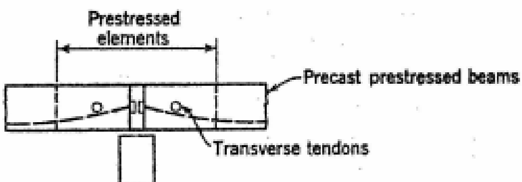
(a) Continuous Tendons Stressed after Erection



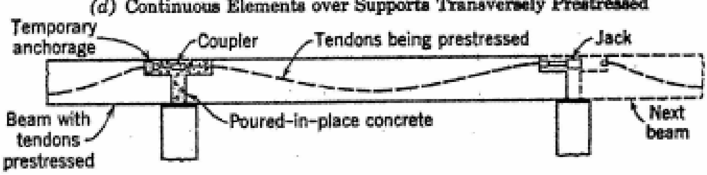
(b) Short Tendons Stressed over Supports



(c) Cap Cables over Supports



(d) Continuous Elements over Supports Transversely Prestressed



(e) Couplers over Supports

Fig. 10-2-3. Layouts for partially continuous beams.

(b) In (b) is shown a layout similar to (a), but with the continuous tendons placed over the supports only. This saves steel but requires more anchorages than the first layout. Moreover, the anchorages are located at intermediate points and are more difficult to tension. Sometimes, non-prestressed mild-steel bars are buried in the concrete in place of the tendons, to provide some amount of continuity.

(c) Another method of supplying continuity over the supports is to add the so-called "cap cables," as in (c). These tendons can be conveniently stressed from the soffit of the beam, but they possess an appreciable curvature and hence corresponding frictional loss. It is not possible to thread big rods through the holes, unless the profile is made into a circular curve and the bars are prebent to a definite curvature.

(d) Still another way to provide continuity is to insert tensile elements over the supports, as in (d), and to attach them to the precast beams by transverse prestressing, which supplies a sort of bolt action clamping the elements together. These tying elements can be made of reinforced- or prestressed-concrete planks, and can be either precast or poured-in-place. Sometimes, the precast elements themselves can be cantilevered at the ends so that they overlap over the supports and transverse prestress is applied to hold them together, thus making them continuous under live load.

(e) Especially applicable to high-tensile bars, but perhaps also to other forms of tendons, is the use of couplers as a means of obtaining continuity,[10-2] (e). This permits the stressing of tendons one span at a time, thus minimizing the frictional loss encountered when prestressing tendons running through several continuous spans. Suppose we erect the beams in (e) successively from left to right. After one beam is fully prestressed, the next beam is erected and its unstressed bar is connected to the stressed bar of the previous beam by a coupler. Then a jack is applied to the right end for tensioning. This method is also applicable to cast-in-place beams, provided the sequence of construction permits the insertion of jacks.

10-3 Analysis, Elastic Theory

Tests on continuous prestressed-concrete beams have shown that the elastic theory can be applied with accuracy within the working range. Since there is little or no tensile stress in the beam under working loads, there are no cracks, and the beam behaves as a homogeneous elastic material, more so than an ordinary reinforced-concrete beam which usually is cracked in certain portions. By making proper

allowance for shrinkage and creep, the elastic theory can be applied for all practical purposes to compute the deflections, strains, and stresses up to cracking. This is true for the effect of prestress as well as of dead and live loads.

The method of analysis presented here is based on the classical elastic theory. Fundamentally, the theory is applicable to all statically indeterminate structures of prestressed concrete, provided that consideration is given to the axial shortening effect in frames and similar structures. However, for simplicity, only fully continuous beams are referred to in the following, although most of the discussions apply to rigid frames, slabs, and partially continuous beams as well.

The analysis and design of continuous prestressed concrete structures are considered by most engineers to present a rather difficult problem. This is an erroneous impression. Undoubtedly, the design is more complicated than that of continuous reinforced-concrete structures or of statically determinate prestressed structures. But the basic theories involved are the same, and the additional complications are limited in nature. Hence a person who understands the analysis of statically indeterminate structures and the design of simple prestressed concrete beams can learn the design of continuous prestressed concrete beams with little difficulty. It is necessary, however, to explain the procedure of design in terms familiar to the majority of engineers in order that it can be more readily understood.

Several methods are available for the analysis of prestressed continuous beams.[10-3,10-4,10-5] All of them are similar to those followed in the analysis of any statically indeterminate structures; they are all based on the same assumptions and yield the same answers. Hence only one method, which the author considers the simplest, will be discussed in this treatise. The method will be based on no involved mathematics, and only the following principles will be utilized:

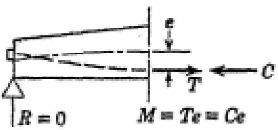
1. Moment and shear diagrams for ordinary continuous beams.
2. Moment distribution method.[10-6,10-7]
3. Location of line of pressure in a prestressed-concrete beam.

Before starting on the method of analysis, let us examine first the difference between a continuous prestressed beam and a simple one. Owing to the application of external loads, the moments in a bonded continuous prestressed beam are computed by the elastic theory, like any other type of statically indeterminate structure. In an unbonded beam, the effect of change in prestress due to beam curvature should be added, although the magnitude is usually small and can be neglected. Owing to the application of prestress, the moments in a

continuous beam are directly affected by the prestress and indirectly by the support reactions induced by the bending of the beam. In a simple beam, no support reactions can be induced by prestressing.

Consider a simple prestressed beam, Fig. 10-3-1 (a). No matter how much the beam is prestressed, only the internal stresses will be affected by prestressing. The external reactions, being determined by statics, will depend on the dead and live load (including the weight of the beam), but are not affected by the prestress. Without load on the beam, no matter how we prestress the beam internally, the external reactions will be zero, hence the external moment will be zero. With no external moment on the beam, the internal resisting moment must be zero, hence the C-line (which is the line of pressure in the concrete) must coincide with the T-line in the steel (which is the c.g.s. line), as in (b). The C-line in the concrete being known, the moment in the concrete at any section can be determined by $M = Te = Ce$.

(a) Beam Elevation



(b) Part of Beam as Free Body

Fig. 10-3-1. Moment in concrete due to prestressing in a simple beam.

Next, let us consider a continuous prestressed-concrete beam, Fig. 10-3-2 (a). When the beam is prestressed, it bends and deflects. The bending of the beam can be such that the beam will tend to deflect itself away from some of the supports, as in (b). If the beam is refrained from deflection at these supports, (c), reactions must be exerted on the beam to hold it there. Thus reactions are induced when a continuous beam is prestressed (unless, by intent or by chance, the prestress has no tendency to deflect the beam from any of its supports). These induced reactions produce moments in the beam, (d). To resist these moments, the C-line must be at a distance a from the T-line, (e), such that the internal resisting moment equals the external moment M caused by the reactions, i.e.,

$$a = \frac{M}{T}$$

Now, let us compare the simple beam with the continuous beam under the action of prestress, neglecting the weight of the beam and all other external loads. In the simple beam, the C-line coincides

with the T-line. In the continuous beam, the C-line usually deviates from the T-line. In the simple beam, the stress distribution in the concrete at any section is given by the location of the T-line. In the

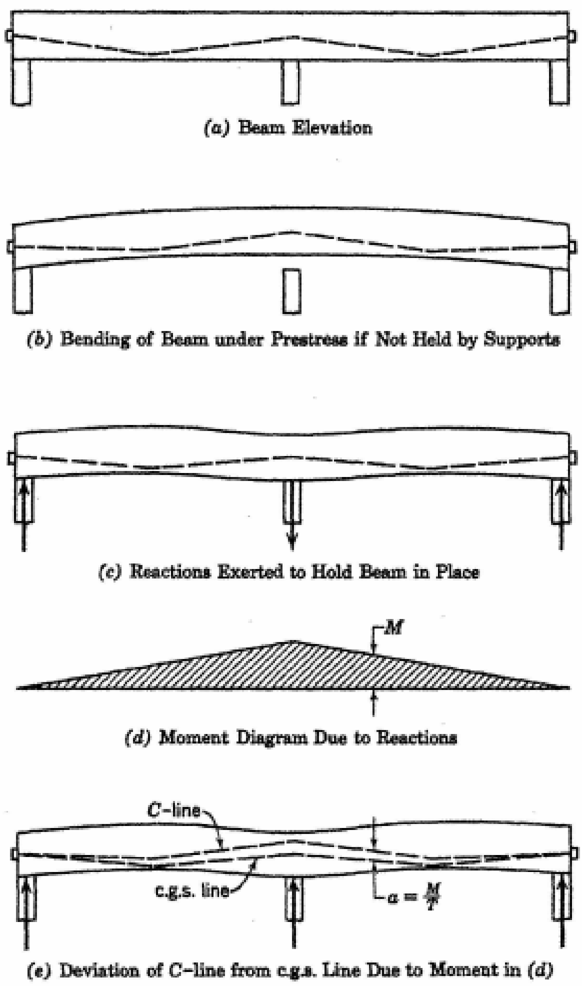


Fig. 10-3-2. Moment in concrete due to prestressing in a continuous beam.

continuous beam, it is given by the location of the C-line which does not coincide with the T-line. The difference between the two beams lies in the presence of external reactions and moments in the contin-

uous beam, produced as a result of prestressing. Since the external moment is solely produced by the reactions, and since the reactions are only applied at the supports, the variation of moment between any two consecutive supports is a linear one. If T remains constant between the supports, then the deviation a, being directly proportional to M, also has to vary linearly, Fig. 10-3-2 (e).

From another point of view, the difference between a simple and a continuous beam under prestress can be represented by the existence of "secondary moments." Once these moments over the supports are determined, they can be interpolated for any point along the beam. These moments are called secondary because they are by-products of prestressing and because they do not exist in a statically determinate beam. The term "secondary" is somewhat misleading, since sometimes the moments are not secondary in magnitude but play a most important part in the stresses and strength of the beam.

From this same point of view, the moment in the concrete given by the eccentricity of the prestress is designated as the primary moment, such as would exist if the beam were simple. On account of such primary moment acting on a continuous beam, the secondary moments caused by the induced reactions can be computed. The resulting moment due to prestress, then, is the algebraic sum of the primary and secondary moments.

The following gives a procedure for computing directly the resulting moments in the concrete sections over the supports, based on the moment distribution method. Once the resulting moments are obtained, the secondary moments can be computed from the relation

$$\text{Secondary moment} + \text{Primary moment} = \text{Resulting moment}$$

It is also possible to consider some reactions as redundant and solve for the values required to produce zero deflections at the supports. This can be done by the classical method of redundant reactions, and sometimes may be simple for a single redundancy. But this method and others will not be discussed here.

Before going any further, it would be well to summarize first the assumptions made for our method of design and analysis. These are the usual assumptions made for continuous prestressed-concrete beams, and their effects on the computed values are known to be negligible in most cases.

1. The eccentricities of the prestressing cables are small compared to the length of the members.

2. Frictional loss of prestress is negligible (where frictional loss is appreciable, it should be taken into account).

3. The same tendons run through the entire length of the member

(varying steel areas can be included with some modifications, which will be evident to the designer once he learns the basic procedure presented herein).

As a result of the above assumptions, analysis can be made on the following bases:

1. The axial component of the prestress is constant for the member and is equal to the prestressing force F.

2. The primary moment M_1 at any section in the concrete is given by

$$M_1 = Fe_1$$

where e_1 is the eccentricity of the c.g.s. with respect to c.g.c.

On these bases, the procedure of analysis can be formulated as follows:

First treat the entire beam as if it had no supports. Plot the moment diagram for the concrete produced by the eccentricity of prestress. Compute the loading on the beam corresponding to that moment diagram; this is the loading produced by the steel on the concrete. Now, with this loading acting on the continuous beam as it is actually supported, compute the resulting moment by moment distribution or other similar method. Referring to Fig. 10-3-3, the various steps will be further outlined as below:

1. Plot the primary moment diagram for the entire continuous beam, as produced only by prestress eccentricity, as if there were no supports to the beam. This is simply given by the eccentricity curve plotted to some suitable scale, as in (b), since $M_1 = Fe_1$, and F is a constant.

2. From the above moment diagram, plot the shear diagram corresponding to it, (c). This can be done either graphically or algebraically.

3. From the above shear diagram plot the loading diagram corresponding to it, (d). This can also be done either graphically or algebraically.

4. Now, for the loading obtained above acting on the continuous beam with the actual supports, and including any singular moments such as might occur at the ends of the beam due to the eccentricity of c.g.s., compute the resulting moments M_2 by moment distribution, (e).

5. The C-line in (a) is now obtained by linearly transforming the c.g.s. line so that it will have new eccentricities e_2 over the supports corresponding to the resulting moments M_2, thus,

$$e_2 = M_2/F$$

Since the C-line deviates linearly from the c.g.s. line, it will have the same intrinsic shape as the c.g.s. line, and can be easily plotted. It

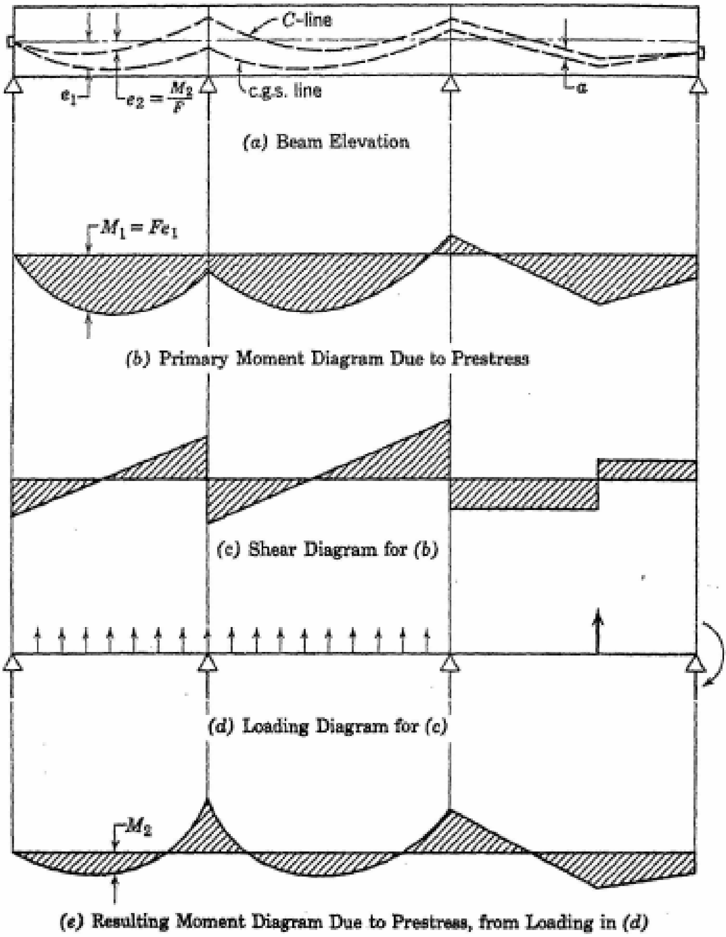


Fig. 10-3-3. Computation of moments due to prestress in continuous beam.

is usually not necessary to compute the secondary moment, which is represented by the deviation between the C-line and the c.g.s. line. If desired, it can be computed by the simple relation

$$\text{Secondary moment} = M_2 - M_1$$

and the deviation a of the C-line from the c.g.s. line is given by

$$a = \frac{M_2 - M_1}{F}$$

Note that the above procedure involves only principles familiar to the engineer except perhaps the plotting of loading and shear diagram from given moment diagrams. While engineers can plot shear from loading diagrams and moment from shear diagrams, which is essentially a process of integration, most are not familiar with the reverse of the process, plotting shear from moment diagrams and loading from shear diagrams, which is essentially a process of differentiation. However, with a little experience, the art can be easily mastered. In fact, very often it is not necessary to plot the shear diagram, since the loading diagram can be obtained directly from the moment diagram.

In order to facilitate the plotting of loading diagrams directly from moment diagrams, the following hints are given for reference, Fig. 10-3-4:

1. At the end of the tendons, the force F from the tendons on the concrete can be resolved into three components:

A. An axial force, $F \cos \theta_1 = F$ (since $\cos \theta = 1$), acting at the end of the anchorage. This usually has no effect on the bending moment in a continuous beam but may produce moments in a rigid frame, owing to the axial shortening effect.

B. A transverse force, $F \sin \theta_1 = F\theta_1 = F \tan \theta$, applied at the support and balanced by the vertical reaction from the support directly beneath. This again produces no moment in a continuous beam, unless it is applied away from the support. Its effect in a rigid frame will be small.

C. A moment, $F \cos \theta_1 e = Fe$, acting at the end of the beam. This will produce moments along the entire length of the continuous beam, and it must be included when following the moment distribution procedure.

2. Along the span of a member, where the c.g.s. or the c.g.c. line of the member bends and curves, transverse loads are applied to the concrete. Two common cases can be considered:

A. When the moment diagram takes the shape of a parabolic or a circular curve (note: owing to the assumption of flat curvature, parabolic and circular curves are considered to have the same effect in producing transverse loads), a uniformly distributed load is applied to the concrete along the length of the curve. The total force for each curve is given by the change in slope between the two end

tangents; thus the total force at θ_2 is given by

$$W = F \sin \theta_2 = F\theta_2$$

For practical purposes, the load W can be considered as uniformly distributed along the length of the curve.

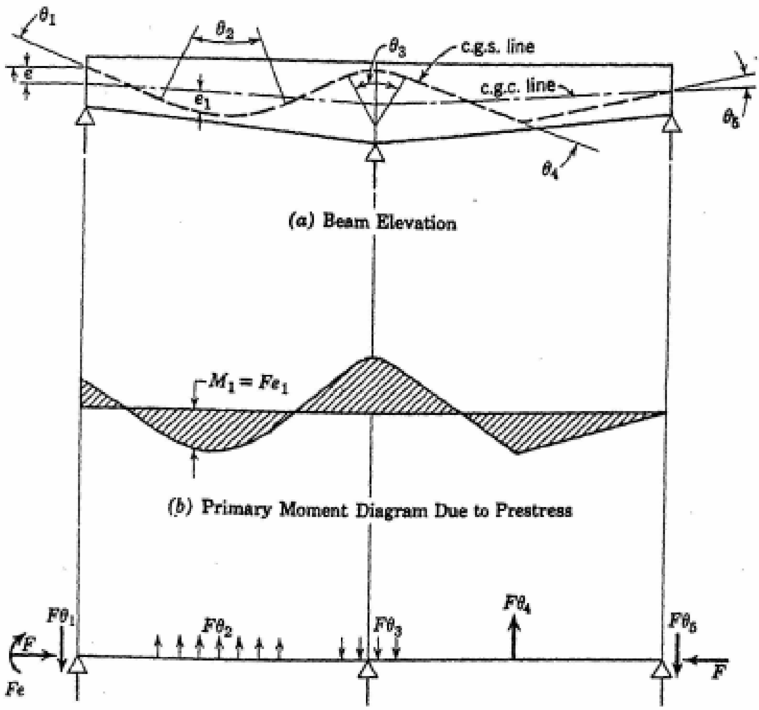


Fig. 10-3-4. Obtaining loading diagrams due to prestress.

B. When the moment diagram changes direction sharply, the force can be considered as concentrated at one point; the amount, at θ_4, for example, is

$$F \sin \theta_4 = F\theta_4$$

3. Over the interior supports, where the moment diagram changes direction, a load is applied directly over these supports. Again two cases can be considered:

A. If the moment diagram curves gradually over the support, again a uniformly distributed load is applied, as shown for θ_3. This will

affect moments in the beam, and the load must be considered in performing the moment distribution.

B. If the moment diagram is bent abruptly over the supports, a concentrated load is applied thereon. Such a concentrated load is directly reacted by the support underneath and produces no moments on the beam. It can be neglected in performing the moment distribution.

Having computed the loads on the concrete, Fig. 10-3-4 (c), we can proceed to determine the bending moments in the concrete, as for any continuous beam. This can be done by any method, but only the moment distribution method will be followed here. The application of moment distribution, which is also based on the elastic theory, necessarily involves other assumptions, such as the validity of Hooke's law, the principle of superposition, and linear variation of strain along the depth of a beam. Although these assumptions may not be exactly correct, the method has been considered accurate enough for reinforced concrete; because of the absence of cracks in prestressed concrete under working loads, the method can be applied with great precision and is considered sufficiently accurate for purpose of design.

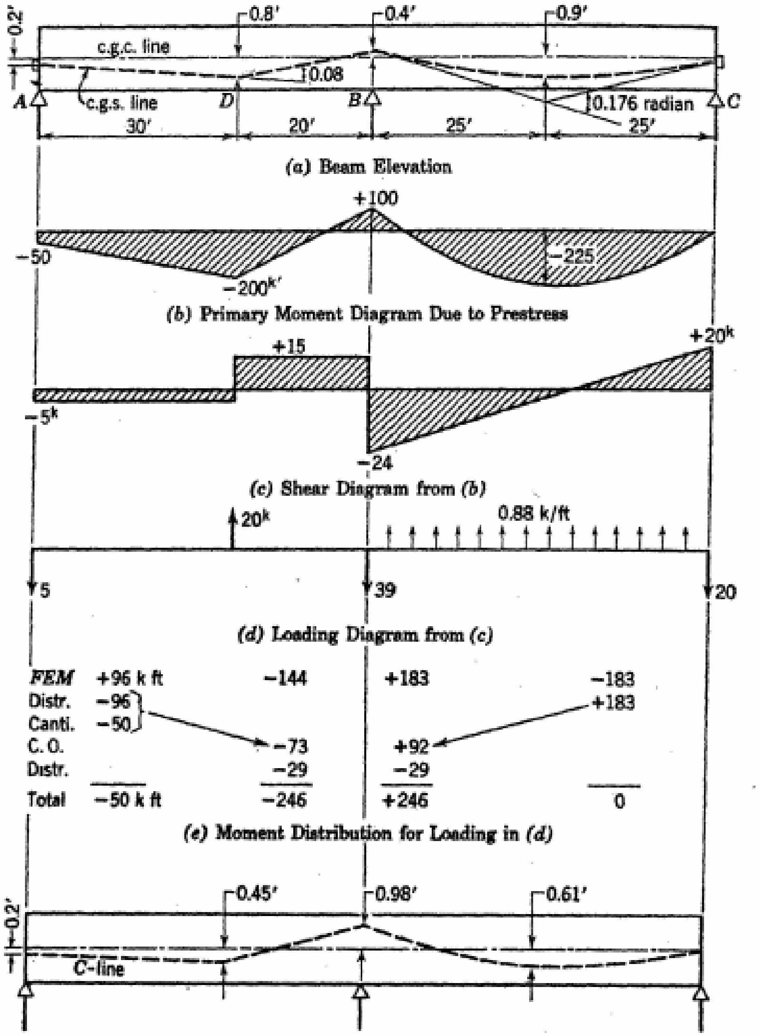
The application of the above method, together with moment distribution, will be illustrated by example 10-3-1. Two points should be noted in the example. First, the example treats of a bonded beam. If the tendons are unbonded, the I of the net concrete area should be used, while the effect of change in prestress in the tendons due to beam curvature should be considered, although the effect is generally small, as previously mentioned. Next, a beam with a straight c.g.c. line is illustrated. Should the beam possess a curved or bent axis, it is only necessary to plot the primary moment diagram by measuring the c.g.s. eccentricity from the curved or bent c.g.c. line instead of from a straight base line.

Example 10-3-1

A continuous prestressed-concrete beam with bonded tendons is shown in Fig. 10-3-5 (a). The c.g.s. has an eccentricity at A, is bent sharply at D and B, and has a parabolic curve for the span BC. Locate the line of pressure (the C-line) in the concrete due to prestress alone, not considering the dead load of the beam. Consider a prestress of 250 kips.

Solution. The primary moment diagram for the concrete is shown in (b). The corresponding shear diagram is computed and shown in (c), from which the loading diagram is drawn in (d). For the loading in (d) acting on the continuous beam, the fixed-end moments are computed: Span AB at A, in addition to 50 k ft singular moment, we have

$$\frac{20 \times 20^2 \times 30}{50^2} = +96 \text{ k ft}$$



(a) Beam Elevation

(b) Primary Moment Diagram Due to Prestress

(c) Shear Diagram from (b)

(d) Loading Diagram from (c)

(e) Moment Distribution for Loading in (d)

(f) C-Line Due to Prestress, from (a) and (e)

Fig. 10-3-5. Example 10-3-1.

at B,

$$\frac{20 \times 30^2 \times 20}{50^2} = -144 \text{ k ft}$$

Span BC,

$$\frac{0.88 \times 50^2}{12} = \pm 183 \text{ k ft at } B \text{ and } C$$

Moment distribution is performed in (*e*). The exterior end moments are first distributed, -96 and $+183$ k ft being obtained. Together with the eccentric moment of -50 k ft at *A*, these are carried over to *B*, obtaining -73 and $+92$ k ft. Now the total unbalanced moment of $+58$ k ft at *B* is distributed, obtaining -29 k ft for each span. The resulting moment is 246 k ft at *B*. The eccentricity of the line of pressure at *B* is, then,

$$246/250 = 0.98 \text{ ft}$$

The line of pressure for the entire beam can be computed by plotting its moment diagram and dividing the ordinates by the value of the prestress. But this is not necessary; since the line of pressure deviates only linearly from the c.g.s. line, it is only necessary to move the c.g.s. line linearly so that it will pass through the points located over the supports, (*f*). (This is known as linear transformation and will be discussed more fully in the next section.) Thus the line of pressure at *D* will be translated upward by the amount of $(0.98 - 0.4)\ 30/50 = 0.35$ ft and is now located at $0.80 - 0.35 = 0.45$ ft below the c.g.c. line. At midspan of *BC*, the line of pressure will be translated upward by the amount of $(0.98 - 0.4)\ 25/50 = 0.29$ ft and is now located at 0.61 ft below the c.g.c. line.

As an exercise, the reader may plot the entire moment diagram for the continuous beam and divide it by the amount of prestress to obtain the line of pressure. Of course it should check exactly with the line here obtained.

If desired, the secondary moment over the center support can be computed as

$$\begin{aligned} M_2 - M_1 &= 246 - 100 \\ &= 146 \text{ k ft} \end{aligned}$$

The above procedure outlines the method for locating the line of pressure due to prestressing in a continuous beam. It is seen that the induced reactions produce moments in a continuous beam, which shift the *C*-line away from the c.g.s. line. Now, when external loads are applied on the beam, additional moments will be produced, and the *C*-line will again be shifted. Two methods of computation are possible:

1. Moments in the continuous beam due to the external loads (including the weight of the beam) are computed by the usual elastic theory, using methods such as moment distribution. These moments are added to the prestressing moments previously calculated, thus yielding the final moments in the beam. This can also be performed by shifting the *C*-line from that obtained for prestressing only. The amount of shifting equals the moments due to external loads divided by the prestress. For simplicity, the effective prestress and the gross concrete area can be used for all computations. This method is generally preferred when there is more than one condition of loading.

2. When there is only one condition of loading to be investigated, it may be easier to consider the effects of prestress and external loading together. Since the effect of prestressing can be reduced to a

system of forces acting on the beam, it is only necessary to add these forces to the external loads to obtain the total loads on the beam. One moment distribution will then be sufficient for the two sets of forces.

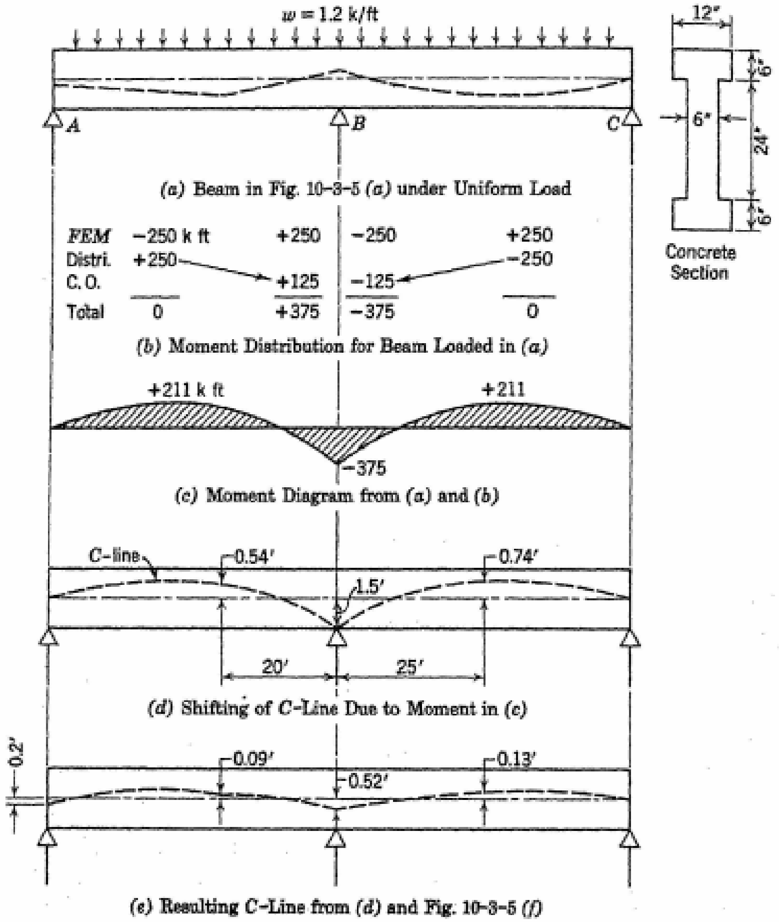


Fig. 10-3-6. Example 10-3-2.

Example 10-3-2

For the prestressed beam in example 10-3-1, a uniform load of 1.2 k ft is applied to the entire length of the two spans (including the weight of the beam itself). Locate the line of pressure in the concrete due to the combined action of the prestress and the external loads. Compute the stresses in concrete at

section B, if the concrete section is as shown in Fig. 10-3-6, with $I = 39{,}700$ in.[4] and $A_c = 288$ sq. in.

Solution. Two methods are possible. Since the moments and line of pressure due to prestress have been obtained in example 10-3-1, we shall now follow the first method, obtaining only the effect of external loads. By a simple moment distribution, see (b), the moment diagram can be plotted as in (c). (Note that for this particular case the moment diagram can be plotted by using ready-made tables from handbooks if desired.) Dividing this moment by the prestress of 250 kips the shifting of C-line due to this external loading is given in (d). Adding (d) to Fig. 10-3-5 (e) of the last example, the final location of the resulting line of pressure for both prestress and external load is given in (e).

The resulting moment in the concrete at section B is $250 \times 0.52 = -130$ k ft, and the stresses are:

$$\text{Top fiber:} \qquad \frac{-250}{288} + \frac{130 \times 12 \times 18}{39{,}700} = -0.867 + 0.707 = -0.160 \text{ ksi}$$

$$\text{Bottom fiber:} \qquad -0.867 - 0.707 = -1.574 \text{ ksi}$$

Note: The reader can try to combine the forces in Fig. 10-3-5 (b) with those in Fig. 10-3-6 (a) and perform one moment distribution to obtain the line of pressure. Obviously he should get the same results.

10-4 Linear Transformation and Concordancy of Cables

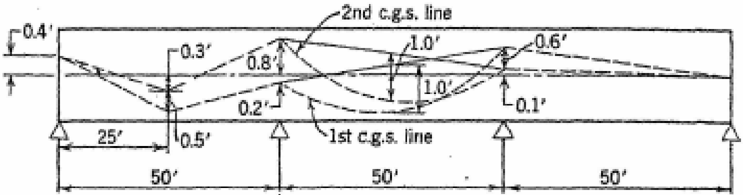
The previous section explains the analysis of prestressed continuous beams; the design of such beams is a more complicated problem. In analysis, the concrete section, the steel, and the location of the steel are already known or assumed. It is only necessary to compute the stresses for the given loading conditions. This is not true in design, which is essentially a trial-and-error process in an effort to reach the best proportions. The designer must be well acquainted with the method of analysis before he can perform efficiently in design. In order to design well, one must be conversant with some of the mechanics of continuous prestressed beams.

In this connection, two terms will be explained first: linear transformation and concordancy of cables.[10-2, 10-6] After a thorough study of this section, the designer should be able to perform linear transformation with ease and skill and to obtain either concordant or non-concordant cables to satisfy the most desirable conditions. First of all, let us define linear transformation:

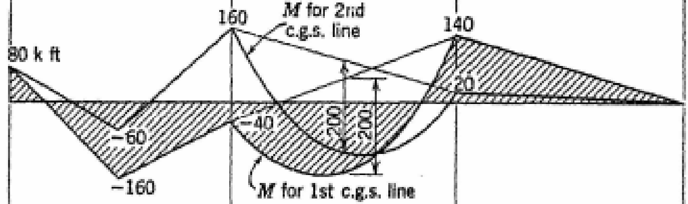
When the position of c.g.s. line or of a C*-line is moved over the interior supports of a continuous beam without changing the intrinsic shape (i.e., the curvature and bends) of the line within each individual span, the line is said to be linearly transformed.* Linear transformation of a c.g.s. line is illustrated in Fig. 10-4-1 (a).

In further explanation of the above definition, attention is called to the following points. First, the position of the line is moved only over

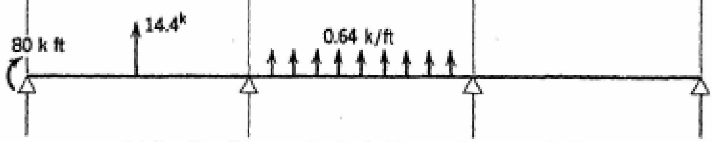
the interior supports whenever desired, but not at the ends of a beam. Strictly speaking, a line can still be termed linearly transformed if it is moved at the ends in addition. However, for purpose of design, linear transformation without involving movement at the ends is much more useful; hence we will define it as such for the sake



(a) Linear Transformation of a c.g.s. Line



(b) Primary Moments Due to Prestress



(c) Loading Diagram for Both Moment Diagrams in (b)

Fig. 10-4-1. Example 10-4-1.

of convenience. Second, by linear transformation, the intrinsic shape of the line within each span remains unchanged; only the amount of bending of the line over the interior supports is changed.

It may be well to remind the reader that one use of linear transformation was described in the previous section, where we stated: "The *C*-line resulting from prestressing a continuous beam is a linearly transformed line from the c.g.s. line," which was explained by the fact that the secondary moment that produces the deviation between the

two lines varies linearly between any two consecutive supports. Now, another interesting theorem concerning linear transformation is that *in a continuous beam, any c.g.s. line can be linearly transformed without changing the position of the resulting* C-*line.* This means that the linear transformation of c.g.s. line does not affect the stresses in the concrete, since the C-line remains unchanged. Thus, the two c.g.s. lines in Fig. 10-4-1 (a) will produce the same C-line and hence the same stresses in the concrete, despite their apparently divergent locations.

The proof of this theorem is as follows. C.g.s. lines having the same intrinsic shape within each individual span will produce primary moment diagrams also having same intrinsic shapes in each individual span. For moment diagrams with the same intrinsic shapes (i.e., the same curvatures and bends), the corresponding loading diagrams along the span are the same, since load is given by the curvature (or second derivative) of the moment. Since the loads are the same, the resulting moments must be the same, which means that the C-lines will have the same position. It must be noted here that, though the resulting moments are the same, the primary moments differ; hence the secondary moments will necessarily differ, since

$$\text{Secondary moment} = \text{Resulting moment} - \text{Primary moment}$$

Any bending of the c.g.s. line over the supports will produce transverse forces acting on the beam which are directly counteracted by reactions from the supports. Hence such bending will not affect the moment along the beam. Since the moment is not affected, the C-line is not affected. Thus, linear transformation involving bending of the c.g.s. line over the interior supports will not change the location of the C-line. On the other hand, any movement of the c.g.s. line at the ends of the beam changes the magnitude of the applied end moments, which do affect the moments along all spans of continuous beam and change the location of the C-line on all spans. Hence linear transformation cannot involve the movement of the c.g.s. line over the ends of the beam or over the exterior support of a cantilever, but can involve movement over the interior supports.

The above theorem, permitting the linear transformation of the c.g.s. line without changing the C-line, offers many possible adjustments in the location of the c.g.s. line which cannot be easily accomplished without that knowledge. Some of these possibilities are evident from the above; others will be discussed later. The validity of this theorem has been proved experimentally within the elastic

range, although such a logical theory hardly needs any experimental proof. The effect of linear transformation of the c.g.s. line on the ultimate strength of continuous beams will be discussed in section 10-6.

Example 10-4-1

The first c.g.s. line in the beam, Fig. 10-4-1 (*a*), is linearly transformed to the second position. Show that the *C*-line in the concrete is the same for both positions. Assume prestress $F = 200$ kips.

Solution. The two moment diagrams are shown in (*b*). The loading diagrams corresponding to those moment diagrams are exactly the same and are both shown in (*c*). Hence the line of pressure must also be the same for both c.g.s. lines. The only forces which are different are those directly over the intermediate and end supports. Since they do not produce any moments in the beam, they do not affect our calculations and are not shown in the figure.

Having defined "linear transformation," let us now define "concordant cable": *A concordant cable in a continuous beam is a c.g.s. line which produces a C-line coincident with the c.g.s. line.*

In other words, a concordant cable produces no secondary moments. Thus, every cable in a statically determinate structure is concordant, because no external reaction is induced, and there is no secondary moment in the structure. For a continuous beam, on the other hand, external reactions will usually be induced by prestressing. These reactions will produce secondary moments in the beam, and the *C*-line will shift away from the c.g.s. line. When this happens, the cable is termed non-concordant. When, by chance or by purpose, no reactions are induced in a continuous beam by prestressing, then there will be no secondary moments and the cable is a concordant one. When a concordant cable is prestressed, it will tend to produce no deflection of the beam over the supports, and hence no reactions will be induced (not considering the weight of the beam). The essential differences between a concordant and a non-concordant cable are shown in Fig. 10-4-2.

One interesting corollary can be observed from the above discussion. When the *C*-line is obtained from a non-concordant cable, that *C*-line itself is one location for a concordant cable, Fig. 10-4-2 (*b*). This is evident from the theory of linear transformation. The *C*-line due to prestressing is itself a line linearly transformed from the c.g.s. line. Thus a new c.g.s. line along that *C*-line will not change the original *C*-line. Hence the new c.g.s. line coincides with its own *C*-line and is a concordant cable.

Besides the fact that a concordant cable line is easier for analysis, there is practically no other necessity for using a concordant one. There were at first some doubts as to the behavior of a non-concordant

cable, whether its C-line would not change with time or with the elastic and plastic properties of concrete. A little thinking on the subject would clear these doubts. If E_c of the entire beam changes uniformly, there will be no change in the secondary moments, since the secondary moments due to prestressing are computed independent of the E_c value, as, for example, by moment distribution. If the E_c of one portion of concrete changes at a different rate from that of another portion, slight changes in the secondary moments might

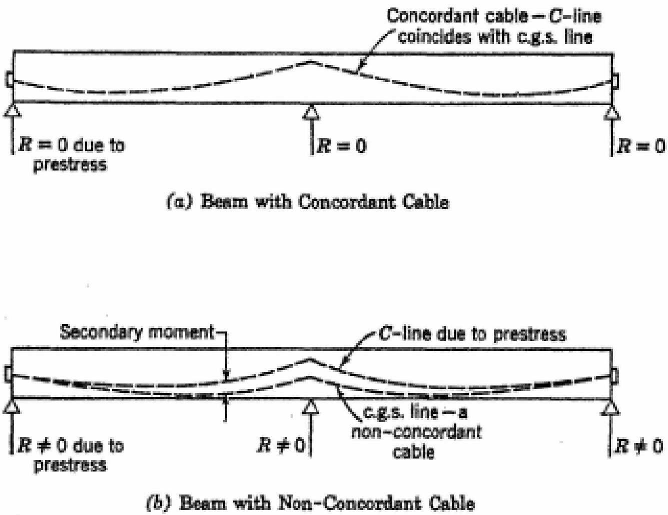


Fig. 10-4-2. Properties of non-concordant cables.

result, but such effects are generally considered negligible, since the elastic theory assuming uniform E_c for a beam is believed to be sufficiently accurate for the analysis of both reinforced and prestressed concrete.

While no significant reason can be given for preferring a concordant cable, there is even less justification for locating a non-concordant cable for the sake of non-concordancy. The real choice of a good c.g.s. location depends on the production of a desirable C-line and the satisfaction of other practical requirements, but not on the concordancy or non-concordancy of the cable. A concordant cable, being somewhat easier to compute, is slightly preferred, other things being equal.

A convenient procedure in design is to obtain a concordant cable that gives good positions of the c.g.s. in resisting the external moment.

If that location falls outside the beam it can be linearly transformed to give a more practical location without changing its C-line. According to this procedure, the finding of locations for concordant cables becomes a useful means to an end.

Several methods have been proposed for obtaining concordant cables, but the author advocates the following method, utilizing only one basic theorem as follows: *Every moment diagram for a continuous beam, produced by any combination of external loadings, whether transverse loads or moments, plotted to any scale, is one location for a concordant cable in that beam.* The application of this theorem is illustrated in example 10-4-2.

Example 10-4-2

For a continuous prestressed-concrete beam loaded as shown in Fig. 10-4-3 (a), obtain some desirable locations for concordant cables to support that loading.

Solution. Note that every moment diagram plotted to any scale is a concordant cable. If we plot the continuous beam moment diagram for the given loading, we obtain (b). Two concordant cable locations are shown in (c) and (d), both proportional to the moment diagram in (b), and hence both are concordant. (f) gives another location of a concordant cable, which is proportional to the moment diagram for loading in (e). Many similar concordant cables can be found by drawing all kinds of moment diagrams. The most desirable concordant cable will be governed by practical requirements of the particular problem as well as by the ability of the cable to resist the applied loads. For example, the location in (c) gives larger resisting arms for the steel but may overstress the concrete if the weight of the beam is light, in which case (d) may be a better location. (f) does not suit this particular loading as well but gives a symmetrical layout and may carry other loadings, such as the beam's own weight, more efficiently.

The above theorem can be easily proved. Since any moment diagram due to the loads on a continuous beam is computed on the basis of no deflection over the supports, and since any c.g.s. line following that diagram will produce a similar moment diagram, that c.g.s. line will also produce no deflection over the supports; hence it will induce no reactions and is a concordant cable. The theorem applies only when the beam is under a constant prestress. If the amount of prestress varies along the beam, the application of the theorem must be modified.

Based on this general theorem, many corollaries can be derived, which will help the designer in selecting proper positions for concordant cables. After the designer has mastered the theorem and its corollaries discussed herein, his work of obtaining a concordant cable is reduced to that of finding a proper moment diagram, which is a

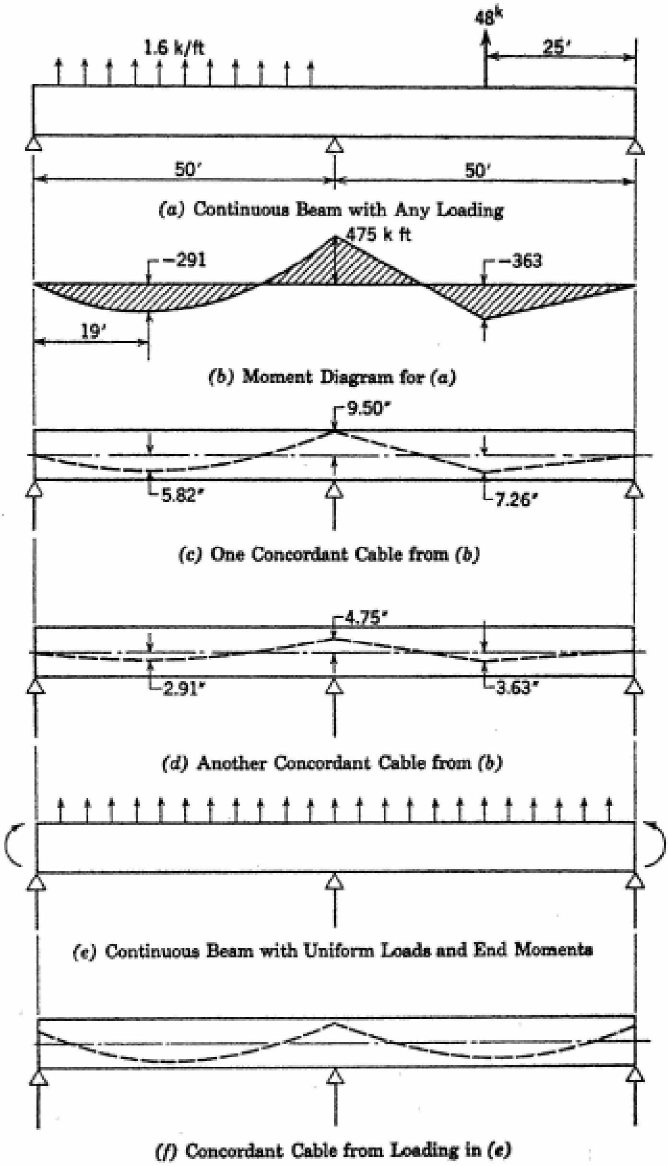


Fig. 10-4-3. Example 10-4-2.

familiar operation with most engineers. Some corollaries will be stated:

1. The reverse of the theorem is also true: The eccentricity of any concordant cable measured from the c.g.c. is a moment diagram for some system of loading on the continuous beam plotted to some scale. This is evident from the fact that any concordant cable coincides with its C-line and any C-line is proportional to a moment diagram produced by some system of loading on the continuous beam.

2. Any C-line is a concordant cable, since it is obtained by computing the moments due to a system of loads on the continuous beam.

3. Superposing two or more concordant cables will result in another concordant cable. Superposing a concordant and a non-concordant cable will result in a non-concordant one.

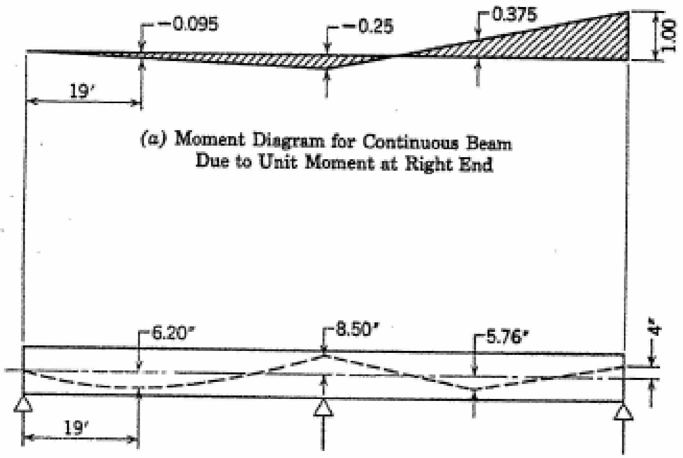
4. When a sudden change in direction is desired, a concentrated load is applied. When a gradual change is desired, a uniform load is applied. One moment diagram can thus be modified into another by the addition of loads. Hence one concordant cable can be easily modified into another.

5. In order to obtain a concordant cable from another by linear transformation involving the moving of eccentricities over the ends of a beam, the following procedure can be used. Apply an end moment on the continuous beam; compute the moment diagram due to that moment. When one end is moved by a given amount, the entire cable must be transformed linearly in proportion to that moment diagram. If the movement of the eccentricities at both ends is desired, apply end moments at both ends proportional to the respective amount of movement, and shift the entire cable in proportion to the moment diagram so obtained. This will yield another concordant cable, as is illustrated in example 10-4-3.

Much ingenuity can be exercised in the location of concordant cables, but it should be left to the skill of the designer after he understands the basic theorem and its main corollaries. When applied to rigid frames, the effect of sidesway and rib shortening should be additionally considered. For varying prestress along the beam, the moment diagram should be divided by the corresponding prestress at each point in order to obtain the location of a concordant cable. Or the tendons may be treated separately. If each individual tendon or group of tendons forms a concordant cable, then, when acting together, they also form a concordant cable.

Like all statically indeterminate structures, it is sometimes desirable to purposely adjust the elevations of the supports in order to produce favorable moments in the beams. The moments so produced are of

a different nature from the secondary moments caused by prestressing. It was previously mentioned that the secondary moments due to prestressing would not change with the value of E_c. Moments due to adjusted support elevations, on the other hand, do change with the value of E_c, because the moments induced by a given displacement are a function of E_c. Hence such moments will change with time as creep takes place and E_c changes. Therefore, when attempts are made to produce moments by support displacements, either the possible change in E_c must be allowed for, or the displacements must be adjusted from time to time. Thus the economy of such a manipulation, though feasible for certain large structures, may be doubtful for small and even medium-sized ones.



(a) Moment Diagram for Continuous Beam Due to Unit Moment at Right End

(b) Concordant Cable from Fig. 10-4-3 (c) with 4 In. Displacement at Right End

Fig. 10-4-4. Example 10-4-3.

Example 10-4-3

Obtain a new concordant cable, with its intrinsic shape the same as that of Fig. 10-4-3 (c), but with the right end of the cable 4 in. above the c.g.c.

Solution. Apply a unit moment at the right end of beam; by the method of moment distribution, plot the moment diagram as in Fig. 10-4-4 (a). The concordant cable in Fig. 10-4-3 (c) can now be linearly transformed in proportion to the moment diagram Fig. 10-4-4 (a), giving a new concordant cable as in Fig. 10-4-4 (b). Note that the same moment diagram (a) can be used to shift the end eccentricity any other amount, not only for the 4 in. illustrated here. Also, owing to the symmetry of the beam, (a) can be similarly used for moving the end eccentricity at the left. A combination of two moment diagrams

due to a moment at each end will permit the simultaneous shifting of both end eccentricities.

An infinite number of concordant cables can be obtained by rotating one concordant cable about the points of inflection, because such rotation simply represents the addition of one concordant cable to another, and should result in a concordant one. The points of inflection in these moment diagrams are called "nodal points" by some European authors.

10-5 Cable Location

Here, again, by cable location is meant the location of the centroid of the tendons, i.e., the c.g.s. line. After the c.g.s. line is determined, the location of the individual position of the various tendons is an easier problem which will not be discussed here.

Designing a continuous prestressed-concrete beam, like that of any other continuous structure, is essentially a procedure of trial and error. Knowledge regarding the analysis of such structures, together with a systematic approach to the solution, will aid greatly in arriving at desired results. The following steps are recommended for designing a continuous prestressed beam:

Step 1. Assume section of members for dead-load computation.

Step 2. Compute maximum and minimum moments at critical points for various combinations of dead, live, and other external loads, Fig. 10-5-1 (*a*). Compute the amount of prestress required for these moments and the corresponding depth of concrete. Modify section of members, and repeat steps 1 and 2 if necessary.

Step 3. Plot top and bottom kern lines for the members, Fig. 10-5-1 (*b*). From the bottom kern line, plot

$$a_{\min} = \frac{M_{\min}}{F} \qquad \text{also} \qquad a_G = \frac{M_G}{F_0}$$

where $M_{\min}$ = the algebraically smallest moment. The distances $a_{\min}$ and a_G should be plotted upward for $-M$ and downward for $+M$.

From the top kern, plot

$$a_{\max} = \frac{M_{\max}}{F} \qquad \text{also} \qquad a_G = \frac{M_G}{F_0}$$

again upward for $-M$ and downward for $+M$.

The shaded area, between the limit of these four lines, obtained by $a_{\max}$ and $a_{\min}$ and a_G, represents the zone in which the line of pressure must lie if no tension is permitted. As in previous discussions on cable location for simple and cantilever beams, when the zone is too wide, an excess of prestress or of concrete section or of girder load is generally indicated. If the limiting line from one kern crosses a

limiting line from another kern, an inadequacy is evident. An ideal layout is obtained when there exists a narrow limiting zone within the beam where the centroid of the cables can be conveniently located.

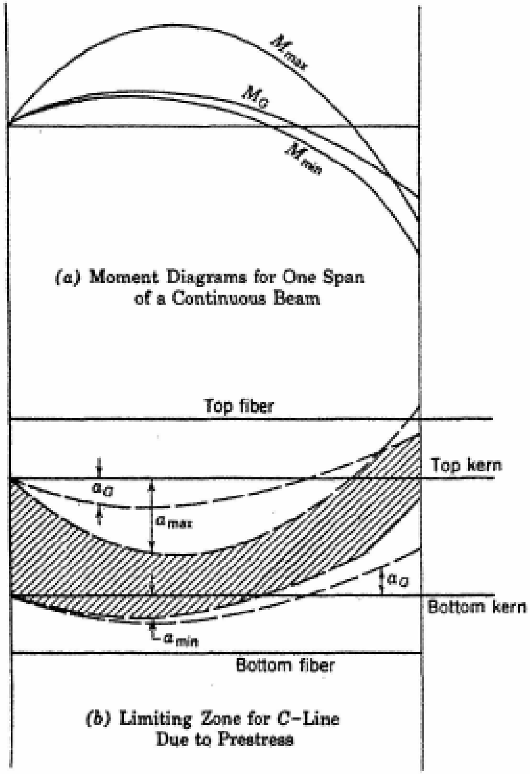


Fig. 10-5-1. Obtaining limiting zone for C-line due to prestress.

Step 4. Select a trial cable location within the above zone. Note that, if the cable follows the shape of some moment diagram, it will be a concordant cable. If this trial location is a concordant cable, it is a satisfactory solution. If it is a non-concordant cable, the C-line can be determined by moment distribution as described in the previous section. If the C-line still lies within the limiting zone, then two locations are possible: either the trial location giving a non-concordant cable, or a new location following the C-line, thus giving a concordant cable. If this C-line lies outside the zone, new cable

locations can be tried. An attempt should be made to get a concordant cable within the zone. It is generally best to try concordant cables, because they coincide with their C-lines and give a more direct solution. Note that, after having obtained one concordant cable, it is much easier to derive from it other concordant cables by the general theorem that any moment diagram for the continuous beam is a concordant cable. Adding to the first concordant cable any form of moment diagram will give another concordant cable. The shape of the added moment diagram can be obtained by applying couples or concentrated and uniform loads anywhere along the continuous beam. Thus it is not a difficult problem to add another moment diagram to the first concordant cable to obtain another concordant one which will lie within the zone.

Step 5. The concordant cable within the limiting zone obtained in step 4 is a good location for resisting the external moment, but it may or may not be a good practical location. For example, it may be desirable to shift the c.g.s. line in order to bring the tendons within the boundaries of the beam. To achieve this without shifting the C-line, the concordant cable can be linearly transformed as desired. This procedure is illustrated in example 10-5-1.

Example 10-5-1

A pedestrian bridge of prestressed-concrete slab (the Harkness Avenue Bridge in San Francisco, California, of the California Division of Highways) has a three-span symmetrical continuous layout as shown, Fig. 10-5-2 (a). The bridge is 9 ft 4 in. wide with a uniform thickness of 13 in. (neglecting curbs). The total effective prestressing force is 1,230,000 lb after deducting a loss of 15%. Design live load is 50 psf. Choose a suitable location for the cable, allowing no tension in the concrete, $f_c' = 5000$ psi.

Solution. Following the procedure described above and considering 1-ft width of slab:

Step 1. The section is already chosen, and the dead load is 162 psf or 162 plf for 1-ft width.

Step 2. The amount of prestress is already chosen; it is 1,230,000/9.33 = 132 kips per ft width of slab for the effective prestress, or 156 kips for the initial prestress.

Step 3. Kern lines for a rectangular section are located at the third points. The maximum and minimum moment diagrams together with the girder moment diagrams are shown in Fig. 10-5-2 (b) for one half of the structure. These diagrams are divided by the respective prestress, F for those with live loads, and F_0 for dead load only. The a values thus obtained are plotted from the kern lines as shown in Fig. 10-5-2 (c), giving the limits for the zone within which the C-line due to prestressing must lie.

Step 4. Using the moment diagrams as guides, select a trial c.g.s. location within the zone as shown in Fig. 10-5-2 (c). For the purpose of illustration, assume the c.g.s. line to possess the following characteristics:

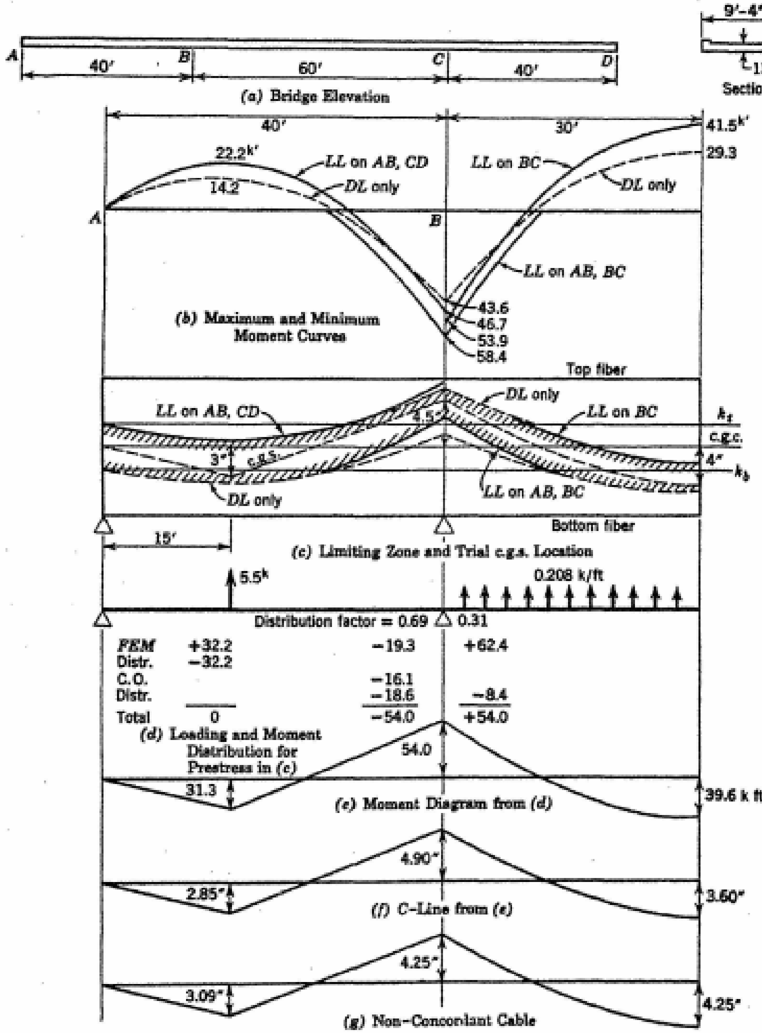


Fig. 10-5-2. Example 10-5-1.

1. Passing through the c.g.c. (mid-depth of slab) at end supports.
2. One sharp bend for each side span.
3. One sharp bend over each intermediate support.
4. A parabolic curve for the center span.

For this c.g.s. location, the corresponding loading on the concrete is shown in (*d*), moment distribution for which gives a moment diagram as in (*e*). Dividing the moment diagram by the prestress yields a *C*-line as shown in (*f*) which

is very close to the trial location and is still within the limiting zone. Hence this C-line is a location for a satisfactory concordant cable.

Step 5. A more practical location for the c.g.s. is shown in (g), affording better protection for the steel. This is obtained by linearly transforming the concordant cable into a non-concordant one. This non-concordant cable will yield the same C-line as the concordant one and hence will serve the same purpose as far as stresses are concerned.

10-6 Cracking and Ultimate Strength

Tests have shown that the elastic theory can be applied to continuous prestressed-concrete beams with great accuracy[10-8] as long as the concrete has not cracked. Occasionally, structures are subjected to overloads beyond the point of cracking or are designed for some permissible cracking under working loads. Then it will be necessary to determine the cracking strength of such structures. In addition, knowledge regarding the ultimate strength of these beams is also of interest in providing criterion for designing. Such strengths will be investigated and discussed in this section.

Since a prestressed structure is nearly a homogeneous material before cracking, the elastic theory can be applied to the calculation of strength up to that point. Even when some cracks have occurred, a prestressed structure is no less homogeneous than a reinforced-concrete structure under working loads. In fact, there is every reason to believe that the elastic theory would be more applicable to prestressed concrete at the start of cracking than reinforced concrete under working loads, since reinforced concrete usually starts to crack at about one-third the working load.

What should be the tensile stress in continuous prestressed-concrete beams at the point of cracking? Some engineers believe that the cracking tensile strength is higher than the modulus of rupture measured from plain concrete specimens. Experiments have shown, however, that the modulus of rupture is a reasonably accurate measure of the start of cracking in continuous prestressed beams. It must be realized that only hair cracks are produced when the modulus of rupture is reached. These cracks, at first, will not be easily visible to the unaided eye but can be detected by strain gages or microscopic examinations.

Before the start of actual cracking, some plastic deformation is usually exhibited in the concrete. Such deformation occurs only in limited regions and does not affect the general behavior of the structure as an elastic body. Hence the validity of the elastic theory can still be counted on, up to and perhaps slightly beyond the cracking of concrete.

Accurate determination of the ultimate strength of a continuous prestressed-concrete beam involves many difficulties. However, for design purposes, the ultimate strength can be estimated on the basis of the limit design theory,[10-9] if plastic hinges are formed at critical points of maximum moment. This is true for under-reinforced sections, which deform extensively before final rupture. For over-reinforced sections, which may suddenly fail in the compressive zone of concrete before any appreciable rotation, a perfect plastic action cannot be expected. The action then is partly elastic and partly plastic.

For the purpose of design, since only an estimate of the ultimate strength is required, the solution need not be complicated. If the beam is under-reinforced, the plastic theory can be applied. If it is over-reinforced, depending upon the degree of over-reinforcing, an interpolation between the plastic and the elastic theories would often give results sufficiently accurate for estimating the factor of safety.

The estimation of the ultimate capacity of a continuous prestressed beam is illustrated in example 10-6-1. The method is based on the plastic hinge theory (or limit design theory) together with the ultimate moment analysis for prestressed-concrete sections (see section 5-6). It is assumed, of course, that only flexural failure, but no shear or bond failure, takes place, and that the sections are under-reinforced.

Example 10-6-1

For the continuous prestressed slab in example 10-5-1, with the c.g.s. located as finally chosen, Fig. 10-5-2 (*g*), compute the ultimate load-carrying capacity for uniform load on all the spans. For effective prestress of 120 ksi, steel area per foot width of slab is 132/120 = 1.1 sq in. f_s' for the steel wire is given as 240 ksi, and the stress-strain diagram is similar to Fig. 2-5-2. $f_c' = 5000$ psi.

Solution. The probable plastic hinge locations are over the intermediate supports, at center of middle span, and near the 0.4 points E and G of the outside spans. These sections are shown in Fig. 10-6-1 (*a*). Let us first determine the ultimate moment capacities of these sections. A little calculation will show that the ultimate strength of steel cannot be developed for these sections. Assume that 90% of ultimate strength of steel is developed for all these sections; the lever arm for the steel at ultimate can be computed as follows:

$$0.90 \times 240 = 216 \text{ ksi}$$

$$216 \times 1.1 = 238 \text{ k}$$

$$k'd = \frac{238}{12 \times 4250}$$

$$= 4.66 \text{ in.}$$

where $4250 = 0.85f_c'$ is the assumed average ultimate compression in concrete. With this computed $k'd$, the strain in steel can be estimated as in (*b*). Corre-

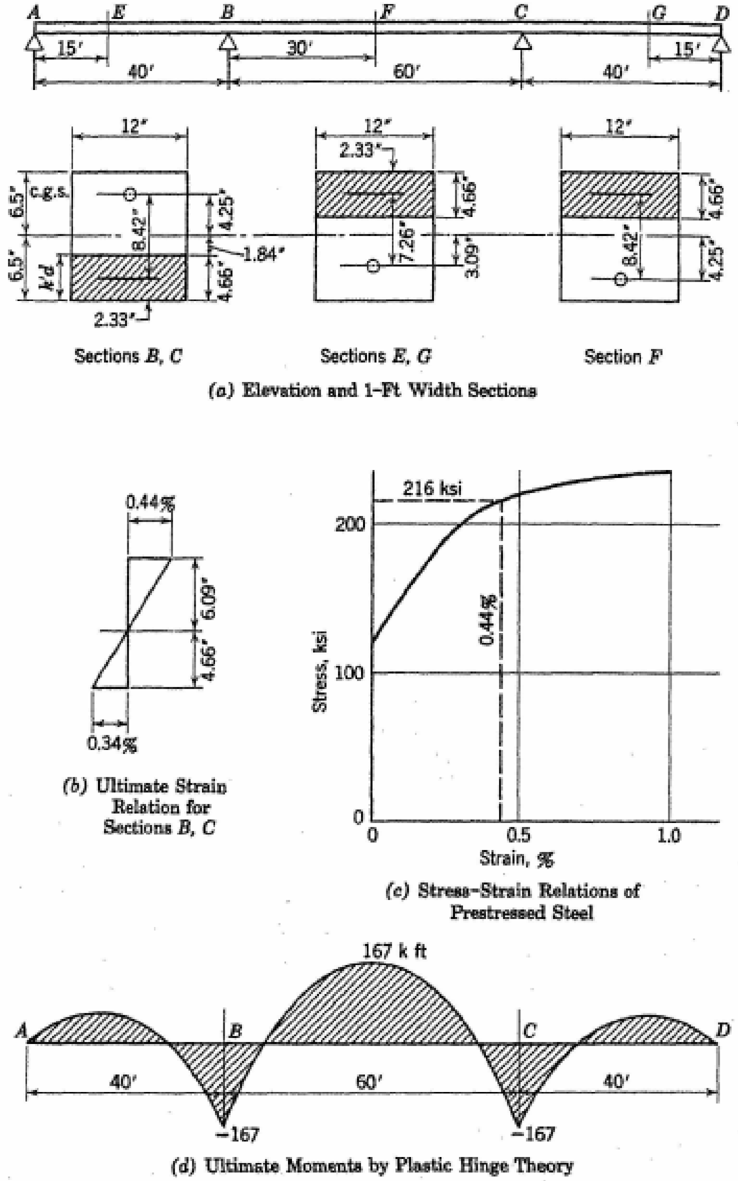


Fig. 10-6-1. Example 10-6-1.

sponding to that strain, the stress in steel, which is about 90% of f_s', can be obtained from (c). No revised computation is necessary. The ultimate capacity of sections B, F, and C is thus

$$238 \times 8.42/12 = 167 \text{ k ft}$$

and of sections E and G, it is

$$238 \times 7.26/12 = 144 \text{ k ft}$$

It must be noted that both of these sections are over-reinforced and will not behave as plastically as under-reinforced sections. According to the elastic theory, maximum moment occurs over the interior supports and is given by

$$-M = 0.0747wL^2$$

$$167 = 0.0747w \times 60^2$$

$$w = 0.621 \text{ klf}$$

By the plastic hinge theory, ultimate failure will occur only when one more hinge than the number of indeterminacy is stressed to the ultimate. For this beam, when hinges form at B, F, and C, the structure will collapse. Corresponding to these hinges, the moment diagram for the center span will be as shown in (d), and the uniform weight on the beam will be

$$\frac{wL^2}{8} = 167 + 167 = 334 \text{ k ft}$$

$$w = \frac{8 \times 334}{3600} = 0.742 \text{ klf}$$

The actual ultimate load may be somewhere between the values given by the two theories, and 700 plf would be a good guess. The factor of safety for both dead and live load together is, hence,

$$700/212 = 3.3$$

It should be noted that according to the plastic theory the ultimate load is affected only by load on the center span and is independent of the load on the side spans in this case.

Tests have been run to prove the validity of the theory of linear transformation in the ultimate range, which can be stated as follows: *Linear transformation of the c.g.s. line does not change the ultimate load-carrying capacity of a continuous beam.* Theoretical proof of this statement has also been made but will not be attempted here. It should be mentioned, however, that the theory is valid in the ultimate range only under the following two conditions:

1. The steel must be sufficiently far from the compressive side of concrete so as not to produce sudden compression failures in the concrete. In other words, the plastic hinges must remain plastic, and the sections must remain under-reinforced.

2. The location of plastic hinges must not be changed as a result of

linear transformation. For uniformly distributed loads and curved cables, linear transformation may change the location of the plastic hinge near midspan and thus modify the ultimate load-carrying capacity. In general, however, such change in location does not affect the strength seriously.

When non-prestressed steel is added to the critical points of a continuous beam, the ultimate moment capacity of these sections is increased. The amount of increase can be figured by some method such as presented in example 9-4-1. Using these increased ultimate strengths, and applying the theory of limit design, or the plastic hinge theory, the ultimate strength of the structure can be figured in the conventional way. Proper addition of non-prestressed steel also helps to distribute the cracks and to increase the shear strength, as well as the fatigue strength of critical sections. Design of continuous beams by ultimate strength and plastic hinge theories is often a simpler procedure than by the elastic theory. However, the elastic theory will still be needed to compute the stresses at transfer, to estimate deflections under working loads, and to compute the cracking loads. The plastic theory will permit a closer estimate of the ultimate load-carrying capacities, and it serves as a good guide for a preliminary design.

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Chapter 11

Slabs

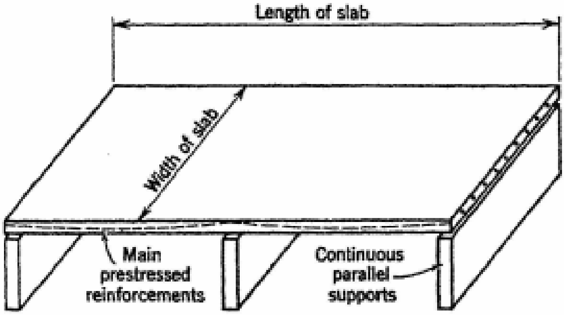
11-1 One-Way Slabs, Transverse Prestress

A one-way slab has main reinforcement only along the length of the slab. All its supports extend the full width of the slab, Fig. 11-1-1, there being no isolated point-supports or supports running along the length of the slab. Occasionally, the supports may be interrupted or stopped before they reach the entire width, in which case the remaining portion will have to be designed for a different condition of support.

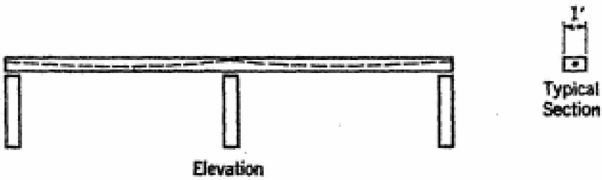
A one-way prestressed slab can be designed similarly to a prestressed beam. The usual procedure is to consider a typical 1-ft width of slab, and treat it as if it were a beam, as is done for a reinforced-concrete one-way slab. Whether the slab is simple, cantilever, or continuous, it is designed like a beam, with identical supports and hinges. Hence all the analysis and design of beams discussed in the previous chapters can be directly applied to slabs without any amplification. For example, the theory of linear transformation and of concordant cables for beams is also valid for one-way slabs.

Although the main prestressing steel runs only along the length of the slab, transverse steel, either prestressed or not, may be added to take care of shrinkage and to distribute any concentration of loads. The design of transverse steel, in both reinforced and prestressed structures, has always been a controversial issue, although the analysis of transverse stresses produced by concentrated loads has been solved both theoretically and experimentally for certain simple cases.[11-1] The main difficulties in design are the choice of a proper amount of concentration, the combination of such concentrations, and the employment of correct allowable stresses in the design. In addition, the simplification of the actual structure in order to suit the conditions of analysis also requires some judgment. But the basic theory followed in the analysis of non-prestressed transverse reinforcement has always been the classical elastic theory, there being no argument about that.

If the transverse reinforcement is prestressed, two additional questions are raised. One question is whether Poisson's ratio effect will have a significant influence on the loss of prestress in a slab prestressed in two directions. If we assume that the concrete has a Poisson's ratio of 0.15 and that the slab is subject to the same prestress in both



(a) **A Two-Span Continuous Prestressed Slab**



(b) **Analysis for Slab in *(a)* Based on an Equivalent Beam 1 Ft Wide**

Fig. 11-1-1. One-way slabs.

directions, corresponding to a loss of prestress due to elastic shortening and creep in concrete of 8%, the decrease in loss due to Poisson's ratio effect will be $0.15 \times 8\% = 1.2\%$, which is not too significant as far as practical design is concerned.

Another question is whether such biaxial prestressing will change the basic strength and strain characteristics of concrete, so that they can no longer be predicted by the application of the elastic theory in conjunction with the properties obtained from ordinary test specimens. Regarding this question, there are two schools of thought. Some engineers believe that concrete as a material is more of a com-

plex "solid-liquid" rather than a simple elastic solid so that the laws of thermodynamics, rather than the laws of elasticity of solid, should be applied.[11-2] This belief has been substantiated by several field tests in France, notably the testing of the prestressed runway at Orly. These tests brought forth the belief that, for a slab prestressed in two directions, the cracking strength might be much higher than the value given by the elastic theory. It was believed that, in a statically indeterminate system, there is no relation between the appearance of cracks and the existence of tension equal to the modulus of rupture.

The second school of thought directly contradicts the first. It is believed that, when concrete is prestressed in two directions, its basic behavior and properties have not been changed. Hence the elastic theory, together with the tested values for modulus of rupture, can be applied to statically indeterminate structures, including slabs prestressed in two directions. This was shown by several laboratory tests conducted at the university of Ghent. In one test on a two-way prestressed slab, the elastic theory was shown to be quite accurate for predicting the cracking load.[11-3]

While the above controversy goes on, it is fortunate indeed that no authoritative engineer thinks that prestressed concrete can possess cracking strength appreciably less than that given by the elastic theory. Hence it is agreed that, pending further evidence justifying the application of any new theories, prestressed slabs up to cracking can be safely designed on the basis of the elastic theory.

For narrow one-way slabs the transverse reinforcements are usually non-prestressed, because short prestressing is neither economical nor accurate. When the width is small compared to the span, any concentrated load is assumed to be carried by the entire width of the slab, and little transverse reinforcing is required for load distribution. Generally, the non-prestressed reinforcement required for shrinkage is sufficient also for load distribution.

For one-way slabs with width greater than about 50% of the span, the deflection of the different slices of the slab may vary considerably under concentrated loads. This indicates heavy transverse bending, which must be resisted by reinforcements whether prestressed or non-prestressed. If non-prestressed, the bending moments can be calculated by the usual elastic theory and the proper amount of steel provided as for any reinforced-concrete design. If economic or other considerations justify the use of transverse prestressing, the moments can also be calculated by the elastic theory and the transverse prestress designed by the ordinary procedure for designing prestressed beam sections. This is the present conventional procedure for design-

ing. According to our knowledge of prestressed concrete at this moment, this is believed to be a safe procedure and, if properly applied, should give reasonable results.

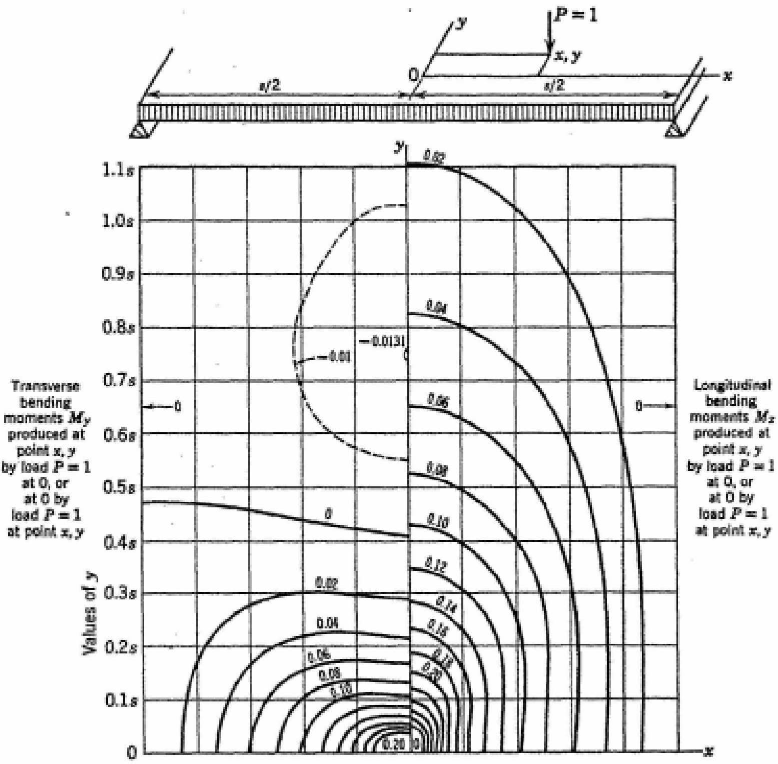
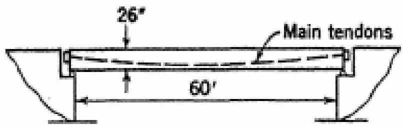


Fig. 11-1-2. Transverse and longitudinal moments in one-way slab due to concentrated load (from reference 11-1).

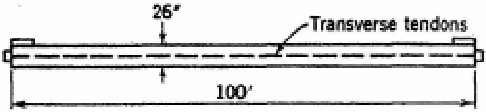
In order to convey some idea of the moments in a wide slab, the longitudinal and transverse moments in a one-way slab under concentrated load[11-1] are given in Fig. 11-1-2. These are based on the elasticity theory as applied to an infinitely wide slab with a Poisson's ratio of 0.15. In such slabs, there also exist torsional moments, but the magnitude is small, and they are not often considered in design.

After the transverse moments to be resisted by prestressing have been computed, the determination of the amount of prestress is a relatively simple matter, it being remembered that, if no tension is allowed, the resisting moment is given by the prestress times its lever

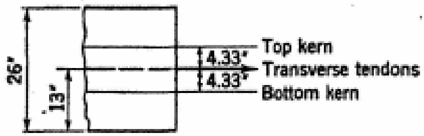
arm measured to the opposite kern point. This simple procedure permits the design of prestressed transverse reinforcements to be made with the same ease as non-prestressed reinforcements. The method is illustrated in example 11-1-1.



(a) Elevation of Bridge



(b) Section of Bridge



(c) Lever Arm for Resisting Moment

Fig. 11-1-3. Example 11-1-1.

Concentric transverse prestressing is often preferred for slabs, although it is not as economical as eccentric prestressing. As can be seen from Fig. 11-1-2, the transverse positive moments are higher than the negative moments. Hence the steel should be positioned farther from the top kern than from the bottom kern, so as to possess a greater lever arm for resisting the positive moments. But such eccentric prestressing will tend to bend the slab transversely, which may be objectionable.

Example 11-1-1

The Bacon Street Highway Bridge in San Francisco has a simple span of 60 ft and a width of 100 ft, Fig. 11-1-3. It is prestressed with an effective prestress of 196,000 lb per ft of width along the 60-ft span. Compute the amount

of concentric transverse prestress required per foot of span. Design for a concentrated load of 16,000 lb and no tensile stress in the concrete.

Solution. Since the width of the bridge is greater than its span, it is close enough to use Fig. 11-1-2 for computing the transverse moments. For a maximum coefficient of 0.20,

$$M = 16{,}000 \times 0.20$$
$$= 3200 \text{ ft-lb/ft width}$$

In order to resist that moment without producing tension in the concrete, the most economical position for the transverse tendons is at the lower kern point. However, if the tendons are located at the lower kern point, the entire slab will be subjected to a negative transverse moment, resulting in a convex bending across the width of the bridge. Moreover, the slab may be subjected to some negative moments under concentrated load, and it would be better to place the cable within the kern. Since the amount of prestress required is small, it will be convenient to locate them through the mid-depth of the slab; then the lever arm to either kern is 4.33 in., Fig. 11-1-3 (*c*). To resist the above moment, the amount of prestress required per foot is

$$\frac{3200 \times 12}{4.33} = 8860 \text{ lb}$$

11-2 Two-Way and Simple Flat Slabs

Though a one-way slab may be prestressed in two directions, it is not a two-way slab, because the transverse prestressing only serves to strengthen the concrete locally but is not intended for carrying any portion of the load to the supports. A two-way prestressed slab is one whose prestressing steels in two perpendicular directions both serve to transfer the load to its supports. Thus a two-way slab generally rests on continuous supports in the form of beams or walls running in two perpendicular directions. When a slab is supported by a network of columns, either with or without capitals, it can properly be called a prestressed flat slab, using that term as in reinforced-concrete construction.

Very little work, actual, experimental or theoretical, has been done on two-way prestressed slabs. The only basis we have for their design is the design of reinforced-concrete two-way slabs, moment coefficients for which are available from building codes on reinforced concrete. When applied to prestressed concrete, the procedure of design can be discussed in two parts: the acting moments due to loads, and the resisting moments provided by the prestressing steel. As far as the load moment is concerned, there is no major difference between reinforced and prestressed two-way slabs. Within the working load, they both behave according to the elastic theory, with the prestressed slabs following it more closely. Although, near the ultimate load, they behave less nearly alike, there is reason to believe that

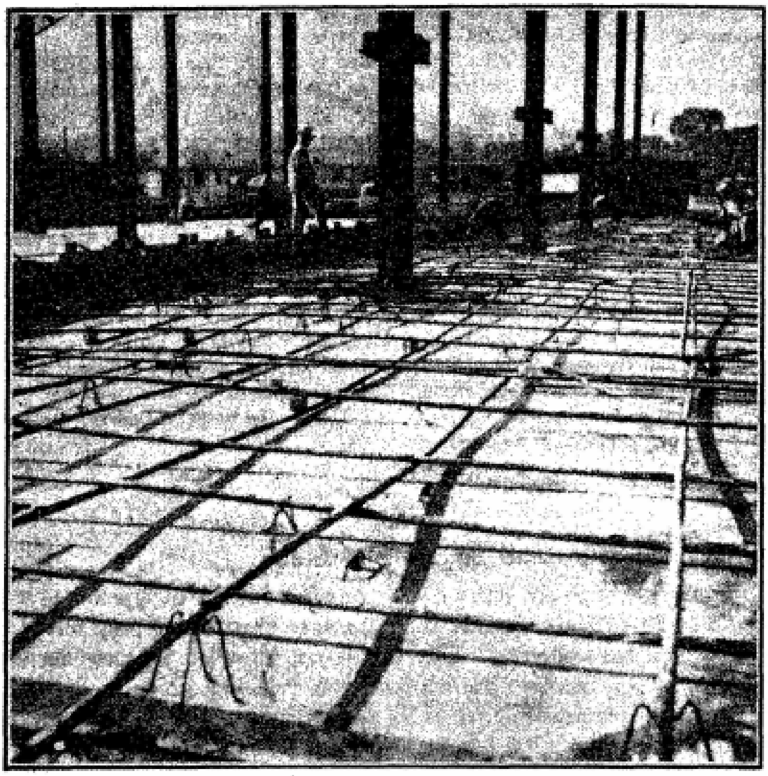


Fig. 11-2-1. Continuous prestressed lift slabs for a state hospital building, Vacaville, California.

the moment coefficients for reinforced concrete, based essentially on elastic analysis, can be used for prestressed concrete without serious adjustments. This does not mean that we are satisfied with these coefficients. In fact, we are not satisfied with them even as applied to reinforced-concrete slabs themselves. However, pending the results of extensive experimental and theoretical investigations, we can with some discretion apply these coefficients to prestressed concrete.

The second part of the problem is to provide the resisting moments. As usual, the resisting moments in prestressed concrete are supplied by the steel acting with a lever arm up to around the kern point. For continuous spans, the resisting couple, instead of being measured from the steel, should be measured from the C-line produced by prestress, the determination of which is a more complicated problem,

although it can be solved by the theory of elasticity or the use of model tests. In this connection, a thorough understanding of the principles discussed in Chapter 10 is essential. Instead of the elastic theory, the application of ultimate design together with proper choice of load factors may also result in satisfactory proportions, although such a method has not yet been developed. It must be remembered that, for prestressed slabs, the initial condition at transfer could be a critical situation that must be examined for overstress in concrete. If the effect of prestress can be reduced to a system of simple loading on the slabs, the theory of elasticity will be found useful for computing stresses in the concrete which are due to prestress.

In contrast to two-way slabs, flat slabs of prestressed concrete supported by a network of columns have already found wide application in this country. This is especially true in connection with lift slabs, where the slabs are cast on the ground and lifted along the columns to their proper height, Fig. 11-2-1. Little experimental or theoretical analysis is available for this type of construction, although many such structures have already been successfully built. It is believed that proper application of the laws of statics and the theory of elasticity, plus a thorough knowledge of prestressed concrete, will enable such slabs to be designed with satisfactory results. It must also be admitted that a great deal of experimental and theoretical investigation is necessary for the purpose of refining our design and arriving at the most economical and safe proportions.

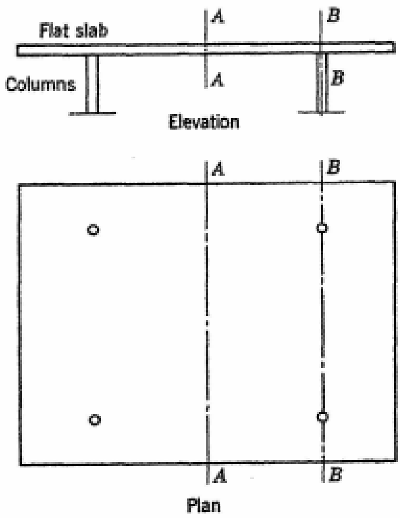


Fig. 11-2-2. A simple flat slab.

Let us first consider a flat slab supported on four columns, Fig. 11-2-2. This is a statically determinate system as far as the reactions are concerned. The total moments across any section, such as A–A or B–B, for example, can be readily determined from statics. But the distribution of the total moments along the length of the section is a problem in elasticity. Such distribution can be obtained theoretically

by the theory of elasticity, or it can be measured experimentally by means of elastic models, such as the Presan method.[11-4] In general, the moments along the columns strip *B–B* will be greater than those along the middle strip *A–A* since strip *B–B* is somewhat stiffer than *A–A*.

The magnitude of the slab moments at each point having been determined, the next step is to provide enough steel to resist the moments. An ideal arrangement would be to provide in both directions exactly the required amount of steel and eccentricity at each point. But this may not be possible in practice, and a reasonably satisfactory solution can be obtained by a good estimation of the distribution of the moments. So long as the total resisting moment equals the external moment, any slight error in distribution is not of serious consequence, since the transverse rigidity of the slab can be depended on to a certain extent to transfer the resistance across the slab. It must be remembered, again, that stresses in the concrete should also be investigated for the initial condition at transfer of prestress.

An example is given in the following, illustrating the computation of steel area for a simple flat slab prestressed in two directions.

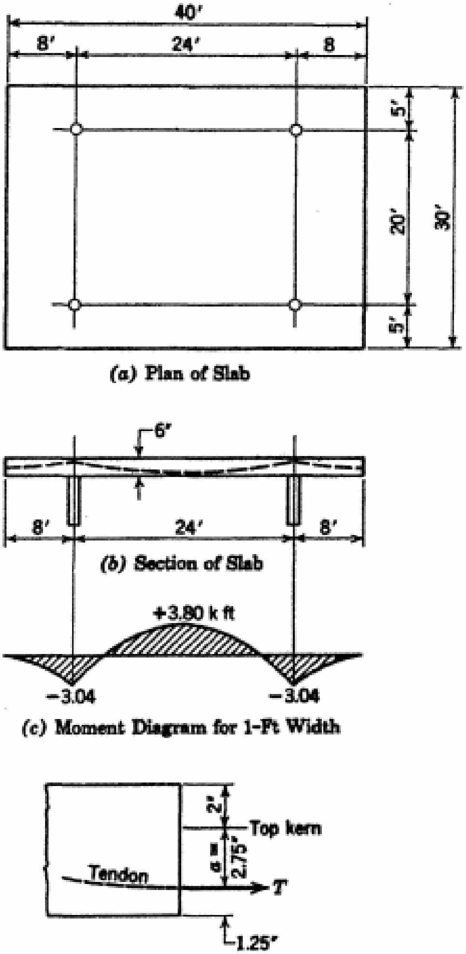
The complete design of such a slab would involve the following:

1. Locating the cable profiles.
2. Spacing the cables.
3. Checking stresses in concrete both at transfer and under working loads.
4. Computation for deflections at various stages, including the effect of plastic flow.
5. Computation of cracking and ultimate loads.
6. Design for end anchorage details.

The reader is referred to other parts of this treatise where these are discussed.

Example 11-2-1

A simple flat slab 40 ft by 30 ft is supported by four columns as shown, Fig. 11-2-3 (*a*). The 6-in. concrete slab weighs 75 psf and carries a roof live load of 20 psf; $f_c' = 4000$ psi; ¼-in. wires grouped in 4 wires per unit are to be used for prestressing in two directions. The cables are greased and wrapped with paper and not bonded to the concrete. Ultimate strength of the wires is 250 ksi, with an initial prestress of 150 ksi and an effective prestress of 125 ksi. Minimum clear coverage for the cables is to be ¾ in., which is equivalent to 1¼-in. protection measured to the center line of the cables. Compute the required number of 40-ft cables per slab.



(*a*) Plan of Slab

(*b*) Section of Slab

(*c*) Moment Diagram for 1-Ft Width

(*d*) Lever Arm *a* for Prestress at Midspan

Fig. 11-2-3. Example. 11-2-1.

Solution.

$$DL = 75 \text{ psf}$$
$$LL = \underline{20 \text{ psf}}$$
$$\text{Total load} = 95 \text{ psf}$$

For the 40-ft direction, the cantilever moment is

$$-wL^2/2 = (95 \times 8^2)/2$$
$$= 3.04 \text{ k ft/ft of width}$$

and the maximum positive moment at midspan is

$$\begin{aligned} wL^2/8 - 3.04 &= (95 \times 24^2)/8 - 3.04 \\ &= 6.84 - 3.04 \\ &= 3.80 \text{ k ft/ft of width} \end{aligned}$$

For the entire width of 30 ft, the moment is

$$3.80 \times 30 = 114 \text{ k ft}$$

The resisting moment is furnished by the steel with a lever arm of 2.75 in. measured to the top kern point, allowing no tension in concrete, Fig. 11-2-3 (d). Hence the total prestress required is, as controlled by the $+M$,

$$(114 \times 12)/2.75 = 497 \text{ k}$$

Each cable has 4 wires with $A = 0.05$ sq in.; hence A_s per cable is 0.20 sq in. For an effective prestress of 125 ksi, each cable has a total prestress of $0.20 \times 125 = 25$ kips. The total number of cables required is

$$497/25 = 19.9$$

Use 20 cables.

11-3 Continuous Flat Slabs

Continuous flat slabs are frequently constructed of prestressed concrete, especially when combined with the lifting process, Fig. 11-3-1. A prestressed slab is lighter than a reinforced one, and its flexibility lends itself to the lifting process. When a reinforced-concrete slab is being lifted, slab levels at the columns must be more carefully controlled to avoid cracking resulting from differential levels. For prestressed concrete, even if cracks did open to some extent, they will be closed up when the lifting is completed. Besides, dead-load deflection in the slab can be largely balanced by the camber produced by prestress. Then there are the saving in formwork and other conveniences inherent in lift-slab construction.

The design of a continuous prestressed slab is based on a knowledge of the design of simple flat slabs as outlined in the previous section. As a result of continuity, two additional problems should be discussed: the negative moments over the interior supports due to loads, and the effect of prestressing a statically indeterminate structure, including the problems of linear transformation and cable concordancy as discussed for continuous beams.

For simple flat slabs, the total moment across any section is definitely known, because all the reactions are statically determinate. The reactions for continuous slabs, however, are statically indeterminate, and hence the total moment across a section cannot be computed from statics alone.

Since prestressed concrete can be treated as a homogeneous and

elastic material in the analysis of moments, the theory of elasticity can be depended upon to yield reasonably accurate results before the cracking of concrete. But to apply a rigid elastic theory to a continuous slab is a very tedious operation which would consume a great deal of time even for a simple case. Hence some easier procedure must be devised for its design.

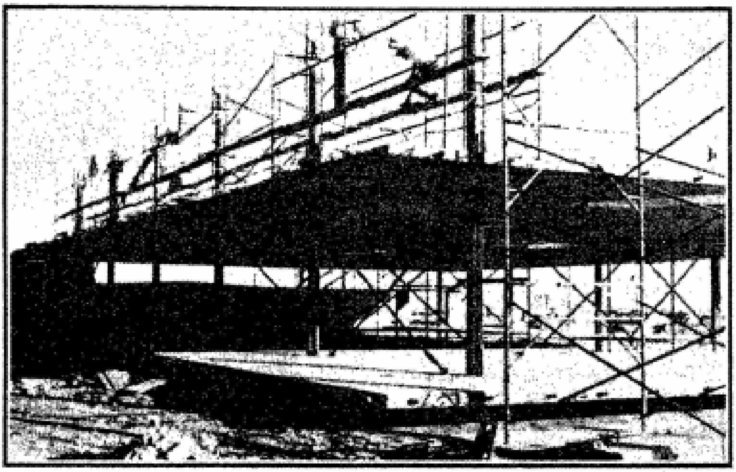
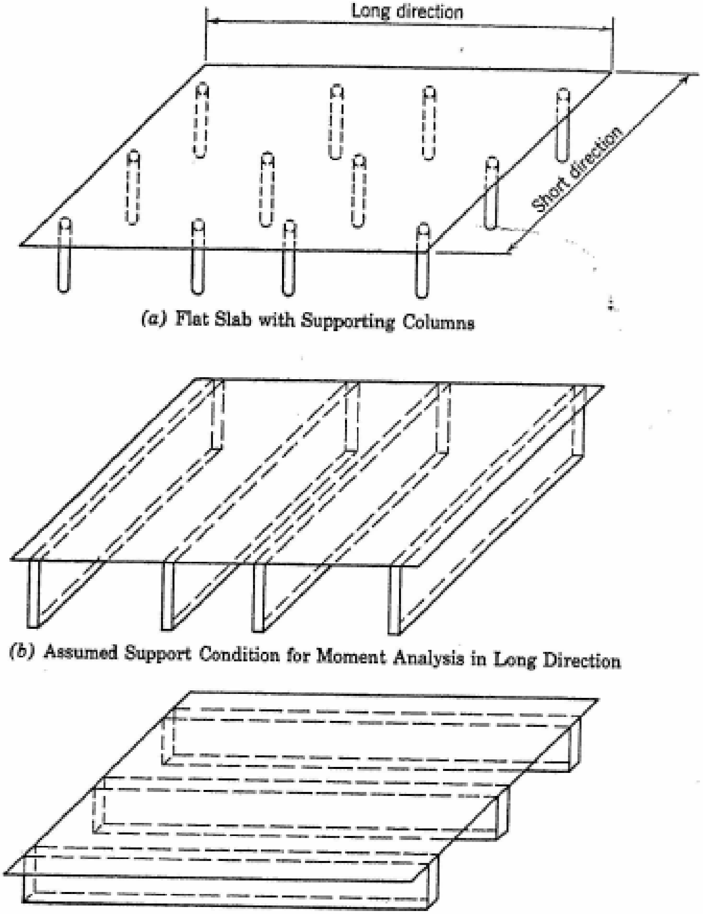


Fig. 11-3-1. Two prestressed slabs being lifted together by electronically controlled equipment, total weight 600 tons (Western Concrete Structures, Los Angeles, California).

No experimental data are available on the analysis of continuous prestressed flat slabs, but some data do exist for continuous reinforced flat slabs, at least to allow semiempirical methods to be used with a sufficient degree of safety and a certain amount of economy. According to most building codes, reinforced-concrete flat slabs can be designed as continuous beams or frames. So far as moment due to external load is concerned, there is as much justification for applying such a method to prestressed flat slabs. The method is illustrated in Fig. 11-3-2, which assumes continuous knife-edge supports along one direction when analysis is being made for moment in the other direction.

A continuous slab having been transformed into a continuous beam, the problem is greatly simplified. The effect of prestressing such a slab can then be computed as for continuous beams. On this assumption, then, it is possible to apply the method of linear transformation and to obtain concordant cables just as is done for a continuous beam.

By analyzing a continuous slab as a continuous beam, the total moment across any section due to loading and the average position of the C-line under prestressing can be obtained. But the distribu-



(a) Flat Slab with Supporting Columns

(b) Assumed Support Condition for Moment Analysis in Long Direction

(c) Assumed Support Condition for Moment Analysis in Short Direction

Fig. 11-3-2. Moment analysis of continuous flat slabs.

tion of the total moment and the variation of the position of C-line along the width of slab still remain to be determined. Approximations have been used, for example, assuming 45% of the total moment to be carried by the middle strip and 55% by the column strip for a flat slab of uniform thickness supported by a regular pattern of col-

umns. The accuracy of such assumptions is not definitely known. Many experimental data and much theoretical investigation are required before we can arrive at rational design criteria.

On the basis of tested high ultimate strength of reinforced-concrete flat slabs, the ACI design code permits using $0.100WL$ instead of $0.125WL$ for the numerical sum of the maximum positive and negative moments in a panel. In other words, it is permissible to design for only 80% of the theoretical moments. When more is known about the ultimate strength of prestressed-concrete flat slabs, it might be possible to economize the design by similar reductions.

Example 11-3-1

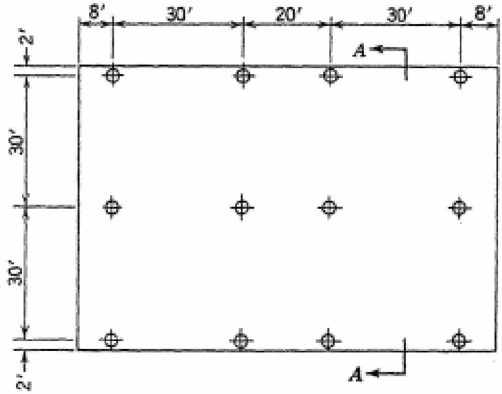
A two-way prestressed lift slab has a plan as shown, Fig. 11-3-3 (*a*) The 7½-in. concrete slab weighs 94 psf and carries a live load of 75 psf; $f_c' = 4000$ psi; ¼-in. wires grouped in 6 wires per unit are to be used for prestressing with ultimate strength of 250 ksi; $f_0 = 150$ ksi; $f_e = 125$ ksi. Minimum coverage for the cables is 1¼ in. measured to the center line. Allowing no tension in the concrete, choose the location for the cables and compute the number of 64-ft long cables required for the slab.

Solution. Assume a cable layout with the maximum possible eccentricities for both the positive and the negative moments, as shown in Fig. 11-3-3 (*b*). This will result in maximum curvature for the cables, and hence the maximum upward force from the cables on the slab. A parabolic cable is used, with an eccentricity of 2.50 in. (corresponding to a concrete protection of 1¼ in.) at the points of maximum positive and negative moments. Note that the maximum $-M$ occurs at 12.4 ft from the exterior supports, which is the lowest point for the parabolic curve. This trial location is not likely a concordant cable but offers the maximum lever arm for the steel at critical points.

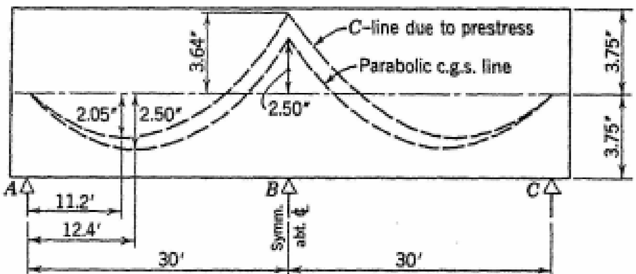
In order to obtain the *C*-line under prestressing for this cable, we can proceed as outlined in section 10-4, for continuous beams. But, for a simple problem like this one, it is not necessary to go through all the steps outlined for the procedure. The *C*-line here can be obtained by inspection after the principles discussed in the previous sections have been mastered. It is noted first that the *C*-line is a curve linearly transformed from the curve of the cable. Then it is seen that, for a parabolic cable on a beam with straight axis, the force from the cable on the slab is a uniformly distributed load. Neglecting the minor effect of the 2-ft cantilevers, the moment diagram for a uniform load on two equal spans is well known, having a value of $wL^2/8$ over the center support and $9wL^2/128$ at 11.2 ft (the ⅜ point) from the exterior supports. This moment diagram, when plotted to proper scale, gives the eccentricity of the *C*-line produced by prestress. Thus the trial parabolic cable is linearly transformed to obtain the *C*-line. A little geometry will show that the position of the cable over the center support is moved upward by 1.14 in. to obtain the *C*-line, and at the ⅜ point is moved up by $\frac{3}{8} \times 1.14 = 0.43$ in., leaving an eccentricity of 2.05 in.

In other words, by the theory of prestressed continuous beams, a cable located as shown in (*b*) will produce the same effect as though it were located through the computed *C*-line. Since it is not practicable to locate the cable through the above *C*-line (too near the top surface over the center support), it is just as

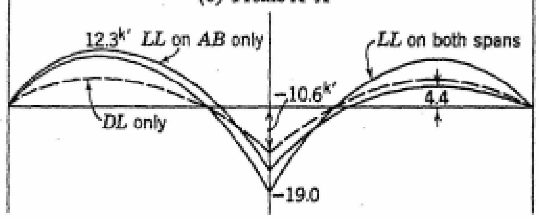
well to place it along the trial line, resulting in the same effect. We will see later that this line of pressure seems to lie in a very desirable location.



(a) Plan of Slab



(b) Profile A–A



(c) Moment Diagrams for DL and LL

Fig. 11-3-3. Example 11-3-1.

Next, let us compute the maximum and minimum moment diagrams, (c). The greatest $+M$ is obtained with live load on its own span only; the greatest $-M$ is obtained with live load on both spans. The smallest $+M$ is obtained with live load on the other span, but the smallest $-M$ with dead load only. For a final location of the cables, the graphical solution explained in section 10-5

should be used. But just to obtain the number of cables, we will compute for the critical points only, i.e., the moments over the center support and near the ⅜ points from the exterior supports.

First, let us design for the total moments. Over the center support, the lever arm available for the resisting couple is measured to the bottom kern point, 1.25 in. below the mid-depth, or $3.64 + 1.25 = 4.89$ in. Hence the effective prestress required is

$$(19.0 \times 12)/4.89 = 46.6 \text{ k/ft of width}$$

For the ⅜ points, the lever arm available is $2.05 + 1.25 = 3.30$ in., and the effective prestress required is

$$(12.3 \times 12)/3.30 = 44.7 \text{ k/ft of width}$$

Hence the moment over the center support controls the design, and a total prestress for the entire slab should be

$$96 \text{ ft} \times 46.6 = 4480 \text{ k}$$

A 6-wire unit of ¼-in. wires will have an effective prestress of $6 \times 0.049 \times 125 = 36.8$ kips; hence the total number of units required is

$$4480/36.8 = 122$$

Note that this number is not too excessive for the $+M$, which would require 117 units, indicating that this is a well-balanced layout.

Now we have to check whether the line of pressure would fall outside the kern under the action of prestress and the minimum moments. The same two critical points as above are chosen for investigation. Over the center support, the minimum DL moment is 10.6 k ft per ft. The initial prestress of the 122 cables will be

$$F_0 = \frac{122 \times 6 \times 0.049 \times 150}{96} = 56.0 \text{ k/ft of width}$$

$$\frac{M_G}{F_0} = \frac{10.6 \times 12}{56.0} = 2.27 \text{ in.}$$

which means that the dead-load moment will bring the C-line from 3.64 in. down to $3.64 - 2.27 = 1.37$ in. above the mid-depth. Since the top kern is only 1.25 in. above the mid-depth, the C-line under dead load only will be $1.37 - 1.25 = 0.12$ in. outside the kern, and some tension will exist in the bottom fiber over the center support under the initial prestress, but the value is evidently small and will be reduced as soon as loss of prestress takes place. Hence this is considered satisfactory.

Now, near the ⅜ points, the minimum moment occurs when live load exists on the other span only, a total moment of 4.4 k ft. Corresponding to the prestress of 56.0 k/ft, this moment will move the C-line upward by the amount of

$$4.4 \times 12/56.0 = 0.94 \text{ in.}$$

This will place the C-line $2.05 - 0.94 = 1.11$ in. below the mid-depth, which is within the kern, and no tension will exist.

Thus, 122 cables with critical points located as above can be considered sufficient. To make a complete design of the slab, it must be remembered that

many related problems, such as those mentioned in section 11-2 for simple flat slabs, must yet be considered. In addition, the sharp bend over the center support may have to be smoothed out. Also note that at the intersection of the two sets of cables in the two directions the maximum lever arm for resisting moment cannot be obtained for both sets.

11-4 Flat Slabs, Some Theoretical Considerations

It must be admitted that at this moment our knowledge concerning the behavior and analysis of prestressed flat slabs is very limited. It will take a great deal of experimental as well as theoretical investigation before we can arrive at the best possible designs. But we do have some basic overall knowledge regarding the computation of the total external moment across a section and the total resisting capacity of that section. Proper use of that knowledge may lead us to approximately correct designs. Some of the more urgent problems which need consideration and investigation are:

A. The degree of accuracy of the continuous beam analysis as applied to flat slabs. Experience has shown that, with the factor of safety we usually have, correct application of the method here presented should yield reasonably satisfactory results. But the degree of accuracy concerning the effect of external loads and of prestress has not been determined either experimentally or theoretically.

B. The proper distribution of the cables among the column and the middle strips. Consider, for example, reinforced-concrete flat slabs. The greater part of the moment is carried by the column strips. How far should that be done for prestressed concrete, especially if there are no drop panels or column capitals?

C. Cracking strength. It is believed that the elastic theory as applied to thin plates can be used to predict the cracking strength of flat slabs, although it was mentioned previously that there are two schools of thought, one for and one against the idea. But it is agreed that the application of the theory of elasticity will not err on the dangerous side. For the use of the designing engineer a simpler method of design is needed, such as the continuous beam analysis. But the accuracy of such an approximate method and its limiting conditions are not yet known.

D. Ultimate strength. The ultimate strength of prestressed flat slabs is little known, although some experiments on simple ones have been performed.[11-3] There are, however, extensive data available on the ultimate strength of reinforced-concrete flat slabs, many of which have been explained by the yield-line theory.[11-5] It is believed that the yield-line theory might be applied to prestressed slabs as well, using the ultimate strength of the prestressed sections. How-

ever, no definite statement can be made until it is confirmed by actual tests.

E. Non-prestressed reinforcements. As in simple and continuous beams, non-prestressed reinforcements in slabs help to distribute the cracks and to increase the ultimate strength. If properly employed, such reinforcements can economically reduce the amount of prestressing. Although few data are available at the present, non-prestressed steel to augment the strength of prestressed slabs at sections of high moments is known to be an economical design for some flat slabs.

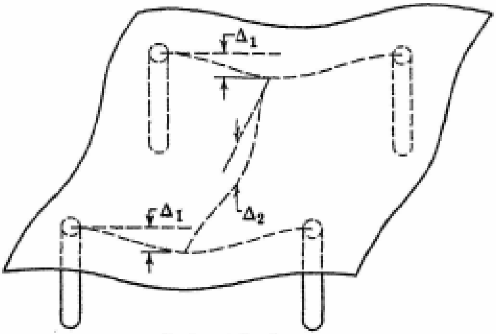


Fig. 11-4-1. Estimating slab deflections.

F. Model analysis. When the layout of the column becomes complicated, it will be almost impossible to apply either the thin plates or the continuous beam theory. Then it may be necessary to resort to model tests, such as the Presan method.[11-4] Such model tests will yield the elastic moments in the slab due to a given system of loads. The moments produced by prestressing can also be obtained, if the eccentricity of prestress can be reduced to a simple system of vertical loads; for example, if the cables are all of the same parabolic shape, they are equivalent to a system of uniform loads on the slab. It is possible to obtain concordant cables for slabs from such model tests. Since every moment diagram is a concordant cable, it follows that the moment diagrams for a slab obtained from the model tests are the basis for concordant cables. In the design of flat slabs, however, concordant cables usually are not the most desirable ones. As is shown in example 11-3-1, the non-concordant cables give a better design. From model tests, it is possible to obtain the C-line for non-concordant cables, provided again that their eccentricities can be reduced to a simple system of vertical loads, which can be conven-

iently used for testing the model. It should be noted that all such tests hold good only within the elastic range, before the cracking of concrete, beyond which ultimate-strength theories[11-5] should be applied.

G. Deflections. Deflections of flat slabs can be obtained by the theory of elasticity, but the time consumed for such an analysis would be enormous. When only approximate results are desired, it is possible to treat strips of the slab as beams and compute the accumulated deflection. For example, the center deflection of a slab is the sum of two deflections, one due to a continuous beam along the columns, another due to a perpendicular continuous beam along the middle, Fig. 11-4-1. If the moments along these two strips are known, the deflections produced by both prestress and external loads can be computed with precision. Model tests may yield more reliable values of elastic deflections. But the effect of plastic flow always has to be considered separately and added to the initial deflections.

11-5 Flat Slabs, Some Practical Remarks on Design

A. Haunched slabs. Up to now, practically all prestressed-concrete flat slabs have been built of uniform thickness. This was largely because they were used in conjunction with lifting. In order to be cast conveniently on the ground with no formwork underneath, it is desirable to employ a flat soffit. If the spans are long, and if the slabs are to be cast in place, it may sometimes be economical to design haunched slabs or slabs with drop panels similar to reinforced-concrete construction.

B. Hollow slabs or waffle slabs. If the spans are long, it often becomes economical to keep the dead load within limits. This is done by hollowing the slab, or by using waffle slabs. The sections are thus either I or T in shape and should be designed accordingly. For area over the columns, these slabs are often made solid in order to carry the heavy shear and the negative moments.

C. Lift collars. Collars for lifting the slabs are of various designs. No tests on them are yet available, and they can be judged only by their past performance or by rational analysis. Figure 11-5-1 shows a collar made by welding angles together.

D. Partition walls for lift slabs. After the prestressed slabs are lifted in position, partition walls may sometimes be constructed beneath them. These partitions actually serve as bearing walls to some extent. The existence of such walls will generally serve to strengthen the slab and to reduce its deflections. When located at odd positions, however, they may tend to increase the moments at certain points, and cracking of the slabs may result.

E. Long slabs. When the continuous slabs are too long in one direction, say much over 100 ft, special problems may arise. First, the friction in the cables may increase appreciably and thus tend to

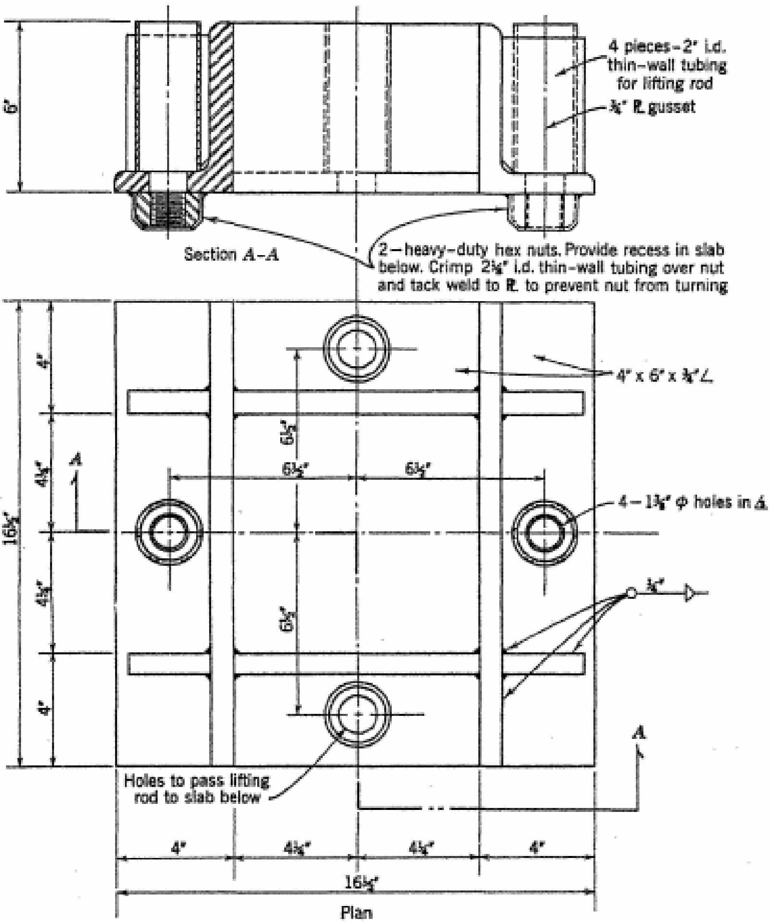


Fig. 11-5-1. Collar for lift slabs.

decrease the effective prestress. Next, there may be excessive shortening of the slab under prestress which may produce bending in the columns if they are rigid. For the steel pipe columns commonly used in lift slab work, such danger is not usually present.

F. Cantilevers. Cantilevering the slabs beyond their exterior row of columns often helps to reduce the maximum bending moment and

saves prestressing steel. But the deflections of such cantilevers under various stages of loading may be excessive and must be studied.

It is evident that this type of construction offers great economies and possibilities, but it is still in its infancy and must be designed with care. Much experience and investigation is needed before the best designs can be worked out. But actual constructions have proved that for the usual proportions the method is practicable and economical if proper care is exercised in the design and execution.

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Chapter 12

Tension and Compression Members

12-1 Tension Members, Elastic Design

Prestressed tension members combine the strength of high-tensile steel with the rigidity of concrete and provide a unique resistance to tension consistent with small deformations that cannot be obtained by either steel or concrete acting alone. The rigidity of prestressed concrete serves well, especially for long tension members such as tie rods for arches or staybacks for wharves and retaining walls. When prestressed, concrete is given strength to resist any local bending and at the same time steel is stiffened and protected.

Several such members have already been designed and constructed both in this country and abroad.[12-1, 12-2] With better understanding of the strength and rigidity of such members, wider application should be found.

The basic behavior of prestressed tension members can be explained from three points of view:

1. The member can be considered as essentially made of concrete which is put under uniform compression so that it can carry tension produced by external loads. If the concrete has not cracked, it is able to carry a total tensile force equal to the total effective precompression plus the tensile capacity of the concrete itself.

2. The member can be considered as essentially made of high-tensile steel which is pre-elongated to reduce its deflection under load. From this viewpoint, the ultimate strength of the member is dependent upon the tensile strength of the steel, but the usable strength is often limited by excessive elongation of the steel which usually takes place at the cracking of the concrete.

3. The member can be considered as a combined steel and concrete member whose strains and stresses before cracking can be evaluated, assuming elastic behavior and taking into account the effect of plastic flow at the same time.

Each of the three points of view furnishes some basic concepts

from which the engineer can visualize his design, but the third viewpoint is most convenient for analysis by the elastic theory and will be explained first.

If the total initial prestress is F_0 and the total effective prestress F, then the stresses in the concrete will be

$$f_c = \frac{F_0}{A_c}$$

for the initial prestress and $f_c = F/A_c$ for the effective prestress.

Because of a load P applied externally, Fig. 12-1-1, both the steel and the concrete will elongate the same amount. Hence the usual

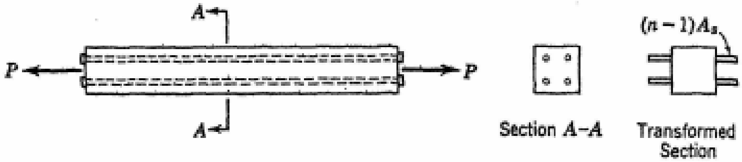

Fig. 12-1-1. Prestressed-concrete tension member.

transformed-section method as applied to reinforced concrete can be applied here. Thus the cross section of the member can be transformed into an equivalent area of concrete equal to

$$A_t = nA_s + A_c \tag{12-1-1}$$

If the gross area of concrete A_g is used, the transformed area can be expressed as

$$A_t = nA_s + A_g - A_s = A_g + (n-1)A_s \tag{12-1-2}$$

This formula is valid only when the section is grouted. Otherwise the hole in the concrete will be greater than A_s and formula 12-1-1 can be more conveniently applied, with A_c referring to the net concrete area.

The stresses produced by P will be, for concrete,

$$f_c = \frac{P}{A_t}$$

and for steel,

$$f_s = \frac{nP}{A_t}$$

In order to be exact, it must be remembered that the value of $n = E_s/E_c$ should be chosen for the proper stress and duration of loading, taking into account the effect of creep if necessary.

Thus the resultant stresses due to the effective prestress plus the external load are, for concrete,

$$f_c = \frac{F}{A_c} + \frac{P}{A_t} \tag{12-1-3}$$

and for steel,

$$f_s = f_e + \frac{nP}{A_t} \tag{12-1-4}$$

If it is desired to determine the load P which will produce zero stress in the concrete, it is only necessary to put $f_c = 0$ in equation 12-1-3, thus

$$\frac{F}{A_c} + \frac{P}{A_t} = 0$$

$$P = -F\frac{A_t}{A_c} = -F(1 + np) \tag{12-1-5}$$

It is seen that, with no stress in the concrete, the load carried by the member is somewhat greater than the effective prestress F. This is because the stress in the steel has been somewhat increased under the action of the external load P.

It is most important to investigate the strains in a prestressed-concrete member, both those due to prestressing and those due to external loads. Under the initial prestress F_0, the stress in the concrete being F_0/A_c, the corresponding instantaneous unit strain will be

$$\delta = \frac{F_0}{EA_c}$$

which will reduce to F/EA_c after the losses have taken place.

Under the action of external load P, the instantaneous strain is given by

$$\delta = \frac{P}{EA_t}$$

In all cases, the value of E must be chosen with regard to the level of stress and the age of concrete, and the effect of creep must be considered.

Let us first compare the magnitude of strains in a prestressed-concrete member with those in an ordinary steel member. For a structural steel member stressed to 20,000 psi, corresponding to a value of $E_s = 30{,}000{,}000$ psi, the unit elongation is

$$\delta = \frac{20{,}000}{30{,}000{,}000} = 0.00067$$

For a prestressed-concrete member, with the stresses in concrete changing from −1000 psi to 0, for an E_c of 4,000,000 psi, the unit strain is

$$\delta = \frac{1000}{4,000,000} = 0.00025$$

which is less than half of the strain in structural steel.

High-tensile steel alone cannot be used for long tension members where elongation must be limited. In order to be stressed to its working strength of 125,000 psi, the unit elongation will be

$$\delta = \frac{125,000}{30,000,000} = 0.00417$$

which is more than 6 times that of structural steel and 16 times that of prestressed concrete in the above example.

Strains in prestressed-concrete members are influenced by several factors. If the precompression in the concrete remains over a period of time, the shortening of that member due to creep could be considerable. Such creep strain, however, would be gradually recovered (though not completely) under the application of an external tension. Since a greater portion of the creep may be eventually recovered, the lengthening of the member under sustained external load may be greater than is indicated by the elastic calculations.

On the other hand, there are ways to limit further the elongation of prestressed-concrete tension members. One obvious method is to increase the cross-sectional area of concrete. For example, if the concrete area is doubled, the stress range will be halved, and so will the strain. However, there is an economical limit to this method, since the area of concrete cannot be indefinitely increased. Another way to control the elongation is to time the application of prestress to the application of the external dead load. If this is carefully done, the elongation due to dead load can really be reduced to a minimum, although practical considerations may not permit such an ideal sequence of application of forces.

Example 12-1-1

A straight concrete member 150 ft long is prestressed with a high-tensile steel strand through the centroid of the section. The strand is anchored to the concrete with end anchorages but separated from it by bond breaking agents along the length. $A_c = 80$ in.^{2}. $A_s = 0.80$ in.^{2}. $f_c' = 4000$ psi, $f_s' = 250,000$ psi, $f_0 = 150,000$ psi, $f_e = 127,500$ psi, $E_c = 4,000,000$ psi, $E_s = 30,000,000$ psi. (*a*) Compute the allowable external load on the member, allowing no tension in the concrete. (*b*) Compute the shortening of concrete due to prestress, assuming a creep coefficient of 1.5. (*c*) Compute the lengthening of the

member due to the external load obtained in (*a*), neglecting creep. (*d*) If the member were designed of structural steel with an allowable stress of 20,000 psi, compute the lengthening under the load. (*e*) Compute the lengthening if the strand is used alone by itself with an allowable stress of 127,500 psi.

Solution. (*a*) From formula 12-1-5

$$\begin{aligned} P &= -F(1 + np) \\ &= 127{,}500 \times 0.80(1 + 7.5 \times 0.80/80) \\ &= 110{,}000 \text{ lb} \end{aligned}$$

(*b*) Under the initial prestress, the shortening of concrete will be

$$\begin{aligned} \frac{F_0L}{E_cA_c} &= \frac{150{,}000 \times 0.80 \times 150 \times 12}{4{,}000{,}000 \times 80} \\ &= 0.675 \text{ in.} \end{aligned}$$

If the effective prestress is considered, the shortening will be

$$0.675 \times \frac{127{,}500}{150{,}000} = 0.573 \text{ in.}$$

If the creep coefficient is based on the effective prestress, the total elastic and creep shortening will be

$$0.573 \times 1.5 = 0.860 \text{ in.}$$

(*c*) Under the external load of 110 kips, for a transformed area of $A_t = 80 + 7.5 \times 0.80 = 86$ in.^{2}, again using $E_c = 4{,}000{,}000$ psi, the lengthening of the member will be

$$\frac{110{,}000 \times 150 \times 12}{4{,}000{,}000 \times 86} = 0.575 \text{ in.}$$

this checks closely with the shortening of the concrete computed in (*b*).

(*d*) For a structural steel stressed to 20,000 psi, the elongation will be

$$\frac{20{,}000 \times 150 \times 12}{30{,}000{,}000} = 1.20 \text{ in.}$$

(*e*) For high-tensile steel stressed to 127,500 psi, the elongation will be

$$\frac{127{,}500 \times 150 \times 12}{30{,}000{,}000} = 7.65 \text{ in.}$$

12-2 Tension Members, Cracking and Ultimate Strengths

The previous section discusses the computation of stresses in a prestressed-concrete tension member, up to zero compression in the concrete. The design of such a member may or may not be made on this basis, depending upon the probable amount of overloading to which the member may be subjected. In order to get a sufficient factor of safety, it may be necessary to design the member so that, under working loads, there will always be some residual compression

in the concrete. This will become evident after a study of the cracking and ultimate strengths of the member. Tension members are one of the typical cases in prestressed concrete where design by the allowable stress method may err very much on the dangerous side and may not yield consistent results.

Generally speaking, prestressed-concrete tension members have a very low reserve strength above the point of zero stress. If the member is not cast as one piece, e.g., if it is made up of blocks, cracking may coincide with zero stress. Then any additional load on the member will be carried by the steel alone. Since the prestressing steel has a relatively small area of cross section, excessive elongation will immediately start at the cracking of concrete, and failure of other parts of the structure may result. For such a member, then, it is evident that a considerable amount of residual compression is necessary in order to ensure safety, the amount being governed by the magnitude of the probable overloads.

If the member is cast as one piece and if shrinkage and other cracks have not occurred, it will be able to take some tension before cracking. The direct tensile strength of concrete is variable and generally ranges from 0.06 to $0.10f_c'$. Thus, for a concrete of 4000 psi, the tensile strength may be from 240 to 400 psi, which may provide a good margin of safety if the strength exists and has not been destroyed. But, once the concrete has cracked, the margin of safety is gone. In fact, failure of the entire structure may result as soon as the concrete cracks, because at this moment the tensile load carried by the concrete in tension is suddenly transferred to the steel. Thus there may be a sudden elongation of steel which may have serious effects, even though the ultimate strength of the steel is far from being reached.

The above discussion must not be construed to mean that such tension members are unsafe. They are just as safe as any other type of tension members and perhaps safer if properly designed. When heavy overloads are possible, they should not be designed on the basis of allowable stresses, but rather on the basis of the cracking or ultimate strength, with proper load factors.

The choice of suitable load factors will be discussed in Chapter 14, but it can be briefly mentioned here that such a choice will depend upon the possibilities of overloading. If dead load predominates in a member, any serious increase in loading is not so likely. Thus, in most buildings and long-span bridges, the load factor required will be smaller than in a short bridge subject to possible heavy overloadings. This is where an engineer must exercise his judgment prudently to get a safe and economical design.

Example 12-2-1

For the tension member in example 12-1-1, what working load can it carry, using a factor of safety of 2.0 against the cracking of concrete, assuming the direct tensile strength in concrete to be $0.08f_c' = 320$ psi? Compute the residual compression in concrete under that working load.

Solution. From formula 12-1-3, for $f_c = 320$ psi,

$$\frac{F}{A_c} + \frac{P}{A_t} = 320$$

From example 12-1-1,

$$F = -102{,}000 \text{ lb}$$
$$A_c = 80 \text{ sq in.}$$
$$A_t = 86 \text{ sq in.}$$

Substituting,

$$\frac{-102{,}000}{80} + \frac{P}{86} = 320$$
$$P = 137{,}000 \text{ lb}$$

which is the cracking load.

For a factor of safety of 2.0, the working load will be

$$137{,}000/2 = 68{,}500 \text{ lb}$$

Though the load factor of 2.0 is not always necessary, the great difference between this answer and the last one of 110 kips should be noticed.

The residual compression can be computed using the same formula, for $P = 68{,}500$ lb,

$$f_c = \frac{F}{A_c} + \frac{P}{A_t} = \frac{-102{,}000}{80} + \frac{68{,}500}{86}$$
$$= -1275 + 795 = -480 \text{ psi}$$

12-3 Column Action due to Prestress

The question is often brought up whether a concrete member under prestress will have a tendency to buckle like an ordinary column under compression. The answer is that, if the prestressing element is in direct contact with concrete all along its length, there will be no "column action" in the member due to prestress.

Consider an ordinary column under an external load, Fig. 12-3-1 (*a*). When the column deflects, additional moment in a section *A–A* is created by the deflection, because the external load now acts with a different eccentricity on that section. This additional moment is the cause of column action. Now consider a member internally prestressed but not externally loaded, (*b*); so long as the steel and con-

crete deflect together, there is no change in the eccentricity of the prestress on the concrete, no matter how the member is deflected. Hence there is no change in moment due to any deflection of the

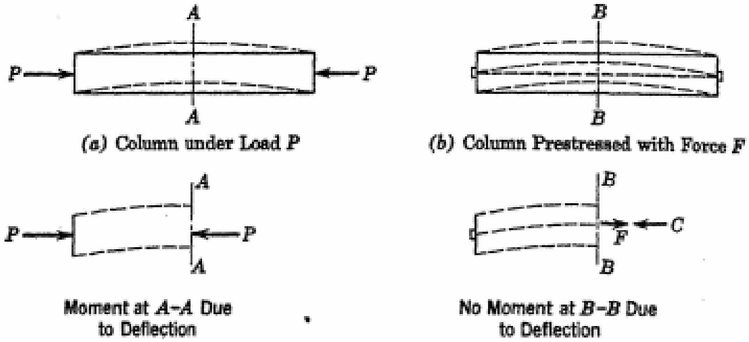

Fig. 12-3-1. Column action due to prestress.

member and no column action. When an external load is applied to a prestressed-concrete column, any deflection of the column will change the moment, and column action will result.

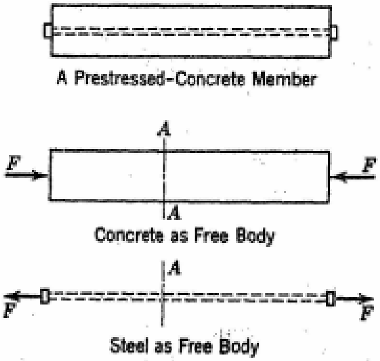

Fig. 12-3-2. Balancing action of concrete and steel.

Another way to look at the problem is to separate the steel from the concrete and treat them as two free bodies, Fig. 12-3-2. Considering the concrete alone, it is a column under direct compression, and any slight bending of the column will result in an eccentricity on a section such as A–A, and hence in a tendency to buckle. But, considering the steel as a free body, there will exist an equal eccentricity with an equal but opposite force, producing a tendency to straighten itself out. The tendency to straighten is exactly equal and opposite to the tendency to buckle, and hence the resulting effect is zero. This is not true, of course, when the member is externally prestressed, say against the abutments, because there will be no balancing effect from the prestressing element, and column action will result.

If the steel and concrete are not in direct contact along the entire

length, the problem will be different, Fig. 12-3-3. The concrete under compression will have a tendency to deflect laterally. That deflection will not at first bring the steel to deflect together with it; hence the eccentricity of prestress on the concrete is actually changed, thus resulting in column action. After a certain amount of deflection, the

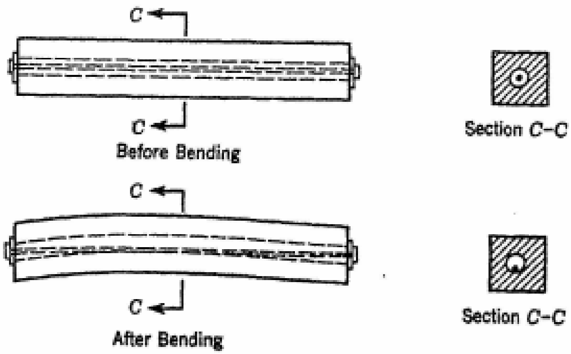

Fig. 12-3-3. Steel and concrete in contact after bending.

steel is brought into contact with the concrete and the two will begin to deflect together. Hence the column action is limited to the differential deflection of the two materials.

If the steel is in contact with the concrete at several points, say at E and F, but not along the entire length, Fig. 12-3-4, then the column action is limited to the length between the points of contact. If such length is short, column action will not be serious.

Next, consider a curved or a bent member subject only to internal prestress, Fig. 12-3-5 (a). If the prestress is concentric at all sections (the c.g.s. line coinciding with the c.g.c. line), then the concrete is behaving like an arch subject to axial force with the exception that the applied force from the steel will move with the deflection of the concrete and will

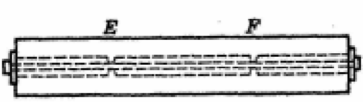

Fig. 12-3-4. Steel and concrete in contact at several points.

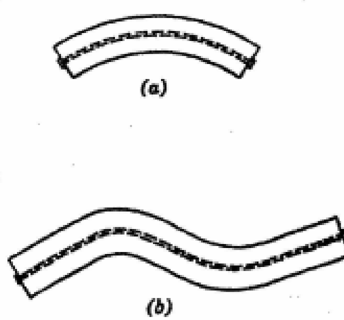

Fig. 12-3-5. Bent members under concentric prestress.

always remain concentric. Hence there is no tendency to buckle as in an ordinary arch under external loads, whose line of pressure is determined by the loads and may not shift together with the deflection of the arch. As an extreme example, even if the member has a reverse curve, Fig. 12-3-5 (*b*), the application of concentric prestress will not tend to straighten the member. If the prestress is eccentric, as on sections *G* and *H*, Fig. 12-3-6, the compression in the concrete is still equal and opposite to the tension in the steel. Any deflection of the member will still displace both of them together, and there will be no column action due to prestress. The effect of an eccentric prestress on the concrete, however, will produce deflection of the member. If the deflection is appreciable, the deflected axis of the member should be used in computing column effects due to external loads.

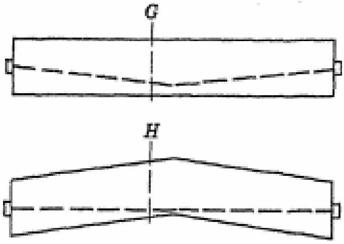

Fig. 12-3-6. Eccentric prestress and column action.

As far as column action is concerned, it is immaterial whether there is any frictional loss along the length of the prestressing tendon, because the tension in the steel is always balanced by the compression in the concrete at any section, whatever frictional losses may occur. Hence, whether there is frictional loss or not, there will be no column action due to prestress.

12-4 Compression Members

A prestressed-concrete compression member is one that carries external compressive load. A member that is simply compressed by its prestress is not a compression member. As explained in the previous section, a prestressed member is not under column action due to its own prestress, but it is subject to column action under an external compressive load just like a column of any other material.

It is seldom that a prestressed-concrete member is utilized to stand compression and is prestressed for compression's sake. Evidently, concrete can carry compressive load better without being precompressed by steel. And it is difficult to conceive of steel wires as adding any appreciable strength to a member carrying axial compression. However, many compression members, besides carrying direct compressive loads, are subject to transverse loads as well. Bending due to these transverse loads may more than offset the axial compressive stress at certain points, so as to produce some resulting tension in the

concrete. Then it will be advisable to reinforce such flanges for possible tension. In other words, some compression members are actually flexural members, and all the advantages of prestressing a beam would apply to the prestressing of those members.

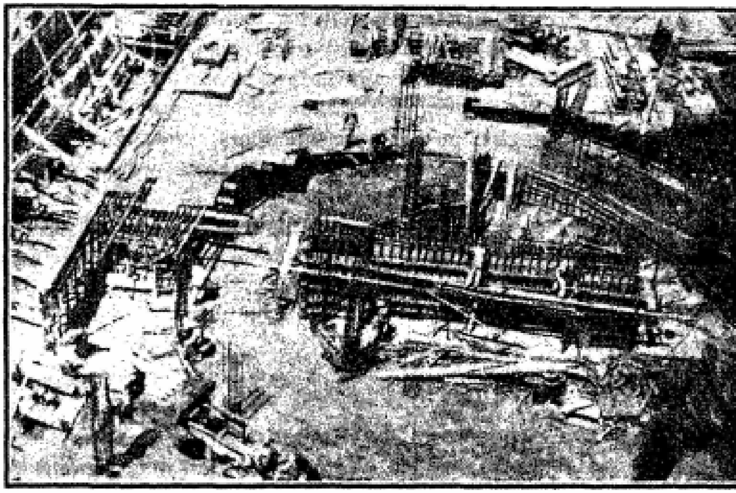

Fig. 12-4-1. Construction of prestressed-concrete pylon to resist earthquake force in a 10-story garage, San Francisco, California. (*Eng. News-Rec.*, March 31, 1955, p. 39.)

Consider an industrial building of one story, for example; the columns or bearing walls may carry only light vertical loads. But they may be subject to bending during handling and erection if they are precast, or they may carry lateral force such as that due to wind and earthquake after the completion of the building.[12-3] Similar conditions may exist in bridges.[12-4] Then it is often feasible to precompress the member so that it can stand a certain amount of bending.

One beneficial effect of prestressing a compression member is the reduction of its deflection under transverse loads. One such pylon, 100 ft high, Fig. 12-4-1, was prestressed to resist an earthquake load of 2450 kips applied horizontally along the pylon. Since the deflection of an uncracked section is about 40% that of a cracked section, a prestressed pylon could be about 2.5 times as stiff as an ordinary reinforced one. In this instance, the reduction of deflection at the top of the pylon minimizes the relative movements between the building floors and saves a tremendous amount of steel otherwise required to reinforce other parts of the building.

Within the working range, the stresses in a prestressed compression member due to both prestress and external loads can be computed from the usual elastic theory. But the design of the member is another question, because the empirical methods for designing reinforced-concrete columns cannot be directly applied to prestressed ones. The stresses ordinarily allowed for reinforced concrete are not applicable to prestressed concrete, partly because the stresses due to internal prestressing are of different nature from those due to external loads, the latter having column action, and the former not. For proper design of prestressed-concrete members, one must go into basic theories of columns and prestress and choose a proper standard for the safety of the structure in each particular case.

If a section of a column is under an effective prestress F with an eccentricity e, and loaded by a concentric load P plus an external moment M, the extreme fiber stresses at that section can be computed by the following formula,

$$f_c = \frac{F}{A_c} \pm \frac{Fec}{I_c} + \frac{P}{A_t} \pm \frac{Mc}{I_t} \tag{12-4-1}$$

If the column is a slender one, the deflection of the member due to both the prestress eccentricity and the external load may significantly affect the magnitude of the external moment M and must be included in it. Computation for the deflection is a problem that can be solved by ordinary column theory and will not be discussed here.

Few data are available on the strength of prestressed-concrete columns.[12-5] An approximate investigation of the effect of prestressing on the ultimate strength can be made, however. Under the action of an external compressive load, the column will shorten and the prestress in the steel will be decreased. If, at the ultimate load, the unit compressive strain in the concrete is of the order of 0.0030, then the pretensioned strain in the steel will be decreased by that same amount, and the remaining prestress at the moment of failure will be less than the original effective prestress.

If the effective prestress is 120,000 psi, the remaining prestress will be only

$$\begin{aligned} f_s &= f_e - 0.0030E_s \\ &= 120{,}000 - 0.0030 \times 30{,}000{,}000 \\ &= 30{,}000 \text{ psi} \end{aligned}$$

In other words, the major part of the prestress may be lost at the ultimate compressive strength of the concrete. This means that the

ultimate load-carrying capacity of the column is not much decreased by prestressing. On the other hand, if the column fails on the tensile side as the result of bending or buckling, the steel on that side can be stressed to near its ultimate strength.

The buckling of the compressive flange of prestressed beams is subject to the same reasoning. There is no danger of flange buckling produced by internal prestress in a beam. For external loads, a tendency to buckle in the flange is governed by the usual theory of elasticity, so long as there are no cracks in the concrete. After cracking or near the ultimate load, little is known about the buckling of the compressive flange in prestressed beams.

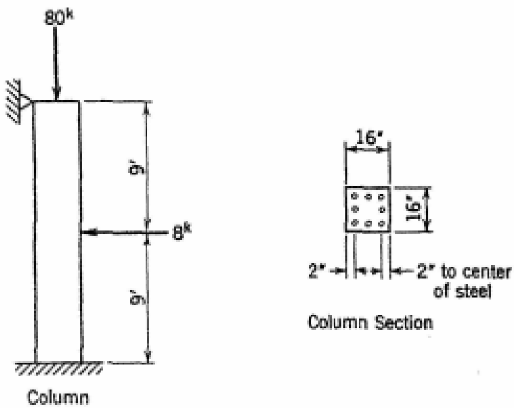

Fig. 12-4-2. Example 12-4-1.

Example 12-4-1

A concrete column 16 in. by 16 in. in cross section and 18 ft high, Fig. 12-4-2, is pre-tensioned with eight ⅜-in. wires, which are end-anchored to the concrete. The effective prestress is 100,000 psi in the steel. For a concentric compressive load of 80 kips and a horizontal load of 8 kips at the midheight of the column, compute the maximum and minimum stresses in the column, assuming it to be hinged at the ends. Investigate the secondary moments in the column due to deflection. Discuss the safety of the column under such loads and also during handling. Assume that $n = 7$, $f_c' = 4000$ psi, $f_s' = 200{,}000$ psi, $E_c = 4{,}000{,}000$ psi.

Solution. Stress in the concrete due to prestress is

$$\frac{F}{A_c} = \frac{8 \times 0.11 \times -100{,}000}{256 - 8 \times 0.11}$$

$$= -344 \text{ psi}$$

Stress due to the axial load of 80 kips, disregarding deflection of column, is

$$\frac{P}{A_t} = \frac{-80{,}000}{256 + (7-1)8 \times 0.11}$$

$$= -80{,}000/261$$

$$= -306 \text{ psi}$$

The maximum bending moment occurs at the midheight of column, and is

$$18 \times 8{,}000/4 = 36 \text{ k ft}$$

The I_t of the transformed section is

$$\frac{16^4}{12} + 6 \times 0.11 \times (7-1) \times 6^2$$

$$= 5460 + 142$$

$$= 5602 \text{ in.}^4$$

The extreme fiber stresses are

$$\frac{Mc}{I_t} = \frac{36.000 \times 12 \times 8}{5602}$$

$$= \pm 616 \text{ psi}$$

The maximum and minimum stresses are hence

$$-344 - 306 - 616 = -1266 \text{ psi compression}$$

$$-344 - 306 + 616 = -34 \text{ psi compression}$$

The maximum deflection of the column due to the horizontal load is

$$\frac{PL^3}{48E_cI_t} = \frac{8000 \times 18^3 \times 12^3}{48 \times 4{,}000{,}000 \times 5602}$$

$$= 0.075 \text{ in.}$$

This will increase the moment due to axial load by the amount of 80,000 × 0.075 = 6000 in.-lb = 0.5 k ft. This moment will produce more deflection and further increase the eccentric moment in the column, but the magnitude is seen to be quite small and may be neglected. Hence the above-computed stresses can be considered sufficiently correct. The maximum compressive stress of 1266 psi would appear high for a reinforced-concrete column but is not excessive for a prestressed member which is more a beam than a column in this example.

The safety of the column can be determined only if we know the ultimate strength of the column under such combined axial and transverse loads and also if we know the possibilities of overloading, i.e., to what extent the axial or the horizontal loads may be increased, and whether eccentricity of the applied axial load may be possible.

For the purpose of investigation, let us assume that both the horizontal and the axial load are increased by 50% while, in addition, there will be an eccentricity of 2 in. for the axial load. Then the stresses will be:

Due to axial load, 1.5 × 306 = −459 psi.

Due to eccentricity of axial load, $1.5 \times 80 \times 2$ in. = 240 k in. = 20 k ft, which will produce stresses of

$$616 \times 20/36 = \pm 342 \text{ psi}$$

Due to horizontal load, $1.5 \times 616 = \pm 924$ psi.

Resulting stress:

$$-344 - 459 - 342 - 924 = -2069 \text{ psi}$$

$$-344 - 459 + 342 + 924 = +463 \text{ psi}$$

Note that the compressive stress of 2069 psi is only about $0.52f_c'$ while the tensile stress is below the modulus of rupture of about $0.12f_c' = 480$ psi. Hence the column would not have cracked, and the midheight deflection can still be computed by the elastic theory to be not more than 0.2 in., which is not a significant value. Thus it can be concluded that the column is safe.

For investigating handling stresses, let us assume that the column is picked up at the midheight.

The moment produced will be

$$\frac{wL^2}{2} = \frac{256 \times (150/144) \times 9^2}{2}$$

$$= 10.8 \text{ k ft}$$

which will produce a maximum tensile stress of

$$\frac{Mc}{I_t} = \frac{10.8 \times 12{,}000 \times 8}{5602} = +185 \text{ psi}$$

This is much less than the precompression of 344 psi, and the column is safe during handling.

References

12-1 A. F. Campbell, "Arch Tie Prestressed to Hold Length," *Eng. News-Rec.*, July 16, 1953, p. 48.

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12-3 F. R. Brewster, "Concrete Building Columns Prestressed," *Eng. News-Rec.*, Nov. 10, 1949, p. 34.

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12-5 R. A. Breckenridge, *A Study of the Characteristics of Prestressed Concrete Columns*, University of Southern California, Los Angeles, 1953.

Chapter 13

Circular Prestressing

13-1 Methods and Applications

The term "circular prestressing" is employed to denote the prestressing of circular structures such as pipes and tanks where the prestressing wires are wound in circles. In contrast to this term, "linear prestressing" is used to include all other types of prestressing, where the cables may be either straight or curved, but not wound in circles around a circular structure. In most prestressed circular structures, prestress is applied both circumferentially and longitudinally, the circumferential prestress being circular and the longitudinal prestress actually linear. For convenience, both types of prestress as they are applied to circular structures will be discussed in this chapter.

The basic theories of circular prestressing are the same as those for linear prestressing; hence practically all the general principles presented in the previous chapters can be applied to circular structures as well, although such application necessarily involves certain details not discussed for linear structures. The practice of circular prestressing differs from linear prestressing in that the techniques of applying the prestress and of anchoring the tendons are often different.

In this chapter, the discussion will be centered on the design of tanks or circular liquid containers. Most of these principles are applicable also to the design of pipes, which will not be discussed in detail. Instead, some citations on prestressed pipes are given to which the reader can refer if he is interested in the subject.[13-1]

Prestressed-concrete pipes in this country can be divided into two types: those with and those without steel cylinders. The construction of those with steel cylinders is now a standardized procedure, as evidenced by the specifications[13-2] approved by the A.W.W.A. in 1952. These specifications cover the manufacture of such water pipes ranging in size from 16 to 54 in. and designed for static loads from 100 to 600 ft of water. A typical longitudinal section of the pipe through the joint is shown in Fig. 13-1-1. The pipe consists of a

continuously welded sheet-steel cylinder with steel joint rings welded to its ends, the cylinder being lined on the inside with dense concrete of suitable thickness. After proper curing of concrete, high-tensile wire is wound around the outside of the steel cylinder at a specified prestress and securely fastened to it at its ends. Then a coating of mortar or concrete is deposited over the cylinder and wire for protection. A self-centering joint with rubber gasket as the sealing element is designed so as to be watertight under all conditions of service.

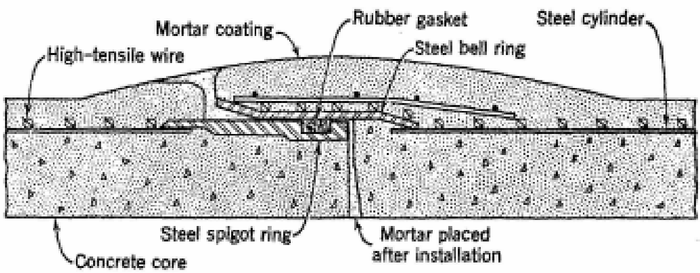


Fig. 13-1-1. Longitudinal section through joint of prestressed-concrete cylinder pipe.

Pipes without steel cylinders are manufactured by simply winding prestressed wires around a concrete core and covering the wires with air-applied mortar. Longitudinal prestress is sometimes provided by pre-tensioning longitudinal wires against the inner steel form.[13-3] In another method, helical wire wrapping is applied in a basket-weave pattern so as to produce a longitudinal component of prestress.[13-4] The concrete core is often cast by the Rocla roller compaction method, by which a rotating mold rolls and places the dense concrete to form a thin-walled pipe. The Rocla firm in Australia applies pre-tensioning techniques to concrete pipes by embedding circular reinforcement in the pipe concrete which immediately after having been placed is subjected to high pressure applied to the inside of the pipe.[13-5] When the concrete hardens, the steel remains stretched; then the inside pressure is relieved, and the concrete becomes compressed.

In this country prestressed-concrete tanks are almost solely constructed by the Preload method, using their wire winding machines, Fig. 1-1-8. Up to 1951, about 700 large tanks with a total capacity of more than 500 million gallons and 300 spherical shell roofs in spans up to 205 ft had been built of prestressed concrete in North America,

using almost exclusively the Preload method of prestressing.[13-6] The Preload procedure consists of the following process. First, the walls for the tanks are built of either concrete or pneumatic mortar, mortar being generally used if the walls are less than 6 in. thick. Often, the walls are poured in alternate vertical slices keyed together. After the concrete walls have attained sufficient strength they are prestressed circumferentially by a self-propelled machine, which winds the wire around the walls in a continuous operation, stressing it and spacing it at the same time. Under favorable conditions, the machine can place the wire up to 7 miles an hour and can complete the horizontal prestressing of an average million-gallon tank in about 2 days.

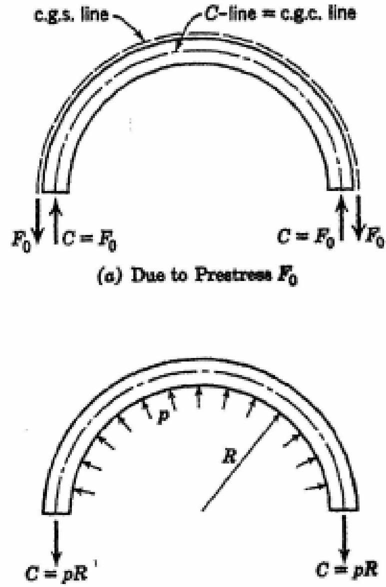
After the circumferential prestressing is completed for each layer, a coat of pneumatic mortar is placed around the tank for protection. Two or more layers of prestressing are used for large tanks. Vertical prestressing for the tanks can be applied using any system of linear prestressing, whichever may be the most economical.

In addition to the Preload procedure, other methods have been applied in this country and abroad, though not as extensively as that using the winding machine. In the 1920's, Hewett in the United States used ordinary bars wrapped around the walls and stressed with turnbuckles.[13-7] But all the prestress in such bars could be lost in the course of time as a result of shrinkage and flow of concrete, although some of the tanks have remained in good service even to the present time. Mautner, of England,[13-8] employed high-strength steel wires wrapped around precast concrete units, between which were inserted jacks which, when extended, stretched the steel wires. Openings left for the jacks were eventually filled with concrete to maintain the compression in the walls. The Freyssinet method of linear prestressing has been applied to tanks with the tendons in equal lengths of portions of a circle.[13-9] The tendons are stressed from both ends and anchored against pylons spaced uniformly around the tank. By staggering the end anchors in adjacent tendons, the frictional loss of prestress is nearly equalized around the circle.

13-2 Circumferential Prestressing

Circumferential prestress in tanks is designed to resist hoop tension produced by liquid pressure. Hence, essentially, each horizontal slice of the wall forms a ring subject to uniform internal pressure. In several senses, such a ring can be regarded as a prestressed-concrete member under tension, and much of the discussion in Chapter 12 on tension members can be applied to the design of circumferential prestressing as well.

Consider one half of a thin horizontal slice of a tank as a free body, Fig. 13-2-1 (a). Under the action of prestress F_0 in the steel, the total compression C in the concrete is equal to F_0. The location of the line of pressure or the C-line in the concrete does not usually coincide with the c.g.s. line. In a circular ring under circular prestress, the C-line always coincides with the c.g.c. line. This is because a closed ring is a statically indeterminate structure, and the theory of linear transformation explained in Chapter 10 for continuous beams is applicable to such a ring. A cable through the c.g.c. is a concordant cable; any other cable parallel to it is simply that line, linearly transformed, whose line of pressure will still remain through the c.g.c. This phenomenon can also be explained by the simple fact that the effect of circular prestress is to produce an initial hoop compression on the concrete, which is always axial irrespective of the point of application of the prestress. Hence, owing to circular prestress, the stress in the concrete is always axial and is given by the formula



(a) Due to Prestress F_0

(b) Due to Internal Pressure p

Fig. 13-2-1. Forces in a horizontal slice of tank. (Half slice as free body.)

$$f_c = -\frac{F_0}{A_c}$$

which reduces to

$$f_c = -F/A_c$$

after the losses in prestress have taken place.

With the application of internal liquid pressure, Fig. 13-2-1 (b), the steel and concrete act together, and the stresses can be obtained by the usual elastic theory. Using the method of transformed section, we have

$$f_c = pR/A_t$$

where p = internal pressure intensity, R = internal radius of the tank, A_t = transformed area = $A_c + (n-1)A_s$.

The resultant stress in the concrete under the effective prestress F and the internal pressure p is

$$f_c = -\frac{F}{A_c} + \frac{pR}{A_t} \tag{13-2-1}$$

In order to be exact, the value of n has to be chosen correctly, considering the level of stress and the effect of creep. In practice, slight variation in the value of n may not affect the stresses very much, and an approximate value will usually suffice. If a coating of concrete or mortar is added after the application of prestress, then the area A_c under prestress may be the core area while the A_c sustaining the liquid pressure may include the additional coating. Such refinements in calculation may or may not be necessary, depending upon the circumstances.

The criteria for designing prestressed tanks vary. The practice in this country has been to provide a slight residual compression in the concrete under the working pressure. This is accomplished by the following procedure of design.

Assume that the hoop tension produced by internal pressure is entirely carried by the effective prestress in the steel; we have

$$F = A_s f_s = pR \tag{13-2-2}$$

thus the total steel area required is

$$A_s = \frac{pR}{f_s} \tag{13-2-3}$$

The total initial prestress is then

$$F_0 = A_s f_0 \tag{13-2-4}$$

For an allowable compressive stress f_c in concrete, the concrete area required to resist the initial prestress F_0 is

$$A_c = -\frac{F_0}{f_c} \tag{13-2-5}$$

From this value of required A_c, the thickness for the tank can be determined.

Corresponding to the adopted value of A_c, the stresses in the concrete and steel under the internal pressure p can be obtained by

$$\text{Stress in concrete} = -\frac{F}{A_c} + \frac{pR}{A_t} \tag{13-2-6}$$

$$\text{Stress in steel} = f_s + nf_c \tag{13-2-7}$$

Since F is equal and opposite to pR, and A_t is always greater than A_c, it can be seen from equation 13-2-6 that there will be some residual compression in the concrete under the working pressure. This residual compression serves as a margin of safety in addition to whatever tension may be taken by the concrete.

Since the serviceability of a tank is impaired as soon as the concrete begins to crack, it is of utmost importance that an adequate margin of safety be provided against cracking. Where overflow pipes are installed for tanks so that there cannot exist any excessive pressure, a smaller margin of safety is required. Thus the English *First Report on Prestressed Concrete*[14-2] recommends a factor of safety of 1.25 against cracking. For pipes that may be subjected to much higher pressure than the working value, a greater factor of safety is necessary. For the design of prestressed-concrete pipes with steel cylinders, the A.W.W.A. specifies that the concrete core should be sufficiently compressed to withstand an internal hydrostatic pressure equal to at least 1.25 times the designed pressure without tensile stress being induced in the core. In addition, the pressure producing elastic limit stresses in the steel cylinder and wire is sometimes required to be 2.25 times the normal operating pressure.[13-1]

The above conventional method of design equating the effective prestress to the hoop tension may or may not provide the necessary factor of safety. If a factor of safety of m against cracking is required, the following procedure of design may be adopted.

Assuming f_t = tensile strength in concrete at cracking (which averages about $0.08f_c'$ but may be zero if the concrete has previously cracked or if precast blocks are used), we may write

$$-\frac{F}{A_c} + \frac{mpR}{A_t} = f_t \tag{13-2-8}$$

At the same time, in order to limit the maximum compression in concrete to f_c, we have

$$A_c = -F_0/f_c$$

Substituting this value of A_c into equation 13-2-8, and noting that $A_t = A_c + nA_s$, $F = f_sA_s$, and $F_0 = f_0A_s$, we have

$$-\frac{f_sA_sf_c}{f_0A_s} + \frac{mpR}{(f_0A_s/f_c) + nA_s} = f_t \tag{13-2-9}$$

Solving for A_s, we have

$$A_s = \frac{mpR}{[f_s - (f_t/f_c)f_0][1 - (nf_c/f_0)]} \tag{13-2-10}$$

After A_s is obtained, F_0 and A_c can be computed using equations 13-2-4 and 13-2-5, and the stresses in the concrete and steel can be evaluated by equations 13-2-6 and 13-2-7.

One of the important items in the design of tanks is the evaluation of the losses of prestress. Although the details of the sources of loss are discussed in Chapter 4, the usual amount of loss occurring and allowed for in prestressed tanks will be mentioned here. Extensive experiments have been made to measure the amount of losses in prestressed tanks.[13-10] The average loss of prestress seems to be about 25,000 psi, resulting chiefly from the shrinkage and creep of concrete. An allowance of 35,000 psi is considered quite conservative, although, under extremely adverse conditions, losses up to 40,000 psi might take place.

Analyzing the principal sources of these losses, it might be estimated that concrete under a constant load of about 600 psi may attain a total elastic and creep deformation of about 0.0006. Since the concrete is under low compression when the tank is full, the amount of creep strain may be much smaller if the tank is kept filled most of the time. The amount of shrinkage will depend chiefly upon the moisture content in the concrete. Although the worst possible shrinkage strain can be as much as 0.0010, there have been tanks whose concrete expanded instead of contracted, thus resulting in a gain of prestress instead of a loss. For example, if a tank is prestressed after the concrete has aged for several months under dry climatic conditions, expansion will take place when it is filled with water.

As a safe average value, the following may be taken:

Elastic and creep strain in concrete	= 0.0005
Shrinkage	= 0.0005
Total loss	= 0.0010

which amounts to about 28,000 psi, taking E_s as 28,000,000 psi. If accurate values are desired, the possible losses must be considered for each individual tank and duly allowed for.

Example 13-2-1

Determine the area of steel wire required per foot of height of a prestressed-concrete water tank 60 ft in inside diameter to resist 20 ft of water pressure. Compute the thickness of concrete required. $f_c' = 3000$ psi, $f_c = 750$ psi, $n = 10$, $f_0 = 150{,}000$ psi, $f_s = 120{,}000$ psi. Neglect the mortar coating in the calculations. Design both steel and concrete on the following two bases:

1. Assuming all hoop tension carried by the effective prestress.
2. For a load factor of 1.25, producing zero stress in concrete.

Solution. 1. Pressure of 20 ft of water

$$p = 20 \times 62.4 = 1248 \text{ psf}$$

Using equations 13-2-3 and 13-2-5,

$$\begin{aligned} A_s &= pR/f_s \\ &= \frac{1248 \times 30}{120{,}000} \\ &= 0.312 \text{ sq in.} \\ A_c &= -F_0/f_c \\ &= \frac{-0.312 \times 150{,}000}{-750} \\ &= 62.5 \text{ sq in.} \end{aligned}$$

For a height of 12 in., the thickness required is 62.5/12 = 5.2 in. Suppose that a thickness of 5.5 in., is adopted; then, under the action of the internal pressure, equation 13-2-6 gives

$$\begin{aligned} f_c &= -\frac{F}{A_c} + \frac{pR}{A_t} \\ &= -\frac{0.312 \times 120{,}000}{5.5 \times 12} + \frac{1248 \times 30}{66 + 10 \times 0.312} \\ &= -567 + 541 \\ &= -26 \text{ psi.} \end{aligned}$$

Note here that, the thicker the concrete, the smaller will be the residual compression under load, unless the amount of wire is proportionately increased.

2. Using equation 13-2-10,

$$\begin{aligned} A_s &= \frac{mpR}{[f_s - (f_t/f_c)f_0][1 - (nf_c/f_0)]} \\ &= \frac{1.25 \times 1248 \times 30}{[120{,}000 + 0][1 - (10 \times -750/150{,}000)]} \\ &= 0.372 \text{ sq in.} \\ A_c &= F_0/f_c \\ &= 0.372 \times 150{,}000/750 \\ &= 74.4 \text{ sq in.} \end{aligned}$$

Thickness required = 74.4/12 = 6.2 in. If a thickness of 6.5 in. is adopted, the resulting stress in the concrete under full water pressure will be

$$\begin{aligned} f_c &= -\frac{F}{A_c} + \frac{pR}{A_t} \\ &= \frac{-0.372 \times 120{,}000}{6.5 \times 12} + \frac{1248 \times 30}{78 + 10 \times 0.372} \\ &= -573 + 458 \\ &= -115 \text{ psi} \end{aligned}$$

which provides a margin of safety of 25% up to zero compression in concrete.

Note that designing by this second method gives heavier sections for both

concrete and steel. The design can be economized if some tension in the concrete is allowed at 25% overload.

13-3 Vertical Prestressing in Tanks

The design of prestressed-concrete structures is based on a knowledge of the behavior of non-prestressed structures plus an understanding of the effect of prestressing. This is as true for the design of tanks as for beams and slabs. Before analyzing the stresses in a prestressed tank, let us consider an ordinary reinforced-concrete tank

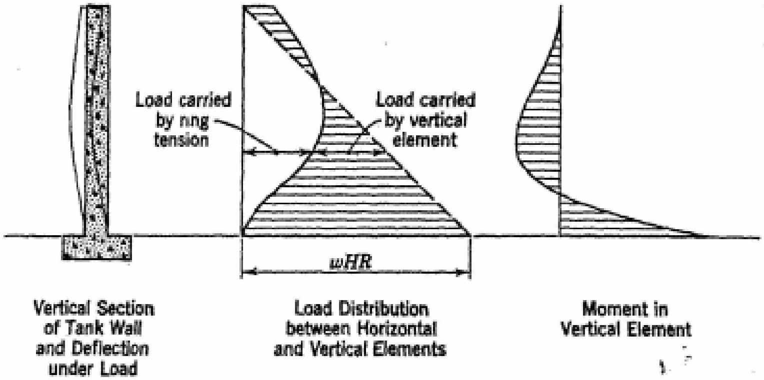


Fig. 13-3-1. Moment and deflection in vertical element of tank wall.

under the action of internal liquid pressure. It is well known that, whereas the horizontal elements of the tank are subject to hoop tension, the vertical elements are under bending, Fig. 13-3-1. The amount and variation of bending in the vertical elements will depend upon several factors:

1. The condition of support at the bottom of the wall, whether fixed, hinged, free to slide, or restrained by friction.
2. The condition of support at the top of the wall, whether fully or partially restrained or free to move.
3. The variation of concrete thickness along the height of the wall.
4. The variation of pressure along the depth, whether triangular or trapezoidal.
5. The ratio of the height of the tank to its diameter.

Theoretical solution for several of these combinations are given by Timoshenko,[13-11] and numerical values, convenient for application, are tabulated in some pamphlets.[13-12] European books give solutions for additional cases, such as walls of varying thickness, and the results are plotted in some publications.[13-13] Readers interested in

the problem are referred to these and to the bibliographies listed in them. To give an idea of such distribution of loads among the horizontal and vertical elements, two graphs are presented in Fig. 13-3-2. It is evident from these graphs that the active pressure on the horizontal elements is not a direct function of depth, but often decreases with it, while the vertical elements may carry a considerable amount of load, especially if the structure is squattier than usual.

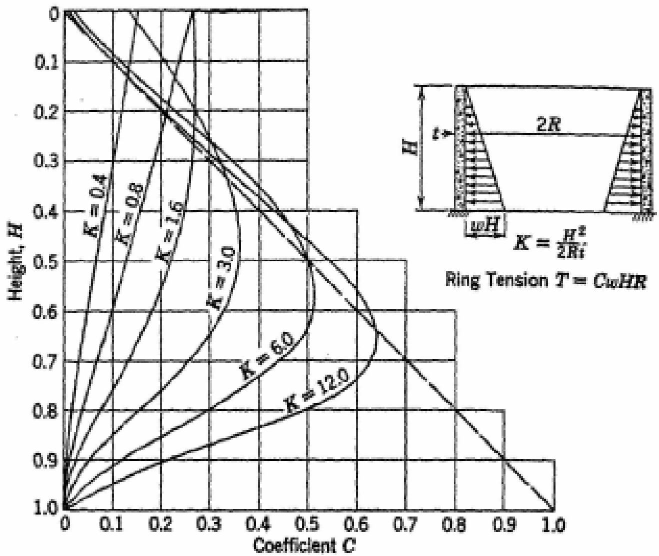
For prestressed-concrete tanks, an additional problem is introduced: the effect of prestressing, both circumferential and vertical. Since horizontal pressure will produce vertical moments in the walls, it is evident that circumferential prestressing will also induce such moments. These vertical moments caused by circumferential prestressing will exist by themselves when the tank is empty and will act jointly with the moments produced by liquid pressure when the tank is filled. To reinforce the wall against these moments, vertical prestressing may be applied. If vertical prestress is concentrically applied to the concrete, only direct compressive stress is produced and the solution is simple. If the vertical tendons are bent or curved, the vertical prestress produces radial components which, in turn, influence the circumferential prestress. Hence the analysis can become quite complicated.

Let us investigate the effect of circumferential prestressing on the vertical moments. If the circumferential prestress varies triangularly from zero at the top to a maximum at the bottom, its effect is equal but opposite to the application of an equivalent liquid pressure. If the circumferential prestress is constant throughout the entire height of the wall, it is the same as the application of an equivalent gaseous pressure. For both cases, tables are available for the computation of vertical moments.[13-12] To obtain the optimum results, the circumferential prestress along the depth of the wall should be varied to suit the variation of the active pressure on the horizontal elements However, the effect of such circumferential prestressing on vertical moments cannot be readily determined.

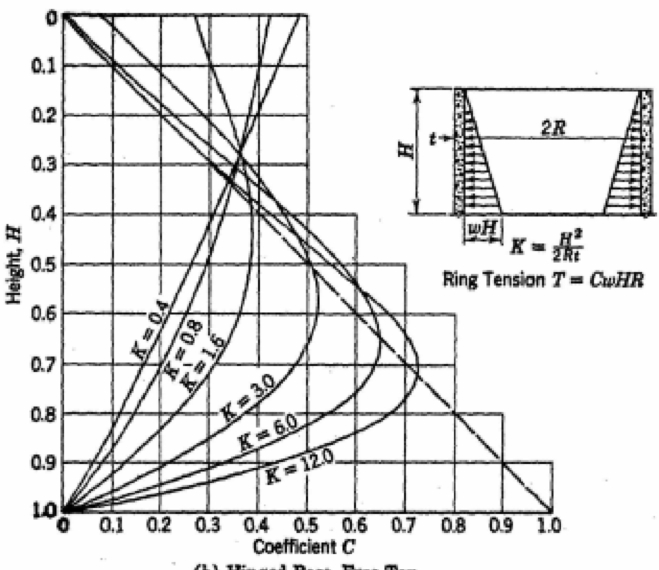
Vertical prestressing should be designed to stand the stresses produced by various possible combinations of the following forces:

1. The vertical weight of the roof and the walls themselves.
2. The vertical moments produced by internal liquid pressure.
3. The vertical moments produced by the applied circumferential prestress.

In addition to the above, stresses may be produced as a result of differential temperature between the inner and outer faces of the wall, and by shrinkage of the concrete walls unless they are enurely free



(a) Fixed Base, Free Top



(b) Hinged Base, Free Top

Fig. 13-3-2. Tension in tank rings, triangular load, uniform wall section.

to slide on the foundation. These forces cannot be easily evaluated and hence are often neglected or provided for indirectly in an overall factor of safety.

It must be noted that the maximum stresses in the concrete usually exist when the tank is empty, because then the circumferential prestress would have its full effect. When the tank is filled, the liquid pressure tends to counterbalance the effect of circumferential prestress and the vertical moments are smaller. Since it is convenient to use the same amount of vertical prestress throughout the entire height of the wall, the amount will be controlled by the point of maximum moment. By properly locating the vertical tendons to resist such moment, a most economical design can be obtained. However, efforts are seldom made to do so, and the amount of prestress as well as the location of the tendons is generally determined empirically rather than by any logical method of design.

Example 13-3-1

A 1-ft vertical element of a water tank is shown in Fig. 13-3-3. It carries 1500 lb of weight from the roof. At a point 20 ft below the top, the vertical moments are: for initial circumferential prestress, $M = 3200$ ft-lb (tension on the inside fibers), which reduces to 2500 ft-lb eventually. For full liquid pressure, $M = 2400$ ft-lb (tension on the outside face). The vertical prestressing wire is located 2¾ in. from the inside face and exerts an initial prestress of 11,000 lb/ft, which reduces to 8000 lb/ft eventually. Compute stresses in the extreme vertical fibers of the concrete under the initial and final conditions, considering both an empty and a full tank.

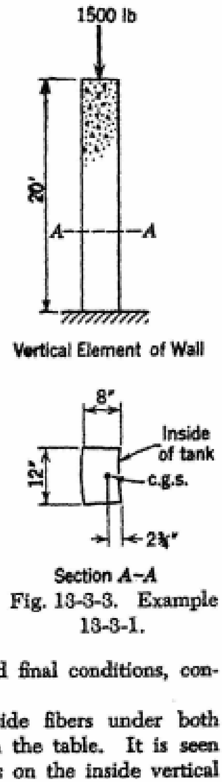


Fig. 13-3-3. Example 13-3-1.

Solution. The stresses for both the inside and outside fibers under both initial and final conditions are computed and listed as in the table. It is seen from the table that a slight tension of 36 to 41 psi exists on the inside vertical fibers when the tank is empty. Otherwise, compressive stresses are obtained throughout. (See table on p. 372.)

13-4 Dome Prestressing

It is beyond the scope of this treatise to discuss the design of domes. Only the general principles and practice of dome prestressing especially as applied to tank roofs will be mentioned here. Readers interested in the subject are referred to other publications for additional details.[13-14–13-16] Generally speaking, for domes with diameter

Computation for Stresses in Concrete, Example 13-3-1

Conditions	Initial		Final	
Fiber	Inside	Outside	Inside	Outside
A. Weight of roof $\dfrac{1500}{8 \times 12} =$	−16	−16	−16	−16
B. Weight of wall $\dfrac{20 \times 150}{144} =$	−21	−21	−21	−21
C. Axial component of vertical prestress $\dfrac{11{,}000}{8 \times 12} =$	−115	−115		
$\dfrac{8000}{8 \times 12} =$			−83	−83
D. Eccentricity of vertical prestress $\dfrac{6M}{bd^2} = \dfrac{6 \times 11{,}000 \times 1.25}{12 \times 8^2} =$	−107	+107		
$\dfrac{6 \times 8000 \times 1.25}{12 \times 8^2} =$			−78	+78
E. Vertical moment due to circumferential prestress $\dfrac{6M}{bd^2} = \dfrac{6 \times 3200 \times 12}{12 \times 8^2} =$	+300	−300		
$\dfrac{6 \times 2500 \times 12}{12 \times 8^2} =$			+234	−234
Total for tank empty	+41	−345	+36	−276
F. Vertical moment due to liquid pressure $\dfrac{6M}{bd^2} = \dfrac{6 \times 2400 \times 12}{12 \times 8^2} =$	−225	+225	−225	+225
Total for tank full	−184	−120	−189	−51

greater than 100 ft, the economy of prestressing should be seriously considered. Domes for tanks up to 230 ft in diameter have been constructed.

The dome roof itself is made of concrete or pneumatic mortar with thickness varying from 2 to 6 in. For domes of large diameter, variable thicknesses may be employed and thicknesses greater than 6 in. are used for the lower portion. Before concreting the dome, some

erection bars are prestressed around the base of the dome. After the hardening of the shell concrete, wires are prestressed around it, Fig. 13-4-1. During this operation, the dome shell rises from its forms as it is compressed, thus simplifying the careful procedure of decentering required for non-prestressed domes.

Methods and formulas, though available for the analysis of dome stresses under uniform loads, are applicable only to points on the

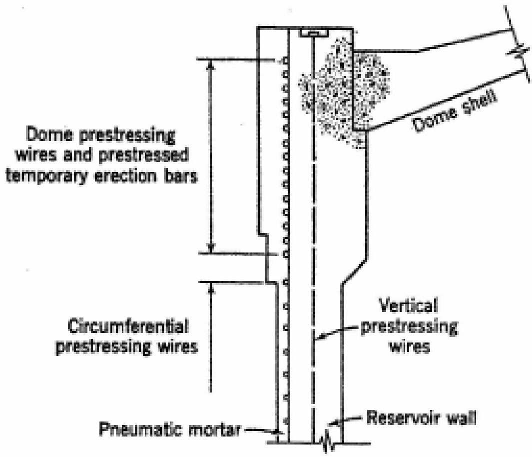


Fig. 13-4-1. Typical section of dome ring for tanks.

domes removed from the discontinuous edge. The computation of stresses in the edge ring becomes a very complicated problem if the edge ring is prestressed. However, for purposes of design, a conventional method is available. It consists of prestressing the ring to induce sufficient compressive stresses to counteract the tensile stresses set up in the ring under the maximum live and dead loads. With this prestress, it is usually possible to raise the dome from its false work, since only the dead load is actually acting on the dome.

Consider a spherical dome carrying loads symmetrical about the axis of rotation, i.e., load with intensity constant along any given latitude, Fig. 13-4-2. If the total load is W, the vertical reaction per foot of length along the edge member will be

$$V = \frac{W}{2\pi R \sin \theta}$$

Since a dome is not supposed to carry any appreciable moment,

the resultant reaction along the edge must be tangent to the surface. Hence the horizontal reaction per foot of length must be

$$H = V \cot \theta = \frac{W \cot \theta}{2\pi R \sin \theta}$$

Assuming this horizontal reaction to be entirely supplied by the pre-

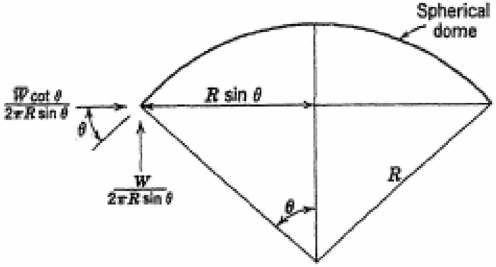


Fig. 13-4-2. Design for prestress in edge ring of dome.

stressing force F acting in hoop tension, then,

$$F = HR \sin \theta$$
$$= \frac{W}{2\pi} \cot \theta \qquad (13\text{-}4\text{-}1)$$

The effective prestressing force F having been determined, the cross-sectional area of the ring concrete can be designed by

$$A_c = \frac{F_0}{f_c} \qquad (13\text{-}4\text{-}2)$$

where F_0 = the initial prestressing force, and f_c = the allowable compressive stress in concrete.

It is desirable to keep f_c at a relatively low value, say about $0.2f_c'$ and not greater than 800 psi. This is necessary in order to minimize excessive strain in the edge ring which might in turn produce high stresses in the shell. It must be further observed that this procedure of design is satisfactory only when there is no possibility of heavy overloads, because the prestressed edge ring does not possess a high factor of safety against overloads, although the factor of safety is sufficient for ordinary roof loading.

Example 13-4-1

A spherical dome, Fig. 13-4-3, carries a total live and dead load of 900 kips. Design the prestress in the edge ring and the cross-sectional area of concrete required for the edge ring. Loss of prestress = 20%. f_c = 600 psi.

Solution. From equation 13-4-1, for $W = 900$ kips and $\theta = 45°$, we have

$$F = (W/2\pi) \cot \theta$$
$$= 900/2\pi$$
$$= 143 \text{ kips}$$

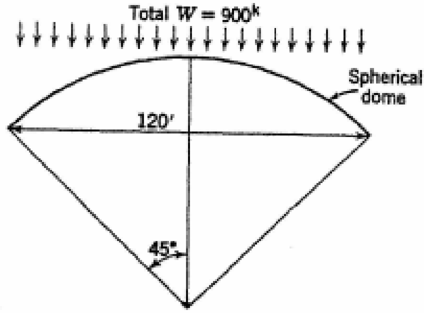


Fig. 13-4-3. Example 13-4-1.

From equation 13-4-2, area of concrete required, for $F_0 = 143/0.8 = 179$ kips, is

$$A_c = F_0/f_c$$
$$= 179/0.6$$
$$= 298 \text{ sq in.}$$

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Chapter

Allowable Stresses and Load Factors

14

14-1 Design Basis—Stress or Strength?

Before proceeding with the determination of allowable stresses and load factors for prestressed concrete, let us first examine our present basis for design of all structures. There are two philosophies of design: one based on stresses, and the other based on strength, known as the allowable stress method and the ultimate strength method, respectively. According to the allowable stress method, a normally maximum service load is specified as the design load, the elastic theory is generally applied in the computation of stresses, and a set of allowable values is set up as the maximum limits. In following such a procedure, a margin of safety is provided almost entirely in the allowable stresses, the choice of whose values determines the degree of safety of the structure and its ability to carry overloads.

In the ultimate strength method of design, allowable stresses are ignored. Instead, the ultimate or rupture strength of the members is obtained and expressed in semiempirical formulas; then the structure is dimensioned so as to possess a specified factor of safety against failure in terms of the service loads. In other words, the specified service loads must be multiplied by a load factor and equated to the ultimate strength of the member. Here, the margin of safety is provided almost entirely in the load factors.

These two different methods of design are being applied to prestressed-concrete structures as they are to other types. Because of the peculiar nature of prestressed concrete, wherein stresses often are not a correct representation of the safety of the structure, checking for the ultimate strength has been found to be indispensable in certain instances. But, at the same time, ultimate design cannot be exclusively applied either, for several reasons. First, the ultimate strength of some types of members is not yet known for prestressed concrete. Second, it is very difficult to choose proper load factors that will yield safe and economical structures. This is especially true when it is realized that

most structures will be entirely unserviceable long before they collapse as a whole. It is because of this range of unserviceable strength below rupture that some engineers are now advocating design based on usable strength instead of on ultimate strength. Third, the behavior of the structure under working loads, such as its local strains and overall deflections, may not be predicted by ultimate design. The effect of repeatedly applied loads producing high local stresses may be detrimental to the structure even though the loads are much lower than the ultimate.

Because of the inadequacy of either the working-stress or the ultimate-strength method of design, it has been the practice in both this country and abroad to adopt a double approach for prestressed concrete. The types of members which, when designed by the working-stress method, will also possess the proper factor of safety against ultimate failure are designed by that method. In fact, for such members, it will be just as well to design them by the ultimate-strength method, since similar dimensions will be arrived at by either method, and the ultimate-strength formulas are usually easier to apply. However, engineers are more used to the application of allowable stresses than load factors; hence the allowable-stress method is still preferred by most people. For other types of members where proportioning by the allowable-stress method will definitely yield inconsistent results, ultimate design is followed, the ultimate strength being based on the cracking strength for certain types of members and structures, such as tension members and tanks, for example. It is important to keep in mind that, whichever method is used, the other one must often be employed as a check.

It should be admitted that, because of this double approach, the design of prestressed-concrete structures is further complicated. But it must also be realized that such a procedure will yield safer and more economical results and is gaining favor for the design of other types of structures as well. Furthermore, with the accumulation of experience, the relation between stresses and strengths will be definitely known for more types of members, and then the application of either the stress or the strength method will suffice, and checking by the other method may eventually become unnecessary.

14-2 Design Loadings and Methods of Computation

Allowable stresses and load factors have no definite meaning unless the design loading for the structure is also specified in conjunction. For example, highway bridges in this country are designed on the basis of the H-loadings, but the actual heavy vehicles over the high-

ways are known to exceed these loadings greatly. The bridges are safe because the allowable stresses used in design are sufficiently low. If the actual heavy vehicles were used in the computation, the allowable stresses could certainly be raised. Since it is beyond the scope of this treatise to discuss the design loadings for various structures, it will be assumed that the present specified loadings for bridges and buildings are acceptable for the design of prestressed-concrete structures. Such an acceptance does not necessarily signify the approval of these loadings presently in use, but it will permit prestressed-concrete structures to be designed on the same basis as other types of construction as far as loadings are concerned. At the same time, it should be kept in mind that, in the event of any radical change in the specified loadings for such structures, the values of allowable stresses and load factors should be correspondingly modified. Outside of bridges and buildings, there are other structures which have no standard loadings. For them, comparable loadings should be worked out by the designer, with a view to the probability of overload and the desired performance of the structures under various conditions.

The method of computation used in obtaining the stresses is another important consideration in the choice of proper allowable stresses. For ordinary types of construction, such as steel or reinforced concrete, methods of computation are well understood and standardized. Even though most of these methods do not yield the actual stresses in structures, proper application will usually result in satisfactory dimensions. For example, the actual stresses in structural rivets are seldom the same as those indicated by the design formulas; the elastic theory in reinforced concrete only occasionally gives correct stresses. Yet, by following such accepted methods of computation, together with the specified loadings and allowable stresses, engineers have designed and built thousands of structures without too many failures. In prestressed concrete, methods of computation for certain stresses are well known and accepted, such as for flexure in beams; it is then relatively easy to specify the allowable values. For other stresses, such as bond at the end of unanchored members, no standardized or logical method is available for computation, and it would be difficult to speak of allowable stresses.

When using the ultimate-strength method of design, it is necessary that formulas for ultimate strengths be derived before the determination of load factors. Ultimate-strength formulas give real values and are not like the formulas for computing stresses, which generally yield only fictitious results. When they are known, they form a solid basis for choosing load factors. However, before deciding on the load

factors, it must be made clear whether the ultimate strength in question means the total collapse of the structure, or its usable limit; whether the strength formulas represent the average or the least values of strength; whether they represent ordinary static strength or failure under repeated loads.

14-3 Choice of Values

A simple initial approach to the establishment of proper allowable stresses and load factors for prestressed concrete is to compare them with those for reinforced concrete. Generally speaking, there is enough similarity between the two types of construction so that one can be conveniently discussed in terms of the other. Yet, specifically, the two types differ in so many respects that it is dangerous to pattern one exactly after the other.

The specific features of prestressed concrete, as distinguished from reinforced concrete, have been discussed elsewhere in this treatise. Their effect on the values of allowable stresses and load factors will now be summarized below:

1. Use of high-strength materials highly stressed. Both high-strength concrete and steel are less ductile than their counterparts in reinforced concrete. Under high stresses, fatigue-resisting properties differ from those at low stresses, elastic and creep strains become more significant, and the Poisson's ratio effect is more pronounced.

2. Effect of prestress on bending. When the design load is exceeded in a prestressed-concrete beam, the neutral axis of the beam shifts appreciably upward as the bending moment is further increased. This shifting of the neutral axis decreases the compressive area of concrete on the one hand but lengthens the lever arm of the tensile steel on the other. Hence, both the compressive stress in the concrete and the tensile stress in the steel are not directly proportional to the external moment, as they are in reinforced concrete.

3. Effect of prestress on shear and principal tension. Shear in reinforced concrete is used as a direct measure of diagonal tension. In prestressed concrete, diagonal tension is greatly reduced by precompression, and the principal tensile plane is shifted to a more nearly horizontal position. The principal tensile stress in prestressed concrete is much smaller than the diagonal tension in reinforced concrete, but it increases rapidly as the amount of shear or of moment is increased at the section.

4. Effect of prestress on direct compression. As stated in section 12-3, column action in a prestressed member generally is not accentuated by the prestressing force. The resultant compressive stress in

prestressed columns is not directly proportional to the external load. For example, when the external load is doubled, the resultant compressive stress is less than doubled, because the compression due to prestress is not increased but somewhat decreased.

5. Bond and bearing stresses. The conditions of bond and bearing stresses at end anchorage in prestressed concrete are not met with in reinforced concrete.

6. Girder load effect. Whereas dead and live loads in a reinforced-concrete beam have similar effects in producing compressive stress in the top fibers, dead load on a prestressed beam applied before the transfer of prestress can be made to result in zero compressive stress or even tensile stress in the top fibers. Although this is an advantage in prestressed concrete from an economic point of view, it renders the compressive fiber stress in prestressed beams more sensitive to the increase of live load.

Because of these differences, allowable stresses and load factors for prestressed concrete cannot be directly based on those for reinforced concrete. They must be chosen with regard to the basic considerations which underlie the choice of such values. These considerations are:

1. The ratio of overloads to design loads. Since it is the practice to design the service loads for the allowable stresses, overloads are to be carried at stresses exceeding the allowable values. Hence the allowable stresses must be determined with respect to the ratio of overloads to design loads. Load factors must be chosen so that the actual overloads will not impair the serviceability of the structure.

2. Frequency and magnitude of repeated loads. Allowable stresses and load factors must be chosen so as to avoid failures under repeated loads.

3. Variation in the properties of materials. Materials having greater variation should be given lower values of allowable stresses and perhaps higher values of load factors.

4. Inaccuracies in design methods and formulas. Whenever the accuracy of the methods and formulas is in doubt, a greater margin of safety will be needed.

5. Variations in the dimensions of materials and possibilities of deterioration. Allowances should differ for different structures.

6. Seriousness and suddenness of failure. Brittle failures are usually given a greater margin of safety. Main members and important structures are also accorded greater safety.

7. Secondary effects. Allowable stresses are sometimes kept suffi-

ciently low to avoid undesirable secondary effects, such as excessive creep and undesirable vibrations.

8. Economics of proportioning. Where safety can be obtained at relatively low cost, it is usually more liberally provided.

Some of the above factors can be analyzed quantitatively; others do not lend themselves to mathematical treatment. Hence the choice of proper values for allowable stresses and load factors often can be based only on good and experienced judgment.

14-4 Values Used in Various Countries

It is not a simple matter to assign a value to an allowable stress or load factor. These values vary with:

1. Types of structures. Allowable stresses and load factors may differ for different types of structures (as is presently true for bridges and buildings). The probability of overloads, the repetition of loadings, the possibilities of deterioration, and the requirement for watertightness differ for different types.

2. Grades of materials. In prestressed concrete, different kinds of reinforcement are employed, including high-tensile wires, strands, and alloy bars. There may be several grades in each of these types. The materials may differ in ductility, creep, and fatigue characteristics. Concrete may have strength varying from 3000 to perhaps 10,000 psi. Beams may be built of blocks cemented by grout or mortar. All these may call for a variety of values for allowable stresses and load factors.

3. Methods of construction. It is known, for example, that under similar conditions a bonded prestressed beam usually has a higher ultimate strength than the unbonded one; post-tensioned members have smaller loss of prestress than pre-tensioned members; precast, cast-in-place, and composite construction may possess different strengths. To be consistent, different values should be assigned for each case.

4. Stages of loading. Load factors and allowable stresses may differ for different stages of loading. For example, at jacking, stresses in steel are rather definite, and there is little danger of overstress; hence a small margin of safety will be sufficient.

A comprehensive set of values must include all the possible variations as described above. Up to now, the most complete set is the German one already adopted as a standard specification.[14-1] In England, main values are recommended in the *First Report*.[14-2] In France, Belgium, and Switzerland, values used by the respective authorities are obtainable.[14-3–14-5] In this country, efforts are now being made by the American Concrete Institute to formulate a set of

recommendations. Some isolated values have been advanced by certain organizations.[14-6,14-7] These representative values are surveyed and listed in Fig. 14-4-1, for the purpose of comparison. However, direct relations among these values cannot always be obtained from the tabulation, essentially because there exist differences between the definitions and standards of the various countries. Some of the variations are:

1. Different bases of concrete strength. The ultimate strength of concrete f_c' is defined as the 6 in. by 12 in. cylinder strength in this country; in European countries, 6-in. and 8-in. cubes are the usual standard. It is known that the ratio of cylinder strength to cube strength ranges from about 0.75 to 0.85, and 0.80 is chosen for the basis of this tabulation.

2. Different definitions of the yield point of high-tensile steel. Although the most commonly accepted definition is the stress at 0.2% set, other definitions, such as 0.1% set, 0.7% elongation, or the approximate proportional limit, are sometimes used. No attempt is made to correct the yield points to a common basis in the table.

3. Different methods of design for non-prestressed reinforcement. No standard method for the design of these reinforcements yet exists, although it is generally assumed that ultimate design should be applied.

4. Different qualifications for conditions of bending. Based on ultimate strengths, German specifications allow 15–20% higher values for bending about two axes as against one axis, another 10% for bending in a T-section as against a rectangular section. Bending in top or bottom fiber and bending alone or combined with torsion are given different allowable values in German specifications.

5. Different methods of construction. Only occasional differentiation is made between allowable stresses for pre-tensioned or post-tensioned, bonded or unbonded, precast or cast-in-place constructions.

6. Different strengths of concrete. Although the allowable stresses are generally expressed as a given percentage of f_c', that percentage should perhaps vary for different strengths of concrete. Practice in this country seems to be favoring constant percentages, but imposing an upper limit to the allowable values. German specifications allow different percentages, but only those for $f_c' = 5000$ psi are listed in Fig. 14-4-1.

7. Different bases for load factors. Some of the load factors are specified for live load only, with no increase in dead load; others are for both live and dead loads. These are noted in the tabulation.

In spite of the above-mentioned differences, it can be observed that

	Material	Nature of Stress	Location of Stress	Stage of Loading	Description	Strength (basic)	Belgium	England	France	Germany	Switz.	U.S.A.
						Structure	General	General	General	General	General	Bridges
						Ref. No.	14-3	14-2	14-5	14-1	14-4	14-6
Allowable stresses in terms of basic strength	Concrete	Compressive	Extreme fiber	Transfer		f'_{ci}	.45	.50	.50	.45	.40	.55→.60
				Design		f'_c	.33	.41	.35	.34	.40	.40
			Axial			f'_c			.35			
		Bearing	Anchorage	Transfer		f'_{ci}	.73	.62	.93	.61		
			Others	Design		f'_c				.31		
		Tensile	Extreme fiber	Transfer	Non-reinf.	f'_{ci}	0→.045	.075	.04	0→.11		.06
					Reinf.	f'_{ci}	.09			.14		
				Design	Non-reinf.	f'_c	0→.045	0→.075	0→.025	0		0
					Reinf.	f'_c	.09			.11		
				Cracking	Non-reinf.	f'_c				.07		.15
					Reinf.	f'_c				.14		
			Axial	Design		f'_c				.03→.04		
				Cracking		f'_c		.04→.06		.06		
			At anchorage	Transfer		f'_{ci}	.045→.09	.075				
			Principal tensile	Design	Non-reinf.	f'_c	.033		.013	.025	.02	.03
					Reinf.	f'_c	.10			.04		
				Ultimate	Non-reinf.	f'_c		.075		.045		.08
					Reinf.	f'_c				.11		
		Bond		Design		f'_c				.025		
				Ultimate		f'_c				.045		
	Pre-stressed rein-forcement	Tensile		Jacking		f_y	.68			.80	.65	
						f'_s	.66			.60	.70	.80
				Transfer		f_y	.80			.75		
						f'_s	.60		.67	.55		
				Design		f'_s			.57			.60
	Non pre-stressed rein-forcement	Tensile	Flange	Transfer Design		f_y			.54	.50→.63		
				Ultimate		f_y				1.00		
			Web	Transfer Design		f_y				.50→.63		
				Ultimate		f_y			1.00	1.00		
Load factors in terms of design load		LL		Cracking	DL factor = 1			1.25 liquid container				
		$LL+DL$								1.35	1.5	
		LL		Ultimate	DL factor = 1				3		2.5	
	Concrete	$LL+DL$						2.5	2	2.60	2.0	
	Rein-forcement							2.0		1.75	2.0	

Fig. 14-4-1. Allowable stresses and load factors for prestressed concrete.

there is a general agreement on the main items, such as the compressive stresses in concrete and tensile stresses in steel. In fact, these main values have been justified by rather extensive application as well as supported by adequate tests and analysis. Other values that differ greatly among different countries must be used with more

precaution and should not be applied in practice unless their background is definitely known to the designer. Values recommended by the U.S. Bureau of Public Roads, 1954, are listed in Appendix D.

14-5 Allowable Stresses for Concrete

1 · Extreme Fiber Compressive Stress. At jacking or at transfer of prestress, the amount of prestress is rather accurately known, there is little likelihood of excessive loading, and there is no danger of fatigue; hence a relatively high stress is permissible. Here the stress is not limited by the consideration of overload capacities but rather by the possibility of excessive creep or other undesirable strains. Values recommended in different countries vary from $0.40f_{ci}'$ to $0.60f_{ci}'$. It is believed that $0.55f_{ci}'$ is not excessive, if camber and creep do not constitute a problem. Where the quality of concrete is under rigid control, as in a precasting factory, it is often permissible to raise the allowable stress to $0.60f_{ci}'$.

When considering the structure under service loads, there is frequently a possibility of fatigue effect and occasional excessive overloads; hence lower values must be allowed. Generally $0.33f_c'$ to $0.40f_c'$ for bridges and $0.40f_c'$ to $0.45f_c'$ for buildings are the maximum. Higher values can be justified only after careful investigations of fatigue and ultimate strength. The value of f_c' is usually based on the 28-day strength but may occasionally be based on the strength of concrete at the time of service if such strength can be assured. Depending upon the shape of the section, the above allowable values will usually yield a factor of safety of about 2.5 to 3, which is ample. Occasionally, a factor of safety of only about 2 is attained, which seems inadequate for concrete except where overloads and repeated loadings are not at all likely.

2 · Axial Compressive Stress. As explained in Chapter 12, axial compressive stress is produced by two sources: prestress and external load. Compressive stress produced by prestress decreases as the external load is increased; that produced by external load generally increases faster than the load, on account of column action. Because of the different nature of these two types of compressive stresses, it will be almost impossible to specify allowable stresses for prestressed columns. When prestressed columns carry more bending than axial loads, the allowable stress can be compared to that for flexure. In addition, the column should be checked for its capacity to carry overloads, as in example 12-4-1.

3 · Bearing Stress. Depending upon the type of anchorage and the methods of stress analysis, allowable bearing stress at the anchorage

has been accorded widely different values. Most conventional methods of analysis yield only fictitious stresses; hence the allowable values must be varied accordingly. Moreover, the strength of concrete in local bearing depends greatly upon the surrounding conditions; e.g., a confined high bearing stress may not be harmful at all. Hence, in general, allowable bearing values are rather high—in fact, so high that more failures in prestressed concrete have occurred in bearing under prestress than any other type of failure. Fortunately, such failures, generally resulting from poor concrete, are noticeable during prestressing, so that means may be immediately taken to remedy the situation in order to ensure safety under service.

For direct bearing under anchorage plates, a value of $0.60f_c'$ is often permitted. This is much higher than that allowed for support bearing in reinforced concrete, but it is justified by the fact that the bearing load comes only from the prestressing steel, and there is no need for providing a greater margin of safety than that given the force in the steel. Since the usual allowable prestress in steel is $0.60f_s'$, the allowable bearing stress in concrete can be as high as $0.60f_c'$. In fact, there is little likelihood of any serious increase in the bearing force on the concrete at the anchorage, even if the load on the beam is greatly augmented. This is especially true for bonded beams.

Bearing stress at places other than the anchorage is similar to that for reinforced concrete, and little reason can be found to allow different values, although it is known that the usual allowable bearing stress for concrete is quite conservative.

4 · Extreme Fiber Tensile Stress. The average modulus of rupture for high-strength concrete is about $0.12f_c'$. Hence an allowable tensile fiber stress such as $0.05f_c'$ is considered to be on the safe side, especially for the stress at transfer when no overload is expected. Under service, structures may be subjected to overloads resulting in cracking of concrete, and objections have been raised as to whether any tensile stress should be allowed in designing, since the concrete would have cracked and would not be able to carry any tension. Actually, whether the concrete has cracked or not has little to do with allowable tensile stress. Consider reinforced concrete, for example. Under the design load concrete always cracks on the tensile side, which means that tensile stress is always allowed in design, although it is neglected in the analysis. If the same criterion is applied to prestressed concrete, high tensile stress can also be allowed provided that it is neglected in the analysis. However, allowing too high a tensile stress in a prestressed beam would tend to lower the ultimate

strength of the beam and to increase its deflection under overloads. Also, cracking in prestressed concrete may expose the wires to corrosion and subject them to possible fatigue failures. These are the real reasons for not allowing high tensile stress in prestressed concrete.

5 · Axial Tensile Stress. The actual direct tensile strength of concrete varies widely, say from $0.06f_c'$ to $0.10f_c'$. For computing cracking load under direct tension, a value of $0.06f_c'$ is recommended by the German code. This appears to be a reasonable value, assuming that the member has not been previously cracked. When designing tension members, some residual compression under design loads is often necessary in order to provide a sufficient margin of safety against cracking. It is considered best not to specify any allowable direct tensile stress. Instead, the member should be designed for its ultimate or serviceable strength with a proper load factor.

6 · Tensile Stress at Anchorage. The English *First Report* recommends half the modulus of rupture as the allowable value. On the European continent, a value of $0.045f_{ci}'$ is often allowed. Where the allowable value is exceeded, reinforcement is designed to carry the total amount of tension.

7 · Principal Tensile Stress. Comparing the allowable principal tensile stress in prestressed concrete to the allowable diagonal tension in reinforced concrete, a value of $0.03f_c'$ seems to be reasonable. It is generally assumed that, if the principal tension under working load does not exceed that allowable value, no web reinforcement is theoretically needed, although some nominal stirrups are often provided. Since the principal tension in a prestressed beam increases rapidly with the increase of the external shear and moment, and since the flexural cracking of concrete will affect its shear resistance, it is believed that ultimate strength would be a better basis for the shear design of structures subject to heavy overloads.

8 · Bond Stress. Bond stress in the beam away from the end anchorage is generally small previous to cracking, but after cracking, it increases very fast. Hence it is again impossible to base the design on allowable stresses within the elastic range. If the bond strengths between concrete and various steels and enclosures are known, it is possible that design for bond will also be formulated on the basis of ultimate strength.

At the end of unanchored pre-tensioned steel, neither the actual bond stress nor the ultimate bond strength is definitely known. Though in Europe general approval is given to the bond strength of small wires, questions have been raised regarding their bond resistance after cracking of concrete near the ends and the minimum length

of anchorage required for such wires. No definite values can be recommended until more data are available.

14-6 Allowable Stresses for Steel

Allowable stresses for steel are expressed as a percentage of either the ultimate strength f_s' or the yield point f_y; sometimes they are expressed in terms of both, and the smaller value thus obtained is used as the allowable. During tensioning and immediately after, the yield point f_y is often a more significant value, because it is an indication of the creep and elastic limits, tensioning near or beyond which might produce excessive creep and plastic deformations. At that same stage, the ultimate strength f_s' is important only as an indication of the existing safety against breakage of the steel. Unless the value of f_y approaches very nearly that of f_s', it should be sufficient to specify the allowable as a percentage of f_y only, disregarding f_s'. However, since the definition of f_y varies so widely and the ratio of f_y/f_s' also differs greatly for various steels, it may be preferable to specify the allowable in terms of f_s'. Current allowable values for steel stress during jacking varies from $0.80f_y$ to $0.88f_y$ and from $0.60f_s'$ to $0.80f_s'$. In order to overcome frictional loss, the Bureau of Public Roads allows a temporary jacking stress of $0.80f_s'$ for post-tensioning. It must be remarked that the chances of breaking the wires during jacking are somewhat increased by high prestressing.

The allowable stress at transfer can be somewhat lower than that at jacking, simply because overtensioning is often exercised during jacking operations (for the purpose of minimizing creep or overcoming friction). For post-tensioning, it is about 5–15% lower than the stress at jacking. For pre-tensioning straight tendons, there is no frictional loss to be overcome and the tendons are jacked and anchored at the same time; hence it is necessary to specify only the stress at jacking, no additional limitation being placed on the so-called stress at transfer.

The allowable stress for design loads is determined by two conditions. First, it is limited by the allowable stress at jacking which, after deducting all losses, becomes automatically the design stress. Hence, often the design stress is not specified but is computed from the stress at transfer by deducting all the losses. There is a second condition, however, which often constitutes a good reason for specifying an upper limit for the design stress. The ratio of the ultimate strength of a member to its design strength is lowered when the design stress is raised in the steel. This explains why some authorities prefer to set a maximum allowable stress for design in addition to one

for the stress at transfer. Furthermore, allowance must be made for the possibility of fatigue and creep effects, which may be more serious at higher stresses. This is another reason for limiting the design stress to a certain level.

The above general discussion has not included possible differentiation between bonded and unbonded reinforcement, since the two types of construction possess different ultimate strengths. Again, for new materials which possess widely different elastic and strength properties from those currently in use, allowable stresses must be necessarily modified. It is more important that the designer understand the basis for fixing the allowable values so that he can exercise his judgment when encountering new situations.

The design of non-prestressed reinforcement can hardly be performed by the allowable stress method (see Chapter 9). The German code calls for design on the ultimate strength basis, using the yield point of mild steel as the maximum stress developed at rupture. A further requirement is to limit the stress under working loads to 50–63% of the yield-point stress as is done for reinforced concrete. This often is not necessary, since the use of proper load factors in ultimate design should automatically yield reasonable stresses under working loads. The design for non-prestressed high-tensile wires must also be based on the ultimate strength method, keeping in mind that such wires will not appreciably increase the cracking strength of the members.

14-7 Load Factors

Load factors are dimensionless values by which the working load must be multiplied in order to be equated to the ultimate strength of a member under design. Instead of using the ultimate strength, the cracking strength is sometimes employed; it should then be stated that the factor is for the cracking strength. Proposals have also been made to use allowable ultimate strengths or usable strengths instead of the tested ultimate strengths[14-10] as the basis for design.

The choice of proper load factors constitutes a most significant problem in the design of prestressed-concrete structures. In the design of more conventional structures, procedures and methods have been so standardized that design by the allowable stress method will result in satisfactory dimensions: ultimate-strength formulas have been either knowingly or unintentionally inserted into and mixed up with the allowable stresses such as column formulas for reinforced-concrete design. In prestressed concrete, because of the nonlinear variation of stresses with loads—steel stress, concrete fiber

stress, bond stress, principal tensile stress, etc.—designers are forced to adopt the ultimate approach at least in part. But no ultimate-strength formulas can be applied until the proper values for load factors have been decided.

The various bases for the choice of load factors have been discussed in section 14-3. The most important item is the ratio of the maximum overload to the design or working load. But, since a structure is rendered unserviceable long before its total rupture, it is evident that the maximum possible overload cannot be equated to the ultimate strength but only to the maximum usable strength. Hence load factors generally can be expressed as a product of two ratios:

$$\frac{\text{Maximum possible load}}{\text{Working load}} \times \frac{\text{Ultimate strength}}{\text{Maximum usable strength}}$$

In addition, they must be chosen so that, in general, no excessive stresses or deflections or fatigue failures will occur under the working load. This is necessary in order that structures designed by the ultimate-strength method will not need serious revisions when checked for stresses, deflections, and fatigue effects.

In prestressed-concrete design, two sets of load factors are often required, one for cracking strength, another for ultimate strength. These load factors are sometimes applied to the live load only; at other times they are applied to the total load, dead and live. At cracking strength, some authorities call for a load factor of 1.35 to 1.5 when applied to the total load. There is, in fact, little justification for checking the cracking strength except for liquid containers and similar structures. In England, a load factor of 1.25 is recommended for them.

Load factors for the ultimate strength are more often specified for the total load. Values recommended in different countries vary from 1.75 to 2.5 for steel and from 2.0 to 2.6 for concrete. Because of the undesirability of brittle failures in concrete, such as shear or compression failures, higher factors are desirable. For the usual range of reinforcement, when failure starts gradually in the elongation of steel and ends in the crushing of concrete, sufficient warning is given before impending collapse so that smaller load factors are considered sufficient. Because of the improbability of any serious increase in dead load, relatively low load factors should be called for when applied to the total load. Speaking in general, a value of 1.75 should suffice for gradual failures and 2.25 for failures without warnings.

When there is a definite possibility of serious overloading, it is best to apply the load factor to the live load only. For highway bridges

in this country the Bureau of Public Roads suggests a load factor of 3.0, although it is known that at present the ratio of maximum possible loading to design loading is about 1.8. As mentioned previously, a margin must be reserved for the strength between the usable and the ultimate, and also for future increase in live load.

It is in a way unfair to prestressed-concrete structures that they alone should be checked for overload capacities, thus sometimes resulting in heavier designs. But it is generally considered a worthwhile safeguard at this stage of its development. The designer must also realize that, when so designed, a prestressed-concrete structure is often stronger than its counterpart designed of other materials by conventional methods.

Besides dead and live loads, the effect of wind, earthquake, and strain loads should also receive consideration and be given proper load factors. When dead load produces stresses opposite to the live load, it is sometimes suggested that only a portion of the dead load should be considered as effective. These related problems are only beginning to get the attention of engineers, since it is more and more realized that ultimate design has to be applied to prestressed-concrete as well as to other types of structures.

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Chapter 15

Economics

15-1 General Considerations

When prestressed concrete was first used in this country in the early 1950's, the problem of the relative economy of this type of construction as compared to others was a controversial issue. Some zealous advocates held an optimistic outlook on its saving in materials; other conservative engineers overestimated the additional labor involved and condemned its popular adoption in this country. That period is now almost concluded. With the numerous prestressed-concrete structures built all over the country, its possible economy is no longer in doubt. Like any new promising type of construction, it will continue to grow as more engineers and builders master its technique. But, like any other type of construction, it has its own limitations of economy and feasibility so that it will suit certain conditions and not others.

The time is also past when one or two specific instances of the relative economy of prestressed concrete as against other types could be cited as positive proof either for or against its adoption. Hence no attempt will be made to refer the reader to the many early articles and discussions on the economy of certain particular prestressed structures. Basic quantity data are now known for prestressed concrete, and the unit price for prestressing is gradually becoming stabilized.[15-1–15-5] Hence a general discussion is possible at this time, although some of the values should be modified with further developments.

It must be realized from the very outset that in prestressed concrete we have materials that are much stronger than those for ordinary reinforced concrete. At the same time, these materials cost more and require more labor and better technique for placement. Speaking in general, the working stress in prestressing steel is 5 to 7 times as high as mild steel, and its unit price in place is 2 to 5 times as much. Concrete is 2 times stronger than reinforced concrete, and costs about

20% more, not including formwork which may cost from 0% to 100% more than that for reinforced concrete. Between the various possible combinations of strength and cost of these materials, it can be readily seen that the net result can be either for or against the use of prestressed concrete.

From an economic point of view, conditions favoring prestressed construction can be listed as follows:

1. Long spans, where the ratio of dead to live load is large, so that saving in weight of structure becomes a significant item in economy. A minimum dead-to-live-load ratio is necessary in order to permit the placement of steel near the tensile fiber, thus giving it the greatest possible lever arm for resisting moment. For long members, the relative cost of anchorages is also lowered.

2. Heavy loads, where large quantities of materials are involved so that saving in materials becomes worth while.

3. Multiple units, where forms can be reused and labor mechanized so that the additional cost of labor and forms can be minimized.

4. Precasting units, where work can be centralized so as to reduce the additional cost of labor and to obtain better control of the products.

5. Pre-tensioning units, where the cost of anchorage, sheathing, and grouting can be saved.

There are other conditions which, at present, are not favorable to the economy of prestressed concrete but which are bound to improve as time goes on. These are:

1. The availability of builders experienced with the work of prestressing. This will stimulate keener competition and supply skilled workmen at smaller cost.

2. The availability of equipments for post-tensioning and of plants for pre-tensioning. This will obviously reduce the unit cost of prestressing.

3. The availability of engineers experienced with the design of prestressed concrete. This will permit more prestressed-concrete structures to be designed and built and thereby their cost will be lowered.

4. The reduction of the cost of materials and installation for prestressed concrete. This has already been taking place, and the trend will continue although at slower pace as new methods and materials are developed and as the demand and supply both grow with time.

5. The promulgation of a logical set of codes and recommendations. This should put prestressed-concrete structures on an equal footing with other types and encourage their design and construction.

It is perhaps unnecessary to repeat that there will always be situa-

tions where prestressed concrete cannot compete economically with other types of construction, whether timber, steel, or reinforced concrete. Each type has its advantages as well as limitations.

15-2 Quantity of Materials

As compared to reinforced concrete, the basic economy of prestressed concrete lies in the saving of materials, because it utilizes concrete and steel of much higher strength. A general comparison of the quantity of materials required for the two types of construction can be made, based on their strength ratios.

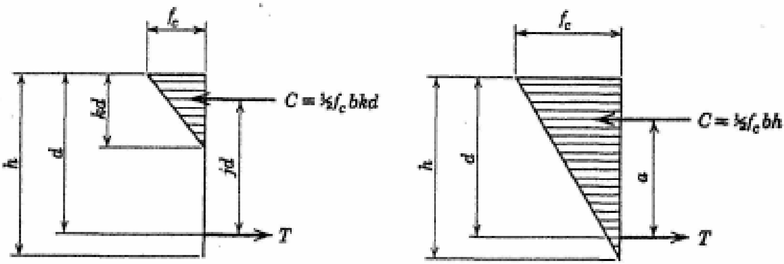

Fig. 15-2-1. Comparison of concrete stress blocks in beams (under working loads).

First, let us compare the quantity of concrete. The economical strength of concrete for ordinary reinforced work is generally 2500 to 3000 psi in this country. The use of higher strength resulting in a smaller section will increase the amount of reinforcing steel and is not economical. In prestressed concrete, the average strength used is 4500 to 5000 psi. Considering 3000 psi for reinforced concrete, the allowable stress at $0.45f_c'$ is 1350 psi; similarly, for 5000 psi in prestressed concrete, the allowable stress at $0.40f_c'$ is 2000 psi. Under working loads, the stress blocks for the two types are shown in Fig. 15-2-1. For reinforced concrete, the resisting moment is given by the well-known formula

$$M = \tfrac{1}{2} f_c kjbd^2$$

Using a value of $k = 0.40$, and of $j = 0.87$, we have

$$M = \tfrac{1}{2} \times 1350 \times 0.40 \times 0.87 \times bd^2$$
$$= 235bd^2 \qquad (15\text{-}2\text{-}1)$$

For prestressed concrete, the resisting moment at working load is given by

$$M = \tfrac{1}{2} f_c bha$$

Assuming that $h = 1.1d$, and $a = 0.6d$, we have

$$M = \tfrac{1}{2} \times 2000 \times b \times 1.1d \times 0.6d$$

$$M = 660bd^2 \tag{15-2-2}$$

Comparing equation 15-2-2 with equation 15-2-1, it is seen that, for the same resisting moment, the ratio of bd^2 required for the two is

$$\frac{bd^2 \text{ for prestressed concrete}}{bd^2 \text{ for reinforced concrete}} = \frac{235}{660} = 0.36$$

If d is kept the same for both sections, the ratio of the areas is 0.36; if b is kept the same, the ratio is $\sqrt{0.36} = 0.6$. Hence it would be proper to say that, as an average, the quantity of concrete required for prestressed is only about one-half of that for reinforced work.

The above comparison does not take into account the fact that the lighter dead weight of the prestressed design would further decrease the quantity required. Also the value of a for the prestressed beam can vary greatly and throw the advantage one way or the other. Again, the above computed saving in concrete is possible with prestressed work only because the principal tension produced by shear is reduced as a result of prestressing. Otherwise, smaller concrete sections might require excessive web reinforcement.

In order to get an optimum value of a for prestressed beams, it is often necessary to use I- or T-sections instead of rectangular ones. This means more complicated formwork. When rectangular sections are used for prestressed beams, the formwork can be a little less than for reinforced beams, since the sectional area is smaller.

The ultimate strength for concrete in reinforced work cannot be used as basis for economic comparison because the factor of safety is unnecessarily high. But the quantity of steel for the two types of construction can be compared on the basis either of working stress or of ultimate strength. On the basis of working stresses, reinforcing bars of intermediate grade are designed for 20,000 psi, and, with a lever arm of $jd = 0.87d$, the resisting moment is

$$\begin{aligned} M &= f_s A_s jd \\ &= 20{,}000 A_s \times 0.87d \\ &= 17{,}400 A_s d \end{aligned} \tag{15-2-3}$$

Taking an average value of design stress for prestressing steel as

110,000 psi, and with $a = 0.7d$ for an average section, the resisting moment is

$$\begin{aligned} M &= f_s A_s a \\ &= 110{,}000 A_s \times 0.7d \\ &= 77{,}000 A_s d \end{aligned} \tag{15-2-4}$$

Comparing equation 15-2-4 with equation 15-2-3, it is seen that the materials required to resist the same moment will be in the ratio of

$$\frac{\text{Prestressed steel}}{\text{Reinforced steel}} = \frac{17{,}400}{77{,}000} = 0.23$$

On the basis of ultimate strength, at the rupture of the beams, prestressed steel will be stressed to about 220,000 psi on the average, while reinforcing bars will be stressed only to about 40,000 psi. Owing to the increase in the lever arm for the prestressing steel at the ultimate range, the ultimate lever arm is about the same for both reinforced and prestressed work; hence the ratio of materials required will be directly proportional to their strength, thus:

$$\frac{\text{Prestressed steel}}{\text{Reinforced steel}} = \frac{40{,}000}{220{,}000} = 0.18$$

Hence an average quantity ratio for the two materials will be

$$(0.23 + 0.18)/2 = 0.20$$

in favor of prestressing steel.

The amount of web reinforcement is not included in the above. Because of the presence of pre-compression, web reinforcement is much less in prestressed beams even though they have smaller concrete sections. However, the ratio of web reinforcement for the two types of construction varies greatly and generalizations are not easy.

It must be noticed that the above two ratios of materials, 0.50 for concrete and 0.20 for steel, apply only to the critical section of a member. For reinforced-concrete beams, the concrete section is often kept constant while the amount of reinforcing steel can be varied along the length of the beam, thus effecting some saving in steel. For prestressed-concrete beams, the area of steel cannot be so conveniently varied, although it is frequently possible to vary the concrete section, saving some concrete at the expense of formwork.

In prestressed concrete, various accessory materials are required which add greatly to the cost. Depending upon the method of prestressing used, the quantities of end anchorages, sheathing, and grouting vary to a great extent. The cost of plant and overhead varies with the amount of work involved; the cost per unit decreases with the

increase in the amount of work. For convenience in discussion, these factors will be analyzed in the next section.

15-3 Unit Cost

The above section describes the saving in material for prestressed as against reinforced concrete because of the higher strength of materials employed. It is only natural that materials of higher strength will cost more per unit volume. Whereas the relation of strength to volume of material is a matter of mechanics, the cost of materials per volume is dependent on time and place. However, it is possible to take an average case as a basis for comparison, keeping in mind that these values must be modified for local and individual conditions if they differ from the so-called average case.

1 · Concrete. On the average, about 2 more bags of cement are required per cubic yard of 5000-psi concrete as compared to 3000-psi concrete. At a cost of $0.80 per bag, this means an additional cost of $1.60 per cu yd. If ready-mixed concrete is available at $12 per cu yd for the ordinary mix, it may cost about $14 for the prestressed concrete. The cost for placing the concrete will also be a little higher for the stronger concrete, since it will be less workable. Assuming $7 per cu yd for the placing of ordinary concrete, $9 may be required for prestressed work. Hence the contract cost will be:

Cost per cu yd of concrete in place, exclusive of forms:
Reinforced concrete at 3000 psi = $19
Prestressed concrete at 5000 psi = $23

Thus the unit cost of concrete for prestressed work is about 20% higher than that for ordinary reinforced work, assuming average conditions.

2 · Formwork and Falsework. The unit cost of formwork for slabs is virtually the same for prestressed and reinforced concrete. For beams, depending upon the shape of the concrete section, the unit cost per square foot of contact area will be 40% to 100% higher than for reinforced work. Assuming $0.50 per sq ft for reinforced concrete, it will average about $0.70 for prestressed beams. The number of square feet of contact area per cubic yard of concrete varies greatly with the size and shape of beams. As an average value, we may assume 40 sq ft per cu yd of reinforced concrete, and 90 sq ft per cu yd of prestressed concrete. Then the average cost of formwork, assuming the forms to be not reusable, will be

Cost of formwork per cu yd of concrete:
Reinforced concrete 40 × $0.50 = $20
Prestressed concrete 90 × $0.70 = $63

If the forms are reusable the cost will be much lower.

The amount of falsework supporting the concrete will vary greatly. For an average job, it will be a little cheaper for prestressed concrete, because less weight is to be supported. But, expressed in terms of dollars per cubic yard, it will be higher for prestressed work, since the total quantity involved is less. When precast elements or composite sections are used for prestressed concrete, the saving may be quite appreciable. Just for the purpose of discussion we may assume:

Cost of falsework per cu yd of concrete:
Reinforced concrete = \$5.00
Prestressed concrete (include cost of lifting if precast) = \$8.00

3 · Steel. For both ordinary reinforced concrete and for non-prestressed reinforcement in prestressed concrete, reinforcing bars cost an average of about \$0.065 per lb of raw material. Adding to it the cost of transportation, fabrication, and installation, the contract cost in place averages about \$0.12 per lb in this country.

High-tensile wires average about \$0.14 per lb of material. Cost of transportation, cutting, straightening, and tensioning may average about \$0.20 per lb. This will yield an average contract cost of \$0.34 per lb for pre-tensioned unanchored wires. Cost of end anchorages may average about \$0.10 per lb of wire, with cost of sheathing and grouting another \$0.06 per lb of wire. Hence post-tensioned wires will average about \$0.50 per lb, a little higher for the bonded and a little lower for the unbonded reinforcement.

Fabricated high-tensile wire strands cost about \$0.20 per lb of material. Cost of end anchorages together with cutting and fitting which are done in the shop may amount to another \$0.16 per lb, depending chiefly upon the length of the strands. Installation, tensioning, and transportation may total another \$0.08 per lb, while sheathing and grouting, if used, may average \$0.06 per lb. Hence the average cost of steel in place for the usual beams may run about \$0.50 per lb for post-tensioned bonded type, while, if used for pretensioning, the cost may not be much more than \$0.33 per lb.

High-tensile bars cost about \$0.22 per lb of material. Cost of end anchorages and splices, if required, may be about \$0.05 per lb. Transporting, installation, and tensioning may total \$0.10 per lb, and sheathing and grouting another \$0.08 per lb. This will yield a total of \$0.45 per lb of bar in place for the post-tensioned bonded type. Bars for anchored pre-tensioning work cost about \$0.30 per lb.

It is evident that the above average values may vary greatly, depending upon the particular conditions, and the designer is advised to obtain his own data for an accurate assessment. It is further nec-

essary to note that while the average unit cost differs for the wires, strands, and bars, when the matter of strength and suitability is taken into account, each of the three materials will have its own field of economy and it is not possible to condemn or to favor one material for all conditions.

If it is desired to express the cost of steel in terms of unit yardage of concrete, we can assume an average amount of 80 lb of high-tensile steel per cubic yard of concrete: For unit cost at \$0.50 per lb, an approximate total cost of \$40 of high-tensile steel in place including accessories is indicated for each cubic yard of post-tensioned work. It must be realized, of course, that values like this should be taken with a grain of salt, since the percentage of steel varies greatly with different designs. The amount of mild steel also varies with each job; a value of \$5 of mild steel per cubic yard of prestressed concrete may be taken as an average.

4 · Accessories. Although the cost of accessories has been included in the last paragraph and expressed in terms of cost per pound of steel, for more accurate estimates it would be desirable to separate such cost from that of steel, since cost does not always vary directly with the weight of steel.

End anchorages for wires cost about \$3 to \$5 apiece, capable of anchoring 6 to 18 wires of about ¼-in. size. End anchorages for bars cost \$1 to \$2 apiece, for ½-in. to 1⅛-in. bars. Those for wire strands cost from \$5 to \$30 apiece, for steel cross-sectional areas of 0.2 sq in. to 2 sq in., including the cost of cutting and swaging but excluding the cost of steel bearing plates. These should be used only for preliminary estimates; more accurate values can be obtained from the respective manufacturers.

Metallic sheathing costs \$0.05 to \$0.20 per ft of length, enclosing from 6 to 32 wires. Plastic sheathing used for unbonded work costs \$0.05 to \$0.10 per ft; paper enclosure costs only \$0.01 to \$0.02 per ft but involves more labor in wrapping. Retractible rubber tubes of about 1½ in. by 2½ in. size cost about \$1.00 per ft but can be reused many times.

The cost of grouting varies markedly with the size of the job. When the plant is set up and the work systematized, grouting does not cost more than \$0.01 to \$0.02 per lb of steel.

Example 15-3-1

Eighty prestressed-concrete roof girders, each 75 ft long, are to be built for a warehouse, using concrete of 5000-psi and wires of 240,000-psi strength. The midspan section of the girders is shown in Fig. 15-3-1. Cables in the web are

to be bent; those in the bottom flange are to remain straight. The girders are to be precast at the building site and to be lifted to place. Allow 100 days for casting and prestressing the girders. Estimate and analyze the cost of the girders: (*a*) using 11 bonded cables of twelve 0.196-in. wires per cable; (*b*)

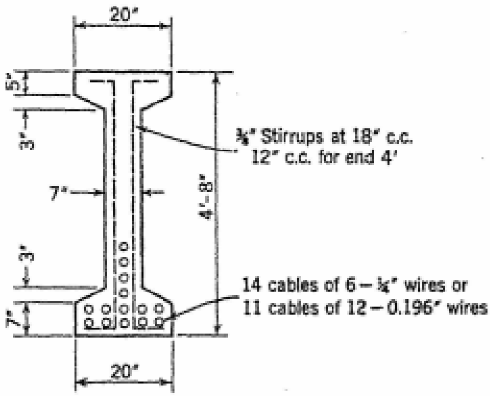

Fig. 15-3-1. Example 15-3-1.

using 14 unbonded cables of six 0.250-in. wires per cable. Assume unit costs as necessary. All cables are to be post-tensioned.

Solution. Quantities of materials required for one girder are estimated as follows:

Concrete	12 cu yd
Wires, 11 cables of 12–0.196-in. wires	1050 lb
14 cables of 6–0.250-in. wires	1100 lb
Mild-steel bars	500 lb
End anchorages, for 11 cables	22 units
for 14 cables	28 units
Metal hose for bonded cables	830 ft
Paper wrapping for unbonded cables	1050 ft
Formwork, bottom contact area	125 sq ft
side contact area	780 sq ft

Contract unit prices for the various items are assumed as follows:

Concrete, 5000 psi, in place	= \$22/cu yd
Wires, material and warehousing	= \$0.17/lb
Straightening, cutting, placing at 0.04 hr/lb and at \$2.50 per man-hour	= \$0.10/lb
Total	\$0.27/lb
Metallic hose, material and labor	= \$0.12/ft
Paper wrapping, and grease	= \$0.03/ft
Anchorage for twelve 0.196-in. wires	= \$4.80/pc

Anchorage for six 0.250-in. wires	= $4.50/pc
Labor for tensioning at $2.50/hr:	
6-wire units @ 1.5 hr per anchorage	= $3.75/pc
12-wire units @ 1.8 hr per anchorage	= $4.50/pc
Equipment and plant, 100 days at $20 a day	= $2000
Grouting, 0.4 hr/cable at $2.50/hr	= $1.00/cable
Formwork, bottom, reused 5 times	= $0.20/sq ft
side, reused 10 times	= $0.15/sq ft

The cost per girder can now be tabulated.

Item	Unit	Quantity	Unit Price	Total Price Design (a)	Total Price Design (b)
Concrete	cu yd	12	$22.00	$264	$264
Wires	Lb	1050	$0.27	$283	
		1100	0.27		$297
Sheathing	Ft	830	0.12	100	
		1050	0.03		32
Anchorage	Piece	22	4.80	106	
		28	4.50		126
Tensioning	Unit	22	4.50	99	
		28	3.75		105
Equipment and plant				25	25
Grouting	Cable	22	1.00	22	
	Cost of steel and prestressing			$635	$585
Formwork, bottom	Sq ft	125	$0.20	$25	$25
side	Sq ft	780	0.15	117	117
Lifting	Girder			50	50
	Cost of formwork and erection			$192	$192
	Total cost per girder			$1091	$1041

The cost analysis is performed as follows:

	Design (a)	Design (b)
Cost per lb of steel	$ 0.605	$ 0.532
Cost per cu yd of concrete		
steel	$53	$49
concrete	22	22
formwork and lifting	16	16
	$91	$87

15-4 Cost Comparison with Reinforced Concrete

On the basis of the unit quantities discussed in section 15-2 and the unit prices in section 15-3, it is possible to make an overall cost comparison between prestressed and reinforced concrete for an average case. Since it is almost impossible to define the exact meaning of the term "average" it must be admitted that such a comparison

merely serves as a qualitative guide, although for the sake of presentation it is expressed in quantitative terms.

The comparison can be made by two methods, depending upon the degree of accuracy desired. The first method of comparison is to consider the three main cost items separately: concrete, formwork and falsework, and steel. The second method, more approximate, is to compare the cost per cubic yard of concrete, everything included.

Method (a). Considering the three main items separately, and referring to sections 15-2 and 15-3, we have:

Concrete:

Average quantity ratio $\dfrac{\text{P.C.}}{\text{R.C.}} = 0.5$

Average unit price ratio $\dfrac{\text{P.C.}}{\text{R.C.}} = \dfrac{23}{19} = 1.2$

Average overall cost ratio $\dfrac{\text{P.C.}}{\text{R.C.}} = 0.5 \times 1.2 = 0.6$

in favor of P.C.

Formwork. Formwork, assuming that, for every cubic yard of P.C., 2 cu yd of R.C. are required:

Average quantity ratio $\dfrac{\text{P.C.}}{\text{R.C.}} = \dfrac{90}{40 \times 2} = 1.12$

Average unit price ratio $\dfrac{\text{P.C.}}{\text{R.C.}} = \dfrac{0.70}{0.50} = 1.4$

Average overall cost ratio $\dfrac{\text{P.C.}}{\text{R.C.}} = 1.12 \times 1.4 = 1.6$

in favor of R.C.

Falsework. Owing to the smaller dead weight of prestressed construction and the possibilities for precasting work, it can be assumed that the average cost ratio will be

$$\frac{\text{P.C.}}{\text{R.C.}} = 0.8 \text{ in favor of P.C.}$$

Steel:

Average quantity ratio $\dfrac{\text{P.C.}}{\text{R.C.}} = 0.20$

Average unit price ratio $\dfrac{\text{P.C.}}{\text{R.C.}} = \dfrac{0.50}{0.12} = 4.2$

Average overall cost ratio $\dfrac{\text{P.C.}}{\text{R.C.}} = 4.2 \times 0.20 = 0.84$

in favor of P.C.

Method (b). When all materials and labor are expressed in cost per cubic yard of concrete, an average case for post-tensioned beams and girders is as follows:

	R.C.	P.C.
Concrete	\$19	\$ 23
Formwork	10	34
Falsework	4	3
Steel	27	50
Total per cu yd	\$60	\$110

The above represents contract cost, including overhead and profit. Since the cost per cubic yard is nearly doubled for prestressed work, while the quantity of concrete required for a structure will be about one-half of that for reinforced beams, the overall cost works out to be a close competition between the two types of construction, with a slight favor for prestressed beams, considering an "average" case. When pre-tensioning can be adopted, the cost of steel in place will be about \$30 per cu yd; and the economy will become more obvious.

15-5 General Economic Considerations in Design

Economy and safety are the two major goals in structural design. The problem of designing prestressed-concrete structures economically is actually discussed in every chapter of this treatise. However, it may be well to summarize the major issues involved so that the designer can always grasp the vital points of economy and not lose himself in a maze of minor details.

The first and foremost decision to be made is whether the structure can be economically designed for prestressed concrete or whether it might be better to employ some other type of construction, be it timber, steel, or reinforced concrete. There are, of course, other problems besides economy, such as esthetic or functional requirements, which might force the choice one way or another, but by far the majority of structures will be decided on an economic basis. There are structures where prestressed concrete would be most suitable, but there are also those where it simply cannot compete with other types. Therefore the first motto for the designer is to employ prestressed concrete only where it belongs and not to use it as a cure-for-all.

The choice for prestressed-concrete construction must be considered together with the possible change in layout of the entire structure. Since most engineers are more familiar with other types of construction, span lengths and proportions are usually laid out with those in mind, the designer not realizing that, with prestressed concrete, radical changes may be possible and often desirable. For

example, longer spans, smaller depth, thinner members, bold cantilevers, precasting procedures, not contemplated for reinforced concrete, may be embodied in the layout when prestressed concrete is used.

Once the use of prestressed concrete has been decided upon, the next important decision concerns the right type of prestressed construction. Should the members be pre-tensioned or post-tensioned? Should they be precast or cast-in-place, or should composite construction be adopted? Is bonded reinforcement necessary for the job, or would the unbonded type suffice for the service? These questions must be answered first, although sometimes not until some preliminary work has been done on the design.

Engineers seldom realize a most important prerequisite in design, namely, that the design loadings for structures must be chosen with care and judgment. Too often the loadings are specified for the structures by certain code requirements, and the engineer simply takes them for granted. Such specified loadings can be either too heavy or too light. Although they may have proved to be satisfactory for other types of construction whose methods of design have been empirically devised to suit such loadings, they may not be directly applicable to a new type of construction like prestressed concrete. If such loadings are used, the designer should exercise his care in employing a suitable set of allowable stresses and load factors so as to attain a proper but not excessive degree of safety in the structure.

Having settled these essential premises, the engineer can now proceed to design the members in detail. The first item to be settled, then, is usually the depth or thickness of the member. This is an individual item which must be determined for each particular structure. But, speaking in general, the economical depth of beams and thickness of slabs would be about 70% to 80% that of reinforced concrete. The depth having been chosen, the concrete section can be designed, taking into account the cost of formwork, ease in concrete placing, as well as stress requirements. At the same time, the strength of concrete for the job can be decided on.

Although prestressing steel is an item peculiar to prestressed work, its choice ranks least in importance when compared to the major considerations mentioned above. When the layout of a member is fixed, there will result a certain amount of prestress required, which is often beyond the control of the designer. Sometimes, the adoption of a particular system of prestressing may modify the sectional dimensions of the members, but such modifications are minor when compared to the major decisions. In this country, it is often unnecessary and

sometimes undesirable to fix a particular system of prestressing for the job, because this will rule out competition which will help to lower the cost. Hence it is best to design a structure so that several systems, using different steels, may fit into it with very little modification, so that the problem of selecting an economic system is left to the competition of the bidders. There are, however, some obvious cases when one system is definitely superior to the others, and then the engineer should design on that basis.

One of the more vital problems in the design of prestress steel lies on the positioning of the steel so as to give it a maximum resisting lever arm. In order that such a position of steel will not overstress the concrete at transfer, it is sometimes necessary to add superimposed weight to the member previous to prestressing, or first to partially tension the steel and to retension it to its final prestress after some superimposed load is put on the structure. Another economic method is to place non-prestressed reinforcements at strategic points in order to increase the ultimate and fatigue strengths. Means such as those must be kept in mind and applied when conditions warrant.

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Chapter 16

Special Problems

16-1 Fire Resistance

The earliest tests on fire resistance of prestressed concrete have been traced back to 1938, when Hoyer carried out several tests on pre-tensioned slabs.[16-1] But the slabs were small, and no load was applied on them during the tests. Billner made some tests in 1943 on a slab using electrical prestressing.[16-2] The slab was loaded and heated to 1900°F for 4 hours; after the fire was extinguished, the slab still carried load up to the elastic limit of steel without failure. More recent tests were reported by Baar of Holland[16-3] on a prestressed beam in 1952, and valuable conclusions were reached. In France[16-4] and Switzerland,[16-5] tests were made around 1949 on high-tensile wires subjected to high temperatures. In England extensive and systematic tests were carried out since 1950.[16-6, 16-7] These tests are still being continued by the British Fire Research Organization with some cooperation from the National Bureau of Standards in Washington, D.C.

One approach to evaluate the fire resistance of prestressed concrete is to compare it with that of reinforced concrete, whose fire-resistance characteristics are better known. From the temperature-strength relations for the two materials, some comparison can be made. Extensive investigations on the thermal conductivity of concrete have been made in the past,[16-8] relative to plain and reinforced concrete. When it is attempted to apply the data to include prestressed concrete, certain problems arise. First, concrete used in prestressed work is denser in texture and is usually mechanically compacted. It possesses higher strength and is under higher compression. Hence such concrete is often believed to be a better conductor of heat. However, there is little evidence to substantiate such an opinion. The little data we have seem to show that the insulating qualities of concrete are as good in prestressed as in reinforced construction.

Regarding the strength of concrete as affected by high temperature,

some tests have showed that mechanically compacted concrete rapidly loses its strength at high temperatures.[16-8] Thus, when the compression side of a beam is heated, there exists the danger of compression failure in concrete, resulting in a sudden collapse of the structure. Although this has happened in isolated cases of testing, there is also evidence to show that sudden failure is not likely if the thickness of the member is more than 2 in. When the tension side is subjected to fire there is a progressive sagging of the member as the prestress is lost in the steel, and ample warning is given before actual collapse.[16-7] When rods of large diameter are used, or when the concrete cover exceeds 3 in. without reinforcing mesh, excessive spalling may take place which will expose the steel to fire and may result in collapse of the member for either reinforced or prestressed concrete.

The sensitivity of high-tensile steel to temperature can be compared with that of mild-steel bars. Results of some tests[16-3–16-5] on the strength of high-tensile wires under high temperature show that, up to about 300°F, there is a slight increase in strength, which begins to drop from there on down to a value of about 50% at 750°F. Since in general there is a factor of safety of 2 in our designs and since the ultimate strength of a prestressed member in bending is proportional to the ultimate strength of steel, that limit of 750°F in the steel may be considered as the point of impending failure, provided that full live load is on the structure. Owing to the loss of prestress in the steel as a result of high temperature, the member will sag considerably more than reinforced concrete, and some of that deflection may not be recoverable.

The strength of structural reinforcing bars at 550°F is about 25% higher than at normal temperature. At 800°F its strength is about the same as at normal temperature, but it drops sharply after that. Since the failure of reinforced-concrete members under flexure usually starts at the yield point of steel, it is more important to determine the variation of the yield point with high temperature than the variation of the ultimate strength. Data in this country seem to indicate that the yield point of mild steel is about halved at 1000°F,[16-9] whereas French tests have indicated that the limit is halved at about 750°F.[16-4] Hence it would be fair to assume that the critical temperature for reinforcing bars is somewhat higher than that for prestressing wires.

The fact that high-tensile steel is slightly more sensitive to heat does not necessarily mean that prestressed-concrete members are less fire resistant, because the resistance of the member will depend upon the thickness of protection afforded by the concrete. Whereas steel

bars for most reinforced work are placed near the surface of concrete with the minimum permissible protection, steel wires for prestressed concrete are distributed along the depth so that, while the exterior layer may be subjected to high temperature, the others may be only slightly heated and may still possess sufficient strength to prevent collapse. This is a point definitely in favor of prestressed concrete, although such an advantage may not exist at the center of a simple slab, where only one layer of steel is employed and it is placed with the minimum concrete protection. Where high fire resistance is necessary, plastering can be effectively employed. According to our National Building Code, ¾ in. of plaster is equivalent to about 1.5 to 2 in. of concrete and is good for fire resistance of about 2 hours. This is also borne out by British tests.[16-7]

Specifications for fire-testing is given by the ASTM Standard E 119–50. Before such tests have been extensively applied to prestressed concrete, the engineer will have to exercise his own judgment guided by available test results and common sense concerning fire resistance. He must understand the practical requirements of fire-resistive construction. First, such construction must serve more or less as a fire screen so as to prevent the spread of fire. Next, the structure must be able to stand a certain intensity and duration of fire without total collapse, and, in case of collapse, sudden failure without sufficient warning to the firemen should be avoided. Then the possibility of reusing the frames after a fire and the cost of repairs necessary for restoration also deserve some consideration.

Even though present data on the fire resistance of prestressed concrete are not sufficient for setting exact fire rating standards, certain tentative recommendations can be proposed from the available test results. For example, it is believed that a fire resistance of 2 hours can be obtained with a concrete cover to steel of about 2 in. For a fire resistance of 4 hours, about ¾ in. of plaster or vermiculite should be added. The addition of a vermiculite-plaster suspended ceiling on metal lath not only increases the fire resistance without adding excessive weight but it also simplifies the subsequent repairs after a fire provided that the basic structure is not damaged.

On the effect of low temperatures on both prestressed and reinforced concrete, tests carried out at the University of Ghent[16-10] indicated the following:

1. The crushing strength of concrete is considerably higher at —40°F than at normal temperature of 68°F, about twice for ordinary strength concrete and 1⅓ times for high-strength concrete.

2. The modulus of rupture of concrete is even more increased,

becoming about 3 times for ordinary-strength concrete, and 2 times for high-strength concrete.

3. The modulus of elasticity is slightly increased to about 1.2 times for ordinary-strength concrete and 1.1 for high-strength concrete.

4. Loading tests on both types of beams confirm the above results; their cracking and ultimate loads were higher at −40°F and their deflection was smaller.

16-2 Fatigue Strength

The fatigue strength of prestressed concrete can be studied from three approaches: that of concrete itself, that of high-tensile steel, and that of the combination. In addition, it will be convenient to use our knowledge on the fatigue strength of reinforced concrete as a guidance in estimating the fatigue properties of prestressed concrete. Although relatively little has been done on the fatigue properties of prestressed concrete, much is known on those of reinforced concrete.[16-11]

Speaking in general terms, the fatigue strengths of concrete are:

1. Under direct compression varying from zero to maximum, the fatigue limit for concrete is about $0.50f_c'$ to $0.60f_c'$.

2. For plain concrete beams under bending, the modulus of rupture under fatigue loads is about one-half that under a single static load.

These fatigue limits seldom worry engineers designing reinforced concrete, for several reasons. First, the stress range is generally small in reinforced-concrete structures, because the major part of the stress may be due to dead load, which does not vary. For both bending and compression members, the factor of safety against compression failure is high, and fatigue failure in concrete under compression is only a remote possibility. Reinforced-concrete structures are not designed against cracking, and hence the fatigue modulus of rupture does not interest design engineers.

In prestressed concrete, the conditions are somewhat different. Compression in the extreme fibers is often zero under dead load and increases to a maximum under live load. Hence the fibers are subjected to a wider stress range. Relatively little is known of the fatigue strength of high-strength concrete, although it is assumed that its ratio to the cylinder strength remains the same as for ordinary concrete.

On the fatigue strength of high-tensile steel, a limited number of tests ha e been performed. One series conducted in Belgium[16-12] on 0.196-in. wires with ultimate static strength of 242,000 psi and 0.275-in. wires with strength of 213,000 psi showed the following:

1. For a stress range of 107,000 to 135,000 psi, no failure will occur along the length of the wires for 2,000,000 repetitions.

2. Owing to stress concentration at the grip of wires, fatigue limit may range from 121,000 to 149,000 psi for a minimum stress of 107,000 psi, depending upon the type of grip.

For prestressed members under the action of design live loads, the stress in steel wires is seldom increased by more than 10,000 psi from their effective prestress of about 120,000 psi. Hence it is seen that there is no danger of fatigue failure along the length of wires if the working load is not exceeded. Fatigue tests on high-tensile bars also indicate that, for the stress range produced by working loads, there is no danger of fatigue failure.[16-13] In fact, it is safe to say that, so long as the concrete has not cracked, there is little possibility of fatigue failure in steel, even though the working load is exceeded.

After the cracking of concrete, high stress concentrations exist in the wires at the cracks. These high stresses may result in a partial breakage of bond between steel and concrete near the cracks. Under repeated loading, either the bond may be completely broken or the steel may be ruptured.

Several tests have been run on prestressed-concrete members, giving considerable data on their fatigue strength.[16-14,16-15] The results of these tests confirm the ability of the combination to stand any number of repeated loads within the working range. Failure started invariably in the wires near the section of maximum moment and directly over the separators where the wires had a sharp change in direction. As an empirical rule, it was found that, when the computed average stress in the wires approached a magnitude of 150,000 psi, when the applied load was 50–100% higher than the working load, fatigue failures in the wires began to occur. This was again confirmed by tests conducted by the author on the fatigue strength of continuous prestressed beams.[16-16]

No tests are available concerning the fatigue bond strength between high-tensile steel and concrete. But, from the results of tests on prestressed-concrete beams, it seems safe to conclude that, if properly grouted, bond between the two materials can stand repeated working loads without failure. This is true because, before the cracking of concrete, bond along the length of the beam is usually low. Tests on railway ties[16-17] seemed to prove that bond for pre-tensioned unanchored steel can stand repeated loads provided there are no cracks in concrete. In actual service, pre-tensioned railway ties may fail in bond, depending upon their conditions of support and the amount of overload.

Fatigue failures at anchorage of end-anchored tendons are hardly known. When the tendons are bonded to the concrete, stress in the tendons near the end is not affected by live load. Hence there is no danger of fatigue failure, even though high localized stresses may exist in the wires at the anchorage. When unbonded, stress near the end of wires will change under live loads, but the range of stress under working loads is small so that fatigue failures are not likely.

Few data on the fatigue strength of unbonded beams are available, but there is some evidence to indicate that the use of mild-steel reinforcement may greatly increase such strength.[16-16] Fatigue tests on partially-prestressed concrete members incorporating non-tensioned wires show that such wires can stand repeated loads somewhat higher than the working loads without decreasing their eventual load-carrying capacity.[16-18] This sounds quite reasonable, since the untensioned wires would be subject to the same stress range as the tensioned ones, while their stress would be at a much lower level and hence there is less danger of fatigue failure.

In this country, fatigue tests on prestressed-concrete beams are being conducted at Lehigh University and at the University of Colorado.[16-19] Because of our lack of knowledge on the static shear strength of prestressed concrete, apparently no work has been done on its shear strength under repeated loads.

16-3 Impact Resistance

The impact resistance of prestressed concrete is of importance when structures are to be designed for suddenly applied load. For ordinary structures such as bridges, the impact effect of wheel loads is treated as a dynamic increment to the static load, expressed as an impact percentage. It is known from experience that such a method of design is sufficiently safe though not accurate. There are, however, also structures which must be designed to resist loads essentially dynamic in nature, such as the sudden drop of a hammer or the sudden application of blast loadings. It will then be necessary to know the resilience or the impact resistance of prestressed concrete. It is not the purpose here to discuss the design for impact or blast in general, but rather to point out the resistance of prestressed concrete to such loadings, how it can be estimated, and how it may compare with other types of construction, say reinforced concrete.

So far as can be ascertained, on only two relatively comprehensive series of impact tests on prestressed concrete are publications available to the public. The first one was conducted by the British Building

Research Station[16-20] on 60 pretensioned beams of 6-ft span and 4 in. wide by 6 in. deep in section. The tests are now being extended to post-tensioned beams on 20-ft spans. Several variables were involved, essentially the strength of concrete, the amount of steel, and the prestress in it. Main conclusions from these tests and some later ones are as follows:

1. There are three types of failures under impact load as under static load: bond, shear, and flexural. Flexural failure may start in steel or in concrete, depending upon the percentage of reinforcement and the prestress in it.

2. Load deflection relations given by static tests can generally be used as a measure of the impact resistance. When failure starts in the steel and ends with the crushing of concrete, this measure is somewhat on the safe side. When failure is due to the breaking of wires or when the wires are non-prestressed, this measure errs a little on the dangerous side.

3. Resistance to repeated blows is not decreased by repetition if cracking or crushing of concrete has not taken place; otherwise, the strength may be decreased, especially when modes of failure other than flexural are developed.

4. For each beam, there appears to be optimum values for the percentage of steel, the prestress in it, and the strength of concrete so far as impact resistance is concerned. For example, either too high or too low a percentage of reinforcement may reduce the impact resistance.

5. The greatest resistance to the impact of a single blow will normally be obtained by the use of reinforced concrete, whereas the greatest resistance to repeated impact may be provided by prestressed-concrete construction.

6. The presence of shear reinforcement in the form of stirrups has an important influence on the impact resistance of reinforced concrete and may be expected to be of similar importance for prestressed-concrete members.

This series of tests also included some reinforced-concrete beams designed for the same static strength as some of the prestressed ones. For all these reinforced beams, failure started at the yield point of steel. And it is pointed out that, as for the prestressed ones, the resistance calculated from the results of the static test slightly underestimated the impact resistance. It is further concluded that, for these beams, the reinforced ones were superior to the prestressed ones under these particular test conditions. This was primarily due to the fact that the ultimate deflections of these prestressed ones were

less than the corresponding reinforced ones, under either static or impact loads.

The second series of tests were conducted at the University of Ghent on post-tensioned members of 13- to 20-ft spans.[16-21] These tests were continued in 1954 on prestressed slabs and beams with and without mild-steel bars, the results of which are not yet published. These tests proved that, because of the big deflections of prestressed beams, the impact resistance of post-tensioned anchored beams is much higher than those of ordinary reinforced concrete of the same static strength.

Although, on the surface, the two series of tests seem to yield conflicting verdicts regarding the relative impact strength of prestressed versus reinforced concrete, upon careful examination it can be seen that they in fact gave the same findings. Both actually agreed that resilience of the beams as given by static tests is a rather good measure of the impact strength. In the first series of tests, the reinforced beams were designed so that they deflected more than the prestressed ones at the ultimate load; in the second series, the prestressed ones deflected more. Whether a beam is reinforced or prestressed, the one with the greater resilience will have the greater impact resistance. In other words, the ability of prestressed beams to stand impact load can be estimated by the usual theory of resilience just as that of reinforced beams.

Even though it is evident that more data on impact resistance are needed before detailed conclusions concerning the impact strength of prestressed beams can be drawn, for the purpose of design, it is nevertheless possible to estimate the energy-absorbing capacity of such beams. The elastic energy that can be absorbed by the beam is measured by the area below its load-deflection curve up to the point of cracking. The total energy that can be absorbed by the beam up to rupture is given by the entire area below the curve. For design, either the elastic or the ultimate basis can be used, provided that a safety factor is properly chosen for the given impact load.

16-4 Corrosion Resistance

Past performance of prestressed-concrete structures has showed definitely that the resistance to corrosion is very high under the usual conditions. It is generally believed that, owing to the absence of cracks, prestressed concrete protects its steel against corrosion better than reinforced concrete.

Several methods are employed for protecting prestressed steel against corrosion. The most common one is to bond the steel to the

concrete, as is done for pretensioning work and for grouted post-tensioning. When steel is surrounded by concrete, oxidation does not take place. Even though the concrete may contain moisture or the grouting may be imperfect, there is enough evidence to show that bonded steel does not corrode under ordinary circumstances.

Unbonded tendons are either galvanized or greased. Like wires used in suspension bridges, galvanized prestressing wires seldom need additional painting. When not galvanized, the tendons are greased and encased in paper or metallic wrapping. Occasionally, galvanized tendons may be greased in addition.

When steel is subjected to high tensile stress, it is known to be more sensitive to corrosion. Hence the term "stress corrosion" is often used to denote the behavior of metals under the combined action of a corrosive environment and high tensile stress. Although the mechanics of stress corrosion is not exactly known, it is surmised that local corrosion may produce high stress concentration which tends to pull the molecules apart and hasten the corroding process. The phenomenon of stress corrosion is identified by corrosion cut directly across the wire or by corrosion splitting along the length of it.

A method for determining the susceptibility of steel to stress-corrosion cracking is described in a German article.[16-22] The method consists in using a corrosive solution containing 60 gm per liter $Ca(NO_3)_2 \cdot H_2O$ with 4 gm per liter NH_4NO_3, maintained at a temperature of approximately 165°F. The wires are stressed by bending to various radii of curvature, and are then suspended into the corrosive media for accelerated corrosion.

John A. Roebling's Sons Company of New Jersey applied this method of testing to different kinds of wires and strands. Their preliminary tests, though not conclusive, seemed to indicate the following (see also reference 16-23):

1. Wires bent to smaller radii, consequently under higher tension, fail in corrosion sooner than straight wires or wires bent to greater radii.

2. Oil-tempered wires are less resistant to corrosion than cold-drawn prestressing wires and strands.

3. Stress-relieved wires and strands are less susceptible to corrosion than non-stress-relieved ones, possibly owing to the lessening of internal stresses in the wires as a result of stress relieving.

So far there has come to the attention of the author only two instances of failure due to stress corrosion. One was in Canada, where prestressed-concrete pipes failed as a result of corrosion in the wires.[16-24] Calcium chloride used in conjunction with Type I and

Type V cements, plus the existence of sulfates in the mixing water, were believed to have been the cause of corrosion. Another instance of corrosion was known, also in connection with the use of calcium chloride.[16-25]

16-5 Special Designs

Only the basic principles of prestressed concrete with applications to common structural designs are presented in this treatise. An engineer who has perused this volume carefully should be able to design conventional structures of prestressed concrete with no special difficulty. However, there are many novel and unusual designs of structures which can be made with prestressed concrete. New and often unexpected problems may arise in connection with special designs, and these can only be solved with good judgment and insight. Such special designs will develop and accumulate as time goes on. Only some of the possibilities together with one or two examples will be briefly mentioned here.

1 · Rigid Frames. In general, the methods discussed in Chapter 10 on continuous beams also apply to rigid frames. There is one additional problem, namely, the shortening of the members under prestress. Non-prestressed rigid frames are subject to small axial stresses; their axial shortening has only a negligible effect on the moments in the frame. Prestressed frames are subject to much higher axial compression due to prestress. When the effect of creep in the concrete is included, shortening of members will be quite appreciable. Such shortening of long-span rigid frames with short supporting columns may produce enormous stresses which can be relieved only by providing sufficient sliding joints or other special devices. To date, prestressed rigid frames have been constructed for both bridges and buildings.[16-26, 16-27]

2 · Trusses. Prestressed-concrete trusses may offer substantial savings for certain structures if we know how to design them. The following points should be considered before such designs can be adopted:

1. Effect of axial shortening of members due to prestress and creep.
2. Secondary moments at the rigid joints, and how to reinforce them.
3. Details at the joints.
4. Design and cost of the formwork.

A concrete bridge over the Seine River in France was prestressed by cables placed in a trusslike arrangement.[16-28] It illustrates the possibilities of novel designs using prestressed concrete.

3 · Thin Shells. The design of prestressed thin shells involves

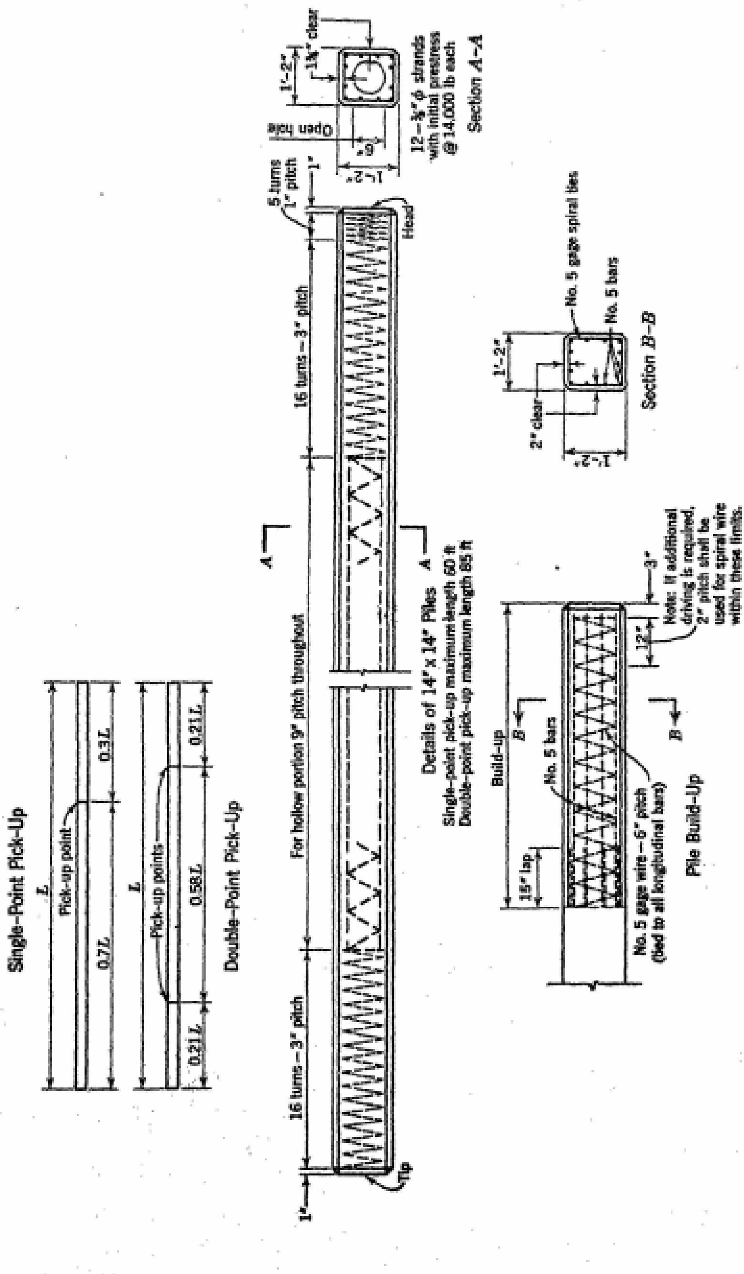

Fig. 16.5.1. Prestressed-concrete piles. (Bridge Division, State Road Department of Florida.)

knowledge of thin-shell design plus an understanding of the principles of prestressed concrete. When the spans are long, efficient combination of thin shell with prestressed construction can lead to great savings in cost, as was exemplified by such a roof built in England.[16-29]

4 · **Piles.** Since piles are subjected to heavy bending stresses during handling and driving, it is often economical to prestress them. The combination of high-strength steel and concrete yields a lighter pile than an ordinary reinforced one. This reduction in weight is significant especially for long piles carrying heavy loads. The absence of cracks in a prestressed-concrete pile gives it high resilience and desirable driving characteristics. A design for a 14-in. precast pretensioned concrete pile is shown in Fig. 16-5-1.

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Appendix A

Definitions, Notations, Abbreviations

Definitions

1 *Pre-tensioning and post-tensioning.* Any method of prestressing concrete members in which the reinforcement is tensioned before (after) the concrete is placed.

2 *Full and partial prestressing.* Degree of prestress applied to concrete in which no tension (some tension) is permitted in the concrete under the working loads.

3 *Circular and linear prestressing.* Circular prestressing refers to prestressing in round members like tanks and pipes; prestressing in all other members is termed linear.

4 *Transfer.* The transferring of prestress to the concrete. For pretensioned members, transfer takes place at the release of prestress from the bulkheads; for post-tensioned members it takes place after the completion of the tensioning process.

5 *Bonded and unbonded reinforcement.* Reinforcement bonded (not bonded) throughout its length to the surrounding concrete.

6 *Anchored and non-end-anchored reinforcement.* Reinforcement anchored at its ends (not anchored) by means of mechanical devices capable of transmitting the tensioning force to the concrete.

7 *Prestressed and non-prestressed reinforcement.* Reinforcement in prestressed-concrete members, which are elongated (not elongated) with respect to the surrounding concrete.

8 *Tendons.* Another name for prestressed reinforcement, whether wires, bars, or strands.

9 *Cables.* A group of tendons, or the c.g.s. of all the tendons.

10 *Concordant and non-concordant cables.* Cables or c.g.s. lines which produce a *C*-line or line of pressure coincident (non-coincident) with the c.g.s. line itself.

11 *Linear transformation.* Moving the position of a c.g.s. line over the interior supports of a continuous beam without changing the intrinsic shape of the line within each individual span.

12 *Girder load, working load, service load, cracking load, and ultimate load.* GIRDER LOAD: The weight of the beam or girder itself plus whatever weight is on it at the time of transfer. WORKING LOAD OR SERVICE LOAD: The normally maximum total load which the structure is specified or expected to carry. CRACKING LOAD: The total load re-

quired to initiate cracks in a prestressed-concrete member. ULTIMATE LOAD: The total load which a member or structure can carry up to total rupture.

13 *Load factor.* The ratio of cracking or ultimate load to the working or service load (sometimes considering only the live load when so specified).

14 *Creep.* Time-dependent inelastic deformation of concrete or steel resulting solely from the presence of stress and a function thereof.

15 *Shrinkage of concrete.* Contraction of concrete due to drying and chemical changes, dependent on time but not directly dependent on stresses induced by external loading.

Notations

Greek Letters

Δ = deflection of beams.
Δ_a = total deformation of anchorage.
Δ_s = total strain in steel.
δ = unit strain.
δ_i = initial unit strain in concrete, due to elastic shortening.
δ_t = final unit strain in concrete, including the effect of creep but not of shrinkage.
δ_s = unit strain in steel.
μ = coefficient of friction.
θ or α = change in angle of tendons; angles in general.

English Letters

A = cross-sectional area in general.
A_c = net cross-sectional area of concrete; or area of precast portion.
A_{c1}, A_{c2} = compressive portion of A_c at transfer or under working load, respectively.
A_g = gross cross-sectional area of concrete.
A_s = cross-sectional area of steel, generally in square inches.
A_{sb} = steel area for a balanced section, in square inches.
A_t = gross cross-sectional area of concrete, including steel transformed by ratio n.
A_v = cross-sectional area of one set of steel stirrups.
a = lever arm between the centers of compression and tension in a beam section.
a' = a at ultimate load.
a_1, a_2, a_G = a at various stages as defined in text.
b = width of beam or its flange.
b' = width of web of beam.
C = center of compressive force, center of pressure, or center of thrust; or carry-over moment, in moment distribution.
C' = C at ultimate load.
C_c = coefficient of creep = δ_t/δ_i.
c = distance from c.g.c. to extreme fiber.

c_b, c_t = c for bottom (top) fibers; c_{b1}, c_{t1}, c_{b2}, c_{t2} for compressive portion at transfer or under working load, respectively; c_b', c_t' for composite sections.
c.g.s. = center of gravity of steel area.
c.g.c. = center of gravity (centroid) of concrete section; c.g.c.′ for composite section.
D = diameter of bars or wires; or distributed moment, in moment distribution.
DL = dead load.
d = depth of beam measured to c.g.s., generally in inches.
e = eccentricity in general.
e_1, e_2, e_b, e_t = various eccentricities as defined locally in text.
e_x, e_y = eccentricities along X-axis (Y-axis).
E = modulus of elasticity in general.
E_c = modulus of elasticity for concrete.
E_s = modulus of elasticity for steel.
F = total effective prestress after deducting losses.
F_a = average prestress in steel for a given length.
F_1, F_2 = total prestress at points 1 and 2, respectively.
F_i = total initial prestress before transfer.
F_0 = total prestress, just after transfer.
FEM = fixed-end moment, in moment distribution.
f = unit stress in general.
f_1, f_2 = unit stresses at stages or points 1 and 2, respectively.
f' = modulus of rupture of concrete.
f_a = average unit stress in steel for a given length.
f_c = unit stress in concrete.
f_c' = ultimate unit stress in concrete, generally at 28 days old.
f_{ci}' = ultimate unit stress in concrete, at time of transfer.
f_{cs} = average concrete stress along the c.g.s. line.
f_i = initial unit prestress in steel before transfer.
f_0 = unit prestress in steel, just after transfer.
f_e = effective unit prestress in steel after deducting losses.
f_s = unit stress in steel, generally; or effective unit prestress in steel after deducting losses.
f_s' = ultimate unit stress in steel.
f_{s1} = initial unit stress in steel to overcome slack.
Δf_s = change in f_s.
f_t, f_b = fiber stress at top (bottom) fibers.
f_t', f_b' = tensile fiber stress at top (bottom) fibers.
f_v = unit stress in steel stirrups.
f_v' = f_v at ultimate load.
f_y = yield point of steel; or f along Y-axis.
f_1, f_2 = unit prestress at stage 1 (2).
f_A, f_B = unit stress at point A (B).
H = horizontal reaction.
h = overall depth of beam; h_1, h_2 for compressive portion at transfer or under working load, respectively.
I = moment of inertia of section; I' for composite section; I_1, I_2 for compressive portion at transfer or under working load, respectively.

IL = impact load.
I_t = I for transformed section.
I_x, I_y = I about Y-axis (X-axis).
j = for resisting lever arm jd in a beam section.
K = coefficient for wobble effect of tendons in a prestressed member.
k = coefficient for depth of compressive area kd in a beam section; or as defined locally.
k' = k at ultimate load.
k_1 = ratio of average stress in ultimate compression area of beam to f_c'.
k_t, k_b = kern distances from c.g.c. for top (bottom) = $r^2/c_b(r^2/c_t)$; k_{t1}, k_{b1}, k_{t2}, k_{b2} for compressive portion at transfer or under working load, respectively; k_t', k_b' for composite section.
L = length of member, or length in general; L_t = length of transfer.
LL = live load.
M = bending moment in general.
M' = ultimate moment.
M_A, M_B = moment at point A (B).
M_C = moment acting on composite section.
M_L = moment due to total live load only.
M_G = moment due to girder load, including any load on the beam or girder at time of transfer.
M_0 = moment at midspan.
M_P = moment on precast portion of composite section.
M_S = moment due to superimposed load.
M_T = moment due to total load.
M_1, M_2 = primary (resulting) moments in a continuous beam.
m = load factor or factor of safety.
m_b, m_t = ratio of section moduli of precast portion to composite section for bottom (top) fiber.
n = modular ratio E_s/E_c.
o = perimeter of bars or wires.
P = concentrated load.
p = percentage of steel, ratio of A_s/A_c; or unit pressure in tank or pipe. For rectangular sections, $p = A_s/bd$.
p_b = value of p for a balanced section = A_{sb}/bd for rectangular sections.
R = radius of curvature, or radius of tanks and pipes.
r = radius of gyration = $\sqrt{I/A}$.
s = stirrup spacing.
S_c = principal compressive stress.
S_t = principal tensile stress.
T = total tension in prestressed steel or center of total tension.
T' = T at ultimate load.
T_1 = T in non-prestressed steel.
T_1' = T' in non-prestressed steel.
t = thickness of beam flange.
u = unit bond stress.
u' = u at ultimate load.
V = total shear in beam.
V_c = total shear carried by concrete.
V_s = total shear carried by steel.

V', V_c', V_s' = V, V_c, V_s at ultimate load, respectively.
W = total weight.
W_0 = total weight carried by plain concrete section up to cracking.
w = load or weight per length.
w' = w at ultimate load.
w_c, w_c' = w and w' for continuous beams, respectively.
w_G = girder load, plf.
w_S = superimposed load, plf.
y = perpendicular distance from c.g.c. line to said fiber.
y_0 = y for certain points, as defined in text.
Z = section modulus I/c.

Abbreviations

cm = centimeter(s)
cu = cubic
F = Fahrenheit
ft = foot (feet)
in. = inch(es)
k = kip(s) = 1000 pounds
ksi = kip(s) per square inch
kg = kilogram(s)
lb = pound(s)
mm = millimeter(s)
plf = pound(s) per linear foot
psf = pound(s) per square foot
psi = pound(s) per square inch
P.C. = prestressed concrete
R.C. = reinforced concrete
sq = square
yd = yard(s)

Appendix B

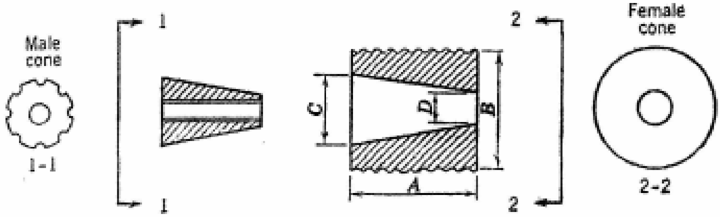
Design Data for Some Prestressing Systems

Data in this appendix are subject to changes made by the companies. The following addresses are given for the convenience of designers who desire more detailed information.

1. Freyssinet System. Freyssinet Co., Inc., 57 Williams St., New York, N. Y.
2. Leonhardt System. The Preload Co., Inc. See item 4.
3. Magnel System. Precompressed Concrete Engineering Co., Ltd., 5012 Western Ave., Montreal, Quebec, Canada.
4. Preload System. The Preload Co., Inc., 211 E. 37 St., New York 16, N. Y.
5. Prescon System (successor to the Strescon System of the Prestressed Concrete Corporation), 16706 S. Garfield Ave., Paramount, Calif.
6. Roebling System. John A. Roebling's Sons Co., 640 S. Broad St., Trenton, N. J.
7. Stressteel or Lee-McCall System. Truscon Division, Republic Steel Corp., Youngstown, Ohio.
8. Texas P. I. System. Prestressing, Inc., 2409 Transit Tower, San Antonio, Texas.
9. Dorland Anchorages for Pre-tensioning. Southwest Structural Concrete Corp., P. O. Box 3247, Hillcrest Station, San Diego, Calif.

B · 1 Freyssinet System

Size of unit, strength, and prestress recommended by the Freyssinet Co. are as shown.



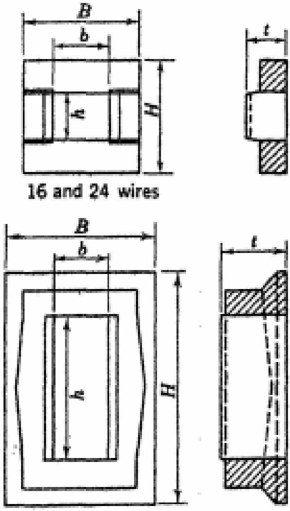
Type of unit		8 × 0.196 in.	10 × 0.196 in.	12 × 0.196 in.	18 × 0.196 in.
Minimum guaranteed ultimate tensile strength for uncoated cables		60,000 lb	75,000 lb	90,000 lb	135,000 lb
Recommended final prestress		34,000 lb	43,000 lb	51,000 lb	77,000 lb
Steel area		0.241 sq in.	0.302 sq in.	0.362 sq in.	0.543 sq in.
Weight per linear foot		0.82 lb	1.03 lb	1.23 lb	1.85 lb
O.D. metal hose		$1\frac{1}{8}$ in.	$1\frac{1}{8}$ in.	$1\frac{1}{4}$ in.	$1\frac{5}{8}$ in.
I.D. metal hose		1 in.	1 in.	$1\frac{1}{8}$ in.	$1\frac{1}{2}$ in.
Dimensions of anchorage	A	$3\frac{15}{16}$	$3\frac{15}{16}$	$3\frac{15}{16}$	$4\frac{23}{32}$
	B	$3\frac{25}{32}$	$3\frac{25}{32}$	$3\frac{25}{32}$	$4\frac{13}{16}$
	C	$1\frac{7}{8}$	$1\frac{7}{8}$	$1\frac{7}{8}$	$2\frac{7}{16}$
	D	$1\frac{3}{32}$	$1\frac{3}{32}$	$1\frac{3}{32}$	$1\frac{15}{32}$

The guaranteed ultimate tensile strength of galvanized cables is 85% of the values above and the recommended final prestress is 76% of that for uncoated cables.

B · 2 Magnel System

Size of units and end anchorages as follows.

Distribution Plates
for 0.200-in.-diameter wire cables

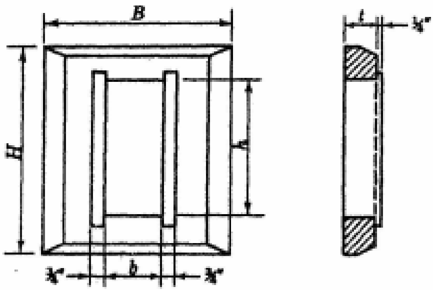


32 to 64 wires
Material, cast steel

No. of Wires in Cable	*H*	*B*	*h*	*b*	*t*	Approximate Weight, lb
16	5″	5.1″	2.16″	2.4″	1.5″	7
24	6.15″	6.3″	3.2″	2.4″	1.58″	11
32	7.75″	6.7″	4.45″	2.4″	1.93″	14
40	9″	7.1″	5.50″	2.4″	2.28″	19
48	10.3″	7.5″	6.55″	2.4″	2.56″	24
56	11.4″	7.9″	7.55″	2.4″	2.8″	30
64	12.4″	8.3″	8.6″	2.4″	2.83″	32

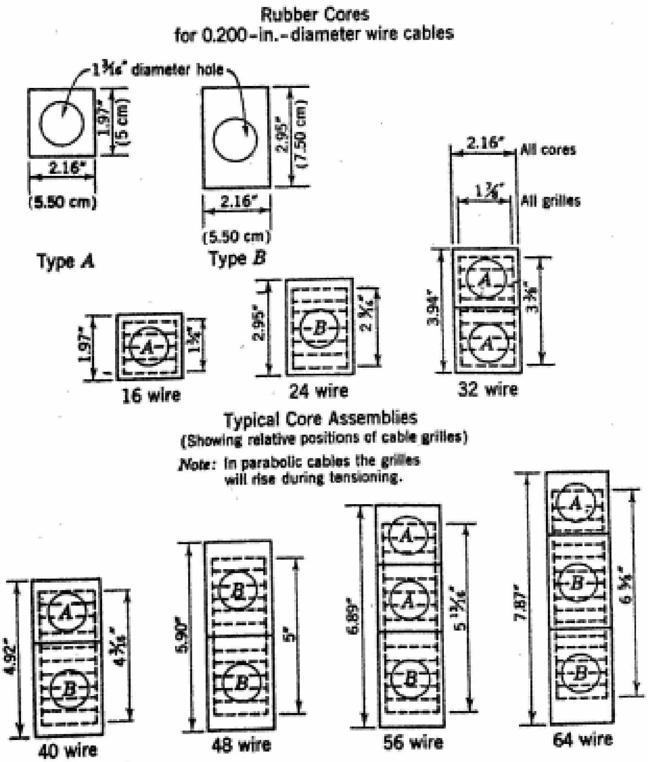
Material, cast steel.

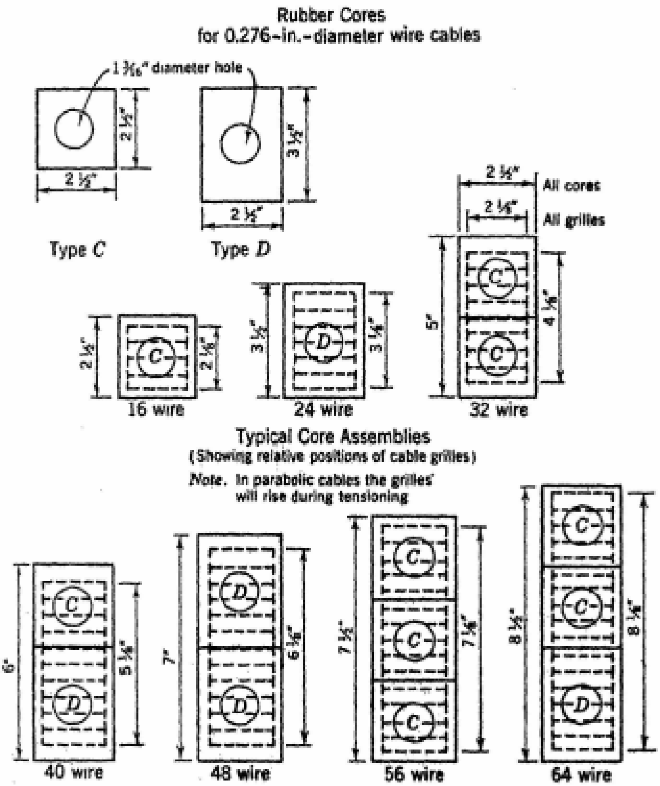
Distribution Plates
for 0.276-in.-diameter wire cables



No. of Wires in Cable	H	B	h	b	t	Approximate Weight, lb
16	5¾″	8″	3″	3⅛″	1¼″	13
24	7¼″	10¼″	4½″	3⅛″	1½″	26
32	9½″	10¼″	5⅞″	3⅛″	1¾″	36
40	11½″	10⅝″	7½″	3⅛″	2″	51
48	13½″	11″	8⅞″	3⅛″	2⅜″	68
56	14½″	11½″	10½″	3⅛″	2½″	76
64	15½″	11¾″	11⅞″	3⅛″	2¾″	90

Material, cast steel.


Rubber Cores
for 0.200-in.-diameter wire cables
1 3/16" diameter hole
1.97" (5 cm)
2.16" (5.50 cm)
Type A
2.95" (7.50 cm)
2.16" (5.50 cm)
Type B
2.16" All cores
1 3/4" All grilles
A
B
1.97"
16 wire
2.95"
24 wire
3.94"
32 wire
Typical Core Assemblies
(Showing relative positions of cable grilles)
Note: In parabolic cables the grilles will rise during tensioning.
4.92"
40 wire
5.90"
5"
48 wire
6.89"
56 wire
7.87"
64 wire


Rubber Cores
for 0.276-in.-diameter wire cables
diameter hole
Type C
Type D
All cores
All grilles
16 wire
24 wire
32 wire
Typical Core Assemblies
(Showing relative positions of cable grilles)
Note. In parabolic cables the grilles will rise during tensioning
40 wire
48 wire
56 wire
64 wire

B·3 Roebling System

7-Wire Uncoated Strands for Pre-Tensioning

Nominal Diameter, in.	Weight per 1000 Feet, lb	Approximate Area, sq in.	Ultimate Strength, lb	Design Load, lb	Tensioning Load, lb
3/16	73	0.0214	5,500	3,080	3,850
1/4	122	0.0356	9,000	5,040	6,300
5/16	198	0.0578	14,500	8,120	10,150
3/8	274	0.0799	20,000	11,200	14,000
7/16	373	0.1089	27,000	15,120	18,900
1/2	494	0.1438	36,000	20,160	25,200

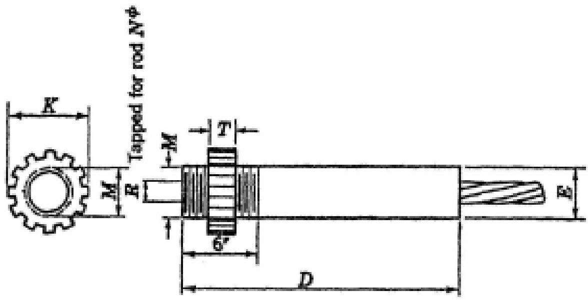
The average modulus of elasticity of the strands in the above table is 27,000,000 psi.

Galvanized Strands for Post-Tensioning

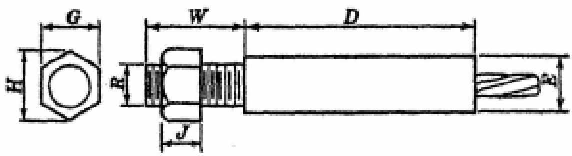
Diameter, in.	Weight per Foot, lb	Area, sq in.	Minimum Guaranteed Ultimate Strength, lb	Design Load, lb
0.600	0.737	0.215	46,000	26,000
0.835	1.412	0.409	86,000	49,000
1	2.00	0.577	122,000	69,000
1 1/8	2.61	0.751	156,000	90,000
1 1/4	3.22	0.931	192,000	112,000
1 3/8	3.89	1.12	232,000	134,000
1 1/2	4.70	1.36	276,000	163,000
1 9/16	5.11	1.48	300,000	177,000
1 5/8	5.52	1.60	324,000	192,000
1 11/16	5.98	1.73	352,000	208,000

The average modulus of elasticity of the strands in the above table is 25,000,000 psi.

Dimension of Anchorage Fittings



Type SDS 34



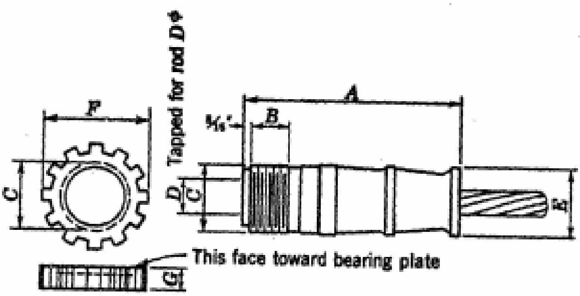
Type SDS 35

Measurements in Inches											Total Weight, lb	
Diameter Strand	*D*	*W*	*E*	*M*	*R*	*G*	*H*	*J*	*K*	*T*	Type SDS34	Type SDS35
0.600	9½	8	1$\frac{11}{16}$	12N	1¼–12N	1$\frac{11}{16}$	1$\frac{15}{16}$	1	2¾	¾	7	9¼
0.835	12½	10	2¼	12N	1⅝–12N	2¼	2$\frac{9}{16}$	1$\frac{5}{16}$	3¾	1	16½	21
1	14⅛	11	2¾	8N	2 – 8N	2$\frac{13}{16}$	3$\frac{3}{16}$	1⅝	4⅜	1¼	24¾	32½
1⅛	16½	11	3	8N	2¼– 8N	3	3$\frac{7}{16}$	1¾	4¾	1½	39½	50

N = American National thread series.

For fittings Type SDS35, standard studs having dimension *W*, shown above, are carried in stock.

Other stud lengths must be fabricated to order. All SDS 34 and 35 fittings are proof-loaded to a stress in excess of the recommended design stress after being attached to the strand



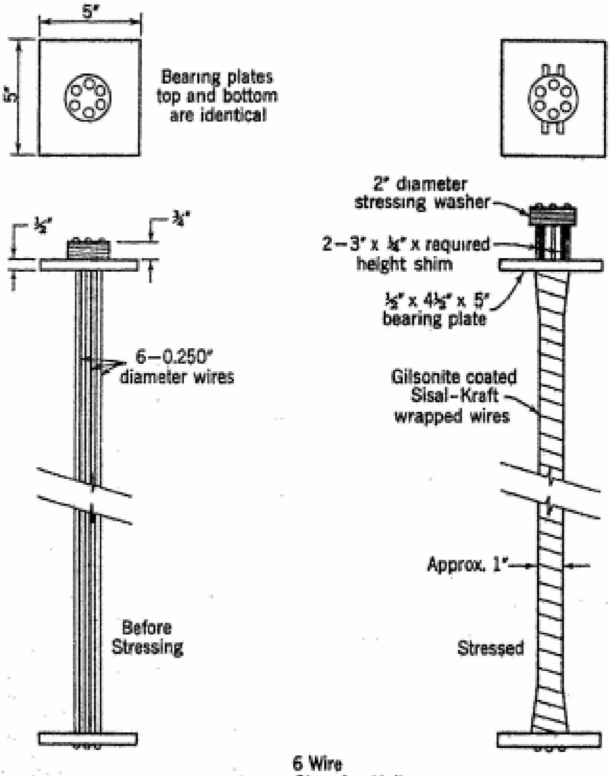
Type SDS 10

Measurements in Inches								Total Weight, lb
Diameter Strand	A	B	E	C	D	F	G	
1¼	14	5	4⅜	4¼–4N	2½–4NC	6⅜	1⁵⁄₁₆	48
1⅜	14¹¹⁄₁₆	5½	4⅞	4¾–4N	3 –4NC	6⅞	1⁵⁄₁₆	57
1½	15⁵⁄₁₆	5½	5⅜	5¼–4N	3 –4NC	7⅝	1¹¹⁄₁₆	78
1⁹⁄₁₆	16³⁄₁₆	6	5⅝	5½–4N	3½–4NC	7⅞	1¹¹⁄₁₆	80
1⅝	16⅜	6	5¾	5⅝–4N	3½–4NC	8	1¹¹⁄₁₆	89
1¹¹⁄₁₆	16¾	6	5⅞	5¾–4N	3½–4NC	8¼	1¾	100

N = American National thread series.
NC = American National coarse-thread series.

B · 4 Prescon System (successor to Strescon System)

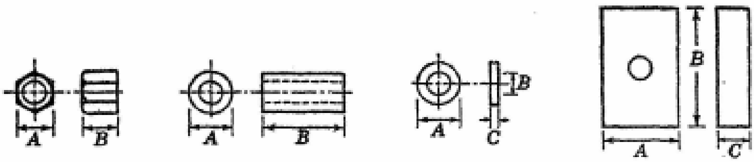
Units consist of 6 or 7 wires of ¼-in. diameter. The 6-wire unit is shown here.



6 Wire
Linear Stressing Unit

B · 5 Stressteel System

Dimensions of end anchorage components are as shown.



Nuts Couplers Washers End Anchorage Plates

TABLE 1

Item	Part No.	For Bar ϕ, in.	Dimensions of Part, in. A	B	C	Weight, lb per 100 pcs
Nuts, high efficiency	HN4	½	1	13/16		12.8
	HN5	⅝	1⅛	1		22.8
	HN6	¾	1⅜	1¼		43.4
	HN7	⅞	1⅝	1 7/16		72.7
	HN8	1	1⅞	1⅝		103.9
	HN9	1⅛	2⅛	1 13/16		139.2
Couplers, high efficiency	HC4	½	1	2⅛		36.6
	HC5	⅝	1¼	2⅝		70.7
	HC6	¾	1½	3⅛		120.0
	HC7	⅞	1¾	3¾		190.8
	HC8	1	2	4¼		289.8
	HC9	1⅛	2¼	4¾		403.5
Washers, thin	TW4	½	1¼	9/16	14G	2.2
	TW5	⅝	1½	11/16	14G	3.2
	TW6	¾	1¾	13/16	14G	4.3
	TW7	⅞	2	15/16	14G	5.5
	TW8	1	2¼	1 1/16	14G	7.0
	TW9	1⅛	2½	1 3/16	14G	8.6
Washers, standard	SW4	½	1¼	9/16	3/16	5.3
	SW5	⅝	1½	11/16	3/16	7.4
	SW6	¾	1¾	13/16	3/16	10.1
	SW7	⅞	2	15/16	3/16	13.0
	SW8	1	2¼	1 1/16	3/16	16.4
	SW9	1⅛	2½	1 3/16	3/16	20.2
Washers, split, thin	STW4	½	1¼	9/16	14G	2.2
	STW5	⅝	1½	11/16	14G	3.2
	STW6	¾	1¾	13/16	14G	4.3
	STW7	⅞	2	15/16	14G	5.5
	STW8	1	2¼	1 1/16	14G	7.0
	STW9	1⅛	2½	1 3/16	14G	8.6

Item	Part No.	For Bar ϕ, in. †	Dimensions of Part, in. A	B	C	Weight, Ea.
Anchorage Plates	*EP1	1 @ ½	2½	2½	⅝	1.1
	*EP2	1 @ ⅝	3	3	¾	1.9
	*EP3	1 @ ¾	3½	3½	⅞	3.0
	EP4	1 @ ⅝	5½	3	1½	7.0
	EP5	1 @ ¾	5½	3	1½	7.0
	EP6	1 @ ⅞	5½	4	1½	9.4
	EP7	1 @ 1	6	4	1½	10.2
	EP8	1 @ 1⅛	6	5	1½	12.7
	EP9	2 @ ⅝	9½	3	1½	12.1
	EP10	2 @ ¾	9½	3	1½	12.1
	EP11	2 @ ⅞	9½	4	1½	16.1
	EP12	2 @ 1	9½	5	1½	20.2
	EP13	2 @ 1⅛	9½	6	1½	24.2
	EP14	3 @ ¾	13½	3	1½	17.4
	EP15	3 @ ⅞	13½	4	1½	22.9
	EP16	3 @ 1	13½	5	1½	28.6
	EP17	3 @ 1⅛	14½	6	1½	36.9

* End plates 1, 2 and 3 require packing under the jack since they are too small to allow its legs to bear against them.

† Plate hole size equals bar diameter plus ¼″.

Properties of Stressteel Tensioning Units

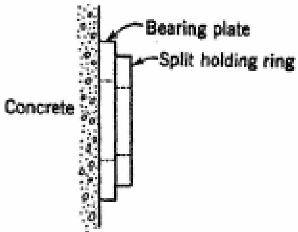
Bar, in.	Area, sq in.	Weight, lb per lin ft	Initial Prestressing Force, lb	Working Force (After Losses of 15%), lb	Ultimate Strength, lb
½	0.196	0.668	19,600	16,680	28,400
⅝	0.306	1.04	30,600	26,050	44,400
¾	0.442	1.50	44,200	37,600	64,100
⅞	0.601	2.04	60,100	51,100	87,200
1	0.785	2.67	78,500	66,800	113,900
1⅛	0.994	3.38	99,400	84,500	144,000

Above table based on following:

Guaranteed minimum ultimate strength:	145,000 psi.
Initial prestress:	100,000 psi.
Working prestress:	85,000 psi.

B · 6 Texas P.I. System

Dimensions of end anchorages are as shown.



No. of ¼-In. Wires	4	6	8	10	12
Bearing plate					
Size	4″ × 5″	5″ × 5″	6″ × 6″	6″ × 6″	6″ × 6½″
Thickness	½″	½″	⅝″	¾″	1″
Diameter of hole	2¼″	2⅝″	2¾″	3¼″	3⅜″
Split holding rings					
Outside diameter	3½′	3½″	4″	4″	4½″
Thickness	⅜″	½″	⅝″	¾″	⅞″
Diameter of hole	1⅝″	1⅞″	2¼″	2¾″	2¾″

B·7 U.S. Steel, American Steel and Wire Division

Strands for Prestressed Concrete

Nominal strand diameter, in.	¼	5/16	⅜	½	⅝	¾	⅞	1
Construction	1 × 7	1 × 7	1 × 7	1 × 7	1 × 7	1 × 19	1 × 19	1 × 19
Approximate weight per 1000 ft-lb	121	205	273	516	812	1160	1610	2060
Area, sq in.	.0352	.0595	.0792	.150	.236	.336	.468	.597
Breaking strength, lb								
Galvanized strand (1)	7,350	12,400	16,500	31,000	49,000	70,000	97,000	123,000
Uncoated strand (2)	8,960	14,900	19,600	35,500	53,400	79,900	108,000	134,000

Usual tensioning loads about 60%, usual design loads about 50% of ultimate.

Mechanical Properties of Wire

(1) For galvanized strands of galvanized bridge wire:
Minimum ultimate tensile strength: 220,000 psi.
Minimum elongation at ultimate strength: 4.0% in 10 in.
Approximate yield strength as determined by 0.7% elongation: 160,000 psi.

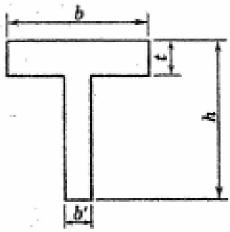
(2) For uncoated strands (bright wire):
Minimum ultimate tensile strength: 238,000–268,000 psi depending on wire size.
Approximate yield strength as determined by 0.7% elongation: 67% of ultimate strength.

Appendix C

Constants for Beam Sections

TABLE 6-2-1

Constants for T-Sections



Section	b'/b	t/h	A*	c_b†	c_t†	I‡	r^2§	k_t†	k_b†
1–a	0.1	0.1	0.19bh	0.714h	0.286h	0.0179bh^3	0.0945h^2	0.132h	0.333h
1–b	0.1	0.2	0.28	0.756	0.244	0.0192	0.0688	0.0910	0.282
1–c	0.1	0.3	0.37	0.755	0.245	0.0193	0.0520	0.0689	0.212
1–d	0.1	0.4	0.46	0.735	0.265	0.0202	0.0439	0.0597	0.165
1–e	0.2	0.1	0.28	0.629	0.371	0.0283	0.1010	0.161	0.272
1–f	0.2	0.2	0.36	0.678	0.322	0.0315	0.0875	0.129	0.272
1–g	0.2	0.3	0.44	0.691	0.309	0.0319	0.0725	0.105	0.234
1–h	0.2	0.4	0.52	0.684	0.316	0.0320	0.0616	0.090	0.195
1–i	0.3	0.1	0.37	0.585	0.415	0.0365	0.0985	0.169	0.237
1–j	0.3	0.2	0.44	0.626	0.374	0.0408	0.0928	0.148	0.248
1–k	0.3	0.3	0.51	0.645	0.355	0.0417	0.0819	0.127	0.231
1–l	0.3	0.4	0.58	0.645	0.355	0.0417	0.0720	0.112	0.203
1–m	0.4	0.1	0.46	0.559	0.441	0.0440	0.0954	0.171	0.216
1–n	0.4	0.2	0.52	0.592	0.408	0.0486	0.0935	0.158	0.229
1–o	0.4	0.3	0.58	0.609	0.391	0.0499	0.0860	0.141	0.220
1–p	0.4	0.4	0.64	0.612	0.388	0.0502	0.0785	0.128	0.205
1–q	1.0	1.0	1.00	0.500	0.500	0.0833	0.0833	0.167	0.167

* Given as a function of bh.
† Given as a function of h.
‡ Given as a function of bh^3.
§ Given as a function of h^2.

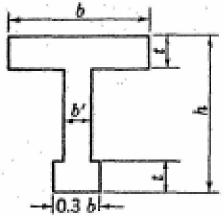


TABLE 6-2-2

Constants for I-Sections

Section	b'/b	t/h	A^*	$c_b\dagger$	$c_t\dagger$	$I\ddagger$	$r^2\S$	$k_t\dagger$	$k_b\dagger$
2–a	0.1	0.1	$0.21bh$	$0.650h$	$0.350h$	$0.0260bh^3$	$0.1236h^2$	$0.190h$	$0.354h$
2–b	0.1	0.2	0.32	0.675	0.325	0.0345	0.1080	0.160	0.332
2–c	0.1	0.3	0.43	0.672	0.328	0.0387	0.0900	0.134	0.274
2–d	0.2	0.1	0.29	0.610	0.390	0.0316	0.1090	0.179	0.280
2–e	0.2	0.2	0.38	0.647	0.353	0.0378	0.0994	0.153	0.282
2–f	0.2	0.3	0.47	0.655	0.345	0.0402	0.0856	0.131	0.248

* Given as a function of bh.
† Given as a function of h.
‡ Given as a function of bh^3.
§ Given as a function of h^2.

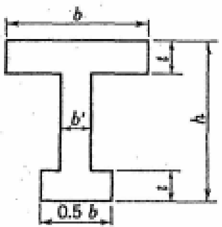


TABLE 6-2-3

Constants for I-Sections

Section	b'/b	t/h	A^*	$c_b\dagger$	$c_t\dagger$	$I\ddagger$	$r^2\S$	$k_t\dagger$	$k_b\dagger$
3–a	0.1	0.1	$0.23bh$	$0.597h$	$0.403h$	$0.0326bh^3$	$0.1420h^2$	$0.238h$	$0.352h$
3–b	0.1	0.2	0.36	0.611	0.389	0.0464	0.1288	0.210	0.331
3–c	0.1	0.3	0.49	0.606	0.394	0.0535	0.1090	0.180	0.274
3–d	0.2	0.1	0.31	0.572	0.428	0.0373	0.1204	0.210	0.282
3–e	0.2	0.2	0.42	0.595	0.405	0.0488	0.1160	0.195	0.286
3–f	0.2	0.3	0.53	0.599	0.401	0.0540	0.1020	0.170	0.254
3–g	0.3	0.1	0.39	0.557	0.443	0.0430	0.1103	0.198	0.250
3–h	0.3	0.2	0.48	0.582	0.418	0.0510	0.1065	0.183	0.255
3–i	0.3	0.3	0.57	0.592	0.408	0.0553	0.0970	0.164	0.238

* Given as a function of bh.
† Given as a function of h.
‡ Given as a function of bh^3.
§ Given as a function of h^2.

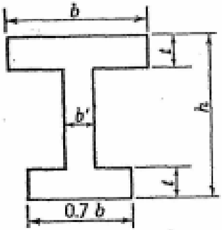


TABLE 6-2-4

Constants for I-Sections

Section	b'/b	t/h	A*	c_b†	c_t†	I‡	r^2§	k_t†	k_b†
4–a	0.1	0.1	$0.25bh$	$0.554h$	$0.446h$	$0.0381bh^3$	$0.1525h^2$	$0.276h$	$0.342h$
4–b	0.1	0.2	0.40	0.560	0.440	0.0560	0.1391	0.248	0.316
4–c	0.1	0.3	0.55	0.557	0.443	0.0651	0.1182	0.212	0.267
4–d	0.2	0.1	0.33	0.540	0.460	0.0425	0.1290	0.239	0.280
4–e	0.2	0.2	0.46	0.552	0.448	0.0578	0.1258	0.228	0.281
4–f	0.2	0.3	0.59	0.553	0.447	0.0657	0.1113	0.202	0.249
4–g	0.3	0.1	0.41	0.534	0.466	0.0467	0.1140	0.214	0.244
4–h	0.3	0.2	0.52	0.546	0.454	0.0598	0.1150	0.210	0.254
4–i	0.3	0.3	0.63	0.550	0.450	0.0663	0.1051	0.191	0.234

* Given as a function of bh.
† Given as a function of h.
‡ Given as a function of bh^3.
§ Given as a function of h^2.

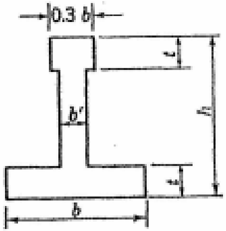


TABLE 6-2-5

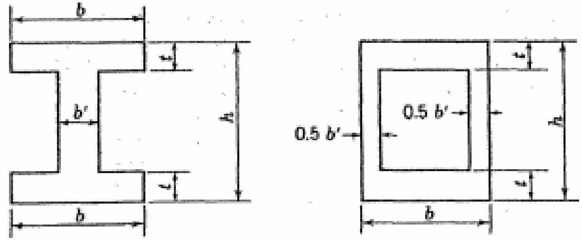
Constants for I-Sections

Section	b'/b	t/h	A*	c_b†	c_t†	I‡	r^2§	k_t†	k_b†
5–a	0.1	0.1	$0.21bh$	$0.350h$	$0.650h$	$0.0260bh^3$	$0.1236h^2$	$0.354h$	$0.190h$
5–b	0.1	0.2	0.32	0.325	0.675	0.0345	0.1080	0.332	0.160
5–c	0.1	0.3	0.43	0.328	0.672	0.0387	0.0900	0.274	0.134
5–d	0.2	0.1	0.29	0.390	0.610	0.0316	0.1090	0.280	0.179
5–e	0.2	0.2	0.38	0.353	0.647	0.0378	0.0994	0.282	0.153
5–f	0.2	0.3	0.47	0.345	0.655	0.0402	0.0856	0.248	0.131

* Given as a function of bh.
† Given as a function of h.
‡ Given as a function of bh^3.
§ Given as a function of h^2.

TABLE 6-2-6

Constants for Symmetrical I- and Box Sections



Section	b'/b	t/h	A*	c_b†	c_t†	I‡	r^2§	k_t†	k_b†
6–*a*	0.1	0.1	0.28bh	0.500h	0.500h	0.0449bh^3	0.160h^2	0.320h	0.320h
6–*b*	0.1	0.2	0.46	0.500	0.500	0.0671	0.146	0.292	0.292
6–*c*	0.1	0.3	0.64	0.500	0.500	0.0785	0.123	0.246	0.246
6–*d*	0.2	0.1	0.36	0.500	0.500	0.0492	0.137	0.274	0.274
6–*e*	0.2	0.2	0.52	0.500	0.500	0.0689	0.132	0.264	0.264
6–*f*	0.2	0.3	0.68	0.500	0.500	0.0791	0.117	0.234	0.234
6–*g*	0.3	0.1	0.44	0.500	0.500	0.0535	0.121	0.243	0.243
6–*h*	0.3	0.2	0.58	0.500	0.500	0.0707	0.122	0.244	0.244
6–*i*	0.3	0.3	0.72	0.500	0.500	0.0796	0.111	0.222	0.222
6–*j*	0.4	0.1	0.52	0.500	0.500	0.0577	0.111	0.222	0.222
6–*k*	0.4	0.2	0.64	0.500	0.500	0.0725	0.113	0.226	0.226
6–*l*	0.4	0.3	0.76	0.500	0.500	0.0801	0.105	0.211	0.211

* Given as a function of bh.
† Given as a function of h.
‡ Given as a function of bh^3.
§ Given as a function of h^2.

Appendix D

Criteria for Prestressed-Concrete Bridges*

Preface

The criteria for prestressed-concrete bridges presented in this pamphlet have been developed in the hope that they may be useful until such time as more complete specifications, covering the subject in far greater detail, may be presented to the civil engineering profession by American specification and code writing bodies.

The Bureau of Public Roads recognized in 1952 that the prestressed method of concrete construction had great possibilities in the building of better and more economical highway bridges of reinforced concrete, and that in many instances prestressed concrete might become a competitor of structural steel also.

There were no American standard codes governing the design of prestressed concrete bridges at that time. In recognition of the need for a guide to design which would provide structures acceptable for Federal-aid projects, the Bureau, in March 1952, prepared and distributed a *Design Criteria for Prestressed Concrete Bridges* (*Post Tensioning*).

Although the scope of the criteria were very limited, the issue attracted considerable attention, and many constructive comments and suggestions were received from American and European engineers engaged in prestressed-concrete design and construction. On the basis of these comments and suggestions, a rough draft of a new and greatly enlarged criteria, covering design, materials, and construction, was prepared and submitted in September, 1953, to a number of authorities in the field both in this country and abroad. Thoroughly revised in light of their comments, the criteria are now issued in this pamphlet, together with supporting discussion and source references.

During the writing of these criteria a joint committee, composed of delegates from the American Society of Civil Engineers and the

* Department of Commerce, Bureau of Public Roads, 1954.

American Concrete Institute, was set up to develop a code of practice for prestressed concrete. When such a code is published, the Bureau's criteria will be reviewed in the light of the Committee's findings.

Design

Temporary Stresses. Temporary stresses before creep and shrinkage shall not exceed the following:*

Concrete:

Compression in extreme fiber	pretensioned	$0.60f_{ci}'$
	post-tensioned	$0.55f_{ci}'$
Tension		$0.05f_{ci}'$
Prestressing Steel: Tension		$0.80f_s'$

Stress under Dead, Live, or Impact Load. Stress after creep and shrinkage under dead, live, or impact load, or any combination of these forces, shall not exceed the following:

Concrete:

Compression in extreme fiber	$0.4f_c'$
Tension in extreme fiber	0

Where the computations show tension in the extreme fiber, unprestressed reinforcement may be used, and designed to take the total tensile stresses, provided that the computed tension in the concrete before the unprestressed steel is added does not exceed $0.08f_c'$.

Prestressing steel	$0.6f_s'$ or $0.8f_y'$, whichever is less

Creep, Shrinkage, and Elastic Deformation. Decrease in prestress in steel due to creep, shrinkage, and elastic deformation shall be assumed to be as follows:

Pretensioned concrete	$6{,}000 + 16f_{cs} + 0.04f_{si}$
Post-tensioned concrete	$3{,}000 + 11f_{cs} + 0.04f_{si}$

In these criteria the efficiency of the anchorage has been assumed to be 100%. The designer should add to the figure given for creep and shrinkage an amount sufficient to allow for the anchorage efficiency, as determined by test.

Light-weight aggregate: An amount to be determined by tests.

Decrease in Prestress Due to Friction. Where the prestressing steel is "draped" and wherever minor irregularities occur in the alinement of the ducts, the stress in the interior of the beam will be somewhat less than that at the jack, due to friction. This loss shall be estimated and

* For notations not defined herein, see Appendix A.

verified in the field as given in the section on construction under the heading "Post-tensioning method" (p. 449). A guide to the estimation of the loss will be found in the discussion.

Ultimate Strength. The ultimate strength must be such as to withstand the following loads without failure:

$$DL + 3(LL + IL) \quad \text{or} \quad 2(DL + LL + IL) \quad \text{whichever is greater}$$

In figuring the ultimate strength, use f_s' and $0.8f_c'$ (see under the heading "Computing ultimate strength of beam," below).

Principal Tensile Stress. The principal tensile stress shall not exceed the following:

Dead, live or impact load, or any combination thereof: $0.03f_c'$ to be carried by the concrete and the excess over $0.03f_c'$ to be carried by properly designed stirrups.

Ultimate loads, without stirrups: $0.08f_c'$. If this stress is exceeded, stirrups shall be designed to take the total principal tensile stresses.

In the case of both working loads and ultimate loads, the maximum shears may be taken at a point 1.5 times the depth of the beam, measured from the nearest support.

End Anchorage Bearing Plates for Prestressing Steel. Bearing plates shall be designed so that the bending stresses in the plates due to dead, live, and impact load do not exceed that allowable for the type of steel used, and the unit pressure on the concrete does not exceed:

$$f_c = 0.4f_c' \sqrt[3]{\frac{a_c}{a_p}} \qquad \text{or} \qquad f_c' \quad \text{whichever is less}$$

where a_c = the maximum area of that portion of the end of the beam which is geometrically similar and concentric to the area of the bearing plate (sq in.).

a_p = bearing area of the anchorage plate (sq in.).

Computing Ultimate Strength of Beam. Unless a more exact method is preferred, the following shall be used:

Where the prestressing elements are bonded to the concrete, the reinforcement shall be assumed "balanced" (i.e., when the steel and concrete fail simultaneously) if:

$$p_b = 0.23 \frac{0.8f_c'}{f_s'}$$

The ultimate moment M' shall be determined as follows: Where p is equal or less than p_b,

$$M' = 0.9A_sf_s'd$$

Where p is greater than p_b,

$$M' = 0.9\sqrt{A_s A_{sb}}\, f_s' d$$

Where the prestressing elements are not bonded to the concrete, the prestressing steel shall be considered as an external force and shall not be figured as reinforcement.

Stirrups. It is recommended that stirrups be used, whether or not computations show that they are needed. The maximum stirrup spacing shall be not more than three-fourths of the depth of the beam. The sum of the cross-sectional areas of the legs of the stirrup should be not less than 0.08% of the cross-sectional area of the prestressed beam for the maximum spacing. Metal mesh of the same cross-sectional area per foot of beam may be substituted for stirrups at the option of the engineer.

Diaphragms. Diaphragm spacing shall be shown on the plans.

Size, Spacing, and Cover of Prestressing Steel. Where tension in the prestressing steel is maintained by bond and there is no adequate end anchorage provided, 0.2 in. shall be the maximum size of wire permitted, where the wires are used singly. Where the wires are used in seven-wire strands, the maximum strand permitted shall be ⅜ in.

The minimum spacing, both vertically and horizontally, shall be 3 times the diameter of the wire or strand, measured center to center. In no case, however, shall the clear spacing between wires or strands be less than 1½ times the maximum size of the coarse aggregate.

The minimum cover distance for all prestressing steel shall be 1½ in. or 1 diameter of bar, strand, or duct, whichever is greater.

Where adequate end anchorage is provided, the above limitations are not applicable, except that a clear spacing horizontally of 1½ times the maximum size of the coarse aggregate shall be maintained.

Composite Construction. Where precast and cast-in-place concrete are designed to act integrally, as when precast beams are used to support a cast-in-place slab, the horizontal shear shall be provided for by positive means, such as keys, and the two types of concrete held firmly together by stirrups extended up into the slab.

Materials

Concrete. Any portland cement and aggregate may be used which is suitable for ordinary concrete.

Prestressing Reinforcement. Prestressing reinforcement shall be high-tensile wire, high-tensile wire strand or rope, or high-tensile alloy bars.

Steel to be bonded to the concrete shall not be galvanized. If the

steel is to be left unbonded, it shall be protected against corrosion as described in the section on construction under the heading "Unbonded steel" (p. 450).

If wire or strand is used, it shall have an elongation at rupture of not less than 3% in 10 in. Bars, if used, shall have an elongation at rupture of not less than 4% in a distance of 20 diameters.

Permissible Variations in Gage of Wire. The dimensions of the wire, on any diameter, shall not vary more than plus or minus 0.003 in. from the specified nominal diameter. The difference between the maximum and minimum diameters, as measured on any given cross section of the wire, shall not be more than 0.003 in.

Finish of Wire. The wire shall be free from injurious defects and shall have a workmanlike finish with smooth surface.

Testing. All wire, strand, or bars to be shipped to the site shall be assigned a lot number and tagged for identification purposes. Anchorage assemblies to be shipped shall be likewise identified.

All samples submitted shall be representative of the lot to be furnished and, in the case of wire or strand, shall be taken from the same master roll.

All of the materials specified for testing shall be furnished free of cost and shall be delivered in time for tests to be made well in advance of anticipated time of use.

Where the engineer intends to require nondestructive testing of one or more parts of the structure, special specifications shall be drawn giving the required details of the work.

The vendor shall furnish for testing the following samples selected from each lot. If ordered by the engineer, the selection of samples shall be made at the manufacturer's plant by the inspector.

Pretensioning Method. For pretensioned strands, samples at least 7 ft long shall be furnished of each strand size. A sample shall be taken from each end of every coil.

Post-Tensioning Method. The following lengths shall be furnished:

For wires requiring heading, 5 ft.

For wires not requiring heading, sufficient length to make up one parallel-lay cable 5 ft long consisting of the same number of wires as the cable to be furnished.

For strand to be furnished with fittings, 5 ft between near ends of fittings.

For bars to be furnished with threaded ends and nuts, 5 ft between threads at ends.

Anchorage Assemblies. Two anchorage assemblies shall be furnished, complete with distribution plates of each size or type to be

used, if anchorage assemblies are not attached to reinforcement samples.

Inspection. An inspector representing the purchaser shall have free entry, at all times while the work on the contract is being performed, to all parts of the manufacturer's works which concern the manufacture of the materials ordered. The manufacturer shall afford the inspector, without charge, all reasonable facilities to satisfy him that the material is being furnished in accordance with these criteria.

Rejection. Material which shows injurious defects during or previous to its installation in the work shall be rejected.

Construction

General. Unless otherwise ordered by the engineer, the contractor shall certify to the engineer that a technician skilled in the prestressing method used will be available to the contractor to give as much aid and instruction in the use of the prestressing equipment and installation of materials as may be necessary to obtain satisfactory results.

Hydraulic jacks shall be equipped with accurately reading calibrated pressure gages. The contractor may elect to substitute screw jacks or other types for hydraulic jacks. In that case, proving rings or other approved devices must be used in connection with the jacks. All devices, whether hydraulic jack gages or other types, shall be calibrated and, if necessary, recalibrated so as to permit the stress in the prestressing steel to be computed at all times. A certified calibration curve shall accompany each device.

Safety measures must be taken by the contractor to prevent accidents due to possible breaking of the prestressing steel or the slipping of the grips during the prestressing process.

Concrete. All concrete shall be handled and placed in accordance with article 2.4.9 of the American Association of State Highway Officials' *Standard Specifications for Highway Bridges*, 1953.

Concrete shall not be deposited in the forms until the engineer has inspected the placing of the reinforcement, conduits, anchorages, and prestressing steel and has given his approval thereof.

The concrete shall be vibrated internally or externally, or both, as ordered by the engineer. The vibrating shall be done with care and in such a manner as to avoid displacement of reinforcing, conduits, or wires.

Steam curing of the concrete will be permitted in lieu of water curing. If the contractor elects to cure with steam or by any other special method, the method and its details shall meet with the approval of the engineer.

Transportation and Storage. Precast girders should be transported in an upright position, and points of support and directions of the reactions with respect to the girder should be approximately the same during transportation and storage as when the girder is in its final position. In the event that the contractor deems it expedient to transport or store precast girders in other than this position, it shall be done at his own risk.

Care shall be taken during storage, hoisting, and handling of the precast units to prevent cracking or damage. Units damaged by improper storing or handling shall be replaced by the contractor at his expense.

Pretensioning Method. The prestressing elements shall be accurately held in position and stressed by jacks. A record shall be kept of the jacking force and the elongations produced thereby. Several units may be cast in one continuous line and stressed at one time. Sufficient space shall be left between ends of units to permit access for cutting after the concrete has attained the required strength. No bond stress shall be transferred to the concrete, nor end anchorages released, until the concrete has attained a compressive stress, as shown by cylinder tests, of at least 3500 psi. The elements shall be cut or released in such an order that lateral eccentricity of prestress will be a minimum.

Post-Tensioning Method. The tensioning process shall be conducted so that the tension being applied and the elongation of the prestressing elements may be measured at all times. The friction loss in the element, i.e., the difference between the tension at the jack and the minimum tension, shall be determined by the formula

$$F_1 - F_2 = 2\left(F_1 - \frac{A_s \Delta_s E_s}{L}\right)$$

where F_1 = observed tension at the jack.

F_2 = minimum tension.

A_s = cross-sectional area of the prestressing element.

Δ_s = observed elongation of the element at the jack when the force at the jack is F_1.

E_s = secant modulus of elasticity of the element for the stress F_1/A_s as determined from the stress-strain diagram of the element.

L = distance from the jack to the point of lowest tension in the element. Where jacking is done from both ends of the member, the point of minimum tension is the center of the beam; where jacking is done from one end only, L is the length of the beam.

A record shall be kept of gage pressures and elongation at all times and submitted to the engineer for his approval.

After tensioning, and wherever practicable, prestressing steel shall be bonded to the concrete.

Bonded Steel. All prestressing reinforcment to be bonded to the concrete shall be free of dirt, loose rust, grease, or other deleterious substances.

Steel installed in holes or flexible metal tubes cast in the concrete preferably shall be bonded, in which case the annular space between the perimeter of the hole or tube and the steel shall be pressure-grouted after the prestressing process has been completed.

The grout shall be made to the consistency of thick paint and shall be mixed in the proportions, by volume, of 1 part portland cement to 0.75 part (max.) of sand passing a No. 30 sieve and 0.75 part (max.) of water. Within the limit specified, the proportions of sand and water shall be varied as required by the engineer. It may be necessary to eliminate the sand from the mix and use neat cement grout.

If aluminum powder is used to expand the grout, it shall be added as follows: From 2 to 4 grams of the powder (about 1 or 2 teaspoons) shall be added for each sack of cement used in the grout. The aluminum powder shall be the unpolished variety. The exact amount of aluminum powder shall be designated by the engineer. The dosage per batch of mortar shall be carefully weighed. A number of weighings may be made in the laboratory and the doses placed in glass vials for convenient use in the mixing operation. The aluminum powder shall be blended with pumicite or other inert powder in the propor tion of 1 part powder to 50 parts pumicite (or other inert powder) by weight. The blend shall be thoroughly mixed with the cement and sand before water is added to the batch, as it has a tendency to float in the water. The amount of the blend used should vary from 4½ ounces per sack of cement for concrete having a temperature of 70°F to 7 ounces for a temperature of 40°F. After all ingredients are added, the batch shall be mixed for 3 minutes. Batches of grout shall be made small enough so that the batch may all be used up in less than 45 minutes, as the action of the aluminum becomes very weak after that period of time.

Except as herein provided, all grout ingredients shall comply with articles 4.1.1, 4.2.1, and 4.3.2 of the AASHO *Standard Specifications for Highway Bridges,* 1953. The final pressure placed on the grout shall be 50 to 100 psi.

Unbonded Steel. Where the steel is to be left unbonded to the

concrete, it shall be carefully protected against corrosion by galvanizing, and, in addition, a coating of tar or other waterproof material shall be applied. If galvanizing is not practicable, another method of protection may be approved by the engineer provided that tests have shown its suitability.

Placing and Fastening Steel. All steel units shall be accurately placed in the position shown in the plans, and firmly held during the placing and setting of the concrete.

Distances from the forms shall be maintained by stays, blocks, ties, hangers, or other approved supports. Blocks for holding units from contact with the forms shall be precast mortar blocks of approved shape and dimensions. Layers of units shall be separated by mortar blocks or other, equally suitable devices. Wooden blocks shall not be left in the concrete.

Wires, wire groups, parallel-lay cables, and any other prestressing elements shall be straightened to ensure proper positioning in the enclosures. Suitable horizontal and vertical spacers shall be provided, if required, to hold the wires in place in true position in the enclosures.

Enclosures. Enclosures for prestressed reinforcement shall be accurately placed at locations shown in the plans or approved by the engineer.

All enclosures shall be water-tight. They shall be metallic, except that the contractor, at his option, may form the enclosures by means of cores or ducts composed of rubber or other suitable material which shall be removed prior to installing the prestressing reinforcement. Enclosures shall be strong enough to maintain their shape under such forces as will come upon them. They shall be ¼ in. larger in internal diameter than the bar, cable, strand, or group of wires which they enclose. Where pressure grouting is specified, cores or ducts shall be provided with pipes or other suitable connections for the injection of grout after the prestressing operations have been completed.

Prestressing. After the concrete has attained the required strength, the prestressing reinforcement shall be stressed by means of jacks to the desired tension and the stress transferred to the end anchorage.

Tensioning of the prestressing reinforcement shall not be commenced until tests on concrete cylinders, manufactured of the same concrete and cured under the same conditions, indicate that the concrete of the particular member to be prestressed has attained sufficient compressive strength.

(For Discussion of the Criteria, see the original pamphlet by the Bureau of Public Roads.)

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