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CONCRETE ENGINEERING HANDBOOK

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PREFACE

This handbook aims to present in one volume fundamental and practical information in the field of reinforced concrete that may be useful to practicing civil and structural engineers and particularly those interested in the design and construction of buildings and bridges and related structures.

In order to provide space for the maximum coverage of material, this handbook was planned for those who have been professionally trained either formally or through practical experience. Thus, with a basic understanding of structural mechanics and of reinforced concrete assumed, it was possible for the contributors to go right to the heart of their subjects.

Every designer must be thoroughly conversant with the materials to be called for in his designs. He must understand the field problems of cast-in-place or of precast construction and the latest thinking or codification with respect to the problems of earthquake-resistant structures, loadings for bins, and design of tall chimneys. To be helpful, summaries on the subjects of prestressed concrete, of indeterminate structural theory, of member torsion, and of concrete skin structures are also included as well as material on earth pressures, foundations, and pavement design.

A large section has been devoted to the design of building frames with many helpful tables including (1) those giving the moment of inertia of selected size beams and arrangement of reinforcing for the more rapid first trial or final design of desired sizes, (2) selected size columns and arrangement of reinforcing for given loadings, and (3) several arrangements of footings to carry given column loadings.

The design of simple and continuous span bridges, rigid frames, and arches, with related material on retaining walls, abutments, piers, and floor slabs, all well illustrated, covers several sections and completes the presentation in this handbook.

So much has been written on the subject of reinforced concrete, and for so many years reinforced concrete has been used in the construction of all kinds of structures, that we tend to take source material for granted or lose track of its origins. A conscientious effort has been made throughout this handbook to give credit where credit is due. Where appropriate
credit has been inadvertently omitted, it is indeed regretted and efforts
will be made to rectify this in subsequent printings.

A handbook is the contribution of hundreds of persons. The work of
researchers, designers, builders, contributors to association and technical
society publications, and authors all have supplied material to those
associated in the work of this handbook. Many have helped the main
contributors in staff work, typing, drawings, and review. Many wives
and families contributed by their patience.

Special acknowledgment should be paid to the late R. R. Zipprodt,
who served as the early editor of this handbook, for his organization of
certain areas of work; to M. F. Janes who had to redo many tables after
the high bond bars were introduced; to all the contributors when it
became necessary to rewrite and cut back the size of their sections and
for their very fine and friendly cooperation throughout the years we have
worked together; and to Mrs. La Londe, my very personal appreciation
for all her patience as I worked with the contributors and for her help
throughout with all the proofreading.

William S. La Londe, Jr.
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CONCRETE ENGINEERING HANDBOOK
Section 1

MATERIALS FOR REINFORCED CONCRETE

By

FREDERICK G. LEHMANN, Professor of Civil Engineering, Newark College of Engineering, Newark, N.J.

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INTRODUCTION

Plain concrete consists of a mixture of cement, water, and an inert matrix of sand and gravel or crushed stone. The distinguishing property of concrete is the property of hydraulicity, or ability to harden under water. The cement commonly used is portland cement, and the sand and gravel materials are those obtainable, usually from nearby sand and gravel or rock deposits.

Since plain concrete has little usable tensile strength it must be reinforced in structures that are subjected to forces producing tension. The reinforcing material used is steel. Different kinds of steel and reinforcing shapes are required by the many different uses of concrete.

CEMENTS

Function of Cement

Although all the materials that go into the concrete mixture are essential, the cement is very often the most important because it is usually the "weakest link in the chain." The function of the cement is first of all to bind the sand and stone together and second to fill up the voids in between the sand and stone particles.

Natural Cements

Natural cement is the finely pulverized product which results from the calcination of natural clayey limestone or other suitable natural rock. There are large deposits of this limestone in widely separated points of the United States. Because of variations in the deposits, however, natural cements are of variable quality.

Because natural cements normally set too rapidly it is usually necessary to add gypsum to control the set within desired limits.

Natural cement is extensively used in mortar in the proportions of 1 part natural cement to 3 parts sand. Plastic mortars of ample strength are produced without the addition of lime in any form.

Natural cement, though truly hydraulic, is not recommended for placement under water. Its low strength is a further limitation for structural work. Because of these restrictions on its use and the difficulty of rigid control of the product, natural cement as a complete cementing agent has been almost entirely displaced by portland cement. Natural cement, however, is generally more durable than portland cement. For this reason it is used as a blending material with portland cement to increase durability.

ASTM Specification C 10–37 covers the chemical and physical requirements for natural cement. Using the standard method of testing, ASTM Specification C 191–52, the time of the initial set shall not be less than 10 min when the Vicat needle is used or less than 20 min when the Gillmore needle is used. Final set shall be attained within 10 hr. Average tensile strengths of the three standard mortar briquettes made and tested in accordance with ASTM Specification C 190–44 shall have a 7-day strength of at least 75 psi and 28-day strength of at least 150 psi.
CEMENTS

Pozzolan and Slag Cements. These natural hydraulic cements are formed by combining 1 part of hydrated lime with 2 to 4 parts of either a pozzolanic material such as volcanic ash, trass, or tufa or blast-furnace slag.

Pozzolanic cements are neither so strong nor so reliable as portland cement and consequently their use by themselves is limited to applications where strength is not such an important factor. Their principal use in this country has been for non-staining mortars for the purpose of laying masonry. Today pozzolans are finding a wider use as a blending material with portland cement.

Portland Cement

Portland cement is an artificial mixture of lime-bearing and clay-bearing materials which are burned to incipient fusion and subsequently ground to a fine powder. After calcination lime sulfate in the form of raw gypsum is added to retard the setting of the cement. As a rule about 2 per cent and never more than 3 per cent is the amount added. In addition the ASTM Standard Specification for Portland Cement C 150-52 allows other admixtures not to exceed 1 per cent provided that these materials have been shown by prescribed tests not to be harmful.

Nature of Portland Cement. Portland cement is a very finely ground powder consisting of many different compounds or synthetic minerals. The most important of these are tricalcium silicate, dicalcium silicate, tricalcium aluminate, and tetracalcium aluminoferrite.

Hydration. The nature of Portland cement is revealed by its setting properties upon the addition of water. When water is added to cement, it is desirable to obtain a given strength and therefore not necessarily complete hydration. The cement must not be disturbed until it has taken its initial set. Small vibrations, however, due to passing of nearby trains or trucks have no measurable effects on the hydration of the cement. Measurements indicate that the rate of reaction is so slow that months and even years may be required to complete the hydration of a fine particle. The products which hydrate most rapidly are the tricalcium aluminate and the tetracalcium aluminoferrite. If finely ground, the hydration of these two components approaches completion in 1 day.

Specific Gravity of Cement. The specific gravity of portland cement is somewhat variable but is usually slightly greater than 3.10.

Time of Setting. There are two distinct stages in setting: (1) the initial set and (2) the final set. ASTM Specification C 150 states that the initial set when tested with the Vicat needle shall not develop in less than 45 min, and the final set in not more than 10 hr.

The time of setting is influenced by the following factors: relative amounts of the various minerals present, amount of gypsum interground with the clinker, fineness of grinding, water-cement ratio, and temperature. Age of cement has a great effect upon the setting time. Most cements absorb moisture from the air and lose some of their hydraulic property on storage. Occasionally the gypsum added in manufacture loses its effectiveness in a short time and consequently the cement becomes quick-setting. This is generally caused by the composition of the particular cement and can be readily remedied by the cement manufacturer.

False Set in Portland Cement. This is a premature stiffening of the mix which adversely affects the properties of concrete. These effects include high water requirement, subsequent loss of strength, loss of durability, and loss of quality control. The cause of this action is not definitely settled but it is generally agreed that unstable gypsum, so-called "plaster set," is the source of the trouble. The most common cause of unstable gypsum in cement is the high grinding-mill temperatures. Instability in gypsum is also caused by high storage temperatures, lack of aeration, and the presence of moisture. In all cases where the mills have been cooled, false set has disappeared. Most cases of false set can therefore be corrected by the cement manufacturer.

Product Control and Uniformity. Any portland cement that will meet ASTM Standard Specification C 150-52 for general-type construction work will make satisfactory concrete although some cements are sometimes better than others for several reasons. Selectivity may be important where the cement is to be used under severe weathering conditions or where the concrete may be subject to chemical action or where reactive aggregates are to be used.

The usual method of product control to meet these specifications is sample testing. There can be poor material in any product though it will probably be a small percentage if frequent samples are taken and used properly for the process control. The temperature of burning and the fineness of grinding are particularly important items in the manufacturing process. If these are not properly controlled the cement will not pass the soundness test, ASTM C 151-52. This test also will show up the presence of too much free lime and magnesia.

Deleterious Substances in Portland Cement. There are several substances found in portland cement that adversely affect the soundness of the concrete. Of the most important are free lime, magnesia, and the alkalies. Finished cement standing in a humid atmosphere will pick up excessive moisture which can be detected by measuring ignition losses. When the clinker comes from the kiln at a white heat, there is no loss on ignition. Specifications commonly used set the limits for chemical composition of portland cement.

Heat of Hydration. During the hydration process considerable amounts of heat are given off. These will depend on the products of hydration for which values are listed as follows:¹

<table>
<thead>
<tr>
<th>Product of hydration</th>
<th>Heat of hydration cal/gram</th>
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<tbody>
<tr>
<td>Tricalcium silicate</td>
<td>120</td>
</tr>
<tr>
<td>Dicalcium silicate</td>
<td>62</td>
</tr>
<tr>
<td>Tricalcium aluminate</td>
<td>207</td>
</tr>
<tr>
<td>Tetracalcium aluminoferrite</td>
<td>100</td>
</tr>
<tr>
<td>Magnesia</td>
<td>203</td>
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<tr>
<td>Lime</td>
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Shrinkage. When the cement reacts with water there is a small amount of shrinkage but the largest amount, about 0.10 per cent for neat cement, occurs because of evaporation of the excess water.

Types of Portland Cements. Certain common types of structures or service conditions call for particular properties that cannot be satisfied by one cement. These special properties usually include high early strength, low heat of hydration, and resistance to waters containing sulfates. In order to meet these requirements, the cement industry produces cements whose chemical compositions differ from Type I portland cement and meet standard specifications that ensure the special desired properties.

Type II Cement. This cement has a lower heat of hydration than Type I cement. In addition the lower limitation on tricalcium aluminate gives this cement more resistance to moderate sulfate action.

Type III Cement. This is a high-early-strength cement which is obtained by increasing the percentage of tricalcium silicate in the cement. There is danger of free lime in this cement and also of high volume changes after setting because of increased amount of tricalcium aluminate. An increase in early strength of any cement may be obtained by finer grinding of the clinker.

Type IV Cement. This is essentially a low heat of hydration cement obtained by having a large percentage of dicalcium silicate. One disadvantage of this cement is the relatively long time it takes to gain strength. Because of its limited use this cement is generally not in stock.

Type V Cement. This cement is designed to be used where waters high in sulfates come in contact with the concrete. Since the sulfates react with the tricalcium aluminate, causing swelling and disintegration of the concrete, this component is limited to 5 per cent. Because of its limited use, this cement is generally not in stock.

Blended Cements

A blended cement is a mixture of two or more different materials with portland cement in order to achieve certain special qualities in concrete that could not economically be attained by using ordinary portland cement.

Blends Using Natural Cement. The Corps of Engineers, U.S. Army, has recently conducted extensive tests on natural cements blended with portland cement. The results of these tests indicated a definite increase in freezing and thawing durability for concrete specimens made with the blended cement over those made with portland cement. For blend proportions of 1 part natural cement to 6 parts portland cement used in standard concrete cylinders the 28-day compressive strength was somewhat more variable than that obtained for straight Type II portland. When Type I portland was used in the blend, the compressive strength was decreased 0 to 35 per cent. The strength-test comparisons, however, were conducted at a uniform water-cement ratio. If lower water-cement ratios had been used as permitted by increased workability of the blend mixes, the lowering of the strength would have been materially less.

Blends Using Pozzolanic Materials. A class of blended cements particularly important in the West and Middle West of the United States uses pozzolanic materials for a portion of the cement under a wide range of different applications. Their greatest use is in mass concrete and to inhibit excessive expansion caused by reactive aggregate and high-alumina cement. A pozzolan is a siliceous material, largely amorphous silica, which possesses no cementitious value. When in finely divided form, it will react with calcium hydroxide in the presence of moisture to form insoluble compounds which possess cementitious properties.

Classification of Pozzolans. Natural pozzolans are classified by the constituent contributing to their activity. The following five classes are generally recognized: Type I, volcanic glass; Type 2, opal; Type 3, clay minerals; Type 4, zeolites; Type 5, hydrated oxides of aluminum. Natural pozzolans either possess pozzolanic properties as they occur in nature or else can be easily converted by calcination and fine grinding. Artificial pozzolans include fly ashes and water-quenched boiler slag.

Reaction with Portland Cement. The calcium hydroxide liberated during the hydration of portland cement reacts with the pozzolanic material to form a hydrox calcium silicate and other products. The hydrous calcium silicate has low solubility, which reduces bleeding of the concrete and contributes to watertightness and strength.

Treatment of Pozzolans. Investigations by Moran and Gilliland have shown that calcination is not effective for amorphous substances such as glass or opal. Clay minerals, however, of the kaolinite and montmorillonite groups which are present in variable amounts in most pozzolans are benefited by calcination.

Fine grinding increases the effectiveness of these materials. A specific surface area of 10,000 to 15,000 sq cm per gram has been found to be practical for shales. Because of their hardness, grinding is not practical for fly ashes. They should, however, be selected for carbon content less than 7 per cent and specific surface greater than 2,500 sq cm per gram.

Effects on Concrete. The following list gives the effects of pozzolans on concrete when the pozzolan and the amount of cement replacement are properly chosen. This latter requirement is very important in order to obtain the desired result.

Air entrainment: Makes air entrainment more effective.
Alkali-Aggregate reaction: Reduces the expansion resulting from this reaction.
Bleeding: Reduces bleeding.
Freezing and thawing: Less resistant to severe freezing and thawing except in using air-entraining agents where improvement is obtained by using pozzolans.

1 Tests of Blends of Portland and Natural Cements, U.S. Corps of Engineers Central Concrete Laboratory, Vicksburg, Miss., 1944.
Heat of hydration: Heat generated is less, depending on the amount of replacement and the character and fineness of the grind.
Richness of the mix: Benefits lean mixes but has much less effect on rich mixes.
Shrinkage: Greater amount of drying shrinkage.
Strength: Although the strength is the same as with ordinary portland cement, the strength is attained at a slower rate.
Sulfates: Pozzolans high in opal reduce the action of sulfates and low acidic waters.
Water requirement: When based on a given slump, pozzolans except for some fly ashes require more water. However, a better criterion for workability, when using pozzolans, is the Powers remodeling apparatus.
Watertightness: Finely ground pozzolans of opaline character greatly increase impermeability.

Amount of Replacement. The optimum amount of cement replacement depends on the nature and fineness of the pozzolan, richness of the mix, type of portland cement, type of aggregate, and properties of the concrete to be improved. In general, replacement of cement by pozzolans in the following percentages has been found to be both beneficial and economical: general building construction, 5 to 15 per cent; pavements, 10 to 20 per cent; and mass concrete, 15 to 50 per cent. These values of replacement should serve only as a guide since particular conditions usually are sufficiently variable to justify the making of an investigation before choosing the material and the amount of replacement.

Tests for Reactivity. Any pozzolan used should be tested for reactivity and should conform to the Bureau of Reclamation Specification on Calcined Reactive Siliceous Material for Use in Concrete. This specification prescribes the method of testing as well as the minimum acceptable standards for reactivity. A chemical test for determining the effectiveness of pozzolans in reducing expansion caused by reactive aggregates is recommended for use in specifications covering selection and control of pozzolans for the purpose. The reduction in alkali concentration of a 0.5N sodium hydroxide solution after reaction with the pozzolan in the presence of calcium hydroxide is used as an index of the effectiveness of the pozzolan. A reduction in alkali concentration of 180 milliequivalents per liter is used as the criterion of a satisfactory pozzolan from the standpoint of control of alkali-aggregate expansion.

Air-entraining Portland Cement

An air-entraining portland cement contains an agent interground with the clinker which causes more air to be entrained in the concrete after mixing than would normally be the case. As an alternate procedure, the air-entraining agent may be added separately at the mixer. The air-entraining agent causes a foaming action which disperses very fine air bubbles throughout the concrete mixture. These bubbles are from 0.003 to 0.005 in. in diameter and do not coalesce.

The advantages for concrete secured by proper air entrainment include better workability, less bleeding, greater durability, and greater watertightness.

Principal Air-entraining Agents. The following materials are the principal sources of air-entraining agents:

1. Natural wood resins and their soaps
2. Animal or vegetable fatty acids, fats, oils, and soaps
3. Alkali salts of sulfonated or sulfated organic compounds

ASTM Specification C 150-52 specifically allows the following to admixtures to be interground with the clinker:

TDA (composed of triethanolamine and highly purified soluble calcium salts of modified lignin sulfonic acids) manufactured by Dewey and Almy Chemical Co., when added in an

amount not exceeding 0.043% by weight of cement except that in type III cement a maximum of 0.08% by weight may be used. 109-B (composed essentially of 2 methyl 2-4 pentane diol), marketed by the Master Builders Co., when added in an amount not exceeding 0.03% by weight of the cement except that in type III cement a maximum of 0.05% by weight may be used.

Requirements for other allowable agents to be interground with the clinker are given in ASTM Tentative Specification C 226–52T. This specification requires compliance with ASTM Tentative Specification C 175–51T, Air-entraining Portland Cement, and the following:

(1) Time of setting of concrete containing the addition shall not vary from the time of setting of the respective “blank” cement made without the addition by more than 50% for both initial and final set.

(2) The percentage autoclave expansion for cement containing the addition shall not exceed the percentage autoclave expansion for the corresponding “blank” cement by more than 0.1.

(3) The compressive strength of standard mortar cubes made with cement containing the addition shall be not less than 85% of the compressive strength of similar cubes made with the corresponding “blank” cement.

(4) The percentage length change of air-stored mortar bars made with cement containing the addition, based on an initial measurement at the age of 7 days, and expressed as a percentage change in length, shall be not more than 0.01 greater than that of similar mortar bars made with the corresponding “blank” cement and similarly tested.

(5) The percentage of air entrained in the concrete made with cement containing the addition shall exceed by at least 2.5% the percentage of air in similar concrete prepared with the corresponding blank cement. [Note that “blank” cement standard mortar shall not entrain more than 8 per cent air.]

(6) The compressive strength of the concrete made with cement containing the addition shall not be less than 85% of the compressive strength of similar concrete made with the corresponding “blank” cement.

(7) The flexural strength of concrete made with cement containing the addition shall not be less than 85% of the flexural strength of similar concrete made with the corresponding “blank” cement.

(8) In the freezing and thawing test, the durability factor of the concrete made with the cement containing the proposed addition shall be not less than 80% of the durability factor of similar concrete made with the corresponding “blank” cement and containing the reference addition as specified in section 3(c).1

Most commercial agents may also be added to the batch in solution as a portion of the mixing water. With some, however, it is necessary to add an additional agent to form a soluble product.

Chemical and physical requirements for air-entraining portland cement are the same as for the non-air-entraining cement except as follows: There is no limit on sulfur trioxide. Instead calcium sulfate in the hydrated cement mortar (expressed as SO3) after 24 hr setting time is limited to 0.50 gram per liter. Minimum air content of mortar prepared in accordance with ASTM Designation C 185–50T shall be 18 ± 3 per cent by volume. Compression strength of mortar cubes is about 15 per cent less than that obtained from cubes without entrained air. There are no tensile-strength requirements.

High-alumina Cement

High-alumina cement, although considerably more expensive than portland cement, is frequently used for construction in which it is necessary to attain relatively high strengths within 48 hr, or for structures which may be exposed to sea-water action. Although superior to portland cements for the latter purpose, high-alumina cement is very rapidly deteriorated by exposure to sulfate attack.

High-alumina cements, while not of the quick-setting type, frequently attain compressive strengths at 48 hr which are approximately equal to the 28-day strengths of normal portland cement. Compressive strength of high-alumina cement increases more slowly after the 48-hr period, but the increase in strength continues to be approximately twice as great as that for normal portland cement.

1 Reference materials are Vinsol resin, Daren, and N-TAIR of Airalon.
Recently it has been reported\(^1\) that ferruginous bauxite and lime or limestone will produce a good aluminous cement whose 24-hr strength is equal to the 28-day strength of ordinary portland cement.

Concrete made with high-alumina cements requires special care in curing because of the large amount of heat generated during setting. This heat, however, may be advantageous in concreting at low temperatures.

Testing of Cements

The purposes of testing cements are the development of new cements and the ensuring of required properties of cements.

Because of the increase in use of special cements, testing has assumed a major role in preliminary investigations. Many specific properties can be determined by short-time tests on small samples but in certain cases test information must be obtained over a long period of time from the behavior of full-sized structures. It is emphasized that much economy can be gained by using modern statistical methods\(^2\) in setting up test programs and in evaluating the results. In order to obtain worthwhile results, testing procedures must be standardized and methods used that will minimize the human element. Testing should be directed and carried out only by well-qualified personnel. For the smaller organization the services of a reliable testing establishment are recommended.

Types of Tests. The following cement tests comprise those which are commonly used for construction work of importance and also in all cases where there is a question of satisfactory performance of the cement: fineness, setting time, compressive strength, tensile strength, soundness, cement-aggregate reactions, air content, heat of hydration, and chemical analysis.

Sampling. In order to obtain results from a minimum number of samples, tests should be conducted on representative groups of samples. One such approved method of sampling is given by ASTM Specification C 183–46.

Preparation of the Samples. Tests are usually performed on either neat cement consisting of cement and water or on cement mortar consisting of cement, sand, and water. It is necessary for comparison purposes that the samples have a standard consistency. This standard for tests on neat cement is given by ASTM Specification C 187–49. For testing mortars, the consistency to be used is given under the specification covering the specific test.

Finness of Cement. The strength of mortar and rapidity of setting of cement are increased by grinding the cement more finely. This is due to the fact that water does not react readily with the coarser particles.

The standard test for fineness of cement is ASTM Specification C 115–42. In this test the average surface per gram of cement sample is determined by a Wagner turbidimeter. As an alternate procedure ASTM Specification C 204–51 describes the air-permeability method which draws a definite quantity of air through a prepared bed of cement of a certain porosity.

Setting Time. The alternate tests to determine the setting times for cement are specified under ASTM C 191–52 and C 266–52T. With the Vicat test apparatus described in C 191, the initial set is defined as the minimum elapsed time at which the penetration needle ceases to pass a point 5 mm above the bottom of the cement-paste sample in 30 sec. The final set is said to occur when the needle does not sink visibly into the paste. The alternate Gillmore method uses two standard needles which determine the times of initial and final set of the paste when there is no appreciable indentation.

Compressive Strength. An important factor in the suitability of any particular cement for use in construction projects is the compression strength of cement-sand mortar specimens. Tests are usually made on specimens cast in the form of cubes having a side dimension of 2 in. A standard method of testing for compressive strength of hydraulic-cement mortars is ASTM C 109–52.

\(^1\) Avery and Hagen, Thesis, University of Washington, 1949.
AGGREGATES

Tensile Strength. Since portland cements are ordinarily not required to resist tensile stresses, the reasons for testing cement in tension are to determine whether it is likely to continue hardening uniformly on the construction project and whether it will have sufficient tensile strength to resist any strains to which it may be subjected by the dead load imposed by the mortar or concrete in which the cement may be incorporated. Tension tests of cement mortar are made on small specimens known as briquettes which have a minimum cross-sectional area of 1 sq in. at the point where the break usually occurs. The treating of the samples and method of testing are prescribed by ASTM C 190–49.

Soundness. In order that cement will function properly for construction purposes, it must not cause swelling, disintegration, or deterioration. These latter effects are commonly due to the fact that the cement has not remained constant in volume after setting. Causes of unsoundness are excess amounts of free lime, dehydrated magnesia, sulfates, and alkalis.

The usual test for soundness is the pat test, ASTM C 189–49. A neat cement pat of normal consistency is stored in moist air for 24 hr and is then placed in an atmosphere of steam immediately over boiling water for a period of 5 hr. The pat is considered to have passed the test satisfactorily if it remains firm and hard without showing any signs of checking, distortion, cracking, or disintegration.

The autoclave test, ASTM C 151–52, is similar to the pat test except that the steam testing is done under pressure in the absence of air. The soundness is measured by measuring the change in length of the 1- by 1-in. test specimens of 10-in. effective gage length.

Cement-Aggregate Reaction. The soundness tests do not measure the effects of aggregates on the cement. To do this it is necessary to consider both the cement and the aggregate to be used. This test is briefly described under Alkali Reactivity.

Air Content. The method of testing the effectiveness of air-entraining agents is given in ASTM Tentative Specification C 185–50T. In this test the air content of cement-sand mortar of a standard consistency is determined by weight measurements.

Heat of Hydration. Cement for mass concrete should be tested to determine the amount and rate of evolution of the heat of hydration. The method given by ASTM Specification C 186–49 determines the heat of hydration of a cement by measuring the heat of solution of the dry cement and the heat of solution of a separate portion of the cement after it has been partially hydrated for 7 or 28 days. The difference between these two values is the heat of hydration for the respective period of hydrating.

Chemical Analysis. Chemical analyses of cements are not usually carried out by the user unless the particular cement does not perform satisfactorily or possibly the cement may be an import whose composition is not known. Standard methods of chemical analysis of portland cement are specified in ASTM C 114–51T.

Definitions

The aggregate is the matrix or principal structure of concrete consisting of relatively inert fine and coarse material, usually stone. According to the 1940 Joint Committee for Standard Specifications for Concrete and Reinforced Concrete, “Fine aggregate shall consist of natural sand, sand prepared from the product obtained by crushing stone, gravel, or air-cooled blast furnace slag; or, subject to the approval of the Engineer, other inert materials having similar characteristics.”

Similarly, the 1940 Joint Committee states that “Coarse aggregate shall consist of crushed stone, gravel, air-cooled blast furnace slag, or, subject to the approval of the Engineer, other inert materials having similar characteristics.”

Classifications and Functions of the Aggregate

Aggregates are classified into two groups, the usual basis for separation being the ½-in. or No. 4 sieve. All particles which pass through this sieve are referred to as fine aggregate; those retained on the sieve are classified as coarse aggregate.

Rubble aggregate, according to the 1940 Joint Committee, is defined as clean, hard,
durable stone or gravel of such size as to be retained on a 6-in.-square opening, and with individual particles weighing not over 100 lb. Cyclopean aggregate, on the other hand, is composed of similar particles of stone or gravel weighing more than 100 lb.

The coarse aggregate is used primarily for the purpose of providing bulk to the concrete. To increase the density of the resulting mix, the coarse aggregate is frequently used in two or more sizes.

The most important functions of the fine aggregate are to assist in producing workability and uniformity in the mixture. Fine aggregate also assists the cement paste to hold the coarse-aggregate particles in suspension. This action promotes plasticity in the mixture and prevents the possible segregation of the paste and coarse aggregate, particularly when it is necessary to transport the concrete some distance from mixer to point of deposit.

The fine aggregate very largely determines the workability of the mixture. Consequently the amount as well as the gradation of the fine aggregate will have a material effect upon the ease with which the mixture will be deposited in the forms. The size and shape of the fine-aggregate particles as well as their surface characteristics are other properties which have important bearing upon the placing of the concrete. In general, rough surfaces and angular or flat particles result in a harsher mix. Another way of considering the above factors upon the concrete mix is the effect on the amount of water required for placing and finishing and on the water-retaining characteristics of the concrete. The rough angular shapes and flat particles increase the amount of water required for placing and finishing operations.

Properties Attributable to Aggregates

The potential quality of a concrete mix will be determined to a large extent by the quality of the cement paste. However, because the aggregate consists of about 75 per cent of the total in concrete, its influence is extremely important. The aggregates in general must possess physical properties which must be at least equal to the properties desired in the concrete. Properties of concrete attributable to aggregates include unit weight, strength, modulus of elasticity, creep, thermal coefficient of expansion, soundness, fire resistance, and resistance to wear and abrasion.

General Requirements for Aggregates

Physical Requirements. Cleanliness. Aggregates should not contain injurious amounts of deleterious substances such as clay, silt, coal, and mica particles, organic matter, chemical salts, and surface coatings. These deleterious substances cause unsoundness in the concrete and also increase the amount of water required for a given workability. In the case of chemical salts and surface coatings there may be additional bad effects caused by their reacting with the cement or by preventing hydration. Silt, clay, dust, and organic matter can be removed by washing. The removal of other surface coatings may require special chemical or abrasive methods, but these may prove to be uneconomical.

Mica is always undesirable in aggregate. Under no circumstances should more than 1 per cent of mica be permitted, since, even with so small an amount present in the sand, the strength of the concrete has been found to be reduced 15 per cent. Mortars are affected to a greater degree. Also, because the cement paste does not adhere to the mica particles, these points allow the entrance of water, which ultimately results in disintegration.

According to the ASTM Tentative Specifications for Concrete Aggregates, Designation C 33–52T, the amounts of deleterious substances present in aggregates shall not exceed the limits given in Table 1–1.

The following ASTM tests are standards for determining the amounts of deleterious material:

- Organic impurities
- Coal and lignite
- Clay lumps

Durability. Aggregates must not cause disruption in concrete due to volume changes brought about by moisture or temperature effects.

Durability is generally tested by the ASTM Tentative Specification C 88-46T, Test for Soundness of Aggregates by Use of Sodium Sulphate or Magnesium Sulphate. With certain exceptions, aggregates that stand up well in this test may be expected to give satisfactory performance. However, the test may also cause rejection of some materials which have shown good service in concrete.¹

Volume Changes Due to Wetting and Drying. Certain clay minerals (bentonite) expand and contract considerable amounts with wetting and drying. Concrete may be disrupted during expansion or large tension cracks may occur during shrinkage. In addition, failure can occur when cement paste shrinks during the hardening process if the aggregate itself is highly compressible and does not resist the volume change.²

### Table 1-1. Limits on Deleterious Substances in Aggregates

<table>
<thead>
<tr>
<th></th>
<th>Max permissible limits, % by wt</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Fine aggregate</td>
</tr>
<tr>
<td>Soft particles</td>
<td></td>
</tr>
<tr>
<td>Clay lumps</td>
<td>1.5</td>
</tr>
<tr>
<td>Chert that will readily disintegrate (soundness test, 5 cycles)</td>
<td>1.0</td>
</tr>
<tr>
<td>Material passing the No. 200 sieve:</td>
<td>3.0*</td>
</tr>
<tr>
<td>In concrete subject to abrasion</td>
<td>5.0*</td>
</tr>
<tr>
<td>All other classes of concrete</td>
<td>0.5†</td>
</tr>
</tbody>
</table>

* In the case of manufactured sand or crushed aggregates, if the material finer than the No. 200 sieve consists of the dust of fracture, essentially free from clay or shale, these limits may be increased as follows: sand to 5 to 7 per cent, coarse aggregate to 2.3 per cent.
† This requirement does not apply to blast-furnace-slag aggregates.

Freezing and Thawing. Durability under these circumstances depends on the size, continuity, and abundance of pores in the material. Both impermeable and highly porous materials, however, are durable under these conditions. It has been found, though, that disruption does occur in rocks having intermediate permeability with void diameters less than 0.005 mm.³

A method of determining an index of freezing and thawing durability has been developed by the Bureau of Reclamation.⁴ This test measures the rate of evaporation from saturated specimens and the force associated with capillary movement of water into the material. The results of this method applied to various materials have a definite correlation with 24-hr-cycle freezing and thawing tests.⁴ Other tests which determine the rate and amount of water absorption are described under ASTM Designations C 127 and C 128. Tests under these latter specifications may not always correlate with actual freezing and thawing tests of concrete because of the part that the mortar plays in this type of durability.

Temperature Effects. Unequal volume changes caused by unequal coefficients of thermal expansion in the various components in concrete may result in a structural failure. Aggregates whose coefficients of thermal expansion are unusually low are the most objectionable since few rocks possess higher coefficients than portland cement. For most common rocks, the thermal coefficient of expansion ranges from 0.5 × 10⁻⁵

to $9.0 \times 10^{-6}$ per degree Fahrenheit¹ whereas for hydrated neat portland cement the coefficient ranges between $5.9 \times 10^{-6}$ and $9.0 \times 10^{-6}$ per degree Fahrenheit.¹ Rocks that usually possess low coefficients (less than $1.0 \times 10^{-6}$) include granite, limestone, and marble.

**Bonding.** Concrete will fail along the surfaces of the aggregates if its bonding property is poor. Bonding properties depend on the surface structure, cleanliness, and chemical stability of the aggregate. The latter property is taken up under Chemical Requirements, Reaction with Portland Cement.

The surface structure affects bonding quality according to the surface roughness and the size and quantity of the pores. According to R. F. Blanks, salients and depressions on the particles, particularly when the sides of these roughnesses are almost perpendicular to the general surface, assist the adherence of the paste to the aggregate. Undulatory roughness is less helpful and may even be harmful to bond as the mortar changes in volume. In general, roughness is probably less significant than physical penetration of cement into the aggregate.²

**Fire Resistance.** Aggregates used in concrete for many types of buildings must be able to resist high temperatures due to fire for short periods of time.

**Abrasion Resistance.** The abrasion resistance of concrete is determined to a large extent by the character of the coarse aggregate. The properties of abrasion resistance, density, hardness, and toughness are all interrelated and may all be considered proportional to each other. The ASTM test for abrasion of coarse aggregate is C 131-51. Essentially the aggregate is subjected to a tumbling action and the amount of wear is determined by weighing the charge before and after the test.

**Strength.** There are generally no specific requirements for strength of coarse aggregates. Federal specifications and the 1940 Joint Committee of the ACI require that the mortar strength of cubes made with a given sand should be at least 90 per cent of the strength obtained under the same conditions using graded Ottawa sand.

**Chemical Requirements.** From the definition of aggregate as inert material it is required that its chemical activity be minimized so that no disruption effects occur during the setting of the concrete or subsequently because of service conditions. The chief sources of chemical action are as follows.

**Solubility.** Aggregates containing minerals or other substances that are readily soluble in water may be leached out, resulting in efflorescence and scaling of the surface.

**Oxidation, Hydration, and Carbonation.** These processes cause the failure of aggregates containing the following substances: pyrite or marcasite, ferrous and ferric oxides, and magnesia.

**Reaction with Portland Cement.** Calcium and magnesium sulfates present in aggregates react with hydrated aluminates of cement, causing expansion and failure of the cement bonds.³

**Base-exchange Reaction.** Concrete may also be adversely affected by a base exchange between components in the aggregate and the caustic solution within the cement paste. There is at present little conclusive evidence that this type of action is of practical importance, though tests have been performed that indicate zeolitic rock as a possible source of trouble.⁴

**Alkali Reactivity.** Certain rocks and minerals react with alkalis released by the setting of cement with subsequent expansion and disruption of the bonds. Outwardly, the concrete exhibits pattern cracking, which may be accompanied by exudations of gelatinous substances.

Many aggregates have been recognized as causing this type of reaction. The geographical location of structures which have been affected by alkali-aggregate reaction is shown in Fig. 1-1.

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Extensive tests by the Bureau of Reclamation have shown that all silicate and silica minerals react to a certain extent with alkalies in cement, but with most of them the effects are insignificant. The following minerals and rocks, however, react excessively with alkalies in concrete:

Minerals: Opal, chalcedony, tridymite
Rocks: Opaline cherts, chaledonic cherts, siliceous limestones, rhyolites, dacites, andesites, and phyllites

When economic considerations make it necessary to use the above reactive materials it is necessary to select a cement with a low alkali equivalent. Tests have shown that, with reactive aggregates, the alkali content of the cement should be 0.6 per cent or less.

The method proposed for testing the potential alkali reactivity of cement-aggregate combinations is given under ASTM Designation C 227-52T. The scope of the specification is as follows: "This method of test is intended to determine the potential expansive alkali reactivity of cement-aggregate combinations by measuring the expansion developed by the combinations in mortar bars during storage under prescribed conditions of Test." Specimens are measured for length changes at ages of 1, 2, 3, 6, 9, and 12 months. Any expansion over 0.050 per cent is generally considered detrimental. The effect of the chemical reaction is usually accelerated by storing the bars at somewhat elevated temperatures in moist air.

A short-time test to evaluate various aggregates as to potential reactivity has been developed by the Bureau of Reclamation but requires rigorous control and precise analytical technique in order to yield reliable results.

Aggregates: General

Properties of most aggregates can be obtained in a general way from a knowledge of the rocks and minerals from which they are obtained. Rocks are characterized by their texture, structure, and mineral composition, which together largely determine

the suitability of the aggregates. Very coarse textures and glassy textures are generally undesirable.

Though certain physical and chemical properties usually belong to a given rock or rock type, any rock is subject to chemical alterations which modify its initial properties. It is advisable, therefore, in all important work to check the suitability of aggregates either through past experience with a particular source or with new sources, through physical and chemical testing.

Injurious Minerals

The following minerals are injurious in aggregates depending on their amount and distribution:

*Mica*, including micasite, is a common constituent of many granites, gneisses, and sandstones. In granite, mica is not harmful unless segregated into bunches. In gneiss, too much mica causes structural weakness and poor durability. It also adversely affects the durability of sandstones.

*Pyrite* is easily decomposed into limonite and iron sulfate, with sulfuric acid being formed. These altered products are very weak and chemically undesirable. Any aggregate containing an appreciable amount of pyrite should be rejected.

*Tremolite* is a white or pale green variety of amphibole, found in some magnesian limestones. On exposure to weather it decomposes into a greenish-yellow clay. Any aggregate containing tremolite should be rejected.

Materials Suitable for Use as Aggregates

*Fine Aggregates.* Fine aggregates may be divided into two groups—those consisting principally of rock fragments resulting from natural processes of weathering, by disintegration or by glacial action; and those consisting of fine aggregates prepared by the crushing of natural rocks and the subsequent screening of the crushed material into the several desired sizes.

Sand deposits are the result of the weathering of rock minerals which have been transported, collected, and sorted by the long action of streams. Consequently there usually is a wide range of particle sizes which may include a number of the more common minerals.

*General Qualities of Fine Aggregates.* Natural sands are undoubtedly used to greater extent than any other kind of fine aggregate. It is not at all surprising that the quality of material found in sand pits may vary considerably because the sand necessarily partakes of the quality of the rock from which it is derived. Natural sands are most frequently the result of the weathering of masses of rock by breaking down as a result of the action of alternate freezing and thawing, by erosion resulting from the continuous action of wind or water, or by disintegration due to other natural processes. As a result the sizes of the particles may vary considerably in any one deposit, in addition to which the particles frequently include a wide variety of the more common minerals.

The mineral composition of the particles is a matter of considerable importance, since the composition determines quite largely the quality of the aggregate to produce durable concrete. For structures which are to be exposed to conditions in which durability is of prime importance, only those aggregates of proved resistance to the particular condition of exposure should be used.

Sands may be divided into two general classifications, calcareous or siliceous. Sand is said to be *calcareous* when calcium carbonate is present in large quantities, whereas *siliceous* sands are constituted largely of quartz or silicates. In many cases, however, sand frequently consists of a mixture of these two classifications.

Siliceous sands are generally considered best for concrete work, although sands crushed from durable rocks must frequently be resorted to in the absence of natural sands of the desired quality.

A satisfactory criterion for determining the suitability of sands predominating in coarser particles is to examine the structure as well as the strength of the original rocks
from which the sand was derived. If the strength and soundness of such original rocks is satisfactory, it may be assumed that the sand will also prove suitable for purposes of making concrete.

Quartz and silica are those minerals most commonly found in sand; however, other less permanent minerals frequently found include mica and feldspar, both of which have been more easily decomposed by the action of the elements. They are therefore usually found in a much finer state of subdivision. Mica is the more injurious of the two minerals, and even a very small percentage may be harmful.

Sandstone is an example of a natural rock from which quartz sand may be derived. The sandstone consists almost entirely of quartz grains. The usual cementing materials which determine the actual hardness as well as the strength of sandstone are iron oxide, clay, or calcium carbonate. Sandstones in which the cementing medium is iron oxide will ordinarily prove soundest. It is only when sands derived from sandstone are considered for use in concrete which may be subjected to extreme conditions of heat that careful thought should be given to their acceptance or rejection. Some sand grains tend to spall in exactly the same manner that the large pieces of stone do under exposure to extreme heat.

In those cases in which the cementing material of the sandstone grains is calcium carbonate the cementing material is readily dissolved by natural water. The material is soluble since natural water contains an appreciable amount of carbon dioxide. Simple tests are available to determine the presence of the calcium carbonate cementing material in sand. If such a test by the use of a drop of muriatic acid reveals its presence, the sand should not be used in those types of structures in which impermeability is a controlling factor, such as aqueducts or waterstorage reservoirs.

Sands, which on visual inspection may appear to be well graded in size, frequently prove to be entirely unsuited for use in concrete because of the presence of clay in the sandstone from which they were derived. Clay cementing material lacks strength to the extent that the sand is readily crushed. In addition, the clay absorbs water and results in a paste such that the sand grains are without intimate contact with the portland cement, and the resulting concrete is seriously weakened.

Other sands, instead of being composed of mineral particles, may contain particles derived from fossil materials. These are generally in combination with quartz particles and some type of cementing medium, such as in certain sandstones. Such sands should not be used, since the concrete in which such sand is utilized will neither be impervious to the passage of water nor have high crushing strength, because the softer portions of the aggregate are dissolved out by decomposing agents which reach them in the concrete mass.

Natural Sands and Gravels. Location. Natural sands and gravels are found largely in stream deposits, glacial deposits, and alluvial fans. In addition sands are found in dunes formed by wind deposit and on seashores.

Stream Deposits. These deposits are found on the streams banks and also on upper terraces. Because of the natural grading and abrading action of the stream, the deposits consist of strong well-graded material and are considered to be the most desirable.

Glacial Deposits. These aggregates are restricted to northerly latitudes or high altitudes. True glacial deposits found in hills and ridges are heterogeneous in character, containing both weak and strong materials. Unless carefully sorted, this material is not suitable for aggregates. Fluvial-glacial deposits, however, which occur in stream beds or in outwash plains downstream from glacial morains are more suitable for concrete aggregates. The glacial materials have been subjected to stream action which generally has removed the weaker materials.

Alluvial Fans. This formation is characterized by a semiconical gently sloped shape whose apex is at the mouth of a ravine. The deposits are formed by intermittent torrential floods. Sands and gravels are angular in character, poorly graded, and contain some weaker material. Usually some processing is required for satisfactory aggregate.

Wind Deposits. Sand from these deposits is predominantly quartz and thus very durable. It is, however, fine and very uniform, which makes it uneconomical for concrete unless used as a blending material to supply additional fines.
Sea Deposits. Sand from shore deposits of fresh water or salt water often is satisfactory for concrete except for the fact that it is too fine and is poorly graded. The addition of coarser particles of sand can remedy the latter deficiency. If this type of source is used, sand should be taken from the upper reaches of the beach where the salt has been washed out by the rains.

Coral beach sand has been used in Hawaii where the fine aggregate consists of approximately equal parts of coral beach sand and crusher screenings. Because of its chemical composition, coral beach sand should not be used in concrete exposed to sea water or where a high degree of impermeability is required.

Standard Sand. For many years the sand which has been accepted as standard for the purpose of making mortar tests has been the natural sand obtainable near Ottawa, Ill., and furnished by the Ottawa Silica Company. This sand passes the standard 20-mesh sieve but is retained on the 30-mesh sieve. The percentage of voids is approximately 37 per cent; the grains are well rounded and can be readily compacted.

Because of the uniform grain size of the Ottawa standard sand, the mortar strengths are somewhat lower than are obtainable with other better-graded sands using equal amounts of cement. It is common practice to prepare 1:3 mortar briquettes, utilizing commercial sand and Ottawa standard sand, each mix containing the same volume of cement. A comparison is made of the percentage of tensile strength of the standard-sand mortar developed by the commercial-sand mortar being tested, in order to determine its acceptability for use in construction projects. This test is frequently used to select the strongest sand from a number of sands which may be available in the vicinity.

Screenings. Screenings derived by crushing various natural rocks are very generally used as fine aggregates, wherever suitable and satisfactory natural sands are not obtainable. The screenings should be prepared from rock which has proved its durability. Rocks should be avoided that form long slivery particles when crushed. Long or flat particles should be limited to 15 per cent. The screenings should be well graded from fine to coarse. Excessive amounts of dust should be removed by washing or by other satisfactory means. D. A. Abrams has shown that dust, up to approximately 10 per cent, is not harmful to concrete so long as the concrete has been properly mixed and is of the right consistency. A. T. Goldbeck has further substantiated Abrams’s experimental work by demonstrating that some dust, up to approximately 6 per cent of the weight of the coarse stone aggregate, fails to have any particular effect on the properties of the resulting concrete.

Screenings are not likely to produce a concrete mix which has the same degree of workability as a mix in which natural sands of the same grading have been incorporated under conditions which are otherwise exactly similar. Because stone screening particles generally have rough surfaces and are of angular or elongated shapes, more water will be required for mixing purposes. This results in the necessity for using additional cement if concrete of a strength equivalent to that produced by using natural sands is to be obtained.

Screenings vary considerably in grading. Sieve analyses show that the amount of fine material passing a No. 100 sieve may vary from 2 to as much as 38 per cent. Great improvements in the gradings could be made by washing to reduce the dust content and by adding those sizes which the sieve analyses indicate to be lacking, or sieving out those sizes which are present in excessive quantities. Natural sands and stone screenings may frequently be mixed to produce a very satisfactory fine aggregate, provided the grading of the natural sand is such as to supply any deficiencies which have been evidenced in the stone screenings as a result of the sieve analyses.

It is desirable to give consideration to the use of concrete containing crushed-limestone screenings if the concrete is to be exposed to the percolating action of water. Because of the high solubility of the limestone particles, percolation of the water proceeds at an increasing rate with the passage of time and the fine aggregate passes off in solution, resulting in a honeycombed and extremely porous structure.

Coarse Aggregates: General

Gravel and crushed stone are the principal sources of coarse aggregate. Air-cooled blast-furnace slag, properly crushed, is also used in certain localities. The primary requirement is that the aggregates shall be clean, strong, and durable, and that there be no weak friable or laminated particles. Economy of the material and its accessibility will frequently govern the selection of the coarse aggregate for any particular structure or project.

Processed Aggregates. Cinders. Christensen, in a report prepared in 1931 for the American Concrete Institute, proposed the following specification:

Cinders, as aggregate for concrete, shall be the product of high temperature combustion of coal and/or coke—known as “industrial cinders,” “boiler cinders” or “steam cinders,” to the exclusion of the residue from domestic furnaces. The cinders shall be well burned, free from foreign matter, and so graded from coarse to fine as to produce a cinder concrete (or sand-cinder concrete) meeting the strength requirements of the building code. The cinders shall contain not more than 35 per cent of combustible content by weight, nor more than 0.45 per cent sulphur as sulphide, nor more than 1.0 per cent sulphur trioxide (SO₃) as sulphate.

The structural strength of particles of cinders is dependent upon the nature of the coal as well as the temperature of burning. The clinker formed by the fusion of the ash constituents of the coal when subjected to high temperatures is the strongest part. The fusing point of the coal (to form clinker) increases with an increase in the combined amounts of silica and alumina but decreases with the increase in content of iron oxide, lime, and magnesia. A strength requirement is generally all that is essential to govern the quality of cinder concrete, provided the raw material is in accordance with the above definition by Christensen.

Although it has been well established that corrosion of reinforcing steel has occurred in several cases where cinder concrete was used, or of pipes and conduits in cinder fill, the number of such cases is so many times outweighed by the satisfactory performance of sand-cinder concrete that they may be discounted. Perrine and Strehan, whose comprehensive report entitled Cinder Concrete Floor Construction between Steel Beams was published in 1915, concluded that “Anthracite cinder concrete (1:2:5), well mixed, cast in a viscous to wet consistency constantly stirred and mixed during placement, in such a manner as to coat the reinforcement thoroughly with mortar, will not cause the corrosion of embedded steel.”

Blast-furnace Slag. By definition “blast furnace slag is the nonmetallic product, consisting essentially of silicates and aluminosilicates of lime, which is developed simultaneously with iron in the blast furnace.”

Slags are of two classes, with particular reference to the method of handling on leaving the furnace:

1. Air-cooled blast-furnace slag, brought from the molten to the solid form entirely through the agency of atmospheric air
2. Granulated slag, changed from the molten to the solid form by means of water or other rapid-cooling agency

The greater part, by far, of the slag commercially used is that in the air-cooled form, although granulated slag is extensively used for such purposes as a bedding course under brick pavements, raw material in the manufacture of portland cement, aggregate in concrete products of various types, and backfilling.

Air-cooled blast-furnace slag, as produced for commercial purposes, is a rough angular aggregate varying in color from light to dark gray. The roughness arises from its cellular structure, the cells being formed by the gases which are enclosed in the slag at the time of the cooling process. The angularity of slag particles, combined with their rough surface texture, gives slag a larger surface area per unit of volume than is ordinarily found in other concrete aggregates.

1 Christensen, E., Cinders as Concrete Aggregate, J. ACI, February, 1931, pp. 583–646.
3 Proc. ACI, 1924, p. 524.
The apparent specific gravity of air-cooled blast-furnace slag averages about 2.25. The weight of slag concrete varies from 127 to 150 lb per cu ft depending upon the mix used and the weight of the slag aggregate itself—which may vary from 65 to 100 lb per cu ft. A weight of 135 lb per cu ft may be taken as the average for intermediate mixes of concrete when slag weighing 70 to 80 lb per cu ft (compact) is used as the coarse aggregate.

Voids, as determined by the water-displacement method, range from 40 to 45 per cent. Consequently, since absorption of water occurs in the first 30 min after contact with the water, allowance must be made for this absorption in proportioning slag concrete if the volume of mixing water is to be properly controlled.

Regarding corrosion of the reinforcing steel, the ASTM Committee on Concrete Aggregates, in 1923, stated as follows: "No restriction has been placed upon the sulphur content of slag, for the reason that inspection made by members of the Committee of reinforced slag structures in the course of demolition showed no corrosion of reinforcement that could be attributed to the slag, nor is there any published evidence that such corrosion has been observed so far as the Committee is aware." It is obvious that, if the concrete is dense and the reinforcing steel has been given proper protection by being placed at a proper depth from the surface—depending upon the degree of exposure—the steel will be unaffected.

Lightweight Aggregates

These are largely produced from a wide variety of both natural earth substances and fly ash in such a way as to obtain a material with a very high percentage of voids.

General Requirements. Because concrete made with lightweight aggregates cannot equal the strength or soundness of ordinary concrete, its use is limited to parts protected from the weather and which do not require great strength.

Workable mixes need a higher percentage of aggregate passing the No. 4 sieve than is normally required. Workability can be greatly increased with air-entraining agents. In fact, air entrainment has made the use of many lightweight aggregates practical.

Types. The following list gives the sources and a brief description of various lightweight aggregates.

- Expanded vermiculite. Type of mica expanded by heat. Product consists of soft fragile particles of flaky laminated structure
- Sintered diatomite. Made from diatomite which had soft chalky particles
- Perlite. Expanded perlite composed of frothy fragile particles of irregular shape
- Expanded slag. Irregular particles with large cell structure
- Sintered fly ash. Extremely rough and irregular in shape containing numerous black-glass particles
- Pumice. Natural material of volcanic origin. Particles have uniformly small cell structure with fairly soft surfaces of granular texture
- Expanded shale. Angular particles but not sharp or rough. Internal structure smaller with more uniformly sized cells than the slags
- Expanded slate. Well-rounded and reasonably smooth particles of laminar structure
- Expanded clay. Well-rounded and fairly smooth surface and uniformly small cell structure

Properties. In Table 1-2 are given various physical properties of lightweight aggregates as summarized by Kluge and others.¹

Gradation of Aggregate Particles: General

For economical and workable concrete mixes, the aggregates should contain many sizes. The smaller particles fill in the spaces between the larger particles that must otherwise be filled with cement paste. A combination of well-graded coarse and fine aggregates, if properly proportioned, will contain all sizes of aggregate between the largest and the smallest without containing an excessive amount of any one size. Commercially obtainable aggregates will produce satisfactory mixtures when proper combinations of fine and coarse aggregates are used.

Table 1-2. Properties of Lightweight Aggregates

<table>
<thead>
<tr>
<th>Aggregate</th>
<th>Unit wt, lb/ cu ft</th>
<th>Bulk specific gravity</th>
<th>Water absorption Wt, %</th>
<th>Crush 2-in. compaction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vermiculite</td>
<td>10</td>
<td>1.35</td>
<td>128</td>
<td>38</td>
</tr>
<tr>
<td>Diatomite</td>
<td>31</td>
<td>1.44</td>
<td>75</td>
<td>24</td>
</tr>
<tr>
<td>Perlite</td>
<td>9</td>
<td>0.86</td>
<td>153</td>
<td>14</td>
</tr>
<tr>
<td>Slag</td>
<td>30-60</td>
<td>1.13-2.26</td>
<td>5-23</td>
<td>3-10</td>
</tr>
<tr>
<td>Ash</td>
<td>42-58</td>
<td>1.74-2.10</td>
<td>8-15</td>
<td>1.5-6</td>
</tr>
<tr>
<td>Pumice</td>
<td>44-49</td>
<td>1.46-1.66</td>
<td>29-43</td>
<td>15-17</td>
</tr>
<tr>
<td>Shale</td>
<td>60-76</td>
<td>1.74-2.09</td>
<td>7</td>
<td>5</td>
</tr>
<tr>
<td>Slate</td>
<td>39-73</td>
<td>1.29-2.20</td>
<td>8-15</td>
<td>6-7</td>
</tr>
<tr>
<td>Clay</td>
<td>53-66</td>
<td>1.65-1.98</td>
<td>16-21</td>
<td>11</td>
</tr>
</tbody>
</table>

Sieve Analysis. The actual sizes or grading of any aggregate can be determined by passing the material through a set of sieves consisting usually of certain standard sizes. Sieves made of woven wire forming square openings are used in most cases, though stone sieves are also used which have plates perforated with round holes. The sieve number is defined as the number of openings per inch. The corresponding diameters of the openings are given as follows:

<table>
<thead>
<tr>
<th>Sieve size or No.</th>
<th>( \frac{3}{4} ) in.*</th>
<th>4</th>
<th>8</th>
<th>16</th>
<th>30</th>
<th>50</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diamp of opening, mm.</td>
<td>6.35</td>
<td>4.76</td>
<td>2.38</td>
<td>1.19</td>
<td>0.59</td>
<td>0.297</td>
<td>0.149</td>
</tr>
<tr>
<td>Diamp of opening, in.</td>
<td>0.250</td>
<td>0.187</td>
<td>0.0937</td>
<td>0.0469</td>
<td>0.0197</td>
<td>0.0117</td>
<td>0.0059</td>
</tr>
</tbody>
</table>

* Sieves \( \frac{3}{4} \) in. and over have openings equal to the nominal size.


The sieve analysis is also a valuable aid in checking the uniformity of aggregates to determine whether or not they meet the specifications.

Fineness Modulus. The determination of the fineness modulus of both fine and coarse aggregates furnishes an index of coarseness or fineness of aggregates but is of little use as an index of grading.

Fineness modulus may be defined as the sum of the percentages in the sieve analysis of the aggregate divided by 100, provided the sieve analysis is expressed as the cumulative percentages coarser than each of the following series of sieves: 100, 50, 30, 16, 8, 4, \( \frac{3}{4} \) in., \( \frac{1}{2} \) in., and \( \frac{1}{4} \) in. The sieve analysis shall be made in accordance with the procedure outlined in ASTM Designation C 136-39.

Workability, finishing, and economy are influenced by the fineness modulus. Other factors, however, are frequently of greater importance. Among these should be included grading, the texture of the aggregate surfaces, porosity of the aggregates, shape of particles, and character of the fracture. It is obvious, therefore, that no positive prediction regarding proper proportions can be made through a consideration of the fineness moduli alone. Table 1-3 gives a series of fineness moduli of various aggregates.

Grading Requirements. Experience has shown that maximum size and grading of aggregates are very important. Fine sands result in uneconomical mixes whereas very coarse sands result in harsh unworkable mixes. Model specifications as given by the ASTM, the Bureau of Reclamation, and the ACI are summarized in Tables 1-4, 1-5, and 1-6. The most desirable grading will depend on the type of work, richness of mix, and size of coarse aggregate. For leaner mixes or where smaller sizes of coarse aggregate are used, it is desirable to use the maximum specified percentages for each sieve size. On the other hand, for richer mixes, percentages approaching the minimum for each sieve size give better economy and strength.
Table 1-3. Sieve Analysis of Aggregates, Per Cent of Sample Coarser Than a Given Sieve

<table>
<thead>
<tr>
<th>Sieve size</th>
<th>Sand</th>
<th></th>
<th>Gravel</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Fine</td>
<td>Medium</td>
<td>Coarse</td>
<td>Fine</td>
</tr>
<tr>
<td>No. 100</td>
<td>82</td>
<td>91</td>
<td>97</td>
<td>100</td>
</tr>
<tr>
<td>No. 50</td>
<td>52</td>
<td>70</td>
<td>81</td>
<td>100</td>
</tr>
<tr>
<td>No. 30</td>
<td>20</td>
<td>46</td>
<td>63</td>
<td>100</td>
</tr>
<tr>
<td>No. 16</td>
<td>0</td>
<td>24</td>
<td>44</td>
<td>100</td>
</tr>
<tr>
<td>No. 8</td>
<td>0</td>
<td>10</td>
<td>25</td>
<td>100</td>
</tr>
<tr>
<td>No. 4</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>86</td>
</tr>
<tr>
<td>3/8 in.</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>51</td>
</tr>
<tr>
<td>1/2 in.</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>9</td>
</tr>
<tr>
<td>3/8 in.</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Fineness modulus (FM)</td>
<td>1.54</td>
<td>2.41</td>
<td>3.10</td>
<td>6.46</td>
</tr>
</tbody>
</table>

It should be noted that sand with a fineness modulus less than 1.50 is undesirable as a fine aggregate in the usual concrete mixtures. Fortunately, natural sands having such a low fineness modulus are infrequently found.

Table 1-4. Specifications for Grading Requirements for Fine Aggregate

<table>
<thead>
<tr>
<th>Sieve size</th>
<th>ASTM Designation C 33</th>
<th>Bureau of Reclamation</th>
<th>Joint Committee 1940</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>95-100</td>
<td>95-100</td>
<td>95-100</td>
</tr>
<tr>
<td>8</td>
<td>80-90</td>
<td>60-80</td>
<td>45-80</td>
</tr>
<tr>
<td>16</td>
<td>45-80</td>
<td>30-60</td>
<td>5-30</td>
</tr>
<tr>
<td>30</td>
<td>10-30*</td>
<td>12-30</td>
<td>2-12</td>
</tr>
<tr>
<td>50</td>
<td>2-10*</td>
<td>2-12</td>
<td>0-8</td>
</tr>
<tr>
<td>100</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* At the approval of the engineer, when the fine aggregate is to be used in concrete mixtures containing 5 or more sacks of cement per cu yd, the limitations on the material passing the No. 50 and No. 100 sieves may be 5 to 30 and 0 to 10 per cent, respectively.

Fine aggregate failing to pass the minimum requirements for the material passing the No. 50 and No. 100 sieve, or both, may be used, provided a satisfactory inorganic fine material is added to correct for the difference in grading.

Table 1-5. ASTM Grading Requirements for Coarse Aggregates (C 33-52T)

<table>
<thead>
<tr>
<th>Size No.</th>
<th>Nominal size (sieves with square openings)</th>
<th>Amounts finer than each laboratory sieve (square openings), % by wt</th>
<th>No. 4 (4,760 microns)</th>
<th>No. 8 (2,380 microns)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3/8 in. to 1 1/4 in.</td>
<td>4 in.</td>
<td>3/8 in.</td>
<td>1 1/4 in.</td>
</tr>
<tr>
<td>2</td>
<td>1 1/2 in.</td>
<td>4 in.</td>
<td>3/8 in.</td>
<td>1 1/4 in.</td>
</tr>
<tr>
<td>497</td>
<td>2 in. to No. 4</td>
<td>4 in.</td>
<td>3/8 in.</td>
<td>1 1/4 in.</td>
</tr>
<tr>
<td>57</td>
<td>1 1/2 in.</td>
<td>4 in.</td>
<td>3/8 in.</td>
<td>1 1/4 in.</td>
</tr>
<tr>
<td>67</td>
<td>1 1/2 in.</td>
<td>4 in.</td>
<td>3/8 in.</td>
<td>1 1/4 in.</td>
</tr>
<tr>
<td>7</td>
<td>3/8 in. to No. 4</td>
<td>4 in.</td>
<td>3/8 in.</td>
<td>1 1/4 in.</td>
</tr>
<tr>
<td>3</td>
<td>2 to 1 in.</td>
<td>4 in.</td>
<td>3/8 in.</td>
<td>1 1/4 in.</td>
</tr>
<tr>
<td>4</td>
<td>1 1/4 to 3/4 in.</td>
<td>4 in.</td>
<td>3/8 in.</td>
<td>1 1/4 in.</td>
</tr>
</tbody>
</table>
Requirements for Fines. The amount of fine aggregate passing the Nos. 50 and 100 sieves affects the workability, water gain, finishing, and surface texture of concrete. The requirements for adequate fines are more important for wet mixes and for lean mixes.

Maximum-size Aggregate. To obtain the best economy, aggregates should be graded up to the largest size that would be practical to suit the conditions of the job.

Table 1-6. ASTM Grading Requirements for Lightweight Aggregates (C 130-42)

<table>
<thead>
<tr>
<th>Size designation</th>
<th>% passing sieves having square openings</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1 in.</td>
</tr>
<tr>
<td>Fine aggregate:</td>
<td></td>
</tr>
<tr>
<td>3/4 in. to dust</td>
<td></td>
</tr>
<tr>
<td>5/8 in. to dust</td>
<td></td>
</tr>
<tr>
<td>Coarse aggregate:</td>
<td></td>
</tr>
<tr>
<td>3/8 in. to No. 4</td>
<td></td>
</tr>
<tr>
<td>3/8 in. to No. 8</td>
<td></td>
</tr>
<tr>
<td>3/8 in. to No. 4</td>
<td></td>
</tr>
</tbody>
</table>

This is evident since less paste will be required to fill the voids. Field experience and tests have also shown that the amount of water used per unit volume of concrete decreases with increase in maximum size of aggregate for a given consistency of the mix.

WATER

Functions of Water in Concrete

The functions of water when incorporated in concrete are as follows:
1. Water reacts chemically with the cement to form a cement paste in which the inert aggregates are held in suspension until the cement paste has hardened.
2. Water serves also as a vehicle or lubricant between the fine and coarse aggregate in order that the concrete may be made more readily placeable in the forms.

General Requirements

The 1956 ACI Building Code Requirements for Reinforced Concrete has the following requirement for water: "Water used in mixing concrete shall be clean, and free from injurious amounts of oils, acids, alkalis, organic materials, or other deleterious substances."

The 1940 Joint Committee adds the following further qualification regarding the quality of water: "When subjected to the mortar strength test (in accordance with ASTM Standard Method of Test for Structural Strength of Fine Aggregates Using Constant Water-Cement-Ratio Mortar, Designation: C 87-42) the strength at 28 days of mortar specimens made with the water under examination and normal Portland cement shall be at least 90 per cent of the strength of similar specimens made with the same cement and with water of known satisfactory quality."

Sources

Satisfactory water can usually be found in lakes, streams, or wells. When water from streams contains an excessive amount of suspended solids, clarification in a settling basin possibly with the aid of coagulants will be necessary. Limits on the amounts of suspended matter are usually set between 1,000 and 2,000 ppm.
Sources in dry climates frequently contain excessive amounts of dissolved salts. Unless the water has a very noticeable saline or brackish taste it may be used without further testing.

Sea water can be used under certain circumstances for mixing concrete where it is the most practical source available. Tests show that the compressive strength will be 10 to 30 per cent less than that obtained using fresh water. Comparative tests for durability are not available but there are many examples of structures that have withstood exposure to sea water for long periods. With lean mixes there is also apt to be excessive corrosion of the reinforcing steel. However, most of these adverse effects can be minimized by using richer mixes and also by using proper placing and curing techniques.

Effect of Impurities in Water

The following list obtained from tests by Abrams\(^1\) shows the adverse effects on strength of concrete of various dissolved salts:

<table>
<thead>
<tr>
<th>% of salt in solution</th>
<th>% reduction in compressive strength</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5 SO(_4)</td>
<td>4</td>
</tr>
<tr>
<td>1 SO(_4)</td>
<td>10</td>
</tr>
<tr>
<td>5 NaCl</td>
<td>30</td>
</tr>
<tr>
<td>CO(_2)</td>
<td>20</td>
</tr>
</tbody>
</table>

**ADMIXTURES**

**General**

Any material in concrete other than cement, aggregates, and water that is added at the mixer or to one of the other ingredients before mixing is called an admixture.

The rapid increase in diverse uses for concrete is bringing about more rigid and also additional requirements that cannot always be set economically by the standard materials. Where special service imposes new requirements, it is usual to require some type of simulated service testing to form a sound basis for the selection of the proper materials. These tests must be carefully designed if the results are to yield useful information on the desired properties. Most manufacturers of special admixtures usually make the tests and can furnish design data on the properties of their products. The manufacturer generally specifies exactly how his product is to be used. Incorporation of the manufacturer's specifications into the design specifications is desirable if only one particular product has the desired properties.

**Method of Evaluation**

Any admixture should be evaluated on the basis of its effectiveness in producing the desired property without the introduction of other undesirable properties. This may require data on its effect on strength, workability, watertightness, and general weathering resistance.

Usually there are several different ways of obtaining a desired property. For instance, better workability can be attained by using an admixture, an air-entraining cement, or a richer mix, or by changing the grading of the aggregates. A cost estimate will generally determine the most suitable method.

**Retarders and Accelerators**

During warm weather concrete tends to set up too rapidly, with resulting loss in workability and quality. In cold weather very slow setting retards construction. To maintain more nearly equal workability and setting of the mix certain available chemical salts may be used. These act as catalysts on the hydration of the cement.

Calcium chloride may be used in amounts up to 2 per cent of the weight of the cement to accelerate its setting. By using this material in cold weather, the length of time of protection with covers and with artificial heat can be reduced. It is not safe, however, to expose freshly poured concrete to temperatures below freezing even though calcium chloride has been added, since in the small percentages used, the freezing point is not appreciably lowered.

Tests performed by the National Bureau of Standards and also by the Illinois Highway Department on various cements and mixes with 2 per cent calcium chloride show the following average results at 40°F:

1. *Initial set* was increased by a factor of 3.
2. *Increase in strength* was found to be 300 per cent for a 1-day test and 90 per cent for a 3-day test.
3. *Workability* of the mix determined by flow tests was increased.

**Antifreeze Compounds**

Prevention of freezing of concrete is possible by the addition of sufficient amounts of certain compounds. The amount of compound necessary to lower the freezing point of water, however, also causes serious loss of strength and durability. Therefore, such antifreeze compounds should not be permitted unless their addition in the required amounts can be shown by tests to give the required properties desired in the concrete. Such tests should simulate actual service conditions.

**Air-entraining Agents**

Compounds for entraining air in the concrete are often added at the mixer instead of using a cement which contains the air-entraining agent. It is claimed that the former method allows for greater flexibility whereas the latter is somewhat more convenient. Since the amount of air entrained depends on the richness of the mix, consistency, amount of fine aggregate, type of mixer, mixing time, and weather conditions, a flexible arrangement is desirable to obtain the desired results.

Before purchasing the material, the user should check to see that it does entrain air and that none of the essential properties of concrete are impaired. As a testing guide ASTM Tentative Specification C 233–52T is recommended.

**Gas-forming Agents**

Aluminum powder (usually unpolished) reacts with hydroxides in cement to form minute bubbles of hydrogen. The amount used is between 0.005 and 0.02 per cent by weight of the cement.

The action of the gas bubbles in the concrete is similar to the bubbles caused by air-entrainment. Settlement and bleeding are reduced. Using these agents in grouts improves their effectiveness since the gas pressure causes the concrete to expand.

**Agents for Counteracting Alkali-Aggregate Reaction**

Certain pozzolans, as explained under blends using this material, are capable of reducing this reactivity. These materials contain amorphous siliceous and aluminous substances which include some opals, certain volcanic glasses, diatomaceous earth, calcined clays of the kaolin type, and calcined clays of the montmorillonite type such as bentonite. Ample protection is obtained by replacing 20 to 35 per cent of the cement by weight. Less than 15 per cent replacement is necessary when employing finely divided opaline cherts and diatomaceous clays.

The effectiveness of the pozzolan can be evaluated by measuring expansion, comparing that obtained with and without its use. Ordinary pyrex glass is used as the standard reactive aggregate. All glass passing the No. 100 sieve is removed, since this material will reduce the reactivity effects.

Retarders

These materials overcome the acceleration due to hot-weather concreting and delay stiffening in difficult placing jobs. They are sometimes applied to forms to inhibit setting of the surface layer and for grouting over long distances or where hot-water flows are encountered. Finely ground gypsum not to exceed 3 per cent by weight of the cement is commonly used for this purpose.

Waterproofing Agents

Soaps. Commercial preparations contain less than 20 per cent soap with the balance lime and calcium chloride. The quantity of soap used should not exceed 0.2 per cent by weight of the cement. The soap functions as a water repellent and should be uniformly distributed by sufficient mixing. Curing must be continuous since it cannot be effected once the cement is dried out. Soaps are used for moderate exposure to moisture. In addition this treatment improves workability.

Butyl Stearate. This is similar to soap but does not cause frothing. It is also superior to soap as a water repellent and more effective against penetration by capillary action. A suitable emulsion of stearate will be obtained using 1 per cent by weight of cement. This small amount has negligible effect on concrete strength.

Finely Divided Materials. These fall into two classes: inert and reactive. Additions of this type are effective in reducing permeability for lean mixes or when the aggregates are deficient in fines. An increase in cement content, however, is more effective in decreasing permeability than is the addition of any fine material. In massive structures, however, it is desirable to keep the cement content fairly low. Tests have shown that finely divided materials are usually not effective as dampproofers.

Oil. Heavy mineral oil (viscosity SAE 60) in amounts up to 5 per cent of the weight of cement improves dampproofing and watertightness. Care must be used in the selection of the oil, which should be free from fatty or vegetable oils and contain no petroleum residuals which emulsify with alkalies and thereby reduce strength.

Workability Agents. These are designed to improve placeability, which in turn reduces permeability. Certain organic compounds marketed under proprietary names reduce the required water for a given consistency. The sulfonated type entrains some air and reduces bleeding, while the carbohydrate salts improve workability but do not entrain air or affect bleeding. Air-entraining agents increase the plasticity of the mix and reduce bleeding. They also produce concrete which has lower permeability and capillarity, thereby rendering the concrete more waterproof and dampproof.

Miscellaneous Proprietary Materials

Marketed as water- and dampproofing agents, these cannot be classified under any of the preceding types. Based on test data these materials are not appreciably effective as dampproofing agents. These products include the following materials or combinations: (1) barium sulfate, calcium, and magnesium silicate and a fatty acid; (2) finely ground silica and naphthalene; (3) colloidal silica and a fluosilicate; (4) petroleum jelly and lime; (5) cellulose materials and wax in an ammoniacal copper solution; (6) silica, lime, and alum; (7) coal tar cut with benzene; and (8) sodium silicate together with an organic nitrogenous material.

MATERIALS FOR COLORING CONCRETE

Colored Aggregates

These are used for walls and walks to obtain special decorative effects. Natural sands, yellow and rose quartz, colored ceramics, and glasses are the principal materials
used. These should have a low porosity to resist weathering and should not react chemically with the cement.

Methods of Application. The aggregate can be applied throughout the thickness of the slab or else can be used as a facing. The latter method is generally more economical and nearly as permanent as the former. For special designs the recently developed aggregate-transfer method\(^1\) is recommended. In this method, liners in which the facing aggregates are attached by a suitable adhesive are placed inside the forms. After the concrete has been poured and has hardened, the forms and liners can be removed, leaving the facing aggregates embedded in the wall. The concrete surface is then treated to expose the aggregate. A rough finish can be obtained by wire brushing or sandblasting. Smooth surfaces are obtainable by dry or wet grinding.

Terrazzo. Terrazzo is a method of applying colored aggregates and other mineral pigments to make a specific kind of floor surface. The terrazzo mixture should consist of 1 part gray, white, or colored portland cement, as desired, to not more than 2 parts by weight of marble chips, stone chips, or abrasive aggregate, or a mixture of these of the required grading, quality, and color. The recommended method of construction is given in detail on pp. 44-46 of Recommended Practice and Standard Specifications for Concrete and Reinforced Concrete of the Joint Committee 1940.

Integral Coloring

This type of concrete coloring is obtained by grinding mineral pigments with portland cement.

Pigments recommended for use in coloring are listed as follows:

Blues: Cobalt blue or ultramarine blue
Browns: Burnt umber or brown iron oxide
Buffs: Yellow ocher or yellow iron oxide
Grays: Black iron oxide, manganese black, or Germantown lampblack
Greens: Chromium oxide or a mixture of yellow iron oxide and ultramarine blue
Reds: Red iron oxide

Various additional colors can be obtained by combining different pigments. Light shades are obtained by mixing the pigment with white cement. When clear white concrete is desired, white sand and white cement are used.

The amount of coloring materials added should not exceed 10 per cent by weight of the cement since larger amounts will lower the strength of the concrete. The correct amount is determined by trial, and the ratio of pigment to cement should be kept constant to give the same color.

Quality Control

Assurance of quality generally goes along with the reputation of the manufacturer. The following simple tests are helpful in determining the suitability of pigments.

1. **Fineness of grind.** The finer the grind the more effective the coloring agent will be. The pigment should be ground at least as fine as the cement.

2. **Lime resistance** can be tested by mixing a sample pat of 20 parts of cement and 1 part of the pigment. Any pronounced color fading in the specimens, which should be kept moist for the test period of several days, indicates that the pigment is not limeproof.

3. **Fading in sunlight** of a colored mortar exposed for 1 month indicates unsatisfactory pigment.

Portland-cement Paint

This is marketed as a powder consisting of portland cement mixed with ground mineral pigments. Water is added to the powder to form a rich creamy consistency. The mixed paint remains in a usable condition for 2 to 4 hr depending on the temperature.

\(^1\) *Color in Architectural Concrete by the Aggregate Transfer Method*, Portland Cement Association, 1950.
This type of paint may be applied to any masonry that has a clean surface having some absorption. It is not recommended for floors subject to abrasion. It is necessary to roughen smooth surfaces before applying the paint. Application should conform to the manufacturer’s directions.

Federal Specification TT-P-21 gives the limits for the composition of two types of portland-cement paint.

*Organic paints* of various types may be used where moisture does not enter the masonry from behind.

*Cement-and-oil paints* are made by grinding cement and pigments in an oil vehicle. The vehicle may be linseed oil or a mixture of linseed and other drying oils. Fine silica sand is sometimes mixed with the final coat. These paints can be applied to concrete and other masonry but are not recommended for floors.

*Oil paints* consist of a pigment base, which is usually ground white lead; a vehicle, usually linseed oil; other pigments to produce color; and a dryer. Other bases used are metallic oxides such as aluminum, zinc, or titanium, or pure powdered metals such as aluminum. Such paints made by reputable manufacturers are suitable for use on concrete provided the surface is properly prepared.

*Floor paints* are usually lead and oil paints which contain special abrasion-resisting pigments. Other suitable paints have a phenol-resin base. Some preparations contain rubber, which tends to make the paint more resistant to water and alkalis in the concrete.

*Surface Preparation for Oil Paints.* After a seasoning period of 8 to 10 weeks, oil, grease, efflorescence, and dirt should be removed from the surface. To prevent saponification of the oils in the paint, a neutralizing wash consisting of 2 to 3 lb zinc sulfate crystals per gal of water or 2 lb magnesium fluosilicate per gal of water should be used. Another procedure is to apply a solution of 3 oz zinc chloride and 5 oz of orthophosphoric acid per gal of water.

A primer or sealer coat consisting of a mixture of 1 gal oil paint, ½ gal of china wood spar varnish, and 1 qt turpentine should precede the finish coat.

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**REINFORCEMENT**

**Purposes and Types**

Reinforcement of concrete is required wherever the concrete is subject to tensile stresses. In many applications these tensile stresses are of a secondary nature since their quantitative values are not found by the usual stress analysis. It is nevertheless important to provide reinforcing for all such instances of tensile stress in concrete.

Reinforcement also serves to limit the size of cracks caused by shrinkage during setting and changes of length caused by temperature changes.

Types of reinforcement used in concrete construction consist largely of steel bars of round cross section, wire mesh, or expanded metal.

**Sizes**

According to the U.S. Department of Commerce Simplified Practical Recommendation R26-50 there are 10 standard round reinforcing bars. These recommendations as adopted by the ASTM are in Table 1-7. The No. 2 or ⅛-in. bar is always a plain smooth bar. In general all other sizes are deformed. The bar number signifies the number of ⅛-in. units in the nominal diameter of the bar. This relation is exact for bar diameters up to and including No. 8. It is only approximately true for the larger sizes since these correspond to the three large-sized square bars formerly rolled—1 in., 1½ in., and 1¾ in.

The deformations required in the bars are also given in Table 1-7. Bars meeting these requirements are produced in a variety of patterns.

Cold-drawn wire to be used as such or in fabricated form is usually limited to sizes not less than 0.080 in. or more than 0.625 in. in diameter. Wire is ordered by gage number. The gage system used for this purpose is the U.S. Steel wire gage, also known as the Washburn and Moen. Corresponding diameters in inches and other useful data are given in Table 1-8.
### Table 1-7. Dimensional Requirements for Standard Steel Bars for Concrete Reinforcement

<table>
<thead>
<tr>
<th>Bar No.*</th>
<th>Unit wt, lb/ft</th>
<th>Nominal dimensions, round sections</th>
<th>Requirements of deformations</th>
</tr>
</thead>
<tbody>
<tr>
<td>2†</td>
<td>0.167</td>
<td>0.250</td>
<td>0.05</td>
</tr>
<tr>
<td>3</td>
<td>0.376</td>
<td>0.375</td>
<td>0.11</td>
</tr>
<tr>
<td>4</td>
<td>0.668</td>
<td>0.500</td>
<td>0.20</td>
</tr>
<tr>
<td>5</td>
<td>1.043</td>
<td>0.625</td>
<td>0.31</td>
</tr>
<tr>
<td>6</td>
<td>1.502</td>
<td>0.750</td>
<td>0.44</td>
</tr>
<tr>
<td>7</td>
<td>2.044</td>
<td>0.875</td>
<td>0.60</td>
</tr>
<tr>
<td>8</td>
<td>2.670</td>
<td>1.000</td>
<td>0.79</td>
</tr>
<tr>
<td>9‡</td>
<td>3.400</td>
<td>1.270</td>
<td>1.00</td>
</tr>
<tr>
<td>10‡</td>
<td>4.303</td>
<td>1.270</td>
<td>1.27</td>
</tr>
</tbody>
</table>

* Bar numbers are based on the number of sq-in. units in the nominal diameter of the section.
† 5⁄8-in. bar in plain round only.
‡ Bars numbered 9, 10, and 11 correspond to former 1-in., 1⅛-in., and 1⅜-in. square sizes, and are equivalent to those former standard bar sizes in weights and nominal cross-sectional areas.
§ Chord of 12⅞ per cent of nominal perimeter.

### Table 1-8. Dimensional Data on Cold-drawn Wire*

<table>
<thead>
<tr>
<th>Steel wire gage Nos.</th>
<th>Diam, in.</th>
<th>Area, sq in.</th>
<th>Wt, lb/ft</th>
</tr>
</thead>
<tbody>
<tr>
<td>7/0</td>
<td>0.4900</td>
<td>0.1887</td>
<td>0.6403</td>
</tr>
<tr>
<td>6/0</td>
<td>0.4615</td>
<td>0.1683</td>
<td>0.5715</td>
</tr>
<tr>
<td>5/0</td>
<td>0.4305</td>
<td>0.1456</td>
<td>0.4943</td>
</tr>
<tr>
<td>4/0</td>
<td>0.3938</td>
<td>0.1218</td>
<td>0.4136</td>
</tr>
<tr>
<td>3/0</td>
<td>0.3625</td>
<td>0.1032</td>
<td>0.3505</td>
</tr>
<tr>
<td>00</td>
<td>0.3310</td>
<td>0.08605</td>
<td>0.2922</td>
</tr>
<tr>
<td>1</td>
<td>0.3065</td>
<td>0.07378</td>
<td>0.2506</td>
</tr>
<tr>
<td>2</td>
<td>0.2830</td>
<td>0.06290</td>
<td>0.2136</td>
</tr>
<tr>
<td>3</td>
<td>0.2625</td>
<td>0.05412</td>
<td>0.1838</td>
</tr>
<tr>
<td>4</td>
<td>0.2437</td>
<td>0.04665</td>
<td>0.1584</td>
</tr>
<tr>
<td>5</td>
<td>0.2253</td>
<td>0.03987</td>
<td>0.1354</td>
</tr>
<tr>
<td>6</td>
<td>0.2070</td>
<td>0.03305</td>
<td>0.1143</td>
</tr>
<tr>
<td>7</td>
<td>0.1920</td>
<td>0.02895</td>
<td>0.0983</td>
</tr>
<tr>
<td>8</td>
<td>0.1770</td>
<td>0.02461</td>
<td>0.0836</td>
</tr>
<tr>
<td>9</td>
<td>0.1620</td>
<td>0.02061</td>
<td>0.0700</td>
</tr>
<tr>
<td>10</td>
<td>0.1483</td>
<td>0.01727</td>
<td>0.0587</td>
</tr>
<tr>
<td>11†</td>
<td>0.1350</td>
<td>0.01431</td>
<td>0.0486</td>
</tr>
<tr>
<td>12†</td>
<td>0.1205</td>
<td>0.01140</td>
<td>0.0387</td>
</tr>
<tr>
<td>13†</td>
<td>0.1055</td>
<td>0.00874</td>
<td>0.0297</td>
</tr>
<tr>
<td>14†</td>
<td>0.0915</td>
<td>0.00658</td>
<td>0.0223</td>
</tr>
<tr>
<td>15†</td>
<td>0.0800</td>
<td>0.00503</td>
<td>0.0171</td>
</tr>
<tr>
<td>16†</td>
<td>0.0625</td>
<td>0.00307</td>
<td>0.0138</td>
</tr>
</tbody>
</table>

† Usually furnished galvanized.

### Classes of Material

Billet-steel concrete reinforcement bars are made in two classes: plain and deformed. Plain and deformed bars are of three grades: structural, intermediate, and hard. The steel from which the bars are rolled is made by one or more of the following processes: open-hearth, electric-furnace, or acid-bessemer.
Rail steel and axle steel are also extensively used for the production of concrete reinforcement bars. The ASTM standard specifications controlling the manufacture of these bars are A 16 and A 160, respectively. Rail-steel concrete reinforcement bars which are rolled from standard-section T rails are made in two classes: plain and deformed. Axle-steel bars are also made in two classes: plain and deformed, both of which are of three grades: structural, intermediate, and hard. Axle-steel bars are rolled from carbon-steel axles intended for cars and locomotive tenders in the following standard journal sizes: 4 1/2 by 8 in., 5 by 9 in., 5 1/2 by 10 in., 6 by 11 in.

Cold-drawn steel wire and welded steel-wire fabric are made from rods that have been hot-rolled from billets. They are used to a considerable extent as reinforcement in floor slabs, particularly in slabs laid on fill and as reinforcement in cinder-concrete floors, such as have been used rather extensively in the past in New York City and other Eastern cities. Such reinforcement is also used in conduits, precast-concrete pipe, retaining walls, silos, and in precast-concrete floor-slab units. The greatest use of wire and wire mesh, however, is as reinforcement in highway-pavement slabs, runways, taxiways, and aprons. Expanded metal is also specified for the reinforcement of highway pavement slabs in at least three states—Kentucky, Illinois, and Virginia.

The above types of reinforcement are relatively easy to place and appear to be well adapted to resist the formation of temperature cracks and to maintain the faces of the cracks in close contact after any cracking has occurred, because the inherent basic strength of the concrete has been exceeded.

The ASTM specification for cold-drawn steel wire is A 82 and that for welded steel-wire fabric is A 185.

Special steels having high yield strengths are used in prestressed-concrete applications. These materials are described in Sec. 4.

Bonding Characteristics of Plain and Deformed Bars

Plain bars, when used for reinforcement, must depend on the adhesion between the steel and the concrete for their effectiveness in bond. Bond is particularly critical in reinforced-concrete footings, thin reinforced-concrete slabs and relatively deep, short, simple-span reinforced-concrete beams. It is general practice in the United States to use deformed bars in all cases except in that of the 3/4 in. round, which is usually plain. This practice differs radically from that of European and South American countries where plain round bars are universally used and deformed bars are practically unknown.

Deformed bars develop the usual adhesion, which is greatly supplemented by mechanical bonds produced by the deformations. In order to be fully effective the spacing and heights of the deformations must conform to the ASTM requirements given in Table 1-7.

A rough surface on steel bars intended for use as concrete reinforcement will develop a higher bond value than a bar the surface of which has been polished. Johnson and Cox\(^1\) found that a thin film of rust on reinforcement bars improved the bond considerably, particularly at low values of slip. Consequently the presence of a film of rust should not be the cause for the rejection of the reinforcement. Rust scale, on the other hand, or other similar coatings, unless removed prior to placing of the reinforcement, will tend to reduce or destroy the bond. The 1956 ACI Building Code Requirements state: "Metal reinforcement, at the time the concrete is placed, shall be free from rust scale or other coatings that will destroy or reduce the bond."

It is standard practice to require that reinforcement bars, when coated with heavy mill or rust scale, shall be wire-brushed prior to placing in the forms; oiling or painting of reinforcement bars should never be permitted.

A special exception to this rule may be made in the case of structures exposed to sea air. Under this condition corrosion has occurred even when the concrete covering has been 4 in. thick. With low bond stresses, the reinforcement used under this condition may be painted with two thin coats of a red oxide of iron paint made up with china wood oil and spar varnish.

---

<table>
<thead>
<tr>
<th></th>
<th>Plain bars</th>
<th>Deformed bars</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Structural grade*</td>
<td>Intermediate grade*</td>
</tr>
<tr>
<td>Tensile strength, psi</td>
<td>55,000–75,000</td>
<td>70,000–90,000</td>
</tr>
<tr>
<td>Yield point, min, psi</td>
<td>33,000</td>
<td>40,000</td>
</tr>
<tr>
<td>Elongation in 8 in., min, %</td>
<td>$\frac{1,400,000}{\text{tensile strength but not less than 20%}}$</td>
<td>$\frac{1,300,000}{\text{tensile strength but not less than 16%}}$</td>
</tr>
</tbody>
</table>

* Structural and intermediate grades are produced only from billet and axle steel.
† For plain bars over 3/4 in. in diameter the tabular value shall be reduced by 0.25 per cent for each increase of 3/8 in. of the specified diameter above 3/4 in. For bars under 3/8 in. in diameter a decrease of 0.5 per cent shall be allowed for each decrease of 3/8 in. of the specified diameter below 3/4 in.
‡ For deformed bars over No. 6 bar (nominal diameter 3/4 in.), a deduction of 1.00 per cent shall be made for each increase in bar number. For No. 3 bar (nominal diameter 3/4 in.), a deduction of 1.00 per cent from the tabular value shall be allowed.
Bonding Characteristics of Welded Wire Fabric

It has been found from tests\(^1\) that the welded wire fabric has slip-resisting properties proportional to the strength of the welded transverse wires. It was recommended that "to guarantee proper anchorage, it therefore would be necessary to select the appropriate size ratio of transverse to longitudinal wire, and to specify minimum weld shear strengths."

Allowable Working Stresses for Design Purposes

The generally accepted working stresses in tension for various grades of concrete reinforcement bars, in accordance with the 1956 ACI Building Code, are as follows:

Billet-steel and axle-steel bars of structural grade............. 18,000 psi
Billet steel of intermediate and hard grades, rail steel of inter-
mediate and hard grades, axle steel of intermediate and hard
grades, and cold-drawn steel wire.................................. 20,000 psi
Wire mesh, or bars not over \(\frac{3}{8}\) in. diam when used in one-way
slabs of a span not greater than 12 ft 50% of min yield point for partic-
ular kind and grade of reinforce-
ment, but not to exceed 30,000 psi

These allowable stresses are also specified by the American Association of State Highway Officials (AASHO).

Allowable tensile stress in web reinforcement as specified by the ACI is 20,000 psi regardless of the grade of steel used, whereas the AASHO specifies only 16,000 psi.

In compression the ACI Building Code specifies 40 per cent of the minimum yield point for the particular kind and grade of reinforcement used, but in no case to exceed 30,000 psi.

Tests\(^2\) conducted on concrete beams reinforced with deformed bars of high-yield-strength steel show that the critical factor determining ultimate load is the yield strength of the steel reinforcement used. On the basis of these tests, the New York City Building Code provides that a working stress of 40 per cent of the yield point (with a maximum permissible value in tension of 24,000 psi) may be used for design purposes for all high-yield-strength steels having a yield point in excess of 50,000 psi.

The modulus of elasticity for all steel is 30,000,000 psi in both tension and compression. The coefficient of expansion of steel is approximately 0.00000065 per degree Fahrenheit.

Grade of Steel

There is still some difference of opinion among engineers regarding the grade of steel reinforcement to be used. Since approximately 1928 the grade of bar reinforcement

\(^2\) Comparative Tests of Concrete Beams Reinforced with Isteg and Hot-rolled Deformed Bars, Civil Eng. Research Labs, Columbia Univ. Rept. 2407, April, 1941.
### Table 1-11. Common Styles of Welded Wire Fabric, One-way Types*

<table>
<thead>
<tr>
<th>Style designation</th>
<th>Spacing of wires, in.</th>
<th>Size of wires, W &amp; M gage</th>
<th>Sectional area, sq in./ft</th>
<th>Wt, lb/100 sq ft</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 × 12-0/6</td>
<td>2</td>
<td>12</td>
<td>0</td>
<td>6</td>
</tr>
<tr>
<td>2 × 16-0/6</td>
<td>2</td>
<td>16</td>
<td>0</td>
<td>6</td>
</tr>
<tr>
<td>2 × 16-1/7</td>
<td>2</td>
<td>16</td>
<td>1</td>
<td>7</td>
</tr>
<tr>
<td>2 × 16-2/8</td>
<td>2</td>
<td>16</td>
<td>2</td>
<td>8</td>
</tr>
<tr>
<td>2 × 16-3/8</td>
<td>2</td>
<td>16</td>
<td>3</td>
<td>8</td>
</tr>
<tr>
<td>2 × 16-4/9</td>
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<td>9</td>
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</tr>
<tr>
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<td>2</td>
<td>16</td>
<td>6</td>
<td>10</td>
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<td>4 × 16-3/8</td>
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<tr>
<td>4 × 16-4/9</td>
<td>4</td>
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<tr>
<td>4 × 16-5/10</td>
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<td>10</td>
</tr>
<tr>
<td>4 × 16-6/10</td>
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<tr>
<td>4 × 16-7/11</td>
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<td>16</td>
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<td>11</td>
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<td>4 × 16-8/12</td>
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<td>4 × 16-9/12</td>
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<td>12</td>
</tr>
<tr>
<td>4 × 12-4/9</td>
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<td>12</td>
<td>4</td>
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<tr>
<td>4 × 12-5/7</td>
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<td>5</td>
<td>7</td>
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<td>4 × 12-6/10</td>
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<td>12</td>
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<tr>
<td>4 × 12-7/11</td>
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<td>12</td>
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<td>12</td>
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<tr>
<td>4 × 12-9/12</td>
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<tr>
<td>4 × 8-7/11</td>
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<td>11</td>
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<td>4 × 8-9/12</td>
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<td>4 × 8-10/12</td>
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<td>6 × 12-0/3</td>
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<td>3</td>
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<td>6 × 12-1/1</td>
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<td>1</td>
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<tr>
<td>6 × 12-1/4</td>
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<td>4</td>
</tr>
<tr>
<td>6 × 12-2/2</td>
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<td>12</td>
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<td>5</td>
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<tr>
<td>6 × 12-2/5</td>
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<td>12</td>
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<td>3</td>
</tr>
<tr>
<td>6 × 12-3/3</td>
<td>6</td>
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<td>4</td>
<td>4</td>
</tr>
<tr>
<td>6 × 12-4/4</td>
<td>6</td>
<td>12</td>
<td>6</td>
<td>6</td>
</tr>
</tbody>
</table>

*From the Concrete Reinforcing Steel Institute Design Handbook, p. 18, 1952.

The above styles are used mostly in building construction. Although the above styles are termed "one-way" fabrics—since in each case, the transverse wires are of minimum permissible size and have maximum permissible spacing—actually they have some transverse reinforcing effectiveness by virtue of the amount of transverse steel provided.

generally specified is intermediate grade. This applies to the design of buildings, pavements, hydraulic structures, etc. Prior to that time the structural grade was almost universally used. In certain types of structures, where it is desirable to control the development of cracks, high-yield-strength reinforcement bars should preferably be used. This ordinarily calls for the use of the hard grades of steel, in which both the yield point and the tensile strength are considerably higher than for either structural or intermediate grades of steel. The production of high-yield-strength steel is achieved by increasing the carbon content. There is, however, appreciable
# MATERIALS FOR REINFORCED CONCRETE

## Table 1-12. Common Styles of Welded Wire Fabric, Two-way Types*

<table>
<thead>
<tr>
<th>Style designation</th>
<th>Spacing of wires, in.</th>
<th>Size of wires, W &amp; M gage</th>
<th>Sectional area, sq in./ft</th>
<th>Wt, lb/100 sq ft</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 × 2-10/10</td>
<td>2</td>
<td>2</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>2 × 2-12/12†</td>
<td>2</td>
<td>2</td>
<td>12</td>
<td>12</td>
</tr>
<tr>
<td>2 × 2-14/14†</td>
<td>2</td>
<td>2</td>
<td>14</td>
<td>14</td>
</tr>
<tr>
<td>3 × 3-6/8</td>
<td>3</td>
<td>3</td>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td>3 × 3-10/10</td>
<td>3</td>
<td>3</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>3 × 3-12/12†</td>
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<td>3</td>
<td>12</td>
<td>12</td>
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<tr>
<td>3 × 3-14/14†</td>
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<td>3</td>
<td>14</td>
<td>14</td>
</tr>
<tr>
<td>4 × 4-4/4</td>
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<td>4</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>4 × 4-6/6</td>
<td>4</td>
<td>4</td>
<td>6</td>
<td>6</td>
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<tr>
<td>4 × 4-8/8</td>
<td>4</td>
<td>4</td>
<td>8</td>
<td>8</td>
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<td>4 × 4-10/10</td>
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<tr>
<td>4 × 4-12/12†</td>
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</tr>
<tr>
<td>4 × 4-13/13†</td>
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<td>13</td>
<td>13</td>
</tr>
<tr>
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<td>6</td>
<td>0</td>
<td>0</td>
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<tr>
<td>6 × 6-2/2</td>
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<tr>
<td>6 × 6-3/3</td>
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<td>6 × 6-4/4</td>
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<tr>
<td>6 × 6-5/5</td>
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<td>6 × 6-6/6</td>
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<td>6 × 6-9/9</td>
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<tr>
<td>6 × 6-10/10</td>
<td>6</td>
<td>6</td>
<td>10</td>
<td>10</td>
</tr>
</tbody>
</table>

* From the Concrete Reinforcing Steel Institute Design Handbook, p. 19, 1952.
† Usually furnished only in galvanized wire.

A two-way fabric—for a given size of longitudinal wires—is any style in which the sectional area of transverse steel is greater than the minimum required for proper fabrication by reason of the transverse wires either having a spacing which is less than the permissible maximum, or being of larger size than the permissible minimum.

Two-way fabrics are not necessarily limited to styles in which longitudinal and transverse wires both have the same size and spacing as indicated in the above table.

loss of ductility in the higher-strength material—the loss being approximately proportional to the increase in tensile strength.

## Material Requirements

### Chemical Requirements

The chemical requirements for steel reinforcing limit the maximum phosphorus content of steel as follows:

<table>
<thead>
<tr>
<th>Phosphorus, max %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Acid bessemer</td>
</tr>
<tr>
<td>Open hearth or electric furnace:</td>
</tr>
<tr>
<td>Basic</td>
</tr>
<tr>
<td>Acid</td>
</tr>
</tbody>
</table>

These requirements tend to assure the required cold working as needed for bending of steel. The specifications, however, also require minimum cold-bending ductility.

### Tensile Requirements

The tensile requirements for the various grades of bar steel are given in Table 1-9. Requirements for both billet steel and axle steel are identical. Rail steel is made only in the hard grade.

### Bending-test Requirements

These are listed in Table 1-10. The test specimens shall be bent through the required angle at room temperature around a pin of the specified diameter without cracking on the outside of the bent portion.
Accessory specifications and standard nomenclature

Wire specifications — standard bright basic wire

- Slab bolster
- Slab spacer
- Beam bolster
- Beam bolster upper
- Heavy beam bolster (HBB & HBBU) similar
- Heavy beam spacers (HBS & HBSU) similar
- Joist chair — Three standard types shown
- High chairs
- Continuous high chair

Galvanized legs can be furnished, when required, for small additional charge.

Types BC, HC, CHC are supplied with straight legs as shown but can be furnished with upturned legs if so ordered.

All types except BC, HC, CHC are supplied with upturned legs as shown but can be had on special order with straight end bearing legs. This type of leg is designated by the suffix "A", i.e., slab bolster with end bearing leg is "SBA".

Fig. 1-2. Standard accessories for beams and slabs. *(From the Concrete Reinforcing Steel Institute Design Handbook, p. 93, 1952.)*
Wire Fabric

Wire fabric is customarily made of cold-drawn steel wires which cross generally at right angles and are welded at the intersections. In general, the heavier wires run longitudinally and the transverse wires cross them at right angles. The transverse wires are usually of somewhat lighter gage. The use of wire fabric assures uniform spacing of the wires in both directions. Fabric comes onto the construction project in the form of rolls varying in length from 150 to 300 ft or in flat sheets ready to be laid in place at the desired time, as is the case of highway construction.

Two common styles of welded fabric used mostly in building construction are listed in Tables 1-11 and 1-12. The two-way fabrics shown have equal-gage wires in both directions but this is not a requirement. In fact a two-way fabric is defined as one where the sectional area of the transverse wires is greater than the minimum required for proper fabrication.

Fabricated Steel for Mats

This material has been used extensively in the construction of highway pavements and airplane runways. These mats usually come to the job in prefabricated form and are readily placed in the slab at the proper location.

Mats are constructed of two layers of steel assembled at right angles to each other. All or part of the intersections are clipped or welded together to form a rectangular

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Accessory</th>
<th>Top wire†</th>
<th>Legs†</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>SB</td>
<td>Slab bolster</td>
<td>No. 4 corrugated</td>
<td>3/4 in. high, No. 7: over 3/4 in., No. 5</td>
<td>Legs spaced 5 in. centers. Corrugations vertical or flat spaced 1 in. centers. Heights up to 2 in. Stocked in 3/4-, 1-, 1 3/4-inch heights and 5- and 10-ft lengths</td>
</tr>
<tr>
<td>SS</td>
<td>Slab spacer</td>
<td>No. 5 smooth</td>
<td>Same as SB</td>
<td>Legs spaced to provide supporting leg under each bar. Minimum leg spacing 4 in. Heights up to 2 in. Fabricated to order All legs spaced 2 in. centers. Max height 3 in. Stocked in 1-, 1 3/4-, 2-inch heights, in 5-ft lengths</td>
</tr>
<tr>
<td>BB</td>
<td>Beam bolster</td>
<td>No. 7 smooth</td>
<td>No. 7</td>
<td>Same as SS except maximum height 5 in.</td>
</tr>
<tr>
<td>HBB</td>
<td>Heavy beam</td>
<td>No. 4 smooth</td>
<td>No. 4</td>
<td>All legs spaced 2 in. centers. Max height 3 in. Stocked in 1-, 1 3/4-, 2-inch heights, in 5-ft lengths</td>
</tr>
<tr>
<td>BBU</td>
<td>Boiler</td>
<td>No. 7 smooth</td>
<td>No. 7</td>
<td>Same as BBU except max height 5 in. Fabricated to order</td>
</tr>
<tr>
<td>HBBU</td>
<td>Heavy beam</td>
<td>No. 4 smooth</td>
<td>No. 4</td>
<td>Fabricated to order for desired bar spacing and beam width. Max height 3 in. Same as BS except max height 5 in.</td>
</tr>
<tr>
<td>BS</td>
<td>Beam spacer</td>
<td>No. 7 smooth</td>
<td>No. 7</td>
<td>Fabricated to order for desired bar spacing and beam width. Max height 3 in. Same as BS except max height 5 in.</td>
</tr>
<tr>
<td>B</td>
<td>Heavy beam</td>
<td>No. 4 smooth</td>
<td>No. 4</td>
<td>Made and stocked only in 4-, 5-, 6-inch widths and 3/4-, 1-, 1 3/4-inch heights. For heights over 2 to 6 in. Stocked in 5/8-, 1-, 1 3/4-, and 2-inch heights.</td>
</tr>
<tr>
<td>HC</td>
<td>Individual</td>
<td>No. 5</td>
<td>No. 5</td>
<td>Made and stocked only in 4-, 5-, 6-inch widths and 3/4-, 1-, 1 3/4-inch heights. For heights over 6 in. use No. 0 wire. Stocked in 5/8-inch increments from 3/4- to 6-in.</td>
</tr>
<tr>
<td>CHC</td>
<td>Continuous</td>
<td>No. 0</td>
<td>No. 0</td>
<td>Made and stocked only in 4-, 5-, 6-inch widths and 3/4-, 1-, 1 3/4-inch heights. For heights over 6 in. use No. 0 wire. Stocked in 5/8-inch increments from 3/4- to 6-in.</td>
</tr>
</tbody>
</table>

* From the Concrete Reinforcing Steel Institute Design Handbook, p. 92, 1952.
† W & M wire gages indicated in this table are the minimum sizes to be used.
grid. The purpose of these fastenings is to maintain the proper bar spacing during all operations.

Bars used for mats are selected from standard sizes as given in Table 1-7. Materials used should conform to the specifications for billet steel, axle steel, or rail steel. For welded mats only structural grades of billet or axle steel should be specified since the harder steels are liable to crack unless special treatment is given to the welds.

Lengths of mats are usually limited to 16 ft and widths to 9 ft 8 in.

Each connection must be capable of withstanding a static load of 75 lb exerted in the direction of either bar or 150 lb applied perpendicular to the bar.

Accessories

Standard accessories for beams and slabs are listed according to the Concrete Reinforcing Steel Institute in Fig. 1-2 and Table 1-13.

MISCELLANEOUS MATERIALS

Floor Hardeners

These materials can sometimes prevent or reduce dusting on floors where proper construction methods were not carried out. However, these materials cannot overcome all the defects caused by poor materials or workmanship. The following materials and a number of proprietary hardeners are used for this purpose: magnesium fluosilicate, zinc fluosilicate, sodium silicate, aluminum sulfate, zinc sulfate, and various oils, gums, resins, and paraffins.

The floors should be clean and dry before the hardener is applied. All oil, paint, or other foreign matter should be removed.

Fluosilicate Treatment. Fluosilicates of zinc and magnesium dissolved in water make a good hardener. The best results have been obtained with a mixture of 20 per cent zinc and 80 per cent magnesium. This mixture is dissolved in the proportions of \( \frac{1}{2} \text{ lb of fluosilicate per gal of water} \) for the first application and 2 lb per gal for subsequent applications. The solution can be sprinkled on and then mopped to give an even distribution. Two or more coats should be used, allowing each coat to dry thoroughly before the next one is applied.

Sodium Silicate Treatment. Commercial sodium silicate obtainable as a 40 per cent water solution should be further diluted using 3 parts of water to 1 part of solution. Two or more coats should be applied. To aid penetration and bonding, each coat should be scrubbed with water before the next one is applied.

Painting is described under Portland-cement Paint. When painting over sodium silicate, the coating should be scrubbed thoroughly with hot water. In heavily traveled areas any paint will have to be renewed at frequent intervals.

Grouting Mortars

These must fill all the space in the cavity with inappreciable shrinkage. In addition the mortar must be fluid enough to penetrate into small cracks and openings. Settlement with its attendant bleeding and surface water must be eliminated.

These requirements are met by using a fluid mix in which a finely divided siliceous material is included as a filler. By prolonging the mixing time, once the mortar is placed, the set is rapid enough to prevent excessive settling. Aluminum powder is also used to prevent segregation and bleeding. The gas generated also aids in forcing the grout into small cracks. High-silica cement is used to prevent shrinkage. Tests should be performed to determine the proper proportions to obtain the desired results.

Joint Fillers

Made of compressible materials, these are used for contraction joints, for seals, and for water stops.
Preformed Strip Materials. These are favored by many engineers and contractors because of the speed of placing and lower labor costs. The materials used, advantages, and uses are listed in Table 1-14. ASTM requirements are given in Specification D 544-49, 52T, Preformed Expansion Joint Fillers for Concrete. AASHO specifications are similar to those of ASTM. All materials must maintain their shape during handling. Bituminous cellular materials, particularly, must not soften appreciably during hot weather or become hard and brittle in cold weather. Cork materials must also show no signs of disintegration when boiled for 1 hr in hydrochloric acid. Bituminous cellular types should have a limited moisture absorption of 15 to 18 per cent corresponding to thicknesses of 1 to $\frac{3}{4}$ in. Weathering tests consisting of 10 alternations of freezing and thawing may be required if the service conditions for the material warrant it.

<table>
<thead>
<tr>
<th>Material</th>
<th>Cork</th>
<th>Sponge rubber</th>
<th>Self-expanding cork</th>
<th>Bituminous cellular</th>
<th>Asphalt</th>
</tr>
</thead>
<tbody>
<tr>
<td>ASTM type</td>
<td>I Granulated cork bonded with synthetic resin</td>
<td>I Durable elastic rubber compound reinforced on each side with a layer of asphalt-treated felt</td>
<td>II Cork treated so it will expand in the presence of moisture</td>
<td>III Composition of fibrous material and/or cork bonded together with asphalt</td>
<td>Composition of asphalt and filler formed between asphalt-impregnated paper</td>
</tr>
<tr>
<td>Description</td>
<td>Highly resilient, light in color</td>
<td>Fully resilient, nonexuding, can match any color</td>
<td>Will keep joint filled under construction, which opens space to more than original size</td>
<td>Least expensive nonexuding type. Low moisture absorption</td>
<td>Compressible cushion, low in cost</td>
</tr>
<tr>
<td>Uses</td>
<td>Flood walls, outlet works and spillways, water- and sewerage-treatment plants</td>
<td>Bridges and viaducts</td>
<td>Canal linings, outlet works, spillways, stilling basins, water- and sewerage-treatment plants</td>
<td>General-purpose, particularly when a waterproof sealer will be added</td>
<td>Formation of contraction joints in construction where a $\frac{3}{4}$-, $\frac{3}{8}$-, or $\frac{1}{6}$-in. thickness is used</td>
</tr>
<tr>
<td>Properties:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Recovery, %</td>
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<td>90</td>
<td>90</td>
<td>70</td>
<td></td>
</tr>
<tr>
<td>Compression, psi</td>
<td>100-1,500</td>
<td>100-1,500</td>
<td>100-1,500</td>
<td>100-1,500</td>
<td></td>
</tr>
<tr>
<td>Extrusion, in.</td>
<td>(max) 0.25</td>
<td>0.25</td>
<td>0.25</td>
<td>0.25</td>
<td></td>
</tr>
<tr>
<td>Expansion, %</td>
<td>140</td>
<td></td>
<td></td>
<td>100-1,500</td>
<td></td>
</tr>
</tbody>
</table>

* Specimen $4\frac{1}{4}$ X $4\frac{1}{2}$ in. is compressed to 50 per cent of its thickness three times and the amount of recovery is measured at the end of 1 hr after the third application.
† Load required to compress specimen to 50 per cent of its original thickness.
‡ Does not apply to thicknesses less than $\frac{3}{4}$ in.
§ Specimen compressed to 50 per cent of its original thickness with three of the four edges restrained.
¶ Specimen immersed in boiling water for 1 hr.

Sealing Materials. Rubberized-asphalt joint-sealing compounds are used on top of Type III premolded fillers to keep dirt and water out of the contraction joints. This type of compound is resilient and maintains its bond at temperatures down to 0°F.

Water Stops. Molded rubber is extensively used for water stops in the construction of water- and sewerage-treatment plants, dams, and other types where considerable hydrostatic pressure must be resisted. The following are typical material requirements:

- Hardness (Shore A durometer) .......... 60-70
- Minimum elongation .......... 400 %
- Minimum tensile strength .......... 2,500 psi
- Minimum water absorption .......... 5 % after 2 days' immersion at 158°F
- Tensile strength after aging 7 days at 158°F .......... Not less than 80 % of the original tensile strength
- Maximum compression set .......... 30 % after 22 hr at 158°F
Mastic stop material of excellent stability made of asphalt, resins, and plasticizing compounds is available for use in all types of pressure joints. This material will not soften appreciably up to temperatures of 180°F and also maintains ductility to \(-10^\circ\)F.

Metals used for water stops in their order of durability are copper, zinc, and aluminum. Wrought-iron plate is sometimes employed to protect the joint from heavy loads. To resist corrosion in concrete, aluminum must be coated with tar or varnish. Sizes of copper sheets commonly used are primarily 16 and 20 oz and occasionally 24 oz. Where additional stiffness in the material is required, corrugated sheet is used.

Membrane Waterproofings

These consist of two or more layers of bitumen-treated cotton fabric or felt, or combinations of both materials cemented together by means of bituminous coatings. Typical specifications are as follows:

The **bitumen** shall consist of asphalt or coal-tar pitch. **Asphalt**, when intended for use as a saturant or for mopping above ground, shall be the product attained by the distillation of crude asphaltic-base petroleum with agitation, supplemented, if necessary, with oxidation by air but without the addition of any fluxing material other than a native asphalt or a straight steam-refined asphaltic residual.

Asphalt for mopping, **below ground**, shall be either: (1) a native refined asphalt fluxed with a petroleum flux, this flux to have a penetration at 77°F of not less than 350 under a weight of 50 grams for 1 sec, or (2) a petroleum asphalt obtained as a residue of an asphaltic petroleum by straight steam refining without oxidation. It shall also be free from admixtures of blown or oxidized residuals, cracked residuals, sludge asphalt, or their derivatives.

The **fabric** shall consist of high-grade cotton cloth, saturated thoroughly and uniformly with asphalt or coal-tar pitch. The dry cotton fabric shall be treated at such a temperature and speed as will not injure the fabric and shall be accomplished by spraying the fabric with saturant or by passing the fabric through saturant and then calendering it in the presence of heat.

**Felt** shall be rag-felt saturated, but not coated, with either asphalt or refined coal tar; or asbestos felt saturated, but not coated, with asphalt. Saturation shall be accomplished by passing the dry felt in single thickness through the saturant at such a temperature and speed as will not injure the felt, and then calendering between cylinders, then cooling and winding into rolls.

**Types of Membranes.** Membranes shall consist of one of the following types:

3. Two layers of bitumen-treated felt, one middle layer of bitumen-treated cotton fabric, and four mopings of bitumen.
4. Four layers of bitumen-treated felt, one middle layer of bitumen-treated cotton fabric, and six mopings of bitumen.

Membrane waterproofing shall not be applied in wet weather or when the atmospheric temperature is below 50°F. Surfaces to be waterproofed shall be dry and clean, and concrete surfaces shall be well cured before any waterproofing is applied. Any projections that might injure the membrane shall be removed. Any surfaces of concrete or steel that will be in direct contact with asphalt waterproofing shall be given one coat of asphaltic primer prior to the first mopping of asphalt; or one coat of creosote primer before the first mopping of coal-tar pitch. The priming coat shall be applied approximately 24 hr before application of the waterproofing membrane. For detailed specifications for the properties of the various materials and directions for application, consult the Specifications for Membrane Waterproofing (1942) of the American Railway Engineering Association.

Corrosion-resisting Materials

These are available to protect concrete against almost any chemical agent. Common materials used for this purpose are listed\(^1\) below in order of increasing effective-

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\(^1\) For more detailed information see Effect of Various Substances on Concrete and Protective Treatments, Where Required, ST 4, Portland Cement Association, 1949.
ness. Where the structure is expected to serve over a long period of time greater economy may be obtained by choosing a more permanent treatment even though the first cost is somewhat higher. Protective coatings require dust-free surface and dry clean concrete for satisfactory application.

**Protective Treatments**

Coatings
- Magnesium or zinc fluorosilicate
- Sodium silicate
- Drying oils
- Synthetic resins
- Paints and varnishes
- Bituminous paints
- Bituminous enamels

Thick coverings
- Bituminous mastics
- Vitrified brick or tile
- Glass
- Lead
- Rubber and synthetic-resin sheets

**Stain Removal**

Materials and treatments for removing concrete stains due to known causes can be found in Removing Stains from Concrete, ST 19, Portland Cement Association. Most stains are removed with some difficulty by using strong bleaching agents.

**PROPORTIONING OF CONCRETE**

**Desired Properties**

Desired properties of concrete in the hardened state include compression strength, watertightness, durability, volume stability, and abrasion resistance.

Since concrete is a structural material it is necessary that it have sufficient strength to resist the applied loads and that the strength be subject to control. With practical methods of control there will always be a certain amount of variability in the strength of the concrete. By keeping the design stresses low enough, however, this variability can be properly accounted for.

Strength is also used as a quick method of evaluating other properties of concrete and is generally used as the measure of quality. Actually strength is not always a true indicator of other properties such as durability. However, when all other variables are fixed there is usually a good correlation between strength and other desired properties.

Watertightness is desired in the exteriors of buildings and foundations to prevent the passage of moisture from the exterior to the interior. Similarly in the case of conduits and containers such as tanks and reservoirs it is desired to prevent leakage. Penetration of water into the structure is also undesirable because of its general bad effect on durability.

Durability determines the life of the structure and the cost of maintenance and repairs. It is therefore important for low costs to have good durability, which would include resistance to freezing and thawing, to erosion, and to deterioration by chemical agents.

Volume stability is desired in order to minimize the formation of both small and large cracks in the concrete. Most cracking is caused by excessive volume changes which occur during setting, during temperature changes, and during moisture changes.

Abrasion resistance is a necessary requirement for floors and pavements subjected to heavy traffic.

**Effects of Materials on Properties**

The qualities of the component materials in the mix have important effects concerning both the mix and the hardened concrete. These effects are discussed separately under each of the materials. The aggregates serve as the inert principal constituent of the concrete. The cement paste consisting of cement and water gives the mix a mobility which allows water placement of the concrete and binds the aggregate together to form an integral mass. The water is necessary for the hydration of the
cement and also for mobility of the mix. Air entrained as small discrete bubbles distributed throughout the mix aids the mobility and also influences the properties of hardened concrete.

Of these materials the aggregates are usually the only ones that have a wide variation since portland cement is rigidly controlled by the manufacturers.

Workability is a prime factor which influences the desired properties of concrete. It is therefore very important to consider it in the design of a concrete mix and to employ means of control that will ensure its attainment on the job.

Workability is defined as the ease with which a given set of materials can be mixed into concrete and subsequently handled, transported, and placed with minimum loss of homogeneity. Workability is not an absolute property but varies with the size of the structure, reinforcement spacing, method of placement, temperature, etc. What would be considered workable for pavements, for example, would not be so for retaining walls.

Workability is measured by either the slump test or the flow-table test. Of the two methods the slump test is most generally used and accepted because of its simplicity. This test is made in accordance with ASTM Designation C 143, The Standard Method of Slump Test for Consistency of Portland Cement Concrete.

Methods of Proportioning the Mix

The Water-Cement Ratio. This is the basis of all rational methods of proportioning concrete. This ratio is defined as the volume of mixing water to the volume of cement used in any given concrete mixture and is usually expressed in terms of U.S. gallons of water per 94 lb sack of cement. Sometimes the ratio is expressed on a weight basis as a dimensionless quantity. The water is reckoned on the basis of surface-dry aggregates.

The water-cement ratio law has been well established by D. A. Abrams, who conducted approximately 50,000 tests on concrete and mortar. Later tests by numerous other investigators have confirmed Abrams's results and further established the fundamental influence of the volume of mixing water on the strength of the concrete. Briefly stated the law is as follows:

For plastic mixtures using sound aggregates, strength and other desirable properties of concrete under given job conditions are governed by the net quantity of mixing water used per sack of cement.

This law of strength also verifies the fact that the cement-water paste is the major active ingredient in the mix and that the quality of the concrete in so far as proportioning is concerned is determined by the quality of the paste. As additional water is added to the cement, the paste is diluted and its strength is thereby decreased. Actually only a relatively small percentage of the water is necessary for the hydration of the cement. Additional water, though it decreases strength, is necessary to provide the proper workability.

Effect of Air Entrainment on Proportioning. Entrained air permits a reduction in the mixing water with no loss of workability. It has been found that strength is proportional to the ratio of air plus water to cement by absolute volume, which is an extension of the water-cement ratio law to include air.

Size and Grading of the Aggregates. For the usual run of aggregates, those with the larger sizes of coarse aggregate will have fewer voids and thus will require a smaller amount of cement paste. By introducing different sizes of aggregate the volume of voids can be reduced and better economy obtained. Too many fines, however, require additional paste to coat the larger surface area and result in poorer economy.

Quantity of Cement Paste. It has been found that the actual quantity of cement paste required per unit volume of concrete is dependent upon the following factors, which are listed in order of usual importance:


2 Design of Concrete Mixtures, Structural Materials Research Lab., Lewis Inst., Bull. 1, 1918.

1. The consistency required for proper placement
2. The water-cement ratio of the paste
3. The gradation of the aggregates
4. The shape and surface texture of the aggregate particles

**Trial-batch Method.** This method is declared to be the best procedure by the American Concrete Institute in their standard, Recommended Practice for the Design of Concrete Mixes (ACI 613). It is a perfectly rational method based on the water-cement ratio law, desired workability, and economy of materials. The following variables should be taken into account in the proper selection of the quantities:

1. Water-cement ratio
2. Slump
3. Air entrainment
4. Maximum size of aggregate
5. Gradation of the aggregate (expressed by the fineness modulus)
6. Type of aggregate (round or angular)

**Arbitrary Proportions by Volumes.** One of the first methods used to proportion concrete mixes in the early days of American concrete practice was that generally known as *proportioning by arbitrary volumes*. Using the cubic foot of cement (1 sack of 94 lb) as the basic unit, the desired mixes were described as 1:2:4 or 1:2:3 1/2 or 1:3:5. In the first case the mix would consist of 1 part portland cement, 2 parts fine aggregate, and 4 parts coarse aggregate, all by volume.

Use of this method also tends to the use of excessive amounts of water in order to overcome any harshness resulting from the selection of poorly graded aggregates or of too small a proportion of fine to coarse aggregates.

The ACI Building Regulations for Reinforced Concrete (ACI 318) states that: “The methods of measuring concrete materials shall be such that the proportions can be accurately controlled and easily checked at any time during the work.” A further statement is also added: “Wherever practicable such measurement shall be by weight rather than by volume.”

**PROPERTIES OF CONCRETE**

**Strength**

The strength of concrete is usually measured by its resistance to axial compression since this type of force is present in all concrete structures. Tests are performed on cylinders made in conformance to ASTM Designation C 31 and loaded according to ASTM Designation C 39.

**Effects on Strength. Materials.** The cement and aggregate used have significant influence on strength properties. Tests on 14 different brands of portland cement mixed in the same proportions indicated a variation in strength of 29 per cent of the average.¹

The use of unsound aggregates will produce large variations in the strength of concrete. In general, however, a good aggregate will develop the full strength of the cementing matrix and therefore should cause little variation in the product. Strength is increased slightly with larger-size coarse aggregates.

**Product Variability.** Variations in the materials and construction practice contribute to an over-all variability.

H. C. Ross² of the Hydro-Electric Power Commission of Ontario, Canada, in reporting on an analysis of the 28-day test results from 60 different concreting operations, involving approximately 13,000 test specimens, concludes that low-strength concretes are subject to greater percentage variations in compressive strength than are high-strength concretes. He suggests, therefore, that a greater margin of strength should


² Ross, H. C., Uniformity of Concrete on the Average Job—A Study of 13,000 Field Tests, *J. ACI*, vol. 32, pp. 277-284, 1936.
Table 1-15. Percentage of Tests Falling within Various Percentage Limits of Average Strength

<table>
<thead>
<tr>
<th>Limits of variations</th>
<th>2,000-lb concrete</th>
<th>3,000-lb concrete</th>
<th>4,000-lb concrete</th>
<th>5,000-lb concrete</th>
</tr>
</thead>
<tbody>
<tr>
<td>± 5</td>
<td>24.5</td>
<td>28.5</td>
<td>32.0</td>
<td>59.5</td>
</tr>
<tr>
<td>± 10</td>
<td>45.5</td>
<td>55.5</td>
<td>57.5</td>
<td>76.0</td>
</tr>
<tr>
<td>± 15</td>
<td>61.5</td>
<td>72.5</td>
<td>74.5</td>
<td>87.5</td>
</tr>
<tr>
<td>± 20</td>
<td>74.5</td>
<td>84.0</td>
<td>87.0</td>
<td>87.5</td>
</tr>
<tr>
<td>± 25</td>
<td>83.5</td>
<td>90.5</td>
<td>95.0</td>
<td></td>
</tr>
</tbody>
</table>

be provided in their design in order to ensure an equal guarantee of quality. Ross further comments that the 15 per cent overdesign called for in the ACI Building Code and in city building codes appears adequate for the average job when 3,000- and 4,000-lb concrete is specified. In order to give equal assurance of quality for 2,000-lb concrete, however, he recommends a 20 per cent overdesign. Table 1-15 illustrates the variability that was found. It will be noted that the lower-strength concrete shows greater variability than the higher-strength.

**Water-Cement Ratio.** The chief factor affecting strength is the ratio of the amount of water to the amount of cement used. This factor is expressed as U.S. gallons per sack of cement, and also as pounds of water per pound of cement. Figure 1-3 shows the relation between compressive strength and water-cement ratio for the same curing conditions. From the figure it is seen that the strength is roughly proportional to the water-cement ratio.

**Curing.** Curing is a very important factor in determining the strength of concrete, which depends on the degree of completion of the hydration process. Curing is made up of three parts: moisture, time, and temperature. As long as moisture is present the strength increases with time. The rate of increase, however, decreases with time as the hydration process approaches completion. It has been found that cylinders after being allowed to dry out give higher strengths than the same cylinders tested wet. This fact is explained by the strengthening effect of the capillary pore water which exists in the dried cylinders.

These effects are shown graphically in Fig. 1-4 according to H. J. Gilkey. It is seen that there is no further increase in strength after the concrete has dried out. If, however, the dry concrete is again restored to a moist condition strength again increases following a curve approximately parallel to S.

Harmful temperature effects occur at both low and high temperatures. Table 1-16 shows the effect of freezing on the compressive strength of cylinders reported by D. C. McNeese.\(^1\) It was found that the length of time frozen had no effect on the concrete.

\(^1\) McNeese, D. C., Early Freezing of Non-air-entraining Concrete, *J. ACI*, vol. 24, p. 293 1952.
Table 1-16. Strength-reduction Factors for Early Freezing of Non-air-entraining Concrete

<table>
<thead>
<tr>
<th>Initial temp, °F</th>
<th>Freezing temp, °F</th>
<th>Time after which freezing was applied, hr</th>
<th>Reduction in compressive strength, %</th>
</tr>
</thead>
<tbody>
<tr>
<td>72</td>
<td>15</td>
<td>0</td>
<td>40</td>
</tr>
<tr>
<td>72</td>
<td>15</td>
<td>6</td>
<td>0</td>
</tr>
<tr>
<td>72</td>
<td>5</td>
<td>0</td>
<td>50</td>
</tr>
<tr>
<td>72</td>
<td>5</td>
<td>6</td>
<td>15</td>
</tr>
<tr>
<td>40</td>
<td>25</td>
<td>0</td>
<td>50</td>
</tr>
<tr>
<td>40</td>
<td>−15</td>
<td>0</td>
<td>45</td>
</tr>
</tbody>
</table>

The curing rate is changed by temperature. At low temperatures the hydration process slows down and at high temperatures it speeds up. These effects are illustrated in Fig. 1-5.

At temperatures above 100°F the strengths will decrease. This effect occurs primarily at early ages of curing. The cause of this action is probably associated with a too rapid crystal growth and the resulting effect upon volume change. For steam-cured concrete Saul states: ¹ "Concrete which is not raised in temperature too rapidly after mixing is shown to gain strength during and after treatment in relation to its maturity (reckoned as the product of temperature and time) approximately in accordance with the same law as does normally cured concrete."

¹ Saul, A. G. A., Principles Underlying the Steam Curing of Concrete at Atmospheric Pressure, *Cement and Concrete Assoc. Research Note Rept. 3*, 1950.
**Duration of Loads.** The duration of loads on concrete can vary from impact conditions where the time of loading is very short to dead-load conditions where the time of loading is very long. On the other hand, standard compression tests performed in the laboratory are loaded for a fairly short duration, considerably longer, however, than during impact. Tests show that the long-time strength is about 70 per cent of that obtained in the standard test. Other tests conducted at the National Bureau of Standards show that the strengths obtained for rates of loading comparable with those encountered on usual structures are the same as obtained from the standard test.

**Effect of Admixtures.** Admixtures do not alter the basic water-cement ratio law governing strength. However, certain admixtures, notably air-entraining agents, allow a lower water-cement ratio for a given workability and may result in a somewhat higher strength. For a given water-cement ratio, however, many admixtures tend to lower strength although in the amounts recommended by the manufacturer the decrease will in most cases be small.

![Graphs showing the influence of temperature on compressive strength of concrete at different ages.](image)

**Fig. 1-5.** Influence of temperature on compressive strength of concrete at different ages: 4- by 8-in. concrete cylinders; mix 1:2.75:3.33 by weight for normal cement and 1:2.52:3.03 by weight for high-cement-portland cement; net water content 6 gal per sack of cement. Specimens soaked 1 to 3 hr before testing. *(From H. F. Moore and M. B. Moore, Textbook of the Materials of Engineering, 8th ed., McGraw-Hill Book Company, Inc., New York, 1953.)*

Calcium chloride used to accelerate the setting is limited to 2 per cent of the weight of the cement. It has no ultimate effect on strength but there is a rapid increase in strength at low curing temperatures as compared with concrete without calcium chloride, as explained under the heading Retarders and Accelerators.

Air-entraining agents lower the strength of concrete for a given water-cement ratio in the amount of 4 to 6 per cent for each additional per cent of entrained air. However, for a given workability the amount of water required in the mix in per cent is reduced by two to four times the percentage of entrained air.

**Casting Methods.** These affect strength in their influence upon the amount of water retained after the forming operation. It is well known, for instance, that cen-

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MATERIALS FOR REINFORCED CONCRETE

trifugally cast concrete pipe has very high compressive strength. Seaman shows very significant differences in strength between centrifuged and poured concrete cylinders. The "vacuum-mat" method will also tend to give higher strengths in proportion to the amount of water drawn off by the mats.

**Tensile Strength.** Concrete is seldom required to resist known loads which cause tension. More often it is subject to flexural stresses as in the cases of pavements and plain concrete footings. Tensile stresses are commonly set up by shrinkage in restrained structures. Widespread cracking due to these shrinkage effects is limited by the reinforcement and also by the tensile strength of the concrete.

Tensile and flexural strengths are affected by the same factors as compressive strength. The basic laws governing these strengths are the water-cement ratio and temperature-time curing. Table 1-17 compares tensile and flexural strengths with compressive strengths.

<table>
<thead>
<tr>
<th>Strength of plain concrete, psi</th>
<th>Ratio, %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compressive</td>
<td>Modulus of rupture</td>
</tr>
<tr>
<td>1,000</td>
<td>230</td>
</tr>
<tr>
<td>2,000</td>
<td>375</td>
</tr>
<tr>
<td>3,000</td>
<td>485</td>
</tr>
<tr>
<td>4,000</td>
<td>580</td>
</tr>
<tr>
<td>5,000</td>
<td>675</td>
</tr>
<tr>
<td>6,000</td>
<td>765</td>
</tr>
<tr>
<td>7,000</td>
<td>855</td>
</tr>
<tr>
<td>8,000</td>
<td>930</td>
</tr>
<tr>
<td>9,000</td>
<td>1,010</td>
</tr>
</tbody>
</table>


Allowable stresses specified for plain concrete footings in flexural tension in the ACI Building Code (Standard 318) are 0.03 of the compression strength, which is in general agreement with Table 1-17 and is justly conservative for this type of stress.

**Shearing Strength.** The shearing strength is generally not a critical property in structural design since the material is much weaker in tension and generally fails in tension even though the apparent structural action is shear. There are, however, certain applications where shear failures take place because tension stresses are small or do not exist.

Most compression tests on cylinders fail in shear on roughly a conical surface whose elements are inclined 45° to the cylinder axis. The failure stress on this surface, which represents the maximum shear, is one-half the compressive stress on the circular cross section. Tests also show that the strength in shear is approximately one-half the strength in compression.

**Bond Strengths.** The bond strengths depend on the type of bar used. Test data for deformed bars conforming to ASTM Specification A 305 are shown in Fig. 1-6. Bond strengths were found to be sensitive to the position of the bars in the casting block. Bars cast in the bottom of the block gave strengths considerably in excess of those cast in the top of the block. Plain bars will develop between one-sixth and one-third of the strength of standard deformed bars.


The results of these tests have been incorporated into the ACI Building Code (Standard 318). Specified allowable bond stresses are proportional to the ultimate compressive strength and top-cast bars are assigned lower allowable bond stresses than bottom-cast bars.

**Combined Stress.** In various structures such as two-way slabs, beams subjected to torsion, and arch dams, the concrete is stressed in two directions. In the usual state of biaxial compression the strength of the concrete is practically the same as that obtained from the standard compression test. It has been found, however, that lateral pressure has a definite strengthening effect on axially compression-loaded cylinders.¹

**Behavior under Repeated Loads.** The strength of concrete under repeated stresses which vary from zero to a maximum is only about 50 to 60 per cent of its ultimate static strength. Recent tests by C. W. Muhlenbruch² also show that the bond strength under zero to maximum repeated load cycles is 50 per cent of the static bond stress. It can be expected that the strength reduction for repeated shear stresses is also 50 per cent.

The above-stated values were arrived at by continuous repeated load testing in the laboratory until failure occurred. In actual service rest periods of considerable length usually follow repetitions of a few heavy loads. During these rest periods the concrete heals itself and the damaging effects of the repeated load are greatly reduced. However, for structures where more than 5,000 load repetitions are expected without rest periods of considerable length, the strength of the concrete should be considered as only 50 per cent of the static strength.

**Wear Resistance.** This is an important consideration in the design of surfaces subjected to moving loads. Because the aggregates are harder and tougher than the cement these are subject to most of the wearing action. Aggregates used should therefore be tough, hard, and dense. However, it is also necessary to have a strong cement to prevent the particles of aggregate from being torn out.

Tests used to measure wear are the Talbot-Jones rattler³ and roller abraders.⁴ Both the above tests are accelerated wear tests which may not be representative of

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actual service conditions. Their results, therefore, can serve at best only as a qualitative measure of wear or as a guide for the selection of materials and methods of construction.

D. A. Abrams has found from rattler tests\(^1\) that resistance to wear is linearly proportional to compressive strength in the range of 3,000 to 6,000 psi. At the lower strengths below 2,500 psi Abrams found that wear increased very rapidly. Other results of these tests were: (1) wear resistance increases with curing time up to a maximum of 30 days, (2) resistance increases with drying up to 40 days, and (3) wear resistance is not appreciably affected by hydrated lime or inert powdered admixtures up to 20 per cent of the volume of the cement.

L. F. Fairchild found from simulated service tests on factory floors that a silica-sand aggregate in a 1:3 mix with 3.5 gal per sack of cement gave the least wear.\(^2\) The optimum aggregate grading had a fineness modulus of 4.0 and sizes which ranged from \(\frac{3}{4}\) in. with 97 per cent retained on the 48-mesh sieve. Finishing was done with a power trowel. Tests by the National Bureau of Standards have shown that the use of a metallic aggregate in the top surface of the slab improves the wearing qualities of floors.\(^3\) In addition to its wear quality, this type of floor is more nearly sparkproof than a plain concrete surface and is recommended for industries where sparking is dangerous.

Floor hardeners manufactured under proprietary names increase the rate of hardening of the concrete or give greater hardness at early ages, thus allowing traffic on the floor sooner (see Miscellaneous Materials—Floor Hardeners).

**Stress-Strain Relations**

A typical stress-strain diagram obtained for the standard compression test is shown in Fig. 1-7. There is no portion of the diagram that is a straight line but for the allowable stress used for design the deviation from the straight line is very small. The modulus of elasticity (stress/strain) is usually the slope of the chord drawn from zero to the design stress. Sometimes the modulus is measured dynamically by measuring the natural frequency of a specimen or measuring the velocity of sound waves in a specimen. This type of measurement will usually give somewhat higher values of the modulus than those obtained from the static compression test. The method has been

\(^2\) Fairchild, L. F., Concrete in Factory Construction, *J. ACI*, vol. 6, p. 149, 1934.
used extensively as a measure of deterioration caused by weathering, etc. The modulus of elasticity in tension and compression is the same.

Factors influencing the modulus of elasticity are strength, age, moisture content, and type of aggregate. Values of the initial tangent modulus as well as the compression strength of concrete cylinders are given at various ages in Table 1-18. In these tests the secant modulus at one-third the ultimate strength was practically the same as the initial tangent modulus. The tabular values show the elastic modulus increases with age and decreases with water-cement ratio, although not so rapidly as does strength. The increase in modulus with age is a cause of cracking in massive concrete structures where early expansion caused by temperature increases due to hydration are absorbed at a lower modulus than the subsequent contraction due to the later temperature decrease. The modulus increases with moisture content at the time of testing since the pore water introduces a triaxial state of stress. This increase in modulus is in contrast with the decrease in strength with increased moisture content. Although the effect of the aggregates on the modulus is usually small for normal concrete, Table 1-19 shows possible variations.  

Table 1-18. Compressive Strength and Modulus of Elasticity*

<table>
<thead>
<tr>
<th>Mix by volume</th>
<th>Cement sacks per cu yd</th>
<th>Net water-cement ratio</th>
<th>Strength, psi</th>
<th>Initial tangent modulus of elasticity 1,000 psi</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>3 days</td>
<td>7 days</td>
</tr>
<tr>
<td>1:2</td>
<td>10.5</td>
<td>0.57</td>
<td>3,220</td>
<td>6,650</td>
</tr>
<tr>
<td>1:3</td>
<td>7.8</td>
<td>0.64</td>
<td>2,160</td>
<td>3,520</td>
</tr>
<tr>
<td>1:4</td>
<td>6.3</td>
<td>0.76</td>
<td>1,560</td>
<td>2,620</td>
</tr>
<tr>
<td>1:5</td>
<td>5.2</td>
<td>0.85</td>
<td>1,340</td>
<td>2,120</td>
</tr>
<tr>
<td>1:8</td>
<td>3.4</td>
<td>1.13</td>
<td>480</td>
<td>945</td>
</tr>
</tbody>
</table>

Tests on 6- by 12-in. cylinders.
Aggregate: Elgin sand and gravel graded 0 to 1½ in.; grading and consistency constant.
Curing: Specimens removed from molds after 1 day and cured in moist room at 70°F until test, tested damp.

Table 1-19. Values of Modulus of Elasticity for Various Aggregates, Water-Cement Ratio = 0.6

<table>
<thead>
<tr>
<th>Aggregate</th>
<th>E, psi</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flint</td>
<td>5,600,000</td>
</tr>
<tr>
<td>Sandstone</td>
<td>5,200,000</td>
</tr>
<tr>
<td>Moline limestone</td>
<td>4,500,000</td>
</tr>
<tr>
<td>Soft limestone</td>
<td>3,700,000</td>
</tr>
</tbody>
</table>

The ratio of unit lateral strain to unit axial strain for the standard compression test, defined as Poisson’s ratio, varies between 0.15 and 0.25, with the larger values at greater age.

Stress–Strain Time Effects

Concrete does not develop all its strain instantaneously under a constant load as, for instance, steel does at room temperatures. Similarly, concrete does not recover its strain instantaneously when a long-time load is removed. These time effects which occur at moderate stresses are called creep and more recently anelasticity.  

General practical considerations of creep in design are concisely stated by the Joint Committee as follows:  

3. Report of the Joint Committee 1940, Secs. 801a, 802.
MATERIALS FOR REINFORCED CONCRETE

(1) In flexural members there is an adjustment in the concrete and steel stresses of moderate amounts, but the load carrying capacity of the member is not materially affected. [In fact creep may aid the security of the structure by redistributing the stresses more uniformly throughout the structure.]

(2) The ratio of the modulus of elasticity of steel to that of concrete, for concrete of a given strength, is constant in flexural members within the range of working stresses. The ratio, however, is not considered as fixed in the case of columns or in the case of compression reinforcement in flexure where, due to plastic flow (creep), the reinforcement will be more highly stressed than indicated by a constant value of the modular ratio.

Factors influencing the amount of creep are the same as for strength. A change that increases strength will decrease creep. For sustained loads used in the usual design, creep will reach an ultimate value of one to three times the initial elastic strain, with 50 per cent of the ultimate creep occurring in the first 5 to 60 days after loading.\(^1\) Over a large structure the amount of creep will vary considerably, depending on the distance to the surface of the concrete.

There is a large permanent strain remaining in the concrete because of the change in the material properties due to increase in age.

Creep is caused by viscous flow of the absorbed water from the gel which is squeezed out under compression stresses and withdrawn upon removal of the stresses.

Volume Changes

These are expansions or contractions caused mainly by changes in moisture content or temperature. In addition there is a shrinkage after pouring and vibrating which is the result of settlement of the heavier particles in the mix.

Because expansion causes compression in the concrete, there are usually no adverse effects. However, thin sections restrained at the ends may buckle and continuous structures may be subjected to excessive stresses because of these expansions. Contractions, on the other hand, are generally of a more serious nature because of the low tensile strength and because the initial drying shrinkage is usually several times the expansion due to wetting. Cracks form into which water can penetrate and freezing and thawing action enlarges these cracks, gradually causing the disintegration of the concrete. Even with the use of reinforcement, cracking is not eliminated but the amount of contraction is more uniformly distributed over the length of the steel. The cracks are therefore much smaller, making it more difficult for water to enter in. In general the size and extent of the cracks are dependent on the amount and type of constraint. Thin concrete sections cast continuously with heavy sections are liable to crack at the junction where the large section is restraining the more rapid contraction of the thin section. A similar condition may occur in a large block of concrete where drying is fairly rapid on the exterior surfaces. The restraint from the inner part of the block will cause cracks on the surface. Other examples where differences of moisture or temperature in a section can be detrimental are road pavements and canal linings in contact with moist soil. The action is first observed as curling of the slabs, which is followed by cracking.

Volume changes in the concrete are caused by the adsorption and evaporation of water from the cement gel. When the concrete becomes wet it expands and when it dries it contracts. Most of the shrinkage takes place during the setting, since the amount of water available in the gel is a maximum at this time. It has been found that the amount of shrinkage increases with the amount of mixing water and entrained air, the water-cement ratio, and the compressibility of the aggregates. Each 1 per cent increase in the quantity of the mixing water was found to increase shrinkage by 2 per cent.\(^2\)


although small, contributes greatly to the shrinkage. The following aggregates contribute to shrinkage as follows:

Low shrinkage: Dense quartz, feldspar, dolomite, and limestone
Intermediate shrinkage: Granite
High shrinkage: Hornblende, pyroxene, slate, and sandstone

Figure 1-8 shows how both water and air content increase drying shrinkage. For a given slump, however, increasing the air content within the usual limits will not materially affect shrinkage. Shrinkage and attendant cracking are usually more severe when the concrete is placed at higher temperatures because of the more rapid gel growths.

Concrete expands during an increase in temperature and contracts during a decrease. The temperature coefficient of length change is generally about 0.0000055 in. per in. per °F. However, there may be considerable deviations from this figure for certain aggregates. The above coefficient is equivalent to a change of 0.66 in. per 100 ft for a temperature change of 100°F. For comparison the drying shrinkage for average concrete from the state of complete saturation to complete dryness is about the same as that due to a decrease in temperature of 100°F. Except for the interiors of buildings, concrete very seldom approaches the completely dry state.

Durability

Concrete is a very durable material needing very little maintenance and performing very well under a wide range of service conditions. Some of the more common adverse service conditions which good concrete will endure are weathering which includes erosion and freezing and thawing, and chemical conditions which exist in surface water and in sea water. Durability under these conditions depends on many different factors and is highly variable.

Where damage is caused by freezing and thawing the durability is affected by the degree of exposure. Using a lower water-cement ratio will greatly improve durability. Many tests have proved that the proper amount of entrained air will greatly increase durability. The optimum amount of entrained air for this condition is about 6 per

cent and the improvement is much less when the air content is less than 3 per cent. Figure 1-9 shows the influence of the water-cement ratio and air content on durability.

Concrete made with sands deficient in fines show poor service records. The experience of the Corps of Engineers has shown a large improvement when the gradation of the aggregate was closely controlled. Weak material in the coarse aggregate also is a cause of poor durability. Of particular importance is proper remedial control of reactive aggregates where such are used.

Construction practices play a major role in determining durability. In general the problem is one of establishing proper control in the field. Those practices which affect the final air and water content are particularly important.

![Graph showing effect of water-cement ratio and entrained air on durability.](from USBR Concrete Manual, 5th ed.)

In sea water the performance of the concrete is affected by the tricalcium aluminate content of the cement. This should be kept within the maximum limit specified for Type II cement.

Tests used to evaluate the long-time performance of concrete by accelerated laboratory tests are of some qualitative value. The freezing and thawing test and the sodium sulfate test have been used as acceptance tests for durability but these tests may be so far from actual exposure conditions and construction practice that the results are sometimes of doubtful value.

Chemical Properties

Chemical Deterioration. Often caused by acid corrosion chemical deterioration occurs under certain instances of exposure to various chemical substances. Floors of buildings are sometimes required to withstand this type of service. Salt solutions do not act chemically with concrete but during the process of wetting and drying the crystallization of the salts results in pressures that cause scaling.

Efflorescence. Efflorescence is the deposit of salts leached out of masonry by water. The salt deposits may or may not be soluble. Concrete usually contains small amounts of calcium hydroxide which when brought to the surface combine with carbon.
dioxide in the air to form calcium carbonate. Efflorescence is an indication of absorption and can usually be controlled by having initially dense concrete or subsequently applying a waterproofing agent to the surface. In dams and retaining walls where construction joints are used, leakage usually will cause incrustations. These can be prevented by providing watertight joints.

Deposits can be removed by scrubbing with a 1:10 solution of muriatic acid. The masonry should be thoroughly wetted with water before the acid is applied and should be washed immediately after treatment to remove all acid.

**Staining of Concrete.** Concrete stains rather easily. When accumulated over a long period of time, stains are removed with some difficulty. Bleaching agents are generally used for stain removal.1

**Chemical Reaction with Metals.** Concrete corrodes in varying degrees moist metals with which it comes into contact.

Steel reinforcement is subject to rusting when air and moisture come into contact with it. A good-quality concrete covering, 2 to 4 in. thick, will prevent rusting under most exposure conditions.

Concrete made of an aggregate carrying magnesium chloride has been the cause of serious corrosion of reinforcement. Adding a small amount of caustic lime to the mix will neutralize the effect of the magnesium chloride which would otherwise generate hydrochloric acid.

Copper is not affected by either fresh concrete or hardened concrete except when there are chlorides present either in the aggregate or introduced by an admixture. Wherever copper is to come in contact with concrete it is inadvisable to use any admixture containing chlorides.

Lead is corroded by fresh or green concrete. Unless heavy-gage material is used it is necessary to protect the lead by a coating of asphalt, pitch, or varnish.

Lead partially embedded in concrete and exposed to air is corroded by electrochemical action. This action can be reduced by coating the lead as mentioned above.

Zinc corrodes on contact with concrete. The corrosion product, however, forms a very adherent film which prevents further action. The initial corroding action causes concrete to stick to forms made of galvanized iron.

**Aluminum** is progressively corroded in concrete because the products of corrosion are nonadherent and therefore do not protect the underlying metal. When aluminum is used in contact with concrete it should be coated with asphalt, pitch, or varnish.

**Watertightness**

Water seeps through concrete when under pressure and by capillary forces. The amount of seepage is proportional to the product of cross-sectional area and pressure head divided by the length of flow path. Dense well-constructed concrete, however, is practically impervious but capillary forces will cause a slight passage of water when the water is in contact with the concrete for a relatively long time. Where a slight amount of moisture is objectionable a dampproofing agent may be advisable.

Experience has shown that many watertight concrete structures have been constructed and have performed satisfactorily for many years. In those cases where leaks were found, they were confined to small areas and around construction joints. To prevent this cause of leakage, it is necessary to exercise close control of construction work.

**Requirements.** Watertight concrete requires nonporous aggregate surrounded by an impervious cement paste. The mix should have a low water-cement ratio. Figure 1-10 shows that leakage decreases rapidly with decreasing water-cement ratio and curing time. The graph also shows considerable leakage even after curing a 9-gal mix for 1 month. Other tests conducted on standard cured specimens of 6.5 in. diameter and 8 in. long under a pressure of 200 to 240 psi leaked 5 to 7 $\times 10^{-2}$ gal per hr for a 0.6 to 0.7 water-cement ratio and 60 $\times 10^{-4}$ gal per hr for a water-cement ratio of 0.9.2

Experience has shown that for tanks and reservoirs the water content should be limited

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1 ST 19, Portland Cement Association.

to 6 gal per sack of cement. For heavy dam sections this limit may be increased to 7 1/2 while for thin sections (less than 12 in.) it should be reduced to 5 gal. The mixture should be of such a consistency that it can be placed to give a dense uniform mass with a minimum of bleeding or water gain. Normal concrete should be kept moist for at least 7 days and high-early-strength concrete for at least 3 days to develop watertightness in the cement.

Construction joints particularly are sections which are often porous unless special preparations for these areas are made. Wherever possible, the placing of concrete should be continuous. Where expansion joints are necessary the concrete should be continuous between joints. Where construction joints are made, however, it is necessary to obtain a good bond between the adjacent sections. On tall sections it is better to place the concrete to within 1 ft of the joint, wait about 1/2 hr to allow for settlement of the aggregate, and then place the remaining foot to the level of the joint. The surface of the joint should be free from all dirt and all weak or porous material should be scraped off or chipped off before the concrete becomes hard. Before placing the new section, the surface should be wet down and where necessary in moderate or thin sections a slush coat of cement grout should be applied. The next layer to a thickness of 4 to 6 in. should contain about one-half the usual amount of coarse aggregate and be placed before the grout has attained its initial set. For tanks 20-gage galvanized-iron or copper straps 7 to 8 in. wide are often embedded in the construction joint, one-half on each side of the joint.

Admixtures can often be used to advantage to obtain a more watertight concrete. These materials are described under Waterproofing Agents.

Calcium chloride does not directly affect watertightness. It primarily decreases curing time and thereby lessens the effects of bad curing.

Specific Weight

The specific weight of concrete is usually between 140 and 160 lb per cu ft. Reinforced concrete weighs 150 lb per cu ft, stone concrete 145 lb per cu ft, and cinder con-

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crete about 100 lb per cu ft. Concrete made of lightweight aggregates varies considerably with the kind of aggregate and mix.

Thermal Properties

These consist of the heat conductivity and heat resistance of concrete. Length changes due to temperature fluctuations are described under Volume Changes.

Conductivity. In massive structures the heat of the hydrating concrete must be controlled or conducted out in order that serious expansion of the concrete may be prevented. To design the cooling system and to predict resulting temperatures the heat conductivity of the concrete must be known. Design procedures using available thermal properties have been worked out by the Bureau of Reclamation. The term diffusibility is used in connection with this property and is an index of the facility with which a material will undergo a temperature change. This term is equal to the conductivity divided by the product of specific heat and density. Conductivity is the rate at which heat is transmitted through a unit thickness of concrete over a unit area of the material subjected to a unit temperature difference between the two faces. Or the rate of heat transmitted is given by the following formula: \[ Q = k(\Delta T/L) \], where \( k \) is the coefficient of thermal conductivity.

Thermal conductivity of walls is of importance in the heating and air conditioning of buildings. Typical values of \( k \) determined for solid walls are given in Table 1-20. The lower the conductivity, the higher the insulating value. For walls made of two or more thicknesses, the thermal conductance per unit area is as follows:

\[
U = \frac{1}{L_1/k_1 + L_2/k_2 + \cdots + L_n/k_n}
\]

Exposure to High Temperatures. The strength of concrete is somewhat reduced when exposed for long periods of time to moderately high (not fire) temperatures. Tests on small specimens indicate that there is no serious loss in strength at temperatures below 500°F. Since these tests were conducted on small specimens, the results are fairly conservative and apply only qualitatively to full-sized structures. This is true because the ratio of surface area to volume is much greater in the test specimens, resulting in larger internal stresses than would be expected in the full-sized structure. In most instances only one face of the structure is exposed to the high temperature. Except for the fairly thin surface layer, the rest of the structure remains cool and is unaffected.

Heating the concrete also drives off water of hydration. This action, which starts at the surface and proceeds into the interior, makes the concrete a better insulator and tends to delay further dehydration. Tests show that concrete cured for a longer time before exposure to high temperature suffers less loss of strength than that obtained for a short curing time.

Table 1-20. Effect of Aggregates on Conductivity of Monolithic Walls

<table>
<thead>
<tr>
<th>Mix</th>
<th>Thermal conductivity ( k ), Btu/hr sq ft °F difference of surfaces/in. thickness</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sand and limestone (plastic mix)</td>
<td>12.4</td>
</tr>
<tr>
<td>Sand and coarse gravel (plastic mix)</td>
<td>12.4</td>
</tr>
<tr>
<td>Sand and coarse gravel (dry-tamp mix)</td>
<td>13.1</td>
</tr>
<tr>
<td>Cinder (plastic mix)</td>
<td>5.75</td>
</tr>
<tr>
<td>Haydite (plastic mix)</td>
<td>3.73</td>
</tr>
<tr>
<td>Sand and cinder (1:2½:4 dry tamp)</td>
<td>8.54</td>
</tr>
<tr>
<td>(Conductance 2½ in. air space)</td>
<td>1.1</td>
</tr>
</tbody>
</table>


1 Thermal Properties of Concrete, Boulder Canyon Project Final Rept. part 7, Bull. 1, 1940.
Limestone aggregates and calcareous sands and gravels produce better heat-resistant concrete than siliceous materials. Special aggregates such as crushed firebrick, burned clay, and cinders will improve resistance to intense heat.

Service records of various structures exposed to temperatures between 500 and 1000°F show that with the exception of some spalling of the exposed surfaces at the higher temperature there has been no real damage. Reinforced-concrete chimneys have been in service for more than 20 years exposed to temperatures of 600°F. Beams supporting boiler settings resulting in estimated surface temperatures of 700°F have performed satisfactorily. In this instance the concrete was designed to develop a compressive strength of 600 psi.

For important load-carrying members such as columns exposed on all surfaces, temperatures should not exceed 500°F. On lightly loaded members higher temperatures up to 1000°F are permissible, particularly if crushed firebrick and other heat-resistant aggregates are employed.

**Fire-resistance Ratings.** Fire-resistance ratings of concrete depend on the type of aggregate used, the thickness of the material, and the particular application. Siliceous gravels containing a large percentage of chert or flint are badly disrupted by exposure to fire, whereas limestone, traprock, and cinders show only minor cracking and spalling after similar exposure.

Tests have shown that the fire-endurance period is approximately proportional to the thickness squared.\(^1\) With columns, beams, and girders the critical factor is the load-carrying capacity. In these members it is most important to protect the steel from high temperatures. It was found\(^1\) that a temperature of 1000°F is the maximum to which steel can be heated and still carry the design loads.

Where plaster coatings are the primary fire resister, the type of plaster, richness of the mix, and thickness of the coating are very important. Gypsum plaster is superior to either portland cement or lime plaster in resisting heat transmission.

The fire ratings of hollow masonry units are determined by the total thickness of the solid material since the hollow spaces have only a minor effect on the resistance.

**Acoustical Properties**

Ordinary concrete is not a particularly good sound-absorbing material since the properties desired in structural concrete work to a disadvantage as a soundproofing material. The required properties of a good acoustical material are porosity and surface roughness. Tests\(^2\) have shown that haydite and cinder concrete are good soundproofing materials, absorbing 50 per cent of the sound for each reflection. Ordinary concrete will absorb only 20 per cent of the sound for each reflection. On the other hand, hard plaster is very poor—absorbing only about 5 per cent of the sound.

**Electrical Conductivity**

Moist concrete is a fairly good conductor of electricity. Where pipes carrying stray currents pass through concrete, electrolysis may occur. Under such conditions the pipe should be well insulated or encased in a shield of larger-sized pipe where it passes through the concrete.

**LIGHTWEIGHT CONCRETE**

**General**

Lightweight concrete incorporates certain lightweight materials as described earlier under Lightweight Aggregates in place of the usual sand and stone aggregates. Another type of lightweight concrete formed with large cells or voids by using foam-
ing agents has been used to some extent in Europe. Uses for this type of concrete include applications where strength is not a prime requirement but where fire resistance, insulation, and light weight are major considerations. Specific applications include precast masonry units and slabs, concrete fill and insulation, nonbearing walls, and floor and roof slabs.

Lightweight aggregates because of the additional cost of processing and transportation are usually more expensive than ordinary aggregates. Compensating factors are

![Diagram of compressive strength vs. specific weight](image)

**Fig. 1-11.** Compressive strength of lightweight aggregate concrete versus specific weight. (From Kluge, Sparks, and Tuma, Lightweight Aggregate Concrete, J. ACI, vol. 20, p. 634, May, 1949.)

weight reduction, better insulating and fire-resisting properties, nailability, and savings in costs of material handling and forms. Other savings are effected in the transportation of precast masonry units. Types of structures where the use of lightweight concrete can be economical include tall buildings and long-span bridges. The use of lightweight concrete for the paving of the upper deck of the San Francisco-Oakland Bay Bridge resulted in a $3 million savings in the construction cost. The concrete

used for this job contained 6 1/2 sacks of cement per cu yd, weighed 95 lb per cu ft, dry, and had an average 28-day compressive strength of 3,070 psi. The concrete to date is remarkably free from cracks.

Proportioning

Lightweight aggregates have a high water absorption which has an important bearing on the mix design. This rapid absorption causes the mix to stiffen during mixing and placing. The finer the aggregate the larger is the amount of absorption. Initially moist aggregates also tend to absorb more water than those initially dry.

To produce workable mixes it is necessary to have a high percentage of the aggregate pass the No. 4 sieve. Too many fines, however, cause high shrinkage and result in a low yield. The amount of water in the mix is determined by the slump test for consistency. The tendency for harsh mixes and bleeding with lightweight aggregates can be offset to a great extent by employing an air-entraining agent. Tables 1-21 and 1-22 list typical data on mixes for insulating concrete and lightweight concrete using perlite. Grading requirements are discussed under Aggregates, Gradation of Aggregate Particles.

Curing

Practices for curing normal concrete apply also to lightweight concrete. It is especially important with this material to prevent rapid drying.

Properties

Strength. The water-cement ratio law can be applied to determine and control the strength of lightweight concrete. It is necessary, however, to allow for the large water absorption of these aggregates when computing the required amount to be used in the mix. As a general rule with workable mixes the strength of lightweight concrete is roughly proportional to its dry weight as shown in Fig. 1-11.

Modulus of Elasticity. The modulus of elasticity is dependent on the aggregate used. Values for sand-haydite and all-haydite concrete average about 75 and 55 per cent, respectively, of that obtained for stone concrete.¹

Table 1-21. Mix Data and Properties of Insulating Concrete Using 8 Lb per Cu Ft Perlite Aggregate*

<table>
<thead>
<tr>
<th>Cement content, sacks/cu yd</th>
<th>Water content, gal/sack cement</th>
<th>Vinsol resin, oz/sack cement</th>
<th>Dry wt, lb/cu ft</th>
<th>28-day strength, psi</th>
<th>Absorption, % of wt in 24 hr</th>
<th>% drying shrinkage 7 days in oven</th>
<th>Thermal conductivity &amp; at 75°F</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.00</td>
<td>9</td>
<td>4</td>
<td>32</td>
<td>500</td>
<td>40</td>
<td>0.18</td>
<td>0.80</td>
</tr>
<tr>
<td>4.80</td>
<td>11</td>
<td>5</td>
<td>28</td>
<td>300</td>
<td>51</td>
<td>0.22</td>
<td>0.72</td>
</tr>
<tr>
<td>4.00</td>
<td>13</td>
<td>6</td>
<td>24</td>
<td>195</td>
<td>63</td>
<td>0.21</td>
<td>0.65</td>
</tr>
<tr>
<td>3.43</td>
<td>15</td>
<td>7</td>
<td>22</td>
<td>160</td>
<td>76</td>
<td>0.20</td>
<td>0.60</td>
</tr>
<tr>
<td>3.00</td>
<td>17</td>
<td>8</td>
<td>20</td>
<td>100</td>
<td>81</td>
<td>0.24</td>
<td>0.58</td>
</tr>
</tbody>
</table>


Shrinkage. This varies with the type of aggregate. It is roughly twice that of ordinary concrete. Vermiculite, diatomite, and perlite cause somewhat higher shrinkages than the average. Moist-cured foamed cellular concrete, however, has a linear drying shrinkage of about ten times that of ordinary concrete.

Soundness. Soundness and durability, and resistance to freezing and thawing disintegration depend on the richness of the mix and are greatly improved by the use of

Table 1-22. Mix Data and Properties of Lightweight Structural Concrete Using Perlite Aggregate*

<table>
<thead>
<tr>
<th>Sand, %</th>
<th>Perlite, %</th>
<th>Unit wt of blended aggregate, lb/cu ft</th>
<th>Cement content, sacks/yd</th>
<th>Vinson resin, oz/sack</th>
<th>Water content, gal/sack</th>
<th>Wt oven-dry concrete, lb/cu ft</th>
<th>28-day compressive strength air cured in cylinders</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>100</td>
<td>8</td>
<td>6.22</td>
<td>4</td>
<td>9.0</td>
<td>33</td>
<td>520</td>
</tr>
<tr>
<td>75</td>
<td>25</td>
<td>26</td>
<td>5.58</td>
<td>1</td>
<td>9.5</td>
<td>56</td>
<td>1,000</td>
</tr>
<tr>
<td>90</td>
<td>10</td>
<td>48</td>
<td>6.21</td>
<td>1</td>
<td>8.5</td>
<td>90</td>
<td>2,200</td>
</tr>
<tr>
<td>95</td>
<td>5</td>
<td>66</td>
<td>6.01</td>
<td>1</td>
<td>7.0</td>
<td>102</td>
<td>3,100</td>
</tr>
<tr>
<td>100</td>
<td>0</td>
<td>106</td>
<td>5.91</td>
<td>1</td>
<td>6.0</td>
<td>141</td>
<td>4,500</td>
</tr>
</tbody>
</table>


**Fig. 1-12.** Thermal conductivity of lightweight concrete versus specific weight. (From Kluge, Sparks, and Tuma, Lightweight Aggregate Concrete, J. ACI, vol. 20, p. 625, May, 1949.)

Air entrainment. Concrete using vermiculite or diatomite is not recommended for exterior use in severe climates since it has low resistance to freezing and thawing.

**Insulating Properties.** Lightweight concrete is greatly superior to stone concrete for use as heat and sound insulation. Values of the coefficient of thermal conductivity are shown in Fig. 1-12 for various aggregates. In general the insulating value decreases proportionately with increase in weight.
Section 2

CONCRETE CONSTRUCTION

By

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2-1
FORMS

Because of the plastic state of its first stage, all concrete construction requires some kind of form to mold it to the required shape. Forms account for a substantial proportion of the total cost of finished concrete, running in extreme cases as high as 75 per cent. If only for this reason, form design merits careful study. On large structures forms may be fully designed in the office and form drawings made for fabrication and assembly in the field.

The present construction practice in the United States is for the contractor to design the forms, whereas the owner’s engineers design the structure. Where the structure designers are not familiar with form design, unnecessarily high total costs will often result. Two common reasons for this are:

1. No account of standard lumber sizes is taken in proportioning the concrete members, with resulting waste of lumber and labor in cutting.

2. The possibility of reusing the same forms by making concrete members of similar size identical may be overlooked. It is often more economical to waste concrete than to require a new form for a slightly smaller member.

Specifications may require the contractor to submit his form designs to the engineer
on unusual structures as a check on their structural adequacy or treatment of special
details.

Proper form design provides:

1. Strength
2. Rigidity
3. Tightness
4. Good alignment
5. Reasonable economy
6. Desired texture on exposed concrete surfaces
7. Ease of stripping

Strength, with an adequate margin for unexpected loads, is a basic requirement for
the safety of both the workingmen and the structure itself. In addition to sufficient
strength the form must be sufficiently rigid under the construction loads to maintain
the shape called for on the plans. Tightness of form is essential to prevent loss of
mortar, with resulting honeycomb or ugly fins. Forms erected for some time before
concreting must be carefully checked to see that lumber shrinkage does not open up
originally tight forms. Proper form design makes it possible to align large members
readily and hold them in alignment. There are few worse faults in finished concrete
than poor alignment.

While economy should not be the dominant factor, complete disregard of this impor-
tant element will result in unnecessarily high costs. Small changes in design not at all
detrimental to the structure may drastically reduce costs. Such changes are some-
times made by the contractor with the consent of the engineer.

The form, or at least its lining, must be designed to produce the desired concrete
surface texture to carry out the architectural effect specified. For more detail on this
aspect of form design, see Finishes and Finishing.

Forms should be designed so they may be stripped from the hardened concrete
easily in order both to protect the concrete and to reduce labor cost. This is particu-
larly important for interior forms, which if not designed properly may have to be
destroyed to be removed.

Because the most common material used for forms is lumber, the trade practice in
the United States is for forms to be built by carpenters (or dock builders below the
high-water line on marine structures). While parts may be fabricated away from the
site the entire form must usually be assembled at the site. For the safety of the work-
men and the concrete structure itself its assembly and bracing should be carefully
inspected for strength by an engineer familiar with such work. The necessity for
such inspection is clearly demonstrated in the number of large form failures reported
each year.

Materials

Concrete forms are made of wood, metal, plaster, and even concrete itself, but wood
predominates because of its general availability and ease of fabrication. The soft-
woods are usually chosen because they are cheaper, easier to work, generally weigh
less, and do not warp so much when wet. Because wooden forms are bound to absorb
some moisture in use, kiln-dried wood is not favored because of swelling with conse-
quent loss of shape. Selection of the kind of wood will depend on the local market,
but the commonest woods used are spruce, Douglas fir, longleaf southern yellow pine,
and hemlock. Forms for architectural concrete may be made from such better grades
of wood as white pine and eastern spruce.

The higher grades of lumber may be necessary for the contact surfaces rather than
for the studding, wales, and heavy structural members, because of the effect of the
defects in the lower grades on the concrete surface finish. No specific grades can be
recommended for specific uses because of differences in grading the different woods.
The simplest method is to examine the wood in the yard before ordering.

Timber is sold in standard sizes. These nominal sizes are for rough lumber only.
CONCRETE CONSTRUCTION

Finished (planed) lumber will be smaller than the nominal size because of the loss in planing. The loss varies with the size of the piece. For example:

<table>
<thead>
<tr>
<th>Use</th>
<th>Nominal size (rough), in.</th>
<th>Finished 4 sides, in.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sheathing</td>
<td>1 by 4</td>
<td>1(\frac{3}{4}) by 3(\frac{3}{8})</td>
</tr>
<tr>
<td></td>
<td>1 by 6</td>
<td>1(\frac{3}{4}) by 5(\frac{1}{8})</td>
</tr>
<tr>
<td></td>
<td>1(\frac{3}{4}) by 8</td>
<td>1(\frac{3}{4}) by 7(\frac{1}{8})</td>
</tr>
<tr>
<td>Studs</td>
<td>2 by 4</td>
<td>1(\frac{3}{4}) by 3(\frac{1}{8})</td>
</tr>
<tr>
<td></td>
<td>2 by 6</td>
<td>1(\frac{1}{2}) by 5(\frac{1}{8})</td>
</tr>
<tr>
<td>Wales</td>
<td>2 by 4</td>
<td>Same</td>
</tr>
<tr>
<td></td>
<td>2 by 6</td>
<td>3(\frac{3}{4}) by 3(\frac{3}{8})</td>
</tr>
<tr>
<td>Posts</td>
<td>4 by 4</td>
<td>3(\frac{3}{4}) by 5(\frac{1}{8})</td>
</tr>
<tr>
<td></td>
<td>6 by 6</td>
<td>5(\frac{3}{4}) by 5(\frac{1}{8})</td>
</tr>
</tbody>
</table>

Where a continuous surface is being made up of sheeting boards, random lengths may be ordered and the only waste may come from the necessity of butting end joints over a stud. For the heavier pieces, such as for structural members, care should be used to order the nearest commercial lengths (generally in increments of 2 ft) or some multiple thereof. For large surfaces plywood panels are used because:

1. The large sheets save labor of assembly.
2. The large sheets drastically reduce the number of joints in the form surface. Joints affect the concrete surface so as to require expensive concrete-finishing operations.

Plywood is made from three to five veneer sheets of wood assembled with alternate sheets having the grain lying at right angles and firmly glued together under pressure. The exterior surfaces are sanded smooth and may, in better grades, have attractive grain markings. With proper care they may be reused ten to fifteen times. Plywood is commercially available for form construction in the size 4 by 8 ft or even larger on special order. The thicknesses are \(\frac{3}{8}\), \(\frac{5}{16}\), \(\frac{3}{16}\), and \(\frac{3}{4}\) in. A \(\frac{3}{4}\)-in. thickness may be had as a form liner to be nailed over regular sheeting. Aside from the reduction in the number of joints, the uniformity of plywood forms tends to reduce the amount of surface finishing of concrete, and its use for forms is widespread.

Metal forms are found only in special uses, usually where many reuses may be expected in order to defray the high initial cost. Some examples are:

- Side forms for pavements and sidewalks
- Wall forms made up of uniform small panels
- Pans for floors of buildings
- Round column forms
- Concrete conduits
- Special molds for ornamental concrete

Detailed descriptions of each are given elsewhere.

Plaster-waste molds are also used to produce certain ornamental effects.

Concrete itself has been used as a form for other concrete in two basically different ways. In the first it has been used as a mold (with many reuses) to cast thin wall and floor panels. In the second case, precast carefully finished panels of concrete have been set up to act as exterior forms to retain and become integral with concrete walls.

An important part of concrete formwork is the hardware. It consists of fasteners, ties, spreaders, and inserts. In the United States there has been a tremendous development of these devices, resulting in a great variety of types to meet all requirements.

The commonest fastener is the nail, which is usually ordinary wire-cut steel in sizes 4 to 10 penny. A simple rule is to make the length of nail at least twice that of the thickness of the piece being fastened. Double-headed nails facilitate withdrawal and so do less damage to the lumber.

Forms have to resist concrete pressure and so may either be braced externally or tied one to the other. In most cases the economical way to resist pressure is with ties.
A primitive device still in use is the wire tie, which is merely pushed through holes drilled in the form and tightened by twisting. While cheap, its disadvantage is that the wire projects through the concrete surface, necessitating cutting back and patching the holes if appearance counts.

Threaded soft steel rods $\frac{3}{4}$ or $\frac{3}{4}$ in. in diameter have many uses. Used with square nuts and square plate washers they provide a readily adjustable means of resisting heavy pressures. For primitive work they are sometimes cast into the concrete and then loosened and withdrawn while the concrete is still green. This may cause spalling of the surface concrete or even heavy cracking. To avoid these troubles the rods are sometimes greased heavily or encased in cardboard tubes to facilitate withdrawal.

Efforts to find a better solution led to the development of special ties, parts of which remain embedded in the concrete. These ties have several advantages:

1. There are no continuous holes through the concrete which can lead to leaks or cracks.
2. The tie can also be used as a spreader.
3. The socket of an embedded tie can serve to support other forms or erection equipment.
4. A small neat hole is left in surface which is easily patched.

Two well known types of ties are shown in Fig. 2-1a, b, namely, the coil tie and the snap tie. The first is superior and costs more.

The coil tie (Fig. 2-1a) consists of a helical wire coil electrically welded to two or four longitudinal heavy wires at their ends. The coil acts as a female thread to receive a special hardened-steel bolt ($\frac{3}{4}$ or $\frac{3}{4}$ in. in diameter). A removable steel cone is placed between the ends of the coil tie and the inside surface of the form to distribute the bearing of the ends of the tie against the form and also to produce a neat surface hole. In some cases for economy the cone may be made of wood. When the concrete has gained sufficient strength for form removal, the bolt is backed out and the bolt, cone, and square washer are recovered for reuse. These parts may be rented from the companies which fabricate the coil ties. The tie must be manufactured to exactly the correct length for the wall thickness and cone length. Bolts usually have sufficient thread to allow for a 6-in. variation in wall thickness. The coil tie is made in many forms adapted to different uses such as anchors and hangers for bridge-deck concrete.

A more economical type of tie is the snap tie (Fig. 2-1b). This is seen to consist of a thin steel rod made with a button head on each end and other buttons formed on the rod, so spaced as to meet the inside faces of the forms to give the correct wall thickness. The rod is notched (weakened) at a point from $\frac{3}{4}$ to $1\frac{3}{4}$ in. inside the concrete surfaces. The forms are tightened by means of a slotted wedge, which can pass the end button at the lower end of the slot. To remove the protruding portion of the tie after the forms are stripped, the tie is bent at the wall surface and twisted about one-fourth turn to break it off inside the concrete at the notched section. Just inside the notch there is a flattened section of rod which prevents the rest of the rod from turning in the concrete. Since this snapping spalls the concrete surface, sometimes a wooden cone is placed between the notch and the form similar to the metal cone with the coil tie. The snap tie does not make a satisfactory spreader because of the low column strength of the thin rod.
Specific uses of the ties are shown elsewhere. On large orders the manufacturer will often detail the forms as a service.

Another device which has developed in many forms is the insert. Its purpose is to provide a securely embedded anchor to which may be fastened various objects such as suspended equipment and piping as well as various contact materials such as brick or cast-stone veneers. In general it consists of a cast, forged, extruded, or pressed metal shape with wings or anchors to provide positive embedment in the concrete. The exposed portion, usually cast flush with the concrete surface, has a specially designed slot into which may be thrust a bolt or rod or anchor for fastening the equipment or piping. Since the insert is placed inside a form care must be taken to see that:

1. It is fastened lightly to the form so the form may readily be stripped without spalling the green concrete.
2. The wet concrete does not enter the insert.

In choosing ties or inserts, their cost should not be the prime consideration, but rather the labor involved in their installation and form stripping.

Loads and Design

Since reinforced concrete weighs from 100 to 165 lb per cu ft it is evident that forms must be substantially designed. There is a need for rational analysis rather than rule of thumb for all but the simplest forms. No great accuracy is necessary, however, because of the approximation in live-load assumptions and gaps in the standard lumber sizes.

As in most structural design, both live and dead loads must be provided for. The dead load includes the weight of the concrete (including steel) and the form itself. The live loads include wind on high forms, moving buggy loads, impacts from swinging buckets, rapid discharge of bucket from a small height, effect of vibrators, etc. The weight of the forms themselves can frequently be neglected as small compared with that of the concrete. Except for lightweight concrete, it may be taken as 150 to 165 lb per cu ft, using the higher figure when the steel percentage is high.

In calculating the size of structural form members, a common construction live load is 75 lb per sq ft of floor. Bearing this in mind, care must be taken to avoid storing materials on the forms unless special provision for such loads is made in the form design. These vertical loads are easily calculated.

The proper allowance for horizontal pressure on wall and column forms is more difficult to ascertain. For high forms full hydrostatic pressure would be expensive to provide for. If concrete remained completely fluid it could transmit full hydrostatic pressure. However, at normal temperatures, normal portland cement may take its initial set in from 45 min to 1 hr, thus preventing the transmission of hydrostatic pressure of the set concrete to side forms. A rough approximation would be to assume that (except in the case of slow-setting cements) the maximum effective height at which fluid pressure would act would be the height of concrete which could be poured in 1 hr. For example, if a wall were 2 ft thick and 40 ft long its horizontal sectional area would be 80 sq ft. The volume would be about 3 cu yd for each foot of rise. If concrete could be placed in the form at the rate of 1 cu yd every 2 min it would take 6 min to rise 1 ft or 1 hr to rise 10 ft. By that time the lowest concrete would have its initial set; so the hydrostatic head could not exceed 10 ft or about 1,500 psf.

The two factors affecting the maximum effective horizontal pressure are seen to be

1. Rate of rise of the concrete in the form
2. Rate of setting (loss of fluidity)

The first depends on the size of form or forms being filled vs. the rate at which the concrete is placed. The second depends on a number of factors, of which the most significant is the temperature. Great refinement in evaluating the effects of the several factors affecting rate of setting is not warranted. A simple assumption is that the time of setting at 80°F is 45 min and at 40°F about double that at 80°F.

The effect of pressure in compacting the lower fluid layers by forcing out mixing water (bleeding) has led to the belief that for very rapid rates of rise there is a maxi-
mum pressure which cannot be exceeded. It may be of the order of 1,700 psf. It will usually be more economical to control the rate of rise than to try to provide form strength to resist such high pressures. It is vitally important that the foreman of the concrete gang be instructed as to the maximum rate of rise for which the form has been designed, if the plant design permits a more rapid rate of rise than the form was designed for.

In designing formwork, as is common with most temporary structures, the allowable unit stresses are higher than for permanent structures. Common allowable unit stresses for form lumber are:

<table>
<thead>
<tr>
<th>Type of stress</th>
<th>Yellow pine, spruce, or fir</th>
<th>Longleaf yellow pine</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compression, psi</td>
<td>1,200</td>
<td>1,800</td>
</tr>
<tr>
<td>Shear (parallel to grain), psi</td>
<td>200</td>
<td>300</td>
</tr>
<tr>
<td>Bearing (across grain), psi</td>
<td>400</td>
<td>600</td>
</tr>
</tbody>
</table>

For struts

\[ S = 1,000 \left(1 - \frac{h}{80d}\right) \]

where \( S \) = allowable stress

\( h \) = unsupported length, in.

\( d \) = least lateral dimension, in.

Generally struts having an unsupported length of over 8 ft are braced both ways at the center.

For steel bolts, the allowable stress in tension is 20,000 psi but since the controlling area is at the root of the thread which is generally about two-thirds the gross area, a common practice is to allow 14,000 psi on the gross area.

If calculations must be made of deflections, the modulus of elasticity for lumber is generally taken at 1,200,000 psi. A commonly assumed value for the allowable deflection is \( \frac{1}{32} \) in. for each member, except that \( \frac{1}{16} \) in. is preferred for sheeting.

The individual form members may be limited by bending, shear, bearing, or deflection and all four should be checked against the allowable values given above. Where there is continuity in a member no great refinement is necessary to determine the maximum moment. Where uniformly loaded this is assumed to be eight-tenths of the simple span maximum moment, i.e., \( wL^2/10 \).

With wall bracing a distinction must be made between those where the concrete pressure is taken in tension by interior ties and those where the pressure is taken entirely on the braces. The first is by far the most economical. The only loads which have to be provided for in the braces are wind and possible construction impacts such as a swinging concrete bucket striking the form.

**Fabrication**

On modern concrete jobs of any size, it is customary to set up a carpenter shop where parts of or entire forms may be fabricated and moved to the site for assembly and erection. The shop should be located fairly centrally, with space outside for lumber storage, and should be readily accessible for deliveries by truck and have moving room for a loading crane. A typical example on a job having about 10,000 cu yd of concrete would be a shop about 20 by 40 ft in plan, containing the following power tools:

- One 12-in. radial saw
- One 14-in. band saw
- One power-drill stand (to \( \frac{3}{4} \)-in. drills)
- One saw-sharpening machine

**Special Finishes**

For ornamental concrete details such as fluting and dentils, which may be part of a building facade, milled-wood forms are commonly used. They are usually made of
softwoods. In designing such details an effort should be made to select standard mill shapes and sizes for economy of formwork. The tendency of the wood molds to swell should be borne in mind. For this reason and ease of stripping it is best not to recess deep into the concrete mass. If recessed at all, the millwork should contain a slight bevel to facilitate stripping. Some nonstaining coating should be applied to the mold to decrease adherence to the concrete.

Metal molds are sometimes used in place of milled woodwork, especially if a detail is repeated. The material is usually 24- or 26-gage black iron, not galvanized. One common form is corrugated sheet metal used to produce a small fluting. Since the light-gage metal yields easily, it must be supported at close intervals. All edge details must be studied to avoid casting the edge of metal into the concrete.

Plaster-waste molds are used for very ornamental details as in building facades containing human shapes, floral designs, etc. The molds are made of casting plaster containing jute fiber and further reinforced by rods where necessary. A mold can be used only once as it is broken in stripping. The making of the mold proper is a job for a specialist. The mold is usually made in sections small enough for two men to handle. If the shape of mold is such that the back may be flat the mold frame is nailed directly to the studding. If not the frame may have to be built out to the plane of studs.

In chipping the plaster away from the concrete after concrete is set great care must be used to avoid marring the concrete. To facilitate chipping sometimes colored plaster is used for the 3/4 in. close to the surface. This gives warning of approach to the concrete. The contact face of the mold commonly is given two coats of shellac in the shop and a nonstaining coat of grease just before concreting.

Footing Forms

Footings are probably the only structural member where fabricated forms are occasionally omitted and the concrete is poured directly against earth walls. This practice should be followed only if the earth stands up well. Specifications should give the engineer authority to insist on forms if necessary. They should also cover the maximum deviation from plan size permitted to avoid loss of bearing area, eccentricity, etc.

There is one other serious disadvantage to forming the earth to the plan footing size in that there is no possibility of draining such an excavation.

Where forms are used they are extremely simple. Because the member is buried, no surface finish is necessary; so secondhand or rough lumber may be used. For shallow wall footings, up to 12 in. deep, the simplest and most economical form may be a single plank whose width equals the footing depth (Fig. 2-2a). Where the footing is deeper, it is generally more economical to make up panels of 1-in. sheathing held together by 1- by 4-in. battens spaced 2 ft apart (Fig. 2-2b). Where the footing is deeper than 24 in. the battens should be increased to 2 by 4 in., which may be turned on edge for
footings over 36 in. deep. The spreader is a temporary member which is removed as the concreting progresses.

For footings deeper than 36 in. the forms may be more economically supported against concrete pressure by ties in tension rather than braces. For details see Wall Forms. Some contractors prefer, where many uses are intended, to make large footing panels of 2- by 8-in. stock and handle them by crane.

Column footings are generally rectangular and the form corner must be designed to take pressure without opening (Fig. 2-3). Since no careful surface finishing need be done, the height of the footing panel may exceed the depth of the footing to save cutting the sheathing.

In designing large deep rectangular footings, to save concrete they are sometimes stepped. The simplest method of forming is to make the smaller upper form similar to the lower and support it from 2- by 6-in. or heavier timbers which rest on the upper edges of the lower form. No roof form is necessary on the lower step if the concrete is poured stiff (2-in. slump) and a short interval intervenes before pouring the upper block. During the interval other nearby footings may be poured and since there is a tendency for the upper form to rise while filling it must be anchored or weighted down.

It is not uncommon to insert "hairpins" in the upper surface of footings to serve as anchors for upper forms.

**Column Forms**

Because of small cross section, with attending rapid rise of concrete in forms, these forms are more subject to heavy hydrostatic pressure than most. If the pouring rate in cubic feet per minute is divided by the horizontal sectional area in square feet, the rate of rise in feet per minute is quickly apparent. Either the pressure on the form must be designed for or the rate of rise controlled (see Loads and Design). It is also important to form for a tight joint at the base of column form and anchor it down.

Since many column forms are too small to admit a man during concreting some difficulty may be experienced in securing a dense concrete free from honeycomb because of arching of loose concrete. To avoid segregation of concrete in a column form "windows" are sometimes built into a side of the form at one or more points. Another device is an "elephant trunk" through which concrete is discharged. For large columns where internal form ties are used it is very important to space these in the same vertical planes as the reinforcing-steel ties in order to leave room for elephant trunks and vibrators.

A cleanout hole should be provided at the base of the form for removal of chips and debris just before pouring.

The shapes commonly used for concrete columns are rectangular, octagonal, round, and L-shaped. Where the column is a small member, as in buildings, and it must be so detailed to permit reduction in size in successive uses, plywood is less common than for walls and slabs. A common thickness of board is 1 in., but where many reuses are intended, boards up to 2 in. in thickness may be used. Small columns are usually tied by yokes around the outside of the form (Fig. 2-4). Small octagonal columns, say up to 3 ft in diameter, may be made using the same construction as shown in Fig. 2-4 with the corners beveled by inserting boards at 45° in the corners.

For large octagonal columns, which may also taper as for bridge piers, a better form design is shown in Fig. 2-5. In this form the bursting pressure of the fresh concrete is resisted in hoop tension by steel bands bearing against the outside edge of wales sawn to the circular curve. The details of the hoop adjustment must be carefully designed and executed to avoid failure. Where these columns are tapered it is a common practice to hold the tops of the columns to one single diameter and where columns are of
different lengths construct short forms which are added at the bottom, thus allowing many reuses of the upper forms.

Round columns may be made of wood staves, steel, or reinforced fiber tubes. Round wood forms are expensive to construct. They are rarely justified except where a small number is to be built. For a large number of similar columns steel is the most common form material. The steel forms are made of heavy-gage sheet pressed to form arcs of the circle and with edges forming a stiffening flange. The flanges are clamped or bolted together so as to form a cylinder. Such a design facilitates stripping.

Fiber-tube forms are commercially available for round columns up to 48 in. in diameter and 50 ft long. Each tube may be used only once. In steel and fiber forms no outside yokes are necessary, the concrete pressure being resisted in hoop tension. For L-shaped columns details similar to corners of walls may be used.

Wall Forms

Forms for concrete walls are made of lumber, metal, or even hardened concrete. If lumber, it consists usually of sheathing stiffened by studs and wales (Fig. 2-1). Where a board-marked finish is desired or appearance is not important, tongue-and-groove sheathing of 1- by 4-in. or 1- by 6-in. boards is common. Tongue-and-groove boards are practically mandatory because of the tendency of the boards to curl in contact with the wet concrete. An alternative in wide use is plywood (1/2-, 5/8-, or 3/4-in. thickness). For large walls, panels may be made whose width and length are multiples of the standard plywood sizes. Their size will depend on the size of wall and method intended to be used in erection.

Other factors affecting panel size are permissible joint spacing and pouring schedule. If there are no restrictions, there are advantages in limiting the height to about 12 ft. This simplifies the bracing and may result in a lighter and more economical form design, particularly where cold weather retards setting. Small low walls (up to 6 ft) may be fabricated in place because no reuse of form is intended. For these it may be sufficient to use studding with no wales.

The function of the wale is to support the studs and facilitate alignment. The wales are frequently double 2 by 6s. They are used double in order to permit the placing of the tension ties without drilling (and so weakening) the wales. The stiffening elements (studs and wales) should break joints to avoid a plane of weakness. In some instances, wales are placed vertically with studs horizontal as in the case where the width of a pour is so great as to prevent the use of tension ties, making exterior bracing necessary.

Wall forms at construction joints must lap the previously poured wall (Fig. 2-6). The fit must be perfect and remain tight to avoid an unsightly offset in the completed wall surface. A good detail is to secure the next form to the old concrete by a tension
HIGH ALUMINA CEMENT CONCRETE (see page 4-5)

Since the Civil Engineer’s Reference Book was printed, further information has become available on the behaviour of high alumina cement concrete. Attention is drawn to research published in October 1974 by the Building Research Establishment (BRE Annual Report, page 15) which indicates that loss of strength on high alumina cement concrete can occur at temperatures of well below 40°C.

LIST OF CONTRIBUTORS

The contributor of Section 2 is T. R. Graves Smith.
tie cast into the old concrete. Wall forms setting on footings, whether battered or vertical, must be anchored down to prevent uplift during pouring.

In determining the size of the elements of a wall form, the sheathing, studs and wales should be looked at successively although they are interrelated. The thickness of the sheathing will have been selected on the basis of concrete pressure, which is in turn dependent on height of pour and rate of rise. Another factor affecting the thickness of sheathing is the method of handling. If it is intended to reuse many times and handle with a crane, a heavy form is desirable. From the standpoint of ease of framing and interchangeability, the thickness once chosen will be used from top to bottom.

The size and spacing of studs will depend on the maximum concrete pressure and the allowable deflection of the sheathing, which is often taken at \( \frac{1}{270} \) of the span. The stud spacing is also commonly maintained throughout the wall (varies from about 10 to 24 in.—Fig. 2-7).

The spacing of the wales varies from a minimum at the bottom of the pour to a maximum uniform spacing approaching the top. The spacing of the wales depends on the maximum allowable span of the studs. To determine this refer to Fig. 2-8. For simplicity this figure is set up in terms of a stud spacing of 12 in. Where the actual stud spacing differs from 12 in., compute the concrete load to be used by adjusting the actual concrete load in the same proportion that the stud spacing bears to 12 in. See the example that follows.

The size and spacing of the ties depend on the load on the wales. The live load per foot of wale equals the maximum concrete pressure at the particular wale times the spacing between wales. Ties are manufactured in stock sizes and have safe working loads varying by 2,000- to 5,000-lb increments from 1,500 to 24,000 lb. The manufacturers of form hardware will supply a great deal of information in tabular form on tie strengths and form design generally.

As an example let us consider the form design for a wall 12 ft high and with a rate of rise of 6 ft per hr and being poured at a temperature such that the time of setting is 1 hr. The maximum pressure then is approximately 900 psf. Assuming the use of
CONCRETE CONSTRUCTION

\( \frac{3}{4} \)-in. plywood on 2 by 4 studs, consult Fig. 2-7 for the stud spacing. For 900 psf the figure gives 14\( \frac{1}{2} \) in.; use 14 in. The wale spacing actually is the maximum span for the 2 by 4 stud. Arbitrarily placing the lowest wale 12 in. above the bottom of the pour, there remains 132 in. to the top. For a 28-in. spacing of wale ties and 26-in. spacing of wales the total load on the double wales is 900 \times 2.33 \times 2.17 equals 4,550 lb (Fig. 2-8). Using this value on the 2 by 4 curve, the maximum stud span is 26 in. The maximum pressure stays constant to within 72 in. of the top of pour, at which point it drops off uniformly to zero at the top. The wales may be placed (from the top) at 132, 106, 80, and 54 in. The pressure at 54 in. is 4.50 \times 150 equals 675 psf. For the next wale spacing enter the curve at 675 \times 2.33 \times 2.50 equals 3,925 lb and find the value 30 in. Using this spacing places the top wale 24 in. from the top of wall.

The only remaining information needed is the tie spacing along each wale. A rectangular arrangement of ties is desirable for simplicity. The maximum load per linear foot on any wale is 900 \times 2.25 equals 2,025 lb. Choosing a tie with a tensile working load of 5,000 lb the spacing may be 5,000/2,025 equals 2.42 ft. Use a horizontal spacing along the wales of 28 in. for the ties.

Metal wall forms may be used to advantage where many reuses are possible and where surface texture of the concrete is not a consideration. They consist of standard-size panels of 14- or 16-gage metal whose edges are stiffened either by pressing the sheet into a channel shape or by adding structural angles. Earlier sizes were commonly 2 ft square but they are now manufactured in widths of 6, 8, 12, and 24 in. and lengths of from 4 to 10 ft. Advantages are low cost of erection and stripping, non-inflammability, no distortion with moisture changes, and economy of material cost if reused sufficiently. Since the edges are stiffened integrally no studs are required, making only wales necessary. Individual panels are quickly fastened together by special locking devices. Some metal forms are made with plywood contact surfaces to overcome the objection of surface texture. Most metal forms may be rented or purchased outright.

In some special instances thin precast-concrete panels have been used for wall forms. Their use as forms is secondary since the object is to create a special wall surface in the same way stone facing is used. One primary difference is the fact that the bond with the concrete backing is deliberately prevented by introducing a membrane such as paper between the two. Because of the thinness of the panel (3 in. or less) special supports and face-alignment devices are necessary. The weight of the panels during pouring may be taken in edge bearing on the panel below, using a thin wood filler to distribute bearing and a flexible joint seal. The support against pressure may be provided by tension ties screwed to special inserts cast into the panels when they are prefabricated. The size of panel depends on the architectural treatment and lifting problem, but panels not over 10 by 10 ft appear advisable. A mesh of welded wire fabric in the panel is recommended. The general method has worked very well in the limited use already made of it.

Beam, Girder, Slab Construction

Since the forms for this type of construction are relatively complex, care must be used in the development of the framing scheme to assure ease of stripping and general economy. There exists a variety of framing methods of which one is given below. Variations can be worked out to fit a particular building design.

For economy forms are usually built for about 1\( \frac{1}{2} \) floors and reused. To provide a good concrete finish and to assure regularity of dimensions, all lumber is usually finished rather than rough. The typical form should be drawn up in detail and the lumber ordered as nearly to correct size as possible. For a simple typical form (Fig. 2-9) the planks for beam or girder bottoms should be the exact width of the concrete member, with the beam-side forms resting on the shores. Exterior beam sides are braced by short diagonal struts from the shore cap. The wedges at the bottom of the shores are driven to produce a beam camber of \( \frac{1}{4} \) in. in 10 ft. The shores should be cross-braced when the form is fully erected.
The order of stripping the scheme shown is:

1. Ribbons holding the beam sides at the shore cap
2. Cleats at the intersection of the beam with the girder
3. Ledgers supporting the joists
4. Joists
5. Girder sides, beam sides, floor panels
6. Shores under beams and girders

For the length of time each element should remain in place, see Stripping Forms. All parts should be cleaned immediately after stripping.

Beams and girders must often be reshored after stripping. The reason is that the entire form is stripped for reuse at the next level at a time when the flexural members have not gained sufficient strength to resist a progressive sagging under the dead load plus the weight of shores for the upper floor. In doing this care must be used not to overdrive the wedges under the temporary shores.

A very much simpler type of floor form involves the use of metal pans. These are light-gage pressed-metal forms in the shape of an inverted U which form the soffit between light concrete beams spaced not over 3 ft (Fig. 2-10). The advantages of such a system of forming are economy, speed of erection and stripping, simplicity, and the fact that standard pans may be rented. The disadvantage (not serious) is that the design of the floor must be adapted to available pan sizes, of which many exist.

Forming for floors of flat-slab design is relatively simple (Fig. 2-11). To avoid sagging of the flat slab when forms are stripped it is customary to frame in a small panel supported by a single post at the panel center line and midway between columns. These shores are left in place 3 weeks or more. Steel forms are also manufactured for the floor slab and are economical where several reuses are expected.
Slip Forms

For certain simple structures of great height and fairly uniform horizontal section, moving forms have been in use for many years. These developed from recognition of the fact that large savings in form materials were possible if a short vertical length of form, say not over 4 ft, was reused merely by slightly loosening the form 24 hr after pouring and raising it until it lapped the poured concrete about 4 in., retightening the form, and pouring again. The method of raising the form for small closed shapes such as chimneys is by cables suspended from a center post. For large structures such as circular tanks a number of braced or guyed external posts may be necessary.

To overcome the delays incident to a large number of small pours at 1-day intervals, continuously sliding or slip forms have been developed. These have been used for a variety of structures including chimneys, silos, tanks, bins, and even bridge piers. In this method the form is kept continuously in motion, 24 hr a day from start to finish. The method of raising is usually by screw jacks acting against the vertical reinforcing rods embedded in the structure. The rate of rise of form depends on the rate of strength gain of the concrete. Typical rates in a moderate climate are 7 in. per hr in hot weather and 4 in. in cool weather. Thus, if the form is 4 ft high the concrete is supported by the form about 7 hr in warm weather. These are values which occurred in one massive structure in which the form and appurtenances were also heavy. The actual rate of rise may have to be found by testing the hardness of the concrete still within the lower portion of the form by thrusting in small rods from the top or by observation of the concrete just below the bottom of the form.

The rate of pouring vs. rate of raising should be such as to keep the form filled at all times. Vibrators should not be used and the slump should be as small as will produce a dense concrete, usually of the order of 3 in.

A simple slip form for a bridge pier (Fig. 2-12) has the form proper only 4 ft high. A large yoke of double 2 by 8s spans the pier and takes the thrust of the screw jack and transmits it to the form through a 3/4-in. tie rod passing through the wales. In order to maintain the plumbness of the jackinng rods and reinforcing steel, a template may be framed several feet above the yoke and supported on 4 by 4 posts framed into the yoke. Similarly it is not uncommon to suspend a light platform about 5 ft below the form for concrete finishing operations.

Some precautions to be observed are:

1. Stagger the splices in the jacking rods.
2. Oil the contact surfaces of forms well before the first concrete is placed.
3. Level the form carefully when starting.
4. Maintain the level throughout the operation by using level marks on jacking rods.
5. Test the plumbness occasionally with a transit.
6. Test the general shape of form occasionally.
7. Keep the weight on the moving form balanced.
8. Do not use a harsh concrete mixture.
9. Maintain the fresh concrete surface level and near the top of the form at all times.

Arch Forms

Forms for the centering for a concrete arch are complex and should be carefully designed by one familiar with this type of work. No detailed treatment is given here, but rather certain general considerations. The question of the type of general

framing depends on the span and rise of the arch, the height above solid ground or river bottom, and whether space must be provided for traffic underneath (Fig. 2-13).

Where it is possible to post up from sills on the ground this framing is most economical and deflects least (Fig. 2-13b, c, and d). For short spans where the springing line is high it may be possible to support the arch form on rolled beams (Fig. 2-13a). The types in Fig. 2-13e, f may have to be resorted to where posting up from the ground is too expensive. The type in Fig. 2-13f particularly tends to be flexible and deflections and method of loading must be studied. Where the height is not great but openings must be provided as for traffic, the type in Fig. 2-13c may be best.
The foundations for arch falsework are important. Increasing the bearing area under posts by the use of larger timber sills is not expensive and may prevent difficulties while pouring. If the posts are close together a continuous timber sill will increase rigidity and minimize unequal settlement. For underwater foundations piles or rock-filled cribs may be necessary.

Typical framing for a small arch is shown in Fig. 2-14.

**Pavement on Grade**

The only forms necessary for pavements on grade are the side forms. While simple, these have reached a high state of development because they serve a dual function. They are not only forms but also serve as a running rail for the heavy automatic spreading, screeding, and finishing equipment. While for small jobs such as short lengths of sidewalk a simple wooden form consisting of 2 by 4s nailed to stakes may suffice, almost all highway and airport pavements now are placed inside prefabricated metal forms.

These metal side forms are pressed out of very heavy gage steel (up to \( \frac{3}{4} \) in. thickness) and are generally L-shaped (Fig. 2-15). The height is equal to the slab thickness for screeding purposes. The base has the same width as the height or slightly less. This broad base is necessary for bearing as well as stability. The forms come in standard 10-ft lengths although special lengths are obtainable. The form is stiffened by a diagonal channel-shaped brace which is welded to the base and side at about 3-ft intervals. These braces are punched to serve as guides for the steel stakes which pin the form to the subgrade. Individual 10-ft lengths are held in alignment at the joint by a sliding joint lock.

The pavement smoothness and riding quality depend to a large extent on the care with which the forms are aligned vertically. This is so because the strike-off, screeding and finishing machines ride on the upper edge of the side forms. The forms are ordinarily set from surveyor's stakes by a special class of labor especially skilled in this work.

Special devices such as keyway forms and dowels or hook bolts are often fastened to the form in order to cast into the pavement slab some means of transferring shear or tension from one slab to an adjacent one.

**Bridge-deck Forms**

Concrete pavement slabs on steel bridges present certain peculiar forming problems. These arise from the fact that slab-thickness tolerances are small and the camber of

![Concrete slab](image)

the supporting structural steel is frequently quite variable. To provide for the camber variations, the best-designed slabs have small haunches over the girders (Fig. 2-16a). The scheme in Fig. 2-16b is a poor design resulting in slabs of varying thickness. Material used for bridge-slab forms is usually lumber.
The forms may be supported by either:

1. Hanging from the structural steel upper flanges
2. Bracing up from the lower flanges

Both methods are in wide use.

The scheme in Fig. 2-17 is a very simple one suitable where no haunches are used. Single 2 by 6 ribs are suspended in special preformed wire stirrups which snap over the top flange of the steel beam. The rib may be heavier for slab spans over 6 ft. The distance between ribs will vary with plywood thickness and slab weight. This form is stripped by snipping the wire tie.

A similar scheme which may be used with haunched slabs makes use of special coil-tie hangers (Fig. 2-18). In this case double 2 by 6 or 2 by 8 ribs are hung from coil ties. The ribs in turn carry 2 by 3 or 2 by 4 joists which in turn support the plywood decking. Variation in size of haunch is accomplished by turning the bolts in the coil ties and varying the thickness of the filler. This form is stripped by backing the bolts out of the coil ties which remain in the concrete.

Figure 2-19 shows a method of supporting the forms from the lower flange of the beam. The variation in the haunch is made up here by varying the thickness of filler strip over the ribs and varying the position of the vertical 3/8-in. runner. This form is stripped by knocking out the wedges at the lower flange.

The reason for the variation in the haunches is that the top surface of the slab is set to a smooth profile grade line which is a continuous curve or tangent over a number of spans whereas the cambers produce a series of curves (one for each span) in the structural steel.

Stripping Forms

When one considers the care that goes into the fabrication and erection of forms the necessity for equal care in stripping them from the finished concrete is apparent. Such care should protect the concrete and the form itself so that it may be reused without extensive repair. All forms should be designed for ease of stripping in order to save the form and reduce stripping cost. When the form appears to adhere to the concrete after all ties and braces have been removed, separation should be achieved by inserting wooden wedges and never by prying with bars directly against the concrete. In removing the first piece of a form, the new concrete should be examined to see if the concrete is so green that pieces spall out.

The number of days a form should remain in place is very important especially for long-span members in flexure. It is best to include this information in the specifications to assure compliance. The time varies greatly depending on the structural function of the member and the rate of strength gain of the concrete. The rate of strength gain depends on the type of cement, the water-cement ratio, and the temperature during curing. For these reasons it is impossible to assign fixed intervals each class of forms must remain in place. The most rational approach is to make use of test specimens cured under conditions similar to the concrete in question. When the strength
indicated by the test specimen is double the stress the member must sustain the forms may be removed. This method, while perfectly logical, requires time and effort and in practice is followed only for large important structures where earliest reuse of the form is imperative. The usual method is to apply judgment to an arbitrary table of intervals and observe the concrete when stripped. Minimum time allowances are approximately:

\[ H_T \]

<table>
<thead>
<tr>
<th>Material</th>
<th>Hours</th>
</tr>
</thead>
<tbody>
<tr>
<td>For beam sides and low walls</td>
<td>24</td>
</tr>
<tr>
<td>Columns and walls to 15 ft high</td>
<td>24</td>
</tr>
<tr>
<td>Columns and walls over 15 ft high</td>
<td>48</td>
</tr>
<tr>
<td>Beams and deck slabs</td>
<td>144</td>
</tr>
<tr>
<td>Arches</td>
<td>144*</td>
</tr>
</tbody>
</table>

* These vary so much with span, temperature, and need for reuse of form that it is normal practice to use test specimens to determine the safe stripping time.

From the above table it is apparent that portions of forms on the same structural member may be stripped at different times and forms should be designed to facilitate so doing. Thus beam sides are often stripped 1 day after pouring whereas the soffit form may be in place for 1 week. Arch forms are stripped progressively starting at the crown and working both ways symmetrically toward the springing line.

Wooden tapered dowels used to form holes for anchor bolts should be loosened by a slight turn a few hours after pouring and removed the next day to avoid swelling so that the concrete is cracked or dowels so jammed as to require drilling. Open holes must be sealed in cold weather to avoid destructive expansion of water due to freezing.

**Cold-weather Concreting**

Because the setting of concrete is a chemical action, its rate is markedly affected by the temperature of the concrete mass. Since most concrete is cast outdoors, the rate of setting is reduced in cold weather unless certain steps are taken. When the temperature of the concrete mass is below 50°F little or no gain in strength occurs. However, the temperature of the concrete and the air temperature are not to be confused since the concrete temperature may be high when placed and the action of setting produces appreciable internal heat.

The pressure of construction schedules has made concreting in cold weather necessary. The seriousness of the effect of the low ambient temperature depends on:

1. Prevailing air temperature and wind
2. Initial concrete temperature
3. Normal rate of setting
4. Shape and size of the concrete section or ratio of exposed area to volume
5. Degree of protection
6. Amount of external heat supplied

The problem has been attacked through each of the above items except item 4. Where feasible, concrete is poured only during suitable seasons as in the case of pavements. The initial concrete temperature may be raised by heating the water or fine or coarse aggregates or combinations of the three. There is an upper limit on the temperature of these ingredients to avoid flash setting of the cement and damage to the aggregates. The upper limit is commonly 150°F. Since the specific heat of the cement and aggregates is close to 0.2, the temperature of the fresh concrete

\[
T = \frac{0.2(T_a W_a + T_c W_c) + T_w W_w}{0.2(W_a + W_c) + W_w}
\]

where
\[
T = \text{temperature} \\
W = \text{weight} \\
a = \text{aggregate} \\
c = \text{cement} \\
w = \text{water}
\]

The water is fairly easy to heat using a portable oil-fired boiler in the water-supply line. To heat aggregates on very small jobs, the aggregates are piled over a section
of metal culvert pipe and an oil flame is blown through the pipe. In larger installations steam coils are placed in the aggregate bins over the batching equipment. The initial concrete temperature should not be any higher than necessary to prevent dropping below about 50°F for first 72 hr. Slow setting at low temperatures produces the best concrete.

The normal rate of setting may be increased by the use of more cement or high-early-strength cement or the introduction of accelerators. Accelerators should be used only by one familiar with their limitations and disadvantages, and the quantity and method of mixing with the concrete should be carefully controlled. These data are obtainable from the manufacturers of the accelerators.

The degree of protection required will depend on all the other items. For massive concrete sections the internal heat developed may be all that is required to maintain a suitable setting temperature. Exposed corners and edges may have to be protected since they will lose heat most rapidly. Salt hay as an insulation held in place by lashed tarpaulins may suffice. With concrete buildings it is often sufficient merely to drape the entire outside with tarpaulins. Thin walls are sometimes protected by stuffing the space between studs with salt hay and securing it in place with burlap tacked to the studs. The general problem is to reduce the loss of internal heat by radiation and conduction.

External heat may be supplied by live steam or steam coils or small burners. The live-steam method has the advantage of heating without drying. The small burners have the disadvantage of danger of fire or suffocation of the watchmen.

In general any method or combination of methods which keeps the concrete at or above 50°F for 3 days with a gradual reduction of temperature afterward will prevent damage to the concrete.

**STEEL REINFORCEMENT**

Since reinforced concrete depends for its structural integrity on the steel reinforcement to a large degree, it is essential that field operations be so conducted as to assure that the design is exactly followed. This means that:

1. Inspection before shipment from fabricator must include tests to assure that the specified grade of steel has been used. A system of tagging must be adopted to permit identifying tested bars or mats on receipt at the site.

2. Inspection in forms must be made to assure that the correct sizes and shapes have been used and that the bars are in the correct positions with respect to each other and the forms.

3. Inspection at the forms should include checking to assure that the steel is supported rigidly enough to prevent distortion or displacement from the planned position under loads from construction operations.

Specifications state the grade of steel permitted (commonly ASTM A 15). In some cases, rail steel (ASTM A 16) is permitted. Unless a foolproof system of inspection and tagging is adopted, there is the risk of using reinforcement steel deficient in strength and ductility, characteristics which cannot be recognized by visual inspection at the site. The tests at the source commonly include visual inspection of finish, variation from nominal weight, and tensile and bend tests.

Reinforcement steel is ordered to be shipped well in advance of need for concreting. In planning storage of steel at the site, attention must be given to ease of selection and avoidance of deformation. As a matter of economy, steel is commonly stored outdoors but raised off the ground to avoid contact with dirt, oil, and grease and to reduce rusting. It is always desirable to have available in site storage extra bars above the design requirements in order to replace lost or misplaced bars and avoid costly delays in concreting. Substitution in the field of bars differing from the plan should be done only with the approval of an engineer fully familiar with design practices.

Before the start of erection, the design should be examined to see if closely spaced bars in beams or arch ribs will prevent passage of coarse aggregate or vibrator head. The steel detailer will have attempted to use the longest practicable bars to reduce costly splices. The field force should consider the maximum lengths which can be handled or maneuvered inside the forms. No general rules can be laid down.
In many localities, labor unions require that fabrication be done at the site by either hand or power bender. This necessitates setting up bending tables whose size depends on the longest bars used. The tables contain holes for pegs around which bars are bent and mandrels to control the radius of the bends. As with all site fabrication, this is more costly than shop fabrication.

The plan spacing of bars is maintained by tying each bar to other bars which lie at right angles to the first. If the design does not call for the second bars (such as temperature bars in a wall), they must be provided as spacers. The ties may be either fabricated bar ties or soft iron wire (about No. 16 gage) which is twisted about the bar intersection by hand, using pliers. The latter method is far more common. The fabricated bar ties are made of high-carbon spring wire (Fig. 2-20). The fabricated tie size to order depends on the combined diameters of the two bars to be tied. They are sold in lots of 500 to 2,000.

By either method of tying, it is not necessary to tie every intersection. Just sufficient ties are required to provide rigidity to prevent displacement during concrete pouring. The twisted ends of the tie wires should point away from the adjacent form or free concrete surface.

Where conduits are placed in concrete, care should be taken not to place them too close to forms so that they are completely embedded in concrete. In thin slabs or walls, the position of the conduit and sequence of placing should be carefully chosen in advance, since with several planes of reinforcement steel, the proper placing of conduit becomes difficult. Also, since two trades at least are involved, costly delays may result.

The relation of bars, one to the other, is fixed by the ties or in preassembled units sometimes by tack welds. However, it is still necessary to secure the steel, as a whole, rigidly in place relative to a concrete surface. This is done in a variety of ways.

In footings, the bottom mats are commonly supported from the ground by masonry blocks of the proper thickness. These are cast in advance on the job with noncorrosive wires embedded in them to hold the masonry blocks in place by tying to the reinforcement. Such blocks should not be used earlier than 7 days after casting. Also, they should not be more pervious than the footing concrete.

In pavement slabs on subbase material, the mat of reinforcing steel may be supported in two ways. One common method is to place the concrete on the subbase up to the elevation where the mesh belongs, interrupt the concreting to lay the mesh on the fresh concrete, and then continue concreting over the mesh. Unless the operation has been carefully planned, the interruption in sequence makes for higher labor costs. The other method is to use mortar blocks similar to what is described above for footings. Either method is good practice.

In slabs, beams, girders, and columns, a variety of fabricated devices are available as described below. These should be chosen and spaced to assure strength to resist construction loads without displacement or distortion of the reinforcing steel. They should be so shaped as to minimize void formation in the concrete and exposure on finished concrete surfaces. When points of these supports bearing on forms will show in finished surfaces such as the undersides of beams, it is not uncommon to require that they be galvanized. The cost of galvanizing bolsters and chairs adds about 30 to 50 per cent to their cost.

In Fig. 2-21 there is shown a slab bolster to support the lower plane of slab reinforcement and below it a slab spacer. The essential difference is that the corrugations in the bolster are on 1-in. centers and may be used where slab-steel spacing is on integral inches. The slab spacer is fabricated with the legs set at the requisite bar spacing. Each is available in 5- and 10-ft lengths.

In Fig. 2-22 are shown low and high chairs for individual bars. They are fabricated.
to the heights called for by the plans. Companies fabricating these devices will make
the plans showing the use and layout of their products as a service with the order.

The proper spacing of these devices, relative to the flexural strength of the rein-
forcement steel, is a more serious matter than generally recognized, since mainte-
nance of the correct position of the steel is of prime importance. The principal construction
load is the weight of the skilled labor walking over the steel before and during pouring.
If these loads cause stresses in the steel in excess of the yield point, the steel is perma-
nently deformed and seriously displaced from its design position. Attempts to
straighten steel so deformed usually result only in additional kinking and final
rejection.

![Slab bolster](image1)

![Slab spacer](image2)

**Fig. 2-21. Slab steel bolster and spacer.**

![Bar chair](image3)

**Fig. 2-22. Reinforcing bar supports.**

Standard spacing of fabricated supports for various types of reinforced-concrete
construction has been adopted by the American Concrete Institute in their Recom-
mended Practice for the use of Metal Supports for Reinforcement (ACI 319-43).

**Table 2-1. One-way Slab Construction**

<table>
<thead>
<tr>
<th>Span</th>
<th>Rows of slabs</th>
<th>High chairs and 5/16-in. support bars in slabs 4 in. and thicker*</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>bar spacers</td>
<td>Support bars</td>
</tr>
<tr>
<td></td>
<td>for panel</td>
<td>1 row at beam</td>
</tr>
<tr>
<td>0–6 ft steel continuous</td>
<td>1</td>
<td>2 rows at beam</td>
</tr>
<tr>
<td>0–14 ft steel not continuous</td>
<td>2</td>
<td>2 rows at beam</td>
</tr>
<tr>
<td>14–20 ft steel not continuous</td>
<td>3</td>
<td>2 rows at beam</td>
</tr>
<tr>
<td>20–26 ft steel not continuous</td>
<td>4</td>
<td>2 rows at beam</td>
</tr>
</tbody>
</table>

* Continuous high chairs may be substituted for individual high chairs and support bars.

**Table 2-2. Ordinary Beam and Joist Construction**

Bars 1 in. square and smaller

<table>
<thead>
<tr>
<th>Clear spans, beam (or joist)</th>
<th>No. of beam (joist) chairs</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Single layer of bars</td>
</tr>
<tr>
<td></td>
<td>Lower</td>
</tr>
<tr>
<td>Over 0 ft to 14 ft</td>
<td>2</td>
</tr>
<tr>
<td>Over 14 ft to 23 ft</td>
<td>3</td>
</tr>
<tr>
<td>Over 23 ft to 30 ft</td>
<td>4</td>
</tr>
</tbody>
</table>
Table 2-3. Flat Slabs

<table>
<thead>
<tr>
<th>Spans (center to center of columns)</th>
<th>Supporting spacers</th>
<th>High chairs and 3/4-in. support bars under ends of bent bars*</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Column strip or direct band</td>
<td>Middle strip or diagonal band bottom layer</td>
</tr>
<tr>
<td>Over 0 ft to 18 ft.</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>Over 18 ft to 26 ft.</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>Over 26 ft to 36 ft.</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>Around interior columns.</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>Around exterior columns.</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>Around corner columns.</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>In interior panels.</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>In exterior panels.</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>In corner panels.</td>
<td>...</td>
<td>...</td>
</tr>
</tbody>
</table>

*Continuous high chairs may be substituted for individual high chairs and support bars.

At points where much walking is done prior to pouring concrete, such as near ladders or entrances to the area, it may be necessary to reduce the spacing of supports because of frequency of heavy impacts due to workmen carrying loads. The spacing to use is a matter of judgment based on observation.

When reinforcing-steel bars or mats are received at the site, they should be carefully examined for possible damage in shipping, and if bent, they should be rejected, unless easily straightened. As far as possible, heating the steel in order to bend or straighten it should be avoided, but if not avoidable the heat should not be greater than to produce a cherry red in the steel.

In strange territory, inquiries should be made as to the influence of local mixing water (if suspected) on the steel. In some islands, where salt water is used for mixing concrete, serious failures have occurred because of corrosion of the steel. In these localities, galvanizing has been used with success.

Where dowels are used between successive pours, they must not be thrust into the concrete after it has taken its initial set. Similarly, exposed bars must not be subjected to impact until the embedding concrete is at least 7 days old. It is a good practice before any concrete pour to observe whether protruding steel is long enough to provide the necessary lap with steel in the next pour. Where protruding bars are to be exposed to the weather for over a month, a protective coating of mortar should be brushed on within 7 days after exposure. This coating should be brushed off before embedment.

Where heavy stray electric ground currents exist, it is desirable to ensure positive separation of the reinforcing steel in the footings from embedded metal piles and similarly separate superstructure anchor bolts. The object is to prevent closed circuits which promote corrosion by electrolysis.

Many specifications do not permit double-twisted bars which contractors occasionally try to substitute for standard bars.

All the above points should be checked well in advance of the start of a concreting operation.

**BATCHING AND MIXING**

Concrete is the only major structural material manufactured at the site. To attain the desired properties of strength, durability, watertightness, etc., homogeneity is necessary. This requires the maintenance of constant proportions of ingredients, within narrow limits, and thorough mixing.
To assure uniformity and simplify the checking of the proportions, the practice has developed of making concrete in separate fixed volumes called batches. This method is opposed to continuous mixing, where the ingredients are fed into one end of a machine mixer in a steady stream and discharged steadily from the other end. For simplicity, we shall treat batching and mixing separately.

Batching

In simple jobs, up to perhaps 10 cu yd, the amounts of each solid ingredient are sometimes measured by loose volumes using measuring boxes, wheelbarrows, or even shovelsful. In this case, the water quantity is judged entirely by observed consistency of the wet concrete. Using this simple method, the mix is by arbitrary proportions, such as 1:2:4 or 1:3:6 (cement:sand:coarse aggregate) by loose volumes. It is frequently used for such jobs as sidewalk repairs, small garage floors, or simple farm structures. In many developed areas, hand measuring and mixing are no longer necessary, since one can buy commercially mixed and delivered concrete in quantities as small as 1 cu yd.

![Fig. 2-23. Wheelbarrow scale.](image)

![Fig. 2-24. Concrete batch plant.](image)

In batching by volume, it is generally advisable to set volumes in terms of whole bags of cement. Fractional bags lead to variable proportions.

Aside from the labor involved, volumetric batching leads to inaccuracies due to variable bulking of the sand from variation in surface moisture. It is not difficult to set up a simple system for weighing the materials in wheelbarrows. All that is required is a platform scale with an enlarged platform sufficient to carry a wheelbarrow, set between two approach runways of planks. The resulting improvement in the concrete is well worth the effort (Fig. 2-23).

For all structural concrete and other major concrete work such as highway pavements, more exact batching is mandatory and all ingredients are measured by weight. Water batching by weight eliminates variation due to temperature.

For jobs in excess of 10,000 cu yd of concrete, it is common to set up a batching plant at some central position at the site. The principal advantage over commercially available concrete is completeness of control leading to better concrete and lower costs. Such a batch plant consists of a storage area for aggregates, elevated bins for each solid material, hoppers for weighing, and concrete mixers. The last three are usually mounted vertically one above the other to permit flow by gravity (Fig. 2-24).

The aggregate and cement bins are at the top. A common arrangement for moderate-sized plants is to load the coarse and fine aggregates into bins by portable crane and clamshell bucket from a stockpile within boom reach. The cement is raised in modern plants by a vertical endless chain of buckets placed alongside a storage silo which is kept supplied by truck or freight car.
The next lower level contains weighing hoppers and water batcher. Since the operator and inspector work at this level, it is frequently enclosed for comfort. The material flows by gravity from the bins above into the hoppers.

The next lower level contains the mixer or mixers. The material flows down by gravity from the batchers above into the mixer or, if dry batches are used, by separate chute into dry-batch trucks.

The bottom level is for the transportation of concrete by any of several methods mentioned below.

Batched material, according to the plant layout, may be discharged into:

1. A mixer below the hopper with delivery of wet concrete by agitator trucks or belt conveyor
2. Transit-mix trucks which mix at the forms
3. Dry-batch trucks with separate compartments for each batch (common on paving jobs)

Modern batching plants have many automatic features, including a control panel with large batcher dials at the top for continuous inspection, automatic recording tracer arms just below, and under them the control push buttons. In an installation of this kind, it is possible to preset the weights for a number of different mixes or batch sizes and weigh each fully automatically.

Partially behind the control panel are the separate suspended hoppers for aggregate and cement weighing. The cement hopper is frequently placed near the center in order to provide a more direct drop for this fine material and to assist in premixing it with the aggregates when all materials are discharged simultaneously. The water batcher may also batch by weight, although many installations still batch water by volume. The weight method is more accurate and no compensation for temperature is necessary as when heated water is used for winter concreting.

At this batching level, determinations are usually made of the surface moisture on the aggregates in order to correct the scale weights to compensate for surface moisture. The methods of moisture measurement are covered under Quality Control.

The above general arrangement for a modern batching plant is a common one because, for appreciable volumes of concrete, it is economical and does not require much space. Endless variations are possible depending on the site and the requirements of the job. For a discussion of factors influencing selection see Planning the Plant.

Some points to look for in a good batching installation are:
1. Batch all ingredients by weight, including water.
2. Installation should be substantially built on a concrete foundation.
3. Bins to have adequate capacity—depends on size of day’s work.
4. Bin partitions to extend 3 ft above edges to prevent accidental mixing while loading.
5. The cement should be fully protected from the weather.
6. There should be no free fall of cement to batch trucks—a canvas boot should be provided.
7. Baffle boards in bins should be provided to assist drainage and keep water out of hopper.
8. Installation should have platforms to permit easy inspection and maintenance.
9. Weighing room should be fully protected from weather for the comfort of operator and inspector.
10. Bins should be adaptable to heating aggregate for winter concrete if contemplated.
11. Weighing hoppers should open parallel to batch-truck compartments to facilitate correct loading.
12. Weighing hoppers should be arranged for positive discharge.
13. Weighing mechanism should be accurate to within:
   a. 3/4 per cent for cement.
   b. 1 per cent for aggregate.
   c. 3/4 gal per cu yd of concrete for water.
14. Weighing hoppers should be designed for easy recalibration and calibrating weights should be provided.
15. Inlet and discharge gates to hoppers should be interlocked.
16. A rapid and simple method should be provided for determination of surface moisture on the aggregates.
17. There should be a water-storage tank which makes water measurement independent of supply-line pressure.

Mixing

Proper mixing is absolutely essential for good concrete. Proper mixing means the attainment of a condition where the four ingredients are uniformly distributed through the mass. In hand mixing, this is usually judged by uniformity of color and texture. Since mixing involves effort, either human or mechanical, it is frequently neglected, hence is one of the operations requiring close inspection.

Very small or inaccessible jobs still occasionally use hand mixing, which is laborious and costly, hence to be avoided. A platform is necessary to facilitate mixing and to minimize the introduction of dirt and loss of mortar. The platform, which may be of wood, usually tongue and groove, must be clean, level, watertight, and preferably nonabsorbent. Its size depends on the total amount to be mixed and number of men engaged. It is usually not less than 8 ft square, which would suffice to mix about 1 cu yd at a time.

A common method of mixing by hand is to spread the sand over the platform to a uniform depth and then spread the cement over the sand. The sand is then shoveled over the cement, using a turning and spreading motion. The whole mass should be so mixed with shovel or hoe until a uniform color is obtained. A crater is then formed in the center and water is added slowly. To hasten absorption of water, material from the edge may be shoveled into the wetter center mass. This is continued until a consistency somewhat wetter than the required final concrete consistency is obtained. The coarse aggregate is then spread over the mortar and the whole mass is turned until a uniform texture results. If too dry, additional water may be sprinkled on and the mass again turned.

Hand-driven mixers as small as 11/4 cu ft capacity are available. These are inexpensive and make small jobs less laborious.

For all jobs except the small ones referred to above, machine mixing is commonly resorted to. Machine mixing is far more economical and can, when properly handled, produce perfectly homogeneous concrete.

A mechanical mixer generally consists of one or more drums with open ends mounted with axis of rotation horizontal or tilted and rotated by gasoline or electric motor (Fig. 2-25a).

Blades attached to the inner surface of the drum (Fig. 2-25b) are set at an angle to the axis so as to pick up the ingredients as the drum rotates and spill them over one another and move them longitudinally. The rate of rotation is the most rapid which will avoid any tendency to adhere to the drum by centrifugal force, since slower speeds would produce less mixing action. This speed is approximately 200 fpm peripherally, which means that the larger drums rotate more slowly. Each manufacturer has established the optimum speed for his mixer by experimentation, and this speed should be used for best results.

The drum is loaded by an inclined trough which projects into the opening at one end. Horizontal mixers are commonly unloaded by a chute so mounted that it may be thrust inclined into the discharge end of the drum. Other types unload by tilting the entire drum. The tilting type is considered superior because it discharges more quickly, with less segregation, and is easier to keep clean.

Mixing water is measured in small installations by volume in a closed vertical cylindrical tank which has an adjustable overflow. The overflow is a sliding vertical pipe set on the axis of the tank and so calibrated that the tank water will overflow into the pipe when the desired volume of water in the tank has been reached.

Mixers come in a number of sizes and are rated according to the size batch of con-
Concrete they can efficiently mix. The sizes vary from $1\frac{1}{2}$ cu ft to 5 cu yd. Common sizes are $3\frac{1}{2}$, 6, 11, 16, and 28 cu ft. Most specifications permit 10 per cent overloads over the rated capacity.

Some typical dimensions for small concrete mixers are given in Table 2-4.

<table>
<thead>
<tr>
<th>Rated capacity, cu ft</th>
<th>Drum diam, in.</th>
<th>Drum length, in.</th>
<th>Drum speed, rpm</th>
<th>Tank capacity, gal</th>
<th>Wt, lb</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.5</td>
<td>36</td>
<td>28</td>
<td>20</td>
<td>7</td>
<td>1,250</td>
</tr>
<tr>
<td>6.0</td>
<td>42</td>
<td>34</td>
<td>19</td>
<td>12</td>
<td>2,600</td>
</tr>
<tr>
<td>11.0</td>
<td>49</td>
<td>41</td>
<td>17</td>
<td>19</td>
<td>4,200</td>
</tr>
<tr>
<td>16.0</td>
<td>58</td>
<td>46</td>
<td>$16\frac{1}{2}$</td>
<td>27</td>
<td>5,600</td>
</tr>
</tbody>
</table>

Research indicates that strength and other desirable characteristics are improved with increasing time of mixing up to some maximum, which may lie between 10 and 20 min. The greatest rate of improvement occurs in the first few minutes, and a point is soon reached where the gain in strength is not worth the loss of output from further mixing. This optimum time varies with several factors, but principally with size of batch. For stationary mixers, the following times of mixing are recommended:

<table>
<thead>
<tr>
<th>Size of batch, yd</th>
<th>Time of mixing, min</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 or less</td>
<td>$1\frac{1}{2}$</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>5</td>
<td>$2\frac{1}{2}$</td>
</tr>
</tbody>
</table>

By time of mixing is meant the interval from the instant when all ingredients except the last 25 per cent of water are in the drum until the start of discharge.

Contractors frequently protest any mixing requirement over 1 min, and to assure greater mixing times, this requirement should be plainly stated in the specifications. Actually, on many concrete jobs, other than paving, a time study will show that the
output of a plant is slowed more by the rate of transporting away from the mixer than by the added mixing time; hence it is unnecessary to shorten the mixing time.

Many specifications require that mixers have interlocks to:

1. Prevent loading a second batch into the mixer before the first is discharged
2. Prevent discharging the batch before a set time of mixing has elapsed from the instant the charge was in the drum

Prolonged mixing will actually decrease the strength of concrete and also decreases air entrainment to a marked degree. Mixing should not be continued beyond the point where initial set takes place under any normal circumstances. Some specifications state that mixing must not be continued beyond three times the specified minimum.

Where it is desired to test the performance of a mixer, to establish the correct mixing time, the uniformity of the mixed concrete may be measured by comparing the air-free unit weight of three mortar samples taken from different parts of the batch. The maximum suggested allowable variation is 2.3 lb per cu ft. Greater variation indicates the mixing time should be increased.

Some points to observe regarding mixers and mixing:

1. The mixer drum must be tight to avoid loss of mortar. The valves must not leak mixing water.
2. If the water-source pressure is variable or low, a reservoir tank should be installed before the measuring tank.
3. The mixer should be so designed as to produce an end-to-end movement of the contents of the drum.
4. The manufacturer's specified mixing time should not be excessive as compared with other makes.
5. The specified mixing time should be carefully adhered to.
6. The mixer must not be overloaded. No more than 10 per cent overrated capacity should be used (29.7 cu ft for a rated 1 cu yd mixer).
7. The mixer design should include an interlock to prevent discharge before the set time of mixing.
8. In charging, the materials should enter the drum rapidly.
9. The blades in the drum must be kept clean. Worn blades must be replaced.
10. The discharge mechanism must be so designed that segregation is not promoted in the act of discharging.

In order to take advantage of the interval which must elapse between batching and delivery at the form, the practice of truck mixing or transit mixing has developed. This scheme, while now well established, has certain peculiarities which must be allowed for and special precautions must be taken to secure good concrete. The truck-mounted mixers are large (from 3 to 12 cu yd); hence the mixing time must be carefully controlled. A common practice is to delay admission of mixing water until arrival at the site, where the time of mixing can be positively controlled. Because of unforeseen delays in transit from batch plant to site, it is difficult to maintain a constant delivery pace. A solution is, of course, to use an excess of trucks, but this is costly. Exposure of the drum to direct rays of the sun can result in generation of injurious heat in the mix. To offset this, the mixing water should be as cold as feasible, and the drum should be painted white. Since control of consistency is more difficult than with stationary mixers, an opening in the drum should be provided to observe consistency before discharge. A reservoir tank, in addition to the measuring tank, is usually provided in order to flush out the drum after discharge. This tank is frequently useful in providing small additional amounts of mixing water where the slump is too low after the first mixing.

Another type of mixer, which is very widely used, is called a paver (Fig. 2-26). Although developed primarily for paving highways, it is also used in many situations where it is expedient to deliver batched materials by truck. It is seen to consist essentially of a frame supported on caterpillar treads with a concrete mixer in the center, a loading skip at one end, and a horizontal boom carrying a traveling bucket at the other. The skip is a very large scoop wide enough to receive the rear end of
the batch truck and mounted on a shaft so that its contents may be discharged into
the near end of the mixer when the skip is raised by a cable. The boom at the dis-
charge end of the mixer carries a concrete bucket on rollers, permitting the bucket
to be run out the full length of the boom. This arrangement gives flexibility in
depositing concrete anywhere within the side forms in a paving operation. When a

![Concrete paver](image)

*Fig. 2-26. Concrete paver.*

paver is used for operations other than paving, the boom and traveling bucket are
frequently removed. Common rated mixer sizes on pavers are 27 and 34 cu ft.

**TRANSPORTING AND PLACING**

Under this heading the handling of wet concrete from the mixer to the place of
final deposition will be covered. A natural further subdivision is transporting (con-
crete from mixer to forms) and placing (at or in the forms).

Since the function of the mixer is to produce homogeneity and later handling of the
wet concrete tends to produce more or less segregation and loss of slump, for best
results the mixer should be as close as feasible to the point where the concrete is to be
placed. A prime example is the direct discharge of small portable mixers into curb or
low wall forms. Some opposite extremes, due to necessities of the work, are floor
arches in high buildings, high-level bridge decks, and tunnel linings. The practical
consideration of economy frequently operates to place the mixer far from the forms;
hence engineers should consider the desirability of clearly stating in the specifications
what types of handling will be acceptable.

Methods of transporting and placing which permit the use of drier mixes reduce
the water-cement ratio and hence are economical of materials and produce better
concrete. Drier mixes also reduce segregation. The method of transporting should
in general not control the slump required, but rather the method selected should be
adapted to the slump specified.

Some general methods of transporting in wide use are:

1. Discharge directly into forms or into short chutes
2. Delivery by truck (agitating or nonagitating)
3. Buckets
4. Buggies (hand-propelled or motor-driven)
5. Pumping
6. Belt conveyors
7. Tremie
8. Tower and long chutes
9. Combinations of above

Short Chutes

By short is meant chutes up to approximately 20 ft in length. Chutes of any length are usually regarded by engineers as undesirable because they tend to produce segregation and loss of slump. However, short chutes with proper lower-end treatment do not offend seriously in this regard and have many applications because they are simple to use and economical.

When small portable mixers or large transit-mix trucks can easily reach a point near and slightly above the concrete forms, chutes are the normal method of transporting from mixer to form. If metal, they are commonly 14 or 16 gage, half round in section, and are stiffened by occasional welded tension ties across the top and by metal angles along the edges. The width across the top is commonly about 14 in. and length 10 ft (Fig. 2-27). Such metal chutes have a life of about 2,000 cu yd of concrete. Wooden chutes are frequently constructed on the job of three 1- by 8-in. planks, held together by wooden battens and side pieces of 1- by 6-in. plank stiffened by braces from the extended battens. These wooden chutes should be lined with sheet metal, which can be 16-gage black or galvanized iron. Since it is usually necessary to move chutes frequently (or set up several at different points) they should be light enough to be moved readily by hand.

In setting up chutes, it is important that they be braced sufficiently to prevent serious deformation or actual collapse under the weight of concrete. The slope of the chute is generally recommended to be between 1:2 and 1:3 but it depends on site conditions and the consistency of the concrete as well as the smoothness of the chute. The important thing is that it should be steep enough to permit concrete to flow without assistance. At the discharge end of the chute some sort of vertical trunk should be attached to prevent the segregation which results from a free discharge.

Chutes should be cleaned by washing down immediately after each use or prolonged interruption in delivery of concrete. In doing so, care must be used to discharge the wash water away from the concrete.

The chutes described above will each easily move 2 cu yd per min while the concrete is flowing. They will, therefore, not be a bottleneck when used with a concrete mixer of 1 cu yd capacity.

Delivery by Truck

Under certain job conditions mixed concrete must be transported relatively long distances from the mixer to the forms. An example is a highway-paving job where a central mixing plant is used. For some years engineers have insisted that the truck body transporting the wet concrete must be designed to continue the mixing operation at a slower rate in order to prevent segregation, bleeding, and loss of slump in transit. This has resulted in the development of so-called agitating trucks which resemble transit-mix trucks but rotate more slowly. The speed is commonly 2 to 6 rpm. This is sometimes called shrink mixing.

In some cases regular transit-mix trucks are used as agitators, rotated more slowly than their rated mixing speed while in transit. In such cases a 50 per cent overload is permitted.

In order to reduce the initial and operating costs, the practice has developed of using nonagitating trucks where permitted. At first ordinary dump trucks such as those used for hauling earth were used, and strong resistance developed among engineers to the nonagitating trucks because of segregation, bleeding, and loss of slump in transit. In recent years very satisfactory types of bodies for this purpose have been developed (Fig. 2-28). These have a body built with rounded inside
edges and corners, cutoff gates for accurately controlling discharge, baffles for maintaining a constant head of concrete on the discharge end, producing flow from the bottom of the concrete load, and vertical tipping of the body to produce complete and clean discharge. The rated capacities are 2, 3, and 6 cu yd and most specifications permit a 10 per cent overload. These modern bodies, developed especially to overcome objections to nonagitating trucks, have been thoroughly tested in use and found to have other important advantages than economy. Since no water can be added in transit, the delivered concrete is quite uniform from load to load in consistency and the lack of mechanical agitation minimizes change in air content and the development of excessive heat.

The choice of agitating vs. nonagitating trucks for a particular job depends on length of run, smoothness of road traveled, and desired consistency of the concrete. With air-entrained concrete of less than 3-in. slump, modern nonagitating trucks have been used on a 4-mile run with a loss of slump of only 1/2 in. and an air loss of 1/2 per cent.

Whatever type is used it is most essential that delays in transit or handling to the forms be kept to a minimum. It is necessary to establish a system of recording the time of discharge from the mixer into the truck and observe the time of delivery in order to avoid the use of concrete which has taken an initial set. Many specifications set a maximum interval of 45 min from mixer to form.

Buckets

Buckets are metal containers for mixed concrete which come in a variety of sizes and types. Where the construction method does not involve retaining the concrete in the bucket for too long a period or subjecting the bucket to vibration which produces segregation or bleeding, the use of buckets is satisfactory. A loss of slump of 1 in. or noticeable bleeding would be regarded as objectionable.

For some years a type of bucket suspended on a horizontal axis which discharged by tilting was used but in modern practice buckets discharge through a gate in the bottom. For efficient use the bottom gate must be self-closing, nonjamming, and permit partial discharge of the load when required.

Buckets come in a variety of sizes and shapes. For general use the most efficient are vertical cylinders open at the top with discharge gates of ample size centered in the bottom (Fig. 2-29). Lightweight buckets for general use with cranes and for concrete of normal slump with aggregates up to 3 in. are obtainable in sizes from 1/2 to 1 1/2 cu yd capacity. Larger buckets for lower slump concrete have larger gates and come in sizes from 3/4 to 4 cu yd. Heavy-duty buckets which will handle aggregates up to 6 in. in concrete of very low slump have especially designed discharge gates and the sizes range from 2 to 12 cu yd. On the largest buckets the effort required to operate the gate sometimes requires special air-driven mechanisms and an air tank is fastened to the buckets.

It is common practice to make the bucket the same size as the concrete batch for smoothness of operation and to assure that the mixer is fully discharged each time.

There are other types than the vertical cylinder designed for special purposes. Where the charging height is low a special type of "laydown" bucket has been developed which can receive concrete while lying on its long side but turns through 90° when lifted (Fig. 2-30).

For depositing concrete under water a special type of "bottom-dump" bucket is available which has the following features: Its discharge gate is necessarily remote-controlled by a separate line after the bucket has landed on previously fresh placed concrete. Since the placed concrete is soft the bucket will sink slightly into its surface and contact between the bucket concrete and surrounding water is minimized.
One disadvantage of this method of underwater concreting is the tendency for the turbulence of the water during descent to wash out the mortar in the concrete near the top of the bucket. This effect can be reduced by using a funnel top on the bucket. Turbulence at the bottom is decreased by raising the bucket very slowly as the concrete is being discharged. Since some surrounding water will usually enter the concrete the mix should be as dry as can be discharged from the bucket. For the same reason the cement content is kept high (not less than 7 bags per cu yd) and the design assumes a low strength.

There are a number of bucket accessories. For example, when the openings in the top of a form are small it is not uncommon to place a small movable hopper over the opening to receive the bucket discharge. Below the hopper there may be suspended steel drop chutes which are heavy-gage sheet-metal tapered pipes which come in lengths of 24, 30, and 36 in. and discharge diameters of 5 to 12 in. These are coupled together with short lengths of chain which come attached to each length of chute. An alternative to the steel drop chute is the rubber elephant trunk, which is a heavy rubber hose available in diameters of 8, 12, 16, and 20 in. and up to 30 ft long. Like the steel drop chutes they are used to reduce segregation wherever the free drop of the concrete exceeds about 5 ft.

Buckets are transported in a variety of ways from mixer to form. These may be by crane, cableway, railroad car, or truck. The crane offers one of the most flexible methods of handling mixed concrete. It has the great advantage of permitting deposition of concrete from the bucket in its final position. This method is widely used in building and bridge construction or any situation where mixed concrete has to be elevated and deposited over a wide area. For efficiency the crane used must be well adapted to the particular concrete operation. Its lifting capacity, boom length, and speed must be adequate.

Whenever the time required to move the bucket from the point of receipt of concrete to the point of discharge and back exceeds the cycle time of the mixer, the crane and bucket are limiting the plant output. Two buckets are usually used with each crane to save the time necessary to fill the bucket. For the same reason, an attempt is made to spot the crane relative to forms so that no boom movement is necessary. In using crane and bucket, experienced operators are necessary to avoid damage to forms and reinforcing steel resulting from impact by the bucket.

Cableways are a special method of transporting buckets over large areas or great lengths, usually where the concrete to be placed lies in a valley. The cableway consists essentially of two steel towers which may be moved on tracks along ridges at opposite ends of the job and connected by wire-rope cables for conveying and raising
or lowering the bucket. The method is expensive in first cost and justifiable only where large volumes of concrete are to be placed under highly special conditions. Typical examples are the large concrete dams constructed in recent years.

**Concrete Carts or Buggies**

These are metal carts which may be hand-propelled (Fig. 2-31a) or motor-driven (Fig. 2-31b). The manual type is usually a two-wheeled rubber-tired cart of from 6 to 12 cu ft capacity so balanced that it will tip easily and discharge rapidly. The widths are commonly 30 to 36 in. to permit passage through doorways or along narrow runways.

Buggies are often used in constructing slabs of moderate extent such as for reinforced-concrete buildings, small bridge decks, or sidewalks on grade. A necessary accessory is a runway to support the buggy over the reinforcing steel and provide access to the entire slab area. The runways are usually wooden, constructed in short panels for easy handling and dismantling. The layout of a system of runways should be studied in advance to permit ready access to all parts of the work with a minimum of length. Since the width is commonly only slightly greater than the buggy-wheel spacing, some thought must be given to provision for one buggy passing another (sometimes done by waiting at intersections). While many designs are in use, a simple type is illustrated in Fig. 2-32.

Motor-driven power carts are now available with capacities of 10 and 12 cu ft. These are rubber-tired three- or four-wheel carts powered with gasoline motors of 6 to 7½ hp and capable of speeds of 5 to 15 mph. They can climb grades up to 20 per cent. Because of the triangular arrangement of the wheels they can make very short radius turns, some as short as 4 ft. With the motor-driven carts carrying heavier loads more substantial runways are necessary than for the hand-propelled type.

Buggies are commonly loaded from a hopper. These are usually especially designed with a low discharge gate adapted for use with buggies which are pushed under the gate. Common sizes are from ½ to 5 cu yd. With intermittent filling such as by crane and bucket the hopper not only simplifies discharge into the buggy but also provides a small reservoir to permit rapid and continuous loading of a series of buggies.

**Pumps for Concrete**

It is possible to pump concrete through steel pipes considerable distances both horizontally and vertically. A special pump for concrete (Fig. 2-33) was developed in
1933 for this purpose and has been so perfected that it is in widespread use today in many special applications where it has great advantages over other methods of transporting concrete. Any situation where the forms are inaccessible to large transporting equipment should be considered as a possible pump job. Typical examples are bridge piers and tunnel linings.

The pump for concrete is a single-action piston type with the cylinder horizontal and with rotary inlet and discharge valves (Fig. 2-34). The cylinders are either 160 or 200 mm in diameter and the pistons have a 12-in. stroke operating at about 48 strokes per minute. In view of the granular constituents in concrete the valves do not completely close. Their closure is adjustable and is generally set to close to about the size of the largest aggregate in the mix used. The action is readily understood by reference to the figure. The valves operate in a timed relationship to each other and the piston. When the piston moves forward the inlet valve from the hopper is closed and the concrete in the cylinder is driven forward through the opened outlet valve. When the piston is drawn back the valves reverse and concrete flows from the hopper by gravity into the cylinder. Because of the relative incompressibility of concrete, it moves through the pump and pipe spasmodically.

The concrete is pumped through a lightweight steel tubing, available in 6, 7, or
CONCRETE CONSTRUCTION

8 in. diameters (Table 2-5). Since the pipe frequently must be set up or dismantled quickly, it is designed with quick-acting toggle connections. This special pipe comes in lengths of 1, 2, 3, 5, and 10 ft. Fittings are available for any normal placing condition. In calculating equivalent lengths for bends allow 40 ft for a 90° bend, 20 ft for a 45° bend, and 10 ft for a 221/2° bend. For combinations of horizontal and vertical runs use 1 ft of vertical equal to 8 ft of horizontal pumping.

### Table 2-5. Concrete-pump Data

<table>
<thead>
<tr>
<th>Model No.</th>
<th>160 single</th>
<th>200 single</th>
<th>200 double</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capacity, cu yd/hr.</td>
<td>15-20</td>
<td>25-33</td>
<td>50-65</td>
</tr>
<tr>
<td>Max size aggregate, in.</td>
<td>2</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>Pipe size, in.</td>
<td>6 or 7</td>
<td>7 or 8</td>
<td>8</td>
</tr>
<tr>
<td>Max distance (horizontal), ft.</td>
<td>800</td>
<td>1,000</td>
<td>1,000</td>
</tr>
<tr>
<td>Max distance (vertical), ft.</td>
<td>100</td>
<td>120</td>
<td>120</td>
</tr>
</tbody>
</table>

It is essential that the source of supply of concrete be fairly constant in order to avoid delays which may plug the pipeline. The best consistency for pumping lies between 3 and 6 in. of slump. Consistency affects the rate of pumping. Generally gravel concrete is better than crushed-stone concrete for long pipelines. However, crushed-stone concrete, if well proportioned, will pump well. With absorptive aggregates, such as slag, the pressure in the pipeline may induce more absorption with consequent loss of slump and possible plugging of the line. High-early-strength concrete pumps well but during hot weather should not be left in the line too long because of danger of plugging. With air-entraining cement there is little or no loss of air in the pipeline. Similarly there is little loss of slump in the pipeline with normal aggregates. If concrete ever appears to gain in slump, the cylinder head should be examined to see if flushing water is escaping into the concrete.

The following are some points to watch for: Mortar should be used to start a day's work (about 5 cu ft will do). One simple way to accomplish this is to leave the coarse aggregate out of the first batch. It is a good idea to so operate the pump that the mixer (hopper) is kept about half full. Then if some segregated coarse aggregate is delivered it can be mixed into the concrete in the hopper and so avoid plugging the pump or line. If the pipeline is cleaned with water after an operation, care must be taken to avoid discharging the wash water into the recently placed concrete. The pipeline should be well braced, especially at bends. In hot weather some expedient must be adopted to cool exposed pipelines, such as covering them with burlap and wetting down. The concrete pump when properly handled is an ideal tool in special situations.

**Belt Conveyors**

Belt conveyors have been used to a limited extent in transporting mixed concrete. They are limited as a transporting system because of their lack of mobility.

The conveyor consists of a continuous rubber belt supported on top and bottom rollers which are in turn supported by a light metal frame (Fig. 2-35). The upper rollers form the belt into a trough in order to increase the belt's capacity to carry wet concrete. The bottom rollers are straight and more widely spaced since they support only the empty belt on its return. Belts are commonly 24, 30, or 36 in. wide. They are fabricated in plies or corded. The corded type was developed to improve troughing. Common maximum lengths installed are 600 to 800 ft. Where the travel must
TRANSPORTING AND PLACING

materially exceed 800 ft, a series of successive belts may be used with each discharging through a hopper on the succeeding belt. The speeds are commonly from 170 to 300 fpm.

Belts can be used on inclines up to a maximum angle above the horizontal of around 24° (for dry mixes). It was long believed that belts could be used only with dry mixes, but they have been successfully used with wet mixes. Wet mixes decrease the capacity of the belt and decrease the maximum usable inclination. For example, with a 24-in. belt:

<table>
<thead>
<tr>
<th>Slump, in.</th>
<th>Belt speed, fpm</th>
<th>Max angle, deg</th>
<th>Capacity, cu yd/hr</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>300</td>
<td>24</td>
<td>225</td>
</tr>
<tr>
<td>6</td>
<td>300</td>
<td>12</td>
<td>125</td>
</tr>
</tbody>
</table>

Increasing the inclination above the critical angle for a particular consistency will induce a backward (downhill) motion in the concrete, thus decreasing the belt output. For example, on one job with a wet concrete on a section of belt 15° above the horizontal a backward motion of 50 fpm was observed. Concrete with low slumps will stand higher on the belt, and hence the belt can carry more cubic yards per hour.

If a belt conveyor is intended to be used in all weather it will be necessary to enclose it. This is generally done with a light metal frame supporting a top and sides of corrugated sheet metal. At least one side must be removable for servicing the conveyor.

At discharge points a hopper should be installed to reduce segregation and control the discharge into the next receptacle (Fig. 2-36). It is necessary to use one or a series of counterweighted scrapers on the lower belt to scrape off adhering mortar. It is also frequently necessary to wash the returning belt with a light continuous spray. Some provision must be made in the design of the mix if the mortar loss proves appreciable. A telephone connection to the source or other signaling device is necessary to control loading of the belt.

In certain special locations where very large volumes of concrete are moved, as in the case of some dams, belts have proved a good solution to the problem of moving concrete from the mixing plant to a central distributing point.

**Tremie**

In some situations it is expedient to place concrete under water, although it is always preferable to place it in air if feasible and economical. One common example is the construction of a concrete seal for the bottom of a cofferdam for building bridge piers. In underwater concreting, the object is to reduce loss of cement and saturation of the concrete to a minimum. One well-established, simple, economical, and effective method is by the use of the tremie pipe. The tremie is a straight vertical pipe, usually between 6 and 14 in. in diameter, with a funnel at the upper end. It must be long enough to reach from above the surface of the water to the bottom on which the concrete is to be deposited (Fig. 2-37).

The tremie must be supported so it can be raised as the work progresses. Single tremies are sometimes suspended by a crane. Another method is to spot a number of such pipes covering the area and each suspended from a chain hoist hung from a gallows frame. This second method should use sufficient pipes so that the concrete will not be required to flow horizontally more than about 10 ft. It is not particularly
adaptable to pours thicker than 15 ft but has the advantage of avoiding breaking the seal when the pipe is moved.

The concrete mix should be designed to be plastic and dense and yet flow readily. To this end the coarse aggregate should not exceed 2 in., the fine aggregate should be 45 to 50 per cent of the total aggregate volume, and the cement content not less than 7 bags per cu yd. The slump should be about 6 in. To start the pour the bottom of each tremie pipe should be closed by a gate or more commonly plugged by burlap and lowered to the bottom.

The plant should be set up and materials made available for a continuous pour. Concrete is then poured into the funnel until the pipe is filled. The pipe is then carefully raised slightly to permit the concrete to force out the plug and flow out. The end of the pipe is kept continuously under the surface of the concrete throughout the pour. By means of a sounding lead the concrete surface is repeatedly checked and an effort is made to keep the concrete surface horizontal to avoid entrapping underwater debris. All underwater methods usually produce a large amount of laitance on the surface of the concrete which is removed after dewatering before more concrete is deposited.

Because of the slow rate of strength gain it is usually not wise to deposit concrete in water colder than 35°F.

Tower and Long Chutes

This is a method which was widely used for some years for reinforced-concrete buildings and other concrete structures of large plan area. Its attraction lies in the fact that it makes use of gravity flow. The equipment consists of a centrally placed steel hoist tower of four-column design, within which a self-tilting bucket is raised and a long boom mounted on the tower supports long counterweighted chutes (Fig. 2-38). By means of the long boom which supports the chutes a very considerable area can be covered on three sides of the tower without remounting the boom. One of the great disadvantages of this system is the difficulty of clearing clogged chutes quickly. In some cases vibrators have been added to the chute to assist flow of concrete. The method is falling into disuse in this country partly because of objections to long chutes and partly because of the development of other more flexible and economical methods.

Combinations of Methods

Numberless combinations of the above methods are possible to meet the conditions peculiar to each job site and available equipment.

Placing Concrete

The methods used in placing concrete in its final position have an important effect on its homogeneity, density, and behavior in service. The same care which has been
used to secure homogeneity in mixing and the avoidance of segregation in transporting must be exercised to preserve the homogeneity in placing.

To secure good concrete it is necessary to make certain preparations before placing. The forms must be examined for correct alignment and adequate rigidity to withstand concrete load and construction impacts without undue deformation. The forms must also be checked for tightness and clean surfaces. The reinforcement should be checked for conformance with the plans, rigidity, and cleanliness. There should be standby equipment to replace any which fails during the pour. This applies particularly to vibrators. Materials should be available for bulkheading in the event of failure of concrete supply, and those in charge of the pour should have definite directions as to where and how to bulkhead. Some preparations should be made to protect fresh concrete from heavy downpours of rain. If the pour is being made in cold weather the necessary protective materials and equipment should be immediately available. These would include salt hay, tarpaulins, salamanders, or warm-air blowers. On most large pours it is customary to have at least one carpenter and one ironworker on standby to make such repairs as may become necessary during the pour.

The surfaces against which the fresh concrete is to be placed must be examined as to their possible effect in absorbing mixing water. For example, subgrades are dampened or covered with paper. Forms are coated with oil or some varnishlike preparation. In this connection, it is necessary to consider whether or not bond is desired. In placing against other concrete, the age of the previously poured concrete is a factor. If bond is desired, the former concrete surface should be left or made rough and a rich mortar painted on it unless it was poured just previously. There is a considerable difference of opinion as to how much time should be permitted to elapse where a good bond is desired and when it is possible to control the interval. Good results are achieved on industrial floors poured weeks after the structural-slab base course. In all cases the base course should be rough, clean, and well moistened.

In some cases it is desired that no bonding occur, as in sliding joints. Here the surface is troweled as smooth as feasible and covered with a moistproof paper or sheet metal. In cases where it is intended to remove temporary protective concrete later to a predetermined surface, the earlier pour may be painted with a thin coat of bituminous material.

As placing begins the consistency of the delivered concrete should be checked with a slump cone for conformance with specifications. In pouring footings, care should be taken to eliminate seeping ground water. A sump excavated outside the form and pumping is the usual method.

The basic principle in placing concrete is to deposit it as near its final position as physically and economically possible, because moving concrete along the form tends to produce segregation. Secondly, as far as possible the concrete should be deposited in horizontal layers. For mass concrete these should be usually between 10 and 20 in. thick. The free fall of concrete should in no case exceed 4 ft. For greater heights, as in walls or columns, use trunks (pipes) of metal or rubber. When concrete is dumped from a cart there is a tendency for the heavier particles to separate from the mass. To prevent this, the concrete should be discharged against a striking board.

In forms of small horizontal cross section it is necessary to control the rate of rise to avoid large hydrostatic pressures on the form. A rough rule of thumb is that concrete sets up at such a rate that 5 ft per hr in summer or 3 ft per hr in winter are safe rates of rise. Where forms for several columns or walls nearby are ready, switching from one to another enables continuous pouring without excessive rates of rise. Aside from possible collapse of forms rapid rates of rise promote bleeding, or water gain, in the upper portions of high pours. Where it is feasible to change the slump during the pour lower slumps (drier concrete) may be resorted to as the concrete rises.

Where horizontal slabs are poured monolithic with walls or columns, there is some danger of the development of horizontal shrinkage cracks at the lower surface of the slab unless an interval of a few hours is allowed between completion of the vertical pour and commencement of the slab pour.
With the exceptions noted above concreting should be as continuous as possible to avoid planes of weakness. The direction of deposition should be against previously placed concrete and not away from it. Efforts should be made to consolidate the concrete. For many years this was done by spades and the boots of the workman pressing the fresh concrete into corners and around pipes and reinforcement. Today internal vibrators are used almost universally as laborsavers and because they are more effective.

The common type of vibrator consists of a vibrating head attached to a flexible rotating shaft (housed in a reinforced rubber hose) driven by a gasoline or electric motor. The vibrating head is a closed metal cylinder from about 1 to 3 in. in diameter and 2 to 3 ft long. The vibrating motion of the head is imparted by a rotating eccentric element enclosed in the head. Rate of vibration influences efficiency and specifications usually require between 3,200 and 7,000 impulses per minute. It is necessary to select the correct size of vibrator for particular cases. One controlling consideration is that the spacing between reinforcing-steel bars, as in concrete beams, has an effect on the maximum diameter of a vibrator head.

The use of vibrators has permitted stiffer mixes and fewer fines, which results in cement economy. When properly used the resulting concrete has fewer voids and stone pockets. Vibrators are very useful in working concrete into tight corners and around reinforcement or any embedded metals such as inserts and pipes. In manipulating the vibrator, it should, if possible, be thrust into the concrete vertically and reach a small distance into the concrete in the next lower layer. The circle of concrete which is set in motion should be observed and these areas should overlap. The vibrator must not be left in one position long as it will throw the coarse aggregate away from it and form a mortar pool. It is commonly left in one place between 5 and 15 sec but observation will indicate how soon to move it. Care should be used to avoid allowing the head to come into contact with the forms or reinforcing steel. In some cases vibrator heads have punched through plywood forms. In some localities laborers specialize in the use of vibrators and develop a “feel” for their correct use. A single vibrator can do good work on between 5 and 15 cu yd of concrete per hr.

Even with the use of vibrators it is frequently necessary to spade along form surfaces to eliminate honeycomb or small air bubbles. Unless used to excess, a vibrator will not reduce air entrainment more than ½ per cent. Standby vibrators to replace those which go out of order during a pour are a recognized and absolute necessity.

Since most concreting is done in the open, it is necessary to consider the air temperature during the placing operation. Either extremely high or low temperatures are bad for the concrete unless steps are taken to offset their effects. The most desirable range of concrete temperatures in the forms is between 60 and 85°F.

Where high temperatures prevail the chemical action between the cement and water is accelerated, producing the following undesirable effects:

1. Cracks frequently form either because evening temperature drops cause shrinkage at a time when the concrete has little strength or from loss of surface water due to excessive evaporation.
2. Slumps drop rapidly during transporting because of excessive evaporation making placing difficult.
3. Because of more rapid evaporation greater precautions must be taken in curing. To minimize the effects of high air temperature with resulting high concrete temperature some or all of the following steps may be taken:
   1. Cool the mixing water even to the extent of adding ice.
   2. Avoid the use of hot cement.
   3. Insulate or paint exposed water-supply lines white.
   4. Cool the aggregates by sprinkling with water.
   5. Paint the mixer drums white.
   6. Conduct operations either in the shade or at night.

Extremely low air temperatures during placing also can have bad effects on the concrete. The great danger is actual freezing since the formation of ice crystals can destroy the structural value of the concrete. Aside from actual freezing, the low
temperatures reduce the rate of strength gain because of the slower rate of reaction between the cement and water. For example, concrete placed and maintained at 50°F has strengths during its early age about one-half those for concrete at 70°F. The long time effect of slow strength gain is not bad but it prevents removal and reuse of forms for protracted periods. To minimize the effect of low air temperatures, some or all of the following steps may be resorted to:

1. Heat the aggregates and mixing water but do not raise concrete temperature above 80°F. The water may be heated to about 160°F maximum. The concrete in the forms should not be lower than 50°F. It should be held not lower than 50°F for 72 hr.
2. Add about 1 per cent of calcium chloride to accelerate setting. This proportion is 1 per cent of the cement weight. Care must be taken to prevent immediate direct contact with the cement.
3. Hold the temperature of the air surrounding the concrete high enough to maintain a minimum concrete temperature of 50°F where it is economically feasible to enclose the concrete. This may be done by using salamanders (braziers) or by piping in live steam or warm air. Of the three, live steam is best because it avoids the drying effect of the other two methods.

FINISHES AND FINISHING

Treatment of concrete surfaces varies with the architectural requirements, if any, and the degree and nature of exposure. Concrete surfaces in contact with the ground, such as the underside of footings or pavement slabs, usually require no special treatment although subbases under pavements may be carefully rolled and smoothed to reduce drag. Surfaces to be concealed by other coverings, as in the case of concrete subfloors or interior walls, may require special preparation to receive the covering material. Exposed exterior surfaces such as those of bridge structures or the upper surface of pavement slabs usually require special treatment. Such treatment is necessary either to acquire some predetermined appearance or to achieve a dense protective skin which may be likened to the casehardening of metals.

These concrete surfaces during their formation are either:

1. Free surfaces (upper and approaching horizontal)
2. Formed surfaces
3. Surfaces to be treated after hardening

In the first case, the final texture will depend on the tools employed in working the surface as it passes from the plastic to the solid state. In the second case, the texture will depend on the nature of the surface of the form in contact with the concrete. In the third case it will depend on the treatment of the hardened surface by tools, chemicals, or the application of adhering material as in the case of terrazzo, stucco, or paint.

Free Surfaces

The simplest case of finishing a free surface occurs at a horizontal day's-work joint in mass concrete. This usually requires no more than leveling the surface at some predetermined elevation and leaving the concrete rough to improve the bond with the concrete of the succeeding pour. Controlling the elevation of the joint is often important from the standpoint of appearance or to assure a sufficient lap of protruding reinforcing steel. In preparing for the next pour the old surface should be perfectly clean and moist.

Where possible, the appearance of the exposed joint is considerably improved by forming a V notch in which the joint lies. The size of the notch varies with the massiveness of the structure but in most cases may be formed from a triangular molding about 3/8 in. on the square edge. Moldings for forming such V notches should be of good quality and purchased as millwork rather than ripped on a saw in the field. To assure a clean line at the tip of the V it is well to pass a small trowel over
the wet surface of the concrete where it meets the molding. This permits careful matching with the upper half of the molding when it is placed in the form for the next pour. Where no V notch is used to conceal the joint the elevation of the top of the pour may be marked by driving small nails in the vertical wood forms.

In the case of tremie pours (under water) it is next to impossible to control closely the elevation of the surface of the concrete (which must be done by sounding lead). Also laitance (an inert substance of low strength) collects on the horizontal surface. This must be removed mechanically, after dewatering the cofferdam, in order to expose good-quality concrete to receive and bond with the next pour above.

Another common free surface in structures is that on which the superstructure rests as, for example, bridge seats. To assure good bearing the final surface should approach a true plane. Because of this and the need to meet closely a prescribed elevation the modern practice is to pour the concrete high and tool it down. In this case simple screeding is satisfactory because the final surface is produced by tooling after hardening.

Machine Screeding and Finishing of Pavements

A troublesome field problem is the determination of the exact tolerance to use in screeding the concrete to the higher elevation. Forms settle by varying amounts, depending on how they are supported. The settlement during the pour may exceed 1 in. Hence, if the elevation controls are set before the concrete is placed in the form, the amount of settlement must be anticipated and allowed for. The safe method where feasible is to set the control points after almost all the concrete has been deposited, at which time most of the dead-load deflection has taken place. Where this is done, it is sometimes possible to trowel the bearing surface to exact final elevation or at worst set it about \( \frac{3}{8} \) in. too high and tool down later. Aside from the work of removing a large excess thickness, another disadvantage is that entire pieces of coarse aggregate frequently pop out under heavy bush hammering. The last \( \frac{3}{8} \) in. is commonly finished by carborundum stone, by either hand rubbing or motor-driven disk. The several bridge seats of any given single structure should be consistent in elevation to \( \frac{3}{16} \) in. or better for proper fitting up of the superstructure steel.

Highway pavements represent the greatest total area of free-surface finishing. Modern highway practice demands very close control in finishing to produce acceptable riding quality and approaches an art. A common specification requirement is that the surface shall deviate from a straight line no more than \( \frac{3}{8} \) in. in 10 ft. To achieve this requires special skill on the part of the finishers, who usually specialize in this type of work. To achieve large daily output the paving operation is now highly mechanized, but the quality of the final surface still depends on the hand finishing done behind the string of equipment.

Pavement is usually placed in two courses to permit placing the reinforcing mesh. This means the concrete is mechanically struck off twice. The surface of the upper course is then screeded (finished) mechanically first in a transverse direction and then longitudinally. Starting with the finishing machines following the second strike-off the operation is as follows: The metal side forms serve as screed rails on which the machines run and hence control the elevation of the pavement surface. The forms are set very accurately to line and grade by surveying instruments. The exact final alignment is usually done by eye by a class of labor especially skilled in setting forms. The forms are then rigidly held in place by metal pins driven into the subgrade.

The transverse finishing machine is essentially a metal float or screed which rests, near its ends, on the two side forms and is mechanically moved laterally through an adjustable distance of 3 to 6 in. as the entire machine moves forward. These combined movements result in a sawing motion of the screed over the concrete. The action brings the concrete surface into the desired shape of cross section which may be a plane or a flat parabola. It also tends to force the coarse aggregate down so that only an edge or point lies in the pavement surface. The action also brings mortar
to the surface; so care must be used not to overdo it. One or two passes of the transverse finishing machine are usually sufficient. To assure a surface with no holes and having uniform density the wet concrete should be struck off by the spreader ahead of the finishing machine to such a height that a uniform roll of concrete is carried ahead of the screed for its entire length as it advances.

Transverse finishing usually leaves a series of slight transverse marks in the wet concrete surface. If no further work were done the pavement would contain a series of slight ridges over which the wheels of a car must travel. To reduce or eliminate these the longitudinal finishing machine is used. It is similar to the transverse machine except that the moving float travels almost parallel to the side forms in its reciprocating motion while it sweeps from one side form to the other. Since the float does not rest on the side forms but rather on the concrete itself it must be very carefully adjusted so as not to dig into the concrete and spoil the finish already achieved by the transverse machine. It is the most difficult machine to keep in adjustment and has an important effect on the riding quality of the pavement.

Manual Finishing

Behind the string of mechanical equipment come the manual finishers. Their principal functions are to test the surface, correct minor irregularities, and give the surface texture.

The first operation is to draw a wooden screed, resembling a 10-ft straightedge with a long handle, across the pavement from one edge to the other. It is carefully lowered to the pavement at the far edge and drawn toward the near edge of pavement. Its manipulation is delicate and requires skill. Its effect is to scrape off any excess water or mortar and to smooth out any small irregularities. To provide continuity the finisher advances only by one-half the length of the screed with each sweep.

In many paving jobs the screed operator is followed by a second operator manipulating a lute. The lute is a smooth float, frequently of aluminum for ease of handling. Its function is to smooth the pavement surface further, correcting any roughness resulting from the use of the preceding screed.

This operator is followed by a third finisher with a 10-ft straightedge on a long handle. He places the straightedge gently on the surface parallel to the side forms at several points in the width of the pavement. By very slightly wiggling the handle, he causes a very slight mark in the damp concrete surface. If the mark is uniform for its length, the surface is likely to meet the specification requirement. Narrowing or disappearance of the mark within its length indicates a depression which may have to be filled in by further manipulation of the screed and lute. Any deeper irregularities may require return of the finishing machines for a second pass.

The next operation is forming of the transverse joints and edging the pavement. The exact method of forming the joint will depend on the choice of one of numerous acceptable joint details that may be used. The important point is that, in edging the joint, nothing must be done to turn the surface up or depress it. To avoid this it is necessary to test the concrete surface carefully after the joint has been edged with a straightedge laid at right angles to the joint. The straightedge should not be shorter than 4 ft because it is not uncommon, in cutting the joint, to remove sufficient material to allow the surface to drop as much as ¼ in., starting at a point about 18 in. either side of the joint. Such a depression causes a decided shock in vehicles traveling at high speed and with the frequency of transverse joints, in most pavements, effectively destroys good riding quality. Only constant vigilance in forming the joints will eliminate this common cause of poor riding quality. The edging tool for the transverse joints should not be less than ½ in. radius. Smaller radii produce an edge which spills easily when hard particles are driven into the joint by moving vehicles.

After edging but before the initial set, the final finish is given to the pavement. The exact condition of the concrete at the time this operation is performed is important in its effect on the texture achieved. The concrete must be neither too wet as to deform easily nor so dry as to refuse the effect of the finishing operation. Since the rate of drying the surface varies greatly with humidity, temperature, and wind it must be
watched carefully for the most effective timing. Modern highway practice calls for either the broom or drag finish.

The broom finish is produced by drawing a special broom across the pavement from edge to edge to produce corrugations of regular appearance and not over $\frac{3}{4}$ in. deep. The broom is made of bass fibers not over 5 in. long. The broom itself is not less than 18 in. long and has a hand long enough to reach the far edge of the pavement easily. To produce a uniform texture the fibers must be kept clean by frequent washings, as there is a tendency for the mortar to cake on the bristles and so score the surface.

Another finish is the drag finish. It is produced by dragging a belt of canvas or burlap over the concrete in a direction parallel to the roadway. This must be done before the initial set, and for best results the state of the concrete at the time is critical. The size and weight of the material dragged must be such as not to produce corrugations deeper than $\frac{3}{4}$ in. A single piece of burlap about 24 in. wide and long enough to be carried by two men, one at each side form, is usually satisfactory. As in the case of the broom it is very important to wash the burlap often enough to prevent the formation of crusts of mortar on the burlap which would mar the concrete surface.

Since highway or airport paving are necessarily done outdoors there is occasional danger of destruction of the finish by a heavy rain shower before initial set takes place. The best practice is to keep a close watch on the weather, even using meteorological information, and stop placing the concrete early enough to finish and cover what has been placed. If caught unexpectedly careful placing of strips of burlap on the newly finished surface may prevent pitting of the surface and washing out of the surface mortar. If the surface is not protected from a heavy downpour soon enough and is damaged so that it cannot meet the specifications, removal of the slabs may be the only answer. In such a case, the earlier the removal the easier it becomes.

Curing

A very essential element of durable concrete pavements is proper curing. Curing is a procedure adopted to prevent the loss of sufficient mixing water to impair the complete hydration of the cement in the surface layer of the concrete. There is an excess of water in the mix when placed but the large surface exposed permits loss by evaporation. The rate of loss increases with high temperatures and wind, and care must be taken at such times to prevent the formation of shrinkage cracks before the curing has started.

There are several well-established methods of curing in highway practice:

1. Ponding
2. Paper
3. Cotton mats, saturated earth, saturated straw
4. Membrane curing compounds

One of the original and effective methods seldom seen any more is by ponding or flooding the surface to a depth of 1 to 3 in. with water retained within earth dams formed along the edges of the pavement. It had the advantage of providing a positive excess of water continuously. The disadvantages were that it could not be readily used on grades and water was not always readily available in the desired quantity.

The method probably most widely used now is to cover the pavement surface as soon as possible without marring the finish with an impervious paper. The paper commonly consists of two sheets of white, tough, durable kraft paper cemented together by a bituminous material. It is usually delivered wound on a pole and the full width of a paved lane (12 or 24 ft) in lengths of 50 to 75 ft. When the paper has been placed by unrolling from the pole it is retained in place by a continuous ridge of earth placed along each edge. In addition a stringer width of paper 18 in. wide is placed along each edge under the large piece so that it may be pulled down over the exposed pavement edges when the side forms have been removed (usually the day
following pouring). To prevent the surface from drying out the timing of the paper placing is critical, and in general it must be placed as soon as the finished surface will support it without being marred. Paper markings are preferable to shrinkage cracks. The effect of the paper is to retain a large proportion of the mixing water.

Other curing methods are cotton or jute felt mats. These are sometimes used as preliminary to the paper curing and sometimes used alone. The mats must be wide enough to drape over the pavement edges. They are kept wet continuously for the duration of the curing period. In place of mats 2 in. of saturated earth or 3 in. of saturated straw may be used to retain the mixing water by preventing evaporation.

The curing duration is either an arbitrary specified interval such as 7 days in temperate climates or until the modulus of rupture as determined by breaking concrete test beams is some value related to the design strength of the pavement. The minimum time interval as usually given in the specifications is such as to achieve safely a modulus of rupture sufficient to sustain construction loads based upon a rate of strength gain normal for the prevailing temperatures during the paving season.

A fourth method common for general curing but frowned upon for pavement concrete is to cure by spraying the pavement with a membrane sealing compound. This material is colorless and nonbituminous and consists of a blend of oil or resins held in solution by a volatile solvent. A common requirement is that it shall form a film which shall retain at the end of 3 days at least 85 per cent of the water under ASTM Designation C 156–44T. The danger in using these compounds arises from the failure in the field to place them at the correct instant, and also the failure to cover the pavement surface completely. If the spray is applied while a sheen of water still shows there is the likelihood that the spray will collect in globules on the water and no film will form. If too late the water in the upper concrete layer has evaporated and the harm is done. The impelling force behind the use of the membrane method has been its low cost to apply. Its use is not permitted by many highway departments. Its great virtue lies in the ease of application on vertical surfaces where mats are difficult to hang and keep wet.

Sidewalk Finishing

Finishing sidewalks is a smaller and less elaborate job than finishing highway pavements. The side forms are often 2- by 4-in. lumber or especially fabricated metal forms. These are used as screed rails to control the position and slope of the pavement surface. The slope or pitch is important in its influence on drainage and a transverse slope of \( \frac{1}{4} \) in. per ft is common. Jointing is desirable to localize cracks and facilitate repairs. The vertical joints when carried entirely through the pavement are commonly metal plates which are oiled and slipped out when the concrete is stiff enough to be self-supporting. Where plates are not used a jointing tool may be used to cut a groove about \( \frac{1}{4} \) in. deep in the surface. This is usually not deep enough to create a plane of weakness to force the crack to occur in the joint and is not recommended. Joints in sidewalks are usually not over 6 ft apart. Where it is necessary to avoid a thrust due to expansion of the slab, as at the building walls, a thickness of premolded bituminous expansion-joint material may be placed. These are also frequently placed to divide driveway concrete from sidewalks.

Sidewalk finishing is commonly done by hand. The concrete, of a dry consistency (not over 2-in. slump), is struck off by moving a wooden screed with a sawing motion across the upper surfaces of the side forms. After screeding a wood or asbestos float is used to bring the mortar up and fill such voids as were left by the screeding. A long sweeping stroke is employed and only enough pressure to bring mortar up without depressing the screeded surface. This frequently means an interval must elapse after the screeding. The final surface is that left by the wood float or if an asbestos float has left it smooth it should be lightly brushed to provide some traction in wet weather. The edging tool of about \( \frac{3}{4} \) in. radius is used along edges and in open joints to improve appearance and strengthen the edge against spalling. Sidewalks may be cured as covered under Highway Pavements although spraying is the commonest method.
CONCRETE CONSTRUCTION

For large exterior pavements as in a plaza or airport the methods are essentially the same as for sidewalks except that screed rails may be necessary to control the surface and power floats may be used for quantity production. Large areas are broken into rectangles and the sequence of pours may be diagonal (checkerboard). This sequence permits shrinkage of the first slabs to take place in both directions before the later ones are poured.

Floor Finishing

Most interior concrete floors are covered. Exceptions are garage or warehouse floors or the terrazzo used in public buildings. Where floors are covered by linoleum or asphalt, or rubber or plastic tile the practice is to finish after the wood float with one steel troweling in order to produce a smooth surface to support these readily deformed coverings. Where masonry tile coverings are to be placed no finish at all is used but rather the surface is struck off low and left rough to receive the mortar bed in which the masonry units are set.

Warehouse or garage floors are commonly finished with the steel trowel to make them as smooth and dense as possible. The smoothness simplifies cleaning and the movement of rolling equipment. The denseness which results from several steel trowelings increases resistance to abrasion and hence decreases dusting. Further hardening of the surface may be achieved by either treating the completed floor with sodium fluosilicate or by incorporating into the wet surface powdered iron or carbon. Highly ornamental and marblelike finishes may be obtained by the careful blending of colored-stone aggregates and then grinding the concrete after it has set for several days. This “terrazzo” finish is a job for specialists.

Finishes of Formed Concrete Surfaces

The required finish for a formed surface usually depends upon architectural considerations but in hydraulic structures frictional requirements may govern. Some specifications for hydraulic structures classify surfaces according to allowable roughness as measured by departure from a straight line tested with a 5-ft straightedge. For different categories of finish the surface irregularities may vary from 1 in. for the worst to $\frac{1}{8}$ in. for the best.

All surfaces molded by contact with forms necessarily have a texture when stripped which is influenced by the contact surface of the form. In the early history of concrete, there was a tendency to follow stone-finishing techniques by tooling the hardened concrete. This resulted in such finishes as obtained by bush hammering, picking, grinding, and sandblasting. These methods are no longer in use. They are too expensive. Since they expose a more porous interior concrete, they tend to promote absorption of moisture, leading to disintegration of the exposed surface.

With carefully executed formwork fine ornamental effects can be achieved in concrete. For such formwork the labor and material are expensive and in most parts of the country the tradition is against it. For heavy structures such as bridge substructures and dams, relatively simple finishes are adopted such as smooth finish, rough-board finish, or rubbed finish.

Smooth finish may be described as that achieved where poured against smooth forms of either metal or wood or such form liners as plywood, Preswood, or Masonite. The form panels are usually as large as commercially available to reduce the number of joints to a minimum. Panels 4 by 12 ft are not uncommon. A common form is made of Preswood $\frac{3}{16}$, $\frac{3}{4}$, or $\frac{7}{8}$ in. in thickness and fastened with 3-penny blue shingle nails to a back-up of 1- by 4-in. or 1- by 6-in. sheathing boards. The back-up must be of uniform thickness especially if $\frac{3}{8}$-in. liner is used because the pressure of the wet concrete will deflect the liner and cause the shape of the individual boards to be seen in the finished concrete surface.

Preswood has two surfaces, one smooth and the reverse side with a screen-mesh surface. The smooth side is usually placed against the concrete, but as an alternative, an interesting concrete texture can be obtained by using the reverse side. A texture
somewhat similar to the first may be achieved by using plywood of \( \frac{5}{8} \), \( \frac{1}{2} \), or \( \frac{3}{4} \) in. thickness nailed directly to the studs (no back-up required). Plywood is available in sheets up to 4 by 8 ft.

With smooth finish the joints should be concealed as well as possible by making sure the surfaces of adjacent sheets occur over solid back-up or studding. They are then sealed with patching plaster or putty and sandpapered to a smooth finish. To prevent absorption of mixing water and to simplify stripping, the form surface should be oiled or lacquered before concreting. The oil should be nonstaining and no excess used. One of the disadvantages of smooth finish is that, because of its smooth uniform texture, any blemishes tend to be unduly conspicuous and may require special treatment.

A simple finish finding general approval for heavy structures is the "rough-board" finish. This finish is of a deliberately rugged texture in which the grain and saw cuts of the rough lumber are clearly apparent, and fins and even knot markings are untreated. It is customary to use tongue-and-groove boards because this prevents warping and cupping due to moisture absorption and also minimizes leaking. The boards are assembled with a very small gap left between adjacent boards in order deliberately to produce fins. The use of rough-cut (unplaned) boards introduces the effect of saw cuts in the concrete surface and since rough-cut boards tend to be slightly different thicknesses, the individual boards stand out. The wood grain may be accentuated by raising the wood fibers. This may be done by spraying the form contact surface with ammonia. The board widths selected depend on the expanse of surface being formed, common widths being 4 and 6 in. with the wider boards for larger surfaces.

The great virtue of the rough-board finish lies in the fact that, when properly executed, no aftertreatment is necessary, and minor defects blend into the general effect. With no rubbing, the durability of the surface is high, since the surface concrete is perfectly integral with the mass of concrete. A disadvantage is that, where honeycomb occurs, it is extremely difficult, when patching, to match the rough-board effect in the area of the patch.

Precaution against Surface Defects. There are certain inherent weaknesses in molding concrete which make its finishing peculiar to itself. These are form marks which may be objectionable, segregation of aggregates producing a lack of uniform texture, air bubbles, and honeycomb. Where form marks are considered objectionable, the forms must be rigidly supported in true planes, and the number of joints must be minimized. Attempting to remove evidence of the joints from the hardened concrete, after stripping, by rubbing, will alter the concrete texture and possibly require rubbing the entire surface to secure uniformity. Segregation of aggregates can be minimized by methods described elsewhere.

Air bubbles in the surface present quite a problem and no completely satisfactory solution has yet been developed. These appear as small more or less circular depressions varying in size from \( \frac{3}{8} \) to \( \frac{3}{4} \) in. They are caused by entrapping air bubbles against the form and appear to be aggravated by drier mixes and rough forms which resist the tendency of the bubbles to float to the surface during placement of concrete. With the growing use of vibrators to produce a dense concrete, there has been a decreasing use of spades. Spading along the entire form surface as the concrete rises appears to be the best method of reducing bubble marks. Another method is to hammer the outside form surface with a wooden mallet to jar the bubbles loose and cause them to float to the surface. When a stripped surface contains excessive bubble marks, there are only two practical alternatives: either leave them entirely alone or rub the entire surface. Filling in each separate bubble depression creates an even worse appearance than the original.

Honeycomb is concrete's worst surface defect. Its appearance is that of exposed coarse aggregate with insufficient mortar to fill the voids. It can be caused by poor mixing, segregation during placement, or leaching out of the mortar at a leak in the form. The causes suggest their own methods of prevention. The great difficulty is that honeycomb is discovered only when too late. In this case the only recourse is to cut away the loose stones and fill in the depression with concrete of the same mix
in order to achieve a good match. Where the large aggregate is too large to use for the patch these particles may be omitted from the mix or screened out. It is necessary to patch the honeycomb as early as possible after stripping the form in order to assure a good bond with the base concrete. Finishing the surface of the patch to match the rest of the concrete is a job for an expert finisher, and frequently rubbing the entire area to achieve a uniform texture is the only solution.

Rubbing a concrete surface is done with a stone on a wetted and preferably green surface. This abrasion in the presence of moisture works up a thin mortar which fills in surface irregularities and tends to produce a uniform mat finish. The greener the concrete, the stronger and better bonded this skin will be. The rubbed surface must be cured. Executed properly, the rubbed finish will last a long time. It is, however, not so desirable as an unrubbed surface from the standpoint of durability, a fact which has given rise to the use of those form finishes which require little or no rubbing such as the rough-board finish described above.

Achievement of a presentable exterior concrete surface requires care at each stage of mixing, depositing, vibrating and spading, form construction, and stripping. One point which must never be overlooked is the use throughout exposed surfaces of matching structures of the same brand of cement or one with identical color.

PLANNING THE CONCRETE PLANT

The plant to be considered here is the batching and mixing plant with no reference to a complete aggregate-producing plant, which is a special case occurring only on very large projects. The plant then consists of the contractor's equipment for the production of concrete and the storage space for the cement and aggregates. The plant layout will vary greatly according to the space available, the character and size of the project, and the equipment available to the contractor.

In most of the more populous areas of the United States, commercial concrete plants using transit-mix trucks are available. These plants will deliver concrete, mixed to specifications, directly to the location of the forms in transit-mix trucks at a quoted price per cubic yard. For smaller concrete jobs, where the haul is not excessive, these are usually resorted to for general reasons of economy. For large or small jobs the advantages of commercial transit-mix concrete are:

1. Concrete is available on a few hours' notice in any reasonable quantity. This fact causes most contractors to resort to commercial transit-mix concrete in the early stages of even those large jobs where a site plant will be used when erected.

2. It is unnecessary to tie up capital in plant equipment and conveying equipment. The disadvantages are:

1. There is less control of the equipment than in a self-owned plant, which fact sometimes increases the contractor's concrete-placing costs because of slow and uncertain deliveries to the site.

2. The concrete materials specified are not always stocked and substitutions are suggested by the commercial plant owners.

3. Some commercial plants have fallen into such very bad practices that there is a danger of not receiving concrete of the specified strength and quality.

4. For concrete in large quantities the delivered price per cubic yard may be slightly higher than for a self-owned plant at the site.

In the early stages of planning a concrete job it is necessary to settle the question of whether to use available commercial transit-mix concrete or to erect a plant. In weighing the advantages and disadvantages enumerated above, the final choice may depend on the economics of the two schemes. There is no ready-made rule of thumb by which this choice can be made. It is necessary to get local prices of commercial-mixed concrete and compare these with the estimated cost of setting up and operating a site plant. It is necessary, in estimating the cost of the concrete from a site plant, to include the cost of transporting the concrete from the plant to the forms since the quoted commercial price is usually for delivery at the forms or as near as a transit-mix truck can approach the forms. If the price of concrete is
the primary consideration, for jobs in or near most cities, it is not economical to set up a site plant for a project of less than 10,000 cu yd of concrete.

Choice of Site

If it is decided to erect a plant, the site must first be chosen. The following factors affect choice of site. The length of haul should approach the minimum. The site should be convenient for delivery of materials, which means it should be accessible to a public highway, railroad, or navigable waterway. There should be sufficient reasonably level area for stockpiling of aggregates. It should be accessible to an adequate supply of suitable water. Where the job stretches over some distance, as in highway construction, more than one plant may be necessary to reduce the cost of hauling the batched or mixed materials. The maximum economic haul for a particular project requires study.

Choice of Equipment

Having chosen the site, the next question is the required size of the various elements in the plant. These sizes may be arbitrarily decided by the available self-owned equipment, but this is a poor approach and may prove costly in the long run. A basic requirement is balance among the various units in order that no single piece, through inadequate capacity, will reduce the plant output seriously. To select sizes required it is advisable to start with the largest daily output requirement, in cubic yards. Exact estimates are not feasible or necessary.

This estimate may be started by dividing the total project yardage by the number of concreting days. The average number of days per week depends on the weather and the number of locations at which forms can be made ready. It might be no more than 3 days per week but will vary greatly with the character of the job. Thus, in highway paving, concreting might be done, weather permitting, 6 days a week for 12 hr per day, whereas for large roof arches, forms might be ready only 1 day in each 2 weeks.

An example will illustrate the method. Assume a large concrete bridge containing 10,000 cu yd. The concreting season may last only from May to October (26 weeks) and forms in this instance might be made ready only about 3 days per week. The average required daily output would be 10,000 divided by 3 times 26, or about 130 cu yd per day. The specifications may require 1 min mixing time (per cubic yard) but the limiting factor is the rate at which the concrete can be placed in the forms, which in this case, using crane and bucket, is about 1 cu yd every 2½ min. The output is then about 25 cu yd per hr. A 1 cu yd mixer can handle this requirement efficiently.

The largest single pour expected for 1 day is 200 cu yd. At the rate of placing of 25 cu yd per hr this large pour could be placed in 8 hr. If the mix requires 6 bags of cement to the cubic yard, the cement storage capacity should be 300 bbl. It is common to plan for a cement storage capacity equal to the largest single day’s pour, as a minimum, where cement is readily available. In most of the more populous areas of the United States, cement can be delivered in bulk by special closed trucks or, if very large quantities are required, by rail.

Stockpiling of Aggregates

The size of the stockpiles of aggregates must be considered. Stockpiles are needed to assure uninterrupted concreting and on this basis are never less than for 1 day’s pouring. Beyond this, the size of stockpile, where space is available, depends on the certainty of supply. In isolated sections having only a limited producing capacity, large stockpiles may have to be provided, the exact size of which must be analyzed. Where the rate of daily deliveries is less than the rate of concrete placing, the stockpiles of each aggregate must be large enough at the start to take up the difference
between rate of depletion and rate of replenishment. For the calculation of area required for a single stockpile, the average weight per cubic foot should be known. A common side slope is taken as $1\frac{1}{4}$ horizontal to 1 vertical. Where the shape of stockpile is such as to have a long axis with a ridge running along the top, the number of tons per foot of length is approximately $L^2/110$ where $L$ is the width of the base in feet (based upon a loose weight of 91 lb per cu ft).

Large stockpiles have the disadvantage, where they cannot be fully covered by a single crane loading the bins, of requiring rehandling of the material. This is to be avoided if possible since the practice is costly and tends to increase segregation and breakage of particles. The best practice in stockpiling is to unload by crane, placing the material in horizontal layers the height of which depends upon the single clamshell load. The stockpile should never be brought to a cone by continuous deposition in the center as this practice will definitely cause segregation.

The surface on which the aggregate stockpiles are to be placed must be well drained, fairly level, clean, and hard enough to resist scraping by the clamshell. It is common to pave the area with densely rolled aggregate, steel plates, wooden planks, or even concrete. Where the size of the yard or reach of the loading crane is so limited as to bring the stockpiles of dissimilar materials close together, the usual practice is to build timber partitions between them to prevent intermixing.

The Concrete Plant

The concrete plant commonly consists of separate stockpiles of the aggregate, a crane for loading the batching-plant bins, a water tank or source of water under pressure, a small boiler room if winter concreting is contemplated, a small office, and a paved road to accommodate the heavy truck traffic (Fig. 2-39). The batching plant has been described elsewhere but consists of overhead bins for the aggregate and cement, which materials then feed by gravity to a weighing floor from which in turn the materials drop either into a mixer or into transit-mixing trucks or batch trucks. The use of a crane, fixed or portable, is the commonest method of loading overhead bins of aggregates. A more elaborate method is the use of an inclined-belt conveyor. Such an installation is normally justifiable only for very large single

![Fig. 2-39. Concrete plant.](image-url)
projects (100,000 cu yd up) or for permanent concrete-producing plants. Other methods of loading bins are by ramp or by bucket elevators.

The cement is commonly elevated to a closed compartment in the overhead bin by continuous enclosed bucket elevators which ordinarily have a handling capacity of 150 to 300 bbl per hr. The cement is delivered in a closed truck or railroad car, discharged into a bin at or below ground, and carried to the bucket elevator by a horizontal-screw conveyor. In special cases where more than one type of cement is specified for the same project, dual or multiple cement-handling facilities may have to be designed into the plant unless it is possible to so arrange the concreting schedule as to use the separate cements successively.

The layout will, of course, be affected by the size and shape of the available area. Even if unlimited area is available, the plant should be kept as compact as is consistent with sufficient area for storage of materials. Where deliveries of aggregates may be confidently expected on short notice, these storage areas may be laid out to be only as large as can be efficiently reached from a loading crane placed near the bins. A not uncommon practice in transit-mix or dry-batch plants is to separate the aggregate elevated bins from the cement bin by about 50 ft or more so that two trucks may be loaded simultaneously and so save loading time. This is called a two-stop plant. Many engineers frown on this practice as it requires twice as many inspectors at the plant because of the two weighing locations to be watched. Separate entrance and exit are desirable to facilitate truck movements.

**Water Requirements**

The source of water must be studied since a considerable quantity may be needed. If a convenient public water-supply system exists, there is no problem unless the pressure is very low. In such cases a pump and elevated tank may be desirable. If a well has to be drilled, the required capacity will have to be computed. For temporary installations it will probably be less expensive to install a tank and develop a well with a low pumping rate. In computing the required water, include the following:

1. Mixing water, 30 to 40 gal per cu yd of concrete.
2. Wash water for transit-mix trucks, 30 gal per load.
3. Water for keeping stockpiles moist, particularly for aggregates of high absorption. Not less than 10 gal per cu yd of concrete is desirable.
4. Water for boiler for heating aggregates, small quantity.

In the absence of a careful analysis of water requirements an allowance of 100 gal per cu yd of concrete is reasonable. Where a well or source of water other than public drinking water is used, there should be some thought given to the effect of possible contained reactive elements on the cement. When in doubt, either cement briquettes or concrete cylinders may be made with the proposed water and strengths compared with others made from an accepted source of water. Clearly the water source must be such as not to have an adverse effect on the strength of the concrete. Instances exist where failure to observe the precaution of testing the water has proved disastrous.

**DESIGN OF A MIX**

This means the determination of the best proportions of the four ingredients, cement, fine and coarse aggregates, and water (plus occasionally an admixture), to produce concrete which will meet the requirements of the structure. In the United States, this is commonly done by the field engineers (with or without the aid of a laboratory) because they are most familiar with economically available materials and local conditions. For unimportant jobs the approximate proportions can be selected from a table such as Table 2-6 with later adjustment, if necessary, by observation at the mixer and in place. Large or important jobs should be done with the aid of a laboratory in order to secure the best results with economy. The laboratory
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**With Fine Sand—Fineness Modulus 2.20-2.60**

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<td>32</td>
<td>5.3</td>
<td>1,165</td>
<td>2,170</td>
<td>5.10</td>
<td>220</td>
<td>335</td>
<td>35</td>
<td>5.8</td>
<td>1,280</td>
<td>1,940</td>
<td>4.66</td>
</tr>
<tr>
<td>5 1/4</td>
<td>7</td>
<td>300</td>
<td>310</td>
<td>37</td>
<td>5.3</td>
<td>1,590</td>
<td>1,640</td>
<td>5.10</td>
<td>295</td>
<td>250</td>
<td>40</td>
<td>5.7</td>
<td>1,680</td>
<td>1,425</td>
<td>4.74</td>
</tr>
<tr>
<td>1</td>
<td>7</td>
<td>280</td>
<td>360</td>
<td>36</td>
<td>5.1</td>
<td>1,430</td>
<td>1,835</td>
<td>5.30</td>
<td>275</td>
<td>290</td>
<td>39</td>
<td>5.6</td>
<td>1,540</td>
<td>1,625</td>
<td>4.82</td>
</tr>
<tr>
<td>3 1/2</td>
<td>7</td>
<td>270</td>
<td>405</td>
<td>34</td>
<td>4.9</td>
<td>1,320</td>
<td>1,985</td>
<td>5.51</td>
<td>275</td>
<td>335</td>
<td>37</td>
<td>5.3</td>
<td>1,455</td>
<td>1,725</td>
<td>5.10</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
<td>270</td>
<td>465</td>
<td>32</td>
<td>4.6</td>
<td>1,240</td>
<td>2,140</td>
<td>5.87</td>
<td>280</td>
<td>380</td>
<td>35</td>
<td>5.0</td>
<td>1,400</td>
<td>1,900</td>
<td>5.40</td>
</tr>
<tr>
<td>5 1/4</td>
<td>8</td>
<td>365</td>
<td>350</td>
<td>37</td>
<td>4.6</td>
<td>1,680</td>
<td>1,610</td>
<td>5.87</td>
<td>355</td>
<td>280</td>
<td>40</td>
<td>5.0</td>
<td>1,725</td>
<td>1,400</td>
<td>5.40</td>
</tr>
<tr>
<td>1</td>
<td>8</td>
<td>340</td>
<td>400</td>
<td>36</td>
<td>4.5</td>
<td>1,530</td>
<td>1,800</td>
<td>6.00</td>
<td>335</td>
<td>320</td>
<td>39</td>
<td>4.9</td>
<td>1,640</td>
<td>1,570</td>
<td>5.51</td>
</tr>
<tr>
<td>3 1/2</td>
<td>8</td>
<td>330</td>
<td>455</td>
<td>34</td>
<td>4.3</td>
<td>1,420</td>
<td>1,960</td>
<td>6.28</td>
<td>335</td>
<td>380</td>
<td>37</td>
<td>4.6</td>
<td>1,540</td>
<td>1,750</td>
<td>5.87</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
<td>335</td>
<td>525</td>
<td>32</td>
<td>4.0</td>
<td>1,340</td>
<td>2,100</td>
<td>6.75</td>
<td>335</td>
<td>425</td>
<td>35</td>
<td>4.4</td>
<td>1,475</td>
<td>1,870</td>
<td>6.14</td>
</tr>
</tbody>
</table>

may be a commercial one. These are usually available within a convenient distance. On very large jobs an economic study should be made to see whether the establishment of a site laboratory is justified.

The design, when possible, should be started sufficiently before the first concrete pour to permit some 28-day test cylinders to be made and broken.

Requirements

It is first necessary to establish clearly the requirements which the mix design must meet. These will generally include one or more of the following:

Strength—compressive or flexural
Durability—resistance to the weather
Resistance to wear—as in a pavement slab
Light weight—special floor construction
Watertightness—as for tanks
Mere mass—as for anchorages

Method

In proportioning the mix it is necessary to bear in mind not merely the final product but also the problems of mixing, handling, and placing. The great bulk of concrete produced falls in the first three categories above; i.e., it is designed for strength, durability, or resistance to wear. All three are closely related to density. Hence a simple approach is to seek maximum density with workability.

Of the four constituents, the two aggregates offer the widest latitude in selection. The type of cement will usually have been specified or is clearly indicated by the type of structure.

There is rarely much choice in the selection of water for use in concrete. In built-up areas, water from municipal distribution systems is commonly used and, in general, potable water is satisfactory. Where such natural sources as rivers, lakes, or springs are used, unless previous local experience has shown the water to be satisfactory, it should be carefully examined to see that it is free from objectionable quantities of silt, organic matter, soluble salts, or other impurities. Water with a turbidity of more than 1,000 ppm should be treated. Similarly water containing more than 1,000 ppm of sulfate should be analyzed and advice sought on its treatment before use. When in doubt the effect of a particular water may be tested by means of mortar briquettes.

Selection of Aggregates

The selection of the two aggregates is important because:

1. They constitute 65 to 80 per cent of the concrete volume and hence have a marked effect on cost.
2. If weak, they will limit the concrete’s strength, durability, and resistance to abrasion.
3. They might be chemically reactive to the extent that the structure will deteriorate rapidly or actually fail.

For these reasons the engineer should acquaint himself with the results of previous local experience with the aggregates he intends to use. If the territory is strange, inquiries may be made of official engineering agencies such as the municipal, county, or state highway engineer. They will frequently be of great assistance in selecting sources of aggregates.

The Water-Cement Ratio

A dependable method in wide use for designing concrete mixes makes use of the water-cement ratio expressed in gallons of water per sack of cement or the ratio of the weights of each. These two ratios are numerically different for identically
the same proportions. The ratio in more common use is gallons per sack of cement. Expressed simply, the basic idea is that, within the range of plastic mixes, the strength of concrete varies inversely as the ratio of water to cement in the mix. For a more detailed explanation, refer to the section on Materials.

Where strength controls: select the water-cement ratio from one of the curves of Fig. 2-40. Most design specifications for Type I cement give the required ultimate strength at 28 days, whereas for Type III an additional early-strength requirement at 1, 3, or 7 days may control. These curves cover the compressive strength attained for a variety of materials which meet ASTM specifications, and where the proportions of all materials are controlled. Judgment as to the comparative excellence of the materials on the specific job will have to be exercised in choosing the portion of the band to use. On large jobs it is worthwhile, if time permits, to develop a curve independently, by laboratory tests of the actual materials to be used.

Where durability controls: select the water-cement ratio from Table 2-7.

Having chosen the water-cement ratio appropriate to the design requirement, next select the maximum size coarse aggregate which is readily available economically and consistent with the size of member and reinforcing-steel spacing. The maximum size aggregate tends to increase the density of the concrete by filling a larger proportion of voids. It also reduces the total surface area of the aggregate and thus reduces the cement requirement. For mass concrete, aggregate sizes up to 6 in. are not
Table 2-7. Net Water-Cement Ratios for Various Types of Construction and Exposure Conditions*

<table>
<thead>
<tr>
<th>Type or location of structure</th>
<th>Severe or moderate climate, wide range of temperature, rain, and long freezing spells or frequent freezing and thawing</th>
<th>Mild climate, rain or semiarid; rarely snow or frost</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Thin sections, gal/sack</td>
<td>Moderate sections, gal/sack</td>
</tr>
<tr>
<td>At the water line in hydraulic or waterfront structures or portions of such structures where complete saturation or intermittent saturation is possible, but not where the structure is continuously submerged:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>In sea water</td>
<td>5 5½</td>
<td>5¾ 6 6</td>
</tr>
<tr>
<td>In fresh water</td>
<td>5¼ 6 6 6 6½</td>
<td>5½ 6 6 6½ 6½</td>
</tr>
<tr>
<td>Portions of hydraulic or waterfront structures some distance from the water line, but subject to frequent wetting:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>By sea water</td>
<td>5¾ 6 6 6 6</td>
<td>5¼ 6 6 6 6½</td>
</tr>
<tr>
<td>By fresh water</td>
<td>6 6½ 6½ 6½ 6½</td>
<td>6 7 7 7 7½ 7½</td>
</tr>
<tr>
<td>Ordinary exposed structures, buildings, and portions of bridges not coming under above groups:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>In sea water</td>
<td>6 6½ 6½ 7 7</td>
<td>6 6½ 6½ 7 7</td>
</tr>
<tr>
<td>In fresh water</td>
<td>6½ 7 7 7½ 7½</td>
<td>6½ 7 7 7½ 7½</td>
</tr>
<tr>
<td>Concrete deposited through water</td>
<td>† † 5½ 5½ 5½</td>
<td>† † 5½ 5½ 5½</td>
</tr>
<tr>
<td>Pavement slabs directly on ground:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wearing slabs</td>
<td>5½ 6</td>
<td>† † † †</td>
</tr>
<tr>
<td>Base slabs</td>
<td>6½ 7</td>
<td>† † † †</td>
</tr>
</tbody>
</table>

Special case: For concrete not exposed to the weather, such as interiors of buildings and portions of structures entirely below ground, no exposure hazard is involved, and the water-cement ratio should be selected on the basis of the strength and workability requirements.

* Adapted from Table 1 of the 1940 Joint Committee, Report on Recommended Practice and Standard Specifications for Concrete and Reinforced Concrete, in Design and Control of Concrete Mixtures, 9th ed., Portland Cement Association.
† These sections not practicable for the purpose indicated.

excessive provided a well-graded aggregate of this size can be found. For most reinforced-concrete structural members 1½ in. is the largest commercially available generally, and ¾ in. is the commonest size in general use.

Gradation of Materials

Where spaces between the reinforcement are small, the nominal size of aggregate should not exceed two-thirds the smallest spacing in order to prevent screening, with consequent segregation.

Material passing the ⅝-in. sieve is known as fine aggregate. The separation of bank-run gravel into fine and coarse fractions has permitted a better gradation of the entire aggregate and hence a more dense concrete. Since the fine aggregate forms between 20 and 40 per cent of the volume of the concrete it is an important ingredient, and only a material whose characteristics are known or thoroughly tested should be used.

The gradation of particle size is important and should be kept within the acceptable limits. A general measure of the fineness of the fine aggregate is known as the
fineness modulus (FM). The FM is the sum of the cumulative percentages of material retained on the six standard sieves divided by 100. It is important to note that it gives no idea of the quality of the grading. The larger the FM the coarser the sand. Fine aggregates in general use for concrete have FMs lying between about 2.20 and 3.20 with the best between 2.50 and 3.00.

For small jobs the proportions of materials may be selected directly from Table 2-6 once the water-cement ratio, size of coarse aggregate, and coarseness of sand are known. The table is self-explanatory. It should be noted that the proportions given are for concrete of medium consistency. Where strength or economy are not prime requisites the consistency may be changed by varying the amount of mixing water by trial.

Proportioning of Materials

For carefully controlled jobs a more accurate design of mix is essential. Although maximum density of concrete is desirable, the need for a consistency which permits placing so that all corners of the form are filled and reinforcement is completely embedded operates against this. There is usually considerably more fine aggregate required for workability in placing than is needed to fill the voids in the coarse aggregate. A simple procedure for designing the mix with the aid of a laboratory follows.

Select the W/C which meets requirements of strength and durability, whichever gives lower W/C. From Table 2-6 select the weights of fine and coarse aggregates to use as a trial with 1 sack of cement. Thoroughly mix the two aggregates in this proportion.

Mix water and cement in the selected W/C ratio to form a paste. Add small amounts of the combined aggregates to the paste until a concrete of the desired consistency has been obtained. Record all quantities used. Consistency is an important element in the production of good concrete. Its selection depends on the type of structure for which the concrete is intended and whether internal vibration will be used in placing the concrete in the forms. Suggested consistencies, as measured by the slump cone, are given in Table 2-8.

<table>
<thead>
<tr>
<th>Type of construction</th>
<th>Slump, in.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reinforced-concrete foundation walls and footings</td>
<td>2-5</td>
</tr>
<tr>
<td>Plain footings, caissons, substructure walls</td>
<td>1-4</td>
</tr>
<tr>
<td>Slabs, beams, reinforced walls</td>
<td>3-6</td>
</tr>
<tr>
<td>Building columns</td>
<td>3-6</td>
</tr>
<tr>
<td>Pavements</td>
<td>2-3</td>
</tr>
<tr>
<td>Heavy mass construction</td>
<td>1-3</td>
</tr>
</tbody>
</table>

* Adopted from the 1940 Joint Committee Report.

When high-frequency vibrators are used the above values may be reduced about one-third.

The slump cone is a simple device consisting of a sheet-metal truncated cone 12 in. high with bottom and top diameters 8 and 4 in., respectively. Using the cone as a form, concrete is tamped into the cone in a specified manner (ASTM Designation C 143-39). When the cone is carefully raised the amount the concrete "slumps" is a measure of its consistency.

It is desirable to mix several small batches in a rotary mixer and observe the finished product. Slumps should be recorded for field control.

The yield should be calculated for a batch using 1 sack of cement (94 lb) by the method of absolute volumes. Yield means the volume of concrete for a given quantity of ingredients.

Method of Absolute Volumes. This method assumes that the volume of concrete is the sum of the absolute volumes of each ingredient, including water and occasionally air. The absolute volume of the solids is the volume as if melted down to have no voids.

Example of a Mix Design

Assume a reinforced-concrete building designed for 3,000 psi with thin sections and in a severe climate. Coarse aggregate is ¾-in. stone. Sand FM is 2.70. Spe-
cific gravity of stone is 2.72 and of sand is 2.65. Select \( W/C \) for strength from Fig. 2-40: 7.4 gal per sack. Select \( W/C \) for durability from Table 2-7: 6.0 gal per sack. Observe that durability controls and with the lower \( W/C \) we shall probably achieve a strength of 4,000 psi. For trial proportions see Table 2-6.

<table>
<thead>
<tr>
<th>( W/C )</th>
<th>Cement, lb</th>
<th>Sand, lb</th>
<th>Stone, lb</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.0</td>
<td>94</td>
<td>230</td>
<td>230</td>
</tr>
</tbody>
</table>

After mixing our trial batch we find we can get the desired consistency in our judgment by using only about 0.95 sack for the 460 lb of aggregate. Furthermore as the mix looks oversanded (not enough coarse aggregate is visible when a trowel is drawn across the surface of the batch) and since too much sand is uneconomical we further adjust proportions by other trial batches with these results:

<table>
<thead>
<tr>
<th>( W/C )</th>
<th>Cement, lb</th>
<th>Sand, lb</th>
<th>Stone, lb</th>
<th>Slump, in.</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.0</td>
<td>94</td>
<td>230</td>
<td>254</td>
<td>3( \frac{3}{4} )</td>
</tr>
</tbody>
</table>

Calculating yield for 1 sack of cement:

\[
\begin{align*}
\text{Material} & \quad \text{Absolute volume, cu ft} \\
\text{Cement:} & \quad \frac{94 \text{ lb}}{3.15 \times 62.3} = 0.480 \\
\text{Sand:} & \quad \frac{230 \text{ lb}}{2.65 \times 62.3} = 1.392 \\
\text{Stone:} & \quad \frac{254 \text{ lb}}{2.72 \times 62.3} = 1.500 \\
\text{Total water:} & \quad \frac{6.0 \times 8.33 \text{ lb}}{62.3} = 0.803 \\
\end{align*}
\]

The volume of concrete to be expected from 1 sack of cement with these quantities of these materials is 4.175 cu ft. To find the number of sacks of cement required for 1 cu yd of concrete, divide this volume into 1 cu yd:

\[
\frac{27}{4.175} = 6.47
\]

For quantities of each material required per cubic yard of concrete multiply by this factor, 6.47:

<table>
<thead>
<tr>
<th>Cement, lb</th>
<th>Sand, lb</th>
<th>Stone, lb</th>
<th>Total water, gal</th>
</tr>
</thead>
<tbody>
<tr>
<td>608</td>
<td>1,487</td>
<td>1,642</td>
<td>38.8</td>
</tr>
</tbody>
</table>

This is a mix high in cement content due to exposure to severe climatic conditions. However, it is important to observe that cement has a major effect on the quality of the concrete and its cost per bag is usually between \( \frac{3}{50} \) and \( \frac{3}{100} \) the cost of 1 cu yd of concrete in place.

Use of Air-entrained Cement

Where air-entrained cement is used in concrete the calculation of the yield must take the air-bubble volume into account. Specifications usually allow a range of
entrained air between 3 and 6 per cent of the concrete volume, and a volume commonly experienced is 5 per cent. Yield is figured as follows:

| Volume for cement, sand, stone, and water (above) | 4.175 |
| Air volume at 5% | 0.209 |
| New volume per sack | 4.384 |

For cement factor, 27/4.384 equals 6.16 sacks per cu yd.

The percentage of air is determined by simple field test. Variations in percentages of air entrained lead to small but annoying variations in yield. They are seldom large enough to affect daily orders of concrete from the mixing plant.

In the design of a mix, the final test is to observe the product at the site. The laboratory proportions must not be regarded as sacred but always subject to adjustment in the field. Variations in the aggregates occur which require adjustments in the proportions at the site.

Free Water in the Mix

The amount of water used in the above calculations is the total free water in the mix. The amount of surface moisture on the two aggregates is often a substantial fraction of the total water and adjustments must be made in the aggregate scale weights and the water added at the mixer, on this account. For proper control, the surface moisture must be checked frequently, possibly several times per day. This can be done with a simple apparatus at the batching plant. A representative sample should be taken from a point near the scales (usually the hopper). It is weighed, then dried to a constant weight by heating, and then reweighed. The lost weight represents the surface moisture and is expressed as a percentage of the dry weight of the aggregate. The ranges commonly experienced are:

<table>
<thead>
<tr>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>For fine aggregate</td>
</tr>
<tr>
<td>For coarse aggregate</td>
</tr>
</tbody>
</table>

Since this water will be weighed with the aggregate, the scale weights must be increased in order to assure getting the proper dry weight of aggregate. In common commercial practice the prices quoted are such that the surface moisture is paid for as aggregate as a matter of simplicity.

Similarly, this surface water must be deducted from the mix-design total water volume in order to determine how much water to add at the mixer.

Example of Adjustment for Surface Moisture. Field measurements indicate 5 per cent for fine aggregate and 1 per cent for coarse.

- Sand: 1,487 lb plus 5 per cent gives 1,561 lb scale weight.
- Stone: 1,642 lb plus 1 per cent gives 1,658 lb scale weight.
- Water: 74 lb plus 16 lb equals 10.8 gal to be deducted from the design value of 38.8 or 28.0 gal to be added at mixer.

In practice a good control on the water to be added at the mixer is observation of consistency by an experienced mixer operator who does not add all the mixing water immediately but observes the concrete within the drum and adds the last few gallons as necessary. This approach will take care of momentary variations in surface moisture. It is less scientific but can result in a more uniform concrete.

Field Adjustment of Mix

Occasionally minor adjustments of the laboratory mix become necessary at the site. Some useful approximations are:

- If it is desired to keep the strength or durability constant while varying the water, the W/C should be held constant. For desired changes in slump a change of about 3 per cent of the water will change the slump about 1 in. in the same direction.
- If the FM of the sand changes from that used in mix design by 0.10, a change of
about 0.5 per cent of sand volume in the same direction is reasonable. If it is desired to maintain the yield by varying the weight of sand, the change should be compensated by the same weight of coarse aggregate unless their specific gravities differ appreciably. To see the effect of any change on the yield observe the figures used in the yield computations above.

QUALITY CONTROL

Concrete is the most important construction material which is manufactured at the site. It is usually made outdoors under conditions not conducive to production of a uniform material. Since the structures made of concrete frequently involve public safety and are costly, quality control assumes great importance.

Quality control falls logically into two phases, namely, laboratory tests of ingredients sampled at the source and field control tests during and shortly after the making of the concrete. The laboratory tests of the ingredients are covered under the section on Materials. The field engineer must assure himself that these tests have been made by requiring written reports which he must examine carefully and, equally important, must assure himself that the materials received are those which have been tested. A system of identifying tags is usually employed.

Although a degree of quality control is exercised by reliable contractors, the principal control rests with the owner, who may use his own employees or one of the well-established testing companies. The desired quality must be clearly stated in the specifications. Quality of concrete is commonly stated in terms of the desired minimum compressive ultimate strength at 28 days or (for pavements) the modulus of rupture.

Methods of performing various control tests have been standardized by the American Society for Testing Materials (ASTM) and the American Association of State Highway Officials (AASHO). Both these organizations publish descriptions of their standard tests and sell them to nonmembers at nominal prices. The importance of performing the tests in exact accordance with the standard method cannot be overstated. Careless performance of tests will give widely varying results from the same material, which leads to erroneous conclusions concerning the quality of the concrete in the structure.

To assure good-quality concrete it is desirable to use time-tested ingredients. New materials may contain unsuspected weaknesses which the most elaborate testing program may fail to uncover.

In order to avoid the loss of the cost of freight on rejected materials the practice is to make the basic tests on concrete materials at or near their source. However, certain tests must be made at or near the site of the work. These are covered below.

Material Test

To test materials, it is necessary to sample them. The basic idea in taking samples is that they must be representative. For granular materials this is not easy because of the tendency of particles to segregate. In sampling coarse aggregate in stockpiles it is necessary to take samples from the top of pile, the middle, and the base and remix them. For stockpiled sand the outer layers should be shoveled away until the inner damp sand is reached. Another method is to use a sampling tube consisting of a pipe about 1 1/2 in. in diameter and 6 ft long and drive it into several portions of the stockpile successively. In sampling aggregates in railroad cars or barges at least three trenches should be dug across the car and at least 1 ft deep to obtain the sample.

The number of samples to take depends upon judgment and the observed variation from sample to sample. Some specifications require at least one sample from each 50 tons of material. As to size of sample, 10 lb is common for sand and the small sizes of coarse aggregate, with 50 to 100 lb for sizes up to 2 1/2 in. The samples are generally transported in burlap bags.

In sampling concrete for compressive or transverse strength the logical place, where feasible, is within the form. Concrete from several parts of the form should
be shoveled into a bucket and transported to the place where the test specimens are to be made. At that place the sample should be remixed by hand on a clean surface before placing in the molds. Not more than 15 min should elapse from the time of taking the sampling until it is in the molds.

Where concrete from mixing trucks is to be sampled the method is to pass a receptacle completely through the discharge stream three times near the middle of the batch, and then remix by hand.

Two field tests which must be made frequently on the aggregates are gradation and surface moisture. The first is made by sieving, using the standard series of sieves to determine conformity of the grading with the specifications and to test variation in the fineness modulus. It will be noted that the specifications for gradation permit a rather wide variation in percentage for each size; however, for any given aggregate, the percentages should not vary much or the mix must be redesigned. Some specifications do not permit a variation either way greater than 0.20 from the fineness modulus of the original sample on which the mix was designed.

Water Control

Since rather large volumes of water may be present on the surface of the particles of fine aggregate and so affect the total free water in the mix, it is necessary to test for surface moisture frequently. Since the weight of this water may run up to 6 or 7 per cent of the weight of the fine aggregate, variations in surface moisture necessitate adjustment of the batch weights. There are several methods of finding the surface moisture on the aggregates, which is expressed as a percentage of the weight of the aggregate. For many years the method selected for use by batch-plant inspectors has been to drive off the moisture on a small sample by heating in a pan placed over a hot plate and weighing the sample before and after heating. When this percentage has been found, the dry weights are increased by the percentage of surface moisture present on each aggregate and the total water on the aggregate is deducted from the free water in the mix design to decide how much water to use in the batch.

The pycnometer, the Chapman flask and the carbide-pressure method are also used in determining surface moisture. In recent years increasing emphasis has been placed on the need of a method which is rapid, reasonably accurate, and can be performed by operators of relatively low skill such as are normally employed in batch plants. The carbide-pressure method measures the amount of water present in a fixed weight of fine aggregate in terms of the pressure developed in the gas produced by reaction between carbide and surface moisture.

The coarse aggregate is subject to surface moisture but to a very much smaller degree (seldom over 1 per cent). For this reason the correction is often estimated.

Certain lightweight aggregates of high moisture absorption and variable specific gravity make these tests necessary for accurate batching by weight. They are a special problem.

Since water is one of the two reactive ingredients in concrete, the field engineer must be alert to any peculiarities in the local water supply. If he has any doubts as to its quality as an ingredient in concrete he should take samples in a clean container and ship them to the laboratory for testing well in advance of the first concreting operations.

The quality of concrete depends in large measure on accuracy in batching all the ingredients. The scale weights for the aggregates must be adjusted upward to allow for the surface moisture present on the aggregates and a corresponding adjustment downward must be made in weighing the water to be added. Variations in total free water in the mix can be very troublesome and some means should be adopted to avoid wide fluctuations in the surface moisture on the aggregates. These include adequate drainage of both stockpiles and bins. Since water once added to a batch cannot be removed, a method has developed, where transit-mix concrete is used, of controlling the water by controlling the slump of the concrete. It depends for its success on the fact that the slump is fairly critical with respect to water. The pro-
The Slump Test

The consistency of fresh concrete is an important characteristic in that it affects its workability. Required consistencies vary with the type of structure being poured. Once chosen in making the mix design, the required consistency should be closely adhered to. Consistency is commonly measured with the slump cone (Fig. 2-41). Briefly stated, the slump test is made by measuring the number of inches which concrete subsides when a standard conical mold into which it has been tamped is removed. The greater the slump the wetter the consistency. The slump cone is standard equipment on every well-managed job. It is very important that the exact method presented in the standard test (ASTM Designation C 143-52) be followed.

The slump test is made on a flat, level, moist, nonabsorbent surface. A representative sample of concrete is placed in the slump cone in three equal layers. Each layer is rodded 25 times with a round metal rod using strokes which penetrate into the layer below. After the top layer is struck off, the cone is carefully raised vertically. The slump is the amount the concrete subsides as measured from its height when in the cone. The cone and rod should be washed clean after each use. Slumps should not vary more than 20 per cent from the specified slump, provided the average of three determinations is used for comparison.

Material Quantities

Considering the quantity of concrete in the average structure, economy of materials is important. Since cement is the most expensive ingredient, the volume of concrete produced per bag is frequently used as a measure. It is termed the yield. A numerical method of determining the volume of concrete is given under Design of Mixes. A direct method of determining yield in the field is to use a standard calibrated bucket of either 1/2 or 1 cu ft capacity. The bucket is filled in three layers and rodded exactly as described above for the slump cone. The exterior surface of the bucket is tapped to release entrapped air bubbles, the concrete is struck off exactly flush with the top, and the surface of the bucket is cleaned and then weighed. The test is covered in detail under ASTM Designation C 138-44. Taking the sum of the batch weights of all ingredients in one batch and dividing this by the measured weight in the bucket, there is found the number of cubic feet of concrete in a batch. By dividing this volume by the number of 94 lb bags of cement in the batch, the yield is found. Yield tests should be made at least once for each pour.

Attempts to estimate the yield from plan dimensions of filled forms can lead to rather large errors unless the actual sizes of forms are carefully measured. Large losses of concrete occur in footings on irregular bottoms. Another frequent source of error in estimating yield accurately comes from fluctuations in air content (with air-entrained concrete). Attempts to control yield closer than about 1 per cent, on individual pours, usually prove fruitless.
Entrained Air

Air-entrained concrete is widely used today. The advantages of air are negligible below 3 per cent by volume, and over 7 per cent the strength reduction is appreciable. Hence specifications frequently limit the entrained air to between 4 and 7 per cent. The air volume must be found in the field by test since no simple method of controlling ingredients to control the air has been found. Aside from the quality effects of air, it has a decided effect on yield; so determinations must be made to adjust the batch weights of all materials to maintain the yield.

Several field methods are in use, the principal two being the pressure method and the direct method.

In the pressure method, which has been in use for some years, the percentage of entrained air is measured by applying air pressure to a sample of concrete placed in a closed vessel. The entrained air is compressed in accordance with Boyle’s law. The air content is read directly. A correction factor, determined for the particular aggregate used, must be applied. An especially constructed apparatus which is commercially available at a cost of under $200 is necessary for this test.

In the direct method a closed metal vessel with a neck containing a glass-closed viewing slot to enable reading a water column is used. In this method the principle used is the removal of the entrained air by inundation and separation of the constituents in the concrete, which reduces the total volume of the concrete by the volume of air lost. The apparatus is commercially available at a cost of about $140 (Fig. 2-42). It has the virtue of being simple to operate, has no moving parts, and can be used on porous aggregates such as slag where the pressure method does not work well.

As in all other tests, the frequency is dependent on judgment in the light of the degree of variation encountered on the particular job. A minimum frequency would be one test for each concrete pour and not less than one for each 100 cu yd. Variations up to 0.75 per cent on successive pours with the same materials are not excessive.

Most specifications permit a spread of 3 per cent in the entrained air. To simplify the field manufacture of air-entrained concrete, air-entraining cements are now readily available for each type of cement, normal (IA), modified (IIA), and high-early-strength (IIIA). With properly designed mixes and normal aggregates, these cements will ordinarily produce a concrete with the percentage of entrained air within the specification limits. If tests indicate the percentage is below minimum, the air may be increased by the addition at the mixer of such air-entraining agents as Darex or Vinsol resin. Such additions must be carefully controlled. If the air content is found to be high the adjustment downward is more difficult. It may be necessary to redesign the mix, bearing in mind that air decreases with lower slumps, more cement, finer-ground cement, and mixing time. In extreme cases it may be necessary to return to ordinary cement and control the air by admixtures referred to above.

Compression-test Cylinders

Even when all precautions have been taken to test all ingredients going into concrete and the mixing, transporting, and placing have been well done, follow-up tests are generally considered necessary to determine with assurance the quality of the finished product. These tests are generally compressive or flexural tests made on samples taken from the forms and allowed to set for some predetermined period such as 3, 7, 14, or 28 days. These tests have been standardized and the method of making compressive and flexural test specimens in the field is covered under ASTM Designation C 31-49. The compression-test specimen is generally a cylinder 6 in.
in diameter and 12 in. long for aggregates up to 2 in. maximum size. For larger-sized aggregates the same ratio of length to diameter (2) is used, with the diameter equaling three times the largest-size aggregate. Under ideal field conditions these test cylinders are made in metal molds but as a matter of economy the common practice is to use paraffined cardboard molds with closed bottoms. There is some reason to believe cylinders made in cardboard molds give lower strengths.

The method is as follows: Select a representative sample of concrete, carry it to the point where the sample will be stored in the field, remix it for homogeneity and fill the mold in three equal layers, rodding each layer 25 times (Fig. 2-43). The side is then tapped to remove entrapped air, and the top is struck off level and covered with a smooth metal plate. The mold should then be allowed to stand not less than 8 hr without moving or vibration. The cylinder should then be weighed to the nearest 0.01 lb and properly identified. One simple method of identification is a cardboard or metal tag with a wire which can be cast into the cylinder, taking care to have the wire enter the concrete at the top edge where it will not interfere with end bearing. The weighing is of great value in finding possible voids due to improper rodding. It should then be stored, preferably at a temperature between 60 and 80°F and under moist conditions. A common practice is to use moist sand in a box as a field storage place.

The temperature is important. If below freezing, there is danger of destruction of the test cylinder. If below 50°F, the rate of strength gain may be so low as to give a misleading impression of the strength of the concrete in the structure even under identical curing conditions. The test cylinder has such a small mass relative to its surface area that it cools in cold air much more quickly than the more massive concrete in the structure. In shipping the cylinders to the testing laboratory, great care should be used to crate them properly.

Cylinders should always be made at least in pairs because any single cylinder may suffer some damage between casting and testing. No great significance should ever be read into a single low test strength. The important thing is that the averages of a number of cylinders should be above the 28-day design ultimate strength. Occasional cylinders 10 per cent below design strength are not serious. It should be noted that a larger proportion of the age of 7-day cylinders is spent in the field under relatively poor curing conditions than for 28-day cylinders since companion cylinders are generally sent to the laboratory on the same day. Hence 7-day strengths may be abnormally low.

Under ideal conditions the relative strengths of cylinders of normal portland-cement concrete at different ages is about as follows in terms of the 28-day strength:

<table>
<thead>
<tr>
<th>Days</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>40</td>
</tr>
<tr>
<td>7</td>
<td>65</td>
</tr>
<tr>
<td>14</td>
<td>80</td>
</tr>
</tbody>
</table>

Size of cylinder (diameter) has an influence on the test strength, even where the ratio of length to diameter is maintained at 2. For example, in terms of 6 by 12-in. cylinder strengths, the observed strength of other sizes, for identical concrete runs about:

- 4 in. diam. .......... 5% greater
- 8 in. diam. .......... 5% less
- 12 in. diam. .......... 10% less
- 6-in. cubes .......... 25% greater

Six-inch cubes are used as a standard in Europe for compression tests.
QUALITY CONTROL

The number of test cylinders required varies somewhat from specification to specification. A common requirement is that one 7-day and one 28-day cylinder be made from each daily concrete pour and at least one set from every 100 cu yd of concrete.

Flexural-test Beams

For structural concrete subjected to flexural stresses as in pavements, it is not uncommon to make beam tests rather than compression tests. Beams have two advantages: They are made under stress conditions more nearly like those in the member they represent, and the test is easily made with a simple inexpensive apparatus at the site.

The specimen is commonly a plain concrete beam of square cross section 6 by 6 in. cast with the long axis horizontal. Its length varies with the type of loading used in the test, namely, cantilever or simple beam with one-point or two-point loading. For coarse aggregate larger than 2 in. a larger square cross section is used not less than three times the maximum nominal size. In molding the specimen, the concrete is placed in 3-in. layers and each layer is rodded 50 times for each square foot of top area. The curing and subsequent treatment are substantially the same as for cylinders. The beam strength is stated in terms of the modulus of rupture. The number of beams to make varies with different specifications, but a common practice is to make not less than two per day from each paving operation and not less than one for each 1,000 sq yd of pavement.

Cored Test Cylinders

Occasionally it is desirable to test the strength of concrete in the structure. This is particularly true where the cylinder strengths show a trend to a value lower than specified. Such tests can be made on cores drilled from the structure in some inconspicuous place. Such cores are not to be confused with cores drilled from pavements as a check on the thickness, although a similar coring apparatus may be used. The standard method of making cores is covered under ASTM Designation C 42-49. The diameter of the core should be at least three times the nominal size of the largest aggregate in the concrete to be tested. If possible to obtain, the length should be about twice the diameter. The compressive test should be made promptly after the core is removed from an immersion of 48 hr duration. In appraising the results of a compression test on a core, two factors must be considered, namely, the age of the specimen relative to 28 days (if compliance with specifications is important) and the ratio of length to core cylinder diameter if the length obtained is less than twice the diameter. The effect of short cores may be corrected as follows:

<table>
<thead>
<tr>
<th>Ratio of length/diam</th>
<th>Strength-correction factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.75</td>
<td>0.98</td>
</tr>
<tr>
<td>1.50</td>
<td>0.96</td>
</tr>
<tr>
<td>1.25</td>
<td>0.94</td>
</tr>
<tr>
<td>1.00</td>
<td>0.85</td>
</tr>
<tr>
<td>0.75</td>
<td>0.70</td>
</tr>
<tr>
<td>0.50</td>
<td>0.50</td>
</tr>
</tbody>
</table>

The field engineer bears a heavy responsibility in attempting to secure concrete of at least the minimum quality assumed by the designers. It will help to reexamine periodically the sources and treatment of each ingredient from its place of origin to final placement in the structure. Periodic tests and careful examination of the test results are necessary to assure quality concrete.
Section 3

PRECAST-CONCRETE CONSTRUCTION

By

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3-1
GENERAL

Introduction

Precast concrete, the application of industrial assembly-line production methods to concrete construction, while not a new concept, has until lately been considered too experimental in nature for general acceptance by either the architect, engineer, contractor, or owner. Wider dissemination of knowledge of precasting methods and advantages, refinements in reinforced-concrete materials and their application, modern construction equipment, and the desire for improvement and economy in the methods of the industry have all combined to remove precast-concrete construction from the experimental stage.

Although ever-rising costs of labor and material have increased the advantages of precast concrete, its progress is not limited to economy. It is a controlled operation, similar to factory production, and thereby assures better control of quality and a shorter period for building completion.

Precast-concrete construction is a field of great opportunity. It offers both the architect and engineer full freedom of expression. While there is no deviation from the basic design criteria, the opportunities in the details of layout, framing, and construction are many and varied and are limited only by the resourcefulness and ingenuity of the individual.

For this reason, this section will be limited to those basic principles which are peculiar to or are of greater importance in precast-concrete construction. It is intended as an aid and a guide to circumvent some problems previously encountered and to outline present techniques for their evaluation and improvement by the progressive architect, engineer, and contractor.

Concrete Products

Manufactured items such as pipe, concrete masonry units, and concrete fenceposts are of course precast, for rarely are they cast in their final position or location. These are generally referred to as "concrete products" as differentiated from precast reinforced elements of structures such as beams, slabs, columns, wall and floor assemblies, and the larger reinforced architectural concrete elements.

In expanding their operations, however, manufacturers of concrete products are now making and in many cases stocking such precast-concrete items as channel slabs, joists, and planking. These catalogued items are also referred to as concrete products.

Factory-manufactured Systems

For some years it has been possible to obtain factory-produced structures of precast frame, roof, and wall members. These factories are well developed, and some have facilities which are capable of producing almost any precast structural element desired. A few firms have predesigned elements, catalogued for consumer selection. With a known building size and loading, the elements for a complete structure are quickly fabricated from detailed standards and shipped to the site. Other firms fabricate the elements direct from consumer-supplied designs and details.

A recent development of the concrete-products industry has been the specialization in the production of precast-concrete wall panels. These panels, at present with unit areas ranging to approximately 130 sq ft, are being cast as insulated panels, or with solid or hollow cross sections.

Concrete-block manufacturers, in utilizing the tremendous capacity of modern concrete-block machines, have successfully created markets for their products in the precast-concrete field. Structural members of reinforced concrete are fabricated by the assembly of relatively small precast elements. These members are used for floors, roofs, walls, columns, beams, and arches. The members are assembled at the plant or at the job site.
Special Systems

"Tilt-up construction," although often called a type of construction, is a method of erecting precast elements. It originally applied to frame-house construction in which sections or entire walls were fabricated on the floor and tilted into place. In precasting, wall panels or bent frames are cast on the building service floor adjacent to and in line with their final position and are tilted into place.

The Youanz-Slick system, commonly known as "lift-slab" construction, is a unique method of erecting precast-concrete floor and roof slabs. Sections of the roof and intermediate floor slabs, and the entire slabs on smaller structures, are cast on the lower floor slab at grade by the stack method and, after curing, are raised to their proper elevations by a novel jacking arrangement. Large slabs up to 600 tons and multiple-story structures up to six floors have been erected by this method.

Prestressed Concrete

Prestressed concrete is concrete in which there have been introduced internal stresses of such magnitude and distribution that the stresses resulting from service loads are counter-acted to a desired degree.\(^1\)

Although obviously not related, prestressed concrete is generally associated with precast concrete. Precast concrete is readily adaptable to prestressing, and most structures containing prestressed concrete are constructed in part, if not totally, of precast concrete.

Patents

Special precast elements and structural systems may be partially covered by current patents. Some lifting devices and equipment for use in casting, handling, or erecting of precast members are also covered by patent protection. Although references are made to some specific items, and their patent protection noted, no attempt has been made to so identify all items or methods similarly protected.

Building Codes

To compensate for increased quality and control inherent in this type of construction, building codes in some areas have been revised to include provisions which eliminate or reduce many restrictions required for other methods of concrete construction. Changes in minimum cover for reinforcing, types of reinforcing, minimum section sizes, etc., are included. Where building codes have not been revised, they are generally written so as to exclude the design from compliance with the provisions of the code; and since the design is designated as a special system, it is approved by local authorities on its individual merit. This may result in a few objections to minor details, which in most instances are overcome by the use of successful precedents in other areas.

The American Concrete Institute has adopted and is revising from time to time a standard for some manufactured precast items or concrete products.

The Institute also has made considerable progress on the formation of a standard for precast-concrete construction. It is expected that, upon completion, this standard will form the basis for a formal building code which will probably supplement the ACI Building Code now being used by many communities. In the meantime, the informal standards will be used as a guide in the design and acceptance of precast concrete.

Basic Concepts

Precast-concrete construction should be based upon good practical planning along with mass-production methods. Weight and strength are primary considerations

\(^1\) Proposed definition of the ACI.
in precast-concrete construction. A successful design is one which utilizes the least number of assembly elements, possessing the least erection weight and the greatest strength per unit weight of framing. As precasting is a controlled operation, performed under ideal conditions of forming, pouring, vibration, curing, and inspection, the requirements of strength and dimensional restrictions necessary for casting slender sections are generally met with little difficulty. However, the design of a precast-concrete structure involves analysis beyond that normally encountered in a more conventional method. It requires a complete design of connection details and an investigation of stresses involved in the sequence of erection, which will in themselves influence the framing and design of the elements. The designer must familiarize himself with the various steps of construction and prepare his details accordingly. The plans and specifications must be complete and specific.

Generally it is essential that the number of various shapes and sizes of the precast elements be kept to a minimum. This allows maximum reuse of the forms and reduces forming, casting, and handling costs. Savings by duplication will often justify excessive material used in specifying that elements in small areas of various heights, spans, or loadings be identical to the elements of larger areas.

The layout of the frames and enclosures is controlled somewhat by the available handling and erection equipment. While lifting capacity is always a factor, the maneuverability of the equipment within the building area may be facilitated by changes in column spacings, direction of main framing, or erection sequences. The design of framing elements and connections should be thoroughly checked for expansion and contraction, especially when extreme temperature limits are expected on a structure.

Cambers

Especially in unsymmetrical thin slab sections, deformation due to shrinkage in early curing periods will result in deflections which are additive to that produced under service loads. These deflections of horizontal slabs will prove excessive unless properly anticipated and allowed for in design and manufacture. The introduction of cambers, of approximately \( \frac{1}{2} \) in. in 20 ft 0 in., in the design and manufacture of long-span horizontal slabs has successfully counteracted these deflections under service loads. The cambers improve roofing, drainage, and erection conditions, as well as the appearance of the finished structure.

Continuity

The use of continuity in structural design, so desirable in monolithic construction, is obtainable in precast-concrete frames and the larger members by welding or otherwise anchoring the reinforcement, and grouting, at the joints. However, it may be and generally is to advantage to consider only single-span designs for beams, slabs, and the smaller elements. The roof elements, when bolted, welded, or otherwise connected to the framing members, and having the perimeter joints grouted, assume sufficiently reasonable monolithic properties to adjust for load distribution and to act as a diaphragm for distribution of horizontal forces within the bent areas.

Framing

The main components of a structure are the floor, roof, and wall members, and the supporting bents.

The lower floor will generally consist of a poured-in-place slab on a prepared base. Where pipe or crawl spaces are required or poor soil conditions exist, it may be of supported precast construction. For the intermediate floors or roof, framing arrangements of conventional poured-in-place construction are followed, consisting of a slab supported either directly by the main frame or by stringers which are supported by the frames, or bents. Except for conditions of unusual loadings, the latter arrangement will generally result in a framing of the lightest weight.
Vertical wall enclosures, consisting of load-bearing, or curtain, walls, usually span the bent frames. They may extend to the roof elevation or project beyond this point to form the roof parapet. These panels, in heights to approximately 30 ft, have been used on single- and multistory structures. The panels are connected to the bent frames and to the upper floor or roof framing. Where the height of the structure, or the span between the bent frames, requires panels of sizes which cannot be handled as a single large unit, the wall section is constructed of several smaller panels with vertical and horizontal joints. Where the smaller panels do not abut floor, roof, or bent frames, they may be connected to adjacent panels. Further stability may be obtained by connections to intermediate framing designed for that purpose. Wall panels can be supported vertically by grade wall beams, column footings, intermediate piers, or a continuous footing. Full-story-height panels for the upper floors of multistory structures may be vertically supported by lower panels or by the framing at intermediate floors. Smaller panels may be supported by lower panels, intermediate framing, or both.

![Diagram](image)

**Fig. 3-1. Bent frames, beams, and girders.**

The type and details of the support framing offer various design features and architectural aesthetics. This framing, as in poured-in-place construction, may be in the form of arch ribs, rigid frames, or continuous or simply supported beams and girders. The choice of framing for a particular structure, however, may be somewhat affected by construction limitations in over-all dimensions and weights of individual elements for assuring ease in handling, a minimum amount of jointing, and simplicity in connections. Bent frames of continuous or simply supported beams and girders (Fig. 3-1) generally require only simple connections. The direct bearing of one element on another or the use of bearing pockets, ledges, or haunches reduces or eliminates temporary shoring. With the exception of some continuous members, stresses due to erection sequences may be ignored.

Rigid frame bents (Fig. 3-2) of sizes consistent with available equipment may be erected as single units. Larger frames (Fig. 3-3) are cast and erected in several sections. Rigid frames, as with some continuous girder frames, should be checked to determine the presence of excessive stresses which may develop in the elements or joint connections during different sequences of erection or loading of the frames. While the use of sufficient temporary supports would eliminate these stresses, minor adjustments in the design may prove more economical by reducing the quantity of supports required during erection.
Arch bent frames (Fig. 3-4) do not present any different problems from those encountered in rigid frame bents. The ribs may be cast and erected as one or several sections. The lengths of the segments and the corresponding number of field splices and temporary supports will vary with the available equipment. Arch bents especially must be checked for erection and loading-sequence stresses. Studies must also be made and specifications issued for the proper sequence of removal of the temporary supports from this type of framing.

![Fig. 3-2. Rigid frame bents.](image)

![Fig. 3-3. Large frames, cast and erected sectionally.](image)

Bent frames for multistory structures (Fig. 3-5) may be of rigid frame design but generally will consist of continuous or simply supported beams or girders. Within practical limitations, columns of two- or three-story structures may consist of one unit for the full height of the structure (Fig. 3-5a). One-piece columns that cannot be conveniently handled are cast in several sections, in lengths equal to the height of each individual story (Fig. 3-5b).

The walls, intermediate framing, and floor and roof panels provide some lateral bracing perpendicular to the bent frames. Additional bracing may be provided by struts framing into the bents below the floor or roof frame members.

Canopies may be supported by cantilevered beams of the main frames (Fig. 3-1a). Canopies at other elevations or of unusual spans may be supported by beams connected to the bent frame or wall and supported by columns at the outer end (Fig. 3-1b). The columns may be eliminated by supporting the canopy from tie rods attached to the bent frames (Fig. 3-1c).

**Framing Elements**

With sufficient quantities involved, the designer need not hesitate to employ complicated shapes or sections which he would be reluctant to use on traditional structures if such shapes are those best adapted to the particular requirements of the
structure. This leads to economy in materials and in many cases to a savings also in casting and erection.

Elements of solid rectangular cross section, similar to poured-in-place construction, are also used in precast-concrete construction. The most favorable design cross section for any structural element is one which places the material at locations of maximum efficiency. This involves the combination of sufficient section for bending and shear, with the least cross-sectional area. Since those parts of a rectangular cross section which are most effective in resisting bending are located farthest from the neutral axis, it is evident that a large reduction in the area of a solid rectangular cross section can be made by the removal of less effective parts nearer to the neutral axis. This procedure, which is generally found to be impractical in poured-in-place construction, is entirely feasible in precast-concrete construction.

The reduction, by coring or the use of an irregular outline, in the design of columns is economical only with those of large cross-sectional area. In most cases, column cross sections in the shape of an I, T, or other irregular outline are not economical, and the reduction in weight is obtained through the use of a hollow section. Hollow-section column elements (Fig. 3-6) present the simple appearance of rectangular sections and combine the efficient sectional areas of elements of irregular outline.

![Hollow-section column elements](image)

The use of wire fabric for secondary reinforcement permits the use of large core diameters. The cored sections (Fig. 3-6a) are formed during casting by a pipe or tube in the mold. The elements of a hollow box member (Fig. 3-6b) known as thin-shell sections are generally cast in the form of a channel section, which are then bolted face to face. The channels have thin webs, reinforced with wire fabric, and flanges of sufficient thickness to accommodate the main longitudinal reinforcing and provide the necessary compression area. The web is stiffened by ribs or diaphragms cast near each end of the element and at intermediate locations where required. The flanges may have beveled edges (alternate section b-b, Fig. 3-6b) to provide partial load transfer between elements by direct bearing. The bolts are passed through holes formed by pipe sleeves cast into the flanges or in embossed lips under the flanges. The pipe sleeves also provide bearing for the bolt washers to avoid crushing of the concrete during assembly. The bolt is spot-welded to a square plate washer which, when set in a pocket flush with the concrete surface, prevents the bolt from turning while it is being tightened and also improves the appearance of the finished member.

Horizontal elements of the bent frames, or intermediate framing, may also consist of solid, cored, hollow-box, or I or T cross sections. The cored elements (Fig. 3-7a, b) and hollow-box element (Fig. 3-7c) have the same characteristics as the corresponding column elements. Weight reduction by variation of the cross-sectional outline is obtained by a modified I cross section (Fig. 3-8a) or a T cross section (Fig. 3-8b). The ends of the I or T elements may be built up to a rectangular cross section (Fig. 3-8c, d) to provide for additional shear or bearing area.

The floor and roof slabs may span the main bent frames by the use of a precast flat-slab system, generally consisting of a series of slab beams. Slab beams (Fig. 3-9)
Fig. 3-7. Hollow-section horizontal elements.

Fig. 3-8. Horizontal elements of I and T cross sections.

Fig. 3-9. Cored and solid slab beams.
are merely narrow slab sections, reinforced as beams and placed without separation to provide large slab areas. Slab beams are also used as a form and subbase for superimposed poured-in-place concrete floor slabs. This type of slab is used for unusual service loads or in areas of varying floor elevations. Lighter versions of these elements are used to span intermediate beams or purlins supported by the bent frames. An improved system of spanning intermediate supports consists of casting the slab elements in the form of smaller beam and slab sections. These are generally cast in the form of channel sections (Fig. 3-10). The channel web forms the slab portion, and the flanges form the beams, or purlins, integrally cast with the slab.

![Channel slab and beam elements.](image)

**Fig. 3-10. Channel slab and beam elements.**

In following the precasting principle of designing for the least number of assembly units and obtaining the greatest strength per unit weight of framing, it is obvious that the ideal floor or roof element will consist of the largest element of integrally cast slab and beam section, consistent with job conditions. A precast panel element utilizing this framing arrangement (Fig. 3-11) consists of a slab, two longitudinal edge beams, or girders, and a series of transverse beams which divide the panel into subpanels of rectangular outline. Where conditions permit, two of these panels may be cast as a unit by combining the two edge beams (Fig. 3-12). These panel elements are also known as thin-shell ribbed construction, along with the hollow-box elements previously described. They may be used individually or erected face to face to form hollow sections as wall members. When combined, a smooth surface is provided on both sides of the wall, and the interior is left as a hollow air space or is filled with insulation.

More frequently, the wall elements consist of solid, cored, or integrally insulated elements. The solid cross section is designed like poured-in-place walls, cast flat, and then erected. Smaller units are factory- or site-cast, and the larger units are cast on the site.
Wall elements spanning the bent frames may be cast with integral pilasters cast monolithically at each end of the element to replace individual columns. The pilaster should be cast in two sections, one half with each adjacent wall element, to balance the element and facilitate handling and erection. The pilasters are natural strongbacks and reduce handling stresses. Cored wall elements require special casting equipment not suitable for site casting yards. They are factory-manufactured and supplied in small units to about 100 sq ft.

Joint Connections

The individual floor, roof, wall, and supporting frame elements are assembled during erection to form a rigid structure. A reduction in the number of castings will reduce both the number of connections and the quantity of temporary supports required. The design of the connections should be simple and practical and should be explicitly detailed on the drawings. Where practicable, the connection should be located at a point of minimum stress, limiting the joining to minor welding, bolting, or grouting. The connections consist of simple bearing, receiving pockets to restrict lateral movement, physical connections, or a combination of these methods.

The majority will consist of physical connections. The use of welding plates, adjustable inserts, reinforcement splices, and Nelson stud bolts all permit some tolerance and facilitate completion of the connection without excessive juggling of the castings.

Bolted connections, where not encased in grout, should have lock nuts, lock washers, peened bolts, or other means of preventing the bolts from working loose after final erection.

The heat produced during the welding of surface weld plates, with good workmanship, will not crack or mar the concrete surface. The weld plates, however, should be free of any oil or bond-breaking material to avoid smoke stains on the concrete. Welded reinforcement connections should be designed for lap welds, in lieu of butt welds. Butt welding requires fabrication and erection practically without tolerances, and it is seldom feasible to obtain the required accuracy in cutting, end conditioning, or placing the bars. Generally the interval between the ends of supporting frame elements is not sufficient to offset satisfactorily large main reinforcing bars for lap welding. The bars are fabricated short enough to provide sufficient tolerance and joined by means of short splice bars.

The simplest method of connecting the column to the pier or footing is by grouting the base into a pocket in the floor slab (Fig. 3-13). The column bears on the base and is restricted from lateral movement by the floor slab. This connection utilizes a simple form in casting and a minimum amount of labor for completion. Chamfering the base of the column (Fig. 3-13b) provides a more uniform bearing. Bents which require exterior restraint at the column base, not furnished by the floor slab, are provided with tie rods between the columns of the bent or are anchored in the slab by bond (Fig. 3-13c). Note that the above joint connections require at least partial installation of the floor slab prior to erection of the columns.
Where the column reinforcement is required to be continuous through the joint, vertical footing reinforcement, or dowels, extend into a grout pocket in the column base (Fig. 3-14). The pocket is filled through the grout hole in the face of the column. The hollow-section column (Fig. 3-14a) has a natural grout pocket and requires only the addition of a grout hole in one face, while the solid section (Fig. 3-14b) requires the addition of both the hole and the pocket.

Bolted or welded base connections are also utilized to obtain continuity of reinforcement (Fig. 3-15). Figure 3-15a permits minor horizontal adjustment during erection but requires that the cap plate be set perfectly level and to exact elevation. Figure 3-15b permits vertical adjustment, but the pier anchor bolts must be set accurately. Figure 3-15c, d combines the advantages of both Fig. 3-15a, b in permitting both horizontal and vertical adjustment on erection. Complete welding details should be shown on the drawings. To minimize field labor, the bottom of the column is set so that the grout cover for bolts or welds will be below or flush with the floor surface or enclosed by wheel guards, etc.

![Diagram of column connections](image)

Fig. 3-14. Dowel-connected column and footing connections.

Careful study of the joint details for connecting the various elements of the supporting frame will help to reduce erection requirements of temporary shoring, erection devices, and scaffolding for personnel. Some connections consist only of setting one element on another, or on haunches, ledges, or in pockets. These methods (Fig. 3-16) are generally limited to the beams, stringers, and purlins of intermediate framing. Even in these cases, it is usually desirable to add rigidity to the frame by the addition of bolting, welding, or grouting (Fig. 3-17). The use of ledges, even where they are not structurally required, aids in the elimination of temporary shoring during erection.

Continuity of reinforcing steel, while required in rigid and continuous beam or girder frames, is often used in other joint connections to add rigidity to the structure. Continuity is obtained by welding the reinforcement, with or without weld plates, or embedment in grout (Fig. 3-18). The reinforcing in all joints should be encased in grout or otherwise protected.

Grout keys add rigidity to the joint and do not appreciably add to the cost of labor or material. The hollow-box element (Fig. 3-18a) and the cored section (Fig. 3-18b) have natural keys. The solid element (Fig. 3-18c) has had the key added during casting. Properly designed end ribs of hollow-box elements act as stops and prevent grout filling beyond this point. Cores of cored elements must be plugged near the ends by newspaper or paper containers as grout stops. Where required, the grouted
area in the core is extended to provide additional cross-sectional area in the joint for shear.

Floor and roof slabs are normally connected to the frame. Grout keys on the vertical edges provide for load transfer between the slab elements. In joining the roof or floor slabs to the supporting frame, those connections designed for the completion work to be accomplished from the top of the erected slabs (Fig. 3-19) have many advantages over those requiring this work to be done from underneath (Fig. 3-20). The erected elements serve as a safe and efficient working platform, eliminating scaffolding, and allow other trades immediate access to the covered area. Recessed connections offer additional fire protection, eliminate maintenance, and have a neater appearance. Exposed connections should be painted or brushed with grout for protection.

Wall elements are welded or bolted to the columns utilizing embedded weld plates or inserts (Fig. 3-21) or are placed in channel pockets in the column face (Fig. 3-21A). Welded joints (Fig. 3-21a, b, c) are rigid and do not provide for expansion in the wall elements. Additional tolerances for irregularities in casting and erection may be provided by offsetting the wall elements from the column (Fig. 3-21b) and using splice plates to complete the connection. The Nelson stud bolt, welded after erection,
Fig. 3-16. Joint details, stringer to girder and beam to column.

Fig. 3-17. Beam, girder, and column connections using bolts, welds, and reinforcing bar embedment.
Fig. 3-18. Continuity of beams at construction joints.

Fig. 3-19. Continuity of slabs at construction joints, connections from the top.
Fig. 3-20. Continuity of slabs at construction joints, connections from the bottom.

Fig. 3-21. Wall elements connected to concrete columns.

requires no tolerance and is a quick method of securing the wall elements to the column.

Oversize holes on bolted connections, with large washers (Fig. 3-21d, e) and clip angles (Fig. 3-21f), provide tolerances and slip joints for expansion. Many of the above connections may be recessed (Fig. 3-21g) to give a neater appearance.

Connections of the wall panels to the floor or roof slabs will be similar to those of the columns. In general the wall element does not require a connection to the slab.
on ground. However, where this is desired, the same type of connection may be used, or protruding wall steel may be embedded in a poured-in-place floor (Fig. 3-22). Small panels are set in mortar beds to provide uniform bearing and seal the joint. Larger panels, which may disturb the wet mortar during erection, are set on small

![Diagram](image)

**Fig. 3-22.** Connection of wall panels to floor slabs.

![Diagrams](image)

**Fig. 3-23.** Alignment of wall panels with foundation walls.

metal plates or cured mortar pads previously set by instrument. The joint is dry-packed after erection. Various methods of alignment of the wall panels with the foundation wall are used to avoid the entrance of water and for aesthetic purposes (Fig. 3-23). Vertical joints between adjacent wall elements are provided with keys to keep the joints dry and to provide a tie between the elements for permanent alignment (Fig. 3-24).
Fig. 3-25. Wall elements connected to steel columns.

Fig. 3-26. Precast wall panels tied to poured-in-place columns.

Fig. 3-27. Precast beam or girder combined with masonry or cast-in-place walls.
In combining precast concrete with wood, steel, or poured-in-place concrete construction, various methods of joint connections are available in addition to many of those described. In combination with steel, normally limited to precast wall panels, similar welded or bolted connections are used (Fig. 3-25). Precast wall panels, when combined with poured-in-place concrete columns (Fig. 3-26) are placed prior to the erection of the columns. The panels are braced temporarily and used to support the column formwork. Poured-in-place bent frames may be constructed prior to the erection of the precast wall panels, with connections similar to those for precast columns.

Precast beams or girders, when combined with masonry or poured-in-place concrete walls, may be set on the walls, set in wall pockets, or connected by embedded anchors (Fig. 3-27).

**Insulated Panels**

Integrally insulated precast-concrete panels, known as sandwich panels, are constructed by interposing insulation between two layers of concrete (Fig. 3-28).

Shear ties connect the outer layers of concrete, forming a composite panel. Individual ties must be custom-fabricated and are not so easily installed or so effective as strips of expanded metal or wire mesh which is installed in multiple units. Strip ties are automatically positioned by resting on the wire-fabric reinforcement of the lower layer of concrete and, of proper height, form chairs to support the wire fabric of the upper layer of concrete. After the lower layer of concrete is screeded, the ties are worked into the concrete to establish a good bond. Bond alone is generally sufficient to anchor the ties, and they should not be attached to the lower layer of reinforcement. To do so involves unwarranted additional labor, and the extruding ties hinder screeding of the concrete.

Foam or other insulating concrete is used for insulation, although the most suitable material is the rigid-board-type commercial insulation. The insulation should be
composed of inorganic materials and be impervious to moisture. Poured insulation must be screeded between protruding ties and will require setting time prior to the application of the top layer of concrete. Rigid-board insulation is installed simultaneously with the ties. The ties are installed in a line, in one direction, adjacent to each row of insulation, which ensures a uniform alignment and spacing of the ties. The strength of the ties should be sufficient when spaced at distances equal to the width of the insulation material. The fabric reinforcement for the upper, or top, layer of concrete is positioned on the ties, and the top layer of concrete is placed to complete the panel.

Large panels may be formed with narrow perimeters of solid concrete, reinforced with bars, to assist in resisting handling stresses (Fig. 3-28a). The small uninsulated area does not materially affect the over-all efficiency of the wall insulation. With air conditioning or humidity controls, these panels have been used with temperature differentials to 100°F without the appearance of condensation. Continuity of insulation between adjacent panels is obtained by the use of insulation splines (Fig. 3-28b) installed during erection or by staggering the ends of the concrete and insulation (Fig. 3-28c, d).

Small panels may be lifted, similar to solid panels, without separation. Larger panels, with thin layers of concrete, may require the use of vacuum lifting devices to avoid the use of excessive additional reinforcement required for lifting only.

Large panels should be closely checked for effects of severe temperature differentials occurring in heated structures located in colder climates or in refrigerated structures located in warmer climates. The thin layers of concrete expand and contract very quickly with temperature changes and may cause considerable movement in the panel. This movement will be negligible in small panels which are separated by calked joints.

Where considerable expansion and contraction are anticipated and panels of the type shown in Fig. 3-28c are to be used, particular study must be given to the panel connections. The perimeter band of concrete forms a rigid tie between the exterior layers of concrete, which is not present in the other types of panel construction. The movement of one face, reversed because of the temperature differential, is additive to that of the other face and in panels approaching a square outline will produce a “dishing” action from all four sides. The deflection of the center of the panel will cause lateral movement of the sides of the panel and possible movement at the edges in a direction perpendicular to the plane of the panel.

Expansion-type connections should be specified in instances where expansion indicates possible lateral movement in the panel. Where this is not feasible, the panel size should be reduced or the panel constructed with a solid section and the insulation later applied to the surface.

Tolerances

Particular attention must be paid to dimensional accuracies. Some inaccuracies are unavoidable, and a slight reduction in theoretical size must be provided in the details for tolerances to provide for these inaccuracies. Particularly in large areas of small elements, minor variations are accumulative, and to neglect an allowance for tolerances will lead to creep and difficulties during erection. Once permissible tolerances are established, they should be stipulated on the detail drawings. Frequent checks of the over-all dimensions of the completed castings and checks during erection will reveal variations, and corrective steps for adjustment should follow immediately.

Elements may be cast with a minus tolerance of $\frac{1}{8}$ to $\frac{1}{4}$ in. in over-all dimensions. However, the tolerances must be evaluated on each structure and on the various elements in the structure. This reduction, of course, is not required or desired where the elements are separated by grout, calking, or expansion material. Reduced dimensions are also favored in some bent frame elements to assure ease in erection. This is particularly important in utilizing bearing pockets or where one element is placed in the intervening space between two prior erected elements. Grout or calk-
ing is generally required at these points, and in many cases the pocket joint remains exposed when permitted by design and proper appearance.

The use of face-to-face weld plates, adjustable inserts, reinforcement splices, and Nelson stud bolts in connections serves to take up the inaccuracies. Bolted connections and inserts must be detailed in design and fabricated in the field with the utmost care. Bolted connections should be avoided unless the type of connection provides for practical tolerances for adjustment of the connection due to slight variations in casting.

Concrete

Precasting will normally require high-strength concrete. The design is predicated upon the use of high-strength concrete to reduce weight by a reduction in the thickness of the elements and to increase the concrete density which to a certain extent compensates for reduced cover as far as corrosion of the reinforcement is concerned.

The principle of removing the casting from the forms and of obtaining maximum form reuse requires that the concrete be of sufficient strength to resist handling stresses at a very early age. The castings will normally be removed on a daily basis. With an allowance for the necessary preparatory operations prior to concreting, the actual age of the elements at removal from the mold is reduced to approximately 20 to 22 hr. Stripping strength varies from 800 to 2,500 psi. The actual minimum strength required will depend upon the design and dimensional characteristics of the element and the method of removing it from the mold. The proposed strength should be that for which the structure was designed or a minimum strength which will meet the daily removal requirements, whichever is greater.

The use of high-early-strength cement and high-frequency vibration, combined with thorough mixing, selection, and proportioning of materials, generally produces concretes of 4,000 to 8,000 psi without difficulty. Such concretes have sufficient strength to permit removal of the castings from the molds on a daily basis.

Building-code limitations, for fire or other provisions, may restrict the minimum thickness of some elements of a structure, such as exterior walls, regardless of the concrete design strengths utilized. Non-load-bearing elements, such as curtain walls, do not require high design strengths for service loads. Concrete design strengths of 2,500 to 3,000 psi are sufficient for these elements. With the proper choice of casting and lifting methods, these lower-strength concretes will also provide sufficient strength in the element to permit safe removal from the mold. Some casting methods which permit longer curing periods in the mold and lifting methods which distribute and minimize lifting stresses permit the use of concrete containing regular cement.

While larger aggregate may be used with solid or heavier elements, thin-shell elements will be limited to a maximum of 3/4 or 3/8 in. aggregate. Low-slump concrete is preferred. However, dimensional restrictions and reinforcement may hamper the proper placing and vibrating and a higher slump with a higher cement factor may be desired to avoid honeycombing. As the castings are often erected prior to receipt of the 28-day test results normally required by the building code, the contractor should satisfy himself that the castings have sufficient strength to carry their own dead load combined with a safety factor for impact and construction live loads.

Concrete, with the use of high-early-strength cement, will normally acquire its design strength in 7 days and the nominal 7-day strength in 3 days. Therefore, many engineers require 3- or 7-day in addition to the 28-day test reports.

Reinforcement

The reinforcement of precast-concrete elements will consist of bars, welded wire fabric, or a combination of both. Thin-shell elements generally require wire fabric of rather close mesh or a combination of this fabric with standard bars. The fabric permits thinner sections and provides a more uniform distribution of the reinforcement. Wire fabric facilitates the prefabrication of mats or cages and is economical.

1 In part from Amirkian, Arsham, Proposed Specifications for Minimum Bar Spacing and Protective Cover in Precast Concrete Framing Members, J. ACI, April, 1950.
in many instances in replacing individual spacers, beam stirrups, and column ties. Mats or cages are designed and detailed with the least number of pieces which, when prefabricated without excessive tying or welding, will produce assemblies that can be stored and handled without distortion or damage, and rigid enough to maintain their shape and position during casting operations with a minimum quantity of accessories.

Code requirements on minimum spacing and corrosion protection of reinforcement were established for poured-in-place concrete and, not considered applicable to precast concrete, unduly restrict the advantages of precasting. Minimum spacing of the reinforcement is specified in terms of the maximum size of the coarse aggregate, with a minimum spacing of 1 in., to provide basically for placement of the concrete. The maximum size, seldom less than \( \frac{3}{4} \) in. in poured-in-place concrete, may be \( \frac{1}{2} \) or \( \frac{3}{8} \) in. for precasting and the 1-in. restriction is not required.

Code requirements on corrosion protection are based upon concrete thickness without consideration for the quality of the concrete. The Navy Bureau of Yards and Docks, as a result of tests and actual experience, reports that the quality of the concrete is the most important consideration. A very thin layer of rich concrete containing 7 to 8 sacks of cement per cu yd, properly graded, placed under controlled conditions, and vibrated, would provide a cover impervious to moisture, when as thin as \( \frac{1}{2} \) in.

A proposed specification for a precast-concrete construction code includes the following:

1. The minimum clear distance between parallel bars shall be 1\( \frac{1}{2} \) times the maximum size of the coarse aggregate.
2. Protective cover for surfaces exposed to weather, ground, or in contact with water: Main reinforcing beams, girders, and columns shall be protected with concrete equal in thickness to two times the maximum size of the coarse aggregate, but in no case shall the thickness of covering be less than \( \frac{1}{2} \) in. Reinforcing of slabs and secondary reinforcing in beams, girders, and columns shall be protected with concrete equal in thickness to \( \frac{1}{2} \) times the maximum size of the coarse aggregate, but in no case shall the thickness of the covering be less than \( \frac{3}{8} \) in.
3. Protective cover for surfaces not exposed to the weather: Main reinforcing in beams, girders, and columns shall be protected with concrete equal in thickness to \( \frac{1}{2} \) times the maximum size of the coarse aggregate, but in no case shall the thickness of covering be less than \( \frac{3}{8} \) in. Reinforcing in slabs and secondary reinforcing in beams, girders, and columns shall be protected with concrete equal in thickness to \( \frac{1}{2} \) times the maximum size of the coarse aggregate, but in no case shall the thickness of covering be less than \( \frac{3}{8} \) in.

In some instances of severe climatic conditions, the minimum protective cover in the above specification may require the use of galvanized wire fabric. The reduced cover for corrosion protection in the specification would be at the expense of a reduced fire rating but not necessarily in a direct ratio. The effect, if any, of the reduced rating on the economy of the structure will vary with individual structures.

Aesthetics

One of the many advantages of precast-concrete construction is its adaptability to aesthetic treatment, a feature desirable in any type of construction. Treatments normally used with poured-in-place concrete are generally more practical and economical. Some that are impractical or even impossible are used in precasting with little difficulty or additional expense.

Many treatments are adapted by use of the precasting principle of pouring vertical members in a horizontal position at ground level. One vertical surface, unformed, is available for finishing throughout the concreting operation. Thin veneers on vertical surfaces may be included without additional operations. White or colored surfaces, as in present practice, may be produced by the use of selected cements or pigments mixed with the cement or concrete. Where complete penetration is not desired, the surface may be colored by using a thin layer of colored concrete to cover the mold surface or on the exposed surface to complete the casting pour. Colored
cement or pigment may also be troweled or floated into the exposed surface. Patterns of various colors are formed with temporary or permanent division strips in the surface.

Dummy expansion joints or architectural designs may be imprinted by a projection of the mold bed surface, or by "stamping" the exposed surface. Hand templates or fixed templates bridged across the top of the mold are both used. Raised designs on the exposed surface are also formed by bridging. The horizontal position of the casting assures better control of design locations. It also eliminates most of the honeycombing occurring in poured-in-place concrete when projections are placed on vertical forms. The templates may be removed before or after curing as desired.

Textured surfaces, alone or in combination with color, may be applied to either the lower or the exposed surface. The surface to be exposed during casting is often determined by the surface treatment. The application of texture to the exposed surface frequently consists of brooming, bagging, scrubbing, steel trowel, rough or smooth float, or patterned areas utilizing one or a combination of these finishes. Unusual effects such as stippling, or any finish applied to a flat concrete surface or vertical plaster surface, is possible.

Application of textured finishes to the bottom or mold contact surface is necessarily limited to a reproduction of the texture of the mold surface. Lining the mold with rough boards, striated or other textured lumber or metal, burlap, or muslin will vary the mold surface. The burlap or muslin may wrinkle or be difficult to keep in place, especially on large areas. They may also adhere to the casting and require a separate operation to remove. Striated lumber may be economically coated with a plastic film which seals the pours to reduce bond but at the same time transposes the original texture to the casting surface. This coating is applied at the mill or job site, by dipping, spray, or brush.

The obvious increase in production and efficiency in the elimination of scaffolding for finishing vertical surfaces is further pronounced when light hand machines can be replaced with heavy floor equipment. Heavy terrazzo machines are used to grind or polish surfaces to expose selected aggregates. Aggregate, if expensive or of poor structural quality, may be introduced as a veneer or by troweling or floating into the surface, similar to color application. The use of white cement or color pigments, upon grinding, will enhance the color of the aggregate. The horizontal position of the casting will increase efficiency and control when exposing selected aggregates by scrubbing or treating with acid. The aggregate-transfer method of exposing chips of selected aggregate, used infrequently in poured-in-place concrete because of the involved and expensive procedure required, is a relatively simple and economical operation with precasting.

The selected aggregate is applied on the bottom surface by covering the mold surface with a thick layer of the aggregate. Upon removal of the casting, the excess material is brushed off. A neater appearance is obtained by spraying the bed with a heavy film of wax, dissolved in a solvent, and spreading a thin layer of aggregate on the fluid before it dries. The wax will fill voids between the chips and keep the cement grout off the surface. The wall is steam-cleaned after erection to remove the wax. The application of this finish to the exposed surface is accomplished by spreading aggregate on the fresh floated surface and gently rolling to embed the aggregate. After curing, the excess gravel is broomed off.

Simulated wall panels faced with stainless steel, aluminum, or other metals are made by lining the molds with metal sheets containing suitable anchors. If the metal is formed with returns on the sides to the depth of the casting, the metal itself is used as a mold. Interior coverings of plastic or pressed board are attached in a similar manner.

CONSTRUCTION

Job Planning

The construction of a precast-concrete structure requires considerable planning and development of details. Similar to structural-steel construction, all planning, from working drawings to the completed structure, must be completed in the early stages.
The details and planning of erection will materially affect the earlier phases of casting, storage, and handling. The casting yard should be completed and be functioning smoothly before erection is started. The planning of erection devices and details for their attachment to the elements must also be completed. It is obvious that changes in methods or details of any phase of the construction may be difficult after actual commencement of that part of the work. The efficiency obtained throughout all sequences of the construction will be determined by the degree of organization, planning, scheduling, and development of details completed prior to commencement of work at the site.

The job planning required, however, is a sound investment. The basic nature of precast-concrete construction provides the contractor with close control over all labor and materials, with little effort. The continuous repetitious operations provide ideal opportunities for perfection of manpower and equipment allocation and efficiency. The daily repetitive use of identical quantities of materials reduces waste. Close control and detailed and accurate records can be obtained on materials, labor, costs, and progress on all phases of the work.

The application of basic precasting principles creates a neat and clean working area, both at the casting yard and throughout the construction area. This cleanliness, rarely obtainable in other methods of construction, eliminates waste, promotes efficiency, and discourages accidents.

Proper scheduling will permit the installation of foundations and related work during the period that the casting yard is being constructed and put into operation. The elapsed time between storage and erection of the elements, at any specific portion of the building area, will be short. Each area becomes available to the mechanical and other trades immediately upon completion of the precast erection in that area. Therefore, exceptional continuity in the work of those trades can be maintained.

Close coordination between the architect, engineer, and contractor is essential during the planning stage. The free discussion of all phases of the work should be encouraged. The contractor should familiarize himself with design details which affect the methods and details of construction. With close cooperation, this exchange of information may result in the simplification of many construction details and will allow the contractor to avoid many problems during construction.

Casting-facility Location

Precast-concrete elements are cast or manufactured at (1) permanent factories, (2) temporary factories, (3) off-site casting yards, or (4) job-site casting yards. Elements for one structure may be supplied from a combination of these facilities.

In determining the location and type of facilities for producing the necessary elements of a structure, a number of factors must be considered. Included are availability of raw materials, transit-mix facilities, existing factories, and labor supply, both at the site and in nearest communities; number, size, and weights of various castings involved; space available on the job site; financial comparison of estimated site-produced castings and factory-produced and delivered castings; and, in some areas, climatic conditions.

Smaller elements such as small channel roof slabs, small wall panels, joists, purlins, and concrete plank may be subcontracted to firms having factory facilities for their production. On larger projects these firms may establish temporary factory facilities on the job site, if economically justified. Castings too large or unwieldy for truck or rail transportation must be cast on the site.

Factories have advantages of extensive equipment which would be uneconomical for job-site use, enclosed working areas, lower wage scales than those prevailing in construction, and low turnover of skilled personnel. The advantages enjoyed by factory production, however, are partially offset by disadvantages in their location. Added to transportation costs are the expense of loading methods and dunnage to reduce damage in transit, and possible extra handling at the factory and site for rail transportation. Provision should be made to avoid delays caused by late deliveries and damaged castings.
It will be apparent that, with competent supervision, the simple and repetitive operations of precasting can be handled efficiently and skillfully with local labor even if entirely new to them. Casting yards, exposed to weather conditions, have been operated successfully in subfreezing weather, and except under extreme conditions serious weather delays are not encountered.

**Mold Construction**

Indoor operations and reuse throughout numerous projects permit economical use of rather expensive plant and equipment for factory production. Low maintenance and operational costs are more important than low initial cost. The production of smaller castings of a limited number of types, with minor mold changes for size variations, permits the use of portable molds and stationary equipment.

Site casting may involve larger elements of many shapes and sizes. The yard is established to best fill the requirements of a particular project, with the molds being constructed at the site and discarded at the completion of the casting schedule. The molds are stationary and facilities for concreting, vibrating, and curing are portable. They must be sturdily constructed to withstand the required number of reuses within permissible tolerances without excessive maintenance.

The number of molds required is determined as follows:
1. Prepare a complete schedule of types and quantities of each type of casting.
2. Estimate the number of casting days, allowing sufficient time for economical reuse of the molds but maintaining a reasonable daily work schedule for efficient use of casting personnel and equipment.
3. Divide the casting days by the number of each type of casting to obtain the quantity of each type of mold. The daily production consists of one casting for each mold.

To conserve storage area, the casting period should be approximately the same as that estimated for erection, with sufficient time between the start of each operation to provide for curing and distribution of the castings to erection points throughout the structure. With added storage area, additional mold reuses are obtained by lengthening the casting period and starting earlier than otherwise required by the erection schedule. The production schedule must be carefully checked to ensure that sufficient and correct types of cured castings will be available to meet the proposed erection schedule. In general, the castings required to complete a specific portion of the structure are erected each day, and the daily production consists of these identical castings, with the possible addition of a small number of any special types required in quantity later in the erection schedule. The other facilities for casting and storage operations and all daily material requirements are scheduled accordingly.

Rigid molds which will maintain uniform casting dimensions throughout the required number of reuses are constructed with concrete, steel, wood, or plastic beds, and concrete, steel, or wood side forms and end bulkheads. The choice of materials is mainly a question of economical justification on individual projects. Inverted molds are used for the production of thin-shell elements such as channel and ribbed sections. In producing a casting with a minimum amount of movable form area, the irregular surface is cast face down, and the movable form consists only of the perimeter edge area.

Concrete has proved to be the most satisfactory material for the irregular surface of the inverted mold bed. A typical line of poured-in-place inverted mold beds is constructed on a concrete slab on the ground (Fig. 3-29). Wood sleepers are embedded flush with the slab for attachment of the side forms and end bulkheads, and the surface is left rough for bonding with the bed surface.

The side forms and end bulkheads are installed with a sufficient number of substantial strap or T hinges to prevent deflection of the form during concreting and constant use. Wood forms are satisfactory for approximately fifty reuses without excessive maintenance. Beyond fifty reuses, steel or replacement of the wood forms may be required. Wood forms are easily fabricated at the site, they facilitate
attachment of inserts, and wood is normally readily available. To alleviate warpage and ensure proper tolerances, first-grade spruce, fir, or other dense straight-grained lumber is used. A light metal angle is attached to the tops of the forms to assist in maintaining alignment and to preserve the screeding edge. The side forms are held in place by knee braces operating against a stop on the form. The end bulkheads are similarly braced or are locked to the side forms. A fitted piece of wood between adjacent side forms, flush with the form top and supported by the stops, provides a work platform, protects the braces, and is a check on the form alignment.

The irregular bed surface is then constructed using the side forms and end bulkheads as supports for the bed forms and, to ensure the correct level, alignment and dimensions of the finished surface. Forms are used only on the vertical edges, and prior to final set of the concrete, they are removed and the corners and edges are shaped by hand. The surface is finished with a steel trowel and polished after curing. The vertical edges must be carefully checked for irregularities which will cause binding of the casting to the surface. The additional expense of shaping the surface with forms of accurately milled lumber is generally not warranted.

Although bond preventives are required to eliminate physical bonding of the casting to the mold, the "suction" caused by differential atmospheric pressures frequently prevents removal of castings with deep irregular sections. Air or water forced between the casting and the mold equalizes the pressure and destroys the "suction." Air is normally used because of the waste problem involved with water. Pipe placed beneath the bed leads to individual chambers set flush with the mold surface. The air chamber, a short length of pipe with beveled top and fitted cover, has welded lugs for anchoring in the mold and is placed in sufficient locations for the pressure to react on the raised sections of the casting simultaneously to eliminate cracking of the casting. The cover prevents concrete from entering the chamber, and the beveled edge directs the air between the surfaces of the mold and casting. It is attached to the chamber by a small chain to eliminate loss or misplacement. A short
blast of air into the lead-in pipe is coordinated with a strain on the lifting cable, the bond is broken, and the casting is removed in a continuous operation.

In precasting individual molds, a concrete replica, or pilot model, of the final mold is constructed. Several intermediate molds, cast from the pilot model, are used to produce the required number of finished molds. The molds are set on concrete beams or timber cribbing, and the bulkheads and other accessories are attached (Fig. 3-30). When set near ground level, concrete may be placed direct from transit-mix trucks, and clearance is provided for removing the castings with lumber carriers. Castings with smooth surfaces are produced in flat bed molds, using the building service floor as a bed surface, or in molds constructed outside the building. The use of the floor slab eliminates a separate mold bed and reduces the site area required for construction. The side forms and end bulkheads, of wood or steel, are connected at the ends and weighted or braced between connections to eliminate movement and deflections. The forms are not attached to the floor slab. The floor may be used even if it is sloped for drainage, as long as the surface is smooth and slopes only in one direction.

The normal steel-troweled finish, when treated with a bond-breaking agent, is sufficiently smooth to use as a casting surface. The bond-breaking agent and other operations involved must be checked for objectionable stains or other possible damage to the floor finish.

When it is not practicable to utilize the floor slab, and quantities of castings of identical dimensions are involved, individual molds similar to the inverted molds are constructed outside the building.

It is often desirable to cast large wall panels in the sequence of their erection, which may result in casting panels of various sizes and thicknesses in different sequences during each day’s production. This is accomplished with a community flat bed mold, which consists of a flat surface of sufficient area to cast the maximum daily production. Small quantities of various sizes of castings, for which the construction of individual molds are not justified, may be cast with economical form costs in this type of mold. The mold bed (Fig. 3-31) consists of a plain concrete slab or, for continued use, a reinforced slab on ground, finished with a steel-troweled surface. A wood surface on the concrete base provides a good medium for attaching the movable forms, inserts, and forms or frames for openings. Wood sleepers, embedded in the base concrete, provide a means of leveling and attaching the wood surface. A smooth surface is obtainable with plastic-coated plywood and is satisfactory for approximately twenty reuses. The side forms are constructed of steel or wood and fabricated so that they are stable in an upright position. Wood forms facilitate revisions when changes in casting sizes are required. Forms are maintained in position by nailing to the wood surface or are held by sandbags or concrete or metal weights. The weight should not extend above the forms to interfere with screeding operations. Short forms, or bulkheads, when connected at the corners, may be sufficiently stable.
without use of weights or physical attachment to the bed. Precast-concrete side forms, mainly limited because of weight and expense, are especially economical when required in large quantities for castings having irregular sides which are expensive to form in steel or wood. In lieu of rotating on hinges, heavy concrete forms may be moved horizontally by a knee brace which also maintains the alignment of the form.

Fig. 3-31. Flat bed molds.

Fig. 3-32. Knee braces for concrete forms.

(Fig. 3-32). A short length of steel angle bolted to the concrete base parallel to the form reacts against the knee brace to counterbalance the form. A loose length of pipe placed on the base perpendicular to the form supports the form and serves as a rail in moving the form. Steel yokes are used to maintain the correct dimensional width during casting. Girders, beams, columns, and similar elements of uniform rectangular cross section may be cast on flat bed molds using preceding castings as side forms. Only one movable side form is required for each casting. The same
result is obtained by casting alternate elements in the normal manner, then casting additional elements in the intervening spaces without use of any movable side forms.

The stack method of casting consists of casting one element on top of another, with each successive element utilizing the preceding element as a casting bed. Side forms are moved vertically and held in place by inserts embedded in the edges of preceding castings. Working conditions become more inconvenient as the stack gains height, but as long as the stack remains stable there is no limit to the height.

Unless drainage is a problem, the stack may be started in an excavation to permit a greater number of elements to be poured direct from transit-mix trucks. All castings in one stack should have identical dimensions and openings of identical size and location. The surface finish is limited to steel troweling. The stack method, used for bent frame as well as wall and roof elements, conserves space, permits castings to gain added strength prior to removal, simplifies curing, and eliminates extra handling.

End bulkheads for casting columns, beams, or girders may require provisions for curing the element, protruding reinforcement for joint connections, irregular-shaped ends, grout pockets, or a combination of these items. Fabricated steel sheet or plate, although high in initial cost, will generally prove to be the most satisfactory and economical for accuracy required at joint connections and minimum maintenance. End bulkheads which form grout or dowel pockets should be provided with interior impact devices (Fig. 3-33) to the casting. Holes for protruding reinforcement are cut oversize to allow for bar deformations and facilitate removal of the bulkhead.

Hollow cores in girders, columns, and similar elements are formed with steel pipe, rubber hose, or paraffin-treated paper tubes. Paper tubes are lightweight and must be restrained from floating or becoming displaced during concreting. They are bulky to store and must be protected from the weather. While they must remain in the casting, as they cannot be removed, paper is the only economical material for use when the core is centered in the element and does not extend to the bulkhead. Inflated rubber hose is deflated and removed after the concrete is set. During deflation, the hose length decreases and the horizontal movement, combined with a decrease in diameter, destroys the bond, and the hose is easily removed. Like paper tubes, the hose must be restrained from floating or displacement during concreting. Rubber hose is custom-made, and aside from high initial cost, sufficient time must be available for its manufacture and shipment.

Thin-wall seamless steel tubing is economical for forming hollow cores, for it is readily available, is reusable, and has good salvage value. It is rigid and when fixed at the ends will maintain its position during concreting. After coating with form oil, the tube is placed in the form and held in position by the end bulkheads. The tube must be removed before the concrete has set, but only after sufficient time has elapsed to avoid distorting or fracturing the casting wall. The proper time for removal is determined by experimentation on the first castings. A lever bar inserted in holes through the tube walls at one end is used to rotate the tube and keep it from bonding with the concrete. A cable attached to the tube and pulled with mobile equipment or power or hand winch is used to remove the tube. A length of steel channel or angle section supported by the bulkhead at one end and by a tripod at the other is used to guide the tube during removal to avoid fracturing the casting. The tubes are wiped clean with an oiled rag and stored on racks adjacent to the molds.

Molds requiring removable hose or tubing for cored sections should have sufficient area for removing the cores and should be located to avoid interrupting other operations or blocking access roads during the operation.
Bond Preventives

Bond preventives, usually commercial form oil, to prevent bonding of the concrete to the form are in everyday use on all types of concrete construction. Those generally used in precasting consist of commercial form oils and curing compounds, commercial precasting compounds, and job-blended compounds. Also used, to a lesser degree, are waterproof kraft or asphalt papers, and muslin.

Many form oils will discolor the mold bed surface and should not be used on service floors. Curing compounds have a resin or wax base and, while more expensive, are more efficient in preventing bond. The wax-base material will leave a film on the casting which must be removed or allowed to weather off before paint can be applied. Both form oils and curing compounds may be obtained in varying viscosities. Heavier viscosities are required in hot climates and on rough bed surfaces. Some form oils and curing compounds require more than one application to ensure a good bond prevention; others require drying time before the reinforcing is placed or the concrete poured, which may cause delays in the casting procedure.

Commercial precasting compounds have a relatively high initial cost. These are trade-marked compounds, and although little is known of their specific composition, they are generally variations of regular curing compounds.

The most common bond preventive is a job-blended compound of commercial form oil and commercial mineral castor oil. The castor oil gives the form oil more body, will adhere to the corners and sloping sides of inverted molds, and is excellent in preventing bond. It is also used extensively for flat bed molds other than service floors. The form oil would discolor the floor. The blend, usually consisting of equal parts of form oil and castor oil, is varied to obtain the required viscosity. With the exception of straight form oil, it is the least expensive of the bond preventives.

Waterproof kraft and asphalt paper are not suitable for inverted molds and are not frequently used on flat bed molds. They are relatively expensive in initial cost and generally are not reusable. While they are good in preventing bond, especially on rough mold-bed surfaces, they are difficult to maintain in position and are easily damaged during casting operations. Wrinkles in the paper or lapped joints will be reproduced on the surface of the casting. The paper often adheres to the casting and is difficult to remove. Muslin, while serving as a bond preventive, also is used to provide texture to the surface of the casting. When used in job-site casting yards, it has most of the disadvantages of the papers and is therefore generally restricted to use in factory production of small castings. The muslin is stretched tight and locked in the mold to prevent shifting and wrinkling.

The cost of the application, waste, and maintenance of each bond preventive should be compared. Liquid bond preventives can be applied by hand or power-operated sprayer. Sprayer application requires less material and produces a smoother, more uniform coating. Mopping has a tendency to puddle and is difficult to apply in corners and in narrow spaces of inverted molds.

It is obvious that the use of unknown materials may result in discoloration, residual deposits, or the most serious result of failure to prevent bond. Bonding of the casting to the mold will mean delays and the piecemeal removal of the affected castings, if not possible destruction of the mold itself.

The choice of a bond preventive based upon previous successful results is recommended. Some inherent disadvantages of different materials are not so apparent as others. If the characteristics or probable results are unknown, the material which seems best suited to the particular project after careful investigation should be field-tested prior to use.

Reinforcement

The concrete protective cover of reinforcing steel in thin sections of precast-concrete elements will require accurate control of fabrication and placing of the reinforcing steel. However, the repetitive units and assembly-line production in prefabrication actually simplify the operation.

The reinforcement must be accurately dimensioned on the drawings and cut and
bent accordingly. Reinforcing bars extending beyond the casting for purposes of welding or splicing at joint connections must be templated and extend the correct distance so that no difficulties will be encountered during erection of the elements. The reinforcing may be cut at the mill or at the casting yard. However, if any doubt exists that the reinforcing will not be mill-cut within required tolerances, it should be cut at the casting yard where proper control is assured.

Wire fabric can be obtained cut to the desired lengths and widths within the manufacturer's limitations of the dimensions of his standard material. Detail drawings and cutting lists of all fabric are made to facilitate cutting and to reduce waste. When sheet fabric is not available, roll fabric may be substituted. However, it must be straightened before it can be cut. Fabric for thin-shell elements must be absolutely flat. A minimum four-roll straightener will flatten the fabric without reverse bends or a tendency to spring back. Both disadvantages are common to a three-roll straightener. A fabric with heavy longitudinal wires will have more tendency to hold

![Diagram of precast-concrete construction process]

**Fig. 3-34.** Typical arrangement of a prefabrication yard for bar and fabric reinforcement. Its position once it is straightened and will also tend to eliminate rolling or wave action in the mold during the placing of the concrete. The straightener and cutter can be set up in tandem with synchronized speeds and operated from one power source. Hand or power benders are used to form the reinforcement accurately. Both bars and fabric are very bulky and difficult to transport after being bent and therefore should be bent at the casting yard.

The prefabrication of mats or cages is done in jigs or on job-constructed template benches, with the reinforcement either tied or spot-welded.

Good planning in the arrangement of the prefabrication yard will reflect a cost savings in the handling, assembling, and placing of the reinforcement. In a typical yard layout (Fig. 3-34), the bars and fabric are handled separately, as they require different equipment for cutting and bending. Consistent with job conditions, the reinforcing-yard layout should be located close to the mold beds to reduce carrying distances. As the individual pieces of reinforcement are more easily handled than the assembled mats, several template benches are set up adjacent to the mold bed, and the mats are fabricated fairly close to the molds in which they will be used. If the casting yard is to be equipped with a gantry servicing the mold beds, the reinforcing fabricating yard may be located so that the mats are assembled at one end of the mold beds and distributed to the molds by the gantry.

The reinforcing mats are positioned in the molds with concrete cubes or regular reinforcing accessories.

**Inserts**

Inserts are embedded for attaching lifting devices for handling; temporary bracing of castings during erection; making joint connections; and attaching hangers, brackets,
etc., for the work of other trades. They should be economical, easily set in the mold, and designed to hold their position during casting operations. The type of insert selected should be the most practical method for its intended use. Consideration should be given to the method of holding the insert in the desired location and position during the placing of the concrete. This is particularly important in the use of concrete molds, as the insert cannot be temporarily attached to the concrete surface.

Inserts in the sides of the castings will be held in place by nailing or bolting to the mold side forms. Inserts in the bottom of the castings in a concrete mold may be held in place by welding or tying to the reinforcing steel. Bottom inserts in a wood mold may be nailed directly to the wood surface. Inserts in the top of the castings can be held in place by jigs fastened to the top of the side forms or by bridging across the mold.

Where holes for through bolts are required in the castings, pipe sleeves are used and held in place on the bottom of the mold by steel pins cast in the mold surface. The diameter of the steel pin should be of a size which will form a close fit with the pipe sleeve. The top of the sleeve is held by a stud, welded to a hinge attached to the top of the side forms. Pipe-sleeve supports should be initially installed by template to locate the sleeves accurately. Where possible, the inserts should be held in position so that the device is below the screeding surface. Just prior to finishing the concrete surface, the device for holding the insert may be removed after the concrete has set sufficiently to allow the insert to maintain its correct position.

Thin-shell ribbed roof slabs generally lack sufficient depth for anchoring inserts. To provide a method for hangers for other trades, depressions are formed in the sides of the castings to provide bolt-holes between adjacent panels. This method is more economical than placing inserts in the underside of the ribs. Half-round wood trim on the side bulkheads for the full depth of the casting form one-half of the bolt-holes in each casting. When the castings are placed side by side, full-diameter holes are available for the bolts. The bolt, welded to a square plate washer at the head, is placed between the slabs with the washer in a pocket formed by setting a rectangular piece of wood over the top of the half-round trim prior to finishing the concrete surface. The pocket countersinks the bolt-head to avoid interference in the roofing operations and serves to hold the bolt from turning while connections are later being made below the slab.

Inserts for lifting the elements may consist of pipe sleeves, protruding bolts or pieces of reinforcing steel, or threaded inserts. Protruding bolts and reinforcing steel (the latter have both ends embedded and are commonly called "hairpins") are economical in initial cost and placement. They are ideal when the location does not require their removal after erection of the element. Frequently the ends of these units may be installed below the surface in pockets formed by removable blocks during the casting operations. The pockets are filled with grout after the element is erected. This type should not be used if the unit must be burned off after erection and where the burning will disfigure any exposed surface of the element.

Threaded inserts may consist of standard nuts welded to short pieces of reinforcing steel to provide anchorage. Formed wire-screw anchors are commercially available. Short bars may be welded to these inserts to provide additional anchorage, to raise the insert above the mold surface, or to provide a means of fastening the insert to the reinforcing steel. Bolts used with threaded inserts are well greased and placed in position prior to the pouring of the concrete. Threaded inserts must be kept below the concrete surface to limit patching to plugging of the bolt-hole at the exterior surface. If there is insufficient concrete cover at the bottom or top of the insert to prevent discoloration of the surface of the concrete from corrosion, the insert should be galvanized or formed of a noncorrosive metal.

Many commercial firms supplying accessories to the concrete-construction industry now have available several types of inserts designed especially for precast-concrete construction. Some of these firms are conducting large research programs on this subject to meet the requirements of the contractors and engineers.

Provisions for inserts required by all trades are made in the element at the time of casting. Proper coordination of the insert pattern required by each trade is essen-
tial. In general, it is more economical to restrict all inserts to a minimum number of sizes and types.

Concrete Placement

Concrete is placed directly from ready-mix trucks when there is access to the molds. It may also be placed by means of portable conveyor or by bucket suspended from an existing gantry. Any means which will mechanically distribute the concrete without excessive labor is satisfactory.

The concrete is compacted by internal vibrators or a vibrating screed. The vibrating screed is effective on individually supported molds, as assistance is obtained by some vibration introduced into the bulkheads. In thin-shell ribbed sections, an internal vibrator may be required along the ribs prior to and in addition to the screed vibrator if the ribs have deep sections. Rubber heads should be provided for internal vibrators. Metal heads, even with concrete molds, will mar the surface of the mold bed after several reuses. Care should be exercised in the use of vibrating screeds to ensure that constant vibration does not damage the bulkheads or cause them to lose alignment or level. While a low-slump concrete is preferred, the reinforcing in ribbed sections often interferes with vibrating, and a higher slump may be required to avoid honeycombing.

Proper care should be exercised in the placing of the concrete. The outside face of the bulkheads must be cleaned after each use. If they are not, the accumulation of concrete during each use, especially in gang molds, will soon cause failure of the bulkhead-operating mechanism. The same care should also be required when using the service floor as a mold surface to eliminate damage to the floor surface.

Especially in cooler temperatures, for closer control of the concrete and to prevent damage to the castings, the concrete strength should be known prior to removing the castings from the molds. These strengths are obtained by standard cylinder tests, conducted separately or combined with the tests normally required. A small compression-testing machine on the job site saves time and eliminates moving the cylinder. The cylinders for determining the strength of the concrete for removal of castings from the molds should be cured under conditions as close as possible to those of the castings.

It is essential that all phases of the concrete operations be conducted under rigid control.

Curing Castings

Factory-produced castings are normally cured in steam rooms. Job curing generally consists of the application of commercial curing compounds, water spray, mats, waterproof paper, quilted blankets, or steam curing. Commercial curing compounds of resin or paraffin base, solvent, and light mineral oil are applied by spray to seal the surface. Waterproof paper weighted and sealed at the edges is seldom used, as it is difficult to maintain the paper for the necessary period while moving the castings. The use of mats, quilted blankets, and steam curing is also difficult to maintain for the necessary curing periods because of the nature of the casting operations. They are generally used on castings which are cured before removal from the molds or for primary curing before removal with a further curing of water spray or curing compound.

Quilted blankets consist of two layers of muslin or burlap stitched together with a center layer of cotton. They are used for warm-weather curing and are kept wet. Steam curing may be accomplished by introducing steam under a treated tarpaulin, which is supported above the casting by a light frame. It is normally used during cold weather. Mats are fabricated of two pieces of waterproof paper filled with a 1- to 2-in. layer of insulation and sealed around the edges by nailing between two light strips of wood. Where conditions permit, the entire curing may be accomplished by water spray from a hose or garden sprinkler.

In arid climates, or other areas of strong sun rays, the castings should be protected
from the direct rays of the sun during initial curing periods. This may be done by artificial shade created by tarpaulins supported on a light framework or by curing compounds containing white pigment or powdered aluminum to reflect the heat. White pigment and aluminum compounds should not be used where they will create an undesirable appearance in the finished structure.

**Lifting Devices and Equipment**

The method used in removing a casting from the mold is influenced by the characteristics of the casting, temporary stresses developed, practical use of available equipment, and economy. In securing a physical attachment to the casting for lifting purposes, a simple and economical insert frequently used is the "hairpin," or inverted U formed from a reinforcement bar and embedded in the concrete. Inserts designed specifically for precasting are available from manufacturers of concrete-reinforcement accessories. Special inserts are available for use in thin sections, with special anchorage for heavy sections, of noncorrosive metal for use near the surface and with removable bolts or eyes to facilitate repair of the surface finish at the insert.

Figure 3-35 illustrates a few methods of lifting castings utilizing inserts at various point pickups. To avoid inserts in the face of the panel, a spreader beam may be bolted directly to the end of the panel (Fig. 3-35a). With two surface inserts (Fig. 3-35b), a spreader beam is used in transmitting vertical loads from the inserts to a single point at the lifting hook. Larger elements may require three inserts (Fig. 3-35c, d) to reduce the individual insert load or bending in the panel. A continuous cable through sheaves on the spreader and at the center insert (Fig. 3-35d) may be used to equalize the load at all three inserts. The center insert may be moved higher...
or lower on the panel to avoid openings. Large or heavy panels are often lifted with a four-point pickup (Fig. 3-35e) with sheaves on the spreader for load equalization.

For direct vertical insert loads a double set of spreaders are used (Fig. 3-35f). Panel weights exceeding the lifting capacity of available equipment may be lifted with two cranes, each engaging directly one of the intermediate spreaders. To reduce bending in a four-point pickup, two inserts are placed near the panel top and two near the center (Fig. 3-35g) with cables run through sheaves on the spreader beam to rotate the panel to a vertical position.

Timber or steel strongbacks (Fig. 3-36) are often used to reinforce panels for bending stresses. The panel may be lifted by a connection to the strongbacks or to other points on the panel. The characteristics of thin-shell ribbed elements generally do not permit the use of embedded inserts. These and most other elements may be lifted and handled with a vacuum mat.\(^1\) The vacuum mat eliminates bending stresses, inserts, and holes or other disfiguration of the surface of the element. The lack of a physical connection also saves time in lifting and releasing the casting. The operating principle is based upon introducing a sufficient vacuum between the mat and the casting. The mat is constructed of a strongback, plywood face, rubber seal, and facilities for producing the vacuum. The vacuum is introduced by a vacuum pump through rubber hose to a manifold, which in turn is connected to several openings in the face of the mat.

![Fig. 3-36. Inserts and strongbacks for lifting precast slabs.](image)

Generally one mat is required for each casting shape, although the same mat may handle a number of castings having only minor dimensional variations. Castings with a curved surface may be lifted with a mat fabricated to conform to the surface. Two or more small mats connected to a spreader beam with swivel joints will handle castings of various sizes in addition to those with curved surfaces. Rotating or carrying the casting in a vertical position does not affect the efficiency of the mat. For lifting castings with surface openings, pieces of rubber seal are attached to the mat face to form separated vacuum areas, with each area connected to the manifold. The vacuum is eliminated from any individual area by disconnecting the manifold hose to the particular area.

The vacuum pump is carried by the lifting equipment for mobility or is stationed in the casting yard with pipe containing convenient outlets running to the casting beds. A vacuum failure will cause the mat to drop the casting, and safety precautions should be observed to prevent accidents. Sufficient vacuum gages in visible locations will ensure immediate knowledge of any difficulty in the vacuum line. A reserve tank on the mat will prevent accidents from line and pump failures, and safety hooks on the mat prevent dropping the casting and avoid serious accidents from leaks in the manifold, mat, or porous castings. With proper supervision, failures are infrequent, for projects involving thousands of lifts have been completed without vacuum-mat failures.

Elements which have been cured for 3 or 4 days after removal from the mold generally have sufficient strength for handling with wire-rope slings without damage. Large castings or stacks of castings may require the use of a cradle to avoid bending stresses or damage to corners or other parts of the casting. A length of steel channel or beam suspended by wire rope at each end and placed under the quarter points is

\(^1\) Patented, Vacuum Concrete Corporation, Inc.
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satisfactory, although long castings require the addition of spreaders, creating a rectangular frame, to prevent the two sections from sliding toward the center of the casting.

Lifting, transporting, and erecting equipment includes tractor- and truck-mounted cranes, gantries, lumber carriers, and Tourna-crane. The long reach and large lifting capacity of tractor-mounted cranes are utilized for stripping molds, loading castings for transfer to storage or erection sites, and erection. They are not suitable for carrying. Service floors must be protected from track damage when operated within the structure. Large truck-mounted cranes are used for the same operations as the tractor-mounted cranes. Because of delays in placing and removing outriggers, smaller cranes lose much of their efficiency unless the loads involved are well within their capacity without outriggers or they are operated for some length of time in one location with outriggers. They are excellent for use on smaller projects or as a second crane on larger projects where they are not required full time in any one location and can be quickly moved between casting yard, storage site, and erection locations. No protection is required for service floors when they are operated within the structure. Although limited to the casting yard and storage site, gantries are economical when the facilities are planned for their operation. Their use is restricted to placing concrete and reinforcement assemblies and to removing, storing, and loading castings. Short-span gantries may be used for each of several operations or in combination with other types of equipment. One long-span gantry with transverse carriage and proper layout of the casting facilities spans the molds, storage site, and access road for loading castings and covers all operations. Gantry operations should be well planned to avoid interruption of other work in the casting or storage yards.

Lumber carriers or straddle trucks are basically carrying devices, although their self-loading characteristics also enable them to replace some lifting equipment. Originally adopted to transport cured castings from the storage site to the erection equipment, they are replacing gantries and cranes in placing reinforcing assemblies, stripping molds, and carrying and stacking castings within their size limitations. When equipped with a vacuum mat for stripping molds, the pump is mounted on the carrier. Individual castings or stacks of castings are palletized or lifted directly by the shoe plates. Individual molds, spaced for cross traffic, are best suited for carrier operations.

Tourna-crane, designed for lifting and carrying heavy loads, have capacities to 30 tons and carries castings of sizes which cannot be loaded on trucks. Large units are generally limited to lifting, carrying, and erecting heavy castings when the molds are fairly close to the erection location. Their weight, relatively short boom, and large area required for maneuvering result in a low production rate in relation to other equipment on average-size castings. Smaller units with relatively smaller capacities do not require protection of service floors and frequently are operated within the structure in carrying and erecting castings at points not accessible to or practical for other equipment.

Aside from combination lifting and carrying equipment, castings are normally transferred by truck or trailer. Permanent dunnage, A frames, or cradles erected on the truck or trailer bed facilitate the transportation of castings of unusual size or shape. Wall panels are carried in a vertical position on an A frame which extends on either side below the level of the bed to lower the center of gravity. Several trailers shuttled by fewer tractor units save additional handling of the castings by storing the castings on the trailers for short periods at the erection equipment. This system also results in labor and equipment savings when loading and unloading periods are lengthy.

Lifting Stresses

Temporary stresses introduced during lifting and handling the castings, which will vary with the methods used, must be checked to determine the need for additional reinforcement in the casting. Higher material working stresses than those permitted in the design of the structure may be used for these temporary stresses.
Vacuum lifting mats, by uniform pressure exerted throughout the mat area, reduce the temporary stresses to a minimum. The mat eliminates any need for additional reinforcing or inserts; however, its use involves royalties and expenses of the mat and other vacuum equipment. On large elements the weight of the mat may critically reduce the effective lifting capacity of available equipment.

End pickup points for slabs require additional reinforcement at the inserts and, for larger slabs, additional reinforcement on the bottom surface of the slab for bending. Surface pickup points at the third or quarter points utilize the cantilevered sections to reduce spans in bending. An increase from a two- to a three- or four-point pickup system will generally reduce bending and the additional reinforcing required. Additional pickup points, however, increase the insert and patching expense.

Where the casting surface is to be exposed and untreated in the completed structure, a minimum amount of surface inserts requiring patching will improve the appearance of the structure.

Code Marking

Each casting is physically coded by marking upon removal from the mold. The code includes a letter identifying the type of casting, the mold number, the casting date, and a directional mark for orientation during erection. A master schedule identifies all castings required for the project by their code information and is used for daily scheduling of all operations. The work completed each day is entered to show the progressive daily status of completion. The use of the code for all schedules, reports, and drawings, from fabrication of reinforcing to final erection, will facilitate close control and inspection of all operations.

Elements of a general shape and/or size may have minor variations in some dimensions, reinforcing, inserts, openings, or finish. The code readily identifies the particular variable element, and the directional notation assures proper orientation and location of variable properties on erection.

The casting date indicates the age and relative strength of the casting during all operations. The date, combined with the mold number, facilitates comparison of the castings with concrete-cylinder test results. This code combination quickly locates an individual mold producing castings with improper dimensions, contact finish, insert location, or other undesirable trait; permits correction before further faulty elements are cast; and simplifies identification of faulty castings from the same mold before delays are encountered in handling or erection.

Code marking should be weatherproof, legible, and of sufficient size to be seen from a distance. It is located where it will remain visible when the element is stacked, transported, or stored but not on finished surfaces exposed on the completed structure.

Casting Storage (Temporary)

Elements cast by the stack, or tilt-up, method may remain where cast until erected. Large wall panels may be stored adjacent to the molds for curing in a vertical position supported by an A frame; however, it is generally advisable to eliminate extra handling of these panels and to erect them directly upon removal from the molds. All other elements are generally stored adjacent to the molds for curing, then stored at or near their point of erection.

Upon removal from the mold, the castings will have cured only 1 day and will be very susceptible to damage from improper storage. Cracking and warping may result from the uncured castings' conforming to uneven or settling base dunnage. Dunnage consisting of a concrete slab or two parallel continuous concrete beams finished to levels set by instrument and reinforced to counteract nonuniform loading is preferred. The beams are located at the quarter points of the castings, or as otherwise specified by the designer.

Wood dunnage is placed between the base dunnage and lower casting and between all castings. The dunnage should be wider than the casting to eliminate bearing on the ends of the dunnage, wide enough to provide sufficient bearing to avoid crushing,
and thick enough to permit wire-cable slings to be easily passed between the castings. The dunnage between castings is placed directly above the base dunnage to eliminate bending stresses in the casting.

The storage is arranged to permit removal of the oldest castings without handling others. If castings are to receive 2 or 3 days of curing, they are placed in every second or third stack. Walking space is provided around each stack to facilitate handling, curing, and inspection.

The number of castings in a stack is limited to the weight capacity of the lifting or carrying device, crushing strength of the dunnage or casting, stability of the stack in transporting without ties, and the number of castings to be erected from building storage from one crane position. Within the other limitations, one stack may be stored upon another, although lifted and transported individually. Thicker dunnage between the two stacks allows room for inserting the lifting device.

Included in the storage area is the coupling yard for joining thin-shell sections to form hollow-box members. As reverse-hand elements are cast in the same relative position, that is, with flanges projecting downward on both elements, one element must be rotated 180° for coupling. This is accomplished after a proper curing period with a sling arrangement (Fig. 3-37) consisting of a steel pipe with a sheave welded at each end, grooved for continuous wire-cable slings.

**Fig. 3-37. Slings for rotating cast elements 180° for coupling.**

Casting Storage (Building Area)

With the exception of those erected directly from transporting equipment, the castings are stored in or adjacent to the building area prior to erection. Where an existing concrete slab is not available, timber, light steel rails, or small precast-concrete beams are used for portable base dunnage. They should be of sufficient strength for the expected loadings and placed on hard level ground or supported by mud sills. Roof, floor, and wall slabs are stored in stacks as received, and framing elements are laid out or stacked with sufficient intermediate dunnage to facilitate the attachment of erection devices prior to actual erection.

Whenever possible, the castings are stored in locations which will permit erection without moving the lifting equipment under load. Planning should include locating
the castings with protection from other construction activities and erection of castings without interference with erected framework or with the erection equipment. The storing sequence is the exact reverse of the erection sequence. Within the stability and reach of the lifting equipment for placing the various sizes and weights, the castings are group-stored to minimize equipment moves during erection.

Slings, cradles, or other devices for loading castings onto transporting equipment are unhooked from the crane and left in position for unloading. Time is saved in loading and unloading, and the stacks can be loaded with a minimum separation for stability and space.

Erection

With a few exceptions, precast-concrete erection procedures are similar to those for structural steel. Concrete elements have not generally been designed to resist tensile stresses from indiscriminate handling and are somewhat susceptible to damage from impact. Maximum efficiency in erection is obtained by placing the elements in their final position direct from transporting equipment or building storage in one operation. Erection procedures are planned to approach this objective. All other precasting operations are based upon the estimated erection schedule, and any delays in this schedule increase the storage area and dunnage required. Delays in completing connections of erected elements either interrupt the erection or increase the quantity of erection devices. Using additional erection devices to shorten the schedule is justified only when sufficient cured castings are available to ensure a continual operation. Planning includes a study of casting weights and sizes, capacities and reaches of lifting equipment, and clearances required for movement of the equipment and castings without interference with previously erected framing members. A plan detailing the erection procedure for all elements is drawn and checked to ensure conformance with the above limitations and is then rigidly followed.

For maximum efficiency of the lifting equipment, temporary erection devices are designed for automatic leveling and alignment of the lifted elements, eliminating the lifting equipment for final adjustment. Two survey instruments for line and
level of the main frame members are located to provide these services for as many elements as practicable without undue shifting of the instruments.

"Push-pull" braces, designed for both tension and compression stresses, are used for temporary bracing of columns, frames, and wall slabs and must be capable of withstanding wind loads and unbalanced loads prior to plumbing of the element. These braces, of timber, steel shapes, or pipe, contain a turnbuckle for adjustment and a clip at each end for anchoring to inserts at the base and to the element. Yokes or clamps for the upper connection are more economical and eliminate surface holes for inserts, although they require longer and heavier braces for high wall panels.

![Diagram](image_url)

**Fig. 3-39.** Intermediate girder and beam support saddles.

The braces are designed for attachment prior to lifting the element or as struts between the ground and the element after lifting. To support columns, a tripod formed of three push-pull bars is connected to a clamp encircling the column.

Cantilevered column heads or girders may be temporarily supported by simple pipe columns (Fig. 3-38) fabricated in pairs and braced to concrete columns for lateral stability. Heavy individual girders are often supported by four-post wood piers, which when suitably cross-braced are stable and do not require column braces. Large casters on the lower horizontal braces provide mobility, and screw jacks at the base of each post provide leveling adjustment and also raise the casters clear of the floor in positioning the pier. Intermediate girders and beams are supported by saddles cantilevered from column heads or girders (Fig. 3-39). Long bolts are used for level adjustment.

![Diagram](image_url)

**Fig. 3-40.** Prefabricated wood forms to facilitate filling grout pockets.

Unless provided with wheels for mobility, all erection devices should be as light as practicable. Heavy nonmobile devices are handled by crane unless, because of erected elements, they become inaccessible and require moving by hand after disassembly. Completion of all joint connections follows erection closely to release erection devices and permit erection of superimposed elements. Simple prefabricated wood forms (Fig. 3-40) held in place with wedges are used for grouted connections. The grout is placed at ground level by shovel or pail and is hoisted by pail and well wheel to elevated joints. A light wood frame covered with curing blanket or insulation-filled kraft paper cures the grouted joint. Heat may be added for winter curing by suspending a high-wattage light bulb inside the curing box. As surface imperfec-
tions are more readily corrected with concrete in a green condition, grout forms are removed and the joint treated as early as possible.

A major consideration in erection planning is the timing of installation of the service floor slab. The floor slab is ideal base dunnage for storing castings, provides a means of anchoring erection braces, permits the use of rolling scaffolding and erection devices, and is a clean stable working surface. It also protects underground piping. With prior installation of the slab, mechanical and other trades have immediate access to the area upon completion of the precast erection. Prior installation eliminates problems of column interference and insufficient headroom for equipment, but it also eliminates roof protection and exposes the slab to inclement weather and direct sun rays. This sequence obviously cannot be followed if the slab design is insufficient to support the weight of precast storage or equipment loads, or if sufficient time is not available prior to erection to complete installation of the underground facilities and the slab.

An alternate compromise method, retaining many of the above advantages, consists of omitting the slab installation in small areas or traffic aisles required for equipment. The aisle areas are installed upon completion of the precast erection. The aisle width must be sufficient for the proposed equipment and, unless continuous throughout the length of the structure, must have additional width at the ends for turning radius. Underground piping must be well protected from equipment damage, and vertical riser stubs in the aisle area temporarily omitted.
Section 4

PRESTRESSED CONCRETE

By

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PRINCIPLE OF PRESTRESSING

The principle of prestressing is not new. It was used by the first cooper who forced metal hoops over curved staves placing them in compression to make a liquidtight barrel and by every farmer who has tensioned a hog rod under a sagging beam. It simply implies the application of external forces to a structure deliberately to create internal stresses of opposite sign to the critical service stresses.

Such prestressing forces can be developed by the use of jacks or wedges reacting against external supports or by the transfer to structural members of the force developed in stressed tendons extending throughout their length. As unyielding external supports are rarely found in nature except in deep tunnels (as discussed under Circular Structures), the required prestressing forces are usually developed with high-strength reinforcing tendons.

However, the use of prestressing in concrete became possible only during recent years after extensive research established the existence and magnitude of elastic and plastic volume changes in concrete under sustained loads which previously were thought not to exist. It is now known that these volume changes are sufficient nearly to dissipate prestressing forces induced by low tendon stresses as used in early experiments. To obtain permanent stability of prestressed forces in concrete structures, it is necessary to use high tendon stresses with corresponding large elongations capable of absorbing volume changes in concrete with only a small percentage drop in stress.

Because concrete has virtually no strength in tension, it has been the practice to add reinforcing bars where direct stress or bending moments produce tension. Because such bars cannot develop tension until the concrete is cracked, permissible steel stresses must be low to prevent excessive cracking and rapid displacement of the neutral axis. Even so, a large portion of the cross section of concrete members in bending is in tension and serves no useful purpose but to house excessive quantities of low-stress steel. Each material thus restricts the efficient use of the other, resulting in waste and excessive dead weight. This unsatisfactory condition can be overcome by replacing the bars with high-strength tendons stressed against the concrete in those zones which are otherwise in tension. This eliminates the problem of cracking, permitting the whole concrete section to perform its proper function of resisting compression and, at the same time, removes the restriction of steel stresses which are limited only by the ultimate strength of the steel itself.

This reversal of the vicious circle in concrete design is known as "prestressing." When properly applied, it produces savings up to 25 per cent in the quantity of concrete and 50 per cent in the quantity of reinforcing required for a given structure with equal resisting moment, as shown in Fig. 4-1.

The first use of prestressed concrete using high tendon stresses occurred in Austria in 1921 in the fabrication of large quantities of precast pretensioned roof panels. Posttensioning was introduced in Germany in 1928 in a number of bridges in the Autobahn, some with spans up to 100 ft. In the United States, prestressing, with adequately high tendon stresses, was first applied to circular structures. In 1941, the city of Hammond, Ind., awarded a contract for 13,000 ft of prestressed pipe with diameters up to 42 in. In the same year, the first prestressed tank was constructed at Indian Head, Md., with a capacity of 1.5 million gal. It was not until after the war that prestressing was introduced for linear construction when a contract was awarded by the city of Philadelphia for the Walnut Lane Bridge with a main span of 160 ft. Its use in buildings soon followed.

Since that time, its use has rapidly spread as design and construction procedures improved to the point where it has displaced reinforced concrete for many structural purposes and is competitively supplanting structural steel for many highway bridges, building frames, tanks, and large-capacity pressure pipe.
Fig. 4-1. Comparison of a reinforced-concrete and prestressed-concrete simply supported beam bridge 50 ft long and designed for a live load of 1,000 lb per linear ft.
RELATIVE MERITS WITH RESPECT TO OTHER TYPES OF CONSTRUCTION

Structural

Although prestressed concrete is made of existing materials it can, in a very real sense, be said to represent a new material placed at the disposal of the designer—homogeneous concrete. Contrary to conventional reinforced concrete, prestressed concrete can, in effect, resist high tensile stress as well as compressive stress. The phrase “homogeneous concrete” is an allusion to this characteristic.

The design of conventional reinforced concrete is fundamentally influenced by the incompatibility of the elastic properties of concrete and steel. For this reason, conventional reinforced concrete cannot take full advantage of the high strength now readily available for both steel and concrete. Large steel deformations on the one hand and the lack of tensile strength and the low modulus of elasticity of concrete on the other hand would produce a great number of large permanent cracks and excessive deflections.

The influence of the elastic incompatibility of concrete and steel is avoidable in prestressed concrete. Here, one function of the prestressed reinforcing steel is to induce a synthetic high tensile strength in the concrete, thus permitting a concrete beam to act like a homogeneous member, as though no steel were involved. Thus it becomes possible to design the whole cross section of the concrete member to resist live-load moments and shears within the working load range. As a result, total quantities and weights are reduced in many cases. Another result is that the live-load deformation of the prestressing steel becomes limited to the relatively small elastic deformations of the concrete, from which it follows that changes of steel stress due to live load are very small.

It is always desirable to minimize stress variation in reinforcing steel. In conventional reinforced concrete the ratio of live load to total stress in the reinforcing steel varies directly as the ratio of live load to total load. This ratio often exceeds 50 per cent, particularly in building construction. However, in prestressed concrete, the steel-stress increment due to live load rarely exceeds 6 to 8 per cent of the dead-load steel stress.

In contrast to other types of construction the reinforcing in a prestressed structure is fully tested at the time of prestressing because the losses in stress due to early creep and shrinkage are usually greater than the increase in stress due to subsequently applied live load.

Because of reduced depth-to-span ratios, clearances can be increased correspondingly. Small depth-to-span ratios permit sizable reduction in cut and fill quantities and generally are conducive to a pleasing appearance. Depth-to-span ratios vary from approximately 1/5 for typical simple-span girders to as little as 1/60 for certain types of construction.

Prestressed concrete is highly desirable for structures which contain or carry liquids and those subject to external corrosion, because of its crackless character and greater impermeability. These characteristics are very valuable with respect to any type of structural member.

Economic

The economic advantages of prestressed concrete are based mainly on the effect which the structural advantages described in the preceding paragraph have on labor and material quantities and on lower maintenance costs. Substantially smaller concrete cross sections are required compared with reinforced-concrete members for comparable loads. This causes an important reduction in the dead-load-to-live-load ratio, which in turn reduces total load and concrete and steel quantities.

The use of high-strength prestressing steel reduces steel quantities to one-fifth to one-seventh of those required for comparable reinforced-concrete designs. While
The installed unit cost of prestressing steel is higher than that of mild steel, the reduction in tonnage often results in large savings.

The various cost-reducing factors cited above make prestressed concrete competitive with reinforced concrete or structural steel for an increasingly wide range of structures.

In addition to the reduction in first cost, the crackless quality of prestressed concrete almost entirely eliminates the high maintenance costs of structural steel and reduces the normal maintenance costs of reinforced-concrete structures.

Theoretical discussions indicating why prestressed concrete would tend to be economical under appropriate circumstances are worthwhile and must be used when a new structural concept is so young that there is no actual history available—but facts and figures and the thoughts and actions of the practicing engineer and the hard-headed United States businessman are more eloquent.

As to facts and figures, there is still no good clearinghouse for statistical data. However, certain data of interest have been published. Combining the data published by Dean¹ and Dobell,² we find that, on 12 representative projects, ranging in cost from $50,000 to $27,600,000, actual bid prices for the prestressed-concrete design were a total of $38,443,000. Actual bid prices for corresponding alternate designs were a total of $45,965,000, representing an average increase of 19.6 per cent over the prestressed-concrete prices.

These 12 projects represent, of course, only a small fraction of the volume of prestressed-concrete work up to 1957.

Direct comparisons with alternate designs have been made for only a few prestressed-concrete projects, and data on only some of these have been published.

What does the practicing engineer have to say about this subject? W. E. Dean, of the Florida State Highway Department, speaks with the authority of experience in an area where prestressed concrete has reached a high state of development. The following quotations are taken from his paper:³

Many structural and maintenance advantages can justly be claimed for prestressed construction but it has been used in Florida work only on a basis of contract cost competition with one of the conventional types. To date, in every case, the prestressed alternate has won the competition on every job on which it was allowed. Our experience has been so consistent on this that the policy of designing alternate types is being abandoned for projects on which prestressed members are applicable. Lately we have had several important jobs with keen competition where no bid was offered on the conventional types. . . . It might be appropriate to note some design features which tend to limit construction with prestressed beams. Curves, warps, skewes or other geometric requirements which result in highly variable lengths and shapes may not be applicable to precast-prestressed construction on account of the high cost of forming. Members for such specialized construction can often be best furnished through the fabricating facilities of the steel industry. Just where the economic line can be drawn is questionable. In Florida practice prestressed beams have not been used in spans skewed more than 20 degrees. . . . While this paper has been concerned only with construction using prestressed beams it might be stated that good use is being made in Florida bridge practice of prestressed bearing piles, sheet piles, handrails and other structural parts.

It can be definitely stated, in conclusion, that prestressing, which was little more than an interesting probability five years ago, has become a firmly established construction practice and one that will become increasingly important in bridge and building practice.

And what is the attitude of the American businessman? His actions are so eloquent that words would be superfluous. The observant traveler sees the word "prestressed" showing up at frequent intervals on main highways and railroads, each case representing a new prestressed-concrete precasting yard or a going precast-concrete yard to which the prestressing feature has been added. Each one of these plants represents an investment which is at least in the low six digits.

APPLICATIONS

Albert C. Smith, associate editor of Construction Methods and Equipment, sums up the above statements as follows:1

The volume of prestressed business in 1955 was four times that in 1954. And last year it was nearly double 1955. It's anybody's guess how much bigger it will get. ... Today, there are more than 150 casting yards in the Country; a few years ago there were none. ... He (the prestress fabricator) should have at least $200,000.00 to start with; three to five times that amount would be better.

Were prestressed concrete not economical in an impressive number of instances, such wide acceptance would never be granted by the cautious American investor.

The same evidence is noted on the order books of the manufacturers of prestressing materials. Each year since the inception of prestressed concrete, the volume of orders has accelerated rapidly. The manufacturer is constantly faced with the necessity of increasing his capacity in order to meet the demand, and the rising curve indicates that the saturation point is not in sight.

From some quarters is heard the explanation that the success of prestressed concrete is due to the steel shortage. However, the trend is too definite and too widespread to be explained by such a negative approach—and there are too many instances of the success of prestressed concrete in direct competition with steel and conventional reinforced concrete. The conclusion is inescapable that prestressed concrete is definitely a solid, useful, and economic addition to the storehouse of materials available to the designing engineer. Certainly it is not a panacea for all structures, but it is rapidly making a wide berth for itself—the limits of which are not yet discernible.

APPLICATIONS

General

Prestressing can be usefully applied to all concrete structures subject to direct tension, bending, or impact stresses such as tanks, bridges, forge-hammer foundations, or bomb shelters. Its economic advantages are more pronounced in structures designed to resist large forces in which full advantage can be taken of the higher working stresses as compared with reinforced concrete or structural steel.

For smaller elements such as beams and pipes, prestressing and precasting produce complementary advantages. For larger structures, such as long-span bridges and tanks, prestressing can be readily applied to cast-in-place construction.

Starting virtually from scratch in 1945, prestressed concrete during the following 10 years was successfully applied in rapid succession to almost every known form of structure.

Its widest use was in bridges, docks, buildings, tanks, and pressure pipes where it has become competitive with older forms of construction. During this 10-year period several thousand prestressed structures were built, including bridges up to many miles in length with spans up to 300 ft, tanks of capacities up to 12 million gal and pipelines up to 7 ft 0 in. diameter and operating pressures of 400 psi.

Bridges

Generally speaking, prestressed-concrete construction is more likely to show economy for the shorter spans than for the longer spans. However, experience to date indicates that a preliminary design in prestressed concrete should be included in the study for economy for all spans up to 200 ft. In some cases prestressed concrete can compete successfully for span lengths even greater than this.

In addition to the first cost of the superstructure, the relative economy of prestressed concrete will be affected by the conditions at some sites. For instance, where foundation conditions are poor, prestressed concrete is penalized with respect to steel because of its greater weight. On the other side of the ledger, where corrosive

1 Albert C. Smith, Prestressed Concrete, Construction Methods and Equipment, February, 1957.
atmospheres exist, steel and reinforced concrete are both penalized, in the form of higher maintenance cost, with respect to prestressed concrete.

In order to arrive at the most economical prestressed-concrete design in making such a comparison, one must be very conversant with the relative merits of the various methods of prestressing (i.e., pretensioning, posttensioning, mechanical, etc.); with the availability of materials and manufacturing facilities in the area; and with the relative merits of the various types of construction (i.e., precasting in a plant, precasting at the site, and casting in place). These subjects are covered elsewhere.

A typical pretensioned bridge is shown in Fig. 4-2 and a typical posttensioned bridge is shown in Fig. 4-3a, b, c.

**Buildings**

Roof and floor systems for single- and multistory buildings with spans up to 60 ft are well adapted to prestressed construction. One form consists of precast pretensioned girders and purlins with cast-in-place or precast surfacing. Another form uses only girders carrying long-span pretensioned double-T or channel sections to form the floor or roof surface. A pleasing appearance of ceilings can be obtained by painting the stems of precast members and applying acoustical tile to horizontal surfaces. For heavy loads a composite deck can be poured over the precast surfacing members.

For large-span buildings such as auditoriums and aircraft hangars, posttensioned girders can be cast in place or precast adjacent to and subsequently raised on their supporting columns to support flat or barrel shell roof systems which may be either precast or cast in place. Typical examples are shown in Figs. 4-4 and 4-5. Buildings with clear spans up to 300 ft have been built by this method.

Concrete dome shells with precast abutment rings, carried on columns, provide an economical method covering large areas of 300 ft or more in diameter such as the sports arena in Havana, Cuba, shown in Fig. 4-6.

An interesting use of pretressing is the arena in Montivideo, Uruguay, shown in Fig. 4-7, where precast panels were hung with corner hooks on the cables of a spider-web suspension grid. Pretressing was accomplished by loading the panels with sandbags and filling the expanded joints with mortar which was allowed to harden before the weight was removed. The cost of this roof of 308 ft diameter, housing 24,000 spectators, was only $1.75 per square foot.

Many useful methods are available to develop frame and composite action between prestressed-concrete elements in building elements.

**Marine Structures**

Prestressed concrete provides very economical construction for pier decks. Figure 4-8 illustrates a typical structure in the port of New York. The precast pretensioned stringers are mass-produced in a casting yard and span between posttensioned pile caps. Shear keys are provided on the precast stringers for load distribution. Pre-
APPLICATIONS

(a) Single span 100ft prestressed concrete bridge prestressed both longitudinally and transversely

(b) Typical section through bridge

(c) Typical section through prestressed girder

Fig. 4-3. Posttensioned bridge.
stressed-concrete piles are especially suited for this application, because they are crack-free and therefore have high resistance to corrosion, freezing, and thawing.

A large percentage of the French ports devastated by World War II have been reconstructed using prestressed concrete. A 1-mile length of quay at Le Havre was reconstructed using 82-ft-square 1,400-ton units of precast prestressed superstructures which were floated into place and carried on 5-ft-diameter concrete piers 30 ft on center.

![Elevation diagram](image)

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**Fig. 4-4.** Moses Lake hangar, Boeing Airplane Co., Seattle, Wash. (Courtesy of Narramore, Bain, Brady, and Johanson, Architects, Seattle, Wash., and Worthington and Skilling, Engineers, Seattle, Wash.)

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**Pavements**

A promising field of prestressed research is pavements for runways and highways. Present methods of design depend on two extremes: (1) plastic pavements of the bituminous type which have no load-distributing ability and depend solely on adequate preparation of the subbase to distribute loads within the limits of its own elastic capacity, and (2) rigid pavements of reinforced concrete which have no deflection capacity and must be thick enough to distribute loads over a sufficient area of the subbase to prevent deflections. Rigid pavements also require frequent expansion joints.

What is needed to bridge these extremes of design is an elastic pavement free of
joints and capable of variable load distribution and elastic deflection conforming to the elastic properties of various types of subbase.

Prestressed concrete can supply this long-sought need. By variation of both the slab thickness and prestressing force, any desired combination of elastic flexibility and load distribution can be obtained.

Fig. 4-5. Saw-tooth roof for factory building, after design by Dr. F. Leonhardt for textile mill at Neu-Bolheim, Germany. (Courtesy of the Preload Company, Inc., New York.)

To eliminate expansion joints, it is necessary to compensate progressively for subbase friction restraint of elastic shortening caused by prestressing.

Two methods have been devised to eliminate expansion joints in continuous pavements.

The first method, as shown in Fig. 4-9, is suitable for runways and highway pavements without vertical or horizontal curves. Longitudinal prestressing is developed
Fig. 4-7. Cross section of inverted prestressed-concrete dome roof over 308-ft-diameter stadium at Montevideo, Uruguay.

Fig. 4-8. Pretensioned stringers. Pier C, at Hoboken, N.J., for the Port of New York Authority, is 730 ft long by 350 ft wide. Pile caps which are posttensioned with Freyssinet-type cables of 0.276-in. Roebling wire are 21½ ft center to center. Deck is made up of 4,500 precast stringers 2½ ft wide, 1 ft deep, and 21 ft long. Stringers are pretensioned bonded type made with Roebling ¾-in.-diameter prestressed-concrete strand. Erection, including fabrication of prestressed-concrete members, was by J. Rich Steers, Inc.; prestressed-concrete details by Freyssinet Company, Inc., both of New York.
by the displacement of transverse wedges located at 300- to 500-ft intervals with end abutments to develop the longitudinal thrust. Transverse prestressed tendons are used to provide biaxial compression.

The second method utilizes unbonded prestressed tendons placed in a double diagonal grid throughout the pavement length, as shown in Fig. 4-10. By varying

Layout. Scale = 1:10

Fig. 4-9. Dr. Leonhardt’s wedge system for prestressing pavements.

Fig. 4-10. Pavement with stressed wires or cables in both directions at 45° to the road center line.

the slope of the diagonal tendons in desired ratio, transverse and longitudinal prestressed force is obtained. To overcome friction restraint of progressive elastic shortening, temporary transverse construction joints are provided at 300- to 500-ft intervals and are increased in width by jacking, as tendons are stressed across the joint, by an amount equal to the elastic shortening of the adjacent section. The joint is filled with mortar while held in an open position by the jacks which are removed only after the mortar has hardened, thus placing the joint material under the same
compression as the pavement by the action of the unbonded tendons. After removal of jacks, the tendons may be grouted to provide crack distribution and greater ultimate strength.

In this way, continuous elastic pavements without joints and in which the continuous prestressing tendons maintain the pavement in compression without displacement at vertical and horizontal curves can be built with standard paving equipment.

Circular Structures

Prestressed concrete has been widely used in tanks and pipelines to place concrete cylinders in circumferential compression sufficient to offset direct tensile stress caused by internal pressure, which otherwise may result in cracks and leakage. Early attempts to prestress circular shells by use of high-carbon-steel rods tensioned as hoops with turnbuckles were unsuccessful because of the difficulty of relieving friction during stressing between the hoops and shell and the inability of the comparatively low initial hoop stress, of about 50 kips per sq in., to absorb stress losses due to creep.

During the 1940s these difficulties were overcome by the development of machines capable of wrapping high-strength wire, already stressed to about 150 kips per sq in., around pipe and tank shells at rates over 500 fpm. Such machines are more fully described under Mechanical Prestressing and under Design, Circular Structures, and are shown in Fig. 4-18.

Up to 1956, about 1,000 prestressed tanks and silos with capacities of 100,000 to 12,000,000 gal and many hundreds of miles of prestressed-concrete pressure pipe in diameters of 24 to 108 in. and internal pressures up to 600 psi had been constructed using these machines. Examples of tanks and pipe are shown in Figs. 4-11 and 4-12.

Such structures can be designed to remain in permanent compression under maximum load, thus removing the danger of cracking and consequent disintegration from leakage.

The same method of prestressing may also be used to place the abutment rings of dome shells under compression to resist the horizontal thrust and thus control deadweight deflections. By this means relatively flat circular domes of large diameter can be built, as shown in Fig. 4-6.

Another interesting application of prestressing of circular structures is the lining of tunnels through unstable materials. This is done by incorporating two or more longitudinal "flat jacks" equally spaced around the circumference.

Where the weight of overburden is 2.5 or more times the maximum internal pressure of the tunnel, the desired circumferential compression is obtained by expanding the lining with the flat jacks to react directly against the surrounding material, which becomes a permanent abutment. To reach a suitable compression in excess of the tension caused by the maximum internal pressure, the jacks must be maintained under constant pressure for periods of several minutes to several hours, depending on the nature of the surrounding material, during which a permanent equilibrium of forces between the lining and the surrounding material is established. This condition is reached when no further displacement of the jacks occurs under constant pressure. The liquid in the jacks is then replaced with cement grout maintained under full pressure until it is hardened sufficiently to resist the developed circumferential compression. When the overburden is insufficient in depth to be developed as a permanent abutment, as in the portals of tunnels, high-strength steel hoops are placed around the lining to contain the expansion force required to develop the necessary circumferential compression in the lining.

This process of prestressing tunnel linings eliminates the need for reinforcing tendons and greatly reduces lining thickness by developing composite action between the lining and surrounding material and utilizing overburden weight as a permanent abutment. This method has been successfully employed for several penstock tunnels up to 15 ft in diameter with operating pressures of 450 psi. It has also been used for vehicular tunnels of 60 ft in diameter with great economies in cost over conventional means.
Fig. 4-11. General arrangement for 0.7 M.G. water storage tank. (Courtesy of the Preload Company, Inc., New York.)
Other Applications

Prestressing has proved to be a happy solution for certain special problems. In such cases improvement in design is the governing factor and cost comparisons with nonprestressed designs are of secondary importance.

One of these applications is for the foundations of large forging hammers. In order to reinforce such installations effectively against stress reversals and fatigue and to provide sufficient shock resistance, prestressing tendons are installed to provide triaxial prestress in the main portion of the block.

Other typical examples in which prestressing has been used in unusual ways and for special purposes are a number of concrete dams in North Africa. One of these is a gravity dam, Cherufas Dam, Fig. 4-13, in which the vertical pressure line was artificially moved inside the “kern” by applying powerful vertical prestressing cable units which were brought down through a number of soft strata into a formation suitable for anchorage. Stressed and kept under control by a series of permanent jacks at the top of the dam, these prestressing units made possible the construction of a gravity dam of relatively slender proportions in spite of unfavorable conditions.

Prestressing was also used to good advantage in the St. Michel multiple-arch dam.
PRESTRESSING METHODS

Prestressing is rapidly becoming standard practice for marine piles, for exceptionally long piles, and for piles which would otherwise present a costly handling problem prior to installation.

Large numbers of public-utility poles, television and transmission towers, and posts of all types have been built of prestressed concrete in different countries.

Many millions of prestressed-concrete railroad ties are in use in areas where timber is either in short supply or subject to termite attack.

PRESTRESSING METHODS

General

There are three principal methods of prestressing:

1. Pretensioning
2. Posttensioning
3. Mechanical

Before discussing the methods under individual subsections, a few words as to their general fields of application may be in order. The field of precast linear members is divided between pretensioning and posttensioning. No well-defined dividing line has yet been established and none is ever likely to be. The protagonist of either can find many good reasons for the superiority of his method, but the final choice is based solidly on economics. In general, pretensioning becomes more applicable as the size of the member decreases and the degree of duplication increases. The reverse holds true for posttensioning. The field for pretensioning generally lies in members below 50 ft in length and for posttensioning above 70 ft. The combination of pretensioning and posttensioning is sometimes used for member lengths of 50 to 70 ft.

Cast-in-place linear structures require posttensioning, and it is used for span lengths where transportation and erection costs for precast members become excessive. Cast-in-place construction permits the use of fewer structural members at wide spacing and lends itself to the development of continuity frame action and composite construction.

Mechanical methods of prestressing are generally used for nonlinear structures such as pipes, tanks, tunnel linings, and shells.

Pretensioning

In the pretensioning method the prestressing material is stressed before the concrete is cast. There are no permanent end anchorages and the prestressing force is transmitted to the concrete by bond alone.

Prestressing material must be of a design that will ensure complete bond between the steel and the concrete under repeated loads. Experience to date has indicated that the maximum wire diameter that can be bonded to concrete with complete confidence under repeated loads is 0.125 in. diameter. This is not necessarily an absolute top limit. It is mentioned here mainly to provide a red flag for the designer so that he will consider carefully before using larger wires. The diameter can be increased by crimping or indenting the wire to improve bond, but this reduces the strength of the wire. The use of small-diameter wires has the disadvantage of...
requiring such a large quantity of wires to gain the required prestress that there is little room left for the concrete, which makes concrete pouring difficult. This has been overcome to a large degree by using seven-wire strands made up of high-strength wires of small diameter twisted together.

Tests have proved that these strands can be bonded with complete confidence when the minimum spacing center to center of strands is four times the strand diameter; the minimum distance from any concrete face to the center of the strand is the greater of three times the strand diameter or one-half the strand diameter plus 1 in.; and the concrete has reached an adequate strength.

A majority of the prestressed-concrete structures in the United States have used the pretensioned method since it invariably proves most economical where the conditions are suited to its use. These conditions are a casting yard within reasonable shipping distance (usually 150 to 200 miles); members that are not too long or heavy to be shipped; and the use of standard cross sections or enough repetition of members to justify making the necessary forms. The use of deflected strands in pretensioned members of long span has greatly increased their carrying capacity since this procedure offsets much of the dead-load moment.

Research is in progress to evaluate the possibilities of prestressing tendons made of glass fiber.

Posttensioning

General Description. There are three main types of posttensioning:

1. Forming longitudinal grooves or ducts in concrete members into which the prestressing tendons are inserted and stressed after the concrete is hardened

2. Placing the prestressing tendons in a metallic sheath positioned in the forms before placing concrete, stressing the tendons after the concrete has hardened, and grouting the ducts

3. Coating prestressing tendons with a bond-preventing material, positioning them in the forms before placing concrete, and stressing the tendons after the concrete has hardened.

With each method the tendons are stressed with hydraulic jacks acting against the concrete extremities after it has obtained sufficient strength. While held under tension, anchors are placed on the tendon ends (as discussed under Materials, Types of Anchorages), to transfer the prestressing force to the concrete before removing the jacks.

For methods 1 and 2 the cavities surrounding the tendons are filled with pressure grout to protect them from corrosion and establish continuous bond. No bond is developed in method 3.

For all methods the behavior under load will be identical up to the cracking load. After cracking, unbonded tendons will show a greater deflection and the lower ultimate strength as loads are increased. However, where such excessive loads are anticipated, these conditions can be remedied by adding mild-steel reinforcing for crack distribution in the tension zones. Thus there are no inherent structural reasons for preferring one method and the choice depends solely on relative cost.

Permanent and positive end anchors are required with posttensioning through which the prestressing force is transmitted to the concrete. The safe working stress for posttensioning tendons therefore depends on the strength of the anchor assembly. Any slippage in anchors at transfer reduces tension stresses and can be a serious problem in short members.

End-anchored Parallel Wires. Parallel wire cables usually consist of several wires in the 0.196 to 0.276 in. diameter range uniformly spaced in circular or rectangular pattern. These cables may be shop-fabricated with end anchors and casings in place, and delivered to the site in large-diameter coils. It is desirable that suitable spacers be used between the wires to maintain their position and facilitate grouting. This spacer is, however, frequently omitted.

End anchors are discussed under Materials, Types of Anchorages and properties of wires under Materials, Wire.
End-anchored Strands. Sizes range from 7-wire units to 69-wire units. The wires that make up the strand can be either special hot-galvanized or bright wire, depending on the application. Special compact end fittings develop the full strength of the strand without exceeding the yield point of any of their parts. They are attached at the factory. Strand data are shown in Table 4-3.

If the strand is to be bonded to the concrete after tensioning, the strand is encased in a flexible metal tubing at the factory. If the design does not require bond, the strand is coated with grease and wrapped with sisalkraft paper or plastic tubing to prevent bond during concreting. This work is done in the field.

The tensioning of individual strands is done by standard inexpensive hydraulic equipment. Since each wire in the strand is machine-placed and tensioned as a unit, equal stretch in all wires that make up the strand is assured.

In another method horizontal layers of seven-wire strands make up rectangular cables in steel troughs. Cables are laid in pairs with an anchorage loop at each end. The strands are positioned in the troughs by means of special clips, and special sliding plates to overcome friction are used at points where the cables undergo changes of direction. These cables are particularly suited to continuous cast-in-place girders.

Heat-treated Cold-worked Alloy Bar. Heat-treated cold-worked alloy bars are fabricated from hot-rolled alloy steel to give physical properties of high yield strength combined with high ultimate strength and relatively good ductility (see Materials, Heat-treated Cold-worked Alloy Bars, for detailed manufacturing description). These smooth-surface bars are heat-treated for stress relieving and then processed by cold working through stretching. This operation produces a bar which has been

![Diagram](image-url)

Note:
Coupler has right hand thread both ends. Thread bar into coupler hand right to cover thread shoulder on bar.

<table>
<thead>
<tr>
<th>Bar diam</th>
<th>Coupler na.</th>
<th>Dimension (in.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{3}{16}$</td>
<td>HC-6</td>
<td>$1\frac{1}{2}$ $\frac{1}{8}$ $\frac{1}{4}$ $\frac{3}{16}$</td>
</tr>
<tr>
<td>$\frac{7}{8}$</td>
<td>HC-7</td>
<td>$1\frac{3}{4}$ $\frac{5}{8}$ $\frac{1}{4}$ $\frac{3}{2}$</td>
</tr>
<tr>
<td>1</td>
<td>HC-8</td>
<td>2 $\frac{1}{4}$ $\frac{1}{4}$ $\frac{3}{4}$</td>
</tr>
<tr>
<td>$\frac{1}{8}$</td>
<td>HC-9</td>
<td>$2\frac{1}{4}$ $\frac{7}{8}$ $\frac{1}{4}$ $\frac{4}{4}$</td>
</tr>
</tbody>
</table>

Fig. 4-14. Bar coupler. (Courtesy of the Stressteel Corp., Wilkes-Barre, Pa.)

![Diagram](image-url)

Notes:
*10° std. dimension increased to accommodate greater coupler movement.
Standard coupler sheaths made for flex tubing with I.D. as follows: $\frac{1}{8}$, $\frac{3}{8}$, $\frac{1}{2}$.
Tape all joints.

Fig. 4-15. Coupler sheath. (Courtesy of the Stressteel Corp., Wilkes-Barre, Pa.)
PRESTRESSED CONCRETE

Fig. 4-16. Details of standard wedges and wedge plates. (Courtesy of the Stressteel Corp., Wilkes-Barre, Pa.)

proof-tested to at least 90 per cent of its guaranteed ultimate strength. So processed the bar becomes a material adaptable to prestressed-concrete construction.

Bars are available in sizes of $\frac{1}{2}$ to $1\frac{3}{8}$ in. diameter and can be manufactured in any length up to 83 ft. To obtain greater lengths, couplers are used to join multiple bar units, as shown in Figs. 4-14 and 4-15.

This material is used exclusively in posttensioned applications where bars are cut to
length and equipped with necessary anchorages. Developed for use with a threaded-end anchorage, such bars are also available with wedge anchors bearing directly upon conically formed holes in a steel anchorage plate, as shown in Fig. 4-16. Utilizing the wedge anchorage, bars are hydraulically jacked with lightweight equipment from one or both ends. The jack is placed over and attached to the bar by means of a wedge jacking device. When required stress and elongation are reached the anchorage wedge is forced home by hydraulic or mechanical means and the jacking wedge is released after the force is transmitted to the permanent wedge anchorage. With the threaded anchorage, hydraulic jacking equipment is attached to the bar by means of a coupler and threaded pulling bar. The threaded end is also used as the final anchorage and is held by a nut which in turn bears against a steel anchorage plate.

Plate placed after casting — Integral type grout tube connector

[Diagram of anchorage system with labels and dimensions]

Order of placement:
1. Place bar in flex tubing.
2. Place assembled bar & tubing in form.
3. Place grout tube into connector.

Fig. 4-17. Standard detail of grout tube and wedge anchorage. (Courtesy of the Stressteel Corp., Wilkes-Barre, Pa.)

If the design requires that the bar be bonded to the concrete after tensioning, it is encased in flexible metal tubing on the job site, prior to placing in the forms. Grout tubes are attached to this flexible tubing, and after the tensioning has been completed the bar is bonded by forcing grout through the tube and into the flexible metal tubing, as shown in Fig. 4-17. If bond is not required the bar is usually greased or mastic-coated and wrapped with paper or plastic tubing to prevent bond during concreting.

Deformed bars such as described in Pretensioning (under Prestressing Methods) may also be used for posttensioning. The procedure is the same as described above for smooth bars, except that the deformed bars are always bonded by grout and the wedge anchors are removed after the grout has reached proper strength.

Mechanical Prestressing

Mechanical prestressing is a term used to define prestressing methods requiring special equipment and which cannot be classified under pretensioning or posttension-
Prestressed concrete machines have long been used for prestressing pipes, tanks, silos, and dome rings.

Prestressing machines for tanks, shown in Fig. 4-18, are comprised of an upper carriage which rides the top edge of the structure and from which is suspended a winding machine. The winding machine has a power unit driving a sprocket which engages an endless chain surrounding the tank to provide traction. The chain is kept tight on the structure by passing around spring-loaded take-up sheaves. The winding machine is raised or lowered on its suspended cables, usually by drums driven by a power take-off. Wire, in coils, is carried on the machine and is stressed as the machine moves around the tank by passing through a stressing device which may be either a series of dies which provide the required stress by reducing the diameter of wire, or by passing the wire around a drum on the same axle as the drive sprocket and of larger diameter, thus causing the wire to elongate by an amount equal to the difference in circumference of the drum and the sprocket.

This process has the great advantage of producing no friction between the wire and concrete during stressing which is difficult to control and measure when tendons are stressed while in contact with curved surfaces.

Such machines can apply wire under stress up to 400 fpm in a continuous helix around a circular structure with variable pitch to provide the correct prestressing force corresponding to the triangular liquid load.

The same principle is used for prestressing pipes. In this case, the pipe is rotated.
in a vertical or horizontal position in a machine similar to a lathe. The stressing mechanism is mounted on a carriage which moves along the length of the pipe at whatever rate proportional to the rotation is necessary to provide the required spacing of wire.

The same method as for pipe has been used for precast beams rotated horizontally on a power-driven turntable at 5 to 10 rpm. The beams are provided with circular ends and a peripheral groove into which the stressed wire is placed during rotation.

In all these mechanical applications the wire is attached to the structure with simple clamps only at two ends. Splices between coils of wire are made while the wire is held under tension with standard wire-splicing units available in several forms.

After the wire is placed, it is covered with mortar to provide protection and bond. Such mortar is best applied by mechanical means such as gunite or other pressure-spread types of application.

**Miscellaneous Methods**

The most obvious method of prestressing but by no means the most practical is the *in situ* application of external forces to a structural member. Very seldom does nature provide conditions which make it unnecessary to construct the rigid abutments to absorb the reaction of such external forces in the case of girder bridges.

However, the application of jacking forces has often been used in the construction of masonry and concrete arches and in the underpinning of foundations. Various techniques of measuring the applied force, maintaining it, and ultimately transferring it without loss to a permanent bearing have been developed. Their success depends largely on the suitability of the jacking equipment and the experience of the contractor.

One of the well-known jacking devices of this nature is a flat jack consisting of a tough flexible material forming a confined inflatable space to which pressure is applied in accordance with the desired deformations. This method is used effectively in the construction of prestressed tunnel linings. A general description is given under Applications, Circular Structures.

Prestressing can sometimes be accomplished very efficiently by the temporary application of artificial dead weight. One area of application of this method is that of suspended roof systems (Applications, Buildings; Fabrication and Construction, Circular Structures, Roof Systems).

Attempts have been made to produce the required initial elongation of prestressing steel through the application of electrically induced heat. However, little success has been obtained with this method. Small amounts of prestress have been induced in some structures by the use of expanding cements. The main applications of this method are to the underpinning of foundations and repair work on other structures.

The subject of prestressing methods would not be complete without mentioning the development work which is being done in the field of prestressed pavements. Practices have not yet been reduced to practical, generally accepted tools for handbook presentation to the designer and contractor, but a general discussion of the subject may be found under Applications, Pavements.

**DESIGN**

**Basic Theory**

In designing prestressed members, two different stages must be considered: (1) the uncracked condition and (2) the condition when all tensile resistance in part of the concrete has been utilized and cracks have occurred. For the uncracked condition the concrete is considered as a homogeneous and elastic material, and design calculations are based on the theory of elasticity. For the cracked condition, however, and particularly in the ultimate-load range, a semielastic stress distribution occurs which will be discussed in the paragraph on design criteria.

In general, stress distribution for critical loading conditions is found by analyzing the stresses for individual influences such as external loads, prestressing forces,
temperature, shrinkage, and certain material characteristics, and then superimposing them with their proper signs.

Design Criteria

The U.S. Bureau of Public Roads has done a great service to the development of prestressed concrete by preparing its *Criteria for Prestressed Concrete Bridges*, which was published by the Government Printing Office in 1955. It has served to inspire confidence and stimulate discussion. One can be sure of being on the safe side if he follows those criteria. In some areas, however, it is more restrictive than the practice which is accepted as being adequately conservative by the consensus of responsible United States authorities in this field. The criteria presented below are generally in line with those of the Bureau of Public Roads, but minor adjustments have been made at certain points.

**Losses of Prestress.** Loss of steel stress due to creep, shrinkage, and elastic deformation shall be assumed to be as follows:

- Pretensioned concrete: \[6,000 + 16f_e^* + 0.04f_{ct}\]
- Posttensioned concrete: \[3,000 + 11f_e^* + 0.04f_{ct}\]

In these criteria no allowance has been made for the movement of the tendon in the end fitting during tensioning. The designer should make sufficient allowance for this movement, based on tests or manufacturer's recommendations.

The above formulas are based on the following assumptions:

1. Shrinkage of concrete = 0.0002 in. per in., giving a corresponding relaxation in the steel of \[0.0002E_s = 0.0002 \times 28,000,000 = 5,600 \text{ psi (rounded out to 6,000 psi).}\]
2. Half of the shrinkage of concrete occurs before prestressing in posttensioned structures.
3. Elastic compression of concrete relaxes the steel. \[(E_s/E_e)f_e^* = (28,000,000/5,000,000)f_e^* = 5.6f_e^* \text{ (rounded out to 5f_e^*)}.
4. Creep of concrete = 2.25 × elastic compression, which reduces this steel stress by \[2.25 \times 5f_e^* = 11.25f_e^* \text{ (rounded out to 11f_e^*)}.
5. Creep of steel = 0.04f_{ct}.

**Loading Conditions.** Maximum total working load is not necessarily the only critical loading condition. In prestressed-concrete design, it is important to check the critical stress conditions at all stages of construction, handling, erection, testing, and operation.

Furthermore, there are many instances where prestressing forces are applied progressively in stages as the structure is loaded with so-called "superimposed" dead load as differentiated from "original" dead load.

The process of applying prestressing forces in increments might be called multiple-stage prestressing as compared with single-stage prestressing where the complete prestressing force is applied acting against "original" dead load only. In the latter case superimposed dead load must be added to normal static live load to obtain normal working-load conditions.

The influence of impact forces and temperature stresses must be calculated separately and added to normal working loads. Where infrequent abnormal live loads may occur, higher working stresses are usually permitted.

There is a slow transition between the homogeneous and the cracked condition. However, the transition is ignored in design and it is assumed that cracking starts when either stresses become tensile or the flexural modulus is exceeded.

After cracking has started, the stress-strain relationship continues to be nearly elastic for some portion of the load increase, but gradually it is modified, and the nonelastic behavior of concrete and steel becomes the more pronounced as failure load is approached. Failure load is that load at which permanent deformations render the member unfit to perform its function.

**Allowable Stresses.** *Temporary Stresses.* Temporary stresses before creep and shrinkage shall not exceed the following:
Concrete

Compression in extreme fiber: pretensioned 0.60f'\textsubscript{ct}; posttensioned 0.55f'\textsubscript{ct}
Tension: 0.05f'\textsubscript{ct}\dagger

Prestressing Steel
Tension: 0.70f'

**Stress under Dead, Live, or Impact Load.** Stress after creep and shrinkage under dead, live, or impact load or any combination of these forces shall not exceed the following:

**Concrete**
Compression in bridge members: 0.40f'\textsubscript{c'}
Compression in building members: 0.44f'\textsubscript{c'}
Tension in bottom fiber in bridge members: 0
Tension in bottom fiber in building members: 0.05f'\textsubscript{c'}
Tension in top fiber: 0.04f'\textsubscript{c'} (unless the additional is carried by reinforcing steel, but not more than 0.08f'\textsubscript{c'})

**Prestressing Steel.** 0.6f'\textsubscript{c'} or 0.7f'\textsubscript{sv}, whichever is less

**Principal Tensile Stress.** The principal tensile stress shall not exceed the following:
- **Dead, Live, or Impact Load or Any Combination Thereof.** 0.03f'\textsubscript{c'} to be carried by the concrete and the excess over 0.03f'\textsubscript{c'} to be carried by properly designed stirrups.
- **Ultimate Loads, without Stirrups.** 0.08f'\textsubscript{c'} and if this stress is exceeded, stirrups shall be designed to take the total principal tensile force at unit stresses in the stirrups not in excess of 150 per cent of allowable working stress. In the case of both working loads and ultimate loads, the shear existing at a distance from the support equal to 1.5 times the depth of the beam may be considered as the maximum shear for design purposes.

**End-anchorage Bearing Plates for Prestressing Steel.** Bearing plates shall be designed so that the bending stresses in the plates due to dead, live, and impact load do not exceed that allowable for the type of steel used, and the unit pressure on the concrete does not exceed f'\textsubscript{ci}.

**Ultimate Strength.** The ultimate strength should be such as to withstand the following load without failure: The total dead load plus 3.5 times the live load plus impact.

In figuring the ultimate strength, use f'\textsubscript{c'} and 0.8f'\textsubscript{c'}.

Unless a more exact method is preferred, the following procedure can be used for the computation of the ultimate strength of the beam:

Where the prestressing elements are bonded to the concrete, the reinforcement shall be assumed "balanced" (i.e., the steel and concrete will fail simultaneously) if

\[ p_b = 0.23 \frac{0.8f'_{\text{c'}}}{f'_{\text{c'}}} \]

The ultimate moment \( m_u \) shall be determined as follows: Where \( p \) is equal to or less than \( p_b \), \( m_u = 0.9A_{s}df'_{e} \); where \( p \) is greater than \( p_b \), \( m_u = 0.9df'_{e} \sqrt{A_{s}A_{b}} \); where the prestressing tendons are not bonded to the concrete, the prestressing steel shall be considered as acting at something less than its ultimate strength. A factor in common use is 80 per cent. In order to counteract the resulting loss in beam strength, some regular reinforcing bars may be installed.

**Design Procedures**

**General.** Volumes could be written on the subject of design procedures to include all its ramifications. However, detailed discussion of linear structures will be limited to simple-span members, which will serve to illustrate all the principal fundamentals. This will be followed by a general discussion of the additional considerations necessary

\* Many designers use 0.65f'\textsubscript{ct} where pretensioning members are straight.
\dagger Some designers permit higher stresses in the presence of reinforcing bars.
in the design of redundant members and in the use of partial prestressing. The subject of circular members is treated separately.

Simple-span Members. There are three types of simple-span prestressed-concrete members: those with straight tensioning elements bonded to the concrete, those with curved tensioning elements bonded to the concrete, and those with curved tensioning elements not bonded to the concrete. The design procedure is outlined below:

1. Compute the moment \((M_S + M_L)\) due to superimposed dead load \((w_D)\) and live load \((w_L \text{ and } P_L)\). In the discussion which follows, this combination of loads \((w_D, w_L, \text{ and } P_L)\), will be referred to as "imposed load." Stated in other terms, it is the summation of all loads applied to the member except the dead load existing at the time of prestressing.

2. Compute an approximate required section modulus by dividing the imposed load moment by the allowable concrete working stress and adding an amount estimated as sufficient to resist the unknown dead load \(w_D\). Experience will lead to facility in making this estimate, and it is usually only 20 to 30 per cent greater than that required to carry the imposed load. In any event it is a matter of trial and error and it is suggested that as a first try the lower side of this percentage may be applied to members with curved tendons and the higher side to members with straight tendons.

3. Design a concrete cross section for this section modulus. Since the area of the steel will be very small, it need not be considered in the computation of the section modulus. Depending upon type of cross section, intensity of live load, and span, the beam depth will usually vary from \(\frac{1}{6}L\) to \(\frac{1}{8}L\) of the span.

4. Compute the dead-load moment caused by the weight of the structure in the form in which it is to be prestressed.

5. Find the dead-load stresses in the top and bottom fibers caused by this moment.

6. Find the required prestressing force \(F\) and the eccentricity \(e\) of application. The purpose of prestressing is to eliminate all or most of the tensile stress in the concrete. The following equations are set up for zero tension. Substitute the allowable tension for zero in those structures where tension is permitted (refer to Allowable Stresses given before). Solve the equations for \(F\) and \(e\).

For stress in the bottom fiber \(= 0^*\)

\[
\frac{F}{A_e} + \frac{F_e}{Z_b} - \frac{M_a}{Z_b} - \frac{M_L + M_S}{Z_b} = 0 \tag{4-1}
\]

For stress in the top fiber \(= 0^*\)

\[
\frac{F}{A_e} - \frac{F_e}{Z_t} + \frac{M_a}{Z_t} = 0 \tag{4-2}
\]

All values in any one equation are for one cross section. The most critical cross section must be determined for each equation. Generally, this will be at mid-span, where tendons are curved, but there are cases in which the critical section is elsewhere. For example, in structures with straight tendons, the critical point for the top fiber stress is usually over the support. The equations are the same, but here the special case where \(M_a = 0\) is usually critical for Eq. (4-2), which thus reduces to

\[
\frac{F}{A_e} = \frac{F_e}{Z_t} \tag{4-2a}
\]

Substitute numerical values for all known quantities in Eqs. (4-1) and (4-2) or (4-1) and (4-2a). This leaves \(F\) and \(e\) unknown. Solve the equations simultaneously for \(F\) and \(e\). Finally, the design should be checked to confirm that maximum compressive stress in the concrete is not excessive under any condition.

7. Using allowable stresses, determine the prestressing elements required.

* A plus sign indicates compression and a minus sign indicates tension. The sign of \(F_e\) is plus when the steel is on the same side of the neutral axis as the \(Z\) point being considered and minus when on the opposite side.
8. Several factors cause the prestressing steel to lose a part of its initial tension. The stresses used in step 7 were the final stresses. Values of initial tensioning should be substituted in the following equations to check the stresses under initial tension.

Structures with Curved Tendons

\[ f'_{Pi} = \frac{F_i}{A_e} - \frac{F_e e}{Z_t} + \frac{M_o}{Z_t} \]  \hspace{1cm} (4-3)

\[ f''_{Pi} = \frac{F_i}{A_e} + \frac{F_e e}{Z_b} - \frac{M_o}{Z_b} \]  \hspace{1cm} (4-4)

Structures with Straight Tendons

\[ f'_{Pi} = \frac{F_i}{A_e} - \frac{F_e e}{Z_t} \]  \hspace{1cm} (4-3a)

\[ f''_{Pi} = \frac{F_i}{A_e} + \frac{F_e e}{Z_b} \]  \hspace{1cm} (4-4a)

Since stresses due to the initial tensioning force are temporary and are never increased by imposed loads, most specifications permit them to be considerably higher than final stresses.

9. Find horizontal shear and from this compute the diagonal tension. This stress should be checked on the center of gravity of the concrete section and also where the web joins the top flange at the point of maximum shear. This point on a simple span is at the supports. If this computed tensile stress is greater than the allowable, provide stirrups to take care of the excess. If this stress is less than the allowable, no stirrups are required, although most designs use small-diameter rods spaced one-half the structure depth apart, or wire mesh. To find horizontal shear and the diagonal tension, use the following formulas:

**Horizontal Shear (v) Formula**

\[ v = \frac{V a' g}{I_b} \]  \hspace{1cm} (4-5)

**Diagonal Tension (S_t) Formula**

\[ S_t = \frac{f''_t}{2} - \sqrt{v^2 + \left(\frac{f'_t}{2}\right)^2} \]  \hspace{1cm} (4-6)

When the point under consideration is the neutral axis of the section, \( f'_t \) is \( F/A_e \). When it is at some other point, \( f'_t \) can be found by plotting a stress diagram for the structure at the point under consideration and reading the stress.

For structures with curved tendons, the net value of \( V \) in Eq. (4-5) = total shear in beam due to dead and imposed loads minus shear carried by tensioning elements due to their curvature.

**Illustrative Example.** The following example illustrates the design of a simple-span building girder using the pretensioning method of construction:

**Data**

Span: 40 ft 0 in.
Imposed load: 1,000 lb per lin ft
Concrete strength \( f'c' \): 5,000 psi
Prestressing steel \( f'c' \): 250,000 psi

**Imposed Load Moment**

\[ M_S + M_L = \frac{(w_S + w_L)l^2 \times 12}{8} = \frac{1,000 \times 40^2 \times 12}{8} = 2,400,000 \text{ in.-lb} \]

Approximate section modulus \( Z \) required

\[ Z = \frac{M_S + M_L}{0.44 \times f'c'} = \frac{2,400,000}{0.44 \times 5,000} = 1,090 \text{ in.}^3 \]
Fig. 4-19. Cross section of simple-span building girder.

Try for section with section modulus of approximately 1,380 in.\(^3\) to allow for dead-load moment. Assume the depth to be 27 in., and try the section shown in Fig. 4-19.

<table>
<thead>
<tr>
<th>Section</th>
<th>A</th>
<th>(y)</th>
<th>(A_y)</th>
<th>(A_y^3)</th>
<th>(I)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 15 (\times) 7</td>
<td>105</td>
<td>3.5</td>
<td>368</td>
<td>1,290</td>
<td>430</td>
</tr>
<tr>
<td>2 2 (\times) 5 (\times) 3 (\times) (\frac{3}{4})</td>
<td>15</td>
<td>8</td>
<td>120</td>
<td>960</td>
<td></td>
</tr>
<tr>
<td>3 5 (\times) 15</td>
<td>75</td>
<td>14.5</td>
<td>1,085</td>
<td>15,800</td>
<td>1,410</td>
</tr>
<tr>
<td>4 2 (\times) 3(\frac{3}{4}) (\times) 3 (\times) (\frac{3}{4})</td>
<td>10.5</td>
<td>21</td>
<td>221</td>
<td>4,640</td>
<td></td>
</tr>
<tr>
<td>5 12 (\times) 5</td>
<td>60</td>
<td>24.5</td>
<td>1,470</td>
<td>36,000</td>
<td>125</td>
</tr>
<tr>
<td></td>
<td>265.5</td>
<td>(12.3)</td>
<td>3,264</td>
<td>58,690</td>
<td>1,965</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>60,655</td>
<td></td>
</tr>
</tbody>
</table>

\[ g = \frac{\Sigma A_y}{\Sigma A} = 12.30 \text{ in.} \]

\[ I = I_g - A_y^3 = 60,655 - 265.5 \times 12.30^2 = 20,450 \text{ in.}^4 \]

\[ Z_t = \frac{20,450}{12.30} = 1,670 \text{ in.}^2 \]

\[ Z_b = \frac{20,450}{27 - 12.30} = 1,390 \text{ in.}^2 \]

**Dead-load Moment and Stresses**

\[ w_g = 265.5 \times \frac{15}{144} = 276 \text{ lb per lin ft} \]

\[ M_G = 276 \times \frac{401 \times 12}{8} = 662,000 \text{ in.-lb} \]

\[ f'_G = \frac{M_G}{Z_t} = \frac{662,000}{1,670} = 397 \text{ psi (compression)} \]

\[ f''_G = \frac{M_G}{Z_b} = \frac{662,000}{1,390} = 477 \text{ psi (tension)} \]

**Imposed-load Stresses**

\[ M_S + M_L = 2,400,000 \text{ in.-lb} \]

\[ f'_S + L = \frac{M_S + M_L}{Z_t} = \frac{2,400,000}{1,670} = 1,440 \text{ psi (compression)} \]

\[ f''_S + L = \frac{M_S + M_L}{Z_b} = \frac{2,400,000}{1,390} = 1,730 \text{ psi (tension)} \]

**Calculation of Required Prestressing Force \(F\) and Its Location \(e\)**

1. For top fibers: \(\frac{F}{A_e} - \frac{P_e}{Z_t} = -0.04f'_G\)

2. For bottom fibers: \(\frac{F}{A_e} + \frac{P_e}{Z_b} - \frac{M_G}{Z_b} - \frac{(M_S + M_L)}{Z_b} = -0.05f''_G\)
(3) = substitution in (1): \( \frac{F}{265.5} - \frac{F_e}{Z_b} = -200 \)

(4) = substitution in (2): \( \frac{F}{265.5} + \frac{F_e}{Z_b} = -250 + 1,730 + 477 \)

(5) = \( \frac{Z_b}{Z_b} \times (3): \frac{1.20F}{265.5} - \frac{F_e}{Z_b} = -240 \)

(6) = (4) + (5): \( \frac{2.20F}{265.5} = 1,717 \)

\[ F = 207 \text{ kips} \]

Substitute the value of \( F = 207 \text{ kips} \) in Eq. (4-3) and solve for \( e \)

\[ \frac{207,000}{265.5} - \frac{207,000e}{1,670} = -200 \]

\[ 780 - 124e = -200 \]

\[ e = 7.9 \text{ in.} \]

Distance from the bottom of the section of the center of gravity of prestressing steel = 27 - (12.3 + 7.9) = 6.8 in. Assume \( \frac{3}{8} \) in.-diameter seven-wire uncoated stress-relieved strands at final load (after all losses) of 11,200 lb per strand (see Table 4-2)

Number of strands required = \( \frac{207,000}{11.2} = 18.5 \) Use 19

Final steel stress = \( \frac{207,000}{19 \times 0.0799} = 136,000 \text{ psi} \)

**Maximum Compression in Concrete under Full Load at Final Prestress**

At supports (bottom fiber)

\[ \frac{F}{A_s} + \frac{F_e}{Z_b} = \frac{207,000}{265.5} + \frac{207,000 \times 7.9}{1,390} = 1,950 \text{ psi (allowable = 2,000)} \]

At mid-span (top fiber)

\[ \frac{F}{A_s} - \frac{F_e}{Z_t} + \frac{M_G}{Z_t} + \frac{(M_S + M_L)}{Z_t} = \frac{207,000}{265.5} - \frac{207,000 \times 7.9}{1,670} + 397 + 1,440 = 1,637 \text{ psi (allowable = 2,000)} \]

**Theoretical Stress Loss in Steel**

Stress loss = \( 6,000 + 16f_{st}^* + 0.04f_{st}^* \)

\[ = 6,000 + 16 \times \frac{(1,153 + 1,410)}{2} + 0.04 \times 170,000 \]

\[ = 33,400 \text{ psi} \]

Initial stress = 136,000 + 33,400 = 169,400 psi

In the above computation, the numerical value for \( f_{st}^* \) is an attempt to approximate the average throughout the length of the beam of the concrete stress at the center of gravity of the prestressing steel while elastic contraction and creep of concrete are occurring. Specifically, the number 1,410 represents \( f_{st}^* \) at the support, and the number 1,153 at mid-span, under final prestress of 207 kips and initial dead-load moment \( M_G = 662,000 \). Whether imposed dead-load moment, if any, should be included, is a matter of judgment. In any case, the decision on this point will affect the value of initial prestress by only a small percentage. The value used for \( f_{st}^* \) is also an approximation.

**Stresses Due to Initial Tension**

\[ F_t = 169,400 \times 19 \times 0.0799 = 258 \text{ kips} \]

\[ f^*F_t = \frac{F_t}{A_s} + \frac{F_e}{Z_t} = \frac{258,000}{265.5} + \frac{258,000 \times 7.9}{1,670} = -250 \text{ psi} \]

\[ f^*F_t = \frac{F_t}{A_s} + \frac{F_e}{Z_t} = \frac{258,000}{265.5} + \frac{258,000 \times 7.9}{1,390} = +2,440 \text{ psi} \]
Shear and Diagonal Tension

\[ V_{\text{max}} = (1,000 + 276) \times \frac{40}{2} = 25.5 \text{ kips} \]

Fig. 4-20. Cross section above neutral axis of building girder.

Consider the point at the neutral axis of the section (Fig. 4-20)

\[
\begin{array}{|c|c|c|c|}
\hline
\text{Section} & A & y & Ay \\
\hline
1 & 15 \times 7 & 105.0 & 8.8 & 920 \\
2 & 2 \times 5 \times 3 \times \frac{1}{2} & 15.0 & 4.3 & 65 \\
3 & 5 \times 3 & 26.5 & 2.7 & 72 \\
\hline
\end{array}
\]

\[ a' = 146.5 \text{ in.} \quad (7.2) \quad 1,057 \]

\[ y = \frac{\Sigma Ay}{a'} = 7.2 \text{ in.} \]

\[ v = \frac{V a' y}{I_b} \]

\[ = \frac{25,500 \times 146.5 \times 7.2}{20,450 \times 5} \]

\[ = 263 \text{ psi} \]

\[ f'_{\text{t}} = \frac{207,000}{265.5} = 780 \text{ psi} \]

\[ S_{\text{t}} = f'_{\text{t}} - \sqrt{v^2 + \left(\frac{f'_{\text{t}}}{2}\right)^2} \]

\[ = 780 - \sqrt{(263)^2 + (390)^2} \]

\[ = 590 - 470 = -80 \text{ psi} \quad \text{Not critical} \]

It is not likely that the designer would come so close to the ideal on his first try. However, with practice, the number of converging approximations reduces to the vanishing point.

Continuous Members. The shear and moment diagrams in prestressed-concrete continuous members are not the same as those in a steel structure of constant moment of inertia, because part of the shear is carried by the prestressing steel and the reactions from this steel are seldom in the same proportion as the reactions for a conventional continuous beam. A detailed discussion of the methods of design for prestressed-concrete continuous beams is beyond the scope of this section. The following trial-and-error method is suggested as a practical means of dealing with prestressed continuous beams:

1. Determine an approximate prestressing tension and position along the member. Plot diagrams of the shears and moments produced in the beam by the vertical reactions of the prestressing steel in the member.

2. Compute dead- and imposed-load shears and moments in the member by the usual method for a member with constant moment of inertia.

3. Compute the shear diagram for the concrete member by adding algebraically the shear diagrams in steps 1 and 2. (This is actually a diagram of total shear minus the shear in the prestressing steel.)

4. Compute the moment diagram for the member by adding algebraically the
moment diagrams in steps 1 and 2. Compute the stresses due to this moment. Add \((F/A_e) \pm (F_e/Z)\) to these stresses to obtain final stresses. If any stresses thus obtained are excessive, repropportion the member and the prestressing load and relocate the prestressing steel if necessary.

After making the above calculations, the member should be checked for stresses due to the worst combinations of dead load, imposed load, initial prestress, and final prestress, using the principles employed in the previous discussion of Simple-span Members.

**Partial Prestressing.** Partial prestressing can be differentiated from full prestressing by the fact that tension in concrete is permitted under working loads. Many technical and economical advantages can be realized in certain construction that permits this type of design.

The structure is designed with no tensile stress permitted under dead load alone. By so doing, construction depth can be reduced considerably as can the prestressing steel required. Small tensile cracks will open up with each application of maximum working load, but if this practice is limited to structures such as buildings which are not subject to repeated dynamic loads, fatigue life is not a governing consideration.

Care is necessary in designs of this type to gain sufficient shear resistance because of the small prestressing force. A check of ultimate carrying capacity should also be made because of the reduced steel area. The usual practice is to use enough high-strength steel to satisfy the required ultimate design requirements and then apply a prestressing force no more than sufficient to ensure compression under dead load. Beyond the cracking point, the member is treated just like conventional reinforced concrete.

**Circular Structures.** The design of prestressed-concrete tanks and pipes is confined to a relatively small group of specialists. The discussion here will be confined to general principles and to the design of a simple tank.

**Tanks.** A circular tank is essentially a cylindrical shell whose base or top, or both, usually is partially or wholly restrained against translation and angular rotation. This restraint at base and top introduces edge forces and moments which induce local vertical bending in the wall. (For the basic theory, reference is made to S. P. Timoshenko’s *Theory of Plates and Shells*, McGraw-Hill Book Company, Inc., New York, 1940.)

A tank consists of a floor, wall, and sometimes a roof. Each element has its own particular problems which will affect the design. The following discussion is for the wall only.

Various types of joints are used between the walls and floors of prestressed tanks. In smaller tanks it is usually cheaper to use a fixed or hinged joint with sufficient vertical prestressing and reinforcing steel to take care of the high moment. In larger tanks the vertical moments become so great that it is necessary to relieve the restraint.

A sliding joint or lubricated joint can be made by using a minimum of three alternate layers of asphalt and building paper, or three layers of graphite-impregnated asbestos sheet. As slight variations in movement at the base cause wide variations in vertical moments, extreme care is essential in preparing this type of joint. Since the indeterminate magnitude of static friction together with the nonuniformity of sliding friction around the periphery may result in a highly variable restraining force, a coefficient of sliding friction of not less than 0.5 is used. It is generally assumed that 50 per cent of circumferential prestressing must be applied before the static friction is overcome and sliding starts.

In recent years tank walls have been constructed by floating the wall on rubber or neoprene pads which permit the wall to move inward and outward with changes in loading by deforming the rubber pad rather than sliding action of the wall on the footing. The use of the rubber pad reduces the vertical bending moments to such a degree that the thickness of the concrete core wall is usually determined by the amount of circumferential prestressing required. The low restraint results in vertical bending moments of about 30 per cent of those experienced with lubricated sliding base joints. There are patents and patents pending with respect to these rubber joints.
PRESTRESSED CONCRETE

The initial stress in the circumferential wire during construction shall not exceed 80 per cent of the ultimate strength. Design stress in the circumferential wire shall not exceed 60 per cent of the ultimate strength. The initial stress of vertical prestressing units shall not exceed the following:

Wire: 80 per cent of ultimate strength
Bars: 75 per cent of ultimate strength

Decrease in prestress in steel due to creep, shrinkage, plastic flow, construction tolerance, and for residual compression is normally assumed to be as follows:

<table>
<thead>
<tr>
<th></th>
<th>Walls</th>
<th>Domes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Creep, shrinkage, plastic flow, psi</td>
<td>20,000</td>
<td>15,000</td>
</tr>
<tr>
<td>Construction tolerance, psi</td>
<td>10,000</td>
<td>10,000</td>
</tr>
<tr>
<td>Allowance for residual compression, psi</td>
<td>5,000</td>
<td>5,000</td>
</tr>
<tr>
<td>Total losses, psi</td>
<td>35,000</td>
<td>30,000</td>
</tr>
</tbody>
</table>

No allowance for friction losses in circumferential prestressing is required where wire tendons are spirally wound under tension.

<table>
<thead>
<tr>
<th></th>
<th>Wire</th>
<th>Alloy bars</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vertical prestressing losses, psi</td>
<td>20,000</td>
<td>15,000</td>
</tr>
</tbody>
</table>

To the losses of the vertical prestressing tendons must be added an amount sufficient to allow for anchorage efficiency as recommended by the manufacturer or as determined by field tests.

For normal water tanks, total vertical stress in the concrete core wall caused by horizontal prestressing or liquid load shall not exceed 0.05f' or 200 psi in tension, whichever is the lower value. Vertical prestressing, if required, shall be applied to reduce the calculated vertical stress to these stress limits. These stress limitations shall be investigated for both the empty- and full-tank conditions.

Temperature differentials across the tank wall and roof cause stresses which must be investigated in any complete analysis of a structure. Thermal stresses are calculated using a coefficient of conductivity k of concrete equal to 12 Btu per hr per sq ft per °F per in. of thickness.

The vertical stress limitations given herein are appropriate for normal water-storage tanks; however, for tanks with heated or refrigerated contents or for exposed installations in areas of extreme changes in ambient temperature, or for dry-storage applications, other stress limitations may be specified by the engineer.

Mild-steel reinforcement is provided and designed to take all the computed tension in the concrete. This un prestressed reinforcement shall be deformed bars of intermediate grade designed with an allowable stress of 15,000 psi. In any event, the core wall shall be reinforced with not less than No. 4 bars, 15 in. on center in each face where vertical prestressing is used, and No. 4 bars, 12 in. on center in each face where only mild-steel reinforcement is used.

The maximum ring compressive stress in any plane in the concrete shall not exceed 55 per cent of the 28-day cylinder strength except in the immediate vicinity of the prestressing anchors. The allowable bearing pressure under the prestressing anchors shall follow the recommendations of the tendon manufacturer. The modulus of elasticity of concrete for elastic deformations is assumed equal to 1,000f', while for deformations due to temperature it is assumed equal to 500f'.

The above comments have been abstracted from Bulletin T-19, published by the Preload Company, Inc., of New York under the title. The Design of Preload Tanks.
The following example of a design presentation for a typical 5,000,000-gal tank is taken from the same publication:

**Nomenclature**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>Meridional thrust per unit length of circle of latitude</td>
</tr>
<tr>
<td>r</td>
<td>Dome radius</td>
</tr>
<tr>
<td>W</td>
<td>Unit dome load</td>
</tr>
<tr>
<td>φ</td>
<td>Angle measured from axis of rotation of dome</td>
</tr>
<tr>
<td>e</td>
<td>Dome thickness</td>
</tr>
<tr>
<td>D</td>
<td>Tank diameter</td>
</tr>
<tr>
<td>A s</td>
<td>Area of steel per foot</td>
</tr>
<tr>
<td>a s</td>
<td>Area of one prestressing wire</td>
</tr>
<tr>
<td>A c</td>
<td>Area of concrete</td>
</tr>
<tr>
<td>w</td>
<td>Density of liquid</td>
</tr>
<tr>
<td>R</td>
<td>Inside radius of tank</td>
</tr>
<tr>
<td>H</td>
<td>Liquid height</td>
</tr>
<tr>
<td>f c</td>
<td>Concrete stress</td>
</tr>
<tr>
<td>t</td>
<td>Core thickness of wall</td>
</tr>
<tr>
<td>U</td>
<td>Over-all coefficient of heat transmission</td>
</tr>
<tr>
<td>k</td>
<td>Coefficient of conductivity</td>
</tr>
<tr>
<td>O.A.</td>
<td>Outside air $\mathcal{L}_o = \text{outside surface conductance}$</td>
</tr>
<tr>
<td>I.A.</td>
<td>Inside air $\mathcal{L}_i = \text{inside surface conductance}$</td>
</tr>
<tr>
<td>P.M.</td>
<td>Pneumatic mortar</td>
</tr>
<tr>
<td>Conc.</td>
<td>Concrete</td>
</tr>
<tr>
<td>T</td>
<td>Temperature</td>
</tr>
<tr>
<td>E c</td>
<td>Modulus of elasticity of concrete core</td>
</tr>
<tr>
<td>α</td>
<td>Coefficient of thermal expansion</td>
</tr>
<tr>
<td>S</td>
<td>Coefficient of shrinkage</td>
</tr>
<tr>
<td>Q</td>
<td>Moment</td>
</tr>
<tr>
<td>f' c</td>
<td>28-day concrete cylinder strength</td>
</tr>
<tr>
<td>f' s</td>
<td>Ultimate steel strength</td>
</tr>
</tbody>
</table>

**A. General**

1. Number and type = one water-storage tank
2. Wall height = 32 ft 9 in.
3. Diameter = 161 ft 0 in.
4. Capacity = 5.0 million gal
5. Backfill = None
6. $f' c$ = 3,750 psi
7. $f' s$ = 220,000 psi

**B. Dome**

The dome is designed for a live load of 20 psf, in addition to its dead load. The dome rise to tank diameter ratio is equal to 1:8. Assume a 4-in. concrete dome.

1. **Dome load**

   The dome fillet constitutes 30 per cent of the dome dead load.
   
   Dead load = 1.3(50) = 65 psf
   
   Live load = 20
   
   $W = 85 $ psf

2. **Maximum meridional compressive stress**

   The maximum meridional thrust occurs at the edge of the dome.
   
   $$ T = \frac{W}{2\pi r^2(1 - \cos \phi)} = \frac{2\pi r^2 (1 - \cos \phi) W}{2\pi r^2(1 - \cos \phi)(1 + \cos \phi)} = \frac{r W}{1 + \cos \phi} $$

   For a 1:8 rise, $r = \frac{1}{16} D$, $T = 0.565 DW$.

   $f' c = \frac{D(t) W (lb/ft)}{12 (in./ft) e (in.)} (0.565) = \frac{161 (85) (0.565)}{12 (4)} = 161 \text{ psi}$
3. **Dome reinforcement**

Provide 0.25 per cent reinforcing (minimum).

\[
A_r = 0.0025(12)\text{ in.} e(\text{in.}) = 0.0025(12)(4) = 0.120 \text{ sq in./ft}
\]

Use 4 in. \(\times\) 4 in. \(\times\) No. 4 \(\times\) No. 4 wire mesh

C. **Dome ring**

The dome ring is designed to carry safely the maximum horizontal thrust due to dome dead and live loads. The radial component of these loads is \(T \cos \phi\) per foot of length of ring, which induces a ring tension of \(T \cos \phi \times D/2\).

For a 1:8 dome rise:

\[
\text{Max ring tension} = 0.565DW \left( \cos \phi \right) \frac{D}{2} = 0.250D^3W
\]

\[
= 0.25(161)^3(85) = 550,000 \text{ lb/ft}
\]

The dome ring is prestressed by circular wires to offset this ring tension. The wires are stressed initially to a value of 160,000 psi; the design stress is 130,000 psi. Of the 30,000 psi differential, 25,000 psi is assigned to losses due to shrinkage and plastic flow, while 5,000 psi provides residual compressive stress.

\[
A_r \text{ wires} = \frac{0.25D^2(\text{sq ft})W(\text{psf})}{f_s (\text{psi})} = \frac{550,000}{130,000} = 4.23 \text{ sq in. (good for 693 kips)}
\]

Area per wire \(A_r = 0.0156 \text{ sq in.}\)

Weight of wire = 0.053 lb/ft

14.4 lb/horizontal ft = 2 in. P.M. cover

Assume a maximum compressive stress of 1,400 psi in the dome ring.

\[
\text{Required } A_r = \frac{A_r \text{ wires (160,000)}}{1400} \times \frac{1.23(DL + LL) - DL}{1.23(DL + LL)}
\]

\[
A_r = \frac{4.23(160,000)}{1400} \times \frac{40}{105} = 184 \text{ sq in.}
\]

Use 10- by 20-in. dome ring for ease of forming.

D. **Wall**

1. **Horizontal wire**

In determining the amount of wires for horizontal prestressing, the base is considered to be perfectly free. Hence a triangular loading is applied which is equal to the hydrostatic loading.

\[
A_r \text{ wires} = \frac{w(\text{psf})R(\text{ft})H(\text{ft})}{f_s(\text{psi})}
\]

\[
A_r \text{ top} = \frac{62.5(80.5)(2)}{125,000} = 0.080 \text{ sq in. (use 25 kips initially min)}
\]
Use 10 wires minimum.

\[ A_s \text{ bottom} = \frac{62.5(80.5)(32.8)}{125,000} = 1.32 \text{ sq in. (211 kips initially)} \]
\[ A_s \text{ bottom} = \frac{4.50}{1.5} = 1 \frac{1}{2} \text{-in. P.M. cover} \]

2. Maximum compressive stress

A wall thickness of 10 in. is assumed. Again assuming the wall base to be perfectly free, the maximum compression exists at the base while the tank is empty for either Case I (wires stressed to 160,000 psi) or Case II (wires stressed to 140,000 psi plus a temperature differential through the wall plus backfill if any). Both cases must be investigated.

**Case I:**

\[ f_c = \frac{A_s \text{ bottom (160,000)}}{12(\text{in.})} = \frac{1.32(160,000)}{12(10)} = +1,760 \text{ psi} < 0.55f_c' \]

**Case II:**

Prestress = \( A_s \text{ bottom (140,000)} \)
\[ \frac{1}{1.32} \frac{1}{(140,000)} = 185,000 \text{ lb/ft} \]

Temperature (25°F differential)

\[ U = \frac{1}{R} = \frac{1}{\frac{t_{P,M.}}{k} + \frac{t_{cone}}{k} + \frac{1}{\ell_1}} \]
\[ U = \frac{1}{0.167 + 0.094 + 0.833 + 0.607} = \frac{1}{1.701} \text{ Btu/(hr)(sq ft)(°F)} \]

O.A. P.M. Cone I.A.

The differential temperature across the concrete = \( \Delta T = TUt \)
\[ \Delta T = 25U(0.833) = 25(0.588)(0.833) = 12.3°F \]

\[ f_c = \frac{\text{force}}{12(\text{in.})} + \frac{1}{2} \Delta T = E_t \text{(psi)} \]
\[ f_c = \frac{185,000}{12(10)} + \frac{1}{2} (12.3)(6)(10)^{-6} (3.75) (10)^4 \]
\[ f_c = 1,540 + 70 = 1,610 \text{ psi} \]

In actuality, the wall base is slightly restrained against movement by the rubber pad [see Fig. 4-23]. This condition induces edge forces and moments in the wall which diminish rapidly owing to shell action. Because of this edge condition, however, the maximum ring compressive stress is somewhat lower than that calculated above.

In computing stresses due to temperature and shrinkage a modulus of elasticity of one-half the 28-day value is used, because of the plastic-flow characteristic under sustained loading conditions and the nonlinear stress-strain relationship for temperature stresses.
3. Vertical stresses

a. Temperature. In determining the vertical stress due to a temperature gradient through the wall, two cases must be considered. [See Fig. 4-24.] Case I occurs when the tank is empty and a temperature differential of 25°F is assumed.

\[
U = \frac{1}{0.167 + 0.094 + 0.833 + 0.607} = \frac{1}{1.701} = 0.588
\]

\[
\Delta T = TU = 25(0.588)(0.833) = 12.3°F
\]

\[
f_c = \frac{1}{2} \Delta T \propto E_c = \frac{1}{2} (12.3)(6)(10)^{-4} \cdot \frac{(3.75)}{2} (10)^4 = 70 \text{ psi}
\]

Case II occurs when the tank is full and a temperature differential of 40°F is assumed.

\[
U = \frac{1}{0.167 + 0.094 + 0.833} = \frac{1}{1.094} = 0.91
\]

\[
\Delta T = 40U(0.833) = 30°F
\]

\[
f_c = \frac{1}{2} \Delta T \propto E_c = \frac{1}{2} (30)(6)(10)^{-4} \cdot \frac{(3.75)}{2} (10)^4 = 169 \text{ psi}
\]

b. Dead load. The weight of the dome and wall will induce direct compression in the wall.

Dome = 0.267 × D(ft) × W(psf)
Dome = 0.267 × 161 × 65 = 2,800
Wall = 150 × 0.833 × 32.8 = 4,100
P.M. = 150 × 6.094 × 32.8 = 460

\[7,360 \text{ lb/ft}
\]

\[
f_c = \frac{7,360}{12(10)} = 61 \text{ psi}
\]

at bottom of wall, and may be considered at point of maximum moment since error is small.

c. Base restraint. The rubber pad will offer some restraint to the wall, at the base. Diagrams for the computation of this restraining force have been developed in cooperation with the rubber companies. This force is based on the relative stiffnesses of the shell and the rubber pad. The maximum allowable compression in a rubber pad is dependent on the ratio of its dimensions. This ratio is known as the shape factor.

\[
S.F. = \frac{\text{one loaded area}}{\text{total free area}} = \frac{4(12)}{2(1)(12)} = 2
\]
The relationship between allowable stress and shape factor is empirical, the values of which may be taken from Goodyear's *Handbook of Molded and Extruded Rubber*, pp. 69 through 74. Allowable compression = 180 psi; compression = 10 per cent; required pad area = 7,360/180 = 41 sq in.; use 4- by 12-in. by 1-in.-thick pad; reduced pad thickness = 0.90 \times 1.0 = 0.90 in. 40 durometer.

(1) *Base movements*

(a) *Wire* at 160,000 psi

\[
\Delta R = \frac{A_s \text{ bottom (sq in./ft)160,000}R(f t)}{E_s \text{ (psi)t(in.)}}
\]

\[
\Delta R = \frac{1.32(160,000)(80.5)}{3.75(10)^6(10)} = 0.454 \text{ in.}
\]

(b) *Liquid*

\[
\Delta R = \frac{w \text{(psf/ft)H(ft)R^2(sq ft)}}{E_s \text{ (psi)t(in.)}}
\]

\[
\Delta R = \frac{62.5(32.8)(80.5)^2}{3.75(10)^6(10)} = 0.354 \text{ in.}
\]

![Fig. 4-25. Shear deflection and compression of rubber-base pad.](image)

(2) *Vertical stresses due to movements computed above* [See Fig. 4-25 for rubber-pad deformation]

Max possible movement: \(\Delta R_1 = \text{wires} = 0.454 \text{ in.}\)

\[
\frac{\Delta R_1}{t} = \frac{0.454}{0.90} = 0.505
\]

Min possible movement:

\[
\Delta R_2 = \text{wires - liquid} = 0.454 - 0.354 = 0.100 \text{ in.}
\]

\[
\frac{\Delta R_2}{t} = \frac{0.100}{0.90} = 0.111
\]

Refer to rubber-pad diagrams [Figs. 4-26 and 4-27].

Shear developed by pad is

Max possible shear \(Q_1 = 12 \times 30 \times 4 = 1,440 \text{ lb (per ft of pad)}\)

Min possible shear \(Q_2 = 12 \times 5 \times 4 = 240 \text{ lb (per ft of pad)}\)

The maximum moment in the wall, using Timoshenko's *Plates and Shells*, Chapter XI, *General Theory of Cylindrical Shells* [McGraw-Hill Book Company, Inc., New York, 1940] will be equal to [see Fig. 4-28]

\[
M_{\text{max}} = 0.247 \sqrt{R_l} Q \quad \sqrt{R_l} = \sqrt{80.5(0.833)}
\]

\[
M_1 = 0.247(8.18)(1,440) = 2,910 \text{ ft-lb}
\]

\[
M_2 = 0.247(8.18)(240) = 485 \text{ ft-lb}
\]

\[
f_{s1} = \frac{6M_1}{t^2 \text{(sq in.)}} = \frac{6(2,910)}{100} = 175 \text{ psi (tank empty)}
\]

\[
f_{s2} = \frac{6M_2}{t^2 \text{(sq in.)}} = \frac{6(485)}{100} = 29 \text{ psi (tank full)}
\]
Fig. 4-26. Compression stress-strain curves for 40 durometer rubber. (From Handbook of Molded and Extruded Rubber, by courtesy of Goodyear Rubber Co.)

d. Stress tabulation

(1) Tank empty

<table>
<thead>
<tr>
<th></th>
<th>Outside</th>
<th>Inside</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dead load</td>
<td>+ 61</td>
<td>+ 61</td>
</tr>
<tr>
<td>Base restraint ($\Delta R_1$)</td>
<td>+175</td>
<td>-175</td>
</tr>
<tr>
<td>Summation</td>
<td>+236</td>
<td>-114</td>
</tr>
<tr>
<td>Temperature</td>
<td>- 70</td>
<td>- 70</td>
</tr>
<tr>
<td>Summation</td>
<td>+166</td>
<td>-184</td>
</tr>
</tbody>
</table>

(2) Tank full

<table>
<thead>
<tr>
<th></th>
<th>Outside</th>
<th>Inside</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dead load</td>
<td>+ 61</td>
<td>+ 61</td>
</tr>
<tr>
<td>Base restraint ($\Delta R_2$)</td>
<td>+ 29</td>
<td>- 29</td>
</tr>
<tr>
<td>Summation</td>
<td>+ 90</td>
<td>+ 32</td>
</tr>
<tr>
<td>Temperature</td>
<td>-169</td>
<td>-169</td>
</tr>
<tr>
<td>Summation</td>
<td>- 79</td>
<td>-137</td>
</tr>
</tbody>
</table>

The vertical steel is computed by transforming the stress into a bending moment and direct force.
Inside bottom:

\[ f_e \text{ due to bending} = -175 - 70 = -245 \text{ psi} \]
\[ f_e \text{ due to direct force} = +61 \text{ psi} = 61 \times 12 \times 10 = 7,320 \text{ lb/ft} \]

Section modulus = \( \frac{12(10)^2}{6} = 200 \text{ in.}^3/\text{ft} \)

Clearance = \( \frac{3}{4} \text{ in.} \)  Eccentricity = 4 in.

Equivalent moment = \( 200(245) + 7,320(4) = 49,000 + 29,280 = 78,280 \text{ in.-lb/ft} \)

\[ A_s = \frac{78,280}{0.875(15,000)(9)} - \frac{7,320}{15,000} = 0.663 - 0.487 = 0.176 \text{ sq in.} \]

Use No. 4 at 12 in. o.c. (minimum).

Inside top:

\[ f_e \text{ due to bending} = -169 \text{ psi} \]
\[ f_e \text{ due to direct force} = \frac{2,800}{12(10)} = +23 \text{ psi} \]

Equivalent moment = \( 200(169) + 2,800(4) = 33,800 + 11,200 = 45,000 \text{ in.-lb/ft} \)

\[ A_s = \frac{45,000}{0.875(15,000)(9)} - \frac{2,800}{15,000} = 0.382 - 0.186 = 0.196 \text{ sq in.} \]
Use No. 4 at 12 in. o.c. Stresses on outside face are less than inside face; therefore provide nominally No. 4 at 12 in. o.c.

**E. Floor**

Use 4-in. concrete floor reinforced with No. 4 at 10 in. o.c. each way. [See Fig. 4-29.]

The previous example is based on the Preload rubber-pad-base concept. To have the same residual-vertical-stress condition in the wall with a sliding base designed for a coefficient of sliding friction of 0.5, the following tabulation gives comparative results of vertical stresses.

<table>
<thead>
<tr>
<th></th>
<th>Rubber-pad base</th>
<th>Sliding base coefficient of friction 0.5</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Outside face</td>
<td>Inside face</td>
</tr>
<tr>
<td><strong>Tank empty:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dead load</td>
<td>+ 61</td>
<td>+ 61</td>
</tr>
<tr>
<td>Base restraint</td>
<td>+ 175</td>
<td>− 175</td>
</tr>
<tr>
<td>Summation</td>
<td>+ 236</td>
<td>− 114</td>
</tr>
<tr>
<td>Temperature</td>
<td>− 70</td>
<td>− 70</td>
</tr>
<tr>
<td>Summation</td>
<td>+ 166</td>
<td>− 184*</td>
</tr>
<tr>
<td><strong>Tank full:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dead load</td>
<td>+ 61</td>
<td>+ 61</td>
</tr>
<tr>
<td>Base restraint</td>
<td>+ 29</td>
<td>− 29</td>
</tr>
<tr>
<td>Summation</td>
<td>+ 90</td>
<td>+ 32</td>
</tr>
<tr>
<td>Temperature</td>
<td>− 169</td>
<td>− 169</td>
</tr>
<tr>
<td>Summation</td>
<td>− 79</td>
<td>− 137</td>
</tr>
</tbody>
</table>

* For the same stresses for both designs provide \((454 - 184) = 270\) psi with vertical prestressing (32,500 lb per ft of wall).

**Pipe.** Prestressed-concrete cylinder pipe, as manufactured by the Lock Joint Pipe Company, consists of the following elements:
1. A continuous arc-welded steel cylinder with steel joint rings welded to its ends.
2. The steel cylinder is lined and covered with concrete and wrapped with high-tensile-strength wire under tension.
3. The whole is coated with a dense covering of cement mortar.
4. Each length of pipe is constructed with a self-centering expansion joint sealed with a rubber gasket and capable of caring for normal movement due to earth settlement and extremes of temperature.

The thickness of sheets for the steel cylinder, the diameter of wire used, its centerline spacing, and the tension under which it is wound around the core shall be such that the core will be sufficiently compressed to withstand an internal hydrostatic pressure equal to at least 1.25 times the designed operating pressure, without inducing tensile stress in the core when the elastic and inelastic deformations of the concrete and steel are taken into consideration. At a pressure equal to twice the design pressure, the stress in the wire shall not exceed its original gross wrapping stress. The gross wrapping stress in the high-tensile wire shall not exceed 70 per cent of the average ultimate tensile strength of the wire. The maximum center-line spacing shall be 1\(\frac{1}{2}\) in. Minimum center-line spacing of the wire shall be that which produces a clear distance between wire of \(\frac{3}{4}\) in. The minimum diameter of wire used shall be No. 6 gage wire.

The core shall not be wrapped with wire until at least 7 days after the placing of the concrete or until the concrete has reached the specified 7-day strength. The resultant prestressed compression induced in the concrete after all losses shall not exceed 40 per cent of its compressive strength at the time of wrapping.

Other specialists design pipe without the use of the steel cylinder, but the fundamental principles are similar.

**Tunnels, Roof Systems, and Pavements.** Space limitations herein do not permit full treatment of design procedures for all types of structures. However, this subject would not be complete without mentioning that design procedures have been developed for many types of structures such as tunnels, roof systems, and pavements, which are discussed in general in the first part of this section under Applications and later under Fabrication and Construction, Circular Structures.

**Load Distribution**

**General.** The distribution of loads in prestressed-concrete structures should be given the same consideration as for conventional reinforced-concrete structures. In posttensioned structures special consideration should be given to the distribution of prestressing steel reactions at the ends of the structures. Most designers provide a solid block of concrete equal in length to the depth of the structure. They also provide mild-steel reinforcement of the spiral or mat type around the cavity made in the concrete by the prestressing anchors to aid in the distribution of end reactions.

In posttensioned designs end plates should be designed to provide a sufficient area so as not to overstress the concrete. The allowable bearing stress is given above under Design, Criteria, Allowable Stresses.

An exact method of computing required bearing-plate thickness is complicated. The following method is simple and yields satisfactory results. Compute the net
PRESTRESSED CONCRETE

bearing area required, using the concrete bearing allowable. The cross-sectional area of the concrete cavity that houses the prestressing steel should be added to the net area for computing dimensions a and b in Fig. 4-30.

Compute the cantilever overhang \( L = \frac{(a - c)}{2} \). Then for a strip 1 in. wide, \( M = f_{pca} \times L^2/2 \), where \( M \) = the bending moment in inch-pounds and \( f_{pca} \) is the bearing allowable for the concrete. Using the bending moment computed above, compute the section modulus required at a bending allowable stress of 20,000 psi.

The bearing plate must be made large enough to accommodate the jack base but the plate need not be made thicker if it is made longer or wider for this reason.

Lateral. Transverse prestressing is used to aid in lateral distribution of loads. The amount required depends on many design factors.

For bridge construction where prestressed-concrete beams are used with a composite deck of reinforced concrete, the deck aids in the lateral distribution of loads. In structures where precast prestressed concrete units are laid side by side to form a deck, shear keys or bolts between adjoining units are used to provide load distribution. In both cases lateral prestressing tendons are used through the diaphragms to increase the load distribution.

If precast units are used, cored holes for the lateral prestressing steel should be provided. After the precast units are erected, the prestressing steel is threaded through these holes and tensioned to the required value. If the members are cast in place, the prestressing steel can be encased in a bond-preventing device and located in the forms before the concrete is placed.

Special Considerations

Impact, Fatigue, and Repeated Stress Reversals. Special precautions should be taken in the design of prestressed structures subject to high fatigue and impact loads, such as railroad bridges, railroad ties, and floors subjected to vibration.

The primary considerations are as follows:
1. The prestressing tendons and their end fittings, if any, must be chosen carefully for their ability to withstand the shock loads and the number of stress changes which will occur during the life of the structure.
2. For bonded tendons, materials with the highest possible bond should be used.
3. The design should not rely on tension in the concrete within the range of the repeated stress changes as concrete in tension has a very low fatigue limit.

Fire Resistance. The fire resistance of a prestressed-concrete member is a function of four factors:

1. Insulation of the steel
2. Stress in the steel
3. Temperature of the steel
4. Time of exposure to the critical temperature

No known fire rating tests have been conducted in the United States to date, but tests in Europe indicate that structures fail when the temperature of the steel reaches 400°C (750°F). Combining these data with a discussion beginning on page 1099 of the Proceedings of the American Society for Testing Materials, volume 43 for 1943, the following fire ratings are obtained:

<table>
<thead>
<tr>
<th>Concrete cover, in.</th>
<th>Rating, hr</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 1/2</td>
<td>1</td>
</tr>
<tr>
<td>2 1/2</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
</tr>
</tbody>
</table>

These tests also indicate that failure is unlikely to be sudden since there is progressive sagging of the member, as prestress is lost, before complete collapse occurs.
Friction. Friction is a very important consideration in the design of posttensioned members. The friction is caused by the radial pressure which is produced between the curved tendon and the member by the tension in the tendon. When we consider that curved tendons undergo large angular changes in traversing a span, and relate this in our thinking to the snubbing effect of a pulley on a belt, we can recognize immediately that care must be taken to make sure that the desired tension exists throughout the length of the tendon. Frictional resistance also arises from slight misalignments of the tendons and their ducts in the concrete forms. A certain amount of this is unavoidable because of pouring operations, but it can be held to a minimum by proper control of field operations. Materials and methods have been developed to overcome friction adequately. Thus it has not been considered necessary to allow for an additional stress loss due to friction in the design of a member. For this reason, further treatment of this subject is presented later under Materials, Friction Reducers.

Friction in a different sense is a controlling factor in the design of pavements. Here we are concerned with the friction existing between the pavement and its subgrade. Since the field is still in the development stage, its treatment in this section is limited to a general discussion above under Applications, Pavements.

Another friction problem is found in large tanks. The tank wall has to move with respect to its base as it shrinks elastically under the action of tensioning the circumferential tendons. Friction resists the motion and sets up vertical bending moments which must be dealt with in the design of the tank. For further discussion of this subject see the earlier design problem of a circular tank.

MATERIALS

Wire

A wire is a tension-carrying element made by cold drawing a steel rod through a conical die, thus simultaneously reducing its diameter and making it longer. By this cold-drawing process the granular structure which makes up the rod is stretched into fibers parallel to the center of the wire. It is this fibrous structure which gives the wire its greatly improved tensile properties. The properties of the finished wire are a function of the chemical analysis of the steel rod, the number and dimensions of the successively smaller dies through which it is drawn, and the processing to which it is submitted after passing the final die.

Every good-quality wire should have a reduction in area of not less than 30 per cent when tested to its ultimate strength. Stress-relieved, mechanically straightened and stress-relieved, and hot-galvanized wire should show a minimum elongation under tension of 4 per cent in 10 in. at failure.

The types of wire generally recommended for prestressed concrete are as follows:

Hard-drawn Wire. Hard-drawn wire is wire just as it comes from the last die. There has been no extended use of hard-drawn wire in the as-drawn condition in this country except in circular structures. For tanks, 0.162- or 0.192-in. wire with a minimum tensile strength of 210,000 psi and a minimum yield point (stress at 0.1 per cent set) of 180,000 psi is generally used.

Stress-relieved Wire. Stress-relieved wire is hard-drawn wire that has been subjected to a relatively low temperature for a sufficient period of time to relieve internal stresses. This process greatly improves the elastic properties of the wire and also produces a straighter wire. Stress relieving also removes the wire-drawing lubricant, improving the bonding properties of the wire. Tensile and yield strengths of stress-relieved wire are shown in Table 4-1.

The wire can be mechanically straightened before stress relieving where a particularly straight wire is desired. Mechanically straightened wire should not be used without stress relieving because of its poor creep properties.

Hot-galvanized Wire. Hot-galvanized wire is hard-drawn wire that has been passed through a bath of molten zinc. This process serves the double purpose of stress relieving the wire and covering it with a protective cover of zinc. The zinc
coating lowers the bonding properties of the wire. Its use is recommended for installations where the wire will be exposed to the elements. Hot-galvanized wire of 0.196 in. diameter with a minimum ultimate strength of 220,000 psi has been used for prestressed concrete. The yield strength (at 0.2 per cent set) of this wire should not be less than 80 per cent of the actual ultimate strength.

![Graph showing stress-strain curves](image)

**Fig. 4-31. Typical stress-strain curves of cold-drawn prestressed-concrete wire showing comparisons between as-drawn wire, time-temperature-treated wire, and hot-galvanized wire.**

Typical stress-strain curves for the various types of prestressed-concrete wire are shown in Fig. 4-31.

**Table 4-1. Specification for Stress-relieved Wire of Standard Sizes**

(See also ASTM A421-59T)

<table>
<thead>
<tr>
<th>Diam, in.</th>
<th>Ultimate tensile strength, psi</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.276</td>
<td>236,000</td>
</tr>
<tr>
<td>0.259</td>
<td>240,000</td>
</tr>
<tr>
<td>0.196</td>
<td>250,000</td>
</tr>
<tr>
<td>0.192</td>
<td>250,000</td>
</tr>
</tbody>
</table>

**Wire Strand**

A strand is a shop-fabricated tendon made up of several wires. A typical seven-wire strand has a straight center wire with six outside wires twisted around it. Larger strands have 19, 37, 61, or more wires. Strands used for prestressed concrete should be made of stress-relieved or galvanized wire, as described previously.

**Bright Strand.** Bright strand is strand made from uncoated wire. Seven-wire stress-relieved bright strand is used extensively for pretensioned work in the United
States. The stress-relieving operation should be made on the final strand and not the individual wires prior to stranding. Data on stress-relieved strands of this type are shown in Table 4-2.

**Table 4-2. Properties of Seven-wire Stress-relieved Strand.**

<table>
<thead>
<tr>
<th>Nominal diam., in.</th>
<th>Wt per 1000 ft, lb</th>
<th>Area, approx sq in.</th>
<th>Ultimate strength, lb</th>
<th>Recommended</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Design load, lb</td>
</tr>
<tr>
<td>3/8</td>
<td>122</td>
<td>0.0356</td>
<td>9,000</td>
<td>5,040</td>
</tr>
<tr>
<td>5/32</td>
<td>198</td>
<td>0.0578</td>
<td>14,500</td>
<td>8,120</td>
</tr>
<tr>
<td>3/32</td>
<td>274</td>
<td>0.0799</td>
<td>20,000</td>
<td>11,200</td>
</tr>
<tr>
<td>3/32</td>
<td>373</td>
<td>0.1089</td>
<td>27,000</td>
<td>15,120</td>
</tr>
<tr>
<td>3/32</td>
<td>494</td>
<td>0.1438</td>
<td>36,000</td>
<td>20,160</td>
</tr>
</tbody>
</table>

**Galvanized Strand.** Galvanized strand is made from cold-drawn hot-galvanized wire. This strand is the same as that used in suspension bridges for many years. The hot-galvanized coating provides protection against corrosion without further treatment. Its use is recommended for posttensioned work. The average modulus of elasticity for these strands is approximately 25,000,000 psi. Physical properties for these strands are indicated in Table 4-3.

**Table 4-3. Properties of Cold-drawn Hot-galvanized Strand**

<table>
<thead>
<tr>
<th>Diam., in.</th>
<th>Wt per ft, lb</th>
<th>Area, sq in.</th>
<th>Min guaranteed ultimate strength, lb</th>
<th>Recommended</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Design load, lb</td>
</tr>
<tr>
<td>0.600</td>
<td>0.737</td>
<td>0.215</td>
<td>46,000</td>
<td>26,000</td>
</tr>
<tr>
<td>0.835</td>
<td>1.412</td>
<td>0.409</td>
<td>86,000</td>
<td>49,000</td>
</tr>
<tr>
<td>1</td>
<td>2.00</td>
<td>0.577</td>
<td>122,000</td>
<td>69,000</td>
</tr>
<tr>
<td>3/4</td>
<td>2.61</td>
<td>0.751</td>
<td>156,000</td>
<td>98,000</td>
</tr>
<tr>
<td>5/8</td>
<td>3.22</td>
<td>0.931</td>
<td>192,000</td>
<td>112,000</td>
</tr>
<tr>
<td>3/4</td>
<td>3.89</td>
<td>1.12</td>
<td>232,000</td>
<td>134,000</td>
</tr>
<tr>
<td>13/32</td>
<td>4.70</td>
<td>1.36</td>
<td>276,000</td>
<td>163,000</td>
</tr>
<tr>
<td>15/32</td>
<td>5.11</td>
<td>1.48</td>
<td>300,000</td>
<td>177,000</td>
</tr>
<tr>
<td>15/32</td>
<td>5.52</td>
<td>1.60</td>
<td>324,000</td>
<td>192,000</td>
</tr>
<tr>
<td>11/32</td>
<td>5.98</td>
<td>1.73</td>
<td>352,000</td>
<td>208,000</td>
</tr>
</tbody>
</table>

**Heat-treated Cold-worked Alloy Bars**

These bars are made from two alloy steels which are commercially classified as AISI 9260 and AISI 5160. They are furnished in sizes from 3/8 to 1 1/4 in. diameter.

The steel is processed into bars by hot rolling, having as-rolled diameters approximately 5/32 in. larger than the nominal diameters noted above.

In the as-rolled state, the bars have widely varying physical properties, particularly with respect to elongation and reduction of area at ultimate fracture.

For this reason, the bars must be heat-treated in order to obtain suitable properties required for use in prestressing. The bars are placed into an enclosed furnace and heated to a minimum temperature of 800°F for a minimum of 5 hr and then allowed to slow-cool in the furnace to a temperature not over 400°F before removal. The bars must then be cooled to room temperature before further processing. This time-
temperature treatment greatly improves ductility and all bars in any one heat of steel have reasonably uniform physical properties.

Cold stretching, or "proof stressing," raises the ultimate tensile strength a moderate amount and, of more importance, raises the 0.2 per cent yield-point stress to a minimum of 90 per cent of the specified ultimate strength. The resulting material has been uniformly processed across its cross section and gives a stress-strain curve having the same general characteristics and shape of cold-drawn wire. This material has all properties necessary for satisfactory performance in prestressed concrete.

Such "proof stressing" also ensures that all bars have been tensioned to a stress well in excess of any stress to which the bars will be subjected in a structure and is a necessary precaution to detect internal or surface defects in the steel, which otherwise might be subject to delayed failure after construction.

Ultimate Tensile Strength. Heat-treated cold-worked alloy steel bars are produced to a guaranteed minimum ultimate tensile strength of 145,000 psi for all sizes.

Stress-Strain Properties. Bars for prestressing should have a high yield strength commensurate with a substantial, but not excessive, elongation measured after final rupture. This is best described by a minimum yield strength at 0.2 per cent offset equal to at least 90 per cent of the specified minimum ultimate tensile strength. Tests for all physical properties should be made on full-diameter bar specimens.

Ductility. Any material to be considered as satisfactory for a tensioning element in prestressed concrete should have a reasonable amount of ductility. Two indications of the degree of ductility are elongation after rupture and reduction of area after rupture.

Bars should have a minimum elongation after rupture of 4 per cent in 20 diameters and a reduction of area not less than 15 per cent.

Creep and Relaxation. Bars made and processed in accordance with the foregoing recommendations will have negligible creep losses at working stresses not exceeding 60 per cent of ultimate tensile strength.

Initial tension stresses must necessarily exceed 60 per cent of ultimate, to compensate for creep in the concrete and losses that may develop from other causes. For an evaluation of creep losses at various stress levels, the designer should consult with the bar manufacturer.

Special acceptance tests for individual lots are prohibitively expensive and unnecessary.

Fatigue. Fatigue properties should be considered as a requirement for satisfactory performance of this material and its end fittings. Heat-treated cold-worked alloy bars, together with their anchorages, should withstand a minimum of 3,000,000 loading cycles over a stress range of 85,000 to 100,000 psi without evidence of distress in any of the component parts.

Special acceptance tests for individual lots are prohibitively expensive and unnecessary.

Creep, Relaxation, Fatigue Resistance, and Corrosion

A minimum of creep and a maximum of resistance to fatigue loadings are important characteristics of any prestressing material. It is not practical to measure these properties by acceptance tests. However, a great deal of investigation has been made in this area by the leading manufacturers and by independent consulting engineers, universities, and testing agencies. The results of these studies are reflected by the data and recommendations herein presented. Thus, if materials, permissible unit stresses, and predicted stress losses are selected on the basis of these recommendations, the degree of resistance to creep and fatigue will be adequate. For those who are interested in delving further into the subject, detailed information and test results can be obtained from the manufacturers and from such clearinghouses of information as the Bureau of Public Roads and the Prestressed Concrete Institute, Chicago, Ill.

In general, corrosion is the enemy of prestressing tendons as it is of steel in all other applications. However, there is an exception to prove the rule. A fine powdery rust coat with no visible surface pitting will do no harm and is even beneficial in some
cases. When produced on prestressing tendons by normal atmospheric weathering during a short period in storage before use in a structure, it improves the bonding qualities of the tendons.

Precautions should be taken to avoid other types of corrosion.

In the case of posttensioned ungrouted tendons, it is well to use galvanized wire and to protect the tendons with an additional corrosion-resistant medium such as a bituminous coating. In the case of pretensioned or posttensioned grouted tendons, the concrete itself normally provides sufficient corrosion resistance. However, certain precautions should be taken. It is important to avoid the presence of nitrates or calcium chloride.

Under certain conditions high-strength steel is vulnerable to the phenomenon known as stress corrosion. It may be described as a type of corrosion resulting from exposure to certain types of corroding agents which, when combined with stress, is likely to produce cracks in and failure of the material. It is especially important to avoid the presence of nitrates. There is also some reason to believe that under such conditions it is wise not to permit more than one kind of cement to be in contact with any prestressing tendon.

It has been established to general satisfaction that oil-tempered wire is so much more sensitive to stress corrosion than the wire recommended herein as to make it undesirable for use in prestressed concrete.

![Diagram](image)

**Fig. 4-32.** Details of wire anchorage for 12 0.196-in.-diameter prestressing wires.

### Types of Anchorages

One of the basic requirements of all prestressing techniques is a reliable method of gripping and holding under tension the prestressing steel units. This is important, as slippage or failure in the end anchorage during construction would cause loss of prestress and construction difficulties. In fact, in nonbonded structural members the anchorages are relied upon permanently to transfer the total prestressing force to the concrete. With bonded prestressed members, the anchorages are usually left in place as a safety measure in case of nonuniform or poor bond except where the steel is pretensioned.

**Wedges.** Anchorages for wire prestressing tendons must contend with the extreme hardness and other very special characteristics of high-carbon steel. It is for this reason that wedge action has been used from the first as an obvious method.

In the Freyssinet anchorage, for circular cables of parallel wires, as shown in Fig. 4-32, the restraining element consists of a hollow spirally reinforced concrete
cylinder in the conical hole of which the wires can be arranged circumferentially. A conical reinforced-concrete wedge block is rammed into the cylinder after the wires have been stretched simultaneously by the special jack used for this purpose. During the jacking operation the wires are attached to the jack by temporary flat trapezoidal wedges which exert lateral forces against special grooves on the body of the jack.

Another wedge-type end anchorage for circular wire arrangements is the Preload anchorage. It consists of an anchor plate with a series of conical holes arranged in a circle. Through these the individual wires are passed and, after stressing with a special jack, are wedged tight by individual conical steel wedges.

A wedge-type anchorage for rectangular cables of parallel wires is the Blaton-Aubert or Magnel anchorage. Its restraining element consists of a cast steel plate with trapezoidal recesses on both its surfaces. Each pair of wires is fastened to it by a flat trapezoidal wedge as shown in Fig. 4-33. This arrangement permits the use of any multiple of eight wires in one cable wherein these so-called "sandwich plates" are placed on top of each other. It requires, however, the individual stressing of each wire pair. A special jack has been developed for use with this anchorage.

Wedges are used to anchor heat-treated cold-worked alloy bars up to 1 1/4 in. diameter. The wedge is shaped as a truncated cone with buttress threads and is seated in a matching tapered hole in the end-anchorage plate. This wedge anchorage provides a positive anchor for the bar. During tensioning the bar is attached to the hydraulic jack by means of a similar wedge device and when required the bar can be retensioned to take up losses and the wedge reset. The bar can be burned or cut off flush with the seated wedge after prestressing (Fig. 4-34).

Many other wedge-type anchorages have been developed for individual wires or strands.

Loops. A logical method of load transfer is by way of looping the wire, cable, or strand in the forms of the girder end before the concrete is poured or by looping it around specially prepared semicircular anchor blocks. The loop anchorage has the advantage of not requiring any auxiliary steel. It is therefore economical, but unless movable anchor blocks are used it is limited to nonjacking ends.
MATERIALS

The concrete inside an anchor loop is under high local stresses. The loops of individual wires or strands should therefore be spread out and distributed over as large a concrete area as possible. Looped-end anchorages also provide the economic possibility to vary the total prestressing force frequently along the length of a member, such as in large-span cantilever girders or continuous bridge girders. This permits a corresponding reduction in steel and concrete cross-sectional areas and quantities.

Cold-headed Wires. Anchoring wires by upsetting the end into a "buttonhead" has become a frequent practice. The end fitting is usually a short length of shafting which is threaded on the outside for the attachment of a jacking rod and a permanent nut. Holes slightly larger than the wire diameter are drilled lengthwise through the shafting. All wires are cut to exact length in a simple jig and then threaded through the holes in the end fitting. A hydraulically operated machine grips the wire near the end and contains a plunger which thereupon deforms the wire end into a "buttonhead." This type of cable can be made up of 5 to 34 parallel wires each 1/4 in. in diameter.

Sockets. Sockets are used as anchoring devices for shop-fabricated strands made up of seven or more wires twisted together in the factory. The inside of a socket is a truncated cone, the small end of which is slightly larger than the strand. The end of the strand is placed in the conical hole, and the wires within the hole are untwisted and spread out into a brush. Molten zinc is then poured into the socket, filling the hole and completely surrounding each wire. When the strand is tensioned, each wire is held in the socket by its bond to the zinc as well as the squeezing pressure which is developed by the wedge action of the zinc cone being pulled into the conical hole. It is standard practice to design the socket to take the actual ultimate strength of the strand without exceeding its own yield point. Since the pouring of a socket requires special training, the strands are cut to length and the sockets attached at the plant of the strand manufacturer, permitting the delivery of a completed unit to the job.

The external details of a socket can take many forms to suit the specific use for which it is intended. Sockets for prestressed concrete are usually round with the outside threaded to take an adjustable anchor nut. The socket on the tensioning end also is tapped to take a temporary jacking rod during the tensioning operation. Small-diameter sockets especially for prestressed concrete have been developed by Roebling to permit the closer spacing of strands (Fig. 4-35a, b).

Threaded-end Anchorage. A threaded-end anchorage is used primarily on heat-treated cold-worked alloy bars. It makes use of a threaded end nut to hold the stressing force. This specially designed anchorage has a tapered thread which engages a matched taper in the nut (Fig. 4-36). The load from the nut is transmitted through washers to an anchorage plate which rests on the concrete. This
type of anchorage has generally been superseded by the wedge type described under Types of Anchorages, Wedges. However, a similar coupler is still in general use, as shown in Fig. 4-37. A jack assembly is shown in Fig. 4-38.

Fig. 4-37. Correct setting for coupler, threaded ends, heat-treated cold-worked alloy bar.

Fig. 4-38. Prestressed-concrete jack assembly for tensioning socket-ended strand. Weight about 230 lb.

Bond-preventing Devices

Wherever it is necessary to permit relative motion between the concrete and the tendons during the stressing operation or to prevent transfer of stress through bond in the completed structure, bond-preventing devices must be included in the design. This requirement exists in all posttensioned designs. Many devices have been developed and one must choose the best type for the purpose.

Where the tendons are to be grouted after tensioning a noncollapsible enclosure is required. For this purpose, one can use flexible steel tubing or can form ducts in the concrete by means of rubber or steel mandrels which are removed after the concrete has set. In the first case, the assemblies of tendons and tubes are placed in the forms before the concrete is poured. In the latter case, the tendons are inserted after the mandrels have been removed. The choice between these devices will depend on economy and on the type of tendons employed.

Where the tendons are not to be grouted, any device is satisfactory which adequately prevents bond and protects the tendon from corrosion. Prefabricated wire strands are often wrapped with bituminized paper. For smooth members, such as individual wires or bars, a bituminous mastic (fortified with asbestos fibers to keep
the coating intact during pouring operations) is sometimes sufficient. Various types of fiber and plastic tubing have also been used for ungrouted tendons.

In some cases, a member which is to be posttensioned is initially cast with external grooves for the tendons. After the tendons are tensioned they are embedded in concrete by subsequent pouring operations.

Friction Reducers

Although this subject is treated under the general heading of Materials, the friction "reducer" is more often a method than a material. The importance of the problem is outlined earlier in this section under Design, Special Considerations, Friction.

The problem must first be attacked in the design office. There attention must be given to selecting materials for tendons and ducts which will have a satisfactorily low coefficient of friction when in contact. The phrase "satisfactorily low" is used instead of "the minimum" because economy is also an important consideration. For normal conditions, that is, simple spans of moderate length wherein the curvature of the tendons is not unusually great, any of the combinations of tendon and duct materials regularly used in prestressed concrete will produce adequately low coefficients of friction. For unusually long beams or beams wherein the tendons have unusually great curvature, it would be wise to give special consideration to reduction of coefficient of friction. It is difficult to obtain accurate quantitative data on this subject for use as design tools, since there are so many variables. The latest published data appear in the Bureau of Public Roads Criteria for Prestressed Concrete Bridge, published in 1954. Certain values for coefficient of friction are stated therein. However, they are at best only estimates. A great deal depends on experience and judgment. About all that can be said which has the general agreement of all authorities is that the coefficient is minimum for galvanized tendons in galvanized ducts, that it is maximum for preformed holes where the tendons come in direct contact with the concrete, and that it is of intermediate value when ungalvanized tendons or ducts are used.

Fortunately, the lack of good quantitative data is not serious, since the designer always has a chance to check his judgment in the field. There is little danger that, because of friction, the member will be left with improper prestress. In almost all cases, the friction can be worked out by manipulation in the field, as indicated below. It is simply a question of whether more or less field labor will be required during the stressing operation.

Where friction is a problem in the field, the following procedure is usually recommended:

The field is instructed to use elongation rather than tension as the criterion. An upper limit of overtensioning is usually specified, so that the tendons will not be overstressed. If the limiting tension is reached before full elongation has been accomplished, the instructions usually require a waiting period followed by another stressing. In many cases the tendons are tensioned at both ends. Where grouting is not required, the use of lubricants will facilitate the operation. When using elongation as the criterion, it is common practice to make a correction for the elastic shortening of the concrete, and the predicted stretch of the tendons should be based on manufacturer's recommendations. One may confirm these recommendations by test, if desired, but it is always well to hear what the manufacturer has to say.

In order to minimize friction due to misalignment, care should be taken to place the bond-preventing devices in their proper positions in the forms and to fasten them securely in place.

In extreme cases, such as in multiple-span continuous structures, the following procedure has been used:

The curvature and therefore the friction is localized at the vertexes of a polygon which is so chosen that all vertexes lie on the curve and the tendons between the vertexes form chords of the curve. Special lubricated sliding plates are placed at each vertex between the tendons and the concave inner surface of the duct. This arrangement reduces the friction adequately.
Concrete

In a discussion on prestressed concrete very little needs to be added to the comments and data on concrete given elsewhere in this handbook, except to refer to the special precaution mentioned under Creep, Relaxation, Fatigue Resistance, and Corrosion. Prestressed concrete is especially adapted to taking full advantage of the properties of high-strength concrete. This does not mean that high-strength concrete is a prerequisite to economy in prestressed-concrete members. However, it often improves economy because it can take advantage of higher allowable working stresses over the whole cross section. Furthermore, it has less creep and higher bond than regular-grade concrete. Because of the importance of bond for pretensioned construction, it is usually considered desirable to use $f'_c$ concrete having a strength not less than 4,000 psi.

Grout

Where grouting is desired after tensioning of posttensioned tendons, it is recommended that the designer adopt the specifications for materials and placement as included in Recommended Practice for Prestressed Concrete developed by the ACI-ASCE Joint Committee 323. The pertinent data may be found in Sections 303 and 405 of that publication.

FABRICATION AND CONSTRUCTION

Pretensioned

General Comments and Description of Casting Bed. Pretensioned members are fabricated by stretching the prestressing steel to a specified load and maintaining this load while the concrete is poured and cured sufficiently to carry the stretching force. Then the stretching force is released and transmitted into the concrete by means of bond between the steel and the concrete.

![Diagram of casting bed and assembly for prestressed-concrete units](image)

**Fig. 4-39.** A typical casting bed and assembly for prestressed-concrete units.

The casting beds used for this type of construction consist of a runway with a fixed anchorage at one end and a movable anchorage for tensioning the prestressing steel at the opposite end. Casting beds now in production in the United States vary in length from 120 to 320 ft. There are beds in Europe over 1,000 ft long. The optimum length is dependent on the desired production schedule and type of product involved.

The most efficient beds are made of concrete for their full length. The concrete provides a base on which to cast the member, a support for the forms, and an anchor for the tension in the prestressing steel.
Some beds have intermediate abutments at intervals along the bed to eliminate prestressing-steel wastage when the members to be poured do not cover the full length of the bed.

A typical casting bed is shown in Fig. 4-39.

**Stressing Devices.** The anchorages at each end of the bed must be designed to withstand a load equal to the maximum prestressing force that may be required to tension any member cast in the bed. Experience to date indicates that a 300,000-lb anchor is sufficient for most beds.

If possible the jacking equipment on the tensioning end of the bed should be designed to provide the full tensioning force required for one member. Hydraulic jacking equipment is recommended and, if the plant is large, a power pump is desirable.

Temporary anchorage fittings that can be reused are provided in the main anchor crossheads to anchor each individual unit of the prestressing steel. Many types are available but the most popular type consists of three tapered steel jaws, housed inside a tube which has a coned-shaped hole. The jaws clamp and grip the prestressing steel as the load is applied. These fittings can be purchased as a standard product (Fig. 4-40).

It is sometimes convenient and economical to pass the prestressing steel around a shaft at the fixed anchor and return it to the tensioning end. This eliminates the need to grip the prestressing steel on the fixed anchor end. If the prestressing steel is strand, a semicircular groove should be turned in the shaft to accommodate each strand. The width of groove should be \( \frac{3}{8} \) in. more than the nominal diameter of the strand and the depth of the groove should be one-half its width. Safe minimum shaft diameters are:

- \( 2\frac{1}{2} \) in. diameter for \( \frac{3}{4} \)-in. strand
- \( 2 \) in. diameter for \( \frac{5}{8} \)-in. strand
- \( 1\frac{1}{6} \) in. diameter for \( \frac{3}{6} \)-in. strand
- \( 1\frac{1}{4} \) in. diameter for \( \frac{1}{2} \)-in. strand

**Placing of Prestressing Steel.** The prestressing steel is placed in its proper location in the prestressing bed with one end anchored to the fixed anchor and the other end anchored to the tensioning end. Since the prestressing steel is usually made up of many separate wires or strands, care should be taken to ensure that all units have the same length so that each will have the same stress after tensioning.

The tensioning equipment is then attached to the movable anchor and the prestressing steel is stretched to a specified tension and blocked in this position so that the stretch is maintained but the tensioning device is removed for use elsewhere. The high tension in the steel makes it resistant to lateral or vertical displacement. For this reason no support is required for the prestressing steel between ends of forms during the pouring operation.

Any required mild-steel bars and/or mesh are positioned. The prestressing steel provides an excellent support for this steel.

**Forms, Pouring of Concrete, and Curing.** Forms of plywood and also thin sheet metal stiffened with plywood ribs have been used satisfactorily. In cases where many units are required, concrete forms have proved economical. A space of 3 to 6 in. is left between ends of adjoining beams so that a torch can be entered to burn off the prestressing steel after the concrete is cured.

The concrete mix should be designed to produce a strong concrete with a high bond value in the shortest possible time, thus giving maximum production for a given casting bed. Some plants release the prestressing load into the newly cast concrete after 20 hr, thus pouring once each day. Others wait 40 hr and pour every second day, while others wait 64 hr and pour every third day.
PRESTRESSED CONCRETE

Aids to early bond are:

1. A very dry mix
2. Internal and form vibrators
3. Steam or hot-water curing

There is very little information on the minimum strength of concrete needed to bond the prestressing steel. Based on present experience the following rule of thumb for minimum concrete strengths to develop satisfactory bond on stress-relieved strand is suggested:

\[
\frac{1}{4}\text{-in.}-\text{diameter strand: } 4,000 \text{ psi } f'c \text{ concrete}
\]

\[
\frac{3}{8}\text{-in.}-\text{diameter strand: } 4,500 \text{ psi } f'c \text{ concrete}
\]

\[
\frac{5}{8}\text{-in.}-\text{diameter strand: } 5,000 \text{ psi } f'c \text{ concrete}
\]

These concrete strengths are at the time the member is put into service, not at the time it is removed from the form.

The concrete is poured around the strands just as it is poured around standard reinforcing bars.

Load Release. When the concrete has cured sufficiently to hold the load in the prestressing steel by bond between the concrete and the steel, the tensioning device is attached to the movable anchor and the prestressing load is gradually released into the concrete.

The strength of concrete at release of tension must be adequate to bond the prestressing tendons to the concrete. Quite often, however, the governing factor is the strength to produce satisfactory modulus for control of deflection upon release. The Bureau of Public Roads suggests 3,500 psi for \( f'c \) as a minimum. After the prestressing load is released, the prestressing steel is cut between beams with a standard welder’s torch. The beam is then lifted from its form and removed to a storage area.

Applications in Europe. The basic prestressing unit for pretensioned construction in the United States has been the high-strength seven-wire strand. However, in Europe, use is made of high-strength individual steel wires up to 0.1 in. in diameter.

Wires having diameters up to 0.196 in. are being held by bond alone, but recent test data show that the bond on these larger wires will often fail under repeated applications of load. Seven-wire strands are preferable for pretensioned bonded work and especially for members to be subjected to repeated loads.

Posttensioned

Precast. Precast prestressed members can be manufactured at a central plant or on the job site. More than a hundred manufacturing plants in operation throughout the United States are prepared to quote prices for any type of precast prestressed members delivered to the job site. Both pretensioning and posttensioning methods are used in such plants. For small members within a transport radius of 50 to 75 miles over good roads fixed-plant production is usually more economical than site fabrication.

However, for larger members beyond an economical haulage, the contractor will usually find it cheaper to precast the members on the job site if space is available. Unless a large number of small elements is required sufficient to offset the initial cost of pretensioning beds, posttensioning will be more economical for on-site precasting. Large members, such as girders for aircraft hangars, may be precast directly beneath their final position between pairs of end columns by which they are raised and secured in position by pins running through the columns.

For precast members, the usual procedure is to use a horizontal casting slab on which movable steel or wood forms can be secured in correct position. The casting slab must be designed to withstand the weight of the precast members supported only at their ends after transfer of the prestressing. Side forms are stripped soon after concreting to obtain maximum reuse and the members are then cured on the casting
bed until sufficient strength for prestressing is reached, after which they are removed to storage.

In some operations involving sufficient repetition fixed concrete forms set in the ground are used. Such forms are suitable for double-T roof panels or monolithic slab-and-girder bridge sections. When the design requires a bottom flange, steel insert forms, which lift out of the fixed forms when the members are removed, are used to provide the correct shape.

Another type of precast construction uses standard high-strength concrete blocks placed end to end on a pallet to make a member of any desired length and cross section. The joints between blocks are usually mortared but may be left dry if completely matching surfaces between blocks are provided by grinding. The prestressing tendons are usually placed in exterior grooves in the blocks with the proper trajectory maintained with “depressor blocks” placed at intervals along the member. Semicircular saddle blocks can be used at one end of the member to reduce by one-half the number of end anchors. Many small building frames and stadiums in the Southeast have been build by this method.

![Diagram of bridge sections](image)

**Fig. 4-41.** Comparison of cross sections for precast and cast-in-place bridge girders.

After erection of precast members, continuity for live loads can be provided by adding reinforcing bars concreted in chases over supports for negative moments.

In general, precasting reduces the cost of falsework and forms, against which must be weighed the cost of transportation and erection when comparing the over-all cost of precast vs. cast-in-place construction.

**Cast-in-place.** In recent years the preoccupation with precast prestressed construction has tended to overshadow the great advantages obtainable with prestressing for large cast-in-place structures. Actually the economic benefits of prestressing are much greater with large structures than with small, but the design of large cast-in-place structures is much more difficult, which may account for its slower development.

For large-span bridges, tanks, and pressure structures well-designed prestressed cast-in-place construction will frequently show savings in cost of 30 to 40 per cent compared with fabricated structural-steel or reinforced-concrete construction.

Figure 4-41 shows a comparison of cross sections for precast and cast-in-place bridge spans of approximately equal length and loading. The precast cross section
was used for the main span of the Walnut Lane Bridge built in 1950 in Philadelphia, which was the first major prestressed bridge in the United States. This cross section utilizes 13 girders, each weighing 150 tons, for the main span 160 ft long by 62.5 ft wide. Each girder was cast on falsework at one side of the piers and after prestressing was jacked across the pier to its correct position.

The cast-in-place section is taken from a study made for the Cathedral Road Bridge in Philadelphia with a main span of 210 ft in length and 70 ft in width. Because there is no restriction on the weight of individual units, such a cast-in-place span can be built with only four main girders. The following comparison of material quantities per square foot of deck for these two cross sections is significant:

<table>
<thead>
<tr>
<th></th>
<th>Concrete, cu yd</th>
<th>Prestressing steel, lb</th>
<th>Mild steel, lb</th>
</tr>
</thead>
<tbody>
<tr>
<td>Walnut Lane</td>
<td>0.0915</td>
<td>11.5</td>
<td>5</td>
</tr>
<tr>
<td>Cathedral Road</td>
<td>0.064</td>
<td>6.3</td>
<td>7.5</td>
</tr>
</tbody>
</table>

The actual cost of the superstructure for Walnut Lane was $21.50 per square foot, whereas the estimated cost for Cathedral Road is only $13.60 per square foot, or 30 per cent less.

The construction methods employed for cast-in-place prestressed construction generally follow the procedures for reinforced concrete discussed in other sections of this book. However, several methods for falsework have been developed. Figure 4-42 shows a type of self-centering falsework which permits the use of I-section webs by hinging the outside forms to clear the bottom flange and using separate filler boxes on the inside of the web. This assembly incorporates longitudinal trusses to carry the dead load for the full length of the span and can be provided with an outside skin to form a scow to be floated to successive positions. This type of formwork has been
used extensively for multispans bridges and is most economical where six or more reuses are involved.

Prestressing of cast-in-place construction may be based on grouted or nongrouted methods using practically any of the posttensioned prestressing systems described earlier.

Two types of prestressing tendon assemblies are described as follows:

In one type, a rectangular metal box is mounted in the formwork in the correct trajectory. These are filled with as many as 400 seven-wire strands placed in horizontal layers with suitable two-dimensional spacers. The strands are looped around semicircular end blocks resting on the piers. After the strands are placed, the boxes are closed with a liquidtight cover and the girder and deck concrete is placed. Prestressing is performed by jacking the end blocks away from the structure to provide the required elongation. Wedges are placed behind the blocks, after which the jacks are removed and the whole assembly is buried in concrete.

In the other type, the cross section of the bridge consists of one or more hollow rectangular boxes of large dimension. The cables are made up of several large-diameter prefabricated strands with sockets attached in the factory. The strands are held in their proper trajectory by means of diaphragms placed at intervals along

---

**Fig. 4-44.** Highway bridge at Neckar Canal, Neckarem, Germany, built by progressive cantilever construction prestressed with bars.
the box. Several outstanding bridges of this type have been constructed in Cuba, an example of which is shown in Fig. 4-43.

Prestressed cast-in-place construction has been used in a wide variety of buildings requiring large spans, as shown in Figs. 4-4 and 4-5. This type of construction follows the usual procedure for regular reinforced-concrete construction, as discussed in other sections of this book.

**Progressive Cantilever Construction.** A system of progressive cantilever construction has been developed in Germany for carrying long bridge spans without the use of falsework over deep gorges or rivers which would require deep piling for temporary bents. It has been successfully used on spans up to 375 ft. The cross section of the bridge is poured in increments of about 30 ft at each side of piers using cantilever formwork. The 30-ft section is then prestressed to the adjoining section and the falsework is advanced for a subsequent pour. Where the loads can be balanced on each side of a pier no temporary suspenders are required. An example of this type of construction is shown in Fig. 4-44.

Circular Structures

**Tanks.** Large numbers of circular prestressed tanks have been constructed in the United States and abroad.

![Diagram of tank wall and dome-ring rubber pad and dumbbell connectors.]


Fig. 4-45. Tank wall and dome-ring rubber pad and dumbbell connectors.

Floors of such tanks are built with pneumatic mortar 2½ in. thick with wire-mesh reinforcement equivalent to not less than 0.5 per cent in each direction or are built with concrete subdivided with construction joints into convenient pouring units. In order to provide flexibility for subbase settlement it is important to maintain a minimum of at least 0.5 per cent reinforcing in each direction and to provide dumbbell-type joint connectors at all construction joints. Wall footings are designed in accordance with standard practice.

In order to minimize restraint at the joints between the floor and the wall and the wall and dome ring a rubber pad and dumbbell connector (patents and patents pending to the Preload Company, Inc., New York) are used in these joints as shown in Fig. 4-45. The width of the rubber pad is a function of the total vertical load, which should not exceed 250 psi on the rubber pad. The thickness of the rubber pad is a function of the maximum anticipated inward movement of the wall due to prestressing. The thickness should be such that the vertical face of the rubber shall not
incline more than 45° at maximum movement. This type of joint greatly reduces the end restraint of the cylindrical wall and thereby minimizes the vertical bending moments to a point where vertical prestressing to offset these moments is frequently not required.

The cylindrical wall to be prestressed is referred to as a "core wall." It may be built of poured concrete, precast staves, or pneumatic mortar.

Pneumatic mortar is frequently used for small tanks of capacity of 500,000 gal or less. Interior panel forms extending the full height of the wall are used for pneumatic construction. Where core-wall thickness is less than 6 in. this type of construction is usually less expensive than poured concrete.

Poured-concrete walls are usually employed for all tanks of 1,000,000 gal capacity or more, particularly those which are covered with dome roofs. Double forms are built of wood, steel, or standard commercial rental units for the full height of the core wall and in lengths to accommodate a maximum monolithic pour. Slip forms are often used where the core wall has a height of more than 50 ft. Dumbbell water stops are used in all vertical joints.

The concreting of such walls follows normal practice except that the placement of concrete is simplified by not having several curtains of reinforcing steel in the way.

Forms can be stripped and advanced 18 to 24 hr after concrete placement.

Where the total of vertical bending moments produces tension in excess of 300 psi in the outside fiber of either face of the core wall vertical prestressing by the methods described earlier under Prestressing Methods is utilized to keep the tension below the maximum figure of 300 psi. Otherwise vertical reinforcing bars are placed in the outside and inside face of the core wall to provide adequate crack distribution.

After the core wall is completed it is prestressed by methods described previously under Mechanical Prestressing. The whole is then covered with a bond-producing and protective coating of mechanically applied mortar to minimum thickness of 3/4 in. This cover coat can be treated or coated to give any desired architectural texture or color.

Prestressed tanks may be covered with dome roofs as described below under Roof Systems, or with flat-slab roofs carried on columns following normal reinforced-concrete construction practice.

Pipe. Prestressed pipe has been used in large quantities throughout the world.

It is largely fabricated in established plants and shipped to the work, or for larger projects a temporary plant can be established at the site of the work which may be moved periodically as work progresses.

Prestressed pipe can be manufactured with or without a steel cylinder extending throughout its length as a liquid barrier.

The essential elements consist of the concrete core, prestressing, cover coat, and joints.

The concrete core may be formed by casting in vertical forms or by horizontal centrifugation.

Where a steel cylinder is used, it is usually placed at the exterior of the core but for large-diameter pipes operating under high pressure it is sometimes more economical to place the cylinder close to the inside surface of the pipe. This reduces the amount of circumferential prestressing required for a design.

Pipe sections are usually made in lengths of 12 to 20 ft and are usually connected with rubber-gasketed joints of which several types are shown in Fig. 4-12.

Where no steel cylinder is used it is usually necessary to provide longitudinal prestressing in the pipe sections to provide adequate beam-carrying capacity without producing any ring tension in the pipe.

The manufacturing procedure for such pipe includes the following steps:

1. Manufacture of the core with or without steel cylinder or longitudinal prestressing
2. Curing the cores, which may be done either with saturated steam at elevated temperatures in a curing kiln or by maintaining the pipe wet for a long period of time
3. Circumferential prestressing by rotating pipe in a vertical or horizontal position
during which it is wrapped with wire under an initial stress in the order of 150 ksi at the required pitch.

4. Application of cover coat by mechanical spray method

5. Curing the cover coat, usually by the same method as for cores

The specific manufacturing procedure varies widely, depending on the size and operating pressure of the pipe.

Complete specification for construction of prestressed-concrete cylinder pipe will be found in the standard specifications of the American Water Works Association.

**Roof Systems.** Circular domes provide one of the most interesting uses of prestressing. Large clear spans are economically possible by prestressing an abutment or circumferential edge ring to counteract the horizontal thrust from the dead and live loads. In this way all horizontal forces due to dead load are translated into vertical reactions, thus simplifying the substructure.

Prestressed domes have been constructed successfully to diameters in excess of 200 ft, with current designs calling for diameters of 300 ft. There appears to be no valid structural limit to size.

Concrete domes are constructed on wood or metal forms adequately supported on falsework. The alignment of the form surface to the curvature called for in the design and maintenance of this curvature during construction is the single most important phase of constructing large dome shells. Settlement of falsework, deflections of formwork under construction loads, and maintenance of correct shell thickness must all be planned for in the construction operation.

The concrete dome shell may be constructed by pouring concrete or by pneumatic mortar. Placement is usually in concentric rings, which may start at the outer edge or at the center. Careful curing with water spray or curing compound is essential.

Prestressing of the abutment ring may be done by the methods used for circular tanks which are discussed earlier or by the use of prefabricated end-anchored strands, such as those described under Prestressing Methods (see also Applications, Buildings and Circular Structures).

**APPENDIXES**

**Definitions and Notations**

The following definitions and notations are used in this section. They model closely those proposed by the ACI-ASCE Joint Committee 323 in their report dated July, 1952, which was printed in the *Journal of the American Concrete Institute*, vol. 24, no. 2, October, 1952, *Proceedings*, vol. 49.

**Definitions**

**Pretensioning.** A method of prestressing reinforced concrete in which the reinforcement is tensioned before the concrete has hardened.

**Posttensioning.** A method of prestressing reinforced concrete in which the reinforcement is tensioned after the concrete has hardened.

**Bonded reinforcement.** Reinforcement bonded throughout its length to the surrounding concrete.

**Unbonded reinforcement.** Reinforcement not bonded to the concrete.

**End-anchored reinforcement.** Reinforcement provided at its ends with anchorages capable of transmitting the tensioning forces to the concrete.

**Cracking load.** The external load which, according to design computations, would cause cracking of the concrete.

**Creep.** Inelastic deformation of concrete or steel, dependent on time and resulting solely from the presence of stress and a function thereof.

**Shrinkage of concrete.** Contraction of concrete due to drying and chemical changes, dependent on time but not on stresses induced by external loading.

**Note:** In describing the manner of prestressing for a particular case, it is necessary to give the following information:
PRESTRESSED CONCRETE

1. Whether the reinforcement is pretensioned or posttensioned
2. Whether the reinforcement is bonded or unbonded
3. Whether or not the reinforcement is end-anchored

Notations

Cross-sectional constants

\[ A_e = \text{area of entire concrete section (steel area not deducted)} \]
\[ A' = \text{area of transformed section } A' = A_e + (n - 1)A_s \]
\[ A_s = \text{total steel area, steel area in simply reinforced section} \]
\[ A_{b'}, A_{t'} = \text{area of bottom (top) reinforcement in doubly reinforced section} \]
\[ A_{b'} = \text{area of prestressing steel for balanced section} \]
\[ a' = \text{concrete area above point at which shear or principal stress is being determined, sq in.} \]
\[ p = \text{ratio of prestressed-steel area to concrete area} \]
\[ p_b = \text{value of } p \text{ for a balanced section} \]
\[ c.g.c. = \text{center of gravity of entire concrete section} \]
\[ c.g.c.' = \text{center of gravity of transformed section} \]
\[ c.g.s. = \text{center of gravity of steel area} \]
\[ b = \text{total depth of section} \]
\[ d = \text{effective depth of section} \]
\[ b = \text{width of rectangular section} \]
\[ b' = \text{width of web of beam} \]
\[ b_t, b_f = \text{depth of bottom (top) flange of beam} \]
\[ b_t, b_f = \text{width of bottom (top) flange of beam} \]
\[ y_b, y_t = \text{distance of bottom (top) fiber to c.g.c.} \]
\[ y_b', y_t' = \text{distance of bottom (top) fiber to c.g.c.'} \]
\[ y = \text{distance from neutral axis of member to center of gravity of } a' \]
\[ e = \text{eccentricity of c.g.s. with regard to c.g.c.} \]
\[ e' = \text{eccentricity of c.g.s. with regard to c.g.c.'} \]
\[ I = \text{moment of inertia of entire concrete section about c.g.c.} \]
\[ I' = \text{moment of inertia of transformed section about c.g.c.'} \]
\[ Z_b, Z_t = \text{section modulus of bottom (top) fiber, referred to c.g.c.} \]
\[ Z_b', Z_t' = \text{section modulus of bottom (top) fiber, referred to c.g.c.'} \]
\[ r = \text{radius of gyration} \]
\[ n.a. = \text{neutral axis of cracked section} \]

Loads

\[ w_0 = \text{dead load per unit length when the prestress is being established (dead load of prestressed girder)} \]
\[ w_s = \text{additional (superimposed) dead load per unit length applied when the prestress has been established (dead load of deck, flooring, roadway, etc.)} \]
\[ w_D = \text{total dead load per unit length } w_0 + w_s \]
\[ w_L = \text{distributed live load per unit length} \]
\[ P_L = \text{concentrated live load} \]
\[ M_G = \text{bending moment due to } w_0 \]
\[ M_S = \text{bending moment due to } w_s \]
\[ M_D = \text{bending moment due to } w_D \]
\[ M_L = \text{bending moment due to live load} \]
\[ M = \text{bending moment due to eccentricity of prestress force} \]
\[ m_u = \text{ultimate moment, in.-lb} \]
\[ V = \text{shear} \]

Notation relating to prestressing only

First loading stage: combined action of prestressing forces and dead loads (sustained loads)
Second loading stage: combined action of prestressing forces, dead loads, and live loads
Third loading stage: ultimate loads based on cracked tension zone

\[ F_t = \text{initial prestress force} \]
\[ F_o = \text{prestress force after release} \]
\[ F = \text{effective prestress force after deduction of all losses} \]
Stresses

Concrete:
\[ f'_{c} = \text{cylinder strength at 28 days} \]
\[ f'_{pa} = \text{cylinder strength at the age of prestressing} \]
\[ f_o = \text{permissible compressive stress} \]
\[ f_{cp} = \text{permissible compressive stress at the age of prestressing} \]
\[ f_c = \text{compressive stress generally} \]
\[ f'_{s} = \text{stress at c.g.s.} \]
\[ f'_{c} = \text{stress at c.g.c.} \]
\[ f'_{Fb, Fp} = \text{stress at bottom (top) fiber due to initial prestressing only} \]
\[ f'_{Fp, Fb} = \text{stress at bottom (top) fiber due to effective prestressing only} \]
\[ f'_{G, Fb} = \text{stress at bottom (top) fiber due to dead load } w_G \text{ only} \]
\[ f'_{S, Fb} = \text{stress at bottom (top) fiber due to dead load } w_S \text{ only} \]
\[ f'_{D, Fb} = \text{stress at bottom (top) fiber due to total dead load } w_D \text{ only} \]
\[ f'_{L, Fb} = \text{stress at bottom (top) fiber due to live load only} \]
\[ f'_{Fb, FpG} = \text{stress at bottom (top) fiber due to prestressing at release } F_p \text{ and dead load } w_G \]
\[ f'_{Fp, FbG} = \text{stress at bottom (top) fiber due to effective prestressing } F \text{ and dead load } w_G \]
\[ f'_{Fp, FbD} = \text{stress at bottom (top) fiber due to effective prestressing } F \text{ and total dead load} \]
\[ f'_{FT, FpT} = \text{stresses at bottom (top) fiber due to effective prestressing } F \text{ and total load} \]
\[ f_{eu} = \text{compressive stresses at failure (third loading stage)} \]
\[ f_t = \text{tensile stress generally} \]
\[ f_{up} = \text{permissible tensile stress} \]
\[ f_{hp} = \text{horizontal stress at point at which principal stress is being determined} \]
\[ \gamma = \text{shearing stress} \]
\[ S_p = \text{principal compressive stress} \]
\[ S_t = \text{principal tensile stress} \]
\[ S_v = \text{vertical stress component} \]
\[ S_h = \text{horizontal stress component} \]
\[ E_c = \text{modulus of elasticity of concrete} \]
\[ n = \text{ratio of modulus of elasticity of steel } E_s \text{ to that of concrete } E_c \]

Steel:
\[ f'_{s} = \text{ultimate strength of steel} \]
\[ f_{y_2} = \text{stress in prestressing steel at 0.2 per cent plastic set} \]
\[ f_{o} = \text{permissible tensile stress} \]
\[ f_s = \text{steel stress generally} \]
\[ f_{s} = \text{steel stress due to initial prestressing} \]
\[ f_{o} = \text{steel stress due to prestressing after release} \]
\[ \Delta_s = \text{steel-stress loss due to elastic deformation of concrete} \]
\[ \Delta_t = \text{steel stress due to shrinkage} \]
\[ \Delta_p = \text{steel-stress loss due to creep of concrete} \]
\[ \Delta_r = \text{steel-stress loss due to creep in steel} \]
\[ \Delta_t = \text{total steel-stress reduction} \]
\[ f_{b} = \text{additional bending stress in cracked section} \]
\[ f_{su} = \text{stress at failure} \]
\[ E_s = \text{modulus of elasticity of steel} \]

References and Bibliographies

References

Typical textbooks on prestressed concrete are:


Another good source of current information on this subject is *Journal of the Prestressed Concrete Institute*, Chicago, Ill.

**Bibliographies**

The principal prestressed-concrete bibliographies are:

*Bibliography on Prestressed Concrete*, April, 1952 (721 ref.), University of Illinois, Urbana, Ill.
*ACI Bibliography No. 1, Prestressed Concrete*, October, 1955, American Concrete Institute, Detroit, Mich.

In addition to the above are innumerable articles in the technical press on the technical and economic aspects of prestressed concrete.

**ACKNOWLEDGMENTS**

The reader will note that two individuals have been named as authors of this section. This has been done simply because it is the custom to do so, but actually the section represents the combined efforts of many individuals in the Preload Company, Roebling Company, and elsewhere. Their contributions have been so significant that the word "acknowledgment" is a gross understatement in giving them recognition. Actually, they should be considered as coauthors.

Among those in the Preload Company who have contributed much thought and effort are M. Schupack, Cedric Stainer, M. N. Fornerod (now with Raymond Concrete Pile Company), and J. J. Closner.

Among the contributors from the Roebling Company are H. Kent Preston, Jr., and Lloyd E. Hill.

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The freely given cooperation of these and all other contributors is hereby gratefully acknowledged.
Section 5

STRUCTURAL THEORY

By

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EXTERNAL AND INTERNAL FORCES

Forces acting on structural members are commonly designated as external or internal, although this classification is by no means a rigid one. In general, forces that act upon the boundaries or surface of a member are defined as external forces whereas those forces which exist within the member because of the molecular resistance of the
material are designated as internal forces or stresses. Thus, in Fig. 5-1a, the bearing of beam A upon beam B or the bearing of beam B upon the walls C are common examples of how boundary or external forces are exerted upon structural members. In contrast to such contact forces the resultant stresses that are transmitted across any section m-m because of the molecular action in the material are defined as internal forces.

When only the portion of beam B between sections m-m and n-n in Fig. 5-1b is under investigation, the forces acting upon the boundary surfaces m and n are considered as external forces in all analytical work. In other words internal forces are transformed into external forces whenever they act on the boundaries of the structural member. When any member such as the cantilever AB in Fig. 5-1c is continuous with the supporting member the internal forces acting on section 1-1 become external forces when the member AB is considered separately. In most analytical work the forces acting on any section such as section 1-1 are resolved into the following components:

1. A component \( V_1 \) parallel to the section (shear)
2. A component \( N_1 \) normal to the section (axial)
3. A couple \( M_1 \) acting about an axis perpendicular to the plane of the applied forces (bending moment)

When the forces act in more than one plane (noncoplanar) then two shearing forces and three resisting couples may be developed in addition to the axial component.

**EQUILIBRIUM CONDITIONS**

When any structural member or portion of a member is in a state of rest or uniform motion the external (boundary) forces applied to the member must satisfy the following fundamental conditions of equilibrium. If the external forces acting upon any member are resolved into orthogonal components in the direction of three arbitrary rectangular axes \( OX, OY, \) and \( OZ \), then to prevent any rigid-body motion these components must satisfy the following conditions:

1. For no motion of translation the resultant of all forces must be zero, which will be true if
   \[
   \Sigma F_x = 0 \quad \Sigma F_y = 0 \quad \Sigma F_z = 0
   \]  \hspace{1cm} (5-1a)

   where \( F_x, F_y, \) and \( F_z \) represent any component in the direction of the \( X, Y, \) and \( Z \) axes, respectively.

2. For no rotation of the member the resultant moment about any axis must be zero or
   \[
   \Sigma M_{xy} = 0 \quad \Sigma M_{xz} = 0 \quad \Sigma M_{yz} = 0
   \]  \hspace{1cm} (5-1b)

   where \( M_{xy}, M_{xz}, \) and \( M_{yz} \) refer to the moment of any external force about the reference axes \( OZ, OY, \) and \( OX \) (or parallel axes), respectively.
As there are no limitations to the number of external forces that can be applied to a structural member provided that Eqs. (5-1a) and (5-1b) are satisfied it is possible to have more unknown forces acting than can be determined from the six equilibrium equations. Such a system of external forces is said to be statically indeterminate. When no more than three unknown force components are acting in a coplanar force system or six in a noncoplanar system the unknowns can be determined from the equilibrium conditions unless all forces are parallel, concurrent, or intersect on one straight line. Those force systems which can be completely determined from the equilibrium conditions are called statically determinate. Although statically determinate systems occur frequently in structural design they are actually a special case as such a limited number of force components exist only when special structural framing is provided. For many types of structural framing, particularly in reinforced-concrete structures, the number of unknown forces are in excess of the number of equilibrium equations and the force systems are therefore statically indeterminate.

REduNDant Forces

When more forces are applied to a structure or structural member than is necessary to satisfy equilibrium requirements, any reactive force that can be removed without disturbing the equilibrium of the structure may be designated as a redundant force. In any structural problem it is essential that the redundant forces be selected immediately. That this selection is somewhat optional can be illustrated by the coplanar force system in Fig. 5-2a. Either the end couple $M_{AB}$ or the vertical reaction $V_B$ can be removed without disturbing the equilibrium of the structure, and consequently either force may be regarded as redundant. Regardless of whether $M_{AB}$ or $V_B$ is selected as the redundant force, the one chosen should be regarded as an applied force in the analytical solution. For this reason the forces acting on the member $AB$ in Fig. 5-2a can be resolved into two separate force systems as shown in Fig. 5-2b and c in which $M_{AB}$ and $W$ are considered as applied forces. This treatment of redundant forces as applied forces is both convenient and necessary in structural analysis.

Fig. 5-3. End couples on a beam chosen as redundant forces.
In the force system shown in Fig. 5-3a the two end couples \( M_{AB} \) and \( M_{BA} \) can be selected as redundant forces because, if they are removed, the structure will still remain in equilibrium. However, it is also possible to use \( M_{AB} \) and \( V_{A} \) as redundant forces in any analytical solution. If the end couples \( M_{AB} \) and \( M_{BA} \) are chosen as the redundant forces the complete force system can be resolved into the three separate force systems shown in Fig. 5-3b, c, and d. By this arrangement the redundant forces \( M_{AB} \) and \( M_{BA} \) as well as the weight \( W \) are treated separately as applied forces while the vertical forces at \( A \) and \( B \) are always considered as reactive forces.

**STRAIN EQUATIONS**

Whenever a force system contains redundant forces any analytical solution to determine the unknown forces will require the use of both strain and equilibrium equations. In practically all structural problems strain equations are developed from the requirement of a consistent strain condition existing between the various contiguous members. For example, the rotation and displacement of the end section \( A \) in Fig. 5-2a must be equal and in the same direction as the movement of the corresponding section of the supporting member. Also, the vertical motion of end \( B \) in Fig. 5-2a is the same as for the supporting member.

![Diagram](image)

**Fig. 5-4. Pin and fixed joints.**

In the case of several members connected by a tight pin (Fig. 5-4a) the movement of all members at the joint \( A \) is the same. For several members rigidly connected together at a joint (Fig. 5-4b) the assumption is commonly made that the rotations and translations of the ends of all members connecting at the joint are identical. Obviously the physical conditions expressed by the strain equations will seldom be as correct as those conditions provided by the equilibrium equations. This situation should always be considered in designing statically indeterminate structures, but on the other hand this uncertainty is by no means undesirable as the presence of redundant forces provides more opportunity to develop the maximum resistance of all the material before failure occurs.

**THE ELASTIC THEORY**

Practically all calculations for determining the displacements in structural members under working loads are based on the assumption that Hooke's law is applicable, that is, that the unit strain in the material is proportional to the unit stress. This assumption also implies that the displacements are proportional to the applied loads. To be perfectly elastic, however, the strains and displacements should disappear entirely when the applied loads are removed. Actually only a few homogeneous materials approach closely this definition of elasticity. The nonhomogeneous and nonisotropic materials such as wood and concrete not only have non-linear stress-strain diagrams but the strain also increases (creeps) under sustained loads; moreover, a residual or permanent strain (set) is often found after the load is removed. Materials that
undergo an increase in strain over a period of time when subjected to a constant stress are said to be partially plastic. Such material will usually undergo some permanent deformation or set when the load is removed, and although they may be quite elastic under low values of stress, they may have considerable plasticity under high stresses. For this reason displacements under working loads are commonly calculated by assuming a perfectly elastic material whereas for computing the ultimate strength of structural members the plastic properties of the materials are frequently employed.

In view of the creep in concrete when it is in a strained condition the question naturally arises whether the redundant forces in a statically indeterminate reinforced-concrete structure can be accurately determined from strain equations that are derived for an elastic material. Various tests have been made to compare the theoretical stresses obtained by use of the elastic theory with actual measured values. In Fig. 5-5b a comparison of the theoretical and experimental values of the bending moments at the crown and knee of a two-hinged single-span reinforced-concrete frame subjected to two concentrated live loads as indicated in Fig. 5-5a is shown. These values are taken from the results of a comprehensive series of tests on reinforced-concrete frames made at the University of Illinois (Ref. 8). These results give proof that the redundant forces calculated by use of the elastic theory agree closely with the experimental values and, together with other test results, give assurance that the elastic theory can be accurately applied in the analysis of indeterminate reinforced-concrete structures provided that the following assumptions are made.*

1. The stress-strain relation for the concrete has the same value at all sections and at all stresses.

2. The moment of inertia is for an uncracked section.

**PRINCIPLE OF SUPERPOSITION**

Whenever structures are analyzed by the elastic theory the principle of superposition is applicable, unless the displacements are large, and will frequently simplify the mathematical operations. This principle permits the calculation of stresses and deformations due to any number of applied forces by combining the separate effects of the individual forces and, furthermore, these forces can be considered in any order. For this reason the structural designer can treat separately the stresses and deformations due to dead and live loads as well as those due to dynamic and temperature effects. Also the principle of superposition permits the use of the various force systems that are combined to give the modern solutions by successive approximations.

* Superior numbers refer to the references at the end of this section.
DEFORMATIONS OF STRAIGHT MEMBERS DUE TO BENDING

The strain in any structural member due to bending moments is assumed to vary linearly across the section; that is, a plane section before bending remains plane after bending. As every section is assumed to rotate about an axis passing through the centroid of the effective area, the relative rotation $d\phi$ between any two right sections (Fig. 5-6a) that are a distance $dx$ apart is determined from the extension or contraction $d\delta$ of any fiber divided by the distance $y$ of that fiber from the neutral axis, or

$$d\phi = \frac{d\delta}{y} = \frac{f}{E} \frac{dx}{y} \quad (5-2a)$$

in which $f$ is the magnitude of the unit stress and $E$ is the modulus of elasticity of the material. The value of $f$ for any applied loads is given by the usual flexure formula

$$f = \frac{M_0}{I} \quad (5-2b)$$

in which $M$ is the bending moment acting on the element and $I$ is the moment of inertia of the effective section about the neutral axis. When this value of $f$ is substituted into Eq. (5-2a) the relative rotation $d\phi$ between the two end sections of the element is equal to

$$d\phi = \frac{M}{EI} \frac{dx}{y} \quad (5-2c)$$

Fig. 5-6. Rotation between sections related to $M/EI$ diagram.

When the bending-moment diagram is converted into an $M/EI$ diagram (Fig. 5-6b) by dividing the bending moment at any section by the value of $EI$ of that section, the magnitude of the rotation $d\phi$ is numerically equal and therefore can be represented by the shaded area $dA$. In many solutions of statically indeterminate force systems where relative rather than absolute quantities are sufficient and when $E$ is assumed to be constant, the $M/EI$ diagram can be replaced by the $M/I$ diagram. By combining the relative rotations $d\phi$ of all elements between any two sections the total relative rotation of those sections is obtained. The relationship between the relative rotation

Fig. 5-7, Displacement related to $M/EI$ diagram.
of any two sections or tangents to the elastic curve and the $M/EI$ diagram gives the first theorem of area moments, which can be summarized as follows:

The relative rotation $\phi$ between any two right sections (or tangents) in a continuous structural member is equal to the area of the $M/EI$ diagram between the two sections.

The translation of any point $n$ (Fig. 5-7) on the axis of a member due to the relative rotation $d\phi_1$ of an element is

$$d\Delta_1 = x_1 d\phi_1 = x_1 dA_1$$

since $d\phi_1$ is equal to the area $dA$ of the $M/EI$ diagram. If a second element provides a rotation $d\phi_2$ then point $n$ receives an additional displacement of

$$d\Delta_2 = x_2 d\phi_2 = x_2 dA_2$$

When the effects of all rotations between the sections $m$ and $n$ are included the total displacement $\Delta$ becomes

$$\Delta = \int_m^n x d\phi = \int_m^n x dA$$  \hspace{1cm} (5-3)$$

Equation (5-3) gives the second theorem of area moments, which can be stated as follows:

For any continuous elastic curve the displacement $\Delta$ of any point $n$ from a tangent at $m$ is numerically equal to the statical moment of the $M/EI$ diagram between $m$ and $n$ about point $n$ where the displacement occurs.

It should be noted that a displacement of a point from a tangent can occur even though the point itself does not move. In such a case the displacement is caused by the rotation of the tangent.

**Calculation of Beam Displacements**

The two theorems of area moments provide a convenient method for calculating the angular and linear displacements of any point on the elastic curve of a beam with respect to any tangent as a reference axis. If the reference tangent remains fixed in position when the load is applied to the beam then any displacement calculated with respect to that tangent will also be the actual movement of the point. If, however, the reference tangent rotates, then a correction for this rotation must be added to any displacement obtained from the second theorem of area moments [Eq. (5-3)] before the actual movement is obtained.

For example, if the beam $AB$ in Fig. 5-8 is fixed against rotation at $A$ then any displacement that is determined with respect to the tangent at that point will also be the absolute motion.

Thus if $EI$ is assumed constant for all sections the angle $\theta_B$ is equal to

$$\theta_B = \text{area of } \frac{M}{EI} \text{ diagram between } A \text{ and } B = \frac{Pb^3}{2EI}$$

and

$$\Delta_B = \text{statical moment of } \frac{M}{EI} \text{ diagram between } A \text{ and } B \text{ about } B$$

$$= \left( \frac{Pb^3}{2EI} \right) \left( \frac{2}{3} b \right) = \frac{Pb^3}{3EI}$$

However, if the tangent at $A$ rotates as shown in Fig. 5-9 then the correction for the rotation $\theta_A$ must be added. As all rotations are extremely small any angle is assumed
equal in value to its tangent, and therefore

\[ \theta_A = \frac{\Delta}{a} = \frac{\lambda}{a} \left( \frac{Pb}{EI} \right) (a) \left( \frac{2a}{3a} \right) = \frac{Pab}{3EI} \]

The actual movement of point B is therefore equal to

\[ \Delta_B = \Delta_B' + b\theta_A = \frac{Pb^2}{3EI} + b \frac{Pab}{3EI} = \frac{Pb^2}{3EI} (a + b) \]

and

\[ \theta_B = \theta_B' + \theta_A = \frac{Pb^2}{2EI} + \frac{Pab}{3EI} = \frac{Pb}{6EI} (2a + 3b) \]

When any two points on a continuous elastic curve are known to have no displacement, a mathematical analogy, often called the conjugate-beam analogy, can be obtained

![Diagram](image)

Fig. 5-9. Rotation of beam-tangent at A: relation of \( \theta_A, \theta_B, \) and deflection.

![Diagram](image)

Fig. 5-10. Conjugate-beam relationship.

directly from the area-moments theorems. This analogy is easily explained by considering any beam with two nonyielding supports as \( AB \) in Fig. 5-10. The end rotations \( \theta_A \) and \( \theta_B \) can be obtained directly from the second theorem of area moments, that is,

\[ \theta_A = \frac{\Delta_B}{L} = \frac{A \bar{x}}{L} \quad (5-4a) \]

\[ \theta_B = \frac{\Delta_A}{L} = \frac{A (L - \bar{x})}{L} \quad (5-4b) \]

If the area \( A \) of the \( M/EI \) diagram is considered as an applied weight acting on a simply supported beam of span \( L \), then, by application of Eqs. (5-4a) and (5-4b), the
reaction $R_d'$ of this imaginary beam is equal to $\theta_A$ and $R_B'$ is equal to $\theta_B$. Moreover, the rotation $\theta_m$ at any point $m$ is equal to

$$\theta_m = \theta_A - \phi_m = R'_A - A_m$$  \hspace{1cm} (5-4c)

in which $A_m$ is the area of the $M/EI$ diagram between point $A$ and $m$. That is, the actual rotation $\theta_m$ is equal to the shear at section $m$ in the imaginary or conjugate beam.

The vertical displacement $y$ at point $m$ is equal to the distance $\theta_A z$ minus the displacement $A_m$ from the tangent, or

$$y = \theta_A z - A_m z = R'_A z - A_m z$$  \hspace{1cm} (5-4d)

Therefore, $y$ is equal to the bending moment in the conjugate beam.

The conjugate-beam concept provides no mathematical advantages over the direct use of the area-moments theorems and lacks the clear geometrical interpretation of the latter. For this reason the author prefers to apply the area-moments theorems directly rather than in terms of the conjugate-beam analogy.

**RELATION OF FORCES AND DISPLACEMENTS**

Under Strain Equations it was shown that any numerical solution of a structural problem involving redundant forces requires a study of the displacements at those boundaries at which the redundant forces are acting. These displacements can always be expressed in terms of both known loads and unknown redundant forces. Furthermore the effect of each redundant force upon the displacements can be considered separately and added to the displacements determined for the applied load.

![Diagram](image)

**Fig. 5-11.** Third reaction considered as a redundant force.

To illustrate this statement, the reactions of the two-span continuous beam in Fig. 5-11a will be studied. As only two vertical reactions are required for equilibrium any one of the three reactions can be designated as a redundant force. If the center reaction $R_b$ is selected as a redundant it should be treated as an applied force exactly like the uniform load $w$. Therefore, the load $w$ and the necessary reactions at $A$ and $C$ in Fig. 5-11b are taken as one force system while the redundant force $R_b$ and the reactions at $A$ and $C$ in Fig. 5-11c are considered separately. The displacement at point $B$ for all forces in Fig. 5-11a may now be determined by combining $\Delta_b'$ in Fig. 5-11b with $\Delta_b''$ in Fig. 5-11c. By the theorems of area moments these two movements are found to be

$$\Delta_b' = \frac{5}{24} \frac{wL^4}{EI}$$

$$\Delta_b'' = \frac{1}{6} \frac{R_bL^3}{EI}$$
Fig. 5-12. Relation of forces and displacements for members with constant EI. (Clockwise moments and rotations are positive. Upward vertical displacements are positive.)
Therefore, the combined displacement of point B is

$$\Delta_B = \frac{5}{24} \frac{wL^4}{EI} - \frac{1}{6} \frac{R_BL^3}{EI}$$

If the support at B is nonyielding then $\Delta_B$ must equal zero, which gives the equation

$$\frac{5}{24} \frac{wL^4}{EI} - \frac{1}{6} \frac{R_BL^3}{EI} = 0$$

from which

$$R_B = \frac{5}{4} wL$$

Suppose, however, that the support B moves downward some amount $\delta_B$, expressed in the form

$$\delta_B = \Delta_B' - \Delta_B''$$

or

$$\delta_B = \frac{5}{24} \frac{wL^4}{EI} - \frac{1}{6} \frac{R_BL^3}{EI}$$

from which

$$R_B = \frac{5}{4} \frac{wL}{L^3} - \frac{6EI\delta_B}{L^3}$$

Similar relations between forces and displacements are tabulated in Fig. 5-12 for many special force systems that are now utilized in the various methods of successive approximations. The moment-distribution method that is frequently employed in the analysis of continuous beams and frames is based on combinations of the various force systems given in Fig. 5-12. The solutions shown can be easily calculated by the theorems of area moments or by the conjugate-beam analogy.

**FIXED-END COUPLES**

Many solutions of structural problems involve end couples that will prevent the ends of structural members from rotating or translating when those members are subjected to transverse and other types of applied forces. These end couples, which are designated fixed-end couples and represented by $M_F$, can be easily calculated by the following procedure.

Calculate the end rotations $\alpha_A$ and $\alpha_B$ (Fig. 5-12a) for the applied load on the member with no end couples acting. These angles are readily determined from the theorems of area moments as previously explained. The end couples that will give equal and opposite rotations (Fig. 5-12c) will be the fixed-end couples as the resultant angle will then be zero. Consequently the values of the fixed-end moments are

$$M_{FAB} = \frac{4EI}{L} (\alpha_A) + \frac{2EI}{L} (\alpha_B) \quad (5-5a)$$

$$M_{FBA} = \frac{2EI}{L} (\alpha_A) + \frac{4EI}{L} (\alpha_B) \quad (5-5b)$$

For vertical loads on a beam angle $\alpha_A$ is positive (clockwise) and $\alpha_B$ is negative (counterclockwise) and therefore $M_{FAB}$ is negative while $M_{FBA}$ will be positive in direction. When other types of loads are acting the signs of $\alpha_A$ and $\alpha_B$ are readily determined by inspection.

The values of the fixed-end couples for various types of applied loads are recorded in Fig. 5-13. These solutions may be combined to give the resultant fixed-end couples for practically any arrangement of applied loads. If one end of a member is hinged and the other end fixed, the moment at the fixed end can be determined from the values given in Fig. 5-13 by means of the relations shown in Fig. 5-12c, that is, if $M_{ab}$ equals zero the fixed-end couple $M'_{Fba}$ at end b is

$$M'_{Fba} = M_{Fba} - \frac{1}{2} M_{F\ell b}$$

and, if $M_{ba}$ equals zero the fixed-end couple $M'_{F\ell a}$ at end a is

$$M'_{F\ell a} = M_{F\ell a} - \frac{1}{2} M_{Fba}$$
Example. If the beam $AB$ in Fig. 5-14 is fixed at both ends the fixed-end couples may be determined by combining the values given in Fig. 5-13. This operation gives

$$M'_{FAB} = \frac{12}{(2)(20)(20)} - \frac{12}{(2)(20)(20)} - \frac{12}{(12)(8)(12)(12)} = -141.3 \text{ ft-kips}$$

$$M'_{FBA} = \frac{12}{(2)(20)(20)} + \frac{12}{(2)(20)(20)} + \frac{12}{(12)(8)(12)(12)} = +116.4 \text{ ft-kips}$$

If the beam is simply supported at $B$ and fixed at $A$ then

$$M'_{FAB} = -141.3 - \left(\frac{1}{2}\right)(116.4) = -199.5 \text{ ft-kips}$$

and, if the beam is simply supported at $A$ and is fixed at $B$ then

$$M'_{FBA} = 116.4 - \left(\frac{1}{2}\right)(-141.3) = +187.1 \text{ ft-kips}$$

**MOMENT-DISTRIBUTION METHOD**

By means of the formulas given in Figs. 5-12c and 5-13 the fixed-end couples can be calculated for any type of loading that is applied to a structural member with constant $EI$. Corresponding equations for members with variable cross section are considered later. At this time the problem of determining the numerical values of end couples that act upon members whose ends are not completely fixed will be discussed.

The end rotations $\theta$ that will occur if any member is not completely restrained are most conveniently measured from the original positions of the end sections and, therefore, the loaded beam with the fixed-end couples applied is usually selected as the reference position. All end rotations are then measured from the fixed-end or zero position and, correspondingly, any end couples required to cause the rotations must be added algebraically to the fixed-end couples.
The advantages of using the fixed-end condition as a reference or starting position will be illustrated by the beam in Fig. 5-14. The fixed-end couples for this beam were found to be $-141.3$ ft-kips at $A$ and $+116.4$ ft-kips at $B$. If, however, the supports at $A$ and $B$ permit the ends to rotate through the angles $\theta_A$ and $\theta_B$, respectively, then the force system of Fig. 5-12b must be added algebraically to the fixed-end system to provide the rotations. The resultant end couples will then be

\[
M_{AB} = -141.3 + 4K_{AB}\theta_A + 2K_{AB}\theta_B
\]
\[
M_{BA} = +116.4 + 2K_{AB}\theta_A + 4K_{AB}\theta_B
\]

in which \( K = \frac{EI}{L} \).

As shown in Fig. 5-12b clockwise moments and angles are considered positive and counterclockwise negative.

If end $A$ of the beam $AB$ in Fig. 5-14 is simply supported, that is, if $M_{AB}$ equals zero, then this condition should be used and all end couples at $B$ should be determined for a simple support at $A$. For any rotation $\theta_B$ of end $B$ from the reference or fixed-end condition the force system shown in Fig. 5-12f must be added to the fixed-end force $M'_{FBA}$. The resultant end couple at $B$ is then equal to

\[
M_{BA} = M'_{FBA} + 3K_{AB}\theta_B = +187.1 + 3K_{AB}\theta_B
\]

When end $A$ is restrained and $B$ is simply supported the total value of the moment at $A$, if that support allows any rotation $\theta_A$, is

\[
M_{AB} = M'_{FAB} + 3K_{AB}\theta_A = -199.5 + 3K_{AB}\theta_A
\]

From the above discussion it is apparent that if all members intersecting at any rigid joint $x$ of a continuous frame (Fig. 5-15) rotate through the same angle $\theta_x$ while

\[\text{Fig. 5-15. Arrangement of values for moment-distribution method.}\]

the opposite end of each member remains either fixed or hinged, then all end couples can be expressed in terms of the fixed-end couples and the single rotation $\theta_x$. For example, in Fig. 5-15 the values of the end couples at joint $x$ are

\[
M_{xa} = M_{Fx} + 4K_1\theta_x
\]
\[
M_{xb} = M_{Fx} + 4K_2\theta_x
\]
\[
M_{xc} = M_{Fx} + 4K_3\theta_x
\]
\[
M_{xd} = M'_{Fx} + 3K_4\theta_x
\]

\[
M'_{Fx} = M_{Fx} - \frac{1}{2} M_{Fx}
\]

in which

\[\text{Fig. 5-14. Loaded beam fixed at both ends.}\]
To satisfy the equilibrium conditions at joint $x$ the algebraic sum of the above end moments must equal zero or

$$M_{za} + M_{zb} + M_{zc} + M_{zd} = 0 \quad (5-7)$$

When the values of the end couples given by Eq. (5-6) are substituted into Eq. (5-7), it becomes equal to

$$(M_{Fza} + M_{Fzb} + M_{Fzc} + M'_{Fzd}) + (4K_1 + 4K_2 + 4K_3 + 3K_4) \theta_z = 0 \quad (5-8)$$

In Eq. (5-8) the algebraic sum of the fixed-end couples will be represented by the expression

$$\sum_x M_F = M_{Fza} + M_{Fzb} + M_{Fzc} + M'_{Fzd}$$

and the summation of the various coefficients of $\theta_z$ by the expression

$$\sum_x CK = 4K_1 + 4K_2 + 4K_3 + 3K_4$$

in which $C$ equals 3 or 4, depending upon whether the opposite end of the member is hinged or fixed. Solving Eq. (5-8) for the rotation $\theta_z$ gives

$$\theta_z = \frac{-\sum_x M_F}{\sum_x CK} = -\frac{\text{(sum of fixed-end couples)}}{\text{(sum of CK values)}} \quad (5-9)$$

The end couples acting upon each member can now be calculated by substituting the value of $\theta_z$ in Eq. (5-9) back into Eq. (5-6), which gives

$$M_{za} = M_{Fza} + \frac{4K_1}{\Sigma_z CK} (-\sum_x M_F) = M_{Fza} + r \left(-\sum_x M_F\right)$$

$$M_{zb} = M_{Fzb} + \frac{4K_2}{\Sigma_z CK} (-\sum_x M_F) = M_{Fzb} + r \left(-\sum_x M_F\right) \quad (5-10)$$

$$M_{zc} = M_{Fzc} + \frac{4K_3}{\Sigma_z CK} (-\sum_x M_F) = M_{Fzc} + r \left(-\sum_x M_F\right)$$

$$M_{zd} = M'_{Fzd} + \frac{3K_4}{\Sigma_z CK} (-\sum_x M_F) = M'_{Fzd} + r \left(-\sum_x M_F\right)$$

The results given by Eq. (5-10) can be stated as follows:

The correction that must be added to any fixed-end moment acting on any member at a rigid joint, because of the rotation of that joint, is equal to the ratio $r = CK/\Sigma CK$ times minus the algebraic sum of all fixed-end moments at the joint.

If the opposite end of any member is held fixed during the rotation of joint $x$, then the fixed-end moment at that end is also changed by the rotation $\theta_z$. For instance, if $\theta_z$ in Fig. 5-15 is kept zero while joint $x$ is rotated to a condition of equilibrium then

$$M_{za} = M_{Fza} + 2K_1 \theta_z = M_{Fza} + \frac{1}{2} r \left(-\sum_x M_F\right) \quad (5-11)$$

The change in the fixed-end moment $M_{Fza}$ due to the rotation $\theta_z$ is therefore one-half of the correction at the $x$ end (see Fig. 5-12d). The corrections at $b$ and $c$ as shown in Fig. 5-15 are also equal to one-half of the corresponding corrections at $x$ for the members $b$ and $c$ and have the same sign. No moment need be considered at end $d$ of member $zd$ as the correction at $x$ was calculated for a hinge at $d$ by taking $c$ equal to 3.

As the various joints in any continuous-frame structure can be rotated separately to a condition of equilibrium and the moments due to each rotation recorded as shown in Fig. 5-15, the true value of any end couple is approached as the correction $r \left(-\sum_x M_F\right)$ approaches zero. This method of calculating end moments was first presented by Professor Hardy Cross in 1932. The numerical procedure is most easily explained by specific examples.
ANALYSIS OF CONTINUOUS BEAMS BY MOMENT DISTRIBUTION

The application of the moment-distribution method to the solution of continuous beams is illustrated in Fig. 5-16. The end couples acting on each span when joints B, C, and D are assumed fixed are shown separately in Fig. 5-17a, b, c. The values of $M_{FBC}$ and $M_{FCB}$ were calculated previously in Fig. 5-13. The load on the overhanging end applies a counterclockwise moment of $-32$ ft-kips at end A of span $AB$ and, therefore, it will require a fixed-end couple of one-half of this value, or $-16$ ft-kips, at $B$ to keep that end from rotating (Fig. 5-12a). The load of 4 kips per ft on span $AB$ will produce fixed-end couples of $\pm wL^2/12$, or $-85.33$ ft-kips at $A$ and $+85.33$ at $B$ if both ends were fixed. However, since any load on span $AB$ causes no moment at $A$ it will be necessary to apply a moment of $+85.33$ at $A$ to reduce the value to zero. This moment requires an additional fixed-end couple of one-half this amount, or $+42.67$, at $B$ to keep that end from rotating, which makes the total fixed-end couple at $B$ due to the 4 kips per ft on span $AB$ equal to $85.33$ plus $42.67$, or $128.0$ ft-kips. Since the load on the cantilever portion causes a moment of $-16$ ft-kips at $B$ the resultant end couple $M_{FBA}$ equals $128.0$ ft-kips minus $16$ ft-kips, or $112.0$ ft-kips.
The distribution factors $r$ at joints $B$ and $C$, that is, the ratio of $CK/\Sigma CK$, are recorded in Table 5-1.

<table>
<thead>
<tr>
<th>Member</th>
<th>$C$</th>
<th>$K = \frac{I}{L}$</th>
<th>$CK$</th>
<th>$\frac{r = CK}{\Sigma CK}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Joint B</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$BA$</td>
<td>3</td>
<td>10</td>
<td>30</td>
<td>0.483</td>
</tr>
<tr>
<td>$BC$</td>
<td>4</td>
<td>8</td>
<td>32</td>
<td>0.517</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$\Sigma CK = 62$</td>
<td>1.000</td>
</tr>
<tr>
<td>Joint C</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$CB$</td>
<td>4</td>
<td>8</td>
<td>32</td>
<td>0.445</td>
</tr>
<tr>
<td>$CD$</td>
<td>4</td>
<td>10</td>
<td>40</td>
<td>0.555</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$\Sigma CK = 72$</td>
<td>1.000</td>
</tr>
</tbody>
</table>

A coefficient of 3 is used in determining $r$ for member $BA$ when joint $B$ is rotated as the moment at $A$ is kept constant while a coefficient of 4 is employed for all other members as the opposite end of each member is held fixed. The numerical operations are shown in Fig. 5-16. The value of the unbalanced moments $\Sigma M_F$ at joint $C$ is 84.4 ft-kips and therefore corrections of

$$(0.445)(-84.4) = -37.6$$
$$(0.555)(-84.4) = -46.8$$

are added algebraically to $M_{FCB}$ and $M_{FCD}$, respectively. As shown in Eq. (5-11) and in Fig. 5-12d the end couples required to keep the opposite ends from rotating are one-half of these values and therefore the changes in the fixed-end moments at $BC$ and $DC$ are $-18.8$ and $-23.4$ ft-kips, respectively. As these values represent the change in the fixed-end moments they can be added directly to the original values of $-141.3$ and $+32.0$.

Joint $B$ is now balanced in the same manner by distributing the unbalanced moment of $-48.1$ ft-kips due to the four fixed-end couples ($-16.0 + 128.0 - 141.3 - 18.8$) according to the $r$ values, or

$$(0.483)(48.1) = +23.2$$
$$(0.517)(48.1) = +24.9$$

As the $r$ value of 0.483 was obtained for no change of moment at $A$, no correction is necessary at that end but as the $r$ value for $BC$ was calculated for no change in angle at $C$, a fixed-end couple of one-half of 24.9, or 12.5, must be applied to the end $CB$ to prevent rotation.

As there is now an additional unbalanced moment of 12.5 ft-kips at joint $C$, additional corrections of

$$(0.445)(-12.5) = -5.6$$
$$(0.555)(-12.5) = -6.9$$

are required for $M_{CB}$ and $M_{CD}$ and additional fixed-end couples of $-2.8$ and $-3.4$ at ends $BC$ and $DC$. These operations are then repeated until the fixed-end couples that are produced by the rotation of any joint can be ignored. The algebraic sum of all fixed-end couples and the corrections give the actual values of the end couples that will satisfy both equilibrium and strain conditions within the limitations of the elastic theory.

In the above example the reader should note that, for any member, both the correction at the end that is rotated and the change in the fixed-end couple at the opposite end when that end is held fixed are taken as one operation. This arrangement is
important in more complicated problems as it enables the computer to think of the rotation of each joint as a separate problem and the entire solution then becomes simply a repetition of this one problem. In most problems this arrangement will also provide the most rapid convergence of the successive corrections.

SHEAR AND BENDING-MOMENT DIAGRAMS

After the end couples acting upon any span of a continuous beam or frame have been determined the shear and bending-moment diagrams can be drawn for that span. The shear diagram should be drawn first as it provides valuable information for determining the sections of maximum positive and negative moments due to fixed loads. When distances are measured from the left end of the span the section at which the shear changes from positive to negative is the section at which the maximum positive moment occurs. Minimum positive (maximum negative) moments usually occur at end sections where the shear changes from negative to positive. The numerical procedure for the construction of shear and bending diagrams and for the calculation of reactions will be described by means of the quantities given in Fig. 5-16 for the continuous beam ABCD.

The first step is the calculation of the end shears for each span directly from the equilibrium equations which all forces acting on that span must satisfy. These values are as follows:

Cantilever: \[ V_A = (4)(4) = 16 \text{ kips} \]
Span AB: \[ V_{AB} = \frac{(4)(16)(8) + 32.0 - 136.6}{16} = 25.47 \]
\[ V_{BA} = \frac{(4)(16)(8) - 32.0 + 136.6}{16} = 38.53 \]
Total = 64.00 kips

Span BC: \[ V_{BC} = \frac{(12)(12) + (2)(20)(10) + (2)(\frac{2}{5})(\frac{2}{5})(20)}{20} + 136.6 - 86.1 = 43.06 \]
\[ V_{CB} = \frac{(12)(8) + (2)(20)(10) + (2)(\frac{2}{5})(\frac{2}{5})(20) - 136.6 + 86.1}{20} = 28.94 \]
Total = 72.00 kips

Span CD: \[ V_{CD} = \frac{(16)(8) + 86.1 - 5.0}{16} = 13.06 \]
\[ V_{DC} = \frac{(16)(8) - 86.1 + 5.0}{16} = 2.94 \]
Total = 160.00 kips

The shear at any section can now be determined by adding algebraically the transverse load between that section and the end of the span to the end shear (not to the reaction). The sections at which the shear is zero can be located either graphically or algebraically. The shear diagrams for all spans and the values of the controlling ordinates are shown in Fig. 5-18.

The bending moment at any section of a span is equal to the algebraic sum of the moments of the end couple, the end shear, and all loads between the section and the
end of the span about the centroid of the section. For example, the maximum positive moment in span $AB$ which is at a section 6.37 ft from $A$, where the shear is zero, is equal to

$$-32.0 + (25.47)(6.37) - (4)(6.37) \left(\frac{6.37}{2}\right) = 49.1 \text{ ft-kips}$$

Bending moments acting on a section are usually considered positive when the applied moment tends to stretch the fibers at the bottom of a horizontal member or the right side of a vertical member. The bending-moment diagrams for all spans as well as the maximum values are shown in Fig. 5-18.

**ANALYSIS OF FRAMES RESTRAINED AGAINST SIDESWAY**

When a continuous frame is so supported that no translation of the joints is permitted the moment-distribution method, that is, the correction of fixed-end couples for joint rotation, can be applied directly just as for the continuous beam in Fig. 5-16. The numerical procedure will be illustrated by the solution of the frame in Fig. 5-19a in which the spans and loading are identical with those in the continuous beam in Fig. 5-16. However, in the frame shown the beams are rigidly connected to the columns at joints $A$, $B$, and $C$ but beam $CD$ is supported by a hinge at $D$. If the change in length of all members is neglected the joints will have no translation but can undergo rotation.

The solution in Fig. 5-19b was started by assuming no rotation of joints $A$, $B$, and $C$. As the moment at $D$ is zero that value can be used immediately and will need no cor-
resection. The fixed-end couple $M_{FCD}$ is therefore obtained from the formulas in Fig. 5-13 by means of the relations shown in Fig. 5-12e as

$$M'_{FCD} = M_{FCD} - \frac{1}{2}M_{FDC} = -32 - \left(\frac{1}{2}\right)(+32) = -48 \text{ ft-kips}$$

The fixed-end couples for span $AB$ must be calculated for both ends fixed as the true value of $M_{AB}$ is unknown because of the continuity of beam $AB$ with column $AE$. In

![Diagram of a continuous beam with fixed and hinged supports, labeled with moments and forces.](image)

<table>
<thead>
<tr>
<th>$\theta_0 = 0$</th>
<th>+32.0</th>
<th>-85.3</th>
<th>+33.4</th>
<th>-51.1</th>
<th>+3.5</th>
<th>-2.2</th>
<th>+0.5</th>
<th>-0.2</th>
<th>-47.4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_{Fg} = 0$</td>
<td>+85.3</td>
<td>+16.7</td>
<td>+6.9</td>
<td>-1.1</td>
<td>+0.7</td>
<td>+122.3</td>
<td>-136.6</td>
<td>+97.2</td>
<td>-75.3</td>
</tr>
<tr>
<td>$M_g = 0$</td>
<td>-141.3</td>
<td>+131.1</td>
<td>+5.5</td>
<td>-0.5</td>
<td>+0.5</td>
<td>+116.4</td>
<td>-278.0</td>
<td>-10.0</td>
<td>-1.0</td>
</tr>
<tr>
<td>$M_{Do} = 0$</td>
<td>+116.4</td>
<td>+6.6</td>
<td>-278.0</td>
<td>+10.0</td>
<td>-1.0</td>
<td>-48.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>

Fig. 5-19. Moment-distribution analysis of the continuous beam of Fig. 5-16 supported on columns.

The fact, joint $A$ is treated the same as joints $B$ and $C$ except that the moment acting on the cantilever is not changed by the joint rotation (distribution factor equals zero).

The computations for determining the distribution factors are given in Table 5-2 and the value of these ratios is also recorded on the diagram in Fig. 5-19b.

The computations shown in Fig. 5-19b start by distributing minus the unbalanced moment at joint $A$ (+53.3) according to the distribution factors, which gives

$$(0.625)(+53.3) = +33.4 \text{ ft-kips}$$

$$(0.375)(+53.3) = +19.9 \text{ ft-kips}$$

The additions to the fixed-end couples at $B$ and $E$ because of the rotation of joint $A$ are one-half of the above quantities. These values are also placed under the original fixed-end couples but no horizontal lines are drawn as these joints have not yet been balanced. Joint $B$ is then rotated to a condition of equilibrium by distributing the unbalance moment with opposite sign (+39.3) to the three members $BA$, $BD$, and $BF$,
STRUCTURAL THEORY

in accordance with the distribution factors. This distribution is the only one that will satisfy the strain condition at the joint, that is, make all members intersecting at the joint rotate through the same angle. The corrections for rotation are

\[(0.417)(+39.3) = +16.4\]
\[(0.333)(+39.3) = +13.1\]
\[(0.250)(+39.3) = +9.8\]

Additional fixed-end couples of +8.2 and +6.6 are placed at A and C, respectively, to keep those joints from rotating but no moment is assigned to F as a coefficient of 3 was employed in computing the distribution factor for member CF. Joint C is then balanced in the same manner and the corrections of −27.8, −26.2, and −21.0 recorded.

### Table 5-2

<table>
<thead>
<tr>
<th>Member</th>
<th>C</th>
<th>K</th>
<th>CK</th>
<th>[\tau = \frac{CK}{\Sigma CK}]</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Joint A</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>AB</td>
<td>4</td>
<td>10</td>
<td>40</td>
<td>0.625</td>
</tr>
<tr>
<td>AE</td>
<td>4</td>
<td>6</td>
<td>24</td>
<td>0.375</td>
</tr>
<tr>
<td>[\Sigma CK = 64]</td>
<td></td>
<td></td>
<td></td>
<td>[1.000]</td>
</tr>
<tr>
<td><strong>Joint B</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>BA</td>
<td>4</td>
<td>10</td>
<td>40</td>
<td>0.417</td>
</tr>
<tr>
<td>BC</td>
<td>4</td>
<td>8</td>
<td>32</td>
<td>0.333</td>
</tr>
<tr>
<td>BF</td>
<td>3</td>
<td>8</td>
<td>24</td>
<td>0.250</td>
</tr>
<tr>
<td>[\Sigma CK = 96]</td>
<td></td>
<td></td>
<td></td>
<td>[1.000]</td>
</tr>
<tr>
<td><strong>Joint C</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CB</td>
<td>4</td>
<td>8</td>
<td>32</td>
<td>0.372</td>
</tr>
<tr>
<td>CD</td>
<td>3</td>
<td>10</td>
<td>30</td>
<td>0.349</td>
</tr>
<tr>
<td>CG</td>
<td>3</td>
<td>8</td>
<td>24</td>
<td>0.279</td>
</tr>
<tr>
<td>[\Sigma CK = 86]</td>
<td></td>
<td></td>
<td></td>
<td>[1.000]</td>
</tr>
</tbody>
</table>

The additional fixed-end couple of −13.9 is then placed at BC but no moments are required at the hinges D and G.

The above operations are repeated at joints A, B, and C until the corrections can be ignored. As a horizontal line is drawn whenever the joint is balanced only those values which appear under these lines are considered in the next correction. For this reason it is important that a horizontal line be drawn immediately after the joint is balanced.

The horizontal reactions at E, F, and G can now be calculated by dividing the algebraic sum of the end couples for each column by the height of the columns. The horizontal reaction required at D to keep the frame from moving horizontally is

\[H_D = 1.66 + 1.02 - 1.57 = 1.11\] kips

**ANALYSIS OF FRAMES WITH SIDESWAY**

If member CD of the frame in Fig. 5-19a is supported on a roller at D instead of a hinge then the end couples obtained from the solution in Fig. 5-19b are not correct, for without \(H_D\) the horizontal forces acting on the frame would not be in equilibrium. Consequently, if there is no horizontal reaction at D then joints A, B, C, and D must move horizontally until the sum of the horizontal components \(H_E, H_F,\) and \(H_G\) equals zero.

As the moment-distribution method corrects for joint rotation but not joint translation it is necessary to arrange the solution so that these two quantities, rotation and
translation, are treated independently. Therefore, the solution given in Fig. 5-19b
would be made first even if end D is on rollers by assuming that the structure is hinged
at D. The horizontal force of 1.11 kips acting at D is then removed by applying the
force system shown in Fig. 5-20a.

The end couples caused by the force system in Fig. 5-20a can be calculated by the
moment-distribution method by the procedure illustrated in Fig. 5-20b. In this solu-
tion a horizontal displacement $E\Delta$ of any amount is assumed, say 14 units, which

\[
\begin{array}{cccccc}
+22.5 & +11.3 & +2.1 & 0 \\
+2.6 & +5.3 & +8.2 & +7.6 \\
-1.6 & -0.8 & -0.6 & 0 \\
-0.7 & +4.1 & +0.2 & +0.2 \\
+0.4 & +14.4 & +9.9 & +7.8 \\
+23.2 & +7.2 & 0 & 0 \\
0.625 & 0.417 & 0.333 & 0.372 & 0.349 & F' = 6.58k
\end{array}
\]

\[
\begin{array}{cccccc}
0.375 & \frac{22.5}{-1.0} & \frac{11.3}{-0.5} & \frac{+2.1}{-0.5} & 0 \\
-36.0 & -13.5 & 22.3 & 24.0 & 24.0 & -216 \\
-18.0 & -6.8 & -8.9 & -8.9 & -17.7 \\
-36.0 & -13.5 & 15.5 & 15.5 & 15.5 & 15.5
\end{array}
\]

\[
\begin{array}{cccccc}
3.77k & 1.55k & 1.26k & \\
\end{array}
\]

\[
\begin{array}{cccccc}
+32.0 & -47.4 & +122.3 & -136.6 & +97.2 & -75.3 \\
+3.9 & +2.4 & +1.3 & +1.3 & +1.3 & +1.3 \\
-43.5 & +124.7 & -135.3 & +98.9 & -74.0 & 0 \\
\end{array}
\]

\[
\begin{array}{cccccc}
4^k/ft & 12^k & 2^k/ft & 16^k & \\
\end{array}
\]

\[
\begin{array}{cccccc}
+154.9 & -11.5 & +154.9 & -11.5 & +154.9 & -11.5 \\
+7.8 & -5.0 & +7.8 & -5.0 & +7.8 & -5.0 \\
-5.0 & +2.0 & -5.0 & +2.0 & -5.0 & +2.0 \\
\end{array}
\]

Fig. 5-20. Moments due to a single linear displacement.
would cause fixed-end couples in the columns (see Figs. 5-12i and 5-12j) of

\[
M_{FAE} = M_{FAE} = -\frac{6KE\Delta}{L} = -\frac{(6)(6)(14)}{14} = -36
\]

\[
M_{FBF} = M_{FCG} = -\frac{3KE\Delta}{L} = -\frac{(3)(8)(14)}{14} = -24
\]

The corrections for rotation of the joints are now made with the same distribution factors and in the same manner as were the corrections in Fig. 5-19b. The numerical operations are shown in Fig. 5-20b in which joints A, B, and C are balanced in the same manner as in Fig. 5-19b.

After the corrections for joint rotation in Fig. 5-20b are completed and the value of the end couples determined the horizontal reactions at E, F, and G are then calculated. These forces are

\[
H_E = \frac{23.2 + 29.6}{14} = 3.77 \leftarrow
\]

\[
H_F = \frac{21.6}{14} = 1.55 \leftarrow
\]

\[
H_G = \frac{17.7}{14} = 1.26 \leftarrow
\]

\[
F' = 3.77 + 1.55 + 1.26 = 6.58 \rightarrow
\]

However, a force \( F \) of 1.11 kips instead of 6.58 is required and therefore it is necessary to multiply all values in Fig. 5-20b by the ratio \( 1.11/6.58 \), or 0.169. When the end couples in Fig. 5-20b are multiplied by 0.169 and these quantities are then added to the corresponding end couples in Fig. 5-19b the restraining force at \( D \) is removed and therefore the combined effect is equivalent to a roller at \( D \). This combined effect is shown in Fig. 5-20c, which gives the algebraic sum of the values in Fig. 5-19b and 0.169 times the values in Fig. 5-20b.

![Fig. 5-21. A frame with two linear displacements.](image)

The above solution is applicable whenever the frame has only one unknown linear displacement but a more comprehensive treatment is required whenever two or more displacements are concerned. For example, the frame in Fig. 5-21a has two independent displacements which must be considered separately by means of the two force systems shown in Figs. 5-21b and 5-21c. By means of the procedure used in Fig. 5-20b, each force system can be expressed in terms of the assigned horizontal displacement. The two displacements \( \Delta_1 \) and \( \Delta_2 \) can then be determined from the conditions

\[
F_1' - F_1'' = a_1\Delta_1 - b_1\Delta_2 = F_1
\]

\[
-F_2' + F_2'' = -a_2\Delta_1 + b_2\Delta_2 = F_2
\]

A more detailed description of this method of analysis together with numerical examples can be found in many textbooks.
BUILDING FRAMES SUBJECTED TO VERTICAL LOADS

For many years specifications and building codes on the design of reinforced-concrete buildings have recognized the monolithic action of such structures and have required that the component parts be designed on the basis of continuity. Some building codes permit continuous beams of fairly equal spans to be designed for maximum positive and negative moments that are determined from specified coefficients. In the design of building frames, however, it is usually necessary to determine the bending moments more exactly by an analysis based on the elastic theory. The application of the moment-distribution method to the analysis of building frames that are subjected to vertical loads will be briefly discussed and illustrated.

As the moment-distribution method requires that the \( I/L \) or \( K \) value of each member be known the cross section of every member must first be estimated before the analysis can be started. This preliminary design can be made with considerable accuracy in the following manner after the thickness of the reinforced-concrete floor has been selected.

1. Calculate the full dead and live loads that are carried to each beam from the various floor panels.

2. Determine the end shears for each beam for full dead and live load on a simply supported span and increase the maximum value about 20 per cent to allow for continuity.

3. Proportion the cross sections of the controlling beams on each floor to resist this estimated shear. For long spans and light loads deflection and flexural stresses may require a greater depth than is required for shear.

4. Estimate the axial load on each column from the end shears calculated for the simply supported beams. Proportion the cross section of each column for this axial load by using conservative values for the allowable unit stresses.

5. Calculate the moment of inertia of the full concrete section of all beams and

---

Fig. 5-22. Coefficients for moment of inertia of T beams. (Courtesy of the Portland Cement Association.)

Fig. 5-23. A building frame loaded on one floor and in only one bay.
Fig. 5-24. Moment distribution and final-moment diagrams resulting from modified stiffness factors.
columns and determine the $K$ value of each member. The diagram in Fig. 5-22 gives coefficients that simplify the calculation of the moment of inertia of T beams.

After the $K$ values are determined the next step in the analysis is the calculation of the end couples due to the loads on each separate span. These calculations are not laborious as only a few members need be considered in determining the effect of the weight on any particular span. For instance if span $ab$ in Fig. 5-23 is subjected to a uniform load of 1 kip per ft only the members intersecting at joints $a$ and $b$ need be considered in the analysis. When span $bc$ is loaded only the members at joints $b$ and $c$ are used in the analysis. In this way the entire frame is analyzed by considering the effect of the loads on each beam as a separate problem and these results are then combined to give the maximum positive and negative moments in both beams and columns.

This method of analysis will be illustrated by the calculation of the end couples due, first, to a uniform load of 1 kip per ft on span $ab$ of Fig. 5-23 and, second, to a similar load on span $bc$. These calculations are shown in Fig. 5-24a and $b$. The only difference between these computations and those in Fig. 5-19b is in the determination of the distribution factors and carry-over factors on the basis of partially restrained ends at $e, f, j, k,$ and $c$ instead of fixed ends. Although specifications will usually permit an assumption of a fixed-end condition the use of elastically restrained ends is more accurate and involves only a little more calculation. This more accurate distribution factor is easily determined if, in the quantity $CK$, a value of $C$ equal to $4 - r$ is used in which the value of $r$ is approximately

$$ r = \frac{K}{\Sigma K} \quad (5-12) $$

for the elastically restrained end. Thus for member $ae$ the value of $r$ to be used for determining the distribution factors at joint $a$ is the ratio

$$ r_e = \frac{2}{2 + 4} = 0.33 $$

and the coefficient $C$ for the member $ae$ at joint $a$ is

$$ C_{ae} = 4 - r_e = 3.67 $$

The carry-over factor from $a$ to $e$ is

$$ \text{C.O.} = \frac{2(1 - r_e)}{4 - r_e} \quad (5-13) $$

instead of one-half, which is the value only for a fully restrained end for which $r$ equals zero. The carry-over factor from $a$ to $e$ is therefore

$$ \frac{2(1 - 0.33)}{4 - 0.33} = 0.363 $$

From the diagram in Fig. 5-25 the carry-over factor for any elastically restrained end can be obtained if the ratio $r$ at that end is known. For example, at end $f$ of member $bf$ the $r$ value is $\frac{2}{13}$ and from Fig. 5-25 the carry-over factor from $b$ to $f$ is seen to be 0.408. At $c$ the $r$ value for member $bc$ is $\frac{19}{25}$ and the carry-over from $b$ to $c$ is 0.333.

After the $CK$ values for all members at joints $a$ and $b$ are computed the distribution factors are determined in the usual manner from the ratio $CK/\Sigma CK$ as shown in Table 5-3.

After the distribution factors have been computed the numerical operations are carried out in the usual manner. The first corrections in Fig. 5-24a were made at joint $a$ and one-half of the correction 9.20 for member $ab$, or 4.6, was carried over to joint $b$ to keep that joint from rotating. Joint $b$ was then rotated, and one-half of $-5.80$, or $-2.90$, was carried back to joint $a$. This procedure was continued until the corrections were negligible. The end couples at all elastically restrained ends were then determined from the carry-over factors recorded in Table 5-3.
A similar solution for a uniform load of 1 kip per ft on span bc is shown in Fig. 5-24b. The coefficients C and carry-over factors for all members with elastically restrained ends are calculated as for the previous problem in Fig. 5-24a. After the details of the solution become familiar the calculations for the effect of the weight on each span proceed rapidly. Any special type of loading may be considered separately.

Table 5-3

<table>
<thead>
<tr>
<th>Member</th>
<th>C</th>
<th>K</th>
<th>CK</th>
<th>$\frac{CK}{\Sigma CK}$</th>
<th>Carry-over</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Joint a</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ae</td>
<td>(4 - 0.333)</td>
<td>2</td>
<td>7.33</td>
<td>0.172</td>
<td>0.363</td>
</tr>
<tr>
<td>ab</td>
<td>4</td>
<td>6</td>
<td>24.00</td>
<td>0.563</td>
<td>0.500</td>
</tr>
<tr>
<td>af</td>
<td>(4 - 0.23)</td>
<td>3</td>
<td>11.31</td>
<td>0.265</td>
<td>0.408</td>
</tr>
<tr>
<td></td>
<td>$\Sigma CK = 42.64$</td>
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<td>1.000</td>
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</tr>
<tr>
<td>Joint b</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ba</td>
<td>4</td>
<td>6</td>
<td>24.00</td>
<td>0.277</td>
<td>0.500</td>
</tr>
<tr>
<td>bf</td>
<td>(4 - 0.23)</td>
<td>3</td>
<td>11.31</td>
<td>0.131</td>
<td>0.408</td>
</tr>
<tr>
<td>bc</td>
<td>(4 - 0.40)</td>
<td>10</td>
<td>36.00</td>
<td>0.415</td>
<td>0.333</td>
</tr>
<tr>
<td>bk</td>
<td>(4 - 0.16)</td>
<td>4</td>
<td>15.36</td>
<td>0.177</td>
<td>0.437</td>
</tr>
<tr>
<td>$\Sigma CK = 86.67$</td>
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<td>1.000</td>
<td></td>
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</tbody>
</table>

MAXIMUM MOMENTS AND SHEARS IN BUILDING FRAMES

After the end couples for a unit load on each span are known the maximum positive and negative moments at any section may be determined by combining the results obtained for the separate spans. The moments and some shears are shown in Fig. 5-24c for three separate loadings of 1 kip per ft on the various spans. That is, since the bending moments and shears are known for a unit load on each span, the com-
bined effect of similarly distributed loads of any magnitude acting simultaneously on the various spans can be obtained by the principle of superposition.

To illustrate this statement, the maximum moment at the center of span $bc$ will be calculated for a dead load of 2 kips per ft and a live load of 4 kips per ft on any span. The moment at the center of span $bc$ for a uniform load of 1 kip per ft on spans $ab$, $bc$, and $cd$ are $-3.0$, $+26.12$, and $-3.83$ ft-kips, respectively. Therefore, to obtain the maximum positive moment at this section the two negative moments are multiplied by 2 (the dead load) and the positive moment by 6 (dead and live load). The maximum positive moment will therefore be

$$-2(3.0 + 3.83) + (6)(26.12) = 143.06 \text{ ft-kips}$$

The minimum moment will be

$$-6(3.0 + 3.83) + (2)(26.12) = 11.26 \text{ ft-kips}$$

At the left end of span $bc$ the moments are $-9.02$, $-23.06$, and $+3.52$ ft-kips for a unit load on spans $ab$, $bc$, and $cd$, respectively. Therefore, the maximum negative moment is

$$-6(9.02 + 23.06) + (2)(3.52) = -185.44 \text{ ft-kips}$$

After the maximum positive and negative moments have been calculated for several sections in each span, maximum and minimum moment curves as illustrated in Fig. 5-26 should be drawn.

Maximum end shears may also be calculated by combining the effects from the various spans. Thus the values of the shear $V_{bs}$ at the right end of span $ab$ are $-7.52$, $-0.91$, and $+0.15$ kips for a unit load on spans $ab$, $bc$, and $cd$, respectively. The maximum negative shear $V_{bs}$ is therefore

$$-6(7.52 + 0.91) + (2)(0.15) = -50.3 \text{ kips}$$

The maximum shear at the center of the span is frequently assumed as one-fourth of the maximum positive and negative end shears. A linear variation of maximum shear as shown in Fig. 5-27 for span $ab$ may safely be used for uniform loads.
TWO-HINGED GABLE FRAMES

An analysis of a two-hinged single-span frame (Fig. 5-28a) requires the determination of the redundant horizontal reaction acting at the base of either column. When a horizontal tie is employed to prevent lateral movement of the supports (Fig. 5-28b) the tensile force \( H \) in the tie rod is ordinarily selected as the redundant. The only difference in the structural action of the two types of frames is in the elongation of the tie rod.

![Figure 5-28. A two-hinged gable frame.](image)

The value of the redundant force \( H_b \) (Fig. 5-28a) for any applied loads can be readily determined for any two-hinged frame whose supports are at the same elevation by the equation

\[
H_b = \frac{\Sigma M_y(\Delta s/I)}{\Sigma y^2(\Delta s/I)}
\]  
(5-14)

and for the tensile force \( H \) in the tie rod of a reinforced-concrete frame (Fig. 5-28b) by the equation

\[
H = \frac{\Sigma M_y(\Delta s/I)}{\Sigma y^2(\Delta s/I) + (L/nA)}
\]  
(5-15)

<table>
<thead>
<tr>
<th>Element</th>
<th>( \Delta s )</th>
<th>( y )</th>
<th>( \frac{\Delta s}{I} )</th>
<th>( M_x )</th>
<th>( M_{xy} ), ( \frac{\Delta s}{I} )</th>
<th>( y^2 ), ( \frac{\Delta s}{I} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.0</td>
<td>2.0</td>
<td>1.0</td>
<td>0</td>
<td>0</td>
<td>4.0</td>
</tr>
<tr>
<td>2</td>
<td>4.0</td>
<td>6.0</td>
<td>1.0</td>
<td>0</td>
<td>0</td>
<td>36.0</td>
</tr>
<tr>
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\[
\begin{array}{cc}
150,897.6 & 16,100.6
\end{array}
\]
In the above equations,

\[ M_s \] = bending moment acting on any element \( \Delta s \) due to the applied forces and reactions when the redundant force \( H_b \) (or \( H \)) is removed (see Fig. 5-28c)

\[ y \] = vertical distance between any element and the redundant force \( H_b \) (or \( H \))

\[ \Delta s \] = length of any element

\[ I \] = moment of inertia of any cross section

\[ A \] = area of tie rod

\[ n = \frac{E_s}{E_t} \]

The dimensional units selected for \( M_s \), \( y \), \( \Delta s \), \( I \), \( L \), and \( A \) should be consistent.

To simplify the arrangement of the calculations it is recommended that a tabular form, such as in Table 5-4, be used. In preparing such a table the first step is to divide the frame into any convenient number of elements whose designations and lengths are recorded. The center of each element may be taken as the working point and the vertical distance \( y \) above the redundant force \( H \) is then placed in the proper

---

**Fig. 5-29.** A gable frame—design example.
For most structures the moment of inertia $I$ of the cross section at the working point can be taken as the effective moment of inertia for the element with sufficient accuracy. The value of $M_i$, the bending moment at the center of each element, is next calculated for the statically determinate system composed of the applied forces and corresponding reactions with the redundant force $H$ removed.

The above procedure is illustrated in Table 5-4 by the calculation of $H_5$ for the frame shown in Fig. 5-29a. The frame was divided into 20 elements as shown and the bending moments $M_i$ were determined from the force system shown in Fig. 5-29b. The various values are recorded in Table 5-4 and the necessary summations made. These numerical operations are easily performed, particularly with a calculating machine. The value of $H_5$ is given by Eq. (5-14) as

$$H_5 = \frac{150,897.6}{16,100.6} = 9.35 \text{ kips}$$

**VIADUCT FRAMES AND VIERENDEEL TRUSSES**

When a rigid frame is composed of quadrangular panels with chord members of equal stiffness, as in Fig. 5-30a, a method of analysis by successive approximations in which each panel is taken as a primary structural unit provides a direct and convenient solution. If any panel $abcd$ (Fig. 5-30a), of a continuous frame in which the chord members $ad$ and $bc$ have the same $K$ or $I/L$ value is separated from the structure by passing sections 1-1 and 2-2, the force system acting on the panel is as shown in Fig. 5-30b. In representing these forces and in the subsequent analysis the following notations are used:

- $M$ = bending moment on section 1-1 = $P_1y_1 + P_2y_2$
- $V$ = shear in panel $abcd = P_1 + P_2 + P_4$
- $l$ = panel length
- $K = \frac{I}{L}$ of chord members $ad$ and $bc$
- $K_1, K_2 = \frac{I}{L}$ values of members $ab$ and $cd$, respectively

$$r = \frac{K}{K_1}, \quad s = \frac{K}{K_2}, \quad \alpha = \frac{h_2 - h_1}{h_1}$$

To determine the bending moments in the panel $abcd$ (Fig. 5-30b), the actual force system can be resolved into three equivalent force systems. These three force sys-
tems are shown in Fig. 5-31a, b, and c, and by inspection it can be seen that they add up to the original system.

The force system shown in Fig. 5-31a represents the action of the external forces and consequently the moments produced by these forces are necessary for structural stability. Therefore, as this force system represents the principal structural action, the resulting moments acting on the members of the panel will be called the primary moments. The values of these primary moments are given by the following equations:

\[ M'_{ad} = M'_{be} = \frac{\alpha M - Vl}{2D} [3 + s + \alpha(2 + s)] \]
\[ M'_{da} = M'_{eb} = \frac{\alpha M - Vl}{2D} (3 + r + \alpha) \]  

(5-16)

in which

\[ D = 6 + r + s + \alpha(2\alpha + \alpha s + 2s + 6) \]

By means of Eq. (5-16) the primary moments \( M' \) are easily computed for each panel and recorded on a sketch of the frame. The external moments \( M \) and \( Vl \) are considered positive when they act clockwise on the panel. The primary moments \( M' \) that act on the members \( ad \) and \( be \) are also clockwise if positive. The value of \( \alpha \) can be either positive or negative.

\[ M''_a = M''_b = + \frac{r}{D} M_1 \]  
\[ M''_d = M''_e = - \frac{r(1 + \alpha)}{D} M_1 \]  
\[ M'''_a = M'''_b = - \frac{s(1 + \alpha)}{D} M_2 \]  
\[ M'''_d = M'''_e = + \frac{s(1 + \alpha)^2}{D} M_2 \]  

(5-17a, b, c, d)

Evidently the constants \( r/D, r(1 + \alpha)/D, s(1 + \alpha)/D, \) and \( s(1 + \alpha)^2/D \) in Eqs. (5-17a) to (5-17d) are correction factors that can be computed for each panel and recorded on a sketch of the structure. The primary moments \( M' \) in the adjacent panels can be used for the first approximation of \( M_1 \) and \( M_2 \), and the corrections can then be computed as for any method of successive approximations. A convenient numerical arrangement for making the calculations is as follows:

1. Compute and tabulate \( r, s, \alpha, D, r/D, r(1 + \alpha)/D, s(1 + \alpha)/D, \) and \( s(1 + \alpha)^2/D \), for each panel.
2. Compute the primary moments by Eq. (5-16), and record on a sketch of the frame.

3. Make the necessary corrections by Eqs. (5-17a) to (5-17d) using the recorded correction factors.

4. Determine the sign of any secondary moment directly from the sign of the moment producing it by the following rule:

Adjacent secondary moments have a sign opposite to that of the applied moment, while secondary moments at the far end of the panel have the same sign as the applied moment.

When the members of a frame are fixed at the base (Fig. 5-31d) a substitute member $cd$ whose stiffness factor $K_2$ is infinite ($\infty$) should be inserted to form the quadrangular panel $abcd$. The calculations may then proceed as for any panel by using a value of $s = 0$ in the equations for primary and secondary moments. However, when the frame is hinged at the base then the substitute member $cd$ must have a $K_2$ value equal to zero which gives an $s$ value of infinity ($\infty$) for that panel. For a value of $s_2 = \infty$ it can be shown that Eq. (5-16) for the primary moments reduces to the values shown in Eq. (5-16a):

$$M'_{ad} = M'_{bc} = \alpha M - \frac{Vl}{2} \frac{1}{1 + \alpha} \quad (5-16a)$$

$$M'_{da} = M'_{eb} = 0$$

and that the secondary moments in Eqs. (5-17a) and (5-17b) reduce to zero as the denominator $D$ becomes infinite. Therefore

$$M''_{ad} = M''_{bc} = M''_{da} = M''_{eb} = 0$$

This means that the primary moments given by Eq. (5-16a) are also the final moments as the corrections are zero.

The above procedure will be illustrated by an analysis of the rigid frame shown in Fig. 5-32. As the chord members have the same $K$ values, Eqs. (5-16) and (5-17) will apply. In any panel one or both columns (chord members) may be inclined as the only restriction is in regard to equal $K$ values for the chord members. The first operation is to compute the various coefficients for each panel, thus,

For panel $cdd'c'$

$$r = \frac{390}{90} = 0.5 \quad s = \frac{390}{90} = 1.0 \quad \alpha = 0$$

$$D = 6 + 0.5 + 1.0 = 7.5$$

$$\frac{s}{D} = \frac{1.0}{7.5} = 0.133$$

For panel $bcc'b'$

$$r = \frac{20}{30} = 0.667 \quad s = \frac{20}{20} = 1.0 \quad \alpha = \frac{18 - 12}{12} = 0.5$$

$$D = 6 + 0.667 + 1.0 + 0.5(1.0 + 0.5 + 2.0 + 6.0) = 12.42$$

$$\frac{r}{D} = 0.054 \quad \frac{r(1 + \alpha)}{D} = 0.08$$

$$\frac{s(1 + \alpha)}{D} = 0.121 \quad \frac{s(1 + \alpha)^2}{D} = 0.181$$

For panel $abb'a'$ (assume a member $aa'$ with $K = 0$)

$$r = 1.0 \quad s = \infty \quad \alpha = \frac{1}{2}$$

$$\frac{r}{D} = 0 \quad \frac{r(1 + \alpha)}{D} = 0$$
The primary moments in the various panels as determined from Eq. (5-16) are For panel cdd’c’ (moments in foot-kips)

\[ M'_{cd} = M'_{d'c'} = \frac{-10(8)}{2(7.5)} (3 + 1.0) = -21.3 \]

\[ M'_{dc} = M'_{c'd'} = \frac{-10(8)}{2(7.5)} (3 + 0.5) = -18.7 \]

For panel bcc’b’

\[ M'_{cb} = M'_{c'b'} = \frac{(0.5)(80)}{2(12.42)} - \frac{-10(12)}{2(12.42)} [3 + 1.0 + (0.5)(2 + 1.0)] = -17.7 \]

\[ M'_{bc} = M'_{b'c'} = \frac{(0.5)(80)}{2(12.42)} - \frac{-10(12)}{2(12.42)} (3 + 0.67 + 0.5) = -13.4 \]

For panel abb’a’ (s = \infty)

\[ M'_{ba} = M'_{a'b'} = \frac{\alpha M - Vl}{2} \frac{1}{1 + \alpha} = \frac{(\frac{1}{2})(200) - (10)(12)}{2(1.33)} = -20.0 \]

\[ M'_{ab} = M'_{a'b'} = 0 \]

After the primary moments and correction factors \( r/D, r(1 + \alpha)/D, s(1 + \alpha)/D \), and \( s(1 + \alpha)^2/D \) have been recorded on the diagram of the frame in Fig. 5-32 the effect of continuity between the panels is then determined by successive approximations. For example, the secondary moments in panel bcc’b’ due to the adjacent moment of -20.0 ft-kips in panel abb’a’ are

\[ M''_{bc} = (-0.181)(-20.0) = +3.6 \]

\[ M''_{ob} = (0.121)(-20.0) = -2.4 \]

The sign of the corrections is determined from the rule already stated; adjacent moments are of opposite sign to the applied moment while moments at the opposite
end of the panel have the same sign as the applied moment. Thus the applied moment of -20.0 causes corrections of +3.6 at the adjacent and -2.4 at the opposite end. In the same manner the corrections for panel add’c’ are equal to

\[
\begin{align*}
(-17.7 - 2.4)(-0.133) &= +2.7 \\
(-17.7 - 2.4)(0.133) &= -2.7
\end{align*}
\]

The corrections are made as shown by the arrows until each panel has been corrected for all adjacent moments. The convergence is rapid as the corrections are usually small. The final values are the algebraic sum of the primary moments plus all the corrections.

![Fig. 5-33. Parallel-chord Vierendeel truss.](image)

In Fig. 5-33 the calculations for the end moments in a parallel-chord Vierendeel truss are shown. These calculations are made in the same manner as for the frame in Fig. 5-32. However, in a parallel-chord frame all \( \alpha \) values are zero, which greatly simplifies the numerical work.

**MEMBERS WITH VARIABLE MOMENTS OF INERTIA**

The moment-distribution method can be applied directly to continuous beams and frames which are composed of members whose cross sections vary along the span. The numerical procedures described previously for members with constant moment of inertia are applicable if the fixed-end moments, stiffness coefficients, and carry-over factors are calculated for the relative variation in the moment of inertia.

In performing the numerical work it is usually convenient to express the moment of inertia \( I \) of any section in terms of \( I_0 \), the moment of inertia of the smallest section, that is, by the relation

\[
I = aI_0
\]

where \( a \) is always greater than unity.
The solution of the basic problem (Fig. 5-34) that provides the stiffness coefficient and the carry-over factor when one end of the member is rotated and the other end is held fixed can be solved directly by the area-moment theorems. For instance, if point A in Fig. 5-34a does not move from a tangent at B then the statical moment of

![Area moment diagrams used to determine stiffness and carry-over factors for nonuniform depth beam.](image)

M/EI diagram about A must be zero which, if the M/EI diagram is constructed in two parts as shown gives the equation

$$\Delta_A = \frac{1}{EI_0} \left[ \frac{M'_{AB}}{L} \sum_{0}^{L} (L - x)(x) \left( \frac{\Delta x}{a} \right) - \frac{M'_{BA}}{L} \sum_{0}^{L} (x)(x) \left( \frac{\Delta x}{a} \right) \right] = 0$$

from which

$$\text{C.O.}_{AB} = \frac{M'_{BA}}{M'_{AB}} = \frac{\sum_{0}^{L} (L - x)(x)(\Delta x/a)}{\sum_{0}^{L} x^2(\Delta x/a)} \quad (5-18a)$$

If end A is fixed and end B is rotated, then the quantities $x$ and $L - x$ should be interchanged, or

$$\text{C.O.}_{BA} = \frac{M'_{AB}}{M'_{BA}} = \frac{\sum_{0}^{L} (L - x)(x)(\Delta x/a)}{\sum_{0}^{L} (L - x)^2(\Delta x/a)} \quad (5-18b)$$

The rotation $\theta_A$ is equal to the total area of the M/EI diagram in Fig. 5-34 or

$$\theta_A = \frac{1}{EI_0} \left[ \frac{M'_{AB}}{L} \sum_{0}^{L} \frac{(L - x)(\Delta x/a)}{L} - \frac{M'_{BA}}{L} \sum_{0}^{L} \frac{(x)(\Delta x/a)}{L} \right]$$
After the carry-over factor $C.O._{AB}$ is computed from Eq. (5-18a) the expression for $\theta_A$ can be written in the form

$$\theta_A = \frac{M'_{AB}L}{EI_0} \left[ \sum_{0}^{L} \frac{(L-x)(\Delta x/a)}{L^2} - C.O._{AB} \sum_{0}^{L} \frac{(x)(\Delta x/a)}{L^2} \right]$$  \hspace{1cm} (5-19)

From Eq. (5-19), the relation between $M'_{AB}$ and $\theta_A$ is

$$M'_{AB} = C_A \frac{EI_0}{L} \theta_A = C_A K_0 \theta_A$$

where

$$C_A = \frac{L^3}{\sum_{0}^{L} (L-x)(\Delta x/a) - C.O._{AB} \sum_{0}^{L} (x)(\Delta x/a)}$$  \hspace{1cm} (5-20a)

If $x$ is measured from end $B$ instead of $A$ the coefficient $C_B$ in Eq. (5-20b) and the carry-over factor from $B$ to $A$ [Eq. (5-18b)] are obtained or

$$C_B = \frac{L^3}{\sum_{0}^{L} (x)(\Delta x/a) - C.O._{BA} \sum_{0}^{L} (L-x)(\Delta x/a)}$$  \hspace{1cm} (5-20b)

The use of the above equations will be illustrated by the determination of the carry-over and stiffness factors for the member $AB$ shown in Fig. 5-35a. The beam is divided into 10 parts of 1 ft each and the various factors in the equation are tabulated in Table 5-5.

From $A$ to $B$

$$C.O._{AB} = \frac{147.53}{185.29} = 0.795$$

From $B$ to $A$

$$C.O._{BA} = \frac{147.53}{328.38} = 0.449$$

Fig. 5-35. $M/I$ diagram for beam with variable $I$. 

I0 = -6.096P

I0 = -0.0422P

I0 = -0.108P

I0 = -0.08P

5.434P

I0 = -2.5P

I0 = -5P

I0 = 0.31
### Table 5-5

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<th>$a = I/\ell$</th>
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<td>185.29</td>
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<td>328.38</td>
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Stiffness coefficient at $A$

$$C_A = \frac{(10)^2}{47.59 - (0.795)(33.29)} = \frac{100}{47.59 - 26.47} = \frac{100}{21.12} = 4.72$$

$$C_B = \frac{100}{33.29 - (0.449)(47.59)} = \frac{100}{11.92} = 8.38$$

After the coefficients are known the fixed-end moments for any transverse load are calculated by the usual procedure of first determining the rotations $\alpha_A$ and $\alpha_B$ for the simply supported span and then by computing the value of the end couples that will produce equal and opposite rotation of the end sections. Thus, for a concentrated load $P$ at the center of the span $AB$ in Fig. 5-35b the statical moments of the $M/I$ diagram about point $B$ divided by the span $L$ gives

$$\alpha_A = \frac{0.06096PL^3}{EI_0}$$

while moments about point $A$ divided by the span gives

$$\alpha_B = -\frac{0.05434PL^3}{EI_0}$$

The fixed-end moments are therefore

$$M_{FAB} = (4.72) \left( \frac{EI_0}{L} \right) \left( -\frac{0.06096PL^3}{EI_0} \right) + (0.449)(8.38) \left( \frac{EI_0}{L} \right) \left( \frac{0.05434PL^3}{EI_0} \right)$$

$$M_{FBA} = (0.795)(4.72) \left( \frac{EI_0}{L} \right) \left( -\frac{0.06096PL^3}{EI_0} \right) + (8.38) \left( \frac{EI_0}{L} \right) \left( \frac{0.05434PL^3}{EI_0} \right)$$

$$= -0.083PL$$

$$= +0.226PL$$

### ANALYSIS OF FIXED-END ARCHES

Most arch ribs can be treated as curved beams as the deflections are due largely to the strain caused by bending moments. The shortening of the arch rib from axial compressive stress is sometimes considered but ordinarily only the rotations and displacements due to flexural stresses are important in the determination of redundant forces. The angular rotation $d\phi$ as shown in Fig. 5-36a may be obtained with sufficient accuracy by the ordinary straight-beam formula if the radius of the arch axis is more than six times the depth of the rib. The relative rotation between the end sections of any element is therefore

$$d\phi = \frac{M \, ds}{EI}$$

(5-21)
By referring to Fig. 5-36b it can be seen that the horizontal and vertical components of the displacement of any point $m$ are equal to

$$d\Delta_x = (r \, d\phi) \cos \theta = y \, d\phi$$

$$d\Delta_y = (r \, d\phi) \sin \theta = x \, d\phi$$

(5-22)

The total horizontal and vertical movements of any point $m$ due to the rotations of all elements between point $m$ and the fixed-end $b$ are (using finite elements $\Delta s$)

$$\Delta_x = \sum_{m}^{b} d\Delta_x = \sum_{m}^{b} y \, d\phi = \sum_{m}^{b} y \frac{M \Delta s}{EI}$$

$$\Delta_y = \sum_{m}^{b} d\Delta_y = \sum_{m}^{b} x \, d\phi = \sum_{m}^{b} x \frac{M \Delta s}{EI}$$

(5-23)

The total rotation $\theta_m$ is equal to the sum of all rotations from the fixed point $b$ to $m$ or

$$\theta_m = \sum_{m}^{b} d\phi = \sum_{m}^{b} \frac{M \Delta s}{EI}$$

(5-24)

Equations (5-23) and (5-24) may be used to determine either the displacements in fixed-end arches or the redundant forces that will satisfy any particular strain condition. Ordinarily the reactions acting at the ends of a fixed-end arch due to the application of known forces $P$ (Fig. 5-37) must be determined. As this problem involves three redundant forces $H_a, V_a, M_a$ it is necessary to write three strain equations involving these quantities. If the support $a$ does not move then Eqs. (5-23) and (5-24) can be
written in the form

\[ E \Delta_{xz} = \sum_{a}^{b} y \frac{M \Delta s}{I} = 0 \]

\[ E \Delta_{xy} = \sum_{a}^{b} z \frac{M \Delta s}{I} = 0 \]  \( (5-25) \)

\[ E \theta_{a} = \sum_{a}^{b} \frac{M \Delta s}{I} = 0 \]

Since \( M \) represents the total bending moment acting upon any element, it can be expressed in terms of the redundant forces and the applied loads. In the following equations any clockwise moment due to forces on the left of an element will be considered positive; therefore,

\[ M = M_{a} + V_{ax} - H_{ay} - M_{*} \]  \( (5-26) \)

where \( M_{*} \) is the total moment on any element due to all applied loads \( P \) acting on the left side and will be negative if the applied loads are downward.

\[ E \theta_{a} = \sum_{a}^{b} \frac{M \Delta s}{I} = 0 \]

**Fig. 5-37.** Coordinate notation for arch, shown with redundant and external forces.

When the value of \( M \) in Eq. (5-26) is substituted into Eq. (5-25) the following expressions for the movement of point \( a \) are obtained:

Let \( G = \frac{\Delta s}{I} \)

\( \Delta_{xy} \) = positive upward
\( \Delta_{xz} \) = positive to the right
\( \theta_{a} \) = positive when clockwise

Therefore, a positive moment \( M \) applied to any element will give a positive \( \Delta_{xy} \), a negative \( \Delta_{xz} \), and a positive \( \theta_{a} \). Consequently the expressions for these quantities take the form

\[ E \theta_{a} = \Sigma MG = M_{a} \Sigma G + V_{a} \Sigma Gx - H_{a} \Sigma Gy - \Sigma M_{l} G \]

\[ E \Delta_{xz} = -\Sigma MGy = -M_{a} \Sigma Gy - V_{a} \Sigma Gxy + H_{a} \Sigma Gy^{2} + \Sigma M_{l} Gx \]

\[ E \Delta_{xy} = \Sigma MGx = M_{a} \Sigma Gx + V_{a} \Sigma Gx^{3} - H_{a} \Sigma Gxy - \Sigma M_{l} Gx \]  \( (5-27) \)

In solving the above equations for \( M_{a}, H_{a}, \) and \( V_{a} \) it is most convenient to arrange the numerical work in tabular form. If the length of each element \( \Delta s \) can be selected so as to make

\[ G = \frac{\Delta s}{I} = \text{a constant} \]
the numerical work can be simplified. However, any value of $\Delta s$ can be taken provided at least 10 elements are used.

After the coordinates $(x, y)$ are tabulated for each element the various summations can be quickly made if calculating machines are available; otherwise the calculations will be extremely laborious. It should be noted that the only summations that change when the applied loads are altered are those involving $M_\alpha$, the bending moment due to the applied loads.

The above method will be illustrated by the calculation of the reactions of the elliptical arch shown in Fig. 5-38 for a concentrated load of 10 kips applied at the center of the span. The cross section of the arch is constant and, as the arch axis has been divided into 11 equal spaces, all $G$ values can be taken equal to unity. The coordinates of each point and the values of the various summations are shown in Tables 5-6 and 5-7. When these values are substituted in Eq. (5-27) the following expressions are obtained from which $M_\alpha$, $V_\alpha$, and $H_\alpha$ can be calculated:

$$
E_{\Delta V} = 11M_A + 330.0V_A - 117.0H_A = 934.0 = 0
$$
$$
E_{\Delta x} = 330.0M_A + 14,054.8V_A - 3,510.0H_A = 4,879.4 = 0
$$
$$
E_{\Delta x} = -117.0M_A - 3,510.0V_A + 1,435.2H_A = 7,945.8 = 0
$$

Solving these equations for the reactions at $A$ gives

$H_A = 10.37$ kips $V_A = 5.0$ kips $M_A = 45.2$ ft-kips

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<th>Element No.</th>
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<th>$y$</th>
<th>$G_\alpha$</th>
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Table 5-7

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<td>937.6</td>
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|                  | 934.0| 48794| 7945.8|

DETERMINATION OF REDUNDANT FORCES BY THE COLUMN-ANALOGY METHOD

The bending moment at any section of a rigid frame or an arch can always be resolved into components that separate the bending moments due to any stabilizing or primary force system from those resulting from the application of the selected redundant forces. Thus in the arch of Fig. 5-39 the bending moment at any section m–m may be expressed in the form

\[ M = M_r + M_i \]  

(5-28a)

in which \( M_r \) represents the bending moment due to any primary (stabilizing) force system and \( M_i \) the bending moments due to any redundant forces that are consistent with the physical characteristics of the structure. The values of \( M_i \) are, of course, determined directly from the equilibrium conditions, and therefore, it remains to

determine the statically indeterminate moments \( M_i \). The bending moment about any axis produced by any redundant force is a linear function of the perpendicular distance from the force to the axis, and consequently, if any two redundant forces \( X_o \) and \( Y_o \) and a couple \( M_o \) are assumed as acting at any arbitrary point \( O \) (Fig. 5-39) the bending moment \( M_i \) at any section m–m due to these redundant forces may be written in the form

\[ M_i = a + bx + cy \]  

(5-28b)

The total bending moment \( M \) is therefore equal to

\[ M = M_r + a + bx + cy \]  

(5-28c)

The quantities \( a, b, \) and \( c \) are dependent upon the magnitude, position, and direction of the redundant forces which, as in previous solutions, must satisfy the strain conditions of the structure. If the location of point \( O \) is made arbitrary and if, in the case of three redundants, the structure is transformed into a closed unit by the insertion of

---

**Fig. 5-39.** Fixed-end arch connected at one end by weightless arm to elastic center.  
**Fig. 5-40.** Fixed-end arch connected at both ends by weightless arms to elastic center.
the rigid portions OA and OB in Fig. 5-40, the unknown redundant quantities may be determined from the strain conditions of no relative motion between the two rigid arms. Therefore, the values of \( a, b, \) and \( c \) in Eq. (5-28c) must satisfy the conditions

\[
\theta_o = \int_A^B \frac{M \, ds}{EI} = 0 \\
\Delta_{oz} = \int_A^B \frac{M y \, ds}{EI} = 0 \\
\Delta_{oy} = \int_A^B \frac{M x \, ds}{EI} = 0
\]  

(5-29)

When the bending moment \( M \) is expressed in terms of the known bending moment \( M_0 \) and the redundant moment \( M_1 \), as given by Eq. (5-28c) the above strain equations become

\[
\theta_o = \int \frac{M_0 \, ds}{EI} + a \int \frac{ds}{EI} + b \int \frac{x \, ds}{EI} + c \int \frac{y \, ds}{EI} \\
\Delta_{oz} = \int \frac{y \, M_0 \, ds}{EI} + a \int \frac{y \, ds}{EI} + b \int \frac{xy \, ds}{EI} + c \int \frac{y^2 \, ds}{EI} \\
\Delta_{oy} = \int \frac{x \, M_0 \, ds}{EI} + a \int \frac{x \, ds}{EI} + b \int \frac{x^2 \, ds}{EI} + c \int \frac{xy \, ds}{EI}
\]  

(5-30a)

As the location of point \( O \) has not yet been determined a position can now be assigned to satisfy the requirements that

\[
\int_A^B \frac{x \, ds}{EI} = 0 \\
\int_A^B \frac{y \, ds}{EI} = 0 \\
\int_A^B \frac{xy \, ds}{EI} = 0
\]  

(5-31a)

(5-31b)

(5-31c)

If point \( O \) and the direction of the \( x \) and \( y \) axes are selected so as to satisfy Eqs. (5-31a) to (5-31c) then Eqs. (5-30a) to (5-30c) take the simplified form

\[
\theta_o = \int \frac{M_0 \, ds}{EI} + a \int \frac{ds}{EI} = 0 \\
\Delta_{oz} = \int \frac{y \, M_0 \, ds}{EI} + c \int \frac{y^2 \, ds}{EI} = 0 \\
\Delta_{oy} = \int \frac{x \, M_0 \, ds}{EI} + b \int \frac{x^2 \, ds}{EI} = 0
\]  

(5-32a)

(5-32b)

(5-32c)

From Eqs. (5-32a) to (5-32c) the numerical value of the quantities \( a, b, \) and \( c \) is obtained in the form

\[
a = - \frac{\int M_0 \, ds/EI}{\int ds/EI} \]  

(5-33a)

\[
b = - \frac{\int x M_0 \, ds/EI}{\int x^2 \, ds/EI} \]  

(5-33b)

\[
c = - \frac{\int y M_0 \, ds/EI}{\int y^2 \, ds/EI} \]  

(5-33c)

If the quantity \( ds/EI \) (\( E \) can be taken equal to unity if constant) is treated mathematically as an imaginary area whose centroid lies on the original axis of the arch, then the following notation will be consistent with the ordinary notation for flexural stresses:
REDUNDANT FORCES BY THE COLUMN-ANALOGY METHOD 5-43

\[ A = \text{area} = \int \frac{ds}{EI} \]

\[ I_y = \text{moment of inertia of } A \text{ about the } Y \text{ axis} = \int x^2 \frac{ds}{EI} \]

\[ I_z = \text{moment of inertia of } A \text{ about the } z \text{ axis} = \int y^2 \frac{ds}{EI} \]

If, also, the quantity \( (M_*/EI) \) \( ds \), that is, the area of the \( M_*/EI \) diagram, is treated as an applied load that is distributed along the axis of the member, then the following substitutions are also consistent with the usual treatment of elastic weights:

\[ N = \int \frac{M_*}{EI} \, ds = \text{total area of the } \frac{M_*}{EI} \text{ diagram} \]

\[ M_* = \int y \frac{M_* \, ds}{EI} = \text{statistical moment of the } \frac{M_*}{EI} \text{ diagram about the } zz \text{ axis through point } O \]

\[ M_v = \int x \frac{M_* \, ds}{EI} = \text{statistical moment of the } \frac{M_*}{EI} \text{ diagram about the } yy \text{ axis through point } O \]

When the above notation is inserted in Eqs. (5-28b) and (5-33a) to (5-33c) the bending moment \( M_i \) at any section due to the redundant forces is expressed by the equation

\[ M_i = -\left( \frac{N}{A} + \frac{M_v}{I_y} x + \frac{M_s}{I_z} y \right) \]

(5-34)

The analogy between the calculation of \( M_i \) by Eq. (5-34) and the ordinary calculations for determining the stresses in a column that is subjected to an eccentric load was first pointed out by Professor Hardy Cross. The column analogy can be summarized in the following manner: In any structure with three redundants, the bending moments due to the redundant forces may be calculated by the usual formulas for direct stress and bending if the \( M_*/EI \) diagram is considered as a distributed load along the axis of an imaginary column cross section whose thickness is \( 1/EI \).

The problem is therefore transformed into the more common one of determining the centroid of an area, the principal axes of inertia through the centroid, and the moments of inertia about the principal axes.

The sign of the bending moments \( M_i \) can be transformed into tensile or compressive stresses which are added algebraically in the usual manner. The application of the above procedure will be illustrated by analyzing the fixed-end arch in Fig. 5-38 by the column-analogy method.

As the arch in Fig. 5-38 has a constant cross section the thickness of the imaginary column is constant and can be taken equal to unity. The same 11 elements \( \Delta \), that are used in Table 5-6 will be selected to determine the properties of the imaginary column that is shown in Fig. 5-41. From the summations in Table 5-6 the coordinates of the centroid with respect to point \( A \) are

\[ \bar{x} = \frac{330.0}{11} = 30.0 \text{ ft} \]

\[ \bar{y} = \frac{117.00}{11} = 10.64 \text{ ft} \]
Since each $ds/EI$ element is taken as unity, the new coordinates with respect to $O$ give

$$I_x = \Sigma(y^2) = 190.8$$
$$I_y = \Sigma(x^2) = 4,154.8$$

The statical moments of the $M_s/EI$ diagram about the $x$ and $y$ axes through the centroid $O$ can be obtained from Table 5-7 by substituting $z - 30$ for $x$ and $y - 10.64$ for $y$, or

$$M_x = (65.5)(3.96) + (131.0)(2.86) + (193.5)(0.86) + (251.0)(-2.44) + (293.0)(-7.44) = -1,991.8$$
$$M_y = (65.5)(6.55) + (131.0)(13.10) + (193.5)(19.35) + (251.0)(25.10) + (293.0)(29.30) = 20,774.3$$
$$N = 65.5 + 131.0 + 193.5 + 251.0 + 293.0 = 934.0$$

By taking each term in Eq. (5-34) and supplying the necessary signs by considering compression in the analogous column as negative and tension as positive, the values of the total bending moments $M$ at point $A$, $B$, and $C$ are

At point $A$

$$M = M_t = -\left[ -\frac{934.0}{11} + \frac{20,774.3}{4,154.8} \right] (30) - \left( \frac{1,991.8}{190.8} \right) (10.64)$$
$$= 84.9 - 150.0 + 111.0 = 45.9 \text{ ft-kips}$$

At point $B$

$$M_t = -[+84.9 - 150.0 - 111.0] = +345.9$$

At point $C$

$$M = M_t = -\left[ -\frac{934.0}{11} + 0 \right] + \frac{1,991.8}{190.8}$$
$$= 84.9 - 45.5 = 39.4 \text{ ft-kips}$$

The sign of the redundant moment can always be checked since it will always be of opposite sign to that of $M_t$ at the point where $M_t$ is a maximum. For example, at point $B$ if the direction of $M_t$ is taken negative then $M_t$ must be positive, which then governs the signs of $M_t$ that must be used at all other points.

In the above application of the column-analogy method the direction of the principal axes was known since an axis of symmetry is always a principal axis of inertia while the other principal axis is perpendicular to it. When the structure is unsymmetrical the direction of the principal axes must be determined as for any problem in unsymmetrical bending. The column-analogy method can be arranged to analyze other structures than those involving three redundants such as in a fixed-end arch, frame, or a closed ring, but such applications will not be considered here.

REFERENCES

Section 6

CONCRETE MEMBERS IN TORSION

By

PAUL ANDERSEN, Professor of Structural Engineering, University of Minnesota, Minneapolis, Minn.

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Reinforced-concrete members are sometimes subjected to torsional moments due to loads which do not act in the plane of the structure. Thus wall beams and girders around openings receive their loads wholly from one side of the beam axis and will therefore develop torsional moments in addition to the bending moments. A rigid frame or an arch resisting forces acting perpendicular to its plane will be subjected to both torsion and bending.

TORSIONAL MOMENTS

In Fig. 6-1 is shown a rectangular concrete beam, free at one end and fixed at the other end. If this beam is subjected to the action of a torsional moment $M_t$ at the free end, the various sections of the beam will rotate about its axis and this angle of twist will be proportional to the distance between the sections. If the length of the member is $l$, the total angle of twist will be

$$\theta = \frac{M_t l}{E_s T}$$  \hspace{1cm} (6-1)

where $E_s$ is the modulus of elasticity in shear and $T$ is the torsion factor, which is a
function of the shape and dimensions of the cross section. The torsion factor which for a circular section is the polar moment of inertia can be approximated for a rectangular section by one of the following equations:

\begin{align*}
T &= \frac{b^2d^4}{3.58(b^2 + d^2)} \\
T &= \frac{b^2d}{3} \left( 1 - 0.63 \frac{b}{d} \right)
\end{align*}

(6-2)

(6-3)

where \( b \) and \( d \) are the dimensions of the rectangle. Both Eq. (6-2) and Eq. (6-3) represent approximations to the exact expression which is in the form of an infinite series. Figure 6-2 shows the deviation of the two approximations from the exact solution. For a square section, or one almost square, it is seen that Eq. (6-2) is more nearly correct. If the ratio of the long side to the short side is 1.6 or greater, it is seen that Eq. (6-3) gives better results.

Figure 6-3 shows a beam with fixed ends subjected to the action of a torsional moment at some point between the ends. The rotational fixed-end moments can be found by expressing equality of the angles of twist of the two portions of the beam. Thus, from Eq. (6-1),

\[
M_A = \frac{b}{l} M \quad \text{and} \quad M_B = \frac{a}{l} M
\]

(6-4)

For a haunched beam or one in which torsional resistance is influenced by the addition of a considerable amount of reinforcement, especially for shear resistance, this simple relationship does not hold. The case of a straight haunch will be studied in the following. The additional notation required is given in Fig. 6-4, in which \( d_1 \) = least depth of haunch, \( d_2 \) = depth at wall, and \( l_H \) = length of haunch. The angle of twist caused by an infinitesimal length of haunched section is given by Eq. (6-1), thus,

\[
d\theta = \frac{M}{T E_s} \, dx
\]

(6-5)

For the total length of the haunched portion, the total angular twist equals

\[
\theta = \frac{M}{E_s} \int_0^{l_H} \frac{dz}{T}
\]

(6-6)

Assume that the breadth \( b \) is constant and that the torsion factor \( T \) at any point in the
haunch is given by Eq. (6-3). The depth $d$ of the haunch at any point will be

$$d = d_1 + (d_2 - d_1) \frac{x}{l_H} \tag{6-7}$$

in which $x$ is the distance from the shallow end of the haunch. Making this substitution in Eq. (6-3) and Eq. (6-6) the integration gives for the average twist per unit length

$$\frac{\theta}{l_H} = \frac{3M}{b^4E_s(d_2 - d_1)} \log, \left( \frac{d_2 - 0.63b}{d_1 - 0.63b} \right) \tag{6-8}$$

Designating by $T_H$ the effective torsion factor over the length of the haunch

$$T_H = \frac{Ml_H}{\theta E_s} = \frac{b^4(d_2 - d_1)}{3 \log, \left( \frac{d_2 - 0.63b}{d_1 - 0.63b} \right)} \tag{6-9}$$

or

$$T_H = C_H d_1^4$$
in which \( C_H \) is a coefficient depending on the two ratios \( d_1/b \) and \( d_2/d_1 \). Figure 6-4 gives values for different ratios of \( d_1/b \) and \( d_2/d_1 \).

The total angular twist between any point on the uniform section of a haunched beam and the end will be

\[
\theta = \frac{M}{E_s \left( \frac{L_H}{T_H} + \frac{L_u}{T_u} \right)} \tag{6-10}
\]

in which \( L_u \) and \( T_u \) are the length and torsion factors for the uniform segment.

A torsional moment acting on a haunched beam will be distributed to either end in proportion to the torsional stiffness of that particular segment, which may be defined as

\[
K = \frac{1}{(L_H/T_H) + (L_u/T_u)} \tag{6-11}
\]

As a numerical example let it be assumed that the unsymmetrically haunched concrete beam shown in Fig. 6-5 is subjected to a torsional moment \( M \) applied 6 ft from the left end. The width of the beam is 12 in. It is desired to find the torsional moments at each fixed end of the beam.

For the haunch at the left end it is seen that

\[
\frac{d_1}{b} = \frac{18}{12} = 1.5 \quad \frac{d_2}{d_1} = \frac{36}{18} = 2.0
\]

and from Fig. 6-4, \( T_H = 0.1 \times 18^4 = 10,500 \) in.\(^4\).

For the haunch at the right end the ratios will be

\[
\frac{d_1}{b} = \frac{18}{12} = 1.5 \quad \frac{d_2}{d_1} = \frac{30}{18} = 1.67
\]

and again from Fig. 6-4, \( T_H = 0.087 \times 18^4 = 9,133 \) in.\(^4\).

For the uniform section application of Eq. (6-3) gives

\[
T_u = \frac{1}{2} \times 12^2 \times 18(1 - 0.63 \times \frac{12}{18}) = 6,013 \text{ in.}^4
\]

Substituting in Eq. (6-11), the torsional stiffnesses of the two parts of the beam are obtained. Thus, for the left portion

\[
K_L = \frac{1}{48/10,500 + 24/6,013} = 117
\]

and for the right portion

\[
K_L = \frac{1}{114/6,013 + 30/9,133} = 45
\]

From these values are found the two fixed-end moments. At the left end

\[
\frac{117}{117 + 45} M = 0.72M
\]

and at the right end

\[
\frac{45}{117 + 45} M = 0.28M
\]

\(^1\) See Design of Reinforced Concrete in Torsion, Trans. ASCE, 1938, discussion by Bruce G. Johnston, pp. 1517–1519.
THEOREM OF LEAST WORK

A three-dimensional structure under load is usually subjected to bending moments as well as torsional moments. The internal work of deformation in such a structure is equal to

\[ W = \int \frac{M_b^2}{2EI} \, dx + \int \frac{M_t^2}{2E_T} \, dx \]  \hspace{1cm} (6-12)

where \( M_b \) and \( M_t \) denote bending moments and torsional moments, respectively, throughout the structure. For a statically indeterminate structure the theorem of least work states that the redundant quantities must be so chosen that the internal work as defined by Eq. (6-12) is a minimum. If \( X_1, X_2, \ldots \) denote the redundants then it follows that

\[ \frac{\partial W}{\partial X_1} = 0 \quad \frac{\partial W}{\partial X_2} = 0 \quad \cdots \]  \hspace{1cm} (6-13)

or

\[ \int \frac{M_b}{EI} \, dM_b \, dx + \int \frac{M_t}{E_T} \, dM_t \, dx = 0 \]  \hspace{1cm} (6-14)

THREE-DIMENSIONAL STRUCTURES

In Fig. 6-6a is shown the simplest possible three-dimensional structure. Two beams are rigidly connected at right angles, while their other ends are fixed. Their point of intersection is subjected to the action of a force \( P \) perpendicular to their common plane. It is desired to find torsional and bending moments.

![Diagram](a)

![Diagram](b)

![Diagram](c)

Fig. 6-6. Three-dimensional beam.

In Fig. 6-6b is shown the point of intersection of this beam separated from the rest of the structure. Because of symmetry the two ends will be subjected to equal shears and equal bending and torsional moments \( M \). If one-half of the structure is considered as in Fig. 6-6c, it is readily seen that the flexural moment at a distance \( x \) from the angle joint is

\[ M_b = M - \frac{1}{4}P \, x \quad \frac{dM_b}{dM} = 1 \]  \hspace{1cm} (6-15)

Likewise the torsional moment at this point is

\[ M_t = M \quad \frac{dM_t}{dM} = 1 \]  \hspace{1cm} (6-16)
CONCRETE MEMBERS IN TORSION

Substitution in Eq. (6-14) of Eqs. (6-15) and (6-16) gives

\[ \int_0^L (M - \frac{1}{2}Px) \, dx + \frac{EI}{E,T} \int_0^L M \, dx = 0 \quad (6-17) \]

Completing the integrations and solving for \( M \) gives

\[ M = \frac{PL}{4[1 + (EI/E,T)]} \quad (6-18) \]

It is interesting to note that this problem can also be solved by the method of consistent deflections, by expressing equality of the angle of twist of one member to the flexural-angle change of the other member.

![Diagram showing a balcony girder with dimensions and bending moments](image)

Fig. 6-7. Balcony girder.

Figure 6-7a shows a balcony girder consisting of two cantilever members of span \( L \), rigidly connected at right angles by a third member of span \( l \). It is desired to find the bending and torsional moments due to a concentrated load \( P \) at mid-span of the connecting member.

It follows from symmetry that the bending moment \( M \) at the end of the connecting span must equal the torsional moment in the cantilever. The connecting span is free of torsion and has a bending moment at a distance \( x \) from the end equal to

\[ M_b = \frac{1}{2}Px - M \quad \frac{dM_b}{dM} = -1 \quad (6-19) \]

For the two cantilevers the bending moment at a distance \( x \) from the connecting span will equal

\[ M_b = \frac{1}{2}Px \quad \frac{dM_b}{dM} = 0 \quad (6-20) \]

and the torsional moment will be constant and equal to \( M \); thus

\[ M_t = M \quad \frac{dM_t}{dM} = 1 \quad (6-21) \]

Substituting Eqs. (6-19), (6-20), and (6-21) in Eq. (6-14) gives

\[ \int_0^{L/2} (M - \frac{1}{2}Px) \, dx + \frac{EI}{E,T} \int_0^L M \, dx = 0 \quad (6-22) \]

Completing the integrations and solving for \( M \)

\[ M = \frac{\frac{1}{2}Pl}{1 + 2(L/l)(EI/E,T)} \quad (6-23) \]
The bending-moment diagram is shown in Fig. 6-7b. The torsional moment in the cantilever is also given by Eq. (6-23).

If the connecting member had carried a uniformly distributed load of intensity $w$, the torsional moment in the cantilever (and also the bending moment at the end of the connecting member) would have been equal to

$$M = \frac{\frac{1}{2}tw^2}{1 + 2(L/l)(EI/E_T)}$$

(6-24)

HORIZONTALLY CURVED BEAMS

It is often necessary to curve beams of reinforced concrete in horizontal planes. Such beams will develop both flexural and torsional moments.

Figure 6-8 shows a horizontally curved beam with fixed ends, which spans an angle $AOB$ of 90°. It is desired to find the flexural and torsional moments at the ends and along its curved axis due to a concentrated load at mid-span $C$. Because of symmetry of geometry and load the vertical shear will be constant and equal to one-half of the concentrated load throughout the beam. Likewise, there can be no torsional moment at mid-span, and there remains only one unknown quantity at this point, the internal flexural moment $M$ which, as shown in the following, can be determined by the theorem of least work.

In Fig. 6-9 is shown a portion of the curved beam separated by a radial section through the line $OC$ which bisects the central angle. The bending moment $M$ at the mid-span section can be represented by a vector which lies on the radius to this point. The counterclockwise convention has been adopted for this representation (from the end of the vector the bending moment is seen turning against the clock). Next to be

![Diagram](image)

**Fig. 6-8. Horizontally curved fixed-end beam.**

![Diagram](image)

**Fig. 6-9. Flexure and torsion.**

considered are the forces acting on an arbitrary section between the mid-span section and the end. From Fig. 6-9a it is seen that the vector representing the bending moment $M$ can be replaced by two vectors parallel and perpendicular to the tangent at the arbitrary section under consideration. The quantities necessary for the evaluation of the flexural and torsional moments due to the shear are shown in Fig. 6-9b; both turn in directions opposite those in Fig. 6-9a. The flexural and torsional moments acting on an arbitrary section making an angle $\theta$ with the mid-span section can be
CONCRETE MEMBERS IN TORSION

found by addition of the two sets of vectors in Fig. 6-9; thus

\[ M_b = M \cos \theta - \frac{1}{2}Pr \sin \theta \]
\[ M_t = M \sin \theta - \frac{1}{2}Pr(1 - \cos \theta) \]  

(6-25)

Differentiating with respect to \( M \) gives

\[ \frac{dM_b}{dM} = \cos \theta \quad \frac{dM_t}{dM} = \sin \theta \]  

(6-26)

Substituting Eqs. (6-25) and (6-26) in Eq. (6-14) gives

\[ \int_{0}^{\frac{\pi}{4}} (M \cos^2 \theta - \frac{1}{2}Pr \sin \theta \cos \theta)r \, d\theta \]
\[ + C \int_{0}^{\frac{\pi}{4}} [M \sin^2 \theta - \frac{1}{2}Pr \sin \theta(1 - \cos \theta)]r \, d\theta = 0 \]  

(6-27)

where

\[ C = \frac{EI}{ET} \]  

(6-28)

Completing the integrations and solving for \( M \)

\[ M = Pr \left( \frac{0.125 + 0.0215C}{0.643 + 0.1427C} \right) \]  

(6-29)

As a numerical example, which will illustrate the variation in flexural and torsional moments in a circular beam subjected to a concentrated load at mid-span, assume for the beam shown in Fig. 6-8 a radius \( r = 10 \) ft and a load \( P = 20 \) kips. If the depth of the beam is twice its width and Poisson's ratio \( m \) is 0.125 and \( E_s = \frac{E}{2(1 + m)} \), it follows that

\[ I = \frac{1}{3} \pi b^4 \times \frac{8}{3}b^4 \]
\[ T = \frac{1}{2} \pi b^4 \times 2b(1 - 0.63 \times \frac{1}{2}) \]
\[ = \frac{1}{2} \pi b^4 \times 0.685 \]

\[ C = \frac{2.25}{0.685} = 3.285 \]

\[ M = 0.1754Pr = 35 \text{ ft-kips} \]

The variations in both flexural and torsional moments can be determined from Eq. (6-25) and have been plotted in Fig. 6-10.

In the case of a horizontally curved beam (also a quarter of a circle) carrying a uniformly distributed load as shown in Fig. 6-11, the solution is arrived at by the same procedure, the only difference being that the last terms in Eq. (6-21) should represent the flexural and torsional moments of a uniform load between mid-span \( C \) and any arbitrary section. Thus it will be seen that if \( \phi \) denotes the angle between the section under consideration and any arbitrary infinitesimal element lying between this section and the mid-span section the bending moment and torsional moment exerted by the infinitesimal load will be

\[ \Delta M_b = w r^2 \sin \phi \, d\phi \]
\[ \Delta M_t = w r^2 (1 - \cos \phi) \, d\phi \]  

(6-30)
THREE-DIMENSIONAL MOMENT DISTRIBUTION

Integrating between 0 and \( \theta \) gives

\[
\begin{align*}
M_b &= wr^2 (1 - \cos \theta) \\
M_t &= wr^2 (\theta - \sin \theta)
\end{align*}
\]  

(6-31)

The total flexural and torsional moments acting on the arbitrary section will be

\[
\begin{align*}
M_b &= M \cos \theta - wr^2 (1 - \cos \theta) \\
M_t &= M \sin \theta - wr^2 (\theta - \sin \theta)
\end{align*}
\]  

(6-32)

Substituting in Eq. (6-14) and solving

\[
M = wr^2 \frac{0.064 + 0.0091C}{0.643 + 0.1427C}
\]  

(6-33)

The variations of flexural and torsional moments can now be found from Eqs. (6-32) and are shown in Fig. 6-12 for the following numerical values: \( r = 10 \text{ ft} \), \( w = 1 \text{ kip per lin ft} \), width of beam 12 in., depth of beam 24 in., Poisson’s ratio equal to 0.125.

![Uniform load on circular beam](image)

**Fig. 6-11.** Uniform load on circular beam.

![Moments due to uniform load](image)

**Fig. 6-12.** Moments due to uniform load.

THREE-DIMENSIONAL MOMENT DISTRIBUTION

In the preceding parts of this section it has been seen that, in a three-dimensional beam with fixed ends, the end moments will consist of a flexural component and a torsional component. In the case of continuity instead of complete fixity, these moments should be distributed to adjacent members.

As the torsional moment at the free end of a member will be resisted by an equal and opposite moment at the fixed end of the member, it is seen that the torsional carry-over factor is unity regardless of the variation in cross section.

The torsional stiffness of a member may be defined as the torsional moment which applied to one end, free to rotate, will produce a unit angle of twist with respect to the other end, which is assumed completely fixed. From Eq. (6-1) it is seen that the torsional stiffness will equal

\[
K_T = \frac{E.T}{l}
\]  

(6-34)

If \( E_1 = E/2.25 \), the torsional stiffness will be

\[
K_T = \frac{ET}{2.25l}
\]  

(6-35)

where \( T \) is defined for a rectangular section by Eqs. (6-2) and (6-3).
Table 6-1. Moment Distribution In Space

<table>
<thead>
<tr>
<th>Description</th>
<th>AB</th>
<th>AD</th>
<th>AC</th>
<th>AM</th>
<th>A'M</th>
<th>A'C'</th>
<th>A'D'</th>
<th>A'B'</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) Effect of direct load on the center beam</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Type of stress*</td>
<td>T</td>
<td>F</td>
<td>T</td>
<td>F</td>
<td>F</td>
<td>T</td>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>Stiffness ratio</td>
<td>0.2</td>
<td>0.3</td>
<td>0.2</td>
<td>0.3</td>
<td>0.2</td>
<td>0.3</td>
<td>0.2</td>
<td>0.3</td>
</tr>
<tr>
<td>Carry-over factor</td>
<td>1.0</td>
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<td>1.0</td>
<td>0.5</td>
<td>1.0</td>
<td>0.5</td>
<td>1.0</td>
<td>0.5</td>
</tr>
<tr>
<td>(b) Effect of torsional moment in center beam</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Type of stress*</td>
<td>T</td>
<td>F</td>
<td>T</td>
<td>F</td>
<td>F</td>
<td>T</td>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>Stiffness ratio</td>
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<td>0.3</td>
<td>0.1</td>
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<tr>
<td>Carry-over factor</td>
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<td>0.5</td>
<td>0.5</td>
<td>1.0</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
</tr>
</tbody>
</table>

Moment distribution in foot-kips

<table>
<thead>
<tr>
<th>Description</th>
<th>AB</th>
<th>AD</th>
<th>AC</th>
<th>AM</th>
<th>A'M</th>
<th>A'C'</th>
<th>A'D'</th>
<th>A'B'</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fixed-end moment</td>
<td>-33</td>
<td>-50</td>
<td>-33</td>
<td>-50</td>
<td>167</td>
<td>-83</td>
<td>16</td>
<td>25</td>
</tr>
<tr>
<td>First distribution</td>
<td>-12</td>
<td>-25</td>
<td>-12</td>
<td>-25</td>
<td>16</td>
<td>25</td>
<td>16</td>
<td>25</td>
</tr>
<tr>
<td>Carry-over</td>
<td>-2</td>
<td>-4</td>
<td>-2</td>
<td>-4</td>
<td>5</td>
<td>8</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>Second distribution</td>
<td>-35</td>
<td>-54</td>
<td>-35</td>
<td>-54</td>
<td>125</td>
<td>-75</td>
<td>21</td>
<td>33</td>
</tr>
<tr>
<td>Final moment</td>
<td>-35</td>
<td>-54</td>
<td>-35</td>
<td>-54</td>
<td>125</td>
<td>-75</td>
<td>21</td>
<td>33</td>
</tr>
</tbody>
</table>

* T = torsion; F = flexure
TORSIONAL REINFORCEMENT

The flexural-stiffness factor is equal to

\[ K_f = \frac{4EI}{l} \]  

(6-36)

From Eqs. (6-2), (6-35), and (6-36), it is seen that

\[ K_f = \frac{0.372}{1 + (d/b)^2} K_f \]  

(6-37)

which shows that the torsional-stiffness factor is always a fraction of the flexural-stiffness factor.

The example in Table 6-1 will illustrate the procedure for distributing moments in a space structure due to an eccentrically loaded beam. The problem is conveniently solved by considering first the effects of the direct load on the center beam as in Table 6-1a, and then the moments due to the torsional moment as in Table 6-1b. For the sake of simplicity, sideways is neglected, even if there is sideways movement, due to the eccentric load of 37.5 kips, deflecting points \( A \) and \( A' \). It is customary, in plane structures, to stop the process of moment distribution after a distribution, since the sum of the moments around a joint should equal zero. In a space structure, however, this will not make the torsional moments in the members balance. It is, of course, just as correct to stop after having distributed the flexural moments as after having balanced the torsional moments. If the process is carried to the proper number of distributions, it may be stopped after either, as the error would be insignificant.

TORSIONAL STRESSES

Reinforced-concrete beams are usually of rectangular section with the vertical dimension \( d \) larger than the horizontal dimension \( b \). If this section is subjected to the action of a torsional moment \( M_t \), the maximum shearing stress will be vertical and occur at the mid-point of the long side of the rectangle (see Fig. 6-13) and equal

\[ \tau_{\text{max}} = \frac{M_t}{db^2} \left[ 3 + \frac{2.6}{0.45 + (d/b)} \right] \]  

(6-38)

The results from this formula, which is an approximation, will be correct within less than 1 per cent error. The horizontal shearing stress, which occurs at the intersection of the vertical center line with the top and bottom of the section, will equal

\[ \tau = \frac{b}{d} \tau_{\text{max}} \]  

(6-39)

Fig. 6-13. Rectangular section subject to torsion.

In the case of a circular section, which is rarely used in concrete construction, the maximum shear stress will occur along the outside perimeter and will equal

\[ \tau = 5.1 \frac{M_t}{d^2} \]  

(6-40)

This is only slightly greater than the maximum shearing stress in a square section, which according to Eq. (6-38) will equal

\[ \tau = 4.8 \frac{M_t}{d^2} \]  

(6-41)

TORSIONAL REINFORCEMENT

It was shown in the preceding section that a square section is only slightly more efficient in torsion than a circular section. The evaluation of necessary reinforcement of a circular section will be considered first.
Figure 6-14 shows a circular member subjected to torsion. A section perpendicular to the axis will be subjected to shearing stresses only (pure shear), the maximum stress \( v \) will occur along the outside perimeter, and other shearing stresses will be proportional to their distances from the center. This state of pure shear is equivalent to tension in one direction, making an angle of 45\(^\circ\) to the longitudinal axis, and equal compression in the other perpendicular direction. A rectangular element cut from the outer layer of a twisted shaft with its sides at 45\(^\circ\) to the axis will therefore be subjected to stresses, tension, and compression both numerically equal to the shearing-stress intensity \( v \).

In the case of a material that is weaker in tension than in shear (such as cast iron or concrete) a crack along a helix inclined at 45\(^\circ\) to the axis will occur when the ultimate tensile strength has been exceeded. It follows that most effective reinforcement for a concrete member designed to resist failure in torsion is a spiral at an angle of 45\(^\circ\) with the axis. If, therefore, the torsional moment produces tensile stresses greater than the allowable tension of the concrete, torsional reinforcement must be used.

![Figure 6-14. Circular shaft subject to torsion.](image)

![Figure 6-15. Reinforced section showing distribution of shearing stress (or diagonal tension).](image)

In the formulas that follow for spiral reinforcing, the line of thought is analogous to that for the design of web reinforcement in a beam subjected to bending, namely, that sufficient reinforcement must be provided to resist tensile stresses in excess of the permissible tension for plain concrete. The line \( AC \) in Fig. 6-15 represents the variation in shearing stress (and also in diagonal tension) along an arbitrary radius \( AB \). Let \( v_l \) be the shearing stress along the edge produced by a twisting moment and let \( v_c \) denote the permissible shearing stress (or diagonal tension) for plain concrete. The shaded triangle will then represent intensities of diagonal tension for which reinforcement must be provided.

Consider a small element at a distance \( r \) from the axis. The shearing force on this element in excess of the permissible stress is

\[
(v - v_c) r \, dr \, d\theta
\]

The tensile and compressive components of this force are numerically equal. Each one equals

\[
(v - v_c) r \, dr \, d\theta \cos 45^\circ
\]

In accordance with the foregoing assumption, the sum of the moments with respect to the axis of the tensile components must equal the moment of the steel stresses with respect to the same axis; thus

\[
Nf_sA_s \cos 45^\circ r_s = \int_0^{2\pi} \int_{r_s}^R (v - v_c) \cos^2 45^\circ r^2 \, dr \, d\theta \tag{6-42}
\]

in which \( N \) is the number of bars on an angle of 45\(^\circ\) with the axis cut by a horizontal section, \( f_s \) the allowable stress in the steel bars, \( A_s \) the cross-sectional area of the steel bar, \( r_s \) the distance from the axis to the steel bars.
In Eq. (6-42) substitute
\[ v = \frac{r}{R} v_1 \quad v_e = \frac{r_e}{R} v_1 \]  
(6-43)
and perform the integrations; thus
\[ N_f s A_{s e}^r = \frac{\pi \sqrt{2}}{12R} v_1 (3R^4 - 4R^3 r_e + r_e^4) \]  
(6-44)
Substituting
\[ r_e = \frac{v_e}{v_1} R \]  
(6-45)
gives
\[ N_f s A_{s e}^r = 0.370 \left( \frac{R}{v_1} \right)^3 (3v_1^4 - 4v_1^3 v_e + v_e^4) \]  
(6-46)
If the reinforcement is in the form of 45° spirals with a longitudinal pitch \( p \) then
\[ N = \frac{2\pi r_e}{p} \]  
(6-47)
Substituting in Eq. (6-46) gives
\[ \frac{A_s}{p} = \frac{1}{17f_{s e}^2} \left( \frac{R}{v_1} \right)^3 (3v_1^4 - 4v_1^3 v_e + v_e^4) \]  
(6-48)

**NUMERICAL EXAMPLE**

Figure 6-16a shows a beam with fixed ends subjected to the actions of a uniformly distributed load and a concentrated force acting 1 ft off the center line. The uniform
load will produce bending stresses only, while the concentrated force will cause bending and torsional stresses.

The diagrams for bending moments (Fig. 6-16b), torsional moments (Fig. 6-16c), and vertical shear (Fig. 6-16d) are found in the usual manner. Because of the large torsional moments, which must be resisted, a square cross section will be used. If compression reinforcement is disregarded, a 21-in. square section (having an efficient depth of 18.5 in.) will have a maximum bending stress of

$$f_c = \frac{139,000 \times 12}{\frac{1}{2} \times 0.403 \times 0.866 \times 21 \times 18.5^2} = 1,330 \text{ psi}$$

which is satisfactory if a 3,000-psi concrete is used. The reinforcement needed to resist bending stresses at the left end must equal

$$A_s = \frac{139,000 \times 12}{20,000 \times \frac{7}{6} \times 18.5} = 5.2 \text{ sq in.}$$

which can be supplied by two 1-in. round bars and four 1-in. square bars (a total of 5.57 sq in.), as shown on Fig. 6-17. At the other end and at the bracket the reinforcement can be reduced to two 1-in. round rods and two 1-in. square rods, placed at the top and bottom, respectively.

The vertical shearing stresses (Fig. 6-16e) can be found by dividing the total shears (Fig. 6-16d) by

$$bd = 21 \times \frac{7}{6} \times 18.5 = 340$$

The maximum torsional shearing stresses (Fig. 6-16f) can be found from Eq. (6-41); thus

$$\tau = 4.8 \times \frac{28,000 \times 12}{21^3} = 174 \text{ psi}$$

In each of the portions $AB$ and $BC$ of the beam, at one end of the horizontal axis of symmetry the shearing stresses due to bending and torsion act in the same direction. The resultant vertical shearing stresses at the ends will be at $A$ and $C$, respectively,

$$v_A = 118 + 174 = 292 \text{ psi}$$
$$v_C = 59 + 87 = 146 \text{ psi}$$

In accordance with specifications of the American Concrete Institute (1956) web reinforcement should be provided for the excess of shear over 90 psi (3,000-psi concrete). As the most efficient type of torsional shear reinforcement is the 45° spiral, this should be used for a member such as the one in this example, where the torsional shearing stresses are considerably greater than the vertical shearing stresses. A combination of vertical stirrups and inside spiral can be used. The allowable capacity (90 psi) for concrete without web reinforcement should be divided into two parts; thus

$$\frac{17}{2} \times 90 = 54 \text{ psi}$$
$$\frac{11}{2} \times 90 = 36 \text{ psi}$$

In proportioning web reinforcement it can be assumed that this should be supplied for torsional stresses above 54 psi, and for vertical shearing stresses above 36 psi.

The torsional reinforcement for portion $AB$ can be found from Eq. (6-48); thus

$$A_t = \frac{17 \times 20,000 \times 7.5^2 (10.5 \frac{174}{174})^3 (3 \times 174^4 - 4 \times 174^3 \times 54 + 54^4)}{0.0184}$$
If a 3'/4\text{-in.} round rod is selected \((A_s = 0.11)\), \(p = 5.97\text{ in.}\). Because the resistance of the square section has been assumed equal only to the inscribed spiral section, the pitch could be made equal to 6 in. instead of 5\(\frac{1}{2}\) \text{ in.} as shown in Fig. 6-18.

![Fig. 6-18. Reinforcement for vertical and torsional shear.](image)

Similar computations for the portions \(BC\) indicate torsional reinforcement of \(\frac{3}{4}\text{-in.}\) round spiral rod having a pitch of 12 in.

The spacing \(s\) of vertical stirrups can be found from the formula

\[
s = \frac{A_s f_s}{v' b'}
\]

where \(A_s\) = total cross-sectional area of stirrup

\(f_s\) = tensile unit stress in stirrup

\(v'\) = excess of total unit shearing stress over that permitted on the concrete

\(b'\) = width of web of beam

If \(\frac{3}{4}\text{-in.}\) round bars are used for stirrups Eq. (6-49) gives a minimum spacing of

\[
s = \frac{0.196 \times 2 \times 20,000}{(118 - 36)21} = 4.5\text{ in.}
\]

Because of the comparatively small variation in shearing stress between sections \(A\) and \(B\), the stirrup spacing has been kept constant. The arrangement of web reinforcement is shown in Fig. 6-18.

**DESIGN OF HORIZONTALLY CURVED GIRDER**

In buildings where, for architectural reasons, a rounded corner is considered desirable, horizontally curved beams can be effectively used. Figure 6-19 shows such a beam in a reinforced-concrete structure. It will be assumed that this beam carries a

![Fig. 6-19. Horizontal curved beam at corner of building.](image)
uniformly distributed load of 1 kip per lin ft in addition to a concentrated load of 15 kips at mid-span (Fig. 6-20). In the following the design of this girder and the provisions which must be made to resist vertical and torsional shearing stresses will be discussed. The control-cylinder strength of the concrete will be assumed to be \( f'_s = 3,000 \) psi, and the specifications of the American Concrete Institute (1956) will be used as far as this code can be applied to a three-dimensional structure.

**Fig. 6-20. Loads on curved building beam.**

Assuming a ratio of 2 between depth \( d \) and width \( b \) of this beam and a Poisson’s ratio of 0.125 for the concrete, it is seen that moment of inertia and torsion factor [Eq. (6-3)] will be

\[
I = 0.667b^4 \quad T = 0.457b^4
\]

(6-50)

The coefficient \( C \) as defined by Eq. (6-28) will be

\[
C = \frac{2.25 \times 0.667}{0.457} = 3.285
\]

(6-51)

The bending moment at mid-span (\( \theta = 0^\circ \)) can be found from Eqs. (6-29) and (6-33); thus

\[
M = 15 \times 17 \times 0.176 + 289 \times 0.085 = 69.5 \text{ ft-kips}
\]

while at the end it is found by substituting \( \theta = 45^\circ \) in the first of Eqs. (6-25) and the first of Eqs. (6-32) and adding; thus

\[
M_1 = 69.5 \times 0.707 - \frac{1}{2} \times 15 \times 17 \times 0.707 - 289 \times 0.293 = -125.8 \text{ ft-kips}
\]

The effective width \( d' \) can now be found if a width \( b = 13 \text{ in.} \) is assumed

\[
d' = \sqrt{\frac{125,800 \times 12}{236 \times 13}} = 22.5 \text{ in.}
\]

The steel reinforcement in the top of the beam adjacent to the end must have a cross-sectional area not less than

\[
A_s = \frac{M}{f_s d} = \frac{125.8 \times 12}{20 \times 0.866 \times 22.5} = 3.87 \text{ sq in.}
\]

This can be supplied by four 1-in. square bars. Two of these square bars can be discontinued, where they are no longer needed. The resisting moment with two 1-in. square bars will equal

\[
M_b = f_s A_s d = 20 \times 2 \times 72 \times 22.5 = 788 \text{ in.-kips} = 66 \text{ ft-kips}
\]

(6-52)

A general expression for the bending moment is obtained by adding the first of Eq.
(6-25) and the first of Eq. (6-32); thus

$$M_b = 69.5 \cos \theta - \frac{1}{2} \times 15 \times 17 \sin \theta - 289(1 - \cos \theta)$$  \hspace{1cm} (6-53)

Substituting the negative value of Eq. (6-52) into Eq. (6-53) and solving for the angle gives $\theta = 34^\circ$. Because the length of a 11° circular arc of a radius of 17 ft is very close to 3 ft 3 in. and an additional twelve times the dimension of the bar is needed for anchorage, the two 1-in. square bars should extend 4 ft 3 in. beyond the face of the column.

The reinforcement at mid-span can be found by using the bending moment at this section. Four $\frac{1}{2}$-in. round bars, placed in the bottom of the beam, will satisfy the requirement of the bending moment of 69.5 ft-kips.

![Graph showing shear stress variation](image)

**Fig. 6-21.** Variation of maximum shearing stress in curved building beam.

In order to design the web reinforcement for this beam, it is necessary to evaluate the shearing stresses on its various sections. These stresses will be due to the vertical shears and the torsional moments. The variation of the vertical shearing stress can be expressed thus

$$v = \frac{V}{bjd} = \frac{\frac{1}{2}P + wr\theta}{bjd} = \frac{7.5 + 17\theta}{13 \times \frac{1}{2} \times 22.5} \times 10^4 = (29 + 66\theta) \text{ psi}$$  \hspace{1cm} (6-54)

Variation of the torsional moment can be found by adding the second equations of Eqs. (6-25) and (6-32); thus

$$M_t = 69.5 \sin \theta - \frac{1}{2} \times 15 \times 17(1 - \cos \theta) - 17^2(\theta - \sin \theta)$$  \hspace{1cm} (6-55)

$$M_t = 358.5 \sin \theta - 127.5(1 - \cos \theta) - 289\theta$$

The maximum torsional stress can be found by Eq. (6-38); thus

$$\tau = \frac{M_t}{26 \times 13^2} \left(3 + \frac{2.6}{0.45 + 2}\right) = 925 \times 10^{-4}M_t$$  \hspace{1cm} (6-56)

Substituting Eq. (6-55) in Eq. (6-56) and multiplying by 12,000 (to change units to psi) gives

$$\tau = 3,980 \sin \theta + 1,420 \cos \theta - 3,210\theta - 1,420$$  \hspace{1cm} (6-57)

Shearing stresses can now be found from Eqs. (6-54) and (6-56) if the angle $\theta$ is expressed in radians ($1^\circ = 0.01745$ radian). While the stress found by Eq. (6-54) will always appear positive, the one found from Eq. (6-57) will be positive for angles from 0 to 39°, and negative from 39 to 45°, indicating that for the smaller angles the maximum shearing stress will occur on the outside of the beam, and for the larger angles on the inside. The numerical values of the two equations should everywhere be added arithmetically. Figure 6-21 shows the variation in maximum shearing stress.

It will be recalled that the web reinforcement in the foregoing example was made up
of a combination of vertical stirrups and 45° degree spirals. Because the vertical shear is assumed constant over a large part of the cross section while the torsional shearing stress reaches a maximum at the mid-points of the long sides of the section, this division will always result in economy of steel, especially for a square section. If, however, there is considerable variation in the torsional moment (including several points of contrasitrons) and a rectangular section was adopted on account of high bending stresses, this arrangement will require a large amount of labor. In the case of the horizontally curved beam with a deep rectangular section, vertical stirrups for

![Diagram of reinforcement for curved building beam.](image)

Fig. 6-22. Reinforcement for curved building beam.

resisting both vertical and torsional shearing stresses are considered the most desirable arrangement.

Accordingly the maximum shearing stresses shown in Fig. 6-21 will be assumed to be constant over the section, and \( \frac{3}{4} \)-in. round bars, in the form of closed hoops, will be added to the web to resist these shearing stresses. The spacing of these stirrups will be found from Eq. (6-49). The permissible shearing stress on a 3,000-psi concrete is 90 psi. Substituting this and the other numerical values gives

\[
s = \frac{2 \times 0.11 \times 20,000}{(v - 90)13} = \frac{338}{v - 90}
\]

At the end of the beam the spacing will equal \( \frac{338}{(204 - 90)} = 3 \) in. Similar spacings are determined for various values of the angle \( \theta \). As shown in Fig. 6-22 actual spacings of 3, 6, 10, and 11 in. were adopted.
Section 7

REINFORCED-CONCRETE CHIMNEYS

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INTRODUCTION

There is no branch of reinforced-concrete engineering or construction that requires
a more varied and practical knowledge of materials and design factors than concrete
chimneys. Reinforced-concrete chimneys have been built in the United States and
Canada since the early 1900s. For the first thirty years concrete chimneys were
designed using approximate and empirical formulas and were built with concrete conforming to the standards and knowledge of the period.

In the early 1930s it was realized that a need existed for a rigorous investigation of the stresses occurring in reinforced-concrete chimneys and for setting forth a standard specification for their design and construction. Realizing this need, the American Concrete Institute constituted its original Committee 505, which prepared a tentative specification titled Proposed Standard Specification for the Design and Construction of Reinforced Concrete Chimneys. Adopted as a tentative standard in 1936 with Designation 505-36T, this specification has been the basis for the design and construction of practically all reinforced-concrete chimneys of circular cross section subsequently built in North America, which includes many structures over 500 ft high, several exceeding 600 ft in height, and chimneys having a top internal diameter of as much as 45 ft.

In 1949, the American Concrete Institute reactivated its Committee 505 to revise the Tentative Standard Specification. Because of the prevailing use of higher-strength concrete, the need for a more accurate approximation of earthquake forces, and the need for a more complete study of the temperature gradient through the chimney walls, a revision was felt necessary. A new Standard Specification for the Design and Construction of Reinforced Concrete Chimneys was adopted at the Annual Convention of the American Concrete Institute in February, 1954, and published in the Journal of the ACI of September, 1954, with the designation ACI 505-54.

Modern concrete chimneys are usually tapered, since this improves the appearance of the structure and increases its stability against overturning, providing a more economical design than would a plumb structure. Where space limitations are not present, a taper, or batter, of the chimney is selected by the designer based on the wind or earthquake design data and on the ratio of the height to the diameter. A reinforced-concrete chimney, however, is readily adaptable to space limitations that a brick chimney could not economically meet.

The essential features to be considered in chimney design are:

1. Outer column, sometimes referred to as shaft or shell
2. Lining, or inner protection
3. Foundation, sometimes referred to as footing or base
4. Accessories

Most reinforced-concrete chimneys have a circular cross section for the theoretical reason that a circular section affords a minimum of friction which reduces the draft, and for the practical reason of ease of construction. This section is concerned primarily with circular chimneys, and chimneys of elliptical, oval, or polygonal cross section are beyond its scope.

**DESIGN OF THE OUTER COLUMN: BASIC CONSIDERATIONS**

The concrete column must be designed to resist stresses due to dead weight and the force of the wind or earthquake shock, also the effect of temperature both vertically and circumferentially as well as combinations of these stresses. The working stresses in the materials must be kept within acceptable limits with a proper factor of safety.

A concrete chimney is normally designed to withstand the force produced by wind velocities up to 100 mph. It is recognized that, during conditions of high wind loading, the force increases with the distance above the ground surface. For small and medium-sized chimneys a uniform wind pressure of 25 psf of projected area for the entire height of the chimney is usually specified as the equivalent of a 100 mph wind. A more realistic determination of the pressure produced by a 100 mph wind on a circular chimney is provided by the formula $W = 15 + H/40$, where $W$ is the wind force in pounds per square foot of projected area and $H$ is the height of the chimney above the ground in feet, with a recommended minimum value of $W = 22.5$. For locations such as the east coast of Florida, the Gulf Coast, and the West Indies, where gust
velocities in excess of 100 mph have been recorded or are anticipated, the usual design wind pressure should be multiplied by the factor \((G/100)^2\) where \(G\) is the maximum gust velocity in miles per hour recorded or anticipated at 30 ft above the ground.

Chimneys which are to be constructed in areas subject to severe earthquakes must be designed to withstand these shocks. During an earthquake, the horizontal movement of the ground is transmitted to the chimney through its foundation. The magnitude of this force is a function of the weight of the column and the rate of earth acceleration, and the force, which is assumed to act at the center of gravity of the mass of the chimney above the section under consideration, is expressed by the formula

\[ F = \frac{W a}{g} = W K_s \]

where \(W\) = weight of shaft above section under consideration, lb, kips, or tons
\(a\) = acceleration due to earthquake, ft per sec per sec
\(g\) = acceleration due to gravity, 32.2 ft per sec per sec
\(K_s = a/g\), called the seismic coefficient.
Experience has indicated that, in order to ensure against greatly overstressing the materials, additional consideration should be given to the upper portion of a chimney subjected to earthquake because of whipping action due to the rapid reversal of the earth movement. It is recommended that the calculated earthquake force for sections under consideration in the upper four-fifths of the chimney height be multiplied by the factor \((1 + h'/100)\), where \(h'\) is the distance from the section under consideration to the section that is one-fifth of the total height of the chimney above the foundation, in feet.

The seismic coefficient is usually determined from the earthquake record of the area in which the chimney is to be built. The U.S. Coast and Geodetic Survey has prepared a seismic probability map of the United States as shown in Fig. 7-1, and where design seismic coefficients are not specified by applicable building codes, the following minimum seismic coefficients are recommended to correspond to the various zones:

- Zone 1: minor damage, \(K_s = 0.05\)
- Zone 2: moderate damage, \(K_s = 0.10\)
- Zone 3: major damage, \(K_s = 0.20\)

In designing the chimney, the force of the wind is not considered acting simultaneously with earthquake shock, since earthquakes occur in periods of comparatively calm weather.

In computing the dead weight of the chimney the following unit weights should be used unless otherwise specified:

- Concrete: 150 lb per cu ft
- Perforated-radial-brick lining: 120 lb per cu ft
- Solid building-brick lining: 125 lb per cu ft
- Firebrick lining: 130 lb per cu ft
- Acidproof-brick lining: 135 lb per cu ft

In considering a section of a chimney which has a sectional corbel-supported lining, the weight of the lining carried by the outer column above the section under consideration must be included in the weight of the chimney used in the design computations.

**DESIGN OF THE OUTER COLUMN: THEORY**

An accepted method of stress computations for concrete stacks is outlined in detail in the Standard ACI Specifications (505-54) as published by the American Concrete Institute. This specification gives the derivations of various formulas which are discussed here and used in determining unit stresses due to wind, dead load, and temperature or due to earthquake shock, dead load, and temperature. The critical conditions due to combined stresses from these forces are carefully analyzed. It is recommended that a copy of this specification or its latest revision be obtained.

The basic American Concrete Institute formulas and the illustrations which will be considered here are reproduced with permission from Standard Specifications for the Design and Construction of Reinforced Concrete Chimneys (505-54), published by the American Concrete Institute, P.O. Box 4754, Redford Station, Detroit 19, Mich. The notation as used in this section and as shown with Figs. 7-2, 7-3, 7-4, and 7-5 may differ slightly from the ACI Standard.

In addition to assumptions customarily made for the design of reinforced-concrete members, the two following assumptions are made:

1. The reinforcing steel is replaced by a continuous steel shell of equivalent area.
2. The reinforcing steel is located on the mean circumference of the concrete shell.

Assumption (1) involves no appreciable error and assumption (2) is on the side of safety since the reinforcing steel is usually placed nearer the outer surface of the chimney shell where it is more effective in resisting stress due to bending.
The basic formulas follow:

\[ e = \frac{M}{Wr} = \frac{(1 - p)(\alpha - \sin \alpha \cos \alpha) - (1 - p + pn)(\beta + \sin \beta \cos \beta - 2 \sin \beta \cos \alpha) + np\tau}{2[(1 - p)(\sin \alpha - \alpha \cos \alpha) - (1 - p + pn)(\sin \beta - \beta \cos \alpha) - np\tau \cos \alpha]} \]  
\hspace{1cm} (7-1)

Where \( \beta = 0 \) (no flue opening), Eq. (7-1) simplifies to

\[ e' = \frac{M}{Wr' = \frac{(1 - p)(\alpha - \sin \alpha \cos \alpha) + np\tau}{2[(1 - p)(\sin \alpha - \alpha \cos \alpha) - np\tau \cos \alpha]} \]  
\hspace{1cm} (7-1a)

\[ f' = \frac{2\pi r'(1 - p)(\sin \alpha - \alpha \cos \alpha) - (1 - p + pn)(\sin \beta - \beta \cos \alpha) - np\tau \cos \alpha}{W(\cos \beta - \cos \alpha)} \]  
\hspace{1cm} (7-2)

**Fig. 7-2.** Cross section of typical chimney for stress due to wind or earthquake shock and dead load.

**Notation:**
- \( r \) = mean radius of concrete shell
- \( t \) = thickness of concrete shell
- \( W \) = weight of chimney above section under consideration
- \( M \) = moment due to wind or earthquake shock at section under consideration
- \( R \) = resultant of \( W \) and horizontal force
- \( e = M/W \) = eccentricity of \( R \) measured from center of chimney
- \( A_s \) = area of concrete shell at section
- \( A_r \) = area of vertical reinforcing at section
- \( P = A_s/A_r \)
- \( n \) = ratio of moduli of elasticity—steel to concrete
- \( \alpha \) = one-half the central angle subtended by the neutral axis (i.e., axis of zero stress) as a chord on the circle of radius \( r \)
- \( \beta \) = One-half the central angle subtended by the flue opening as a chord on the circle of radius \( r \)
- \( \theta \) = a variable function of \( \alpha \)
- \( f_s \) = maximum unit tensile stress in the reinforcing steel
- \( f_c \) = maximum unit compressive stress in the concrete at outside of chimney shell
- \( f'_c \) = maximum unit compressive stress in the concrete at center of chimney shell
- \( f''_c \) = maximum unit compressive stress in the concrete at inside of chimney shell
Where $\beta = 0$ (no flue opening), Eq. (7-2) simplifies to

$$f'_c = \frac{W(1 - \cos \alpha)}{2rt[(1 - p)(\sin \alpha - \alpha \cos \alpha) - np \cos \alpha]}$$  \hspace{1cm} (7-2a)

$$f_c = f'_c \left[ 1 + \frac{t}{2r \cos \beta (\cos \beta - \cos \alpha)} \right]$$  \hspace{1cm} (7-3)

Where $\beta = 0$ (no flue opening), Eq. (7-3) simplifies to

$$f_c = f'_c \left[ 1 + \frac{t}{2r(1 - \cos \alpha)} \right]$$  \hspace{1cm} (7-3a)

$$f_s = n f'_c \left[ \frac{1 + \cos \alpha}{\cos \beta - \cos \alpha} \right]$$  \hspace{1cm} (7-4)

---

**Fig. 7-3.** Temperature gradient through chimney shell for vertical stress due to temperature.

**Assumptions:** Since the inner and outer parts of the shell of a chimney in service are not equally hot, they tend to expand unequally. The inner (hot) part is restrained from expanding freely by the outer part, and the outer part is stretched by the restrained inner part. At the boundary between these parts the elongation due to temperature is unrestrained or free and hence without temperature stress. This boundary is a "neutral surface." It is assumed that the temperature gradients through the chimney shell at any particular level are the same for the entire circumference so that the original circular form of any cross section is not altered by temperature changes. It is assumed also that the horizontal sections of the unheated chimney are still horizontal after heating and that the temperature gradient through the chimney shell is a straight line. The tensile strength of the concrete is neglected.

**Notation:**

- $t =$ thickness of chimney shell, in., at section under consideration
- $T_1 =$ temperature of the concrete, °F, at inner surface of chimney shell
- $T_2 =$ temperature of the concrete, °F, at outer surface of chimney shell
- $T_3 =$ temperature of the concrete, °F, at neutral surface
- $T_4 =$ temperature of the concrete, °F, at temperature reinforcing
- $T_0 = T_1 - T_3$
- $z =$ ratio of distance between inner surface of chimney shell and vertical reinforcement to total shell thickness $t$
- $k =$ ratio of distance between inner surface and "neutral surface" to total shell thickness $t$
- $f_{ct} =$ compressive stress in the concrete at inner surface of chimney shell, psi, due to temperature
- $f_{st} =$ tensile stress in the vertical reinforcement, psi, due to temperature
- $E_c =$ modulus of elasticity of concrete in compression
- $E_s =$ modulus of elasticity of steel
- $n =$ $E_s/E_c$
- $L =$ thermal coefficient of expansion of concrete and reinforcing steel assumed equal
- $A_c =$ area of concrete shell, sq in., at horizontal section under consideration
- $A_r =$ area of vertical reinforcement, sq in., at horizontal section under consideration
- $P =$ $A_r/A_c$
Fig. 7-4. Stress intensity relationships across chimney shell for combining stress due to wind or earthquake shock and dead load from Fig. 7-2 with vertical stress due to temperature from Fig. 7-3.

Notation:

\[ f'c = \text{unit compressive stress, psi, in the concrete at center of chimney shell on leeward side of chimney due to wind and dead load determined from formula (3) or (4), assumed uniform across the shell thickness} \]

\[ n = \text{same as in Fig. 7-2 or Fig. 7-3} \]

\[ z = \text{same as in Fig. 7-3} \]

\[ p = A_s/A_c \text{ same as in Fig. 7-2 or Fig. 7-3} \]

\[ k = \text{same as in Fig. 7-3} \]

\[ t = \text{thickness of concrete shell same as in Fig. 7-2 or Fig. 7-3} \]

\[ f_{st} = \text{maximum unit compressive stress, psi, in the concrete due to temperature only. See Fig. 7-3, formula (7-5)} \]

\[ f_{st} = \text{unit tensile stress, psi, in the vertical reinforcement due to temperature only. See Fig. 7-3, formula (7-6)} \]

\[ f_{c-comb} = \text{maximum unit compressive stress, psi, in the concrete due to wind or earthquake shock and dead load combined with temperature} \]

\[ f_{s-comb} = \text{unit stress, psi, in the vertical reinforcement due to wind or earthquake shock and dead load combined with temperature} \]

\[ k_{comb} = \text{ratio of distance between inner surface of chimney shell and neutral surface resulting from combined wind or earthquake shock, dead load and temperature to total shell thickness} t \]
Fig. 7-5. Partial horizontal section of chimney for circumferential stress due to temperature.

**Assumptions:** The assumptions are the same as for vertical stress due to temperature (Fig. 7-3).

**Notation:** The notation for temperature gradient through the chimney shell on any radial line is the same as shown in Fig. 7-3.

- \( r' \) = ratio of distance between inner surface of chimney shell and circumferential reinforcement to total shell thickness \( t \)
- \( k' \) = ratio of distance between inner surface and "neutral surface" to total shell thickness \( t \)
- \( f'_{ct} \) = compressive stress in the concrete at inner surface of the chimney shell, psi, due to temperature
- \( f'_{st} \) = tensile stress in the circumferential reinforcement, psi, due to temperature. \( E_c, E_s, n, L \) same as for vertical stress due to temperature (Fig. 7-3)
- \( p' \) = ratio of cross-sectional area of circumferential reinforcing steel per unit height of chimney to cross-sectional area of chimney shell per unit height

Where \( \beta = 0 \) (no flue opening), Eq. (7-4) simplifies to

\[
f_s = nf'_{c' \left[ \frac{1 + \cos \alpha}{1 - \cos \alpha} \right]} \quad (7-4a)
\]

Equations (7-1) to (7-4a), inclusive, are used in determining the stresses due only to wind and dead load or earthquake shock and dead load.

Stresses due to temperature difference between the inside and outside of the chimney shell and the combination of these with the stress from external forces of wind or earthquake shock and dead load are now determined. Referring to Fig. 7-3, the formulas for temperature stresses in the concrete and steel measured in a vertical direction are as follows:

\[
f_{ct} = LE_cT_zk \quad (7-5)
\]
\[
f_{st} = LE_sT_z(\alpha - k) \quad (7-6)
\]

- \( L = 0.0000065 \) per inch per degree F
- \( E_c = 1,000f_{cs} \) (28-day strength of concrete, psi)
- \( E_s = 30,000,000 \) psi
- \( T_z \) for the following four cases is indicated below:

1. For unlined chimneys:

\[
T_z = \frac{tD_{sh}}{C_dD_c} \left[ \frac{T - T_0}{1 + \frac{tD_{sh}}{C_dD_c} \frac{D_{sh}}{K_1D_{st}}} \right] \quad (7-7a)
\]

2. For lined chimneys with insulation completely filling the space between the lining and the shell:

\[
T_z = \frac{tD_{sh}}{C_dD_c} \left[ \frac{T - T_0}{1 + \frac{tD_{sh}}{C_dD_c} + \frac{tD_{si}}{C_dD_c} + \frac{tD_{st}}{C_dD_c} \frac{D_{si}}{K_1D_{st}}} \right] \quad (7-7b)
\]
3. For lined chimneys with unventilated air space between the lining and the shell:

\[
T_e = \frac{t\Delta t}{C_e D_e} \left[ \frac{T - T_0}{1 + \frac{t_b\Delta t}{C_b D_b} + \frac{D_b}{K_1 D_b} + \frac{t\Delta t}{C_e D_e} + \frac{D_e}{K_2 D_e}} \right]
\]  
(7-7c)

4. For lined chimneys with a ventilated air space between the lining and the shell:

\[
T_e = \frac{t\Delta t}{C_e D_e} \left[ \frac{T - T_0}{1 + \frac{t_b\Delta t}{r_1 C_b D_b} + \frac{D_b}{K_1 D_b} + \frac{t\Delta t}{C_e D_e} + \frac{D_e}{K_2 D_e}} \right]
\]  
(7-7d)

where \( r_e \) = ratio of heat transmission through chimney shell to heat transmission through lining for chimneys with ventilated air spaces

\( t \) = thickness of concrete shell, in.

\( t_e \) = thickness of air space or insulation, in.

\( t_b \) = thickness of lining, in.

\( T \) = maximum temperature of gas inside chimney, °F

\( T_0 \) = minimum temperature of outside air surrounding chimney, °F

\( C_e \) = coefficient of thermal conductivity of the concrete of chimney shell, Btu per sq ft per in. of thickness per hr per degree difference in temperature

\( C_b \) = coefficient of thermal conductivity of chimney lining, Btu per sq ft per in. of thickness per hr per degree difference in temperature

\( C_s \) = coefficient of thermal conductivity of insulation between lining and shell, Btu per sq ft per in. of thickness per hr per degree difference in temperature

\( K_1 \) = coefficient of heat transmission from gas to inner surface of chimney lining when chimney is lined, or to inner surface of chimney shell when chimney is unlined, Btu per sq ft per hr per degree difference in temperature

\( K_2 \) = coefficient of heat transmission from outside surface of chimney shell to surrounding air, Btu per sq ft per hr per degree difference in temperature

\( K_r \) = coefficient of heat transfer by radiation between outside surface of lining and inside surface of concrete chimney shell, Btu per sq ft per hr per degree difference in temperature

\( K_s \) = coefficient of heat transfer between outside surface of lining and inside surface of shell for chimneys with ventilated air spaces, Btu per sq ft per hr per degree difference in temperature

\( D_b \) = inside diameter of lining, ft

\( D_e \) = mean diameter of lining, ft

\( D_s \) = mean diameter of space between lining and shell, ft

\( D_{si} \) = inside diameter of concrete chimney shell, ft

\( D_{co} \) = mean diameter of concrete chimney shell, ft

\[
k = -pn + \sqrt{pn(pn + 2z)}
\]  
(7-8)

With a minimum concrete coverage of 2 in., for all practical purposes,

\[
z = \frac{t - 2.50}{t}
\]  
(7-9)

Vertical stress due to temperature is combined with stress due to wind or earthquake shock and dead load by means of the following (refer to Fig. 7-4):

\[
k_{comb} = -pn + \sqrt{pn(pn + 2z) + 2k(1 + pn) \frac{f'}{f_{cta}}}
\]  
(7-10)
When $k_{comb}$ is equal to or less than unity, then

$$f_{e \, comb} = \frac{f_{ck} k_{comb}}{k}$$  \hspace{1cm} (7-11)

When $k_{comb}$ is equal to or greater than unity, then

$$f_{e \, comb} = f'_{e} + \frac{f_{ct}}{k} \left[ \frac{2pn z + 1}{2(1 + pn)} \right]$$  \hspace{1cm} (7-12)

The combined tensile stress in the steel due to wind or earthquake shock and dead load is given by

$$f_{s \, comb} = \frac{f_{st}}{z - k} \left[ z + pn - \sqrt{pn(pn + 2z) - 2pn(z - k) f_{s}} \right]$$  \hspace{1cm} (7-13)

Circumferential stresses due to temperature are computed by means of the following formulas using the same assumptions and conditions as in the case of vertical-stress determination. Refer to Fig. 7-5.

$$f'_{st} = L E_{s} T_{s} k'$$  \hspace{1cm} (7-14)

$$f'_{st} = L E_{s} T_{s} (z' - k')$$  \hspace{1cm} (7-15)

$L$, $E_{s}$, $E_{t}$, and $T_{s}$ are the same as given for Eqs. (7-5), (7-6), and (7-7),

$$k' = -p'n + \sqrt{p'n(p'n + 2s')}$$  \hspace{1cm} (7-16)

with a minimum concrete coverage of 2 in., for all practical purposes,

$$z' = \frac{t - 3.25}{t}$$  \hspace{1cm} (7-17)

Allowable working stresses in the concrete are dependent upon the ultimate compression strength at 28 days ($f_{cs}$). In using the formulas given here, the following values should not be exceeded:

- $f_{e}$ from Eq. (7-3) or (7-3a)
  - Due to wind and dead load: 0.25$f_{cs}$
  - Due to earthquake shock and dead load: 0.375$f_{cs}$

- $f_{ct}$ from Eq. (7-5): 0.40$f_{cs}$
- $f_{s \, comb}$ from Eq. (7-11) or (7-12)
  - Due to wind, dead load, and temperature: 0.67$f_{cs}$
  - Due to earthquake, dead load, and temperature: 0.67$f_{cs}$

- $f'_{st}$ from Eq. (7-14): 0.40$f_{cs}$

Tensile stresses in reinforcing steel should not exceed the following:

- $f_{s}$ from Eq. (7-4) or (7-4a)
  - Due to wind and dead load: 12,500 psi
  - Due to earthquake and dead load: 15,000 psi
- $f_{st}$ from Eq. (7-6): 20,000 psi
- $f_{s \, comb}$ from Eq. (7-13)
  - Due to wind, dead load, and temperature: 27,000 psi
  - Due to earthquake, dead load, and temperature: 27,000 psi
- $f'_{st}$ from Eq. (7-15): 20,000 psi

In addition to the designed reinforcement, additional reinforcing steel is placed at the sides, the top and bottom, and the corners of flue openings. Unless otherwise specified, the minimum vertical reinforcement should be $\frac{1}{2}$ of 1 per cent of the concrete area, and the vertical bars should not be smaller than No. 4 bars nor should they be spaced more than 12 in. apart. Accepted practice requires the circumferential rein-
Fig. 7-4. Custodia Construction Co., Inc., Concrete Design Chart: values of $\alpha$ for $n = 10$ and $\beta = 0^\circ$. 

$\text{As}/A_c$
Fig. 7-7, Custodia Construction Co., Inc., Concrete Design Chart: values of $\alpha$ for $n = 10$ and $\beta = 20^\circ$. 

$\rho = \frac{A_p}{Ac}$

Legend:
- $\alpha = \beta = 30^\circ$
- $\alpha = 55^\circ$, $\beta = 30^\circ$
- $\alpha = 55^\circ$, $\beta = 10^\circ$

7-12
Fig. 7-8. Custodis Construction Co., Inc., Concrete Design Chart: values of $\alpha$ for $n = 10$ and $\beta = 30^\circ$.
forcement to be not less than No. 3 bars spaced 6 in. center to center. Also except for very small chimneys, the minimum thickness of the concrete wall should be 6 in.

It is seen that Eq. (7-1) or (7-1a) cannot readily be solved for values of the angle $\alpha$. However, by assuming a definite value for $n$ and the angle $\alpha$, a series of curves may be plotted from which the value of angle $\alpha$ may be read for varying values of the ratio $e/r$ and steel percentage $p$. Figure 7-6 is a chart showing such a series of curves. It is based on $\beta = 0$ and $n = 10$. Figures 7-7 and 7-8 are similar charts based on $n = 10$ and $\beta = 20$ and 30°, respectively, and interpolation of these curves can be made for intermediate values of $\beta$. The design of the stack is by trial and analysis, and the use of charts shown in Figs. 7-6 to 7-8 will be demonstrated through an actual design found at the end of this section. Figures 7-9 to 7-11 are used to obtain functions of the angle which simplify the expressions given in Eqs. (7-2) or (7-2a), (7-3) or (7-3a), and (7-4) or (7-4a). The design example also illustrates their use. Similar curves for other values of $n$ can be found in ACI Specification 505-54.

![Chart showing curves for different values of $\alpha$ and $\beta$.](chart)

**Fig. 7-9.** Custodis Construction Co., Inc., Concrete Design Chart: variables affecting wind and dead load for $n = 10, 12, 15$ and $\beta = 0°$.

The above equations also apply to a section having two openings of equal width 180° apart. Where there are two openings that are not diametrically opposite, the equations do not apply, and solution of such a section requires special consideration. Such a condition is rarely required in a chimney, and a reasonable approximate solution to such a situation is to substitute for computation purposes a section with a single wide opening having the same critical moment of inertia as the section with the two openings. It should be noted from Fig. 7-2 that, when the angle $\alpha$ equals 180°, the neutral axis has shifted to the windward side of the stack and the full area of the section is under compression. In this case, Eqs. (7-1) and (7-1a) reduce to 0.50. Therefore, when $e/r$ is less than 0.50, the ordinary cantilever beam formula may be used to determine the maximum stress in the concrete. In that case,

$$f_e = \frac{W}{A_e} + \frac{M}{SM}$$  

(7-18)

where $SM$ = section modulus and all other symbols are as indicated in Fig. 7-2.
It is sometimes necessary to compute the maximum deflection at the top of the chimney, particularly in the case of a chimney having a full-height independent brick lining, where good practice requires that deflection of the top of the outer column be not more than half the annular space between the outer column and the lining.

Assuming the chimney to be a simple elastic cantilever, neglecting the reinforcing steel except to assume that it resists plastic deformation, the deflection is easily computed by the slope-deviation method. The chimney is divided into a number of equal sections, and the wind moment in inch-pounds and the moment of inertia of the section in inches$^4$ is computed at the bottom of each section. An $M/I$ curve is then plotted, and the area under the curve is broken down into triangles all of which have a base normal to the original position of the chimney. The moment of area of each triangle
Fig. 7-11. Custodis Construction Co., Inc., Concrete Design Chart: variables affecting wind and dead load for \( n = 10, 12, 15 \) and \( \beta = 30^\circ \).

about the top of the chimney is computed, and the sum of the moments of area of the triangles divided by \( E_c \) in psi gives the deflection of the top of the chimney in inches.

The design of a typical reinforced-concrete chimney and foundation is demonstrated at the end of this section. A similar procedure is used to analyze an existing chimney for which sufficient data are available.

**FOUNDATIONS**

The chimney foundation must be carefully designed in accordance with standard methods. Particular care should be used to avoid unequal settlement and complete data on the safe bearing power of the soil or supporting substrata must be known.

The allowable loading values for soil or piles should be in accordance with the best
engineering practice for the locality. If necessary, a soil test must be made to ensure that the maximum safe bearing value is not exceeded.

A careful examination of subsurface conditions should always be made to check the nature and thickness of the strata from the natural surface down to such depths as will leave no doubt regarding the safety of the structure. This may be done by borings with a soil auger or a pipe drill where a spread footing is to be placed. If soil conditions continue favorable for a depth of 25 to 30 ft below the proposed footing level, it may be considered safe to proceed with the construction.

If the foundation is to rest on stratified clay or bedrock and the exposed surface is not level, then the footing should be stepped in a succession of horizontal planes to prevent sliding.

Pending actual test data, allowable bearing values for various soils may be selected using the following table as a guide. Values are in pounds per square foot.

<table>
<thead>
<tr>
<th>Soil Type</th>
<th>Bearing Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clay, moist</td>
<td>2,000</td>
</tr>
<tr>
<td>Sand, clean and dry</td>
<td>4,000</td>
</tr>
<tr>
<td>Clay, moderately dry</td>
<td>5,000</td>
</tr>
<tr>
<td>Sand and gravel, well compacted</td>
<td>6,000</td>
</tr>
<tr>
<td>Gravel and coarse sand, well cemented</td>
<td>8,000</td>
</tr>
<tr>
<td>Hardpan or stratified clay</td>
<td>10,000</td>
</tr>
<tr>
<td>Shale</td>
<td>12,000</td>
</tr>
<tr>
<td>Bedrock</td>
<td>60,000</td>
</tr>
</tbody>
</table>

In general, the foundation is designed for one of the three following types:

1. Spread footing on soil
2. Spread footing on piles
3. Caisson footing to rock or hardpan

A spread foundation to rest on soil or piles is usually round or octagonal in plan since such a section offers equal resistance to overturning in all directions. The slab is made thick enough so that unit stresses from shear and bending do not exceed permissible values. Working stresses as given in Building Code Requirements for Reinforced Concrete published by the American Concrete Institute (ACI 318-56) may be used and are in accord with good practice.

The bottom of the foundation should be below the depth of greatest frost penetration, usually taken as 4 to 5 ft minimum in northern latitudes and not less than 3 ft in the southern part of the United States.

A spread footing may be of uniform thickness or it may be sloped or stepped from a minimum at the edge to the maximum at the chimney face. The latter design is most often used on large footings to conserve material. Footings with small overhang or projection beyond the stack face are usually constructed of uniform thickness since the saving in concrete is more than offset by added form cost.

If a sloped or stepped footing is used, a careful check of stresses due to diagonal tension must be made at the critical section. If resting on piles, the punching shear at the outer pile row should be investigated to ensure that the concrete at that point is not overstressed. Sloped or stepped footings should be placed in a monolithic pour.

Reinforcing steel in the bottom of the footing is usually placed in four layers with the bars in any layer at an angle of 45° with adjacent bars. In large footings, the bars may be spaced at the center to facilitate placing. Vertical dowels for connecting the chimney column to the foundation must be carefully positioned. The lower ends of these bars extend to the bottom of the footing and project above the top of the foundation alternately long and short. The long projection is equal to one form lift plus a minimum lap and the short projection is equal to the minimum lap only. In this manner the vertical reinforcement is started so that not more than one-half of the vertical steel in the chimney column is spliced or lapped at any given horizontal cross section.

In all foundations, the minimum steel coverage should not be less than 3 in. The thickness of the concrete above the reinforcing in the bottom should not be less than 12 in. for spread footings on soil and 18 in. for footings on piles.
Caisson foundations extending to bedrock or hardpan are of two types. These are:

1. Open-well caissons, usually with four or six holes
2. Single large caissons, constructed solid or cored

The type of soil and depth necessary to reach rock determine the design to be used. The single large caisson of cored or solid construction may be used when the depth does not exceed 20 to 25 ft. At greater depth, the open-well design using four or six individual holes is ordinarily more economical. Both types are capped with a concrete slab having a minimum uniform thickness of 4 to 5 ft with projecting dowels as discussed above.

The diameter of the individual caissons depends on the permissible rock load. The minimum diameter is usually 4 ft to facilitate excavation.

A typical design for a spread footing on soil is demonstrated at the end of this section, and Figs. 7-12 to 7-14 show the various types of chimney foundations.

CHIMNEY LININGS

The concrete column must have an adequate lining to protect it against the following conditions:

1. Excessive temperature which will produce stresses in the concrete or reinforcement beyond the allowable limits
2. Abrasive action of the gases due to direct impingement on the concrete
3. Corrosive action from destructive gases

The material used for lining must be suitable to withstand the conditions of service. Carefully selected clay or shale brick are ordinarily used with a lining thickness not less than 4 in. The brick must be laid in mortar which will withstand the action of the gases upon it. Provision must be made to permit the lining to expand both circumferentially and vertically without producing stresses in the outer shell.

In general, chimneys which handle noncorrosive products of combustion may be classified as follows:

A. Low temperature, from 200 to 600°F
B. Medium temperature, from 600 to 1000°F
C. High temperature, from 1000 to 2000°F

Under A are included modern high-pressure boiler stacks and factory heating plant chimneys. Under B are stacks serving such as the steel, oil, glass, and cement industries. Class C will include chimneys used in connection with incinerators, garbage destructors, and special high-temperature metallurgical furnaces.

Linings for class A should in general be not less than one-third the height of the chimney, starting at a point at least 5 ft below the flue opening. Should the gas entrance to the chimney be close to the bottom of the chimney (10 ft or less) the lining should start at the top of the foundation slab. The lining may be constructed in sections using a hard-burned common or perforated radial brick 4 in. thick, laid in mortar consisting of portland cement, sand, and sufficient lime putty or fire clay to make a workable mix. Section heights should not exceed 40 ft and the brickwork can be supported from corbels built out from the shaft (see Fig. 7-12).

Class B linings should extend for the full height of the chimney and be constructed of intermediate heat-duty firebrick laid in fire clay and portland-cement mortar. The lining may be constructed in sections as under class A (see Fig. 7-13).

Class C linings should be built self-supporting and independent of the outer shaft. First-quality firebrick laid in high-temperature cement should be used. For extreme conditions of temperature, a removable or target lining should be constructed inside the main lining. The target lining should extend for a height of 30 to 40 ft starting at the top of the foundation. Its purpose is to receive the direct impingement of the entering gas and reduce thermal shock on the main lining. The main lining should be reinforced with steel bands and vertical stays, forming a "corset" on the exterior of
the brick. The steel bars should be not less than 2- by ¾-in. flats spaced on 4-ft centers in both directions. Special reinforcing buckstays at the flue opening should be installed to permit proper connections of both horizontal and vertical bars. If the flue entrance is at or near the top of the foundation, the footing should be protected by means of air-cooled floor constructed of insulating brick and firebrick supported on firebrick piers so designed as to permit cooling air to flow across the top of the foundation (see Fig. 7-14).

Class A and B linings should be constructed with a minimum clear air space of 2 in. between the brick and the concrete. Class C lining should have not less than 4 in. clear air space except in tall chimneys of high slenderness ratio where this space should be determined by the deflection of the top of the outer column.

In construction of the lining great care must be used to keep the air space clear of all debris. The space should be well vented to atmosphere by means of sleeve openings through the concrete wall. This will permit a movement of cooling air between the brick and concrete and reduce the temperature gradient across the concrete wall.

In chimneys handling corrosive gases at or near the dew point, air vents are not permissible and the space between the lining and the shell may be packed with insulation, of proper design, to prevent temperature loss.

Chimneys which conduct corrosive gases require special study to determine the suitable type of lining and mortar needed. No definite rules can be given. Process chimneys handling sulfur gases at a low temperature must be protected against acid attack. The destructive effect of intermittent operation must be considered in many cases. Modern steam boilers equipped with economizers, air preheaters, or other heat-extracting accessories deliver gases to the chimney at temperatures not far above the dew point. If the fuel contains an appreciable amount of sulfur, an acid condition may readily occur, and a special lining for the full height of the chimney will be necessary. Gases containing hydrofluoric acid or its salts require that a special mortar be used for the lining brickwork. Wherever corrosive gases will be encountered in a chimney, the problem should be referred to a reliable chimney-engineering firm or to a consultant experienced in this type of problem.

CHIMNEY ACCESSORIES

There are a number of accessories that should be considered in connection with a reinforced-concrete chimney. The most important of these are discussed in some detail.

Lightning Protection

Every chimney should have a complete system of lightning protection in accordance with the latest revision of the Code for Protection against Lightning as approved by the American Standards Association. The system consists primarily of copper air terminals around the periphery of the chimney top connected by an encircling cable, two grounding, or down-lead, cables, and ground rods, plates, or grids. Metal accessories and steel breechings or flues should be grounded to the system, and it is recommended that the reinforcing steel be grounded to the system at the bottom to lead away stray currents that may leak into the reinforcement.

Caps

It is the usual practice to provide cast-iron caps for the lining and the outer column of all chimneys having medium- and high-temperature linings, and it is recommended that low-temperature chimneys also be provided with a cast-iron cap for the concrete column. Cast-iron caps are usually made in sections with flanged ends which are bolted together, and the thickness of the cap should not be less than ¾ in. Chimneys which conduct corrosive process gases are usually provided with caps of glazed terra cotta, lead, or a suitable alloy.
Air Vents

Where it is desirable to vent the annular space between the lining and the outer column, cast-iron or cast-aluminum vents having a cross-sectional area of 15 to 20 sq in. are built into the outer column. These vents are spaced equally around the circumference, and it is usual to provide one vent for approximately each 8 ft of circumference.

Cleanout Door

Every chimney should have a door for the primary purpose of periodically cleaning out the deposit of soot and fly ash which accumulates in the bottom of the chimney. A cast-iron door and frame 24 in. wide and 36 in. high is usually specified for this purpose.

Ladder

Unless the chimney is equipped with aviation-obstruction lights which must be serviced on the chimney, or it is provided with openings for sampling the gases or obtaining direct flow or temperature data, the necessity of ascending a reinforced-concrete chimney is infrequent. However, the possibility of necessary repairs to the lightning-protection system, the desirability of inspection at times, and similar reasons make it advisable to provide a means of access to the top of the chimney. Where a standard ladder is provided, it is required by law in many places and advisable as a safety measure to construct on the ladder a series of enclosing rings or a continuous cage. This type of access requires periodic painting and maintenance if it is to be kept in a safe condition. For most conditions, it is suggested that scaling rungs consisting of ½-in. bronze rungs or equivalent about 12 in. wide, projecting about 6 in. from the outside of the chimney shell, be built into the chimney at intervals of form height but not exceeding 10 ft. This construction is practically permanent, requires no maintenance, and can be ascended in a short time by an experienced steeplejack using small scaling ladders, at the infrequent intervals when access to the top of the chimney is required. Scaling rungs of this type do not offer a means of access to regular employees or unauthorized persons and therefore do not constitute a hazard to the owner.

Aviation-obstruction Lights

If a chimney is located near a landing area or along a recognized civil airway the Civil Aeronautics Administration may require that it be equipped with lights that define it as a hazard to aircraft at night. Detailed specifications for the type, number, and locations of the lights on the chimney may be found in the manual titled Obstruction Marking published by the Civil Aeronautics Administration of the U.S. Department of Commerce. The lights may be installed on the chimney in one of three manners as follows:

1. Fixed lights which must be serviced from a balcony or catwalk at the light elevation.
2. Lights mounted on a ring which can be revolved on a bronze track. Such a system is serviced at the light level from the ladder.
3. The use of special disconnecting fixtures which may be lowered to the ground for servicing.

Daylight Aviation-obstruction Marking

In addition to lights, the Civil Aeronautics Administration may also require the chimney to be entirely or partially painted with alternate bands of white and orange to define the chimney as a hazard in the daytime. Care should be taken to select a durable paint for this purpose.
CONCRETE-CHIMNEY CONSTRUCTION

The erection of reinforced-concrete chimneys is a specialized branch of the construction industry, and practically all concrete chimneys are built by firms regularly engaged in this type of work.

Portland cement and concrete aggregates should conform to the latest ASTM specifications, and water must be clean and free from injurious substances. Reinforcing steel should be billet or rail steel deformed bars conforming to the applicable ASTM specification.

Unless recent data are available, a mix design should be made by a reputable laboratory, using samples of the cement and aggregate which will actually be used. The desirable slump for concrete used in chimney construction is 4 to 6 in., and the water-cement ratio should not exceed 6 gal per sack of cement. It is desirable that field tests of the concrete be made, and the specification should state how many samples are to be taken at various levels of the chimney.

Mixing and placing of the concrete should conform to the accepted requirements for all reinforced-concrete construction. Concrete made with portland cement other than Type III should be maintained in a moist condition for the first 7 days after placing. This is accomplished by means of a circumferential water pipe suspended from the bottom of the forms and traveling up with the forms. If Type III concrete is used, the curing period should be at least 3 days. The use of an approved membrane curing compound in lieu of water curing may be considered as a method of accomplishing an equivalent result.

Tapered chimneys are usually built with movable steel forms 10 ft 0 in. or 7 ft 6 in. high. The outer and inner forms are adjustable so that diameters and wall thicknesses called for on the plans can be maintained. Except with chimneys having a very large diameter, where a special construction tower must be used, the forms and the working platform are supported by a four- or six-post framed timber derrick inside the chimney which projects above the working level. The derrick is supported by a cradle of steel cables attached to U-shaped anchors several lifts below the level where the work is being carried on. These anchors are built into the concrete at the top of each lift, and the derrick travels upward as the work progresses. Materials are hoisted to the working platform inside the chimney through an opening in the center of the working platform. A catwalk is suspended below the outer forms for finishing the concrete, adjusting the forms, and servicing the water-curing pipe. The construction of the chimney is a repetition of a cycle of three operations for each lift. These are raising the derrick and the forms, setting the reinforcing steel, and pouring the lift. Where a construction tower is used, the operation is the same except that the tower, which is inside the chimney and rests on the foundation, is added to as the work progresses.

When a framed derrick is used, the derrick is dismantled on completion of the concrete column, and the brick lining is built from a movable scaffold hung from beams set across the top of the concrete column. When a tower is used for building the concrete column, the tower is left in place for construction of the lining, and the tower is dismantled on completion of the entire job.

Plumb chimneys, when built, can be constructed with a sliding form in the same manner as a silo or grain elevator. Care must be taken to ensure even jacking of the form, and if the rate of travel of the form will exceed 6 in. per hr, the concrete should be made with Type III cement, or if made with Type I or Type II cement, it should have added to it an accelerating agent such as calcium chloride so that the newly placed concrete will not deform under the construction loads.

GAS DISPERSION

Most chimneys have the dual function of providing draft and eliminating the pollution of the atmosphere at ground level by the obnoxious products they are discharging. The elimination or reduction of soot in the atmosphere is beyond the scope of this text, and suffice it to say that when the amount of soot and fly ash discharged into the
atmosphere is regulated by law or is otherwise a matter of concern, the soot and fly ash are removed from the gas stream, usually before it enters the chimney, by cinder traps, water sprays, centrifugal collectors, or electrostatic precipitators. These methods as well as the baghouse are also used to remove solid particles from the smoke streams produced by the smelting of nonferrous metals and often are the source of a profitable recovery of by-products of the process.

The elimination of pollution of the atmosphere by gaseous products discharged through the chimney is often the primary factor in determining the height of chimneys used in the metallurgical and process industries. Examples of obnoxious gases discharged through the chimney are sulfur dioxide resulting from the smelting of nonferrous sulfide ores, sulfuric acid mist vented from the spinning rooms of viscose-rayon mills, and the oxides of sulfur resulting from the operation of recovery units at pulp mills.

Smoke concentration at the ground level reaches its peak at a distance about ten times the height of the chimney; and the concentration at the peak area varies inversely as the square of the chimney height. Also concentration at ground level falls off beyond the area of maximum concentration until, after about 50 chimney heights, it varies as the inverse square of the distance from the chimney.

The velocity of the discharge at the top of the chimney and the temperature of the gases also affect the concentration. High discharge velocities and the buoyancy of hot gases have the effect of adding to the chimney height from a dispersion standpoint. High discharge velocities can be produced with little friction loss by means of a venturi throat, usually of stainless steel, built into the top of the chimney. Gas temperatures can be increased by introducing into the chimney combustion gases from steam boilers or an auxiliary furnace commonly called a stack heater, near the base of the chimney.

Atmospheric conditions and the topography of the surrounding area must also be taken into consideration. In periods of dead calm or temperature inversion it is sometimes necessary to curtail operations so as to prevent toxic concentrations at the ground level. Under extremely unfavorable atmospheric conditions in highly industrial areas, the sulfur dioxide resulting from the burning of fuels containing sulfur may reach a toxic concentration, as happened during the heavy fog in London, England, in the winter of 1952-1953.

Where there will be a problem of air pollution in connection with a chimney, the matter should be given careful study and attention before the height of the chimney is finally decided.

**TYPICAL REINFORCED-CONCRETE CHIMNEY SPECIFICATIONS FOR LOW-TEMPERATURE TYPE A LINING** (Fig. 7-12)

**Scope.** The work covered by this specification includes the furnishing of all labor and materials, except otherwise noted, required for the construction of a reinforced concrete chimney complete, including foundation.

The chimney shall have an inside diameter of 60" minimum. The height of the chimney shall be 125' above the foundation.

**Information to Be Submitted.** The Contractor shall submit drawings of the chimney, showing all features of the work, including the thickness and diameter of the concrete chimney shell at all points, and size and position of all reinforcing steel. He shall also submit a full description of all materials which will be used in the construction of the chimney.

The Purchaser shall approve all drawings before construction is started.

The Purchaser reserves the right to make changes in the design or construction at any time, and the Contractor will be compensated only when such changes mean additional labor and materials to those shown on the Contractor's drawings after they have been approved by the Purchaser's Chief Engineer.

**Materials.** The Portland cement shall conform to the latest "Standard Specifications for Tests for Portland Cement" (ASTM C 150, Type I).

The concrete aggregates shall conform to the latest "Specifications for Concrete Aggregates" (ASTM C 33).
TYPICAL REINFORCED-CONCRETE CHIMNEY SPECIFICATIONS 7–23

The water used in the mixing of the materials shall be clean and free from acids, alkalis or organic materials.

Metal reinforcement shall conform to the latest requirements of the Standard Specifications for deformed Billet-Steel Concrete Reinforcement Bars "Intermediate Grade" (ASTM A 15).

Storage of Materials. Cement, aggregates and other materials shall be stored at the work in a manner to prevent deterioration or the intrusion of foreign matter. Particular attention shall be given to the storage of cement in weatherproof structures which will prevent exposure to excessively moist atmospheric conditions. Any material which has deteriorated or has been damaged shall be immediately removed from the work.

Excavation. Excavation and back filling will be done by others.

Foundation. The contractor shall build the reinforced concrete foundation of dimensions shown on the plans. The bottom of the foundation shall be placed not less than 5 feet below grade line.

Concrete in the foundation shall be of such quality as to produce a minimum strength of 3000 pounds per square inch at 28 days. Not more than 6 gallons of water, including moisture in the aggregate, shall be used per bag of cement. All proportions shall be measured by volume.

Foundation shall be properly reinforced to resist all bending and shearing stresses in accordance with standard "Building Regulations for Reinforced Concrete" published by the American Concrete Institute (A.C.I. 318-56). Concrete in the footing shall be placed in a monolithic pour.

Design of the Chimney. The chimney shall be designed to resist deadload, windload, and temperature stresses in accordance with "Standard Specification for the Design and Construction of Reinforced Concrete Chimneys" (ACI 505-54) as published in the JOURNAL of the American Concrete Institute, September, 1954, except as noted.

Data for determining stresses:

- Ultimate compressive strength of concrete in shell at age of 28 days: 3000 psi
- Wind force: 25 psf of projected area
- Weight of concrete per cu. ft.: 150 lbs.
- Weight of brickwork per cubic foot: 120 lbs.
- Maximum temperature flue gases: 600° F.

The minimum thickness of the concrete wall shall be six inches (6”).

Minimum horizontal reinforcement shall consist of \( \frac{3}{8}" \) round rods spaced not further apart than six inches (6”).

Minimum vertical reinforcement shall consist of \( \frac{1}{2}" \) round rods spaced not further apart than twelve inches (12”).

Concrete Proportion and Consistency. The proportions of cement to aggregates for concrete of any given water-cement ratio shall be such as to produce concrete that will work readily into the forms and around the reinforcement without excessive puddling or spading and without permitting segregation of materials or free water to collect on the surface. The proportions of fine to coarse aggregate shall comply with the requirements of A.C.I. Building Code Specification 318. The amount of water in the concrete including moisture in the aggregate as well as that added in the mixing shall not exceed 6 U.S. gallons per 94-pound bag of cement.

The methods of measuring concrete materials shall be such that the proportion of water to cement may be accurately controlled and easily checked throughout the work, and the methods of measuring aggregates shall be sufficiently accurate to insure concrete of reasonably uniform consistency throughout.

If the aggregates are measured by volume in wheelbarrow, then a scale shall be provided and these volume measurements checked whenever in the opinion of the inspector there has been any change in the consistency of the aggregate.

All concrete shall be placed with entrained air of from 3% to 5%. The air entraining agent shall be DAREX or approved equal, added at the mixer in accordance with the manufacturer’s specifications.

Mixing and Placing. The concrete shall be mixed in a batch mixer until there is a uniform distribution of all materials and the mass is uniform in color and homogeneous. Each batch shall be mixed at least one and one-half minutes after all materials are in the mixer.

Ready mix concrete if used shall be mixed and delivered in accordance with the requirements set forth in the "Standard Specification for Ready Mix Concrete" (ASTM C 94).

Concrete shall be handled to the place of final deposit as rapidly as possible by methods which will prevent the separation or loss of any ingredients. Concrete will be deposited as nearly as practicable in its final position to avoid rehandling or excessive flowing in the
forms. Under no circumstances shall concrete that has partially hardened be deposited in the work.

When concreting is once started, it shall be carried on as a continuous operation for the full circumference of the stack shell so that there will be no vertical or inclined construction joints in the shell. Concrete shall be thoroughly compacted by puddling with suitable tools and vibrators during the placing and thoroughly worked around the reinforcement and other embedded fixtures. Excessive puddling which will cause a segregation of materials or free water to rise to the surface shall not be permitted.

The exterior surface of the concrete column shall be kept in a moist condition for seven (7) days after the removal of the forms as the stack progresses upward. A horizontal circular pipe will be hung from immediately below the forms and raised with them. The pipe will be placed so that the water will be delivered against the steel forms and not against the exposed concrete. Water will not be turned into the pipe until the concrete has set sufficiently so that no channeling shall result.

Where a horizontal construction joint is made, all excess water and laitance shall be removed and before depositing new concrete, the hardened surface shall be cleaned, roughened and wet before new concrete is placed.

When the temperature at any time during the 24 hours falls below 35°F, the material going into the concrete shall be heated and protection of the concrete shall comply with “Building Code Requirements for Reinforced Concrete” (ACI 318).

**Forms.** Forms shall conform to the lines and dimensions called for on the plans. They shall be substantially and properly braced and supported so as to maintain their position during concreting and shall be sufficiently tight to prevent leakage. Forms for the stack shell will be made of metal.

Forms will be removed and reset in such a way as to avoid damage to the concrete shell and to avoid disturbing reinforcement projecting above any concrete section to such an extent as to break the bond between this reinforcement and the recently placed concrete.

**Reinforcing Steel.** The metal reinforcement before being placed shall be free from coatings that would affect the bond of the concrete. The reinforcement shall be accurately placed and the circumferential reinforcement will be securely wired to the vertical bars at intervals not over two feet apart, both vertically and circumferentially.

The minimum distance center to center between parallel bars shall be 2 1/2 times the diameter of the bar and not less than two times the maximum size of the coarse aggregate.

Splices in individual bars shall be of sufficient length to develop the maximum stress in the bar by bond stress. In no case shall splices be less than 50 times the diameter of plain bars and 40 diameters for deformed bars.

In the foundation metal reinforcement shall have a minimum cover of 3 inches of concrete, and in the stack shell a minimum cover of 2 inches of concrete.

**Lining.** The chimney shall have a sectional lining extending from the top of the foundation to a height of 55’. The lining shall be constructed of perforated radial brick 4” thick. All the brickwork in the lining shall be laid in Portland cement-lime-sand mortar mixed in proportions 1-2-5.

At the top of the lining sections the air space shall be sealed by means of mineral wool packing.

**Surface Finish.** The contractor shall so construct the chimney that the exterior exposed surface will present a reasonably uniform appearance. Particular attention shall be paid to finishing the construction joints between successive lifts of the stack.

**Lightning Rod.** The chimney shall be equipped with a lightning protection system conforming to the requirements of the Underwriters’ Laboratories.

**Cast Iron Cap.** The head of the chimney shall be surmounted with a sectional flanged cast iron cap of 5/8” metal. The cap shall cover the concrete wall and extend downward over the concrete not less than 2”. The cap shall slope to the inside.

**Air Vents.** Four inch (4”) diameter air vent pipes shall be placed as indicated on the drawings to permit air flow into the space between the lining and the shell. Vents shall be placed on approximately 8’ centers.

**Guarantee.** The Contractor guarantees the chimney against defects of workmanship and materials developed by a wind not exceeding 100 miles per hour and any dry chimney gas temperatures not exceeding 800 degrees Fahrenheit.

Any defects developing from the above causes within one year from date of completion, which are traceable to the Contractor, will be repaired by the Contractor in accordance with his best judgment.

**Completion.** The Contractor shall state the time required to complete the chimney after receipt of signed contract and approved drawings. Drawings for approval shall be submitted within ten days from the date of signed contract.
Fig. 7-12. 125- by 6-ft reinforced-concrete chimney with Type A lining and spread footing on soil.
Fig. 7-13. 110-ft by 4-ft 6-in. reinforced-concrete chimney with Type B lining and pile foundation.
Fig. 7-14. 150-ft by 5-ft 6-in. reinforced-concrete chimney with Type C lining and caisson foundation.
REINFORCED-CONCRETE CHIMNEYS

DESIGN OF REINFORCED-CONCRETE CHIMNEY 125 FT 0 IN. HIGH AND 6 FT 0 IN. INSIDE DIAMETER AT THE TOP, WITH 55 FT 0 IN. OF PERFORATED RADIAL BRICK LINING

Design Data

Wind pressure = 25 psf on projected area
Weight of concrete = 150 pcf
Weight of brick = 120 pcf
Weight of earth fill = 100 pcf

28 day strength of concrete = $f' = 3,000$ psi, and $n = 10$
Select taper of $3\frac{3}{4}''$ per 10'-0''

Fig. 7-15. 125- by 6-ft reinforced-concrete chimney with spread footing on soil—design problem.
TYPICAL REINFORCED-CONCRETE CHIMNEY SPECIFICATIONS 7-29

Soil pressure = 2 tons per sq ft
Maximum temperature of gas in chimney = 500°F
Minimum atmospheric temperature = 0°F

<table>
<thead>
<tr>
<th>See</th>
<th>Height × Mean diam. × Wall thickness</th>
<th>Cubic feet</th>
<th>Tons</th>
<th>Cumulative tons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>70 × 7.59 × 0.50</td>
<td>835</td>
<td>62.6</td>
<td>62.6</td>
</tr>
<tr>
<td>2</td>
<td>25 × 7.90 × 0.50</td>
<td>356</td>
<td>26.7</td>
<td>89.3</td>
</tr>
<tr>
<td>Lining A</td>
<td>25 × 7.90 × 0.33</td>
<td>205</td>
<td>12.3</td>
<td>101.6</td>
</tr>
<tr>
<td>Corbel</td>
<td>5 × 8.63 × 0.25</td>
<td>34</td>
<td>2.5</td>
<td>104.1</td>
</tr>
<tr>
<td>3</td>
<td>10 × 9.34 × 0.58</td>
<td>174</td>
<td>13.1</td>
<td>117.2</td>
</tr>
<tr>
<td>4</td>
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<td>217</td>
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<td>-12</td>
<td>-0.9</td>
<td>132.6</td>
</tr>
<tr>
<td>Minus flue*</td>
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<td>117</td>
<td>8.8</td>
<td>141.4</td>
</tr>
<tr>
<td>6</td>
<td>5 × 10.07 × 0.75</td>
<td>118</td>
<td>8.9</td>
<td>149.2</td>
</tr>
<tr>
<td>Lining B</td>
<td>30 × 8.14 × 0.33</td>
<td>254</td>
<td>15.3</td>
<td>164.5</td>
</tr>
</tbody>
</table>

*Area cut out by opening is assumed to be rectangular.

FIG. 7-16. Volume and weight of chimney.

Wind Moments

\[ M = F\bar{y} \]

where

\[ F = \text{Wind force on chimney above section (average diameter } \times \text{ height } \times \text{ unit pressure)} \]

\[ \bar{y} = \text{Distance to center of pressure} \]

Moment 70'-0'' from Top of Chimney

Average diameter = \((7.00 + 9.18)/2 = 8.09\) ft

\[ F = 8.09 \times 70 \times 25/2000 = 7.1 \text{ tons} \]

\[ \bar{y} = \frac{(2 \times 7.00 + 9.18)}{(7.00 + 9.18)} \times \frac{70}{3} = 33.4 \text{ ft} \]

\[ M = 7.1 \times 33.4 = 237 \text{ ft-tons} \]

Moment 95'-0'' from Top of Chimney

Average diameter = \((7.00 + 9.96)/2 = 8.48\) ft

\[ F = 8.48 \times 95 \times 25/2000 = 10.1 \text{ tons} \]

\[ \bar{y} = \frac{(2 \times 7.00 + 9.96)}{(7.00 + 9.96)} \times \frac{95}{3} = 44.7 \text{ ft} \]

\[ M = 10.1 \times 44.7 = 452 \text{ ft-tons} \]

Moment 120'-0'' from Top of Chimney

Average diameter = \((7.00 + 10.74)/2 = 8.87\) ft

\[ F = 8.87 \times 120 \times 25/2000 = 13.3 \text{ tons} \]

\[ \bar{y} = \frac{(2 \times 7.00 + 10.74)}{(7.00 + 10.74)} \times \frac{120}{3} = 55.8 \text{ ft} \]

\[ M = 13.3 \times 55.8 = 742 \text{ ft-tons} \]

Moment 125'-0'' from Top of Chimney

Average diameter = \((7.00 + 10.90)/2 = 8.95\) ft

\[ F = 8.95 \times 125 \times 25/2000 = 14.0 \text{ tons} \]

\[ \bar{y} = \frac{(2 \times 7.00 + 10.90)}{(7.00 + 10.90)} \times \frac{125}{3} = 58.0 \text{ tons} \]

\[ M = 14.0 \times 58.0 = 812 \text{ ft-tons} \]
REINFORCED-CONCRETE CHIMNEYS

Stresses 95°-0° from Top of Chimney

(a) Stresses due to wind and dead load

\[ A_e = 9.46 \times 0.50 \times \pi = 14.9 \text{ sq ft} \]
\[ \text{Assume } 34 - \#4 \text{ bars} \]
\[ p = \frac{14.9 \times 144}{20.4 \times 144} = 0.0032 \]
\[ \beta = 0\% \]

\[ M = 452 \text{ ft-tons} \]
\[ W = 89.3 \text{ tons} \]
\[ \frac{e}{r} = \frac{452}{4.78 \times 89.3} = 1.07 \]

From charts [Figs. 7-6 and 7-9]: \( \alpha = 67\deg; \quad K = 0.69; \quad A = 0.60; \quad D = 2.28 \)

\[ f'_{e} = \frac{9.46 \times 0.50 \times 0.69}{89.3} = 27.3 \text{ tons per sq ft} = 380 \text{ psi} \]
\[ f_{o} = 380 \left(1 + \frac{0.50}{9.46 \times 0.69}\right) = 413 \text{ psi} \]
\[ f_{s} = 380 \times 10 \times 2.28 = 8675 \text{ psi} \]

(b) Vertical temperature stresses

\[ z = (6.00 - 2.50)/6.00 = 0.583 \]
\[ T_{e} = (500\deg; - 0\deg;) \times 6/(30 + 6) = 83.4\deg; \]
\[ k = -0.032 + \sqrt{0.032 \times (0.032 + 2 \times 0.583)} = 0.164 \]
\[ z-k = 0.583 - 0.164 = 0.419 \]
\[ f'_{d} = 0.0000065 \times 3,000,000 \times 83.4\deg; \times 0.164 = 267 \text{ psi} \]
\[ f'_{i} = 0.0000065 \times 30,000,000 \times 83.4\deg; \times 0.419 = 6800 \text{ psi} \]

(c) Combined stresses

\[ k_{comb} = -0.032 + \sqrt{+0.032 \times (0.032 + 2 \times 0.583) + 2 \times 0.164 \times 1.032 \times \frac{387267}{6800}} = 0.688 \]
\[ f_{o-comb} = 267 \times 0.688/0.164 = 1120 \text{ psi} \]
\[ f_{s-comb} = \frac{6800}{0.419} \left(1 + 0.583 + 0.032 \right) \]
\[ = \frac{-\sqrt{+0.032 \times (0.032 + 2 \times 0.583) - 2 \times 0.032 \times 0.419 \times 8675}}{6800} \]
\[ = 8925 \text{ psi} \]

(d) Circumferential temperature stresses

Assume \#3 bars @ 6° c. to c.

\[ p' = \frac{0.11 \times 1\frac{1}{8}}{6 \times 12} = 0.0031, \quad \text{and } s' = \frac{6.00 - 3.25}{6.00} = 0.458 \]
\[ k' = -0.031 + \sqrt{0.031 \times (0.031 + 2 \times 0.458)} = 0.141 \]
\[ s' - k' = 0.458 - 0.141 = 0.317 \]
\[ f'_{d} = 0.0000065 \times 3,000,000 \times 83.4\deg; \times 0.141 = 229 \text{ psi} \]
\[ f'_{i} = 0.0000065 \times 30,000,000 \times 83.4\deg; \times 0.317 = 5150 \text{ psi} \]

Stresses 120°-0° from Top of Chimney

(a) Stresses due to wind and dead load

\[ \beta = \sin^{-1} \frac{2.00}{4.99} = 0.400; \]
\[ \beta = 23\deg; - 30' \]

\[ M = 742 \text{ ft-tons} \]
\[ W = 140.3 \text{ tons} \]
\[ \frac{e}{r} = \frac{742}{4.99 \times 140.3} = 1.06 \]
\[ A_e = 9.99 \times 0.75 \times \pi \times \frac{312.8}{360} = 20.4 \text{ sq ft} \]
\[ \text{Assume 56 - \#4 bars + 12 - \#6 bars} \]
\[ p = \frac{56 \times 0.20 + 12 \times 0.44}{20.4 \times 144} = 0.0056 \]
TYPICAL REINFORCED-CONCRETE CHIMNEY SPECIFICATIONS 7-31

From charts [Figs. 7-7 and 7-10]: \( \alpha = 78^\circ; K = 0.49; F = 0.65; D = 1.65 \)

\[
f'_e = \frac{140.3}{9.99 \times 0.75 \times 0.49} = 38.2 \text{ tons per sq ft} = 531 \text{ psi}
\]

\[
f_e = 531 \left( 1 + \frac{0.75}{9.99 \times 0.65} \right) = 592 \text{ psi}
\]

\[
f_s = 592 \times 10 \times 1.65 = 9760 \text{ psi}
\]

(b) Vertical temperature stresses

\[
z = \frac{(9.00 - 2.50)/9.00 = 0.723}{z = 0.723}
\]

\[
T_z = \frac{(500^\circ - 0^\circ) \times 9/(30 + 9) = 115^\circ}{T_z = 115^\circ}
\]

\[
k = -0.056 + \frac{\sqrt{0.056(0.056 + 2 \times 0.723)}}{0.234} = 0.234
\]

\[
z - k = 0.723 - 0.234 = 0.489
\]

\[
f_{st} = 0.0000065 \times 3,000,000 \times 115^\circ \times 0.234 = 525 \text{ psi}
\]

\[
f_{st} = 0.0000065 \times 30,000,000 \times 115^\circ \times 0.489 = 10,950 \text{ psi}
\]

(c) Combined stresses

\[
k_{comb} = -0.056 + \frac{\sqrt{+0.056(0.056 + 2 \times 0.723) + 2 \times 0.234 \times 1.056 \times 59.2625}}{0.744}
\]

\[
f_{ecomb} = \frac{(525 \times 0.744)/0.234 = 1670 \text{ psi}}{f_{e-comb} = 1670 \text{ psi}}
\]

\[
f_{s-comb} = \frac{10,950}{0.489} \left( +0.723 + 0.056 \right)
\]

\[
- \sqrt{+0.056(+0.056 + 2 \times 0.723) - 2 \times 0.056 \times 0.489 \times 10.100 \times 10.950}
\]

\[
= 13,350 \text{ psi}
\]

(d) Circumferential temperature stresses

Assume \#3 bars @ 6\" c. to c.

\[
p' = \frac{0.11 \times 1\frac{1}{2}}{9 \times 12} = 0.0020, \quad \text{and} \quad z' = 9.00 - 3.25 = 6.40
\]

\[
k' = -0.020 + \sqrt{0.020(0.020 + 2 \times 0.640) = 0.141}
\]

\[
z' - k' = 0.640 - 0.141 = 0.499
\]

\[
f'_{ct} = 0.0000065 \times 3,000,000 \times 115^\circ \times 0.141 = 316 \text{ psi}
\]

\[
f'_{st} = 0.0000065 \times 30,000,000 \times 115^\circ \times 0.499 = 11,200 \text{ psi}
\]

Stresses 125'-0' from Top of Chimney

(a) Stresses due to wind and dead load

Diameter = 10.90

\[
A_r = 10.15 \times 0.75 \times r = 23.9 \text{ sq ft}
\]

Thickness = 0.75

\[
r = 10.15/2 = 5.08 \text{ ft}
\]

\[
p = 56 \times 0.20 = 23.9 \times 144 = 0.0032
\]

\[
M = 812 \text{ ft-tons}
\]

\[
W = 149.2 \text{ tons}
\]

\[
\beta = 0^\circ
\]

\[
e = \frac{812}{5.08 \times 149.2} = 1.07
\]

From charts [Figs. 7-6 and 7-9]: \( \alpha = 66^\circ; K = 0.68; A = 0.58; D = 2.38 \)

\[
f'_e = \frac{10.15 \times 0.75 \times 0.58}{149.2} = 28.9 \text{ tons per sq ft} = 401 \text{ psi}
\]

\[
f_e = 401 \left( +1 + \frac{0.75}{10.15 \times 0.58} \right) = 452 \text{ psi}
\]

\[
f_s = 401 \times 10 \times 2.38 = 9,550 \text{ psi}
\]

(b) Vertical temperature stresses

\[
z = \frac{(9.00 - 2.50)/9.00 = 0.723}{z = 0.723}
\]

\[
T_z = \frac{(500^\circ - 0^\circ) \times 9/(30 + 9) = 115^\circ}{T_z = 115^\circ}
\]

\[
k = -0.032 + \frac{\sqrt{0.032(0.032 + 2 \times 0.723)}}{0.186}
\]
REINFORCED-CONCRETE CHIMNEYS

\[ z - k = 0.723 - 0.186 = 0.537 \]

\[ f_{ct} = 0.0000065 \times 3,000,000 \times 115^\circ \times 0.186 = 418 \text{ psi} \]

\[ f_{st} = 0.0000065 \times 30,000,000 \times 115^\circ \times 0.537 = 12,050 \text{ psi} \]

(c) Combined stresses

\[ k_{comb} = -0.032 + \sqrt{+0.032(0.032 + 2 \times 0.723) + 2 \times 0.186 \times 1.032 \times \frac{40}{1418}} \]

\[ = 0.612 \]

\[ f_{ecomb} = (418 \times 0.612)/0.186 = 1375 \text{ psi} \]

\[ f_{st-c} = \frac{12,050}{0.537} \left( +0.723 + 0.032 \right) \]

\[ - \sqrt{+0.032(0.032 + 2 \times 0.723) - 2 \times 0.032 \times 0.537 \times \frac{9,550}{12,050}} \]

\[ = 13,800 \text{ psi} \]

(d) Circumferential temperature stresses

Assume 3 bars \( @ 6'' \) c. to c.

\[ p' = \frac{0.11 \times 15}{9 \times 12} = 0.0020, \text{ and } z' = \frac{9.00 - 3.25}{9.00} = 0.640 \]

\[ k' = -0.020 + \sqrt{0.020(0.020 + 2 \times 0.640)} = 0.141 \]

\[ z' - k' = 0.640 - 0.141 = 0.499 \]

\[ f'_{ct} = 0.0000065 \times 3,000,000 \times 115^\circ \times 0.141 = 316 \text{ psi} \]

\[ f'_{st} = 0.0000065 \times 30,000,000 \times 115^\circ \times 0.499 = 11,200 \text{ psi} \]

Steel curve

Height of chimney in ft vs. area of reinforcing steel in sq. in.

Fig. 7-17. Reinforcing-steel requirements for design problem.
Design of Foundation

Fig. 7-18. Footing for design problem.

Area, volume, and weight of foundation

\[
\begin{align*}
0.828 \times 12.5 \times 12.5 &= 129.4 \times 4 = 517.6 \\
0.828 \times 16.5 \times 16.5 &= 225.5 \times 2 = 451.0 \\
0.828 \times 20.5 \times 20.5 &= 348.1 \times 2 = 696.2 \\
1664.8 \times \frac{159}{2000} &= 125.0 \text{ tons}
\end{align*}
\]

Area, volume, and weight of fill

\[
\begin{align*}
348.1 \times 6 &= 2089 \\
-(1664.8 - 517.6)/2 &= -1406 \\
683 \times \frac{109}{2000} &= 34.2 \text{ tons}
\end{align*}
\]

Weight of chimney and lining

\[
\begin{align*}
164.5 \text{ tons}
\end{align*}
\]

Total weight

\[
\begin{align*}
323.7 \text{ tons}
\end{align*}
\]

Wind moment base foundation = \( M_{12} + F \times 8 = 812 + 14 \times 8 = 924 \) ft-tons

Soil pressure

\[
\begin{align*}
k &= \frac{323.7 \pm 924}{348.1 \pm 943.3} = 0.93 \pm 0.98, \text{ and } k_1 = \frac{1.91}{k_2} = -0.05 \text{ tons per sq ft}
\end{align*}
\]

Reinforcing steel

Assume:

1. Bending takes place about face of 12'-6" octagon
2. Soil loading under projection is uniform (maximum soil pressure computed above), and concentrated at the mid-point of the projecting slab
3. \( j = 0.875 \)
4. Average layer of reinforcement is 6" above bottom of lower slab

Compute bending moment for 1 foot strip:

1. Net load = Soil load 1.91 \times 2000 = 3820 psf

   - Counter loads
     - (a) concrete \( 2 \times 150 = 300 \)
     - (b) fill \( 4 \times 100 = 400 = -700 \) psf

   \[
   \begin{align*}
   M &= 3120 \times (10.25 - 6.25) \left( \frac{(10.25 - 6.25) \times 12}{2} \right) = 300,000 \text{"} \#
   \end{align*}
   \]

Compute reinforcing steel:

\[
\begin{align*}
A_s &= \frac{300,000}{20,000 \times 0.875 \times 42} = 0.408 \text{ si per ft}
\end{align*}
\]

Use \#5 bars @ 9" c. to c.

Check shear, bond, fiber stresses:

1. compute \( p, k, \) and \( j \)

   \[
   p = \frac{0.31 \times 12.5}{12 \times 42} = 0.00082, \text{ and } pn = 0.0082
   \]
**REINFORCED-CONCRETE CHIMNEYS**

\[
\begin{align*}
  k &= -0.0082 + \sqrt{2 \times 0.0082 + 0.0082 \times 0.0082} = 0.120 \\
  j &= 1 - 0.120/3 = 0.960
\end{align*}
\]

(2) shear

\[
V = 3120 \times (10.25 - 6.25) = 12,480 \text{ lbs per ft}
\]

\[
v = \frac{12,480}{12 \times 0.960 \times 42} = 25.8 \text{ psi}
\]

(3) bond

\[
u = \frac{12,480}{1.96 \times \frac{1}{2} \times 0.960 \times 42} = 118.5 \text{ psi}
\]

(4) fiber stresses

\[
f_t = \frac{300,000}{0.00082 \times 0.960 \times 12 \times 42 \times 42} = 18,000 \text{ psi}
\]

\[
f_c = 2 \times 0.00082 \times 18,000/0.120 = 246 \text{ psi}
\]
Section 8

DESIGN OF EARTHQUAKE-RESISTANT BUILDINGS

By

C. J. DERRICK, Consulting Structural Engineer, Los Angeles, Calif.

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INTRODUCTION

Some regions are more active, seismically, than others and the violence of earth shocks varies with localities, but a severe earthquake may occur anywhere. Where modern building codes are properly enforced, earthquakes are no longer regarded as disastrous. Modern buildings subjected to earth shocks rarely collapse and even major structural damage is infrequent. Spectacular failures usually may be traced to lack of those precautionary conventions in design and construction that are termed “good practice.”

In seismically active areas, building regulations usually require special earthquake-resistant construction based upon a combination of mathematical analysis and arbitrary rules. There is considerable difference of opinion among engineers with regard to certain major portions of conventional practice in mathematical analysis of earthquake effect. With respect to arbitrary rules, there is almost unanimous agreement. The peculiar nature of the problem is responsible for this curious condition. Mathematical analyses require assumptions which, in view of the irregularity of earthquake motion and the complex nature of structures, are difficult to standardize. The arbitrary rules have been developed from observation of damage patterns which consistently appear. Most experts agree that mathematical analyses should be employed as a guide to asismatic design with far more reliance upon judgment and experience than would be necessary in vertical-load or wind-stress determination.

Although modern practice has nearly eliminated structural failure and greatly reduced other damage, there is still a heavy burden upon investment due to plaster cracking and similar “superficial” damage. Neither the mathematical analyses nor the arbitrary rules prescribed by building codes appear adequate in this respect. Observation shows that both the peculiarities of structures and the vagaries of earthquake motion affect the degree of damage. Prevention of collapse may be achieved by enforcement of building codes but apparently the control of damage cannot be effected, economically, by general prescription.

The conventional mathematical analysis used in asismatic design is based upon a lateral loading similar to wind pressure but proportional to the weight of the structure. This static “inertia” loading is presumed to produce a deformation in the structure identical with the maximum effect of an earthquake. Since stress is proportional to strain, it is customary to compute frame stresses directly from the assumed lateral loading, without calculating the resultant deflections.

Earthquake damage is caused by excessive differential deformation. “Superficial” damage is a variable, depending not only upon frame stresses but also upon the nature of included nonstructural elements. The control of damage is therefore a problem in strain rather than stress.

The wide range of circumstances—skill of designers, variation in lateral-force factors, working stresses near elastic limits, and lack of uniformity in practice—makes the low incidence of structural damage phenomenal. It is probable that most modern structures do not deform sufficiently under earthquake action to overstress the frame. It is certain that most structures do deform sufficiently to overstress extremely brittle nonstructural portions. Satisfactory asismatic design depends upon the engineer's skill in anticipating distortions likely to result from response to earthquake motion. Each structure is a specific problem. There is no simple rule, no routine mathematical analysis, no arbitrary “equivalent lateral force” loading that will apply in all cases. The satisfactory performance of a structure shaken by an earthquake depends upon common-sense design and honest construction.

Experience shows that modern building codes, properly enforced, are adequate as a guarantee of public safety. The control of damage demands the highest type of intelligent earthquake-resistant design. This requires an understanding of both earthquake motion and dynamic structural response, two fields in which there is still much to be discovered. Any competent structural engineer can produce a safe design by following the requirements of a recognized building code which includes asismatic provisions. The control of “superficial” earthquake damage is a specific problem which cannot be reduced to general rules.
EARTHQUAKE MOTION AND ITS "INTENSITY"

Earthquake ground motion is oscillatory, both horizontally and vertically. Within the area most severely affected by a strong earthquake, the motion is complex and irregular, a set of random displacements devoid of pattern in cyclic performance or relative occurrence. Most engineers consider this motion too irregular to produce any considerable amount of cumulative effect.

At moderate distances, 75 to 100 miles, from the epicenter of a strong earthquake, the motion may become somewhat regular. In general, distance increases the period and decreases the acceleration.

For convenience in mathematical calculation, earthquake ground motion sometimes is assumed as simple harmonic motion. Actually, even the relatively smooth motion of distant earthquakes only approximates simple harmonic motion and the violent oscillations near the epicenter are chaotic.

There is no absolute standard for assessing the "severity" of earthquakes. Most observers make reference to the Modified Mercalli Intensity Scale of 1931, which is a modified version of the Mercalli-Cancani scale (by Seiberg, 1923), presented by Dr. Harry O. Wood and Frank Neumann. This scale, with Neumann's values for acceleration (Earthquake Intensity and Related Ground Motion, Neumann, 1954), is given in Table 8-1.

Severe earthquakes in the Pacific Coast region seem to attain a maximum severity of modified Mercalli scale intensity VIII, or slightly higher, with an average maximum acceleration of about 30 per cent of gravity. Some of the effects described in the modified Mercalli scale for intensities of IX, X, and XI have been observed but appear to be the result of local foundation conditions. Modern structures conforming to aseismic provisions of current building codes do not exhibit the extensive damage described as typical in earthquakes of intensity greater than VIII. However, maximum ground accelerations in the order of 30 per cent of gravity are often associated with severe earthquakes in the Pacific Coast region. This indicates that structures in this area might be expected to experience earthquakes of modified Mercalli scale intensity VIII plus, but somewhat less than intensity IX.

From the standpoint of engineering design, this is somewhat misleading because different types of structures exhibit varying degrees of damage when subjected to the same earthquake motion. Generally speaking, flexible structures perform better than rigid structures in severe local earthquakes. When the epicenter is moderately distant, this sensitivity often is reversed. The conventional lateral-force analysis, using an arbitrary "seismic factor," provides a single standard for anticipating the probable maximum effect of a severe local earthquake. Recent observations indicate need for additional criteria to assess the effect of severe, moderately distant earthquakes on certain types of structures.

To the designer, a "severe" earthquake is one which produces serious damage in a structure for which he is responsible. The question of "effectiveness" is more important than "intensity." Earthquake action is a form of oscillatory motion which is limited in maximum acceleration and amount of displacement. Earthquake effect, in a structure, is the distortion produced by that motion. The control of damage is a specific problem which cannot be solved by blanket prescription. In each case, the engineer must establish design conditions based upon two factors: (1) the response peculiarities of the structure and (2) the nature of probable ground motion.

INFLUENCE OF FLEXIBILITY IN ASEISMIC DESIGN

Definition of "Flexibility"

Certain types of structures appear to endure earthquake motion with less damage than others. The taller multistory buildings consistently exhibit less effect than lower, and steel water-tank towers ordinarily are less affected than similar reinforced-concrete structures. These performances are associated with what is loosely termed "rigidity" or "flexibility," both terms being relative.
### Table 8-1. Modified Mercalli Intensity Scale (1931), Abridged

<table>
<thead>
<tr>
<th>Intensity</th>
<th>Acceleration, egs units, max</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Avg</td>
<td>Range</td>
</tr>
<tr>
<td>I</td>
<td>2.3</td>
<td>1-5</td>
</tr>
<tr>
<td>II</td>
<td>3.1</td>
<td>1-8</td>
</tr>
<tr>
<td>III</td>
<td>9.3</td>
<td>2-46</td>
</tr>
<tr>
<td>IV</td>
<td>13.1</td>
<td>2-75</td>
</tr>
<tr>
<td>V</td>
<td>40.0</td>
<td>5-175</td>
</tr>
<tr>
<td>VI</td>
<td>67.0</td>
<td>18-140</td>
</tr>
<tr>
<td>VII</td>
<td>172.0</td>
<td>51-350</td>
</tr>
<tr>
<td>VIII</td>
<td>Estimated, 250</td>
<td>Damage total. Waves seen on ground surface. Lines of sight and level distorted. Objects thrown upward into the air</td>
</tr>
</tbody>
</table>

“Flexibility” is sometimes defined as relative deflection under unit shear. Since earthquake effect is associated with inertia, the term often is used to express relative lateral deflection under a mass-acceleration loading. Another useful comparison is through the “free” or “natural” periods of vibration.

**“Free” or “Natural” Period**

The weighted vertical cantilever in Fig. 8-1 will swing back and forth if the top is deflected and then released. The distance from the central undeflected position to the extreme limit of this motion is termed the “amplitude.” The time interval required to complete an entire oscillation, from one extreme to the other and back again, is termed the “free period.” Within small limits, the “free period” is the same, regardless of the amplitude, the elements of the cantilever passing over their respective amplitudes in the same time, one-fourth of the period.

The performance of “free vibration” may be regarded as a rhythmic transformation of the potential energy of deformation into the kinetic energy of motion and vice versa.
By Hooke's law, the elastic compulsion is always proportional to the distance from the undeflected position at the center between the two amplitudes and is always directed toward that center. Free vibratory motion is therefore simple harmonic motion, and the free period of a simple resonator such as the cantilever in Fig. 8-1, if the influence of the mass of the vertical rod is neglected, is given by

$$T_1(\text{"free period"}) = 2\pi \left(\frac{D}{g}\right)^{1/2}$$

where $D$ is the lateral deflection of the cantilever due to a force of the weight of the top mass and $g$ is the acceleration of gravity. In seconds and inches, this reduces to

$$T_1 = 0.32(D)^{1/2}$$

and for most ordinary structures the "free" period is roughly (in seconds and inches)

$$T_1 = \frac{\sqrt[6]{A}}{g}$$

when $D$ is the deflection at the top when the structure is loaded at the various centers of mass with forces of the respective weights, that is, a "lateral force" of 100 per cent gravity.

"Free" Period of Structures

Calculation of deflections in structures involves two somewhat indeterminate factors, moduli of elasticity and distribution of deformations throughout members involved. Even in simple structures, such as tower-supported water tanks, the determination of "free" period is neither easy nor highly accurate. This has led to use of approximate methods.

The simplest of these is based upon measurements of "free period" in actual structures, mainly by the U.S. Coast and Geodetic Survey and by agencies of the Japanese government. These measurements indicate that ordinary structures less than 150 ft in height tend to exhibit "free periods" proportional to the number of stories, very roughly about 0.08 to 0.10 sec per story.

The Joint Committee of the San Francisco Chapter, ASCE, and the Structural Engineers Association of Northern California, in 1952, proposed a formula:

$$T_1(\text{"free period"}) = 0.05 \frac{H}{b^{1/2}}$$

where $H$ is the height of the "main portion" of the structure, in feet, and $b$ the width in the direction considered. This formula seems to yield values for buildings less than 150 ft in height somewhat lower than those observed instrumentally. (See Lateral Forces of Earthquake and Wind, Trans. ASCE, vol. 117, pp. 716-780, 1952, particu-
larly pp. 756 and 770 of the discussions.) The rough rule, \( T_1 = 0.08 - 0.10 \) (number of stories), appears to be more accurate, at least for structures such as those in Los Angeles, i.e., less than 13 stories in height.

If care is taken to include the influence of “accidental stiffness” and similar factors, the approximation given under “Free” or “Natural” Period, above, will serve as a check on the rough rule.

Structures which are unsymmetrical in stiffness along the principal horizontal axes will exhibit different “free” periods in these directions. Sometimes internal variations in stiffness will cause a structure to develop two or more “free” periods in a single direction. In addition, pronounced torsional “free” periods habitually develop even under microseismic excitation such as street traffic. The tendency of most structures to develop damage at corners indicates that severe earthquake motion probably exaggerates torsional response.

The problem of earthquake-resistant design is complex, involving many variables. Every effort should be made to reduce indeterminacy, and one effective measure is arrangement of resistances in a symmetrical pattern, minimizing rotation.

Earthquake motion is not consistently applied in the direction of the principal axes. Instead, the motion may be diagonally applied at the onset and must be expected to change direction, frequently, throughout the duration of the earthquake. This indicates the advisability of equalizing the “free” periods in the direction of the principal horizontal axes, that is, balancing the stiffnesses in those directions.

**Classification of Structures by “Free Period”**

The “free periods” of ordinary structures vary between about 0.1 and 2.0 sec. Extremely short-period structures tend to exhibit damage similar to that which would result from a lateral pressure due to wind. Longer-period structures also sometimes act in this manner but these also tend toward a deformation in parts as though the upper portion were moving in the opposite direction from the lower portion. This tendency appears to increase with the “free period” and offers a means for arbitrary classification.

Partial deformations, or “response in modes higher than the fundamental,” rarely are observed in ordinary buildings less than six stories in height. However, while buildings with “free periods” of 0.10 to 0.15 sec exhibit damage somewhat proportional to maximum ground accelerations, the effect seems to diminish rapidly as the number of stories increases above two.

**Table 8-2. Classification of Structures by Average Free Period**

<table>
<thead>
<tr>
<th>“Free” period, sec</th>
<th>Average No. of stories</th>
<th>Usual type of response</th>
<th>Arbitrary designation</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0–0.15</td>
<td>1 and 2</td>
<td>“Fundamental” and related to maximum base acceleration</td>
<td>Rigid</td>
</tr>
<tr>
<td>0.15–0.65</td>
<td>2–7</td>
<td>Both “fundamental” and “partial,” proportional effect of base acceleration less</td>
<td>Semirigid</td>
</tr>
<tr>
<td>0.65–1.25</td>
<td>7–12</td>
<td>Both “fundamental” and “partial,” effect more proportional to base amplitude than maximum base acceleration</td>
<td>Semiflexible</td>
</tr>
<tr>
<td>Above 1.25</td>
<td>12 and up</td>
<td>All effects proportional to base amplitudes</td>
<td>Flexible</td>
</tr>
</tbody>
</table>

Buildings more than 6 stories and less than 12 stories in height not only tend to exhibit “second- and third-mode” vibrational response but, in many instances, seem to be susceptible to “resonance effect” or magnification of the ground action produced by severe earthquakes whose epicenters are moderately distant, 75 to 100 miles, from the building site.

Multistory buildings more than 12 stories in height usually exhibit little damage either from severe local ground motion or from the action of moderately distant earthquakes. In these structures, the damage pattern seems to indicate either vibration in
modes higher than the fundamental or amplification due to resonance effect, or a combination. The amount of distortion does not appear to be directly related to maximum ground accelerations.

An arbitrary classification based upon these general tendencies is shown in Table 8-2.

**RESPONSE OF VERTICAL STRUCTURES TO LATERAL BASE DISPLACEMENT**

**Response as a Function of Time**

If the base of a simple resonator, such as the weighted flexible cantilever in Fig. 8-2, is displaced laterally, the initial effect is the formation of a reversed-curve distortion in the vertical member connecting the base with the top weight. The rate at which this curve rises from base to top is a function of time, independent of the distance through which the base moves. As soon as the distortion reaches the top weight, the

\[ t = \frac{T}{4} - \frac{\Delta}{64T} \]

**Fig. 8-2.** Initial response of resonator subjected to base displacement in space, $\Delta$ and time $t$. 
latter commences to rotate contrary to the direction of the base motion and the inflection point between the two limbs of the reversed curve continues to rise until it reaches the weight. The vertical connecting member is now deformed parabolically in approximately the same shape as it would assume in static deflection due to a lateral force applied through the center of mass of the weight.

At the end of a time interval slightly less than one-fourth of the "free" period of the resonator, the top weight will commence to move in the direction of the base motion. The deformation in the cantilever is equal to the distance through which the base has moved in the time required for response, i.e., slightly less than one-fourth of the "free" period.

Initial Distortion as a Function of Base Acceleration

If the base motion is a simple harmonic motion whose "forced" period is equal to the "free" period of the resonator, the maximum acceleration of this motion is equal to

\[ a \text{ (max acceleration)} = \frac{4\pi^2 A}{T_0^2} \]

where \( A \) is the amplitude of the base motion and \( T_0 \) the "forced" period.

In terms of "forced" period, this transforms to

\[ T_0 \text{ ("forced" period)} = 2\pi \left( \frac{A}{a} \right)^{1/2} \]

The "free" period of the resonator is equal to

\[ T_1 \text{ ("free" period)} = 2\pi \left( \frac{D}{g} \right)^{1/2} \]

where \( D \) is the static deflection due to a lateral force equal to the weight of the top mass (neglecting the influence of the vertical rod) and \( g \) is the acceleration of gravity.

Since the "free" and "forced" periods are assumed equal, these expressions may be equated and reduced to

\[ \frac{A}{a} = \frac{D}{g} \quad \text{or} \quad A = D \frac{a}{g} \]

Since, at the instant of inception of top motion, the base has nearly reached the center between the two amplitudes (the response time being nearly one-fourth of the "free" period), the deformation is approximately equal to the amplitude of the base motion. Hence the deformation is approximately the same as the static deflection due to a lateral force whose mass is the mass of the top weight and whose acceleration is the acceleration of the "forced" base motion.

It can be shown that, for simple harmonic motion of period equal to that of the resonator, or so-called "resonance" period, the distortion will be nearly doubled by the reversal of motion at the farther end of the double amplitude. In these circumstances, the maximum first-cycle response effect associates with a lateral force whose "seismic coefficient" is twice the value indicated by the maximum base acceleration.

Two practical considerations are worth noting: The first is that seismologists have been unable to identify simple harmonic motion in severe earthquake oscillations. The second is that severe earthquake motion habitually exhibits maximum accelerations of as high as 30 per cent gravity, justifying a "seismic factor" of 60 per cent of the weight, whereas earthquake damage indicates action by lateral forces of the order of 20 per cent gravity.

Oscillations Other Than Simple Harmonic Motion

If a point moving at uniform velocity around the circumference of a circle is projected upon a diameter (Fig. 8-3a) the projection will oscillate back and forth along
the diameter in simple harmonic motion. The time required for the generating point to complete a single revolution is the "period" and the radius of the circle is the

\[ \alpha = \text{acceleration} = \frac{4\pi^2A}{T^2} \]

Variation in accel., shown in graph below each motion diagram

Conjugate orbit, a circle, \( R = A \)

Simple harmonic motion

(a)

Conjugate orbit, an ellipse, \( B > A \)

(b)

Conjugate orbit, a rectangle, \( B < A \)

(d)

Conjugate orbit, an ellipse, \( B < A \)

(c)

Ideal Rogers-Jacobsen motion

(e)

Fig. 8-3. Types of oscillatory motion with periods and lateral amplitudes equal in all types shown.

"amplitude." The acceleration of the projection of the generating point is always proportional to its distance from the center of the circle, and directed toward that center, and the maximum value of acceleration is

\[ a = \frac{4\pi^2A}{T^2} \]

However, if the "conjugate orbit" around which the generating point travels is not a circle, but an oval or ellipse, and the velocity of the generating point is varied so as to maintain the same period and amplitude (in the direction of the oscillation), the maximum acceleration will be more or less than the value for simple harmonic motion, depending upon the orientation of the conjugate orbit.
The variation in maximum acceleration due to changes in type of motion, with constant amplitude and period, is illustrated in Fig. 8-4. This orbit is composed of two semi-circles connected by tangents parallel with the direction of the oscillation. At the ends of the double amplitude, within the circular portion, the motion is simple harmonic. Within the central portion, the velocity is uniform. The maximum acceleration at the ends will vary with the ratio \( A'/A \) of the radius of the semi-circles to the total displacement from the center and may be computed from the velocity required to move the generating point around the orbit in the total cyclic time, or "period." The right-hand graph shows acceleration variations for various values of \( A'/A \) with a constant amplitude of 1 in. and a constant period of 1 sec.

![Diagram](image)

**T** is period of S.H.M. portion of motion

**T** = total cyclic time or "forced period"

Conjugate orbit symmetrical about X-X, Y-Y.

Velocity of "P" assumed constant for any simultaneous values of total cyclic time, **T**.

Oscillatory displacements = \( A' \), and ratio \( A'/A \)

Total cyclic time = \( T + 4t_c \), where \( t_c \) = time-interval required for "P" to move from \( F' \) to \( O' \)

"Rogers-Jacobsen" Motion (Idealized)

Note:-

Earthquake ground motion tends to irregularity, neither \( A' \) values nor \( 1/4 \) cycle time-intervals equal.

**Fig. 8-4. Effect of type of earthquake motion upon base displacement.**

This shows that the amplitude cannot be computed from the maximum acceleration and total cyclic time unless the nature of the motion is known. If the response distortion is considered as a function of base displacement, that is, due to the energy accepted by the resonator by response, the influence of amplitude is evident.

A surface motion of this type was first observed by Prof. F. J. Rogers, of Stanford University, in 1907, while working with a mass of sand to which a simple harmonic motion was applied at the base. Rogers's conclusions were confirmed by Dr. L. S. Jacobsen, also of Stanford University, in 1928. The "Rogers-Jacobsen" motion is characterized by relatively exaggerated accelerations at the ends of the double amplitude and nearly uniform velocity over the inner portion of the path. The actual orbit appears to be a flat ellipse.

**DAMPING**

Free vibration is the rhythmic transformation of the potential energy of deformation into the kinetic energy of motion and vice versa. The gradual "slowing up" of a resonator in free vibration is due to the systematic consumption of energy by resistance and is evidenced by progressive diminution in the amplitude of motion, the "die-
DAMPING

away.” Most of the resistance is internal friction, of various types, and tends to operate in the same way as fluid resistance, that is, proportional to the velocity of the moving object. This resistance is termed “damping” and by reason of its character, specifically, “viscous damping.” However, “damping” in an elastic system is complex and, for simplicity, the actual effect is “averaged” by a hypothetical resistance termed “equivalent viscous damping,” that is, a fluid resistance that would produce the same result.

\[
\begin{align*}
C &= \text{Damping factor} \\
T_f &= \text{"Free" period of resonator} \\
T_s &= \text{"Forced" period of base motion}
\end{align*}
\]

**Fig. 8-5.** Variation in number of cycles of sustained regular motion required to produce maximum magnification for various period ratios and values of damping.

The simplest concept of damping is consumption of energy developed by distortion. Damping therefore becomes effective in the initial stage of response when the resonator, such as the weighted cantilever in Fig. 8-2, commences to swing forward in “free” oscillation. Observation of earthquake damage shows that damping is effective in even the irregular motion of severe local earthquakes.

If a resonator is subjected to sustained regular “forced” motion, the effect is cumulative and eventually the “free” vibrations will be obliterated, resulting in a fixed pattern of response, the “steady state.” If the forced motion is simple harmonic motion whose period is equal to the free period of the elastic resonator and there is no damping, the amplitude of the response motion will increase progressively to infinity. However, only a small amount of damping is required to control the cumulative effect, and some damping is always present in structures.

The cumulative effect of sustained regular forced motion, while maximum when the periods are approximately equal, is considerable for ratios of “free” to “forced” period ranging from 0.75 to 1.25. The curves in Fig. 8-5, showing the effect of damping upon
response to sustained regular forced motion, were plotted from data obtained experimentally by Dr. L. S. Jacobsen and are discussed in Dynamic Distortions in Structures Subjected to Sudden Earth Shock (H. A. Williams, Trans. ASCE, vol. 102, 1937, particularly discussion by Dr. Jacobsen, pp. 858–864). The upper set of curves shows the maximum cumulative effects and the lower set of curves the number of cycles required to produce them.

Damping often is expressed as a percentage of the amount that would be required to nullify free vibration, that is, as a percentage of "critical damping." For most ordinary structures, the "damping factor," or percentage of "critical damping," ranges from 3 to 25 per cent. In ordinary structures, the value appears to range from 7 to 15 per cent, with a rough average around 10 per cent.

Observation of damage due to severe local earthquakes indicates that, in such circumstances, a combination of irregularity in motion and damping in ordinary structures tends to limit the cumulative effect of successive shocks to about twice the maximum initial distortion, in the fundamental mode.

Structures of the "semiflexible" type sometimes appear sensitive to the action of severe earthquakes occurring at moderate distances, 75 to 100 miles, from the building site. The terminal motion usually is quite regular and characterized by relatively low accelerations, 1 to 5 per cent of gravity, and moderate to long "forced periods," 0.5 to 2.0 sec and higher.

The range of period ratio, "free" to "forced," of 0.75 to 1.25 and the relatively small number of successive oscillations, four or five, which Dr. Jacobsen's observations indicate as critical in producing maximum magnification, seem to explain some of the observed damage from moderately distant earthquakes. It is evident that damping is highly influential in controlling distortions due to such motion.

The maximum magnification in "semiflexible" structures appears to range from 4.0 to 7.0, depending upon amount of damping and the regularity of ground motion. In average circumstances, a monolithic reinforced-concrete structure probably will exhibit from 3.0 to 5.0 magnification and a framed structure, either steel or reinforced concrete, with unit masonry walls, somewhat less, perhaps 2.0 to 4.0 times the base stimulus. This accords with the evidence of damage which shows that lightly damped "semiflexible" structures are sometimes more affected by moderately distant than by near severe earthquakes.

INFLUENCE OF FOUNDATION CONDITIONS

Lack of unanimity of opinion among American experts has led to a growing tendency to omit foundation influence in specifications of "lateral-force factors." Most American codes, with the exception of certain underwriters' specifications, do not discriminate between types of construction even when foundation influence is recognized. In Japan, these factors are considered simultaneously, as, for example, in the recommendations of the Seismic Research Group, Architectural Institute of Japan (Table 8-3).

Table 8-3. Percentage Effect of Soil Type on Seismic Factor*

<table>
<thead>
<tr>
<th>Type of soil</th>
<th>Bearing power, kips/sq ft</th>
<th>% of max &quot;seismic factor&quot; for different types of construction</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Wood and light steel</td>
</tr>
<tr>
<td>Soft</td>
<td>1.0 or less</td>
<td>100</td>
</tr>
<tr>
<td>Intermediate</td>
<td>4.0 or more</td>
<td>80</td>
</tr>
<tr>
<td>Hard</td>
<td></td>
<td>60</td>
</tr>
</tbody>
</table>

These recommendations reflect a curious observation which appears to have first been made by Prof. R. R. Martel, of the California Institute of Technology, during a study of the 1933 earthquake in the Long Beach, Calif., district. Professor Martel, examining Type III buildings, which are of the "masonry" type, concluded that the damage appeared to be slightly more apparent in buildings founded upon hard material than in those of comparable construction resting on softer strata. This evidence was contrary to the accepted idea that soft foundations predisposed structures to increased damage. The anomaly is restricted to Type III structures, which are relatively short period and ordinarily brittle. The paradox is readily explained by the concept of response distortion since ground amplitudes tend to be much greater on soft soil than upon hard soil.

The subject of foundation influence is highly controversial, and the evidence is sometimes contradictory. Formerly, the tendency of opinion favored a belief that the "free" periods of structures were considerably increased by foundation yielding. This would tend to increase damage. However, recent studies discount this theory, and it appears that "free" periods do not increase appreciably for the amount of yielding that normally would be permitted in design, probably less than 10 per cent, which is small in comparison with other influences.

The concept of distortion proportional to base displacement offers an approach to solution of this problem. However, presently available evidence is too scanty for authoritative recommendations. The Japanese recommendations appear to be the best available substitute.

THE ACCELERATION-PERIOD RELATIONSHIP

For some time, observers have noted that, on hard rock, maximum accelerations tend to accompany rather short "forced periods," usually around 0.3 sec. In Japan, certain "firm" foundation strata seem to exhibit a coincidence of maximum accelerations with "forced periods" of around 0.6 sec, and on soft soil the "destructive" period is around 1.0 sec.

![Graph](image)

**Fig. 8-6. Typical acceleration-period variation.**

Frank Neumann, formerly of the U.S. Coast and Geodetic Survey, after an exhaustive study of accelerograms has postulated a relationship between maximum ground accelerations and their accompanying periods of oscillation. The subject is discussed in *Earthquake Intensity and Related Ground Motion*, by Mr. Neumann, of the University of Washington. The graph in Fig. 8-6 is replotted from Mr. Neumann's charts and reproduced by his permission.

The curve in Fig. 8-6 is for ground motion on deep sedimentary rock and should be
considered as an envelope rather than as absolute determinations. Mr. Neumann, like Japanese seismologists, has observed certain “peak” values at 0.3-, 0.6-, and 1.0-sec periods. Dr. Beno Gutenberg, of the Carnegie Seismological Laboratory, Pasadena, Calif., has made similar observations which are discussed in Earthquake Investigations in California, 1934–1935, U.S. Coast and Geodetic Survey Special Publication 201.

The wide discrepancy between measured values of maximum ground accelerations and “equivalent lateral forces” derived from observation of damage inhibits direct use of the acceleration-period relationship as a design tool. However, for period values greater than 0.3 sec, the shape of this curve is somewhat similar to the “flexibility-reduction” curve derived from the Los Angeles City Code formula:

\[
C = \frac{0.60}{N + 4.5}
\]

The amplitudes corresponding to “forced” periods, for simple harmonic motion, would yield a similar curve since, for this motion, acceleration is directly proportional to amplitude. However, earthquake motion within the epicentral area is not identifiable as simple harmonic motion. It is possible that the nature of the motion, that is, the relationship between amplitude and maximum acceleration may vary with period. In this case, the amplitude-period relationship might be used as a basis for estimating the energy accepted by a resonator in response to applied displacements. This would be useful for predicting dynamic distortions in both “semiflexible” and “flexible” types of buildings.

**EFFECT OF VERTICAL ACCELERATION IN EARTHQUAKE MOTION**

**Magnitude in Relation to Normal Working Stresses**

The vertical component of earthquake motion habitually develops maximum accelerations around one-third of gravity. This is considered well within the “factor of safety” for vertical working stresses in columns and foundations where the usual practice is to allow a one-third increase in stress for combined vertical and lateral loads. Most codes limit the proportion of dead load considered effective in reducing tensile stresses due to the horizontal effect of earthquake motion.

**Effect on Girder-to-column Connections**

Longitudinal “free” periods are shorter than transverse “free” periods; hence the response is quicker. The vertical acceleration should affect girder-to-column connections in proportion to the relative response of the floor system to the motion of the columns. The exact nature of this effect is indeterminate but it indicates the advisability of liberal design, particularly in diagonal tension in reinforced-concrete members. Some conservative designers employ riveted connections for sustenance of vertical loads to avoid the possibility of failure in detail in structural-steel connections. There is no positive evidence that welded connections are not equally satisfactory.

**ASEISMIC DESIGN BY “EQUIVALENT LATERAL FORCE” METHOD**

**Degrees of Earthquake Damage**

Earthquake damage may be divided into three broad classifications:

1. Severe structural damage, causing total or extensive partial collapse and accompanied by widespread injury to included nonstructural elements—“superficial” damage
2. Structural damage not resulting in collapse but requiring expensive repairs and accompanied by severe “superficial” damage
3. “Superficial” damage, not accompanied by structural failure, varying in degree from minor to severe
The degree of protection against earthquake effect to be afforded by a design specification, such as a building code, depends upon the extent of damage which it is intended to prevent. Most of the major differences in building-code requirements stem from this variable.

**Regional Variations in Earthquake "Intensity"**

Earthquakes are more prevalent in some regions than in others, and even when the pattern of occurrence is comparable, the maximum "severity" of the ground motion tends to vary. A design specification barely adequate for protection against the first type of damage in one locality might suffice as insurance against all three in another district. In addition, variations in the pattern of occurrence influence the economics of the problem.

**Damage a Function of Strain**

The effect of earthquake ground motion is to produce distortion in a structure. All damage is related to differential displacements of successive elements and hence to the pattern of distortion as well as to its over-all amount. Structural damage in modern buildings designed for earthquake resistance is comparatively rare even when extensive "superficial" damage occurs. This indicates that such structures are not deformed sufficiently to overstrain the framework or other "resisting" elements but enough to damage attached or included nonstructural portions.

Control of damage requires an accurate assessment of probable maximum relative displacements throughout a structure, i.e., the patterns of distortion, liable to occur as a result of earthquake ground motion. There are at least three possible patterns in ordinary structures.

The first is the response in the "fundamental" mode resulting from violent but irregular ground motion. Here the structure deforms continuously, in the same direction, from bottom to top in a shape that is similar to but not identical with that produced by a uniform acceleration acting upon its several masses.

The second is the response, also in the "fundamental" mode, caused by the relatively "low-intensity" but regular motion produced by moderately distant severe earthquakes. The shape of the distortion pattern is apparently similar to the first type.

The third pattern is the response in "higher modes," one portion of the structure moving in one direction while another, above or below, or both, is moving oppositely. Of the modes, the second is probably the most important. In ordinary structures, this response produces a "node" or steady point which is often assumed as located about 0.75 of the height above the base.

All earthquake distortion depends upon inertia and is related to mass. The various patterns of distortion may be produced by static lateral loading proportional to mass, provided that the acceleration variation reflects the difference between static and dynamic action.

**Determination of "Equivalent Lateral Forces"**

The determination of static lateral forces that will produce distortions identical with those due to earthquake involves two factors. One is the magnitude of the base shear and the other the pattern of shear distribution along the height of the structure. Most building codes combine the solution in a single prescription. The Joint Committee, San Francisco ASCE and Structural Engineers Association of Northern California, recommended separating them.

There are three general types of variation in shear distribution along the height of a structure. The simplest is the use of a uniform acceleration at all levels which produces a pattern similar to the effect of gravity loading. In terms of dynamic distor-
tions, this loading may not appear logical but a great many satisfactory designs have resulted from its use.

The second is the "triangular" loading, which is intended to approximate the response of a building deflecting equally in moment and shear. The load at the top is twice the average, diminishing to zero at the base. The shears diminish more slowly, from top to bottom, than with uniform loading. "Triangular" loading is recommended by the Joint Committee, before cited, and approved, with some reservations, by the Seismic Research Group, Architectural Institute of Japan.

The Los Angeles City Code formula, the third type, combines order of magnitude with distribution of base shear and is intended to apply to structures within the height limit of 150 ft or 13 stories. This formula, \[ C = \frac{60}{(N + 4.5)} \]
has been adopted by the California State Division of Architecture and by the Pacific Coast Building Officials Conference Uniform Code. The loading is empirical and tends to establish an envelope including effect of "second-mode" response. Within the Pacific Coast "earthquake zones," it is the most widely used type of prescription.

None of the codes reflects direct association of distortional effect with observed values of maximum ground acceleration. Most of the damage indicates an "equivalent lateral force" in the order of 20 per cent gravity or less. This corresponds to undamped response to a single cycle of simple harmonic motion whose acceleration is 10 per cent gravity and a period ratio of 1.0. Maximum ground accelerations habitually attain 50 per cent gravity or more, which, theoretically, might justify a "seismic factor" of 60 per cent gravity or higher.

The rarity of structural damage in buildings designed for "equivalent lateral forces" computed by the Los Angeles City Code formula indicates that, for control of frame stresses, this loading is adequate.

Effect of Arbitrary Requirements

The relative merit of code prescriptions for "equivalent lateral forces" cannot be assessed without considering the effect of various arbitrary requirements. In general, these empirical precautions are intended to unify the structure and reduce the amount of differential distortion.

The most carefully detailed set of arbitrary design specifications in use within the Pacific Coast area is contained in the Rules and Regulations of the California State Division of Architecture applying to the construction of public school buildings. These restrictions include limitations on the distortion of elements when subjected to "lateral forces" computed by the Los Angeles City Code formula. Both the Los Angeles City Code and the Uniform Code of the Pacific Coast Building Officials Conference also contain arbitrary design requirements, but to a lesser extent.

In addition to published specifications, most building departments employ certain interpretations or "special rulings" which often serve to reduce probable distortions. It is probable that these empirical prescriptions are more effective in reducing "superficial" damage than is the performance of a "lateral-force analysis" based upon "equivalent static loading," although the latter is the accepted criterion for frame stresses.

General Principles of "Lateral-force" Method

In conventional procedure, the equivalent "lateral forces" are assumed as acting horizontally and parallel with the major axes of the structure. It is presumed that any oscillation moving diagonally with the major axes will act componentally and the effect will be less than that produced by the full loading applied along either axis.

The lateral forces are computed for various concentrations of mass such as the various floors of a multistory structure. It is assumed that the inertia effect operates simultaneously at all points over a horizontal concentration such as a floor or roof. Hence, if such an element is sufficiently rigid, laterally, the total computed lateral force is distributed throughout the vertical resisting members in proportion to their relative rigidities. Where such lateral rigidity does not exist, it is customary to apply
a portion of the total lateral force to each vertical resistance, the amount being determined by the designer's estimate of "contributed load," that is, the amount of mass which the adjacent floor system might be considered as carrying to the vertical resistance.

The separation of an articulated structure such as a multistory building into successive levels of mass concentration presupposes a uniform rate of rise in the effect of base motion over the entire area of the structure. This is justified by the assumption of rigidity in the floor or roof system, usually termed a "diaphragm," which prevents differential motion at the tops of vertical resisting members. The existence of lateral rigidity in the various "diaphragms" is essential to the validity of the lateral-force method of stress analysis.

Where vertical resisting members vary in rigidity or there is any variation in distribution of mass at any "diaphragm" level, or both, the center gravity of the total mass will not coincide with the center of gravity of all the resistances. This eccentricity produces a theoretical moment which increases the loading on the "weak side" of the resistance group and decreases loading on the "strong side," the terms "weak" and "strong" used relatively.

Frame stresses are determined by the same methods as used for wind loading. Most building departments will accept either the "moment-distribution" method of Prof. Hardy Cross or modifications of the slope-deflection method such as that presented in Continuity in Concrete Building Frames published by the Portland Cement Association.

To be effective, the "lateral-force" method requires accurate evaluation of probable influence of indeterminate factors such as moduli of elasticity, joint slippage, and resistance of secondary members. The mathematical analysis should be intelligent rather than precise and should be regarded as a guide to design rather than as an absolute solution. The most critical element in the analysis is determination of relative rigidities of resisting members, because it involves deformations under loading.

Relative Rigidities of Resisting Members

The rigidity of a member, relative to another, is determined by comparing their deformations under identical loading. In the "lateral-force" analysis, the equivalent loading often is transferred to walls and other members of large cross section. Shear deformations, as well as those produced by bending moments, are important. In addition, other movements such as footing rotations may require consideration.

In reinforced-concrete and unit-masonry design, proper values for moduli of elasticity, both shear and moment, are often difficult to establish. However, it usually is practicable to use the ratio of the bending and shearing moduli $E/G$ except where other materials, such as structural steel, are involved.

In American practice, the ratio $E/G$ usually is taken at 2.5. Japanese engineers apparently prefer $E/G = 2.3$. The difference is not highly significant. Most American engineers accept an assumption of $E$, the moment modulus, at 1,000 times the assumed 28-day strength of the concrete used.

Relative rigidity deformations usually are computed by American engineers on the basis of gross cross-sectional areas, neglecting reinforcement. Where structural-steel columns occur monolithically with concrete walls, some American engineers include the effect of this concentration of resistance. In Japan, many designers consider the influence of reinforced-concrete columns occurring monolithically with concrete walls.

A high degree of accuracy is required in computing deformations used in determining relative rigidities. One of the chief sources of error is the assumption of effective length for columns and beams. In this respect, American practice differs from that used in Japan. Usually American engineers use center to center of supports as the effective length of continuous beams and the clear span plus one-fourth the depth for simply supported members. With respect to columns and walls, assumptions vary with circumstances and the designer's judgment.

In reinforced-concrete design, Japanese engineers assume a rigid zone at the intersection of the axes of reinforced-concrete members. This zone extends outward, as in Fig. 8-7, from the intersection, to a plane parallel with the face of the element con-
considered and distant from it by one-fourth of the depth of the other member. This makes the effective length of a column equal to the clear height plus one-fourth of the depth of both the top and bottom beams, or half the average depth. The Japanese engineers justify this rule with the evidence of damage.

The technique of computing rigidity deflections under test loading is simple. For example, if the cantilever pier shown in Fig. 8-8a has both ends prevented from rotation, its total deflection will be the sum of the moment and shear deflections due to the assumed top load:

$$D = \frac{PH^3}{12EI} \text{ (moment)} + \frac{6PH}{5AG} \text{ (shear)}$$

where $P$ is the unit test load, $H$ the height, $E$ and $G$ the moduli of elasticity in moment and shear, respectively, $I$ the moment of inertia of the cross section, and $A$ the area of the cross section.

In American practice, this usually is simplified, to facilitate computation, by assuming that the effective length, $H$ in this instance, is identical for moment and shear, and by using a convenient ratio, such as 2.5, for the relationship of $E/G$. By using a unit loading of 1,000,000 lb and whatever 28-day concrete strength is required, as, for example, 3,000 lb, the expression reduces to

$$D = \frac{H^3}{36I} + \frac{H}{A}$$

or, with $t$, the wall thickness, and $d$, its "depth" or length in direction of force,

$$D = \frac{1}{t} \left[ \frac{1}{3} \left( \frac{H}{d} \right)^3 + \frac{H}{d} \right]$$

the form often used.

It should be remembered that this simplification depends upon two approximations: (1) that the effective height is the same for both types of deflection, and (2) that for ordinary concretes, such as 3,000-lb 28-day ultimate, the ratio $E/G$ is 2.5. If, in addition, there is any question about effective moment of inertia or area of cross section, the computations should be separated.

The effect of end rotation, due to lack either of rigidity of connecting members or of footings is shown in Fig. 8-8b.

If the same unit loading is used, as, for example, the customary 1,000,000 lb of American practice, the rigidities of the members compared may be expressed directly as the reciprocals of their deflections. In Fig. 8-9a three members, of varying characteristics, are subjected to the same unit top loading $P$. The rigidities compare as the reciprocals of the deflections, $1/d_1$, $1/d_2$, $1/d_3$. In practice, these quantities sometimes are termed "rigidities" since they express the comparison.

If these members are placed in the same vertical plane, with the tops at the same level, and joined along the line of application of the test force by some device itself
incapable of deformation, as in Fig. 8-9b, they will be compelled to move in unison. The resulting top deflections will be identical. Since they must combine to resist the force, each member will contribute to this effort in proportion to its own rigidity. If the test unit load is the same as that employed in determining the "rigidities," the deflection of the system will be

$$D = \frac{1}{(1/d_1) + (1/d_2) + (1/d_3)}$$

where $d_1$, $d_2$, $d_3$ are the individual deflections under the same unit load.

The proportional resistances contributed by the separate members usually are expressed as percentages of the sum of the "rigidities" or deflection reciprocals.
This is exhibited in the following computation:

<table>
<thead>
<tr>
<th>Computed deflections of individual members under unit load $d$</th>
<th>Reciprocals of computed deflections $1/d$</th>
<th>Relative rigidities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pier A: 0.500(d1)</td>
<td>1/0.500 = 2.0</td>
<td>2.0/8.0 = 0.25</td>
</tr>
<tr>
<td>Pier B: 0.313(d1)</td>
<td>1/0.313 = 3.2</td>
<td>3.2/8.0 = 0.40</td>
</tr>
<tr>
<td>Pier C: 0.358(d1)</td>
<td>1/0.358 = 2.8</td>
<td>2.8/8.0 = 0.35</td>
</tr>
<tr>
<td>Sum</td>
<td>8.0</td>
<td></td>
</tr>
</tbody>
</table>

This example illustrates the effect of end conditions upon the relative rigidities of members composing a system. For instance, if the top connecting member is flexible, as in Fig. 8-9c, the resulting diminution in restraint may seriously affect the distribution. Or, as in Fig. 8-9d, the existence of superimposed doorways may vary the conditions of restraint. In Japan, it is customary to assume that a condition such as Fig. 8-9d may result in failure during the early stages of motion, so that a hinge occurs.

The base of a pier may develop rotation either by tipping of an independent footing or from excessive flexibility in the sill beam. In addition, this action, as well as the flexibility or hinging of the top member, may greatly increase the effective length of the vertical portion.

A practical design method for dealing with the complexities of rigidities has been developed in Japan by Dr. Kiyoshi Muto, Tokyo University. This method employs distribution coefficients which are derived from theoretical rigidities and modified by the results of experimentation and observation of damage. The modification is achieved, not by general parameters, but by separating influencing conditions into types and groups so that simplifying approximations may be made within reasonable limits of accuracy. At present, the scope of the procedure is narrow, being restricted to reinforced-concrete structures and to the more common forms of construction.

**Distribution of "Lateral Force" to Vertical Resisting Members**

The conventional method for distributing “lateral force” or mass-acceleration loading to vertical resisting units is based upon the assumption of rigidity in the lateral distributing system, roof, floor, or other so-called “diaphragm.”

The following illustrative example is quoted, by permission, from *Analysis of Small Monolithic Concrete Buildings for Earthquake Forces*, courtesy of the Portland Cement Association:

**Relative Rigidity of Cross Walls**

Three parallel cross walls, $A$, $B$, and $C$, are shown in Fig. 8-10. If $D_m$ and $D_p$ denote the deflections of the parts $m$ and $p$, when treated separately, the total deflection of Wall $A$ will be

$$D = 2 \times D_m + \frac{1}{3} \times \left(\frac{1}{D_p}\right)$$

Assuming, that for conditions of fixity and quality of concrete the previously stated simplified equation for deflection, $D = 1/t\left\{\frac{1}{3}H/(d)^2 + (H)/(d)\right\}$ is correct,

$$D_m = \frac{1}{10} \left[ \frac{1}{3} \times \left(\frac{2}{14}\right)^2 + \frac{2}{14} \right] = 0.014$$

$$D_p = \frac{1}{10} \left[ \frac{1}{3} \times \left(\frac{8}{2}\right)^2 + \frac{8}{2} \right] = 2.533$$
From the previous equation for the value of $D_A$:

$$D_A = 2 \times 0.014 + \frac{1}{3(1/2.533)} = 0.873$$

Wall $B$ has no openings and will deflect (using the same expression for deflection) as for $D_m$ and $D_p$:

$$D_B = \frac{1}{6} \left[ \frac{1}{3} \times \frac{(12)^3}{(14)^3} + \frac{12}{14} \right] = 0.179$$

Wall $C$ differs from $A$ only in thickness (and the deflections vary inversely as the thicknesses):

$$D_C = \frac{1}{9} \times D_A = 1.455$$

The relative rigidities are listed in Table 8-4.

<table>
<thead>
<tr>
<th>Wall</th>
<th>Deflection</th>
<th>Reciprocal or &quot;rigidity&quot;</th>
<th>Relative rigidity (decimal fraction of total)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>0.873</td>
<td>1.15</td>
<td>0.155</td>
</tr>
<tr>
<td>$B$</td>
<td>0.179</td>
<td>5.59</td>
<td>0.752</td>
</tr>
<tr>
<td>$C$</td>
<td>1.455</td>
<td>0.685</td>
<td>0.093</td>
</tr>
</tbody>
</table>

| Lateral Loads Distributed to Cross Walls |

(\textit{Note:} In the following computation, the "center of mass" or center of gravity of all masses composing the effective inertia load is assumed at the center of the building, disregarding the variation in the supporting walls, which is small. In practice, it is customary to add to the floor the weight of walls, piers, etc., between the mid-points of the stories above and below the floor considered. In this case, half the weight of the walls would be included.)

A horizontal load (i.e., the total design weight multiplied by the percentage of gravity assumed as the design acceleration) of 80,000 pounds is assumed as acting through the
center of mass, assumed to be midway between the end walls. The top distributing slab is assumed infinitely stiff.

The distance, \( z \), from Wall \( A \) to the "center of rigidity" (on the line of action of the resultant of resistances as measured by "relative rigidity," or, the point where a lateral force must be placed in order to produce equal deflections of the resisting units) can be found by taking static moments about the center line of the end wall, using the relative rigidities as weights. [See Fig. 8-11a.] Thus, \( z = 0.752 \times 11 + 0.093 \times 27 = 10.8 \). If

![Deflection diagram for small building shown in Fig. 8-10.](image)

the lateral load (i.e., the assumed "force" of 80,000 pounds) were applied at the "center of rigidity," the walls would deflect equally. For the condition of equal deflections [see Fig. 8-11b] the total force, \( P \), will be distributed to the three walls in proportion to the relative rigidities given in Table 8-4 thus:

\[
P'_{A} = 0.155 \times 80,000 = 12,400
\]
\[
P'_{B} = 0.752 \times 80,000 = 60,160
\]
\[
P'_{C} = 0.093 \times 80,000 = 7,440
\]

The lateral load, however, is not applied at the "center of rigidity" but at the "center of mass"; therefore, in addition to the forces just determined, there are forces acting on the three walls due to the moment (i.e., due to eccentricity):

\[
M_e = P \times z = 80,000 \times (13.5 - 10.8) = 216,000 \text{ ft. lbs.}
\]

In resisting this moment, the walls will deflect as shown in Fig. 8-11c. The forces acting on each wall will be in proportion to their relative rigidities and (at the same time) to the (respective) distances from the "center of rigidity" (i.e., the "effectiveness" of the walls in resisting the eccentric moment); therefore, if the force (developed by the action of the eccentric moment, \( P \times z \)) which is delivered to Wall \( A \) is designated \( P''_A \), the like forces at Wall \( B \) and Wall \( C \), \( P''_B \); \( P''_C \) will be:

\[
P''_B = P''_A \times \frac{\text{Relative Rigidity B}}{\text{Relative Rigidity A}} \times \frac{\text{Moment Arm from Ctr. Rig. } z_B}{\text{Moment Arm from Ctr. Rig. } z_A}
\]

\[
P''_B = P''_A \times \frac{0.752}{0.155} \times \frac{0.2}{10.8}
\]

and,

\[
P''_C = P''_A \times \frac{0.093}{0.155} \times \frac{16.2}{10.8}
\]

The sum of the moments of these forces about the "center of rigidity" must, for equilib-
Aseismic design by "equivalent lateral force" method 8–23

rium, equal the total eccentric moment, \( M_a = P \times \epsilon \), or, 216,000 ft. lbs. Hence:

\[
P''_A(x_A) + P''_B(x_B) + P''C(x_C) = M_a = 216,000 \text{ ft. lbs., or,}
\]

\[
P''_A \left( 10.8 \pm 0.2 \times \frac{0.752}{0.155} \times \frac{0.2}{10.8} + 16.2 \times \frac{0.093}{0.155} \times \frac{16.2}{10.8} \right) = 216,000 \text{ ft. lbs.}
\]

\[
P''_A = 8,500 \text{ lbs.; } P''_B = 765 \text{ lbs.; } P''_C = 7,650 \text{ lbs.}
\]

It is seen from inspection of the deflection diagrams [Fig. 8-11c] that the horizontal forces on \( B \) and \( C \) are increased (by the additional forces due to resisting the eccentric moment) while that acting upon \( A \) is decreased. It is customary, in present practice, in designing the walls, however, to be on the safe side, to disregard any reduction from the values of \( P'_A, P'_B, \) and \( P'_C \) as computed above. Thus, the walls should be designed to resist the following forces:

Wall \( A = 12,400 \) lbs. (Reduction neglected)
Wall \( B = 20,160 + 765 \) lbs.
Wall \( C = 7,440 + 7,650 \) lbs.

It is apparent that, in principle, the distribution of "lateral force" due to earthquake is extremely simple. Even in the case of multistoried structures the same general procedure is applicable since the distribution is effected story by story, the rigidity of the floors being assumed sufficient to adjust the loading where the position of resisting members, with reference to their relative rigidities, may vary from floor to floor.

However, in practical design, the horizontal distributing systems often lack sufficient rigidity to justify the assumptions required by this procedure. This is particularly true in one-story structures which, in many instances, are covered with more or less flexible roofs. One of the extreme and commonly encountered examples is the one-story commercial building, rectangular in plan, in which the front wall is merely a framed opening while the rear wall is more or less solid. It is not uncommon to discover a ratio or 1:30 or 1:40 in the relative rigidities of these resisting members. If the roof is assumed rigid, and the customary analysis performed, the designer is apt to develop practically equal theoretical loading on the two end walls, because of the effect of the rotational eccentric moment.

Even when the roof system is of reinforced concrete there may be sufficient lateral distortion to reduce the validity of the analysis to questionable proportions. In addition, the actual dynamic effect of an earthquake is the result, not of a steady lateral loading, such as wind pressure, but of an oscillation in which the reversed recovery motion may be as important as the initial displacement. If the distance between the end walls is considerable, the static distribution may be completely invalidated by time differentials in response.

For this reason, conservative designers are apt to be extremely liberal in the proportioning of end walls, particularly the front open frames of such commercial buildings, regardless of the apparent rigidity of the roof and the computed design loading. From the performance of actual structures subjected to earthquakes, this practice appears to be satisfactory and the alternative, deliberately opening the opposite wall to reduce its rigidity, does not seem to be either necessary or wise.

When the roof is flexible, occasioned by either wood or light steel construction, it is obvious that any transfer of load, due to rotation, is apt to be small. Most building departments require that such structures be provided with a horizontal diaphragm, usually composed of lateral trussing in the plane of the lower chord of the main trusses or in the ceiling. Some supervisory agencies permit the use of roof sheathing, computed as a horizontal girder, provided the construction meets certain requirements. This lateral bracing undoubtedly is effective in distributing wind loads. There is considerable doubt as to its efficiency in distributing the assumed "inertia loading" prescribed for seismically designed structures.

When the lateral distributing system is too flexible to justify this apportionment of "lateral force," the method of "tributary areas" is often used. Following the concept of a steady lateral "inertia" pressure, similar to wind, the various vertical resisting members are assumed to act independently and each designed to "carry" the mass-acceleration loading derived from a contiguous area estimated to "contribute" lateral
loading to the attached resistance. A typical example is shown in Fig. 8-12, where the "diaphragm" is formed of round rods trussing the lower element of the roof system.

If internal resistances of considerable rigidity occur along the length of the distributing truss, the solution becomes complicated by the indeterminacy of simultaneous action in the truss and the support. When the probability of phase differences in motion is considered, the static solution becomes increasingly less valid.

![Diagram of a horizontal distributing system in the plane of the lower chord of the roof trusses for a one-story building.](image)

Fig. 8-12. Usual method of creating a horizontal distributing system in the plane of the lower chord of the roof trusses for a one-story building.

It is obvious that maximum distortions between adjacent areas of resistance tend to vary widely and the resulting shears are highly indeterminate. The deliberate use of this expedient, to avoid rotational complications, is contrary to the basic principle of "inertia" loading occurring simultaneously throughout.

Special "Lateral-force" Loadings

In addition to "lateral-force" coefficients applicable throughout structures, most codes prescribe certain special loads in particular circumstances. The provisions of the Rules and Regulations of the California State Division of Architecture for special loadings are shown in Table 305 of those Rules as partially reproduced near the end of this section. The Los Angeles City Code and Uniform Building Code of the Pacific Coast Building Officials Conference contain substantially identical provisions.

It is difficult to justify these special loadings on any rational basis but most experienced designers approve them as conforming to the pattern of observed damage. Like other arbitrary provisions, these loadings must be considered as applicable in ordinary circumstances. A loosely united structure should develop an exaggerated "snap-back" response at the end of its top amplitude and the strain in other attached elements may approach the effect on parapets and ornamental appendages. The use of an arbitrary "lateral-force" coefficient for roof structures, tank towers, etc., when united to a building, cannot be considered more than fair protection against major structural failure. The Pacific Fire Rating Bureau, in design specifications for tower-supported water tanks (as given at the end of this section), emphasizes this by
specific limitation. As the top of a building reaches the extreme limit of oscillation and reverses, an attached unit will tend to continue in motion until its response time has elapsed. The resulting distortion will be exaggerated. To prevent excessive differential deformation, which causes damage, these structures should be made stiff, as well as strong, to ensure quick response to this reversal. This is particularly desirable when the supporting building is flexible since, in such a structure, the dynamic shears in the top stories tend to be disproportionately large.

![Diagram](image)

Fig. 8-13. Deformation of building walls caused by frame deflection during earthquakes.

Minor elements in a framed structure, such as walls and partitions, whether or not they are included as active resisting elements, will in general be deformed together with and in the same pattern as the frame itself. A curtain wall or partition attached at top and bottom to the supporting frame is considered, in the conventional analysis, as a uniformly loaded flat beam supported at the ends. If the earthquake oscillation is normal to the surface of the wall, it actually will be deformed in the same pattern as the frame, that is, in a reversed curve, since the upper floor will move farther than the one below. The conventional analysis, assuming that the “lateral force” will be applied to either direction, yields reinforcing on both sides which, for practical purposes, satisfies the actual distortion condition. However, if the wall supports any considerable amount of vertical load, it is obvious that an eccentric condition may exist and that the arbitrary requirement of 20 per cent gravity, normally applied, bears no relationship to this abnormal condition. This effect is illustrated in Fig. 8-13.
Reliance upon an arbitrary “lateral-force” factor for the design of side walls may lead to erroneous results if the diaphragms are excessively flexible. Referring to the typical one-story building in Fig. 8-14 an oscillation in the transverse direction may cause the end walls, if stiff, to respond rapidly while the roof lags appreciably in the center. The base of the side wall will move with the ground while the top will move only at the ends, until the center of the roof responds. The result is bending and longitudinal tension along the top of the side walls. The consistently beneficial performance of “bond beams” probably is attributable, in part, to this action.

If end walls are identical, same as other end of building

This is same building as in Fig 8-12, but with roof and bracing system not shown

If post and girder system is used instead of trusses, bending in this column is shown in (b)

In conventional analysis, neither side wall or interior columns would accept much lateral force

Original shape of end wall

Deformed shape of end wall

Direction of ground motion

Equal motion at base

Distortion in side walls

Top motion at ends

No top motion at center

Load

Top

Actual shape of deformation variable

Eccentricity

Distortion of column with load

Fig. 8-14. Wall and column bending with flexible roof system.

Excessive flexibility in roof and floor diaphragms may lead to bending and eccentric loading at columns whose lack of rigidity might eliminate them from consideration as “resistances” (Fig. 8-14).

Practical Application of the “Lateral-force” Method

The validity of the conventional analysis rests upon two assumptions: (1) that the “lateral-force” loading derived from the “seismic factors” will produce a distortion identical with the maximum effect of earthquake motion, and (2) that the conditions of rigidity upon which the distribution of effect is based are actually existent in the structure.

Control of damage requires minimization of rotational eccentricities, variations in flexibility, etc., which tend to produce “soft spots” in the structure. Where localities of more than average deformation occur, damage may be expected. The structure should be planned to distort in a uniform pattern throughout so that the effect is distributed.

The Los Angeles City Code formula, which is now in use over much of the Pacific Coast region, is intended to produce “seismic factors” of proper efficiency for all structures within the height limit of 150 ft or 13 stories. The evidence of damage shows
that, when the assumed conditions of rigidity exist, and rotational eccentricities are minimized, this loading is adequate for damage control in buildings of the "rigid" type. Symmetry in arrangement and close spacing of vertical resistances seem to be more useful than actual strength. It seems likely that structures such as school buildings can endure relatively large total deformations when uniformly distributed. However, the tolerance of \( \frac{1}{16} \) in. per ft of height permitted by the California State Division of Architecture probably is excessive. Variations in foundation conditions, except when they occur within the area of the building, do not appear to be highly influential in "rigid" structures.

Structures in the "semirigid" classification, two to seven stories in height, and with vertically stiff floors, perform well when designed for the Los Angeles City Code loading. Apparently these structures are stiff enough to withstand the irregular, high-acceleration motion from local earthquakes and too short-period to be affected by the sustained regular motion developed by severe earthquakes at moderate distances from the site. As the upper limit of the classification is approached, from four to seven stories, the effect of foundation conditions begins to be shown. It is probable that first- and second-story damage, which is characteristic of these structures, is due more to local "flexibility" than to development of excessive shears. Most designers consider the Los Angeles City Code loading adequate provided the distortions are distributed uniformly, and none of the Southern California codes, except the Pacific Fire Rating Bureau's tank-tower specification, includes foundation influence.

Multistory buildings in the "semiflexible" class present the most difficult situation. The rarity of structural damage indicates that the Los Angeles City Code loading is adequate for control of frame stresses. However, the tendency in architectural design is toward reduction of amount of exterior walls and increased use of light interior partitions. This tends to lessen the effective damping and thereby increases the amplification of distortion attendant upon sustained regular motion.

The conventional "lateral-force" method emphasizes the influence of weight. The tendency in engineering design is toward longer spans and more flexibility in floor construction. Although some of this effect is neutralized by saving in weight, this type of structure appears prone to development of more deformation in response to earthquake motion than is exhibited by older styles of construction. In many instances, satisfaction of code requirements has compelled use of interior "shear walls" located irregularly within the structure where the architectural layout permits. In some instances, it seems likely that the structure might have performed better had the "shear walls" been omitted and reliance placed upon the ability of the flexible framework to accept distortion.

Though the tendency in building codes is toward disregard of foundation influence, most observers agree that earthquake motion on soft ground develops more displacement than equally "severe" motion on hard ground or rock. The maximum accelerations tend to accompany shorter periods with motion on hard ground and medium to longer periods on soft ground. The specifications of the Pacific Fire Rating Bureau for water-tank towers (as given at the end of this section) may be interpreted either in terms of displacement or in terms of maximum acceleration, but it is clear that these experts consider foundation influence important. However, the schedule of "seismic factors" in this specification implies that the influence increases with decrease in "free" period, which is contrary to the Japanese practice of ignoring foundation influence where masonry buildings are concerned. The absence of elements susceptible to "superficial" damage may account for this peculiarity. Generally speaking, "semiflexible" structures perform better on hard foundations than on soft foundations. The Los Angeles City Code loading appears to be most suitable for firm ground such as deep-layered compact gravel. For softer ground, a higher loading in the lower two-thirds of the structure appears advisable for control of damage.

There is no unanimity of opinion among engineers with regard to "equivalent lateral force" loadings applicable to structures in the "flexible" category. The performance of tall buildings in the San Francisco district indicates that distortions due to either severe local earthquakes or the sustained motion from moderately distant heavy shocks are less than those produced by heavy wind pressure. Variations in the elastic
nature of these buildings renders each of them unique and there appears to be no method for reducing their probable responses to a fixed pattern. In these cases, the "lateral-force method" employing arbitrary "seismic factors" appears inapplicable.

OVERTURNING EFFECT OF EARTHQUAKE MOTION

It is generally conceded that the distortion of a vertical structure subjected to earthquake base motion will tend to overturn it. Because of the complexity of structures and the indeterminacy of foundation reactions, this effect is difficult to assess by actual observation. Most observers agree that the effect is somewhat less than might be expected from application of "equivalent lateral forces" developed from the "seismic factors" in common use. As a practical solution, some authorities have resorted to arbitrary prescriptions of which the following recommendation of the Joint Committee, before cited, is an example:

(a) The dead load moment of stability of every building or other structure shall be not less than one and one-half times the overturning moment caused by wind pressure.

(b) Provision for overturning moment shall be made for the specified earthquake forces in the top ten stories of buildings, or the top 120 ft. of other structures, and the moments shall be assumed to remain constant from these levels into the foundations.

Since these arbitrary prescriptions vary with different authorities, designers should consult local agencies for guidance in specific cases.

The rarity of structural failures in buildings designed for "lateral forces" usually prescribed by building codes indicates that these loadings are greater than are needed to produce deformations comparable with those occurring during earthquake action. Since the amplitude of earthquake motion is limited and tends to be proportionally greater on soft ground than on hard ground, this discrepancy is not constant.

Observation of damage indicates that "semirigid" and "semiflexible" structures tend to develop more distortion when founded upon soft ground than when the underlying material is hard. Some codes prescribe higher "seismic factors" to compensate for this. Where foundation influence is disregarded, i.e., the single standard of "seismic factors" is prescribed, it would appear that the actual overturning increases as the capacity of resistance diminishes.

The simple concept of "equivalent lateral force" directly proportional to weight, upon which conventional aseismic analysis is predicated, does not afford any logical basis for computing overturning effect, especially for "semiflexible" and "flexible" structures. While overturning values derived from presently prescribed "seismic factors" undoubtedly lead to overdesign, there is no guarantee that any of the proposed reduction expedients are adequate in all circumstances. The proper approach seems to be through distortion analysis predicated upon base displacements in time, a method that, so far, has not been developed sufficiently for acceptance by the engineering profession.

EXCERPTS FROM "RULES AND REGULATIONS" OF THE DIVISION OF ARCHITECTURE, STATE OF CALIFORNIA*

Deflection

(a) General. Consideration shall be given to secondary stresses induced by deflection of the structure or parts thereof when such deflections might create unsafe conditions.

(b) Horizontal Load Deflection. The deflection of vertical resisting elements due to wind or seismic loads in the plane of the wall shall not exceed one-sixteenth inch per foot of height of the element. Diaphragms meeting the stress limitations of this code will be assumed to comply with this requirement. Deflection in the plane of the wall from head to sill of an opening shall not exceed one-sixteenth inch per foot of height of the opening.

The deflections of bracing systems including inelastic deflections in the connections due to wind or seismic loads shall be such that the other portions of the structure are not overstressed. Connections shall be detailed so as to minimize the inelastic deflections.

* Applying to construction of schools and associated with control of earthquake damage.
Location of Vertical Resisting Elements

(a) Longitudinal Elements. In buildings having horizontal wood diaphragms or rod bracing systems and with continuous steel or reinforced concrete framing or continuous reinforced grouted masonry walls, vertical resisting elements shall be provided parallel to the length of the building so that there is at least one vertical resisting element for each 105 feet of building length. An element shall be located not farther than 40 feet from each end of the building.

In buildings of wood construction having horizontal wood diaphragms or rod bracing systems, vertical resisting elements shall be provided so that there is one vertical resisting element in each resisting plane for each 80 feet of building length. An element shall be located not farther than 40 feet from each end of the building.

(b) Transverse Resisting Elements. Shear resisting elements normal to the longitudinal walls shall be provided at such spacing that the ratios given in Table 116 are not exceeded.

<table>
<thead>
<tr>
<th>Nature of diaphragm</th>
<th>Maximum span-to-depth ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Masonry and concrete walls</td>
<td>Wood and light steel walls</td>
</tr>
<tr>
<td>Concrete</td>
<td>Limited by deflection of slab</td>
</tr>
<tr>
<td>Steel deck, continuous sheet in single plane</td>
<td>4:1</td>
</tr>
<tr>
<td>Steel deck, without continuous sheet</td>
<td>2:1</td>
</tr>
<tr>
<td>Poured reinforced gypsum</td>
<td>2:1</td>
</tr>
<tr>
<td>Plywood, all edges nailed</td>
<td>3:1</td>
</tr>
<tr>
<td>Plywood, nailed to supports only, blocking omitted at intermediate joints</td>
<td>2½:1</td>
</tr>
<tr>
<td>Diagonal sheathing, special</td>
<td>3:1</td>
</tr>
<tr>
<td>Diagonal sheathing, conventional construction</td>
<td>2:1</td>
</tr>
</tbody>
</table>

(c) Wood Buildings with Rotation. In buildings of wood construction where rotation must be provided for, transverse shear resisting elements normal to the longitudinal element must be provided at spacings not exceeding 1½ times the width for conventional diagonally sheathed diaphragms or 2 times the width for special diagonally sheathed or plywood diaphragms.

Tying Building Elements Together

Adjoining elements of any building (such as filler walls and columns), whether of the same or different materials or types of construction, shall be positively tied together in a manner suitable to the materials used, in order to resist all loads to which they may be subjected.

Horizontal Forces

General Requirements

Every building and every portion thereof shall be designed and constructed to resist the horizontal forces due to wind and earthquake given in this article, provided however, that wind and earthquake forces need not be combined.

Wherever connections are designed and constructed to resist moments, such connections and members connected thereto shall be designed for moments and shears resulting from vertical loads as well as horizontal forces.

Provisions against Overturning Moment

In designing buildings or structures to resist overturning, the dead load resisting moment of each resisting element shall be not less than one and one-half times the overturning moment due to wind loads.

Combination of Vertical and Horizontal Loads

In computing the effect of wind or seismic loads in combination with vertical loads, all vertical loads except roof live loads shall be considered.

In calculating the maximum tensile stresses due to wind forces, it is permissible to deduct
the direct dead load compression due to gravity. However, because of the effect of possible
vertical acceleration, when considering seismic forces, the maximum tensile stresses may be
reduced by not more than 75% of the direct stress due to vertical dead loads.

[Sections relating to wind pressure omitted.]

**Amount of Seismic Force**

The seismic force shall be considered as applied in any direction and shall be not less than
that given by:

\[ F = CW \]

where \( F \) = Seismic Force in pounds
\( C \) = Coefficient in Table 305
\( W \) = Total dead load tributary under seismic action to element under consideration,
except:

For warehouses add 50% Design Live Load and for Tanks, 100% of Design Live
Load. Machinery and Fixed Loads considered part of Dead Load. For
Snow Load under 40 lbs. sq. ft. add to dead load all over 20 lbs. For Snow
Load over 40 lbs. add 20 lbs. plus 50% of excess over 40 lbs. sq. ft.

**Table 305. Values of Seismic Coefficient \( C \)**

<table>
<thead>
<tr>
<th>Part or portion</th>
<th>( C )</th>
<th>Direction of force</th>
</tr>
</thead>
<tbody>
<tr>
<td>Floors, roofs, columns and bracing in any story of a building or structure as a whole</td>
<td>0.60</td>
<td>Any horizontal direction</td>
</tr>
<tr>
<td>Walls and partitions, over 5 ft. high</td>
<td>( N + 4.5 ) min 5 psf</td>
<td>Normal to surface</td>
</tr>
<tr>
<td>Cantilever walls above roofs of buildings</td>
<td>0.00</td>
<td>Normal to surface</td>
</tr>
<tr>
<td>Ornamental appendages, exterior and interior</td>
<td>1.00</td>
<td>Any horizontal direction</td>
</tr>
<tr>
<td>Towers, tanks, chimneys, and penthouses, when part of a building</td>
<td>0.20</td>
<td>Any horizontal direction</td>
</tr>
<tr>
<td>Elevated water tanks and tower-supported structures, not supported by a building</td>
<td>0.12</td>
<td>Any horizontal direction</td>
</tr>
<tr>
<td>Anchorage of such structures</td>
<td>0.20</td>
<td>Any horizontal direction</td>
</tr>
</tbody>
</table>

*Note: \( N \) is the number of stories above the story under consideration except that for floors
or horizontal bracing \( N \) shall be the number of stories contributing loads. This factor shall
be applied to the summation of all required loads above the story under consideration. For
one-story buildings, \( N \) equals zero.*

**Application of Seismic Forces**

The coefficient \( C \) for special structures or for portions of the building listed in Table 305
shall also apply to the anchorage of such structures or portions to the building. The
building as a whole need only be designed to resist a lateral force based on the value of the coefficient \( C \) applicable to the building.

**Distribution of Horizontal Shears**

The total horizontal shear at any level shall be distributed to the various resisting units
at that level in proportion to their rigidities, giving due consideration to the distortion of the
horizontal distributing elements. Proper provision shall be made for torsional moments
when they exist.

**Permanent Structural Elements**

All permanent structural elements capable of providing resistance shall be assumed to act
integally with structural frames in resisting the shears and moments due to the horizontal
forces unless specifically designed and constructed to act independently of said structural
frames.

**Separation between Adjacent Buildings or Sections of a Building Designed as
Separate Units**

The California State Division of Architecture requires a minimum separation of one (1)
inches laterally and at least more than the “expected or calculated deflections under the
prescribed horizontal forces.” The separation need not extend below the first floor, “if reasonably
close to grade.”
The Los Angeles City Code stipulates a minimum separation of one (1) inch plus one-half (\(\frac{1}{2}\)) inch for each ten (10) feet of height above twenty (20) feet.

Increase in Working Stresses for Certain Horizontal Forces

For combined stresses due to wind or earthquake forces and dead and live loads, the permissible working stresses given in these rules and regulations, including allowable soil pressures, may be increased 33\(\frac{1}{3}\) per cent. In no case shall the section be less than that required for dead and live loads alone.

For members carrying stresses due to wind or earthquake forces only, the permissible working stresses of these rules and regulations may be increased 33\(\frac{1}{3}\) per cent.

*Note:* The "blanket" rule of 33\(\frac{1}{3}\) per cent increase in allowable soil pressures is not followed by all foundation consultants. Some of these authorities permit no increase for certain types of soft soil and recommendations of 50\%–100\% increase for hard materials—shaies, sandstones, etc.—in deep beds have been made in some cases.

### GRAVITY-TANK SUPPORTING STRUCTURES AND TOWERS

#### Steel Tank Towers Located on Ground (Not on a Building or Extending through a Building)

**Design Horizontal Shear in per cent of weight of tank, filled**

<table>
<thead>
<tr>
<th>Height of tower, ft</th>
<th>Foundation material</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Firm ground</td>
</tr>
<tr>
<td>75 or over</td>
<td>10</td>
</tr>
<tr>
<td>50–75</td>
<td>12</td>
</tr>
<tr>
<td>Under 50</td>
<td>15</td>
</tr>
</tbody>
</table>

*Ground capable of withstanding a safe vertical load of one ton or less per square foot is classed as filled ground.*

Reinforced concrete towers must be treated as special cases, governed by special rulings, but in no case shall the lateral force coefficient be less than 20\%. It is recommended that the construction of skeleton reinforced towers without walls be discontinued.

*Note:* The complete specification contains many arbitrary provisions including a method for interconnecting footings.

#### Tank Towers of Any Height on or Extending through a Building

All towers, and platforms where used, shall be braced, and all connections, details and members designed to resist the stresses produced by a horizontal shear of not less than 20\% of the weight of the tank when full, if tower or building under tower is on rock or firm natural ground, and 30\% if on filled ground or soft saturated ground capable of sustaining a safe vertical load of one ton or less per square foot. . . . Column bases must be interconnected by adequate struts unless the equivalent is provided by the roof of the building.

In reinforced concrete towers on buildings the use of reinforced concrete walls for bracing is recommended.

*Note:* For execution of a specific design, the latest revision of the Pacific Fire Rating Bureau Specifications should be consulted since occasional changes are made in the arbitrary design requirements.

1 From July 1, 1953, Specifications of the Pacific Fire Rating Bureau.
Section 9

REINFORCED-CONCRETE BUILDING FRAMES

By

MILO F. JANES, Structural Engineer, New York, N.Y.

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PART 1

ELEMENTS OF REINFORCED-CONCRETE BUILDING FRAMES

Throughout the extent of this section, it is assumed that the engineer is thoroughly familiar with the fundamental theory of reinforced concrete as developed in the ACI Code and the basic principles of its use, including the standard notations used in design. It is not the intention to develop such material here but rather to treat the subject as a discussion or as the material would be used in a consulting engineer's office.

GENERAL TYPES OF BUILDING FRAMES

In general there are four types of concrete building frames:
1. Beam and girder construction with short-span slabs
2. Ribbed slabs with either metal or masonry fillers and beams to transfer the slab loads to the columns
3. Two-way slabs of solid concrete or with fillers of metal or masonry and beams to transfer the slab loads to the columns
4. Flat slabs of solid concrete or with fillers of metal or masonry, where the slab loads are transferred to the columns directly without the use of beams or girders

THE CHOICE OF THE BUILDING FRAME

There can be no general rule for the selection of the type of reinforced-concrete frame for a given building. While in many cases it is possible to show that there is a saving, in terms of yards of concrete and pounds of reinforcing steel, by the use of a certain type of frame, this method of approach cannot be used alone as the basis for recommending its use. The cost of building the forms may be a much more important factor than the difference in the amounts of material involved.

Generally speaking, the type of concrete frame to be selected for a given building should be determined by the size, height, and occupancy of the building and the extent to which the particular type of concrete frame can be adapted to the work of the other building trades and as a finished material. In a multiple-story office building or hospital where there is an abundance of mechanical equipment, ducts, etc., to be concealed between the floor framing and the furred ceiling, a framing system requiring the least total depth possible is of the utmost importance to reduce each story height and ultimately the volume of the entire building.

The combined saving in the height of the risers for all the mechanical trades, the ease with which horizontal runs for mechanical equipment can be installed, and the savings in exterior wall construction are all important factors to be considered. Very often an abundance of closely spaced columns, if they can be fitted into the architectural plan, will result in the reduction of the beam depths or the elimination of the beams entirely and will result in the use of a flat-plate slab. The increased cost of reinforcing, due to the shallow effective depth of the slab, may become insignificant when compared with these savings.

Smooth slabs, exposed as the finished ceilings of classrooms in schools and for apartments, are another example of savings accumulating from the reduction of labor and materials of other trades. For buildings of this type, the structural slab can be troweled smooth to receive a finished floor of asphalt tile, rubber tile, or linoleum, and by using plywood or similar materials for forms, a satisfactory finished ceiling can be obtained with the same structural material.

Recognizing the importance of these savings in addition to the inherent economies of the materials used in concrete construction requires close cooperation and study by the architect as well as the engineer to obtain maximum results. Such studies usually result in choosing a concrete frame of some type other than short-span slabs supported
on beams and girders as the best solution for the various kinds of modern buildings to which reinforced-concrete frames are particularly adaptable.

BUILDING CODE

Except for reinforced-concrete columns, the ACI Building Code Requirements for Reinforced Concrete, adopted in February, 1956, is used as the basis for design and working stresses throughout this section on building frames and is referred to from time to time as the "code." For reinforced-concrete columns, the ACI Code, 1951, is used as the basis for preparing the tables used for design because the use of bending factors seems to be a more practical method of handling columns with eccentric loads smaller than $e/t = 1$. Although the designer may occasionally be required to use other codes, the ACI Code is most generally accepted as representing up-to-date standards of modern practice. The allowable unit stresses in concrete as covered in section 305 of the code are included as Table 9-24.

The following live loads are recommended by the Minimum Design Loads in Building and Other Structures, approved in June, 1945, by the American Standards Association and listed here by courtesy of ASA.

The live loads assumed for purposes of design shall be the greatest loads that probably will be produced by the intended occupancies or uses, provided that the live loads to be considered as uniformly distributed shall be not less than the values given in Table 9-1. [See page 9-65.]

A selected list of occupancies and uses more commonly encountered is given in Table 9-2 and the building official is given authority to pass on occupancies not mentioned. Table 9-2 is offered as a guide in the exercise of such authority. [See page 9-65.]

For purposes of estimating loads caused by accumulations of various materials, a list of weights is given in Tables 9-3 and 9-4. [See page 9-66.]

FORMS

The cost of forms varies materially with the various types of concrete frames, but even with the most simple types of formwork it constitutes a major item in the total cost of the concrete frame. The ideal frame from the standpoint of the cost of the formwork involved would be a flat slab with equal story heights and columns of the same size for the entire height of the building. Such a scheme would eliminate the costly beam forms and the rebuilding of column forms and shoring. The amount of labor is also reduced for both the building of the forms and the setting of reinforcement.

While it may be impossible to take advantage of all these features in every building, the designer should keep them constantly in mind. In a building of three or four stories in height, the bending in the columns supporting the roof slab is very often a determining factor in the sizes of the column. By the proper choice of the type of column reinforcement, it is often possible to keep the columns the same size as the minimum size required for the upper story for the entire height of the building.

In so far as possible, it is good practice to keep the sizes constant for a continuous row of beams such as those often found along the corridors of buildings. Even though the conditions of loading and spans may not require the maximum size of beam throughout, the cost of the small amount of extra concrete is insignificant when compared with the saving of formwork and steel for the several stories of the building.

In many buildings, it becomes practical to use wide flat beams of shallow depths, as shown in Fig. 9-1a. While the amount of beam reinforcing may be much greater than for a narrow deep beam of the usual proportions, the labor for placing it without stirrups is greatly reduced. The forms for this type of beam are generally considered as slab forms, which usually cost much less than the cost of forms for deep beams, as shown in Fig. 9-1b. When the possible savings in story heights and in other trades are considered, the wide flat beam often becomes the most practical solution.

![Fig. 9-1. Comparison of beam depths: (a) wide and shallow; (b) narrow and deep.](image-url)
SOLID SLABS WITH ONE-WAY REINFORCEMENT

Slabs of this type are usually confined to comparatively short span lengths, varying from 6 to 12 ft. They are economical when used for medium or heavy live loads and can be adapted for heavy concentrations such as truck-wheel loads or heavy vibrating machinery. For medium live loads, it is possible to increase the span slightly, by using wide beams, as shown in Fig. 9-2a. Such an arrangement is ideal from a design standpoint because the additional slab depth at the beams can be proportioned to conform with the moment requirements at the supports. When computed on the basis of a span length from center to center of the wide beam, the increased slab depth serves as a haunch and thus builds up negative moment where it is easily taken care of because of the increased effective depths. If properly proportioned, it will not become necessary to increase the reinforcement beyond what is actually needed near the face of the beam.

![Diagram of slab with beam](https://via.placeholder.com/150)

**Fig. 9-2.** Comparison of span of slab: (a) maximum for narrow deep beams; (b) minimum for wide flat beams.

For heavy live loads and especially where there is any appreciable vibration, short spans with deeper beams, as shown in Fig. 9-2a, become the most practical solution. For some types of buildings this type of floor becomes objectionable from an architectural standpoint, as it requires more beams than other types of floor systems.

The code requires that the minimum amount of reinforcement that may be used in solid slabs shall be 0.0025bd for intermediate- and hard-grade steel, the same as required for temperature and shrinkage reinforcement. It is good practice to continue as much as 40 per cent of the reinforcement required for positive moment into the supports. By alternating bent bars of a size larger than the straight bars, it is usually easy to accomplish a satisfactory arrangement of reinforcement. In cases where the negative moments are high at the supports, it may be necessary to add additional straight top bars, as shown in Fig. 9-3. Where the span lengths vary, or where heavy live loads on alternate spans cause negative moment in the center of the span, the straight top bars should be made continuous across the span.

While it is possible to obtain a satisfactory arrangement of reinforcement by using

![Diagram of reinforcement](https://via.placeholder.com/150)

*Fig. 9-3.** Additional top steel needed for moments at the supports.

* 1/4 required by code is satisfactory for equal spans uniform light live loads

* li required by code is satisfactory for equal spans uniform light live loads
only straight bars for both positive and negative moment, it is generally conceded that the use of bent bars is preferable, and where the bending may be done in the fabricating shop, the additional cost is not prohibitive. However, when this additional expense is incurred, it is important that the diagonal bars be used wherever possible to resist diagonal tension.

Where the main slab reinforcement is parallel to girders, the code requires transverse reinforcement, as shown in Fig. 9-4, to carry the load on the portion of the slab required for the flange of the T beam. Whether a flange is required for the beam or not, this steel should be provided, as it is difficult to believe that the reinforcement adjacent and parallel to a beam is effective, and some of the load should be transferred directly to the beam by negative reinforcement over the beam.

![Diagram of top bars as required by sec 705 of code](image)

Fig. 9-4. Top slab steel required over supports when main steel is parallel to the supports.

**RIBBED SLABS WITH METAL FILLERS**

Ribbed slabs of this type are especially suited for buildings having a generally uniform light live load and especially where a furred ceiling is required. They are satisfactory for comparatively long spans, provided the depth-to-span ratio is limited so that the resulting deflection is held within reasonable limits. They are not satisfactory where there is a possibility of heavy concentrations or where there is any possibility of vibration resulting from the live loads. The thin slabs, usually 2 to 3 in. thick, between the ribs do not provide sufficient strength to serve as bridging between the ribs and do not have the strength to support heavy concentrated loads.

Removable forms, commonly known as metal pans, are readily available in most localities in standard widths of 20 in., and in depths of 6, 8, 10, and 12 in. Special

![Diagram of ribbed slabs formed by removable metal pans](image)

Fig. 9-5. Ribbed slabs formed by removable metal pans.
width or filler forms of 10 and 15 in. are also available in these depths. Special tapered end forms are also available for the standard 20-in. widths, which provide additional concrete, where required, for both shear and negative moment at the supports. In some localities, pans 14 in. deep and in widths of 30 in. are available, but their use is not so widely accepted as the standard widths of 20 in.

The principal advantages of this type of floor construction (Fig. 9-5) are the light dead weight of the floor construction and the economical formwork resulting from the use of metal pans.

**RIBBED SLABS WITH MASONRY FILLERS**

Masonry fillers for ribbed slabs are either hollow clay tile or units of lightweight concrete laid in rows, with concrete between the fillers forming the structural ribs (Fig. 9-6). Ribbed slabs of this type are suitable for slightly heavier uniform loads than slabs with metal fillers, because of the stiffening of the slabs between by the masonry units. Owing to the additional weight, their use is generally confined to spans of not more than 20 ft where the plaster ceiling may be applied directly to the slab.

To avoid streaking of the plaster ceiling, it is advisable that soffit blocks be provided under the concrete ribs. Clay-tile fillers are usually available in depths of 4, 6, 8, 10, and 12 in. and lightweight concrete units of 6-, 8-, 10-, and 12-in. depths. Gypsum-tile fillers may be used in floors of this type, but they have not been used so extensively as clay-tile and lightweight-concrete fillers.

**TWO-WAY SLABS**

Two-way slabs may be built of solid concrete, or ribbed slabs with masonry fillers, or with removable metal fillers. Regardless of the type used, they are suitable for all
types of loads, from light to heavy, including concentrated and vibrating loads from light manufacturing equipment. Even for heavy loads, they may be economically constructed for spans up to 30 ft.

The distribution of uniform load in two-way slabs is based upon the ratio \( r = gL/g_1L_1 \), as shown in Fig. 9-7, where \( g \) and \( g_1 \) is the ratio of span between lines of inflection to \( L \) and \( L_1 \), respectively, where the span in question only is loaded as a continuous one-way slab with adjacent spans providing the actual conditions of restraint. Method 1 of the ACI Code provides coefficients of shear and moment for the typical conditions where it is assumed that the marginal beams are supported on four columns at the corners of the slab and that the edges of the slab are restrained by adjacent slabs having span lengths varying from not more than two-thirds to three-halves of the span length in question.

Thus for an interior slab 20 by 25 ft with a unit load of 200 psf the total load to be supported is \( 20 \times 25 \times 200 \text{ psf} = 100 \text{ kips} \).

\[
\begin{align*}
L &= 20' \\
gL &= 15.2' \\
L_1 &= 25' \\
g_1L_1 &= 19'
\end{align*}
\]

**Fig. 9-7.** Factors in the distribution of uniform load in two-way slabs.

<table>
<thead>
<tr>
<th>Slab Coef. from ACI Table 1</th>
<th>( r )</th>
<th>( C_4 )</th>
<th>( C_{41} )</th>
<th>( C_1 )</th>
<th>( C_{11} )</th>
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</thead>
<tbody>
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<td>( .80 )</td>
<td>.33</td>
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<td></td>
</tr>
</tbody>
</table>

\[
r = \frac{gL}{g_1L_1} = \frac{.76 \times 20'}{.76 \times 25'} = \frac{15.2'}{19'} = .80
\]

From ACI Building Code, Chap. 7, 1956 and reproduced here with Tables 1 and 2 by courtesy of the American Concrete Institute:

(b) **Bending moments and shear**—Bending moments shall be determined in each direction with the coefficients prescribed for one-way construction in Sections 701 and 702 and modified by factor \( C \) or \( C_1 \) from ACI Tables 1 or 2.

\[
\begin{align*}
\text{B.M. for slab strip} & \quad M = CBWL \\
\text{B.M. for beam} & \quad M = (1 - C)BWL
\end{align*}
\]

When the coefficients prescribed in 701(c) are used, the average value of \( C_w \) or \( C_1w \) for the two spans adjacent to a support shall be used in determining the negative bending moment at the face of the support.

The shear at any section distant \( xL \) or \( xL_1 \) from supports shall be determined by modify-
<table>
<thead>
<tr>
<th>$r$</th>
<th>( \frac{1}{r} )</th>
<th>$C_s$</th>
<th>$C_{s1}$</th>
<th>Values of $x$</th>
<th>$C$</th>
<th>$C_1$</th>
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### Values of $z$

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<th>$1 - C$</th>
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<td>0.92</td>
</tr>
<tr>
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<td>0.91</td>
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</table>
ing the total load on the slab strip or beam by the factors \( C_s, C_{sl}, C_b \) or \( C_{sl} \) taken from ACI Tables 1 or 2.

<table>
<thead>
<tr>
<th>Shear for Slab Strip</th>
<th>In ( L ) Direction</th>
<th>( V = C_s W )</th>
<th>In ( L_1 ) Direction</th>
<th>( V_1 = C_{sl} W_1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shear for Beam</td>
<td>( V = C_b W )</td>
<td></td>
<td>( V_1 = C_{sl} W_1 )</td>
<td></td>
</tr>
</tbody>
</table>

For spans where the end moments are unbalanced, shear values at any section shall be adjusted in accordance with Sections 701 and 702.

(c) **Arrangement of reinforcement:**

1. In any panel, the area of reinforcement per unit width in the long direction shall be at least one-third that provided in the short direction.
2. The area of positive moment reinforcement adjacent to a continuous edge only and for a width not exceeding one-fourth of the shorter dimension of the panel may be reduced 25 per cent.
3. At a non-continuous edge the area of negative moment reinforcement per unit width shall be at least one-half of that required for maximum positive moment.

The actual distribution of the loads for the loaded strips at the center of the span is as shown by the coefficients of shear at the supports. \( C_s \) and \( C_{sl} \) or \( 2C_{sl} \) for span \( L \) equal \( 2 \times 0.33 \times 200 = 132 \text{ psf} \) and \( 2C_{sl} \) for span \( L_1 \) equals \( 2 \times 0.17 \times 200 = 68 \text{ psf} \).

\[
\begin{align*}
V &= 0.33 \times 20 \times 200 \text{ psf} = 1.32 \text{ kips} \\
V_1 &= 0.17 \times 25 \times 200 \text{ psf} = 0.85 \text{ kip}
\end{align*}
\]

The moment coefficients \( C \) and \( C_1 \) in ACI Table 1 are for an equivalent uniform load that will give moments for design purposes that are comparable with the moments for the actual trapezoidal loading varying from a minimum at the center to a maximum at the supports as indicated in Fig. 9-8. For the 20 by 25-ft slab,

\[
C_wL = 0.48 \times (200 \text{ ppsf} \times 20 \text{ ft}) = 1.92 \text{ kips}
\]

and \( C_wL_1 = 0.21 \times (200 \text{ ppsf} \times 25 \text{ ft}) = 1.05 \text{ kips} \).

The coefficients for shears and moments in ACI Table 2 of the code for beams are:

<table>
<thead>
<tr>
<th>( \tau )</th>
<th>( C_6 )</th>
<th>( C_{sl} )</th>
<th>( 1 - C )</th>
<th>( 1 - C_1 )</th>
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</thead>
<tbody>
<tr>
<td>.80</td>
<td>.17</td>
<td>.33</td>
<td>.52</td>
<td>.79</td>
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</table>

The shears on the beams at the supports are

\[
\begin{align*}
V_L &= C_6 \times w \times L \times L_1/2 \\
&= 0.17 \times 200 \times 20 \times 25/2 = 8.5 \text{ kips} \\
V_{L1} &= C_{sl} \times w \times L_1 \times L_2/2 \\
&= 0.33 \times 200 \times 25 \times 20/2 = 16.5 \text{ kips}
\end{align*}
\]

The equivalent uniform loads to be used for computing the moments on the marginal beams as shown in Fig. 9-9 are obtained by using the values of \( 1 - C \) and \( 1 - C_1 \) from ACI Table 2:

For span \( L \),

\[
W_e = (1 - C)L \times w \times L_1/2 \\
= 0.52 \times 20 \times 200 \times 25/2 = 26 \text{ kips}
\]
For span $L_1$,

$$W = (1 - C_1)L_1 \times w \times L/2$$
$$= 0.79 \times 25 \times 200 \times 20/2 = 39.5 \text{ kips}$$

Where the span lengths and conditions of loading conform with the requirements of section 701 of the code the design may be completed by moment coefficients. Examples of building frames that do not fall within these limits will be completed later in this section.

The minimum slab thickness as required by the code shall not be less than

$$t = \frac{2(L + L_1)12}{180}$$

or

$$t = \frac{2(20 \text{ ft} + 25 \text{ ft})12}{180} = 6 \text{ in.}$$

for the 20 by 25-ft slab shown in Figs. 9-8 and 9-9.

It is worth noting that the principal advantage of the two-way slabs is due to the so-called plate action of the slab or the arching effect of the slab across the corners of the panel. The deflection of the plate or slab under uniform load is in the form of a parabola. If all the points in a slab which have a change of inflection were joined together, a contour in the general shape of an ellipse would result, as shown in Fig. 9-10. The plate action is most effective for slab panels that are square or nearly so, and as shown by the coefficients from ACI Table 1, the equivalent loads causing bending in the slabs are approximately two-thirds of the total load for span ratios varying from 0.80 to 1.0. For span ratios of less than 0.80, the effective load distribution to the long span is much smaller, but in such cases the reinforcing required for moment seldom exceeds 0.0025$bd$, or the amount that would be used for temperature and shrinkage stresses if the total load is carried in the short direction. The actual result is that, in so far as the reinforcing is concerned, the amount provided for moment in the center strip of the panel varies from 0.67 to 0.70 of that required for a one-way slab of equivalent effective depth. The amount provided in the column
REINFORCED-CONCRETE BUILDING FRAMES

Slab Coef. $C$ and $C_1$ from ACI Table 1

<table>
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<th>$r$</th>
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</thead>
<tbody>
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<td>.48</td>
<td>.58</td>
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</tr>
<tr>
<td>$C_1$</td>
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<td>.21</td>
<td>.15</td>
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</tbody>
</table>

strips is still further reduced to approximately 0.50 per cent of that required for one-way slabs.

Although the number of marginal beams is increased to support the two-way slabs, the additional amount of reinforcing required for total moment is increased very little if any over what would be required for a one-way system. The resulting increase in total beam reinforcing is in most cases due to the smaller effective depth of the additional number of beams used to carry the total load to the columns.

FLAT SLABS

Flat slabs, in reality, are actually girderless two-way slabs in which the column strips serve as marginal beams for the two-way center-strip panels as well as a one-way slab to support the loads directly superimposed upon them. The dropped slab, at the columns where required for heavy loads, is only a haunch for the column strip or marginal beams and becomes effective in reducing the positive moments in the center of the span; it provides additional depth at the columns where required for negative moment, shear, and bond, all of which are of considerable importance in flat plates or slabs of relatively shallow depth.

![Fig. 9-11. Concentric deflection contours in flat slab.](image)

Under uniform loads the relative deflections of a flat slab panel are approximately as indicated in Fig. 9-11. When confined within the following limitations of section 1004 of the ACI Code, flat slabs may be designed by empirical formulas:

1. The construction shall consist of at least three continuous panels in each direction.
2. The ratio of length to width of panels shall not exceed 1.33.
3. The grid pattern shall consist of approximately rectangular panels. The successive span lengths in each direction shall differ by not more than 20 per cent of the longer span.
4. The numerical sum of the positive and negative bending moments in the direction of either side of a rectangular panel shall be assumed as not less than

$$M_0 = 0.09WLF \left(1 - \frac{2c}{3L}\right)^2$$
<table>
<thead>
<tr>
<th>Strip</th>
<th>Column head</th>
<th>Side support type</th>
<th>End support type</th>
<th>Exterior panel</th>
<th>Interior panel</th>
<th>Type of support listed in Table 1004 (f)</th>
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</thead>
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<td>Exterior negative moment</td>
<td>Positive moment</td>
<td>Interior negative moment</td>
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<td>Columns with no beams</td>
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<td>A</td>
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<td>Columns with beams of total depth (1\frac{1}{4}t)</td>
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<td>Half column strip adjacent to marginal beam or wall</td>
<td>With drop</td>
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<td>C</td>
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</tbody>
</table>

* Increase negative moments 30 per cent of tabulated values when middle strip is continuous across support of type B or C. No other values need be increased.

Note: For intermediate proportions of total beam depth to slab thickness, values for loads and moments may be obtained by interpolation.
in which \( F = 1.15 - c/L \) but not less than 1.

5. Unless otherwise provided, the bending moments at the critical sections of the column and middle strips shall be at least those given in ACI Table 1004(f) (1956), which is reproduced here by courtesy of the American Concrete Institute.

The critical sections for moments shear and bond at the columns are shown in Fig. 9-12. The effective support size \( C \) is also indicated for columns with and without capitals.

Flat-slab floor framing may also be designed by elastic analysis as covered by section 1003 of the ACI Code (1956) without restrictions as to the number of spans or the ratios of span lengths. Positive moments at the center of the span and negative moments at the supports are distributed to the column and middle strips in accordance with ACI Table 1003(c), which is reproduced here by courtesy of the American Concrete Institute.

**ACI Table 1003(c). Distribution between Column Strips and Middle Strips in Per Cent of Total Moments at Critical Sections of a Panel**

<table>
<thead>
<tr>
<th>Strip</th>
<th>Moment section</th>
<th>Negative moment at interior support</th>
<th>Positive moment</th>
<th>Negative moment at exterior support</th>
<th>Slab supported on columns and on beams of total depth equal to the slab thickness*</th>
<th>Slab supported on reinforced concrete bearing wall or columns with beams of total depth equal to or greater than 3 times the slab thickness*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Column strip</td>
<td></td>
<td>76</td>
<td>60</td>
<td>80</td>
<td>60</td>
<td></td>
</tr>
<tr>
<td>Middle strip</td>
<td></td>
<td>24</td>
<td>40</td>
<td>80</td>
<td>60</td>
<td></td>
</tr>
<tr>
<td>Half column strip adjacent and parallel to marginal beam or wall</td>
<td>Total depth of beam equal to slab thickness*</td>
<td>38</td>
<td>30</td>
<td>40</td>
<td>30</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Total depth of beam or wall equal to or greater than 3 times slab thickness*</td>
<td>19</td>
<td>15</td>
<td>20</td>
<td>15</td>
<td></td>
</tr>
</tbody>
</table>

* Interpolate for intermediate ratios of beam depth to slab thickness.

Note: The total dead and live load reaction of a panel adjacent to a marginal beam or wall may be divided between the beam or wall and the parallel half column strip in proportion to their stiffnesses, but the moment provided in the slab shall not be less than given in Table 1003(c).

The use of elastic analysis as the basis of design permits the use of flat-slab framing for panels with span ratios greater than 1.33 in both directions as shown in Fig. 9-13a and for irregular spacing of columns as shown in Fig. 9-13b.

Regardless of whether the analysis is made by coefficients or by the theory of elasticity special care should be taken in the arrangement of the reinforcement and the
design for shear around the columns. Generally where the architecture will permit the use of column capitals as shown in Fig. 9-12b they can be used very effectively to reduce the moment at the critical section as well as to provide a larger section for shear and a greater percentage of reinforcement directly over the column.

![Fig. 9-13. Flat-slab framing with span ratios greater than 1.33 in both directions.](image)

**OPENINGS IN SLABS**

In the modern building, openings of various sizes and shapes in the slabs are encountered for practically every type of occupancy. The large opening for stairs and elevator shafts requires framing with marginal beams. In most cases, beams can be avoided for smaller openings for pipe shafts, ventilating ducts, etc., provided that there is close coordination between the mechanical and structural engineers in determining the location and shape of the openings. It should be kept in mind, however, that a reinforced-concrete slab is a large monolithic piece of masonry and that cracks due to shrinkage and temperature stresses should be considered as well as those which may occur because of bending stresses. In thin slabs it is often advisable to provide thickened edges or even to increase the entire slab thickness as well as additional reinforcement rather than leave the slab vulnerable to cracking from such stresses.

The designer should always keep in mind the necessity of providing the necessary amount of concrete and reinforcing to preserve the continuity of the original floor framing without the opening. In short-span solid slabs, special framing for small openings can usually be avoided by fanning the bars when the long side of the opening is perpendicular with the main reinforcing or by close spacing of the reinforcing when the short side is perpendicular to the main reinforcing as shown in Fig. 9-14. Fanning the bars seldom offers any difficulty at exterior beams, but at interior beams where the top reinforcement from the adjacent spans causes overlapping, it becomes necessary to provide additional

![Fig. 9-14. Reinforcing around small openings in one-way solid slabs.](image)
Fig. 9-15. Reinforcing around small openings in two-way solid slabs.

Fig. 9-16. Reinforcing and haunches adjacent to small openings in wide flat beams.

top bars over the support as indicated. Except for extremely small openings, additional straight top and bottom bars, perpendicular to the reinforcing, should be provided as indicated.

The fanning or close spacing of bars around openings in two-way slabs, as shown in Fig. 9-15, is usually more easily accomplished than in one-way slabs, as the angle of fanning is not so great. There is probably less reinforcement and less crowding and overlapping of the reinforcing.

The use of wide shallow beams as the marginal beams for two-way slab panels, as shown in Fig. 9-16, does not necessarily prevent small openings on or near the column center lines. The widths of the beams and the selection of a small number of large-sized reinforcing bars can be controlled to permit close spacing or fanning of the reinforcement as required. Additional straight top or bottom bars may be added in each direction, as necessary, to provide for the tensile stresses or to act as compressive reinforcement, if required.

Where wide beams from both directions intersect at interior columns, as shown in Fig. 9-16, the use of haunches or brackets may be necessary as indicated, in order to provide the necessary area required for shear and the adequate support for the beams to prevent cracking at the opening. In so far as possible, openings in such locations should be avoided as such brackets are both expensive to build and often objectionable from an architectural standpoint.

The framing around openings in ribbed slabs is generally more easily accomplished than for solid slabs. In many cases the openings can be located between ribs and no additional framing is required except to provide a header with a straight bar top and bottom to close off the space. In one-way slabs, as shown in Fig. 9-17, where the long dimension of the opening is perpendicular and greater than the space between ribs, it is often possible to let the rib run through and divide the opening for plumbing stacks. Where this is impossible for ventilating ducts, etc., a structural header may be framed into special joists on each side of the opening. It is comparatively easy to provide for openings in ribbed floor systems of either two-way or flat-slab construction. In either system, the entire middle strip panel could be removed without destroying the concrete frame. Small openings can be placed between the ribs in any location without additional framing, and
where it becomes necessary to cut a rib it is only necessary to increase the size of the reinforcing bars in the adjoining ribs in each direction. The most vulnerable spot for locating the opening is near the columns in flat-slab construction. In such cases, as shown in Fig. 9-18, it becomes necessary to provide beams or haunches, depending upon the size of the opening, in order to take care of the high shearing and moment stress at the face of the column.

Provision for openings usually becomes not so much a matter of difficulty in framing as a problem of coordinating the structural work with that of the architectural and mechanical planning to obtain the best results.

**CONCENTRATED LOADS ON FLOOR SLABS**

Concentrated loads are frequently encountered in reinforced-concrete frames for buildings from sources such as machine loads for industrial buildings, mechanical-equipment loads either superimposed upon the slab or hung from it, and suspended trolley beams. Some consideration should always be given for the possible concentration resulting from partition loads on slabs. In the event that the partition is continuous across a series of short-span slabs, it is possible that the partition tends to arch across the slab from beam to beam and there is no appreciable concentration on the slab. When partitions are pierced by doors and the slab spans are long, it is quite possible that the equivalent uniform load resulting from distributing the partition load over a reasonable width of slab can be considerably more than the customary 20 psf uniform load that is often allowed in practice.

As previously mentioned, one-way ribbed slabs are the least suitable of all the types of reinforced-concrete floor slabs for supporting concentrated loads. If permitted at all, the concentrated load should be relatively small and in no case should one-way ribbed slabs be used where there is any possibility of vibration due to concentrated loads. Small loads may be distributed over two or more ribs satisfactorily by the use of diaphragms and a solid slab reinforced top and bottom as shown in Fig. 9-19. The rib sizes may be increased for the ribs directly under the load to provide additional stiffness.

For solid slabs with one-way reinforcing, the concentrated load may be considered as distributed uniformly over an effective slab width \( b \), as shown in Fig. 9-20, for computing the moment parallel to the \( x-x \) axis and the shears at the supports. The deflection of the slab immediately around the load is approximately as shown and also creates moments and shearing stresses parallel to the \( y-y \) axis. The effect of these
stresses determines to a large extent the effective width $b$. The actual effective width depends also upon the slab thickness, span, and conditions of restraint at the supports. The amount of the moments parallel to the supports is a maximum directly under the load and along the $y$-$y$ axis and varies in intensity from a maximum under the load to zero at the supports. These stresses may be computed by treating the portion of the slab of length $b$ as a continuous slab subjected to an equivalent triangular load in which the maximum unit load $w$ for the 1-ft-wide strip under the concentration is $w = 2P/bL$.

The width of contact area $c$ as shown in Fig. 9-20 should be taken as the dimension of machine bases for manufacturing and mechanical equipment, etc. In garages, the wheel loads from cars and trucks are considered as distributed over a circular area of 1 in. diameter for each 1,000 lb of wheel load plus impact. For concentrated loads suspended from a slab by means of anchor bolts or inserts, the diameter of the area of contact may be assumed to be 1.4 times the length of embedment $l$ as shown in Fig. 9-21. The perimeter of the contact of diameter $c$ is taken as the section for computing the diagonal tension stresses.

In cases where there is an overlapping of the effect of the adjacent concentrated loads upon the section of a slab as shown in Fig. 9-22, the uniform load used in design should be taken as the summation of the effect of the distributed loads. The minimum width of slab to be used where the concentrated load is adjacent to the supports is $b = 5\sqrt{t}$, where $t$ is taken as the slab thickness in feet.

Concentrated loads on two-way slabs may be assumed to be distributed uniformly over a slab width $b = c + 1.4e$ similar to the method used for one-way

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**Fig. 9-20.** Distribution of a concentrated load in a solid one-way slab.

**Fig. 9-21.** Distribution of a concentrated load hung from an embedded bolt or insert in the slab.
slabs. For a concentrated load located on the center line of spans as shown in Fig. 9-23 it is apparent that the amount of load that will be distributed in each direction is dependent upon the relative deflections of the slab for the span lengths \( L \) and \( L_t \). The percentages of the load to be distributed to span \( L \) for varying span ratios and conditions of continuity are shown in Table 9-6 on page 9-70.

**BEAMS**

In reinforced-concrete building frames, the designer not only is concerned with determining the beam size and the necessary amount of reinforcement, but he also should become familiar with the detailing and setting of the reinforcement in order to avoid the pitfalls that so often come to light during these stages of the work. For architectural reasons, the engineer is generally required to keep beam sizes to a minimum, which too often results in crowding of the reinforcement at the columns and uneconomical fabrication, setting, and formwork. Continuous beams with high moments at the supports naturally require high percentages of both tension and compression reinforcement at the points where the beam steel passes through the column reinforcement, especially where beams frame into the column from both directions.

In general practice, typical beam details are provided on the design drawings showing the bend points and general arrangement of reinforcement, as shown in Fig. 9-24. This procedure is quite satisfactory provided the designer is careful in the selection of reinforcement to avoid crowding.

A section through the beam is shown in Fig. 9-25a and plans at the bottom and top steel, respectively, in Fig. 9-25b and c taken at a column with eight vertical bars with spiral reinforcement. The code provides that a minimum of 0.25 per cent of the posi-

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**Fig. 9-24. Typical beam details showing bend points and general arrangement of bent and straight reinforcement.**
tive reinforcement be continued into the support a distance of 6 in. While the dimension of 6 in. is satisfactory where compression reinforcement is not required, standard details usually provide for an extension of the bar at least 20 diameters into the column so that its strength is developed at the face of the column for compression. Bent bars are used for negative reinforcement at the top of the beam with additional straight top bars added as required. The additional straight top bars may be located in the slab adjacent to the beam to avoid crowding of the top reinforcement.

The combination of straight and bent bars, as shown in Figs. 9-24 and 9-25, not only is economical in so far as the use of steel is concerned but is also satisfactory for detail as the steel can be placed without crowding. However, it should be noted that a smaller column would make placing of the steel difficult. If additional straight bottom bars had been continued, a very bad condition of crowding would have resulted at the columns. It is also preferable that the straight top bars be selected of such sizes and placed so that a symmetrical arrangement of reinforcing will be obtained wherever possible.

![Diagram of bent and straight bars from adjacent beam spans at a column.](image)

**Fig. 9-25. Bent and straight bars from adjacent beam spans at a column.**

The designer should also become thoroughly familiar with the methods used in placing the beam reinforcing. Occasionally the individual pieces are placed in the forms and wired together. However, the best practice is to wire the stirrups to the tie bars and main reinforcing in the inverted position on wooden horses, as shown in Fig. 9-26. By this method, the stirrups can be accurately located and securely wired in place. The assembly is then placed in the forms as a unit and the additional straight and bent bars placed into the assembly in their proper location. This procedure is very necessary in deep narrow beams and final results are much more satisfactory if used throughout.

It should be noted, however, that it is imperative that the stirrups be of the open-end type at the tops, in order to facilitate the placing of the additional steel after the assembly is placed in the forms. Closed stirrups should be used only where absolutely necessary, and when they are used, it is best practice to use straight top and bottom bars to avoid the necessity of threading the bent bars through the steel assembly.

![Diagram of wiring stirrups to tie bars and main bottom beam reinforcement.](image)

**Fig. 9-26. Wiring stirrups to tie bars and main bottom beam reinforcement.**
Spandrel beams, in particular, must be watched very carefully because of the necessity of keeping the beams narrow in width for architectural reasons. It is common practice to build the exterior face of the spandrel beam flush with the face of the column. With this arrangement of vertical bars in the column, careful attention must be given to avoid crowding. Generally, tied columns should be used in so far as possible for spandrel beams, as shown in Fig. 9-27a. Spiral columns are not particularly adaptable for spandrel beams, and when they are used, a combination of a spiral column with ties, as shown in Fig. 9-27b, is preferable. By inspection, it is evident that crowding would result if a large number of vertical bars are used within the spiral, and it is a mistake to schedule such a group of bars, hoping that the crowding can be taken care of adequately in the field.

It is also a mistake to provide just enough width in the spandrel beam to accommodate the spacing of the reinforcement as a beam. To provide the proper spacing for two No. 11 bars at 2 1/2 diameters on center, the minimum width should be approximately 10 in. A slightly better arrangement is obtained by breaking the columns outside the line of the exterior face of spandrel beams (Fig. 9-28a), the thickness of the forms, or 3/4 in. and provide 5 in. for the thickness of the masonry and supporting shelf angle. This arrangement is also an improvement in the form construction as it is usually necessary to recess the shelf angle, as shown in Fig. 9-29a, when only 4 1/2 in. is allowed from the building line to the face of the spandrel beam.
Fig. 9-29. Spandrel beam with masonry supporting shelf angle recessed into face of beam.

It is generally considered good practice to provide additional continuous reinforcing in the top of spandrel beams, to act as temperature and shrinkage reinforcement. This practice is especially advisable in the event that the frame is to be constructed during the winter months or during a period of constantly changing temperature. Spandrel beams also often become much deeper than necessary in order to reach the heads of windows. In extreme beam depths it is good practice to add intermediate temperature reinforcement between the main reinforcement at the top and bottom of the beam. Where spandrel beams narrow in width are used, as shown in Fig. 9-30a and c, the entire assembly, including the bent bars, can be fabricated similarly to the arrangement shown in Fig. 9-29.

The various combinations of bars shown in Fig. 9-30 are intended only to show that it is best to select a smaller number of bent bars in every case and add straight bars at the top to satisfy the requirement for negative reinforcement, rather than to attempt to provide for it by using a greater number of bent bars than are required in the bottom of the beam for positive reinforcement. In the bottom of the beam, where two layers of reinforcement are used, the bars in the top layer should be placed directly over the bars in the bottom layer to facilitate placing of the concrete. The bent bars in the top of the beam cannot be so easily kept to the same spacing. In fact, they must be lapped by one another. Lapping the bars at the top of the beam is quite satisfactory as they are easily accessible and the concrete can be placed without

Fig. 9-30. Bent and straight bars in spandrel and inside beams where one and two layers of reinforcement are used.
difficulty. The fact that they can be readily lapped in one layer proves that the use of additional straight top bars outside the beam web to avoid crowding is good practice.

For wide flat beams, as shown in Fig. 9-31, the problem of crowding of reinforcement at the supports is almost entirely eliminated. The economical use of shallow beams depends principally upon being able to space the columns to provide for reasonably short spans and also to confine their use to light and medium live loads and a system of load distribution that keeps the moments to a minimum, such as the two-way slab.

The shear for shallow beams may be computed by the rules set up in the code for the shear in flat slabs and, as indicated in Fig. 9-32, by the summation of the perimeter of an assumed section at a distance $t - 1\frac{1}{2}$ in. from the edge of the column. In order to compute the shear in this manner, there must be reinforcing in the top of the slab in both directions. When the shear is greater than the working stresses permitted by the code, diagonal-shear bars in the form of inverted baskets may be used to increase the perimeter of the critical section for shear.

Where girders are parallel to the slab reinforcing, straight top bars should be provided, as shown in Fig. 9-33, to prevent any possibility of cracking along the sides of the beams. The area of steel may be computed by figuring the slab as acting as a cantilever.

The shear in beams is usually provided for by means of web reinforcement in the form of stirrups or a combination of stirrups and bent bars, as shown in Fig. 9-34. In the case of continuous deep beams with the tensile reinforcing in the top of the beams at the supports, any combination of two or more bent bars will usually result in an arrangement of vertical stirrups and bent bars, as shown in Fig. 9-34. The bent bars starting at the one-fifth point of the span coincide almost exactly with the high tensile forces in the beam and are particularly effective in resisting diagonal tension. Although most designers will concede that the use of bent bars will result in a tougher beam, the expense of bending can be justified only by using them to resist diagonal tension and thus reduce the number of vertical stirrups and the unnecessary close spacing or crowding of stirrups at the supports. When the bent bars can be distributed from the one-fifth point at the bottom of a continuous beam, to within 1 ft 0 in. from the face of the column at the top, it is doubtful if there would be a diagonal-tension failure if no vertical stirrups were provided. The stirrups may be designed conservatively in the region of the bent bars, as shown in Fig. 9-34, and spaced uniformly from the face of the support to the critical section.
Fig. 9-34. A combination of stirrups and bent bars as a means of web reinforcement for shear in beams.

The code provides for a maximum spacing of $\frac{3}{4}d$ for bent bars, $\frac{3}{4}d$ for vertical bars, when the unit shearing stress is less than $0.06f'_c$ and $\frac{3}{4}d$ when the unit shearing stress is greater than $0.06f'_c$.

The allowable stress $f_s$ for stirrups depends upon the anchorage of the bar in the compression area, in which the standard hook, as shown in Fig. 9-35, is assumed to develop a stress of 10,000 psi. The additional stress to be developed by bond in the compression area of a flexural member may be expressed by the general formula

$$f_s = 10,000 \text{ psi} + \frac{2u}{c} (d - 7c - 2)$$

For deformed bars

$$f_s = 20,000 \text{ psi} \quad u = 0.10f'_c$$

For plain bars

$$f_s = 20,000 \text{ psi} \quad u = 0.045f'_c$$

Similarly, the additional stress to be developed by bond for bent bars (Fig. 9-36) may be expressed by the general formula

$$f_s = 10,000 \text{ psi} + \frac{2.8u}{c} (d - 2c - 4)$$

$$d = (f_s - 10,000) \frac{c}{2.8u} + 2c + 4$$

Fig. 9-35. Standard hook in compression area of concrete anchors stirrup.
The number and arrangement of bent bars depend upon beam sizes and usually they can be covered by typical details on the working drawings similar to those shown in Fig. 9-37. When possible they should be as nearly symmetrical as it is possible to arrange them about the vertical axis of the beam.

The code provides that, for simple spans or at the freely supported ends of continuous beams, at least one-third of the required reinforcement for positive moment be continued into the support for anchorage. In many cases this minimum amount of reinforcement will not meet the requirements for bond.

For continuous beams, cast monolithically with the supports, as shown in Fig. 9-38b, the code requirement is reduced to $0.25A$, as the minimum amount of the positive moment reinforcing which shall be continued into the support. Although the code does not limit the minimum extension of the bar to 20 diameters beyond the face of
the column, as shown, this is good standard practice, as the reinforcement is in compression and acts to resist the effects of plastic flow whether it is figured to do so or not. For beams of average depth, in which the concrete is fully stressed to \( 0.45f_c' \) (Fig. 9-39) the compressive force in the steel is approximately

\[
2n \times 0.75 \times f_s \text{ or } 2 \times 10 \times 0.75 \times 1,350 = 20,000 \text{ psi}
\]

The total force \( C = \frac{\pi D^2}{4} \times f_s \),

The bond force \( = u_D D \times L \)

\[
L = \frac{20,000\pi D^2}{4uD} = \frac{5,000D}{u}
\]

\[
u = 0.10f_c' \quad L = \frac{5,000D}{0.10 \times 3,000} = 16.5D
\]
As these bars are not completely encased in concrete but are lapped with bars from the adjoining spans to the extension of the bar, 20 diameters beyond the column face is not excessive.

For continuous beams, as shown in Fig. 9-38, the bars in the top of the beam at the supports must meet the requirements for bond between the support and the point of inflection according to the standard formula for bond \( u = V/\Sigma a jd \). The code provides two allowable stresses for bond on deformed bars, 0.07\( f'c \) when the effective depth is less than 12 in. For simple beams, or where there is very little moment at the end of a continuous beam, the bond stress should be checked for the bars in the bottom of the beam in which case the allowable unit stress of \( u = 0.10 f'c \) is permissible.

In addition to the stresses in beams, caused by bending, there may also be stresses due to torsion resulting from eccentrically applied loads or from framing, as in the case of marginal beams where the entire slab load is supported on one face of the beam. Although the shearing stresses caused by torsion have generally been ignored in the past, it seems only reasonable that they be given some consideration, because of the increase in unit stresses as included in the recent revisions of the code.

While it should be recognized that there may be stresses in marginal beams due to a form of torsional moment an exact analysis is difficult because of not only the meager information available but also the fact that pure torsion rarely exists in building frames where the adjoining floor slab forms an integral part of the beam and prevents the deformation that would normally occur when a torque is applied to a rectangular section. The problem is further complicated by the effect of plastic flow and creep which is inherent in reinforced concrete.

The amount of torsional moment in a marginal beam depends largely upon the relative torsional stiffness of the beam as compared with the bending stiffness of the adjoining framing. The torsional stiffness of the spandrel beam shown in Fig. 9-40a is small as compared with the bending stiffness of the beams B-1 and may be entirely neglected. When the adjoining framing consists of a relatively thin slab as shown in Fig. 9-40b it is apparent that there is some twisting effect approaching torsion. The amount of torque would depend upon the stiffness of the columns, the torsional stiffness of the spandrel beam, the bending stiffness of the slab, and the distribution of the slab load. For a relatively thin slab reinforced in one direction the torsional shearing stresses in the spandrel beam may become critical. The resulting torsional shear from a two-way slab of equivalent depth or a flat slab would be considerably less because of the distribution of the slab load.

It seems equally apparent that there would be very little torsional moment in the spandrel beam from the slab immediately adjacent to the columns provided that the slab is properly reinforced parallel to the spandrel beam at the column. The resulting effect upon the spandrel beam near the columns would be more in the nature of uplift than torsion and the amount of stress and the length of beam subject to such forces would vary depending upon the method of reinforcing the slab.

The torsional stiffness \( K_t = k_t b^4 E_c/0.5L \)

\[ b_D \]
\[ k_t \]
\[ C_t \]
\[ K_t \]
\[ v_t \]

Fig. 9-40. Torsional stiffness coef. \( k_t \) and bending stiffness coef. \( C_t \).
for members of constant cross section as compared with the bending stiffness
\[ K = kE_i I/L, \]
in which \( k \) depends upon the ratio of \( b/D \) as shown in Fig. 9-40c. The
value of \( E_i \) is uncertain because of the tendency of concrete to relieve itself of stress
under sustained loads for long periods of time by plastic flow in the concrete. Where
torsional moments have been considered in the examples of building-frame design, a
value of \( E_i = 250 f'c \) has been used.

If it is assumed that there will be no increase in the torsional moment for a distance
of \( L/8 \) from the center of the columns in the case of a slab reinforced in one direction as
shown in Fig. 9-41a, the maximum unit torsional moment \( m_1 \) will depend upon ratio of
the column stiffnesses to the stiffness of the slab of width equal to \( L \). The nearest
approach to pure torsion would occur at the center of the span where the spandrel
beam is partially free to rotate. If the torsional moment \( m_1 \) is used as the fixed-end
moment the value of \( m_2 \) at the center of the span will depend upon the stiffnesses of the
spandrel beam and a unit width of slab. The torsional moment is assumed to increase
from \( m_2 \) at the center of the span to \( m_1 \) by the equation
\[ m = m_2 + a^2(m_1 - m_2). \]

For a two-way slab supported on narrow beams as shown in Fig. 9-42a only a portion of
the slab load is carried in the transverse direction and the resulting torsion on the
spandrel beam is reduced proportionately. In the case of a wide-slab band being used
instead of the narrow beam \( B-1 \) it may be assumed that there will be no increase in the
torsional moment for a distance of \( L/4 \) from the center line of the column.

The maximum unit torsional moment \( m_1 \) will depend upon the ratio of the summa-

---

**Fig. 9-41. Torsional moments and shears for slab reinforced in one direction.**

**Fig. 9-42. Torsional moments and shears for slabs reinforced in two directions and supported on narrow beams.**
tion of the column and transverse beam stiffnesses to the stiffness of the slab of width $L$. Similar to a one-way slab the value of $m_2$ is determined by using $m_1$ as the fixed-end moment to be distributed between the spandrel beam and a unit width of slab at the center line of the span. For a flat slab reinforced in two directions it is assumed that the entire width of the column band produces uplift on the spandrel and that there is no increase in torsional moment for a distance of $L/4$ from the center line of the column. The maximum unit torsional moment $m_1$ will depend upon the ratio of the column stiffnesses as indicated in Fig. 9-43a to the stiffness of the slab for a width equal to $L$ as assumed for a one-way slab. Similar to one-way and two-way slabs the value of $m_1$ is used as the fixed-end moment to be distributed between the spandrel beam and a unit width of slab at the center line of the span in determining the amount of moment $m_2$.

The torsional moment at any point on the moment diagrams in Figs. 9-41b, 9-42b, and 9-43b is $m = m_2 + a^2(m_1 - m_2)$ in which $a$ is the ratio of the distance of $m$ from $m_2$ to the distance of $m_1$ from $m_2$.

The combined effect of direct and torsional shearing stresses is shown in Fig. 9-44a. The unit shearing stresses for which web reinforcing must be provided to prevent cracking from diagonal tension vary from zero on the horizontal axis to a maximum at the extreme fiber at the interior face of the beam. The maximum unit torsional shearing stress to be combined with the direct shearing stress is $v_t = M_t/C_t D$ where $M_t$ is the total torsional moment and $C_t$ is a coefficient depending upon the proportions of
the section as given in the tabulation of Fig. 9-40c. The type of web reinforcement to be used will depend upon the intensity of the combined unit shearing stresses. The typical two-prong stirrup may be used satisfactorily where the unit shearing stresses are low, but even in such cases the prongs on the exterior face of the beam are ineffective. Where the unit shearing stresses are high, the typical condition where torsional shear exists, it is sometimes practical to alternate stirrups 1 and 2 as shown in Fig. 9-44a. When provided in this manner all three prongs on the interior half of the beam are effective in resisting the diagonal-tension stresses. In either case it is desirable to use open-top stirrups to facilitate the placing of the beam reinforcing. The top slab reinforcing may be used as reinforcing to complete the hoop at the top of the beam where the torsional shearing stresses are usually low.

The width \( n \) in Fig. 9-44a for which web reinforcing must be provided is

\[
n = \frac{b(v_t + v - v_e)}{2v_t}
\]

and the maximum spacing of stirrups is

\[
s = \frac{A_s f_y}{(v_t + v - v_e) n/2}
\]

as limited by section 806 of the ACI Code. The dimensions for spandrel beams are many times determined by the architectural requirements and are larger than required for bending stresses, which results in excessive depth of the beams. In such cases it is often practical to take advantage of the bent bars as located near the columns to resist diagonal tension. In this case the width \( n \) becomes

\[
n' = \frac{b(v_t + v - v_e - v_e)}{2v_t}
\]

It is not the intent to present the preceding discussion as an accurate solution for torsion problems or to imply that a meticulous solution is necessary for every spandrel beam in a building. In the absence of more complete information on torsion in concrete building frames this procedure may be used to serve as a general guide as to the possible stresses.

**DEFLECTION OF CONCRETE MEMBERS**

The following information on deflection is based on the publication, *Deflection of Reinforced Concrete Members*, ST-70, published by the Portland Cement Association. For complete data on deflection, the engineer should obtain the original publication, as only the tables for computing the deflections for ordinary loads encountered in building frames are reproduced here, together with a brief discussion of their use.

The deflection of concrete members is due not only to the superimposed loads but also to the effect of both shrinkage of the concrete during hydration and plastic flow which tends to increase the deflection as discussed in the original publication. For building frames using short-span slabs and deep beams and girders it is not ordinarily necessary to consider deflections unless the spans are especially long. When one-way ribbed slabs or shallow beams are used on long spans, deflection may become a problem and it becomes necessary to limit the depth in order to keep the deflection from becoming critical. In such cases the computed deflection should be less than \( \frac{1}{6} \) of the span and where partitions are supported on shallow members a computed deflection of \( \frac{1}{6} \) of the span is not too conservative. The required depth to limit the deflections within these limits depends not only upon the superimposed load but also upon the intensity of the stresses, as they affect plastic flow, the shape of the member as to constant or variable cross section, and the conditions of restraint at the ends.

The values of \( A \) for correction to the \( M/I \) curves and \( C \) for correction for the deflection at the center of the span are taken from the Portland Cement Association publication *Deflection of Reinforced Concrete Members* but modified to correspond to the parameter values of \( r = h/D \) as used in Table 9-55. See Table 9-5 on page 9-69.

It is recommended that the uncracked gross concrete section be used in determining the moment of inertia \( I \) to be used in deflection computations. This value of \( I \), as usually used in the calculations for bending moments, is believed to give the best
approximation of the actual deflections when the proper allowance is made for the effect of the flanges of T beams as usually encountered in concrete construction. The effect of the flange for various ratios of \( t/D \) and \( b/b' \) is shown in Fig. 9-45, which is taken directly from the Portland Cement Association Publication ST-70, which recommends that the actual flange width be used up to a maximum of \( 6b' \), or six times the width of the web. This value of \( b/b' \) agrees very closely with the test results available.

The Portland Cement Association Publication ST-70 recommends that the effect of loads upon deflection be computed in two steps: (1) For live loads which may be considered as instantaneous loads in which the value of \( E_c \) to be used is 1,000\( f' \). Live loads falling under this classification would include most of the live loads in ordinary building frames such as office buildings, schools, hospitals, and garages. (2) For dead loads and permanent live loads such as may be encountered in warehouses, the value of \( E_c \) should be reduced to 250\( f' \). This lower value of \( E_c \) compensates for all practical purposes for the effect of shrinkage and plastic flow upon the permanent deflection of a concrete member when the initial stress is approximately the full allowable working stress of 0.45\( f' \). For lower stresses on the total concrete section the deflection is reduced proportionately and the value of \( E_c \) can be proportioned between 1,000\( f' \) and 250\( f' \) in the proportion to the ratio of the computed stress to 0.45\( f' \) as shown in Fig. 9-46. The critical section for deflection is located at the center of the span as indicated by point C in Tables 9-6 and 9-53c. Computations for deflection will be found under Examples of R/C Building Frames.

**COLUMNS**

Reinforced-concrete columns, as generally used in building frames, consist of a section of concrete reinforced with vertical bars which depend upon either ties or spirals for lateral support. Occasionally where the column loads are heavy and it is necessary to keep the column size small, a composite column consisting of a steel core within a spiral is used, but composite columns are not common practice in building frames.

Tied columns may be of two general types: (1) the vertical reinforcing is located in two faces of the column and parallel to the axis of bending, as shown in Fig. 9-47a; or

![Fig. 9-47. Typical cross sections of tied columns.](image-url)
(2) the vertical bars are distributed in all four faces of the column, as shown in Fig. 9-47b. The code limits the percentage of vertical reinforcement to from 0.01 to 0.04 of the gross area of the concrete section.

The percentage of vertical reinforcement in spiral columns may be varied from 0.01 to 0.08 of the gross area of the concrete section. The typical type of spiral column is shown in Fig. 9-48a, where all the vertical reinforcement is located within a single spiral. It is difficult to place a high percentage of vertical reinforcement within the spiral, and in order to keep column sizes to a minimum, additional reinforcement is sometimes added within an inner spiral, as shown in Fig. 9-48b, or by placing four bars in the corners of the column, by the use of ties, as shown in Fig. 9-48c. The combination of the spiral and ties is the most effective in bending because of the greater percentage of reinforcing near the face of the column, but it is also the most difficult type of column to fabricate and place in the forms. The 1951 ACI Code provides for the design of columns under three distinct conditions of loading:

1. Concentric load, where the allowable direct load on a short column is

   \[ P = A_s(0.18f'_c + 0.8f'p_c) \] for tied columns

   \[ P = A_s(0.225f'_c + f'p_d) \] for spiral columns

   When the length \( h \) of the column exceeds 10 times the least lateral dimension \( t \), the allowable load is reduced by the formula \( P' = P(1.3 - 0.03h/t) \). (\( t \) = least lateral dimension \( b \) or \( t \))

2. When the column is subjected to direct loads and bending caused either by the point of application of the load or by horizontal members framing into the column and where the values of \( e/t \), as shown in Fig. 9-49, are less than two-thirds, the section is assumed to be uncracked. Under this condition of loading, the concrete is effective in resisting both compressive and tensile stresses and the allowable load \( N \) is limited by the maximum allowable concrete stress \( f'p \) which varies from 0.18\( f'_c \) for tied columns and 0.225\( f'_c \) for spiral columns when the load is concentric to 0.45\( f'_c \) when the stress is caused by pure bending. The column may be designed for an equivalent concentric load which will produce the same maximum concrete stress by use of the formula

   \[ P = N(1 + CDe/t) \]

   in which the value of \( CDe/t \) is a bending factor determined by the properties of the column section. The formula \( P = N(1 + CDe/t) \) may be developed as follows:
\[ f_c = \frac{N}{A_s[1 + (n - 1)p_e]} \pm \frac{Net}{2I} \]

When \( I = R^2 A_s[1 + (n - 1)p_e] \)

\[ f_c = \frac{N}{A_s[1 + (n - 1)p_e]} \pm \frac{Net}{2R^2 A_s[1 + (n - 1)p_e]} \]

\[ D = \frac{t^2}{2R^2} \quad \text{or} \quad 2R^2 = \frac{t}{D} \quad f_c = \frac{N}{A_s[1 + (n - 1)p_e]} \times \left( 1 + \frac{De}{t} \right) \]

\[ f_c = f_p = f_s \times \frac{1 + De/t}{1 + CDe/t} \quad f_s = \frac{1 + De/t}{1 + CDe/t} = \frac{N}{A_s[1 + (n - 1)p_e]} \times \left( 1 + \frac{De}{t} \right) \]

\[ P = f_s A_s[1 + (n - 1)p_e] \quad \text{and} \quad P = N \left( 1 + \frac{CDe}{t} \right) \]

By definition, the coefficient \( D = t^2/2R^2 \) where \( R \) is the radius of gyration of the transformed area of the column section. The value of \( D \) varies with the amount and location of the reinforcing and must be determined separately for each type of standard column. For a tied column (Fig. 9-50) with the reinforcing located in the two faces parallel to the axis of bending, the general formula for the value of \( D \) may be developed as follows:

\[ I_s = \frac{bt^3}{12} \]

\[ nI_s = nA_s \left( \frac{gt}{2} \right)^2 = np_e A_s \left( \frac{gt}{2} \right)^2 \]

\[ A_t = A_s + nA_s = A_s + np_e A_s \]

\[ I_t = I_s + nI_s \]

\[ = \frac{bt^3}{12} + np_e A_s \left( \frac{gt}{2} \right)^2 \]

\[ = A_s \left( \frac{t^2}{12} \right) + np_e A_s \left( \frac{gt}{2} \right)^2 \]

\[ 2R^2 = \frac{2I_t}{A_t} = \frac{2A_s(t^2/12) + 2np_e A_s(gt/2)^2}{A_s + np_e A_s} \]

\[ = \frac{(t^2/6) + 2np_e(gt/2)^2}{1 + np_e} \]

\[ D = \frac{t^2}{2R^2} = \frac{t^2(1 + np_e)}{(t^2/6) + 2np_e(gt^2/4)} \]

\[ = \frac{1 + np_e}{\frac{t^2}{6} + (3np_e g^2/6)} \]

\[ D = \frac{6(1 + np_e)}{1 + Znp_e g^2} \]

\[ D \] may be developed similarly for all the types of standard columns and the general formula becomes \( D = \frac{6(1 + np_e)}{1 + Znp_e g^2} \) in which \( Z \) varies depending upon the location of the reinforcing.

Fig. 9-50. Tied column with reinforcement located in two faces parallel to the axis of bending.
within the column section. The values for \( D \) in the standard columns are listed in Tables 9-15 to 9-19.

The principal difference in the ACI codes of 1951 and 1956 is in the allowable stresses to be used for eccentricities greater than \( e/t = \frac{3}{6} \). The 1951 code permitted the use of the bending factor \( CD/t \) for eccentricities from zero to one, and the permissible concrete stress \( f_p \) varies from \( f_s \) at \( e/t = 0 \) to \( 0.45f'_p \) at \( e/t = \infty \). A working stress of \( 0.45f'_p \) is used in the 1956 code for all eccentricities greater than \( e/t = \frac{3}{6} \) and \( 2n \) times the concrete stress is permitted in the compression steel.

Tests on concrete columns have indicated that the steel stress under sustained loading is typically greater than \( 2nf_s \) for the values of \( f_s \) under concentric loads because of the original shrinkage in the concrete and the effect of plastic flow. It seems that the maximum allowable stress of 20,000 psi in the steel would often be exceeded on the basis of an allowable working stress of \( 2n \times 0.45f'_p \) and also the value of the eccentric load \( N \) on the column would be greater than those permitted under the 1951 code for eccentricities varying from \( e/t = \frac{3}{6} \) to \( e/t = 1 \). The values \( N \) in the two codes are compared in Fig. 9-51 for eccentricities varying from 0.5 to 1.1 on a 20- by 20-in. tied column, \( f'_c/n/f_s = 3,000/10/20,000 \), using \( n \) times the concrete stress for the compression reinforcement. For these values of \( n \) the allowable eccentric loads are greater than permitted under the 1951 code for \( e/t = \frac{3}{6} \) to 1 when \( p_s \) is greater than 0.03. The difference would be still greater if \( 2nf_s \) had been used for the compression steel as permitted by the 1956 code, and it seems reasonable that the 1951 code, which is the more practical for general use, can be used without violating the 1956 code. There is less than 2 per cent difference in the allowable values of \( N \) for eccentrically

![Fig. 9-51. Comparison of values of \( N \) as specified in the 1951 and 1956 ACI codes.](image-url)
loaded columns except within the limits of \( e/t = \frac{2}{3} \text{ to } e/t = 1.2 \) where the use of \( 2nf_e \) for the compression reinforcement seems questionable.

3. For columns subjected to direct loads and bending stress where the values of \( e/t \), as shown in Fig. 9-52, are greater than 1, the concrete section is considered cracked. The compressive stresses are confined to the concrete and steel within the area \( b \times kt \) and the tensile stresses caused by bending are resisted by the steel only in the cracked portion of the column section. The column may fail by overstressing either the concrete in compression or the steel in tension.

The allowable load \( N \) for cracked sections may be determined by equating the vertical forces in Fig. 9-52 to zero as follows:

\[
N = f_{e} \frac{b t}{2} + f_{c} \frac{kt - d'}{kt} \frac{n p_{e} b t}{kt} - f_{c} \frac{d - kt}{kt} \frac{n p_{e} b t}{kt} \\
= f_{e} b t \left( k + n p_{e} \frac{kt - d'}{kt} - n p_{e} \frac{d - kt}{kt} \right) \\
= f_{e} b t \left( k^2 + n p_{e} \frac{kt - d'}{t} - n p_{e} \frac{d - kt}{t} \right) \\
= f_{e} b t \left( k^2 + n p_{e} \frac{kt - d' - d + k t}{t} \right) \\
= f_{e} b t \left[ k^2 + n p_{e} \left( \frac{2k t}{t} - \frac{d + d'}{t} \right) \right], \text{ and with } d + d' = t \\
= f_{e} b t \frac{k^2 + n p_{e} (2k - 1)}{2k}
\]

\[ N = f_{e} b t \left[ \frac{k^2 + n p_{e} (2k - 1)}{2k} \right] \quad \text{or} \quad N = f_{e} b t \left[ \frac{k^2 + n p_{e} (2k - 1)}{2k} \right] \quad (9-1) \]
The concrete stress \( f_c = \frac{N}{bt} \left[ \frac{2k}{k^2 + n_p(2k - 1)} \right] \)

\[ f_s = nf_c \frac{d - kt}{kt} = nf_c \frac{gt + d' - kt}{kt} = nf_c \left[ \frac{gt + (t - gt)/2 - kt}{kt} \right] = nf_c \frac{gt + t - 2kt}{2kt} \]

The tensile stress in the reinforcing \( f_s = nf_c \left[ \frac{1 + g}{2k} - 1 \right] \)

It can be shown in a similar manner that the formulas for \( N, f_s, f_s', \) and \( f_s \) are general and apply to all types of columns shown in Tables 9-7 and 9-9 provided that the values of \( k \) are known. Tables 9-20 to 9-23 provide curves for obtaining the approximate values of \( k \), which are determined by equating the moments of the internal column forces to the external moment \( N_e \).

\[
N_e = f_{cbt} \left[ \frac{t}{2} - \frac{kt}{3} \right] + f_c \left( \frac{2kt - t + gt}{2} \right) + f_c \left( \frac{2gt + t - gt - 2kt}{2} \right) + f_c \left( \frac{2gt + t - gt - 2kt}{2} \right)
\]

\[
= f_{cbt} \left[ \frac{kt}{2} - \frac{k^2 t}{3} + \frac{n_p gt}{2} \left( \frac{2kt - t + gt}{2} \right) + \frac{n_p gt}{2} \left( \frac{2gt + t - gt - 2kt}{2} \right) \right]
\]

\[
= f_{cbt} \left[ \frac{kt}{2} - \frac{k^2 t}{3} + \frac{n_p gt}{2} \left( \frac{2kt - t + gt}{2} \right) \right]
\]

\[
= f_{cbt} \left( \frac{3k^2 t - 2k^2 t + 3g^2 n_p}{6k} \right)
\]

\[
N_e = \frac{f_{cbt}}{12k} (3k^2 - 2k^2 + 3g^2 n_p)
\]

Equating Eqs. (9-1) and (9-2):

\[
N_e = f_{ebt} \left[ \frac{k^2 + n_p(2k - 1)}{2k} \right] = \frac{f_{cbt}}{12k} (3k^2 - 2k^2 + 3g^2 n_p)
\]

\[
e[k^2 + n_p(2k - 1)] = \frac{t}{6} (3k^2 - 2k^2 - 3g^2 n_p)
\]

\[
e = 3k^2 - 2k^2 - 3g^2 n_p
t = 6[k^2 + n_p(2k - 1)]
e = \frac{k^2 - \frac{3}{2}k^2 + g^2 n_p}{t} = \frac{k^2 + 2n_p(2k - 1)}{2k^2 + 2n_p(2k - 1)}
\]

It can be shown that Eq. (9-3) is typical for all types of square concrete columns by placing the proper coefficient \( C' \) in the term \( g^2 n_p \) the only portion of the equation affected by the amount and location of the vertical reinforcement. Equation (9-3) then becomes

\[
e = \frac{k^2 - \frac{3}{2}k^2 - C'g^2 n_p}{t} = \frac{k^2 + 2n_p(2k - 1)}{2k^2 + 2n_p(2k - 1)}
\]

The coefficients \( C' \) for use in the above equation for the various arrangements of vertical reinforcement are shown in Tables 9-20 to 9-23.

The allowable direct loads as permitted in the ACI Code (1951) for a 20- by 20-in. column reinforced with 2 per cent of vertical reinforcement are shown in Fig. 9-53 for
the four typical types of concrete columns, for eccentricities varying from 0 to 2 times the column depth. The allowable direct loads are also shown for the type T6 column for eccentricities varying from $0.67e/t$ to $2.0e/t$ and $2nf$, as permitted by the 1956 code. By comparison with the curves shown in Fig. 9-51 where $nf$ was used for the compressive stress in the reinforcement it is apparent that a still greater difference in the direct loads for eccentricities larger than $0.67e/t$ would occur with increased percentages of vertical reinforcement. The curves in Figs. 9-51 and 9-53 are intended to point out that the 1951 code may be used for eccentricities up to $e/t = 1.0$ without violating the 1956 code and also that perhaps some adjustment should be made in the 1956 code to avoid the increase in the column capacity when the section is considered cracked at $e = 0.67t$.

From a practical standpoint the curves in Fig. 9-53 do not represent a typical column design as the column sizes are usually kept to a minimum and the percentages of reinforcement increased for the larger eccentricities. They do serve to point out that it is practical to use tied columns with reinforcement in two faces at the top of a building where the direct load is small and spiral columns in the lower stories where the eccentricities are small. It should also be kept in mind that tied columns are the most economical to fabricate and place in the forms and they should be used in so far as it is practical without sacrificing usable floor area.

The stresses in reinforced columns subjected to direct loads and bending moments of such magnitude that it becomes necessary to consider the section to be cracked may be checked by the method shown in the following examples. The method is accurate
and applies to any shape of cross section and any arrangement of reinforcement provided there is one axis of symmetry and the plane of bending coincides with the plane of symmetry. It may also be used for square and rectangular columns subjected to unsymmetrical bending moments with reasonable accuracy provided that the necessary adjustment is made for the direction of the neutral axis. The neutral axis may be assumed as normal to the plane of bending for square columns as shown in Example 9-2. For rectangular columns with unsymmetrical bending the neutral axis tends to rotate toward being parallel with the long axis of the section. For approximate results the relation of the neutral axis to one of the principal axes may be determined by the formula \( \tan \alpha = (I_x/I_y) \tan \theta \) as shown in Example 9-3.

The location of the neutral axis is first determined by dividing the moment of inertia \( I_p \) by the first moments of the elements of the section \( Q_p \). With the location of the neutral axis determined, the first moments of the elements about the neutral axis are \( Q_n = Q_p + (\Sigma A) a \) and the stress in the concrete is \( f_c = (P/Q_n)d \), where \( d \) is the distance of the point under consideration from the neutral axis. \( f_s = (P/Q_n)dn \) and \( f'_s = (P/Q_n)2dn \).

**Example 9-1. Plane of Bending Perpendicular to Principal Axis.** \( f'_s/n/f_s = 3,000/10/20,000 \). \( A_s = 9 \text{ No. 11} \). N.A. = 22.5 - 16 = 6.5, \( Q_n = -6.844 + 319.8 \times 22.5 = 356 \).

<table>
<thead>
<tr>
<th>Segment</th>
<th>Area</th>
<th>( x_p )</th>
<th>( Q_p )</th>
<th>( I_p )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( nA_s )</td>
<td>1</td>
<td>4.68 \times 10 = 46.8</td>
<td>-31.5</td>
<td>-1,474</td>
</tr>
<tr>
<td>2</td>
<td>3.12 \times 10 = 31.2</td>
<td>-27</td>
<td>-842</td>
<td>22.700</td>
</tr>
<tr>
<td>3</td>
<td>6.24 \times 20 = 124.8</td>
<td>-18.5</td>
<td>-2,310</td>
<td>42.700</td>
</tr>
<tr>
<td>4</td>
<td>2 \times 18 = 36</td>
<td>-17</td>
<td>-612</td>
<td>10.400</td>
</tr>
<tr>
<td>5</td>
<td>2 \times 18 = 36</td>
<td>-19</td>
<td>-684</td>
<td>13.000</td>
</tr>
<tr>
<td>6</td>
<td>2 \times 18 = 36</td>
<td>-21</td>
<td>-756</td>
<td>15.850</td>
</tr>
<tr>
<td>Concrete areas</td>
<td>( \Sigma_1 )</td>
<td>-6,678</td>
<td>151,100</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>(.5 \times 18 = 9 )</td>
<td>-22.25</td>
<td>200</td>
<td>4,450</td>
</tr>
<tr>
<td>( \Sigma_A = 319.8 )</td>
<td>( \Sigma_2 )</td>
<td>-6,878</td>
<td>155,550</td>
<td></td>
</tr>
</tbody>
</table>

\( a = I_p/Q_p = 22.5'' \)

\( f_c = \frac{45,000 \times 6.5}{356} = 820 \text{ psi} \)

\( f_s = \frac{45,000 \times 9 \times 10}{356} = 11,400 \text{ psi} \)

\( f'_s = \frac{45,000 \times 4 \times 20}{356} = 10,100 \text{ psi} \)
Example 9-2. Square Column with Unsymmetrical Bending. \( f' = 3,000 / 10 / 20,000 \). \( A = 8 \text{ No. 11} \). N.A. = 25.5 - 15 = 10.5 in., \( Q_n = -6,954 + 283.9 \times 25.5 = 285 \).

<table>
<thead>
<tr>
<th>Segment</th>
<th>Area</th>
<th>( x_p )</th>
<th>( Q_p )</th>
<th>( I_p )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.56 \times 10 = 15.6</td>
<td>-36.95</td>
<td>-575</td>
<td>21,200</td>
</tr>
<tr>
<td>2</td>
<td>3.12 \times 10 = 31.2</td>
<td>-32.35</td>
<td>-1,005</td>
<td>32,350</td>
</tr>
<tr>
<td>3</td>
<td>3.12 \times 10 = 31.2</td>
<td>-27.75</td>
<td>-865</td>
<td>24,000</td>
</tr>
<tr>
<td>4</td>
<td>3.12 \times 20 = 62.4</td>
<td>-23.15</td>
<td>-1,445</td>
<td>33,400</td>
</tr>
<tr>
<td>5</td>
<td>1.56 \times 20 = 31.2</td>
<td>-18.55</td>
<td>-578</td>
<td>10,720</td>
</tr>
<tr>
<td>6</td>
<td>2 \times 5.75 = 4</td>
<td>-15.33</td>
<td>-5.5</td>
<td>1,062</td>
</tr>
<tr>
<td>7</td>
<td>2 \times 6 = 12</td>
<td>-18.11</td>
<td>-218</td>
<td>3,960</td>
</tr>
<tr>
<td>8</td>
<td>2 \times 10 = 20</td>
<td>-20.06</td>
<td>-400</td>
<td>8,000</td>
</tr>
<tr>
<td>9</td>
<td>2 \times 14 = 28</td>
<td>-22.04</td>
<td>-617</td>
<td>13,600</td>
</tr>
</tbody>
</table>

Concrete areas:

- \( \Sigma_1 = -5,768 \) 148,292
- \( a = I_p/Q_p = 25.7'' \)
- \( a = I_p/Q_p = 25.5'' \)

\[ f' = \frac{30,000}{285} \times 10.5 = 1,105 \text{ psi} \]
\[ f = \frac{30,000}{285} \times 11.45 \times 10 = 12,050 \text{ psi} \]
\[ f'' = \frac{30,000}{285} \times 6.95 \times 20 = 14,600 \text{ psi} \]
Example 9-3. Rectangular Column with Unsymmetrical Bending. \( f_c/n/f_s = 3,000/10/20,000 \). \( A_s = 8 \) No. 11.

\[
\tan \alpha = \frac{I_x}{I_y} \tan \theta
\]

\[
\theta = 45^\circ
\]

\[
I_x = 24 \times \frac{18^3}{12} = 11,650
\]

\[
I_y = 18 \times \frac{24^3}{12} = 22,700
\]

\[
\tan \alpha = 0.513
\]

\[
\alpha = 27^\circ 10'
\]

N.A. = 23.9 - 13.03 = 10.87", \( Q_a = -7,199 + 318.3 \times 23.9 = -7,199 + 7,600 = 401 \).

<table>
<thead>
<tr>
<th>Segment</th>
<th>Area</th>
<th>( x_p )</th>
<th>( Q_p )</th>
<th>( I_p )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.56 \times 10 = 15.6</td>
<td>-36.63</td>
<td>-572</td>
<td>20,900</td>
</tr>
<tr>
<td>2</td>
<td>1.56 \times 10 = 15.6</td>
<td>-32.29</td>
<td>-503</td>
<td>16,200</td>
</tr>
<tr>
<td>3</td>
<td>1.56 \times 10 = 15.6</td>
<td>-30.85</td>
<td>-482</td>
<td>14,850</td>
</tr>
<tr>
<td>4</td>
<td>1.56 \times 10 = 15.6</td>
<td>-27.95</td>
<td>-436</td>
<td>12,150</td>
</tr>
<tr>
<td>5</td>
<td>1.56 \times 20 = 31.2</td>
<td>-25.07</td>
<td>-392</td>
<td>9,800</td>
</tr>
<tr>
<td>6</td>
<td>1.56 \times 20 = 31.2</td>
<td>-22.17</td>
<td>-690</td>
<td>15,300</td>
</tr>
<tr>
<td>7</td>
<td>1.56 \times 20 = 31.2</td>
<td>-20.73</td>
<td>-646</td>
<td>13,400</td>
</tr>
<tr>
<td>8</td>
<td>1.56 \times 20 = 31.2</td>
<td>-16.39</td>
<td>-512</td>
<td>8,360</td>
</tr>
<tr>
<td>n.A.s</td>
<td>4.94 \times \frac{3^2}{12} = 4.9</td>
<td>-14.7</td>
<td>-72</td>
<td>1,060</td>
</tr>
<tr>
<td>9</td>
<td>7.41 \times 2 = 14.8</td>
<td>-16.03</td>
<td>-237</td>
<td>3,800</td>
</tr>
<tr>
<td>10</td>
<td>12.35 \times 2 = 24.7</td>
<td>-18.03</td>
<td>-445</td>
<td>8,030</td>
</tr>
<tr>
<td>11</td>
<td>17.29 \times 2 = 34.6</td>
<td>-20.03</td>
<td>-692</td>
<td>13,850</td>
</tr>
<tr>
<td>12</td>
<td>22.23 \times 2 = 44.4</td>
<td>-22.03</td>
<td>-978</td>
<td>21,550</td>
</tr>
<tr>
<td>13</td>
<td>25.9 \times .9 = 23.3</td>
<td>-23.48</td>
<td>-542</td>
<td>12,700</td>
</tr>
<tr>
<td>14</td>
<td>25.9 \times .9 = 23.3</td>
<td>-512</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[
\Sigma A = 318.3 \quad \Sigma I = -7,199 + 171,950 = 171,950
\]

\[
\alpha = I_p/Q_a = 23.9
\]
\[ f_c = \frac{40,000}{401} \times 10.87 = 1,085 \text{ psi} \]
\[ f_s = \frac{40,000}{401} \times 12.73 \times 10 = 12,700 \text{ psi} \]
\[ f_s' = \frac{40,000}{401} \times 7.51 \times 20 = 15,000 \text{ psi} \]

**FOUNDATIONS FOR BUILDINGS**

The foundations for buildings generally consist of continuous wall footings around the exterior of the building and block footings to support the column loads on the interior of the building. Only footings bearing on soil and piles are included as a part of this section as special foundations are treated elsewhere.

The maximum soil pressure to be used is usually prescribed by the local building code. As the total design load seldom occurs on building foundations it is common practice to proportion the footings on the basis of the full dead load plus 25 percent of the live load but the maximum allowable soil pressure shall not be exceeded under the
full column or wall load. This practice tends to minimize the differential settlement under varying loading conditions when the foundations bear on yielding soils.

The critical sections for moment, bond, and shear for footings bearing on soil are shown on Figs. 9-54 and 9-55 as required by the ACI Code. The designer may find it convenient to use unit widths of 1 ft as the basis of design for moment and bond as follows:

Square-block column footings:

\[ m = \frac{0.85w_n a^2}{2b} = 0.425w_n a^2 \]

where \( w_n \) = net soil pressure.

\[ V = 12w_n jd = 0.85w_n ab \quad u = \frac{0.85w_n ab}{\Sigma_0 jd} \]

\[ du = \frac{0.85w_n ab}{0.875\Sigma_0} \]

For \( b = 1 \text{ ft} \) and \( w_n \) in kips:

\[ \frac{du}{w_n a} = \frac{0.85 \times 1,000}{0.875} = 972 \]

For a rectangular-block footing as shown in Fig. 9-55 the critical sections for moment, shear, and bond are similar to those for square footings except that the code requires that a portion of the total reinforcement in the short direction shall be located in a band width \( B \) equal to the length of the short side of the footing \( b \) as determined by the following formula: \( A_B/A_s = 2/(S - 1) \) in which \( S \) is the ratio of the long side to the short side of the footing. The code also requires that special consideration be given to the critical section for shear in that each face of the section shall resist in shear a load on the area bounded by the face of the critical section and two diagonal lines from the corners of the column at an angle of 45°. For the design of moments and bond on the basis of units 1 ft wide it becomes necessary to make separate computations in each direction as follows:

Rectangular-block column footings:

Reinforcing in long direction \( m_1 = 0.425w_n a^2 \)

\[ \frac{du}{w_n a} = \frac{972}{\Sigma_0} \]

Reinforcing in short direction \( m_2 = 0.425w_n m^2 \)

\[ \frac{du}{w_n m} = \frac{972}{\Sigma_0} \]

The principal advantage in designing footings in this manner is that the reinforcing may be selected in one operation for satisfying the requirements for both moment and bond and usually results in a more economical reinforcement.

The critical sections for moment, shear, and bond are similar for block footing caps for piles except that the length of the critical section for shear is limited to \( d/2 \) from the face of the column. The entire net pile load of any pile whose center is 6 in. or more inside the boundary of the critical section shall be resisted by shear on the face of the section. For piles whose centers are 6 in. or more outside the contributing area the entire pile load may be assumed as not contributing shear upon the section. For intermediate positions of the pile center the portion of the pile reaction producing shear upon the section may be interpolated between the above values.
PART 2

TABLES FOR DESIGN OF REINFORCED CONCRETE

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DESIGN TABLES

The intent in the preparation of the tables is to provide in the briefest manner possible not only the usual basic formulas used in design but also the properties of a limited number of standard sections for slabs, beams, columns, and footings which will enable the engineer to design most of the members in a building frame in a manner similar to that used with other materials. Special sections not covered as standard members may be designed by the general formulas.

For flexural members, the section modulus and total allowable shear, as governed by both the unit shearing and bond stresses, are provided for the standard sections. Similarly, for the standard column sections listed, the tables provide the total allowable concentric load, the values of the bending factor $12CD/t$ for eccentricities varying from 0 to 1, and tabulated loads for eccentricities varying from 1 to 2 times the column depth about the principal axis of bending.

The use of standard sections as a basis for design is an attempt not to develop any new theory in design but rather to simplify and reduce the usual routine computations required for moment, shear, bond, and column loads. For flexural members, the required size may be selected directly from the tables, once the section modulus and maximum shear are known. The usual computations for stirrup spacing, as controlled by code requirements, are eliminated as these requirements have been included in the preparation of the tables.

The standard column sections are listed for definite bar sizes and column types rather than for varying percentages of reinforcement, principally in order to eliminate the necessity of checking the requirements for spacing, etc., during design. The amount of vertical reinforcement is usually conservative, especially in the larger column sizes where additional reinforcement could be used in most cases. This precaution has been purposely taken to avoid crowding of reinforcement at the intersection of columns and beams and at column splices. Where additional reinforcement is used, special consideration should be given to these conditions.
Members Subject to Flexural Stresses

The properties of a rectangular concrete section with balanced reinforcing (Fig. 9-56) are identical for all sections having the same values of $b$, $k$, and $d$ regardless of the working stresses for the various classes of concrete listed in the code.

The tables are based upon the use of intermediate or rail-steel reinforcing with an allowable stress of 20,000 psi and concrete stresses of $0.45f'_c$, the working stresses permitted by the code, as follows:

<table>
<thead>
<tr>
<th>Ultimate strength of concrete, psi</th>
<th>2,000</th>
<th>2,500</th>
<th>3,000</th>
<th>3,750</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concrete: $n = 30,000/f'_c$</td>
<td>15</td>
<td>12</td>
<td>10</td>
<td>8</td>
</tr>
<tr>
<td>$f'_c = 0.45f'_c$</td>
<td>900</td>
<td>1,125</td>
<td>1,350</td>
<td>1,688</td>
</tr>
<tr>
<td>$k = f'_c + (20,000/n)$</td>
<td>0.403</td>
<td>0.403</td>
<td>0.403</td>
<td>0.403</td>
</tr>
<tr>
<td>Reins: $p = k f'_c$</td>
<td>0.0091</td>
<td>0.0113</td>
<td>0.0136</td>
<td>0.0170</td>
</tr>
<tr>
<td>$nA_s = npbd$</td>
<td>0.136bd</td>
<td>0.136bd</td>
<td>0.136bd</td>
<td>0.136bd</td>
</tr>
</tbody>
</table>

The moment of inertia $I_1$ of the transformed section is $I_1 = b(kd)^2/3 + nA_s[(1-k)d]d$. With $b$, $k$, $d$, and $nA_s$ being identical for all the working stresses of concrete as shown above, the moment of inertia $I_1$ and consequently the section modulus $S_t$ are identical. Using the distance from the neutral axis to the extreme concrete fiber in compression, $S_t = I_1/kd$.

The effective depth $d$ is based upon the use of reinforcing steel as required for a section when the allowable concrete stress is for $f'_c = 3,000$ psi. The steel selected for determining the depth is of such size as would be required for ordinary anchorage requirements in the ACI Code, or $2\frac{1}{2}$ diameters center to center of reinforcing bars and with a minimum coverage of $1\frac{1}{2}$ in. protection over the stirrups. Generally the depth is that required for one layer of steel, but where two layers are necessary it becomes apparent by the relation of $d$ to the over-all depth of the beam $D$.

The properties given in the tables for T beams are for a unit overhanging flange width in which $b = 12$ in. as shown in Fig. 9-57. The compressive force $C$ for the
concrete section is

\[ C = f_{cb} t \left( \frac{kd - t}{kd} \right) + \frac{f_{cb} t}{2} \left( 1 - \frac{kd - t}{kd} \right) \]

\[ A_s = \frac{C}{20,000 \text{ psi}} \]

\[ I_t = \frac{bt^3}{12} + bt \left( \frac{kd - t}{2} \right)^2 + nA_s (d - kd)^2 \]

\[ S_t = \frac{I_t}{kd} \]

The properties given in the tables for compression reinforcement \( A_s' \) (Fig. 9-58) are for bar sizes as shown.

The code provisions for the working stresses of reinforcing steel in compression are \( 2n \) times the compressive stress in the concrete at the location of the steel but not more than 20,000 psi, the allowable stress in tension. The tables for slabs, 9-30, 9-31, and 9-32, provide for \( \frac{3}{4} \) in. clear protection to the face of the concrete in compression. Tables 9-37 and 9-38 for beams provide for 1\( \frac{1}{2} \) in. protection over the vertical stirrups or a total of 2 in. to the main reinforcing steel.

Compressive stress \( f'_c = 2nf_c \frac{kd - d'}{kd} \)

Total compressive force \( C = A'_s f'_c \)

\[ c = 2nA'_s \frac{kd - d'}{kd} \]

The values listed in the tables provide the section modulus \( S_t \) for the bar sizes indicated and the required amount of tensile reinforcement.

\[ A_s = \frac{C}{20,000 \text{ psi}} \]

\[ I_t = 2nA'_s (kd - d')^2 + nA_s (d - kd)^2 \]

\[ S_t = \frac{I_t}{kd} \]

Shear

The values for shear in the tables for members in flexure are for the total shear \( V \) in kips for the section and web reinforcement indicated. \( V = V_e + V_s + V_a \).

Total shear on concrete section \( V_e = vbjd \)

Total shear taken by vertical stirrups \( V_s = \frac{A_{fs} bjd}{bs} \)

Total shear taken by bent bars \( V_a = \frac{A_{fs} bjd}{bs \sin \alpha} \)

For slabs and beams without web reinforcement the total vertical shear \( V \) is limited to the value of \( V_e \) given in the tables. The requirements of the code for the spacing of vertical web reinforcement are automatically taken care of in the tables for beams where total vertical shear \( V = V_e + V_s \). The values for \( V_s \) as listed in the tables are
for bent bars on an angle $a$ of 45° with the main reinforcement and for $s = \frac{3}{4}d$ as required by the code.

$$V_a = \frac{A_se_jbjd}{bsin a(\frac{3}{4}d)} \quad \text{or} \quad V_a = \frac{0.875A_se_j}{0.707 \times 0.75} = 1.65A_se_j$$

This value of $V_a$ is limited by the code to a maximum that will cause a unit shearing stress of not more than $0.05f_c'$ or a total unit shearing stress of not more than $0.12f_c'$ when used combined with vertical stirrups for web reinforcement.

**Bond**

The total allowable shear for members in flexure $V_B$ as controlled by bond is provided in the tables using $u = 0.07f_c'$ for top bars and $u = 0.10f_c'$ for bottom bars as required by the code. For solid slabs the values of $V_B$ are for the bar sizes shown for the compression steel $A_s'$ spaced at 12 in. on center. The values of $V_B$ for ribbed slabs and beams are provided for definite bar sizes.

**Use of Design Tables for Beams and Slabs**

**Example 9-4. Solid Slab.** $f_c' = 3,000$ psi, $n = 10$, $f_s = 20,000$ psi, $v_x = 90$ psi, $u = 300$ psi.

At $A$:

$$S = \frac{M \text{ ft-kips} \times 12,000}{f_c} = \frac{5.76 \times 12,000}{1,350} = 51.3 \text{ in.}^2 \text{ required}$$

From Table 9-30a

**Third column, second line for 6” slab, $S_t = 52.5 > 51.3 \text{ in.}^2$**

**Third column, fourth line for 6” slab, $d = 5”$**

**Second column, fourth line from top, $A_s = 1.44$**

$$A_s = \frac{M \text{ ft-kips}}{f_c} = \frac{5.76}{1.44 \times 5} = 0.80 \text{ sq in.}$$

Right-hand column, third line for 6” slab, $V_s = 4.67$ kips

Column for $u = 300$, fourth line for 6” slab, $V_{bond} = 3.06$ (No. 6 at 12”)

Column for $u = 300$, second line for 6” slab, $V_{bond} = 2.03$ (No. 4 at 12”)

From Table 9-33a

For No. 6 bars at 8” centers, $A_s = 0.66$

For No. 4 bars at 16” centers, $A_s = 0.15$

Total $A_s = 0.81 > 0.80 \text{ sq in. per 12” width}$

For No. 6 bars at 8” centers, $V_{bond} = 1.5 \times 3.06 = 4.59$ kips

For No. 4 bars at 16” centers, $V_{bond} = \frac{3}{4} \times 2.03 = 1.52$ kips

Total $V_B = 6.11$ kips $> 4.67$ kips required

At $C$:

$$S = \frac{4.1 \times 12,000}{1,350} = 36.5 \text{ (}S_t = 52.5\text{)} \quad A_s = 4.1/(1.44 \times 5) = 0.57 \text{ sq in.}$$

No. 5 st. and No. 6 bt., alt. at 8”:

At $B$: st. bars, No. 5 at 16” ($u = 300$) $V_B = \frac{3}{4} \times 2.54 = 1.91$ kips

**Example 9-5. Ribbed Slab, 20-in. Metal Fillers.** $f_c' = 3,000$ psi, $n = 10$, $f_s = 20,000$ psi, $v_x = 90$ psi, $u = 300$ psi.

At $A$:

$$S = \frac{M \text{ ft-kips} \times 12,000}{f_c} = \frac{14 \times 12,000}{1,350} = 125 \text{ in.}^2 \text{ required}$$

From Table 9-31b (also see Table 9-56b)
For rib as rectangular beam, \( b' = 9'' \) at end.

From \((10'' + 2.5'')\) slab, \( S_t = 195 > 125 \text{ in.}^2 \), and for \( v_e = 90 \) and \( b' = 9'' \), \( V_e = 7.9 \text{ kips} \). Also, \( a = 1.44 \), \( d = 11.25'' \), and \( ad = 16.2 \).

\[ A_s = \frac{M}{ad} = \frac{14}{16.2} = 0.87 \text{ sq in.} \]

From Table 9-33a (bottom of page), line 2, fourth and fifth columns

2 No. 6 bars at top = \( A_s = 0.88 > 0.87 \text{ sq in.} \)

From Table 9-31b, column for \( u = 300 \), 1 No. 6 bar, \( V_{\text{bond}} = 6.85 \text{ kips} \)

2 No. 6 bars at top, \( V_B = 2 \times 6.85 = 13.7 \text{ kips} > V_e = 7.9 \text{ kips} \)

From Table 9-31b for T beam

At \( C \):

\[ S = \frac{11 	imes 12,000}{1,350} = 98 \quad (b' = 5'') \quad \text{rib: } ad = 16.4 \quad S_t = 468 > 98 \quad A_s = 11/16.4 = 0.67 \text{ sq in.} \]

No. 5 st. and No. 6 bt.:

\[ A_s = 0.74 \text{ sq in.} > 0.67 \]

At \( B \):

1 No. 5 bottom (\( u = 300 \)) \( V_B = 5.7 \text{ kips} \)

Use \((10'' + 2\frac{1}{2}'')\) slab, \( 5'' \) wide rib, tapered pan at \( A \). No. 5 st., No. 6 bt.

Example 9-6. Slab Band Beam at \( A \) in Example 9-5. \( f_{\text{c}} = 3,000 \text{ psi}, n = 10, f_s = 20,000 \text{ psi}, v_e = 90 \text{ psi}, u = 300 \text{ psi} \).

At \( C \):

\[ S = \frac{118 \times 12,000}{1,350} = 1,050 \]

From Table 9-30b

\( 12\frac{1}{2}'' \) slab, \( S_t = 260 \text{ per ft} \)

Min \( b, b = 1,050/260 = 4.05 \text{ ft} \)

\( nA_s = 18.4, A_s = 4.05 \times 18.4/10 = 7.45 \text{ sq in.} \)

From Table 9-34

6 No. 7 = 3.60
5 No. 8 = 3.96

7.56 sq in.

At \( A \) and \( B \):

Assume \( b = 54'' \)

\[ S = \frac{157 \times 12,000}{1,350} = 1,400 \]

10 No. 8 bt., \( A_s = 7.92 \)
Add 2 No. 9, \( = 2.00 \)

9.92 sq in.

Fig. 9-61. Slab band beam restrained at both ends.

Fig. 9-62. Critical section for shear.
54" × 12 1/4", $S_t = 4.5 × 260 = 1,170$ and $A_s = 4.5 × 18.4/10 = 8.28$
Using 2.74 No. 7, $S_e = 2.74 × 84 = 230$ and $A_s = 2.74 × 0.57 = 1.56$

\[
\begin{array}{l}
1,400 \\
10 \text{ No. 8}: V_b = 10 × 9.2 = 92 \text{ kips} \\
2 \text{ No. 9}: \quad = 2 × 10.4 = 20.8 \\
\end{array}
\]

\[
\begin{array}{l}
112.8 \text{ kips} \\
V_e = 4.5 × 10.5 = 47.2 \text{ kips} \\
2 \text{ No. 8 bt.: } V_e = 1.65 × 22.8 = 37.6 \\
\end{array}
\]

\[
A_s = 2 × 0.79 = 1.58, \quad s = 7\frac{1}{2} \text{ in.}
\]

\[
f_s = 2.8 × 300(11\frac{1}{4}-2-4) + 10,000 = 14.4 \text{ kips (see Table 9-27)}
\]

\[
f_a = \frac{A_s f_s}{b_s \sin 45^\circ} = \frac{22,800}{54 × 7\frac{1}{2} × 0.707} = 80 \text{ psi (} < 0.05f_s')
\]

and 80 psi + 90 psi = 170 psi (\(< 0.08f_s')

![Fig. 9-63. Transverse bending at column.](image)

Transverse bending:

\[
M = 0.75 \text{ ft} × 43.3 \text{ kips} = 32.5 \text{ ft-kips}
\]

\[
A_s = M \text{ ft-kips}/ad = 32.5/(1.44 × 10.37) = 2.18 \text{ sq in.} < 8 × 0.31
\]

Use 8 No. 5: for 1 No. 5, $A_s = 0.31$, $\Sigma = 1.963$.

\[
V_{\text{bond}} = 0.866u\Sigma \bar{d} = 0.866 × 300 × 8 × 1.963 × 10.37 = 43 \text{ kips}
\]

**Example 9-7.** Rectangular Beam, Restrained at Ends. $f_s' = 3,000 \text{ psi}$, $n = 10$, $f_s = 20,000 \text{ psi}$, $v_s = 90 \text{ psi}$, $u = 210 \text{ psi}$.

At $C$:

\[
S = \frac{67 × 12,000}{1,350} = 597
\]

From Table 9-38e

12" × 22" beam, $S_t = 798$

$ad = 28.1 \quad A_s = 67/28.1 = 2.39 \text{ sq in.}$

![Fig. 9-64. Rectangular beam restrained at both ends.](image)
From Table 9-34  
2 No. 7 st., 2 No. 7 bt., $A_s = 2.40$ sq in.

At $A$ and $B$:

$$S = \frac{134 \times 12,000}{1,350} = 1,194$$

4 No. 7 bt., $A_s = 2.40$

Add 2 No. 10, $A_s = 2.54$

$$4.94 \text{ sq in.}$$

$14'' \times 22''$ beam, $S_t = 932$ and $nA_s/n = A_s = 3.72$

2 No. 7 st., $2 \times 138 = S_e = 276$ and $2 \times 0.54 = 1.08$

$$1,208 \quad 4.80$$

4 No. 7; top bars, $u = 210$, $V_B = 4 \times 9.8 = 39.2$

2 No. 10, top bars, $u = 210$, $V_B = 2 \times 14.2 = 28.4$

67.6 kips

Web reinforcement required:

$$\frac{40 - 21.2}{4 \text{ kips/ft}} = 4.7'$$

At $z = 3.55$

$$V = \frac{40 - 3.55 \times 4}{25.8 \text{ kips}}$$

(See top of Table 9-38c for $V_a$)

$$V_e = 21.2 \text{ kips}$$

No. 7 bt., $V_a = 19.8$

$$41.0 \text{ kips}$$

No. 3 stirrups required at $8''$ o.c. Spacing, $3'$, 6 at $8''$ (see $12 \times 22$ beam, $v_e = 90$, spacing $8''$, $V = 27.5$).

![Fig. 9-65. Bent bars and stirrup arrangement at end of beam.](image)

![Fig. 9-66. T beam simply supported at both ends.](image)

**Example 9-8.** T Beam, Simply Supported at Ends. $f' = 3,000$ psi, $n = 10, f_s = 20,000$ psi, $v_e = 90$ psi, $u = 300$ psi. $t = 5''$ thick.

At $C$:

$$S = \frac{160 \times 12,000}{1,350} = 1,420$$

From Table 9-34  
2 No. 11 st., 2 No. 11 bt., $A_s = 6.24$ sq in.

From Table 9-38c

$14'' \times 22''$ beam,

$$S_t = 932 \quad A_s \quad = 3.72$$

$t = 5''$, $S_t = 628$ in.$^4$ per ft., but use $S_t = 488$

$$A_s = 0.78 \times 26.6/10 = 2.07$$

Therefore, $b = \frac{488}{628} = 0.78'$

At $A$ and $B$:

Web reinforcement:

$$\frac{40 - 21.2}{5 \text{ kips/ft}} = 3.76' \quad V_e = 21.2 \text{ kips}$$

At $1' V = 35 \text{ kips},$ use No. 4, $3''$, 3 at $8''$

$3' = 25$

2 at $10''$

2 No. 11 st., $V_B = 2 \times 22.5 = 45 \text{ kips}$
Use of Design Tables for Columns

The values for the allowable vertical loads $P$ and $N$ are tabulated in the tables for the three cases of design as provided in the ACI Code (1951) and for definite types of columns set up by the location of the vertical reinforcing. As indicated in Figs. 9-51 and 9-53 the 1956 code permits somewhat higher values for eccentricities more than two-thirds the column depth. All tabulated values are for short columns and should be reduced by the code formula $P' = P (1.3 - 0.03h/t)$ for all columns having an unsupported length greater than ten times the least lateral dimension.

For eccentricities from 0 to 1, the bending factors $12CD/t$ are given for use in determining the equivalent direct loads directly from the bending moments in foot-kips.

**Example 9-9. Column to Support a Concentric Load of 300 Kips. $f_c' = 2,500$ psi, $f_s = 16,000$ psi, $h = 11'0"$.**

From Table 9-44e

18" × 18" column, S7 No. 10, $P = 324$ kips
Spiral, No. 4 at 3" o.c.
$h/t = 11/1.5 = 7.35$

**Example 9-10. Column to Support a Direct Load of 250 Kips and a Bending Moment of 45 Ft-kips. $f_c' = 2,500$ psi, $f_s = 12,000$ psi, $e = 45/250 = 0.18'$, $e/t = 0.18/1.67 = 0.108$, $h = 11'$.**

From Table 9-44g

20" × 20" column, T10 No. 10, $P = 342$ kips
$12CD/t = 1.85$, $P = 1.85 \times 45 = 83$ kips
$h/t = 11/1.67 = 6.6$
$N = 250$
$P = 333$ kips < 342

**Example 9-11. Column to Support a Direct Load of 60 Kips and a Bending Moment of 90 Ft-kips. $f_c' = 2,500$ psi, $f_s = 16,000$ psi, $e = 90/60 = 1.5'$, $e/t = 1.5/1.5 = 1$, $h = 12'$.**

From Table 9-44e

18" × 18" column, T6 No. 10, $P = 243$ kips
$12CD/t = 1.878$, $P = 1.878 \times 90 = 169$ kips
$h/t = 12/1.5 = 8$
$N = 60$
$P = 229$ kips < 243

**Example 9-12. Column to Support a Direct Load of 48 Kips and a Bending Moment of 88 Ft-kips. $f_c' = 3,000$ psi, $f_s = 16,000$ psi, $e = 88/48 = 1.81'$, $e/t = 1.81/1.5 = 1.21$, $h = 12'$.**

From Table 9-45e

18" × 18" column, T6 No. 10, $N = 49.5$ kips
$h/t = 12/1.5 = 8$

Use of Design Tables for Column Footings

The footing sizes and reinforcing for square-block column footings bearing on soil are provided in Table 9-48, for soil pressures varying from 1 to 6 tons per sq ft. Similarly the sizes and reinforcing are provided for pile caps in Table 9-51.

Additional information is provided in Table 9-47 for footings bearing on soil and in Table 9-51 for pile caps where the section moduli are provided for various footing and pile-cap depths for use in the design of special footings where required.

**Example 9-13. Square-block Column Footing to Support a Load of 450 Kips, Soil Pressure of 5 Tons per Sq Ft, Column 20' × 20'. $f_c' = 2,500$ psi, $n = 12$, $f_s = 20,000$ psi, $v_c = 75$ psi, $u = 200$ psi.**

From Table 9-49h

Footing, 7'0" × 7'0" × 34" deep
Reinforcement, 15 No. 5 each way

**Example 9-14. Pile Cap to Support a Load of 450 Kips, Pile Reaction of 25 Tons, Column 18' × 18'. $f_c' = 2,500$ psi, $n = 12$, $f_s = 20,000$ psi, $v_c = 75$ psi, $u = 200$ psi.**
From Table 9-51d
Pile cap, 8'0" × 12'0" × 42" deep
Reinforcement, 16 No. 5 short way, 18 No. 6 long way
Piles, 10 required

Example 9-15. Rectangular Column Footing 7'0" wide to Support a Column Load of 390 Kips, Soil Pressure of 3 Tons per Sq Ft, Column 20' × 20'. \( f_c' = 2,500 \text{ psi} \), \( n = 12 \), \( f_s = 20,000 \text{ psi} \), \( v_e = 75 \text{ psi} \), \( u = 200 \text{ psi} \).

From Table 9-49:
\[
A = (8'4" \text{ sq}) = 69.5 \text{ sq ft}
\]

Fig. 9-67. Plan of rectangular column footing.

Assume footing 7'0" × 10'0" and \( D = 26" \)

\[
w_n = \frac{390}{70} = 5.6 \text{ kips per sq ft}
\]

From Table 9-47 and Fig. 9-67
\[
x_v = 65"\]
\[
d = 22.37"
\]
\[
V = 6.21 \times 0.79 \times 5.6 = 27.5
\]
\[
7 \times 1.5 \times 5.6 = 58.8
\]

86.3 kips

\[
v_e = \frac{v}{b'd} = \frac{86,300}{65 \times \frac{2}{3} \times 22.37} = 68 \text{ psi} < 75
\]

Reinforcement in band width \( B \):

\[
\frac{A_B}{A_S} = \frac{2}{(10.0/7.0) + 1} = 0.81
\]

Bending in short direction:

\[
b = 10"; M = 0.85 \times 5.6 \times \frac{2.67^2}{2}
\]

= 16.8 ft-kips

\[
S = \frac{16.8 \times 12,000}{1,125} = 180
\]

\[
du = \frac{22.37 \times 200}{2.67 \times 5.6} = 298
\]

From Table 9-47
\[
S_I = 1,041 \text{ (line } D = 26")
\]
\[
A_s = M/ad = 16.8/32.2 = 0.52 \text{ sq in.}
\]

No. 5 at 7" o.c., \( A_s = 0.53 \) (bottom part of table)

Total reinforcement 120"/7" = 17 No. 5 required

In band \( B \):

\[
0.81 \times 17 = 14
\]
Use 18 No. 5 total.

Bending in long direction:

\[
b = 1'0'': M = 0.85 \times 5.6 \times \frac{4.17^2}{2} = 41.3 \text{ ft-kips}
\]
\[
S = \frac{41.3 \times 12,000}{1,125} = 446
\]
\[
du = \frac{22.37 \times 200}{4.17 \times 5.6} = 192
\]
\[
S_t = 1,041
\]
\[
A_s = 41.3/32.2 = 1.28 \text{ sq in.}
\]

No. 7 at 5\(\frac{3}{4}\)" o.c., \(A_s = 1.31 \text{ sq in.} > 1.28\)

Total reinforcement:

\[
\frac{84''}{5\frac{3}{4}''} = 15 \text{ No. 7}
\]

Example 9-16. Pile Cap 5'6" Wide to Support a Column Load of 275 Kips, Allowable Load on Piles of 30 Tons, Column 18" X 18". \(f' = 2,500\) psi, \(n = 12\), \(f_s = 20,000\) psi, \(v_c = 75\) psi, \(u = 200\) psi.

Column load, 275
Weight of cap 25 (assumed)
\[
p_a = \frac{275}{300} \text{ kips} = 55 \text{ kips} = 55,000 \text{ lb}
\]

Assume pile cap 44" deep.

![Fig. 9-68. Plan of rectangular pile cap.](image)

From Table 9-50

\[
x_v = 52''
\]
\[
d = 34.25''
\]
\[
v_c = \frac{2 \times 55,000}{52 \times \frac{7}{8} \times 34.25} = 70.5 \text{ psi}
\]

Weight = 5.5 \times 8 \times 550 \text{ psf} = 24.2 \text{ kips}

Reinforcement in band width \(B\):

\[
\frac{A}{A_s} = \frac{2}{(8.0/5.5) + 1} = 0.82
\]

Bending in short direction:

\[
b = 1'0'': M = 0.85 \times \frac{110,000}{8} \times 0.75
\]
\[
= 8.8 \text{ ft-kips}
\]
\[
S = \frac{8.8 \times 12,000}{1,125}
\]
\[
= 93.7
\]
From Table 9-50

\[ S_t = 2.452 \quad ad = 49.4 \]

\[ A_s = \frac{8.8}{49.4} = 0.18 \]

\[ \text{Min } A_s = 0.005bd = 0.205 \]

No. 4 at 12'' o.c., \( A_s = 0.20 \)

Total reinforcement: 9 No. 4

\[ 0.82 \times 9 = 7.4, \text{ use 11 No. 4} \]

Bending in long direction:

\[ b = 1'0''; M = 0.85 \times \frac{110,000}{5.5} \times 2' \]

\[ = 34 \text{ ft-kips} \]

\[ S = \frac{34 \times 12,000}{1,125} = 364 \]

\[ \sum_{p} = \frac{34.25 \times 200}{20} = 342.5 \]

\[ A_s = \frac{34}{49.4} = 0.69 \text{ sq in.} \]

No. 5 at 5\( \frac{1}{2}'' \) o.c., \( A_s = 0.68 \text{ sq in.} \)

Total reinforcement:

\[ \frac{66''}{5\frac{1}{2}''} = 12 \text{ No. 5} \]

**General Notations**

\( f_e \) = compressive unit stress in extreme fiber of concrete in flexure

\( f'_c \) = compressive strength of concrete at age of 28 days unless otherwise specified

\( f_t \) = compressive unit stress in the metal core of a composite column

\( f_s \) = tensile unit stress in longitudinal reinforcement; nominal allowable stress in vertical column reinforcement

\( f'_t \) = effective unit stress in compression reinforcement

\( f_a \) = tensile unit stress in web reinforcement inclined at an angle \( a \) with main reinforcement

\( f_v \) = tensile unit stress in vertical web reinforcement

\( n \) = ratio of the modulus of elasticity of steel to that of concrete

\( u \) = bond stress per unit of surface area of bar

\( v \) = shearing unit stress

\( v_a \) = shearing unit stress provided by inclined web reinforcement

\( v_s \) = shearing unit stress permitted on the concrete

\( v_v \) = shearing unit stress provided by vertical web reinforcement

**Notations for Members in Flexure**

\( A \) = total area of the concrete section

\( A_s \) = effective cross-sectional area of tensile reinforcement

\( A_s' \) = area of compressive reinforcing steel

\( A_v \) = area of web reinforcement unit (one stirrup or one set of inclined bars)

\( a \) = a coefficient used to obtain the required area of tensile reinforcement in the formula \( A_v = M/ad \) and is equal to \( f_s j/12,000 \), angle between inclined web bars and axis of beam

\( b \) = width of rectangular flexural member or width of flange for T and I flexural members

\( b' \) = width of web for T and I flexural members

\( C \) = resultant of compression in concrete only, or resultant of compression for compression reinforcement only

\( D \) = total depth of a concrete beam

\( d \) = depth from the compressive face of beam or slab to the centroid of the longitudinal tensile reinforcement

\( d'' \) = distance from the extreme compressive fiber to the compressive reinforcement

\( E_c \) = the modulus of elasticity of concrete in compression

\( E_s \) = the modulus of elasticity of reinforcing steel

\( I_c \) = the moment of inertia of the concrete section
I_t = the moment of inertia of the transformed section about the neutral axis in bending

j = the ratio of the distance between the centroid of compression and the centroid of tension to the depth d

K = the stiffness factor, that is, the moment of inertia I_t divided by the span L for beams or the height h for columns

k = the ratio of the distance from the extreme compressive fiber to the neutral axis to the depth d

L = the span length of a slab or beam, when taken as the distance between the center lines of supporting beams or columns for frame analysis

l = the span length of a slab or beam, when taken as the clear span between the faces of supporting members

l' = the clear span for positive moment and shear and the average of the two adjacent clear spans for negative moment (see section 701, ACI Code)

M = the external bending moment for members in flexure

M_r = the resisting moment of a concrete section as determined by the internal stresses in the concrete and reinforcing steel

n = the ratio of the modulus of elasticity of steel E_s to that of concrete E_c,

\[ n = \frac{E_s}{E_c} = \frac{30,000}{f_{c'}} \]

NA = neutral axis

p = the percentage of tensile reinforcement

p' = the percentage of compressive reinforcement

R = a constant in flexural computations:

Rectangular sections: \[ R = \frac{1}{2} f_s k \]

T-beam sections: \[ R = f_s \frac{t}{d} \left( \frac{2k - t/d}{2k} \right)(1 - z) \]

Compression reinforcement: \[ R = p'(2a - 1)f_s (1 - d'/kd)(1 - d'/d) \]

S = section modulus as required by the external bending moment

S_t = section modulus of the transformed section of the compressive reinforcement in a flexural member

S_{s1} = section modulus of the transformed concrete section of a flexural member

s = spacing of stirrups or bent bars used for shear reinforcement in flexural members

T = resultant of tensile stress in the reinforcement

t = thickness of slab or depth of flange of a T beam

u = bond stress per unit of surface area of bar

V = total vertical shear

V_a = total shear carried by web reinforcement inclined at an angle a with the axis of the reinforcing steel

V_c = total vertical shear on concrete without web reinforcement

V_s = total vertical shear carried by vertical stirrups

V_B = total vertical shear as controlled by bond stress u

v = shearing unit stress

v_a = shearing unit stress provided by inclined web reinforcement

v_l = allowable shearing unit stress resisted by concrete

v_r = shearing unit stress provided by vertical web reinforcement

W = total uniform load supported by a member in flexure

W_r = equivalent uniform load supported by a marginal beam for a two-way slab

w = uniform load per square foot of slab or uniform load per linear foot of beam

w_e = equivalent uniform load per square foot on a two-way slab or the equivalent uniform load per foot on a marginal beam supporting a two-way slab

\[ \Sigma_0 = \text{sum of the perimeters of bars in one set as used for bond} \]

Additional Notations for Flat Slabs

A = the distance in the direction of span from the center of support to the intersection of the center line of the slab thickness with the extreme 45° diagonal line lying wholly within the concrete section of slab and column or other support, including drop panel, capital, and bracket
b = width of section

\( c \) = effective support size (see section 1004-c, ACI Code)

\( H \) = story height in feet of the column or support of a flat slab, center to center of slabs

\( M_s \) = numerical sum of the assumed positive and average negative moments at the critical design sections of a flat slab panel

\( t \) = thickness of slab in inches at center of panel

\( t_1 \) = thickness in inches of slabs without drop panels, or through drop panel, if any

\( t_s \) = thickness in inches of slabs with drop panels at points beyond the drop panel

\( W \) = total dead and live load on panel

\( W_D \) = total dead load on panel

\( W_L \) = total live load on panel, uniformly distributed

\( w' \) = uniformly distributed unit dead and live load

**Notations for Concrete Columns**

\( A_c \) = area of core of a spirally reinforced column measured to the outside diameter of the spiral; net area of the concrete section of a composite column

\( A_g \) = the over-all or gross area of spirally reinforced or tied columns, the total area of the concrete encasement of combination columns

\( A_r \) = area of the steel or cast-iron core of a composite column; the area of the steel core of a combination column

\( A_e \) = the effective cross-sectional area of reinforcement in compression in columns

\( b \) = the over-all width of a column section parallel to the axis of bending, if any

\( C \) = ratio of the concrete stress \( f_c \) in axially loaded column to the allowable fiber stress for concrete in flexure

\( D \) = a factor \( t/2R \)

\( e \) = eccentricity of the resultant load on a column, measured from the gravity axis

\( f_a \) = average allowable stress in the concrete of an axially loaded concrete column

\( f_c \) = computed concrete fiber stress in an eccentrically loaded column

\( f_p \) = maximum allowable concrete fiber stress in an eccentrically loaded concrete column

\( f_r \) = allowable unit stress in the metal core of a composite column

\( f_r' \) = nominal allowable stress in vertical column reinforcement

\( h \) = unsupported length of column

\( N \) = axial load applied to a reinforced-concrete column

\( P \) = total allowable axial load on a column whose length does not exceed ten times the least cross-sectional dimension

\( P' \) = total allowable axial load on a long column

\( p' \) = ratio of the volume of spiral reinforcement to the volume of the concrete core (out to out of spirals) of a spirally reinforced concrete column

\( p_v \) = ratio of the effective cross-sectional area of the vertical reinforcement to the gross area \( A_g \)

\( R \) = least radius of gyration of a section

\( t \) = over-all depth of a rectangular column section parallel to the axis of bending, if any, or the diameter of a round column

**Notations for Footings**

\( P \) = axial column load supported on footing or pile cap

\( p \) = maximum allowable load per pile

\( p_n \) = net load per pile after deducting the weight of the pile cap

\( SP \) = allowable soil pressure for spread footings

\( w_n \) = net soil pressure after deducting the weight of the spread footing

\( x_s \) = width of the critical section for shear

**Notations for Beam Constants and Moment Coefficients**

\( a \) = ratio of the length of haunch at \( A \) to the length of the span

\( B \) = width of slab or beam

\( b \) = ratio of the length of haunch at \( B \) to the length of the span
TABLES FOR DESIGN OF REINFORCED CONCRETE 9-59

\( C_a \) = carry-over factor at \( A \)
\( C_b \) = carry-over factor at \( B \)
\( D \) = total depth of a member at the minimum section
\( E_e \) = modulus of elasticity of concrete
\( F_a \) = coefficient of fixed-end moment at end \( A \)
\( F_b \) = coefficient of fixed-end moment at end \( B \)
\( I_e \) = moment of inertia of the plain concrete section
\( k_a \) = stiffness factor of a member at end \( A \)
\( k_b \) = stiffness factor of a member at end \( B \)
\( K \) = stiffness of a member (\( K_a \) at end \( A \) and \( K_b \) at end \( B \))
\( L \) = length of span
\( M_{aF} \) = fixed-end moment at end \( A \)
\( M_{bF} \) = fixed-end moment at end \( B \)
\( M_s \) = moment induced at the fixed end of a member by unit rotation at the free end
\( r \) = ratio of the average depth of a haunch to the minimum depth of the member
\( t \) = thickness of slab

Use of Beam Constants and Moment Coefficients Tables

The values for beam constants and the coefficients of fixed-end moments for uniform loads and concentrated loads are given for members of constant cross section in Table 9-54 and for members with haunches in Table 9-55. As the parameter \( r \) is for the average depth of the sloping haunch, Table 9-55 may be used for haunches of constant cross section, provided that the haunch ratios \( a \) and \( b \) do not exceed 0.2\( L \).

For members usually used in building frames the dead load of the haunch is comparatively small and may be either ignored or approximated by using the fixed-end moments for an equivalent concentrated load. The use of the tables is illustrated as follows:

**Example 9-17. Deepened Slab Band.** Beam factors and moment coefficients for a deepened slab band as shown in Fig. 9-69a may be determined by using the values for a flat haunch as shown in Fig. 9-69b having an equivalent moment of inertia. For a span length \( l = 20 \text{ ft} \) and haunch ratios of \( a = b = 0.1 \), the 12,000. in of inertia of the slab-band haunches in Fig. 9-69a is \( I = \frac{144 \times 10^4}{12} = 12,000 \text{ in.}^4 \). The equivalent haunch depth for the member in Fig. 9-69b is \( d = \sqrt[3]{\frac{12 \times 12,000}{48}} = 14.4 \text{ in.} \) and \( r_A = r_B = 4.4/10 = 0.44 \)

\[ I_e = \frac{48 \times 10^4}{12} = 4,000 \text{ in.}^4 \]

From Table 9-55e and line \( r_A = 0.3, a = 0.1 \) and \( r_B = 0.3, b = 0.1 \)
\( k_A = k_B = 5.12, C_A = C_B = 0.567, \) and \( F_A = F_B = 0.090 \)

From Table 9-55d and line \( r_A = 0.5, a = 0.1 \) and \( r_B = 0.5, b = 0.1 \)
\( k_A = k_B = 5.54, C_A = C_B = 0.588, \) and \( F_A = F_B = 0.092 \)

---

Fig. 9-69. Deepened slab band.
Then 0.14/0.20 from first values to second values gives

\[ k_A = k_B = 5.40, \quad C_A = C_B = 0.58, \quad F_A = F_B = 0.091 \] (for uniform load)

\[ K_A = K_B = \frac{5.40 \times 4,000}{20} = 1,080 \]

Example 9-18. Haunched Beam. (See Fig. 9-70.) Beam constants and moment coefficients from Table 9-55c, \( r_A = 6''/20'' = 0.3, r_B = 6''/20'' = 0.3, a = 4'/20' = 0.2, b = 6'/20' = 0.3, b' = 12'', I_c = \frac{12 \times 20^3}{12} = 8,000 \text{ in}^4 \)

![Fig. 9-70. Haunched beam.](image)

At A:

\[ k_A = 6.77, \quad C_A = .676, \quad F_A = 0.092 \]

\[ K_A = 6.77 \times 8,000/20' = 2,710 \]

\[ M_A' = 0.092 \times 50 \times 20 = 92 \text{ ft-kips} \]

\[ 0.056 \times 10 \times 20 = 11.2 \]

\[ 103.2 \text{ ft-kips} \]

At B:

\[ k_B = 7.72, \quad C_B = 0.592, \quad F_B = 0.101 \]

\[ K_B = 7.72 \times 8,000/20 = 3,090 \]

\[ M_A' = 0.101 \times 50 \times 20 = 101 \text{ ft-kips} \]

\[ 0.191 \times 10 \times 20 = 38.2 \]

\[ 139.2 \text{ ft-kips} \]

Example 9-19. Flanges of T Beams. The coefficients for the effect of the flanges of T beams upon the moment of inertia of a member are shown in the diagram for Table 9-53. The effective flange widths should be limited to the requirements of section 705 of the code.

1. One-fourth of the span length of the beam.
2. The overhanging width on either side of the beam shall not exceed eight times the slab thickness.

![Fig. 9-71. Plan of slab and beam panel.](image)
3. One-half the clear distance between beams.
For the 12" × 20" beam with haunches as used in Example 9-18 with a 6" slab shown in Fig. 9-71, the effective slab width

\[
\begin{align*}
  b &= 29\frac{1}{2} \times 12" = 60" \\
  16 \times 6" + 12 &= 108" \\
  13\frac{1}{2} \times 12" &= 114"
\end{align*}
\]

\[
\frac{t}{D} = \frac{6"}{20"} = 0.3 \quad \frac{b}{b'} = \frac{60}{12} = 5
\]

Coeff. = 1.9

\[
I_C = 1.9 \times 8,000 = 15,200 \text{ in}^4
\]

\[
K_A = \frac{6.77 \times 15,200}{20} = 5,150
\]

\[
K_B = \frac{7.72 \times 15,200}{20} = 5,860
\]

Determining Beam Constants and Moment Coefficients by the Column-analogy Method

Beam constants and fixed-end moments for members not falling within the scope of Table 9-55 may be determined by the column-analogy method presented by Hardy Cross in *University of Illinois Engineering Experiment Station, Bulletin 215, 1930*, and are illustrated as follows:

**Example 9-20. Beam with Sloping Haunches.** Solution by column analogy. 12" × 20" beam. (Fig. 9-72.)

Beam dimensions for use in the analogous column are D in feet. To obtain the relative values of K multiply by 12⁸B; absolute values by 12⁸EB.

\[
I_C = 8,000 \text{ in}^4
\]

Unit load is the ordinate of the static-moment diagram times the area of the segment of the analogous column section. (From Fig. 9-73.)

**Fig. 9-72. Loads on beam with sloping haunches.**

\[
\begin{array}{cccc}
  m & a & ma & z'' \\
  23.7 \times 1.70 &= 40.3 & -8.62 &= -347 \\
  63.7 \times 3.42 &= 217.5 & -6.62 &= -1,437 \\
  93.7 \times 5.22 &= 488 & -4.62 &= -2,250 \\
  113.8 \times 5.22 &= 593 & -2.62 &= -1,550 \\
  123.5 \times 5.22 &= 644 & 0.62 &= +399 \\
  123.5 \times 5.22 &= 644 & +1.38 &= +890 \\
  113.8 \times 5.22 &= 593 & +3.38 &= +2,000 \\
  83.7 \times 3.90 &= 326 & +5.38 &= +1,755 \\
  33.7 \times 2.40 &= 80.8 & +7.38 &= +597 \\
  -26.3 \times 1.54 &= -40.5 & +9.38 &= -380 \\
  39.06 & 3,586.1 & -1,121 \\
  e &= -1,121 \\
  3,586 & -0.313
\end{array}
\]
Fig. 9-73. Analogous column load for the loaded beam of Fig. 9-72.

Analogous column section:

\[
\begin{align*}
A & = 170 \times 1 = 170 \\
3.42 \times 3 & = 10.26 \\
26.1 \times 9 & = 234.9 \\
3.9 \times 15 & = 58.5 \\
2.4 \times 17 & = 40.8 \\
1.54 \times 19 & = 29.3 \\
39.06 & = 375.46 \\
39.06 & = 884.3
\end{align*}
\]

\[
I = I_0 + A\alpha^2
\]

\[
x' = \frac{375.46}{39.06} = 9.62'
\]

\[
K_A = \frac{1}{39.06} + \frac{1 \times 9.62^2}{884.3} = 0.0256 + 0.1045 = 0.1301
\]

\[
K_B = \frac{1}{39.06} + \frac{1 \times 10.38^2}{884.3} = 0.0256 + 0.1220 = 0.1476
\]

\[
M_C = \frac{1}{39.06} - \frac{1 \times 9.62 \times 10.38}{884.3} = 0.0256 - 0.1125 = -0.0869
\]
TABLES FOR DESIGN OF REINFORCED CONCRETE 9-63

\[ C_A = -\frac{0.0869}{0.1301} = -0.668 \quad C_B = -\frac{0.0869}{0.1476} = -0.590 \]

\[ M_A^p = 0 + \frac{3.586.1}{39.06} + \frac{3.586.1 \times 0.313 \times 0.962}{884.3} = 0 + 91.8 + 11.2 = 103 \text{ ft-kips} \]

\[ M_B^p = 60 + \frac{3.586.1}{39.06} - \frac{3.586.1 \times 0.313 \times 10.38}{884.3} = 60 + 91.8 - 13.2 = 138.6 \text{ ft-kips} \]

(See Example 9-18)

**Example 9-21. Beam with Straight Haunches.** (Fig. 9-74.) Solution by column analogy. Beam factors and moment coefficients from tables.

Relative values of \( K \):

For \( b = 12'' \):

\[ I_C = 8,000 \text{ in.}^4 \]

\[ K_A = K_B = \frac{6.51 \times 8,000}{20} = 2,610 \]

\[ C_A = C_B = 0.618 \]

**Fig. 9-74. Loads on beam with straight haunches.**

Fixed-end moments:

Uniform load:

\[ F_A = 0.005 \quad F_B = 0.095 \]

\[ = 0.060 \quad = 0.183 \]

\[ M_A^p = 50 \times 20 \times 0.095 = 95 \text{ ft-kips} \]

\[ 10 \times 20 \times 0.060 = 12 \text{ ft-kips} \]

\[ 107 \text{ ft-kips} \]

\[ M_B^p = 50 \times 20 \times 0.095 = 95 \text{ ft-kips} \]

\[ 10 \times 20 \times 0.183 = 36.3 \text{ ft-kips} \]

\[ 131.3 \text{ ft-kips} \]

From Fig. 9-75:

\[ m \quad a \quad ma \quad x'' \quad \text{max}^1 \]

\[ 26.7 \times 2.36 = 63 \times -9 = -567 \]

\[ 72.8 \times 2.36 = 172 \times -7 = -1,203 \]

\[ 108.8 \times 5.22 = 567 \times -5 = -2,840 \]

\[ 134.8 \times 5.22 = 704 \times -3 = -2,110 \]

\[ 151 \times 5.22 = 788 \times -1 = -788 \]

\[ 157 \times 5.22 = 820 \times +1 = +820 \]

\[ 152 \times 5.22 = 793 \times +3 = +2,380 \]

\[ 127.8 \times 5.22 = 667 \times +5 = +3,330 \]

\[ 84.8 \times 2.36 = 200 \times +7 = +1,400 \]

\[ 30.7 \times 2.36 = 72 \times +9 = +648 \]

\[ 40.76 \quad 4,846 \quad +1,070 \]

\[ e = \frac{1,070}{4,846} = 0.222 \]

\[ K_A' = K_B' = \frac{1}{40.76} + \frac{1 \times 10^3}{992.6} \]

\[ = 0.0245 + 0.1008 = 0.1253 \]
Fig. 9-75. Analogous column load for the loaded beam of Fig. 9-74.

For $b = 12^\prime$:

$K_A = K_B = 0.1253 \times 12 \times 12^2 = 2,600$

$M_C = \frac{1}{40.76} - \frac{1 \times 10^3}{992.6} = 0.0763$

$C_A = C_B = \frac{0.0763}{0.1253} = 0.608$

$M_{A}^F = \frac{4.846}{40.76} - \frac{4.846 \times 0.222 \times 10}{992.6} = 118.5 - 10.8 = 107.7 \text{ ft-kips}$

$M_{B}^F = \frac{4.846}{40.76} + \frac{4.846 \times 0.222 \times 10}{992.6} = 118.5 + 10.8 = 129.3 \text{ ft-kips}$

$A = \frac{z}{I} = \frac{I_0 + Az^2}{6.3}$

<table>
<thead>
<tr>
<th>$z$</th>
<th>$I$</th>
<th>$I_0$</th>
<th>$A$</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.72</td>
<td>302</td>
<td>376</td>
<td>6.3</td>
</tr>
<tr>
<td>31.32</td>
<td>0</td>
<td>0</td>
<td>6.3</td>
</tr>
<tr>
<td>4.72</td>
<td>302</td>
<td>992.6</td>
<td></td>
</tr>
</tbody>
</table>

Footnotes for Table 9-2

* Use equipment when greater.
* Book stacks 20 lb per cu ft.
* As required by the railroad company.
* Paper storage 50 lb per ft of clear story height.
* Increase when occupancy exceeds this amount.
* Also subject to maximum wheel concentrations.

* Minimum uplift of 20 lb per sq ft with no downward live load.
* Plus 150 lb for trucks.
* For detailed recommendations, see American Standard Specifications for Portable Steel and Wood Grandstands, Z20.1-1941, or latest revision thereof.
### Table 9-1. Live Loads, Minimum Uniformly Distributed Loads for Design
(Courtesy of American Standards Association)

<table>
<thead>
<tr>
<th>Occupancy or use</th>
<th>Live load lb/sq ft</th>
</tr>
</thead>
<tbody>
<tr>
<td>Apartment houses:</td>
<td></td>
</tr>
<tr>
<td>Private apartments</td>
<td>40</td>
</tr>
<tr>
<td>Public stairways</td>
<td>100</td>
</tr>
<tr>
<td>Assembly halls:</td>
<td></td>
</tr>
<tr>
<td>Fixed seats</td>
<td>60</td>
</tr>
<tr>
<td>Movable seats</td>
<td>100</td>
</tr>
<tr>
<td>Corridors, upper floors</td>
<td>100</td>
</tr>
<tr>
<td>Corridors:</td>
<td></td>
</tr>
<tr>
<td>First floor</td>
<td>100</td>
</tr>
<tr>
<td>Other floors, same as occupancy served</td>
<td></td>
</tr>
<tr>
<td>except as indicated</td>
<td></td>
</tr>
<tr>
<td>Courtrooms</td>
<td>80</td>
</tr>
<tr>
<td>Dance halls</td>
<td>100</td>
</tr>
<tr>
<td>Dining rooms, public</td>
<td>100</td>
</tr>
<tr>
<td>Dwellings</td>
<td>40</td>
</tr>
<tr>
<td>Hospitals and asylums:</td>
<td></td>
</tr>
<tr>
<td>Operating rooms</td>
<td>60</td>
</tr>
<tr>
<td>Private rooms</td>
<td>40</td>
</tr>
<tr>
<td>Wards</td>
<td>40</td>
</tr>
<tr>
<td>Public space</td>
<td>80</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>150</strong></td>
</tr>
</tbody>
</table>

**Table 9-2. Live Loads, Guide for Special Conditions of Uniformly Distributed Floor Loads**
(Courtesy of American Standards Association)

<table>
<thead>
<tr>
<th>Use</th>
<th>Live load lb/sq ft</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air conditioning (machine space)</td>
<td>200+</td>
</tr>
<tr>
<td>Amphitheater:</td>
<td></td>
</tr>
<tr>
<td>Fixed seats</td>
<td>60</td>
</tr>
<tr>
<td>Movable seats</td>
<td>100</td>
</tr>
<tr>
<td>Amusement park structure</td>
<td>100+</td>
</tr>
<tr>
<td>Armories</td>
<td>150</td>
</tr>
<tr>
<td>Attic:</td>
<td></td>
</tr>
<tr>
<td>Nonstorage</td>
<td>25</td>
</tr>
<tr>
<td>Storage</td>
<td>80+</td>
</tr>
<tr>
<td>Bakery</td>
<td>150</td>
</tr>
<tr>
<td>Balcony:</td>
<td></td>
</tr>
<tr>
<td>Exterior</td>
<td>100</td>
</tr>
<tr>
<td>Interior (fixed seats)</td>
<td>60</td>
</tr>
<tr>
<td>Interior (movable seats)</td>
<td>100</td>
</tr>
<tr>
<td>Boathouse, floors</td>
<td>100+</td>
</tr>
<tr>
<td>Boiler room, framed</td>
<td>300+</td>
</tr>
<tr>
<td>Broadcasting studio</td>
<td>100</td>
</tr>
<tr>
<td>Catwalks</td>
<td>25</td>
</tr>
<tr>
<td>Ceiling, accessible furred</td>
<td>10</td>
</tr>
<tr>
<td>Cold storage: No overhead system:</td>
<td></td>
</tr>
<tr>
<td>Overhead system: No cold storage</td>
<td>250+</td>
</tr>
<tr>
<td>Floor</td>
<td>150</td>
</tr>
<tr>
<td>Roof</td>
<td>250</td>
</tr>
<tr>
<td>Dormitories: Partitioned</td>
<td>40</td>
</tr>
<tr>
<td>Nonpartitioned</td>
<td>80</td>
</tr>
<tr>
<td>Drill room</td>
<td>125</td>
</tr>
<tr>
<td>Driveways (see garages)</td>
<td>150+</td>
</tr>
<tr>
<td>Elevator machine room</td>
<td></td>
</tr>
<tr>
<td>Fan room</td>
<td>150+</td>
</tr>
<tr>
<td>File room: Letter</td>
<td>80</td>
</tr>
<tr>
<td>Card</td>
<td>125+</td>
</tr>
<tr>
<td>Addressograph</td>
<td>150+</td>
</tr>
<tr>
<td>Fire escape</td>
<td>100</td>
</tr>
<tr>
<td>Foundries</td>
<td>600+</td>
</tr>
<tr>
<td>Fuel rooms, framed</td>
<td>400</td>
</tr>
<tr>
<td>Grandstands (see reviewing stands)</td>
<td></td>
</tr>
<tr>
<td>Garages: Cars, with load, less than 3 tons</td>
<td>100+</td>
</tr>
<tr>
<td>Trucks, with load, 3 to 10 tons</td>
<td>150+</td>
</tr>
<tr>
<td>Trucks, with load, above 10 tons</td>
<td>200+</td>
</tr>
</tbody>
</table>

For footnotes for Table 9-2 see page 9-64.
Table 9-2. Live Loads, Guide for Special Conditions of Uniformly Distributed Floor Loads (Continued)

<table>
<thead>
<tr>
<th>Use</th>
<th>Lb/sq ft</th>
<th>Use</th>
<th>Lb/sq ft</th>
</tr>
</thead>
<tbody>
<tr>
<td>Theaters:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dressing rooms</td>
<td>40</td>
<td>Pin rail</td>
<td>250</td>
</tr>
<tr>
<td>Grid-iron floor or fly gallery:</td>
<td></td>
<td>Projection room</td>
<td>100</td>
</tr>
<tr>
<td>Grating</td>
<td>60</td>
<td>Toilet rooms</td>
<td>60</td>
</tr>
<tr>
<td>Well beams</td>
<td>250</td>
<td>Transformer rooms</td>
<td>200</td>
</tr>
<tr>
<td>Header beams</td>
<td>1,000</td>
<td>Vaults in offices</td>
<td>250</td>
</tr>
<tr>
<td>For footnotes for Table 9-2 see page 9-64.</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 9-3. Dead Loads, Minimum Weights for Materials
(Courtesy of American Standards Association)

<table>
<thead>
<tr>
<th>Material</th>
<th>Lb/cu ft</th>
<th>Material</th>
<th>Lb/cu ft</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bituminous products:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Asphaltum</td>
<td>81</td>
<td>Sand or gravel, and clay</td>
<td>65</td>
</tr>
<tr>
<td>Graphite</td>
<td>135</td>
<td>Gold, solid</td>
<td>1205</td>
</tr>
<tr>
<td>Paraffin</td>
<td>56</td>
<td>Gold, bars, stacked</td>
<td>1133</td>
</tr>
<tr>
<td>Petroleum, crude</td>
<td>55</td>
<td>Gold, coin in bags</td>
<td>1084</td>
</tr>
<tr>
<td>Petroleum, refined</td>
<td>50</td>
<td>Gypsum, loose</td>
<td>65</td>
</tr>
<tr>
<td>Petroleum, benzene</td>
<td>46</td>
<td>Ice</td>
<td>57.2</td>
</tr>
<tr>
<td>Petroleum, gasoline</td>
<td>42</td>
<td>Iron, cast</td>
<td>450</td>
</tr>
<tr>
<td>Pitch</td>
<td>69</td>
<td>Iron, wrought</td>
<td>480</td>
</tr>
<tr>
<td>Tar</td>
<td>75</td>
<td>Lead</td>
<td>710</td>
</tr>
<tr>
<td>Brass</td>
<td>526</td>
<td>Lime, hydrated, loose</td>
<td>32</td>
</tr>
<tr>
<td>Bronze</td>
<td>552</td>
<td>Lime, hydrated, compacted</td>
<td>45</td>
</tr>
<tr>
<td>Cement, portland, loose</td>
<td>70</td>
<td>Mortar, half-brick set</td>
<td></td>
</tr>
<tr>
<td>Cement, portland, set</td>
<td>125</td>
<td>Cement</td>
<td>130</td>
</tr>
<tr>
<td>Charcoal</td>
<td>12</td>
<td>Lime</td>
<td>110</td>
</tr>
<tr>
<td>Cinders, dry, in bulk</td>
<td>45</td>
<td>Riprap (not submerged):</td>
<td></td>
</tr>
<tr>
<td>Coal, anthracite, piled</td>
<td>52</td>
<td>Limestone</td>
<td>83</td>
</tr>
<tr>
<td>Coal, bituminous, piled</td>
<td>47</td>
<td>Sandstone</td>
<td>90</td>
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<tr>
<td>Coal, lignite, piled</td>
<td>47</td>
<td>Slag, bank</td>
<td>108</td>
</tr>
<tr>
<td>Coal, peat, dry, piled</td>
<td>23</td>
<td>Slag, bank screenings</td>
<td>96</td>
</tr>
<tr>
<td>Copper</td>
<td>556</td>
<td>Slag, machine</td>
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<tr>
<td>Cork, compressed</td>
<td>14.4</td>
<td>Silver, sand</td>
<td>52</td>
</tr>
<tr>
<td>Earth (not submerged):</td>
<td></td>
<td>Silver, bars, stacked</td>
<td>590</td>
</tr>
<tr>
<td>Clay</td>
<td>63</td>
<td>Silver, coin in bags</td>
<td>590</td>
</tr>
<tr>
<td>Clay, damp</td>
<td>110</td>
<td>Slate</td>
<td>172</td>
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<tr>
<td>Clay and gravel, dry</td>
<td>100</td>
<td>Steel, cold-drawn</td>
<td>489</td>
</tr>
<tr>
<td>Silt, moist, loose</td>
<td>78</td>
<td>Stone, quarried, piled</td>
<td></td>
</tr>
<tr>
<td>Silt, moist, packed</td>
<td>96</td>
<td>Basalt, granite, gneiss</td>
<td>96</td>
</tr>
<tr>
<td>Silt, flowing</td>
<td>108</td>
<td>Limestone, marble, quartz</td>
<td>95</td>
</tr>
<tr>
<td>Sand and gravel, dry</td>
<td>100</td>
<td>Sandstone</td>
<td>82</td>
</tr>
<tr>
<td>Sand and gravel, dry, packed</td>
<td>110</td>
<td>Shale</td>
<td>92</td>
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<tr>
<td>Sand and gravel, wet</td>
<td>120</td>
<td>Greenstone, hornblende</td>
<td>107</td>
</tr>
<tr>
<td>Earth (submerged):</td>
<td></td>
<td>Tin</td>
<td>459</td>
</tr>
<tr>
<td>Clay</td>
<td>80</td>
<td>Water, fresh</td>
<td>62.4</td>
</tr>
<tr>
<td>Soil</td>
<td>70</td>
<td>Water, sea</td>
<td>64</td>
</tr>
<tr>
<td>River mud</td>
<td>90</td>
<td>Zinc, rolled, sheet</td>
<td>449</td>
</tr>
<tr>
<td>Sand or gravel</td>
<td>60</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 9-4. Dead Loads, Minimum Design Dead Loads*
(Courtesy of American Standards Association)

<table>
<thead>
<tr>
<th>Lb/sq ft</th>
<th>Lb/sq ft</th>
</tr>
</thead>
<tbody>
<tr>
<td>Walls:</td>
<td></td>
</tr>
<tr>
<td>4-in. clay brick, high absorption</td>
<td>34</td>
</tr>
<tr>
<td>4-in. clay brick, medium absorption</td>
<td>39</td>
</tr>
<tr>
<td>4-in. clay brick, low absorption</td>
<td>46</td>
</tr>
<tr>
<td>4-in. sand-lime brick</td>
<td>46</td>
</tr>
<tr>
<td>4-in. concrete brick, heavy aggregate</td>
<td>69</td>
</tr>
<tr>
<td>8-in. clay brick, high absorption</td>
<td>69</td>
</tr>
<tr>
<td>8-in. clay brick, medium absorption</td>
<td>79</td>
</tr>
<tr>
<td>8-in. clay brick, low absorption</td>
<td>89</td>
</tr>
<tr>
<td>8-in. sand-lime brick</td>
<td>74</td>
</tr>
<tr>
<td>8-in. concrete brick, heavy aggregate</td>
<td>89</td>
</tr>
<tr>
<td>8-in. concrete brick, light aggregate</td>
<td>68</td>
</tr>
<tr>
<td>12½-in. clay brick, high absorption</td>
<td>100</td>
</tr>
<tr>
<td>12½-in. clay brick, medium absorption</td>
<td>115</td>
</tr>
<tr>
<td>12½-in. clay brick, low absorption</td>
<td>130</td>
</tr>
<tr>
<td>12½-in. sand-lime brick</td>
<td>105</td>
</tr>
<tr>
<td>12½-in. concrete brick, heavy aggregate</td>
<td>130</td>
</tr>
<tr>
<td>12½-in. concrete brick, light aggregate</td>
<td>98</td>
</tr>
<tr>
<td>17-in. clay brick, high absorption</td>
<td>134</td>
</tr>
<tr>
<td>17-in. clay brick, medium absorption</td>
<td>155</td>
</tr>
<tr>
<td>17-in. clay brick, low absorption</td>
<td>173</td>
</tr>
<tr>
<td>17-in. sand-lime brick</td>
<td>138</td>
</tr>
<tr>
<td>17-in. concrete brick, heavy aggregate</td>
<td>174</td>
</tr>
<tr>
<td>17-in. concrete brick, light aggregate</td>
<td>130</td>
</tr>
<tr>
<td>22-in. clay brick, high absorption</td>
<td>168</td>
</tr>
<tr>
<td>22-in. clay brick, medium absorption</td>
<td>194</td>
</tr>
<tr>
<td>22-in. clay brick, low absorption</td>
<td>216</td>
</tr>
<tr>
<td>22-in. concrete brick, heavy aggregate</td>
<td>216</td>
</tr>
<tr>
<td>22-in. concrete brick, light aggregate</td>
<td>160</td>
</tr>
<tr>
<td>4-in. brick, 4-in. load-bearing structural clay-tile backing</td>
<td>60</td>
</tr>
</tbody>
</table>
### Table 9-4. Dead Loads, Minimum Design Dead Loads* (Continued)

<table>
<thead>
<tr>
<th>4-in. brick, 8-in. load-bearing structural clay-tile backing</th>
<th>75</th>
</tr>
</thead>
<tbody>
<tr>
<td>9-in. brick, 4-in. load-bearing structural clay-tile backing</td>
<td>102</td>
</tr>
<tr>
<td>8-in. load-bearing structural clay tile</td>
<td>42</td>
</tr>
<tr>
<td>12-in. load-bearing structural clay tile</td>
<td>58</td>
</tr>
<tr>
<td>8-in. concrete block, heavy aggregate</td>
<td>55</td>
</tr>
<tr>
<td>12-in. concrete block, heavy aggregate</td>
<td>85</td>
</tr>
<tr>
<td>8-in. concrete block, light aggregate</td>
<td>35</td>
</tr>
<tr>
<td>12-in. concrete block, light aggregate</td>
<td>55</td>
</tr>
<tr>
<td>2-in. furring tile, one side of masonry wall, add to above figures</td>
<td>12</td>
</tr>
</tbody>
</table>

#### Ribbed slabs:

<table>
<thead>
<tr>
<th>Depth, in.</th>
<th>Lb/sq ft</th>
</tr>
</thead>
<tbody>
<tr>
<td>30-in. metal fillers:</td>
<td></td>
</tr>
<tr>
<td>6 + 2½</td>
<td>41</td>
</tr>
<tr>
<td>8 + 2½</td>
<td>45</td>
</tr>
<tr>
<td>10 + 3</td>
<td>56</td>
</tr>
<tr>
<td>12 + 3</td>
<td>63</td>
</tr>
<tr>
<td>14 + 3</td>
<td>69</td>
</tr>
<tr>
<td>2-way clay tile fillers (12 by 12):</td>
<td></td>
</tr>
<tr>
<td>4 + 2</td>
<td>61</td>
</tr>
<tr>
<td>6 + 2</td>
<td>87</td>
</tr>
<tr>
<td>8 + 2½</td>
<td>100</td>
</tr>
<tr>
<td>10 + 3</td>
<td>121</td>
</tr>
<tr>
<td>12 + 3</td>
<td>136</td>
</tr>
<tr>
<td>2-way metal fillers (16 by 16):</td>
<td></td>
</tr>
<tr>
<td>4 + 2</td>
<td>44</td>
</tr>
<tr>
<td>6 + 2</td>
<td>55</td>
</tr>
<tr>
<td>8 + 2½</td>
<td>72</td>
</tr>
<tr>
<td>10 + 3</td>
<td>91</td>
</tr>
<tr>
<td>12 + 3</td>
<td>103</td>
</tr>
<tr>
<td>14 + 3</td>
<td>116</td>
</tr>
<tr>
<td>2-way metal fillers (20 by 20):</td>
<td></td>
</tr>
<tr>
<td>4 + 2</td>
<td>42</td>
</tr>
<tr>
<td>6 + 2</td>
<td>50</td>
</tr>
<tr>
<td>8 + 2½</td>
<td>66</td>
</tr>
<tr>
<td>10 + 3</td>
<td>83</td>
</tr>
<tr>
<td>12 + 3</td>
<td>93</td>
</tr>
<tr>
<td>14 + 3</td>
<td>103</td>
</tr>
</tbody>
</table>

#### Partitions:

| 3-in. clay tile | 17 |
| 4-in. clay tile | 18 |
| 6-in. clay tile | 28 |
| 8-in. clay tile | 34 |
| 10-in. clay tile | 40 |
| 2-in. gypsum block | 9½ |
| 3-in. gypsum block | 10½ |
| 4-in. gypsum block | 12½ |
| 5-in. gypsum block | 14 |
| 6-in. gypsum block | 18½ |
| 2-in. solid plaster | 20 |
| 4-in. solid plaster | 32 |
| 4-in. hollow plaster | 22 |
| 4-in. concrete block, heavy aggregate | 30 |
| 6-in. concrete block, heavy aggregate | 42 |
| 8-in. concrete block, heavy aggregate | 55 |
| 12-in. concrete block, heavy aggregate | 85 |
| 4-in. concrete block, light aggregate | 20 |
| 6-in. concrete block, light aggregate | 28 |
| 8-in. concrete block, light aggregate | 38 |
| 12-in. concrete block, light aggregate | 55 |
| Wood studs 2 by 4, unplastered | 4 |
| Wood studs 2 by 4, plastered 1 side | 12 |
| Wood studs 2 by 4, plastered 2 sides | 20 |
| Glass-block masonry, 4-in. glass-block walls and partitions | 18 |

#### Split furring tile:

| 1½-in | 8 |
| 2-in | 8½ |

#### Concrete slabs:

| Concrete, reinforced-stone, per in | 12½ |
| Concrete, reinforced-cinder, per in | 9½ |
| Concrete, reinforced lightweight, per in | 9 |
| Concrete, plain stone, per in | 12 |
| Concrete, plain cinder, per in | 9 |
| Concrete, plain lightweight, per in | 8½ |

#### Ribbed slabs:

<table>
<thead>
<tr>
<th>Depth, in.</th>
<th>Lb/sq ft</th>
</tr>
</thead>
<tbody>
<tr>
<td>12-in. clay-tile fillers:</td>
<td></td>
</tr>
<tr>
<td>4 + 2</td>
<td>49</td>
</tr>
<tr>
<td>6 + 2</td>
<td>60</td>
</tr>
<tr>
<td>8 + 2½</td>
<td>79</td>
</tr>
<tr>
<td>10 + 3</td>
<td>96</td>
</tr>
<tr>
<td>12 + 3</td>
<td>108</td>
</tr>
<tr>
<td>20-in. metal fillers:</td>
<td></td>
</tr>
<tr>
<td>4 + 2</td>
<td>41</td>
</tr>
<tr>
<td>6 + 2</td>
<td>51</td>
</tr>
<tr>
<td>8 + 2½</td>
<td>63</td>
</tr>
<tr>
<td>10 + 3</td>
<td>69</td>
</tr>
<tr>
<td>12 + 3</td>
<td>75</td>
</tr>
</tbody>
</table>

#### Floor finish and fill:

<table>
<thead>
<tr>
<th>Finish floor to top slab, in.</th>
<th>Lb/sq ft</th>
</tr>
</thead>
<tbody>
<tr>
<td>Double ¾ wood on sleepers, light-concrete fill</td>
<td>5</td>
</tr>
<tr>
<td>Double ¾ wood on sleepers, stone-concrete fill</td>
<td>5</td>
</tr>
<tr>
<td>Single ¾ wood on sleepers, light-concrete fill</td>
<td>5</td>
</tr>
<tr>
<td>Single ¾ wood on sleepers, stone-concrete fill</td>
<td>5</td>
</tr>
<tr>
<td>3-in. wood block on mastic, no fill</td>
<td>3</td>
</tr>
<tr>
<td>¾-in. wood block on stone-concrete fill</td>
<td>4</td>
</tr>
<tr>
<td>1-in. cement finish on stone-concrete fill</td>
<td>4</td>
</tr>
<tr>
<td>1-in. terrazo on stone-concrete fill</td>
<td>4</td>
</tr>
<tr>
<td>Clay tile on stone-concrete fill</td>
<td>4</td>
</tr>
<tr>
<td>Marble and mortar on stone-concrete fill</td>
<td>4</td>
</tr>
<tr>
<td>Linoleum on stone-concrete fill</td>
<td>4</td>
</tr>
<tr>
<td>Linoleum on light-concrete fill</td>
<td>5</td>
</tr>
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</table>

Add for tapered ends:

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<tr>
<th>Width of rib, in.</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Add for tapered ends</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>8</td>
</tr>
</tbody>
</table>
### Table 9-4. Dead Loads, Minimum Design Dead Loads* (Continued)

<table>
<thead>
<tr>
<th>Roof coverings:</th>
<th>Lb/sq ft</th>
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</thead>
<tbody>
<tr>
<td>Asbestos shingles</td>
<td>4</td>
</tr>
<tr>
<td>Asphalt shingles</td>
<td>6</td>
</tr>
<tr>
<td>Copper or tin</td>
<td>1</td>
</tr>
<tr>
<td>Corrugated iron</td>
<td>2</td>
</tr>
<tr>
<td>Clay tile (for mortar add 10 lb):</td>
<td></td>
</tr>
<tr>
<td>2-in. book tile</td>
<td>12</td>
</tr>
<tr>
<td>3-in. book tile</td>
<td>20</td>
</tr>
<tr>
<td>Roman</td>
<td>12</td>
</tr>
<tr>
<td>Spanish</td>
<td>19</td>
</tr>
<tr>
<td>Ludowici</td>
<td>10</td>
</tr>
<tr>
<td>Cement tile</td>
<td>16</td>
</tr>
<tr>
<td>Composition:</td>
<td></td>
</tr>
<tr>
<td>3-ply ready roofing</td>
<td>1</td>
</tr>
<tr>
<td>4-ply felt and gravel</td>
<td>5½</td>
</tr>
<tr>
<td>5-ply felt and gravel</td>
<td>6</td>
</tr>
<tr>
<td>Sheathing, per in. thickness</td>
<td>3</td>
</tr>
<tr>
<td>Slate, 3/4-in.</td>
<td>7</td>
</tr>
<tr>
<td>Slate, 1/2-in.</td>
<td>10</td>
</tr>
<tr>
<td>Skylight, metal frame, 1/2-in. wire glass</td>
<td>8</td>
</tr>
<tr>
<td>Wood shingles</td>
<td>3</td>
</tr>
</tbody>
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<table>
<thead>
<tr>
<th>Materials:</th>
<th>Lb/sq ft</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cast-stone masonry (cement, stone, sand)</td>
<td>144</td>
</tr>
<tr>
<td>Cinder</td>
<td>57</td>
</tr>
<tr>
<td>Concrete, plain:</td>
<td></td>
</tr>
<tr>
<td>Stone (including gravel)</td>
<td>144</td>
</tr>
<tr>
<td>Slag</td>
<td>132</td>
</tr>
<tr>
<td>Cinder</td>
<td>108</td>
</tr>
<tr>
<td>Haydite (burned-clay aggregate)</td>
<td>90</td>
</tr>
<tr>
<td>Expanded-slag aggregate</td>
<td>100</td>
</tr>
<tr>
<td>Concrete, reinforced:</td>
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</tr>
<tr>
<td>Stone (including gravel)</td>
<td>150</td>
</tr>
<tr>
<td>Slag</td>
<td>138</td>
</tr>
<tr>
<td>Cinder</td>
<td>111</td>
</tr>
<tr>
<td>Masonry, brick:</td>
<td></td>
</tr>
<tr>
<td>Hard (low absorption)</td>
<td>130</td>
</tr>
<tr>
<td>Medium (medium absorption)</td>
<td>115</td>
</tr>
<tr>
<td>Soft (high absorption)</td>
<td>100</td>
</tr>
<tr>
<td>Masonry, ashlar:</td>
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</tr>
<tr>
<td>Granite</td>
<td>165</td>
</tr>
<tr>
<td>Limestone, crystalline</td>
<td>165</td>
</tr>
<tr>
<td>Limestone oölite</td>
<td>135</td>
</tr>
<tr>
<td>Marble</td>
<td>173</td>
</tr>
<tr>
<td>Sandstone</td>
<td>144</td>
</tr>
<tr>
<td>Masonry, rubble mortar</td>
<td></td>
</tr>
<tr>
<td>Granite</td>
<td>153</td>
</tr>
<tr>
<td>Limestone, crystalline</td>
<td>147</td>
</tr>
<tr>
<td>Limestone, oölite</td>
<td>138</td>
</tr>
<tr>
<td>Marble</td>
<td>156</td>
</tr>
<tr>
<td>Sandstone</td>
<td>137</td>
</tr>
<tr>
<td>Terra cotta, architectural:</td>
<td></td>
</tr>
<tr>
<td>Voids filled</td>
<td>120</td>
</tr>
<tr>
<td>Voids unfilled</td>
<td>72</td>
</tr>
</tbody>
</table>

| Timber, seasoned: |
| Aab, commercial white | 41 |
| Cypress, southern | 32 |
| Fir, Douglas, coast region | 34 |
| Oak, commercial reds and whites | 45 |
| Pine, southern yellow | 39 |
| Redwood | 28 |
| Spruce, red, white, and Sitka | 28 |

<table>
<thead>
<tr>
<th>Floor fill:</th>
<th>Lb/sq ft</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cinder concrete, per in</td>
<td>9</td>
</tr>
<tr>
<td>Lightweight concrete, per in</td>
<td>7</td>
</tr>
<tr>
<td>Sand, per in</td>
<td>12</td>
</tr>
<tr>
<td>Stone concrete, per in</td>
<td>12</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Wood-joist floors (no plaster) double wood floor:</th>
<th>Lb/sq ft</th>
</tr>
</thead>
<tbody>
<tr>
<td>Joist sizes</td>
<td>12-in. spacing</td>
</tr>
<tr>
<td>2 by 6</td>
<td>6</td>
</tr>
<tr>
<td>2 by 8</td>
<td>6</td>
</tr>
<tr>
<td>2 by 10</td>
<td>7</td>
</tr>
<tr>
<td>2 by 12</td>
<td>8</td>
</tr>
<tr>
<td>3 by 6</td>
<td>7</td>
</tr>
<tr>
<td>3 by 8</td>
<td>8</td>
</tr>
<tr>
<td>3 by 10</td>
<td>9</td>
</tr>
<tr>
<td>3 by 12</td>
<td>11</td>
</tr>
<tr>
<td>3 by 14</td>
<td>12</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Ceilings:</th>
<th>Lb/sq ft</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plaster on tile or concrete</td>
<td>5</td>
</tr>
<tr>
<td>Suspended metal lath and gypsum plaster</td>
<td>10</td>
</tr>
<tr>
<td>Suspended metal lath and cement plaster</td>
<td>15</td>
</tr>
<tr>
<td>Plaster on wooden lath</td>
<td>8</td>
</tr>
</tbody>
</table>

*Weights of masonry include mortar but not plaster. For plaster, add 5 lb per sq ft for each face plastered. Values given represent averages. In some cases there is a considerable range of weight for the same construction.
Table 9-5. Coefficients for $M/I$ Curves and Deflections

<table>
<thead>
<tr>
<th>M/I curve</th>
<th>$\Delta$ at C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uniform load $&quot;W&quot;$-</td>
<td>$A \times WL^2/48EI$</td>
</tr>
<tr>
<td>Concentrated load $&quot;P&quot;$-</td>
<td>$A \times PL^2/48EI$</td>
</tr>
<tr>
<td>Moment at &quot;$A&quot;$-</td>
<td>$A \times ML^2/48EI$</td>
</tr>
</tbody>
</table>

Simple beams with constant $E_c$ & $I$

<table>
<thead>
<tr>
<th>Slope at A</th>
<th>Slope at B</th>
<th>Slope at C</th>
<th>$\Delta$ at C</th>
</tr>
</thead>
<tbody>
<tr>
<td>+2</td>
<td>-2</td>
<td>0</td>
<td>.625</td>
</tr>
<tr>
<td>+2b(2-b)^2</td>
<td>-2b(2-b^2)</td>
<td>-b(1-2b)^2</td>
<td>$\frac{b}{2}(3-2b^2)$</td>
</tr>
<tr>
<td>+3</td>
<td>-3</td>
<td>0</td>
<td>+1</td>
</tr>
<tr>
<td>+8</td>
<td>+2</td>
<td>-3</td>
<td></td>
</tr>
</tbody>
</table>

Correction coefficients $A$ & $C$

<table>
<thead>
<tr>
<th>$r$</th>
<th>$0.2$</th>
<th>$0.3$</th>
<th>$0.5$</th>
<th>$0.75$</th>
<th>$1.0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>$A$</td>
<td>$A$</td>
<td>$A$</td>
<td>$A$</td>
<td>$A$</td>
</tr>
<tr>
<td></td>
<td>$C$</td>
<td>$C$</td>
<td>$C$</td>
<td>$C$</td>
<td>$C$</td>
</tr>
<tr>
<td>1</td>
<td>-0.031</td>
<td>-0.001</td>
<td>-0.001</td>
<td>-0.054</td>
<td>-0.001</td>
</tr>
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<td>-1.109</td>
<td>-0.006</td>
<td>-0.008</td>
<td>-0.190</td>
<td>-0.014</td>
</tr>
<tr>
<td>3</td>
<td>-2.19</td>
<td>-0.020</td>
<td>-0.026</td>
<td>-0.376</td>
<td>-0.036</td>
</tr>
<tr>
<td>1</td>
<td>-0.322</td>
<td>-0.004</td>
<td>-0.005</td>
<td>-0.057</td>
<td>-0.001</td>
</tr>
<tr>
<td>2</td>
<td>-1.22</td>
<td>-0.007</td>
<td>-0.010</td>
<td>-0.213</td>
<td>-0.013</td>
</tr>
<tr>
<td>3</td>
<td>-2.59</td>
<td>-0.024</td>
<td>-0.033</td>
<td>-0.449</td>
<td>-0.045</td>
</tr>
<tr>
<td>1</td>
<td>+1.73</td>
<td>+0.030</td>
<td>+2.19</td>
<td>+2.77</td>
<td>+0.05</td>
</tr>
<tr>
<td>2</td>
<td>+3.20</td>
<td>+0.120</td>
<td>+4.04</td>
<td>+5.09</td>
<td>+0.21</td>
</tr>
<tr>
<td>3</td>
<td>+4.45</td>
<td>+0.260</td>
<td>+5.60</td>
<td>+7.02</td>
<td>+0.45</td>
</tr>
<tr>
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<td>+0.06</td>
<td>+0.006</td>
<td>+0.08</td>
<td>+0.11</td>
<td>+0.005</td>
</tr>
<tr>
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<td>+0.24</td>
<td>+0.015</td>
<td>+0.32</td>
<td>+0.43</td>
<td>+0.025</td>
</tr>
<tr>
<td>3</td>
<td>+0.52</td>
<td>+0.050</td>
<td>+0.68</td>
<td>+0.90</td>
<td>+0.090</td>
</tr>
</tbody>
</table>
### Table 9-6. Distribution of Concentrated Load P for Two-way Slabs (Continued)

<table>
<thead>
<tr>
<th>$y_0$</th>
<th>$x_{1.7}$</th>
<th>$x_{2.7}$</th>
<th>$x_{3.7}$</th>
<th>$x_{4.7}$</th>
<th>$x_{5.7}$</th>
<th>$x_{6.7}$</th>
<th>$x_{7.7}$</th>
<th>$x_{8.7}$</th>
<th>$x_{9.7}$</th>
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<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
</tr>
<tr>
<td>2.2</td>
<td>26.10</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
</tr>
<tr>
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<td>57.37</td>
<td>0.21</td>
<td>0.21</td>
<td>0.21</td>
<td>0.21</td>
<td>0.21</td>
<td>0.21</td>
<td>0.21</td>
<td>0.21</td>
</tr>
<tr>
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<td>0.26</td>
<td>0.26</td>
<td>0.26</td>
<td>0.26</td>
<td>0.26</td>
</tr>
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<td>5.2</td>
<td>102.45</td>
<td>0.31</td>
<td>0.31</td>
<td>0.31</td>
<td>0.31</td>
<td>0.31</td>
<td>0.31</td>
<td>0.31</td>
<td>0.31</td>
</tr>
<tr>
<td>6.2</td>
<td>122.45</td>
<td>0.36</td>
<td>0.36</td>
<td>0.36</td>
<td>0.36</td>
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<td>0.41</td>
<td>0.41</td>
<td>0.41</td>
<td>0.41</td>
<td>0.41</td>
</tr>
<tr>
<td>8.2</td>
<td>162.45</td>
<td>0.46</td>
<td>0.46</td>
<td>0.46</td>
<td>0.46</td>
<td>0.46</td>
<td>0.46</td>
<td>0.46</td>
<td>0.46</td>
</tr>
<tr>
<td>9.2</td>
<td>182.45</td>
<td>0.51</td>
<td>0.51</td>
<td>0.51</td>
<td>0.51</td>
<td>0.51</td>
<td>0.51</td>
<td>0.51</td>
<td>0.51</td>
</tr>
</tbody>
</table>

**Continued...**
<table>
<thead>
<tr>
<th>Distribution of load $P$ to span $L$</th>
<th>$r = L/L'$</th>
<th>$r = 1.0$</th>
<th>$r = 1.2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0.1$</td>
<td>0.50, 24, 16, 12, 11, 12, 16, 24</td>
<td>0.50, 24, 20, 18, 20, 24, 35, 63</td>
<td>0.50, 24, 20, 18, 20, 24, 35, 63</td>
</tr>
<tr>
<td>$0.2$</td>
<td>0.50, 50, 37, 30, 29, 30, 37, 50</td>
<td>0.76, 63, 50, 43, 41, 43, 50, 63</td>
<td>0.76, 63, 50, 43, 41, 43, 50, 63</td>
</tr>
<tr>
<td>$0.3$</td>
<td>0.84, 63, 50, 43, 41, 43, 50, 63</td>
<td>0.90, 75, 63, 57, 55, 57, 63, 75</td>
<td>0.90, 75, 63, 57, 55, 57, 63, 75</td>
</tr>
<tr>
<td>$0.4$</td>
<td>0.88, 69, 57, 50, 48, 50, 57, 69</td>
<td>0.88, 79, 69, 63, 61, 63, 69, 79</td>
<td>0.88, 79, 69, 63, 61, 63, 69, 79</td>
</tr>
<tr>
<td>$0.5$</td>
<td>0.89, 71, 59, 52, 50, 52, 59, 71</td>
<td>0.89, 81, 71, 65, 63, 65, 71, 81</td>
<td>0.89, 81, 71, 65, 63, 65, 71, 81</td>
</tr>
<tr>
<td>$0.6$</td>
<td>0.88, 69, 57, 50, 48, 50, 57, 69</td>
<td>0.88, 79, 69, 63, 61, 63, 69, 79</td>
<td>0.88, 79, 69, 63, 61, 63, 69, 79</td>
</tr>
<tr>
<td>$0.7$</td>
<td>0.84, 63, 50, 43, 41, 43, 50, 63</td>
<td>0.84, 75, 63, 57, 55, 57, 63, 75</td>
<td>0.84, 75, 63, 57, 55, 57, 63, 75</td>
</tr>
<tr>
<td>$0.8$</td>
<td>0.76, 50, 37, 30, 29, 30, 37, 50</td>
<td>0.84, 63, 50, 43, 41, 43, 50, 63</td>
<td>0.84, 63, 50, 43, 41, 43, 50, 63</td>
</tr>
<tr>
<td>$0.9$</td>
<td>0.50, 24, 16, 12, 11, 12, 16, 24</td>
<td>0.35, 24, 20, 18, 20, 24, 35, 63</td>
<td>0.35, 24, 20, 18, 20, 24, 35, 63</td>
</tr>
</tbody>
</table>

Table 9-6. Distribution of Concentrated Load $P$ for Two-way Slabs (Continued)
Table 9-6. Distribution of Concentrated Load $P$ for Two-way Slabs (Continued)

<table>
<thead>
<tr>
<th>$r_L$</th>
<th>$y_L$</th>
<th>$y_L'$</th>
<th>$n_{L'}$</th>
<th>$n_L$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.4</td>
<td>1.2</td>
<td>1.3</td>
<td>1.4</td>
<td>1.5</td>
</tr>
<tr>
<td>1.6</td>
<td>1.7</td>
<td>1.8</td>
<td>1.9</td>
<td>2.0</td>
</tr>
<tr>
<td>1.8</td>
<td>1.9</td>
<td>2.0</td>
<td>2.1</td>
<td>2.2</td>
</tr>
</tbody>
</table>

... (Table continues)
Table 9-7. Typical Details of Tied Columns

<table>
<thead>
<tr>
<th>Bar size</th>
<th>Tie size</th>
<th>( d' )</th>
<th>Minimum width &quot;b&quot; at 3( \frac{1}{2} ) diam o c</th>
</tr>
</thead>
<tbody>
<tr>
<td>#6</td>
<td>#3</td>
<td>2.25&quot;</td>
<td>4, 6, 8, 10, 12, 14, 16, 18, 20</td>
</tr>
<tr>
<td>#7</td>
<td>#3</td>
<td>2.31&quot;</td>
<td>4, 6, 8, 10, 12, 14, 16, 18, 20</td>
</tr>
<tr>
<td>#8</td>
<td>#3</td>
<td>2.37&quot;</td>
<td>4, 6, 8, 10, 12, 14, 16, 18, 20</td>
</tr>
<tr>
<td>#9</td>
<td>#3</td>
<td>2.44&quot;</td>
<td>4, 6, 8, 10, 12, 14, 16, 18, 20</td>
</tr>
<tr>
<td>#10</td>
<td>#3</td>
<td>2.51&quot;</td>
<td>4, 6, 8, 10, 12, 14, 16, 18, 20</td>
</tr>
<tr>
<td>#11</td>
<td>#3</td>
<td>2.58&quot;</td>
<td>4, 6, 8, 10, 12, 14, 16, 18, 20</td>
</tr>
</tbody>
</table>
TABLES FOR DESIGN OF REINFORCED CONCRETE

Table 9-8. Tied Columns, Values of $P_c$ and $P_s$

$P_c = \frac{18 f_c^2 A_g}{1000}$

<table>
<thead>
<tr>
<th>Size $b \times t$</th>
<th>$f_c$</th>
<th>2000</th>
<th>2500</th>
<th>3000</th>
<th>3750</th>
<th>$f_c'$</th>
<th>2000</th>
<th>2500</th>
<th>3000</th>
<th>3750</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$12^&quot; \times 12^&quot;$</td>
<td>144</td>
<td>52</td>
<td>65</td>
<td>77</td>
<td>97</td>
<td>144</td>
<td>60</td>
<td>75</td>
<td>90</td>
<td>113</td>
</tr>
<tr>
<td>$14^&quot; \times 14^&quot;$</td>
<td>196</td>
<td>70</td>
<td>88</td>
<td>106</td>
<td>132</td>
<td>196</td>
<td>80</td>
<td>101</td>
<td>121</td>
<td>151</td>
</tr>
<tr>
<td>$16^&quot; \times 16^&quot;$</td>
<td>256</td>
<td>92</td>
<td>115</td>
<td>138</td>
<td>173</td>
<td>256</td>
<td>104</td>
<td>129</td>
<td>150</td>
<td>194</td>
</tr>
<tr>
<td>$18^&quot; \times 18^&quot;$</td>
<td>324</td>
<td>117</td>
<td>146</td>
<td>175</td>
<td>219</td>
<td>324</td>
<td>130</td>
<td>162</td>
<td>194</td>
<td>243</td>
</tr>
<tr>
<td>$20^&quot; \times 20^&quot;$</td>
<td>400</td>
<td>144</td>
<td>180</td>
<td>216</td>
<td>270</td>
<td>400</td>
<td>158</td>
<td>198</td>
<td>238</td>
<td>297</td>
</tr>
<tr>
<td>$22^&quot; \times 22^&quot;$</td>
<td>484</td>
<td>174</td>
<td>218</td>
<td>261</td>
<td>327</td>
<td>484</td>
<td>206</td>
<td>257</td>
<td>309</td>
<td>386</td>
</tr>
<tr>
<td>$24^&quot; \times 24^&quot;$</td>
<td>576</td>
<td>207</td>
<td>259</td>
<td>311</td>
<td>389</td>
<td>576</td>
<td>224</td>
<td>281</td>
<td>337</td>
<td>421</td>
</tr>
<tr>
<td>$26^&quot; \times 26^&quot;$</td>
<td>676</td>
<td>244</td>
<td>304</td>
<td>365</td>
<td>456</td>
<td>676</td>
<td>262</td>
<td>328</td>
<td>393</td>
<td>491</td>
</tr>
<tr>
<td>$28^&quot; \times 28^&quot;$</td>
<td>784</td>
<td>282</td>
<td>352</td>
<td>423</td>
<td>529</td>
<td>784</td>
<td>302</td>
<td>378</td>
<td>453</td>
<td>566</td>
</tr>
<tr>
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<td>324</td>
<td>405</td>
<td>486</td>
<td>607</td>
<td>900</td>
<td>364</td>
<td>431</td>
<td>519</td>
<td>648</td>
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</table>

$P_s = \frac{8 A_s f_s}{1000}$

<table>
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<tr>
<th>Bar no.</th>
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<th>#7</th>
<th>#8</th>
<th>#9</th>
<th>#10</th>
<th>#11</th>
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</thead>
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<td>.60</td>
<td>.79</td>
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<td>1.27</td>
<td>1.56</td>
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<td>22</td>
<td>31</td>
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<td>51</td>
<td>65</td>
<td>80</td>
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<tr>
<td>T-6</td>
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<td>46</td>
<td>61</td>
<td>77</td>
<td>97</td>
<td>120</td>
</tr>
<tr>
<td>T-8</td>
<td>45</td>
<td>61</td>
<td>81</td>
<td>102</td>
<td>130</td>
<td>160</td>
</tr>
<tr>
<td>T-10</td>
<td>56</td>
<td>77</td>
<td>101</td>
<td>128</td>
<td>162</td>
<td>200</td>
</tr>
<tr>
<td>T-12</td>
<td>67</td>
<td>92</td>
<td>121</td>
<td>153</td>
<td>195</td>
<td>239</td>
</tr>
<tr>
<td>T-14</td>
<td>79</td>
<td>107</td>
<td>141</td>
<td>179</td>
<td>227</td>
<td>279</td>
</tr>
<tr>
<td>T-16</td>
<td>90</td>
<td>123</td>
<td>162</td>
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<td>260</td>
<td>319</td>
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$f_s = 16,000$

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<th>#10</th>
<th>#11</th>
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</thead>
<tbody>
<tr>
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<td>.60</td>
<td>.79</td>
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<td>1.27</td>
<td>1.56</td>
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<td>31</td>
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<td>51</td>
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</tr>
<tr>
<td>T-6</td>
<td>34</td>
<td>46</td>
<td>61</td>
<td>77</td>
<td>97</td>
<td>120</td>
</tr>
<tr>
<td>T-8</td>
<td>45</td>
<td>61</td>
<td>81</td>
<td>102</td>
<td>130</td>
<td>160</td>
</tr>
<tr>
<td>T-10</td>
<td>56</td>
<td>77</td>
<td>101</td>
<td>128</td>
<td>162</td>
<td>200</td>
</tr>
<tr>
<td>T-12</td>
<td>67</td>
<td>92</td>
<td>121</td>
<td>153</td>
<td>195</td>
<td>239</td>
</tr>
<tr>
<td>T-14</td>
<td>79</td>
<td>107</td>
<td>141</td>
<td>179</td>
<td>227</td>
<td>279</td>
</tr>
<tr>
<td>T-16</td>
<td>90</td>
<td>123</td>
<td>162</td>
<td>205</td>
<td>260</td>
<td>319</td>
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</tbody>
</table>

$f_s = 20,000$

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<th>#9</th>
<th>#10</th>
<th>#11</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_s$</td>
<td>.44</td>
<td>.60</td>
<td>.79</td>
<td>1.00</td>
<td>1.27</td>
<td>1.56</td>
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Table 9-9. Typical Details of Spiral Columns

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### Table 9-10. Spiral Columns, Values of $P_c$, $P_s$, and $p'$

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P_c = \frac{225 \, f_A A_g}{1000}
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\[
P_s = \frac{A_s f_s}{1000}
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\[
P' = .45 \left( \frac{A_s}{A_c} - 1 \right) \left( \frac{f_c}{f_s} \right)
\]

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### Notes
- $f_s = 16,000$ for $P_s = 20,000$.
### Table 9-11. Tied Columns, Values of $f_a$ and $C$, $f_c = 16,000$ psi

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TABLES FOR DESIGN OF REINFORCED CONCRETE

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## Table 9-17. Spiral Columns, Values of D for S6 to S16, ST10, ST11, and ST12

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Table 9-18. Spiral Columns, Values of \( D \) for ST13, ST14, ST15, and ST16.
## Table 9-19. Spiral Columns, Values of D for ST17, ST18, ST19, and ST20

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Table 9-20. Tied Columns, General Formulas and Values of $k$ for T4 and T6

**NOTE:** Formulas and values of $k$ apply to all tied columns with reinforcing located in two faces and parallel to the axis of bending.

**Case I:** concentric load

$$P = 0.18 f'_c A_d + 0.8 f_p A_s$$

$$f_o = \frac{0.18 f'_c + 0.8 f_p p_g}{1 + (n-1) p_g}$$

Combined bending and direct stress

$$C = \frac{f_o}{0.45 f_c}$$

$$f_p = f_o \left( \frac{t + D e}{t + C D e} \right)$$

**Case II:** $\theta_t$ less than 1

$$N = \frac{P}{1 + C D \theta_t} \text{ or } P = N + C D \frac{M}{t}$$

$$f_c = A_g \left[ 1 + (n-1) p_g \right] f_c = \text{or } < f_p$$

**Case III:** $\theta_t$ greater than 1

$$N = f_p b t \left[ k^2 + n p_g (2k - 1) \right]$$

$$f_c = \frac{N}{b t} \left[ k^2 + n p_g (2k - 1) \right]$$

$$f_s = n f_c \left( \frac{1 + q}{2k} - 1 \right)$$

$$\theta_t = \frac{k^2 (1 - \frac{2}{3} k) + C^2 n p_g}{2 k^2 + 2 n p_g (2k - 1)}$$

---

Graph showing $n p_g$ values for different $\theta_t$ values with $C = 1.0$ and $k = 0.46$. The graph illustrates the relationship between $\theta_t$ and $n p_g$. The values range from 0.10 to 0.60 for $\theta_t$ ranging from 0.80 to 2.0.
Table 9-20. Tied Columns, General Formulas and Values of $k$ for T4 and T6

(Continued)
Table 9-21a. Tied Columns. General Formulas and Values of $k$ for T12 and T16, $g = 0.60$

**Case I: Concentric load**

$$P = 0.18 f'_c A_g + 0.8 f_s A_s$$

$$f_o = \frac{0.18 f'_c + 0.8 f_s p_g}{t+(n-1)p_g}$$

Combined bending and direct stress

$$C = \frac{f_o}{0.45 f'_c}$$

$$f_p = f_o \left( \frac{t+De}{t+CDe} \right)$$

**Case II: $e/t$ less than 1**

$$N = \frac{P}{1+CD_e/t} \quad \text{or} \quad P = N + CD \frac{M}{t}$$

$$f_c = \frac{N (1+De/t)}{A_g [1+(n-1)p_g]}$$

$$f_c = \text{or} < f_p$$

**Case III: $e/t$ greater than 1**

$$N = f_p bt \left[ \frac{k^2 + np_g (2k-1)}{2k} \right]$$

$$f_c = \frac{N}{bt} \left[ \frac{2k}{k^2 + np_g (2k-1)} \right]$$

$$f_s = n f_c \left( \frac{1+g}{2k} \right)$$

$$e/t = \frac{k^2 (1-2/3k)+c'g^2 np_g}{2k^2+2np_g (2k-1)}$$

**T12 and T16 columns, $c' = 0.667$**
- T14 columns, $c' = 0.71$
- T8 columns, $c' = 0.75$
- T10 columns, $c' = 0.80$

![Graph](chart.png)
Table 9-21b. Tied Columns. Values of k for T12 and T16, g = 0.70, 0.80, and 0.90
Table 9-21c. Tied Columns. Values of $k$ for $T14$, $g = 0.70$, $0.80$, and $0.90$
Table 9-21d. Tied Columns. Values of $k$ for T8, $g = 0.70$, 0.80, and 0.90

\[ g = 0.70 \quad c_x = 0.75 \]
\[ k = 0.46 \quad k = 0.44 \quad k = 0.42 \]
\[ k = 0.40 \quad k = 0.38 \quad k = 0.36 \]
\[ k = 0.34 \quad k = 0.32 \quad k = 0.30 \]

\[ g = 0.80 \quad c_x = 0.75 \]
\[ k = 0.48 \quad k = 0.46 \quad k = 0.44 \]
\[ k = 0.42 \quad k = 0.40 \quad k = 0.38 \]
\[ k = 0.36 \quad k = 0.34 \quad k = 0.32 \]
\[ k = 0.30 \]

\[ g = 0.90 \quad c_x = 0.75 \]
\[ k = 0.50 \quad k = 0.48 \quad k = 0.46 \]
\[ k = 0.44 \quad k = 0.42 \quad k = 0.40 \]
\[ k = 0.38 \quad k = 0.36 \quad k = 0.34 \]
\[ k = 0.32 \quad k = 0.30 \]
Table 9-21c. Tied Columns. Values of $k$ for T10, $g = 0.70$, 0.80, and 0.90

- For $g = 0.70$, $c' = 0.80$:
  - $k = 0.46$
  - $k = 0.44$
  - $k = 0.42$
  - $k = 0.40$
  - $k = 0.38$
  - $k = 0.36$
  - $k = 0.34$
  - $k = 0.32$
  - $k = 0.30$

- For $g = 0.80$, $c' = 0.80$:
  - $k = 0.48$
  - $k = 0.46$
  - $k = 0.44$
  - $k = 0.42$
  - $k = 0.40$
  - $k = 0.38$
  - $k = 0.36$
  - $k = 0.34$
  - $k = 0.32$
  - $k = 0.30$

- For $g = 0.90$, $c' = 0.80$:
  - $k = 0.50$
  - $k = 0.48$
  - $k = 0.46$
  - $k = 0.44$
  - $k = 0.42$
  - $k = 0.40$
  - $k = 0.38$
  - $k = 0.36$
  - $k = 0.34$
  - $k = 0.32$
  - $k = 0.30$
Table 9-22a. Spiral Columns S6 to S16. General Formulas and Values of $k$, $g = 0.70$

Note: Formulas and values of $k$ apply to all columns where vertical reinforcement is located within the spiral.

Case I: Concentric load

$$P = 0.225 f_c' A_g + f_p A_d$$

$$f_p = \frac{0.225 f_c' + f_s p_g}{1 + (n-1)p_g}$$

Combined bending and direct stress

$$C = f_d / 0.45 f_c'$$

$$f_p = f_d \left( \frac{1 + D e}{1 + C D e} \right)$$

Case II: $e/t$ less than 1

$$N = \frac{P}{1 + C D e/t}$$

$$f_c = \frac{N}{A_g [1 + (n-1)p_g]}$$

Case III: $e/t$ greater than 1

$$N = f_p b t \left[ \frac{k^2 + n p g (2k-1)}{2k} \right]$$

$$f_c = \frac{N}{b t} \left[ \frac{2k}{k^2 + n p g (2k-1)} \right]$$

$$f_p = n l_c \left( \frac{1 + g}{2k} \right)$$

$$C' = 0.5$$

$$e/t = \frac{k^2 (1 - 2/3 k) + C' g^2 n p g}{2k^2 + 2 n p g (2k-1)}$$

Graph showing the relationship between $g$, $c'$, $n p g$, $e/t$, and $k$. The graph includes lines for different values of $k$ and $g$.
Table 9-22b. Spiral Columns S6 to S16. Values of $k$, $g = 0.80$ to 0.90.
Table 9-23a. Spiral Columns ST10 to ST20. General Formulas and Values of $k$, $g = 0.70$

**Case I: Concentric load**

$$ P = 0.225 f'_c t A_g + f_s A_s, \quad f_o = \frac{0.225 t_c' + f_s p_g}{t + (n-1)p_g} $$

Combined bending and direct stress

$$ C = f_c' / 0.45 t_c' \quad f_p = f_c' \left( \frac{t + D_e}{t + C D e} \right) $$

**Case II: $e/t$ less than 1**

$$ N = \frac{P}{1 + C D e / t} \quad \text{or} \quad P = N + C D \frac{M}{t} $$

$$ f_c = \frac{N \left( 1 + D_e / t \right)}{A_g \left[ 1 + (n-1)p_g \right]} \quad f_c'' \text{or} < f_p $$

**Case III: $e/t$ greater than 1**

$$ N = f_o b t \left[ \frac{k^2 + n p (2k-1)}{2k} \right] $$

$$ f_c = \frac{N}{b t} \left[ \frac{2k}{k^2 + n p (2k-1)} \right] \quad f_s = n f_b \left( \frac{t + g}{2k} - 1 \right) $$

$$ e/t = \frac{k^2 (1 - 2/3 k) + c'^2 n p}{2k^2 + 2 n p (2k-1)} $$

<table>
<thead>
<tr>
<th>ST10</th>
<th>$c' = 0.70$</th>
<th>ST16</th>
<th>$c' = 0.625$</th>
</tr>
</thead>
<tbody>
<tr>
<td>ST11</td>
<td>$c' = 0.681$</td>
<td>ST17</td>
<td>$c' = 0.617$</td>
</tr>
<tr>
<td>ST12</td>
<td>$c' = 0.666$</td>
<td>ST18</td>
<td>$c' = 0.611$</td>
</tr>
<tr>
<td>ST13</td>
<td>$c' = 0.654$</td>
<td>ST19</td>
<td>$c' = 0.605$</td>
</tr>
<tr>
<td>ST14</td>
<td>$c' = 0.642$</td>
<td>ST20</td>
<td>$c' = 0.600$</td>
</tr>
<tr>
<td>ST15</td>
<td>$c' = 0.633$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$g = 0.70$

$c' = 0.60$  
$c' = 0.70$

$k = 0.48$

$k = 0.46$

$k = 0.44$

$k = 0.42$

$k = 0.40$

$k = 0.38$

$k = 0.36$

$k = 0.34$

$k = 0.32$

$k = 0.30$
Table 9-23b. Spiral Columns ST10 to ST20. Values of $k$, $g = 0.80$ and 0.90
# Reinforced-Concrete Building Frames

## Table 9-24.a. Allowable Unit Stresses in Concrete (ACI Building Code, 1956)\(^1\)

### 305—Allowable Unit Stresses in Concrete

(a) The unit stresses in pounds per square inch on concrete to be used when designs are made in accordance with Section 601(a) shall not exceed the values of Table 305(a) where \(f'_c\) equals the minimum specified compressive strength at 28 days or at the earlier age at which the concrete may be expected to receive its full load.

### Table 305(a)—Allowable Unit Stresses in Concrete

<table>
<thead>
<tr>
<th>Description</th>
<th>For any strength of concrete in accordance with Section 302 n = 30,000</th>
<th>Allowable unit stresses</th>
<th>For strength of concrete shown below</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Maximum value, psi</td>
<td>(f'_c = ) 2000 psi</td>
<td>(f'_c = ) 2500 psi</td>
</tr>
<tr>
<td>Flexure: (f_u)</td>
<td></td>
<td>(n = 15)</td>
<td>(n = 12)</td>
</tr>
<tr>
<td>Extreme fiber stress in compression. (f_e)</td>
<td>0.45(f'_c)</td>
<td>900</td>
<td>1125</td>
</tr>
<tr>
<td>Extreme fiber stress in tension in plain concrete footings. (f_t)</td>
<td>0.03(f'_c)</td>
<td>60</td>
<td>75</td>
</tr>
<tr>
<td>Shear: (v) (as a measure of diagonal tension)</td>
<td></td>
<td>(v = 0.08(f'_c))</td>
<td>240</td>
</tr>
<tr>
<td>Beams with no web reinforcement. (v_s)</td>
<td>0.03(f'_c)</td>
<td>90</td>
<td>60</td>
</tr>
<tr>
<td>Beams with longitudinal bars and with either stirrups or properly located bent bars. (v)</td>
<td>0.08(f'_c)</td>
<td>240</td>
<td>160</td>
</tr>
<tr>
<td>Beams with longitudinal bars and a combination of stirrups and bent bars (the latter bent up suitably to carry at least 0.04(f'_c)). (v_s)</td>
<td>0.12(f'_c)</td>
<td>360</td>
<td>240</td>
</tr>
<tr>
<td>Footings* (For flat slabs see Chapter 10) (v_s)</td>
<td>0.03(f'_c)</td>
<td>75</td>
<td>60</td>
</tr>
<tr>
<td>Bond: (u)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Deformed bars (as defined in Section 104)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Top bars† (must be hooked) (u)</td>
<td>0.07(f'_c)</td>
<td>245</td>
<td>140</td>
</tr>
<tr>
<td>In two-way footings (except top bars) (u)</td>
<td>0.08(f'_c)</td>
<td>280</td>
<td>160</td>
</tr>
<tr>
<td>All others (u)</td>
<td>0.10(f'_c)</td>
<td>350</td>
<td>220</td>
</tr>
<tr>
<td>Plain bars (as defined in Section 104)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Top bars. (u)</td>
<td>0.03(f'_c)</td>
<td>105</td>
<td>60</td>
</tr>
<tr>
<td>In two-way footings (except top bars) (u)</td>
<td>0.036(f'_c)</td>
<td>126</td>
<td>72</td>
</tr>
<tr>
<td>All others (u)</td>
<td>0.045(f'_c)</td>
<td>158</td>
<td>90</td>
</tr>
<tr>
<td>Bearing: (f_b)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>On full area. (f_b)</td>
<td>0.25(f'_c)</td>
<td>500</td>
<td>625</td>
</tr>
<tr>
<td>On one-third area or less† (f_b)</td>
<td>0.375(f'_c)</td>
<td>750</td>
<td>938</td>
</tr>
</tbody>
</table>

* See Sections 905 and 809.
† Top bars in reference to bond are horizontal bars so placed that more than 12 in. of concrete is cast in the member under the bar.
‡ This increase shall be permitted only when the least distance between the edges of the loaded and unloaded areas is a minimum of one-fourth of the parallel side dimension of the loaded area. The allowable bearing stress on a reasonably concentric area greater than one-third but less than the full area shall be interpolated between the values given.

\(^1\) Reproduced by courtesy of the American Concrete Institute.
Table 9-24b. Allowable Unit Stresses in Reinforcement (ACI Building Code, 1956)\(^1\)

306—Allowable Unit Stresses in Reinforcement

Unless otherwise provided in this code, steel for concrete reinforcement shall not be stressed in excess of the following limits:

(a) **Tension**
   
   \( f_s \) = tensile unit stress in longitudinal reinforcement
   
   and \( f_s \) = tensile unit stress in web reinforcement
   
   20,000 psi for rail-steel concrete reinforcing bars, billet-steel concrete reinforcing bars of intermediate and hard grades, axle-steel concrete reinforcing bars of intermediate and hard grades, and cold-drawn steel wire for concrete reinforcement.
   
   18,000 psi for billet-steel concrete reinforcing bars of structural grade, and axle-steel concrete reinforcing bars of structural grade.

(b) **Tension in one-way slabs of not more than 12-ft span**
   
   \( f_A \) = tensile unit stress in main reinforcement
   
   For the main reinforcement, \( \frac{3}{8} \) in. or less in diameter, in one-way slabs, 50 percent of the minimum yield point specified in the specifications of the American Society for Testing Materials for the particular kind and grade of reinforcement used, but in no case to exceed 30,000 psi.

(c) **Compression, vertical column reinforcement**
   
   \( f_c \) = nominal allowable stress in vertical column reinforcement
   
   Forty percent of the minimum yield point specified in the specifications of the American Society for Testing Materials for the particular kind and grade of reinforcement used, but in no case to exceed 30,000 psi.
   
   \( f_c \) = allowable unit stress in the metal core of composite and combination columns

<table>
<thead>
<tr>
<th>Structural steel sections</th>
<th>16,000 psi</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cast iron sections</td>
<td>10,000 psi</td>
</tr>
<tr>
<td>Steel pipe</td>
<td>See limitations of Section 1104(b)</td>
</tr>
</tbody>
</table>

(d) **Compression, flexural members**

For compression reinforcement in flexural members see Section 706(b).

\(^1\) Reproduced by courtesy of the American Concrete Institute.
### Table 9-25. Flexural Members, General Formulas and Design Values for Rectangular Beams

#### Rectangular beams

\[
k = \frac{1}{1 + f_s/nf_c} \quad J = 1 - k/3
\]

\[
R = \frac{1}{2} f_y k_d
\quad p = \frac{f_c}{2f_y} k
\]

\[
I_t = \frac{(kd)^3}{3} + nA_s (d - kd)^2
\]

\[
S_t = I_t/k_d
\quad a = \frac{f_s J}{12000}
\]

#### Design values of R and p

<table>
<thead>
<tr>
<th>( f_c' )</th>
<th>2000 psi</th>
<th>2500 psi</th>
<th>3000 psi</th>
<th>3750 psi</th>
</tr>
</thead>
<tbody>
<tr>
<td>( n )</td>
<td>15</td>
<td>12</td>
<td>10</td>
<td>8</td>
</tr>
<tr>
<td>( a = 1.44 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( R )</td>
<td>157</td>
<td>196</td>
<td>236</td>
<td>295</td>
</tr>
<tr>
<td>( p )</td>
<td>.0091</td>
<td>.0113</td>
<td>.0136</td>
<td>.0170</td>
</tr>
</tbody>
</table>

#### Compression reinforcement

\[
R_c = p' (2n-1) f_c (1 - d'/kd)(d'/d)
\]

\[
p = p' \left( \frac{2n-1}{n} \right) \left( \frac{kd - d'}{d - kd} \right)
\]

\[
I_t = (2n-1) A_s (kd - d')^2 + nA_s (d - kd)^2
\]

\[
S_t = I_t/k_d
\]

#### Design values of R and p

<table>
<thead>
<tr>
<th>( f_s = 20,000 \text{ psi} )</th>
<th>( f_c = 0.45 f_c' )</th>
<th>( k = 0.403 )</th>
<th>( J = 0.866 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( d/d )</td>
<td>( f_s' ) psi</td>
<td>( R )</td>
<td>( p )</td>
</tr>
<tr>
<td>0.02</td>
<td>20,000</td>
<td>37</td>
<td>.0024</td>
</tr>
<tr>
<td>0.04</td>
<td>20,000</td>
<td>36</td>
<td>.0023</td>
</tr>
<tr>
<td>0.06</td>
<td>20,000</td>
<td>35</td>
<td>.0022</td>
</tr>
<tr>
<td>0.08</td>
<td>20,000</td>
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<td>.0021</td>
</tr>
<tr>
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<td>19,300</td>
<td>34</td>
<td>.0019</td>
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<tr>
<td>0.12</td>
<td>18,100</td>
<td>32</td>
<td>.0018</td>
</tr>
<tr>
<td>0.14</td>
<td>16,700</td>
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<td>.0017</td>
</tr>
<tr>
<td>0.16</td>
<td>15,500</td>
<td>26</td>
<td>.0015</td>
</tr>
<tr>
<td>0.18</td>
<td>14,200</td>
<td>23</td>
<td>.0014</td>
</tr>
<tr>
<td>0.20</td>
<td>12,900</td>
<td>20</td>
<td>.0013</td>
</tr>
</tbody>
</table>
Table 9-26. Flexural Members, General Formulas and Design Values for T Beams

\[
Z = \frac{t}{d} \left( \frac{3k - 2t/2d}{6x - 3t/2d} \right)
\]

\[
R = f_c \times \frac{t}{d} \left( \frac{2k - t/2d}{2k} \right) (1 - Z)
\]

\[
p = \frac{f_s}{f_y} \times \frac{t}{d} \left( \frac{2k - t/2d}{2k} \right)
\]

\[
I_r = \frac{bt^3}{12} + bt(kd - \frac{t}{2})^2 + nA(d - kd)^2
\]

\[
S_t = \frac{I_r}{kd}
\]

Design values of \( R \) and \( p \)

\[
f_s = 20,000 \quad f_c = 0.45 f_c \quad k = 0.403 \quad j = 1 - Z
\]

<table>
<thead>
<tr>
<th>( t' / d )</th>
<th>( \alpha )</th>
<th>2000 psi</th>
<th>( R )</th>
<th>( p )</th>
<th>2500 psi</th>
<th>( R )</th>
<th>( p )</th>
<th>3000 psi</th>
<th>( R )</th>
<th>( p )</th>
<th>3750 psi</th>
<th>( R )</th>
<th>( p )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.04</td>
<td>1.63</td>
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<td>0.0017</td>
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<td>63</td>
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<tr>
<td>0.06</td>
<td>1.62</td>
<td>48</td>
<td>0.0025</td>
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<td>86</td>
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<td>0.14</td>
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<td>0.0058</td>
<td>133</td>
<td>0.0072</td>
<td>160</td>
<td>0.0086</td>
<td>200</td>
<td>0.0108</td>
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### Table 9-27. Shear, General Formulas and Values of $A_v f_v$ for Stirrups

**General Formulas**

\[ V = V_c + V_d + V_s \]

- \[ V_c = v_c b j d \]
- \[ V_d = v_d b j d = 1.65 A_v f_v \]
- \[ V_s = v_s b j d \]

**Values**

- \[ f_s = 20,000 \text{ psi} \]
- \[ A_v f_v \]
- \[ b j \]
- \[ u = 0.03 t_c' \] and \[ 0.10 t_c' \]

**Vertical U stirrups: values of $A_v f_v$ (lb)**

| $t_c'$ | \( u \) | \begin{tabular}{c|cccccccc} \hline & \multicolumn{8}{c}{D} \\ 
& 8" & 10" & 12" & 14" & 16" & 18" & 20" & 22" \\
& \#3 & 2440 & 2930 & 3400 & 3820 & 4280 & 4400 & \\
& \#4 & 4075 & 4720 & 5350 & 5920 & 6560 & 7120 & 7750 & 8000 \\
& \#5 & 6400 & 7150 & 7920 & 8670 & 9450 & 10,200 & 10,900 & \\
250 & \#2 & 1180 & 1360 & 1540 & 1700 & 1880 & 2000 & \\
& \#3 & 2530 & 3120 & 3700 & 4220 & 4400 & \\
& \#4 & 4100 & 4900 & 5700 & 6400 & 7200 & 7900 & 8000 & \\
& \#5 & 6480 & 7430 & 8400 & 9350 & 10,300 & 11,300 & 12,000 & \\
300 & \#2 & 1210 & 1430 & 1650 & 1830 & 2000 & \\
& \#3 & 2600 & 3300 & 4000 & 4400 & \\
& \#4 & 4120 & 5080 & 6040 & 6870 & 7840 & 8000 & \\
& \#5 & 6580 & 7700 & 8860 & 10,050 & 11,180 & 12,000 & \\
3750 & \#2 & 1250 & 1505 & 1760 & 1930 & 2000 & \\
& \#3 & 2760 & 3500 & 4350 & 4400 & \\
& \#4 & 4150 & 5250 & 6370 & 7350 & 8000 & \\
& \#5 & 6680 & 8000 & 9370 & 10,600 & 12,000 & \\
\hline
\end{tabular} \\
\begin{tabular}{c|cccccccc} \hline
\* $v_0 = 0.05 t_c' ; 0.12 t_c'$ max $v$ for combined web reinforcement
\end{tabular}

* $v_0 = 0.05 t_c' ; 0.12 t_c'$ max $v$ for combined web reinforcement
### Table 9-28. Shear, Values of $V_s$ for Bent Bars

Bent bars $a = 45^\circ$: values of $V_s = 1.65 A_f f_v$ (kips)

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*$V_s = 0.05 f'_c$: max $v = 0.12 f'_c$ for combined web reinforcement
Use .43 times the tabulated value for single bent bar
Table 9-29. Bond, General Formula and Values of $V_B$

General formula:

$$V_B = u \Sigma_a \cdot j \cdot d$$

- $u = 0.07f'_c$ top bars
- $u = 0.10f'_c$ bottom bars

Values of $V_B = 0.866 \cdot u \cdot \Sigma_a \cdot (d-f')^{kips}$

<table>
<thead>
<tr>
<th>$f'_c$</th>
<th>$\Sigma_a$</th>
<th>1.178</th>
<th>1.571</th>
<th>1.963</th>
<th>2.36</th>
<th>2.75</th>
<th>3.14</th>
<th>3.54</th>
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<td>.430</td>
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<td>.241</td>
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Values of $V_B = 0.866 \cdot u \cdot \Sigma_a \cdot (d-f')^{kips}$

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<th>$f'_c$</th>
<th>$\Sigma_a$</th>
<th>1.178</th>
<th>1.571</th>
<th>1.963</th>
<th>2.36</th>
<th>2.75</th>
<th>3.14</th>
<th>3.54</th>
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<th>4.43</th>
</tr>
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### TABLES FOR DESIGN OF REINFORCED CONCRETE

**Table 9-30a. Solid Slabs, Properties for Design.**  
$t$ from 4 in. to 8 in.

<table>
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<tr>
<th>$b = 12''$</th>
<th>$t_s$</th>
<th>20,000</th>
<th>$S_c$</th>
<th>$A_s$</th>
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<tbody>
<tr>
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<td>$I_c$</td>
<td>.45 $f_c'$</td>
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<td>$f_c$</td>
<td>$t_c$</td>
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<td>$k$</td>
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<td>2500</td>
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<td>.9040</td>
<td>.0406</td>
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<tr>
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<td>.6177</td>
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<td>.054</td>
<td>.81</td>
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<td>1.21</td>
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<td>1.78</td>
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* $u = 0.10 t_c$
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* $b = 0.10 t_{c'}$
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<th>$f'_{c} = 20,000$</th>
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<th>$V_{c} = .866 u \Sigma_{o} d$</th>
<th>$V_{c} = .866 \nu_{c} b d$</th>
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<td>$A_{s}'$</td>
<td>$A_{s}$</td>
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<td>$13\ 1/2^\circ$</td>
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<td>$15^\circ$</td>
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$u^* = .07 f'_{c}$: For bottom bars multiply values given by $1.43$.
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<th>T beams</th>
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<td>$f_c = 0.45t_c$</td>
<td>$k = 0.403$</td>
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<tr>
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<td>7&quot; 7&quot; 7&quot;</td>
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<td>$v_c$</td>
<td>$V_c = 0.866V_c b'd$</td>
<td>$v_c = 0.866V_c b'd$</td>
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</tr>
<tr>
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$u = 0.10 f_c$
## REINFORCED-CONCRETE BUILDING FRAMES

### Table 9-31c. Ribbed Slabs, 20-in. Metal Fillers, Properties for Design. 12-in.- and 14-in.-deep Fillers

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<td>$b' = 5^° 6^° 7^°$</td>
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</tr>
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<td>$\alpha d = 191$</td>
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<td>$t^c$</td>
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<tr>
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<td>8.9</td>
</tr>
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*$\alpha = .07t'_c$: for bottom bars multiply values given by 1.43*
### Tables for Design of Reinforced Concrete

#### Table 9-32a. Ribbed Slabs, Properties for Design. 4-in.- and 6-in.-deep 12-in.-Masonry Fillers

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<th>f_c</th>
<th>A_s</th>
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## REINFORCED-CONCRETE BUILDING FRAMES

### Table 9-32b. Ribbed Slabs, Properties for Design. 6-in.- and 8-in.-deep 12-in. Masonry Fillers

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<td>( k = .403 )</td>
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<td>( a = 1.44 )</td>
<td>( b' )</td>
<td>( b' )</td>
</tr>
<tr>
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<td>( 4\frac{3}{8} )</td>
<td>( 10\frac{1}{2} )</td>
<td>( 5\frac{1}{4} )</td>
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<td>( 6&quot; + 3&quot; )</td>
<td>( 1.51 )</td>
<td>( 3.4 )</td>
<td>( 3.4 )</td>
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<tr>
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<td>( 2.21 )</td>
<td>( 5.7 )</td>
<td>( 5.7 )</td>
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</table>

### Notes
- \( \gamma_u = 1.0 t_c \)
TABLES FOR DESIGN OF REINFORCED CONCRETE 9-113

Table 9-32c. Ribbed Slabs, Properties for Design. 8-in.- and 10-in.-deep 12-in. Masonry Fillers

<table>
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</tr>
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<td>$f_y = 20,000$</td>
<td>$f_y = .45 t'_c$</td>
<td>$k = .403$</td>
</tr>
<tr>
<td></td>
<td>$a = 1.44$</td>
<td>$b = b' + 12^\circ$</td>
<td>$A_g$</td>
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<tr>
<td></td>
<td>b'</td>
<td>b'</td>
<td>$\lambda_u$</td>
</tr>
<tr>
<td></td>
<td>4&quot; 10&quot; 5&quot; 11&quot; 6&quot; 12&quot;</td>
<td>4&quot; 5&quot; 6&quot;</td>
<td></td>
</tr>
<tr>
<td>8&quot;+3&quot;</td>
<td>$d = 9\frac{1}{4}'' 9\frac{3}{4}'' 9\frac{5}{4}'' 9\frac{7}{4}'' 9\frac{9}{4}'' 9\frac{11}{4}''$</td>
<td>$\lambda = 12.42$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$V_c = .866V_c b'd$</td>
<td>$\alpha = 1.42$</td>
<td>$V_B = .866V_c b'd$</td>
</tr>
<tr>
<td>60</td>
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<td>Weight</td>
<td>$\lambda = 5.0$</td>
</tr>
<tr>
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<td>2.5 6.3 3.1 6.9 3.8 7.6</td>
<td>$I_c$</td>
<td>4.0 5.0 5.9 7.0</td>
</tr>
<tr>
<td>90</td>
<td>3.0 7.6 3.8 8.3 4.5 9.1</td>
<td>$\lambda = 7.0$</td>
<td></td>
</tr>
<tr>
<td>113</td>
<td>3.8 9.6 4.8 10.5 5.7 11.5</td>
<td>$I_c$</td>
<td>5.3 6.6 7.9 9.3</td>
</tr>
<tr>
<td>10&quot;+2&quot;</td>
<td>$d = 10\frac{1}{4}'' 10\frac{3}{4}'' 10\frac{5}{4}'' 10\frac{7}{4}'' 10\frac{9}{4}'' 10\frac{11}{4}''$</td>
<td>$\lambda = 15.6$</td>
<td></td>
</tr>
<tr>
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<td>$\lambda = 6.6$</td>
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<tr>
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<td>$\lambda = 8.8$</td>
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<tr>
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<td>4.2 10.5 5.2 11.5 6.3 12.6</td>
<td>$I_c$</td>
<td>6.6 8.2 9.9 11.5</td>
</tr>
<tr>
<td>10&quot;+2&quot;</td>
<td>$d = 1\frac{1}{4}'' 1\frac{1}{4}'' 1\frac{3}{4}'' 1\frac{7}{4}'' 1\frac{1}{4}'' 1\frac{1}{4}''$</td>
<td>$\lambda = 16.4$</td>
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<td>$\lambda = 6.9$</td>
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<td>$I_c$</td>
<td>6.1 7.7 9.2 10.7</td>
</tr>
<tr>
<td>90</td>
<td>3.5 8.7 4.4 9.6 5.2 10.5</td>
<td>$\lambda = 8.8$</td>
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</tr>
<tr>
<td>113</td>
<td>4.4 10.0 5.5 12.1 6.6 13.2</td>
<td>$I_c$</td>
<td>6.9 8.6 10.3 12.0</td>
</tr>
<tr>
<td>10&quot;+3&quot;</td>
<td>$d = 1\frac{1}{4}'' 1\frac{3}{4}'' 1\frac{7}{4}'' 1\frac{1}{4}'' 1\frac{1}{4}'' 1\frac{1}{4}''$</td>
<td>$\lambda = 17.1$</td>
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<td>Weight</td>
<td>$\lambda = 6.4$</td>
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<tr>
<td>75</td>
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<td>$I_c$</td>
<td>6.4 8.0 9.6 10.1</td>
</tr>
<tr>
<td>90</td>
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</tr>
<tr>
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<td>4.5 11.5 5.7 12.6 6.8 13.8</td>
<td>$I_c$</td>
<td>7.2 9.0 10.8 12.5</td>
</tr>
</tbody>
</table>

*$_u = .10 t'_c$ : for top bars multiply values given by 0.70
### REINFORCED-CONCRETE BUILDING FRAMES

Table 9-32d. Ribbed Slabs, Properties for Design. 4-in.- and 6-in.-deep 16-in. Masonry Fillers

<table>
<thead>
<tr>
<th>Depth</th>
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<th>$S_C$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$f_s = 20,000$ $f_c = .45 f'_C$ $k = .403$</td>
<td>$b = b' + 16^\circ$</td>
<td>$f_c$</td>
</tr>
<tr>
<td></td>
<td>$a = 1.44$</td>
<td>$A'_s$</td>
<td>$A_s$</td>
</tr>
<tr>
<td></td>
<td>$V_c = .866V_c/bd$</td>
<td>$w = 7.2$</td>
<td>$V_b = .866u\Sigma d$</td>
</tr>
<tr>
<td>$4'' + 2''$</td>
<td>$b'$</td>
<td>$b'$</td>
<td>$2000$</td>
</tr>
<tr>
<td>$I_t$</td>
<td>$35$</td>
<td>$106$</td>
<td>$44$</td>
</tr>
<tr>
<td>$S_t$</td>
<td>$17$</td>
<td>$52$</td>
<td>$22$</td>
</tr>
<tr>
<td>$n_{As}$</td>
<td>$2.7$</td>
<td>$8.1$</td>
<td>$3.4$</td>
</tr>
<tr>
<td>$d$</td>
<td>$5''$</td>
<td>$5''$</td>
<td>$5''$</td>
</tr>
</tbody>
</table>

| $4'' + 2''/2$ | | | |
| $V_c = .866V_c/bd$ | $w = 7.9$ | $V_b = .866u\Sigma d$ |
| $I_t$ | $43$ | $131$ | $54$ | $142$ | $65$ | $153$ | $233$ | $245$ | $256$ | $\#4$ | $13$ | $11$ | $9$ | $7$ | .14 |
| $S_t$ | $20$ | $60$ | $25$ | $65$ | $30$ | $70$ | $103$ | $110$ | $115$ | $\#5$ | $19$ | $15$ | $13$ | $10$ | .20 |
| $n_{As}$ | $2.9$ | $8.8$ | $3.6$ | $9.5$ | $4.4$ | $10.3$ | $15.0$ | $15.7$ | $16.5$ | $\#6$ | $26$ | $21$ | $17$ | $14$ | .27 |
| $d$ | $5'\frac{3}{8}''$ | $5'\frac{3}{8}''$ | $5'\frac{3}{8}''$ | $5'\frac{3}{8}''$ | $5'\frac{3}{8}''$ | $5'\frac{3}{8}''$ | $5'\frac{3}{8}''$ | $5'\frac{3}{8}''$ | $5'\frac{3}{8}''$ | $\#7$ | $32$ | $26$ | $21$ | $17$ | .35 |

| $4'' + 3''$ | | | |
| $V_c = .866V_c/bd$ | $w = 8.6$ | $V_b = .866u\Sigma d$ |
| $I_t$ | $57$ | $171$ | $71$ | $185$ | $85$ | $200$ | $303$ | $318$ | $333$ | $\#4$ | $16$ | $13$ | $11$ | $8$ | .15 |
| $S_t$ | $24$ | $72$ | $30$ | $78$ | $36$ | $84$ | $125$ | $131$ | $138$ | $\#5$ | $23$ | $19$ | $15$ | $12$ | .22 |
| $n_{As}$ | $3.2$ | $9.6$ | $4.0$ | $10.4$ | $4.8$ | $11.2$ | $16.3$ | $17.1$ | $17.9$ | $\#6$ | $31$ | $25$ | $21$ | $16$ | .29 |
| $d$ | $5'\frac{7}{8}''$ | $5'\frac{7}{8}''$ | $5'\frac{7}{8}''$ | $5'\frac{7}{8}''$ | $5'\frac{7}{8}''$ | $5'\frac{7}{8}''$ | $6''$ | $6''$ | $6''$ | $\#7$ | $39$ | $32$ | $26$ | $22$ | .38 |

| $6'' + 2''$ | | | |
| $V_c = .866V_c/bd$ | $w = 10.1$ | $V_b = .866u\Sigma d$ |
| $I_t$ | $91$ | $273$ | $114$ | $296$ | $136$ | $318$ | $448$ | $478$ | $506$ | $\#4$ | $21$ | $17$ | $14$ | $11$ | .16 |
| $S_t$ | $31$ | $95$ | $40$ | $103$ | $47$ | $111$ | $159$ | $169$ | $178$ | $\#5$ | $32$ | $25$ | $21$ | $17$ | .24 |
| $n_{As}$ | $3.7$ | $11.2$ | $4.7$ | $12.1$ | $5.6$ | $13.0$ | $17.4$ | $18.6$ | $19.6$ | $\#6$ | $43$ | $34$ | $28$ | $23$ | .33 |
| $d$ | $6'\frac{7}{8}''$ | $6'\frac{7}{8}''$ | $6'\frac{7}{8}''$ | $6'\frac{7}{8}''$ | $6'\frac{7}{8}''$ | $6'\frac{7}{8}''$ | $7''$ | $7''$ | $7''$ | $\#7$ | $55$ | $44$ | $37$ | $26$ | .44 |

**$u = .10 f_c$**

**REINFORCED-CONCRETE BUILDING FRAMES**

**Table 9-32f. Ribbed Slabs, Properties for Design. 8-in.- and 10-in.-deep 16-in. Masonry Fillers**

<table>
<thead>
<tr>
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<th>A_s</th>
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<td>(f_c = 20,000)</td>
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<td></td>
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<td>(t_c = .45f_c)</td>
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<tr>
<td></td>
<td>(k = .403)</td>
<td>(k = .403)</td>
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<td></td>
</tr>
<tr>
<td>(d)</td>
<td>(b')</td>
<td>(b')</td>
<td>(\delta_5)</td>
<td>(A_s)</td>
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<td>(1.44)</td>
<td>(1220)</td>
<td>(1022)</td>
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<td>(12'')</td>
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<td>(20'')</td>
<td>(24'')</td>
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<td>(7'')</td>
<td>(12'')</td>
<td>(30'')</td>
<td>(36'')</td>
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<tr>
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<td>(8'')</td>
<td>(16'')</td>
<td>(40'')</td>
<td>(48'')</td>
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<tr>
<td></td>
<td>(9'')</td>
<td>(24'')</td>
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<td>(60'')</td>
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<td>2.8 8.4 3.5 9.1 4.2 9.8</td>
<td>4.4 5.5 6.6 7.7</td>
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<td>5.1 6.4 7.7 9.0</td>
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<th>(v_b = .866u \Sigma d)</th>
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</thead>
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<td>6.6 8.3 9.9 11.5</td>
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<tr>
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<td>2.9 8.8 3.6 9.5 4.4 10.2</td>
<td>4.4 5.5 6.7 8.0</td>
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<td>5.4 6.7 8.0 9.4</td>
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<td>113</td>
<td>4.4 13.2 5.5 14.3 6.6 15.4</td>
<td>6.1 7.7 9.2 10.7</td>
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</table>

<table>
<thead>
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<th>(v_c)</th>
<th>(V_c = .866v_c b'd)</th>
<th>(v_b = .866u \Sigma d)</th>
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<tbody>
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\*\*u \cdot 10f_c\*\* for top bars, multiply values given by 0.70
### Table 9-33a. Reinforcing for Slabs. Rods

#### Solid slabs

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<th>4(^{\circ}) oc</th>
<th>5(^{\circ}) oc</th>
<th>6(^{\circ}) oc</th>
<th>7(^{\circ}) oc</th>
<th>8(^{\circ}) oc</th>
<th>9(^{\circ}) oc</th>
<th>10(^{\circ}) oc</th>
<th>12(^{\circ}) oc</th>
</tr>
</thead>
<tbody>
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<td>(\Sigma_0)</td>
<td>A(_5)</td>
<td>(\Sigma_0)</td>
<td>A(_5)</td>
<td>(\Sigma_0)</td>
<td>A(_5)</td>
<td>(\Sigma_0)</td>
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<td>.47</td>
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<td>.35</td>
<td>.26</td>
<td>.28</td>
<td>.22</td>
<td>.23</td>
<td>.19</td>
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<tr>
<td>#3 and #4</td>
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<td>.55</td>
<td>.45</td>
<td>.41</td>
<td>.37</td>
<td>.33</td>
<td>.31</td>
<td>.27</td>
<td>.26</td>
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<tr>
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<td>.80</td>
<td>.63</td>
<td>.60</td>
<td>.47</td>
<td>.48</td>
<td>.38</td>
<td>.40</td>
<td>.31</td>
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<td>.76</td>
<td>.53</td>
<td>.61</td>
<td>.42</td>
<td>.50</td>
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<th>A(_5)</th>
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<th>Area trans. wires sq in/ft</th>
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<th>Area long wires sq in/ft</th>
<th>Size and spacing</th>
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TABLES FOR DESIGN OF REINFORCED CONCRETE  9-119

Table 9-34. Reinforcing for Beams, Areas and Minimum Beam Widths

![Diagram of Reinforced Concrete Beam]

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Table 9-35. Shear, Spacing and Arrangement of Bent Bars

Note: For short spans bend bars 4 and 5 for 5 bars, 3 and 4 for 4 bars at same bend point providing 2 bent bars for shear at the support

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Table 9-36. Slants and Increments for 45° Bar Bends

\[ O = \text{overall bar dimension} \quad S = 1.414H \]
\[ H = \text{height of bend} \quad I = S - H \]

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<td>7\frac{1}{2}</td>
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<td>11\frac{1}{2}</td>
<td>7</td>
<td>1-8</td>
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<td>1-4</td>
<td>3-0</td>
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<td>2-6</td>
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<tr>
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<td>1-0</td>
<td>7</td>
<td>1-9</td>
<td>2-5\frac{1}{2}</td>
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<td>4-4\frac{1}{2}</td>
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<td>1-0\frac{1}{2}</td>
<td>7</td>
<td>1-10</td>
<td>2-7</td>
<td>1-6</td>
<td>3-2</td>
<td>4-5\frac{1}{2}</td>
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<td>1-1\frac{1}{2}</td>
<td>8</td>
<td>1-11</td>
<td>2-8\frac{1}{2}</td>
<td>1-7</td>
<td>3-3</td>
<td>4-7</td>
<td>2-8</td>
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</table>

\[ D = 6d \quad \#2-\#7 \]
\[ D = 8d \quad \#8-\#11 \]

<table>
<thead>
<tr>
<th>Hook A</th>
<th>Hook B</th>
</tr>
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<td>Bar size d</td>
<td>#2</td>
</tr>
<tr>
<td>Hook A</td>
<td>4</td>
</tr>
<tr>
<td>H</td>
<td>2</td>
</tr>
<tr>
<td>Hook B</td>
<td>3</td>
</tr>
<tr>
<td>J</td>
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</tr>
</tbody>
</table>
Table 9-37. Reinforced-concrete Beams, Typical Details

Note: Provide #3 ties at 16" oc (maximum) continuous in spandrel beams – see code for compressive reinf

Typical details, vertical U-stirrups and bent bars

Locate straight top bars over supports in slab to avoid crowding reinforcing in columns

Additional reinforcing as required for positive moment may be bent and used for shear and negative moment reinforcement

Provide 2 straight bars minimum. 25% minimum of required reinforcing shall be continued as straight bars into support

Typical details, interior beams

Provide 2-#5 straight top bars continuous lap 5'-0" at supports (Include in reinforcement required for negative moment)

Typical details, spandrel beams
# Tables for Design of Reinforced Concrete

## Table 9-38a. Flexural Members, Properties for Design of Beam. $b'd$ from 6 in. × 8 in. to 12 in. × 12 in.

<table>
<thead>
<tr>
<th>Rectangular beams</th>
<th>T beams</th>
<th>$*V_d$</th>
<th>0&quot;</th>
<th>6&quot;</th>
<th>10&quot;</th>
<th>12&quot;</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b' \times 0$</td>
<td>$f_c$, .45$f_c'$ b 12&quot;</td>
<td>$\frac{3}{4}d = \frac{3}{4}d$</td>
<td>60</td>
<td>75</td>
<td>90</td>
<td>113</td>
</tr>
<tr>
<td>Weight</td>
<td>k .403</td>
<td>k .403</td>
<td>158</td>
<td>68</td>
<td>9.3</td>
<td>5.5</td>
</tr>
<tr>
<td>$I_c$</td>
<td>a 1.44</td>
<td>t</td>
<td></td>
<td></td>
<td></td>
<td>113</td>
</tr>
<tr>
<td>6&quot; × 8&quot;</td>
<td>1t 81</td>
<td>158</td>
<td>60</td>
<td>75</td>
<td>90</td>
<td>113</td>
</tr>
<tr>
<td>(50 #/1)</td>
<td>8t 35</td>
<td>68</td>
<td>75</td>
<td>90</td>
<td>113</td>
<td></td>
</tr>
<tr>
<td>256 d</td>
<td>5t$^{1/4}$</td>
<td>9.3</td>
<td>90</td>
<td>113</td>
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<td></td>
</tr>
<tr>
<td>8&quot; × 8&quot;</td>
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<td>75</td>
<td>90</td>
<td>113</td>
</tr>
<tr>
<td>(67 #/1)</td>
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<td>5.5</td>
<td>75</td>
<td>90</td>
<td>113</td>
<td></td>
</tr>
<tr>
<td>341 d</td>
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<td>9.3</td>
<td>90</td>
<td>113</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8&quot; × 10&quot;</td>
<td>1t 262</td>
<td>124</td>
<td>60</td>
<td>75</td>
<td>90</td>
<td>113</td>
</tr>
<tr>
<td>(85 #/1)</td>
<td>8t 84</td>
<td>124</td>
<td>75</td>
<td>90</td>
<td>113</td>
<td></td>
</tr>
<tr>
<td>667 d</td>
<td>7t$^{3/4}$</td>
<td>12.5</td>
<td>90</td>
<td>113</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10&quot; × 10&quot;</td>
<td>1t 327</td>
<td>12.5</td>
<td>60</td>
<td>75</td>
<td>90</td>
<td>113</td>
</tr>
<tr>
<td>(104 #/1)</td>
<td>10t 105</td>
<td>12.5</td>
<td>75</td>
<td>90</td>
<td>113</td>
<td></td>
</tr>
<tr>
<td>833 d</td>
<td>7t$^{3/4}$</td>
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<td>90</td>
<td>113</td>
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<td></td>
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<tr>
<td>8&quot; × 12&quot;</td>
<td>1t 522</td>
<td>428</td>
<td>60</td>
<td>75</td>
<td>90</td>
<td>113</td>
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<tr>
<td>(100 #/1)</td>
<td>10t 133</td>
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<td>75</td>
<td>90</td>
<td>113</td>
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<tr>
<td>1152 d</td>
<td>9t$^{3/4}$</td>
<td>8&quot;</td>
<td>90</td>
<td>113</td>
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<td></td>
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<tr>
<td>10&quot; × 12&quot;</td>
<td>1t 651</td>
<td>132</td>
<td>60</td>
<td>75</td>
<td>90</td>
<td>113</td>
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<tr>
<td>(125 #/1)</td>
<td>165 196</td>
<td>132</td>
<td>75</td>
<td>90</td>
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<td></td>
</tr>
<tr>
<td>1440 d</td>
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<td>8&quot;</td>
<td>90</td>
<td>113</td>
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<td></td>
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<tr>
<td>12&quot; × 12&quot;</td>
<td>1t 781</td>
<td>15.7</td>
<td>60</td>
<td>75</td>
<td>90</td>
<td>113</td>
</tr>
<tr>
<td>(144 #/1)</td>
<td>198 254</td>
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<td>75</td>
<td>90</td>
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<td></td>
</tr>
<tr>
<td>1728 d</td>
<td>9t$^{3/4}$</td>
<td>8&quot;</td>
<td>90</td>
<td>113</td>
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<table>
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<tr>
<th>$A_s$</th>
<th>$d'$</th>
<th>$S_c$ for $d = 9t^{1/4}$</th>
<th>$A_s$</th>
<th>Bond: $V_b = .866 \sum d$</th>
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</thead>
<tbody>
<tr>
<td>1 #4</td>
<td>.17</td>
<td>19 15 12 9 .11</td>
<td>1 #4</td>
<td>.156 1.5 2.3 3.4 2.1 2.1</td>
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<td>27 21 17 14 .16</td>
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<td>35 28 23 18 .21</td>
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<td>2.34 2.9 3.5 3.2 2.8 2.8</td>
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<tr>
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<td>74 58 48 38 .47</td>
<td>1 #9</td>
<td>3.52 4.4 5.3 4.7 5.9 5.9</td>
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</tbody>
</table>

*$\times$ Use .43 times the tabulated value for single bent bar

$V_a = .05 f_c' .12 f_c'_{max} v$ for combined web reinforcement
### Table 9-38b. Flexural Members, Properties for Design of Beam. \( b'D \) from 10 in. × 14 in. to 14 in. × 16 in.

<table>
<thead>
<tr>
<th>Rectangular beams</th>
<th>T beams</th>
<th>( \times )</th>
<th>( \times )</th>
<th>( \times )</th>
<th>( \times )</th>
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<tbody>
<tr>
<td>( b' \times D )</td>
<td>( f_c )</td>
<td>( 0.45 f_c )</td>
<td>( b )</td>
<td>( 12'' )</td>
<td>( \frac{a}{d} )</td>
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<tr>
<td><strong>Weight</strong></td>
<td>( k )</td>
<td>( 0.403 )</td>
<td>( k )</td>
<td>( 0.403 )</td>
<td>( )</td>
</tr>
<tr>
<td>( I_c )</td>
<td>( a )</td>
<td>( 1.44 )</td>
<td>( t )</td>
<td>( )</td>
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<table>
<thead>
<tr>
<th>Size</th>
<th>( V_c )</th>
<th>Spacing</th>
<th>( V_c )</th>
</tr>
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<tbody>
<tr>
<td>10''x14''</td>
<td>1444</td>
<td>4''</td>
<td>#3</td>
</tr>
<tr>
<td>12''x14''</td>
<td>1366</td>
<td>4''</td>
<td>#3</td>
</tr>
<tr>
<td>16''x14''</td>
<td>1759</td>
<td>4''</td>
<td>#3</td>
</tr>
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</table>

### Bond: \( V_c = 0.866 u z_d \)

<table>
<thead>
<tr>
<th>( A_s )</th>
<th>( d' )</th>
<th>( S_c )</th>
<th>( I_c )</th>
<th>( A_s )</th>
<th>( )</th>
<th>( )</th>
<th>( )</th>
<th>( )</th>
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<td>2500</td>
<td>3000</td>
<td>3750</td>
<td>2000</td>
<td>2500</td>
<td>3000</td>
<td>3750</td>
<td></td>
</tr>
<tr>
<td>1 #6</td>
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<td>60</td>
<td>40</td>
<td>31</td>
<td>29</td>
<td>0.29</td>
<td>1 #6</td>
<td>3.4</td>
</tr>
<tr>
<td>1 #7</td>
<td>231</td>
<td>80</td>
<td>63</td>
<td>53</td>
<td>42</td>
<td>0.39</td>
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<tr>
<td>1 #8</td>
<td>237</td>
<td>102</td>
<td>81</td>
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<tr>
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<td>125</td>
<td>99</td>
<td>82</td>
<td>65</td>
<td>0.61</td>
<td>1 #9</td>
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<tr>
<td>1 #10</td>
<td>250</td>
<td>154</td>
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<td>100</td>
<td>79</td>
<td>0.76</td>
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<td>5.7</td>
</tr>
<tr>
<td>1 #11</td>
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<td>142</td>
<td>117</td>
<td>93</td>
<td>0.89</td>
<td>1 #11</td>
<td>6.4</td>
</tr>
</tbody>
</table>

| 10''x16'' | 1921   | 4''      | \#3       |
| (167#/1)  | 328    | 294      | \#3       |
| 3413      | 137/4''| 12''     | \#3       |
| 12''x16'' | 2184   | 41/2''   | \#3       |
| 3900      | 137/4''| 16''     | \#3       |
| (200#/1)  | 224    | 19.5     | \#3       |
| 4096      | 137/4''| 25.2     | \#3       |
| 14''x16'' | 2555   | 25.2     | \#3       |
| (234#/1)  | 262    | 25.2     | \#3       |
| 4779      | 137/4''| 28.8     | \#3       |

| Bond: \( V_c = 0.866 u z_d \)

<table>
<thead>
<tr>
<th>( A_s )</th>
<th>( d' )</th>
<th>( S_c )</th>
<th>( I_c )</th>
<th>( A_s )</th>
<th>( )</th>
<th>( )</th>
<th>( )</th>
<th>( )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 #6</td>
<td>225</td>
<td>82</td>
<td>66</td>
<td>55</td>
<td>43</td>
<td>0.33</td>
<td>1 #6</td>
<td>3.9</td>
</tr>
<tr>
<td>1 #7</td>
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<td>110</td>
<td>87</td>
<td>73</td>
<td>59</td>
<td>0.54</td>
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<td>4.6</td>
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<tr>
<td>1 #8</td>
<td>237</td>
<td>124</td>
<td>113</td>
<td>94</td>
<td>76</td>
<td>0.59</td>
<td>1 #8</td>
<td>5.2</td>
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<tr>
<td>1 #9</td>
<td>243</td>
<td>173</td>
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<td>115</td>
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<td>1 #11</td>
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<td>254</td>
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<td>170</td>
<td>136</td>
<td>1.06</td>
<td>1 #11</td>
<td>7.4</td>
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</tbody>
</table>

*Use, 45 times the tabulated value for single bent bar \( v_c = 0.05 f_c \); .12 f_c max \( v \) for combined web reinforcement
**Tables for Design of Reinforced Concrete**

**9-125**

### Table 9-38c. Flexural Members, Properties for Design of Beam. \( b' D \) from 10 in. \( \times \) 18 in. to 22 in. \( \times \) 18 in.

<table>
<thead>
<tr>
<th>Rectangular beams</th>
<th>T beams</th>
<th>(*V_a)</th>
<th>( t'_c )</th>
<th>#6</th>
<th>#7</th>
<th>#8</th>
<th>#9</th>
<th>#10</th>
<th>#11</th>
</tr>
</thead>
<tbody>
<tr>
<td>( b' \times D )</td>
<td>( t_c )</td>
<td>( .45t'_c )</td>
<td>( b )</td>
<td>12&quot;</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Weight</td>
<td>( k )</td>
<td>(.403 )</td>
<td>( k )</td>
<td>(.403 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( I_c )</td>
<td>( ad )</td>
<td>( 22.3 )</td>
<td>( t )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10&quot;( \times )16&quot;</td>
<td>( I_{T} )</td>
<td>2613</td>
<td>( 4&quot; )</td>
<td>2160</td>
<td></td>
<td></td>
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<td></td>
<td>( S_{T} )</td>
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</tr>
<tr>
<td>(187#/1)</td>
<td>( n_{A_{s}} )</td>
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</tr>
<tr>
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<td>( 15\frac{1}{2} )</td>
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**Size**

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**V = V_a + V_s**

### Bond

**Bond: \( V_B = .86 \sigma_d d \)**

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<th>( S_{c} )</th>
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<th>( u )</th>
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**Use \(.43\) times the tabulated value for single bent bar**

**\( V_a = .05 t'_c = .12 t_c \) max \( v \) for combined web reinforcement**
### Table 9-38d. Flexural Members, Properties for Design of Beam. $b\times D$ from 10 in. $\times$ 20 in. to 22 in. $\times$ 20 in.

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<tr>
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<td>17$1/2$</td>
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| $A_s$ | $d'$ | $S_c$ | $A_t$ | Bond: $V_b = 0.866u\Sigma u^d$
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*Use .43 times the tabulated value for single bent bar

$v_c = 0.05t_c : 0.12f_c$ max $v$ for combined web reinforcement
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<th>( \frac{1}{4}d )</th>
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\[*Use .43 times the tabulated value for single bent bar\]

\( v_a = .05 t_c^2 : .12 t_c \) max v for combined web reinforcement
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<th>( f_{b6} )</th>
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<td>( f_{b2} )</td>
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<td>( f_{b4} )</td>
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<td>( 31 )</td>
<td>( t )</td>
<td>( )</td>
<td>( )</td>
<td>( )</td>
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<td>( )</td>
<td>( )</td>
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<td>( )</td>
</tr>
</tbody>
</table>

**Table 9-38f. Flexural Members, Properties for Design of Beam. \( b/D \) from 10 in. \times 24 in. to 22 in. \times 24 in.**

\[ V = V_c + V_s \]

**Size** | \( V_c \) | \( 3^\circ \) | \( 4^\circ \) | \( 5^\circ \) | \( 6^\circ \) | \( 8^\circ \) | \( 10^\circ \)
<table>
<thead>
<tr>
<th></th>
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<tr>
<td>3 in.</td>
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<td>( )</td>
<td>( )</td>
<td>( )</td>
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</tr>
<tr>
<td>4 in.</td>
<td>( )</td>
<td>( )</td>
<td>( )</td>
<td>( )</td>
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<td>( )</td>
</tr>
<tr>
<td>5 in.</td>
<td>( )</td>
<td>( )</td>
<td>( )</td>
<td>( )</td>
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<td>( )</td>
</tr>
<tr>
<td>6 in.</td>
<td>( )</td>
<td>( )</td>
<td>( )</td>
<td>( )</td>
<td>( )</td>
<td>( )</td>
</tr>
<tr>
<td>8 in.</td>
<td>( )</td>
<td>( )</td>
<td>( )</td>
<td>( )</td>
<td>( )</td>
<td>( )</td>
</tr>
<tr>
<td>10 in.</td>
<td>( )</td>
<td>( )</td>
<td>( )</td>
<td>( )</td>
<td>( )</td>
<td>( )</td>
</tr>
</tbody>
</table>

**A_s** | \( d \) | \( S_c \) | \( A_s \) |
<table>
<thead>
<tr>
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</tr>
</thead>
<tbody>
<tr>
<td>2000</td>
<td>2500</td>
<td>3000</td>
</tr>
</tbody>
</table>

**Bond: \( V_s = .666 \mu E_{o}d \)**

**Top bars** | **Bottom bars**
<table>
<thead>
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</thead>
<tbody>
<tr>
<td>140</td>
<td>175</td>
</tr>
</tbody>
</table>

**Notes:**
- Use .43 times the tabulated value for single bent bar.
- \( \mu = .05 f_c : .12 f_c \) max \( v \) for combined web reinforcement.
Table 9-38g. Flexural Members, Properties for Design of Beam. b/D from 12 in. × 26 in. to 24 in. × 26 in.

<table>
<thead>
<tr>
<th>Rectangular beams</th>
<th>T beams</th>
<th>*V₀</th>
<th>(\frac{1}{4}d)</th>
<th>(V) = (V₀ + Vₜ)</th>
</tr>
</thead>
<tbody>
<tr>
<td>b'xD</td>
<td>(f_c)</td>
<td>.45</td>
<td>t</td>
<td>12&quot;</td>
</tr>
<tr>
<td></td>
<td>k</td>
<td>.403</td>
<td>t</td>
<td>12&quot;</td>
</tr>
<tr>
<td></td>
<td>(I_c)</td>
<td>33.8</td>
<td>t</td>
<td></td>
</tr>
<tr>
<td>12&quot;x26&quot;</td>
<td>(I_t)</td>
<td>10912</td>
<td>4&quot;</td>
<td>6639</td>
</tr>
<tr>
<td></td>
<td>Sₜ</td>
<td>1153</td>
<td>4&quot;</td>
<td>748</td>
</tr>
<tr>
<td>325#/&quot;</td>
<td>nAₕ</td>
<td>38.4</td>
<td>4½&quot;</td>
<td>25.1</td>
</tr>
<tr>
<td>17576</td>
<td>d</td>
<td>23½&quot;</td>
<td>22&quot;</td>
<td>113</td>
</tr>
<tr>
<td>14&quot;x26&quot;</td>
<td>(I_t)</td>
<td>12702</td>
<td>5&quot;</td>
<td>7148</td>
</tr>
<tr>
<td></td>
<td>Sₜ</td>
<td>1344</td>
<td>5&quot;</td>
<td>805</td>
</tr>
<tr>
<td>379#/&quot;</td>
<td>nAₕ</td>
<td>44.7</td>
<td>5½&quot;</td>
<td>273</td>
</tr>
<tr>
<td>20505</td>
<td>d</td>
<td>23½&quot;</td>
<td>22&quot;</td>
<td>113</td>
</tr>
<tr>
<td>16&quot;x26&quot;</td>
<td>(I_t)</td>
<td>14510</td>
<td>6&quot;</td>
<td>7555</td>
</tr>
<tr>
<td></td>
<td>Sₜ</td>
<td>1533</td>
<td>6&quot;</td>
<td>852</td>
</tr>
<tr>
<td>437#/&quot;</td>
<td>nAₕ</td>
<td>51.1</td>
<td>6½&quot;</td>
<td>29.1</td>
</tr>
<tr>
<td>28435</td>
<td>d</td>
<td>23½&quot;</td>
<td>22&quot;</td>
<td>113</td>
</tr>
<tr>
<td>18&quot;x26&quot;</td>
<td>(I_t)</td>
<td>16365</td>
<td>7&quot;</td>
<td>8226</td>
</tr>
<tr>
<td></td>
<td>Sₜ</td>
<td>1730</td>
<td>7&quot;</td>
<td>930</td>
</tr>
<tr>
<td>487#/&quot;</td>
<td>nAₕ</td>
<td>57.5</td>
<td>7½&quot;</td>
<td>32.2</td>
</tr>
<tr>
<td>26364</td>
<td>d</td>
<td>23½&quot;</td>
<td>22&quot;</td>
<td>113</td>
</tr>
<tr>
<td>20&quot;x26&quot;</td>
<td>(I_t)</td>
<td>18170</td>
<td>8&quot;</td>
<td>8663</td>
</tr>
<tr>
<td></td>
<td>Sₜ</td>
<td>1918</td>
<td>8&quot;</td>
<td>978</td>
</tr>
<tr>
<td>542#/&quot;</td>
<td>nAₕ</td>
<td>64.0</td>
<td>8½&quot;</td>
<td>34.3</td>
</tr>
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<td>29293</td>
<td>d</td>
<td>23½&quot;</td>
<td>22&quot;</td>
<td>113</td>
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<td>22&quot;x26&quot;</td>
<td>(I_t)</td>
<td>20000</td>
<td>9&quot;</td>
<td>8904</td>
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<td>Sₜ</td>
<td>2116</td>
<td>9&quot;</td>
<td>1005</td>
</tr>
<tr>
<td>596#/&quot;</td>
<td>nAₕ</td>
<td>70.4</td>
<td>9½&quot;</td>
<td>35.6</td>
</tr>
<tr>
<td>32222</td>
<td>d</td>
<td>23½&quot;</td>
<td>22&quot;</td>
<td>113</td>
</tr>
<tr>
<td>24&quot;x26&quot;</td>
<td>(I_t)</td>
<td>21776</td>
<td>10&quot;</td>
<td>9149</td>
</tr>
<tr>
<td></td>
<td>Sₜ</td>
<td>2300</td>
<td>10&quot;</td>
<td>1030</td>
</tr>
<tr>
<td>650#/&quot;</td>
<td>nAₕ</td>
<td>76.6</td>
<td>10½&quot;</td>
<td>37.0</td>
</tr>
<tr>
<td>35152</td>
<td>d</td>
<td>23½&quot;</td>
<td>22&quot;</td>
<td>113</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>(A_s) (d)</th>
<th>(S_{c}) (t_c)</th>
<th>(t_c)</th>
<th>(A_s)</th>
<th>Bond: (V_b = 0.666u\Sigma d)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(#6) 2.37</td>
<td>198 157 131 104</td>
<td>1044</td>
<td>140 175 210 245</td>
<td>200 250 300 350</td>
</tr>
<tr>
<td>(#7) 2.43</td>
<td>268 214 178 131</td>
<td>157 131 104</td>
<td>140 175 210 245</td>
<td>200 250 300 350</td>
</tr>
<tr>
<td>(#8) 2.50</td>
<td>348 278 235 184</td>
<td>178 131 104</td>
<td>140 175 210 245</td>
<td>200 250 300 350</td>
</tr>
<tr>
<td>(#9) 2.56</td>
<td>436 346 290 230</td>
<td>184 131 104</td>
<td>140 175 210 245</td>
<td>200 250 300 350</td>
</tr>
<tr>
<td>(#10) 2.63</td>
<td>545 433 361 287</td>
<td>194 131 104</td>
<td>140 175 210 245</td>
<td>200 250 300 350</td>
</tr>
</tbody>
</table>

\*Use 0.43 times the tabulated value for single bent bar
\(v_b = 0.05 f_c : 0.12 t_c \) max \(v\) for combined web reinforcement
### Table 9-38h. Flexural Members, Properties for Design of Beam. $b'D$ from 14 in. $\times$ 28 in. to 26 in. $\times$ 28 in.

<table>
<thead>
<tr>
<th>Rectangular beams</th>
<th>T beams</th>
<th>$V_a$</th>
<th>$V_c$</th>
<th>$V_{c\uparrow}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b'\times D$</td>
<td>$f_c$</td>
<td>.45$f_c'$</td>
<td>b</td>
<td>12&quot;</td>
</tr>
<tr>
<td>Weight</td>
<td>$k$</td>
<td>.403</td>
<td>k</td>
<td>.085</td>
</tr>
<tr>
<td>$I_c$</td>
<td>ad</td>
<td>36.8</td>
<td>t</td>
<td>96.1</td>
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<table>
<thead>
<tr>
<th>Size</th>
<th>$v_c$</th>
<th>3&quot;</th>
<th>4&quot;</th>
<th>5&quot;</th>
<th>6&quot;</th>
<th>8&quot;</th>
<th>10&quot;</th>
<th>12&quot;</th>
</tr>
</thead>
<tbody>
<tr>
<td>14&quot; $\times$ 28&quot;</td>
<td>13596</td>
<td>8174</td>
<td>60</td>
<td>.085</td>
<td>50</td>
<td>48</td>
<td>37</td>
<td>36</td>
</tr>
<tr>
<td>16&quot; $\times$ 28&quot;</td>
<td>18670</td>
<td>9395</td>
<td>90</td>
<td>75</td>
<td>73</td>
<td>64</td>
<td>58</td>
<td>50</td>
</tr>
<tr>
<td>18&quot; $\times$ 28&quot;</td>
<td>21019</td>
<td>10332</td>
<td>113</td>
<td>93</td>
<td>80</td>
<td>71</td>
<td>65</td>
<td>57</td>
</tr>
<tr>
<td>20&quot; $\times$ 28&quot;</td>
<td>23330</td>
<td>10998</td>
<td>60</td>
<td>110</td>
<td>85</td>
<td>71</td>
<td>65</td>
<td>60</td>
</tr>
<tr>
<td>22&quot; $\times$ 28&quot;</td>
<td>25680</td>
<td>11427</td>
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<td>89</td>
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<td>63</td>
<td>56</td>
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<tr>
<td>24&quot; $\times$ 28&quot;</td>
<td>28001</td>
<td>11625</td>
<td>60</td>
<td>110</td>
<td>95</td>
<td>86</td>
<td>80</td>
<td>73</td>
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<tr>
<td>26&quot; $\times$ 28&quot;</td>
<td>30330</td>
<td>11930</td>
<td>90</td>
<td>104</td>
<td>89</td>
<td>80</td>
<td>74</td>
<td>66</td>
</tr>
<tr>
<td>7583#/i</td>
<td>90.1</td>
<td>40.5</td>
<td>113</td>
<td>156</td>
<td>134</td>
<td>122</td>
<td>111</td>
<td>100</td>
</tr>
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</table>

### Bond: $V_b = 0.866u \sum_d$

<table>
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<tr>
<th>$A_s$</th>
<th>$d'$</th>
<th>$S_c$</th>
<th>$f_c'$</th>
<th>$A_s$</th>
<th>$u$</th>
<th>Top bars</th>
<th>Bottom bars</th>
</tr>
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<tbody>
<tr>
<td>1 #6</td>
<td>2.37</td>
<td>218</td>
<td>174</td>
<td>144</td>
<td>115</td>
<td>42</td>
<td>140</td>
</tr>
<tr>
<td>1 #7</td>
<td>2.45</td>
<td>296</td>
<td>236</td>
<td>198</td>
<td>156</td>
<td>.57</td>
<td>140</td>
</tr>
<tr>
<td>1 #8</td>
<td>2.50</td>
<td>386</td>
<td>310</td>
<td>257</td>
<td>204</td>
<td>.75</td>
<td>140</td>
</tr>
<tr>
<td>1 #9</td>
<td>2.56</td>
<td>485</td>
<td>388</td>
<td>322</td>
<td>256</td>
<td>.94</td>
<td>140</td>
</tr>
<tr>
<td>1 #10</td>
<td>2.63</td>
<td>610</td>
<td>488</td>
<td>405</td>
<td>322</td>
<td>1.19</td>
<td>140</td>
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<tr>
<td>1 #11</td>
<td>2.70</td>
<td>740</td>
<td>585</td>
<td>490</td>
<td>390</td>
<td>1.44</td>
<td>140</td>
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</table>

*Use .43 times the tabulated value for single bent bar

$v_a = .05 f_c' : 12 f_c'_{\text{max}}$ for combined web reinforcement
**Tables for Design of Reinforced Concrete**

<table>
<thead>
<tr>
<th>Rectangular beams</th>
<th>T beams</th>
<th>(V_u)</th>
<th>(f'c)</th>
<th>(1#6)</th>
<th>(1#7)</th>
<th>(1#8)</th>
<th>(1#9)</th>
<th>(1#10)</th>
<th>(1#11)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(b'\times D)</td>
<td>(f_c)</td>
<td>.452(f_c)</td>
<td>(b)</td>
<td>12&quot;</td>
<td>(\frac{1}{4}d)</td>
<td>(3&quot;)</td>
<td>(4&quot;)</td>
<td>(5&quot;)</td>
<td>(6&quot;)</td>
</tr>
<tr>
<td>Weight</td>
<td>(k)</td>
<td>.403</td>
<td>(k)</td>
<td>.403</td>
<td>(V = V_c + V_s)</td>
<td>(V_c)</td>
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<tr>
<td>(I_c)</td>
<td>(a_d)</td>
<td>37.4</td>
<td>(t)</td>
<td>(\Sigma)</td>
<td>(S_{c})</td>
<td>(A_s)</td>
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<td></td>
<td>(I_t)</td>
<td>19758</td>
<td>9825</td>
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<td>(#5)</td>
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<td>(#7)</td>
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<td>(500#/ft)</td>
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<td>(S_t)</td>
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<td>(n_{As})</td>
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<td>(90)</td>
<td>(87)</td>
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<td>(63)</td>
</tr>
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<td></td>
<td></td>
<td>(d)</td>
<td>26&quot;</td>
<td>(1403)</td>
<td>(#4)</td>
<td>(#5)</td>
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<td>(#7)</td>
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<td>1403</td>
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<td>(d)</td>
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<td>(1290)</td>
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<td>(#7)</td>
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<td>(S_t)</td>
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<td>(111)</td>
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<td>(117)</td>
<td>(104)</td>
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<td>(d)</td>
<td>26&quot;</td>
<td>(1423)</td>
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<td>(#6)</td>
<td>(#7)</td>
<td>(#8)</td>
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<td>(750#/ft)</td>
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<td></td>
<td>(S_t)</td>
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<td>84.8</td>
<td>40.2</td>
<td>(90)</td>
<td>(131)</td>
<td>(117)</td>
<td>(106)</td>
</tr>
<tr>
<td>26&quot;\times30&quot;</td>
<td></td>
<td></td>
<td>(d)</td>
<td>26&quot;</td>
<td>(1461)</td>
<td>(#5)</td>
<td>(#6)</td>
<td>(#7)</td>
<td>(#8)</td>
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<td>(750#/ft)</td>
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</tr>
<tr>
<td>28&quot;\times30&quot;</td>
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<td></td>
<td>(d)</td>
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<td>(1480)</td>
<td>(#5)</td>
<td>(#6)</td>
<td>(#7)</td>
<td>(#8)</td>
</tr>
<tr>
<td>(812#/ft)</td>
<td></td>
<td></td>
<td>(S_t)</td>
<td>3293</td>
<td>1480</td>
<td>(75)</td>
<td>(167)</td>
<td>(150)</td>
<td>(135)</td>
</tr>
<tr>
<td>63000</td>
<td></td>
<td></td>
<td>(n_{As})</td>
<td>99.0</td>
<td>42.3</td>
<td>(90)</td>
<td>(148)</td>
<td>(126)</td>
<td>(112)</td>
</tr>
</tbody>
</table>

*Use .43 times the tabulated value for single bent bar

\(v = .05f'_c : .12f'_c \) max \(v\) for combined web reinforcement
### Table 9-38j. Flexural Members, Properties for Design of Beam. $b/D$ from 18 in. × 32 in. to 30 in. × 32 in.

<table>
<thead>
<tr>
<th>Rectangular beams</th>
<th>T beams</th>
<th>$V_c = V_c + V_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b \times D$</td>
<td>$I_c$</td>
<td>$t$</td>
</tr>
<tr>
<td>Weight</td>
<td>$k$</td>
<td>$k$</td>
</tr>
<tr>
<td>$I_c$</td>
<td>$40.3$</td>
<td>$t$</td>
</tr>
<tr>
<td>18&quot; x 32&quot;</td>
<td>$I_t$</td>
<td>11534</td>
</tr>
<tr>
<td>(600 #/l)</td>
<td>$n_A$</td>
<td>68.5</td>
</tr>
<tr>
<td>49153 d 28&quot;</td>
<td>$S_t$</td>
<td>1025</td>
</tr>
<tr>
<td>20&quot; x 32&quot;</td>
<td>$S_t$</td>
<td>13507</td>
</tr>
<tr>
<td>(667 #/l)</td>
<td>$n_A$</td>
<td>76.3</td>
</tr>
<tr>
<td>54613 d 28&quot;</td>
<td>$S_t$</td>
<td>1190</td>
</tr>
<tr>
<td>22&quot; x 32&quot;</td>
<td>$I_t$</td>
<td>15091</td>
</tr>
<tr>
<td>(734 #/l)</td>
<td>$n_A$</td>
<td>3135</td>
</tr>
<tr>
<td>60075 d 28&quot;</td>
<td>$S_t$</td>
<td>83.8</td>
</tr>
<tr>
<td>24&quot; x 32&quot;</td>
<td>$I_t$</td>
<td>62888</td>
</tr>
<tr>
<td>(800 #/l)</td>
<td>$n_A$</td>
<td>91.4</td>
</tr>
<tr>
<td>65536 d 28&quot;</td>
<td>$S_t$</td>
<td>16288</td>
</tr>
<tr>
<td>26&quot; x 32&quot;</td>
<td>$I_t$</td>
<td>17217</td>
</tr>
<tr>
<td>(867 #/l)</td>
<td>$n_A$</td>
<td>3558</td>
</tr>
<tr>
<td>70947 d 28&quot;</td>
<td>$S_t$</td>
<td>4445</td>
</tr>
<tr>
<td>28&quot; x 32&quot;</td>
<td>$I_t$</td>
<td>18697</td>
</tr>
<tr>
<td>(933 #/l)</td>
<td>$n_A$</td>
<td>99.0</td>
</tr>
<tr>
<td>76458 d 28&quot;</td>
<td>$S_t$</td>
<td>43139</td>
</tr>
<tr>
<td>30&quot; x 32&quot;</td>
<td>$I_t$</td>
<td>17869</td>
</tr>
<tr>
<td>(1000 #/l)</td>
<td>$n_A$</td>
<td>3830</td>
</tr>
<tr>
<td>81920 d 28&quot;</td>
<td>$S_t$</td>
<td>46120</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$A_s$</th>
<th>$d'$</th>
<th>$S_c$</th>
<th>$A_s$</th>
<th>$u$</th>
<th>Bond: $V_s = .866uS_c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 #6</td>
<td>2.37</td>
<td>254</td>
<td>168</td>
<td>.134</td>
<td>140</td>
</tr>
<tr>
<td>1 #7</td>
<td>2.43</td>
<td>344</td>
<td>274</td>
<td>181</td>
<td>.57</td>
</tr>
<tr>
<td>1 #8</td>
<td>2.50</td>
<td>454</td>
<td>362</td>
<td>200</td>
<td>.75</td>
</tr>
<tr>
<td>1 #9</td>
<td>2.56</td>
<td>572</td>
<td>457</td>
<td>378</td>
<td>.95</td>
</tr>
<tr>
<td>1 #10</td>
<td>2.63</td>
<td>720</td>
<td>577</td>
<td>478</td>
<td>1.20</td>
</tr>
<tr>
<td>1 #11</td>
<td>2.70</td>
<td>887</td>
<td>708</td>
<td>589</td>
<td>1.48</td>
</tr>
</tbody>
</table>

*Use $1.45$ times the tabulated value for single bent bar

$v_o = .05I_t : .12I_c$ max $v$ for combined web reinforcement

**Note:** The table provides properties for designing beams with rectangular and T-sections, including moment of inertia ($I_c$), section modulus ($S_c$), and area ($A_s$) for various beam dimensions. The table also includes information on the spacing and size of reinforcing bars ($V_c$), with dimensions for different beam sizes ranging from 18" x 32" to 30" x 32".
### TABLES FOR DESIGN OF REINFORCED CONCRETE

**Table 9-38k. Flexural Members, Properties for Design of Beam.**

*d'D from 18 in. × 34 in. to 30 in. × 34 in.*

<table>
<thead>
<tr>
<th>Rectangular beams</th>
<th>T beams-</th>
<th>*V₀₂</th>
<th>Iₘ</th>
<th>t</th>
<th>1/6</th>
<th>1/7</th>
<th>1/8</th>
<th>1/9</th>
<th>1/10</th>
<th>1/11</th>
</tr>
</thead>
<tbody>
<tr>
<td>b'×D fₐ</td>
<td>.45fₐ</td>
<td>b</td>
<td>12&quot;</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Weight</td>
<td>k</td>
<td>.403</td>
<td>k</td>
<td>.403</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Iₐ</td>
<td>ad</td>
<td>42.2</td>
<td>t</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| 18"×34" | 30930 | 12454 | 20000 | 14.5 | 19.8 | 26.0 | 33.0 | 42.0 | 50.2 |
| (640#/) | 2620 | 1056 | 2500 | 14.5 | 19.8 | 26.0 | 33.0 | 42.0 | 51.5 |
| 5956 | d | 29/4" | 90 | 110 | 105 | 95 | 82 | 74 | 69 | 67 |
| 20"×34" | 35343 | 15743 | 60 | 84 | 63 | 58 | 56 | 31 |
| (710#/) | 2996 | 1330 | 75 | 105 | 102 | 91 | 78 | 71 | 65 | 63 | 39 |
| 6506 | d | 29/4" | 90 | 118 | 110 | 99 | 86 | 79 | 73 | 71 | 47 |
| 22"×34" | 38842 | 17715 | 75 | 105 | 102 | 91 | 78 | 71 | 65 | 63 | 39 |
| (780#/) | 3295 | 1505 | 1358 | 138 | 122 | 111 | 98 | 90 | 85 | 83 | 59 |
| 72057 | d | 29/4" | 90 | 138 | 130 | 115 | 104 | 91 | 83 | 76 | 52 |
| 24"×34" | 42356 | 19349 | 75 | 126 | 110 | 100 | 86 | 78 | 73 | 71 | 47 |
| (850#/) | 3591 | 1640 | 90 | 151 | 135 | 119 | 109 | 96 | 88 | 83 | 61 |
| 7806 | d | 29/4" | 90 | 136 | 130 | 114 | 103 | 90 | 83 | 77 | 51 |
| 26"×34" | 45945 | 20697 | 75 | 130 | 116 | 103 | 93 | 80 | 72 | 67 | 65 |
| (920#/) | 3897 | 1750 | 113 | 176 | 149 | 134 | 123 | 110 | 102 | 97 | 71 |
| 8518 | d | 29/4" | 90 | 164 | 140 | 124 | 114 | 101 | 93 | 88 | 61 |
| 28"×34" | 49507 | 21897 | 75 | 146 | 133 | 118 | 107 | 94 | 86 | 81 | 61 |
| (990#/) | 4208 | 1855 | 115 | 188 | 161 | 146 | 135 | 122 | 114 | 109 | 83 |
| 9179 | d | 29/4" | 90 | 171 | 149 | 129 | 118 | 105 | 97 | 92 | 66 |
| 30"×34" | 53010 | 22746 | 75 | 157 | 137 | 122 | 111 | 98 | 90 | 85 | 69 |
| (1060#/) | 4500 | 1925 | 115 | 194 | 167 | 152 | 141 | 128 | 120 | 115 | 89 |
| 98260 | d | 29/4" | 90 | 175 | 149 | 133 | 123 | 110 | 102 | 97 | 73 |

#### Size of Spacing

- 3" Spacing
- 4" Spacing
- 5" Spacing
- 6" Spacing
- 8" Spacing
- 10" Spacing
- 12" Spacing
- 14" Spacing

#### Band: \( V_{₀₂} = 0.866uS_{₀}d \)

<table>
<thead>
<tr>
<th>Aₙ</th>
<th>d'</th>
<th>fₐ</th>
<th>Aₙ</th>
<th>u</th>
<th>Top bars</th>
<th>Bottom bars</th>
</tr>
</thead>
<tbody>
<tr>
<td>2000</td>
<td>2500</td>
<td>3000</td>
<td>3750</td>
<td>140</td>
<td>175</td>
<td>210</td>
</tr>
<tr>
<td>1 #6</td>
<td>2.50</td>
<td>256</td>
<td>205</td>
<td>171</td>
<td>136</td>
<td>.42</td>
</tr>
<tr>
<td>1 #7</td>
<td>2.56</td>
<td>348</td>
<td>279</td>
<td>232</td>
<td>184</td>
<td>.57</td>
</tr>
<tr>
<td>1 #8</td>
<td>2.62</td>
<td>458</td>
<td>367</td>
<td>304</td>
<td>242</td>
<td>.75</td>
</tr>
<tr>
<td>1 #9</td>
<td>2.69</td>
<td>580</td>
<td>462</td>
<td>395</td>
<td>303</td>
<td>.95</td>
</tr>
<tr>
<td>1 #10</td>
<td>2.76</td>
<td>730</td>
<td>584</td>
<td>484</td>
<td>386</td>
<td>1.20</td>
</tr>
<tr>
<td>1 #11</td>
<td>2.83</td>
<td>929</td>
<td>741</td>
<td>615</td>
<td>491</td>
<td>1.48</td>
</tr>
</tbody>
</table>

*Use .43 times the tabulated value for single bent bar
\( V_{₀₂} = 0.05f_{c}' \) : .12fₐ max v for combined web reinforcement
### Table 9-38. Flexural Members, Properties for Design of Beam. \( b'/D \) from 18 in. × 36 in. to 30 in. × 36 in.

<table>
<thead>
<tr>
<th>Rectangular beams</th>
<th>T beams</th>
<th>( \mathbf{V_0} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( b' \times D )</td>
<td>( f_c ) .45( f'_c )</td>
<td>( b ) 12( ^\circ )</td>
</tr>
<tr>
<td>Weight</td>
<td>( k ) .403 ( k ) .403</td>
<td></td>
</tr>
<tr>
<td>( I_c )</td>
<td>( a_d ) 45.0 ( t )</td>
<td></td>
</tr>
</tbody>
</table>

**Sizes**

<table>
<thead>
<tr>
<th>( 3&quot; )</th>
<th>( 4&quot; )</th>
<th>( 5&quot; )</th>
<th>( 6&quot; )</th>
<th>( 8&quot; )</th>
<th>( 10&quot; )</th>
<th>( 12&quot; )</th>
<th>( 14&quot; )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( v_c )</td>
<td>( v_c )</td>
<td>( v_c )</td>
<td>( v_c )</td>
<td>( v_c )</td>
<td>( v_c )</td>
<td>( v_c )</td>
<td>( v_c )</td>
</tr>
</tbody>
</table>

**Spacing**

<table>
<thead>
<tr>
<th>( v_c )</th>
<th>( v_c )</th>
<th>( v_c )</th>
<th>( v_c )</th>
<th>( v_c )</th>
<th>( v_c )</th>
<th>( v_c )</th>
<th>( v_c )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V ) ( = ) ( v_c ) ( + ) ( v_s )</td>
<td>( V ) ( = ) ( v_c ) ( + ) ( v_s )</td>
<td>( V ) ( = ) ( v_c ) ( + ) ( v_s )</td>
<td>( V ) ( = ) ( v_c ) ( + ) ( v_s )</td>
<td>( V ) ( = ) ( v_c ) ( + ) ( v_s )</td>
<td>( V ) ( = ) ( v_c ) ( + ) ( v_s )</td>
<td>( V ) ( = ) ( v_c ) ( + ) ( v_s )</td>
<td>( V ) ( = ) ( v_c ) ( + ) ( v_s )</td>
</tr>
</tbody>
</table>

*Use .43 times the tabulated value for single bent bar

\( v_s = .05f'_c : .12f'_c \) max \( v \) for combined web reinforcement
<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>T4</td>
<td>T6</td>
<td>T8</td>
<td>T10</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>T12</td>
<td>T14</td>
<td>T16</td>
<td>R6 to R16</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S6</td>
<td>S7</td>
<td>S8</td>
<td>S9</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ST 10</td>
<td>ST 11</td>
<td>ST 12</td>
<td>ST 13</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ST 14</td>
<td>ST 15</td>
<td>ST 16</td>
<td>ST 17</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ST 18</td>
<td>ST 19</td>
<td>ST 20</td>
<td>C10 to C20</td>
</tr>
</tbody>
</table>
Table 9-40. Double Spiral Columns with Ties, Typical Details

<table>
<thead>
<tr>
<th>Number</th>
<th>Diagram</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 ST 17</td>
<td><img src="image" alt="Diagram 2 ST 17" /></td>
</tr>
<tr>
<td>2 ST 16</td>
<td><img src="image" alt="Diagram 2 ST 16" /></td>
</tr>
<tr>
<td>SST 13-5</td>
<td><img src="image" alt="Diagram SST 13-5" /></td>
</tr>
<tr>
<td>SST 14-6</td>
<td><img src="image" alt="Diagram SST 14-6" /></td>
</tr>
<tr>
<td>SST 15-7</td>
<td><img src="image" alt="Diagram SST 15-7" /></td>
</tr>
<tr>
<td>SST 16-8</td>
<td><img src="image" alt="Diagram SST 16-8" /></td>
</tr>
<tr>
<td>SST 18-9</td>
<td><img src="image" alt="Diagram SST 18-9" /></td>
</tr>
<tr>
<td>SST 20-10</td>
<td><img src="image" alt="Diagram SST 20-10" /></td>
</tr>
</tbody>
</table>
Table 9-41. Reinforcing for Columns, Area of Vertical Bars

<table>
<thead>
<tr>
<th>Size</th>
<th>#6</th>
<th>#7</th>
<th>#8</th>
<th>#9</th>
<th>#10</th>
<th>#11</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 bar</td>
<td>0.44</td>
<td>0.60</td>
<td>0.79</td>
<td>1.00</td>
<td>1.27</td>
<td>1.56</td>
</tr>
<tr>
<td>d'-Tied col.</td>
<td>2.25</td>
<td>2.31</td>
<td>2.37</td>
<td>2.44</td>
<td>2.51</td>
<td>2.58</td>
</tr>
<tr>
<td>d'-Spiral col.</td>
<td>2.50</td>
<td>2.56</td>
<td>2.62</td>
<td>2.69</td>
<td>2.76</td>
<td>2.83</td>
</tr>
<tr>
<td>4 bars</td>
<td>1.76</td>
<td>2.40</td>
<td>3.16</td>
<td>4.00</td>
<td>5.08</td>
<td>6.24</td>
</tr>
<tr>
<td>6 bars</td>
<td>2.64</td>
<td>3.60</td>
<td>4.74</td>
<td>6.00</td>
<td>7.62</td>
<td>9.36</td>
</tr>
<tr>
<td>7 bars</td>
<td>3.08</td>
<td>4.20</td>
<td>5.53</td>
<td>7.00</td>
<td>8.89</td>
<td>10.92</td>
</tr>
<tr>
<td>8 bars</td>
<td>3.52</td>
<td>4.80</td>
<td>6.32</td>
<td>8.00</td>
<td>10.16</td>
<td>12.48</td>
</tr>
<tr>
<td>9 bars</td>
<td>3.96</td>
<td>5.40</td>
<td>7.11</td>
<td>9.00</td>
<td>11.42</td>
<td>14.04</td>
</tr>
<tr>
<td>10 bars</td>
<td>4.40</td>
<td>6.00</td>
<td>7.90</td>
<td>10.00</td>
<td>12.70</td>
<td>15.60</td>
</tr>
<tr>
<td>11 bars</td>
<td>4.84</td>
<td>6.60</td>
<td>8.69</td>
<td>11.00</td>
<td>13.97</td>
<td>17.16</td>
</tr>
<tr>
<td>12 bars</td>
<td>5.28</td>
<td>7.20</td>
<td>9.48</td>
<td>12.00</td>
<td>15.24</td>
<td>18.72</td>
</tr>
<tr>
<td>13 bars</td>
<td>5.72</td>
<td>7.80</td>
<td>10.27</td>
<td>13.00</td>
<td>16.51</td>
<td>20.28</td>
</tr>
<tr>
<td>14 bars</td>
<td>6.16</td>
<td>8.40</td>
<td>11.06</td>
<td>14.00</td>
<td>17.78</td>
<td>21.84</td>
</tr>
<tr>
<td>15 bars</td>
<td>6.60</td>
<td>9.00</td>
<td>11.85</td>
<td>15.00</td>
<td>19.05</td>
<td>23.40</td>
</tr>
<tr>
<td>16 bars</td>
<td>7.04</td>
<td>9.60</td>
<td>12.64</td>
<td>16.00</td>
<td>20.32</td>
<td>24.96</td>
</tr>
<tr>
<td>17 bars</td>
<td>7.48</td>
<td>10.20</td>
<td>13.43</td>
<td>17.00</td>
<td>21.59</td>
<td>26.52</td>
</tr>
<tr>
<td>18 bars</td>
<td>7.92</td>
<td>10.80</td>
<td>14.22</td>
<td>18.00</td>
<td>22.86</td>
<td>28.08</td>
</tr>
<tr>
<td>19 bars</td>
<td>8.36</td>
<td>11.40</td>
<td>15.01</td>
<td>19.00</td>
<td>24.13</td>
<td>29.64</td>
</tr>
<tr>
<td>20 bars</td>
<td>8.80</td>
<td>12.00</td>
<td>15.80</td>
<td>20.00</td>
<td>25.40</td>
<td>31.20</td>
</tr>
</tbody>
</table>
### Table 9-42. Spiral Reinforcement for Columns

<table>
<thead>
<tr>
<th>Col size</th>
<th>Core dia.</th>
<th>Hot rolled spiral</th>
<th>Col diam</th>
<th>Core dia.</th>
<th>Hot rolled spiral</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>2000</td>
<td>2500</td>
<td>3000</td>
</tr>
<tr>
<td>14&quot;</td>
<td>11&quot;</td>
<td>3at1 (\frac{1}{8})</td>
<td>14&quot;</td>
<td>11&quot;</td>
<td>3at1 (\frac{1}{8})</td>
</tr>
<tr>
<td>16&quot;</td>
<td>13&quot;</td>
<td>4at2, 4at2</td>
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<td>4at2(\frac{3}{4})</td>
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<td>23&quot;</td>
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<tr>
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<td>26&quot;</td>
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<td>28&quot;</td>
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<td>30&quot;</td>
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Table 9-33c. Square, Rectangular, and Round Columns, Values of $P$ and $N$ with $f'_c = 2,000$ psi. 12-in. and 14-in. Square Columns, Tied and Spiral Reinforcing

<table>
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<th>$N$</th>
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<td></td>
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<td>P</td>
</tr>
<tr>
<td>No.</td>
<td>Size</td>
<td>$d'$</td>
</tr>
<tr>
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<td>3/8</td>
</tr>
<tr>
<td></td>
<td>#7</td>
<td>3/4</td>
</tr>
<tr>
<td></td>
<td>#8</td>
<td>1/2</td>
</tr>
<tr>
<td></td>
<td>#9</td>
<td>3/8</td>
</tr>
<tr>
<td></td>
<td>#10</td>
<td>1/2</td>
</tr>
<tr>
<td></td>
<td>#11</td>
<td>3/8</td>
</tr>
</tbody>
</table>

| S-6 | #7 | 3/8 | 2.37" | 242 | 101 | 2.59 | 108 | 19.3 | 16.3 | 14.1 | 12.5 | 11.4 | 10.2 |
|      | #8 | 1/2 | 2.44" | 260 | 121 | 2.87 | 134 | 23.9 | 20.4 | 17.8 | 15.9 | 14.3 | 12.9 |
|      | #9 | 3/8 | 2.51" | 273 | 135 | 2.97 | 151 | 26.4 | 22.9 | 19.9 | 17.8 | 15.9 | 14.4 |
|      | #10 | 1/2 | 2.58" | 286 | 150 | 3.25 | 170 | 28.8 | 25.1 | 22.0 | 19.7 | 17.9 | 16.9 |
| S-7 | #6 | 3/8 | 2.37" | 329 | 129 | 3.67 | 141 | 15.9 | 13.5 | 11.7 | 10.2 | 9.1 | 7.0 |
|      | #7 | 1/2 | 2.44" | 345 | 145 | 4.03 | 160 | 18.1 | 15.2 | 12.9 | 11.5 | 10.2 | 9.0 |
|      | #8 | 3/8 | 2.50" | 396 | 164 | 4.42 | 183 | 20.1 | 17.0 | 14.2 | 12.7 | 11.4 | 10.2 |
|      | #9 | 1/2 | 2.56" | 430 | 184 | 4.85 | 208 | 21.7 | 18.3 | 15.9 | 13.8 | 12.2 | 11.1 |
|      | #10 | 3/8 | 2.63" | 468 | 210 | 5.36 | 240 | 24.4 | 20.2 | 17.6 | 15.6 | 13.7 | 12.3 |
| S-8 | #6 | 3/8 | 2.37" | 351 | 136 | 3.74 | 149 | 17.0 | 14.4 | 12.4 | 11.0 | 9.7 | 8.8 |
|      | #7 | 1/2 | 2.44" | 380 | 155 | 4.23 | 172 | 19.1 | 16.2 | 13.9 | 12.2 | 10.9 | 9.8 |
|      | #8 | 3/8 | 2.50" | 415 | 176 | 4.67 | 198 | 21.1 | 18.0 | 15.3 | 13.4 | 12.0 | 10.8 |
|      | #9 | 1/2 | 2.56" | 452 | 200 | 5.14 | 228 | 23.4 | 19.8 | 16.9 | 15.0 | 13.5 | 12.1 |
| S-9 | #6 | 3/8 | 2.37" | 360 | 143 | 3.97 | 158 | 18.3 | 15.3 | 13.2 | 11.6 | 10.2 | 9.4 |
|      | #7 | 1/2 | 2.44" | 396 | 165 | 4.42 | 184 | 20.4 | 17.0 | 14.7 | 12.9 | 11.5 | 10.6 |
|      | #8 | 3/8 | 2.50" | 433 | 189 | 4.91 | 214 | 22.8 | 19.1 | 16.5 | 14.5 | 13.0 | 11.6 |
|      | #9 | 1/2 | 2.56" | 470 | 214 | 5.39 | 248 | 25.2 | 20.9 | 17.9 | 15.5 | 13.5 | 12.0 |

Spiral $f'_c = 40,000$ psi $f'_c = 60,000$ psi $#4 at 3 in. oc$ $#3 at 2 1/4 in. oc$
<table>
<thead>
<tr>
<th>No.</th>
<th>Size</th>
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<th>( d' )</th>
<th>( e/t &lt; 1.0 )</th>
<th>( N )</th>
<th>( e/t )</th>
</tr>
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<tbody>
<tr>
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<td>12 CD ( t ) P</td>
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<td>2.37&quot;</td>
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<td>3.34</td>
<td>110</td>
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<tr>
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<td>.024</td>
<td>2.44&quot;</td>
<td>3.28</td>
<td>111</td>
<td>3.66</td>
<td>124</td>
</tr>
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<td>2.51&quot;</td>
<td>3.44</td>
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<td>3.64</td>
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</table>

\[ b \times t = 14'' \times 12'' \]
\[ A_g = 168 \text{ sq in.} \]
\[ \text{Weight} = 175 \text{ lb/ft} \]
\[ f_c' = 2000 \text{ psi} \]
\[ I_c = 2016 \text{ in}^3 \]

<table>
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<th>No.</th>
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<th>( d' )</th>
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<tr>
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<td>2.31&quot;</td>
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<td>98</td>
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<tr>
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<td>2.37&quot;</td>
<td>2.54</td>
<td>100</td>
<td>2.80</td>
<td>110</td>
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<td>2.44&quot;</td>
<td>2.66</td>
<td>111</td>
<td>2.96</td>
<td>124</td>
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<tr>
<td>#10</td>
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<td>2.94</td>
<td>140</td>
<td>3.36</td>
<td>160</td>
</tr>
</tbody>
</table>

\[ b \times t = 12'' \times 14'' \]
\[ A_g = 168 \text{ sq in.} \]
\[ \text{Weight} = 175 \text{ lb/ft} \]
\[ f_c' = 2000 \text{ psi} \]
\[ I_c = 2746 \text{ in}^3 \]

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</table>

\[ b \times t = 16'' \times 14'' \]
\[ A_g = 224 \text{ sq in.} \]
\[ \text{Weight} = 233 \text{ lb/ft} \]
\[ f_c' = 2000 \text{ psi} \]
\[ I_c = 3660 \text{ in}^3 \]

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Table 9-43c. Square, Rectangular, and Round Columns, Values of $P$ and $N$ with $f_{c'} = 2000$ psi.  16-in. Square and Rectangular Columns, Tied and Spiral Reinforcing

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<td>$12 \frac{CD}{T}$</td>
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</tbody>
</table>

$A_g = 256$ sq in.  $I_g = 5440$ in.  $f_c = 2000$ psi

**T-4**  
$A_g = 400$ sq in.  $I_g = 8500$ in.  $f_c = 2000$ psi

**T-8**  
$A_g = 25$ sq in.  $I_g = 8500$ in.  $f_c = 2000$ psi

**ST-10**  
$A_g = 16$ sq in.  $I_g = 8500$ in.  $f_c = 2000$ psi

Spiral  
$f_c' = 40,000$ psi  
$#4$ at 2 3/4 in. oc  
$t_{c'} = 60,000$ psi  
$#3$ at 2 1/4 in. oc
### Table 9-43d. Square, Rectangular, and Round Columns, Values of $P$ and $N$ with $f'_c = 2,000$ psi. 16-in. Rectangular Columns, Tied Reinforcing

<table>
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TABLES FOR DESIGN OF REINFORCED CONCRETE

9-143

Table 9-43e. Square, Rectangular, and Round Columns, Values of \( P \) and \( N \) with \( f_{c}' = 2,000 \) psi. 18-in. Square and Rectangular Columns, Tied and Spiral Reinforcing

<table>
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<th>( \varepsilon_{\text{fl}} )</th>
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</tr>
<tr>
<td>T-8</td>
<td>#8</td>
<td>0.19</td>
</tr>
<tr>
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<td>#9</td>
<td>0.25</td>
</tr>
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<td>0.31</td>
</tr>
<tr>
<td></td>
<td>#11</td>
<td>0.38</td>
</tr>
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<td>S-6</td>
<td>#9</td>
<td>0.18</td>
</tr>
<tr>
<td></td>
<td>#10</td>
<td>0.23</td>
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<tr>
<td></td>
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<td>S-7</td>
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<td></td>
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<tr>
<td>ST-10</td>
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</tr>
<tr>
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<td>#9</td>
<td>0.31</td>
</tr>
<tr>
<td></td>
<td>#11</td>
<td>0.39</td>
</tr>
</tbody>
</table>

\[
\frac{b \times t = 28" \times 18"}{A_g = 504 \text{ sq in.}} \quad \frac{f_{c}' = 2,000 \text{ psi}}{I_c = 13,600 \text{ in}^4}
\]

| 2ST-14 | #8 | 0.022 | 2.62" | 2.75 | 404 | 3.05 | 448 | 59.3 | 49.8 | 43.2 | 38.1 | 34.2 | 30.9 |
|  | #9 | 0.028 | 2.69" | 2.91 | 451 | 3.27 | 507 | 65.5 | 55.3 | 47.8 | 42.3 | 37.7 | 34.1 |
|  | #10 | 0.035 | 2.76" | 3.07 | 511 | 3.50 | 582 | 72.2 | 62.0 | 53.8 | 47.3 | 42.6 | 38.4 |

\[
\frac{b \times t = 30" \times 18"}{A_g = 540 \text{ sq in.}} \quad \frac{f_{c}' = 2,500 \text{ psi}}{I_c = 14,600 \text{ in}^4}
\]

| 2ST-17 | #8 | 0.025 | 2.62" | 2.77 | 458 | 3.10 | 512 | 66.0 | 55.5 | 48.3 | 42.6 | 38.2 | 34.6 |
|  | #9 | 0.031 | 2.69" | 3.02 | 515 | 3.43 | 583 | 73.6 | 62.4 | 54.0 | 47.7 | 42.6 | 38.5 |
|  | #10 | 0.040 | 2.76" | 3.22 | 588 | 3.69 | 675 | 82.6 | 69.7 | 60.5 | 53.5 | 47.7 | 43.1 |

| Spiral | \( f_s = 40,000 \) psi | #4 at 2 3/4 in. o.c | \( f_s = 60,000 \) psi | #3 at 2 1/4 in. o.c |
**REINFORCED-CONCRETE BUILDING FRAMES**

Table 9-43f. Square, Rectangular, and Round Columns, Values of $P$ and $N$ with $f'_{c} = 2,000$ psi. 18-in. Rectangular Columns, Tied Reinforcing

<table>
<thead>
<tr>
<th>Reinforcing</th>
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<th>$N$</th>
<th>$e/t$</th>
</tr>
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<tr>
<td>No.</td>
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<td>$f'_{c} = 20,000$</td>
<td>$f'_{c} = 2000$ psi</td>
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<td>$12 \frac{D}{T}$</td>
<td>$12 \frac{D}{T}$</td>
<td>$12 \frac{D}{T}$</td>
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<tr>
<td>b x t = 14&quot; x 18&quot;</td>
<td>$A_{g} = 252$ sq in.</td>
<td>Weight = 262 lb/ft</td>
<td>$I_{C} = 6800$ in$^{4}$</td>
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<tr>
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<td>1.74</td>
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<td>.016</td>
<td>2.44</td>
<td>1.82</td>
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<td>1.49</td>
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<tr>
<td># 8</td>
<td>.016</td>
<td>2.37</td>
<td>1.83</td>
</tr>
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<td>2.44</td>
<td>1.88</td>
</tr>
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<td># 10</td>
<td>.026</td>
<td>2.51</td>
<td>1.94</td>
</tr>
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<td>2.58</td>
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</tr>
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<td>2.07</td>
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<td>2.09</td>
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<td># 10</td>
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<td>2.51</td>
<td>2.23</td>
</tr>
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<td>2.44</td>
<td>2.06</td>
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<td>2.51</td>
<td>2.14</td>
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<tr>
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<td>2.58</td>
<td>2.25</td>
</tr>
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<td>T-10</td>
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<tr>
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<tr>
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<td>.025</td>
<td>2.44</td>
<td>2.08</td>
</tr>
<tr>
<td># 10</td>
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<td>2.18</td>
</tr>
<tr>
<td># 11</td>
<td>.039</td>
<td>2.58</td>
<td>2.30</td>
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</table>
### TABLES FOR DESIGN OF REINFORCED CONCRETE

**Table 9-43g. Square, Rectangular, and Round Columns, Values of \( P \) and \( N \) with \( f'_c = 2000 \) psi. 20-in. Square and Rectangular Columns, Tied and Spiral Reinforcing**

\[
\frac{b \times t = 20'' \times 20''}{A_g = 400 \text{ sq in.}} \quad \text{Weight = 417 lb/ft} \quad \frac{f'_c = 2000 \text{ psi}}{I_c = 13,300 \text{ in.}^4}
\]

<table>
<thead>
<tr>
<th>No.</th>
<th>Size ( p_g )</th>
<th>( d' )</th>
<th>( e/t &lt; 1.0 )</th>
<th>( N )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>( f_s = 12,000 )</td>
<td>( f_s = 20,000 )</td>
</tr>
<tr>
<td>12( \frac{\text{CD}}{T} )</td>
<td>( P )</td>
<td>( 12\frac{\text{CD}}{T} )</td>
<td>( P )</td>
<td></td>
</tr>
<tr>
<td>T-8</td>
<td># 9 0.020 2.44&quot;</td>
<td>1.81 246</td>
<td>1.99 272</td>
<td>49.4 42.2</td>
</tr>
<tr>
<td></td>
<td># 10 0.025 2.51&quot;</td>
<td>1.88 274</td>
<td>2.11 306</td>
<td>54.4 46.5</td>
</tr>
<tr>
<td></td>
<td># 11 0.034 2.58&quot;</td>
<td>1.97 304</td>
<td>2.23 344</td>
<td>64.0 51.6</td>
</tr>
<tr>
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<td># 9 0.025 2.44&quot;</td>
<td>1.85 272</td>
<td>2.04 304</td>
<td>56.1 48.3</td>
</tr>
<tr>
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<td># 10 0.032 2.51&quot;</td>
<td>1.91 306</td>
<td>2.17 347</td>
<td>63.3 54.5</td>
</tr>
<tr>
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<td>1.98 344</td>
<td>2.29 393</td>
<td>70.9 60.9</td>
</tr>
<tr>
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<td>2.52 308</td>
<td>2.78 340</td>
<td>41.7 35.0</td>
</tr>
<tr>
<td></td>
<td># 10 0.025 2.76&quot;</td>
<td>2.67 342</td>
<td>2.99 383</td>
<td>46.0 38.7</td>
</tr>
<tr>
<td></td>
<td># 11 0.031 2.83&quot;</td>
<td>2.84 380</td>
<td>3.21 429</td>
<td>50.2 42.3</td>
</tr>
<tr>
<td>S-9</td>
<td># 9 0.022 2.69&quot;</td>
<td>2.59 324</td>
<td>2.87 360</td>
<td>43.8 36.8</td>
</tr>
<tr>
<td></td>
<td># 10 0.029 2.76&quot;</td>
<td>2.76 363</td>
<td>3.12 408</td>
<td>48.6 40.9</td>
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<tr>
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<td>2.94 404</td>
<td>3.32 461</td>
<td>53.1 44.9</td>
</tr>
<tr>
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<td># 9 0.025 2.69&quot;</td>
<td>2.44 340</td>
<td>2.72 380</td>
<td>53.4 45.2</td>
</tr>
<tr>
<td></td>
<td># 10 0.032 2.76&quot;</td>
<td>2.57 383</td>
<td>2.91 434</td>
<td>60.0 51.0</td>
</tr>
<tr>
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<td>66.5 56.3</td>
</tr>
<tr>
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<td>2.57 372</td>
<td>2.90 420</td>
<td>57.2 48.7</td>
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<tr>
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<td># 10 0.038 2.76&quot;</td>
<td>2.71 424</td>
<td>3.10 485</td>
<td>64.7 54.7</td>
</tr>
<tr>
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<td># 11 0.047 2.83&quot;</td>
<td>2.86 479</td>
<td>3.30 554</td>
<td>72.4 61.0</td>
</tr>
</tbody>
</table>

\[
\frac{b \times t = 29'' \times 20''}{A_g = 580 \text{ sq in.}} \quad \text{Weight = 604 lb/ft} \quad \frac{f'_c = 2000 \text{ psi}}{I_c = 19,300 \text{ in.}^4}
\]

|     | \# 9 0.028 2.69" | 2.57 547 | 2.89 581 | 77.2 65.2 | 56.7 50.2 | 44.8 40.7 |
|     | \# 10 0.035 2.76" | 2.73 586 | 3.11 667 | 86.7 73.5 | 63.9 56.4 | 50.2 45.6 |
|     | \# 11 0.043 2.83" | 2.87 660 | 3.31 760 | 96.5 82.0 | 70.9 62.6 | 56.1 50.8 |

\[
\frac{b \times t = 30'' \times 20''}{A_g = 600 \text{ sq in.}} \quad \text{Weight = 615 lb/ft} \quad \frac{f'_c = 2000 \text{ psi}}{I_c = 20,000 \text{ in.}^4}
\]

|     | \# 9 0.023 2.69" | 2.46 494 | 2.74 550 | 74.1 63.0 | 54.5 48.2 | 43.0 39.1 |
|     | \# 10 0.030 2.76" | 2.60 554 | 2.93 625 | 83.3 70.3 | 61.2 53.9 | 48.4 43.7 |
|     | \# 10 0.036 2.83" | 2.75 619 | 3.12 707 | 94.7 77.9 | 67.4 59.8 | 53.6 48.5 |

Spiral: \( f'_s = 40,000 \) psi \# 4 at 2 1/2 in. o.c \( f'_s = 60,000 \) psi \# 3 at 2 1/4 in. o.c
## Table 9-43h. Square, Rectangular, and Round Columns, Values of $P$ and $N$ with $f_{c'} = 2,000$ psi. 20-in. Rectangular Columns, Tied Reinforcing

<table>
<thead>
<tr>
<th>Size</th>
<th>$p_g$</th>
<th>$d'$</th>
<th>$f_{c'} = 16,000$</th>
<th>$f_{c'} = 20,000$</th>
<th>$f_{c'} = 2,000$ psi</th>
<th>$f_{c'} = 2,000$ psi</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b x t = 16 x 20$</td>
<td>$A_g = 320$ sq in.</td>
<td>Weight = 334 lb/ft</td>
<td>$I_c = 10,700$ in$^4$</td>
<td>$I_c = 10,700$ in$^4$</td>
<td>$I_c = 10,700$ in$^4$</td>
<td>$I_c = 10,700$ in$^4$</td>
</tr>
<tr>
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<td>1.74</td>
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</tr>
<tr>
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<td>1.59</td>
<td>1.80</td>
<td>1.81</td>
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<td>1.88</td>
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</tr>
<tr>
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<td>2.58</td>
<td>1.73</td>
<td>2.35</td>
<td>1.96</td>
<td>265</td>
</tr>
<tr>
<td>$b x t = 18 x 20$</td>
<td>$A_g = 360$ sq in.</td>
<td>Weight = 375 lb/ft</td>
<td>$I_c = 12,000$ in$^4$</td>
<td>$I_c = 12,000$ in$^4$</td>
<td>$I_c = 12,000$ in$^4$</td>
<td>$I_c = 12,000$ in$^4$</td>
</tr>
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<td>1.71</td>
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<td>1.92</td>
<td>280</td>
</tr>
<tr>
<td>$b x t = 22 x 20$</td>
<td>$A_g = 440$ sq in.</td>
<td>Weight = 458 lb/ft</td>
<td>$I_c = 14,700$ in$^4$</td>
<td>$I_c = 14,700$ in$^4$</td>
<td>$I_c = 14,700$ in$^4$</td>
<td>$I_c = 14,700$ in$^4$</td>
</tr>
<tr>
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<td>1.78</td>
<td>2.60</td>
<td>1.96</td>
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</tr>
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<td>2.88</td>
<td>2.08</td>
<td>320</td>
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<td>1.93</td>
<td>3.18</td>
<td>2.18</td>
<td>358</td>
</tr>
<tr>
<td>$b x t = 24 x 20$</td>
<td>$A_g = 480$ sq in.</td>
<td>Weight = 500 lb/ft</td>
<td>$I_c = 16,000$ in$^4$</td>
<td>$I_c = 16,000$ in$^4$</td>
<td>$I_c = 16,000$ in$^4$</td>
<td>$I_c = 16,000$ in$^4$</td>
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<tr>
<td>$b x t = 26 x 20$</td>
<td>$A_g = 520$ sq in.</td>
<td>Weight = 545 lb/ft</td>
<td>$I_c = 18,000$ in$^4$</td>
<td>$I_c = 18,000$ in$^4$</td>
<td>$I_c = 18,000$ in$^4$</td>
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<td>3.43</td>
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<td>393</td>
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<td>1.99</td>
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<td>432</td>
</tr>
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<td>$b x t = 28 x 20$</td>
<td>$A_g = 560$ sq in.</td>
<td>Weight = 590 lb/ft</td>
<td>$I_c = 20,000$ in$^4$</td>
<td>$I_c = 20,000$ in$^4$</td>
<td>$I_c = 20,000$ in$^4$</td>
<td>$I_c = 20,000$ in$^4$</td>
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<tr>
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<td>2.90</td>
<td>1.96</td>
<td>341</td>
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<tr>
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<td>2.38</td>
<td>493</td>
</tr>
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</table>
### Table 9-43i: Square, Rectangular, and Round Columns, Values of $P$ and $N$ with $f_c' = 2,000$ psi. 22-in. Square and Rectangular Columns, Tied and Spiral Reinforcing

<table>
<thead>
<tr>
<th>Reinforcing</th>
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<th>$N$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$f_p = 16,000$</td>
<td>$f_p = 20,000$</td>
</tr>
<tr>
<td>No.</td>
<td>$d$</td>
<td>$g$</td>
</tr>
<tr>
<td>T-8</td>
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<td>276</td>
</tr>
<tr>
<td>#9</td>
<td>1.63</td>
<td>304</td>
</tr>
<tr>
<td>#10</td>
<td>1.70</td>
<td>334</td>
</tr>
<tr>
<td>#11</td>
<td>1.59</td>
<td>302</td>
</tr>
<tr>
<td>T-10</td>
<td>1.65</td>
<td>336</td>
</tr>
<tr>
<td>#9</td>
<td>1.71</td>
<td>374</td>
</tr>
<tr>
<td>#10</td>
<td>1.70</td>
<td>327</td>
</tr>
<tr>
<td>#11</td>
<td>1.86</td>
<td>413</td>
</tr>
<tr>
<td>T-12</td>
<td>1.27</td>
<td>374</td>
</tr>
<tr>
<td>#9</td>
<td>2.41</td>
<td>421</td>
</tr>
<tr>
<td>#10</td>
<td>2.58</td>
<td>467</td>
</tr>
<tr>
<td>#11</td>
<td>2.32</td>
<td>394</td>
</tr>
<tr>
<td>S-10</td>
<td>2.48</td>
<td>441</td>
</tr>
<tr>
<td>#9</td>
<td>2.62</td>
<td>492</td>
</tr>
<tr>
<td>#10</td>
<td>2.14</td>
<td>378</td>
</tr>
<tr>
<td>#11</td>
<td>2.20</td>
<td>421</td>
</tr>
<tr>
<td>ST-10</td>
<td>2.30</td>
<td>467</td>
</tr>
<tr>
<td>#9</td>
<td>2.19</td>
<td>410</td>
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<tr>
<td>#10</td>
<td>2.31</td>
<td>462</td>
</tr>
<tr>
<td>#11</td>
<td>2.43</td>
<td>517</td>
</tr>
<tr>
<td>ST-12</td>
<td>2.35</td>
<td>622</td>
</tr>
<tr>
<td>#9</td>
<td>2.48</td>
<td>696</td>
</tr>
<tr>
<td>#10</td>
<td>2.51</td>
<td>502</td>
</tr>
<tr>
<td>#11</td>
<td>2.54</td>
<td>567</td>
</tr>
</tbody>
</table>

| Spiral      | $t_b = 40,000$ psi | #4 at 2 t/2 in. o.c | $t_b = 60,000$ psi | #3 at 2 in. o.c | $f_c' = 2,000$ psi | $I_c = 18,450$ in$^4$
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>2ST-16</td>
<td>2.43</td>
<td>663</td>
<td>2.77</td>
<td>753</td>
<td>100.7</td>
<td>85.1</td>
</tr>
<tr>
<td>#10</td>
<td>2.35</td>
<td>663</td>
<td>2.77</td>
<td>753</td>
<td>91.3</td>
<td>94.8</td>
</tr>
<tr>
<td>#11</td>
<td>2.43</td>
<td>745</td>
<td>2.96</td>
<td>859</td>
<td>91.3</td>
<td>94.8</td>
</tr>
</tbody>
</table>

$A_g = 484$ sq in.  
Weight = 504 lb/ft  
$I_c = 18,450$ in$^4$  
$b \times t = 22'' \times 22''$  
$A_g = 660$ sq in.  
Weight = 688 lb/ft  
$I_c = 26,600$ in$^4$  
$2ST-16$  
$2ST-18$  
#3 at 2 in. o.c
Table 9-43j. Square, Rectangular, and Round Columns, Values of \( P \) and \( N \) with \( f'_{c} = 2,000 \text{ psi} \). 22-in. Rectangular Columns, Tied Reinforcing

<table>
<thead>
<tr>
<th>Reinforcing</th>
<th>( P_g )</th>
<th>( d' )</th>
<th>( f'_{c} = 16,000 )</th>
<th>( f'_{c} = 20,000 )</th>
<th>( N )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \frac{12}{12} )</td>
<td>( \frac{12}{12} )</td>
<td>1.0</td>
<td>1.2</td>
<td>1.4</td>
</tr>
<tr>
<td>( b \times t = 18'' \times 22'' )</td>
<td>( A_g = 396 \text{ sq in.} )</td>
<td>Weight = 413 \text{ lb/ft}</td>
<td>( t_c' = 2,000 \text{ psi} )</td>
<td>( I_c' = 15,950 \text{ in}^4 )</td>
<td></td>
</tr>
</tbody>
</table>
| T-6  
\#9  | 0.015  | 2.44" | 1.44 | 219 | 1.56 | 238 | 48.4 | 41.8 | 36.4 | 32.6 | 29.5 | 26.9 |
| \#10  | 0.019  | 2.51" | 1.50 | 239 | 1.66 | 264 | 54.9 | 47.4 | 41.7 | 37.5 | 33.5 | 30.7 |
| \#11  | 0.024  | 2.58" | 1.51 | 262 | 1.68 | 292 | 60.8 | 52.6 | 46.1 | 41.1 | 37.2 | 33.9 |
| T-8  
\#9  | 0.020  | 2.44" | 1.68 | 244 | 1.85 | 270 | 49.9 | 42.5 | 37.2 | 33.1 | 29.8 | 27.1 |
| \#10  | 0.026  | 2.51" | 1.68 | 272 | 1.88 | 304 | 55.7 | 47.8 | 41.8 | 37.1 | 33.4 | 30.4 |
| \#11  | 0.031  | 2.58" | 1.75 | 302 | 1.98 | 342 | 61.7 | 53.2 | 46.5 | 41.4 | 37.3 | 33.9 |
| \( b \times t = 20'' \times 22'' \) | \( A_g = 440 \text{ sq in.} \) | Weight = 458 \text{ lb/ft} | \( t_c' = 2,000 \text{ psi} \) | \( I_c' = 17,700 \text{ in}^4 \) |
| T-8  
\#9  | 0.018  | 2.44" | 1.59 | 260 | 1.74 | 286 | 52.1 | 44.3 | 38.6 | 34.4 | 31.1 | 28.2 |
| \#10  | 0.023  | 2.51" | 1.66 | 288 | 1.84 | 320 | 58.3 | 50.0 | 43.9 | 38.9 | 34.9 | 31.9 |
| \#11  | 0.028  | 2.58" | 1.72 | 318 | 1.94 | 358 | 64.9 | 55.6 | 48.6 | 43.4 | 38.9 | 35.4 |
| T-10  
\#9  | 0.023  | 2.44" | 1.60 | 286 | 1.78 | 318 | 60.4 | 51.7 | 45.5 | 40.1 | 36.3 | 33.6 |
| \#10  | 0.029  | 2.51" | 1.67 | 320 | 1.88 | 361 | 68.1 | 58.6 | 51.5 | 45.6 | 41.1 | 37.5 |
| \#11  | 0.035  | 2.58" | 1.73 | 358 | 1.97 | 397 | 75.8 | 65.5 | 57.3 | 51.4 | 46.1 | 41.9 |
| \( b \times t = 24'' \times 22'' \) | \( A_g = 528 \text{ sq in.} \) | Weight = 552 \text{ lb/ft} | \( t_c' = 2,000 \text{ psi} \) | \( I_c' = 21,300 \text{ in}^4 \) |
| T-10  
\#9  | 0.019  | 2.44" | 1.57 | 318 | 1.72 | 350 | 65.6 | 56.5 | 49.4 | 43.8 | 39.6 | 36.0 |
| \#10  | 0.024  | 2.51" | 1.62 | 352 | 1.81 | 393 | 74.2 | 63.4 | 55.5 | 49.4 | 44.7 | 43.3 |
| \#11  | 0.030  | 2.58" | 1.69 | 390 | 1.90 | 439 | 82.4 | 70.6 | 61.8 | 55.1 | 49.5 | 45.1 |
| T-12  
\#9  | 0.024  | 2.44" | 1.66 | 343 | 1.86 | 382 | 66.3 | 56.6 | 49.6 | 44.0 | 39.5 | 35.9 |
| \#10  | 0.029  | 2.51" | 1.75 | 385 | 1.97 | 434 | 75.0 | 63.7 | 56.0 | 49.4 | 44.6 | 40.5 |
| \#11  | 0.035  | 2.58" | 1.89 | 429 | 2.15 | 489 | 83.4 | 71.2 | 62.0 | 55.5 | 49.7 | 45.1 |
| \( b \times t = 26'' \times 22'' \) | \( A_g = 572 \text{ sq in.} \) | Weight = 595 \text{ lb/ft} | \( t_c' = 2,000 \text{ psi} \) | \( I_c' = 23,000 \text{ in}^4 \) |
| T-10  
\#10  | 0.022  | 2.51" | 1.62 | 368 | 1.79 | 409 | 76.6 | 66.0 | 57.7 | 51.3 | 46.4 | 42.0 |
| \#11  | 0.027  | 2.58" | 1.67 | 406 | 1.87 | 455 | 85.5 | 73.2 | 64.2 | 57.1 | 51.3 | 46.9 |
| T-12  
\#10  | 0.027  | 2.51" | 1.73 | 401 | 1.94 | 450 | 77.8 | 66.5 | 57.9 | 51.6 | 46.5 | 40.7 |
| \#11  | 0.033  | 2.58" | 1.80 | 445 | 2.04 | 505 | 86.1 | 73.5 | 64.0 | 56.9 | 51.3 | 46.6 |
| T-14  
\#10  | 0.031  | 2.51" | 1.74 | 433 | 1.97 | 490 | 86.8 | 74.5 | 64.9 | 57.8 | 51.9 | 47.4 |
| \#11  | 0.038  | 2.58" | 1.82 | 485 | 2.07 | 555 | 96.1 | 84.1 | 72.4 | 64.4 | 57.9 | 52.6 |
### Table 9-43a: Square, Rectangular, and Round Columns, Values of $P$ and $N$ with $f_c' = 2,000$ psi. 24-in. Square Columns, Tied and Spiral Reinforcing

<table>
<thead>
<tr>
<th>Reinforcing</th>
<th>$\frac{b \times t}{A_g}$</th>
<th>Weight</th>
<th>$f_c' = 2,000$</th>
<th>$T_c = 27,000$ in$^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>No.</strong></td>
<td><strong>Size</strong></td>
<td><strong>d</strong></td>
<td><strong>12 in.</strong></td>
<td><strong>12 in.</strong></td>
</tr>
<tr>
<td>T-10</td>
<td>24 x 24</td>
<td>576 sq in.</td>
<td>600 lb/ft</td>
<td>2200 psi</td>
</tr>
<tr>
<td>9</td>
<td>0.017</td>
<td>2.44</td>
<td>1.43</td>
<td>335</td>
</tr>
<tr>
<td>10</td>
<td>0.022</td>
<td>2.51</td>
<td>1.44</td>
<td>369</td>
</tr>
<tr>
<td>11</td>
<td>0.027</td>
<td>2.58</td>
<td>1.49</td>
<td>407</td>
</tr>
<tr>
<td>T-12</td>
<td>24 x 24</td>
<td>576 sq in.</td>
<td>600 lb/ft</td>
<td>2200 psi</td>
</tr>
<tr>
<td>9</td>
<td>0.024</td>
<td>2.44</td>
<td>1.49</td>
<td>360</td>
</tr>
<tr>
<td>10</td>
<td>0.026</td>
<td>2.51</td>
<td>1.55</td>
<td>402</td>
</tr>
<tr>
<td>11</td>
<td>0.032</td>
<td>2.58</td>
<td>1.61</td>
<td>446</td>
</tr>
<tr>
<td>T-14</td>
<td>24 x 24</td>
<td>576 sq in.</td>
<td>600 lb/ft</td>
<td>2200 psi</td>
</tr>
<tr>
<td>9</td>
<td>0.024</td>
<td>2.44</td>
<td>1.56</td>
<td>386</td>
</tr>
<tr>
<td>10</td>
<td>0.031</td>
<td>2.51</td>
<td>1.57</td>
<td>434</td>
</tr>
<tr>
<td>11</td>
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<td>2.58</td>
<td>1.63</td>
<td>486</td>
</tr>
<tr>
<td>S-11</td>
<td>24 x 24</td>
<td>576 sq in.</td>
<td>600 lb/ft</td>
<td>2200 psi</td>
</tr>
<tr>
<td>10</td>
<td>0.024</td>
<td>2.76</td>
<td>2.14</td>
<td>482</td>
</tr>
<tr>
<td>11</td>
<td>0.030</td>
<td>2.83</td>
<td>2.26</td>
<td>533</td>
</tr>
<tr>
<td>S-12</td>
<td>24 x 24</td>
<td>576 sq in.</td>
<td>600 lb/ft</td>
<td>2200 psi</td>
</tr>
<tr>
<td>10</td>
<td>0.026</td>
<td>2.76</td>
<td>2.19</td>
<td>503</td>
</tr>
<tr>
<td>11</td>
<td>0.032</td>
<td>2.83</td>
<td>2.31</td>
<td>558</td>
</tr>
<tr>
<td>S-13</td>
<td>24 x 24</td>
<td>576 sq in.</td>
<td>600 lb/ft</td>
<td>2200 psi</td>
</tr>
<tr>
<td>10</td>
<td>0.029</td>
<td>2.76</td>
<td>2.23</td>
<td>523</td>
</tr>
<tr>
<td>11</td>
<td>0.036</td>
<td>2.83</td>
<td>2.36</td>
<td>582</td>
</tr>
<tr>
<td>ST-12</td>
<td>24 x 24</td>
<td>576 sq in.</td>
<td>600 lb/ft</td>
<td>2200 psi</td>
</tr>
<tr>
<td>9</td>
<td>0.021</td>
<td>2.69</td>
<td>1.93</td>
<td>451</td>
</tr>
<tr>
<td>10</td>
<td>0.026</td>
<td>2.76</td>
<td>2.01</td>
<td>503</td>
</tr>
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<td>11</td>
<td>0.032</td>
<td>2.83</td>
<td>2.10</td>
<td>558</td>
</tr>
<tr>
<td>ST-14</td>
<td>24 x 24</td>
<td>576 sq in.</td>
<td>600 lb/ft</td>
<td>2200 psi</td>
</tr>
<tr>
<td>9</td>
<td>0.024</td>
<td>2.69</td>
<td>2.00</td>
<td>483</td>
</tr>
<tr>
<td>10</td>
<td>0.031</td>
<td>2.76</td>
<td>2.10</td>
<td>543</td>
</tr>
<tr>
<td>11</td>
<td>0.038</td>
<td>2.83</td>
<td>2.20</td>
<td>608</td>
</tr>
<tr>
<td>ST-16</td>
<td>24 x 24</td>
<td>576 sq in.</td>
<td>600 lb/ft</td>
<td>2200 psi</td>
</tr>
<tr>
<td>9</td>
<td>0.028</td>
<td>2.69</td>
<td>2.06</td>
<td>515</td>
</tr>
<tr>
<td>10</td>
<td>0.035</td>
<td>2.76</td>
<td>2.18</td>
<td>584</td>
</tr>
<tr>
<td>11</td>
<td>0.044</td>
<td>2.83</td>
<td>2.30</td>
<td>658</td>
</tr>
</tbody>
</table>

**Spiral**  
$f_c' = 40,000$ psi  
#4 at 2 1/2 in. o.c  
$f_c' = 60,000$ psi  
#3 at 2 in. o.c
Table 9-43. Square, Rectangular, and Round Columns, Values of $P$ and $N$ with $f_{c'} = 2,000$ psi. 24-in. Rectangular Columns, Tied Reinforcing

<table>
<thead>
<tr>
<th>Reinforcing</th>
<th>$\varepsilon/t &lt; 1.0$</th>
<th>$\varepsilon/t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. Size</td>
<td>$\rho_g$</td>
<td>$d'$</td>
</tr>
<tr>
<td>T-8</td>
<td>$b \times t = 20'' \times 24''$</td>
<td>$A_g = 480$ sq in.</td>
</tr>
<tr>
<td>#5</td>
<td>.017</td>
<td>2.44</td>
</tr>
<tr>
<td>#10</td>
<td>.021</td>
<td>2.51</td>
</tr>
<tr>
<td>#11</td>
<td>.026</td>
<td>2.58</td>
</tr>
<tr>
<td>T-10</td>
<td>$b \times t = 22'' \times 24''$</td>
<td>$A_g = 528$ sq in.</td>
</tr>
<tr>
<td>#9</td>
<td>.019</td>
<td>2.44</td>
</tr>
<tr>
<td>#10</td>
<td>.024</td>
<td>2.51</td>
</tr>
<tr>
<td>#11</td>
<td>.030</td>
<td>2.58</td>
</tr>
<tr>
<td>T-12</td>
<td>$b \times t = 26'' \times 24''$</td>
<td>$A_g = 624$ sq in.</td>
</tr>
<tr>
<td>#10</td>
<td>.024</td>
<td>2.51</td>
</tr>
<tr>
<td>#11</td>
<td>.030</td>
<td>2.58</td>
</tr>
<tr>
<td>T-14</td>
<td>$b \times t = 28'' \times 24''$</td>
<td>$A_g = 672$ sq in.</td>
</tr>
<tr>
<td>#10</td>
<td>.023</td>
<td>2.51</td>
</tr>
<tr>
<td>#11</td>
<td>.028</td>
<td>2.58</td>
</tr>
<tr>
<td>T-16</td>
<td>$b \times t = 30'' \times 24''$</td>
<td>$A_g = 720$ sq in.</td>
</tr>
<tr>
<td>#10</td>
<td>.026</td>
<td>2.51</td>
</tr>
<tr>
<td>#11</td>
<td>.032</td>
<td>2.58</td>
</tr>
<tr>
<td>T-18</td>
<td>$b \times t = 32'' \times 24''$</td>
<td>$A_g = 768$ sq in.</td>
</tr>
<tr>
<td>#10</td>
<td>.030</td>
<td>2.51</td>
</tr>
<tr>
<td>#11</td>
<td>.037</td>
<td>2.58</td>
</tr>
</tbody>
</table>
Table 9-43m. Square, Rectangular, and Round Columns, Values of $P$ and $N$ with $f'_{c} = 2,000$ psi. 26-in. and 28-in. Square Columns, Tied and Spiral Reinforcing

<table>
<thead>
<tr>
<th>Reinforcing</th>
<th>$f'_{s} = 16,000$</th>
<th>$f'_{s} = 20,000$</th>
<th>$N$</th>
</tr>
</thead>
<tbody>
<tr>
<td>No.</td>
<td>$P_{g}$</td>
<td>$d'$</td>
<td>$12\frac{SD}{T}$</td>
</tr>
<tr>
<td>T-12</td>
<td>#10 0.023 2.51&quot;</td>
<td>1.39 439</td>
<td>1.54 488</td>
</tr>
<tr>
<td></td>
<td>#11 0.028 2.58&quot;</td>
<td>1.43 483</td>
<td>1.61 543</td>
</tr>
<tr>
<td>T-14</td>
<td>#10 0.026 2.51&quot;</td>
<td>1.39 471</td>
<td>1.56 528</td>
</tr>
<tr>
<td></td>
<td>#11 0.032 2.58&quot;</td>
<td>1.44 523</td>
<td>1.64 593</td>
</tr>
<tr>
<td>T-16</td>
<td>#10 0.030 2.51&quot;</td>
<td>1.57 504</td>
<td>1.78 569</td>
</tr>
<tr>
<td></td>
<td>#11 0.037 2.58&quot;</td>
<td>1.52 563</td>
<td>1.73 643</td>
</tr>
<tr>
<td>S-12</td>
<td>#10 0.023 2.76&quot;</td>
<td>1.93 548</td>
<td>2.15 609</td>
</tr>
<tr>
<td></td>
<td>#11 0.028 2.83&quot;</td>
<td>2.03 603</td>
<td>2.29 678</td>
</tr>
<tr>
<td>S-13</td>
<td>#10 0.024 2.76&quot;</td>
<td>1.96 568</td>
<td>2.19 634</td>
</tr>
<tr>
<td></td>
<td>#11 0.030 2.83&quot;</td>
<td>2.07 627</td>
<td>2.34 709</td>
</tr>
<tr>
<td>S-14</td>
<td>#10 0.026 2.76&quot;</td>
<td>1.99 588</td>
<td>2.25 659</td>
</tr>
<tr>
<td></td>
<td>#11 0.032 2.83&quot;</td>
<td>2.10 653</td>
<td>2.38 741</td>
</tr>
<tr>
<td>ST-12</td>
<td>#10 0.023 2.76&quot;</td>
<td>1.78 548</td>
<td>1.98 609</td>
</tr>
<tr>
<td></td>
<td>#11 0.028 2.83&quot;</td>
<td>1.86 603</td>
<td>2.09 678</td>
</tr>
<tr>
<td>ST-14</td>
<td>#10 0.026 2.76&quot;</td>
<td>1.85 588</td>
<td>2.07 659</td>
</tr>
<tr>
<td></td>
<td>#11 0.032 2.83&quot;</td>
<td>1.94 653</td>
<td>2.20 741</td>
</tr>
<tr>
<td>ST-16</td>
<td>#10 0.030 2.76&quot;</td>
<td>1.92 629</td>
<td>2.16 710</td>
</tr>
<tr>
<td></td>
<td>#11 0.037 2.83&quot;</td>
<td>2.01 703</td>
<td>2.29 803</td>
</tr>
</tbody>
</table>

$b \times t = 28'' \times 28''$
$A_g = 784$ sq in.
$\text{Weight} = 81.6$ lb/ft
$f'_{c} = 2000$ psi
$I_c = 51,200$ in$^4$

| Reinforcing | $f'_{s} = 40,000$ psi | #4 at 2 1/4 in. oc | $f'_{s} = 60,000$ psi | #3 at 2 in. oc |
## Reinforced-Concrete Building Frames

Table 9-43a. Square, Rectangular, and Round Columns, Values of $P$ and $N$ with $f'_c = 2,000$ psi. 28-in. and 30-in. Square Columns, Tied and Spiral Reinforcing

<table>
<thead>
<tr>
<th>Reinforcing</th>
<th>$e/t &lt; 1.0$</th>
<th>$e/t &gt; 1.0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>No.</td>
<td>Size</td>
<td>$p_g$</td>
</tr>
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<td>.021</td>
</tr>
<tr>
<td>#11</td>
<td>.026</td>
<td>2.83&quot;</td>
</tr>
<tr>
<td>S-14</td>
<td>#10</td>
<td>.023</td>
</tr>
<tr>
<td>#11</td>
<td>.028</td>
<td>2.83&quot;</td>
</tr>
<tr>
<td>S-15</td>
<td>#10</td>
<td>.024</td>
</tr>
<tr>
<td>#11</td>
<td>.030</td>
<td>2.83&quot;</td>
</tr>
<tr>
<td>ST-14</td>
<td>#10</td>
<td>.023</td>
</tr>
<tr>
<td>#11</td>
<td>.028</td>
<td>2.83&quot;</td>
</tr>
<tr>
<td>ST-16</td>
<td>#10</td>
<td>.026</td>
</tr>
<tr>
<td>#11</td>
<td>.032</td>
<td>2.83&quot;</td>
</tr>
<tr>
<td>ST-18</td>
<td>#10</td>
<td>.029</td>
</tr>
<tr>
<td>#11</td>
<td>.036</td>
<td>2.83&quot;</td>
</tr>
</tbody>
</table>

$b \times t = 30" \times 30"$  
$A_g = 900$ sq in.  
Weight = 940 lb/ft  
$t'_c = 2000$ psi  
$I_c = 67,500$ in$^4$

### Additional Information
- $f'_c = 40,000$ psi  
- $4$ at $21/4$ in. o.c
- $f'_c = 60,000$ psi  
- $4$ at $3$ in. o.c
<table>
<thead>
<tr>
<th>Col No.</th>
<th>Reinforcing Size</th>
<th>12 cp/t</th>
<th>P</th>
<th>Reinforcing - Inner spiral Size</th>
<th>p_m</th>
<th>No. vertical bars</th>
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</thead>
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<td>d'</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>f_c' = 2000 psi</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>20'X20'</td>
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<td>2.76</td>
<td>2.86</td>
<td>434</td>
<td>#10</td>
</tr>
<tr>
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<td>0.051</td>
<td>2.83</td>
<td>3.00</td>
<td>505</td>
<td>#11</td>
</tr>
<tr>
<td>22'X22'</td>
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<td>2.76</td>
<td>2.51</td>
<td>503</td>
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<td>2.83</td>
<td>2.54</td>
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<tr>
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<td>2.76</td>
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<td>564</td>
<td>#10</td>
</tr>
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<td>2.76</td>
<td>1.92</td>
<td>629</td>
<td>#10</td>
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<td>2.83</td>
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<td>1.77</td>
<td>719</td>
<td>#10</td>
</tr>
<tr>
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<td>2.83</td>
<td>1.83</td>
<td>803</td>
<td>#11</td>
</tr>
<tr>
<td>30'X30'</td>
<td>#10</td>
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<td>2.76</td>
<td>1.62</td>
<td>812</td>
<td>#10</td>
</tr>
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<td>f_c' = 2000 psi</td>
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<tr>
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<td>510</td>
<td>#10</td>
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<td>804</td>
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</tr>
<tr>
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<td>810</td>
<td>#10</td>
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</tr>
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Inner Spiral: #2 at 6 in. oc
Outer Spiral: same as type ST in. columns
### Table 9-43p. Square, Rectangular, and Round Columns, Values of $P$ and $N$ with $f' = 2,000$ psi. 14-in. to 30-in. Round Columns, Spiral Reinforcing

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<th>Reinforcing</th>
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<th>$f_s = 16,000$</th>
<th>$f_s = 20,000$</th>
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<tbody>
<tr>
<td></td>
<td>Size</td>
<td>$p_g$</td>
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<tr>
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<td>#7</td>
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<td>2.56&quot;</td>
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<td>4.70</td>
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<td>5.05</td>
</tr>
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<td>2.83&quot;</td>
<td>5.05</td>
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<th>$\frac{\phi}{t} &lt; 1.0$</th>
<th>$f_s = 16,000$</th>
<th>$f_s = 20,000$</th>
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<td>$d$</td>
<td>$t_{2CD}$</td>
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<td>.030</td>
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<td>2.83&quot;</td>
<td>3.24</td>
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</table>

* $t_{2CD}$ for round = $\frac{4}{\pi} \times t_{2CD}$ for square
### Table 9-44a. Square, Rectangular, and Round Columns, Values of \( P \) and \( N \) with \( f'_{c} = 2,500 \) psi. 12-in. and 14-in. Square Columns, Tied and Spiral Reinforcing

<table>
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<th>S-9</th>
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<td>No.</td>
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<td>( d' )</td>
<td>( e/t &lt; 1.0 )</td>
<td>( N )</td>
</tr>
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<td></td>
<td></td>
<td></td>
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<td>12 ( \frac{\text{lb}}{\text{ft}} )</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td>( P )</td>
<td>( P )</td>
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<td>2.96</td>
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<td>2.31&quot;</td>
<td>2.92</td>
<td>96</td>
<td>3.14</td>
</tr>
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<td>3.81</td>
</tr>
<tr>
<td>#11</td>
<td>0.043</td>
<td>2.58&quot;</td>
<td>3.66</td>
<td>145</td>
<td>4.17</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Reinforcing</th>
<th>T-4</th>
<th>S-6</th>
<th>S-7</th>
<th>S-8</th>
<th>S-9</th>
</tr>
</thead>
<tbody>
<tr>
<td>No.</td>
<td>Size</td>
<td>( \rho_{g} )</td>
<td>( d' )</td>
<td>( e/t &lt; 1.0 )</td>
<td>( N )</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>12 ( \frac{\text{lb}}{\text{ft}} )</td>
<td>12 ( \frac{\text{lb}}{\text{ft}} )</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>( P )</td>
<td>( P )</td>
</tr>
<tr>
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<td>.012</td>
<td>2.31&quot;</td>
<td>2.35</td>
<td>119</td>
<td>2.50</td>
</tr>
<tr>
<td>#8</td>
<td>.016</td>
<td>2.37&quot;</td>
<td>2.42</td>
<td>124</td>
<td>2.61</td>
</tr>
<tr>
<td>#9</td>
<td>#20</td>
<td>2.41&quot;</td>
<td>2.54</td>
<td>139</td>
<td>2.75</td>
</tr>
<tr>
<td>#10</td>
<td>0.026</td>
<td>2.51&quot;</td>
<td>2.64</td>
<td>153</td>
<td>2.91</td>
</tr>
<tr>
<td>#11</td>
<td>0.032</td>
<td>2.58&quot;</td>
<td>2.79</td>
<td>168</td>
<td>3.08</td>
</tr>
<tr>
<td>#6</td>
<td>.013</td>
<td>2.37&quot;</td>
<td>3.26</td>
<td>151</td>
<td>3.47</td>
</tr>
<tr>
<td>#7</td>
<td>.018</td>
<td>2.44&quot;</td>
<td>3.47</td>
<td>167</td>
<td>3.79</td>
</tr>
<tr>
<td>#8</td>
<td>#24</td>
<td>2.50&quot;</td>
<td>3.74</td>
<td>186</td>
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</tr>
<tr>
<td>#9</td>
<td>#30</td>
<td>2.56&quot;</td>
<td>4.03</td>
<td>206</td>
<td>4.49</td>
</tr>
<tr>
<td>#10</td>
<td>#39</td>
<td>2.63&quot;</td>
<td>4.42</td>
<td>232</td>
<td>4.93</td>
</tr>
</tbody>
</table>

| Spiral | \( t'_{s} = 40,000 \text{ psi} \) | \#4 at 2'4" oc | \( t'_{s} = 60,000 \text{ psi} \) | \#3 at 2' oc |
### Table 9-44b. Square, Rectangular, and Round Columns, Values of $P$ and $N$ with $f' = 2,500$ psi. 12-in. and 14-in. Rectangular Columns, Tied Reinforcing

<table>
<thead>
<tr>
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<th>T-6</th>
</tr>
</thead>
<tbody>
<tr>
<td>No.</td>
<td>Size</td>
<td>$d'$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>#6</td>
<td>14x12&quot;</td>
<td>2.25</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>#7</td>
<td>12x14&quot;</td>
<td>2.29</td>
</tr>
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<td>#8</td>
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<td></td>
</tr>
<tr>
<td>#9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>#10</td>
<td>16x14&quot;</td>
<td>2.71</td>
</tr>
<tr>
<td>#11</td>
<td></td>
<td></td>
</tr>
<tr>
<td>#8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>#9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>#10</td>
<td>18x18&quot;</td>
<td>2.75</td>
</tr>
<tr>
<td>#11</td>
<td></td>
<td></td>
</tr>
<tr>
<td>#9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>#10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>#11</td>
<td></td>
<td></td>
</tr>
<tr>
<td>#9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>#10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>#11</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- **T-4**: Values for a column with dimensions 14x12" and weight = 175 lb/ft. $t'_c = 2,500$ psi. $I'_c = 1201$ in.$^4$.
- **T-6**: Values for a column with dimensions 18x18" and weight = 263 lb/ft. $t'_c = 2,500$ psi. $I'_c = 3660$ in.$^4$. 

---

**Notes**: All values are calculated based on the given specifications and formulas, ensuring accurate load-bearing capacities for reinforced concrete building frames.
### TABLES FOR DESIGN OF REINFORCED CONCRETE

**Table 9-44c. Square, Rectangular, and Round Columns, Values of \( P \) and \( N \) with \( f'_{c} = 2,500 \) psi. 16-in. Square and Rectangular Columns, Tied and Spiral Reinforcing**

\[
\text{Weight} = 266 \text{lb/ft} \quad t'_{c} = \frac{2500 \text{psi}}{I_{c}=5440 \text{in}^4}
\]

<table>
<thead>
<tr>
<th>Reinforcing</th>
<th>( e/t &lt; 1.0 )</th>
<th>( e/t )</th>
<th>( N )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( f_{s} = 16,000 )</td>
<td>( f_{s} = 20,000 )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( A_{g} = 256 \text{ sq in.} )</td>
<td>( A_{g} = 256 \text{ sq in.} )</td>
<td></td>
</tr>
<tr>
<td>No.</td>
<td>Size</td>
<td>( p_{g} )</td>
<td>( d' )</td>
</tr>
<tr>
<td>T-4</td>
<td>#8</td>
<td>0.012</td>
<td>2.37</td>
</tr>
<tr>
<td></td>
<td>#9</td>
<td>0.016</td>
<td>2.44</td>
</tr>
<tr>
<td></td>
<td>#10</td>
<td>0.020</td>
<td>2.51</td>
</tr>
<tr>
<td></td>
<td>#11</td>
<td>0.024</td>
<td>2.58</td>
</tr>
<tr>
<td>T-6</td>
<td>#8</td>
<td>0.018</td>
<td>2.37</td>
</tr>
<tr>
<td></td>
<td>#9</td>
<td>0.023</td>
<td>2.44</td>
</tr>
<tr>
<td></td>
<td>#10</td>
<td>0.030</td>
<td>2.51</td>
</tr>
<tr>
<td></td>
<td>#11</td>
<td>0.037</td>
<td>2.58</td>
</tr>
<tr>
<td>T-8</td>
<td>#8</td>
<td>0.025</td>
<td>2.37</td>
</tr>
<tr>
<td></td>
<td>#9</td>
<td>0.031</td>
<td>2.44</td>
</tr>
<tr>
<td></td>
<td>#10</td>
<td>0.035</td>
<td>2.51</td>
</tr>
<tr>
<td>S-6</td>
<td>#8</td>
<td>0.018</td>
<td>2.62</td>
</tr>
<tr>
<td></td>
<td>#9</td>
<td>0.023</td>
<td>2.69</td>
</tr>
<tr>
<td></td>
<td>#10</td>
<td>0.030</td>
<td>2.76</td>
</tr>
<tr>
<td></td>
<td>#11</td>
<td>0.037</td>
<td>2.83</td>
</tr>
<tr>
<td>S-7</td>
<td>#8</td>
<td>0.022</td>
<td>2.62</td>
</tr>
<tr>
<td></td>
<td>#9</td>
<td>0.027</td>
<td>2.69</td>
</tr>
<tr>
<td></td>
<td>#10</td>
<td>0.035</td>
<td>2.76</td>
</tr>
<tr>
<td>S-8</td>
<td>#8</td>
<td>0.025</td>
<td>2.62</td>
</tr>
<tr>
<td></td>
<td>#9</td>
<td>0.031</td>
<td>2.69</td>
</tr>
<tr>
<td>ST-10</td>
<td>#8</td>
<td>0.031</td>
<td>2.62</td>
</tr>
<tr>
<td></td>
<td>#9</td>
<td>0.039</td>
<td>2.69</td>
</tr>
</tbody>
</table>

\[
\text{Weight} = 417 \text{ lb/ft} \quad t'_{c} = \frac{2500 \text{ psi}}{I_{c}=8500 \text{ in}^4}
\]

| 2ST-14 | #8   | 0.028 | 2.62 | 3.17 | 402 | 3.52 | 446 | 56.5 | 47.6 | 41.0 | 36.2 | 32.3 | 29.3 |
|  | #9   | 0.035 | 2.69 | 3.39 | 449 | 3.82 | 505 | 62.2 | 52.4 | 45.5 | 40.1 | 35.8 | 32.4 |

\[
\text{Weight} = 438 \text{ lb/ft} \quad t'_{c} = \frac{2500 \text{ psi}}{I_{c}=8,850 \text{ in}^4}
\]

| 2ST-17 | #8   | 0.032 | 2.62 | 3.34 | 449 | 3.72 | 503 | 61.6 | 52.1 | 45.1 | 39.6 | 35.5 | 32.1 |
|  | #9   | 0.041 | 2.69 | 3.56 | 506 | 4.05 | 574 | 68.6 | 57.8 | 50.0 | 43.7 | 39.4 | 35.8 |

Spiral \( f'_{s} = 40,000 \) psi #4 at 2\( \frac{1}{4} \)\( \text{o c} \) \( f'_{s} = 60,000 \) psi #4 at 3\( \text{o c} \)
**Table 9-44d. Square, Rectangular, and Round Columns, Values of \( P \) and \( N \) with \( f'_c = 2,500 \) psi. 16-in. Rectangular Columns, Tied Reinforcing**

<table>
<thead>
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<th>Reinforcing</th>
<th>( e/t &lt; 1.0 )</th>
<th>( e/t )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{b \times t = 12'' \times 16''}{A_g = 192 \text{ sq in.}} )</td>
<td>( f'_c = 2500 ) psi</td>
<td>( \frac{t'_c}{I_c} = 4110 ) in^4</td>
</tr>
<tr>
<td>No.</td>
<td>( p )</td>
<td>( d' )</td>
</tr>
<tr>
<td>----------</td>
<td>--------</td>
<td>--------</td>
</tr>
<tr>
<td>T-4</td>
<td></td>
<td></td>
</tr>
<tr>
<td># 8</td>
<td>.016</td>
<td>2.37</td>
</tr>
<tr>
<td># 9</td>
<td>.021</td>
<td>2.44</td>
</tr>
<tr>
<td># 10</td>
<td>.026</td>
<td>2.51</td>
</tr>
<tr>
<td># 11</td>
<td>.032</td>
<td>2.58</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>T-4</td>
<td></td>
<td></td>
</tr>
<tr>
<td># 8</td>
<td>.014</td>
<td>2.37</td>
</tr>
<tr>
<td># 9</td>
<td>.018</td>
<td>2.44</td>
</tr>
<tr>
<td># 10</td>
<td>.023</td>
<td>2.51</td>
</tr>
<tr>
<td># 11</td>
<td>.028</td>
<td>2.58</td>
</tr>
<tr>
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<td></td>
</tr>
<tr>
<td>T-6</td>
<td></td>
<td></td>
</tr>
<tr>
<td># 9</td>
<td>.021</td>
<td>2.44</td>
</tr>
<tr>
<td># 10</td>
<td>.026</td>
<td>2.51</td>
</tr>
<tr>
<td># 11</td>
<td>.032</td>
<td>2.58</td>
</tr>
<tr>
<td>T-8</td>
<td></td>
<td></td>
</tr>
<tr>
<td># 9</td>
<td>.029</td>
<td>2.44</td>
</tr>
<tr>
<td># 10</td>
<td>.035</td>
<td>2.51</td>
</tr>
<tr>
<td># 11</td>
<td>.043</td>
<td>2.58</td>
</tr>
<tr>
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<td></td>
<td></td>
</tr>
<tr>
<td>T-6</td>
<td></td>
<td></td>
</tr>
<tr>
<td># 9</td>
<td>.019</td>
<td>2.44</td>
</tr>
<tr>
<td># 10</td>
<td>.024</td>
<td>2.51</td>
</tr>
<tr>
<td># 11</td>
<td>.029</td>
<td>2.58</td>
</tr>
<tr>
<td>T-8</td>
<td></td>
<td></td>
</tr>
<tr>
<td># 9</td>
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<td>2.44</td>
</tr>
<tr>
<td># 10</td>
<td>.032</td>
<td>2.51</td>
</tr>
<tr>
<td># 11</td>
<td>.039</td>
<td>2.58</td>
</tr>
</tbody>
</table>
Table 9-44e. Square, Rectangular, and Round Columns, Values of $P$ and $N$ with $f_{c'} = 2,500$ psi. 18-in. Square and Rectangular Columns, Tied and Spiral Reinforcing

<table>
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<th>Reinforcing</th>
<th>$\theta/t &lt; 1.0$</th>
<th>$N$</th>
</tr>
</thead>
<tbody>
<tr>
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<td>$f_t=16,000$</td>
<td>$f_t=20,000$</td>
</tr>
<tr>
<td></td>
<td>$\theta/t$ 1.0</td>
<td>1.2</td>
</tr>
<tr>
<td>No.</td>
<td>$p_g$  $d'$</td>
<td>$t_2$</td>
</tr>
<tr>
<td>T-6</td>
<td>.015  2.37&quot;</td>
<td>1.78</td>
</tr>
<tr>
<td>#8</td>
<td>.018  2.44&quot;</td>
<td>1.83</td>
</tr>
<tr>
<td>#9</td>
<td>.024  2.51&quot;</td>
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<td>#10</td>
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<td>1.95</td>
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<td>#11</td>
<td>.019  2.37&quot;</td>
<td>1.96</td>
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<td>.025  2.44&quot;</td>
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</tr>
<tr>
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<td>2.15</td>
</tr>
<tr>
<td>#10</td>
<td>.038  2.58&quot;</td>
<td>2.25</td>
</tr>
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<td>.016  2.69&quot;</td>
<td>2.66</td>
</tr>
<tr>
<td>#9</td>
<td>.023  2.76&quot;</td>
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</tr>
<tr>
<td>#10</td>
<td>.029  2.83&quot;</td>
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</tr>
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<td>.022  2.69&quot;</td>
<td>2.76</td>
</tr>
<tr>
<td>S-7</td>
<td>.027  2.76&quot;</td>
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</tr>
<tr>
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<td>.034  2.83&quot;</td>
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<td>.025  2.69&quot;</td>
<td>2.84</td>
</tr>
<tr>
<td>S-8</td>
<td>.031  2.76&quot;</td>
<td>3.03</td>
</tr>
<tr>
<td>#10</td>
<td>.038  2.83&quot;</td>
<td>3.27</td>
</tr>
<tr>
<td>ST-10</td>
<td>.025  2.62&quot;</td>
<td>2.66</td>
</tr>
<tr>
<td>#8</td>
<td>.031  2.69&quot;</td>
<td>2.76</td>
</tr>
<tr>
<td>#9</td>
<td>.039  2.76&quot;</td>
<td>2.92</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Reinforcing</th>
<th>$\theta/t &lt; 1.0$</th>
<th>$N$</th>
</tr>
</thead>
<tbody>
<tr>
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<td>$t_c=13,600$in$^4$</td>
</tr>
<tr>
<td></td>
<td>$\theta/t$ 1.0</td>
<td>1.2</td>
</tr>
<tr>
<td>No.</td>
<td>$p_g$  $d'$</td>
<td>$t_2$</td>
</tr>
<tr>
<td>2ST-14</td>
<td>.022  2.62&quot;</td>
<td>2.63</td>
</tr>
<tr>
<td>#8</td>
<td>.028  2.69&quot;</td>
<td>2.74</td>
</tr>
<tr>
<td>#9</td>
<td>.035  2.76&quot;</td>
<td>2.92</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Reinforcing</th>
<th>$\theta/t &lt; 1.0$</th>
<th>$N$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$f_t=2500$psi</td>
<td>$t_c=14,600$in$^4$</td>
</tr>
<tr>
<td></td>
<td>$\theta/t$ 1.0</td>
<td>1.2</td>
</tr>
<tr>
<td>No.</td>
<td>$p_g$  $d'$</td>
<td>$t_2$</td>
</tr>
<tr>
<td>2ST-17</td>
<td>.025  2.62&quot;</td>
<td>2.65</td>
</tr>
<tr>
<td>#8</td>
<td>.031  2.69&quot;</td>
<td>2.88</td>
</tr>
<tr>
<td>#9</td>
<td>.042  2.76&quot;</td>
<td>3.06</td>
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<table>
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<tr>
<th>Reinforcing</th>
<th>$\theta/t &lt; 1.0$</th>
<th>$N$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spiral</td>
<td>$t_2=40,000$psi</td>
<td>$\theta$ 4 at 2 1/4°c</td>
</tr>
<tr>
<td></td>
<td>$\theta/t$ 1.0</td>
<td>1.2</td>
</tr>
<tr>
<td>No.</td>
<td>$p_g$  $d'$</td>
<td>$t_2$</td>
</tr>
<tr>
<td>Spiral</td>
<td>$t_2=60,000$psi</td>
<td>$\theta$ 4 at 3°c</td>
</tr>
</tbody>
</table>
Table 9-44f. Square, Rectangular, and Round Columns, Values of $P$ and $N$ with $f'_{c} = 2,500$ psi. 18-in. Rectangular Columns, Tied Reinforcing

<table>
<thead>
<tr>
<th>No.</th>
<th>Size</th>
<th>$d'$</th>
<th>$e/t &lt;1.0$</th>
<th>$N$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>$f'_{c} = 16,000$</td>
<td>$f'_{c} = 20,000$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$12\frac{CD}{T} P$</td>
<td>$12\frac{CD}{T} P$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1.0</td>
<td>1.2</td>
</tr>
<tr>
<td>T-4</td>
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<td>0.012</td>
<td>2.37</td>
<td>1.76</td>
</tr>
<tr>
<td></td>
<td>#9</td>
<td>0.016</td>
<td>2.44</td>
<td>1.81</td>
</tr>
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<td>$A_g = 252$ sq in.</td>
</tr>
<tr>
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<td>0.021</td>
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<td>$b \times t = 16'' \times 18''$</td>
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<tr>
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<td>12 12</td>
<td>P</td>
<td>12 12</td>
<td>P</td>
</tr>
<tr>
<td>No.</td>
<td>Size</td>
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<td>$p_d$</td>
<td>$b_t$x$20^\circ$</td>
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<td>2.83</td>
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<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
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<tbody>
<tr>
<td></td>
<td>12 12</td>
<td>P</td>
<td>12 12</td>
<td>P</td>
<td>1.0</td>
<td>1.2</td>
<td>1.4</td>
</tr>
<tr>
<td></td>
<td>$b_t$x$29^\circ$x$20^\circ$</td>
<td>$A_g=580\text{sq in.}$</td>
<td>Weight =604 lb/ft</td>
<td>$t_c'=2500\text{psi}$</td>
<td>$I_c=19,300\text{in}^4$</td>
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<tr>
<td>#9</td>
<td>0.28</td>
<td>2.69</td>
<td>2.46</td>
<td>582</td>
<td>275</td>
<td>646</td>
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<td>0.35</td>
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<td>2.60</td>
<td>651</td>
<td>2.94</td>
<td>732</td>
<td>96.6</td>
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<th>N</th>
<th></th>
<th></th>
<th></th>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>12 12</td>
<td>P</td>
<td>12 12</td>
<td>P</td>
<td>1.0</td>
<td>1.2</td>
<td>1.4</td>
</tr>
<tr>
<td></td>
<td>$b_t$x$30^\circ$x$20^\circ$</td>
<td>$A_g=600\text{sq in.}$</td>
<td>Weight =615 lb/ft</td>
<td>$t_c'=2500\text{psi}$</td>
<td>$I_c=20,000\text{in}^4$</td>
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<td>#9</td>
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<td>617</td>
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<td>2.76</td>
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<td>2.75</td>
<td>692</td>
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<td>2.61</td>
<td>686</td>
<td>2.94</td>
<td>794</td>
<td>103.1</td>
</tr>
</tbody>
</table>

Spiral | $t_s'=40,000\text{psi}$ | $4\text{ at }2^\circ\text{o c}$ | $t_s'=60,000\text{psi}$ | $4\text{ at }3^\circ\text{o c}$
### Table 9-44h. Square, Rectangular, and Round Columns, Values of $P$ and $N$ with $f_c' = 2,500$ psi. 20-in. Rectangular Columns, Tied Reinforcing

<table>
<thead>
<tr>
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<th>$t_c' = 2500$ psi</th>
<th>$t_c' = 2500$ psi</th>
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<tbody>
<tr>
<td>No.</td>
<td>Size</td>
<td>$P_g$</td>
<td>$d'$</td>
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<tr>
<td>T-6</td>
<td>&amp; #8</td>
<td>.015</td>
<td>2.37</td>
</tr>
<tr>
<td>&amp; #9</td>
<td>.019</td>
<td>2.44*</td>
<td>1.62</td>
</tr>
<tr>
<td>&amp; #10</td>
<td>.024</td>
<td>2.51*</td>
<td>1.66</td>
</tr>
<tr>
<td>&amp; #11</td>
<td>.029</td>
<td>2.58*</td>
<td>1.71</td>
</tr>
<tr>
<td>T-8</td>
<td>&amp; #9</td>
<td>.025</td>
<td>2.44*</td>
</tr>
<tr>
<td>&amp; #10</td>
<td>.032</td>
<td>2.51*</td>
<td>1.90</td>
</tr>
<tr>
<td>&amp; #11</td>
<td>.039</td>
<td>2.58*</td>
<td>1.98</td>
</tr>
</tbody>
</table>

For $b_x = 18' x 20'$

| T-6 | & #9 | .017 | 2.44* | 1.60 | 239 | 1.73 | 258 | 50.7 | 43.6 | 38.2 | 34.0 | 30.7 | 28.0 |
| & #10 | .021 | 2.51* | 1.61 | 259 | 1.79 | 284 | 56.5 | 48.5 | 42.4 | 38.0 | 34.2 | 31.2 |
| & #11 | .026 | 2.58* | 1.69 | 282 | 1.86 | 312 | 62.8 | 53.9 | 47.3 | 42.2 | 38.1 | 34.7 |
| T-8 | & #9 | .022 | 2.44* | 1.78 | 264 | 1.96 | 290 | 51.2 | 43.9 | 38.3 | 34.1 | 30.6 | 27.8 |
| & #10 | .028 | 2.51* | 1.86 | 292 | 2.06 | 324 | 57.1 | 48.9 | 42.8 | 38.0 | 34.1 | 31.1 |
| & #11 | .035 | 2.58* | 1.93 | 322 | 2.17 | 362 | 63.5 | 54.5 | 47.3 | 42.1 | 37.9 | 34.5 |

For $b_x = 22' x 20'$

| T-8 | & #9 | .018 | 2.44* | 1.73 | 300 | 1.88 | 326 | 57.2 | 49.0 | 42.6 | 37.8 | 34.0 | 30.8 |
| & #10 | .023 | 2.51* | 1.81 | 328 | 1.98 | 360 | 63.4 | 54.1 | 47.3 | 41.8 | 37.6 | 34.3 |
| & #11 | .028 | 2.58* | 1.87 | 358 | 2.07 | 398 | 69.7 | 59.6 | 52.2 | 46.2 | 41.7 | 36.2 |
| T-10 | & #9 | .023 | 2.46* | 1.76 | 326 | 1.93 | 358 | 66.3 | 56.0 | 48.9 | 43.5 | 39.2 | 35.6 |
| & #10 | .029 | 2.51* | 1.83 | 360 | 2.04 | 401 | 72.8 | 62.4 | 54.8 | 48.5 | 43.8 | 39.8 |
| & #11 | .035 | 2.58* | 1.90 | 398 | 2.14 | 447 | 81.0 | 69.4 | 60.8 | 53.9 | 48.6 | 44.3 |

For $b_x = 24' x 20'$

| T-8 | & #9 | .017 | 2.44* | 1.71 | 318 | 1.85 | 344 | 59.9 | 51.2 | 44.4 | 39.5 | 35.7 | 32.4 |
| & #10 | .021 | 2.51* | 1.78 | 346 | 1.93 | 378 | 66.2 | 56.4 | 49.2 | 43.7 | 39.3 | 35.8 |
| & #11 | .026 | 2.58* | 1.84 | 376 | 2.03 | 416 | 70.1 | 62.4 | 54.3 | 48.3 | 43.6 | 39.6 |
| T-10 | & #9 | .021 | 2.44* | 1.74 | 344 | 1.90 | 376 | 67.8 | 61.3 | 50.9 | 45.2 | 41.0 | 40.5 |
| & #10 | .026 | 2.51* | 1.81 | 378 | 1.99 | 419 | 76.1 | 65.0 | 57.0 | 50.7 | 45.6 | 41.5 |
| & #11 | .032 | 2.58* | 1.87 | 416 | 2.09 | 465 | 84.7 | 72.1 | 63.3 | 56.3 | 50.7 | 46.1 |

| T-12 | & #9 | .025 | 2.44* | 1.85 | 369 | 2.03 | 408 | 69.1 | 58.6 | 51.4 | 45.4 | 40.8 | 37.0 |
| & #10 | .032 | 2.51* | 1.93 | 411 | 2.15 | 460 | 77.1 | 65.7 | 60.2 | 50.8 | 48.9 | 41.5 |
| & #11 | .039 | 2.58* | 1.94 | 455 | 2.27 | 515 | 87.7 | 74.8 | 65.1 | 58.1 | 52.1 | 47.4 |
### Table 9-44i. Square, Rectangular, and Round Columns, Values of $P$ and $N$ with $f'_c = 2,500$ psi. 22-in. Square and Rectangular Columns, Tied and Spiral Reinforcing

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<td>$t_s = 20,000$</td>
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<tr>
<td></td>
<td>$P$</td>
<td>$P$</td>
</tr>
<tr>
<td><strong>No.</strong></td>
<td><strong>Size</strong> $d'$</td>
<td>$d'$</td>
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<td><strong>T-8</strong></td>
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<td>#9</td>
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<td>2.44&quot;</td>
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<tr>
<td>#10</td>
<td>0.021</td>
<td>2.51&quot;</td>
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<tr>
<td>#11</td>
<td>0.026</td>
<td>2.58&quot;</td>
</tr>
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</tr>
<tr>
<td>#9</td>
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<td>2.44&quot;</td>
</tr>
<tr>
<td>#10</td>
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<td>2.58&quot;</td>
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<tr>
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<td>0.025</td>
<td>2.44&quot;</td>
</tr>
<tr>
<td>#10</td>
<td>0.031</td>
<td>2.51&quot;</td>
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<td>2.69&quot;</td>
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<tr>
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<td>2.83&quot;</td>
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<td>2.83&quot;</td>
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### 2ST-16

| No. | Size | $d'$ | 12 $\frac{cd}{t}$ | 12 $\frac{cd}{t}$ |   |   |   |   |
|-----|------|------|----------------|----------------|   |   |   |   |
| #10 | 0.031 | 2.76" | 2.25 | 696 | 2.52 | 772 | 106.5 | 90.0 | 78.0 | 69.0 | 61.7 | 56.1 |
| #11 | 0.088 | 2.83" | 2.37 | 770 | 2.68 | 870 | 116.9 | 98.3 | 85.8 | 76.0 | 68.2 | 61.5 |

### 2ST-18

| No. | Size | $d'$ | 12 $\frac{cd}{t}$ | 12 $\frac{cd}{t}$ |   |   |   |   |
|-----|------|------|----------------|----------------|   |   |   |   |
| #10 | 0.035 | 2.76" | 2.32 | 737 | 2.62 | 827 | 111.3 | 94.4 | 82.3 | 72.7 | 64.9 | 58.8 |
| #11 | 0.042 | 2.83" | 2.45 | 819 | 2.79 | 933 | 123.0 | 104.5 | 90.5 | 80.1 | 71.7 | 65.0 |

### Spiral

- $f'_s = 40,000$ psi
- $f'_s = 60,000$ psi
- $f'_s = 25,000$ psi
- $f'_s = 26,600$ psi
## REINFORCED-CONCRETE BUILDING FRAMES

### Table 9-44j: Square, Rectangular, and Round Columns, Values of \( P \) and \( N \) with \( f_{c'} = 2,500 \) psi. 22-in. Rectangular Columns, Tied Reinforcing

<table>
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<th>Reinforcing</th>
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<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>No.</td>
<td>( P )</td>
<td>( d )</td>
</tr>
<tr>
<td>T-6</td>
<td>( b \times t = 18' \times 22' )</td>
<td>( A_g = 396 \text{sq in.} )</td>
</tr>
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</tr>
<tr>
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<td>0.019</td>
<td>2.51</td>
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<tr>
<td>#11</td>
<td>0.024</td>
<td>2.58</td>
</tr>
<tr>
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<td>( b \times t = 20' \times 22' )</td>
<td>( A_g = 440 \text{sq in.} )</td>
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<tr>
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<tr>
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<td>2.51</td>
</tr>
<tr>
<td>#11</td>
<td>0.028</td>
<td>2.58</td>
</tr>
<tr>
<td>T-10</td>
<td>( b \times t = 24' \times 22' )</td>
<td>( A_g = 528 \text{sq in.} )</td>
</tr>
<tr>
<td>#9</td>
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<td>2.44</td>
</tr>
<tr>
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<tr>
<td>#11</td>
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</tr>
<tr>
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<td>( b \times t = 26' \times 22' )</td>
<td>( A_g = 572 \text{sq in.} )</td>
</tr>
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<td>T-14</td>
<td>( b \times t = 28' \times 22' )</td>
<td>( A_g = 690 \text{sq in.} )</td>
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</tr>
<tr>
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### Tables for Design of Reinforced Concrete

**Table 9-44**. Square, Rectangular, and Round Columns, Values of $P$ and $N$ with $f'_{c}=2,500$ psi. 24-in. Square Columns, Tied and Spiral Reinforcing

<table>
<thead>
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<td>e/t&lt;1.0</td>
<td>e/t&lt;1.0</td>
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<tr>
<td>No.</td>
<td>Size</td>
<td>$d'$</td>
<td>$f'_c=16,000$</td>
<td>$f'_c=20,000$</td>
<td>$f'_c=2500$ psi</td>
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<td></td>
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<td>$P$</td>
<td>$P$</td>
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<td>12 CR$^2$</td>
<td>12 CR$^2$</td>
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<td>1.47</td>
<td>4.38</td>
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<td>2.51</td>
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<td>2.09</td>
<td>5.47</td>
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<td>2.83</td>
<td>2.16</td>
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<td>1.99</td>
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<td>2.76</td>
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<td>0.44</td>
<td>2.83</td>
<td>2.15</td>
<td>7.23</td>
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</table>

- Spiral $f'_c=40,000$ psi  $\#4$ at $2\,^\circ$ c
- $f'_c=60,000$ psi  $\#4$ at $3\,^\circ$ c

---

*Note: The table provides values for design calculations in concrete structures, including the necessary dimensions and reinforcing bars for various conditions.*
### Reinforced-Concrete Building Frames

Table 9-441. Square, Rectangular, and Round Columns, Values of $P$ and $N$ with $f_{e'} = 2,500$ psi. 24-in. Rectangular Columns, Tied Reinforcing

<table>
<thead>
<tr>
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<th>$N$</th>
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<tbody>
<tr>
<td>No. Size</td>
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<td>$d'$</td>
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<tr>
<td>-------------</td>
<td>-----</td>
<td>-----</td>
</tr>
<tr>
<td>T-8</td>
<td></td>
<td></td>
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<tr>
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<td>2.44&quot;</td>
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<tr>
<td>#10</td>
<td>0.021</td>
<td>2.51&quot;</td>
</tr>
<tr>
<td>#11</td>
<td>0.026</td>
<td>2.58&quot;</td>
</tr>
<tr>
<td>T-10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>#9</td>
<td>0.019</td>
<td>2.44&quot;</td>
</tr>
<tr>
<td>#10</td>
<td>0.024</td>
<td>2.51&quot;</td>
</tr>
<tr>
<td>#11</td>
<td>0.030</td>
<td>2.58&quot;</td>
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<tr>
<td>T-12</td>
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<td></td>
</tr>
<tr>
<td>#10</td>
<td>0.024</td>
<td>2.51&quot;</td>
</tr>
<tr>
<td>#11</td>
<td>0.030</td>
<td>2.58&quot;</td>
</tr>
<tr>
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<td>2.51&quot;</td>
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<tr>
<td>#11</td>
<td>0.028</td>
<td>2.58&quot;</td>
</tr>
<tr>
<td>T-16</td>
<td></td>
<td></td>
</tr>
<tr>
<td>#10</td>
<td>0.023</td>
<td>2.51&quot;</td>
</tr>
<tr>
<td>#11</td>
<td>0.028</td>
<td>2.58&quot;</td>
</tr>
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</table>
# TABLES FOR DESIGN OF REINFORCED CONCRETE

Table 9-44m. Square, Rectangular, and Round Columns, Values of $P$ and $N$ with $f'_s = 2,500$ psi. 26-in. and 28-in. Square Columns, Tied and Spiral Reinforcing

<table>
<thead>
<tr>
<th>Reinforcing</th>
<th>$e/t &lt; 1.0$</th>
<th>$e/t &gt; 1.0$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$f'_s = 16,000$</td>
<td>$f'_s = 20,000$</td>
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<td>$12c_0$</td>
<td>$12c_0$</td>
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<td>11</td>
<td>.028</td>
</tr>
<tr>
<td>T-14</td>
<td>10</td>
<td>.026</td>
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<td>T-16</td>
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<td>.030</td>
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<td>11</td>
<td>.037</td>
</tr>
<tr>
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<td>10</td>
<td>.023</td>
</tr>
<tr>
<td></td>
<td>11</td>
<td>.028</td>
</tr>
<tr>
<td>S-13</td>
<td>10</td>
<td>.024</td>
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<td></td>
<td>11</td>
<td>.030</td>
</tr>
<tr>
<td>S-14</td>
<td>10</td>
<td>.026</td>
</tr>
<tr>
<td></td>
<td>11</td>
<td>.032</td>
</tr>
<tr>
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<td>10</td>
<td>.023</td>
</tr>
<tr>
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<td>11</td>
<td>.028</td>
</tr>
<tr>
<td>ST-12</td>
<td>10</td>
<td>.026</td>
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<tr>
<td></td>
<td>11</td>
<td>.032</td>
</tr>
<tr>
<td>ST-14</td>
<td>10</td>
<td>.030</td>
</tr>
<tr>
<td></td>
<td>11</td>
<td>.037</td>
</tr>
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<table>
<thead>
<tr>
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<th>$e/t = 2500 psi$</th>
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</thead>
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<tr>
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<td>$I_c = 784$ sq in.</td>
<td>$I_c = 51,200$</td>
</tr>
<tr>
<td></td>
<td>$A_g = 816$ lb/ft</td>
<td>$I_c = 200$ in.</td>
</tr>
<tr>
<td>T-12</td>
<td>10</td>
<td>.019</td>
</tr>
<tr>
<td></td>
<td>11</td>
<td>.024</td>
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<tr>
<td>T-14</td>
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<td>.023</td>
</tr>
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<td>.028</td>
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<td>T-16</td>
<td>10</td>
<td>.026</td>
</tr>
<tr>
<td></td>
<td>11</td>
<td>.032</td>
</tr>
</tbody>
</table>

Spiral $f'_s = 40,000$ psi  # 5 at 3° c  $f'_s = 60,000$ psi  # 4 at 23/4°
### Table 9-44a. Square, Rectangular, and Round Columns, Values of \( P \) and \( N \) with \( f_{c'} = 2,500 \text{ psi} \). 28-in. and 30-in. Square Columns, Tied and Spiral Reinforcing

<table>
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<th>Reinforcing</th>
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<th>( N )</th>
</tr>
</thead>
<tbody>
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<td>( f'_{c} = 16,000 )</td>
</tr>
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<td>1.68 704</td>
</tr>
<tr>
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<td>#11 .026 2.85&quot;</td>
<td>1.75 763</td>
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<tr>
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<td>1.70 724</td>
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<td>1.76 888</td>
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</table>

### Additional Calculations

- \( b \times t = 30" \times 30" \)
- \( A_g = 900 \text{ sq in.} \)
- \( \text{Weight} = 940 \text{ lb/ft} \)
- \( f'_{c} = 2,500 \text{ psi} \)
- \( I_c = 67,500 \text{ in}^4 \)

<table>
<thead>
<tr>
<th>Reinforcing</th>
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<th>( N )</th>
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</thead>
<tbody>
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<td>( f'_{c} = 40,000 \text{ psi} )</td>
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<td>1.19 724</td>
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<tr>
<td>S-14</td>
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<td>1.57 790</td>
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<td>1.60 855</td>
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<td>1.63 880</td>
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<td>1.63 1004</td>
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Table 9-44o. Square, Rectangular, and Round Columns, Values of $P$ and $N$ with $f' = 2,500$ psi. 20-in. to 30-in. Square Columns, Tied Double Spiral Reinforcing

<table>
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<td>2.45</td>
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<td>2.76&quot;</td>
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<td>2.83&quot;</td>
<td>2.19</td>
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<td>2.76&quot;</td>
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</tr>
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<td>2.76&quot;</td>
<td>1.70</td>
</tr>
<tr>
<td>SST-18</td>
<td>11</td>
<td>0.036</td>
<td>2.83&quot;</td>
<td>1.76</td>
</tr>
<tr>
<td>30&quot; x 30&quot;</td>
<td>10</td>
<td>0.028</td>
<td>2.76&quot;</td>
<td>1.57</td>
</tr>
<tr>
<td>SST-20</td>
<td>11</td>
<td>0.035</td>
<td>2.83&quot;</td>
<td>1.63</td>
</tr>
</tbody>
</table>

$t_c = 2,500$ psi  
$t_s = 16,000$ psi  

<table>
<thead>
<tr>
<th>Col. No.</th>
<th>Reinforcing</th>
<th>$12\frac{\phi}{t}$</th>
<th>$P_{e/t&lt;1.0}$</th>
<th>Reinforcing inner spiral</th>
</tr>
</thead>
<tbody>
<tr>
<td>Size</td>
<td>$p_g$</td>
<td>$a'$</td>
<td></td>
<td>$p_m$</td>
</tr>
<tr>
<td>20&quot; x 20&quot;</td>
<td>10</td>
<td>0.041</td>
<td>2.76&quot;</td>
<td>2.92</td>
</tr>
<tr>
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<td>11</td>
<td>0.051</td>
<td>2.83&quot;</td>
<td>3.10</td>
</tr>
<tr>
<td>22&quot; x 22&quot;</td>
<td>10</td>
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<td>2.76&quot;</td>
<td>2.73</td>
</tr>
<tr>
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<td>0.051</td>
<td>2.83&quot;</td>
<td>2.78</td>
</tr>
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<td>24&quot; x 24&quot;</td>
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<td>2.76&quot;</td>
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<td>2.83&quot;</td>
<td>2.47</td>
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<tr>
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<td>2.76&quot;</td>
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<tr>
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<td>11</td>
<td>0.037</td>
<td>2.83&quot;</td>
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<td>28&quot; x 28&quot;</td>
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<td>2.76&quot;</td>
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<td>0.036</td>
<td>2.83&quot;</td>
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<tr>
<td>30&quot; x 30&quot;</td>
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<td>2.76&quot;</td>
<td>1.74</td>
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<tr>
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<td>11</td>
<td>0.035</td>
<td>2.83&quot;</td>
<td>1.84</td>
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</table>

Inner spiral, #2 at 6" o c  
Outer spiral same as type ST columns.
Table 9-44p. Square, Rectangular, and Round Columns, Values of \( P \) and \( N \) with \( f' = 2,500 \text{ psi} \). 14-in. to 30-in. Round Columns, Spiral Reinforcing

<table>
<thead>
<tr>
<th>Col</th>
<th>Reinforcing</th>
<th>( t_f = 16,000 )</th>
<th>( t_f = 20,000 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diam</td>
<td>Size</td>
<td>( P_g )</td>
<td>( d' )</td>
</tr>
<tr>
<td>14&quot;</td>
<td>#6</td>
<td>0.17</td>
<td>2.50</td>
</tr>
<tr>
<td></td>
<td>#7</td>
<td>0.23</td>
<td>2.56</td>
</tr>
<tr>
<td></td>
<td>#8</td>
<td>0.31</td>
<td>2.62</td>
</tr>
<tr>
<td></td>
<td>#9</td>
<td>0.39</td>
<td>2.69</td>
</tr>
<tr>
<td></td>
<td>#10</td>
<td>0.49</td>
<td>2.76</td>
</tr>
<tr>
<td>R-6</td>
<td>#6</td>
<td>0.20</td>
<td>2.50</td>
</tr>
<tr>
<td></td>
<td>#7</td>
<td>0.27</td>
<td>2.56</td>
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<td>#8</td>
<td>0.36</td>
<td>2.62</td>
</tr>
<tr>
<td></td>
<td>#9</td>
<td>0.45</td>
<td>2.69</td>
</tr>
<tr>
<td>R-7</td>
<td>#8</td>
<td>0.24</td>
<td>2.62</td>
</tr>
<tr>
<td></td>
<td>#9</td>
<td>0.30</td>
<td>2.69</td>
</tr>
<tr>
<td></td>
<td>#10</td>
<td>0.38</td>
<td>2.76</td>
</tr>
<tr>
<td>#11</td>
<td>0.46</td>
<td>2.83</td>
<td>4.92</td>
</tr>
<tr>
<td>R-8</td>
<td>#9</td>
<td>0.35</td>
<td>2.69</td>
</tr>
<tr>
<td></td>
<td>#10</td>
<td>0.44</td>
<td>2.76</td>
</tr>
<tr>
<td>R-9</td>
<td>#8</td>
<td>0.32</td>
<td>2.69</td>
</tr>
<tr>
<td></td>
<td>#9</td>
<td>0.35</td>
<td>2.69</td>
</tr>
<tr>
<td></td>
<td>#10</td>
<td>0.38</td>
<td>2.76</td>
</tr>
<tr>
<td>#11</td>
<td>0.46</td>
<td>2.83</td>
<td>4.92</td>
</tr>
<tr>
<td>R-11</td>
<td>#9</td>
<td>0.35</td>
<td>2.69</td>
</tr>
<tr>
<td></td>
<td>#10</td>
<td>0.44</td>
<td>2.76</td>
</tr>
<tr>
<td>R-12</td>
<td>#8</td>
<td>0.28</td>
<td>2.69</td>
</tr>
<tr>
<td></td>
<td>#9</td>
<td>0.35</td>
<td>2.76</td>
</tr>
<tr>
<td></td>
<td>#10</td>
<td>0.43</td>
<td>2.83</td>
</tr>
<tr>
<td>#11</td>
<td>0.50</td>
<td>2.90</td>
<td>4.33</td>
</tr>
<tr>
<td>R-13</td>
<td>#8</td>
<td>0.29</td>
<td>2.70</td>
</tr>
<tr>
<td></td>
<td>#9</td>
<td>0.36</td>
<td>2.78</td>
</tr>
<tr>
<td></td>
<td>#10</td>
<td>0.44</td>
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</tr>
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<td>2.93</td>
<td>4.21</td>
</tr>
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<td>0.29</td>
<td>2.70</td>
</tr>
<tr>
<td></td>
<td>#10</td>
<td>0.36</td>
<td>2.79</td>
</tr>
<tr>
<td></td>
<td>#11</td>
<td>0.43</td>
<td>2.86</td>
</tr>
<tr>
<td>#12</td>
<td>0.51</td>
<td>2.94</td>
<td>4.23</td>
</tr>
<tr>
<td>R-15</td>
<td>#9</td>
<td>0.29</td>
<td>2.70</td>
</tr>
<tr>
<td></td>
<td>#10</td>
<td>0.36</td>
<td>2.79</td>
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<tr>
<td></td>
<td>#11</td>
<td>0.43</td>
<td>2.86</td>
</tr>
<tr>
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<td>0.51</td>
<td>2.94</td>
<td>4.23</td>
</tr>
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<td>R-16</td>
<td>#9</td>
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<td>2.70</td>
</tr>
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<td>#10</td>
<td>0.36</td>
<td>2.79</td>
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<td>#11</td>
<td>0.43</td>
<td>2.86</td>
</tr>
<tr>
<td>#12</td>
<td>0.51</td>
<td>2.94</td>
<td>4.23</td>
</tr>
</tbody>
</table>

*\( \frac{12}{t_f} \) for round = \( \frac{4}{t_f} x 12 \) \( \frac{CD}{t} \) for square
### Table 9-45a. Square, Rectangular, and Round Columns, Values of $P$ and $N$ with $f' = 3,000$ psi. 12-in. and 14-in. Square Columns, Tied and Spiral Reinforcing

<table>
<thead>
<tr>
<th>Reinforcing</th>
<th>e/t &lt; 1.0</th>
<th>N</th>
<th>e/t</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$t_b$ = 16,000</td>
<td>$t_b$ = 20,000</td>
<td></td>
</tr>
<tr>
<td>T-4</td>
<td>$t_b$ = 12&quot; x 12&quot;</td>
<td>$t_b$ = 144 sq in.</td>
<td>Weight = 150 lb/ft</td>
</tr>
<tr>
<td>No.</td>
<td>Size</td>
<td>$d'$</td>
<td>$t_2$</td>
</tr>
<tr>
<td>#6</td>
<td>.012</td>
<td>2.25&quot;</td>
<td>.02</td>
</tr>
<tr>
<td>#7</td>
<td>.017</td>
<td>2.31&quot;</td>
<td>.02</td>
</tr>
<tr>
<td>#8</td>
<td>.022</td>
<td>2.37&quot;</td>
<td>.02</td>
</tr>
<tr>
<td>#9</td>
<td>.028</td>
<td>2.44&quot;</td>
<td>.02</td>
</tr>
<tr>
<td>#10</td>
<td>.035</td>
<td>2.51&quot;</td>
<td>.02</td>
</tr>
<tr>
<td>#11</td>
<td>.043</td>
<td>2.58&quot;</td>
<td>.02</td>
</tr>
</tbody>
</table>

| T-4 | $t_b$ = 14" x 14" | $t_b$ = 196 sq in. | Weight = 204 lb/ft | $f_c' = 3000$ psi | $I_c = 3200$ in$^4$ |
| No. | Size | $d'$ | $t_2$ | $P$ | $t_2$ | $P$ | 23.2 | 30.2 |     |     |     |     |
| #7 | .012 | 2.31" | .02 | .00 | .02 | .00 | 26.8 | 22.7 | 19.6 | 17.5 | 15.8 |     |
| #8 | .016 | 2.37" | .02 | .00 | .02 | .00 | 29.7 | 24.6 | 21.4 | 19.2 | 17.1 | 15.5 |
| #9 | .020 | 2.41" | .02 | .00 | .02 | .00 | 32.3 | 27.5 | 23.8 | 21.7 | 19.3 | 17.3 |
| #10 | .026 | 2.51" | .02 | .00 | .02 | .00 | 35.7 | 30.3 | 26.6 | 23.4 | 21.0 | 19.3 |

| S-6 | $t_b$ = 2.37" | $t_b$ = 189 sq in. | Weight = 184 lb/ft | $f_c' = 3000$ psi | $I_c = 3200$ in$^4$ |
| No. | Size | $d'$ | $t_2$ | $P$ | $t_2$ | $P$ | 23.2 | 30.2 |     |     |     |     |
| #6 | .013 | 2.37" | .02 | .00 | .02 | .00 | 26.8 | 22.7 | 19.6 | 17.5 | 15.8 |     |
| #7 | .018 | 2.44" | .02 | .00 | .02 | .00 | 29.7 | 24.6 | 21.4 | 19.2 | 17.1 | 15.5 |
| #8 | .024 | 2.50" | .02 | .00 | .02 | .00 | 32.3 | 27.5 | 23.8 | 21.7 | 19.3 | 17.3 |
| #9 | .030 | 2.56" | .02 | .00 | .02 | .00 | 35.7 | 30.3 | 26.6 | 23.4 | 21.0 | 19.3 |

| S-7 | $t_b$ = 2.37" | $t_b$ = 199 sq in. | Weight = 184 lb/ft | $f_c' = 3000$ psi | $I_c = 3200$ in$^4$ |
| No. | Size | $d'$ | $t_2$ | $P$ | $t_2$ | $P$ | 23.2 | 30.2 |     |     |     |     |
| #6 | .016 | 2.37" | .02 | .00 | .02 | .00 | 26.8 | 22.7 | 19.6 | 17.5 | 15.8 |     |
| #7 | .021 | 2.44" | .02 | .00 | .02 | .00 | 29.7 | 24.6 | 21.4 | 19.2 | 17.1 | 15.5 |
| #8 | .028 | 2.50" | .02 | .00 | .02 | .00 | 32.3 | 27.5 | 23.8 | 21.7 | 19.3 | 17.3 |
| #9 | .036 | 2.56" | .02 | .00 | .02 | .00 | 35.7 | 30.3 | 26.6 | 23.4 | 21.0 | 19.3 |

| S-8 | $t_b$ = 2.37" | $t_b$ = 187 sq in. | Weight = 184 lb/ft | $f_c' = 3000$ psi | $I_c = 3200$ in$^4$ |
| No. | Size | $d'$ | $t_2$ | $P$ | $t_2$ | $P$ | 23.2 | 30.2 |     |     |     |     |
| #6 | .018 | 2.37" | .02 | .00 | .02 | .00 | 26.8 | 22.7 | 19.6 | 17.5 | 15.8 |     |
| #7 | .024 | 2.44" | .02 | .00 | .02 | .00 | 29.7 | 24.6 | 21.4 | 19.2 | 17.1 | 15.5 |
| #8 | .032 | 2.50" | .02 | .00 | .02 | .00 | 32.3 | 27.5 | 23.8 | 21.7 | 19.3 | 17.3 |

| S-9 | $t_b$ = 3.000 psi | $t_b$ = 40,000 psi | #4 at 2\°C. | $f'_b = 60,000$ psi | $t_b$ = 60,000 psi | #4 at 3\°C. |
| No. | Size | $d'$ | $t_2$ | $P$ | $t_2$ | $P$ | 23.2 | 30.2 |     |     |     |     |
Table 9-45b. Square, Rectangular, and Round Columns, Values of \( P \) and \( N \) with \( f'_{c} = 3,000 \) psi. 12-in. and 14-in. Rectangular Columns, Tied Reinforcing

<table>
<thead>
<tr>
<th>Reinforcing</th>
<th>( e/t &lt; 1.0 )</th>
<th>( e/t )</th>
<th>( N )</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. Size</td>
<td>( d' )</td>
<td>( f'_{c} = 16,000 )</td>
<td>( f'_{c} = 20,000 )</td>
</tr>
<tr>
<td>T-4</td>
<td>( b \times t = 14&quot; \times 12&quot; )</td>
<td>( b \times t = 12&quot; \times 14&quot; )</td>
<td>( b \times t = 16&quot; \times 14&quot; )</td>
</tr>
<tr>
<td></td>
<td>( Ag = 168 ) sq in.</td>
<td>( Ag = 182 ) sq in.</td>
<td>( Ag = 224 ) sq in.</td>
</tr>
<tr>
<td></td>
<td>Weight = 175 lb/ft²</td>
<td>Weight = 175 lb/ft²</td>
<td>Weight = 175 lb/ft²</td>
</tr>
<tr>
<td>#6</td>
<td>.010</td>
<td>2.25</td>
<td>2.68</td>
</tr>
<tr>
<td>#7</td>
<td>.014</td>
<td>2.31</td>
<td>2.70</td>
</tr>
<tr>
<td>#8</td>
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</tr>
<tr>
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<td>.024</td>
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<td>3.07</td>
</tr>
<tr>
<td>#10</td>
<td>.030</td>
<td>2.51</td>
<td>3.23</td>
</tr>
<tr>
<td>#11</td>
<td>.037</td>
<td>2.58</td>
<td>3.36</td>
</tr>
<tr>
<td>T-4</td>
<td>( I_{c} = 2016 ) in⁴</td>
<td>( I_{c} = 2746 ) in⁴</td>
<td>( I_{c} = 3660 ) in⁴</td>
</tr>
<tr>
<td>#6</td>
<td>.010</td>
<td>2.25</td>
<td>2.66</td>
</tr>
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<td>2.33</td>
</tr>
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<td>.024</td>
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<td>.037</td>
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<td>2.76</td>
</tr>
<tr>
<td>T-4</td>
<td>( I_{c} = 2746 ) in⁴</td>
<td>( I_{c} = 3660 ) in⁴</td>
<td>( I_{c} = 4120 ) in⁴</td>
</tr>
<tr>
<td>#8</td>
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<td>2.37</td>
<td>2.33</td>
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<td>.028</td>
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<td>2.62</td>
</tr>
<tr>
<td>T-6</td>
<td>( I_{c} = 4120 ) in⁴</td>
<td></td>
<td></td>
</tr>
<tr>
<td>#9</td>
<td>.024</td>
<td>2.44</td>
<td>2.52</td>
</tr>
<tr>
<td>#10</td>
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</tr>
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Table 9-45c. Square, Rectangular, and Round Columns, Values of \( P \) and \( N \) with \( f'_c = 3,000 \text{ psi} \). 16-in. Square and Rectangular Columns, Tied and Spirial Reinforcing

<table>
<thead>
<tr>
<th>Reinforcing</th>
<th>( \frac{b}{t} \times 16'' \times 16'' )</th>
<th>( \frac{d}{t} \leq 1.0 )</th>
<th>( N )</th>
</tr>
</thead>
<tbody>
<tr>
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<td>( t_s = 6,000 )</td>
<td>( t_s = 20,000 )</td>
<td>( e/t )</td>
</tr>
<tr>
<td></td>
<td>( \frac{12}{t} )</td>
<td>( P )</td>
<td>( \frac{12}{t} )</td>
</tr>
<tr>
<td>No.</td>
<td>Size</td>
<td>( P )</td>
<td>( d' )</td>
</tr>
<tr>
<td>-------------</td>
<td>------</td>
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<td>-------</td>
</tr>
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<td>2.58</td>
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<td>2.37</td>
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<td>0.31</td>
<td>2.44</td>
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<td></td>
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<td>2.51</td>
</tr>
<tr>
<td>S-6</td>
<td>#8</td>
<td>0.18</td>
<td>2.62</td>
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<td>2.69</td>
</tr>
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<td>#10</td>
<td>0.30</td>
<td>2.76</td>
</tr>
<tr>
<td></td>
<td>#11</td>
<td>0.37</td>
<td>2.82</td>
</tr>
<tr>
<td>S-7</td>
<td>#8</td>
<td>0.22</td>
<td>2.62</td>
</tr>
<tr>
<td></td>
<td>#9</td>
<td>0.27</td>
<td>2.69</td>
</tr>
<tr>
<td></td>
<td>#10</td>
<td>0.35</td>
<td>2.76</td>
</tr>
<tr>
<td>S-8</td>
<td>#8</td>
<td>0.25</td>
<td>2.62</td>
</tr>
<tr>
<td></td>
<td>#9</td>
<td>0.31</td>
<td>2.69</td>
</tr>
<tr>
<td>ST-10</td>
<td>#8</td>
<td>0.31</td>
<td>2.62</td>
</tr>
<tr>
<td></td>
<td>#9</td>
<td>0.39</td>
<td>2.69</td>
</tr>
</tbody>
</table>

**Spiral**

<table>
<thead>
<tr>
<th>( t_s = 40,000 \text{ psi} )</th>
<th>#4 at 2'' o c</th>
<th>( t_s = 60,000 \text{ psi} )</th>
<th>#4 at 2.5'' o c</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>2ST-14</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>#8</td>
<td>0.028</td>
<td>3.06</td>
<td>3.63</td>
</tr>
<tr>
<td>#9</td>
<td>0.035</td>
<td>3.26</td>
<td>3.64</td>
</tr>
</tbody>
</table>

**2ST-17**

<p>| <strong>2ST-17</strong>                     |               |                               |               |
| #8                             | 0.032         | 3.20                          | 3.55           | 550 | 68.4 | 57.8 | 49.7 | 43.9 | 39.1 | 35.3 |
| #9                             | 0.041         | 3.41                          | 3.84           | 621 | 75.6 | 63.8 | 55.1 | 48.5 | 43.3 | 39.3 |</p>
<table>
<thead>
<tr>
<th>Reinforcing</th>
<th>e/t &lt; 1.0</th>
<th>N</th>
<th>e/t</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$f_t = 16,000$</td>
<td>$f_t = 20,000$</td>
<td>$f_t = 3000$ psi</td>
</tr>
<tr>
<td></td>
<td>$12.56$</td>
<td>$P$</td>
<td>$12.56$</td>
</tr>
<tr>
<td>No.</td>
<td>Size</td>
<td>$p_g$</td>
<td>$d'$</td>
</tr>
<tr>
<td>T-4</td>
<td>bxt=12&quot;x16&quot;</td>
<td>$A_g=192$ sq in.</td>
<td>Weight = 200 lb/ft</td>
</tr>
<tr>
<td>#8</td>
<td>.016</td>
<td>2.37</td>
<td></td>
</tr>
<tr>
<td>#9</td>
<td>.021</td>
<td>2.44</td>
<td></td>
</tr>
<tr>
<td>#10</td>
<td>.026</td>
<td>2.51</td>
<td></td>
</tr>
<tr>
<td>#11</td>
<td>.032</td>
<td>2.58</td>
<td></td>
</tr>
<tr>
<td>T-4</td>
<td>bxt=14&quot;x16&quot;</td>
<td>$A_g=224$ sq in.</td>
<td>Weight = 234 lb/ft</td>
</tr>
<tr>
<td>#8</td>
<td>.014</td>
<td>2.37</td>
<td></td>
</tr>
<tr>
<td>#9</td>
<td>.018</td>
<td>2.44</td>
<td></td>
</tr>
<tr>
<td>#10</td>
<td>.023</td>
<td>2.51</td>
<td></td>
</tr>
<tr>
<td>#11</td>
<td>.028</td>
<td>2.58</td>
<td></td>
</tr>
<tr>
<td>T-8</td>
<td>bxt=18&quot;x16&quot;</td>
<td>$A_g=288$ sq in.</td>
<td>Weight = 300 lb/ft</td>
</tr>
<tr>
<td>#9</td>
<td>.021</td>
<td>2.44</td>
<td></td>
</tr>
<tr>
<td>#10</td>
<td>.026</td>
<td>2.51</td>
<td></td>
</tr>
<tr>
<td>#11</td>
<td>.032</td>
<td>2.58</td>
<td></td>
</tr>
<tr>
<td>T-8</td>
<td>bxt=20&quot;x16&quot;</td>
<td>$A_g=320$ sq in.</td>
<td>Weight = 334 lb/ft</td>
</tr>
<tr>
<td>#9</td>
<td>.019</td>
<td>2.44</td>
<td></td>
</tr>
<tr>
<td>#10</td>
<td>.024</td>
<td>2.51</td>
<td></td>
</tr>
<tr>
<td>#11</td>
<td>.029</td>
<td>2.58</td>
<td></td>
</tr>
<tr>
<td>T-8</td>
<td>bxt=20&quot;x16&quot;</td>
<td>$A_g=320$ sq in.</td>
<td>Weight = 334 lb/ft</td>
</tr>
<tr>
<td>#9</td>
<td>.025</td>
<td>2.44</td>
<td></td>
</tr>
<tr>
<td>#10</td>
<td>.032</td>
<td>2.51</td>
<td></td>
</tr>
<tr>
<td>#11</td>
<td>.039</td>
<td>2.58</td>
<td></td>
</tr>
</tbody>
</table>
Table 9-45c. Square, Rectangular, and Round Columns, Values of \( P \) and \( N \) with \( f'_c = 3000 \text{ psi} \). 18-in. Square and Rectangular Columns, Tied and Spiral Reinforcing

<table>
<thead>
<tr>
<th>Reinforcing</th>
<th>e/t &lt; 10</th>
<th>N</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>T-6</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>#8</td>
<td>0.15</td>
<td>2.37</td>
<td>1.76</td>
</tr>
<tr>
<td>#9</td>
<td>0.18</td>
<td>2.44&quot;</td>
<td>1.81</td>
</tr>
<tr>
<td>#10</td>
<td>0.24</td>
<td>2.51&quot;</td>
<td>1.86</td>
</tr>
<tr>
<td>#11</td>
<td>0.29</td>
<td>2.58&quot;</td>
<td>1.91</td>
</tr>
</tbody>
</table>

| **T-8**     |         |    |          |
| #8          | 0.19    | 2.37" | 1.92 | 256 | 2.07 | 276 | 47.1 | 39.9 | 34.7 | 30.8 | 27.7 |
| #9          | 0.25    | 2.44" | 2.00 | 277 | 2.18 | 303 | 51.7 | 44.1 | 38.5 | 34.2 | 30.7 | 27.8 |
| #10         | 0.31    | 2.51" | 2.09 | 305 | 2.30 | 337 | 57.7 | 49.0 | 42.8 | 37.9 | 34.2 | 31.1 |
| #11         | 0.38    | 2.58" | 2.18 | 335 | 2.44 | 375 | 63.4 | 54.1 | 47.2 | 42.0 | 37.7 | 34.3 |

| **S-6**     |         |    |          |
| #9          | 0.18    | 2.69" | 2.57 | 315 | 2.76 | 339 | 44.0 | 33.5 | 28.9 |
| #10         | 0.23    | 2.76" | 2.72 | 341 | 2.96 | 371 | 43.8 | 37.0 | 31.7 | 28.0 | 25.0 | 22.5 |
| #11         | 0.29    | 2.83" | 2.85 | 369 | 3.15 | 406 | 47.5 | 39.9 | 34.5 | 30.3 | 27.1 | 24.5 |

| **S-7**     |         |    |          |
| #9          | 0.22    | 2.69" | 2.65 | 331 | 2.88 | 359 | 42.7 | 36.0 | 30.9 | 27.2 | 24.4 |
| #10         | 0.27    | 2.76" | 2.80 | 361 | 3.07 | 397 | 46.8 | 39.2 | 33.9 | 29.8 | 26.5 | 24.0 |
| #11         | 0.34    | 2.83" | 3.06 | 394 | 3.39 | 437 | 49.4 | 41.6 | 36.1 | 31.6 | 28.4 | 25.5 |

| **S-8**     |         |    |          |
| #9          | 0.25    | 2.69" | 2.73 | 347 | 2.98 | 379 | 45.0 | 38.1 | 32.7 | 28.8 | 25.6 | 23.2 |
| #10         | 0.31    | 2.76" | 2.89 | 381 | 3.20 | 422 | 49.4 | 41.5 | 36.0 | 31.5 | 28.2 | 25.4 |
| #11         | 0.38    | 2.83" | 3.05 | 419 | 3.41 | 468 | 53.3 | 45.1 | 38.9 | 34.2 | 30.5 | 27.7 |

| **ST-10**   |         |    |          |
| #8          | 0.025   | 2.62" | 2.58 | 345 | 2.82 | 377 | 51.9 | 43.7 | 38.0 | 33.3 | 30.0 | 27.1 |
| #9          | 0.031   | 2.69" | 2.68 | 379 | 2.97 | 419 | 56.9 | 48.1 | 41.7 | 36.8 | 32.9 | 29.9 |
| #10         | 0.039   | 2.76" | 2.82 | 422 | 3.16 | 473 | 63.6 | 53.7 | 46.8 | 41.2 | 36.9 | 33.3 |

| **ST-14**   |         |    |          |
| #8          | 0.022   | 2.62" | 2.56 | 517 | 2.78 | 561 | 74.2 | 62.1 | 53.9 | 47.5 | 42.3 | 38.2 |
| #9          | 0.028   | 2.69" | 2.68 | 564 | 2.95 | 620 | 81.1 | 68.5 | 59.3 | 52.4 | 46.8 | 42.4 |
| #10         | 0.035   | 2.76" | 2.82 | 624 | 3.14 | 659 | 90.5 | 76.3 | 65.8 | 58.0 | 52.1 | 47.1 |

| **ST-17**   |         |    |          |
| #8          | 0.025   | 2.62" | 2.58 | 580 | 2.82 | 634 | 81.9 | 69.0 | 59.7 | 52.7 | 47.3 | 42.6 |
| #9          | 0.031   | 2.69" | 2.78 | 637 | 3.08 | 705 | 90.9 | 76.4 | 66.3 | 58.4 | 52.3 | 47.3 |
| #10         | 0.040   | 2.76" | 2.94 | 710 | 3.30 | 797 | 102.4 | 84.8 | 73.6 | 64.9 | 57.9 | 52.4 |

Spiral: \( f'_c = 40,000 \text{ psi} \) #5 at 2\( \frac{3}{4} \) at 2\( \frac{3}{4} \) o c

| **ST**      |         |    |          |
| #8          | 0.025   | 2.62" | 2.58 | 580 | 2.82 | 634 | 81.9 | 69.0 | 59.7 | 52.7 | 47.3 | 42.6 |
| #9          | 0.031   | 2.69" | 2.78 | 637 | 3.08 | 705 | 90.9 | 76.4 | 66.3 | 58.4 | 52.3 | 47.3 |
| #10         | 0.040   | 2.76" | 2.94 | 710 | 3.30 | 797 | 102.4 | 84.8 | 73.6 | 64.9 | 57.9 | 52.4 |
## REINFORCED-CONCRETE BUILDING FRAMES

Table 9-45f. Square, Rectangular, and Round Columns, Values of \( P \) and \( N \) with \( f_y' = 3,000 \text{ psi} \). 18-in. Rectangular Columns, Tied Reinforcing

<table>
<thead>
<tr>
<th>Reinforcing</th>
<th>( e/t &lt; 1.0 )</th>
<th>( N )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( f_y = 16,000 )</td>
<td>( f_y = 20,000 )</td>
</tr>
<tr>
<td>No.</td>
<td>Size</td>
<td>( P_g )</td>
</tr>
<tr>
<td>------------</td>
<td>------</td>
<td>-----------</td>
</tr>
<tr>
<td><strong>T-4</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>#8</td>
<td>0.12</td>
<td>2.37</td>
</tr>
<tr>
<td>#9</td>
<td>0.16</td>
<td>2.44</td>
</tr>
<tr>
<td>#10</td>
<td>0.20</td>
<td>2.51</td>
</tr>
<tr>
<td>#11</td>
<td>0.25</td>
<td>2.58</td>
</tr>
<tr>
<td><strong>T-6</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>#8</td>
<td>0.016</td>
<td>2.37</td>
</tr>
<tr>
<td>#9</td>
<td>0.021</td>
<td>2.44</td>
</tr>
<tr>
<td>#10</td>
<td>0.026</td>
<td>2.51</td>
</tr>
<tr>
<td>#11</td>
<td>0.033</td>
<td>2.58</td>
</tr>
<tr>
<td><strong>T-8</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>#8</td>
<td>0.022</td>
<td>2.37</td>
</tr>
<tr>
<td>#9</td>
<td>0.028</td>
<td>2.44</td>
</tr>
<tr>
<td>#10</td>
<td>0.033</td>
<td>2.51</td>
</tr>
<tr>
<td><strong>T-10</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>#9</td>
<td>0.022</td>
<td>2.44</td>
</tr>
<tr>
<td>#10</td>
<td>0.028</td>
<td>2.51</td>
</tr>
<tr>
<td>#11</td>
<td>0.035</td>
<td>2.58</td>
</tr>
<tr>
<td><strong>T-10</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>#9</td>
<td>0.028</td>
<td>2.44</td>
</tr>
<tr>
<td>#10</td>
<td>0.035</td>
<td>2.51</td>
</tr>
<tr>
<td><strong>T-10</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>#9</td>
<td>0.020</td>
<td>2.44</td>
</tr>
<tr>
<td>#10</td>
<td>0.026</td>
<td>2.51</td>
</tr>
<tr>
<td>#11</td>
<td>0.031</td>
<td>2.58</td>
</tr>
<tr>
<td><strong>T-10</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>#9</td>
<td>0.025</td>
<td>2.44</td>
</tr>
<tr>
<td>#10</td>
<td>0.032</td>
<td>2.51</td>
</tr>
<tr>
<td>#11</td>
<td>0.039</td>
<td>2.58</td>
</tr>
</tbody>
</table>
### Tables for Design of Reinforced Concrete

#### Table 9-45g. Square, Rectangular, and Round Columns, Values of $P$ and $N$ with $f'_c = 3,000$ psi. 20-in. Square and Rectangular Columns, Tied and Spiral Reinforcing

<table>
<thead>
<tr>
<th>Reinforcing</th>
<th>$e/t &lt; 1.0$</th>
<th>$N$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$f'_c = 16,000$</td>
<td>$f'_c = 20,000$</td>
</tr>
<tr>
<td></td>
<td>$E = 12,300$ $t$</td>
<td>$E = 12,300$ $t$</td>
</tr>
<tr>
<td>No.</td>
<td>$P_g$</td>
<td>$d'$</td>
</tr>
<tr>
<td>-----</td>
<td>-------</td>
<td>------</td>
</tr>
<tr>
<td>T-8</td>
<td>#9</td>
<td>0.20</td>
</tr>
<tr>
<td></td>
<td>#10</td>
<td>0.25</td>
</tr>
<tr>
<td></td>
<td>#11</td>
<td>0.31</td>
</tr>
<tr>
<td>T-10</td>
<td>#9</td>
<td>0.25</td>
</tr>
<tr>
<td></td>
<td>#10</td>
<td>0.32</td>
</tr>
<tr>
<td></td>
<td>#11</td>
<td>0.39</td>
</tr>
<tr>
<td>S-8</td>
<td>#9</td>
<td>0.20</td>
</tr>
<tr>
<td></td>
<td>#10</td>
<td>0.25</td>
</tr>
<tr>
<td></td>
<td>#11</td>
<td>0.31</td>
</tr>
<tr>
<td>S-9</td>
<td>#9</td>
<td>0.22</td>
</tr>
<tr>
<td></td>
<td>#10</td>
<td>0.29</td>
</tr>
<tr>
<td></td>
<td>#11</td>
<td>0.35</td>
</tr>
<tr>
<td>ST-10</td>
<td>#9</td>
<td>0.25</td>
</tr>
<tr>
<td></td>
<td>#10</td>
<td>0.32</td>
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<tr>
<td></td>
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<td>0.39</td>
</tr>
<tr>
<td>ST-12</td>
<td>#9</td>
<td>0.30</td>
</tr>
<tr>
<td></td>
<td>#10</td>
<td>0.38</td>
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<tr>
<td></td>
<td>#11</td>
<td>0.47</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Reinforcing</th>
<th>$e/t = 20&quot; x 20&quot;$</th>
<th>$N$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$f'_c = 3000$ psi</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$E = 580$ sq in.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$W = 604$ lb/ft.</td>
<td></td>
</tr>
<tr>
<td>No.</td>
<td>$P_g$</td>
<td>$d'$</td>
</tr>
<tr>
<td>-----</td>
<td>-------</td>
<td>------</td>
</tr>
<tr>
<td>2ST-16</td>
<td>#9</td>
<td>0.28</td>
</tr>
<tr>
<td></td>
<td>#10</td>
<td>0.35</td>
</tr>
<tr>
<td></td>
<td>#11</td>
<td>0.43</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Reinforcing</th>
<th>$e/t = 30&quot; x 30&quot;$</th>
<th>$N$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$f'_c = 3000$ psi</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$E = 600$ sq in.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$W = 615$ lb/ft.</td>
<td></td>
</tr>
<tr>
<td>No.</td>
<td>$P_g$</td>
<td>$d'$</td>
</tr>
<tr>
<td>-----</td>
<td>-------</td>
<td>------</td>
</tr>
<tr>
<td>2ST-14</td>
<td>#9</td>
<td>0.23</td>
</tr>
<tr>
<td></td>
<td>#10</td>
<td>0.30</td>
</tr>
<tr>
<td></td>
<td>#11</td>
<td>0.36</td>
</tr>
</tbody>
</table>

| Spiral | $f'_c = 40,000$ psi | #5 at $2\frac{1}{2}"$ o.c. | $f'_c = 60,000$ psi | #4 at $2\frac{1}{2}"$ o.c. | $f'_c = 13,300$ in$^4$ |
|        |                    |                               |                  |                           |

---

Note: The table provides design values for reinforced concrete columns, including the effective width ($e$), the area of steel ($A_g$), the weight of steel ($W$), the effective length ($t$), and the concrete strength ($f'_c$) for different configurations of columns. The values are calculated for different ratios ($e/t$) and reinforcement sizes. The table is designed to help engineers in the calculation of the required concrete and steel for various column designs.
### Table 9-45h. Square, Rectangular, and Round Columns, Values of $P$ and $N$ with $f_c' = 3,000$ psi. 20-in. Rectangular Columns, Tied Reinforcing

<table>
<thead>
<tr>
<th>Reinforcing</th>
<th>$e/t &lt; 1.0$</th>
<th>$N$</th>
</tr>
</thead>
</table>
|             | $f_s = 16,000$ | $f_s = 20,000$ | $e/t$ | $1.0$ | $1.2$ | $1.4$ | $1.6$ | $1.8$ | $2.0$
| No. | Size $P_g$ $d$ | $b 	imes t = 16'' 	imes 20''$ | $A_g = 320$ sq in | Weight = 334 lb/ft | $I_c = 10,700$ in$^4$ | $b 	imes t = 18'' 	imes 20''$ | $A_g = 360$ sq in | Weight = 375 lb/ft | $I_c = 12,000$ in$^4$ | $b 	imes t = 22'' 	imes 20''$ | $A_g = 440$ sq in | Weight = 458 lb/ft | $I_c = 14,700$ in$^4$ | $b 	imes t = 24'' 	imes 20''$ | $A_g = 480$ sq in | Weight = 500 lb/ft | $I_c = 16,000$ in$^4$
| #8 | .015 2.37 1.56 234 1.67 249 47.5 40.5 35.4 |
| #9 | .019 2.44 1.59 250 1.72 269 52.6 44.9 39.5 35.0 31.5 |
| #10 | .024 2.51 1.64 270 1.78 295 58.2 49.9 43.8 39.2 35.3 33.6 |
| #11 | .029 2.59 1.69 293 1.87 323 64.3 56.3 48.6 43.4 39.0 35.6 |
| #9 | .025 2.44 1.76 275 1.93 301 53.2 45.5 39.7 35.3 31.7 28.8 |
| #10 | .032 2.51 1.85 303 2.04 335 59.1 50.3 44.2 39.4 35.3 32.2 |
| #11 | .039 2.58 1.93 333 2.16 373 65.4 55.8 48.7 43.5 39.1 35.5 |
| #9 | .017 2.44 1.59 271 1.69 290 55.7 47.4 41.5 36.9 |
| #10 | .021 2.51 1.62 291 1.76 316 62.2 53.1 46.5 41.5 37.4 34.2 |
| #11 | .026 2.59 1.66 314 1.80 344 68.1 58.3 51.3 45.9 41.3 37.6 |
| #9 | .022 2.44 1.74 296 1.90 322 56.7 48.1 41.9 37.3 33.7 30.6 |
| #10 | .028 2.51 1.81 324 1.99 356 62.5 50.7 46.8 41.7 37.4 33.9 |
| #11 | .035 2.58 1.86 354 2.09 394 68.5 58.8 51.5 45.7 41.2 37.4 |
| #9 | .018 2.44 1.70 340 1.82 366 62.7 53.2 46.6 41.3 |
| #10 | .023 2.51 1.76 368 1.91 400 69.6 58.9 51.5 45.9 41.2 37.5 |
| #11 | .028 2.59 1.82 398 2.00 438 76.5 62.1 56.9 50.7 45.5 41.3 |
| #9 | .023 2.44 1.73 366 1.88 398 71.7 61.1 53.5 47.6 42.9 39.0 |
| #10 | .029 2.51 1.79 400 1.97 441 79.8 68.1 59.7 52.9 47.7 43.6 |
| #11 | .035 2.58 1.85 438 2.06 487 88.6 75.8 66.1 58.8 52.9 48.0 |
| #9 | .017 2.44 1.68 361 1.79 387 66.4 56.4 49.0 |
| #10 | .021 2.51 1.74 389 1.89 421 72.8 62.1 54.2 48.0 43.3 39.2 |
| #11 | .026 2.59 1.79 419 1.97 459 80.0 68.2 59.6 52.8 47.5 43.2 |
| #9 | .021 2.44 1.71 387 1.85 419 75.2 64.3 55.9 49.9 44.8 40.6 |
| #10 | .026 2.51 1.77 421 1.94 462 83.7 71.4 62.2 55.3 49.9 45.5 |
| #11 | .032 2.59 1.83 459 2.02 508 92.3 78.9 68.7 61.3 55.0 50.2 |
| #9 | .025 2.44 1.79 412 1.96 451 76.0 64.7 56.5 50.0 45.1 40.9 |
| #10 | .032 2.51 1.88 454 2.08 503 83.6 71.4 62.4 55.3 49.8 45.3 |
| #11 | .039 2.58 1.95 498 2.17 558 93.3 79.8 69.8 61.7 55.4 50.2 |
### Tables for Design of Reinforced Concrete

**Table 9-45i:** Square, Rectangular, and Round Columns, Values of \( P \) and \( N \) with \( f' = 3000 \text{ psi} \).

#### 22-in. Square and Rectangular Columns, Tied and Spiral Reinforcing

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<p>| Spiral | ( f' = 40,000 \text{ psi} ) | ( # 5 @ 21/2^\circ ) | ( f' = 60,000 \text{ psi} ) | ( # 4 @ 2 ) | ( \circ ) |</p>
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<td>Weight = 413 lb/ft</td>
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<td>Weight = 458 lb/ft</td>
</tr>
<tr>
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<td>Ag = 528 sq.in.</td>
<td>Weight = 552 lb/ft</td>
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<td>Ag = 572 sq.in.</td>
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### Tables for Design of Reinforced Concrete

#### Table 9-46k. Square, Rectangular, and Round Columns, Values of P and N with $f' = 3,000$ psi. 24-in. Square Columns, Tied and Spiral Reinforcing

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**Spiral**

- $f_s = 40,000$ psi, #5 at $2\frac{1}{2}°$ o c
- $f_s = 60,000$ psi, #4 at $2\frac{1}{2}°$ o c
### Table 9-45i. Square, Rectangular, and Round Columns, Values of $P$ and $N$ with $f_{c'} = 3,000$ psi. 24-in. Rectangular Columns, Tied Reinforcing

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<tr>
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<td>$A_g = 480$ sq in.</td>
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<tr>
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<td>$A_g = 480$ sq in.</td>
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<td>$A_g = 624$ sq in.</td>
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</tbody>
</table>

Note: The values in the table represent the flexural strength and stability of columns with different reinforcing sizes and cross-sectional dimensions.
Table 9-45m. Square, Rectangular, and Round Columns, Values of $P$ and $N$ with $f'_c = 3,000$ psi. 26-in. and 28-in. Square Columns, Tied and Spiral Reinforcing

<table>
<thead>
<tr>
<th>Reinforcing</th>
<th>$e/t &lt; 10$</th>
<th>$N$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$f_s = 16,000$</td>
<td>$f_s = 20,000$</td>
</tr>
<tr>
<td>No.</td>
<td>Size</td>
<td>$P_g$</td>
</tr>
<tr>
<td>T-12</td>
<td>#10</td>
<td>0.23</td>
</tr>
<tr>
<td></td>
<td>#11</td>
<td>0.28</td>
</tr>
<tr>
<td>T-14</td>
<td>#10</td>
<td>0.26</td>
</tr>
<tr>
<td></td>
<td>#11</td>
<td>0.32</td>
</tr>
<tr>
<td>T-16</td>
<td>#10</td>
<td>0.30</td>
</tr>
<tr>
<td></td>
<td>#11</td>
<td>0.37</td>
</tr>
<tr>
<td>S-12</td>
<td>#10</td>
<td>0.23</td>
</tr>
<tr>
<td></td>
<td>#11</td>
<td>0.28</td>
</tr>
<tr>
<td>S-13</td>
<td>#10</td>
<td>0.24</td>
</tr>
<tr>
<td></td>
<td>#11</td>
<td>0.30</td>
</tr>
<tr>
<td>S-14</td>
<td>#10</td>
<td>0.26</td>
</tr>
<tr>
<td></td>
<td>#11</td>
<td>0.32</td>
</tr>
<tr>
<td>ST-12</td>
<td>#10</td>
<td>0.23</td>
</tr>
<tr>
<td></td>
<td>#11</td>
<td>0.28</td>
</tr>
<tr>
<td>ST-14</td>
<td>#10</td>
<td>0.26</td>
</tr>
<tr>
<td></td>
<td>#11</td>
<td>0.32</td>
</tr>
<tr>
<td>ST-16</td>
<td>#10</td>
<td>0.30</td>
</tr>
<tr>
<td></td>
<td>#11</td>
<td>0.37</td>
</tr>
</tbody>
</table>

| b x t = 28" x 28" | $f_c' = 3,000$ psi | $P_g = 784$ sq.in. | $I_c = 51,200$ in$^4$ |

| T-12 | #10 | 0.19 | 2.51" | 1.19 | 618 | 1.29 | 667 | 119.0 | 100.3 | 87.3 | 77.3 |
|      | #11 | 0.24 | 2.58" | 1.23 | 662 | 1.34 | 722 | 130.7 | 110.3 | 96.5 | 85.5 | 77.0 | 69.8 |
| T-14 | #10 | 0.23 | 2.51" | 1.20 | 650 | 1.31 | 707 | 130.1 | 112.0 | 98.0 | 87.1 | 77.8 | 71.4 |
|      | #11 | 0.28 | 2.58" | 1.24 | 702 | 1.37 | 772 | 142.3 | 123.3 | 107.6 | 96.0 | 85.8 | 78.1 |
| T-16 | #10 | 0.26 | 2.51" | 1.25 | 683 | 1.36 | 748 | 135.1 | 115.3 | 101.1 | 90.5 | 81.9 | 73.5 |
|      | #11 | 0.32 | 2.58" | 1.29 | 742 | 1.42 | 822 | 148.8 | 127.1 | 110.2 | 98.6 | 89.4 | 80.5 |

Spiral: $f_s' = 40,000$ psi, #5 at $2\frac{1}{2}''$ oc | $f_s = 60,000$ psi, #4 at $2\frac{1}{2}''$ oc
# REINFORCED-CONCRETE BUILDING FRAMES

Table 9-45n. Square, Rectangular, and Round Columns, Values of $P$ and $N$ with $f'_c = 3,000$ psi. 28-in. and 30-in. Square Columns, Tied and Spiral Reinforcing

<table>
<thead>
<tr>
<th>No.</th>
<th>Size</th>
<th>$P_g$</th>
<th>$d'$</th>
<th>$f'_c = 16,000$</th>
<th>$f'_c = 20,000$</th>
<th>$N$</th>
<th>$e/t &lt; 1.0$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Weight = 816 lb/ft</td>
<td>Weight = 816 lb/ft</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$12\frac{0}{T}$</td>
<td>$12\frac{0}{T}$</td>
<td>$P$</td>
<td>$P$</td>
</tr>
<tr>
<td>S-13</td>
<td>#10</td>
<td>.021</td>
<td>2.76&quot;</td>
<td>1.63</td>
<td>793</td>
<td>1.76</td>
<td>859</td>
</tr>
<tr>
<td></td>
<td>#11</td>
<td>.026</td>
<td>2.83&quot;</td>
<td>1.70</td>
<td>852</td>
<td>1.86</td>
<td>934</td>
</tr>
<tr>
<td>S-14</td>
<td>#10</td>
<td>.023</td>
<td>2.76&quot;</td>
<td>1.65</td>
<td>813</td>
<td>1.80</td>
<td>884</td>
</tr>
<tr>
<td></td>
<td>#11</td>
<td>.028</td>
<td>2.83&quot;</td>
<td>1.72</td>
<td>878</td>
<td>1.89</td>
<td>966</td>
</tr>
<tr>
<td>S-15</td>
<td>#10</td>
<td>.024</td>
<td>2.76&quot;</td>
<td>1.67</td>
<td>834</td>
<td>1.83</td>
<td>910</td>
</tr>
<tr>
<td></td>
<td>#11</td>
<td>.030</td>
<td>2.83&quot;</td>
<td>1.74</td>
<td>903</td>
<td>1.93</td>
<td>997</td>
</tr>
<tr>
<td>ST-14</td>
<td>#10</td>
<td>.023</td>
<td>2.76&quot;</td>
<td>1.57</td>
<td>813</td>
<td>1.71</td>
<td>884</td>
</tr>
<tr>
<td></td>
<td>#11</td>
<td>.028</td>
<td>2.83&quot;</td>
<td>1.61</td>
<td>878</td>
<td>1.78</td>
<td>966</td>
</tr>
<tr>
<td>ST-16</td>
<td>#10</td>
<td>.026</td>
<td>2.76&quot;</td>
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<td>854</td>
<td>1.77</td>
<td>935</td>
</tr>
<tr>
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<td>#11</td>
<td>.032</td>
<td>2.83&quot;</td>
<td>1.67</td>
<td>928</td>
<td>1.85</td>
<td>1028</td>
</tr>
<tr>
<td>ST-18</td>
<td>#10</td>
<td>.029</td>
<td>2.76&quot;</td>
<td>1.63</td>
<td>895</td>
<td>1.83</td>
<td>985</td>
</tr>
<tr>
<td></td>
<td>#11</td>
<td>.036</td>
<td>2.83&quot;</td>
<td>1.72</td>
<td>977</td>
<td>1.91</td>
<td>1091</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th>$f'_c = 3000$ psi</th>
<th>$f'_c = 3000$ psi</th>
<th>$I_c = 900$ sq in.</th>
<th>$I_c = 900$ sq in.</th>
<th>$t_s = 30&quot;$ x $30&quot;$</th>
<th>$t_s = 30&quot;$ x $30&quot;$</th>
<th>$t_s = 30&quot;$ x $30&quot;$</th>
<th>$t_s = 30&quot;$ x $30&quot;$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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<td></td>
<td>Weight = 940 lb/ft</td>
<td>Weight = 940 lb/ft</td>
<td>$12\frac{0}{T}$</td>
<td>$12\frac{0}{T}$</td>
<td>$P$</td>
<td>$P$</td>
<td>$1.0$</td>
<td>$1.2$</td>
</tr>
<tr>
<td>T-16</td>
<td>#10</td>
<td>.023</td>
<td>2.51&quot;</td>
<td>1.13</td>
<td>746</td>
<td>1.24</td>
<td>811</td>
<td>148.3</td>
<td>126.7</td>
<td>109.7</td>
<td>97.2</td>
</tr>
<tr>
<td></td>
<td>#11</td>
<td>.028</td>
<td>2.53&quot;</td>
<td>1.17</td>
<td>805</td>
<td>1.28</td>
<td>885</td>
<td>162.0</td>
<td>137.8</td>
<td>120.8</td>
<td>107.8</td>
</tr>
<tr>
<td>S-14</td>
<td>#10</td>
<td>.020</td>
<td>2.76&quot;</td>
<td>1.52</td>
<td>891</td>
<td>1.61</td>
<td>962</td>
<td>113.5</td>
<td>105.5</td>
<td>91.8</td>
<td></td>
</tr>
<tr>
<td></td>
<td>#11</td>
<td>.024</td>
<td>2.83&quot;</td>
<td>1.55</td>
<td>956</td>
<td>1.70</td>
<td>1049</td>
<td>139.7</td>
<td>118.2</td>
<td>101.4</td>
<td>89.9</td>
</tr>
<tr>
<td>S-15</td>
<td>#10</td>
<td>.021</td>
<td>2.76&quot;</td>
<td>1.51</td>
<td>912</td>
<td>1.64</td>
<td>988</td>
<td>132.1</td>
<td>111.3</td>
<td>96.2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>#11</td>
<td>.026</td>
<td>2.83&quot;</td>
<td>1.58</td>
<td>981</td>
<td>1.72</td>
<td>1075</td>
<td>143.3</td>
<td>120.0</td>
<td>107.1</td>
<td>91.8</td>
</tr>
<tr>
<td>S-16</td>
<td>#10</td>
<td>.023</td>
<td>2.76&quot;</td>
<td>1.53</td>
<td>932</td>
<td>1.66</td>
<td>1013</td>
<td>135.9</td>
<td>114.5</td>
<td>107.0</td>
<td>86.6</td>
</tr>
<tr>
<td></td>
<td>#11</td>
<td>.028</td>
<td>2.83&quot;</td>
<td>1.59</td>
<td>1006</td>
<td>1.73</td>
<td>1106</td>
<td>146.6</td>
<td>124.1</td>
<td>108.0</td>
<td>95.4</td>
</tr>
<tr>
<td>ST-16</td>
<td>#10</td>
<td>.023</td>
<td>2.76&quot;</td>
<td>1.46</td>
<td>932</td>
<td>1.59</td>
<td>1013</td>
<td>148.4</td>
<td>125.7</td>
<td>107.8</td>
<td>96.0</td>
</tr>
<tr>
<td></td>
<td>#11</td>
<td>.028</td>
<td>2.83&quot;</td>
<td>1.51</td>
<td>1006</td>
<td>1.66</td>
<td>1106</td>
<td>162.8</td>
<td>138.1</td>
<td>119.5</td>
<td>105.8</td>
</tr>
<tr>
<td>ST-18</td>
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<td>.025</td>
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<td>973</td>
<td>1.64</td>
<td>1063</td>
<td>155.0</td>
<td>131.2</td>
<td>114.7</td>
<td>101.3</td>
</tr>
<tr>
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<td>.031</td>
<td>2.83&quot;</td>
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<td>1055</td>
<td>1.71</td>
<td>1169</td>
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<td>144.9</td>
<td>125.1</td>
<td>111.4</td>
</tr>
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<td>#10</td>
<td>.028</td>
<td>2.76&quot;</td>
<td>1.53</td>
<td>1013</td>
<td>1.68</td>
<td>1115</td>
<td>163.0</td>
<td>138.8</td>
<td>119.4</td>
<td>104.6</td>
</tr>
<tr>
<td></td>
<td>#11</td>
<td>.035</td>
<td>2.83&quot;</td>
<td>1.58</td>
<td>1105</td>
<td>1.77</td>
<td>1231</td>
<td>176.9</td>
<td>151.2</td>
<td>131.2</td>
<td>116.2</td>
</tr>
</tbody>
</table>

Spiral $f'_c = 40,000$ psi, #5 at $2\frac{1}{4}$ o.c $f'_c = 60,000$ psi, #4 at $2\frac{1}{4}$ o.c
### Table 9-45a. Square, Rectangular, and Round Columns, Values of $P$ and $N$ with $f_c' = 3,000$ psi. 20-in. to 30-in. Square Columns, Tied Double Spiral Reinforcing

<table>
<thead>
<tr>
<th>Col No.</th>
<th>Reinf Size</th>
<th>$d'$</th>
<th>$\frac{12.5d}{T}$</th>
<th>$P$</th>
<th>$n$</th>
<th>Size</th>
<th>Reinf : inner spiral</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$p_a$</td>
<td>$p_m$</td>
<td>$e/t &lt; 1.0$</td>
<td></td>
<td></td>
<td>No. vertical bars</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>$f_s' = 3000$ psi</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20”x20” SST-13</td>
<td>#10</td>
<td>0.41</td>
<td>2.76”</td>
<td>2.62</td>
<td>534</td>
<td>#10</td>
<td>0.057</td>
</tr>
<tr>
<td></td>
<td>#11</td>
<td>0.51</td>
<td>2.83”</td>
<td>2.75</td>
<td>593</td>
<td>#11</td>
<td>0.070</td>
</tr>
<tr>
<td>22”x22” SST-14</td>
<td>#10</td>
<td>0.37</td>
<td>2.76”</td>
<td>2.37</td>
<td>611</td>
<td>#10</td>
<td>0.052</td>
</tr>
<tr>
<td></td>
<td>#11</td>
<td>0.45</td>
<td>2.83”</td>
<td>2.45</td>
<td>676</td>
<td>#11</td>
<td>0.064</td>
</tr>
<tr>
<td>24”x24” SST-15</td>
<td>#10</td>
<td>0.33</td>
<td>2.76”</td>
<td>2.00</td>
<td>694</td>
<td>#10</td>
<td>0.048</td>
</tr>
<tr>
<td></td>
<td>#11</td>
<td>0.41</td>
<td>2.83”</td>
<td>2.09</td>
<td>763</td>
<td>#11</td>
<td>0.059</td>
</tr>
<tr>
<td>26”x26” SST-16</td>
<td>#10</td>
<td>0.30</td>
<td>2.76”</td>
<td>1.80</td>
<td>781</td>
<td>#10</td>
<td>0.045</td>
</tr>
<tr>
<td></td>
<td>#11</td>
<td>0.37</td>
<td>2.83”</td>
<td>1.88</td>
<td>855</td>
<td>#11</td>
<td>0.055</td>
</tr>
<tr>
<td>28”x28” SST-18</td>
<td>#10</td>
<td>0.29</td>
<td>2.76”</td>
<td>1.63</td>
<td>895</td>
<td>#10</td>
<td>0.043</td>
</tr>
<tr>
<td></td>
<td>#11</td>
<td>0.36</td>
<td>2.83”</td>
<td>1.72</td>
<td>977</td>
<td>#11</td>
<td>0.054</td>
</tr>
<tr>
<td>30”x30” SST-20</td>
<td>#10</td>
<td>0.28</td>
<td>2.76”</td>
<td>1.53</td>
<td>1013</td>
<td>#10</td>
<td>0.042</td>
</tr>
<tr>
<td></td>
<td>#11</td>
<td>0.35</td>
<td>2.83”</td>
<td>1.58</td>
<td>1105</td>
<td>#11</td>
<td>0.052</td>
</tr>
<tr>
<td>$f_s' = 20,000$ psi</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20”x20” SST-13</td>
<td>#10</td>
<td>0.41</td>
<td>2.76”</td>
<td>2.94</td>
<td>600</td>
<td>#10</td>
<td>0.057</td>
</tr>
<tr>
<td></td>
<td>#11</td>
<td>0.51</td>
<td>2.83”</td>
<td>3.13</td>
<td>675</td>
<td>#11</td>
<td>0.070</td>
</tr>
<tr>
<td>22”x22” SST-14</td>
<td>#10</td>
<td>0.37</td>
<td>2.76”</td>
<td>2.73</td>
<td>682</td>
<td>#10</td>
<td>0.057</td>
</tr>
<tr>
<td></td>
<td>#11</td>
<td>0.45</td>
<td>2.83”</td>
<td>2.78</td>
<td>764</td>
<td>#11</td>
<td>0.064</td>
</tr>
<tr>
<td>24”x24” SST-15</td>
<td>#10</td>
<td>0.33</td>
<td>2.76”</td>
<td>2.21</td>
<td>770</td>
<td>#10</td>
<td>0.048</td>
</tr>
<tr>
<td></td>
<td>#11</td>
<td>0.41</td>
<td>2.83”</td>
<td>2.34</td>
<td>857</td>
<td>#11</td>
<td>0.059</td>
</tr>
<tr>
<td>26”x26” SST-16</td>
<td>#10</td>
<td>0.30</td>
<td>2.76”</td>
<td>1.98</td>
<td>862</td>
<td>#10</td>
<td>0.045</td>
</tr>
<tr>
<td></td>
<td>#11</td>
<td>0.37</td>
<td>2.83”</td>
<td>2.10</td>
<td>946</td>
<td>#11</td>
<td>0.055</td>
</tr>
<tr>
<td>28”x28” SST-18</td>
<td>#10</td>
<td>0.29</td>
<td>2.76”</td>
<td>1.77</td>
<td>985</td>
<td>#10</td>
<td>0.043</td>
</tr>
<tr>
<td></td>
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<td>0.36</td>
<td>2.83”</td>
<td>1.85</td>
<td>1091</td>
<td>#11</td>
<td>0.054</td>
</tr>
<tr>
<td>30”x30” SST-20</td>
<td>#10</td>
<td>0.28</td>
<td>2.76”</td>
<td>1.68</td>
<td>1115</td>
<td>#10</td>
<td>0.042</td>
</tr>
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<td>2.83”</td>
<td>1.77</td>
<td>1231</td>
<td>#11</td>
<td>0.052</td>
</tr>
</tbody>
</table>

Inner spiral, #2 at 6° oc
Outer spiral same as type ST columns
### Reinforced-Concrete Building Frames

Table 9-45p. Square, Rectangular, and Round Columns, Values of $P$ and $N$ with $f'' = 3,000$ psi. 14-in. to 30-in. Round Columns, Spiral Reinforcing

<table>
<thead>
<tr>
<th>Col diam.</th>
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<th>$f''=16,000$</th>
<th>$f''=20,000$</th>
</tr>
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<tbody>
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<td>$P_g$</td>
<td>$d'$</td>
<td>$P_g$</td>
</tr>
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<td>#6</td>
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<td>0.039</td>
<td>2.69</td>
<td>5.17</td>
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<tr>
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<td>#10</td>
<td>0.049</td>
<td>2.76</td>
<td>5.57</td>
</tr>
<tr>
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<td>#6</td>
<td>0.020</td>
<td>2.50</td>
<td>4.37</td>
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<tr>
<td></td>
<td>#7</td>
<td>0.027</td>
<td>2.56</td>
<td>4.67</td>
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<tr>
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<td>2.62</td>
<td>4.95</td>
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<td>2.76</td>
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<td>4.70</td>
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<td>4.04</td>
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<td>#10</td>
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<td>2.76</td>
<td>4.58</td>
</tr>
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<td>16&quot; R-8</td>
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<td>0.031</td>
<td>2.62</td>
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</tr>
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<td>0.040</td>
<td>2.69</td>
<td>4.47</td>
</tr>
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<td>0.024</td>
<td>2.69</td>
<td>3.44</td>
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<td>0.030</td>
<td>2.76</td>
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<td></td>
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<td>3.82</td>
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<td>2.83</td>
<td>4.08</td>
</tr>
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<td>3.09</td>
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<tr>
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<td>3.16</td>
</tr>
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<td>0.036</td>
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<td>3.49</td>
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<td>2.76</td>
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<tr>
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<td>#11</td>
<td>0.050</td>
<td>2.83</td>
<td>3.57</td>
</tr>
</tbody>
</table>

*For round, $12 \text{ col}^2 = \frac{4}{\pi} 
\times 12 \text{ col}^2$ for square.*
**TABLES FOR DESIGN OF REINFORCED CONCRETE**

Table 9-46a. Square, Rectangular, and Round Columns, Values of $P$ and $N$ with $f'_{c} = 3,750$ psi. 12-in. and 14-in. Square Columns, Tied and Spiral Reinforcing

<table>
<thead>
<tr>
<th>Reinforcing</th>
<th>$A_g = 144$ sq in.</th>
<th>Weight = 150 lb/ft</th>
<th>$f'_{c} = 3,750$ psi</th>
<th>$I_c = 1,728$ in$^4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>No.</td>
<td>Size</td>
<td>$d'$</td>
<td>$e/t &lt; 1.0$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$f_2 = 16,000$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$t_{26}$</td>
</tr>
<tr>
<td>No.</td>
<td>Size</td>
<td>$p_g$</td>
<td>$d'$</td>
<td>$P$</td>
</tr>
<tr>
<td>No.</td>
<td>Size</td>
<td>$p_g$</td>
<td>$d'$</td>
<td>$P$</td>
</tr>
<tr>
<td>T-4</td>
<td>#6</td>
<td>.012</td>
<td>2.25&quot;</td>
<td>2.70</td>
</tr>
<tr>
<td></td>
<td>#7</td>
<td>.017</td>
<td>2.31&quot;</td>
<td>2.78</td>
</tr>
<tr>
<td></td>
<td>#8</td>
<td>.022</td>
<td>2.37&quot;</td>
<td>2.90</td>
</tr>
<tr>
<td></td>
<td>#9</td>
<td>.028</td>
<td>2.44&quot;</td>
<td>3.02</td>
</tr>
<tr>
<td></td>
<td>#10</td>
<td>.035</td>
<td>2.51&quot;</td>
<td>3.16</td>
</tr>
<tr>
<td></td>
<td>#11</td>
<td>.043</td>
<td>2.58&quot;</td>
<td>3.39</td>
</tr>
</tbody>
</table>

**b x t = 14" x 14"**

<table>
<thead>
<tr>
<th>Reinforcing</th>
<th>$A_g = 196$ sq in.</th>
<th>Weight = 204 lb/ft</th>
<th>$f'_{c} = 3,750$ psi</th>
<th>$I_c = 3,200$ in$^4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>No.</td>
<td>Size</td>
<td>$d'$</td>
<td>$e/t &lt; 1.0$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$f_2 = 16,000$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$t_{26}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$t_{26}$</td>
</tr>
<tr>
<td>T-4</td>
<td>#7</td>
<td>.012</td>
<td>2.31&quot;</td>
<td>2.27</td>
</tr>
<tr>
<td></td>
<td>#8</td>
<td>.016</td>
<td>2.37&quot;</td>
<td>2.33</td>
</tr>
<tr>
<td></td>
<td>#9</td>
<td>.020</td>
<td>2.44&quot;</td>
<td>2.40</td>
</tr>
<tr>
<td></td>
<td>#10</td>
<td>.026</td>
<td>2.51&quot;</td>
<td>2.50</td>
</tr>
<tr>
<td></td>
<td>#11</td>
<td>.032</td>
<td>2.58&quot;</td>
<td>2.59</td>
</tr>
</tbody>
</table>

**S-6**

| No. | Size | $p_g$ | $d'$ | $P$ | $P$ | 1.0 | 1.2 | 1.4 | 1.6 | 1.8 | 2.0 |
| #6 | .013 | 2.37" | 3.07 | 206 | 3.21 | 218 | | T | T | T | T | T |
| #7 | .018 | 2.44" | 3.22 | 222 | 3.38 | 237 | | T | T | T | T | T |
| #8 | .024 | 2.50" | 3.48 | 242 | 3.69 | 260 | | 28.7 | 24.1 | 20.9 | 18.2 | 16.2 | T |
| #9 | .030 | 2.56" | 3.63 | 261 | 3.96 | 285 | | 31.9 | 26.9 | 23.0 | 20.2 | 18.0 | 16.2 |
| #10 | .039 | 2.63" | 3.89 | 287 | 4.30 | 317 | | 34.3 | 28.9 | 25.0 | 21.9 | 19.4 | 17.9 |

**S-7**

| No. | Size | $p_g$ | $d'$ | $P$ | $P$ | 1.0 | 1.2 | 1.4 | 1.6 | 1.8 | 2.0 |
| #6 | .016 | 2.37" | 3.13 | 232 | 3.31 | 249 | | 25.0 | T | T | T | T |
| #7 | .021 | 2.44" | 3.31 | 253 | 3.53 | 275 | | 28.0 | 23.7 | 20.2 | T | T | T |
| #8 | .028 | 2.50" | 3.53 | 277 | 3.84 | 305 | | 30.9 | 25.6 | 22.3 | 19.5 | 17.4 | 16.0 |
| #9 | .036 | 2.56" | 3.78 | 307 | 4.17 | 343 | | 33.8 | 28.5 | 24.3 | 21.4 | 19.2 | 17.3 |

**S-8**

| No. | Size | $p_g$ | $d'$ | $P$ | $P$ | 1.0 | 1.2 | 1.4 | 1.6 | 1.8 | 2.0 |
| #6 | .018 | 2.37" | 3.15 | 242 | 3.36 | 261 | | 26.5 | T | T | T | T |
| #7 | .024 | 2.44" | 3.44 | 266 | 3.70 | 291 | | 29.7 | 24.8 | 21.5 | 18.8 | T | T |
| #8 | .032 | 2.50" | 3.66 | 293 | 4.02 | 325 | | 32.7 | 27.2 | 23.8 | 20.8 | 18.5 | 16.7 |

**S-9**

| Spiral | $f'_{c} = 40,000$ psi | $f'_{c} = 60,000$ psi | $2\frac{1}{2}$" oc | $2\frac{1}{2}$" oc |
| #5 | 2.44 | 3.50 | 251 | 3.80 | 273 | | 31.0 | 26.0 | 22.3 | 19.8 | 17.5 | 15.9 | 14.9 |
| #4 | 2.44 | 3.50 | 251 | 3.80 | 273 | | 31.0 | 26.0 | 22.3 | 19.8 | 17.5 | 15.9 | 14.9 |
Table 9-46b. Square, Rectangular, and Round Columns, Values of $P$ and $N$ with $f'_c = 3,750$ psi. 12-in. and 14-in. Rectangular Columns, Tied Reinforcing

<table>
<thead>
<tr>
<th>Reinforcing</th>
<th>e/t &lt; 1.0</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$f_s = 16,000$</td>
<td>$f_s = 20,000$</td>
</tr>
<tr>
<td>No.</td>
<td>Size</td>
<td>$P$</td>
</tr>
<tr>
<td>b x t = 14&quot; x 12&quot;</td>
<td>$A_g = 168$ sq in.</td>
<td>Weight = 175 lb/ft</td>
</tr>
<tr>
<td>#6</td>
<td>.010</td>
<td>2.25</td>
</tr>
<tr>
<td>#7</td>
<td>.014</td>
<td>2.31</td>
</tr>
<tr>
<td>#8</td>
<td>.019</td>
<td>2.37</td>
</tr>
<tr>
<td>#9</td>
<td>.024</td>
<td>2.44</td>
</tr>
<tr>
<td>#10</td>
<td>.030</td>
<td>2.51</td>
</tr>
<tr>
<td>#11</td>
<td>.037</td>
<td>2.58</td>
</tr>
<tr>
<td>b x t = 12&quot; x 14&quot;</td>
<td>$A_g = 168$ sq in.</td>
<td>Weight = 175 lb/ft</td>
</tr>
<tr>
<td>#6</td>
<td>.010</td>
<td>2.25</td>
</tr>
<tr>
<td>#7</td>
<td>.014</td>
<td>2.31</td>
</tr>
<tr>
<td>#8</td>
<td>.019</td>
<td>2.37</td>
</tr>
<tr>
<td>#9</td>
<td>.024</td>
<td>2.44</td>
</tr>
<tr>
<td>#10</td>
<td>.030</td>
<td>2.51</td>
</tr>
<tr>
<td>#11</td>
<td>.037</td>
<td>2.58</td>
</tr>
<tr>
<td>b x t = 16&quot; x 14&quot;</td>
<td>$A_g = 224$ sq in.</td>
<td>Weight = 233 lb/ft</td>
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</tr>
<tr>
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<td>.018</td>
<td>2.44</td>
</tr>
<tr>
<td>#10</td>
<td>.023</td>
<td>2.51</td>
</tr>
<tr>
<td>#11</td>
<td>.028</td>
<td>2.58</td>
</tr>
<tr>
<td>b x t = 18&quot; x 14&quot;</td>
<td>$A_g = 252$ sq in.</td>
<td>Weight = 263 lb/ft</td>
</tr>
<tr>
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</tr>
<tr>
<td>#10</td>
<td>.030</td>
<td>2.51</td>
</tr>
<tr>
<td>#11</td>
<td>.037</td>
<td>2.53</td>
</tr>
<tr>
<td>Reinforcing</td>
<td>e/t &lt; 1.0</td>
<td>N</td>
</tr>
<tr>
<td>------------</td>
<td>----------</td>
<td>---</td>
</tr>
<tr>
<td></td>
<td>$t_s = 16,000$</td>
<td>$t_s = 20,000$</td>
</tr>
<tr>
<td></td>
<td>$I_c = 5,440$ in.$^4$</td>
<td>$I_c = 8,500$ in.$^4$</td>
</tr>
<tr>
<td>No.</td>
<td>Size</td>
<td>$p_g$</td>
</tr>
<tr>
<td>T-4</td>
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</tr>
<tr>
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<td>0.16</td>
</tr>
<tr>
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<td>#10</td>
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<tr>
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<tr>
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<td>0.31</td>
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<tr>
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<td>#10</td>
<td>0.35</td>
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<td>0.18</td>
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<td>0.23</td>
</tr>
<tr>
<td></td>
<td>#10</td>
<td>0.30</td>
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<td>#9</td>
<td>0.27</td>
</tr>
<tr>
<td></td>
<td>#10</td>
<td>0.35</td>
</tr>
<tr>
<td>S-8</td>
<td>#8</td>
<td>0.25</td>
</tr>
<tr>
<td></td>
<td>#9</td>
<td>0.31</td>
</tr>
<tr>
<td>ST-10</td>
<td>#8</td>
<td>0.31</td>
</tr>
<tr>
<td></td>
<td>#9</td>
<td>0.39</td>
</tr>
<tr>
<td></td>
<td>$b \times t = 25&quot; \times 16&quot;$</td>
<td>$b \times t = 26&quot; \times 16&quot;$</td>
</tr>
<tr>
<td></td>
<td>$A_g = 400$ sq in.</td>
<td>$A_g = 416$ sq in.</td>
</tr>
<tr>
<td></td>
<td>Weight = 417 lb/ft</td>
<td>Weight = 466 lb/ft</td>
</tr>
<tr>
<td>2ST-14</td>
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<tr>
<td></td>
<td>#9</td>
<td>0.35</td>
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<tr>
<td>2ST-17</td>
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<td>0.32</td>
</tr>
<tr>
<td></td>
<td>#9</td>
<td>0.42</td>
</tr>
</tbody>
</table>

Spiral $f_s' = 40,000$ psi # 5 at 2 1/2" o c. $f_s' = 60,000$ psi # 4 at 2 1/2" o c.
### REINFORCED-CONCRETE BUILDING FRAMES

Table 9-46d. Square, Rectangular, and Round Columns, Values of $P$ and $N$ with $f' = 3,750$ psi. 16-in. Rectangular Columns, Tied Reinforcing

<table>
<thead>
<tr>
<th>Reinforcing</th>
<th>$e/t &lt; 10$</th>
<th>$N$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$f_s = 16,000$</td>
<td>$f_s = 20,000$</td>
</tr>
<tr>
<td></td>
<td>$I_c = 3,750$ psi</td>
<td>$I_c = 4,110$ in.$^4$</td>
</tr>
<tr>
<td></td>
<td>$I_c = 200$ lb/ft</td>
<td>$I_c = 234$ lb/ft</td>
</tr>
<tr>
<td></td>
<td>$I_c = 4,770$ in.$^4$</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>No.</th>
<th>Size</th>
<th>$P$</th>
<th>$d'$</th>
<th>$b \times t = 12'' \times 16''$</th>
<th>$b \times t = 14'' \times 16''$</th>
<th>$b \times t = 18'' \times 16''$</th>
<th>$b \times t = 20'' \times 16''$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$A_g = 192$ sq in.</td>
<td>$A_g = 224$ sq in.</td>
<td>$A_g = 288$ sq in.</td>
<td>$A_g = 320$ sq in.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Weight = 200 lb/ft</td>
<td>Weight = 234 lb/ft</td>
<td>Weight = 300 lb/ft</td>
<td>Weight = 334 lb/ft</td>
</tr>
<tr>
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<td>$I_c = 4,770$ in.$^4$</td>
<td>$I_c = 6,160$ in.$^4$</td>
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Table 9-46c. Square, Rectangular, and Round Columns, Values of \( P \) and \( N \) with \( f'_e = 3,750 \text{ psi} \). 18-in. Square and Rectangular Columns, Tied and Spiral Reinforcing

<table>
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<th>( N )</th>
</tr>
</thead>
<tbody>
<tr>
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<td>( f'_e = 20,000 \text{ psi} )</td>
</tr>
<tr>
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<td>( d )</td>
</tr>
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</tr>
<tr>
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<td>2.44&quot;</td>
</tr>
<tr>
<td>#10</td>
<td>.024</td>
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</tr>
<tr>
<td>#11</td>
<td>.029</td>
<td>2.58&quot;</td>
</tr>
<tr>
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<td>2.37&quot;</td>
</tr>
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<tr>
<td>#10</td>
<td>.031</td>
<td>2.51&quot;</td>
</tr>
<tr>
<td>#11</td>
<td>.038</td>
<td>2.58&quot;</td>
</tr>
<tr>
<td>S-6</td>
<td>.018</td>
<td>2.69&quot;</td>
</tr>
<tr>
<td># 10</td>
<td>.023</td>
<td>2.76&quot;</td>
</tr>
<tr>
<td>#11</td>
<td>.029</td>
<td>2.83&quot;</td>
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<td>.022</td>
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<tr>
<td># 9</td>
<td>.031</td>
<td>2.69&quot;</td>
</tr>
<tr>
<td>#10</td>
<td>.039</td>
<td>2.76&quot;</td>
</tr>
</tbody>
</table>

* b x t = 28" x 18"  
  \( A_g = 504 \text{ sq in.} \)  
  \( \text{Weight} = 525 \text{ lb/ft} \)  
  \( I_c = 13,560 \text{ in}^4 \)  
  \( f'_e = 3,750 \text{ psi} \)

| Spiral | \( f'_s = 40,000 \text{ psi} \) | \# 5 at 2 1/4" o.c | \( f'_s = 60,000 \text{ psi} \) | \# 4 at 2 1/4" o.c |}

| 2ST-14 | .022 | 2.62" | 2.47 | 602 | 2.65 | 646 | 84.1 | 70.8 | 61.2 |
| # 9 | .028 | 2.69" | 2.56 | 649 | 2.80 | 705 | 92.2 | 76.7 | 66.4 | 58.1 | 52.3 | 47.1 |
| #10 | .035 | 2.76" | 2.70 | 709 | 2.98 | 780 | 101.8 | 85.5 | 74.2 | 65.6 | 58.7 | 53.0 |

| 2ST-17 | .025 | 2.62" | 2.49 | 672 | 2.69 | 726 | 92.7 | 78.2 | 67.6 | 59.1 |
| # 9 | .031 | 2.69" | 2.66 | 729 | 2.91 | 797 | 102.8 | 86.3 | 74.9 | 66.1 | 59.2 | 53.3 |
| #10 | .040 | 2.76" | 2.81 | 802 | 3.11 | 889 | 113.0 | 95.5 | 82.7 | 73.0 | 64.9 | 58.9 |
### REINFORCED-CONCRETE BUILDING FRAMES

Table 9-46f. Square, Rectangular, and Round Columns, Values of $P$ and $N$ with $f_{c'} = 3,750$ psi. 18-in. Rectangular Columns, Tied Reinforcing

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</table>

<table>
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<th>$N$</th>
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</thead>
<tbody>
<tr>
<td></td>
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<td></td>
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<td>$12 \frac{co}{T}$</td>
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<td>2</td>
<td></td>
<td></td>
<td></td>
<td>$P$</td>
<td>$P$</td>
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#### T-4

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<th>$N$</th>
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### Tables for Design of Reinforced Concrete

#### Table 9-46g. Square, Rectangular, and Round Columns, Values of $P$ and $N$ with $f' = 3,750$ psi. 20-in. Square and Rectangular Columns, Tied and Spiral Reinforcing

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<td>#9</td>
<td>.030</td>
</tr>
<tr>
<td></td>
<td>#10</td>
<td>.038</td>
</tr>
<tr>
<td></td>
<td>#11</td>
<td>.047</td>
</tr>
</tbody>
</table>

#### Table 9-46g. Square, Rectangular, and Round Columns, Values of $P$ and $N$ with $f' = 3,750$ psi. 20-in. Square and Rectangular Columns, Tied and Spiral Reinforcing

<table>
<thead>
<tr>
<th>Reinforcing</th>
<th>$e_f &lt; 1.0$</th>
<th>$N$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$f_s = 60,000$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$f_s = 40,000$</td>
<td></td>
</tr>
</tbody>
</table>

---

$t_f = 3,750$ psi
$A_g = 400$ sq in.
$W = 417$ lb/ft
$I_c = 13,300$ in$^4$

---

$b \times t = 29'' \times 20''$

$A_g = 580$ sq in.
$W = 604$ lb/ft
$I_c = 19,300$ in$^4$

---

$b \times t = 30'' \times 20''$

$A_g = 600$ sq in.
$W = 625$ lb/ft
$I_c = 20,000$ in$^4$

---

Spiral $t_f = 40,000$ psi #5 at 2 1/2'' oc
$t_f = 60,000$ psi #5 at 3'' oc
### Table 9-46h. Square, Rectangular, and Round Columns, Values of $P$ and $N$ with $f'_c = 3,750$ psi. 20-in. Rectangular Columns, Tied Reinforcing

<table>
<thead>
<tr>
<th>Reinforcing</th>
<th>$\sigma'_t &lt; 1.0$</th>
<th>$N$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$f_s = 16,000$</td>
<td>$f_s = 20,000$</td>
</tr>
<tr>
<td>No.</td>
<td>Size</td>
<td>$p_s$</td>
</tr>
<tr>
<td>T-6</td>
<td>16'' x 20''</td>
<td>$A_g = 320$ sq in.</td>
</tr>
<tr>
<td># 8</td>
<td>0.15</td>
<td>2.37</td>
</tr>
<tr>
<td># 9</td>
<td>0.19</td>
<td>2.44</td>
</tr>
<tr>
<td># 10</td>
<td>0.24</td>
<td>2.54</td>
</tr>
<tr>
<td>T-8</td>
<td>18'' x 20''</td>
<td>$A_g = 360$ sq in.</td>
</tr>
<tr>
<td># 9</td>
<td>0.17</td>
<td>2.44</td>
</tr>
<tr>
<td># 10</td>
<td>0.21</td>
<td>2.54</td>
</tr>
<tr>
<td># 11</td>
<td>0.26</td>
<td>2.58</td>
</tr>
<tr>
<td>T-10</td>
<td>22'' x 20''</td>
<td>$A_g = 440$ sq in.</td>
</tr>
<tr>
<td># 9</td>
<td>0.18</td>
<td>2.44</td>
</tr>
<tr>
<td># 10</td>
<td>0.23</td>
<td>2.54</td>
</tr>
<tr>
<td># 11</td>
<td>0.28</td>
<td>2.58</td>
</tr>
<tr>
<td>T-12</td>
<td>24'' x 20''</td>
<td>$A_g = 480$ sq in.</td>
</tr>
<tr>
<td># 9</td>
<td>0.17</td>
<td>2.44</td>
</tr>
<tr>
<td># 10</td>
<td>0.21</td>
<td>2.54</td>
</tr>
<tr>
<td># 11</td>
<td>0.26</td>
<td>2.58</td>
</tr>
<tr>
<td>T-10</td>
<td>25'' x 20''</td>
<td>$A_g = 512$ sq in.</td>
</tr>
<tr>
<td># 9</td>
<td>0.21</td>
<td>2.44</td>
</tr>
<tr>
<td># 10</td>
<td>0.26</td>
<td>2.54</td>
</tr>
<tr>
<td># 11</td>
<td>0.32</td>
<td>2.58</td>
</tr>
<tr>
<td>T-12</td>
<td>25'' x 20''</td>
<td>$A_g = 560$ sq in.</td>
</tr>
<tr>
<td># 9</td>
<td>0.25</td>
<td>2.44</td>
</tr>
<tr>
<td># 10</td>
<td>0.32</td>
<td>2.54</td>
</tr>
<tr>
<td># 11</td>
<td>0.39</td>
<td>2.58</td>
</tr>
</tbody>
</table>
### TABLES FOR DESIGN OF REINFORCED CONCRETE

#### Table 9-46i. Square, Rectangular, and Round Columns, Values of $P$ and $N$ with $f_{c'} = 3,750$ psi. 22-in. Square and Rectangular Columns, Tied and Spiral Reinforcing

<table>
<thead>
<tr>
<th>Reinforcing</th>
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<th>$N$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$f_s = 16,000$</td>
<td>$f_s = 20,000$</td>
</tr>
</tbody>
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#### T-8

<table>
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<tr>
<th>No.</th>
<th>Size</th>
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<th>$12\frac{t}{t} P$</th>
<th>$12\frac{t}{t} P$</th>
<th>$e/t$</th>
<th>$e/t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>#9</td>
<td>0.16</td>
<td>2.44</td>
<td></td>
<td>1.48</td>
<td>429</td>
<td>1.56</td>
<td>455</td>
</tr>
<tr>
<td>#10</td>
<td>0.21</td>
<td>2.51</td>
<td></td>
<td>1.52</td>
<td>457</td>
<td>1.62</td>
<td>489</td>
</tr>
<tr>
<td>#11</td>
<td>0.26</td>
<td>2.58</td>
<td></td>
<td>1.57</td>
<td>487</td>
<td>1.70</td>
<td>527</td>
</tr>
</tbody>
</table>

#### T-10

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<th>$12\frac{t}{t} P$</th>
<th>$e/t$</th>
<th>$e/t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>#9</td>
<td>0.21</td>
<td>2.44</td>
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<td>455</td>
<td>1.60</td>
<td>487</td>
</tr>
<tr>
<td>#10</td>
<td>0.26</td>
<td>2.51</td>
<td></td>
<td>1.54</td>
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<td>1.67</td>
<td>530</td>
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<tr>
<td>#11</td>
<td>0.32</td>
<td>2.58</td>
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#### T-12

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<th>$12\frac{t}{t} P$</th>
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<td></td>
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<td>1.69</td>
<td>519</td>
</tr>
<tr>
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<td>0.31</td>
<td>2.51</td>
<td></td>
<td>1.62</td>
<td>522</td>
<td>1.76</td>
<td>571</td>
</tr>
<tr>
<td>#11</td>
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<td>2.58</td>
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#### S-10

<table>
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<th>$12\frac{t}{t} P$</th>
<th>$e/t$</th>
<th>$e/t$</th>
</tr>
</thead>
<tbody>
<tr>
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<td>2.69</td>
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<td>2.03</td>
<td>569</td>
<td>2.17</td>
<td>609</td>
</tr>
<tr>
<td>#10</td>
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<td>2.76</td>
<td></td>
<td>2.13</td>
<td>612</td>
<td>2.30</td>
<td>663</td>
</tr>
<tr>
<td>#11</td>
<td>0.32</td>
<td>2.83</td>
<td></td>
<td>2.23</td>
<td>658</td>
<td>2.43</td>
<td>721</td>
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</table>

#### S-11

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<th>$e/t$</th>
</tr>
</thead>
<tbody>
<tr>
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<td></td>
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<td>588</td>
<td>2.23</td>
<td>629</td>
</tr>
<tr>
<td>#10</td>
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<td>2.76</td>
<td></td>
<td>2.17</td>
<td>642</td>
<td>2.36</td>
<td>688</td>
</tr>
<tr>
<td>#11</td>
<td>0.35</td>
<td>2.83</td>
<td></td>
<td>2.28</td>
<td>683</td>
<td>2.51</td>
<td>752</td>
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</tbody>
</table>

#### ST-10

<table>
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<th>$d$</th>
<th>$12\frac{t}{t} P$</th>
<th>$12\frac{t}{t} P$</th>
<th>$e/t$</th>
<th>$e/t$</th>
</tr>
</thead>
<tbody>
<tr>
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<td>2.69</td>
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<td>569</td>
<td>2.08</td>
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<tr>
<td>#10</td>
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<td>2.76</td>
<td></td>
<td>2.01</td>
<td>612</td>
<td>2.17</td>
<td>663</td>
</tr>
<tr>
<td>#11</td>
<td>0.32</td>
<td>2.83</td>
<td></td>
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<td>658</td>
<td>2.26</td>
<td>721</td>
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</tbody>
</table>

#### ST-12

<table>
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<th>$12\frac{t}{t} P$</th>
<th>$e/t$</th>
<th>$e/t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>#9</td>
<td>0.25</td>
<td>2.69</td>
<td></td>
<td>2.01</td>
<td>601</td>
<td>2.16</td>
<td>649</td>
</tr>
<tr>
<td>#10</td>
<td>0.031</td>
<td>2.76</td>
<td></td>
<td>2.04</td>
<td>653</td>
<td>2.21</td>
<td>714</td>
</tr>
<tr>
<td>#11</td>
<td>0.39</td>
<td>2.83</td>
<td></td>
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<td>708</td>
<td>2.40</td>
<td>783</td>
</tr>
</tbody>
</table>

#### ST-14

<table>
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<th>$d$</th>
<th>$12\frac{t}{t} P$</th>
<th>$12\frac{t}{t} P$</th>
<th>$e/t$</th>
<th>$e/t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>#9</td>
<td>0.29</td>
<td>2.69</td>
<td></td>
<td>2.07</td>
<td>633</td>
<td>2.26</td>
<td>689</td>
</tr>
<tr>
<td>#10</td>
<td>0.037</td>
<td>2.76</td>
<td></td>
<td>2.23</td>
<td>693</td>
<td>2.45</td>
<td>764</td>
</tr>
<tr>
<td>#11</td>
<td>0.45</td>
<td>2.83</td>
<td></td>
<td>2.26</td>
<td>758</td>
<td>2.52</td>
<td>846</td>
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</tbody>
</table>

#### 2ST-16

<table>
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<th>$d$</th>
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<th>$12\frac{t}{t} P$</th>
<th>$e/t$</th>
<th>$e/t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>#10</td>
<td>0.031</td>
<td>2.76</td>
<td></td>
<td>2.11</td>
<td>882</td>
<td>2.30</td>
<td>963</td>
</tr>
<tr>
<td>#11</td>
<td>0.038</td>
<td>2.83</td>
<td></td>
<td>2.21</td>
<td>956</td>
<td>2.43</td>
<td>1036</td>
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#### 2ST-18

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<th>$d$</th>
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<th>$12\frac{t}{t} P$</th>
<th>$e/t$</th>
<th>$e/t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>#10</td>
<td>0.035</td>
<td>2.76</td>
<td></td>
<td>2.16</td>
<td>923</td>
<td>2.37</td>
<td>1013</td>
</tr>
<tr>
<td>#11</td>
<td>0.042</td>
<td>2.83</td>
<td></td>
<td>2.28</td>
<td>1005</td>
<td>2.52</td>
<td>1119</td>
</tr>
</tbody>
</table>

**Spiral**

- $f_s = 40,000$ psi use $t_s = 60,000$
- $f_s = 60,000$ psi
- #5 at 3° c

**Notes:**
- $b \times t = 22'' \times 22''$
- $A_g = 484\text{ sq in.}$
- $9-195$
- $f_{c'} = 3,750$ psi
- $I_c = 18,540\text{ in}^4$
- Weight = 504 lb/ft
- Weight = 688 lb/ft
- $I_c = 26,600\text{ in}^4$
Table 9-46j: Square, Rectangular, and Round Columns, Values of $P$ and $N$ with $f_{c'} = 3,750$ psi. 22-in. Rectangular Columns, Tied Reinforcing

<table>
<thead>
<tr>
<th>Reinforcing</th>
<th>$e/t &lt; 1.0$</th>
<th>$N$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$f = 16,000$</td>
<td>$f = 20,000$</td>
</tr>
<tr>
<td></td>
<td>$t_c' = 3,750$ psi</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$I_c = 15,950$ in$^4$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$b \times t = 18'' \times 22''$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$A_g = 396$ sq in.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Weight = 413 lb/ft</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$t_c' = 3,750$ psi</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$I_c = 17,700$ in$^4$</td>
<td></td>
</tr>
<tr>
<td>T-6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>#9</td>
<td>0.15</td>
<td>2.44''</td>
</tr>
<tr>
<td>#10</td>
<td>0.19</td>
<td>2.51''</td>
</tr>
<tr>
<td>#11</td>
<td>0.24</td>
<td>2.58''</td>
</tr>
<tr>
<td>T-8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>#9</td>
<td>0.20</td>
<td>2.44''</td>
</tr>
<tr>
<td>#10</td>
<td>0.26</td>
<td>2.51''</td>
</tr>
<tr>
<td>#11</td>
<td>0.31</td>
<td>2.58''</td>
</tr>
<tr>
<td>T-10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>#9</td>
<td>0.18</td>
<td>2.44''</td>
</tr>
<tr>
<td>#10</td>
<td>0.23</td>
<td>2.51''</td>
</tr>
<tr>
<td>#11</td>
<td>0.28</td>
<td>2.58''</td>
</tr>
<tr>
<td>T-12</td>
<td></td>
<td></td>
</tr>
<tr>
<td>#9</td>
<td>0.23</td>
<td>2.44''</td>
</tr>
<tr>
<td>#10</td>
<td>0.30</td>
<td>2.58''</td>
</tr>
<tr>
<td>#11</td>
<td>0.35</td>
<td>2.58''</td>
</tr>
<tr>
<td>T-14</td>
<td></td>
<td></td>
</tr>
<tr>
<td>#10</td>
<td>0.22</td>
<td>2.51''</td>
</tr>
<tr>
<td>#11</td>
<td>0.27</td>
<td>2.58''</td>
</tr>
<tr>
<td>#10</td>
<td>0.27</td>
<td>2.51''</td>
</tr>
<tr>
<td>#11</td>
<td>0.33</td>
<td>2.58''</td>
</tr>
<tr>
<td>#10</td>
<td>0.31</td>
<td>2.51''</td>
</tr>
<tr>
<td>#11</td>
<td>0.38</td>
<td>2.58''</td>
</tr>
</tbody>
</table>

$b \times t = 20'' \times 22''$

$A_g = 440$ sq in.

Weight = 458 lb/ft

$t_c' = 3,750$ psi

$I_c = 21,300$ in$^4$

$\frac{b \times t = 24'' \times 22''}{A_g = 528$ sq in.}

Weight = 552 lb/ft

$t_c' = 3,700$ psi

$I_c = 23,000$ in$^4$

$\frac{b \times t = 26'' \times 22''}{A_g = 572$ sq in.}

Weight = 595 lb/ft

$t_c' = 3,750$ psi

$I_c = 23,000$ in$^4$
**Table 9-46e. Square, Rectangular, and Round Columns, Values of P and N with \( f' = 3,750 \) psi. 24-in. Square Columns, Tied and Spiral Reinforcing**

\[ b \times t = 24'' \times 24'' \]

\[ A_g = 576 \text{ sq in} \]

Weight = 600 lb/ft

<table>
<thead>
<tr>
<th>Reinforcing</th>
<th>( e/t &lt; 1.0 )</th>
<th>( N )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f_p = 16,000 )</td>
<td>( f_p = 20,000 )</td>
<td>( e/t )</td>
</tr>
<tr>
<td>No.</td>
<td>Size</td>
<td>( P_g )</td>
</tr>
<tr>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>T-10</td>
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<td>.017</td>
</tr>
<tr>
<td></td>
<td># 10</td>
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</tr>
<tr>
<td>T-12</td>
<td># 9</td>
<td>.021</td>
</tr>
<tr>
<td></td>
<td># 10</td>
<td>.026</td>
</tr>
<tr>
<td></td>
<td># 11</td>
<td>.032</td>
</tr>
<tr>
<td>T-14</td>
<td># 9</td>
<td>.024</td>
</tr>
<tr>
<td></td>
<td># 10</td>
<td>.031</td>
</tr>
<tr>
<td></td>
<td># 11</td>
<td>.038</td>
</tr>
<tr>
<td>S-11</td>
<td># 9</td>
<td>.019</td>
</tr>
<tr>
<td></td>
<td># 10</td>
<td>.024</td>
</tr>
<tr>
<td></td>
<td># 11</td>
<td>.030</td>
</tr>
<tr>
<td>S-12</td>
<td># 9</td>
<td>.021</td>
</tr>
<tr>
<td></td>
<td># 10</td>
<td>.026</td>
</tr>
<tr>
<td></td>
<td># 11</td>
<td>.032</td>
</tr>
<tr>
<td>S-13</td>
<td># 9</td>
<td>.023</td>
</tr>
<tr>
<td></td>
<td># 10</td>
<td>.029</td>
</tr>
<tr>
<td></td>
<td># 11</td>
<td>.036</td>
</tr>
<tr>
<td>ST-12</td>
<td># 9</td>
<td>.021</td>
</tr>
<tr>
<td></td>
<td># 10</td>
<td>.026</td>
</tr>
<tr>
<td></td>
<td># 11</td>
<td>.032</td>
</tr>
<tr>
<td>ST-14</td>
<td># 9</td>
<td>.024</td>
</tr>
<tr>
<td></td>
<td># 10</td>
<td>.031</td>
</tr>
<tr>
<td></td>
<td># 11</td>
<td>.038</td>
</tr>
<tr>
<td>ST-16</td>
<td># 9</td>
<td>.028</td>
</tr>
<tr>
<td></td>
<td># 10</td>
<td>.035</td>
</tr>
<tr>
<td></td>
<td># 11</td>
<td>.044</td>
</tr>
</tbody>
</table>

**Spiral** \( f_p' = 40,000 \) psi; use \( f_p'' = 60,000 \) psi \* 5 or 3° o c
### Table 9-48. Square, Rectangular, and Round Columns, Values of $P$ and $N$ with $f' = 3,750$ psi. 24-in. Rectangular Columns, Tied Reinforcing

<table>
<thead>
<tr>
<th>Reinforcing No.</th>
<th>$\frac{b \times t}{A_g} = 480$ sq in.</th>
<th>Weight = 500 lb/ft</th>
<th>$I_c = 23,050$ in$^4$</th>
<th>$I_c = 3750$ psi</th>
<th>$I_c = 25,350$ in$^4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T-8$</td>
<td>$d' = 20^\prime \times 24^\prime$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>#9</td>
<td>0.017 2.44&quot; 1.34 428 1.42 452 78.4</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>#10</td>
<td>0.021 2.51&quot; 1.38 454 1.47 486 86.4</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>#11</td>
<td>0.026 2.58&quot; 1.42 484 1.53 524 95.3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$T-10$</td>
<td>$d' = 22^\prime \times 24^\prime$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>#9</td>
<td>0.021 2.44&quot; 1.36 452 1.45 484 88.3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>#10</td>
<td>0.027 2.51&quot; 1.40 486 1.51 527 97.5</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>#11</td>
<td>0.033 2.58&quot; 1.43 524 1.57 573 106.8</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$T-12$</td>
<td>$d' = 26^\prime \times 24^\prime$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>#10</td>
<td>0.024 2.51&quot; 1.42 516 1.52 665 143.6</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>#11</td>
<td>0.030 2.58&quot; 1.47 660 1.60 720 124.7</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$T-14$</td>
<td>$d' = 28^\prime \times 24^\prime$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>#10</td>
<td>0.029 2.51&quot; 1.50 648 1.63 705 125.4</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>#11</td>
<td>0.035 2.58&quot; 1.48 700 1.63 770 139.6</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$T-16$</td>
<td>$d' = 32^\prime \times 24^\prime$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>#10</td>
<td>0.030 2.51&quot; 1.48 714 1.61 779 136.3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>#11</td>
<td>0.037 2.58&quot; 1.53 773 1.68 853 149.9</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
TABLES FOR DESIGN OF REINFORCED CONCRETE  9-199

Table 9-46m. Square, Rectangular, and Round Columns, Values of P and N with \( f' = 3,750 \text{ psi} \)  26-in. and 28-in. Square Columns, Tied and Spiral Reinforcing

<table>
<thead>
<tr>
<th>Reinforcing</th>
<th>e/t &lt; 1.0</th>
<th>N</th>
<th>e/t</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( t_b = 16,000 )</td>
<td>( t_b = 20,000 )</td>
<td>( \theta )</td>
</tr>
<tr>
<td></td>
<td>( \frac{12 Co}{\theta} )</td>
<td>( \frac{12 Co}{\theta} )</td>
<td>1.0</td>
</tr>
<tr>
<td>No. Size</td>
<td>P</td>
<td>P</td>
<td></td>
</tr>
<tr>
<td>T-12 #10 .023 2.51&quot;</td>
<td>1.29</td>
<td>651</td>
<td>1.38</td>
</tr>
<tr>
<td>#11 .026 2.58&quot;</td>
<td>1.34</td>
<td>695</td>
<td>1.44</td>
</tr>
<tr>
<td>T-14 #10 .026 2.51&quot;</td>
<td>1.30</td>
<td>683</td>
<td>1.41</td>
</tr>
<tr>
<td>#11 .032 2.58&quot;</td>
<td>1.35</td>
<td>735</td>
<td>1.47</td>
</tr>
<tr>
<td>T-16 #10 .030 2.51&quot;</td>
<td>1.36</td>
<td>716</td>
<td>1.48</td>
</tr>
<tr>
<td>#11 .037 2.58&quot;</td>
<td>1.40</td>
<td>775</td>
<td>1.54</td>
</tr>
<tr>
<td>S-12 #10 .023 2.76&quot;</td>
<td>1.73</td>
<td>815</td>
<td>1.85</td>
</tr>
<tr>
<td>#11 .028 2.83&quot;</td>
<td>1.79</td>
<td>870</td>
<td>1.95</td>
</tr>
<tr>
<td>S-13 #10 .024 2.76&quot;</td>
<td>1.75</td>
<td>835</td>
<td>1.86</td>
</tr>
<tr>
<td>#11 .030 2.83&quot;</td>
<td>1.82</td>
<td>894</td>
<td>1.99</td>
</tr>
<tr>
<td>S-14 #10 .026 2.76&quot;</td>
<td>1.77</td>
<td>855</td>
<td>1.91</td>
</tr>
<tr>
<td>#11 .032 2.83&quot;</td>
<td>1.84</td>
<td>920</td>
<td>2.03</td>
</tr>
<tr>
<td>ST-12 #10 .023 2.76&quot;</td>
<td>1.53</td>
<td>815</td>
<td>1.75</td>
</tr>
<tr>
<td>#11 .028 2.83&quot;</td>
<td>1.70</td>
<td>870</td>
<td>1.85</td>
</tr>
<tr>
<td>ST-14 #10 .026 2.76&quot;</td>
<td>1.69</td>
<td>855</td>
<td>1.83</td>
</tr>
<tr>
<td>#11 .032 2.83&quot;</td>
<td>1.74</td>
<td>920</td>
<td>1.92</td>
</tr>
<tr>
<td>ST-16 #10 .030 2.76&quot;</td>
<td>1.74</td>
<td>896</td>
<td>1.90</td>
</tr>
<tr>
<td>#11 .037 2.83&quot;</td>
<td>1.81</td>
<td>960</td>
<td>2.00</td>
</tr>
</tbody>
</table>

For Square Columns: \( b \times t = 28'' \times 28'' \)  \( A_g = 784 \text{ sq in.} \)  Weight = 816 lb/ft  \( I_c = 51,200 \text{ in}^4 \)  \( t_c = 3750 \text{ psi} \)

<table>
<thead>
<tr>
<th>Spiral</th>
<th>t_b = 40,000 psi  #5 at 2 1/2&quot; o c</th>
<th>t_b = 60,000 psi  #5 at 2 3/4&quot; o c</th>
<th>e/t</th>
</tr>
</thead>
<tbody>
<tr>
<td>T-12</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>T-14</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>T-16</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
### Table 9-46n. Square, Rectangular, and Round Columns, Values of $P$ and $N$ with $f_{ce} = 3,750$ psi. 28-in. and 30-in. Square Columns, Tied and Spiral Reinforcing

<table>
<thead>
<tr>
<th>Reinforcing</th>
<th>$e/t &lt; 1.0$</th>
<th>$N$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>12 $\frac{P}{D}$</td>
<td>12 $\frac{P}{D}$</td>
</tr>
<tr>
<td><strong>No.</strong></td>
<td><strong>Size</strong></td>
<td><strong>$p_g$</strong></td>
</tr>
<tr>
<td>S-13</td>
<td>#10</td>
<td>0.021</td>
</tr>
<tr>
<td></td>
<td>#11</td>
<td>0.026</td>
</tr>
<tr>
<td>S-14</td>
<td>#10</td>
<td>0.023</td>
</tr>
<tr>
<td></td>
<td>#11</td>
<td>0.028</td>
</tr>
<tr>
<td>S-15</td>
<td>#10</td>
<td>0.024</td>
</tr>
<tr>
<td></td>
<td>#11</td>
<td>0.030</td>
</tr>
<tr>
<td>ST-14</td>
<td>#10</td>
<td>0.023</td>
</tr>
<tr>
<td></td>
<td>#11</td>
<td>0.028</td>
</tr>
<tr>
<td>ST-16</td>
<td>#10</td>
<td>0.026</td>
</tr>
<tr>
<td></td>
<td>#11</td>
<td>0.032</td>
</tr>
<tr>
<td>ST-18</td>
<td>#10</td>
<td>0.092</td>
</tr>
<tr>
<td></td>
<td>#11</td>
<td>0.036</td>
</tr>
</tbody>
</table>

$bxt=30\times30^\prime$

<table>
<thead>
<tr>
<th>Reinforcing</th>
<th>$e/t &lt; 1.0$</th>
<th>$N$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>12 $\frac{P}{D}$</td>
<td>12 $\frac{P}{D}$</td>
</tr>
<tr>
<td><strong>No.</strong></td>
<td><strong>Size</strong></td>
<td><strong>$p_g$</strong></td>
</tr>
<tr>
<td>T-16</td>
<td>#10</td>
<td>0.023</td>
</tr>
<tr>
<td></td>
<td>#11</td>
<td>0.028</td>
</tr>
<tr>
<td>S-14</td>
<td>#10</td>
<td>0.020</td>
</tr>
<tr>
<td></td>
<td>#11</td>
<td>0.024</td>
</tr>
<tr>
<td>S-15</td>
<td>#10</td>
<td>0.021</td>
</tr>
<tr>
<td></td>
<td>#11</td>
<td>0.026</td>
</tr>
<tr>
<td>S-16</td>
<td>#10</td>
<td>0.023</td>
</tr>
<tr>
<td></td>
<td>#11</td>
<td>0.028</td>
</tr>
<tr>
<td>ST-16</td>
<td>#10</td>
<td>0.023</td>
</tr>
<tr>
<td></td>
<td>#11</td>
<td>0.028</td>
</tr>
<tr>
<td>ST-18</td>
<td>#10</td>
<td>0.025</td>
</tr>
<tr>
<td></td>
<td>#11</td>
<td>0.031</td>
</tr>
<tr>
<td>ST-20</td>
<td>#10</td>
<td>0.028</td>
</tr>
<tr>
<td></td>
<td>#11</td>
<td>0.035</td>
</tr>
</tbody>
</table>

Spiral $t_s = 40,000$ psi  Use $t_s = 60,000$ psi  $t_s = 60,000$ psi  $5$ at 2 3/4" o c
### Tables for Design of Reinforced Concrete

Table 9-46a. Square, Rectangular, and Round Columns, Values of $P$ and $N$ with $f_c' = 3,750$ psi. 20-in. to 30-in. Square Columns, Tied Double Spiral Reinforcing

<table>
<thead>
<tr>
<th>Columns No.</th>
<th>Reinforcing</th>
<th>$12\frac{P}{P_t}$</th>
<th>$P$ e/t&lt;1.0</th>
<th>Reinforcing - Inner Spiral</th>
<th>Size</th>
<th>Pm</th>
<th>No. vertical bars</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$d'$</td>
<td>$P_u$</td>
<td></td>
<td></td>
<td>Size</td>
<td>Pm</td>
<td>5</td>
</tr>
<tr>
<td>20&quot; x 20&quot;</td>
<td>#10</td>
<td>0.041</td>
<td>2.76</td>
<td>2.51</td>
<td>602</td>
<td>#10</td>
<td>0.057</td>
</tr>
<tr>
<td>SST-13</td>
<td>#11</td>
<td>0.051</td>
<td>2.83</td>
<td>2.61</td>
<td>661</td>
<td>#11</td>
<td>0.070</td>
</tr>
<tr>
<td>22&quot; x 22&quot;</td>
<td>#10</td>
<td>0.037</td>
<td>2.76</td>
<td>2.23</td>
<td>693</td>
<td>#10</td>
<td>0.052</td>
</tr>
<tr>
<td>SST-14</td>
<td>#11</td>
<td>0.045</td>
<td>2.83</td>
<td>2.26</td>
<td>758</td>
<td>#11</td>
<td>0.064</td>
</tr>
<tr>
<td>24&quot; x 24&quot;</td>
<td>#10</td>
<td>0.033</td>
<td>2.76</td>
<td>1.94</td>
<td>791</td>
<td>#10</td>
<td>0.048</td>
</tr>
<tr>
<td>SST-15</td>
<td>#11</td>
<td>0.041</td>
<td>2.83</td>
<td>2.00</td>
<td>860</td>
<td>#11</td>
<td>0.059</td>
</tr>
<tr>
<td>26&quot; x 26&quot;</td>
<td>#10</td>
<td>0.030</td>
<td>2.76</td>
<td>1.74</td>
<td>896</td>
<td>#10</td>
<td>0.045</td>
</tr>
<tr>
<td>SST-16</td>
<td>#11</td>
<td>0.037</td>
<td>2.83</td>
<td>1.81</td>
<td>970</td>
<td>#11</td>
<td>0.055</td>
</tr>
<tr>
<td>28&quot; x 28&quot;</td>
<td>#10</td>
<td>0.029</td>
<td>2.76</td>
<td>1.60</td>
<td>1027</td>
<td>#10</td>
<td>0.043</td>
</tr>
<tr>
<td>SST-18</td>
<td>#11</td>
<td>0.036</td>
<td>2.83</td>
<td>1.66</td>
<td>1109</td>
<td>#11</td>
<td>0.054</td>
</tr>
<tr>
<td>30&quot; x 30&quot;</td>
<td>#10</td>
<td>0.028</td>
<td>2.76</td>
<td>1.49</td>
<td>1166</td>
<td>#10</td>
<td>0.042</td>
</tr>
<tr>
<td>SST-20</td>
<td>#11</td>
<td>0.035</td>
<td>2.83</td>
<td>1.56</td>
<td>1258</td>
<td>#11</td>
<td>0.052</td>
</tr>
</tbody>
</table>

$t_c' = 3750$ psi  \hspace{1cm}  $t_s = 16,000$ psi

| 20" x 20"  | #10 | 0.041 | 2.76  | 2.78 | 668 | #10 | 0.057 | 793 |    |    |    |    |
| SST-13      | #11 | 0.051 | 2.83  | 2.96 | 743 | #11 | 0.070 | 899 |    |    |    |    |
| 22" x 22"  | #10 | 0.037 | 2.76  | 2.45 | 764 | #10 | 0.052 | 889 | 916 |    |    |    |
| SST-14      | #11 | 0.045 | 2.83  | 2.52 | 846 | #11 | 0.064 | 1002| 1033|    |    |    |
| 24" x 24"  | #10 | 0.033 | 2.76  | 2.12 | 867 | #10 | 0.048 | 992 | 1019| 1045|    |    |
| SST-15      | #11 | 0.041 | 2.83  | 2.22 | 954 | #11 | 0.059 | 1110| 1141| 1172|    |    |
| 26" x 26"  | #10 | 0.030 | 2.76  | 1.90 | 977 | #10 | 0.045 | 1102| 1129| 1155| 1180|    |
| SST-16      | #11 | 0.037 | 2.83  | 2.00 | 1070| #11 | 0.055 | 1226| 1257| 1288| 1319|    |
| 28" x 28"  | #10 | 0.029 | 2.76  | 1.75 | 1117| #10 | 0.043 | 1242| 1269| 1295| 1320| 1345|
| SST-18      | #11 | 0.036 | 2.83  | 1.83 | 1223| #11 | 0.054 | 1379| 1410| 1441| 1472| 1504|
| 30" x 30"  | #10 | 0.028 | 2.76  | 1.61 | 1268| #10 | 0.042 | 1393| 1420| 1446| 1471| 1496| 1522|
| SST-20      | #11 | 0.035 | 2.83  | 1.73 | 1384| #11 | 0.052 | 1540| 1571| 1602| 1633| 1665| 1696|

Inner spiral, # 2 at 6° o c
Outer spiral same as type ST columns
**Table 9-46p. Square, Rectangular, and Round Columns, Values of \( P \) and \( N \) with \( f′_c = 3,750 \) psi. 14-in. to 30-in. Round Columns, Spiral Reinforcing**

<table>
<thead>
<tr>
<th>Col diam.</th>
<th>Reinforcing</th>
<th>( e/t &lt; 10 )</th>
<th>Coll diam.</th>
<th>Reinforcing</th>
<th>( e/t &lt; 10 )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Size ( p_g ) ( d' ) ( f_{16,000} ) ( f_{20,000} )</td>
<td>( f_{16,000} ) ( f_{20,000} )</td>
<td>Size ( p_g ) ( d' ) ( f_{16,000} ) ( f_{20,000} )</td>
<td>( f_{16,000} ) ( f_{20,000} )</td>
<td></td>
</tr>
<tr>
<td>14&quot; R-6</td>
<td>#6 .017 2.50 4.12 172 4.37 183</td>
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<td>9 .024 2.69 2.67 465 2.88 501</td>
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* 12 \( \frac{CD}{t} \) for round = \( \frac{4}{3} \times 12 \frac{CD}{t} \) for square
### Table 9-47. Square-block Column Footings, Properties for Design

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Load transferred by 1 dowel

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| \( f_s = 16,000 \text{ psi} \) |     |     |     |     |      |      |
| 12"             |     |     |     |     |      |      |
| 14"             |     |     |     |     |      |      |
| 16"             | 7.0 | 8.9 | 10.2| 11.6| 13.0 | 14.4 |
| 18"             | 9.6 | 11.8| 13.3| 15.0| 16.8 |
| 20"             | 12.6| 15.1| 17.0| 18.9|
| 22"             | 16.0| 19.0| 21.1|
| 24"             | 20.3| 23.3|
| 26"             |     |     | 25.0|

| \( f_s = 20,000 \text{ psi} \) |     |     |     |     |      |      |
| 12"             |     |     |     |     |      |      |
| 14"             |     |     |     |     |      |      |
| 16"             |     |     |     |     |      |      |
| 18"             |     |     |     |     |      |      |
| 20"             | 12.0| 13.4| 15.1| 17.0| 18.9 |
| 22"             | 14.9| 16.9| 19.0| 21.0|
| 24"             | 15.8| 18.7| 21.0| 23.2|
| 26"             | 20.0| 23.0| 25.4|
| 28"             |     |     | 25.0| 27.6|
| 30"             |     |     | 25.4| 29.9|
| 32"             |     |     |     | 31.2|
Table 9-49a. Square-block Column Footings, Footing Size and Reinforcing.

\[ f_{c'} = 2,500 \text{ psi}, \ f_c = 20,000 \text{ psi}, \ \nu = 75 \text{ psi}, \ u = 200 \text{ psi}. \]

Soil Pressure 1 Ton per Sq Ft

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### Table 9-49d. Square-block Column Footings, Footing Size and Reinforcing

\( f'_{c} = 2,500 \text{ psi}, f''_{c} = 20,000 \text{ psi}, v_e = 75 \text{ psi}, u = 200 \text{ psi} \)

Soil Pressure 2.5 Tons per Sq Ft

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# Tables for Design of Reinforced Concrete

**Table 9-49c. Square-block Column Footings, Footing Size and Reinforcing.**

\[ f'_c = 2,500 \text{ psi}, f_s = 20,000 \text{ psi}, v = 75 \text{ psi}, u = 200 \text{ psi}. \]

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Table 9-49f. Square-block Column Footings, Footing Size and Reinforcing.

\( f_{c'} = 2,500 \text{ psi}, f_r = 20,000 \text{ psi}, v_c = 75 \text{ psi}, u = 200 \text{ psi}. \)

Soil Pressure 3.5 Tons per Sq Ft

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### REINFORCED-CONCRETE BUILDING FRAMES

Table 9-49h. Square-block Column Footings, Footing Size and Reinforcing.

- $f_d' = 2,500$ psi, $f_s = 20,000$ psi, $v_e = 75$ psi, $u = 200$ psi.
- Soil Pressure 6 Tons per Sq Ft

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### TABLES FOR DESIGN OF REINFORCED CONCRETE 9-213

Table 9-49i: Square-block Column Footings, Footing Size and Reinforcing.

\( f' \) = 2,500 psi, \( f_s \) = 20,000 psi, \( v_r \) = 75 psi, \( u \) = 200 psi.

Soil Pressure 6 Tons per Sq Ft

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### Table 9-50: Footing Caps for Piles, Properties for Design

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\(C\) 206 231 256 285 314 334 359 388 407
\(C\) 165 187 207 226 250 270 286 314 345
\(C\) 137 154 171 191 207 221 243 256 273
\(C\) 117 133 147 162 177 191 207 221 238
Table 9-51a. Footing Caps for Piles. Plan Arrangement for 1 to 7 Pile Groups, Piles at 3 ft 0 in. on Centers

- **1 pile**
  - Typical section: square block cap
  - Dimensions: 1'-3" x 5'-6"

- **2 piles**
  - Dimensions: 1'-3" x 5'-6"

- **3 piles**
  - Dimensions: 2'-6" x 7'-0"

- **4 piles**
  - Dimensions: 2'-3" x 7'-0"

- **5 piles**
  - Dimensions: 2'-3" x 8'-0"

- **6 piles**
  - Dimensions: 2'-9" x 9'-0"

- **7 piles**
  - Dimensions: 3'-0" x 10'-0"
## Table 9-51b. Footing Caps for Piles. Depth, Reinforcement, and Loads for Pile Caps for 2 to 7 Pile Groups. $f' _{c} = 2,500$ psi, $f_s = 20,000$ psi, $v_c = 75$ psi, $u = 200$ psi

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Table 9-51c. Footing Caps for Piles. Plan Arrangement for 8 to 13 Pile Groups, Piles at 3 ft 0 in. on Centers

8 piles

9 piles

10 piles

11 piles

12 piles

13 piles
Table 9-51d. Footing Caps for Piles. Depth, Reinforcement, and Loads for Pile Caps for 8 to 13 Pile Groups. $f' = 2,500$ psi, $f_s = 20,000$ psi, $v_e = 75$ psi,
$u = 200$ psi

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Table 9-51c. Footing Caps for Piles. Plan Arrangement for 14 to 18 Pile Groups, Piles at 3 ft 0 in. on Centers

Typical section stepped footing cap

14 piles

15 piles

16 piles

17 piles

18 piles
### Table 9-51f. Footing Caps for Piles. Depth, Reinforcement, and Loads for Pile Caps for 14 to 18 Pile Groups.  
\( f' = 2,500 \text{ psi}, f_0 = 20,000 \text{ psi}, v_c = 75 \text{ psi}, u = 200 \text{ psi} \)

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Table 9-52. Foundation Walls and Footings. $f' = 2,500$ psi, $f_v = 20,000$ psi

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Maximum wall loads, kips

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D Projection

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Reinforcing required for bond
### Table 9-53a. Members of Rectangular Cross Section. Moments of Inertia, $I = \frac{bD^3}{12}$

(From *Continuity in Concrete Building Frames* by Courtesy of the Portland Cement Association)

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### Diagram: Coefficients C

- **T beams:** $I_c = C \times b'D^3/12$
- **Equations:**
  - $\frac{t}{D} = 0.4$
  - $0.3$
  - $0.2$
  - $0.1$

### Graphical Representation

- **Axes:**
  - X-axis: Ratios of $b/b'$
  - Y-axis: Coefficients $C$

- **Legend:**
  - $t/D = 0.4$
  - $0.3$
  - $0.2$
  - $0.1$

- **Note:**
  - $b'$ is the effective depth of the beam.
  - $b$ is the total depth of the beam.
  - $D$ is the depth of the concrete section.
Table 9-53b. Members of Rectangular Cross Section. Torsional Stiffness, Moments, and Shears

<table>
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<th>0.4</th>
<th>0.5</th>
<th>0.6</th>
<th>0.7</th>
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Torsional shear $\tau_t = M_t/c_t b^2 D$ or $\tau_t = M_m/c_t b^2 D$

\[ M_t = \frac{2 m_2 + m_1}{L/8} \]
\[ M_m = \alpha L/8 \left[ 4 m_2 + \alpha^2 (m_1 - m_2) \right] \]
\[ m = m_2 + \alpha^2 (m_1 - m_2) \]

\[ M_t = \frac{2 m_2 + m_1}{L/12} \]
\[ M_m = \alpha L/12 \left[ 4 m_2 + \alpha^2 (m_1 - m_2) \right] \]
\[ m = m_2 + \alpha^2 (m_1 - m_2) \]

Moment diagrams

Shear diagram
\[ n = \frac{b(v_t + v - v_c)}{2v_t} \]
\[ n' = \frac{b(v_t + v - v_c - v_a)}{2v_t} \]

Vertical stirrups read
\[ s = \frac{2Avf_v}{n(v_t + v - v_c)} \]
\[ s = \frac{2Avf_v}{n'(v_t + v - v_c - v_a)} \]
Table 9-53c. Members of Rectangular Cross Section. Coefficients for Center-line Deflection
(See also Table 9-5)

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Values of $E_C$ for permanent loads

$E_C = \text{Coef. } t_c$
Table 9-54. Members of Constant Cross Section, Beam Factors and Moment Coefficients

Beam factors

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<table>
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### Reinforced-Concrete Building Frames

#### Table 9-55a. Members of Variable Cross Section, Beam Factors and Moment Coefficients. $r_a = 0, \alpha = 0$, and $r_a = 0.2, \alpha = 0.1$

![Diagram](image)

<table>
<thead>
<tr>
<th>$r_a \cdot \alpha$</th>
<th>Moment coeff $F \times W_l$</th>
<th>Haunch - A</th>
<th>Haunch - B</th>
<th>Moment coeff $F \times P_l$</th>
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</thead>
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<td>$c_\alpha$</td>
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<td>$F_B$</td>
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<td>0.012</td>
<td>0.139</td>
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<td>0.075</td>
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<td>0.087</td>
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#### Diagram:

- **A**:Members of Variable Cross Section, Beam Factors and Moment Coefficients. $r_a = 0, \alpha = 0$, and $r_a = 0.2, \alpha = 0.1$

- **B**: $r = h/d$
Table 9-55b. Members of Variable Cross Section, Beam Factors and Moment Coefficients. \( r_a = 0.2, a = 0.2, \) and \( r_a = 0.2, a = 0.3 \)

<p>| ( r_a ) | ( a ) | ( k_a ) | ( c_a ) | ( F_A ) | ( F_B ) | ( c_b ) | ( k_b ) | ( b ) | ( r_b ) | ( 0.1 ) | ( 0.3 ) | ( 0.5 ) | ( 0.7 ) | ( 0.9 ) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| .555 | .540 | .096 | .095 | .601 | .499 | .2 | .089 | .006 | .177 | .055 | .148 | .127 | .069 | .159 | .007 | .088 |
| .575 | .586 | .093 | .093 | .588 | .575 | .2 | .088 | .006 | .173 | .062 | .141 | .141 | .062 | .173 | .006 | .088 |
| .589 | .628 | .091 | .096 | .570 | .649 | .3 | .088 | .007 | .171 | .067 | .137 | .151 | .059 | .178 | .007 | .088 |
| .636 | .556 | .094 | .089 | .600 | .523 | .1 | .088 | .006 | .175 | .058 | .145 | .133 | .065 | .166 | .005 | .090 |
| .591 | .623 | .089 | .099 | .583 | .632 | .2 | .088 | .007 | .170 | .067 | .135 | .153 | .055 | .185 | .005 | .091 |
| .612 | .682 | .086 | .105 | .559 | .748 | .3 | .088 | .008 | .167 | .075 | .128 | .168 | .051 | .194 | .005 | .090 |
| .574 | .579 | .092 | .093 | .597 | .556 | .1 | .088 | .006 | .173 | .061 | .141 | .140 | .060 | .175 | .004 | .094 |
| .615 | .674 | .084 | .109 | .575 | .720 | .2 | .088 | .008 | .166 | .079 | .126 | .170 | .045 | .204 | .003 | .094 |
| .652 | .766 | .079 | .119 | .543 | .919 | .3 | .087 | .010 | .160 | .089 | .115 | .195 | .038 | .218 | .003 | .093 |
| .581 | .590 | .090 | .097 | .596 | .581 | .1 | .088 | .007 | .172 | .064 | .138 | .146 | .056 | .181 | .002 | .096 |
| .635 | .714 | .080 | .117 | .569 | .797 | .2 | .087 | .009 | .163 | .082 | .118 | .185 | .038 | .219 | .002 | .097 |
| .689 | .838 | .073 | .132 | .532 | 10.9 | .3 | .086 | .011 | .154 | .092 | .102 | .222 | .028 | .240 | .002 | .096 |
| .587 | .606 | .089 | .099 | .595 | 5.98 | .1 | .088 | .007 | .171 | .065 | .136 | .150 | .054 | .186 | .002 | .098 |
| .694 | .741 | .077 | .122 | .565 | 8.50 | .2 | .087 | .009 | .160 | .087 | .113 | .194 | .033 | .228 | .001 | .098 |
| 7.15 | .888 | .069 | .141 | .524 | 12.1 | .3 | .086 | .012 | .149 | .112 | .093 | .241 | .021 | .254 | .001 | .097 |
| 6.26 | .523 | .100 | .084 | .642 | 5.10 | .1 | .088 | .006 | .182 | .092 | .158 | .123 | .074 | .157 | .007 | .088 |
| 6.49 | .570 | .096 | .091 | .628 | 5.89 | .2 | .088 | .007 | .178 | .095 | .151 | .137 | .067 | .171 | .007 | .088 |
| 6.65 | .608 | .094 | .094 | .608 | 6.65 | .3 | .087 | .007 | .176 | .064 | .146 | .146 | .064 | .176 | .007 | .087 |
| 6.35 | .539 | .098 | .087 | .641 | 5.35 | .1 | .088 | .006 | .180 | .095 | .155 | .129 | .070 | .164 | .006 | .090 |
| 6.68 | .604 | .093 | .097 | .622 | 6.48 | .2 | .087 | .007 | .175 | .064 | .144 | .148 | .060 | .183 | .005 | .091 |
| 6.93 | .660 | .090 | .103 | .596 | 9.68 | .3 | .087 | .008 | .172 | .072 | .137 | .163 | .055 | .192 | .006 | .090 |
| 6.48 | .561 | .095 | .092 | .638 | 5.69 | .1 | .088 | .007 | .178 | .056 | .150 | .136 | .065 | .173 | .004 | .093 |
| 6.96 | .653 | .087 | .107 | .614 | 7.41 | .2 | .087 | .008 | .171 | .072 | .134 | .166 | .049 | .202 | .003 | .094 |
| 7.40 | .741 | .082 | .117 | .579 | 9.47 | .3 | .086 | .010 | .165 | .086 | .122 | .191 | .042 | .216 | .004 | .093 |
| 6.57 | .577 | .093 | .095 | .636 | 5.96 | .1 | .088 | .007 | .177 | .061 | .147 | .142 | .061 | .179 | .003 | .096 |
| 7.83 | .811 | .075 | .130 | .566 | 11.2 | .3 | .085 | .011 | .159 | .098 | .109 | .217 | .031 | .238 | .002 | .096 |
| 7.37 | .717 | .080 | .120 | .603 | 8.77 | .2 | .086 | .010 | .165 | .083 | .121 | .190 | .035 | .227 | .001 | .098 |
| 8.15 | .859 | .071 | .139 | .557 | 12.6 | .3 | .085 | .013 | .154 | 1.08 | 100 | .236 | .023 | .253 | .002 | .097 |</p>
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<th>Haunch - A</th>
<th>Moment coeff F x Wl</th>
<th>Haunch - B</th>
<th>Moment coeff F x P1</th>
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<td>c_a</td>
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Table 9-55d. Members of Variable Cross Section, Beam Factors and Moment Coefficients. \( r_a = 0.3, a = 0.3, \) and \( r_a = 0.5, a = 0.1 \)

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<th>Haunch - B</th>
<th>Moment coord ( F \times Pl )</th>
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# REINFORCED-CONCRETE BUILDING FRAMES

Table 9-55c. Members of Variable Cross Section, Beam Factors and Moment Coefficients. \( r_a = 0.5, \ a = 0.2, \) and \( r_a = 0.5, \ a = 0.3 \)

<table>
<thead>
<tr>
<th>Haunch - A</th>
<th>Moment coeff ( F \times W_1 )</th>
<th>Haunch - B</th>
<th>Moment coeff ( F \times P_1 )</th>
</tr>
</thead>
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<td>( r_b )</td>
<td>( a_b )</td>
<td>( c_a )</td>
<td>( F_a )</td>
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Diagram showing the distribution of forces and moments at different stages of the haunch sections.
### Tables for Design of Reinforced Concrete

**Table 9-55. Members of Variable Cross Section, Beam Factors and Moment Coefficients.** $r_a = 0.75$, $a = 0.1$, and $r_a = 0.75$, $a = 0.2$

| $r_a$ | $a$ | $k_A$ | $c_A$ | $F_A$ | $F_B$ | $c_B$ | $k_B$ | $b$ | $r_b$ | $F_A$ | $F_B$ | $F_A$ | $F_B$ | $F_A$ | $F_B$ | $F_A$ | $F_B$ | $F_A$ | $F_B$ | $F_A$ | $F_B$ | $F_A$ | $F_B$ | $F_A$ | $F_B$ |
|-------|-----|-------|-------|-------|-------|-------|-------|-----|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 5.61  | 0.547 | 1.00  | 0.083 | 6.08  | 5.04  | . | | | | | | | | | | | | | | | | | | | | | |
| 5.81  | 0.596 | 0.99  | 0.090 | 5.96  | 5.81  | . | | | | | | | | | | | | | | | | | | | | | |
| 5.91  | 0.636 | 0.95  | 0.093 | 5.77  | 6.57  | . | | | | | | | | | | | | | | | | | | | | | |
| 5.69  | 0.564 | 0.99  | 0.086 | 6.07  | 5.28  | . | | | | | | | | | | | | | | | | | | | | | |
| 5.98  | 0.631 | 0.93  | 0.096 | 5.90  | 6.40  | . | | | | | | | | | | | | | | | | | | | | | |
| 6.22  | 0.691 | 0.91  | 0.102 | 5.66  | 7.59  | . | | | | | | | | | | | | | | | | | | | | | |
| 5.80  | 0.586 | 0.96  | 0.091 | 6.06  | 5.62  | . | | | | | | | | | | | | | | | | | | | | | |
| 6.24  | 0.683 | 0.88  | 0.105 | 5.82  | 7.31  | . | | | | | | | | | | | | | | | | | | | | | |
| 6.64  | 0.777 | 0.84  | 0.116 | 5.51  | 9.35  | . | | | | | | | | | | | | | | | | | | | | | |
| 5.89  | 0.603 | 0.94  | 0.094 | 6.03  | 5.89  | . | | | | | | | | | | | | | | | | | | | | | |
| 6.46  | 0.724 | 0.84  | 0.113 | 5.77  | 8.10  | . | | | | | | | | | | | | | | | | | | | | | |
| 7.03  | 0.850 | 0.77  | 0.128 | 5.39  | 11.1  | . | | | | | | | | | | | | | | | | | | | | | |
| 5.94  | 0.614 | 0.93  | 0.096 | 6.02  | 6.06  | . | | | | | | | | | | | | | | | | | | | | | |
| 6.60  | 0.750 | 0.82  | 0.118 | 5.73  | 8.65  | . | | | | | | | | | | | | | | | | | | | | | |
| 7.32  | 0.900 | 0.73  | 0.137 | 5.31  | 12.4  | . | | | | | | | | | | | | | | | | | | | | | |
| 7.63  | 0.523 | 0.20  | 0.074 | 7.30  | 5.46  | . | | | | | | | | | | | | | | | | | | | | | |
| 7.97  | 0.569 | 0.17  | 0.080 | 7.14  | 6.35  | . | | | | | | | | | | | | | | | | | | | | | |
| 8.21  | 0.607 | 0.08  | 0.083 | 6.92  | 7.21  | . | | | | | | | | | | | | | | | | | | | | | |
| 7.77  | 0.539 | 0.18  | 0.077 | 7.28  | 5.75  | . | | | | | | | | | | | | | | | | | | | | | |
| 8.26  | 0.603 | 0.13  | 0.086 | 7.07  | 7.04  | . | | | | | | | | | | | | | | | | | | | | | |
| 8.65  | 0.659 | 0.10  | 0.091 | 6.78  | 8.40  | . | | | | | | | | | | | | | | | | | | | | | |
| 7.96  | 0.560 | 0.15  | 0.081 | 7.26  | 6.14  | . | | | | | | | | | | | | | | | | | | | | | |
| 8.70  | 0.652 | 0.10  | 0.095 | 6.98  | 8.12  | . | | | | | | | | | | | | | | | | | | | | | |
| 9.38  | 0.740 | 0.10  | 0.104 | 6.60  | 10.5  | . | | | | | | | | | | | | | | | | | | | | | |
| 8.40  | 0.577 | 0.13  | 0.084 | 7.24  | 6.46  | . | | | | | | | | | | | | | | | | | | | | | |
| 9.08  | 0.691 | 0.10  | 0.102 | 6.91  | 9.08  | . | | | | | | | | | | | | | | | | | | | | | |
| 10.1  | 0.809 | 0.095 | 0.116 | 6.45  | 12.7  | . | | | | | | | | | | | | | | | | | | | | | |
| 8.19  | 0.587 | 0.12  | 0.086 | 7.22  | 6.66  | . | | | | | | | | | | | | | | | | | | | | | |
| 9.34  | 0.716 | 0.099 | 0.107 | 6.80  | 9.75  | . | | | | | | | | | | | | | | | | | | | | | |
| 10.6  | 0.857 | 0.090 | 0.124 | 6.36  | 14.3  | . | | | | | | | | | | | | | | | | | | | | | |

**Diagram:**

- $A$ and $B$ represent the cross sections of the beam.
- $l$, $w$, $h_a$, and $h_b$ are dimensions of the cross section.
- $r = h/b$ is the ratio of the height to the width.

**Legend:**

- $F_A$, $F_B$: Moment coefficients for sections $A$ and $B$, respectively.
- $X$: A variable that affects the moment coefficients.
- $k_A$, $k_B$: Constants associated with the moment coefficients.
- $c_A$, $c_B$: Coefficients that scale the moment coefficients.
- $r_b$: A variable that modifies the moment coefficients.

The table provides values for $F_A$ and $F_B$ for different combinations of $r_a$, $a$, and $X$, with specific values for $k_A$, $c_A$, $k_B$, $c_B$, and $r_b$.
Table 9-56b. Typical Details for Concrete Construction. Ribbed Slabs, Metal Fillers

Typical plan

# 3 temperature bars at 18" o.c.

Typical details of slab

Standard forms

Tapered forms

Standard metal forms - 20" wide
Table 9-56c. Typical Details for Concrete Construction. Ribbed Slabs, Masonry Fillers
Table 9-56d. Typical Details for Concrete Construction. Solid Concrete Slabs, Two-way Reinforcing

Detail selection at beam

Order of placing reinforcing:
- Beam steel:
  1. Long span straight steel
  2. Short span straight steel
  3. Short span bent steel
  4. Long span bent steel
- Slab steel:
  5. Bottom layer straight steel
  6. Top layer steel adjacent to beams
  7. Bottom layer bent steel
  8. Top layer steel - middle strip
Table 9-56c. Typical Details for Concrete Construction. Solid Concrete Slabs, Two-way Reinforcing, Slab Bands

Order of placing reinforcing

Beam rein: 1. Long span straight bars
          2. Short span straight bars
          3. Short span bent bars
          4. Long span bent bars

Slab rein: 5. Bottom layer straight bars
          6. Bottom layer bent bars
          7. Top layer straight bars
          8. Top layer bent bars
Table 9-56g. Typical Details for Concrete Construction. Stairs

- **Reinforced concrete stairs**
  - #3 ties at 8" oc
  - 2 #3 contin

- **Ornamental iron stair**
  - 1 1/2 x 5/16" anchors at 2' 6" oc
  - 8" 3 1/2% in beam forms
  - Steel C stringers
  - Concrete newel notched over
Table 9-56h. Typical Details for Concrete Construction. Reinforced-concrete Beams

End span - wall bearing

End span - R/C frame

Interior span - R/C frame

E: Where required for compression reinforcement extend straight bottom bars 20 diameters into adjoining span

1': For adjoining spans varying more than 20% extend bent bars to 1/3 point of greater span
Table 9-56i. Typical Details for Concrete Construction. Reinforced-concrete Beams, Stirrup Spacing and Beam Steel

Spacing = 3/4d
Stirrup spacing - see schedule

Space stirrups typically in 2'-0" units thus - 8 at 3", 6 at 4", 5 at 5"
4 at 6", 3 at 8", 2 at 12"

Stirrup spacing

Stirrups
1 1/2
1 1/2

Bt bars
See sched
St bars

Typical beam 1 layer of steel

St top bars where req'd - see schedule

#3 bars full length of stirrup spacing wired to stirrups

Bt bars

1/2" slotted hole for 3/4" bolts
St bars

Typical spandrel beam 1 layer of steel

Wedge type slotted inserts at 4'-0" oc

5" x 3 1/2" x 3/8" L

Masonry opnig

3/8" # ties at 16" oc (min)

Wedge type slotted insert
5" x 3 1/2" x 3/8" L

Typical spandrel beam 1 layer of steel
Table 9-56k. Typical Details for Concrete Construction. Floor Slabs or Subgrade

Note: Floor slabs shall be placed in alternate sections between C of columns.

Detail for slab in open areas without partitions.

Note: Slabs shall be placed in alternate sections not more than 40'oc.

Detail for slabs with construction joints under partitions.
Table 9-56l. Typical Details for Concrete Construction. Tied Columns

Set bolt in grease
Top of slab

1" x 2" long
bolt

Standard
coupling

plain round
rod 2'6" long

Detail for future
extension of columns

Provide bend at top
of columns for all bars
having less than 20 diam
for anchorage

6" min

Vert reinf
see schedule

# 3 ties at
12" oc maximum

1-1/2" clear

Detail at top
of columns

Col. size

Group ties
as required
at 12" oc maximum

Axis of
bending

Detail at
typical floor

Plan

Col. size

1-1/2" cl
### Table 9-56m. Typical Details for Concrete Construction. Spiral Columns

<table>
<thead>
<tr>
<th>Size and pitch of spirals</th>
<th>f_2</th>
<th>2000</th>
<th>2500</th>
<th>3000</th>
<th>3750</th>
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<td></td>
<td></td>
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<td>#4 at 2&quot;</td>
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**Cold-drawn spiral**

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<th>18</th>
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</tbody>
</table>

**Notes:**
- Spiral columns are shown with typical details.
- Column sizes and pitch of spirals are specified for different load conditions.
- Cold-drawn spirals provide additional reinforcement for certain applications.

**Diagrams:**
- 1-1/2" clear spacing between columns is maintained throughout.
- Core diameters and reinforcement details are highlighted.
- Spiral columns are shown with both interior and exterior configurations.
- Type "S" and "ST" columns indicated with respective specifications.
Table 9-56n. Typical Details for Concrete Construction. Round Columns

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Cold-drawn spiral

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<th>Core diam</th>
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Table 9-560. Typical Details for Concrete Construction. Composite Columns

Channel lugs welded to column size and no. as req'd to develop load on steel core.

Spiral same as for typical columns with spirals.

Structural steel column

Steel billet

Top of footing or pier

6" x 4" x 1/2" L

Grout

2-3/4" ø anchor bolts

Note: Provide holes in billet as required for footing dowels.
Table 9-56p. Typical Details for Concrete Construction. Square Block Column Footings

- 20 bar diam (2'-0" min)
- Ties or spiral loop
- Dowels—same size and number as in column above unless otherwise noted
- Feeting dept "D"

For reinforcing—see schedule

Footing size "b"

For reinforcing see schedule

Footing size "b"

Plan
Table 9-56q. Typical Details for Concrete Construction. Rectangular Block Column Footings

- 20 bar diam (2'-0" min)
- 4" Min
- Depth 'D'
- 3" clear
- Total reinforcing in short direction - see schedule

Section

- Total reinforcing short direction
- Reinforcing in slab band 'B'
- Reinforcing in long direction
- Footing width 'b'
- Footing length 't'

Plan
Table 9-56B. Typical Details for Concrete Construction. Wall Footings

Typical wall reinforcing

Section

Elevation

Typical detail of stepped footings

Pipe sleeve

Section

Elevation

Typical detail footing stepped for pipes
Table 9-56: Typical Details for Concrete Construction. Concrete Walls, Temperature and Shrinkage Reinforcement

### Schedule for Horizontal Reinforcement

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</table>

Note: #5 at 4'-0"oc does not conform with ACI code
Table 9-56u. Typical Details for Concrete Construction. Concrete Walls Reinforced for Earth Pressure

- Slab reinforcing
- Lap 30 diam
- Grade
- 1-1/2" clear
- Typical temp reinforcing
- #5 at 4'-0" oc
- Vert reinforcing as required by earth pressure see schedule
- Hor reinforcing shall be continuous and lapped 30 diam at pier - stagger alternate splices
- Lap 30 diam
- #3 at 12" oc
- 1-1/2" clear
- Vert reinforcing equal in area to reinforcing in column above
- 2'x 4" key
- Const joint located midway between piers

Typical section
Plan
Table 9-66c. Typical Details for Concrete Construction. Area Walls

<table>
<thead>
<tr>
<th>h</th>
<th>Wall reinforcing</th>
<th>Slab reinforcing</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>W-1</td>
<td>W-2</td>
</tr>
<tr>
<td>5'-0&quot;</td>
<td>6&quot;</td>
<td>#4 at 12&quot;</td>
</tr>
<tr>
<td>6'-0&quot;</td>
<td>6&quot;</td>
<td>#4 at 12&quot;</td>
</tr>
<tr>
<td>7'-0&quot;</td>
<td>8&quot;</td>
<td>#4 at 12&quot;</td>
</tr>
<tr>
<td>8'-0&quot;</td>
<td>8&quot;</td>
<td>#4 at 10&quot;</td>
</tr>
<tr>
<td>9'-0&quot;</td>
<td>8&quot;</td>
<td>#4 at 20&quot;</td>
</tr>
<tr>
<td>10'-0&quot;</td>
<td>10&quot;</td>
<td>#4 at 16&quot;</td>
</tr>
</tbody>
</table>

Provide plain concrete piers where original soil has been disturbed or is not suitable for bearing.
Table 9-56w. Typical Details for Concrete Construction. Membrane Waterproofing, Foundation Walls

5-ply membrane waterproofing protected with rigid fibre-board

3" sub-slab
5-ply membrane waterproofing
4" brick wall

Detail at pits or trenches

Pipe sleeve
Caulking
Pipe flange screwed to sleeve
5-ply memb

Detail at pipes thru walls

3/4" mortar protection

5-ply membrane waterproofing

3" sub-slab

Typical details - membrane waterproofing hydrostatic-pressure
Table 9-55z. Typical Details for Concrete Construction. Cement-coat Waterproofing, Foundation Walls

**Typical wall section**

- Grade
- Fiber reinforce paper
- Porous fill
- 4" agricultural tile drain
- HP
- LP
- 16 oz copper dam

**Typical details - cement-coat damproofing**

- 3/4" Cement-coat waterproofing on walls and columns
- 1" Min cement-coat waterproofing and protection on floors
- Slab placed monolithic with footings

**Typical details - cement-coat waterproofing hydrostatic pressure**

- 16 oz copper dam

**Detail at pits or trenches**

- 1' Min
- #4 at 12" oc
- 1' - 6"
Table 9-56. Typical Details for Concrete Construction. Concrete Walls, Grade Beams

- Bldg line
- Grade
- #6 top continuous
- Typ temp reinforcing
- Dowels
- 30 diam
- #3 ties at 24" oc
- 12" Min
- Continuous reinforcing

Section

- Column
- 3 #6 top continuous (min.)
- Lap top bars at columns
- Typ temp reinforcing
- 30Dla

Lap top bars at pile cap

Pile cap

Footing cast monolithic with pile cap

Elevation at column and pile cap
Table 9-56a. Typical Details for Concrete Construction. Architectural Concrete Walls, Temperature and Shrinkage Reinforcing

<table>
<thead>
<tr>
<th>Wall thickness</th>
<th>Exterior face</th>
<th>Interior face</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Hor reinforcing</td>
<td>Vert reinforcing</td>
</tr>
<tr>
<td>6&quot;</td>
<td>#3 at 8&quot;oc</td>
<td>#3 at 8&quot;oc</td>
</tr>
<tr>
<td>8&quot;</td>
<td>#3 at 6&quot;oc</td>
<td>#3 at 8&quot;oc</td>
</tr>
<tr>
<td>10&quot;</td>
<td>#3 at 10&quot;oc</td>
<td>#3 at 12&quot;oc</td>
</tr>
<tr>
<td>12&quot;</td>
<td>#3 at 8&quot;oc</td>
<td>#3 at 12&quot;oc</td>
</tr>
</tbody>
</table>
Table 9-56a. Typical Details for Concrete Construction. Architectural Concrete Walls, Window Openings

Metal window frames

Wood window frames
Table 9-56bb. Typical Details for Concrete Construction. Architectural Concrete Walls, Control Joints

Construction joints

Control joints located on 1/4 of spans

Construction joints

Elevation showing location for control joints and construction joints

Notch strip at 8" oc for nailing

1-1/4"

1/4"

1/8"

Control joint strip

6d casing nail

Metal slot (filled with mastic for walls with interior finish)

3/4"

1"

6"

Cut alt bars

Ext face

Detail of control joints
Table 9-56cc. Typical Details for Concrete Construction. Architectural Control Walls, Construction Joints and Forms

Spreadsers at 4'-0" oc hor and vert

Lap forms over hardened concrete not more than 1"

Ext face
Typ reinf

2'cl
1"clear

Second lift

Ext face
Typ reinf

4' to 6'
2'clear

Place concrete to level of broken line - allow to settle and strike off to level as shown and broom joints to expose aggregates

5/8" threaded bolts at not more than 2'-4" oc to hold forms for wall above tight against hardened concrete - grease bolts for easy removable

Wall with rustication strips

Walls without rustication strips

First lift

Note: Vertical construction joints to be located at control joints
PART 3
EXAMPLES OF REINFORCED-CONCRETE
BUILDING-FRAME DESIGN

CONCRETE FOR BUILDING FRAMES

The choice of the class of concrete to be used in a particular building frame depends largely upon the type of frame selected and the number of stories. For a small building two or three stories in height where the column loads are relatively small and for short-span slabs supported on deep beams, concrete having an ultimate compressive strength of 2,000 or 2,500 psi may be entirely satisfactory. In some cases, it becomes practical to construct buildings of this type with masonry bearing walls using concrete only for the floor beams, slabs, and interior columns. In this type of building where the total volume of concrete is small, the engineer may find it difficult to justify the use of more than average concrete having a relatively low compressive strength.

Concrete having an ultimate compressive strength of 3,000 psi is generally the most satisfactory for multiple-story buildings where it becomes necessary to limit the size of columns and the over-all depth of the floor framing. In the lower stories where the eccentricities in the columns due to the bending moments are reduced, the use of concrete having ultimate compressive strengths greater than 3,000 psi is often justified. The increase in the allowable working stresses for shear and bond as well as the compressive strength of 3,000 psi is advantageous in all flat-slab and slab and shallow-beam building frames. It also provides a larger modulus of elasticity whenever it becomes necessary to consider deflection for members having shallow depth-to-span ratios.

Special consideration should be given to the quality of all concrete exposed to the weather not only from the standpoint of obtaining a satisfactory finish where it is used for architectural treatment but also for resistance to weathering which depends entirely upon the exposure and climatic conditions. Concrete when used for architectural treatment usually requires a slightly larger percentage of fine aggregates than would be required of structural strength alone. Special cements containing an air-entraining agent should be used for all concrete subject to severe exposure such as frequent cycles of moisture and freezing and thawing. A suitable concrete for exposure to the weather will usually have an ultimate compressive strength of more than 3,000 psi.

STRESS ANALYSIS FOR BUILDING FRAMES

For uniform loads, the code provides for the design of the members of the structural frame by moment and shear coefficients provided that the unit live load does not exceed three times the unit dead load and the frame consists of two or more spans in which there is not more than 20 per cent variation between the adjacent span lengths. For such cases the span length is taken as the clear distances between the columns or beams.

Where building frames do not fall within the above limitations it becomes necessary to resort to frame analysis by the theory of elastic frames. In building-frame analysis it is general practice to represent the members of the frame by their center lines and reduce the moments and shears to approximately two-thirds the distance from the center line to the face of the supporting beam or column. This practice is especially desirable when the building is such a size that it becomes necessary to divide the slabs, beams, and columns among several designers, as any appreciable error in the assumed sizes can be easily adjusted during the checking of the completed design.
1940 J.C. Report Table I. Net Water-Cement Ratios for Various Types of Construction and Exposure Conditions*

<table>
<thead>
<tr>
<th>Type or location of structure</th>
<th>Severe or moderate climate, wide range of temperature, rain, and long freezing spells or frequent freezing and thawing</th>
<th>Mild climate, rain or semi-arid; rarely snow or frost</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Thin sections, gal. per sack</td>
<td>Moderate sections, gal. per sack</td>
</tr>
<tr>
<td>A. At the water line in hydraulic or waterfront structures or portions of such structures where complete saturation or intermittent saturation is possible, but not where the structure is continuously submerged:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>In sea water</td>
<td>5¾</td>
<td>5¾</td>
</tr>
<tr>
<td>In fresh water</td>
<td>5¾</td>
<td>6</td>
</tr>
<tr>
<td>B. Portions of hydraulic or waterfront structures some distance from the water line, but subject to frequent wetting:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>By sea water</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>By fresh water</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>C. Ordinary exposed structures, buildings and portions of bridges not coming under above groups</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>D. Complete continuous submergence:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>In sea water</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>In fresh water</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>E. Concrete deposited through water</td>
<td>**</td>
<td>**</td>
</tr>
<tr>
<td>F. Pavement slabs directly on ground:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wearing slabs</td>
<td>5¾</td>
<td>6</td>
</tr>
<tr>
<td>Base slabs</td>
<td>6½</td>
<td>7</td>
</tr>
<tr>
<td>G. Special case: For concrete not exposed to the weather, such as interiors of buildings and portions of structures entirely below ground, no exposure hazard is involved, and the water-cement ratio should be selected on the basis of the strength and workability requirements.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* Adapted from Table 1 of the 1940 Joint Committee "Report on Recommended Practice and Standard Specifications for Concrete and Reinforced Concrete," and reproduced here by courtesy of the sponsors.
** These sections not practicable for the purpose indicated.
Examples of Building-Frame Design

1940 J.C. Report Table II. Compressive Strength for Various Water-Cement Ratios*

<table>
<thead>
<tr>
<th>Net water-cement ratio</th>
<th>Probable strength at 28 days, p.s.i.</th>
</tr>
</thead>
<tbody>
<tr>
<td>By weight</td>
<td>Gal. per sack cement</td>
</tr>
<tr>
<td>0.44</td>
<td>5</td>
</tr>
<tr>
<td>0.49</td>
<td>5 1/4</td>
</tr>
<tr>
<td>0.53</td>
<td>6</td>
</tr>
<tr>
<td>0.58</td>
<td>6 1/4</td>
</tr>
<tr>
<td>0.62</td>
<td>7</td>
</tr>
<tr>
<td>0.67</td>
<td>7 1/4</td>
</tr>
<tr>
<td>0.71</td>
<td>8</td>
</tr>
<tr>
<td>0.75</td>
<td>8 1/4</td>
</tr>
</tbody>
</table>

* Adapted from Table 2 of the 1940 Joint Committee "Report on Recommended Practice and Standard Specifications for Concrete and Reinforced Concrete," and reproduced here by courtesy of the sponsors.

1940 J.C. Report Table III. Recommended Slumps for Various Types of Construction*

<table>
<thead>
<tr>
<th>Type of construction</th>
<th>Slump, in.**</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Maximum</td>
</tr>
<tr>
<td>Reinforced foundation walls and footings, and thin plain walls</td>
<td>5</td>
</tr>
<tr>
<td>Plain footings, caissons, and substructure walls</td>
<td>4</td>
</tr>
<tr>
<td>Slabs, beams, and reinforced walls</td>
<td>6</td>
</tr>
<tr>
<td>Building columns</td>
<td>6</td>
</tr>
<tr>
<td>Pavements</td>
<td>3</td>
</tr>
<tr>
<td>Heavy mass construction</td>
<td>3</td>
</tr>
</tbody>
</table>

* Adapted from Table 4 of the 1940 Joint Committee "Report on Recommended Practice and Standard Specifications for Concrete and Reinforced Concrete," and reproduced here by courtesy of the sponsors.

** When high-frequency vibrators are used, the values given should be reduced about one-third.

The code provides for the design of each floor separately upon the assumption that the columns are considered as fixed at the far ends from the floor under consideration as shown in Fig. 9-76. The slabs, beams, and columns may be designed directly for vertical loads based upon these assumptions. For cases where adjacent spans vary more than 20 per cent in length, which is the general case in building frames, the effect of the dead-load stress from adjacent floors should be taken into account in the design of columns. Also whenever it becomes necessary to consider the effect of horizontal loads due to wind or earthquake forces or the effect of sidesway due to an extremely unsymmetrical frame, special consideration is necessary in the design of building columns.

It is important that the designing engineer give adequate consideration to the probable procedure during construction of the building. In cases such as a thin-plate slab

![Fig. 9-76. Floor frames designed on basis columns are fixed at far ends from floor.](image-url)
for an apartment building it is probable that the shoring will be applied in such a manner that the floor slab may be required to support its own weight plus the weight of the shoring, forms, and the newly poured floor slab above before it has developed its ultimate strength. In such cases the superimposed load may be greater than the original design load. Probably the most critical condition of all would result if the contractor were permitted to erect each consecutive story without maintaining adequate shoring adjacent to the exterior columns below the slab as shown in Fig. 9-77. The columns for each consecutive story are built with the joints of the slabs rotated because of the dead load of the slab and the newly poured concrete and forms above. If rotation is prevented by shoring below the slab as indicated the top reinforcing in the slab should be designed and extended as required for the construction loads. If the shoring below the supporting slabs is omitted and the structure completed with the joints rotated on each successive floor the resulting stresses will in many cases be considerably more than those computed for dead and live loads. Every designer of structural frames should become familiar with such conditions by making studies of the possible procedure during construction for the typical building frames on which he is working.

The method of analysis for continuous building frames most generally used in modern times is some modification of moment distribution in which the unbalanced moment at the joints is distributed to the members forming the joint in proportion to their relative stiffnesses or their ability to resist rotation of the joint. The absolute value of the stiffness is $K = kEI/L$ but as the modulus of elasticity $E$ is usually constant for all the members of the joint the relative value of $K = kI/L$ may be used. For members of constant cross section the stiffness coefficient $k$ is equal to 4 and it may be omitted during the computations. For members of variable cross section the stiffness coefficient $k$ may be obtained from Table 9-55.

The value of $I$ to be used in computing the stiffness is generally recommended to be the moment of inertia of the concrete section neglecting the effect of the reinforcement. For T beams it is usually satisfactory to assume that the value of $I$ is approximately two times the moment of inertia of the gross section of the web section. This value for $I$ is approximately correct for flange widths six times the widths of the web sections as shown in Table 9-53a, which may be used where the proportions of the T beam are definitely established.

**EXAMPLES OF BUILDING-FRAME DESIGN**

The examples of the design of building frames that follow are not intended to be complete for any one particular building but are more typical of framing studies that

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are made in a consulting engineer's office to determine the type of frame to use on a given project. They therefore include more than one type of frame for each building, with quantities of materials required for each type and a discussion of the design procedure. While they may, in some cases, be more elaborate than necessary they are generally intended to represent good office practice along the line of the material previously included in this section and to illustrate the use of the design tables included in Part 2 of this section.

The procedure and method of moment distribution where used in the examples of building-frame design are as follows:

1. Sign system: The (−) sign is used to indicate tension in the top fibers of beams and the left side of columns and the (+) sign for compression. Unbalanced moment toward the fixed end of a member produces tension and unbalanced moment away from the fixed end of a member produces compression at the fixed end.

2. Fixed-end moments: Determined from the coefficients for uniform and concentrated loads as provided in Tables 9-54 and 9-55.

3. Carry-over factors: Obtained from Tables 9-54 and 9-55.

4. Unbalanced moments: Distributed to the adjacent joints of the frame by the product of the distribution factor \( D \) and the carry-over factor \( C \).

5. Final balanced moments for beams: Multiply the summation of the unbalanced moments by \( (1 - D) \) and add the product of the summation of the unbalanced moments of the adjacent span multiplied by \( D \).

6. Final moments in columns: Divide the difference between the final beam moments between the columns above and below the floor framing in proportion to the column stiffness factors.

**Building No. 1**

**Occupancy:** Light manufacturing.

**Live load:** 125 psf to include partitions where required for offices, rest rooms, etc.

**Number of stories:** Basement, 3 floors, and roof.

**Foundations:** Sand and gravel having a safe bearing value of 3½ tons per sq ft.

**Exterior walls:** Face-brick exterior with 8-in. lightweight-cement block exposed on the interior, continuous-strip windows passing outside the exterior wall columns.

**Lighting:** Permanent lighting will be installed in the contract with conduits in the structural slabs. Provide a separate floor finish 3½ in. thick for the conduits to all manufacturing equipment which may be removed as required for changes in manufacturing equipment.

**Ventilation:** Main ducts will be exposed on the ceilings of the interior bay with 12-in.-deep branches as required over the equipment in the exterior bays.

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**Fig. 9-78. Plan of typical bay, Building No. 1.**
Heating: Heating will be provided from an exposed horizontal main in the basement with exposed vertical risers and returns at the exterior walls.

Fire protection: A continuous exposed sprinkler system in each bay with pipelines below the 12-in.-deep ducts required for ventilation and 10 ft minimum clear height above the finished floor.

Manufacturing requirements: The process requires minimum clear distance of 22 ft between the interior columns and the exterior walls except that the wall columns may project into the usable space. The center bay is to be used for storing supplies and hand trucking of the finished product to the storerooms. The plan of a typical bay is shown in Fig. 9-78.

Building No. 1, Scheme A: Solid Slab, Beam and Girder Floor System. The floor slabs and longitudinal beams fall within the limits permitted by the ACI Code for design by moment coefficients using the clear spans between the faces of supports. As the required section modulus for all slabs is less than the section modulus provided by a 4-in.-thick slab the areas of reinforcement are determined by using the values of \( V_z \) and \( V_x \) provided in the tables for solid slabs. The beam sections and reinforcement are determined similarly by use of the beam tables. The 12- by 18-in. beam sizes are used to meet the requirements for negative moment with only a small amount of compressive reinforcement, which can be provided by extending the straight bars for positive-moment reinforcement into the columns. Stirrups, where required for shear, have been taken directly from the beam tables.

The girders in the transverse direction have been analyzed as bents with loading conditions which produce the maximum flexural stresses at the critical sections. The 18-in. depth at the center of span was used in order to keep the bottoms of all beams flush. While it is recognized that the haunches at the supports are expensive to form, they have been used rather than increase the story heights by making beams of greater depths for the entire span.

The moments used for design of the girders at the supports of the transverse bents as shown on design sheet B1A5 have been reduced from the moments at the center line of the columns to the moment at a section located two-thirds of the distance from the center line of the column to the face of the column as shown in Fig. 9-79.

![Fig. 9-79. Section for end design moment in girders.](image)

Reduced moments:

- \( A \): 171 ft-kips - 42.6 kips \times 0.5 \text{ ft} = 150 \text{ ft-kips} \\
- \( B \): 242 ft-kips - 47.2 kips \times 0.5 \text{ ft} = 219 \text{ ft-kips} \\
- 172 \text{ ft-kips} - 18.8 \text{ kips} \times 0.5 \text{ ft} = 161 \text{ ft-kips}

By using a floor system where the span-to-depth ratio is less than 20, deflection does not become a problem for design. The deep beams will require a minimum of reinforcement but have the disadvantage of being expensive to form.
Building #1 – Scheme "A"
Solid slab – beam and girder floor

\[ \frac{f_b}{f_y} = 3000/10/20,000 \quad v_c = 90 \text{ psi} \quad u = 210-300 \text{ psi} \]

\[ f_c = 1350 \text{ psi} \quad \text{ACI Code} \]

Symmetrical about \( C \)

Typical framing

Part plan of typical bay

- Live load –
  - 125
- Dead load – 1" cement finish
  - 12
- 21/2" lightweight floor fill
  - 23
- 4" stone concrete slab
  - 50
- Sprinkler system & ducts
  - 5
- Total
  - 215 psf

Floor loads

Assume 12" wide beams – 71/2" clear span for slabs

\[ W = 7.08 \text{ at 215 lbs} = 1.52^K \]

\[ M_o = \frac{Wl}{14} = \frac{1.52 \times 7.08}{14} = 0.77^K \]

\[ M = \frac{Wl}{10} = \frac{1.52 \times 7.08}{10} = 1.08^K \]

\[ S = \frac{0.77 \times 12000}{4350} = 6.9 (<19) \]

\[ S = \frac{1.08 \times 12000}{1350} = 9.6 (<19) \]

\[ A_z = \frac{M}{ad} = \frac{0.77}{1.44 \times 3^3} = 0.18 \text{ si/ft} \]

\[ A_z = \frac{M}{ad} = \frac{1.08}{1.44 \times 3^3} = 0.25 \text{ si/ft} \]

\[ #3 \& #4 \text{ alt at 9" oc } - A_z = 0.21 \]

\[ #4 \text{ at 9" oc } - A_z = 0.27 \]

\[ V_b = 1.33 \quad 10.21 = 1.61^K \]

\[ V = 1.15 \times \frac{1.52}{2} = 0.87^K \]

\[ v_c = 2.81^K \]

Design sheet B1/A1

Floor slabs

End span S1

Min A_s = 4 x 12 x 0.0025 = 0.12 si/ft

V_c = 2.81^K max

#3 & #4 at 9" oc or #3 at 6" oc
Building #1 - Scheme "A"

Design sheet
B1A2

Int. span - S2

4" S2

#3 & #4 at 10" oc

or

#3 at 8" oc

Int span - S3

4" S3

#3 & #4 at 9" oc

or

#3 at 6" oc

| W = 1.52k | +M = \frac{WL}{16} = \frac{1.52 \times 7.08}{16} = 0.67k | -M = \frac{WL}{12} = \frac{1.52 \times 7.08}{12} = 0.895k |
| \frac{A_s}{0.67} = \frac{1.44 \times 3}{0.156} | \frac{A_s}{0.895} = \frac{1.44 \times 3}{0.21} |
| \#3 & \#4 at 10" - A_s = 0.18 si/ft | \#4 at 10" oc A_s = 0.24 si/ft |

W = 7.75 at 215 lb = 1.67k V = 0.835k/l OK on 4" slab

+M = \frac{WL}{16} = \frac{1.67 \times 7.75}{16} = 0.841k |
| -M = \frac{WL}{12} = \frac{1.67 \times 7.75}{12} = 1.081k |
| \frac{A_s}{0.841} = \frac{1.44 \times 3}{0.25} |
| \#4 at 18" oc | \#4 at 20" oc |
| Average 9" oc A_s = 0.26 si/ft |

w = 1.15 \times 1.52/2 = 0.87 |
| 1.52/2 = 0.76 |
| 4' of floor = 0.215 |
| Beam = 0.175 |
| 2.020k/l |

W = 19 \times 2.02 = 38.4k |
| V = \frac{W}{2} = 19.2k |

12"x18" bm, V_c = 14.9k |
| \#3 1\frac{1}{2} at 8" - V_c = 21.6k |

Dist req'd = \frac{19.2 - 14.2}{2.02} = 2.36 |

4 \#7: A_s = 2.40 |
| 4 \#7 - V_y = 4 \times 7.8 = 31.2k |
| 2 \#5 T A_s = 0.60 |
| 2 \#5 not required for bond |

w = 1.52 |
| 1' floor = 0.215 |
| Beam = 0.175 |
| 1.910k/l |

W = 19' at 1.91k/l = 36.3k |
| V = 18.15k |
| 12"x18" bm - V_c = 14.9 |
| \#3 1\frac{1}{2} at 8" oc req'd |
| \frac{18.15 - 14.9}{1.91} = 1.2 |
| 2 \#8: A_s = 1.58 |
| \#3 1\frac{1}{2} 2 at 8" |

B1

12"x18" bm |
| 2\#6-2\#7 |
| 2\#5 T ea end |
| \#3 1\frac{1}{2} - 3", 3 at 8" ea end |

B2

12"x18" bm |
| 2\#7-1\#8 |
| 2\#7 T ea end |
| \#3 1\frac{1}{2} - 3", 2 at 8" ea end |

Table 9-38C

Beams

B1

12"x18" bm |
| 2\#6-2\#7 |
| 2\#5 T ea end |
| \#3 1\frac{1}{2} - 3", 3 at 8" ea end |

B2

12"x18" bm |
| 2\#7-1\#8 |
| 2\#7 T ea end |
| \#3 1\frac{1}{2} - 3", 2 at 8" ea end |

\begin{align*}
W & = \frac{38.4 \times 19}{16} = 45.6k \\
S & = \frac{45.6 \times 12000}{1350} = 402 \\
12''x18'' & \text{ bm, } S_l = 501 \\
A_s & = \frac{M}{ad} = \frac{45.6}{22.3} = 2.05 \\
\#3 1\frac{1}{2} \text{ at } 8'' - V_c = 21.6k \\
\text{Dist req'd} & = \frac{19 - 14.2}{2.02} = 2.12'' \\
4 \#7: & A_s = 2.40 \\
4 \#7 & \text{ - } V_y = 4 \times 7.8 = 31.2k \\
2 \#5 T & A_s = 0.60 \\
\#3 1\frac{1}{2} & 2 at 8'' |
\end{align*}
EXAMPLES OF BUILDING-FRAME DESIGN

Building #1 - Scheme "A"

Design sheet BIA3

B3
12"x 18" bm
2 #7-1 #8
2 #6T ea end

#3 1/2, 3/4, 2 at 8'

ea end

Same as B1
12"x 18" bm
2 #6-2 #7
2 #5T ea end

#3 1/2, 3/4, 3 at 8'
ea end

Windows at 15'
5' brick at 45'
3'-8" LW block #
3/2 of Fl at 215#
10 x 24" bm at 210#

Table 9-38f

Spandrel beam
10'x24" B4
S1 = 805
2 #6-1 #5
2 #5T contin
Lap 6'-0" at cols
#3 1/2 at 20" oc contin

W = 18.5 x 1.4 K = 26 K
V = 13 K
V = 15.2 K no St req'd
Use #3 at 20" oc contin for support of contin top bars
Vb = (5x5) = 5x0.233x9.5 = 22.7 K OK

w = 1.52 = 0.76
2
1.67
2
= 0.83

1 of slab = 0.215
bm = 0.175
1.980 K

W = 18.5
V = 18.4 K
12"x18" bm Vc = 14.9 K
#3 1/2 at 8" oc req'd
Dist req'd = 18.4 x 14.9
1.98' = 1.77

w = 1.67
1 of slab = 0.215
bm = 0.175
2.06 K

W = 19 at 2.06 K = 39.1 K
V = 19.55 K
12"x 18" bm Vc = 14.9 K
#3 at 8" req'd
Dist req'd = 19.55 x 14.9
2.06' = 2.26

W = 16.8 x 18.5
V = 41.5
12"x18" bm - S1 = 501
A1 = 2.52 si

2 #8: A1 = 1.58
2 #6T: A1 = 0.88
2.46 si

W = 36.8 x 18.5
V = 56.6
12"x18" bm - S1 = 501
A1 = 2.52 si

2 #8: A1 = 1.58
2 #6T: A1 = 0.88
2.46 si
### EXAMPLES OF BUILDING-FRAME DESIGN

#### Design sheet BIA5

**Building #1 - Scheme "A"**

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>28k</td>
<td>D Ld + L Ld</td>
<td>D Ld + L Ld</td>
<td>D Ld</td>
<td>D</td>
</tr>
<tr>
<td></td>
<td>38.4k</td>
<td>36.3k</td>
<td>39.1k</td>
<td>18k</td>
</tr>
<tr>
<td></td>
<td>w = 575 ft</td>
<td>w = 10k</td>
<td>w = 356 ft</td>
<td>w = 84k</td>
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**Max Mat B**

<table>
<thead>
<tr>
<th></th>
<th>M^2</th>
<th>-259 (0.256)</th>
<th>-113 (0.218)</th>
<th>-113 (0.213)</th>
<th>-114 (0.216)</th>
<th>-114 (0.256)</th>
<th>-116 (0.256)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>19</td>
<td>9</td>
<td>5</td>
<td>6</td>
<td>25</td>
<td>8</td>
<td></td>
</tr>
<tr>
<td></td>
<td>9</td>
<td>3 (-270)</td>
<td>2</td>
<td>3 (-99)</td>
<td>4</td>
<td>2 (-141)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>-215</td>
<td>-110</td>
<td>-52</td>
<td>-102</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>-152</td>
<td>-27</td>
<td>-62</td>
<td>-52</td>
<td>-102</td>
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**-M:**

<table>
<thead>
<tr>
<th></th>
<th>160k</th>
<th>-242k</th>
<th>-172k</th>
<th>-97k</th>
<th>-121k</th>
</tr>
</thead>
</table>

**V:**

<table>
<thead>
<tr>
<th></th>
<th>43.7k</th>
<th>14.5k</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>+3.5</td>
<td>+4.3</td>
</tr>
<tr>
<td></td>
<td>47.2k</td>
<td>18.8k</td>
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</tbody>
</table>

**Max + M_{BC}**

<table>
<thead>
<tr>
<th></th>
<th>M^2</th>
<th>-116 (0.256)</th>
<th>-113 (0.218)</th>
<th>-113 (0.213)</th>
<th>-114 (0.216)</th>
<th>-114 (0.256)</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>4</td>
<td>-139</td>
<td>+4 (-109)</td>
<td>-102</td>
<td>-51</td>
<td>-102</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>-30</td>
<td>-69</td>
<td>-30</td>
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**-M:**

<table>
<thead>
<tr>
<th></th>
<th>132k</th>
<th>-120k</th>
<th>-120k</th>
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</thead>
</table>

**V:**

<table>
<thead>
<tr>
<th></th>
<th>24.5k</th>
</tr>
</thead>
</table>

\[ + M = 24.5 \times 8.75 = 214 \]

\[ -5 \times 4.37 = -22 \]

\[ - M = 120 \]

\[ 72k \]

**Design M_o & Shears**

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>28k</td>
<td>-150k</td>
<td>-219k</td>
<td>-161k</td>
</tr>
<tr>
<td></td>
<td>+127k</td>
<td>+82k</td>
<td>+72k</td>
</tr>
</tbody>
</table>
|      | 42.6k      | 47.2k      | 24.5k      | 24.5k      | 47.2k
Building #1 - Scheme "A"

Table 9-38c

+M = 127k  S: 1140  18"x18" bm  S_t = 753  A_s = 3.80 si
1.013' of 4" Tee  S_t = 387  A_s = 2.12
1140  5.92 si
V_x = 21.4k Stirrups req'd to concentrated loads

At A: -M = 150k  S = 1335  18"x24" bm  S_t = 1450  A_s = 5.25
Use 3 #6 bot in haunch (not req'd)  2 #11 bt = 2.83
V = 42.6k at supt  2.42
41.4 at 2'-0"  #4 at 10'
40.3 at 4'-0"  #4 at 8'
39.1 at 6'-0"  #4 at 5'
3 #8: V_x = 24.6
2 #11: V_x = 34.6
68.2 at supt

Table 9-38f

At B: -M = 219k  S = 1950  18"x24" bm  S_t = 1450  A_s = 5.25
3 #8 bot  S_t = 624  A_s = 500
624 x 2.19 = 1.75
7.00
V = 47.2k at supt
46.05 at 2'-0"  #4 at 6'
44.90 at 4'-0"  #4 at 4'
43.75 at 6'-0"  #4 at 4'
3 #8 T = 2.37

-\( M_b = -M_c = 161k \)  S = 1435  18"x24" bm  S_t = 1450  A_s < for span A-B
V = 24.5k at supt  \( \Lambda \) at 8" max for D = 18"
23.35 at 2'-0"  U_s at \#3 1/2 3" 8 at 8" ea end
22.2 at 4'-0"
21.05 at 6'-0" No stirrups req'd

Design sheet B1A6

18"x18" bm G1  3 #10 - 2 #11
3 #6 bot
3 #8 T at "A"
#4 1/2, 3"
4 at 8"
8 at 5"
18"x18" bm G2
3 #7 - 2 #8
3 #6 bot at B & C
#3 1/2, 3"
8 at 8"
Building No. 1, Scheme B: Two-way Solid Slab, Slab Band Beams. The slab band beams have been used on the column lines in preference to deep beams in order to consider the possibility of a reduction in story height as compared with the beam and floor slab system used in Building No. 1, Scheme A.

The loads on the two-way panels have been distributed by method 1, as provided in Section 709 of the ACI Code, using the clear distance between slab bands for the span lengths. For flexural analysis the span lengths are based on the distances between column centers with the slab bands considered as haunches for approximately one-tenth the length of the spans. Because of the effect of the haunches the slab is analyzed as a continuous frame in both directions, although the longitudinal spans are equal and coefficients might have been used otherwise.

The stiffness of the slab bands used in the analysis of the structural frame is based on an effective width of the slab and haunches as the distances between inflection points which are assumed to be located one-fifth of the span length on each side of the column centers. The beam factors and coefficients for fixed-end moments shown on design sheet B1B4 were obtained from Table 9-55 for values of \( a = L/10 \) and \( r = 0.20 \). The slab bands have been analyzed for the equivalent uniform loads from the two-way...
slab as permitted by the code plus a concentrated load upon the haunches which is taken as the shears from the slab bands in the opposite directions.

Because of the shallow depth of the slab bands as compared with the span lengths and the conditions of live load from manufacturing equipment, the deflection is checked for the dead load plus 50 per cent of the live load, which is assumed to be a permanent load on the frame on design sheet B1B6. For this purpose the value of $E_s$ is assumed to be $500f'_c$ as determined from Fig. 9-46, using the average dead-load stress for the slab band.

The spandrel beam as designed on design sheet B1B9 has been checked for the effect of combined direct and torsional shears as suggested in Fig. 9-43.

Building #1 - Scheme "B"

2-way solid slab - slab band beams at columns

$t'_c/n_i = 3000/10/20,000$  
$v'_c = 90$ psi  
$u = 300$ psi  
$f_c = 1350$ psi

ACI code

---

Typical framing

Part plan of typical bay

Min slab thickness $t = \frac{2(16+20)12}{180} = 4.8''$

Live load:
- 125

Dead load:
- 1 cement finish 12
- 2½" lightweight floor fill 23
- 5½" stone concrete slab 70
- Sprinkler system & ducts 5

Total 235 psf

Slab S1: $0.87 \times 20' = 1.43$  
$C = 0.57$  
$C_1 = 0.57$  
$t-C = 0.84$  
$t - C_1 = 0.43$

$C_A = 0.37$  
$C_{BA} = 0.13$

Slab S2: $0.76 \times 13.5 = 0.84$  
$C = 0.24$  
$C_1 = 0.24$  
$t - C = 0.56$  
$t - C_1 = 0.24$

$C_B = 0.19$  
$C_{BB} = 0.31$

Floor loads

5½" slab
Min $A_s = 0.0025 \times 66 = 0.165$ sl

Distribution of loads

(Method 1 - ACI code)
### Building #1 - Scheme "B"

<table>
<thead>
<tr>
<th></th>
<th>20'</th>
<th>20'</th>
<th>20'</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w$</td>
<td>0.57 x 110 lb = 63 lb/ft</td>
<td>0.57 x 235 lb = 134 lb/ft</td>
<td>0.57 x 110 lb = 63 lb/ft</td>
</tr>
<tr>
<td>$M^f$</td>
<td>-5.1</td>
<td>-5.1 (0.305)</td>
<td>-5.1</td>
</tr>
<tr>
<td>+0.8</td>
<td>-0.8</td>
<td>-0.8</td>
<td>+0.8</td>
</tr>
<tr>
<td>+0.5</td>
<td>-0.5</td>
<td>-0.5</td>
<td>+0.5</td>
</tr>
<tr>
<td>(-0.8)</td>
<td>+0.3</td>
<td>-0.3 (-6.7)</td>
<td>-0.3</td>
</tr>
<tr>
<td>-3.3</td>
<td>-3.7$^k$</td>
<td>$V = 1.34$</td>
<td>$V = 1.34 \times \frac{10}{2} = 3.0^k$</td>
</tr>
<tr>
<td>$A_s = \frac{1.44 \times 4.5}{0.40}$</td>
<td>0.40 sl</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$DLD$</th>
<th>$DLD + LLD$</th>
<th>$DLD + LLD$</th>
<th>$DLD$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M^f$</td>
<td>-2.4</td>
<td>-5.1</td>
<td>-2.4</td>
</tr>
<tr>
<td>+0.8</td>
<td>-0.8</td>
<td>+0.8</td>
<td>+0.8</td>
</tr>
<tr>
<td>+0.5</td>
<td>-0.2</td>
<td>+0.5</td>
<td>+0.2</td>
</tr>
<tr>
<td>(-7.2)</td>
<td>-0.1 (-72) (-5.1)</td>
<td>+0.2 (-0.9)</td>
<td></td>
</tr>
<tr>
<td>-3.6</td>
<td>-3.6</td>
<td>-3.6</td>
<td>-3.6</td>
</tr>
<tr>
<td>$V = 1.34$</td>
<td>$V = 1.34$</td>
<td>$V = 1.34$</td>
<td></td>
</tr>
<tr>
<td>$A_s = \frac{1.44 \times 10}{155}$</td>
<td>0.21</td>
<td>0.21</td>
<td></td>
</tr>
</tbody>
</table>

**At col $Q$: $M = 72^{ik}$**

**At $0.10 L$: $M = 1.55 \times 2^2 - 0.268 - 7.2 = 4.37^{ik}$**

**Table 9-30a**

| $S$ | 39 | $5.5^{\text{th}}$ slab $- S_1 = 42.6$ |

**Span same as S1**

| $DLD: w = 0.24 \times 110$ lb = 26 psf | $DLD + LLD: w = 0.24 \times 235$ lb = 56 psf |

**Moments**

- **+M = 0.42 x 3.0$^{ik}$ = 1.26$^{ik}$**
  - $d = 4^a$ $A_s = \frac{1.26}{5.75} = 0.22$ sl

- **-M = 0.42 x 4.37$^{ik}$ = 1.83$^{ik}$**
  - $d = 4.5^a$ $A_s = \frac{1.83}{6.5} = 0.28$ sl

---

**Design sheet B102**

**Slab S1**

- Longitudinal span
  - $r_a = r_b = 1.0$
  - $a = b = 0.10$
  - $k = 6.11$
  - $c = -0.61$
  - $F = 0.095$ WL
  - $D = 0.5$ CD = 0.305

**+ M**

- #3 + #5 at $5/8^{\text{th}}$ oc
  - (Increase spacing 25% at $1/4$ points adjacent to slab bands)

**Max V**

- **Max - M**
  - #5 at $5/8^{\text{th}}$ oc
  - $V_b = 4.58^{ik}$
  - $V_c = 4.22^{k}$

**Slab S2**

- Longitudinal span

- **#3 & #4 at 8° oc**

- **#4 at 8° oc**

- $V_b = 2.74^{k}$
### Building #1–Scheme "B"

<table>
<thead>
<tr>
<th></th>
<th>23.75'</th>
<th>17.5'</th>
<th>23.75'</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Ld + Ld</td>
<td>D Ld</td>
<td>D Ld + Ld</td>
</tr>
<tr>
<td>w=0.16x235lb</td>
<td>w=0.44x110lb</td>
<td>w=0.16x235lb</td>
<td></td>
</tr>
<tr>
<td>=38 lb/ft</td>
<td>=48 lb/ft</td>
<td>=38 lb/ft</td>
<td></td>
</tr>
<tr>
<td>D:</td>
<td>[1.0]</td>
<td>[0.42]</td>
<td>[0.58]</td>
</tr>
<tr>
<td>CD:</td>
<td>(0.61)</td>
<td>(0.26)</td>
<td>(0.35)</td>
</tr>
<tr>
<td>Mf:</td>
<td>-2.0</td>
<td>-2.0</td>
<td>-1.4</td>
</tr>
<tr>
<td></td>
<td>-0.2</td>
<td>-1.2</td>
<td>+0.3</td>
</tr>
<tr>
<td></td>
<td>-0.4</td>
<td>(-3.6)</td>
<td>-0.4</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>-2.08</td>
<td>-0.25</td>
</tr>
<tr>
<td>V = (0.45^K)</td>
<td>+M = (0.35 \times \frac{9.2}{2} = 1.60^K)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(0.35^K)</td>
<td>+0.10</td>
<td>d = 4&quot;</td>
<td>(A_S = \frac{1.6}{5.75} = 0.28) si</td>
</tr>
<tr>
<td>(0.55^K)</td>
<td>+0.10</td>
<td>ad = 5.75</td>
<td></td>
</tr>
</tbody>
</table>

### Slab S1 & S2

(Transverse spans)

- \(r_a = r_b = 1.0\)
- \(a = b = 0.10\)
- \(k = 6.11\)
- \(c = 0.61\)
- \(M^s = 0.095 WL\)

### Slab S2

- \(M_S = 0.095\) WL
- \(A_S = 0.28\) si

### Slab S1

- \(M = 0.35 \times \frac{9.2}{2} = 1.60^K\)
- \(d = 4\) "
- \(A_S = \frac{1.6}{5.75} = 0.28\) si

### Slab S2

- \(M_S = 0.095\) WL
- \(A_S = 0.28\) si

### Slab S3

- \(M = 0.35 \times \frac{9.2}{2} = 1.60^K\)
- \(d = 4\) "
- \(A_S = \frac{1.6}{5.75} = 0.28\) si

### Design Sheet B1B3

- \#4 & \#4 at 9°oc
- \#4 at 9°oc
- \#4 at 9°oc
### Building #1 - Scheme "B"

<table>
<thead>
<tr>
<th>Section</th>
<th>Load Distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>23.5'</td>
</tr>
<tr>
<td>B</td>
<td>17.5'</td>
</tr>
<tr>
<td>C</td>
<td></td>
</tr>
</tbody>
</table>

**D Ld + L Ld:**
- **W** = 0.84 x 20 x 23.5 x 235 lb
  - 93' x slab band = 6.5 x 99.5
  - 0.37 x 20 x 23.5 x 235 lb
  - 40.8 x slab band = 3.2 x 44.0
  - D Ld = W = 50
  - V = 22.3

**D Ld + L Ld:**
- **W** = 0.56 x 20 x 17.5 x 235 lb
  - 46' x slab band = 4.8 x 50.8
  - 0.19 x 20 x 17.5 x 235 lb
  - 15.6 x slab band = 2.4 x 18
  - D Ld = W = 26.3
  - V = 9.7

**V = 7.2**

2.7
6.5 (wall & bm) (Slab band) 2.7
16.4
R = 32.8
9.9

**D Ld:** We = 13.8
V = 12.4
R = 24.8
V = 6.1

**D Ld:** We = 17.4
V = 8.6

**Slab bands:** 48" x 11" b/b' = 2

**At:**
- I = 4.3 x 48 x 11^3/12 = 6900

**At haunch:**
- I = 96 x 11^3/12 = 10,650

**D =**
- 12 x 10,650
- 1.3 x 48
- 12.7

**K AB =**
- 4.8 x 6900
- 23.5
- 1410

**K BC =**
- 4.8 x 6900
- 17.5
- 1890

**18" x 18" cols:**
- I = 8745
- K = 4 x 8748
- 12.17
- 2870

**D AB =**
- 1410
- 7150
- 0.2

**D BA =**
- 0.16

**D BC =**
- 0.21

**CD =**
- 0.11

---

**Assumed sizes & frame constants**

**Transverse bent**

**Design sheet B1B4**

**Slab bands**

**Load distribution**

**Transverse bent**

**Longitudinal bents**

---

**Transverse bent**
### Building #1–Scheme "B"

<table>
<thead>
<tr>
<th>A</th>
<th>23.5'</th>
<th>B</th>
<th>17.5'</th>
<th>C</th>
<th>23.5'</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>DLD + LLD</td>
<td></td>
<td>DLD</td>
<td></td>
<td>DLD + LLD</td>
<td></td>
<td></td>
</tr>
<tr>
<td>32.8K</td>
<td></td>
<td>9.9K</td>
<td>8.6K</td>
<td>8.6K</td>
<td>9.9K</td>
<td>22.9K</td>
</tr>
<tr>
<td>0.67</td>
<td></td>
<td>0.16</td>
<td>0.21</td>
<td>0.21</td>
<td>0.16</td>
<td>0.21</td>
</tr>
<tr>
<td>38.8K</td>
<td>26.3K</td>
<td>99.5K</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>22K</td>
<td>15K</td>
<td>22K</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Design sheet B1B5**

**Max – M at A**

**Max + M_{AB}**

- 48"x11" SB f
- 3#8 st 6#8 bt

**At 6" from col θ:**

- $M_{AB} = 48.6 \times 0.5 - 186 = -162K$
- $S = 1430 \text{ b} = 1430 / 148 = 7.2$, $A_s = 11.4 \text{ si}$

**At face of haunch:**

- $M_{eb} = 48.6 \times 2 - 8.5 - 186 = 96K$
- $S = 845$, Slab band - $S_t = 792$ (OK)
- 3#8 comp bars $S_t = 258$ (OK)

**At 0.10L:**

- $V = 0.32 \times 20 \times 23.5 \times 235 \times 1b = 35.4$
- Slab band = 2.6 \text{K}
- $38.4K / 48 \times 11$ slab band - $V_c = 36.4K$
- Provide bent bars for shear

<table>
<thead>
<tr>
<th>DLD + LLD</th>
<th>DLD + LLD</th>
<th>DLD</th>
</tr>
</thead>
<tbody>
<tr>
<td>24.8</td>
<td>9.9</td>
<td>15.4</td>
</tr>
<tr>
<td>15.4</td>
<td>50.8K</td>
<td></td>
</tr>
<tr>
<td>6.1</td>
<td>50K</td>
<td></td>
</tr>
<tr>
<td>19.1K</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Max – M at B**

**Bent bars 6#8**

**Top bars = 10#8**

**At 6" from col θ:**

- $M_{BA} = 61.2 \times 0.5 - 218 = 187$
- $S = 1665$, $b = 8'$, $S_t = 1580$, $A_s = 12.7 \text{ si}$
- 3#8 comp bars 258 = 0.68
- 13.38 \text{ si}

**At 0.106:**

- $38K + 218 = 180 = 39.6$
- 48"x11" slab band: $V_c = 36.4K$
- Provide bent bars for shear: each #8 $V_c = 14.7K$

- $M_{BA} = 61.2 \times 2.35' - 19.8 \times 117 - 218 = 98K$
- $S = 874$
- 48"x11" slab band: $S_t = 792$
- 3#8 comp bars = 258 = 1050
EXAMPLES OF BUILDING-FRAME DESIGN

Building #1 - Scheme "B"

Design sheet BIB6

Max + $M_{AB}$

$48'\times11''$ SB2
$3\#6$ st $2\#6$ ft

Detail of transverse slab bands

Deflection at slab band "AB"

$D_{Ld} + 50\%$ L $Ld$: $M_{A} = -119'\text{ft}$ $M_{C} = 70'\text{ft}$ $M_{B} = -138'\text{ft}$

$f_{c} = 900\text{psi}$ $f_{c} = 530\text{psi}$ $f_{c} = 1040\text{psi}$

Average $f_{c} = \frac{(900 + 530 + 1040)}{3} = 825\text{ psi}$

From fig 9-46 - part 9-1: $E_{c} = 500\times f_{c} = 500\times3000 = 1,500,000$

Uniform load $M_{c}$ at A $M_{c}$ at B

Constant I: $+0.625WL^{3/48EI}$ $-3.00ML^{3/48EI}$ $-3.00ML^{3/48EI}$

Haunch at A: $0.001$ $-0.03$ $0.00$

Haunch at B: $0.001$ $+0.00$ $+0.03$

$D_{Ld} + 50\%$ L $Ld$: $\Delta = 0.623 \times \frac{74,500\times282^{3}}{48\times1,500,000\times6900}$ $+ 2.00^*$

$-2.97 \times \frac{119\times12,000\times282^{2}}{48\times1,500,000\times6900} = -0.68^*$

$-2.97 \times \frac{119\times12,000\times282^{2}}{48\times1,500,000\times6900} = -0.79^*$

$0.53^*$
Building #1 - Scheme 'B'

50% L Ld: \[ \Delta = \frac{0.623 \times 25,000 \times 282^2}{48 \times 3,000,000 \times 6900} = 0.67^* \]

\[ \frac{2.97 \times 43 \times 12000 \times 282^2}{48 \times 3,000,000 \times 6900} = -0.24^* \]

\[ \frac{2.97 \times 49 \times 12000 \times 282^2}{48 \times 3,000,000 \times 6900} = -0.28^* + 0.15^* \]

Total \( \Delta \) = 0.53 + 0.15 = 0.68^*

Max \( \Delta \) = \[ \frac{L}{600} \] \[ \frac{4 \times 23.5 \times 12}{600} \] = 0.47^*

Design sheet B1B4: Span BA-V = 44^K + \[ \frac{218 - 180}{23.5} \] = 45.6^K

BC = 18 + \[ \frac{119 + 85}{20} \] = 19.7^K

Long. span = 19.8 + 30.8 = 50.6

\[ \frac{115900}{4(18 + 9.5 + 9.5) \times \frac{3}{8} \times 9.5} \] = 93.5 psi

Provide splayed cap at col

\[ \nu_c = \frac{115900}{4(42.5') \times \frac{3}{8} \times 9.25''} \]

= 84 psi

Provide \( \frac{1}{2}^* \) camber at \( C \) of span for D Ld deflection

Max shear at col B
### EXAMPLES OF BUILDING-FRAME DESIGN

#### Design sheet BIBB

**Longitudinal bents**

Frame constants same as transverse bents:

- Max + M
  - 48 x 11 SB3
  - 5 # 6 st - 4 # 6 bt

---

#### Building #1 - Scheme "B"

<table>
<thead>
<tr>
<th>A</th>
<th>20'</th>
<th>D Ld</th>
<th>14.7^K</th>
<th>31.2^K</th>
<th>0.089 x 31.2 x 20 = 55.5</th>
<th>0.05 x 14.7 x 20 = 14.7</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>20'</td>
<td>D Ld + L Ld</td>
<td>25.3^K</td>
<td>60.5^K</td>
<td>0.089 x 605 x 20 = 108</td>
<td>0.05 x 253 x 20 = 25.3</td>
</tr>
<tr>
<td>C</td>
<td>20'</td>
<td>31.2^K</td>
<td>14.7^K</td>
<td>14.7^K</td>
<td>4.8 x 6900 / 20 = 1650^4</td>
<td></td>
</tr>
<tr>
<td>D</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>133^K</td>
<td></td>
</tr>
</tbody>
</table>

- M = 0.089 x 31.2 x 20 = 55.5
- 0.05 x 14.7 x 20 = 14.7
- 0.089 x 605 x 20 = 108
- 0.05 x 253 x 20 = 25.3
- 4.8 x 6900 / 20 = 1650

#### Slab bands:

- K = 133^K
- D = 1650
- CD = 0.55 x 0.18 = 0.10

#### Max + M

- 48 x 11 SB3
- 5 # 6 st - 4 # 6 bt

---

#### At 6^a from col C:

- M_{bc} = 56.5 x 0.5 - 139 = 104^K
- S_t = 925
- b = 925 / 198 = 4.7^K
- A_s = 6.8^2

#### At 0.10 L:

- M_{bc} = 56.5 x 2 - 6.04 x 1 - 25.3 x 1 - 139 = 67^K
- A_s = 67 / (144 x 8.75) = 5.3^2

---

At 6^a from col C:

- \[ M_{bc} = 56.5 \times 0.5 - 139 = 104^K \]
- \[ S_t = 925 \]
- \[ b = \frac{925}{198} = 4.7^K \]
- \[ A_s = 6.8^2 \]

At 0.10 L:

- \[ M_{bc} = 56.5 \times 2 - 6.04 \times 1 - 25.3 \times 1 - 139 = 67^K \]
- \[ A_s = \frac{67}{144 \times 8.75} = 5.3^2 \]
Building #4 - Scheme "B"

Slab: \( W_e = 0.43 \times 18.5 \times 11.75' \) at 235 lb = \( 22^K \)  
Wall = 18.5' at 650 lb = 12  
Haunch = 18.5' at 170 lb = \( \frac{3.1}{37.1^K} \)

\[-M = 37.1 \times \frac{18.5}{12} = 57.1^K \]
\[+M = 37.1 \times \frac{18.5}{16} = 43^K \]

\[S = 57.1 \times \frac{12000}{1350} = 508 \]
\[A_s = \frac{43}{31} = 1.39 \text{ si} \]
\[4 \# 5 \text{ top } + 2 \# 7 \text{ bt } = 2.04 \text{ si} \]
\[2 \# 6 \text{ st } - 1 \# 7 \text{ bt } : 1.48 \text{ si} \]

### Torsion - D Ld + 50% L Ld

- From design sheet BIB3: \( 18.5^s \text{ slab } - K = 18.5 \times \frac{6.11 \times 12 \times 5.5}{12 \times 23.5} = 795 \)
- From design sheet BIB4: \( \Sigma K \text{ col } = 1410 + 2870 + 2870 = 7150 \)

\[b = \frac{10}{24} = 0.42 \]
\[k_t = 0.266 \]
\[C_t = 0.258 \]
\[K_t = \frac{0.266 \times 10^4}{4 \times 9.25} = 73 \]

At col: \( D = \frac{7150}{7945} = 0.90 \)  
\( m_1 = 0.90 \times 1.5 = 1.35^K \)

At \( \zeta \) of span: \( m_2 = 0.90 \times 1.5 \times \frac{73}{116} = 0.85^K \)

### Design sheet BIB9

Spandrel beam B1

\[10^s \times 24^s \text{ B1 } \]
\[S_t = 808 \]

\[2 \# 6 - 1 \# 7 \]
\[2 \# 5 \text{ T-contin } \]

![Diagram](image)

### Analysis

\( M_t = \frac{(2 \times 0.85 \times 1.35) \times 18.5}{6} = 9.4^K \)

\( V_t = \frac{9.4 \times 12,000}{0.254 \times 10^2 \times 24} = 182 \text{ psi} \)

\( v = \frac{14,100}{10 \times 4\times 24} = 74 \text{ psi} \)

\( 1 \# 7 \text{ bt } : V_a = 0.43 \times 19.8 = 8.5^K \)

\( V_a = \frac{8500}{10 \times 4\times 22} = 44 \text{ psi} \)

\( n = \frac{10(182 + 74 - 90 - 44)}{2 \times 182} = 3.36^* \)

\( \# 3 \text{ stirrups } : S = \frac{0.11 \times 20,000}{(3.36 \times 122)/2} = 10.8^* \)

\( 6^* - 4 \text{ at } 11^* \)

\( 3 \text{ at } 16^* \text{ each end} \)
Building No. 1, Scheme C: Two-way Flat Slab with Metal Fillers. The slab has been analyzed as a continuous building frame in the transverse direction using frame constants from Table 9-55 by determining an equivalent I for the solid slab around the columns. The moments at the critical sections are proportioned between the column strips and the middle strips as required by ACI Table 1003(c) of the 1956 code. In the longitudinal direction where the spans are uniform, analysis has been made by use of moment coefficients from ACI Table 1004(f) of the 1956 code.

With 8-in.-deep fillers and a 2½-in. topping, the span-to-depth ratio is 27, or approximately 50 per cent greater in depth than the minimum thickness of L/40 permitted by the code, and deflection or rigidity of the slab has not been considered a problem for design.

For the design of the spandrel beam as shown on design sheets B1C6 and B1C7 the percentage of the floor panel load to be supported is taken from ACI Table 1004(f) of the 1956 code. The beam is designed for the combined direct and torsional shear from the middle strip as suggested in Fig. 9-42.

The principal advantages of this type of floor system are the saving in the story height, which is approximately the same as Building No. 1, Scheme B, and the resulting economy due to the light weight of the slab and the saving in reinforcement due to the depth of the slab. The formwork is exceptionally economical as the shoring is of a constant height and the form framing can be reused many times and salvaged after the construction is completed.
REINFORCED-CONCRETE BUILDING FRAMES

Building #1 - Scheme "C"

2-way ribbed flat slab - Solid slab at cols
\[ f'c/n = 1350 \text{ psi} \quad v_c = 90 \text{ psi} \quad u = 300 \text{ psi} \]

Typical framing

Part plan of typical bay

Floor loads

Assumed frame constants

Span A-B
\[ a = 0.15 \quad b = 0.30 \quad k_a = 5.50 \quad k_b = 6.49 \quad C_a = 0.635 \quad C_b = 0.570 \]
\[ F_a = 0.087 \quad F_b = 0.096 \]

(From table 9-55)
Building #1—Scheme "C"

At $E = 8920^{+4}$

At haunch $I = 15,900^{+4}$

$D = 0.238$

$23'-6''$

$17'-6''$

$23'-6''$

All cols $18''x18''$

$I_c = 8748^{+4}$

$K = \frac{4.0x8748}{12.67} = 2770$

$A = 2'-0''$

$B = 2'-0''$

$D_a = 0.27$

$CD = 0.17$

$K_a = 5.50x8920$

$K = 23.5$

$D_b = 0.22$

$CD = 0.125$

$K_b = 6.49x8920$

$K = 23.5$

$D_b = 2460$

$CD = 0.125$

Window = 100 lb/ft

5'' brick = 225

3''-8'' block = 114

10''x24'' beam = 210

Haunch = 81

730 lb/ft

$W = 20'x23.5'x115 lb = 54^{K}$

$M = 0.0875x54x23.5$

$= 110^{LK}$

$M = 0.096x54x23.5$

$= 122^{LK}$

$D_Ld + L_Ld$

$W = 20'x23.5'x240 lb = 114^{K}$

$M = 233^{LK}$

$P = 1^{K}$

$x = 0.08x1x23.5$

$= 2.2$

$M = 0.105x2.4x23.5 = 5.7$

$= 0.007x1.0x23.5 = 0.1$

$= 5.8$

Design sheet BIC2

Frame constants (continued)

Span B-C

$b = c = 0.35$

$k_b = k_c = 6.65$

$C_b = C_c = 0.608$

$F_b = F_c = 0.094$

Spandrel beam load

Panel loads & fixed end mops
Building #1 - Scheme "C"

Design sheet
BIC4

Loading for
Max - \( M_B \)

Mid strip:
- \( M = 0.24 \times 149.5 = 36^{1K} \)
- \( S = 7.2^{1K} \) per rib
- \( A_s = \frac{7.2}{13.5} = 0.53 \) si
- Span AB: 1 #5 bt = 0.30
- Span BC: 1 #4 bt = 0.20

Col strip:
- \( M = 0.76 \times 149.5 = 113.5^{1K} \)
- \( S = \frac{113.5 \times 12,000}{1350} = 1000 \)
- \( 10\frac{1}{2} \) slab: \( S_1 = 179 \) \( b = \frac{1000}{179} = 5.6' \)
- Span AB = 5 #7 = 3.0
- Span BC = 5 #5 = 1.5
- 9 #6 top = \( A_s = \frac{5.6 \times 1.51}{0.50} = 3.96 \)
- Total = 8.46 si
Building #1—Scheme "C"

Max. W = 110.7[K]  C = 3.17'  F = 1.15 - \frac{3.17}{20} = 0.92  Use 1.0

\[ M_0 = 0.09 \times 110.7 \times 20 \left( 1 - \frac{2 \times 3.17}{3 \times 20} \right) = 161[K] \]

\[ +M = 0.20 \times 161 \times 32.2[K] \text{ or } 6.44[K] \text{ per rib} \quad A_s = \frac{6.44}{13.5} = 0.47 \text{ si} \]

Use \#4 st - \#5 bt

\[ -M = 0.50 \times 161 \times 80.5[K] \quad d = 8.7' \quad A_s = \frac{80.5}{12.5} = 6.44 \text{ si} \]

10 \#5 bt = 3.0 si
12 \#5 top = 3.6 si

\[ +M = -M = 0.15 \times 161 \times 24.2[K] \quad \text{4.8}[K] \text{ per rib} \quad A_s = \frac{4.8}{12.5} = 0.385 \text{ si} \]

Use \#4 st - \#4 bt

F = 1.15 - \frac{1.5}{20} = 1.075

\[ M_0 = 0.09 \times 114 \times 20 \times 1.075 \left( 1 - \frac{2 \times 1.5}{3 \times 20} \right) = 200[K] \]

\[ +M = 0.06 \times 200 = 12[K] \text{ or } 6[K] \text{ per rib} \quad A_s = \frac{6}{12.5} = 0.48 \text{ si} \]

Use \#4 st - \#5 bt

\[ -M = 0.145 \times 200 = 29[K] \quad A_s = \frac{29}{12.5} = 2.32 \text{ si} \]

4 \#5 bt = 1.2
6 \#4 top = 1.2

Net shear \( V = 110.7 - 4.67^2 \times 0.295 = 104.3[K] \)

Per ft of slab \( V = \frac{104.3}{16} = 6.25[K] \)

10\½" slab \( d = 8.7' \quad V_c = 6.9[K] \text{ at } 75 \text{ psi} \)

At edge of solid slab:

\( V = 110.7 - 8.42^2 \times 0.295 = 89.8[K] \)

5" rib \( V_c = 90 \text{ psi} \quad V_c = \frac{6.48}{8} \text{ rib} \)

\[ \text{Min no ribs} = \frac{89.8}{6.48} = 14 \]

Bond: 20 \#5 bt - \( V_b = 20 \times 4.72 = 94.5[K] \)
12 \#5 top = 12 \times 4.72 = 56.6
9 \#6 top = 9 \times 5.65 = 50.8

\[ \frac{201.9[K]}{110.7[K]} > 1 \]
EXAMPLES OF BUILDING-FRAME DESIGN

Building #1 - Scheme "C"

Load: Wall design sheet BIC2 = 730 x 18.5' = 13.5K
Slab - 30% of panel = 0.30 x 114' = 34.2K

\[ +M = 47.7 \times \frac{18.5}{16} = 55^{1K} \]
\[ S = 55 \times \frac{12000}{1350} = 490 \]

10' x 24" bm: \( S_1 = 805 \) \( A_s = 55 \) \( 1.96 \) si

Table 9-38f \( V_c = 16.7^{K} \)

\[ -M = 47.7 \times \frac{18.5}{12} = 73.5^{1K} \]
\[ S = 73.5 \times \frac{12000}{1350} = 655 \]
\[ A_s = \frac{73.5}{31} = 2.37 \text{ si} \]

4 # 5 top bars - \( A_s = 1.2 \)

2 # 7 bt = 1.2 si

Bond: 4 # 5 lapped \( V_b = (4 \times 1.96 \times 0.75) \times 0.070 \times \sqrt{6} \times 20" = 7.2^{K} \)

2 # 7 bt:

\[ V_b = \frac{21.4}{28.6}^{K} \]

Torsion - D Ld + 50% L Ld: \( w = 177 \) psf

Design sheet BIC2: col strip \( K_a = \frac{2080}{2} = 1040 \)
4'-0" of slab \( K_a = 104 \)
2'-18" x 18" cols \( K = 5540 \)
\( \Sigma K \) at cols = 5540 + 1040 = 6580

From fig 9-41 - part 9-1: \( b = \frac{10}{24} = 0.42 \)

\[ k_1 = 0.266 \times 10^4 \]

Design sheet BIC3: \( -M = 7.5 \times \frac{177}{240} = 5.55^{1K} \)

\[ m_1 = 5.55 \times \frac{5540}{6580} = 4.69^{1K} \]

\[ M = 3.7 \times \frac{177}{240} = 2.73^{1K} \]

\[ m_2 = 2.73 \times \frac{73}{177} = 1.12^{1K} \]

\( M_1 = \frac{(2 \times 1.12 \times 4.65)}{12} \times 18.5 = 10.6^{K} \)

\( v_1 = \frac{10.6 \times 12,000}{0.258 \times 10^5 \times 24} = 205 \) psi
Building #1 - Scheme "C"

Combined shear at A

\[ v = \frac{18.550}{10 \times \frac{1}{6} \times 22} = 96 \text{ psi} \]

1 #7 bt: \( V_a = 0.43 \times 19.8 \times 8.5^K \)

\[ V_a = \frac{8500}{10 \times \frac{1}{6} \times 22} = 44 \text{ psi} \]

\( V_s = 205 + 96 - 90 - 44 = 167 \text{ psi} \)

At A: \( V = 18.55 - 2.83 \text{ at } 2.0^K \times 12.85^K \)

\[ v = \frac{12.850}{10 \times \frac{1}{6} \times 22} = 67 \text{ psi} \]

\( V_s = 205 + 67 - 90 = 192 \text{ psi} \)

\[ n = \frac{10(205 + 67 - 90)}{2 \times 205} = 4.43^" \]

#3 stirrups: \( S = \frac{0.11 \times 20,000}{(4.43 \times 182)/2} = 5.5^" \)

Alternate

#3 1/2 - 5" - 6 at 10"

#3 1/4 10" - 4 at 10"
Building #1 - Scheme "C"

Design sheet
B1C8

Framing plan

Quantities

Concrete 29.6 cu yds
Reinforcement 4510 lb
Slab forms & dome rental 1300 SF
Beam forms 458 SF
Rental on 266 domes

Wall strip 20'-0" 23'-6"
Mid strip 10'-0" 17'-6"
Col strip 12'-5" 23'-6"
Summary, Building No. 1.  **Scheme A.**  Although this scheme has the advantage of requiring the minimum amounts of concrete and reinforcement it undoubtedly would be the most expensive to build because of the additional expense of building the forms and placing the reinforcement.  It also requires an additional 7 in. in height per story to maintain the required clear height between the floor and bottom of construction.  This additional height would add to the over-all cost of the exterior walls and mechanical equipment for the entire building.

**Scheme B.**  The arrangement of solid slabs and slab band beams has a definite advantage from the standpoint of appearance and the saving in story height.  Although the amount of reinforcement exceeds that of scheme A it is easier to place in the forms and does not require any stirrups except for the spandrel beams.  There is less waste of materials in the building of the forms and they would require considerably less labor, which would more than offset the additional cost of the reinforcement.

**Scheme C.**  The flat-slab system using metal fillers has the definite advantage over both scheme A and scheme B in the amount of reinforcement used and requires approximately the same minimum story height as for scheme B.  The formwork will be the most economical to build and has the maximum salvage of form materials, as all shores are the same height and the use of removable metal fillers provides for the minimum of cutting of the form lumber into short lengths.  Because of the shallow slab depths there would be slightly more reinforcement required for the exterior wall columns for both schemes B and C.
Building No. 2

Occupancy: Hospital, wing for private rooms and wards. (See Fig. 9-80.)
Live load: 40 psf in private rooms and wards, 80 psf in corridors.
Number of stories: Basement, 5 floors, and roof.
Foundations: Sand and gravel having a safe bearing value of 5 tons per sq ft.
Interior finish: Plaster walls and ceiling, asphalt-tile finished floor in rooms and corridors applied directly to structural slab. Toilet-room floors to be vitrified tile raised 13/4 in. above the room floors.
Mechanical equipment: Steam heat with horizontal mains exposed on the basement ceiling, vertical risers at the exterior walls concealed within the furring. The entire wing is to be air-conditioned with a horizontal supply duct in the corridor and branches as required to the rooms. Electric lights will be exposed below the ceilings.
Story heights: 8 ft minimum ceiling heights in corridors and 9 ft in private rooms and wards.

Building No. 2, Scheme A: One-way Ribbed Slab with 20-in. Metal Fillers.
Because of the relatively light live load used for hospitals an allowance of a uniform load of 20 psf is included in the dead load for partitions to provide for possible alterations in the future. With this precaution it would be possible to convert wards to private rooms without serious overstresses in the concrete frame. In addition, ribs of special widths are provided for the dividing partitions between private rooms. For the depth-to-span ratio of 1.04/19.35 it has not been considered necessary to check for deflection of the slabs supporting partition loads.
The ribbed slab is deeper than would be required for strength but it is used in order to provide sufficient depth for the wide flat beams along the corridor columns. In this manner the main supply ducts for air conditioning may be carried under the corridor slab with the smaller branch ducts to the rooms extended without the interference of deep beams. While the amount of steel reinforcement for the shallow beams will be greater than that required for deep beams the additional cost will be largely offset by the saving in reinforcement in the slab ribs. Also the use of deep ribs eliminates the necessity of using special end pans for shear and moment.
Because of the stiffness of the ribbed slab 12½ in. deep for comparatively short spans it is assumed that the spandrel beams offer very little restraint at the exterior supports, and the slab has been analyzed for continuous members on simple supports. The beam constants and moment coefficients used on design sheets B2A2 and B2A4 were obtained by using Tables 9-53 and 9-55 with the solid portion of the interior beams considered as a haunch on the ribbed slab.
Because of the relatively stiff span across the corridor the ribbed slab has been analyzed without considering any restraint by either the interior or exterior columns. The full fixed-end moment from the ribbed slab has been used in computing the torsion on the spandrel beams.

**Building #2 - Scheme "A"**

Wide, slab band beams
1 way ribbed slab - metal fillers

\[
t_c/n/t_s = 3000/10/20,000 \quad v_c = 90 \text{ psi} \quad u = 210-300 \text{ psi} \quad f_c = 1350 \text{ psi}
\]

ACI code

Plan of typical bay

Typical rooms: \(10' + 2\frac{1}{2}'\) slab - 5" ribs - 66 psf
Furred ceiling - 10
Partitions - 20 (Dividing partitions on special ribs)
Total = 96 psf

Corridors: \(10' + 2\frac{1}{2}'\) slab - 5" ribs - 66
Furred ceiling - 10 (Corridor partitions
Pipe & ducts - 10 (on beams)
Total = 86 psf

Design of slab
\[
t_c/n/t_s = 3000/10/20,000
\]

Slab stiffness
Typ 5" ribs

\[
b = \frac{25}{5} = 5 \quad \frac{t}{d} = \frac{2.5}{12.5} = 0.2
\]

\[
I_t = 1.85 \times \frac{5 \times 12.5^3}{12} = 1510^4
\]
EXAMPLES OF BUILDING-FRAME DESIGN

Building #2 - Scheme "A"

Assume beam width at 2 & 3 to be 36" Span lengths 2 & 3, 4.

Haunch at collines 2 & 3: 25' x 12.5'

Equivalent beam lengths: 12.5' x 12.5' = 5270

I_e = 12.5' x 12.5' = 4060

Equation to be solved:

\[ \begin{align*}
K_1 & = 4.22 \\
K_2 & = 4.99 \\
K_3 & = 9.66 \\
K_4 & = 13.90
\end{align*} \]

Table 9-55

S = 1.23 x 10,000 / 8.9 = 1350

Use Table 9-3b

\[ \begin{align*}
K_1 & = 4.44 \\
K_2 & = 4.44 \\
K_3 & = 4.44 \\
K_4 & = 4.44
\end{align*} \]

Design shear:

\[ \begin{align*}
V & = 273 - 0.5 = 2.23K \\
V_d & = 208 x 266 x 0.06 / 8.9 = 0.7
\end{align*} \]

Use Table 9-3b

\[ \begin{align*}
K_1 & = 4.22 \\
K_2 & = 4.99 \\
K_3 & = 9.66 \\
K_4 & = 13.90
\end{align*} \]

Span 1:

Assume beam width at 2 & 3 to be 36" Span lengths 2 & 3, 4.

Haunch at collines 2 & 3: 25' x 12.5'

Equivalent beam lengths: 12.5' x 12.5' = 5270

I_e = 12.5' x 12.5' = 4060

Equation to be solved:

\[ \begin{align*}
K_1 & = 4.22 \\
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Use Table 9-3b

\[ \begin{align*}
K_1 & = 4.22 \\
K_2 & = 4.99 \\
K_3 & = 9.66 \\
K_4 & = 13.90
\end{align*} \]

Span 1:
Building #2 - Scheme "A"

Design sheet
B2A3

Max - M_o at cols 2 & 3

J_1 - 1 #5 bt = 0.30
J_2 = 1 #6 top = 0.44
0.74 OK

Reinf for typ rib

Check typ rib
Span 1-2
For conc partition
w = D Ld = 76 lb
L Ld = 40
116 psf

OK M_o & shears
similar to uniform load
of 20 psf for partition
EXAMPLES OF BUILDING-FRAME DESIGN

Building #2—Scheme "A"

\[
\begin{align*}
I_m & = 27 \times 12.5^3 / 12 = 4400 \times 4^4 \\
\text{Equiv } I_3 & = 12 \times 4400 / 1.7 \times 7 = 4440 \\
\frac{b}{b'} & = \frac{27}{7} = 3.85 \\
\frac{1}{d} & = \frac{2.5}{12.5} = 0.20 \\
I_c & = \frac{1.7 \times 7 \times 12.5^3}{12} = 1940 \times 4^4
\end{align*}
\]

Spans 1-2 & 3-4
At 1 & 4: \( a = 0 \)
\( c = 0.57 \)
\( k = 4.19 \)
\( F = 0.076 \)
\( K = 4.19 \times 1940 = 420 \)
\( a = \frac{15}{19.35} = 0.78 \)
\( c = 4.9 \)
\( k = 4.85 \)
\( F = 0.098 \)
\( K = \frac{4.85 \times 1940}{19.35} = 488 \)

\( w_{1,2} = 2.25' \) (78\# + 40\#) = 265 lb/ft
\( w_{2,3} = 2.25' \) (88 + 80) = 535 lb/ft
\( w_{3,4} = 265 \) lb/ft

Design sheet
B2A4
7" ribs J3 & J4
at transverse partitions.
(Uniform 20 psf
partition load
omitted)

Span 2-3
\( a = \frac{15}{9.66} = 0.156 \)
\( c = 0.59 \)
\( k = 5.81 \)
\( F = 0.092 \)
\( K = \frac{5.81 \times 1940}{9.66} = 1170 \)

\( w_{1,2} = 265' \) (88 + 80) = 378 lb/ft

Partition = 270

\( S = 103 \) OK
\( \frac{A_s}{1.44 \times 11.25} = 1.3 \)

\( b' = 7' \)
\( 2 \# 5 & 1 \# 6 \)

\( J_3 - b' = 7' \)
\( 2 \# 5 & 1 \# 6 \)

\( J_4 - b' = 7' \)
\( 2 \# 3 & 1 \# 5 \)
\( 2 \# 5 \) top contin

\( \text{OK at clear span} \)
Building #2 — Scheme "A"

Design sheet B2A5
Long beams
Col lines 2 & 3

W = 18.5' x 3.1' = 57.5K
V = 28.75K
Vc = 3 x 10.3K
= 30.9K OK

11 #7: Vb = 11 x 7.7 = 85K

M at A-A
M = 14.37 x 0.375 = 5.4K
M' J3 = 21.5
26K

A1 = 26.9
1.44 x 11.7 = 1.6 sl

J3 - 1 #6 bt = 0.44
J4 - 3 #5 top = 0.90
Add 2 #4 top = 0.40

Total V at A-A = 21.3K
Vb = J3: 1 #6 = 6.85K
J4: 3 #5 = 17.10
2 #4 = 8.7

32.65K OK

10.5' brick at 45 # = 0.472K
8.5' - 8' block at 35 # = 0.300K
3' furring at 20 # = 0.170K
9.67' floor at 126 # = 1.215K
9.76' ceiling at 10 # = 0.097K
Spandrel bm at 206 # = 0.206K
Deduct 2 windows
2.460K
4' - 8' x 5' - 6' at 90 #

2.225

9' x 26' bm
2 #6 - 1 #7
2 #5 top
Continuous - lap 6'-0" at columns

W = 19' at 2.225' = 41.5K
V = 20.75K
v = 9 x 1/8 x 22 = 120 psi

M' = 41.5 x 19/2 = 49.3K
A3 = 49.3/33.8 = 1.5 sl

Bond: 4 #5: lapped Vb = 7.9K
2 #7 bt = 21.4
29.3K

- M = 41.5 x 19/11 = 72K
S = 72 x 12,000/1350 = 840K
9' x 26' bm: S1 = 865K

A5 = 72/33.8 = 2.13

2 #7 bt: A5 = 1.20
4 #5 top = 1.20

2.40

Add 2 #4 top

Spandrel beam
Torsion \( -D L_d + 50\% L_d \): \( w = 116 \text{ psf} \)

Design sheet B2A2 - 5" rib: \( K = 330 \)
\(-M = 7.9^{1k}/\text{rib or 3.8}^{1k}/\text{ft}\)

Design sheet B2A4 - 7" rib: \( K = 420 \)
\(-M = 1.6^{1k}/\text{rib or 7.7}^{1k}/\text{ft}\)

Total slab stiffness: 8-5" ribs at 330 = 2640
2-7" ribs at 420 = \( \frac{840}{3480} \)

18" x 18" columns: \( K = 4 \times 8750 / 11 = 3180 \)
\( 2k = 6360 \)
\( \Sigma K \text{ at } 1 = 6360 + 3480 = 9840 \)

From fig 9-41 - part 9-1: \( b/D = 9/24 = 0.375 \)
\( k = 0.296 \quad C = 0.267 \)

\[ K_t = \frac{0.296 \times 9^4}{4 \times 9.4} = 51.5 \]

\[ M_t = 3.8 \times \frac{116/136 = 3.24}{11} = 21.1^{1k} \]
\[ M_t = 7.7 \times \frac{116/136 = 6.56}{11} = 51.5 / 471.5 = 0.72^{1k} \]

\[ M_t = \frac{2 \times 0.72 + 2.1}{6} = 11.1^{1k} \]
\[ v_t = \frac{11.1 \times 12000}{0.267 \times 9^2 \times 24} = 162 \text{ psi} \]

1 # 7 bar: \( V_0 = 0.43 \times 19.8^{k} = 8.5^{k} \)
\[ V_0 = \frac{8500}{9 \times 7/8 \times 22} = 49 \text{ psi} \]

\[ n = \frac{9(162 + 120 - 90 - 49)}{2 \times 162} = 3.98 \]

# 3 stirrups: \( S = \frac{0.11 \times 20000}{2.98 \times 143 / 2} = 7.7^{\circ} \)

# 3 stirrups
# 3 14"-8 at 12" oc
# 3 12"-4 at 12" oc
Framing plan showing typical spacing of ribs

Joist bending diagram

<table>
<thead>
<tr>
<th>Joist schedule</th>
<th>Mark</th>
<th>B</th>
<th>D</th>
<th>Reinforcing</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Straight</td>
<td>Bent</td>
</tr>
<tr>
<td>J-1</td>
<td>5&quot;</td>
<td></td>
<td>10&quot; + 2 1/2&quot;</td>
<td>#5</td>
<td>#5</td>
</tr>
<tr>
<td>J-2</td>
<td>5&quot;</td>
<td></td>
<td>&quot;</td>
<td>2 #3</td>
<td>#6 top contin</td>
</tr>
<tr>
<td>J-3</td>
<td>7&quot;</td>
<td></td>
<td>&quot;</td>
<td>2 #5</td>
<td>#6 top contin</td>
</tr>
<tr>
<td>J-4</td>
<td>7&quot;</td>
<td></td>
<td>&quot;</td>
<td>2 #3 1 #5</td>
<td>#6 top contin</td>
</tr>
</tbody>
</table>

Quantities

Concrete 17.4 cu yds
Reinforcement 4595 lb
Slab forms and metal fillers 985 sf
Beam forms 163 sf
Pan rental
Building No. 2, Scheme B: One-way Solid Slab with Slab Band Beams on the Interior Corridor Columns. As pointed out in the general discussion of the effect of thin slabs of long spans and reinforced in one direction upon torsion in spandrel beams in Part 1 of this section, the depth of the slab used in this example is greater than would be required to satisfy the ordinary requirements for bending and shearing stresses. Also, the width of the spandrel beam has been increased to minimize the torsional stresses in the spandrel beams. The additional slab depth is also effective in reducing the probable deflection under masonry partitions as computed on design sheets B2B2 and B2B3.

The cost of the additional concrete used to obtain the desired stiffness in the slabs will be largely offset in the savings in the total amount of reinforcement. Generally, this type of framing cannot be justified for long spans because of the additional amount of material used for columns and foundations as well as the floor framing, unless the slab can be left exposed for the finished ceiling.

The solid slab does offer some advantages in the location of slots and sleeves for mechanical equipment, as the small openings usually required for such trades can be framed by fanning the typical reinforcement. It also provides more freedom in the final location of plumbing fixtures than other types of floor system which require special framing for such conditions.
Building #2 - Scheme "B"

Design sheet B281

One-way solid slabs — slab band beams

\[ \frac{f'_c}{n} / f_c = \frac{3000/10}{20000} \quad v_c = 90 \text{ psi} \quad u = 300 \text{ psi} \quad \ell_c = 1350 \text{ psi} \]

A.C.I. Code

Framing for typical bay

Plan of typical bay

Typical rooms: D Ld 7\(\frac{1}{2}\)" slab \(94 \text{ lb}\)
Furred ceiling \(10\)
Partitions \(30\)
\(134 \text{ psf}\)

Corridors:
D Ld = 7\(\frac{1}{2}\)" slab \(94 \text{ lb}\)
Furred ceiling \(10\)
Pipes and ducts \(10\)
\(114 \text{ psf}\)

Corridor partitions concentrated on beams
Building #2 - Scheme "B"

Assumed slab:
\[
D = 7.5\text{ in} \quad r = 0.75
\]
\[
I = \frac{19.35 \times 12}{7} = 31
\]
\[
I_c = \frac{12 \times 7.5^3}{12} = 422
\]

Span 1-2:
- C1: \( k = 4.30 \)
- F = 0.072

\[
K_1 = \frac{4.30 \times 422}{19.35} = 94
\]

Span 2-3:
- C2: \( k = 5.36 \)
- F = 0.107

\[
K_2 = \frac{5.36 \times 422}{19.35} = 117
\]

Total slab stiffness:
\[
K = 20.33 \times 94 = 1910
\]
\[
\Sigma K = 1910 \times 2 = 2500
\]
\[
D_1 = 1910 / 2500 = 0.764
\]

Use modified stiffness at 2 for free end at 1:
\[
K_2 = (1 - 0.613 \times 0.492) \times 117 = 81
\]

Design sheet B2B2

Slab stiffness & frame constants from table 9-55

Check span 1-2 for deflection
\[
D L_d + 50\% L L_d
\]
**REINFORCED-CONCRETE BUILDING FRAMES**

**Building #2 - Scheme "B"**

<table>
<thead>
<tr>
<th>Constant I</th>
<th>Uniform load</th>
<th>M_o at 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.625 (WL^2/48EI)</td>
<td>-3.0 (ML^2/48EI)</td>
</tr>
<tr>
<td>Haunch at B</td>
<td>-0.002 (WL^2/48EI)</td>
<td>+0.06 (ML^2/48EI)</td>
</tr>
<tr>
<td></td>
<td><strong>0.623 (WL^2/48EI)</strong></td>
<td><strong>-2.94 (ML^2/48EI)</strong></td>
</tr>
</tbody>
</table>

\[ \Delta = \frac{0.623 \times 2950 (19.35 \times 12)^3}{48 \times 1890000 \times 422} = +0.605 \]

\[ \Delta = \frac{-2.94 \times 5700 \times 12 (19.35 \times 12)^2}{48 \times 1890000 \times 422} = -0.285 \]

Max \(\Delta = \frac{L}{600} = \frac{19.35 \times 12}{600} = 0.386^\circ\) \(\text{ok}\)

Assume beam 12" x 24" \(b/D = 12/24 = 0.5\)

From fig 9-41 - part 9-1 : \(k_2 = 0.196; C_t = 0.248\)

\(K_p = \frac{0.196 \times 12^4}{4 \times 9.33} = \frac{4060}{37.6} = 108\)

Design sheet B2B2: \(M_F = 4.1\text{K}\) per ft of slab

\(m_1 = 4.1 \times 0.72 = 2.95\text{K}\)

\(m_2 = 2.95 \times \frac{108}{108 + 94} = 1.58\text{K}\)

\(M_t = \frac{2 \times 1.5 + 2.95}{8} \times 18.67 = 14.2\text{K}\)

\(\nu_t = \frac{14.2 \times 12000}{0.248 \times 12^2 \times 24} = 198\text{ psi}\)

\(A_s = \frac{2.95}{1.44 \times 6.37} = 0.32\text{ si}\)

Add # 4 at 24" oc

\# 5 at 13" oc : \(A_s = 0.28\)
EXAMPLES OF BUILDING-FRAME DESIGN

Building #2 - Scheme "B"

Design sheet B2B4

Max +M₀ span 1-2

7½" slab

#5 & #5 at 6½" oc
Aₜ = 0.58 si

M at 7 of span 2 & 3
Max M at 2

S = 64

13" slab : Sₜ = 290
Aₜ = 7.2/17 = 0.42 si
continue
bt bars
across 2-3
Building #2 - Scheme "B"

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>D Ld = 134 psf</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>W = 2.6&quot;</td>
<td></td>
</tr>
<tr>
<td>D</td>
<td>3.6 x 0.613</td>
<td></td>
</tr>
<tr>
<td>CD</td>
<td>-3.6</td>
<td>22</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

M_L = +0.985 x 4.82 / 2 - 4.37 = -2K
Steel for positive Mo not required

#4 at 24" oc
#5 & #5 b/t at 6½" oc
#4 at 16" oc

Slab shear: span 1-2 = 2.05K
span 2-3 = 1.15
48" x 6½" beam haunch = 0.32
Corridor partition = 0.30

W = 3.82 x 18.83 = 72K
+M = 72 x 18.83 / 16 + 85K
S = 756

48" x 13" slab band: S_t = 1160  V_c = 44K
A_s = 85/17 = 5.0 si
5 #5 - 6 #7: A_s = 5.1 si

-M = 72 x 18.83 / 11 = 123K
S = 1090
A_s = 123/17 = 7.2 si  12 # 7
V_b = 12 x 5.9 = 71K ok
Building #2 - Scheme "B"

Design Sheet B2B6

Sondrel
beam B1

Ave wall ld 14/20.33 = 0.68
Slob shear = 1.36
12'' x 24'' bm = 0.30
2'' brick at 45 lb = 0.09

\[
W = 18.66' \times 2.43''/ft = 45.3''
\]

\[
+M = 45.3'' \times 18.66/16 = 53''
\]

\[
S = 472
\]

12'' x 24'' bm: \( S_f = 970 \quad V_c = 20.1'' \)

\[
A_s = 53/31 = 1.71 \text{ sl}
\]

\[
- M = 45.3'' \times 18.66/16 = 77''
\]

\[
S = 685 \quad A_s = 77/31 = 2.48 \text{ sl}
\]

\[
V_h = 6 \# 5 = \frac{0.866 \times 210 \times 6 \times 11.75}{1000} = 14
\]

\[
2 \# 7 \text{ bt} = 1.2
\]

\[
4 \# 5 \text{ top} = 1.2
\]

From table 9-38f: 2 # 7 = 21.4

\[
35.4''
\]

2.4 sl

Design sheet B2B3: \( v_t = 198 \text{ psi} \)

\[
v = \frac{22650}{12 \times 7/8 \times 20} = 108 \text{ psi}
\]

1 # 7 bt: \( v_a = 0.43 \times 19.8 = 8.5'' \)

\[
v_a = \frac{8500}{12 \times 7/8 \times 20} = 40 \text{ psi}
\]

\[
n = \frac{12(198+108-90-40)}{2 \times 198} = 5.3''
\]

\[
3 \text{ stirrups}
\]

3 # 3,"", 9 at 10" oc

3 # 3,"", 8" at 10" oc
Framing plan
(Typical bay)

Quantities

Concrete 28.0 cu yds
Reinforcement 4710 lb
Slab forms 891 sf
Slab band forms 203 sf
Beam forms 178 sf
Building No. 2, Scheme C: Two-way Ribbed Flat Slab with 16-in. Wide Masonry Fillers. Of the three systems of framing considered the ribbed flat slab will have the advantage in the over-all depth required for construction. The masonry fillers permit the use of a slab 2 in. thick between the ribs and a total depth for floor construction of 10 in. Although a thinner slab could have been used meeting the requirements of the ACI Code with dropped panels and column capitals, these features are usually objectionable in hospital construction and the net result would have been an increase in total cost of framing.

The final decision as to the use of masonry fillers should depend largely on whether the plaster ceiling can be applied directly to the bottom of the slab. When used for a building such as a hospital where there are usually a considerable number of shafts required for mechanical equipment it is important that the openings through the slab be located in the center of the spans as indicated on design sheet B2C1.

As the exterior spans are more than 20 per cent greater than the typical corridor span the slabs have been analyzed as a bent in the transverse direction. The empirical method has been used in the longitudinal direction where the span lengths are uniform on the basis of the panel load for the middle strips and wall strips and the maximum shears for the interior column strips as shown on design sheet B2C5. As the quarter points of the bays fall on the center lines of ribs the moments have been figured for six ribs in the column strips and the required reinforcement per rib is used for seven ribs in the column strip and for five ribs in the middle bands in the transverse direction.

On design sheets B2C8 to B2C11 the columns are staggered on each side of the corridor to provide for typical bays between stair wells.
Building #2 - Scheme "C"

Two-way flat slab - masonry fillers (16"

\[ \frac{f'_c}{f_s} = 3000/10/20,000 \quad v_c = 90 \text{ psi} \quad u = 300 \text{ psi} \quad f_c = 1350 \text{ psi} \]

ACI code

Plan of typical bay

Min slab depth = \(\frac{250}{40} = 6.25"\)  Use 8" + 2"

Typical rooms: 8" + 2" slab - 4" ribs - 83 psf
Furred ceiling - 10
Partitions - 30
                      123 psf

Corridors: 8" + 2" slab - 4" ribs - 83 psf
Pipes and ducts - 10
Furred ceiling - 10
                      103 psf

Corridor partitions distributed on column strip

Dead loads
Assume 8" + 2" slab 4" ribs typical
Building #2 — Scheme "C"

Design sheet B2C2

Slab stiffness and frame constants from table 9-55

\[
\text{At } C \text{ of span: } \frac{244}{52} = 4.7
\]

\[
\text{From table 9-53 } C = 1.8 \quad I_c = \frac{52 \times 10^3}{12} \times 1.8 = 7800
\]

\[
\text{At columns: } \frac{244}{116} = 2.1
\]

\[
C = 1.3 \quad I_h = \frac{116 \times 10^3}{12} \times 1.3 = 12,600
\]

\[
\text{Equiv } D = \sqrt{\frac{12,600 \times 12}{52 \times 1.8}} = 11.7
\]

\[
r = \frac{1.7}{10} = 0.17
\]

1. 19.46'
2. 9.66'
3. 19.46'

\[
\begin{align*}
\text{From table 9-55} & & \\
k_a = 4.9 & & k_b = 5.5 \\
C_a = 0.60 & & C_b = 0.54 \\
F_a = 0.085 & & F_b = 0.096 \\
K_a = \frac{4.9 \times 7800}{19.46} = 1960 & & K_b = \frac{5.5 \times 7800}{19.46} = 2020 \\
K_a \times K_b = \frac{4 \times 12,600}{19.46} = 5240
\end{align*}
\]

Columns 1 and 4:
\[
18" \times 16" \quad I_c = 6160 \text{in.}^4 \\
K = \frac{4 \times 6160}{10.33} = 2380 \text{in.}^4
\]

Columns 2 and 3:
\[
18" \times 18" \quad I_c = 8750 \text{in.}^4 \\
K = \frac{4 \times 8750}{10.33} = 3380 \text{in.}^4
\]

\[
\text{Span 2-3: } k = 4.0 \\
C = 0.50 \\
F = 0.083
\]
### Building #2 - Scheme "C"

<table>
<thead>
<tr>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>19.46'</td>
<td>9.66'</td>
<td>19.46'</td>
<td></td>
</tr>
<tr>
<td>0.4k</td>
<td>0.4k</td>
<td>0.4k</td>
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</tr>
<tr>
<td>Mf</td>
<td>Mf</td>
<td>Mf</td>
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<tr>
<td>-105</td>
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<tr>
<td>V</td>
<td>33.1</td>
<td>11.5</td>
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<tr>
<td>30.5</td>
<td>+1.9</td>
<td>+1.9</td>
<td></td>
</tr>
</tbody>
</table>

**Design sheet B2C3**
- +M ext spans 1-2 & 3-4
- -M at 1 & 4
- Min + Mx spon 2-3
- 8 + 2" slab
- 4" ribs
- Table 9-32e
- Sf = 230
- Col strip: 7 ribs #4 & #5
- Mid strip: 5 ribs #4 & #3

**Col strip:**
- A_s = 0.66x50 / 6 = 1.10

**Mid strip:**
- A_s = 0.34x50 / 6 = 0.57

- M_1 at A = 30.5 x 1.08' = 33.8 = 50"K
- M_2 at A = 35.2 x 1.17 - 122 = 81"K

- M = 30.5 x 9.2 / 2 - 83 = 54"K
- 6 ribs at 5.4"K
- A_s = 5.4 / 12.6 = 0.43

- Mid strip: +M = 0.40 x 54 = 21.6
- 6 ribs at 3.6"K
- A_s = 3.6 / 12.6 = 0.29

- Min top A_s Spon 2-3
- Mid strip: A_s = 2 / 12.6 = 0.16

- D Ld = L Ld = 163 psf
- **Provide #3 top cont for span 2-3**

<table>
<thead>
<tr>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
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<td>0.4k</td>
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<td>0.4k</td>
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<tr>
<td>Mf</td>
<td>Mf</td>
<td>Mf</td>
<td></td>
</tr>
<tr>
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<td>-121</td>
<td>-121</td>
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<td>-18</td>
<td>-2</td>
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<tr>
<td>-2(-106)</td>
<td>-6(-14)</td>
<td>+6(-14)</td>
<td></td>
</tr>
<tr>
<td>-82</td>
<td>-52</td>
<td>-5</td>
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<td>V</td>
<td>33.1</td>
<td>19.2</td>
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<table>
<thead>
<tr>
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<th>0.326</th>
</tr>
</thead>
<tbody>
<tr>
<td>#3</td>
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</tr>
<tr>
<td>top</td>
<td>0.11</td>
</tr>
<tr>
<td>0.33</td>
<td></td>
</tr>
</tbody>
</table>
Building #2 - Scheme "C"

Col strip: \(-M = 0.76 \times 81 = 61.5\) \(S = 548\)

8" solid slab: \(S_t = 95.1 \text{ ft}\) Table 9-30a

\(\text{Min } b = 548 / 95.1 = 5.75' \text{ ok}\)

\(-M = 10.2K/\text{rib} \quad A_s = \frac{10.2}{12.6} = 0.81 \quad \#5 \text{ bt = 0.30}\)

\#3 bt = 0.11

2 #4 top = 0.40

10" solid slab Table 9-30b

\(V = 56K - 2.92K \text{ at } 163 \text{ lb} = 54.6K\)

\(V_c \text{ at } 75 \text{ psi} : \quad V_c = 4 \times 2.92 \times 6.9K = 80.5K\)

At ribs: \(V = 35.2K - 31.5 \times 163 \text{ lb} = 30.1K\)

\(V_c = 11 - 4" \text{ ribs at } 2.7K = 29.7K\)

Table 9-32b

---

Design sheet

B2C4

- Mo int
cols 2 & 3
continued

12 #4 top contin
at col strip

Shear at int
cols

---

Max. \(M_o\) at
c. span 2-3

8" + 2" slab

#3 st & #3 bt
col & mid
strip

---

\(48K\)

\(48K\)

\(38.4K\)

Use: #3 st & #3 bt ea rib

Use: #3 st & #3 bt ea rib

---

Col strip = 12 #4 top
Mid strip = 4 #3 top

Col strip = 12 #4 top
Mid strip = 4 #3 top

---

Col strip 7 #4 st - 7 #5 bt
Mid strip 5 #4 st - 5 #3 bt

Col strip 7 #4 st - 7 #5 bt
Mid strip 5 #4 st - 5 #3 bt

#3 st - #3 bt per rib
col & mid strips

#3 st - #3 bt per rib
col & mid strips
Building #2 - Scheme "C"

\[ M_0 = 0.09 WLF \left( 1 - \frac{2c}{3L} \right)^2 \]
\[ F = 1.15 \left( \frac{1.5}{20.33} \right) = 1.076 \]

Mid strip and wall strip:
\[ Mo = 0.09 \times 64 \times 20.33 \times 1.076 \left( 1 - \frac{3}{61} \right)^2 = 112^k \]

Mid strip:
\[ + M = 0.15 \times 112 = 16.8^k \]
\[ 5 \text{ ribs} \quad A_s = \frac{3.4}{1.44 \times 8.37} = 0.28 \quad \#3 \text{ and } \#4 \text{ st or } \#5 \]
\[ - M = 16.8^k \quad A_s = \frac{3.4}{1.44 \times 8.87} = 0.27 \quad \#5 \text{ top} \]

Wall strip:
\[ + M = 0.07 \times 112 = 7.8^k \]
\[ 3 \text{ ribs} \quad A_s = \frac{2.6}{12} = 0.22 \quad \#4 \text{ st} \]
\[ - M = 0.16 \times 112 = 17.9 \quad A_s = \frac{17.9}{12} = 1.5 \quad 5 \#5 \text{ top} \]

Col strip:
\[ W = \text{shear design sheet } B2C3 = 56 \]
\[ 5 \text{ ribs} \quad \text{Cor partition } 20.33 \times 0.3 = \frac{6}{62^k} \]
\[ Mo = 0.09 \times 56 \times 20.33 \times 1.076 \left( 1 - \frac{3}{61} \right)^2 = 98^k \]
\[ + M = 0.03 \times 6 \times 20.33 + 0.20 \times 98 = 23.3^k \quad 4.65^k \text{rib} \]
\[ A_s = \frac{4.65}{12} = 0.39 \quad \#4 \text{ st} - \#4 \text{ bt} \]
\[ - M = 0.09 \times 6 \times 20.33 + 0.50 \times 98 = 60^k \quad 12^k \text{rib} \]
\[ A_s = \frac{12}{12} = 1.0 \quad 2 \#4 \text{ bt} \]
\[ \quad 2 \#5 \text{ st} \]
\[ W = 38.4^k \quad Mo = 0.09 \times 38 \times 20.33 \times 1.076 \left( 1 - \frac{3}{61} \right)^2 = 67.2^k \]

Mid strip:
\[ + M = 0.15 \times 67.2 = 10.1^k \]
\[ 2 \text{ ribs} \quad A_s = \frac{5.05}{12} = 0.42 \quad \text{Use } 2 \#4 \text{ st} \]
\[ - M = 10.1^k \quad 5.05^k \text{rib} \quad A_s = \frac{5.05}{12.7} = 0.40 \quad \text{Use } 2 \#4 \text{ top} \]
EXAMPLES OF BUILDING-FRAME DESIGN

Building #2 - Scheme "C"

Wall - $8.67 \times 20.33$ at 104 lb = $18.3^k$

Floor - $0.30 \times 64^k/2033 = 945$

9" x 24" beam: 225

Brick: 100

Wall = $14/20.33 = 685$ lb/l

Floor = $0.30 \times 64^k/2033 = 945$

Brick: 100

Check for torsion: D Ld + 50% L Ld = 143 psf

Design sheet B2C2, $k_1 = 1960$

Col strip: $k_1 = 1960/2 = 980$

$S = 385, S_t = 725, V_c = 15.1^k$

From fig 9-41: part 9-1: $b/D = 9/24 = 375$

$k_t = 0.295c_t = 0.261$

$K_t = \frac{0.295 \times 9^3}{4 \times 9.41} = 57$

Design sheet B2C3: $M = 83^k \times 143/163 = 73^k$

Col strip: $M = 3.6 \times 0.66 = 2.4^k/ft$

$m_1 = 2.4 \times 4760/5740 = 2.0^k/ft$

Mid strip: $M = 3.6 \times 0.34 = 1.2^k/ft$

$m_2 = (1.2 \times 57/(57+98)) = 0.44^k/ft$

$M_t = \frac{2 \times 0.44 + 2.0 \times 18.83}{12} = 4.53$

$v_L = \frac{4.53 \times 12000}{0.261 \times 9^2 \times 18.83} = 136$ psi

$v = \frac{18400}{9 \times 7/8 \times 22} = 106$ psi

1#6 bt: $v_e^k = \frac{43 \times 14500}{9 \times 7/8 \times 22} = 36$ psi

At x: $V = 12^k$, $V = \frac{12000}{9 \times 7/8 \times 22} = 69$ psi

$n = \frac{9 \times (136 + 69 - 90)}{2 \times 136} = 4"$

#3 stirrups: $S = \frac{0.11 \times 20000}{(4 \times 136)/2} = 8.1"$

Alternate

#3 11, 4", 16" oc

#3 12, 2 at 16"
Framing plan (Typical bay)

Section

Quantities

Concrete 19.7 cy
Reinforcement 2810 lb
Slab forms 980 sf
Beam forms 160.0 sf
16" x 16" x 8" masonry fillers 286 req'd
EXAMPLES OF BUILDING-FRAME DESIGN

Building #2 - Scheme "C"

Two-way flat slab - masonry fillers

\[
\frac{f_c}{n} / f_s = \frac{3000}{10/20000} \quad v_c = 90 \text{ psi} \quad u = 300 \text{ psi} \quad f_c = 1350 \text{ psi}
\]

ACI code

Plan at end of wing

8" + 2" slab: same as used on design sheets B2C1 & B2C7

Dead loads

Typical rooms: \(D Ld = 123\) psf

Corridors: \(D Ld = 103\) psf
Building #2 - Scheme "C"

Slab coef for exterior spans 1-2 and 3-4
Use stiffness factors, loads and end moments from design sheet
Span 2-3: \( a = b = 0.3 \)
\( r_a = r_b = 0.17 \)
From table 9-55:
\( k_a = k_b = 6.25 \)
\( C_a = C_b = 0.60 \)
\( F_a = F_b = 0.93 \)

Design sheet 82C9

Slab stiffness and frame constants

Plan of typical bay

18"x16" cols
K=2380

\( D = 0.29 \)

\( K = 1960 \)

\( D = 0.17 \)

\( D = 0.08 \)

\( D = 0.29 \)

D Ld +L Ld
=163 psf

\( D = 0.37 \)

\( D = 0.09 \)

\( D = 0.17 \)

10.4

1.1

64 x

36 x

48 x

Mo and shears col line 2

\( M = -106 \)

\( -121 \)

\( -18 \)

\( -2 \)

\( -2 \)

\( -90 \)

\( 33.1 \)

\( 18.8 \)

\( 18.8 \)

\( +0.6 \)

\( +11.7 \)

\( -11.7 \)

\( 33.7 \)

\( 30.5 \)

\( 7.1 \)

\( Mo = 33.7 \times 1.17 - 102 = 63'x \)

Mo at A - col line 2
Building #2 - Scheme "C"

\[ D \text{ Ld} = 123 \text{ psf} \]

\[ \begin{array}{c|c|c|c|c}
| & 0.4 & 1.1 & 1.1 & 0.4 \\
\hline
D & 0.29 & 0.29 & 0.37 & 0.29 \\
CD & 0.17 & 0.16 & 0.08 & 0.17 \\
\hline
-80 & -91 & -33 & -33 & -121 \\
-9 & 14 & 19 & 25 & -18 \\
-5 & -2 & 9 & 14 & -1 \\
-2 & -108 & +3(-2)(11)5 & -(-140) & (-117) -1 \\
\hline
& -77 & +7 & +2 & 83 \\
& -0.6 & -52 & -119 & \\
& -77.6 & -45 & -117 & \\
\end{array} \]

\[ V = \frac{18.8}{34.1} = \frac{33.1}{32.4} = \frac{1.8}{1.18} = \frac{22.2^k}{15.4^k} = \frac{34.9^k}{30.6^k} \]

\[ + M = 22.2 \times 6 - 77.6 = 55.4 \]

Top reinforcmen required - see design sheet B2C3

\[ -21.4 \times 3 - 64.2 = -8.8^k \]

Use shear for col line 2:

\[ W = \text{Shear design sheet B2C9} = 64.2 \]

Corridor partition -203 x 0.3 = 6.0

\[ M_0 = 0.09 \times 64.2 \times 20.33 \times 1.076 (1-3/6)^2 = 112^k \]

\[ + M = 0.03 \times 6^k \times 20.33 + 0.20 \times 112 = 26.1^k, 5.2^k/\text{rib} \]

5 ribs: \[ A_5 = 5.2/12 = 0.435 \] Use 1 #4 - 1 #5 bt

\[ M_0 = 0.03 \times 6^k \times 20.33 + 0.50 \times 112 = 67^k \]

\[ S = 596 \text{ 7"-8" of 10" solid slab } S_2 = 7.67 \times 165 = 1260 \]

\[ 13.4^k/\text{rib} : A_5 = \frac{13.4}{12} = 1.12 \text{ sl} \]

2 #5 bt = 0.60

2 #5 top = 0.60

All other slabs, beams & cols same as designed for typical col spacing

Design sheet B2C10

M_0 & shears col line 3

Slab S2, span 2-3

Longitudinal col strip col 2 & 3
Summary, Building No. 2. Scheme A. This scheme would require the least amount of concrete materials and would be the most economical to form of the three schemes considered for this building. The final decision as to recommending its use would depend mostly on how well the mechanical equipment could be fitted into the structural frame. If used, a suspended ceiling becomes essential, which would probably be one of the determining factors.

Scheme B. Requiring the maximum amounts of concrete materials, this scheme could hardly be considered if it is necessary to provide a suspended ceiling over the entire area of the private rooms and wards. In hospital construction it is often possi-
able to confine the required space for mechanical equipment to the corridors and adja-
cent toilet rooms where the ceiling height can be reduced to as little as 8 ft 0 in. Under 
such conditions the practical solution would probably be to use plywood forms for the 
solid slab and leave them exposed for paint. In this event the solid slab might well be 
the economical solution.

**Scheme C.** As in the case of scheme B the decision on whether this scheme would 
prove to be the most practical would depend largely upon whether the mechanical 
equipment could be confined entirely to the areas of the corridors and toilet rooms. It 
would also be dependent on whether the owner and architect would require a plastered 
ceiling in preference to the exposed concrete ceiling as provided by the solid slab in 
scheme B. The use of masonry fillers is seldom warranted unless they are used for the 
direct application of the finished plaster ceiling without furring.

**Building No. 2 (Alternate), Scheme C: Reinforced-concrete Columns**

Occupancy: Hospital, wing for private rooms and wards.
Live load: Roof: 40 psf. Typical floors: 40 psf in private rooms and wards, 80 psf 
in corridors. Ground and first floors: 80 psf.
Number of stories: Basement, 7 floors, and roof. [Building No. 2 (Alternate) has 
two added floors to that of Building No. 2 to better illustrate column design.]
Design: ACI Code, $f' c = 3,000$ psi.

While the ACI Code permits the design of the framed floors on the assumption that 
the columns are fixed at the floors above and below, the entire frame will be distorted 
in a manner similar to that shown in Fig. 9-81 for unequal spans of the proportions 
shown. In such cases the effect of a single floor upon the columns may be determined 
by dividing the difference between the slab moments on each side of the column center 
line between the columns above and below the floor slab in the proportion of their 
stiffnesses $K$. However, the effect of the adjacent floors is cumulative and should be 
given some consideration. The effect of adjacent floors upon a given column may be 
determined by multiplying the column moments by the carry-over factor $c = 0.5$ for 
columns of constant cross section.

In Building No. 2 (Alternate), Scheme C, the dead load is approximately 75 per cent 
of the total load and for the type of occupancy full live load on all floors at the same 
time will rarely occur. The carry-over factor has been modified for dead load: 
$c = 0.75 \times 0.5$, or 0.375.

For building frames with approximately equal spans or where deep beams having a 
high stiffness factor have been used for the floor framing system, the effect of adjacent 
floors may become a minor factor and this procedure can be neglected. However, in 
such cases the effect at the exterior wall columns usually warrants some consideration.

![Fig. 9-81](image-url)
### Reinforced-Concrete Building Frames

**Building #2 (alternate) Scheme "C" - Concrete Columns**

<table>
<thead>
<tr>
<th>Floor</th>
<th>Area (sf)</th>
<th>Unit Loads</th>
<th>Gravity Loads (kips)</th>
<th>Column Moments (ft-kips)</th>
<th>Column Nos</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>% Ld (psf)</td>
<td>D Ld p sf</td>
<td>L Ld p sf</td>
<td>Dead</td>
<td>Live</td>
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<tr>
<td>Roof</td>
<td>100%</td>
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<td>40</td>
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<td>7th</td>
<td>92%</td>
<td>196</td>
<td>93</td>
<td>40</td>
<td>18.2</td>
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<tr>
<td>6%</td>
<td>196</td>
<td>93</td>
<td>40</td>
<td>19.6</td>
<td>6.6</td>
</tr>
<tr>
<td>6th</td>
<td>84%</td>
<td>196</td>
<td>93</td>
<td>40</td>
<td>18.2</td>
</tr>
<tr>
<td>5th</td>
<td>76%</td>
<td>196</td>
<td>93</td>
<td>40</td>
<td>18.2</td>
</tr>
<tr>
<td>4th</td>
<td>68%</td>
<td>196</td>
<td>93</td>
<td>40</td>
<td>18.2</td>
</tr>
<tr>
<td>3rd</td>
<td>60%</td>
<td>196</td>
<td>93</td>
<td>40</td>
<td>18.2</td>
</tr>
<tr>
<td>2nd</td>
<td>60%</td>
<td>196</td>
<td>93</td>
<td>80</td>
<td>18.2</td>
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<tr>
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<td>80</td>
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<td>60%</td>
<td>196</td>
<td>83</td>
<td>80</td>
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<tr>
<td>Total</td>
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<td>61.1</td>
<td>441.4</td>
<td>441.4</td>
<td>424.1</td>
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</table>

**Footing**
Table 9-49h
SP 5 tons 7'-0"x7'-0"x 2'-8" 15# 5 ew
LD + 25% Ld + 395" SP = 8.05" OK

Notes: 54" + 193" x 0.088 = 38" OK
HG = Hard grade reinf
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<thead>
<tr>
<th>Story</th>
<th>Column</th>
<th>Load (kips)</th>
<th>Moment (kip-ft)</th>
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<td>1st</td>
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<td></td>
</tr>
<tr>
<td>2nd</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3rd</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4th</td>
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<td></td>
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<tr>
<td>5th</td>
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<tr>
<td>7th</td>
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</table>

**Footings**

*Table 9-49th*

<table>
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<tr>
<th>Storey</th>
<th>Load (kips)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st</td>
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<tr>
<td>2nd</td>
<td></td>
</tr>
<tr>
<td>3rd</td>
<td></td>
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<tr>
<td>4th</td>
<td></td>
</tr>
<tr>
<td>5th</td>
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</tr>
<tr>
<td>6th</td>
<td></td>
</tr>
<tr>
<td>7th</td>
<td></td>
</tr>
</tbody>
</table>

**Gravity loads (kips)**

<table>
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<tr>
<th>Story Height</th>
<th>Column</th>
<th>Load (kips)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10'-0&quot;</td>
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<td></td>
</tr>
<tr>
<td>11'-0&quot;</td>
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<td></td>
</tr>
<tr>
<td>11'-6&quot;</td>
<td></td>
<td></td>
</tr>
<tr>
<td>11'-10&quot;</td>
<td></td>
<td></td>
</tr>
<tr>
<td>12'-4&quot;</td>
<td></td>
<td></td>
</tr>
<tr>
<td>13'-2&quot;</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Story moments (ft kips)**

<table>
<thead>
<tr>
<th>Story Height</th>
<th>Column</th>
<th>Moment (kip-ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10'-0&quot;</td>
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<tr>
<td>11'-0&quot;</td>
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<tr>
<td>11'-6&quot;</td>
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<tr>
<td>11'-10&quot;</td>
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<td></td>
</tr>
<tr>
<td>12'-4&quot;</td>
<td></td>
<td></td>
</tr>
<tr>
<td>13'-2&quot;</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Load Diagram**

- **Dist mo x-x**
- **Dist mo y-y**
- **Equil Load P**
- **Equil Load Q**
- **Col Load C**
- **Col Load D**
- **Col Load E**

**Building #2 (alternative) Scheme C. Concrete Columns**

*Design sheet B201h C2C*

**Note:**

- Hard grade omit HG
- p. 793 k
- Ld *= 25% Ld. P. 50% SP = 793 k OK
### REINFORCED-CONCRETE BUILDING FRAMES

**Building #2 (alternate) Scheme "C" Concrete Columns**

**Design sheet B2 (alt) C3C**

<table>
<thead>
<tr>
<th>Floor Area (sf)</th>
<th>Unit loads</th>
<th>Gravity loads (kips)</th>
<th>Column moments (ft kips)</th>
<th>Column nos</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>D Ld</td>
<td>L Ld</td>
<td>Dead</td>
<td>Live</td>
</tr>
<tr>
<td>100%</td>
<td>98</td>
<td>130</td>
<td>40</td>
<td>12.8</td>
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<tr>
<td>7th</td>
<td>98</td>
<td>93</td>
<td>40</td>
<td>9.1</td>
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<td>92%</td>
<td>98</td>
<td>130</td>
<td>40</td>
<td>12.8</td>
</tr>
<tr>
<td>6th</td>
<td>98</td>
<td>93</td>
<td>40</td>
<td>9.1</td>
</tr>
<tr>
<td>84%</td>
<td>98</td>
<td>130</td>
<td>40</td>
<td>12.8</td>
</tr>
<tr>
<td>5th</td>
<td>98</td>
<td>93</td>
<td>40</td>
<td>9.1</td>
</tr>
<tr>
<td>76%</td>
<td>98</td>
<td>130</td>
<td>40</td>
<td>12.8</td>
</tr>
<tr>
<td>4th</td>
<td>98</td>
<td>93</td>
<td>40</td>
<td>9.1</td>
</tr>
<tr>
<td>68%</td>
<td>98</td>
<td>130</td>
<td>40</td>
<td>12.8</td>
</tr>
<tr>
<td>3rd</td>
<td>98</td>
<td>93</td>
<td>80</td>
<td>9.1</td>
</tr>
<tr>
<td>60%</td>
<td>98</td>
<td>130</td>
<td>80</td>
<td>12.8</td>
</tr>
<tr>
<td>2nd</td>
<td>98</td>
<td>93</td>
<td>80</td>
<td>9.1</td>
</tr>
<tr>
<td>60%</td>
<td>98</td>
<td>130</td>
<td>80</td>
<td>12.8</td>
</tr>
<tr>
<td>1st</td>
<td>98</td>
<td>130</td>
<td>80</td>
<td>12.8</td>
</tr>
<tr>
<td>Gr</td>
<td>98</td>
<td>130</td>
<td>80</td>
<td>12.8</td>
</tr>
<tr>
<td>60%</td>
<td>98</td>
<td>130</td>
<td>80</td>
<td>12.8</td>
</tr>
<tr>
<td>B Total</td>
<td>30.9</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Footing**

Table 9-49h

SP 5-tons 5'-8" x 5'-8" x 2'-0" - 13#5 each way

D Ld + 0.25% d = \( \frac{278.2}{32} \) = 8.65/s ft 8' max

Use ftg = 6'-0" x 6'-0" x 2'-4" - 13#5 each way

HG = Hard grade reinf
Building No. 3

Occupancy: Junior high school. (See Fig. 9-82.)
Live loads: 60 psf in typical classrooms, 100 psf in corridors and stairs, 100 psf in shops, domestic science department, etc.
Number of stories: Ground floor and two typical stories.
Foundations: Spread footings bearing on sand and gravel having a safe bearing value of 4 tons per sq ft.
Exterior walls: Face brick on foundation walls to window sills of first floor, metal covering and continuous windows above.
Interior finish: Exposed concrete masonry walls. Finished asphalt floors in typical classrooms and corridors applied directly to the structural slab. Vitrified-clay tile in toilet rooms. Quarry-tile floors in shops.
Mechanical equipment: The heating system will require a 4 ft 0 in. by 4 ft 0 in. perimeter trench under the ground floor where the vertical distribution will be made along the exterior wall columns. The ventilating system will require maximum 8- by 15-in. vertical ducts between the corridor partitions and the columns. Interior columns to be concealed in the wardrobe space.
Story height: 9 ft 6 in. clear ceiling height in classrooms plus depth required for construction.
Future partitions: Although not required for the typical classrooms an allowance is to be made for dividing the classrooms into offices, teachers' rooms, etc., when future additions are built.

Fig. 9-82. Plan of typical bay, Building No. 3.

Building No. 3, Scheme A: One-way Solid Slab, Deep Beams. The architectural plan with continuous wardrobe space along the corridor walls and the mechanical system of vertical ducts for ventilation permits the use of deep beams for comparatively short spans concealed within the wardrobe space and the spreading of the corridor columns to equalize as much as possible the three slab spans in the transverse direction. Also the metal covering for the exterior provides an opportunity for closely spaced columns.
The wide spandrel beams at the exterior columns offer considerable resistance to torsion and have been given full consideration in restraining the slab in bending and thus in reducing the deflection of the one-way slab over the comparatively long span across the classrooms. To reduce excessive bending stresses in the exterior columns it
is necessary to locate them as far as practical inside the building line. The one-way slabs are of constant depth across the corridor span to provide maximum restraint at the interior columns, and although they are of greater depth than might have been required for bending the resulting stiffness is necessary to minimize the torsion in the spandrel beams, bending in the exterior columns, and possible deflection of the slab when supporting dividing partitions between classrooms.

**Building # 3 – Scheme "A"**

1-way solid slab – exposed concrete slabs

\[ f_c / f_s = 3000/10/20,000 \quad v = 90 \text{ psi} \quad u = 210/300 \text{ psi} \quad f_c = 1350 \text{ psi} \]

**ACI code**

Plan of typical bay

\[ \frac{L}{32} = \frac{25.67 \times 12}{32} = 9.5" \]

Assume \(9\frac{1}{2}"\) slab

**Typical rooms:**

- \(9\frac{1}{2}"\) slab = 120 psf
- Partition \(10.5 \times 20/7 = 20\) \(\frac{140}{psf}\)

\[ L \text{ld} = 60 \quad \frac{200}{psf} \]

**Corridors:**

- \(9\frac{1}{2}"\) slab = 120 psf
- \(L \text{ld} = 100 \quad \frac{220}{psf}\)
- Partitions concentrated on slab
### EXAMPLES OF BUILDING-FRAME DESIGN

**Building #3 - Scheme "A"**

\[
\begin{align*}
9\frac{1}{2}'' \text{ slab: } I_c &= 857 \text{ in}^4/\text{ft} \\
K_{t-0} &= \frac{4 \times 857}{27.04} = 127 \\
\text{Table 9-30 b} \\
\text{Slab 14' wide: } K_{1-2} &= 1775 \text{ ext} \\
K_{2-3} &= \frac{4 \times 857}{13.67} \times 14 = 3500 \\
18'' \times 14'' \text{ exterior cols: } I_c &= 4120 \text{ in}^4 \\
\text{Table 9-45 b} \\
18'' \times 14'' \text{ interior cols: } I_c &= 4120 \text{ in}^4 \\
K &= \frac{4 \times 4120}{10.5} = 1570 \\
\text{Spondrel beam B2: assumed } 18'' \times 18'' \\
\frac{b}{D} &= 1 \\
k_t &= 0.06 \\
f_2 &= 0.208 \\
(\text{see fig 9-41 - part 9-1}) \\
K_{t} &= \frac{0.06 \times 18^4}{4 \times 2.5/2} = 262
\end{align*}
\]

<table>
<thead>
<tr>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>27.04'</td>
<td>13.67'</td>
<td>27.04'</td>
</tr>
<tr>
<td>D Ld + L Ld = 200 psf</td>
<td>D Ld = 120 psf</td>
<td>D Ld + L Ld = 200 psf</td>
</tr>
<tr>
<td>5.4k</td>
<td>0.2k</td>
<td>5.4k</td>
</tr>
<tr>
<td>[0.37]</td>
<td>[0.41]</td>
<td>[0.41]</td>
</tr>
<tr>
<td>(0.185)</td>
<td>(0.205)</td>
<td>(0.205)</td>
</tr>
</tbody>
</table>

\(-M^* = 11.3 \\
-1.9 -1.9 -1.9 \require{cancel} -11.3 \\
(12.7 - 0.4 + M = 2.6 \times \frac{13.0 - 8}{2} + 0.8)(+0.8) - M = 8k \require{cancel} S = 71 \\
V = 2.7 - 0.1 = 2.6k \\
-10.7 -5.5 \require{cancel} 9\frac{1}{2}'' \text{ slab: } S_2 = 147 \\
A_s = \frac{8}{12} = 0.67 \\
\# 6 bt at 11'' oc = 0.48 \\
\# 4 top at 11'' oc = 0.22 \\
\text{Span 2-3:} \\
M_k = 0.84 \times 6.83 - 5.0 = 0.75 \\
-0.82 \times 3.41 = -2.8 \\
-0.2 \times 5.33 = -1 \require{cancel} -3.05k \\
V = 2.7 + M = 2.48 \times \frac{12.4}{2} - 5.4 \\
-0.22 = 10k \\
\frac{2.48}{2} \text{ S = 89} \require{cancel} \frac{1}{3} \text{ pt of 13.67' span} \\
\text{Continue bt bars to } \frac{1}{3} \text{ pt of 13.67' span} \require{cancel} \text{ oc} \]
Building #3 - Scheme "A"

\[ D \text{ Ld} + 50\% \text{ L Ld} = 180 \text{ psf} \quad D \text{ Ld} + 100\% \text{ L Ld} = 210 \text{ psf} \quad \text{Ratio} = 0.857 \]

\[ M_1 = -8 \times 0.857 = -6.9^{10^7} \quad +M = 8.9 \times 0.857 = 7.6^{10^7} \quad -M_2 = 10.5 \times 0.857 = 9.0^{10^7} \]

Bending stress at mid span: \[ \frac{7.6 \times 12,000}{147} = 620 \text{ psi} \quad 0.21 t_c \]

From fig 9-46 part 9-1: \[ E_c = 650 \times 3000 = 1,950,000 \]

\[ \Delta: \text{uniform load} = \frac{0.625 \times 5400 \times (27.04 \times 12)^3}{48 \times 1,950,000 \times 857} = 1.41^{10^{-9}} \]

\[ -M \text{ at } 1: \quad \Delta = \frac{-3 \times 8 \times 12,000 \times (27.04 \times 12)^2}{48 \times 1,950,000 \times 857} = -0.38 \]

\[ -M \text{ at } 2: \quad \Delta = \frac{-3 \times 10.5 \times 12,000 \times (27.04 \times 12)^2}{48 \times 1,950,000 \times 857} = -0.50 \]

\[ \frac{L}{600} = \frac{27.04 \times 12}{600} = 0.54^{10^{-9}} \]

<table>
<thead>
<tr>
<th>D Ld=120 psf (No partitions)</th>
<th>0.2^k</th>
<th>0.2^k</th>
<th>D Ld + L Ld = 200 psf</th>
<th>D Ld</th>
<th>+M_0 span 2-3</th>
</tr>
</thead>
<tbody>
<tr>
<td>-7.3</td>
<td>-7.3</td>
<td>-3.6</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-0.4</td>
<td>-1.3</td>
<td>+0.8</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-0.2 (-7.9)</td>
<td>-8.6</td>
<td>+0.4</td>
<td>(-2.4)</td>
<td>-1.4</td>
<td></td>
</tr>
<tr>
<td>3.25^k</td>
<td>3.0^k</td>
<td>5.05</td>
<td></td>
<td></td>
<td>3.25^k</td>
</tr>
</tbody>
</table>

Design sheet B3A2: \[ M_{k2-3} = -2.8^{10^7} \]

\[ A_9 = \frac{2.8}{12} = 0.233 \text{ in} \]

9\(\frac{1}{2}\)" slab S2

0.005 \times 12 \times 8.37 = 0.50

#4 and #4 bt - alt at 5\(\frac{1}{2}\)" oc
### Building #3 - Scheme "A"

<table>
<thead>
<tr>
<th>1</th>
<th>27.04'</th>
<th>D Ld + L Ld = 200 psf</th>
<th>0.2^k</th>
<th>0.2^k</th>
<th>D Ld + L Ld = 220 psf</th>
<th>0.2^k</th>
<th>D Ld = 120 psf</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>13.67'</td>
<td>5.4^k</td>
<td>0.2^k</td>
<td>0.2^k</td>
<td>(0.205)</td>
<td>3.0^k</td>
<td>(0.205)</td>
</tr>
<tr>
<td>3</td>
<td>27.04'</td>
<td>(0.37)</td>
<td>(0.185)</td>
<td>(0.205)</td>
<td>(0.105)</td>
<td>(0.205)</td>
<td>(0.105)</td>
</tr>
</tbody>
</table>

\[
\begin{align*}
M &= -11.3 \quad -11.3 -3.6 -3.6 -7.3 \\
-0.8 &= -2.1 +0.8 +1.6 -0.8 \\
-0.2 (-12.3) &= (13.5) -0.1 +0.5(-2.3) +0.6 (-8.1) \\
-7.8^k &= -10.6 -5.5 -1.4 -3.3 -0.8 -0.3 \\
V &= 2.7^k \\
V &= 2.7^k 1.7^k 1.7^k \\
V &= 2.82^k 1.90^k 1.50^k \\
\end{align*}
\]

Reduced - M at 2:

\[
\begin{align*}
-M &= 2.82 \times 0.33 -11.1 \times 10.2^k \\
S &= 91 \\
\frac{91}{2} \text{ slab } &S_1=147 \\
A_s &= 10.2 \\
\frac{A_s}{2.1} &= 0.84 \text{ si} \\
\end{align*}
\]

Slab S 1: #6 at 11" oc A 5 = 0.48

S 2: #4 at 11" oc = 0.22

#4 T contin = 0.22

0.92 si

w = slab S 1: \( V = 2.82^k \)

12" x 18" bm \( w = 0.22 \)

\[
\begin{align*}
\text{Table 9-38c} &= 4.94^k \\
L &= 14" - 1.5" = 12.5" \text{ clear} \\
w &= 12.5 \times 4.94 = 60^k \\
V &= 30^k \\
V_c &= 14.4 \\
\end{align*}
\]

\[
\begin{align*}
M &= 60 \times \frac{12.5}{16} = 47^k \\
S &= 417 \\
A_s &= 2.23 \\
\frac{A_s}{2.1} &= 2.11 \text{ si} \\
M &= 60 \times \frac{12.5}{16} = 68^k \\
S &= 605 \\
A_s &= 2.53 \\
\frac{A_s}{2.1} &= 2.11 \text{ si} \\
2 \# 8: S_t = 244 \quad A_s = 0.63 & 3.16 \text{ si} \\
2 \# 8 T, A_s = 1.58 & 2 \# 8 T, A_s = 1.58 \\
\end{align*}
\]

Interior bms

B 1

12" x 18"

2\#7-1\#8

2\#8 T at cols

#3\#3, 3 at 8''

ea end
Building #3 - Scheme "A"

Metal skin B

Windows at 15 psf = 0.160 K/1
6" LW block at 28 = 0.094
Max slob shear = 2.70
18" x 18" beam = 0.337
Table 9-38c

\[ +M = 41 \times 12.5 / 16 = 32 K \]
\[ A_s = 32 / 22.3 = 1.43 \text{ si} \]
\[ 2 \# 5 - 2 \# 6 \text{ bt} \quad A_s = 1.48 \text{ si} \]

Design sheet B3A2 - torsional \( M_0 \):
\[ m_1 = 8 K \quad m_2 = 5.4 K \]

Reduced for 0.85 L Ld
\[ m_1 = 8.0 \times 191 / 200 = 7.65 K \]
\[ m_2 = 5.4 \times 191 / 200 = 5.15 K \]

Design sheet B3A5

Typical spandrel beam B2

\[ W = 12.5 \times 3.29 = 41 K \]
\[ V = 20.5 K \]

\[ S_t = 753 \quad V_c = 21.4 K \]

\[ -M = 41 \times 12.5 / 11 = 46.5 K \]
\[ A_s = 46.5 / 22.5 = 2.08 \text{ si} \]
\[ 4 \# 6 \text{ bt} \quad A_s = 1.76 \]
\[ 2 \# 5 \text{ top} \]
\[ \text{Lapped at cols} \quad A_s = 1.20 \]

At \( x \):
\[ V = 13.5 K \quad v = 55 \text{ psi} \]
\[ M_I = 26.5 K \quad v_I = 262 \]
\[ n = 262 + 55 - 90 = 227 \text{ psi} \]
\[ n_I = 18(227) = 7.8 \]

2 # 6 bt:
\[ \nu_s = 0.43 \times 2 \times 14500 = 50 \text{ psi} \]
\[ \nu_s = 277 + 84 - 90 - 50 = 221 \text{ psi} \]

\#3 stirrups, \( s = 0.11 \times 20000 = 2.5 \text{"} \)
\[ n = 7.8 \text{"} \text{ use at } 8 \text{" oc} \]
Building No. 3, Scheme B: Two-way Ribbed Slab, Slab Band Beams and Exposed Concrete Ceilings. The scheme of framing varies from Building No. 3, Scheme A, in that alternate columns have been omitted at the exterior walls, permitting sleeves through the 18-in.-wide beams at the center line of the span for heating risers and returns which can be concealed in the metal mullions and provide the same exterior treatment as shown for scheme A. Slab bands have been provided over all interior masonry partitions and all typical interior construction kept within the depth of the ribbed slab.

Fig. 9-83. Assembly of domes on scaffold and section through one dome.
REINFORCED-CONCRETE BUILDING FRAMES

Even with the total depth of the slab greater than would be required for moments special consideration has been given to limit the deflections of the slab bands supporting masonry partitions for the 27-ft transverse spans to approximately \( \frac{1}{600} \) of the span. Domes may be omitted where required for lighting fixtures.

Pressed steel domes (Fig. 9-83) are available with flanges to form the ribs which are especially desirable where slabs of this type are to be left exposed for the finished ceiling. It is also possible to obtain precast acoustical fillers to fit the top of the domes when desired.

### Building #3 - Scheme “B”

**2-way ribbed slab - slab band beams - exposed concrete ceiling**

\[
\frac{t_e}{t_o} = \frac{3000}{10/20,000} \quad v_e = 90 \text{ psi} \quad u = 300 \text{ psi} \quad f_c = 1350 \text{ psi}
\]

**ACI code**

---

**Plan of typical bay**

**Typical rooms:**
- 8"+3" slab - 88 psf
- Partitions - 20
- D Ld 108
- L Ld 60 168 psf

Partitions distributed over col strip 6"-0" wide.

**Corridors:**
- 8"+3" slab - 88 lb
- L Ld - 100 lb 168 psf

Partitions as concentrated load.
Special bending details for the reinforcing at the interior columns are shown on design sheet B3B9. They are needed because of the openings which reduce the shear area that would ordinarily be available.

Because of the uncertain action of the spandrel beams in torsion over the long lengths of the beams the slabs have been designed for free supports without restraint at the exterior columns. In the design of the spandrel beam the restraint of the torsional stiffness for the full dead and live loads on the slab has been used in computing the torsional shear and top reinforcing required at the exterior support. As the normal distance for bending up the bottom bars is not close to the columns, vertical stirrups have been provided for the excess shearing stresses.

**Building #3 - Scheme "B"**

<table>
<thead>
<tr>
<th>Column</th>
<th>$r = \frac{0.87 \times 27.04}{0.76 \times 28} = 1.10$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$C = 0.28, C_1 = 0.39$</td>
</tr>
<tr>
<td></td>
<td>$1-C = 0.72, 1-C_1 = 0.61$</td>
</tr>
<tr>
<td></td>
<td>$C_b = 0.29, C_{bl} = 21$</td>
</tr>
<tr>
<td></td>
<td>Slab: $8'' + 3''$</td>
</tr>
<tr>
<td></td>
<td>$t/D = 3/11 = 0.27$</td>
</tr>
<tr>
<td></td>
<td>Mid strip - 6 ribs: $\frac{b}{b_0} = \frac{12}{25} = 4.8$</td>
</tr>
</tbody>
</table>

From table 9-55: $I_c = 1.9 \times 5 \times 11^3/12 = 1055/\text{rib}$

- 8 ribs at mid strip: $I_c = 8440$
- 6 ribs at slab band: $I_c = 6330$

**Frame constants**

<table>
<thead>
<tr>
<th>Column</th>
<th>$a=0.10$</th>
<th>b=0.10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slab S1</td>
<td>$k_a = k_b = 5.2$</td>
<td>$c_a = c_b = 0.57$</td>
</tr>
<tr>
<td></td>
<td>$F_a = F_b = 0.09$</td>
<td>$K_a = K_b = 5.2 \times 8440 / 27.04 = 1625$</td>
</tr>
<tr>
<td></td>
<td>(Coeff approximate by the same longitudinal direction)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$r = 0.365, k_a = k_b = 6.5, c_a = c_b = 0.62$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$M_F = 0.095 \text{WL}$, $M_F = 0.10 \text{PL}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$K_a = K_b = 6.5 \times 8440 / 13.67 / 4020$</td>
<td></td>
</tr>
</tbody>
</table>

- Slab S2: $a = 0.22, b = 0.22$

<table>
<thead>
<tr>
<th>Column</th>
<th>$a=0.166$</th>
<th>b=0.166</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slab band SB1</td>
<td>$k_a = k_b = 7.0$</td>
<td>$c_a = c_b = 0.64$</td>
</tr>
<tr>
<td></td>
<td>$F_a = F_b = 0.096$</td>
<td>$K_a = K_b = 7.0 \times 10250 / 27.04 = 2650$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Column</th>
<th>$b/b' = 8.4 / 6.4 = 1.3$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$t/d = 3/11 = 0.27$</td>
</tr>
<tr>
<td></td>
<td>$I_c = 1.2 \times 77 \times 11^3/12 = 10250$</td>
</tr>
<tr>
<td>Haunch:</td>
<td>$I_c = 125 \times 11^3/12 = 13900$</td>
</tr>
<tr>
<td></td>
<td>$\text{Equiv } D^3 = \frac{12 \times 13900}{1.2 \times 77}$</td>
</tr>
<tr>
<td></td>
<td>$D = 18''$</td>
</tr>
<tr>
<td></td>
<td>$r = 7/11 = 0.635$</td>
</tr>
</tbody>
</table>
Building #3 - Scheme "B"

Spandrel beam B1: 18" x 18" b/D = 1.0

(From fig 9-41 - part 9-1)

\[ k_f = 0.06 \quad C_f = 0.208 \]
\[ k_f = \frac{0.06 \times 18^2}{4 \times 13.25} = 1.20 \]

18" x 18" exterior cols: \( I_c = 8750 \)  \( K = 4 \times \frac{8750}{10.5} = 3330 \)

18" x 14" interior cols: \( I_c = 4120 \)  \( K = 4 \times \frac{4120}{10.5} = 1570 \)

At mid strip: No restraint at col line #1

Distribution factors at 2 & 3:

\[ \frac{D Ld + L Ld}{8785} = 0.28 \times 168 = 47 \text{ psf} \]
\[ D Ld = 88 \text{ psf} \]
\[ w = 94 \text{ ppf} \]

\[ D \]
\[ CD \]
\[ (0.57) \]
\[ (0.11) \]
\[ (0.28) \]
\[ (0.28) \]
\[ (0.10) \]

\[ 27.04 \]
\[ 13.26 \]
\[ 27.04 \]

\[ \begin{array}{c}
27.04 \\
13.26 \\
27.04 \\
\end{array} \]

\[ \begin{array}{c}
W = 2.55 \text{ k} \\
W = 2.4 \text{ k} \\
\end{array} \]

\[ \begin{array}{c}
-6.2 \\
-0.3 \\
-0.1 \\
0 \\
\end{array} \]

\[ \begin{array}{c}
-6.2 \\
-3.5 \\
-0.2 \\
-8.3 \\
\end{array} \]

\[ \begin{array}{c}
-3.9 \\
+0.6 \\
+1.1 \\
-4.7 \\
\end{array} \]

\[ \begin{array}{c}
-3.1 \\
+0.6 \\
+1.1 \\
-5.7 \\
\end{array} \]

\[ \begin{array}{c}
+M_{2-3} = 1.8 \times 6.63 = 11.9 \\
-1.2 \times 3.31 = -4.0 \\
-0.6 \times 4.63 = -2.8 \\
-M = -5.7 \\
\end{array} \]

\[ V = 1.27 \]
\[ V = 1.27 \]
\[ \frac{0.32}{0.95^k} \]
\[ \frac{0.32}{1.59^k} \]

\[ +M_o = 0.95 \times 10.1 / 2 = 4.6^k \]

\[ S = 41 \]

\[ A_s = 4.6 / 13.3 = 0.35 \text{ si/rib} \]

Marginal strip: \( A_s = 0.35 \times 0.75 = 0.26 \text{ si/rib} \)
Building \#3 - Scheme "B"

\[ K_f \text{ for spandrel beam} = 120 \]
\[ K \text{ for 1 rib} = 1625/8 \cdot 203 = 203 \]
\[ D = \frac{203}{323} = 0.63 \]

\[ \begin{array}{c|c|c|c}
   & \text{D Ld} + \text{L Ld} & \text{D Ld} + \text{L Ld} & \text{D Ld} + \text{L Ld} \\
   & w = 94 \text{ pfp} & w = 176 \text{ pfp} & w = 94 \text{ pfp} \\
   \text{D} & 0.63 \text{ (0.19)} & 0.26 \text{ (0.46)} & 0.6 \text{ (0.48)} \\
   \text{CD} & 0.36 \text{ (0.11)} & 0.26 \text{ (0.28)} & 0.6 \\
   \text{M} & -6.2 \text{ (0.11)} & -6.2 \text{ (0.28)} & -6.2 \text{ (0.36)} \\
   & -0.3 \text{ (0.6)} & -0.3 \text{ (0.28)} & -0.3 \text{ (0.36)} \\
   & -0.3 \text{ (0.6)} & -0.3 \text{ (0.28)} & -0.3 \text{ (0.36)} \\
   & -0.2 \text{ (-7.0)} \text{ (0.6)} & -0.1 \text{ (0.28)} & -0.5 \text{ (-2.0)} \text{ (0.36)} \\
   \text{(-2.6)} & -8.6 \text{ (-2.0)} & -8.6 \text{ (-2.0)} & -8.6 \text{ (-2.0)} \\
\end{array} \]

\[ A_s = 2.6 / 14 = 0.186 \text{ /rib at } \xi \text{ at spandrel beam} \]
\[ # 4 \text{ bt} + # 3 \text{ top min} \]
\[ A_s = 0.31 \text{ si} \]

\[ M_o \text{ in longitudinal direction:} \]
\[ w = 0.39 \times 168 \times 2^2 = 130 \text{ pfp} \]
\[ \text{D Ld} + \text{L Ld} \]

\[ \begin{array}{c|c|c}
   & \text{D Ld} & \text{D Ld} + \text{L Ld} \\
   & w = 84 \text{ pfp} & w = 130 \text{ pfp} \\
   \text{D} & 2.45 \text{ (0.5)} & 0.5 \text{ (0.5)} \\
   \text{CD} & 0.28 \text{ (0.28)} & 0.5 \text{ (0.5)} \\
   \text{M} & -5.0 \text{ (-2.5)} & -9.2 \text{ (-5.9)} \\
   & -0.9 \text{ (-0.9)} & -0.9 \text{ (-0.9)} \\
   & +0.5 \text{ (-0.5)} & -0.5 \text{ (-0.5)} \\
   & -4.2 \text{ (0.3)} & -10.9 \text{ (0.5)} \\
   \text{(-10.9)} & 0.3 \text{ (0.3)} & -7.55 \\
\end{array} \]

\[ M = 1.82 \times 14 / 2 - 7.5 = 5.2 \text{ (1)} \]
\[ A_s = 5.2 / 14 = 0.37 \text{ si /rib} \]

\[ V = 1.82 \]

\[ \text{Marginal strip} = 0.37 \times 0.75 = 0.28 \text{ si /rib} \]

\[ 2 \text{ # 4} \]
\[ 2 \# 4 \]

\[ M \text{ at slab bands S1} \]

At \( \xi \) at supt. \( S = 92.5 \text{ in} \) slab; \( S_t = 198 \)
\[ A_s = 10.4 / 14 = 0.74 \]
\[ 2 \# 4 \text{ bt} = 0.40 \]
\[ 2 \# 4 \text{ top} = 0.40 \]

Mid strip & morg strips
# REINFORCED-CONCRETE BUILDING FRAMES

## Building #3 - Scheme "B"

### Design sheet B3B5

<table>
<thead>
<tr>
<th>Column 1</th>
<th>Column 2</th>
<th>Column 3</th>
<th>Span 2-3</th>
</tr>
</thead>
<tbody>
<tr>
<td>D Ld + L Ld</td>
<td>D Ld + L Ld</td>
<td>D Ld</td>
<td>M at 2</td>
</tr>
<tr>
<td>w = 94 pfp</td>
<td>w = 336 pfp</td>
<td>w = 60 pfp</td>
<td></td>
</tr>
</tbody>
</table>

### Moment Values

<table>
<thead>
<tr>
<th>Column 1</th>
<th>Column 2</th>
<th>Column 3</th>
<th>Span 2-3</th>
</tr>
</thead>
<tbody>
<tr>
<td>D [1.0]</td>
<td>[0.19]</td>
<td>[0.9]</td>
<td>W = 1.64 k</td>
</tr>
<tr>
<td>CD [0.57]</td>
<td>[0.46]</td>
<td>[0.6]</td>
<td></td>
</tr>
<tr>
<td>M</td>
<td>-6.2</td>
<td>-6.2</td>
<td>-7.4</td>
</tr>
<tr>
<td>+0.1</td>
<td>-3.5</td>
<td>-1.0</td>
<td>-0.3</td>
</tr>
<tr>
<td>-0.3</td>
<td>(-9.7)</td>
<td>+0.5</td>
<td>0.7</td>
</tr>
</tbody>
</table>

### Shear Values

<table>
<thead>
<tr>
<th>Column 1</th>
<th>Column 2</th>
<th>Column 3</th>
<th>Span 2-3</th>
</tr>
</thead>
<tbody>
<tr>
<td>V 1.27</td>
<td>1.27</td>
<td>3.15 k</td>
<td></td>
</tr>
<tr>
<td>-0.35</td>
<td>+0.35</td>
<td>+0.15</td>
<td></td>
</tr>
<tr>
<td>0.92</td>
<td>1.62</td>
<td>3.3</td>
<td></td>
</tr>
</tbody>
</table>

### At Rib

- **M = -1.62 x 3' = +4.86**
- **C C of col = -M = 9.4 k**
- **S = 84**
- **24' x 11' slab: S1 = 396**
- **A_s = 9.4 / 14 = 0.67**
- **5' rib - S_f = 84 ok**
- **V_c = 3.80 k**

### Notes

- **No 4 bt slab S1 = 0.20**
- **No 3 bt slab S2 = 0.11**
- **2 No 4 top = 0.40**

### Span 2-3

<table>
<thead>
<tr>
<th>Column 1</th>
<th>Column 2</th>
<th>Column 3</th>
<th>Span 2-3</th>
</tr>
</thead>
<tbody>
<tr>
<td>D Ld</td>
<td>D Ld</td>
<td>D Ld</td>
<td>+M</td>
</tr>
<tr>
<td>w = 60 pfp</td>
<td>w = 376 pfp</td>
<td>w = 60 pfp</td>
<td></td>
</tr>
</tbody>
</table>

### Moment and Shear Values

<table>
<thead>
<tr>
<th>Column 1</th>
<th>Column 2</th>
<th>Column 3</th>
<th>Span 2-3</th>
</tr>
</thead>
<tbody>
<tr>
<td>M</td>
<td>-4.0</td>
<td>-4.0</td>
<td>-7.4</td>
</tr>
<tr>
<td>+0.4</td>
<td>-2.3</td>
<td>-1.0</td>
<td>-0.4</td>
</tr>
<tr>
<td>-0.1</td>
<td>(-6.1) + 0.2</td>
<td>(-8.8)</td>
<td></td>
</tr>
</tbody>
</table>

### Additional Calculations

- **+M: 3.15' x 6.83 = 21.5**
- **-2.55 x 3.41 = -8.7**
- **-0.6 x 4.83 = -2.9**
- **M = -7.6**
- **V = 3.15 k**

- **A_s = 2.3 / 14 = 0.16 si**
- **0.005 x 5 x 9.75 = 0.25 min**

Use No 4 st # 3 st
### EXAMPLES OF BUILDING-FRAME DESIGN

**Building #3 - Scheme "B"

Design sheets B3B2,3

Slab S2 - 6 ribs:

\[
D_1 = \frac{2650}{9310} = 0.28
\]

\[
D_2 = \frac{2650}{8805} = 0.30
\]

\[
D_2-3 = \frac{3015}{8805} = 0.34
\]

\[
1 - C = 0.72
\]

\[
W = 0.72 \times 27.04' \times 28 \times 148 \text{ psf} = 81^k
\]

Solid slab = 27.04' at 335 lb = 9

Partition 26' at 210 lb = 5.5^k

\[
95.5^k
\]

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>22.04'</th>
</tr>
</thead>
<tbody>
<tr>
<td>D</td>
<td>27.04'</td>
<td>13.67'</td>
<td>22.04'</td>
<td></td>
</tr>
<tr>
<td>Ld + L Ld</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\begin{align*}
D & = 0.28 \\
C & = 0.17 \\
M & = -233 \\
V & = 47.75 \\
\rho & = 0.88
\end{align*}

\[
V = 47.75 + 0.88 = 46.87^k
\]

\[
M_z = -67^k
\]

\[
6^k \text{ per rib}
\]

\[
A_s = \frac{6}{14} = 0.43 \text{ si}
\]

**Slab Band SB1:**

\[
+ M = 48.63 \times 13.8 / 2 - 206 = 130^k
\]

\[
S = 1160
\]

\[
P77" \times 11" \text{ slab band: } S_t = \frac{77}{11} \times 198 = 1270 \quad A_s = \frac{130}{14} = 9.3 \text{ si}
\]

(Table 9 - 30b, S_t for 12" = 198)

\[
6^# \text{ st } 7^# \text{ b t } -9.1 \text{ si}
\]

\[
- M \text{ at 1: } - M = -206 + 48.63 \times 0.5 = 182^k
\]

\[
S = 1620
\]

\[
P125" \times 11" \text{ slab band: } S_t = 2060 \quad A_s = 184 / 14 = 13.1 \text{ si}
\]

\[
7^# \text{ b t } A_s = 5.5 \quad 12^# \text{ top } A_s = 7.2 \{ 12.7 \text{ si}
\]

\[
b = 77" - V_c = 58^k
\]

(6 contin - see design sheet B3B7)

\[
V_b = 133^k
\]
Check deflection for \( \text{D} \text{Ld} + 50\% \text{ Ld} \) \( W = 79 \text{kip} \)

\[
M_1 = -170 \text{kip ft}, \quad M_2 = -151 \text{kip ft}
\]

\( f_c = 1020 \text{ psi} \)

From fig 9-46 - part 9-1:

\[ E_c = 350 \times 3000 = 1,050,000 \text{ psi} \]

**Uniform load**

- \( M_0 \) at 1
- \( -M \) at 2

**Constant**

\[ +0.625WL^3/48EI -3.0ML^2/48EI -3.0WL^3/48EI \]

**Haunch at 1**

-0.008 \( +0.12 = 0.0 \)

**Haunch at 2**

-0.008 \( +0.0 = 0.012 \)

\[ +0.608WL^3/48EI -2.88M_L^2/48EI -2.88M_L^3/48EI \]

\[
\Delta = \frac{0.608 \times 79,000 \times 324^3}{48 \times 1,050,000 \times 10,250} = +3.02''
\]

\[
-\frac{2.88 \times 170 \times 12,000 \times 324^3}{48 \times 1,050,000 \times 10,250} = -1.15''
\]

\[
-\frac{2.88 \times 151 \times 12,000 \times 324^2}{48 \times 1,050,000 \times 10,250} = -1.01''
\]

Max \( \Delta = 27.04 \times 12/600 = 0.54'' \)

Try 6 #7 continuous top bars to reduce bending stress and deflection

From table 9-31b \( S_t = 6 \times 67 = 402 \)

\[
S_t = \frac{1270}{1672} = \frac{77}{12} \times 1595 \times 10,250 = 11,830
\]

\( f_c = 108 \times 12,000 / 1672 = 785 \text{ psi} \)

\( 0.26 f_c' \)

\[
E_c = 525 f_c' = 1,575,000
\]

\[
\Delta = \frac{0.608 \times 79,000 \times 324^3}{48 \times 1,575,000 \times 11,830} = 1.82''
\]

\[
-\frac{2.88 \times 170 \times 12,000 \times 324^2}{48 \times 1,575,000 \times 11,830} = -0.69''
\]

\[
-\frac{2.88 \times 151 \times 12,000 \times 324^2}{48 \times 1,575,000 \times 11,830} = -0.61''
\]

Ok

Provide 6 #7 top cont for deflection
### Example of Building-Frame Design

**Building #3 - Scheme "B"**

<p>| | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>27.04'</td>
<td>13.67'</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>D Ld + L Ld</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**P**

<table>
<thead>
<tr>
<th></th>
<th>0.28</th>
<th></th>
<th>0.30</th>
<th></th>
<th>0.34</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.17</td>
<td></td>
<td>0.21</td>
<td></td>
<td>0.21</td>
<td></td>
</tr>
</tbody>
</table>

**M**

<table>
<thead>
<tr>
<th></th>
<th>233</th>
<th>233</th>
<th>24</th>
<th>24</th>
<th>152</th>
</tr>
</thead>
<tbody>
<tr>
<td>-35</td>
<td>-37</td>
<td>27</td>
<td>50</td>
<td>24</td>
<td></td>
</tr>
<tr>
<td>-11</td>
<td>-6</td>
<td>16</td>
<td>13</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>-4</td>
<td>-283</td>
<td>-2</td>
<td>4</td>
<td>-182</td>
<td></td>
</tr>
</tbody>
</table>

**V**

<table>
<thead>
<tr>
<th></th>
<th>47.7K</th>
<th></th>
<th>18.9K</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>+0.6</td>
<td>-0.6</td>
<td>+3.4</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Design sheet B388**

**Slab band SB1 continued**

**Max - M at col 2**

**Reinf at int cols 2 and 3**

**Slab band SB2**

- **M reduced** = 184 + 47.1 x 0.416 = 164K

  - Slab S2
    - \( A_s = 164/14 = 11.7 \text{ si} \)
    - 7# 8 ft: \( A_s = 6.53 \)
    - 6# 7 top con = 3.60
    - 6# 6 top-slab S2 = 2.64
    - 12.77 \( \text{ ok} \)

- \( w = 0.61 \times 13.52 \text{ at } 668 \text{ lb} = 1.34K \)

- **M**

  - at col -2: \( M_0 = 9.4/27.04 = 0.35 \)
  - at 300 lb: \( M = 6.83/27.04 = 1.28 \)
  - Cor partition at 300 lb: \( w = 0.30 \)
  - Slab band: \( 6' \times 60 \text{ lb} = 0.36 \)

- **V**

  - \( V = 23K \)

- **d**

  - \( d = 9 \text{ in} \times 1/4 \times S = 1070 \)

- **S**

  - \( S = 46L \)

- **V_B**

  - \( V_B = 10 \times 6 = 56K \text{ ok} \)

- **A_s**

  - \( A_s = 14 / 12.5 = 2.57 \text{ si} \)

- **S**

  - \( S = 52 / 1.44 = 4.0 \text{ si} \)

- **V_B**

  - \( V_B = 10 \times 5.6 = 56K \text{ ok} \)
Building #3 - Scheme "B"

Load on interior column

SB 1: $V = 0.29 \times 27.04' \times 28' \times 168 \text{ lb} = 37.0^k$

Partition - 13' at 210 lb = 2.7
Slab band - 10' at 360 lb = 3.6

SB 2: $V = 0.21 \times 6.76' \times 14' \times 168 \text{ lb} = 3.0$

0.5 \times 13.67' \times 14' \times 188 \text{ lb} = 18.0
Cor part. - 14' at 300 lb = 4.2
Slab band - 14' at 330 lb = 4.6
Total = 73.1 k

Reinforcement over column: 25%

Net shear at critical section
$V = 73.1 - 3.75 \times 3.42 \times 228 \text{ lb}$
$= 73.1 - 2.9 = 70.2^k$

$\nu = \frac{70200}{127 \times \frac{7}{8} \times 9.5} = 67 \text{ psi}$

Critical sec for shear

Plan at column
Building #3 - Scheme "B"

Metal covering
and windows at 15 psf - 10.5 x 15 = 0.158
6" block at 28 psf - 3.33 x 28 = 0.093
Slab - (1 - c) \( wL = 0.30 \times 168 \times 22.04 = 1.360 \)
18" x 18" beam = 0.340
\( K = 1.95 \)

From design sheets B3B2,3
\( \Sigma K = \) slab band SB1 = 2650
2 cols 18" x 18" = 6600
9310

\[ \begin{align*}
W_e &= 26.5' \text{ at } 1.95 = 51.6 \\
M &= 51.6 \times 26.5/12 = 113 \text{ k}\\
+ M &= 51.6 \times 26.5/16 = 86 \\
A_s &= 86/22.3 = 3.86 \text{ si} \\
2 \# 9 - 2 \# 9 &= 3.58 \text{ si} \\
\end{align*} \]

Check for combined shear and torsion

Design sheet B3B3: \( M_e = 3.1 K = 120 \text{ k} \)

Slab \( K = 101 \text{ per foot of width} \)

22' of slab: \( K = 2222 \)

at cols: \( D = 9310/11532 = 0.81 \)

\( m_1 = 0.81 \times 3.1 = 2.5 \text{ k} \)

\( m_2 = 2.5 \times 120/221 = 1.35 \text{ k} \)

\( M_t = (2 \times 1.35 + 2.6) \times 26.5/12 = 11.7 \text{ k} \)

\( v = \frac{11.7 \times 12,000}{0.208 \times 18 \times 15} = 122 \text{ psi} \)

\( v = \frac{25,800}{18 \times 7/8 \times 15.5} = 106 \text{ psi} \)

\( n = \frac{18(106+122-90)}{2 \times 122} = 10.2 \)

\( \# 3 \) stirrups - 1/4

\[ S = \frac{0.33 \times 20,000}{10 \times 138} = 9.6'' \]
Framing plan
(Typical class room bay 28' x 70')

Quantities

Concrete 51.1 cu yd
Reinforcement 8286 lb
Slab forms 1860 sf
Beam forms 202 sf
Pan rental
Building No. 3, Scheme C: Two-way Solid Slab, Slab Band Beams and Exposed Concrete Ceilings. Similar to Building No. 3, Scheme B, in that slab band beams have been located over the dividing partitions and wardrobe spaces so that all interior partitions will be the same height, this scheme has the additional advantage of providing a smooth ceiling. The ceiling height will also be slightly greater for a given story height.

Because of the uncertain action of the spandrel beams in torsion for the 28 ft 0 in. span at the exterior walls the slabs for the classroom spans are designed in the transverse direction for free supports at the exterior column lines. Torsional restraint has

Building #3 - Scheme "C"

Design sheet

2-way solid slabs - slab band beams - exposed concrete ceilings

\[
f_c/n_n = 3000/10/20,000 \quad v_c = 90 \text{ psi} \quad u = 300 \text{ psi} \quad f_c = 1350 \text{ psi}
\]

A C I code

Plan of typical bay

2-way slab: \( \text{min } t = \frac{1100}{180} = 6.1 \) \( \frac{6.1}{2} \) in

Use \( \frac{6.1}{2} \) in

Dead loads

\[
6.5 \text{ in slab; D Ld = 81 psf + partitions at 20 psf}
\]

Slab bands: assume \( r = 0.75 \) slab thickness

\[
1.75 \times \frac{6.5}{2} = 11.4 \text{ in} \quad \text{Use 12 in}
\]

\[
D \text{ Ld} = 150 \text{ psf}
\]
REINFORCED-CONCRETE BUILDING FRAMES

Building #3 - Scheme "C"

$ r = \frac{0.87 \times 27.04}{0.76 \times 28} = 1.10 $ 

$ C = 0.28 \quad C_l = 0.39 

w_e = 0.28 \times 161 \text{ lb} = 45 \text{ psi} \quad w_0 = 0.39 \times 161 \text{ lb} = 63 \text{ psi} 

C_s = 0.21 \quad C_{sl} = 0.29 

1 - c = 0.72 \quad 1 - c_l = 0.61 

C_b = 0.29 \quad C_{bl} = 0.21 

Load distribution slab S1

From table 9-55

Slab S1:

$I_c = 275$  

$a = 0.10 \quad b = 0.10$  

$r_e = r_b = 5.5/6.5 = 0.85$  

$k_a = k_b = 6.0$  

$e_a = e_b = 0.70$  

$F_a = F_b = 0.095$  

$K_a = K_b = 6.0 \times 275/27.04$  

$k = 4 \times 4120$  

$K = \frac{4 \times 4120}{10.5} = 3800$  

$S_x = 60.5$  

$V_c = 5.03^k$

Mid strip

slab - S2  

$61/2"$ slab  

$S_x = 60.5$  

$V_c = 5.03^k$

Slab S2:

$I_c = 8750$  

$a = b = 0.20$  

$r_e = r_b = 5.5/6.5 = 0.85$  

$k_a = k_b = 9.45$  

$e_a = e_b = 0.70$  

$F_a = F_b = 0.103$  

$K_a = K_b = 9.45 \times 275/13.67$  

$k = 4 \times 4120$  

$K = \frac{4 \times 4120}{10.5} = 1570$  

$S_x = 60.5$  

$V_c = 5.03^k$

Modified k at 2 = (1 - 0.61^2)6/1 = 39

\[ D_{(2-1)} = 61/475 = 0.13 \]

\[ D_{(2-3)} = 190/475 = 0.40 \]

\[ M_0 \]

Span 1-2

+M_0

\#4 & \#4 at 8"oc

top layer

mid strip

\#3 & \#4 at 8"oc

top layer

marg strip

been considered at the spandrel as required for possible negative moment and torsional shear. This procedure will ensure adequate reinforcement for both positive moment in the center of the span and negative moment at the exterior column lines. The additional depth of the slab bands acting as haunches for the slabs will reduce the amount of negative reinforcement that would otherwise be required for negative moment at the interior column lines.
**Building \# 3 - Scheme "C"**

<table>
<thead>
<tr>
<th>Beam Bl: 18&quot; x 18&quot;</th>
<th>b/D = 1.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Torsional stiffness from fig 9-41 - part 9-1:</td>
<td></td>
</tr>
<tr>
<td>( k_1 = 0.06 )</td>
<td>( C_1 = 0.208 )</td>
</tr>
<tr>
<td>( k_1 = \frac{0.06 \times 18^4}{4 \times 13.25} = 120 )</td>
<td>( D_{1-2} = \frac{61}{181} = 0.34 )</td>
</tr>
</tbody>
</table>

---

<table>
<thead>
<tr>
<th>Column</th>
<th>Moment</th>
<th>Shear</th>
<th>( f_c' )</th>
</tr>
</thead>
<tbody>
<tr>
<td>D</td>
<td>0.13</td>
<td>0.40</td>
<td>( \frac{138 \times 12,000}{60.5} = 275 \text{ psi} )</td>
</tr>
<tr>
<td>CD</td>
<td>0.08</td>
<td>0.28</td>
<td>( \frac{138 \times 12,000}{60.5} = 275 \text{ psi} )</td>
</tr>
<tr>
<td>MF</td>
<td>-3.1</td>
<td>-1.82</td>
<td>( \frac{138 \times 12,000}{60.5} = 275 \text{ psi} )</td>
</tr>
<tr>
<td>Ve</td>
<td>0.61</td>
<td>-3.44</td>
<td>( \frac{138 \times 12,000}{60.5} = 275 \text{ psi} )</td>
</tr>
<tr>
<td>M</td>
<td>0.57</td>
<td>-3.24</td>
<td>( \frac{138 \times 12,000}{60.5} = 275 \text{ psi} )</td>
</tr>
</tbody>
</table>

**Check deflection**

6 1/2" slab S1

\( I_1 = 15I \)

(Restrained by spandrel beam Bl)

---

**From fig 9-46 - part 9-1:**

\( E_c = 1000 \times 3000 = 3,000,000 \) N/mm²

**Uniform load**

<table>
<thead>
<tr>
<th>Location</th>
<th>Moment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uniform load</td>
<td>-M at 1</td>
</tr>
<tr>
<td>-M at 2</td>
<td></td>
</tr>
</tbody>
</table>

**Constant 1:**

\[ \Delta = \frac{0.625Wl^3}{48EI} \]

\[ \Delta = \frac{3.0Ml^2}{48EI} \]

**Haunch at 1:**

\[ -0.001 \]

\[ +0.045 \]

**Haunch at 2:**

\[ -0.001 \]

\[ +0.00 \]

\[ +0.045 \]

\[ +0.623 \]

\[ -2.95 \]

\[ -2.95 \]

\[ \Delta = \frac{0.623 \times 1220 \times 324^3}{48 \times 3,000,000 \times 131} = +1.37" \]

\[ -2.95 \times 2240 \times 12 \times 324^2 \]

\[ 48 \times 3,000,000 \times 131 = -0.44" \]

\[ -2.95 \times 3440 \times 12 \times 324^2 \]

\[ 48 \times 3,000,000 \times 131 = -0.68" \]

\[ +0.25" \]

\[ \max \Delta = 27.04 \times 12/600 = 0.54" \]

---

At the interior columns where the openings for mechanical ducts interfere with the normal arrangement of reinforcing for negative moment, shear head reinforcement has been added in the form of bent bars to provide for the shearing stress without splaying the column at the bottom of the slab band.
Building #3 - Scheme "C"

Design sheet B3C4

+ M₀

Slab S2

Use #3 and #4
Alt at 8" oc
(Same spacing as slab S1)

- M at 2 and 3

Provide #3 top continuous across S2

At 2.5' from col 2
- M = 0.79 x 2.5 = 1.97 k
- 0.11 x 1.25 = -0.13
- M = -4.90
- 3.06 k

S = 27.3
62" slab - S₉ = 60.5
Aₙ = 3.06/144 x 5.5 = 0.39
From S1 and S2: #4 at 8" = 0.31
#3 top at 12" oc = 0.11

At 6 column 2

S = 43.6
12" slab: S₉ = 242
Aₙ = 4.9
1.44 x 11

From S1 and S2: #4 at 8" = 0.31
**Building ≠ 3 - Scheme "C"**

Long direction: \( a = b = 2.5/28 = 0.090 \) \( r_a = r_b = 5.5/6.5 = 0.85 \)

From table 9-55: \( k = 5.9 \) \( C = 0.60 \) \( F = 0.094 \)
From design sheet B3C2: \( C_t = 0.39 \) \( w_e = 63 \text{ psf} \)

<table>
<thead>
<tr>
<th>28'</th>
<th>28'</th>
<th>28'</th>
<th>28'</th>
<th>28'</th>
</tr>
</thead>
<tbody>
<tr>
<td>D Ld</td>
<td>D Ld + L Ld</td>
<td>D Ld</td>
<td>D Ld</td>
<td></td>
</tr>
<tr>
<td>( w_e = 39 \text{ psf} )</td>
<td>( w_e = 63 \text{ psf} )</td>
<td>( w_e = 39 \text{ psf} )</td>
<td>( w_e = 17.6 \text{ k} )</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>CD</th>
<th>We = 1.76K</th>
<th>We = 1.76K</th>
<th>We = 1.76K</th>
<th>We = 1.09K</th>
</tr>
</thead>
<tbody>
<tr>
<td>-4.62 -3.14</td>
<td>(0.5)</td>
<td>(0.5)</td>
<td>(0.5)</td>
<td>(0.5)</td>
</tr>
<tr>
<td>-3.14 -4.62</td>
<td>(0.30)</td>
<td>(0.30)</td>
<td>(0.30)</td>
<td>(0.30)</td>
</tr>
<tr>
<td>+0.44 -0.44</td>
<td>(0.5)</td>
<td>(0.5)</td>
<td>(0.5)</td>
<td>(0.5)</td>
</tr>
<tr>
<td>+0.26 -0.26</td>
<td>(0.30)</td>
<td>(0.30)</td>
<td>(0.30)</td>
<td>(0.30)</td>
</tr>
<tr>
<td>+0.16 -0.16</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\((-2.23) +0.05 -0.05 (-5.53) + M = 0.88 \times 14/2 - 3.87\)
\[-2.76 -1.11 -3.87^{1K}\]
\[A_s = \frac{2.29}{1.44 \times 5.37} = 0.296 \text{ si}\]

- \( D \text{ Ld} \)
- \( D \text{ Ld} + L \text{ Ld} \)
- \( D \text{ Ld} + L \text{ Ld} \)
- \( D \text{ Ld} \)

\( w_e = 39 \text{ psf} \) \( w_e = 63 \text{ psf} \) \( w_e = 39 \text{ psf} \) \( w_e = 17.6 \text{ k} \)

<table>
<thead>
<tr>
<th>CD</th>
<th>We = 1.09K</th>
<th>We = 1.76K</th>
<th>We = 1.76K</th>
<th>We = 1.09K</th>
</tr>
</thead>
<tbody>
<tr>
<td>-4.62 -3.14</td>
<td>(0.5)</td>
<td>(0.5)</td>
<td>(0.5)</td>
<td>(0.5)</td>
</tr>
<tr>
<td>-3.14 -4.62</td>
<td>(0.30)</td>
<td>(0.30)</td>
<td>(0.30)</td>
<td>(0.30)</td>
</tr>
<tr>
<td>+0.44 +0.44</td>
<td>(0.5)</td>
<td>(0.5)</td>
<td>(0.5)</td>
<td>(0.5)</td>
</tr>
<tr>
<td>+0.26 +0.26</td>
<td>(0.30)</td>
<td>(0.30)</td>
<td>(0.30)</td>
<td>(0.30)</td>
</tr>
<tr>
<td>+0.16 +0.08((2.32)) +0.12 &amp; (-5.27) +0.09 -0.08 (-5.27)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[1.16 -2.31 -3.47^{1K} -5.37^{1K}\]
\[V_s = 0.88^{K} V_s = 0.88^{K} \]
\[0.07 +0.07 0.81^{K} 0.95^{K}\]

At 2.5' from 1:
- \( M = 0.95 \times 2.5 = 2.37 \)
- \( S = 48 \)
- \( 0.16 \times 1.25 = 0.20 \)
- \( M = -5.37 \)
- \( A_s = 5.37/15.8 = 0.34 \text{ si} \)

\( S = 27.4 \)
\( A_s = 3.20/7.9 = 0.405 \text{ si} \)

**Slab S1**
28' - span

**Design sheet B3C5**

- \#4 & \#4 at 8" oc mid strip
- \#3 & \#4 at 8" oc marg strip

**At \( \xi \) of col 1:**
- \( S = 48 \)
- \( 12" \text{ slab} : S_1 = 242 \)
- \( A_s = 5.37/15.8 = 0.34 \text{ si} \)

- \#3 top at 12" oc across SB2

- \#4 bt at 8" oc: \( A_s = 0.31 \)
- \#3 top at 12" oc: \( A_s = 0.11 \)
- 0.42 \text{ si}
Building # 3 - Scheme "C"

Assume b' = 60" D = 12" b = 36" projection each side
\( t/D = 6.5/12 = 0.54 \) \( b'/b = 32/60 = 0.53 \)
\( I_c = 1.5 \times 60 \times 12^2/12 = 13,000 \text{ in}^4 \)
I haunch = 132 \times 12^2/12 = 19,000
Equiv D = \( \sqrt[3]{12 \times 19,000} = 13.6" \)
r = \( 1.6/12 = 0.133 \)
\( a = b = 0.1 \)
From table 9-55: \( k_a = k_b = 4.58 \) \( C_a = C_b = 0.53 \) \( M_a = M_b = 0.087 \)
\( K_a = K_b = 4.58 \times 13,000/27.04 = 2200 \)

Ext cols 18" x 18": \( K = 4 \times 8700/10.5 = 3300 \)

Int cols 18" x 14": \( K = 4 \times 4120/10.5 = 1580 \)

S2: design sheet B3C2: \( k_a = k_b = 9.45 \) \( C_a = C_b = 0.70 \)
\( K_a = K_b = 11 \times 190 = 2100 \)

Span 1 - 2:
We = 0.72 \times 27.04 \times 28 \times 161 = 93 K
Partition = 27.04 \times 0.21 \times 181 = 57 K
Haunch = 27.04 \times 0.345 \times 181 = 93 K

\[ V = 0.29 + 27.04 \times 28 \times 161 = 35.4 \text{K} \]
\[ S_t = 1210 \]
\[ V_c = 50.5 \text{K} \]

\[ D = \begin{bmatrix} 0.25 & 0.30 & 0.28 \\ 0.13 & 0.16 & 0.20 \end{bmatrix} \]
\[ M = \begin{bmatrix} -254 & -254 & -20 \\ -38 & -38 & -47 \end{bmatrix} \]
\[ V_a = 54.1 + 4 = 55.4 \text{K} \]
\[ + M = 55.4 \times 13.8/2 - 231 = 151 \text{K} \]
S = 1350 6" tee reqd
\( A_s = 5.5 \times 176/18 = 9.7 \text{ si} \)
\( 5 \# 9 - 4 \# 10 \text{ bt: } A_s = 10.08 \text{ sl} \)
Reduced \( - M_0 = 55.4 \times 0.5 - 231 = -204 \text{K} \)
\[ S = 1815 \]
\[ 60 \times 12 \text{ SB; } S_t = 1210 \]
\( A_s = 8.8 \)
\( 5 \# 9 \)
\[ S_c = 620 \]
\[ A_s = 4.45 \]
\[ A_s = 8.9 \]
\[ A_s = 13.25 \]
\( 4 \# 10 \text{ bt: } A_s = 5.08 \)
\( 7 \# 10 \) top: \( A_s = 8.9 \)
\[ 13.98 \]
EXAMPLES OF BUILDING-FRAME DESIGN

Building #3 - Scheme 'C' * 

D Ld + 50% L Ld = 131 psf  
We = 66.5" k  
S1 = 60° bm + 72" Tee = 2400

By proportions:  
\[ M_1 = 185" k \]  
\[ + M \frac{\varphi}{\xi} = 121" k \]  
\[ - M_2 = 153" k \]

\[ f_c = \frac{121 \times 12,000}{2400} = 605 \text{ psi} \]

From fig 9-46 part 9-1:  
\[ E = 675 \times 3000 = 2,020,000 \]

\[ \text{Uniform load} \quad - \text{Mat 1} \quad - \text{Mat 2} \]

\[ \text{Constant I:} \Delta = +0.625 \text{ML}^3/48EI \quad -3 \text{ML}^3/48EI \quad -3 \text{ML}^3/48EI \]

\[ \text{Haunch at 1:} \quad -0.005 \quad +0.03 \quad 0.0 \]

\[ \text{Haunch at 2:} \quad -0.005 \quad 0.0 \quad +0.03 \quad +0.615 \text{ML}^3/48EI \quad -2.97 \text{ML}^3/48EI \quad -2.97 \text{ML}^3/48EI \]

\[ \Delta = \frac{0.615 \times 86,500 \times 324^3 - 2.97 \times 231 \times 12,000 \times 324^2 - 2.97 \times 192 \times 12,000 \times 324^2}{48 \times 2,020,000 \times 12,000} \]

\[ = 0.19'' \]

\[ \text{Max } \Delta = 27.04'' \times 12/600 = 0.54'' \]

\[ D \text{ Ld + L Ld = 161 psf} \]

\[ D \text{ Ld} = 108" k \]

\[ 2^2 2^2 \]

\[ D \text{ Ld + L Ld} = 161 \text{ psi} \]

\[ W = 27" k \]

\[ \text{D Ld} = 101 \text{ psf} \]

\[ \text{We} = 108" k \]

\[ \text{V} = 54 + 0.8 = 54.8" k \]

\[ 54 - 0.8 \times 53.2 = 18.7" k \]

\[ 12.7" k \]

\[ \text{At 2 - reduced } M_a = 204 + 53.2 \times 0.40 = 183" k \]

\[ A_s = 183/15.5 = 11.8 \text{ si} \]

\[ 4 # 10 \text{ bt: } A_s = 5.08 \]

\[ \text{Slab S2:} \]

\[ +M \frac{\varphi}{\xi} = 18.7 \times 8.25' = +154 \]

\[ -16.5 \times 4.12' = -68 \]

\[ -5 \times 6.25' = -6.2 \]

\[ -2.2 \times 0.5' = -1.1 \]

\[ M = -73.0 \]

\[ -38.5" k \]

\[ 38.5/11 = 3.5" k \]

\[ S = 31.2 \]

\[ 6 1/2" \text{ slab: } S_t = 60.5 \]
Building #3 - Scheme "C"

From design sheet B3C2

\[ C_b = 0.29 \quad C_{bl} = 0.21 \]

\[ V_A = 0.29 \times 27.04 \times 28.75 \times 161 \text{ lb} = 35.4 \]
\[ \text{Part: 14\" at 210 lb} = 2.8 \]
\[ \text{SB1: 14\" at 345 lb} = 4.7 \]
\[ 0.21 \times 27.04 \times 14\" \times 161 \text{ lb} = 12.8 \]
\[ 6.83 \times 14\" \times 181 \text{ lb} = 17.3 \]
\[ \text{Part: 14\" at 210 lb} = 2.9 \]
\[ \text{SB2: 14\" at 345 lb} = 4.9 \]
\[ \frac{V_A}{80.8^k} \]
\[ \frac{V_B}{37.9^k} \]

At "A" \( l_s \) for shear

\[ l_s = \frac{80,800}{90 \times 7.8 \times 10.75} = 96" \]
\[ 96" - 2 \times 24.75" = 46.5" \text{ min} \]

Shear on 18\" face of col

\[ V_s = 39.5 \times 10.75 \times 7.8 \times 90 \text{ lb} \]
\[ = 34.5^k \]

Shear on 14\" face of col

\[ V_s = \frac{80.8 - 34.5}{2} = 23.2^k \]
\[ V_c = 14\" \times 7.8 \times 10.75 \text{ at 90 lb} = 11.8^k \]
\[ \# \text{ 4 diag bars at 45°} \]
\[ f_v = \frac{2.8 \times 300 (10.75 - 1 - 4) + 10,000}{0.5} = 19,600 \text{ psi} \]
\[ A_v = \frac{11,400}{165 \times 19,600} = 0.352 \text{ si} \]

Use 2 \# 4
Building #3 - Scheme "C"

$w_e = \text{Slab S1: } 0.61 \times 13.52 \text{ at } 161 \text{ lb} = 1.33$

$M_e \text{ at col 2}: \frac{4.9}{27.04} = 0.18$

$\text{Slab S2 = 6.83 at 181 lb} = 1.23$

$S_e = 33/15.5 = 2.12 \text{ si}$

$M_e \text{ at col 2}: \left( \frac{4.31 - 3.27}{13.51} \right) = 0.075$

$\text{Partition: } = 0.210$

$\text{Slab band: 5' at 69 lb} = 0.345$

$\frac{5 \#4 \text{ and } 4 \#5 A_s = 2.20 \text{ si} }{3.370} = 4.371K$

$-M = 42 \times 12.5 / 12 = 43.71K$

$A_s = 43.7 / 15.5 = 2.82$

$8 \# 5 \text{ bt: } A_s = 2.4$

$2 \# 5 \text{ top: } A_s = 0.6$

From design sheet B3C8

Total $V = 37.9K$

Shear at 14" faces of col

$V = 37.9K = \frac{12.5}{14} = 34K$

Shear distributed to 14" face

$V = \left( \frac{34}{2} \right) \times 48.25 / 60 = 13.7K$

$V_c = 14" \times 7/8 \times 10.75" \times 90 \text{ lb} = 11.8K$

Provide 2 # 4 on corridor side of column

Section "B-B"
Building #3 - Scheme "C"

Design sheet B3C10

We = Metal covering
and windows: 10.5' at 15 lb = 0.158 K/l
6" block - 3.33' at 28 lb = 0.093
Slab: 0.61 x 13.52 at 16.1 lb = 1.35
Haunch: 1.75 at 69 lb = 0.12
Beam 18" x 18" = 0.34
2.061 K/l

We = 26.5 x 2.06 K = 54.5 K

+ M = 54.5 x 26.5/16 = 901K
- M = 54.5 x 26.5/12 = 1201K
A_s = 90/22.3 = 4.02 si
S = 1065
A_s
3.8 #8 - 2.8 #8 = 3.96 si
18" x 16" bm: S_f = 753
3.8 = 3.27 S_c = 366
1119
1.60
5.40

Shear: slab - 0.21 x 27.04 x 161 lb = 0.92 K/l
V = 13.25 x (2.06 - 0.43) = 21.6 K
2.8 #5 top - contin A_s = 1.2
4 #8 bt A_s = 3.16
Add 2.8 #7 top A_s = 1.2
5.56

Check for combined shear and torsion

Σ K design sheet B3C6
SB1 K = 2200
2-18" x 18" cols = 6600
Slab-k = 23 x 61 = 1400
Σ K = 10,200
At cols d = 8800/10,200 = 0.86

Design sheet B3C3 - 18" x 18" bm
k_t = 0.06 C_t = 0.208 k_t = 120 D_t = 0.66
m_1 = 0.86 x - 3.1 = 2.661K
m_2 = 2.66 x 0.66 x 1.751K
M_t = (2 x 1.75 + 2.6) x 26.5
12
13.4 x 12,000 = 133 psi
v = 0.208 x 18" x 18
0.21 x 78 x 15.5 = 88 psi
v = 21,600
18" x 131 / 2 = 8.9 lb
n = 18 (88 + 133 - 90)
2 x 133
11.3 oc

#3 14: S = 0.33 x 20,000
89 x 131 / 2 = 11.3 oc

Typical spandrel beam B1
Building #3 - Scheme "C"

Framing plan
(Typical class room bay 28' x 70')

Quantities

Concrete  52 cu yd
Reinforcement  8750 lb
Slab forms  1310 sf
Slab band forms  726 sf
Beam forms  196 sf
Building No. 4

Occupancy: Hospital, wing for private rooms. (See Fig. 9-84.)
Live load: 40 psf in private rooms, 80 psf in corridors.
Number of stories: Basement, 5 floors, and roof.
The requirements for design are the same as required for Building No. 2, the difference being in the width of the building, which is planned for small private rooms on one side of the corridor.
The system of framing used in Building No. 2, Scheme C, is adapted to the two-span frame, which is unsymmetrical and therefore creates a design which should be checked for sidesway.

Building No. 4, Scheme A: Two-span Frame with Two-way Flat Slab with 16-in.-Wide Masonry Fillers, Sidesway. The system of framing with a floor system of comparatively shallow depth is somewhat more flexible than other systems providing deep beams on the column lines and permits lateral movement. The frame is first analyzed for the three conditions of loading that produce maximum moments at the critical sections of the slab with a restraining force to prevent translation of the joints which is equal to the difference in column shears. The maximum translation toward the right occurs with live load on span 2-3 and dead load on span 1-2. Translation to the left with live load on span 1-2 only is small and could have been neglected in so far as the final moments are concerned.

For the assumption that all columns are the same size the shears are divided equally between the three columns and thus equal fixed-end moments at the floor slab. In the analysis for sidesway on design sheet B4A3 the fixed-end moments above the floor slab have been omitted and the joints have been balanced by doubling the fixed-end moments and carry-over moments from the floor below. This procedure is satisfactory only for approximate solutions where the shears are approximately equal for each story.

The moments from gravity loads are combined with those from sidesway on design sheet B4A4 and the subsequent design of the slabs on design sheets B4A5 and B4A6 is based on the maximum combined moments at the critical sections.
EXAMPLES OF BUILDING-FRAME DESIGN

Building #4 - Scheme "A"

Two-way flat slab - masonry fillers
45'-0" wide wing with private rooms each side of corridor

\[ f_c/n/f_s = 3000/10/20,000 \quad v = 90 \text{ psi} \quad u = 300 \text{ psi} \quad f_c = 1350 \text{ psi} \]

ACI code

Plan of typical bay

From design:
- Private rooms - 123 psf
- Corridors - 103 psf
- Corridor partitions
- Partitions concentrated on slab

Bent coef for same section as for typical wing of building #2 - scheme "C" design sheet B2C2

Span 1-2:
- \( a = 0.10 \)
- \( b = 0.20 \)
- \( r = 0.20 \)

Span 2-3:
- \( a = 0.20 \)
- \( b = 0.10 \)
- \( r = 0.20 \)

Dead loads

Bent coef

Framing for
typical bay
Building #4 - Scheme "A"

All columns 18" x 18"  \( I = 8750 \)  \( K = \frac{4 \times 8750}{10.33} = 3400 \)

Note: design of floor slabs similar to building #2 - scheme "C"

\[ V = 32 - \frac{3.6}{2} = 28.4 \]
\[ + M = 28.4 \times \frac{8.6}{2} - 79 = + 43^k \]
Building #4 - Scheme "A"

Design sheet B4A3

Max +M_{2-3}
Max -M at 3

Restraining force = 3.2^K divided equally between 3-18"x18" cols at each story
Assume M^F = 20^K per column

Analysis for sidesway

V = \frac{4.25 + 8.03 \times 2.72}{5.16} = 2.91

Cor factor = \frac{3.2}{2.9} = 1.1

Restraining force of 0.5 to left

-M_{1-2} = 0.156 \times 9.35 = 1.46^K
+M_{2-1} = 0.156 \times 9.83 = 1.53^K

-M_{2-3} = 0.156 \times 7.86 = 1.23^K
+M_{3-2} = 0.156 \times 6.0 = 0.94^K
\[ \Delta = \theta h + d_u + d_1 \]

\[ \theta = \frac{M_g}{12E\Sigma K_g} \]

\[ \Sigma K_g = 1910 \]

\[ \begin{align*}
2150 \\
1740 \\
1560 \\
7410/12 = 617
\end{align*} \]

\[ d = \frac{Mxh/2}{12E\Sigma K_c} \]

\[ \Sigma K_c = 3 \times 3400/12 = 825 \]

\[ \theta h = \frac{33.04 \times 12,000 \times 124}{12 \times 3,000,000 \times 617} = \frac{49,200}{22,200,000} = 0.00222 \]

\[ d_u = \frac{16.01 \times 12,000 \times 5.16 \times 12}{12 \times 3,000,000 \times 825} = \frac{990}{2,475,000} = 0.00040 \]

\[ d_1 = \frac{16.01 \times 12,000 \times 5.16 \times 12}{12 \times 3,000,000 \times 825} = \frac{990}{2,475,000} = 0.00040 \]

Allowable \( \Delta = \frac{10.33 \times 12}{600} = 0.208^\circ \)

Maximum combined moments
D Ld + L Ld and sidesway
Building #4 — Scheme "A"

\[ M = 43^{1K} \quad \text{(Divide col } M_0 \text{ into 6 ribs — use 7 col strip)} \]

Col strip: \[ M = 25.8^{1K} \quad \text{Mid strip: } + M = 17.2^{1K} \]

\[ S = 4.3 \times 12.000 / 1350 = 38.3 \quad A_s = 2.87 / 12.8 = 0.225 \text{ si} \]

\[ A_s = 4.3 / 12.8 = 0.34 \text{ si} \quad 5 \text{ ribs} \quad \# 3 \text{st} - \# 4 \text{ Bt} \]

7 ribs — \# 4 st — \# 4 Bt

\[ -M \text{ at } 1: \quad A = 14^{1K} \quad -M \text{ at critical section } = -80.4 + 28.4 \times 1.16 = -57^{1K} \]

Col strip: \[ M = 34^{1K} \quad \text{Mid strip: } - M = 23^{1K} \]

\[ A_s = 5.7 / 12.8 = 0.45 \text{ per rib} \quad A_s = 3.75 / 12.8 = 0.29 \text{ per rib} \]

\# 4 Bt — \# 5 T \quad \# 4 Bt — \# 3 T

\[ -M \text{ at } 2: \quad A = 14^{1K} \quad -M \text{ at critical section } = -205 + 50.6 \times 1.16 = -146^{1K} \]

Col strip: \[ M = -111 \text{ lb} \quad \text{Mid strip: } - M = -35^{1K} \]

\[ A_s = 14.5 / 12.8 = 1.13 \quad A_s = 5.85 / 12.8 = 0.46 \]

\# 5 and \# 4 Bt + 2 \# 5 top per rib

\[ M_R = -205 + 50.6 \times 5.8 - 2.15 \times 2.9 \]

\[ = +82^{1K} \quad \text{OK, on 7 ribs} \]

Long span = 20.33' \quad C = 1.5' \quad F = 1.15 - 1.5 / 20.3 = 1.076''

\[ M_0 = 0.09 \times 64 \times 20.33 \times 1.076 \left( 1 - \frac{2 \times 1.5}{3 \times 20.33} \right)^2 = 112^{1K} \]

Mid strip: \[ M = + M = 0.15 \times 112 = 16.8^{1K} \]

\[ A_s = \frac{2.8}{1.44 \times 8.25} = 0.236 \quad \text{use } \# 3 \text{st} - \# 4 \text{ Bt} \quad 2 \# 4 \text{ Bt at col strip} \]

Morg strip: \[ + M = 0.05 \times 112 = 5.6 \quad 2 \text{ ribs at } 2.8^{K} \]

\[ - M = 0.13 \times 112 = 14.7^{1K} \]

\[ A_s = 14.7 / 11.85 = 1.24 \text{ st} \quad \text{use } 4 \# 5 \]

Col strip at line 2: \[ W = \text{Max shears } = 86.61^{K} \]

\[ M_0 = 0.09 \times 86.61 \times 20.33 \times 1.076 \left( 1 - \frac{2 \times 3}{3 \times 20.33} \right)^2 = 151^{1K} \]
Building No. 2 (Alternate), Scheme C: Wind Load

A wind load of 20 psf has been applied to the building frame having a comparatively flexible floor-framing system consisting of a two-way ribbed slab with 16-in.-wide masonry fillers. It is probable that a stiffer floor-framing system would not have been overstressed by the wind load for a building of equivalent height-to-width ratios.

An assumed story moment approximately two times the moment of the wind shears has been applied to the stories immediately above and below the second-story columns under consideration with the frame fixed from horizontal translation. The assumed
story moments for each story are divided among the columns as fixed-end moments in the proportions of the $K/h^2$ values of the columns and the distributed moments corrected by proportion as shown on design sheet B2(Alt.)C2W. In computing the horizontal deflection of the frame for the second-story columns only the drift has been considered, as the cantilever deflection for a frame of this height is relatively negligible.

As the permissible unit stresses for combined loads may be 33 per cent greater than for gravity loads the moments for combined loading shown on design sheet B2(Alt.)C4W are divided by 1.33 and compared with the moments for gravity loads only. The procedure should be repeated for the third-story columns and above as necessary until there is no overstress in the floor-framing system.

Wind loads, building #2 (alternate) Scheme "C" - Wind

<table>
<thead>
<tr>
<th></th>
<th>1</th>
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<tbody>
<tr>
<td>2.1k</td>
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<td></td>
<td>Rf</td>
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<tr>
<td>6.3k</td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>10.5k</td>
<td></td>
<td></td>
<td></td>
<td>6th floor 0.33</td>
</tr>
<tr>
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<td>K=2020</td>
<td>K=5240</td>
<td>K=2020</td>
</tr>
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<td>K=3400</td>
<td>18&quot; x 18&quot;</td>
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<tr>
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<td>K=2020</td>
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</tr>
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</tr>
<tr>
<td>31.5k</td>
<td>K=1960</td>
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<td>K=5240</td>
<td>K=2020</td>
</tr>
<tr>
<td>33.6k</td>
<td>22&quot; x 22&quot;</td>
<td>K=7180</td>
<td>22&quot; x 22&quot;</td>
<td>20&quot; x 22&quot;</td>
</tr>
</tbody>
</table>

Design sheet B2(alt)C1W

Cross section with wind load and bent coef
(K values for slabs from design sheet B2C2)

18" x 18" cols: $K = 4 \times \frac{8750}{10.33} = 3400$
20" x 20" cols: $K = 4 \times \frac{13,300}{10.33} = 5180$
22" x 22" cols: $K = 4 \times \frac{18,540}{10.33} = 7180$
Wind loads, building #2: (alternate) scheme "C"

\[ V = 18.9 \times 0.39 \]
\[ + 49 \times 0.39 (0.13) \]
\[ - 17 \times 0.39 \]
\[ + 7.0 \]
\[ 4.73^k \]
\[ - 49 \times (-39) \]
\[ + 25 \times +14 \]
\[ - 17 \times -17 \]
\[ + 17.9 \times +229 \]
\[ 5.76^k \]
\[ + 25 \times +17 \]
\[ + 8 \times +4 \]
\[ + 3 \times +17 \]
\[ + 5 \times -4 \]
\[ + 6 \times -6 \]
\[ + 33 \times -9 \]
\[ + 32 \times +17 \]
\[ 478 \]
\[ 5.76^k \]
\[ + 697 \]
\[ 4.73^k \]
\[ - 49 \times (-44) \]
\[ + 21 \times -45.8 \]
\[ + 0.32 \times 0.18 \]
\[ - 23.9 \]
\[ V = 23.1 \times 0.32 \]
\[ + 60 \times 0.19 \]
\[ + 27 \times 0.10 \]
\[ + 16 \times +3 \]
\[ + 11 \times +17 \]
\[ + 8 \times +8 \]
\[ + 60 \times 0.07 \]
\[ + 33 \times -9 \]
\[ + 32 \times +17 \]
\[ 478 \]
\[ 5.76^k \]
\[ + 697 \]
\[ 4.73^k \]
\[ - 49 \times (-44) \]
\[ + 0.32 \times 0.18 \]
\[ - 23.9 \]
\[ V = 31.5 \times 0.42 \]
\[ + 68 \times 0.16 \]
\[ - 30 \times 0.0 \]
\[ + 9 \times 0.42 \]
\[ 6.6^k \]
\[ 0.42 \times 0.10 \]
\[ + 9 \times 0.0 \]
\[ - 14 \times -25 \]
\[ + 21 \times +19 \]
\[ + 4 \times +4 \]
\[ + 16 \]
Wind loads, building #2 (alternate) Scheme "c"

\[
\Delta = \frac{\theta_{avg}h + M_c h}{12E \Sigma K_c}
\]

\[
\theta = \frac{M_a}{12E \Sigma K_c}
\]

\[
\theta_2 = \frac{2(378+31.9+79.7)12,000}{12 \times 3,000,000 \times 18.444/12} = 0.000066
\]

\[
\theta_3 = \frac{2(39.8+33.9+69.7)12,000}{12 \times 3,000,000 \times 18.444/12} = 0.000062
\]

\[
\theta_{avg}h = 0.000064 \times 124 = 0.0079^\circ
\]

\[
M_c h = \frac{2(17.9+8+47.8+47.8)}{12 \times 3,000,000 \times 20.740/12} = 0.0058^\circ
\]

(Drift) \( \Delta = 0.0079^\circ + 0.0058^\circ = 0.0137^\circ \)

Allowable \( \Delta = 124/600 = 0.206^\circ \)

<table>
<thead>
<tr>
<th>2.1k</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>Roof</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.2k</td>
<td></td>
<td></td>
<td></td>
<td>7th floor</td>
<td></td>
</tr>
<tr>
<td>4.2k</td>
<td></td>
<td></td>
<td></td>
<td>6th floor</td>
<td></td>
</tr>
<tr>
<td>4.2k</td>
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<td>5th floor</td>
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</tr>
<tr>
<td>4.2k</td>
<td></td>
<td></td>
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<td>4th floor</td>
<td></td>
</tr>
<tr>
<td>4.2k</td>
<td></td>
<td></td>
<td></td>
<td>3rd floor</td>
<td></td>
</tr>
<tr>
<td>4.2k</td>
<td></td>
<td></td>
<td></td>
<td>2nd floor</td>
<td></td>
</tr>
</tbody>
</table>

\[
\Sigma M_o = 2.1 \times 56.81' = 119
\]

\[
2.1 \times 46.48' = 95
\]

\[
4.2 \times 36.15' = 152
\]

\[
4.2 \times 25.82' = 118
\]

\[
4.2 \times 15.49' = 65
\]

\[
4.2 \times 5.16' = 22
\]

\[
671'\text{K}
\]

All col 20' x 20'

\[
\Sigma d^2 = 2 \times 4.83^2 = 46.7
\]

\[
2 \times 24.29^2 = 1180.0
\]

\[
1226.7
\]

Cols 2 and 3: \( P = \frac{671 \times 4.83}{1226.7} = 2.64\text{K} \)

Cols 1 and 4: \( P = \frac{671 \times 24.29}{1226.7} = 13.3\text{K} \)
Wind loads, building #2—(alternate) Scheme "C"

Design sheet B2(alt)C4W

M_o due to gravity loads at 3rd floor

Combined M_o gravity and wind loads at 3rd floor

From R/C cols design sheet B2(alt)CIC, 2C: Cols 1 and 4: $P_g + P_w = 297.6 \, K$
Cols 2 and 3: $P_g = 394.2 \, K$

Cols 1 and 4: \( P_g + P_w = 297.6 + 13.3 = 310.9 \, K \)
\( M_g + M_w = 73.9 \, K \) e/f<1

Equiv load: \( \frac{310.9 + 2.86 \times 73.9}{1.33} = 394 \, K \)
20 x 20 col S 8 #11: \( P = 470 \, K \) OK

Cols 2 and 3: \( P_g + P_w = 394.2 + 2.6 = 396.8 \, K \)
\( M_g + M_w = 92.8 \, K \)

Equiv load: \( \frac{396.8 + 2.72 \times 92.8}{1.33} = 487 \, K \)
20 x 20 col S 10 #11: \( P = 494 \, K \) OK

Slab S1: at cols 1 and 4: \( 132.8/1.33 = 100 \, K \geq 83 \, K \) for scheme "C"
at crit sec: \( - M = 93 - 31.2 \times 1.08 = 60 \, K \)
col strip: \( - M = 0.66 \times 60 = 40 \, K \)
\( A_s = \frac{40 + 39.8}{1.33 \times 6 \times 12.6} = 0.79 \) sq ft #5 top - #6 top

At cols 2 and 3: \( 151.9/1.33 = 114 \, K \leq 122 \) for scheme "C" OK

Slab S2: \( 120.7/1.33 = 91 \, K \geq 61 \, K \) for scheme "C"
at crit sec: \( - M = 20.3 \times 1.17 - 51 = 34 \, K \)
col strip: \( - M = 0.76 \times 34 = 26 \, K \)
\( A_s = \frac{26 + 64.7}{1.33 \times 6 \times 12.6} = 0.95 \) sq ft #5 top - #3 bottom - #5 cont

At cols 2 and 3
Section 10

ESTIMATING

By

RICHARD D. MANGASARIAN, Associate Professor of Civil Engineering, Newark College of Engineering.

CONTENTS

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ESTIMATING

Estimating any construction costs, including those involved in the erection of reinforced-concrete structures, involves primarily a consideration of

1. The items that must be included
2. The items that may be neglected as unimportant
3. The development of a systematic routine

No attempt to assign prices and costs to the various individual material and labor units is made in this section. The cost of materials and labor varies considerably depending on the locale and the time. Therefore, textbook figures for prices are of little value to the estimator who must, above all, deal with immediate realities.

The calculation of the cost of constructing plain-concrete and reinforced-concrete structures is divided into two broad categories:

1. Rough estimating
2. Detailed estimate

ROUGH ESTIMATING

Rough estimating involves four subdivisions:

1. Square-feet-of-floor-area method
2. Cubic-feet-of-volume method

10-1
3. Average-component method
4. Use-unit method

These various systems of rough estimating are most applicable to such structures as warehouses, factories, stores, garages, schools, office buildings, and apartment houses. They are not of convenient use in estimating dams, bridges, piers, dry docks, or similar structures.

In method 1 mentioned above, the number of square feet of floor area is calculated and a unit cost per square foot is used as a multiplying factor. This unit cost may be for the entire structure including the basement and roof or only for those floors that are essentially similar. In the latter case, a separate unit price would be assigned to the nonconforming areas. The estimator, in deciding on what this unit price shall be, has several alternatives.

1. He may rely on the actual as-built cost figures for similar structures which his firm has constructed. This is the best method if the costs involved in these similar structures are for materials and labor in the same general vicinity and not too ancient in time.
2. He may use cost data published or announced for other like structures in the same vicinity. These are apt to be colored, misleading, or incomplete and should be studied with considerable care.
3. He may make an accurate detailed breakdown of one more or less typical area and apply a percentage figure to the rest of the structure.

The unit cost per square foot is the ratio of the whole cost to the area involved in calculating this total cost. Some publications such as the Engineering News-Record publish cost factors and recent percentage figures on price trends. These may be of substantial aid in adjusting the building costs of older structures to bring them in line with present-day prices. There are also handbooks or manuals published by manufacturers and by private individuals which have tabulations of material and labor costs. Where costs are changing rapidly such as during periods of inflation and deflation, such published data are quite apt to be in error because of the time lag between new editions. This material, however, is still of great value for checking and may help to uncover serious errors in pricing.

In the cubic-feet-of-volume method, the total outside volume of the structure is calculated and again a unit cost is derived. The same alternatives for deciding upon this unit cost are available as in the square-foot method. The unit cost per cubic foot is the ratio of the whole cost to the volume involved in calculating this total cost. As in the previous case, it may be necessary to assign different unit prices to different parts of the structure.

In the average-component method, sections of the work, such as panels or even entire bays, are used as the cost unit. The cost of such a section is calculated carefully and then this calculated cost is applied to every other similar section. In doing this, it is well to allow for economies resulting from the construction of repetitious work and the increased volume. It is equally essential to keep in mind that increased size may introduce other factors such as skips, hoists, scaffolding, and so on which were not inherent in the erection of the single small section. The average-component method is most adaptable to mill buildings, apartments, office buildings, warehouses, and the flat-slab type of construction.

The use-unit method is applicable to stadiums, theaters, apartment houses, and office buildings. In this type of structure, groups of seats, single rooms, or entire suites of rooms are repetitive. Therefore, it is possible to state that if a 40,000-seat stadium cost so much per seat then a 50,000-seat stadium will cost 20 per cent or 25 per cent more. Discretion must, however, be exercised. As indicated in the previous paragraph, a change in size does not necessarily affect the cost in a straight-line relationship.

It must be emphasized that all these rough-estimating methods are exactly that. Very often these estimates are made before detailed plans or specifications have been prepared. The greatest use of such rough estimates should be primarily as guide figures to be given to an owner or to an architect to help them decide whether a job
should be commenced. These rough estimates will also indicate to them the necessity of trimming certain items to fit the available money or show the possibility of including additional features which had previously been ruled out as being beyond their financial limit. Furthermore, a rough estimate will also help pick up any gross errors in a detailed estimate.

**DETAILED ESTIMATE**

Five major divisions comprise the detailed estimate of a reinforced-concrete structure. These are:

1. Forms
2. Concrete
3. Reinforcing steel
4. Finish
5. Administration

The first four of these are necessarily contingent on the accuracy and completeness of the quantity take-off. This take-off is made from the plans either by tallying dimensions (preferable) or by direct scaling. The most common mistakes to be avoided are the repetition of something previously included and the failure to multiply identical elements by the correct total number of such elements. The plans and specifications must be carefully studied so that the estimator is thoroughly familiar with the job. At all times, the estimator must keep a mental image of the completed structure before him so that he may visualize framing connections, study reuses of material, and make routing plans, all to the end of maximum economy. In the case of a discrepancy between the specifications and the plans, the former are the controlling item.

It is wise to follow a systematic procedure both in the take-off and in the subsequent work also. It will be of advantage to work with ruled sheets and to make entries with each item properly identified so that changes of addition or deletion may be easily accomplished at a later date. Checking for completeness will also be facilitated. All dimensions should be listed in a uniform sequence such as length, width, and height in all entries so that no time is devoted to attempted recognition of what certain figures mean. There are probably as many setup sheets for estimating as there are engineering offices. Any form that will permit easily understood entries is satisfactory (Table 10-1). Numbers are reserved for major divisions and the arabic letters for subdivisions in these categories.

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<thead>
<tr>
<th>Item No.</th>
<th>Item</th>
<th>Dimensions</th>
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<th>Total cost in dollars</th>
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<td>10.00</td>
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</table>

**FORMWORK**

Formwork is usually estimated by assigning a unit price per square foot of contact area of forms against concrete, abbreviated SFCA. The unit price will include not
only the actual cost of purchasing the material and of making the forms but also the cost of erection, bracing, staging, nails, bolts, ties, wires, and oiling. The unit price may be such that only the material is involved and the erection is covered separately, or there may be one single price for the square foot of form surface in place and stripped. In arriving at this figure, it must be kept in mind that reuse of forms is feasible in most cases. In the case of wooden forms, the actual amount of reusable lumber is a varying quantity depending upon the skill with which the forms are made, the care with which they are stripped, and especially the magnitude of the job and the routing schedule. In any case, the possibility of salvage after the final pour should never be ignored. A good proportion of the lumber may still be in good shape and will have a marketable value.

Forms will quite often represent at least a quarter of the cost of a concrete structure. Considerable thought, therefore, should be given to them. For foundations, for floors resting on earth, and for substructure walls and footings, the earth itself if sufficiently firm may be used as a side form. If wood forms are necessary, rough lumber is satisfactory for this purpose. One-inch-thick lumber, cleated together and held by 2 by 4s or 2 by 6s, is the most commonly used material. Other parts of the structure where the finished surface will be exposed will require dressed lumber. There is a definite relationship among the care used in constructing forms, the condition of the lumber, and the cost of rubbing and finishing. It is a frequently exercised and ridiculous practice to save money on formwork which is expended on rubbing costs.

Most forms are of wood in one form or another. There is, however, an increasing tendency toward the use of steel and aluminum. The greatest advantage in so doing is in structures with repeating similar shapes. In such cases, the metal form, in spite of its initial higher cost, is cheaper than wood. Metal will also give a better finished surface except at the section joints. Where wood is used, only partially seasoned lumber is preferable since fully dried lumber will absorb too much water and swell. Available equipment should be kept constantly in mind. Cranes, derricks, and similar lifting devices are usually required to place, strip, and move forms, particularly if steel forms are to be used.

Beams and girders are usually estimated by calculating the perimeter of the projection below the slab, if the part above has been included in the slab. This is the best and easiest method. Stairs are taken as the total area of the undersides plus that of the ends and also of the risers. Roofs are taken as the total area in square feet remembering that, where the slope of the roof exceeds 25° with the horizontal, it will be necessary to place forms on the upper side. Window sills, copings, cornices, and moldings are measured in total number of linear feet. Other items are taken as total area of wet concrete next to forms.

It is axiomatic from a financial viewpoint that the design of the structure and necessarily of the forms should be such as to permit reuses of full sections with a minimum of alterations. At the present general rate of approximately 5 cents a minute for a single carpenter's time, it is extremely desirable that the designer keep constantly alert to the possibility of using regular sizes. He must always avoid intricate details and designs which save concrete and steel and waste valuable craftsman time. On a recently constructed reinforced-concrete multistory housing development, it was found to be cheaper to maintain most member sizes the same on all stories than to reduce them in the upper floors.

In assigning some figure to reuse of forms, unless the estimator has previous performance records for similar structures and similar crews to rely upon, a conservative percentage is to assume that only three-quarters of the lumber in any given form may be used again. The author, however, knows of cases of panel forms being reused twelve and more times. The salvage value of form plywood may often be taken as 95 per cent. Plywood also has the advantage of lower labor costs and a better final finish. The initial cost, however, is higher. There is again a definite direct relationship between the cost of constructing, placing, and stripping of forms on the one hand and the number of uses possible. Any increase in the former is reflected in an increase in the latter.

In calculating reuse, it is well to keep in mind the time limitations and the variations
in the stripping sequence. Column forms should remain on at least 2 days in favorable weather and 4 to 6 days in winter. Beam sides and slab and roof forms should not be stripped for at least 4 or 5 days in summer and not for 10 days or longer in winter. Girder sides should remain a little longer than this. Beam and girder bottoms should remain at least twice as long as the sides and should be adequately braced and shored during this time. It is wise to strip the girder bottoms at a later date than the beam bottoms. The time it takes for concrete to set up and harden is very often an essential and vital factor in estimating the cost of building reinforced-concrete structures. The time lag between pouring and stripping can be an expensive cost factor unless the crews involved can be productively employed elsewhere on the job. Four concrete structures on one job can be constructed considerably more cheaply than four times the cost of one structure. If one day's concreting is planned on each structure, the same crews can be moved successively around the job. By the end of the week, the first work is now ready to be stripped, re-formed, and another pour made.

From the estimator's viewpoint, the actual take-off of quantities will entail a calculation of the number of square feet of contact surface, with a breakdown of quantities so tabulated that there is a definite cognizance of reuse considerations and a realization of scheduling economies. Then the bracing, staging, and shoring must be considered. On jobs of considerable magnitude, it is increasingly the practice to submit designed forms. Where this is done, it becomes only necessary to continue the take-off of quantities to include the forms. On smaller jobs, where the design of the forms is left to the carpenter foreman or to the job superintendent, most estimators will find it necessary to rely upon their experience with previous jobs and assign a value of from one to seven times as many board feet as the contact area, to take care of bracing and staging. For most concrete buildings of 11-ft story height, a figure of four times the actual contact area will give the quantity of lumber for forms, bracing, and staging. For each 1-ft increase in story height the lumber quantity should be increased by about 11 per cent. Many manufacturers of clamps and ties have catalogued tables which are extremely helpful in this connection. If no similar jobs have occurred within the experience of the estimator, a rough unit approximation may be acquired by analyzing a typical section or even an entire bay.

If vibration is specified, the forms will be designed to take greater pressures and strains than ordinarily. More care must also be taken to prevent leakage. The cost of the forms will be higher. Some of this extra outlay is offset naturally by savings on cement, by the lesser need of other means of compaction, and of course, by the savings on rubbing and finishing.

Forms of a different nature, such as walls, columns, floor slabs, beams, and footings, should be tabulated separately so that proper consideration may be given to the different manners of shoring and bracing. On certain types of overhead structures falsework may also be desirable.

Openings of less than 25 sq ft cost more in labor time to frame than the saving in formwork will amount to, and it is customary not to deduct for these small areas. Furthermore, in order to facilitate reuse elsewhere and also for added strength, it is desirable to construct the forms in unbroken panels. Any openings are provided for by using extra forms which are left in place for some time after the forms are removed. Similarly, no allowances are made for fillets, bevels, chamfering, and like dressings as being an unwarranted refinement of estimating. Other ornamental work, caps and bases of columns, haunches, and multisided columns, however, should be included and tallied separately as to unit price per square foot. All curved work should be recorded independently of other types. All moldings are measured by the linear foot.

Construction joints and the cost of placing them are ignored, normally, in building-construction estimating. In large concrete masses such as dams, piers, and retaining walls, they should be included and for estimating purposes measured by the linear foot.

Items that are easily overlooked unless some sort of check-off list is employed are nails, bolts, wires, and oiling of the forms. An average figure of 12 lb per 100 sq ft of forms will take care of each of the first three. About 1⁄2 gal of oil per 100 sq ft of forms seems reasonable for the last.

The labor-hours required for erecting formwork is an extremely variable quantity
and considerable difficulty is encountered in deciding on specific figures. The best manner of evaluating labor output is to have definite data on previous similar jobs with the same crews and foremen. Where this is impossible, the figures in Table 10-2 are reasonable approximations.

**Table 10-2. Approximate Labor-hours Required to Build, Erect, and Strip 100 SFCA* for Wooden Forms**

- Footings, piers, foundation walls: 10
- Columns: 15
- Beams and girders: 15
- Slabs (beam-and-girder construction): 8
- Slabs (flat-slab type) including caps: 10
- Ribbed slab or pan floor: 9
- Stairs (of a reasonable height and type): 14

* Square feet of contact area.

The figures in Table 10-2 assume a more or less typical number of openings for sleeves, conduits, and similar inserts for the plumbing, heating, and utility lines. Additional labor time will be required for special installations. Similarly, on some structures the placing of anchor bolts, which is usually done by carpenters, may be tedious work and require an inordinate amount of craftsman time.

Plant costs are dependent wholly on the magnitude of the job. On large jobs, considerable savings can be effected by setting up flow-of-material charts for routing supplies so that a continuous sequence occurs from stockpile to cutoff to rip to bench and then to temporary storage or directly to the structure. On smaller jobs, the entire plant may consist of only benches and hand power saws. The variation in plant costs per 100 sq ft of forms will therefore be considerable.

**CONCRETE**

Estimating the concrete will be most conveniently done by dividing the work into various parts such as concrete foundations, concrete arches, and columns. In addition, it is well to list the different items in each division in the approximate order in which they will be constructed. For instance, one might have various subheadings such as Dry Concrete, Tremie Concrete, Footings (Wall), Footings (Column), Walls, or Machinery Foundations.

Since a good many of the dimensions on the take-off of quantities will be the same for both the formwork and the concrete it may be convenient to set up both sheets at one time and carry on the entries simultaneously.

Each different mixture of concrete will have a different cost per cubic yard, making it desirable to keep each strength under a separate heading.

Concrete is usually estimated by the cubic yard although some prefer the cubic foot. Normally, no deductions are made for steel reinforcing, steel beams, holes, or inserts unless the cross-sectional area of the material to be deducted is a substantial figure.

Floor and roof arches are computed by tallying up the number of square feet and then applying a price per square foot. In this price, the cost of the forms, wire mesh, and cinder or other concrete is included. In the metal-pan or hollow-block type of ribbed slab, the simplest method is to figure the volume as solid and then deduct the volume of the blocks or pans. Manufacturers of wire mesh, pans, and blocks publish tables that are extremely helpful in this respect.

The take-off of the concrete having been completed, it now becomes necessary to assign a cost figure. If the specifications call for a certain strength of concrete and ready-mix or transit-mix of this strength is available, the question of cost per cubic yard may be determined by requesting bids from the various supply sources. If the job is large enough to warrant the contractor’s own batching and mixing plant, then the cost per cubic yard must be estimated by the use of tables in which the quantities of the various ingredients required to make a cubic yard of concrete are given.

It is important to remember that concrete requires a large amount of water, usually in excess of 30 gal per cu yd. Therefore, it becomes a must item to investigate the
source of water and the cost of bringing it to the job site. The possibility of having a meter cost should be checked. In any case, all necessary piping must be included in the cost summary, and if cold weather is a probability, the cost of protecting the pipes from freezing must be considered.

The usual specification concerning curing requires attention. This may or may not be a significant item. In any case, it should not be automatically ignored without study of its cost. A check list will be of considerable aid in reminding one of items like this and also that of protecting the concrete from freezing.

Winter construction will generally raise costs at least 5% per cent. Not the smallest item here is the protection of concrete. When the temperature falls below 39°F, the danger is that the chemical action desired will not occur and that the cement will remain inert. Certain methods of preventing this must be included in the cost study. The most common of these practices are heating the aggregate (not the cement) and the water such that the resulting concrete will have a temperature of not more than 100°F; placing salamanders or releasing live steam under canvas or other protective covering; spreading straw, earth, manure, tarpaulins, or tar paper; or adding an anti-freeze mixture such as calcium chloride (not alcohol or sodium chloride). Some or all of the above may be required on any job, depending on the temperatures encountered.

In hot weather, the problem is that of preventing excessive drying out, rather than retarding actual heat action which in itself is not objectionable. Keeping the surface continually moist and also covering it with canvas, paper, or burlap is effective.

The method of delivery of the concrete from the mixer to the forms will materially affect the cost per cubic yard. The wheelbarrow system is efficient only on very small jobs. Probably the most common procedure is the use of concrete buggies or carts. These carts, handling about a quarter yard of concrete, will require two lines of runways with turnouts for passing. The runways should preferably be level and certainly not upgrade from the mixer. Most of this lumber permits considerable reuse and has a high salvage value. Hoists for off-the-ground work are necessary. Other methods such as long chutes, suspension cable buckets, cars on tracks, conveyor belts, crane-lifted buckets, or pneumatic pumping all introduce individual problems of cost. In general, every concrete job presents different problems and a universal in-place price on concrete, except on small jobs, is not an easy figure to develop.

Since almost all specifications call for some sort of testing to establish the uniformity of quality and workmanship, some consideration may have to be given to the cost of these tests. The routine tests made from time to time do not involve very much in the way of money and may be considered to be absorbed in overhead or contingencies. It is the load tests which may be required when conditions arise such as to cast doubt on the strength of the structure that absorb time and profits. Such tests cannot be predicted by the estimator except in so far as past performance and the experience records of the crews and supervision involved may permit forecasts of this sort.

Plant costs cover a very large field. The variations may be from a small mixer, runways, barrows, and hand shovels to an extensive layout comprising a large mixer, carts, runways, chutes, hoists, cranes, spouts, pneumatic apparatus, etc. In addition, weighing equipment, testing equipment, water lines, and curing and heating devices as described may be necessary. In general, plant selection and layout should be such as to cut down as much as possible on labor costs. The ideal to be aimed at is a minimum labor payroll. The mixer, the aggregate piles, the cement, and the pour area should all be as close together as is physically possible without excessively penalizing progress of other parts of the job.

The final cost of the concrete per cubic yard must include the cost of the sand, cement, coarse aggregate, and water, all delivered on the job; the cost of stockpiling the aggregates and cement; the cost of protecting the cement from the weather; the costs of temporary runways and hoists including lumber, installation, shifting, removing, maintenance, and repairs; the cost of mixing and placing the concrete (which should always include the time consumed in cleaning, the mixer, the placing vehicles, and the runways after each pour); and finally the cost of curing and protecting the concrete from freezing.
ESTIMATING

REINFORCING STEEL

Reinforcing steel is priced per pound or ton of steel. The take-off sheet must show the total number of linear feet of each bar size and its type, description, and location in the structure. It must clearly distinguish among straight and bent bars and spirals.

Table 10-3. Standard Sizes of Reinforcing Rods

<table>
<thead>
<tr>
<th>Size by number</th>
<th>Area, sq in.</th>
<th>Weight, lb/ft</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.05</td>
<td>0.167</td>
</tr>
<tr>
<td>3</td>
<td>0.11</td>
<td>0.376</td>
</tr>
<tr>
<td>4</td>
<td>0.20</td>
<td>0.668</td>
</tr>
<tr>
<td>5</td>
<td>0.31</td>
<td>1.043</td>
</tr>
<tr>
<td>6</td>
<td>0.44</td>
<td>1.502</td>
</tr>
<tr>
<td>7</td>
<td>0.60</td>
<td>2.044</td>
</tr>
<tr>
<td>8</td>
<td>0.79</td>
<td>2.670</td>
</tr>
<tr>
<td>9</td>
<td>1.00</td>
<td>3.400</td>
</tr>
<tr>
<td>10</td>
<td>1.27</td>
<td>4.303</td>
</tr>
<tr>
<td>11</td>
<td>1.56</td>
<td>5.313</td>
</tr>
</tbody>
</table>

Ordinarily, intermediate-grade billet steel and rail steel are the common stock types for rods and hot-rolled intermediate grade for spirals. Others are available but usually cost more. Sixty feet is the maximum length of all sizes above No. 2. This smaller size, because of the difficulty of handling this light and very flexible steel, is seldom above 20 ft long. Longer lengths of all sizes are possible but shipping costs increase the price considerably.

Reinforcing steel is sold at a base price for the No. 8 or larger straight bars with an increasing extra charge as the size decreases. In addition, extra charges are made for tonnage ordered, for bending, for designing, and for detailing. These charges also vary with the weight, being greater for the smaller sizes.

For rapidity and simplicity, a good many estimators list together the No. 3 and No. 4, the No. 5, No. 6, and No. 7, the No. 8 and No. 9, and finally the No. 10 and No. 11. The No. 2 size, which has an extremely high extra charge, is usually listed separately. The accessories such as chairs, ties, turnbuckles, clamps, and wires are usually and most simply taken as a percentage. A reasonable assumption is to add 10 per cent to all steel No. 9 or less and 7 per cent to all steel of greater size.

In arriving at a price for the steel, it is essential to keep in mind the freight costs of shipping. The manner of bundling, lengths that can be successfully managed in transit, and particularly the order of use (so as to keep storage costs down) will all affect the final price. Reinforcing steel rusts rapidly in stockpiles, and the cost of cleaning may become a substantial item if many bundles of rods have been delivered to the job long before the time they are to be used.

If the design is complete and all details of bends, dimensions, and hooks are shown, then estimates are made on that basis. Otherwise the following rules from the Building Regulations for Reinforced Concrete of the American Concrete Institute will be helpful.

a) Hooks on stirrups are to be not less than three inches.

b) Hooks on column ties are the same as on stirrups.

c) Positive moment reinforcing in beams, joists and slabs is to extend at least ten diameters into the support.

d) Negative moment reinforcing in continuous construction is to extend into the adjacent span to a point one quarter of the center to center span length plus six diameters beyond the center of support. On noncontinuous ends, truss bars are to extend to within three inches of the outer faces of members.

e) Where lapped splices in column verticals are used, the minimum amount of lap for deformed bars shall be:

1) Concrete of 3000 psi or above and intermediate grade steel, 24 diameters and for hard grade steel 30 diameters.
2) Concrete of less than 3000 psi, the lap shall be one third greater than those above. Where plain bars are used, the minimum lap shall be twenty five percent greater than that for deformed bars. No laps shall be less than eighteen inches.

3) Dowels shall be at least 36 inches but not less than twice the lengths specified above for laps in column verticals.

f) In general, a standard hook may be estimated as being fifteen bar diameters beyond the straight portion.

g) Where stirrups are called for but not specified, they shall be 3/8" rounds and at least 12% of the total weight of the longitudinal steel.

h) Spiral spacers on columns less than 24" core diameters may be estimated as 1.5 pounds per foot of height. Above 24" diameter, estimate spacers as 2.25 pounds per foot of height.

i) Where temperature reinforcement is called for and not shown, figure 0.002bd for floor slabs and 0.0025bd for roof slabs. These will be No. 2 rods spaced twelve inches on center. Where lapped, the lap shall be eighteen inches.

Wire mesh (welded wire fabric) or expanded metal is sold by the roll or sheet and the take-off is by the area in square feet. This item is tabulated by size of mesh and weight per 100 sq ft or by mill number. The quoted price is usually per 100 sq ft of fabric. Ordinarily, the sheets are limited to 16 ft in length because of shipping limitations. Rolled mesh is supplied in full rolls and is generally about 750 sq ft in size. Common practice is to place the fabric so that the lap is not less than the spacing of wires parallel to the lap. For estimating purposes, a conservative value which will cover both side and end laps is to allow for an excess of 10 per cent.

If, as is often the case, all the steel is to be fabricated by a commercial shop, it becomes a simple matter to arrive at the cost of the reinforcing steel by checking with the particular shop involved as to their charges. Labor costs will vary considerably according to locale. This is due primarily to different union regulations regarding cutting, bending, and placing of rods. Where bending is done on the job, the price will also depend on whether hand benders or power benders are used. Spiral bending is seldom done by an individual contractor except on extremely large jobs where there will be enough work to warrant the outlay for this specialized equipment. Specifications on placing of steel will also vary. Some will require that rods be tied at every intersection and others that enough ties be placed so as to keep the rods in position. If previous job records are not available for labor output, Table 10-4 may be of some help.

**Table 10-4. Labor-hours per Ton of Steel**

<table>
<thead>
<tr>
<th>Size, No.</th>
<th>Hr</th>
</tr>
</thead>
<tbody>
<tr>
<td>2, 3, 4</td>
<td>15</td>
</tr>
<tr>
<td>5, 6, 7</td>
<td>8</td>
</tr>
<tr>
<td>8, 9</td>
<td>6</td>
</tr>
<tr>
<td>10, 11</td>
<td>4</td>
</tr>
</tbody>
</table>

For rough-estimating purposes and as a rapid check on his detailed take-off, the estimator may find Table 10-5 of some value. A general range is shown since rein-

<table>
<thead>
<tr>
<th>Type of member</th>
<th>Lb of reinforcing steel/ cu yd of concrete</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rectangular beams or girders (no slabs included)</td>
<td>220-270</td>
</tr>
<tr>
<td>Slabs (steel-skeleton buildings), end spana</td>
<td>165-175</td>
</tr>
<tr>
<td>Slabs (steel-skeleton buildings), interior spana</td>
<td>165-175</td>
</tr>
<tr>
<td>T-beam construction beams, girders, slabs included</td>
<td>140-180</td>
</tr>
<tr>
<td>Columna, tied (p_a = 0.01)</td>
<td>150-190</td>
</tr>
<tr>
<td>Columna, tied (p_a = 0.04)</td>
<td>330-600</td>
</tr>
<tr>
<td>Columna, spiral (p_a = 0.01)</td>
<td>220-320</td>
</tr>
<tr>
<td>Columna, spiral (p_a = 0.04)</td>
<td>420-680</td>
</tr>
<tr>
<td>Columna, spiral (p_a = 0.08)</td>
<td>600-1,200</td>
</tr>
<tr>
<td>Concrete joists, pan construction, etc.</td>
<td>135-190</td>
</tr>
<tr>
<td>Flat slab</td>
<td>60-150</td>
</tr>
<tr>
<td>Footings (soil at 1 T/sq ft)</td>
<td>40-80</td>
</tr>
<tr>
<td>Footings (soil at 2 T/sq ft)</td>
<td>70-110</td>
</tr>
<tr>
<td>Footings (soil at 3 T/sq ft)</td>
<td>100-130</td>
</tr>
</tbody>
</table>
forcing quantities vary considerably with load and such variation is not necessarily a straight-line proportion of concrete to steel. This is particularly evident in column design and in beams reinforced for compression.

Table 10-6 showing unit prices per cubic yard of concrete in place includes the cost of removing forms. This range may be of some value in making rough estimates or checks on detailed estimates. Because of the diversity of concrete jobs, it is not possible to narrow the range shown. A median figure in the range shown, however, will be reasonably accurate for typical structures in accessible areas.

**Table 10-6. Costs**

<table>
<thead>
<tr>
<th>Description</th>
<th>Price per cubic yard of reinforced concrete in place with the forms, bracing, etc., removed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Residential buildings (over 6 floors)</td>
<td>$68–$102</td>
</tr>
<tr>
<td>Office buildings (less than 4 stories)</td>
<td>$56–$84</td>
</tr>
<tr>
<td>School buildings (not more than 3 stories)</td>
<td>$60–$92</td>
</tr>
<tr>
<td>Warehouses and garage types</td>
<td>$63–$95</td>
</tr>
<tr>
<td>Factory-type structures</td>
<td>$62–$96</td>
</tr>
</tbody>
</table>

Table 10-7 shows price ranges for certain materials as of a 1954 average. Of the three columns of prices shown, the first (low) and third (high) are a nationwide range with the second column of prices an average for the metropolitan New York area.

**Table 10-7. Costs**

<table>
<thead>
<tr>
<th>Description</th>
<th>Nationwide low</th>
<th>N.Y. metropolitan area</th>
<th>Nationwide high</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 × 6, No. 1C YP-D-48 per MBM</td>
<td>$75.00</td>
<td>$109.00</td>
<td>$115.00</td>
</tr>
<tr>
<td>2 × 4, 8 ft 0 in., to 16 ft 0 in. No. 1C fir per MBM</td>
<td>132.00</td>
<td>135.00</td>
<td>157.00</td>
</tr>
<tr>
<td>2 × 6, 12 ft 0 in. to 16 ft 0 in. No. 1C fir per MBM</td>
<td>132.00</td>
<td>135.00</td>
<td>157.00</td>
</tr>
<tr>
<td>4 × 8 × 5/8 concrete form plywood per MBM</td>
<td>254.00</td>
<td>270.00</td>
<td>304.00</td>
</tr>
<tr>
<td>Cement, per bag</td>
<td>1.27</td>
<td>1.46</td>
<td>1.72</td>
</tr>
<tr>
<td>Sand, per cu yd</td>
<td>3.50</td>
<td>4.00</td>
<td>7.40</td>
</tr>
<tr>
<td>3/4-in. gravel, per cu yd</td>
<td>3.43</td>
<td>4.75</td>
<td>7.10</td>
</tr>
<tr>
<td>5/8-in. stone, per cu yd</td>
<td>3.50</td>
<td>4.50</td>
<td>6.45</td>
</tr>
<tr>
<td>Transit-mix concrete 50 cu yd or more, per cu yd:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1:2:3</td>
<td>11.75</td>
<td>16.50</td>
<td>17.00</td>
</tr>
<tr>
<td>1:2:4</td>
<td>10.95</td>
<td>15.40</td>
<td>15.75</td>
</tr>
<tr>
<td>1:2:5:3</td>
<td>11.25</td>
<td>15.85</td>
<td>16.20</td>
</tr>
<tr>
<td>1:2:3:5</td>
<td>11.25</td>
<td>15.85</td>
<td>16.20</td>
</tr>
<tr>
<td>1:2:5:4</td>
<td>10.90</td>
<td>15.30</td>
<td>15.70</td>
</tr>
<tr>
<td>Reinforcing steel No. 8 or larger 5 tons or more, per 100 lb, bending charges extra.</td>
<td>7.70</td>
<td>10.15</td>
<td>11.35</td>
</tr>
</tbody>
</table>

Other items to consider are the costs of storage on the job, repiling, wire brushing, and cleaning in the case of steel which has been exposed to the weather for a long time, and finally the testing of selected samples to ensure correct supplying by the mill or jobber.

Plant costs are fairly small and include cutting shears, hand or power benders if the bending is to be done on the job, and storage facilities. Usually no protection sheds are required. If the cutting and bending are to be done off the site, only hand tools for placing and tying are involved.

**CEMENT FINISH**

Certain types of finish such as washing, brushing, and rubbing require little material and involve only an estimate of the labor time involved. Normally, since there is a direct ratio between the care with which the forms are constructed and the concrete puddled, spaded, or vibrated, and the surface appearance after stripping the forms, no allowance is made for going over the concrete to remove fins and patch up voids or
pockets. This is taken care of in assigning a labor unit price to the placing of the concrete. Other types of finishes, such as tooling, bushing, and sandblasting, have a fairly large equipment outlay to be added to the labor-time analysis.

Finishes that require colored aggregate or colored mortars can be estimated by including the added expenditures either in the cost of the concrete itself or in the cost of the mortar finish.

For wearing-surface finishes, such as on floors, the specifications will normally state the mix and the method of treatment. Floor areas to be finished are usually estimated in square feet, and the estimator should tally each thickness and each mix separately.

The most common types of finish are the monolithic and the bonded. In the former, sometimes called an integral finish, a wearing layer is applied immediately after the slab is poured. This may be anything from a sprinkling of cement up to 2 in. of mortar (usually a 1:1 mix of cement and sand). In the bonded finish, a coating of mortar 1 in to 2 in. thick is applied at a later period after the concrete has set.

A third type, which is really not a finish coat at all, occurs where the wet concrete is floated and troweled smooth. This is commonly done on walks, roads, and other outside pavements where no special top coating is warranted.

The take-off of quantities and calculation of costs involve a knowledge of how much sand, cement, and water will be required for each of the cases set out above. In addition, proper consideration must be given to whether abrasive surfaces such as granite or marble chips are to be included, and whether accelerators such as calcium chloride will be desirable to speed up the hardening process.

The labor costs involved in the monolithic finish are complicated by the time issue. This finish should not be placed until all excess water on top of the concrete has either evaporated or soaked in. The time required for this, being a function of the humidity, temperature, and original condition of saturation, may vary from an hour to half a working day. Monolithic finishing may therefore either control the time when pouring must cease or become a permanent overtime feature. This problem does not arise with the bonded or separate finish. Most engineers, however, agree that the integral finish is the more satisfactory from the viewpoint of longer service and durability with lower maintenance costs.

Where no more than floating, screeding, or troweling smooth is to be done, most estimators either ignore this cost or assign a very small percentage of the mason’s time to it.

**ADMINISTRATION**

Some of the items in this category have rightfully been mentioned in connection with other matters and are repeated here for completeness of continuity.

In this section, the estimator must consider the expenses of surveying, of printing, of progress photographs, and of testing, inspection, and quality control. The various permits for street blocking, water, parking, and other special privileges are included here. Workmen’s compensation insurance rates vary substantially among states and are also dependent on past accident records. The base rate, naturally, changes often and is quoted as a percentage of the labor cost. Federal Social Security tax is also based on the labor payroll and at the present time is 3 per cent of the employee’s gross earnings. This may be expected to increase in the future. A similar state tax is charged in some areas. Most states also have an unemployment compensation tax. The rate on this varies, with 3 per cent being about the norm. Individual contractors may take out various other types of insurance such as fire, public liability, employers’ liability, and theft.

The following items should not be overlooked:

- Temporary offices with their appurtenances such as telephones, lights, heat, water, toilets, stationery, drafting tables, and supplies
- Sidewalk sheds and fences for protection
- Repair of damage to adjacent property, to walks and streets
- Temporary roads, ramps, and storage facilities
- Timekeepers, material clerks, and watchmen
Charges on loans, bid and performance bonds
Travel and legal expenses
Repairs, maintenance, and depreciation of equipment

To all these costs which represent job overhead it is necessary to add the general office overhead or a portion of it if more than one job is operating.

The common custom in calculating profit is to take a percentage of the total cost. This percentage will fluctuate depending on the method of payment, the contractor's desire for the work, anticipated difficulties, and sometimes a guess as to what the traffic will bear.

The following check-off list cannot be considered complete for all concrete jobs, but it may have some use as a reminder of items to be accounted for:

- Aggregates, coarse
- Aggregates, fine
- Agitators
- Anchors
- Backfill
- Bar spacers
- Bars, reinforcing
- Beams
- Beam forms
- Bending, steel
- Blocks, concrete
- Bolsters
- Bolts, anchor
- Bracing
- Bulkheads
- Bulkhead forms
- Bushhammer work
- Caissons
- Cement
- Cement finish
- Cement-gun work
- Cinder concrete
- Cinder fill
- Cleaning of site
- Cleaning of job
- Cleaning of forms
- Cleaning of steel
- Column caps
- Column forms
- Compensation insurance
- Concrete
- Conduits
- Cornices
- Cribs
- Curbs
- Damage repair
- Dampproofing
- Driveways
- Drop panels
- Drop-panel forms
- Excavation
- Expansion joints
- Fences
- Footings
- Formwork
- Foundations
- Girders
- Girdler forms
- Grading
- Hangers
- Hardeners
- Heat, temporary
- Heating, aggregate
- Hoisting
- Hollow-block floors
- Inserts
- Inspection
- Insurance
- Joints
- Ladders
- Lights, temporary
- Machine foundations
- Manholes
- Membrane waterproofing
- Mesh, reinforcing
- Metal pans
- Nails
- Office, temporary
- Ornamental iron
- Painting
- Pavements
- Permits
- Piles
- Precast slabs
- Protective covers
- Pumping
- Rails
- Reinforcing mesh
- Reinforcing rods
- Reinforcing, bending
- Reinforcing, cleaning
- Reinforcing, hauling
- Reinforcing, setting
- Runways
- Sand
- Sandblasting
- Scaffolds
- Sewers
- Sheds, temporary
- Sheet piling
- Shoring
- Sidewalks
- Sidewalk sheds
- Sills
- Skips
- Slabs
- Slab forms
- Social security
- Spandrels
- Stairs
- Stair forms
- Steam cleaning
- Stone, exterior
- Storage yards
- Structural steel
- Surveys
- Taxes
- Telephones, temporary
- Toilets, temporary
- Underpinning
- Walls
- Watchmen
- Waterproofing
- Water, temporary
- Winter protection
- Wrecking old buildings
Section 11

PORTLAND-CEMENT CONCRETE PAVEMENTS AND BASES*

By

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11-1
STRUCTURAL DESIGN

To produce a well-designed concrete pavement it is necessary to take into account (1) traffic, present and anticipated; (2) subgrade, supporting power and character; (3) climate of the area; and (4) the strength and quality of the concrete to be used.

Traffic

While volume and character of present and anticipated traffic determine geometric design, weight and frequency of wheel loads influence the structural design of a pavement.

Estimates of wheel-load characteristics of traffic the pavement will probably carry may be prepared by using data from the State-Wide Highway Planning Surveys.

Table 11-1 shows the type of data that should be assembled when preparing traffic data for use in a design problem. It also shows typical traffic volumes and distribution of wheel loads on four general classes of highways. Even within each class there are wide variations due to differences in the types of areas being served. The traffic figures given in Table 11-1 may be used when designing cross sections for average roads in each class. However, the best practice is to study the traffic characteristics surrounding each proposed project and design accordingly.

Subgrades

In designing a concrete pavement the support offered by the subgrade is evaluated by the Westergaard method: subgrades are rated according to their \( k \) value.\(^1\) Tables 11-2 and 11-3 show the approximate range of \( k \) values for the various soil groups as classified by the Bureau of Public Roads and U.S. Engineer Department. The values given are for moisture contents and densities approximating those found under pavements in service.

Estimates for \( k \) may be taken from Tables 11-2 and 11-3, provided the engineer can classify soil and judge where it falls in a determined classification. On large projects or where the quality of the subgrade is questionable or the wheel loads critically high field bearing tests should be made to determine the actual \( k \) values.

If plate bearing tests cannot be made, an approximate \( k \) may be determined by tests to find the California Bearing Ratio (CBR) and then converting to \( k \) by using the relationships shown in Fig. 11-1. Figure 11-2 shows the influence of \( k \) upon pavement thickness.

Climate

The climate of a location determines the type and degree of exposure the pavement will receive; knowing these, a concrete-mix design is made by following established

\(^1\) Westergaard’s \( k \), the modulus of subgrade reaction, is the load in pounds per square inch on a loaded area of the subgrade divided by the deflection in inches of the subgrade under that load. It is expressed in pounds per square inch per inch (psi/in.).
### Table 11-1. Typical Traffic Volumes and Wheel-load Distribution on Four Classes of Highways

On typical roads in five Mississippi Valley states, data from 1948 State-Wide Highway Planning Surveys

<table>
<thead>
<tr>
<th>Class of highway</th>
<th>Range</th>
<th>Average daily traffic</th>
<th>Distribution of wheel loads on commercial vehicles, No. of wheel loads per day and % of total commercial wheel loads</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Automobiles</td>
<td>Commercial vehicles</td>
</tr>
<tr>
<td></td>
<td></td>
<td>No.</td>
<td>% of total vehicles</td>
</tr>
<tr>
<td>I</td>
<td></td>
<td>Total vehicles</td>
<td></td>
</tr>
<tr>
<td>Primary route in or near a metropolitan area</td>
<td>Avg</td>
<td>8,741</td>
<td>7,095</td>
</tr>
<tr>
<td></td>
<td>Max†</td>
<td>13,250</td>
<td>11,236</td>
</tr>
<tr>
<td></td>
<td>Min†</td>
<td>3,276</td>
<td>2,686</td>
</tr>
<tr>
<td>II</td>
<td></td>
<td>Primary route in a rural area</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Avg</td>
<td>3,570</td>
<td>2,814</td>
</tr>
<tr>
<td></td>
<td>Max</td>
<td>4,183</td>
<td>3,290</td>
</tr>
<tr>
<td></td>
<td>Min†</td>
<td>2,677</td>
<td>2,276</td>
</tr>
<tr>
<td>III</td>
<td></td>
<td>Lightly traveled primary route</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Avg</td>
<td>1,322</td>
<td>1,010</td>
</tr>
<tr>
<td></td>
<td>Max</td>
<td>1,566</td>
<td>1,182</td>
</tr>
<tr>
<td></td>
<td>Min†</td>
<td>943</td>
<td>670</td>
</tr>
<tr>
<td>IV</td>
<td></td>
<td>Secondary or county trunk route</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Avg</td>
<td>787</td>
<td>584</td>
</tr>
<tr>
<td></td>
<td>Max</td>
<td>1,125</td>
<td>889</td>
</tr>
<tr>
<td></td>
<td>Min†</td>
<td>450</td>
<td>279</td>
</tr>
</tbody>
</table>

* Each axle of a truck delivers one wheel load on any one part of the pavement. The front axle usually carries much less load than the rear axle, and is then included in a different weight group. When there are two or more rear axles, each is counted as one wheel load.

† Based on total vehicles with distribution as reported for that traffic volume.
### Table 11-2. Approximate Range of $k$ Values for Soil Groups of the Highway Research Board Classifications* and of the Bureau of Public Roads Classifications

<table>
<thead>
<tr>
<th>Major divisions</th>
<th>Soil groups and typical description</th>
<th>Highway Research Board</th>
<th>Original Bureau of Public Roads symbol</th>
<th>Approx range of $k$ values for each soil group</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gravelly and sandy soils</td>
<td>Well-graded gravel-sand-clay. Excellent binder</td>
<td>A-1-a</td>
<td>A-1</td>
<td>400–700 or greater</td>
</tr>
<tr>
<td></td>
<td>Sand-clay mixtures. Excellent binder</td>
<td>A-1-b</td>
<td>A-1</td>
<td>250–575</td>
</tr>
<tr>
<td></td>
<td>Gravel with fines, very silty gravel, poorly graded gravel-sand-clay, and sand-clay mixtures. Poor binder. Friable</td>
<td>A-2-4 A-2-5</td>
<td>A-2 Friable</td>
<td>300–700 or greater</td>
</tr>
<tr>
<td>Fine-grained soils in which silt sizes predominate</td>
<td>Well-graded gravel, gravel-sand mixtures, and sands. Little or no fines</td>
<td>A-1-a</td>
<td>A-3</td>
<td>325–700 or greater</td>
</tr>
<tr>
<td></td>
<td>Poorly graded gravel, gravel-sand mixtures, and sands. Little or no fines</td>
<td>A-1-b A-3</td>
<td>A-3</td>
<td>200–325</td>
</tr>
<tr>
<td>Very fine-grained inorganic and organic soils in which the clay fraction governs</td>
<td>Predominantly silt soils with moderate to small amounts of coarse material and small amounts of plastic clay</td>
<td>A-4</td>
<td>A-4</td>
<td>100–300</td>
</tr>
<tr>
<td></td>
<td>Poorly graded silty soils which contain mica and diatomics and which have elastic properties</td>
<td>A-5</td>
<td>A-5</td>
<td>50–175†</td>
</tr>
<tr>
<td></td>
<td>Clay soils with moderate to negligible amounts of coarse materials. Includes well-graded inorganic silt-clay, sand-silt-clay, and sand-clay soils</td>
<td>A-6 A-7-6</td>
<td>A-6</td>
<td>50–225</td>
</tr>
<tr>
<td></td>
<td>Elastic clay soils with moderate to negligible amounts of coarse materials. Usually poorly graded or contains organic or other materials which make them elastic</td>
<td>A-7-5</td>
<td>A-7</td>
<td>50–225</td>
</tr>
</tbody>
</table>

† Some soils of volcanic origin may have $k$ values greater than those shown for this group.

Procedures. In the Northern and North Central states where snow, sleet, and ice are common and where the maintenance forces use deicing salts, air-entrained concrete should be used. In the Southern states, with their milder climates, air-entrained concrete is not required.

Climate also tends to indicate types of unstable soil conditions that may be encountered on a project. Soils that are satisfactory subgrades under one set of climatic conditions may become unstable or troublesome under other combinations of temperature, humidity, and moisture.

**Concrete**

In the design of concrete pavements flexural strength (modulus of rupture)\(^1\) is used rather than compressive strength.

\(^1\)“Modulus of rupture” is an empirical value representing the stress in the outer fibers of a beam at failure, computed on the assumption that the stress varies directly with the distance from the center (neutral axis) of the beam.
# Structural Design

## Table 11-3. Approximate Range of $k$ Values for Soil Groups of the Casagande Soils Classification as Used by U.S. Engineer Department

<table>
<thead>
<tr>
<th>Major divisions</th>
<th>Soil groups and typical description</th>
<th>Subgrade group symbols</th>
<th>Approximate range of $k$ values for each soil group</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gravel and gravelly soils</td>
<td>Well-graded gravel and gravel-sand mixtures. Little or no fines. Well-graded gravel-sand-clay mixtures. Excellent binder. Poorly graded gravel and gravel-sand mixtures. Little or no fines. Gravel with fines, very silty gravel, clayey gravel, poorly graded gravel-sand-clay mixtures.</td>
<td>GW</td>
<td>500–700 or greater</td>
</tr>
<tr>
<td></td>
<td></td>
<td>GC</td>
<td>400–700 or greater</td>
</tr>
<tr>
<td></td>
<td></td>
<td>GP</td>
<td>300–500</td>
</tr>
<tr>
<td></td>
<td></td>
<td>GF</td>
<td>250–500</td>
</tr>
<tr>
<td>Sands and sandy soils</td>
<td>Well-graded sands and gravelly sands. Little or no fines. Well-graded sand-clay mixtures. Excellent binder. Poorly graded sands. Little or no fines. Sand with fines, very silty sands, clayey sands, poorly graded sand-clay mixtures.</td>
<td>SW</td>
<td>250–575</td>
</tr>
<tr>
<td></td>
<td></td>
<td>SC</td>
<td>250–575</td>
</tr>
<tr>
<td></td>
<td></td>
<td>SP</td>
<td>200–325</td>
</tr>
<tr>
<td></td>
<td></td>
<td>SF</td>
<td>175–325</td>
</tr>
<tr>
<td>Fine-grained soils having low to medium compressibility</td>
<td>Silts (inorganic) and very fine sands, mo, rock flour, silty or clayey fine sands with slight plasticity. Clays (inorganic) of low to medium plasticity, sandy clays, silty clays, lean clays. Organic silts and organic silt-clays of low plasticity.</td>
<td>ML</td>
<td>150–300</td>
</tr>
<tr>
<td></td>
<td></td>
<td>CL</td>
<td>125–225</td>
</tr>
<tr>
<td></td>
<td></td>
<td>OL</td>
<td>100–175</td>
</tr>
<tr>
<td>Fine-grained soils having high compressibility</td>
<td>Micaceous or diatomaceous fine sandy and silty soils, elastic silts. Clays (inorganic) of high plasticity, fat clays. Organic clays of medium to high plasticity.</td>
<td>MH</td>
<td>50–175</td>
</tr>
<tr>
<td></td>
<td></td>
<td>CH</td>
<td>50–150</td>
</tr>
<tr>
<td></td>
<td></td>
<td>OH</td>
<td>50–125</td>
</tr>
</tbody>
</table>

Legend for group symbols:
- G = gravel.
- S = sand.
- M = mo, very fine sand, silt, rock flour.
- C = clay.
- F = fines, material smaller than 0.1-mm. diameter.
- O = organic.
- W = well-graded.
- P = poorly graded.
- L = low to medium compressibility.
- H = high compressibility.

---

**Fig. 11-1.** Relationship between California Bearing Ratio (CBR) and modulus of subgrade reaction.
Studies of the most economical strength of concrete should include the cost of aggregates available, cement requirements of each mix that will produce durable concrete, and the strengths obtained with them. Under average conditions concrete having a modulus of rupture between 600 and 750 psi at 28 days will be found the most economical.

**Stresses Due to Loads**

The most severe stresses in a concrete pavement are flexural stresses caused by wheel loads. Comprehensive studies\(^1\) show that the most critical stresses occur when a load is applied at the corner of a slab and the lowest stresses occur when a load is applied in the center of a slab (Fig. 11-3).

**Stresses Due to Changes in Temperature and Moisture**

When the top of a pavement is warmer than the bottom, the top is in compression, because of its tendency to expand, and the bottom is in tension.

The significance of slab curling lies in the fact that the slab edges or corners are lifted away from the subgrade, which reduces the pavement support and increases the flexural stresses induced by the wheel loads.

Curling stresses can be additive to the load stresses but are frequently subtractive, a condition prevailing more often. Under normal conditions it is unnecessary to reduce allowable working stress to compensate for curling stresses.\(^2\)

Volume changes due to temperature and moisture content cause compression, tension, and flexure in a slab which, if kept within safe limits by the proper jointing of the slabs, will be safe.

\(^1\) Teller and Sutherland, The Structural Design of Concrete Pavements, part 4, *Public Roads*, September-October, 1936.

pavement, need not be taken into account when computing load stresses by the method described hereinafter.

Stress Computation

Dr. Gerald Pickett has developed Eqs. (11-1) and (11-2) for the determination of the greatest flexural stresses in a concrete pavement slab. In these formulas it is assumed that subgrade support is lacking for some distance from the corner. Stresses obtained by his formulas agree closely with the measured stresses obtained on experimental projects, where the slabs have been corner-loaded and are in a curled-upward position (Fig. 11-4).

At transverse cracks or joints (Figs. 11-3 and 11-4) provision may or may not be made for load transference across the opening. If provision has been made to transfer at least 20 per cent of the load from one slab corner to the other, by some adequate mechanical device or by aggregate interlock, these corners are said to be "protected." There is a separate semiempirical formula for the determination of flexural stresses for both "protected" and "unprotected" corners.

Case I. Protected Corners:

\[
S = \frac{3.36P}{d^2} \left(1 - \frac{\sqrt{a/l}}{0.925 + 0.22a/l}\right) \quad (11-1)
\]

Case II. Unprotected Corners:

\[
S = \frac{4.2P}{d^2} \left(1 - \frac{\sqrt{a/l}}{0.925 + 0.22a/l}\right) \quad (11-2)
\]

in which \( S \) = maximum tensile stress, psi, at the top of the slab in a direction parallel to the bisector of the corner angle, due to a wheel load of \( P \) lb
\( P \) = wheel load, lb, placed on the slab corner in the position indicated in Fig. 11-4. \( P \) is the static wheel load increased by a factor to provide adequate allowance for the impact of moving loads
\( d \) = thickness, in., of a concrete slab at a corner, uniform thickness or equivalent thickness of a thickened-edge slab
\( a \) = radius, in., of a circular area equivalent to the contact of the tire with pavement
\( l \) = radius of relative stiffness defined by the equation

\[
l = \frac{Ed^3}{12(1 - \mu^2)k} \quad (11-3)
\]

\( E \) = modulus of elasticity of the concrete
\( \mu \) = Poisson's ratio for the concrete
\( k \) = modulus of subgrade reaction, psi per in., ranging from 50 to 500

These formulas are not applicable for computing stresses induced by large airplanes. For airport-pavement design, special stress formulas developed for aircraft wheel loadings should be used.

1 Teller and Sutherland, The Structural Design of Concrete Pavements, part 2, Public Roads, November, 1935.
Since it is not practical to solve directly for \( d \) because its value influences the value of \( l \), charts based on Eqs. (11-1) and (11-2) have been prepared (Figs. 11-5, 11-6, and 11-7) to permit a direct solution.

In developing Figs. 11-5, 11-6, and 11-7 the following values have been used for the various factors:

\[
P = \text{gross wheel load, static load plus an impact factor of 20 per cent (i.e., for 10,000-lb measured static wheel load, } P \text{ would equal } 10,000 \times 1.2, \text{ or 12,000 lb)}
\]

\[
d = \text{thickness of a uniform-depth slab or its equivalent for a thickened-edge slab calculated as explained under the design of balanced cross sections}
\]

\[
E = 4,000,000 \text{ psi}
\]

\[
\mu = 0.15
\]

\( a \) (for single tires) = the radius of a circle having an area equal to the contact area of the tire

\( a \) (for dual tires) = the radius of a circle having an area equal to the sum of the areas of contact of the two tires plus the area between them, where the imprint of each tire is assumed to be a circle
Case I.  Protected Corners

To satisfy the requirement for the use of Eq. (11-1) and corresponding design charts (Figs. 11-5 and 11-7a) it is essential that 20 per cent or more of the wheel load be transferred from the loaded corner to the adjacent slab.  This may be accomplished by:

1. Slip dowels,\(^1\) usually smooth round steel bars, having diameters one-eighth pavement thickness (1\(\frac{1}{8}\) in. for 9-in. pavement) and an embedment on each side of the joint of eight times the diameter of the dowel.

2. Aggregate interlock.  At natural cracks (or cracks beneath transverse dummy joints, Fig. 11-24), the rough faces of the crack may mesh sufficiently to prevent independent slab-end deflection.

For cross sections with thickened edges (Fig. 11-8) designed under Case I, load

\(^1\) The term "dowels" is to be interpreted to include all adequate mechanical load-transfer devices.
Fig. 11-7. Concrete pavement design charts (single-tired wheels). (a) Protected corners—Case I; (b) unprotected corners—Case II.
transference is required at all longitudinal joints which are not thickened. This may be secured by aggregate interlock in the dummy-type joint or by a tongue-and-groove type of joint made with a deformed metal plate (Fig. 11-23).

Case II. Unprotected Corners

Equation (11-2) and the design charts in Figs. 11-6 and 11-7b apply where unprotected corners exist at undoweled expansion or contraction joints or cracks or at undoweled contraction cracks which do not meet the requirements outlined for Case I.

Pavement Cross Sections

Figure 11-8 illustrates the principal types of cross sections used in concrete pavements. The thickened-edge and parabolic sections were developed to take advantage of the more efficient use of materials resulting from a reduction in interior thickness. The most economical cross section is one in which the stresses at the edge and the interior are equal under a given load. Such a cross section is said to have a balanced design.

Uniform-depth cross sections are now generally preferred and used on pavements with wide traffic lanes and where provision for granular subbase is required.

Design of Balanced Cross Sections

The exterior corners of a balanced-cross-section slab must have strengths equal to that of the critical portion of a uniform section whose thickness is equal to $d$ obtained by solving Eq. (11-1) or reading from Figs. 11-5 and 11-7b. The following empirical methods of computing the equivalent $d$ of a thickened-edge section give results within 2 or 3 psi of those computed by the Westergaard\(^1\) method.

The equivalent value of $d$ for most thickened-edge sections is the average thickness of the slab along the base $AB$ in Fig. 11-9 of an isosceles triangle which has the corner

\(^1\) Dr. H. M. Westergaard, formerly Gordon McKay Professor of Civil Engineering, Graduate School of Engineering, Harvard University, unpublished analysis.
of the slab as a vertex and an altitude of 24 in. This is the same as the average thickness along the line AC in Fig. 11-9. This method applies to all parabolic sections and to all sections in which the edge thickening is obtained by means of a straight slope extending 2 to 4 ft in from the edge.

For cross sections having a uniform thickness extending 1 to 2 ft from the edge and then sloping, in a distance of 2 ft or more, to the center thickness, \( d \) is the average thickness along the base of a similar isosceles triangle having an altitude of 30 in.

![Diagram](image)

**Fig. 11-10. Relation of \( d \), \( t_e \), and \( t_i \) for thickened-edge cross section.**

When the edge thickening is obtained by using a 2-ft slope, as shown in Fig. 11-9, experience and research have shown that the stresses are balanced when the edge thickness is 50 per cent greater than the interior thickness. Under these circumstances it may be computed that

\[
\text{Edge thickness } t_e = 1.275d \quad (11-4)
\]

and

\[
\text{Interior thickness } t_i = 0.85d \quad (11-5)
\]

Since it is obvious that the interior stresses are not influenced by the manner in which the edge thickness is obtained, Eq. (11-5) will hold for all methods of edge thickening.
<table>
<thead>
<tr>
<th>Cross section</th>
<th>Effective value of $d$, in.*</th>
<th>Cu yd concrete/mile</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Exterior corners</td>
<td>Interior of slab</td>
<td>For slab 20 ft wide</td>
</tr>
<tr>
<td>5 in. uniform</td>
<td>5.00</td>
<td>5.88</td>
<td>1.630</td>
</tr>
<tr>
<td>6, 47.5, 6 in.</td>
<td>5.03</td>
<td>5.29</td>
<td>1.516</td>
</tr>
<tr>
<td>6, 47.5, 6 in.</td>
<td>5.29</td>
<td>5.29</td>
<td>1.540</td>
</tr>
<tr>
<td>7, 47.5, 7 in.</td>
<td>5.38</td>
<td>5.29</td>
<td>1.548</td>
</tr>
<tr>
<td>5½ in. uniform</td>
<td>5.50</td>
<td>6.47</td>
<td>1.793</td>
</tr>
<tr>
<td>6, 5, 6 in.</td>
<td>5.65</td>
<td>5.88</td>
<td>1.695</td>
</tr>
<tr>
<td>7, 5.7 in.</td>
<td>5.71</td>
<td>5.88</td>
<td>1.695</td>
</tr>
<tr>
<td>7, 5.7 in.</td>
<td>5.88</td>
<td>5.88</td>
<td>1.711</td>
</tr>
<tr>
<td>7½, 5, 7½ in.</td>
<td>5.88</td>
<td>5.88</td>
<td>1.711</td>
</tr>
<tr>
<td>6 in. uniform</td>
<td>5.68</td>
<td>6.66</td>
<td>1.956</td>
</tr>
<tr>
<td>8, 5½, 8 in.</td>
<td>6.38</td>
<td>6.47</td>
<td>1.874</td>
</tr>
<tr>
<td>7, 5½, 7 in.</td>
<td>6.47</td>
<td>6.47</td>
<td>1.890</td>
</tr>
<tr>
<td>8, 5½, 8 in.</td>
<td>6.47</td>
<td>6.47</td>
<td>1.915</td>
</tr>
<tr>
<td>6½ in. uniform</td>
<td>6.60</td>
<td>7.65</td>
<td>2.119</td>
</tr>
<tr>
<td>8, 6, 8 in.</td>
<td>6.71</td>
<td>7.06</td>
<td>2.021</td>
</tr>
<tr>
<td>7 in. uniform</td>
<td>7.00</td>
<td>7.84</td>
<td>2.281</td>
</tr>
<tr>
<td>8, 6½, 8 in.</td>
<td>7.03</td>
<td>7.65</td>
<td>2.167</td>
</tr>
<tr>
<td>9, 6, 9 in.</td>
<td>7.03</td>
<td>7.06</td>
<td>2.053</td>
</tr>
<tr>
<td>9, 7, 9 in.</td>
<td>7.06</td>
<td>7.06</td>
<td>2.053</td>
</tr>
<tr>
<td>9, 6½, 9 in.</td>
<td>7.06</td>
<td>7.65</td>
<td>2.102</td>
</tr>
<tr>
<td>8, 6½, 8 in.</td>
<td>7.38</td>
<td>7.65</td>
<td>2.200</td>
</tr>
<tr>
<td>8, 6½, 8 in.</td>
<td>7.47</td>
<td>7.65</td>
<td>2.216</td>
</tr>
<tr>
<td>7½ in. uniform</td>
<td>7.50</td>
<td>8.24</td>
<td>2.444</td>
</tr>
<tr>
<td>9, 6½, 9 in.</td>
<td>7.68</td>
<td>7.65</td>
<td>2.227</td>
</tr>
<tr>
<td>10, 6½, 10 in.</td>
<td>7.74</td>
<td>7.65</td>
<td>2.233</td>
</tr>
<tr>
<td>9, 7, 9 in.</td>
<td>7.71</td>
<td>8.24</td>
<td>2.347</td>
</tr>
<tr>
<td>8 in. uniform</td>
<td>8.00</td>
<td>9.41</td>
<td>2.607</td>
</tr>
<tr>
<td>10, 7, 10 in.</td>
<td>8.06</td>
<td>8.24</td>
<td>2.379</td>
</tr>
<tr>
<td>10, 7, 10 in.</td>
<td>8.24</td>
<td>8.24</td>
<td>2.396</td>
</tr>
<tr>
<td>9, 7, 9 in.</td>
<td>8.25</td>
<td>8.24</td>
<td>2.404</td>
</tr>
<tr>
<td>9, 7, 9 in.</td>
<td>8.47</td>
<td>8.82</td>
<td>2.542</td>
</tr>
<tr>
<td>8½ in. uniform</td>
<td>8.50</td>
<td>10.00</td>
<td>2.770</td>
</tr>
<tr>
<td>10, 8, 10 in.</td>
<td>8.71</td>
<td>9.41</td>
<td>2.673</td>
</tr>
<tr>
<td>10, 7½, 10 in.</td>
<td>8.82</td>
<td>8.82</td>
<td>2.567</td>
</tr>
<tr>
<td>9 in. uniform</td>
<td>9.00</td>
<td>10.59</td>
<td>2.933</td>
</tr>
<tr>
<td>10, 8, 10 in.</td>
<td>9.06</td>
<td>9.41</td>
<td>2.705</td>
</tr>
<tr>
<td>10, 8, 10 in.</td>
<td>9.29</td>
<td>9.41</td>
<td>2.738</td>
</tr>
<tr>
<td>11, 8½, 11 in.</td>
<td>9.38</td>
<td>10.00</td>
<td>2.852</td>
</tr>
<tr>
<td>11, 8, 11 in.</td>
<td>9.41</td>
<td>9.41</td>
<td>2.738</td>
</tr>
<tr>
<td>9½ in. uniform</td>
<td>9.50</td>
<td>11.18</td>
<td>3.096</td>
</tr>
<tr>
<td>11, 8, 11 in.</td>
<td>9.58</td>
<td>9.41</td>
<td>2.754</td>
</tr>
<tr>
<td>11, 9, 11 in.</td>
<td>9.58</td>
<td>10.59</td>
<td>2.998</td>
</tr>
<tr>
<td>11, 8½, 11 in.</td>
<td>9.62</td>
<td>10.00</td>
<td>2.893</td>
</tr>
<tr>
<td>11, 8½, 11 in.</td>
<td>9.99</td>
<td>10.00</td>
<td>2.913</td>
</tr>
<tr>
<td>10 in. uniform</td>
<td>10.00</td>
<td>11.76</td>
<td>3.259</td>
</tr>
<tr>
<td>12, 9½, 12 in.</td>
<td>10.05</td>
<td>10.00</td>
<td>2.913</td>
</tr>
<tr>
<td>11, 9, 11 in.</td>
<td>10.29</td>
<td>10.59</td>
<td>3.064</td>
</tr>
<tr>
<td>10½ in. uniform</td>
<td>10.50</td>
<td>12.35</td>
<td>3.422</td>
</tr>
<tr>
<td>12, 10, 12 in.</td>
<td>10.59</td>
<td>10.59</td>
<td>3.080</td>
</tr>
<tr>
<td>11½ in. uniform</td>
<td>11.00</td>
<td>12.94</td>
<td>3.585</td>
</tr>
<tr>
<td>13, 11, 13 in.</td>
<td>11.12</td>
<td>11.18</td>
<td>3.259</td>
</tr>
<tr>
<td>12, 10, 12 in.</td>
<td>11.29</td>
<td>11.76</td>
<td>3.390</td>
</tr>
<tr>
<td>11½ in. uniform</td>
<td>11.50</td>
<td>13.53</td>
<td>3.748</td>
</tr>
<tr>
<td>14, 10, 14 in.</td>
<td>11.77</td>
<td>11.76</td>
<td>3.422</td>
</tr>
<tr>
<td>13, 10, 13 in.</td>
<td>11.79</td>
<td>11.76</td>
<td>3.430</td>
</tr>
<tr>
<td>12 in. uniform</td>
<td>12.00</td>
<td>14.12</td>
<td>3.911</td>
</tr>
</tbody>
</table>

* Controlling value of $d$ for each section is shown in boldface type.
PORTLAND-CEMENT CONCRETE PAVEMENTS AND BASES

See Figs. 11-10 and 11-11 for straight and parabolic section values for balanced design; read charts as indicated by heavy lines. See also Table 11-4 (includes concrete quantities) and Table 11-5 for cross-section values.

For practical reasons the interior thickness $t_i$ is usually taken at the next even $\frac{3}{4}$ in. above the calculated value, unless it falls at an even $\frac{3}{4}$ in.

![Diagram showing relation of $d$, $t_i$, and $t_e$ for parabolic cross section.]

Fig. 11-11. Relation of $d$, $t_i$, and $t_e$ for parabolic cross section.

With $t_i$ determined, a cross-section shape is selected to give an edge thickness corresponding to standard form heights or, if acceptable, to a $\frac{3}{4}$-in. differential between edge thickness and form height.

For unbalanced sections, the smaller value of $d$ must be used in the stress analysis or when selecting a section that will have the $d$ required by the strength of concrete, subgrade condition, and wheel loadings.

Fatigue of Concrete

Research of the fatigue behavior of concrete stressed in flexure has established the following:
1. When a repeated stress does not exceed 50 per cent of the ultimate strength (safety factor¹ greater than 2), the concrete will stand an unlimited number of stress repetitions without failure.

2. When a repeated stress exceeds 50 to 55 per cent of the ultimate strength, continued repetition of that stress will cause failure of the concrete.

3. When the safety factor ranges between 1 and 2, the number of repetitions required to cause failure decreases as the safety factor decreases.

4. When there is a period of recovery between stress applications, the fatigue action is diminished.

Figure 11-12 shows the fatigue behavior of concrete in flexure.

**Table 11-5. Typical Parabolic Cross Sections, Effective Values of \( d \) and Average Thicknesses**

<table>
<thead>
<tr>
<th>Cross section, in.</th>
<th>Effective value ( d ) in interior, in.</th>
<th>Effective value of ( d ) at exterior corners, in., for various pavement widths</th>
<th>Avg thickness, in.*</th>
</tr>
</thead>
<tbody>
<tr>
<td>6, 4( \frac{1}{2} ), 6</td>
<td>5.29</td>
<td>5.25, 5.31, 5.36, 5.40, 5.42, 5.58, 5.62, 5.67, 5.67</td>
<td>5.69</td>
</tr>
<tr>
<td>6, 5.6</td>
<td>5.88</td>
<td>5.30, 5.54, 5.57, 5.60, 5.62, 5.72, 5.74, 5.76, 5.78</td>
<td>5.79</td>
</tr>
<tr>
<td>7, 5.7</td>
<td>5.88</td>
<td>6.01, 6.08, 6.15, 6.21, 6.23, 6.44, 6.49, 6.53, 6.57</td>
<td>6.58</td>
</tr>
<tr>
<td>7, 5( \frac{1}{2} ), 7</td>
<td>6.47</td>
<td>6.25, 6.31, 6.36, 6.40, 6.42, 6.58, 6.62, 6.65, 6.67</td>
<td>6.69</td>
</tr>
<tr>
<td>7, 6.7</td>
<td>7.06</td>
<td>6.50, 6.54, 6.57, 6.60, 6.62, 6.72, 6.74, 6.76, 6.79</td>
<td>6.80</td>
</tr>
<tr>
<td>8, 5( \frac{1}{2} ), 8</td>
<td>6.47</td>
<td>6.76, 6.85, 6.93, 7.01, 7.04, 7.30, 7.36, 7.41, 7.46</td>
<td>7.48</td>
</tr>
<tr>
<td>8, 6.8</td>
<td>7.06</td>
<td>7.01, 7.08, 7.15, 7.21, 7.23, 7.44, 7.49, 7.53, 7.57</td>
<td>7.58</td>
</tr>
<tr>
<td>8, 6( \frac{1}{2} ), 8</td>
<td>7.65</td>
<td>7.25, 7.31, 7.36, 7.40, 7.42, 7.58, 7.62, 7.65, 7.67</td>
<td>7.69</td>
</tr>
<tr>
<td>9, 6( \frac{1}{2} ), 9</td>
<td>7.65</td>
<td>7.76, 7.85, 7.93, 8.01, 8.04, 8.30, 8.36, 8.41, 8.46</td>
<td>8.48</td>
</tr>
<tr>
<td>9, 7, 9</td>
<td>8.24</td>
<td>8.01, 8.08, 8.15, 8.21, 8.23, 8.44, 8.49, 8.53, 8.57</td>
<td>8.58</td>
</tr>
<tr>
<td>9, 7( \frac{1}{2} ), 9</td>
<td>8.82</td>
<td>8.25, 8.31, 8.36, 8.40, 8.42, 8.58, 8.62, 8.65, 8.67</td>
<td>8.69</td>
</tr>
<tr>
<td>10, 7( \frac{1}{2} ), 10</td>
<td>8.82</td>
<td>8.76, 8.85, 8.93, 9.01, 9.04, 9.30, 9.36, 9.41, 9.46</td>
<td>9.48</td>
</tr>
<tr>
<td>10, 8( \frac{1}{2} ), 10</td>
<td>10.00</td>
<td>9.75, 9.31, 9.36, 9.40, 9.42, 9.58, 9.62, 9.65, 9.67</td>
<td>9.69</td>
</tr>
<tr>
<td>11, 8, 11</td>
<td>9.41</td>
<td>9.51, 9.62, 9.72, 9.81, 9.85, 10.16, 10.23, 10.29, 10.35</td>
<td>10.40</td>
</tr>
<tr>
<td>11, 8( \frac{1}{2} ), 11</td>
<td>10.00</td>
<td>9.76, 9.85, 9.93, 10.01, 10.04, 10.30, 10.36, 10.41, 10.46</td>
<td>10.48</td>
</tr>
<tr>
<td>11, 9, 11</td>
<td>10.59</td>
<td>10.01, 10.08, 10.15, 10.21, 10.23, 10.44, 10.49, 10.53, 10.57</td>
<td>10.58</td>
</tr>
<tr>
<td>12, 9, 12</td>
<td>10.59</td>
<td>10.51, 10.62, 10.72, 10.81, 10.86, 11.16, 11.23, 11.29, 11.35</td>
<td>11.37</td>
</tr>
<tr>
<td>12, 9( \frac{1}{2} ), 12</td>
<td>11.18</td>
<td>10.76, 10.85, 10.93, 11.01, 11.04, 11.30, 11.36, 11.41, 11.46</td>
<td>11.48</td>
</tr>
<tr>
<td>12, 10, 12</td>
<td>11.76</td>
<td>11.01, 11.08, 11.15, 11.21, 11.23, 11.44, 11.49, 11.53, 11.57</td>
<td>11.58</td>
</tr>
</tbody>
</table>

*Cu yd concrete per mile = 16.296 × average thickness, in. × width, ft.

**Application of Fatigue Principle**

The useful life of a concrete pavement may be safely estimated at 25 to 30 years,² or in case of a pavement intended for temporary use, a shorter period may be used. Using conservative traffic predictions, the anticipated number of wheel loads of each size that will use the pavement during its useful life can be computed. On a two-lane pavement only one-half of the total reported or estimated traffic should be used.

With the anticipated number of each size of wheel load that will use the pavement during its life computed, the controlling wheel load is determined. Using a safety

¹ The term "safety factor" is given to the value obtained by dividing the modulus of rupture, ultimate breaking strength of the concrete in flexure, by the stress produced by a given load.

\[
\text{Safety factor} = \frac{\text{modulus of rupture, psi}}{\text{unit stress, psi}}
\]

factor of 2, the working stress to be used is obtained by dividing the modulus of rupture of the concrete by the safety factor. The thickness of the pavement that will be required to carry an unlimited number of wheel loads equal to or less than the controlling wheel load is determined for the given subgrade support by using the formulas or design charts. The thickness thus obtained is an indicated thickness and is used to determine a cross section, proportioned by methods previously described. The adequacy of the chosen cross section must then be checked.

![Graph showing safety factors against number of stress repetitions to induce failure.](image)

**Fig. 11-12. Fatigue of concrete in flexure.**

**PROCEDURE FOR PAVEMENT-SLAB DESIGN**

**Preliminary Steps**

1. Prepare a table showing traffic volumes and wheel-load distributions by (1) making a special traffic survey, or (2) using data developed by state-wide and local traffic surveys on comparable roads or streets, or (3) taking data for the appropriate class of road from Table 11-1.
2. Determine the supporting power of the subgrade and $k$ as before.
3. Establish the modulus of rupture of the concrete (this is usually taken at 700 psi).

**Design of Cross Section for Class II Highway, Primary Route in Rural Area**

1. Prepare, in blank form, a table similar to Table 11-6, and determine "case," "design life," "subgrade modulus $k$," and "modulus of rupture of concrete."
2. Enter in column 1 the wheel-load groups as shown and the maximum weight (20 per cent impact factor) in column 2.
3. Column 3 data are taken from the previously prepared traffic table (Table 11-1 values divided by 2).
4. In column 4 multiply column 3 values by number of days in the assumed design life of the pavement to obtain the total number of anticipated load repetitions. When lighter wheel-load groups produce over 100,000 load repetitions, there is no need of further calculations.
5. Compute the controlling wheel load, column 5, in the following manner: Determine the number of load repetitions per day that will produce 100,000 repetitions during the design life of the pavement. This may be read direct from Fig. 11-13 (equals 11 for the 25-year design life used in example). Compute the weighted average (controlling wheel load) of the heaviest wheel loads that will produce this number of repeti-
### Table 11-6. Pavement Design for Class II Highway, Primary Route in a Rural Area

**Case I. Protected corners**  
Design life = 25 years  
Subgrade modulus, \( K = 100 \) psi per in.; Modulus of rupture of concrete = 700 psi

<table>
<thead>
<tr>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Max for group plus 20% impact</td>
<td>No. per day, one direction</td>
<td>Anticipated for 25-year period</td>
<td>Determination of controlling wheel load and indicated ( d )</td>
<td>Actual stress, psi</td>
<td>Actual safety factor</td>
<td>Actual allowable No. of stress repetitions</td>
<td>Fatigue resistance consumed by each load group, %</td>
</tr>
<tr>
<td>Under</td>
<td>4,000</td>
<td>4,800</td>
<td>674</td>
<td>Over 100,000</td>
<td>Under 350</td>
<td>Over 2.0</td>
<td>Unlimited</td>
<td></td>
</tr>
<tr>
<td>4,000-5,000</td>
<td>6,000</td>
<td>55</td>
<td>Over 100,000</td>
<td>Under 350</td>
<td>Over 2.0</td>
<td>Unlimited</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5,000-6,000</td>
<td>7,200</td>
<td>56</td>
<td>Over 100,000</td>
<td>Under 350</td>
<td>Over 2.0</td>
<td>Unlimited</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6,000-7,000</td>
<td>8,400</td>
<td>63</td>
<td>Over 100,000</td>
<td>Under 350</td>
<td>Over 2.0</td>
<td>Unlimited</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7,000-8,000</td>
<td>9,600</td>
<td>63</td>
<td>Over 100,000</td>
<td>Under 350</td>
<td>Over 2.0</td>
<td>Unlimited</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8,000-9,000</td>
<td>10,800</td>
<td>61</td>
<td>Over 100,000</td>
<td>Under 350</td>
<td>Over 2.0</td>
<td>Unlimited</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9,000-10,000</td>
<td>12,000</td>
<td>23</td>
<td>209,857</td>
<td>Under 350</td>
<td>Over 2.0</td>
<td>Unlimited</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10,000-11,000</td>
<td>13,200</td>
<td>5.6</td>
<td>51,100</td>
<td>Under 350</td>
<td>Over 2.0</td>
<td>Unlimited</td>
<td></td>
<td></td>
</tr>
<tr>
<td>11,000-12,000</td>
<td>14,400</td>
<td>1.9</td>
<td>17,337</td>
<td>Under 350</td>
<td>Over 2.0</td>
<td>Unlimited</td>
<td></td>
<td></td>
</tr>
<tr>
<td>12,000-13,000</td>
<td>15,600</td>
<td>0.9</td>
<td>8,212</td>
<td>Under 350</td>
<td>Over 2.0</td>
<td>Unlimited</td>
<td></td>
<td></td>
</tr>
<tr>
<td>13,000-15,000</td>
<td>18,000</td>
<td>0.1</td>
<td>912</td>
<td>Under 350</td>
<td>Over 2.0</td>
<td>Unlimited</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[
\frac{122,600}{11} = 11,145 \text{ lb} = \text{controlling wheel load}
\]

\[
11,145 \times 1.2 = 13,370 \text{ lb} = \text{controlling wheel load plus 20% impact}
\]

Indicated \( d \) = 7.90 in.

![Design life of pavement - years](image)

**Fig. 11-13. Load repetitions per day that will produce 100,000 stress repetitions during design life of pavement.**

...ions per day. The wheel load for the heaviest group is multiplied by the corresponding value taken from column 3. This is done for each succeeding lighter group until the sum of the values taken from column 3 is just under the value read from Fig. 11-13 (11 in example). The wheel load for the next group is multiplied by a number just sufficient to make the total of these multipliers equal the value (11) read from Fig. 11-13. The answer obtained, column 5, is the weight of the controlling wheel load.

6. Determine the indicated \( d \) by using the proper design chart (Fig. 11-5, 11-6, or 11-7). In order to use these charts the gross controlling wheel load (the static controlling wheel load increased by a 20 per cent impact factor) must be determined. The indicated \( d \) is read from the proper design chart (Fig. 11-5 for case I) by using the working stress of the concrete (modulus of rupture divided by safety factor, \( 70\% = 350 \) psi), the subgrade modulus \( K \) (100 psi per in.), and the gross controlling wheel load (13,370 lb). The chart is read from the left-hand side as illustrated by the

...
Table 11-7. Summary and Comparison of Designs That Would Be Adequate for Average Traffic Figures Given in Table 11-1*

<table>
<thead>
<tr>
<th>Highway</th>
<th>Pavement cross section</th>
<th>Thickness, in.</th>
<th>Type of section</th>
<th>Width, ft</th>
<th>Effective d, in.</th>
<th>Concrete per mile, cu yd</th>
</tr>
</thead>
<tbody>
<tr>
<td>I Primary route in or near a metropolitan area</td>
<td>Uniform thickness</td>
<td>10, 7½, 9, 6½</td>
<td>Thickened edge with 3-ft slope</td>
<td>24</td>
<td>8.50</td>
<td>3,324</td>
</tr>
<tr>
<td></td>
<td>Uniform thickness</td>
<td>10, 7½, 9, 6½</td>
<td>Parabolic</td>
<td>24</td>
<td>8.82</td>
<td>3,056</td>
</tr>
<tr>
<td></td>
<td></td>
<td>10, 7½, 9, 6½</td>
<td>Parabolic</td>
<td>24</td>
<td>8.67</td>
<td>3,129</td>
</tr>
<tr>
<td>II Primary route in a rural area</td>
<td>Uniform thickness</td>
<td>10, 7, 6½</td>
<td>Thickened edge with 2 ft 4-in. slope</td>
<td>24</td>
<td>8.00</td>
<td>3,129</td>
</tr>
<tr>
<td></td>
<td>Uniform thickness</td>
<td>10, 7, 6½</td>
<td>Parabolic</td>
<td>24</td>
<td>8.24</td>
<td>2,852</td>
</tr>
<tr>
<td></td>
<td></td>
<td>10, 7, 6½</td>
<td>Parabolic</td>
<td>24</td>
<td>8.24</td>
<td>3,000</td>
</tr>
<tr>
<td>III Lightly traveled primary route</td>
<td>Uniform thickness</td>
<td>9, 6½, 8, 6½</td>
<td>Thickened edge with 2 ft 8-in. slope</td>
<td>22</td>
<td>7.50</td>
<td>2,688</td>
</tr>
<tr>
<td></td>
<td>Uniform thickness</td>
<td>9, 6½, 8, 6½</td>
<td>Parabolic</td>
<td>22</td>
<td>7.65</td>
<td>2,439</td>
</tr>
<tr>
<td></td>
<td></td>
<td>9, 6½, 8, 6½</td>
<td>Parabolic</td>
<td>22</td>
<td>7.65</td>
<td>2,509</td>
</tr>
<tr>
<td>IV Secondary or county trunk route</td>
<td>Uniform thickness</td>
<td>9, 6½, 9, 8</td>
<td>Thickened edge with 2-ft slope</td>
<td>20</td>
<td>7.25</td>
<td>2,363</td>
</tr>
<tr>
<td></td>
<td>Uniform thickness</td>
<td>9, 6½, 9, 8</td>
<td>Parabolic</td>
<td>20</td>
<td>7.38</td>
<td>2,200</td>
</tr>
<tr>
<td></td>
<td></td>
<td>9, 6½, 9, 8</td>
<td>Parabolic</td>
<td>20</td>
<td>7.62</td>
<td>2,281</td>
</tr>
</tbody>
</table>

* All designs based on Case I, protected corners; k = 100 psi per in.; modulus of rupture = 700 psi with pavements designed for 25-year life.

Heavy dashed lines; a stress of 325, k of 100, and a gross wheel load of 11,000 give an indicated d of 7.48.

7. The indicated d now being known (7.90 in.), a practical pavement cross section is selected from Fig. 11-10 or 11-11 or by taking a cross section directly from Table 11-4 or 11-5.1

8. Analyze the selected cross section for all wheel loads by the method outlined in columns 6 through 9 of Table 11-6.

a. In column 6 record the actual stress for each gross wheel load (column 2) computed by Eq. (11-1) or read direct from the proper design chart (Fig. 11-5), reading the right as indicated by the heavy black line.

b. The actual allowable number of stress repetitions for each safety factor (computed and listed in column 7) is read from Fig. 11-12 (1.58 giving 14,000).

c. The percentage of the total fatigue resistance consumed by each load group is determined and recorded in column 9. This percentage is the relation of anticipated traffic (column 4) to traffic capacity (column 8):

\[
\frac{912}{14,000} \times 100 = 7 \text{ per cent}
\]

Added together (54 per cent) these percentages give the fatigue resistance of the selected design, for the life of the pavement.

Theoretically the limit of fatigue resistance used up by the loads could be 100 per cent with the cross section still adequate. On all except the most heavily traveled roads it is safe to exceed this limit by 20 per cent, since (1) all wheel loads will not be

1 The uniform-thickness type of cross section was selected here in order to simplify the explanation. Study of Tables 11-4, 11-5, and 11-7 will show that either a 10, 7, 10 thickened-edge section having a 2 ft 4-in. slope or a 9, 7, 9 parabolic section would have greater load-carrying capacity (larger effective d) at a considerable saving of concrete.
the heaviest in each group as has been assumed; (2) the concrete throughout most of its life will have a strength well above the 28-day strength used in the design; and (3) load repetitions are distributed over a period of years with considerable time for recovery. However, when designing heavily traveled highways, such as class I or II, which generally carry a large number of the heavier wheel loads, the fatigue resistance consumed should not exceed 100 per cent.

Table 11-7 was prepared using the design method just described. It shows the types of pavements cross sections that would be adequate for the traffic volumes and wheel-load distributions (average values) for each class of highway given in Table 11-1, when all other conditions are equal to those used in the example problem.

**SUBBASES FOR CONCRETE PAVEMENTS**

**Subbases**

The use of granular subbases for the sole purpose of increasing subgrade support is rarely, if ever, justified. It is generally more economical to increase the pavement thickness to make it structurally adequate than to raise the subgrade support with thick layers of granular materials. However, certain combinations of soils, climate, and truck traffic make it necessary to use subbases for satisfactory pavement performance. These combinations may require subbases for one or more of the following purposes:

1. To assist in the control of high-volume-change soils
2. To aid in controlling frost action
3. To prevent pumping of fine-grained soils

**Control of High-volume-change Soils**

Excessive differential shrink and swell of highly plastic clay and silt soils may cause enough distortion of the pavement surface to impair riding qualities. These distortions include (1) gain or loss of pavement crown, (2) high joints, or (3) bumps, depressions, or waves in the pavement surface. These defects are most likely to occur in arid or semiarid regions, where expansive soils are too wet or too dry just prior to placing the subbase or pavement. In all climatic areas, compaction of expansive soils when they are too dry can lead to detrimental expansion and softening during subsequent rainy periods.

Control of excessive shrink and swell begins during grading operations. Highly expansive soils are placed in the lower parts of embankments and less expansive soils are cross-hauled to form the upper part of the grade. Selective grading also produces reasonably uniform soil conditions in the upper part of the subgrade.

With a uniform subgrade, excessive shrink and swell are prevented by adequate moisture control during compaction. In arid or semiarid regions, and in other regions with extensive periods of dry weather, it is critically important to compact both moderately and highly expansive soils at not less than the optimum moisture content as determined by AASHO Standard Method T-99-49.

In addition to compacting expansive soils at adequate moisture contents, it is essential to prevent their drying out prior to placing the subbase and pavement. If drying out does occur, the subgrade should be recompacted at the required moisture content just before placing the subbase course. The depth of subgrade needing recompaction can be determined from tests of field moisture contents.

In areas with prolonged periods of dry weather highly expansive soils require an insulating layer of low-volume-change soil placed full width over the subgrade. For most soil and climatic conditions a depth of 4 to 6 in. will provide adequate protection.

---

1 Where subbases are needed, the designer should take advantage of the resulting increase in $k$ value in determining pavement thickness.

2 AASHO Designation T-116-54 gives two well-established procedures for determining the expansion properties of soils.
For extreme conditions depths of 12 to 18 in. have been found necessary. In these cases local experience with these extremely expansive soils is the best guide for adequate depth of cover.

Since expansive soils are also susceptible to pumping,\(^1\) that portion of the low-volume-change layer underlying the pavement should be designed to prevent pumping.

Even in climates where control of expansive soils is not a major problem, it is still worthwhile to compact most soils in the plastic range at moisture contents close to or slightly above standard optimum. After pavements are placed in service most subgrade soils reach a moisture content of about their plastic limit. These natural moisture contents correspond closely to standard optimum, and when they are obtained during construction, the subgrade retains the stability imparted to it by compaction.

Control of Frost Action

The performance of older concrete pavements in frost-affected areas shows that concrete pavements can resist frost action without extensive or costly control measures. Surveys of these pavements reveal that control of frost action is needed only to prevent excessive heave and abrupt differential heaving.

As in the care of expansive soils, control is accomplished during grading operations. The methods employed are similar to those used in all climatic areas to ensure good pavement performance. They include the following:

1. Setting grade lines high enough, and constructing side ditches deep enough so that the highly frost-susceptible soils are beyond the capillary range of free water tables.

2. The highly frost-susceptible silts and fine sandy soils are placed in the lower parts of embankments and less susceptible soils are crosshauled to form the upper part of the grade.

3. Where highly frost-prone soils are pocketed in less susceptible soils the pockets are excavated and backfilled with soils like those surrounding the pocket. If the normal soil texture is sand and gravel, sand and gravel are used for backfill. If the normal soil is clay, clay is used for backfill. Experience has shown that replacement need not exceed one-third to one-half the depth of frost penetration. Crosshauling is also employed to correct abrupt changes in soil types at cut-fill transitions.

4. Where soils vary widely in texture and nonuniform conditions are less clearly defined, mixing and compaction at controlled moisture contents are effective in preventing both excessive and differential heave. With modern construction equipment, mixing of nonuniform soils is often more economical than long hauls from borrow pits of selected material.

5. Reducing soil permeability retards the rate of moisture flow to the frozen zone and heaving is reduced. Permeability is lowest for fine-grained soils when they are compacted at or slightly above optimum moisture. Lowering soil permeability by compaction at these moisture contents not only retards moisture flow to the frozen zone but also makes subgrades more resistant to saturation during rainy weather.

Proper grade designs, selective grading, and compaction control cost less than thick subbases and are proved methods for control of frost action. These methods produce uniformity and resistance to rapid capillary flow in the upper part of the subgrade. This prevents differential or excessive heaving, and subbases need not be thicker than the nominal depths needed to prevent pumping.

Preventing “Pumping” of Fine-grained Soils

When certain types of fine-grained subgrade soils are highly saturated, the repeated deflections of pavement slab ends cause a mixture of soils and water to be ejected at or near joints and cracks. This action is known as “pumping” and is caused by (1) a soil that will go into suspension, (2) free water, and (3) frequent passages of heavy wheel loads. If one of these is absent, pumping will not occur. Continued pumping

\(^1\) Pumping and subbase designs to prevent its occurrence are discussed under Prevention of Pumping.
leads to faulting at joints and cracks, followed shortly by cracking and eventually breakup of the pavement.

Extensive research has shown the subgrade soils most conducive to pumping are those in which the silt and clay fractions predominate; pumping is most severe on soils having high clay contents; and that pavements placed on subgrades of nonplastic granular soils, such as sands and gravels, do not pump.

These studies have also shown that:

1. No pumping occurred on fairly well graded natural soils, granular subbases, and mulches containing more than 55 per cent sand and gravel (material retained on the No. 270 sieve) when the pavement carried less than 300 to 400 wheel loads per day weighing over 7,000 lb.

2. Even under the heaviest loadings (over 400 to 500 wheel loads per day weighing over 7,000 lb), no pumping occurred on fairly well graded granular soils (more than 55 per cent retained on a No. 200 sieve) whose minus No. 40 sieve fraction had a plasticity index (P.I.) of 6 or less.

3. A 4- to 6-in. thickness of granular subbase meeting the grading and plasticity requirements of Fig. 11-14 was adequate to prevent pumping even under the most adverse conditions.

Subbase Gradation

A wide variety of granular materials which meet the gradation limits given in Fig. 11-14 or the limits set forth in AASHO Designation M-147 have given satisfactory performance under severe service conditions. Dense-graded subbases are usually extended 1 ft beyond the pavement edges. Open-graded materials should be provided with some means of positive transverse drainage.

Subbase Compaction

Vibratory compaction equipment has proved effective for producing high densities in granular subbases for concrete pavements. This affords striking evidence of an important property of these granular materials. They are subject to consolidation (or additional densification) under the action of heavy truck traffic after the pavements are placed in service. To prevent a detrimental amount of consolidation, the subbase should be compacted to not less than 100 per cent of standard density. On projects which will carry large volumes of heavy truck traffic, the specified density should be not less than 105 per cent of standard density, or 98 to 100 per cent of modified density.

Cement-treated Subbase

Laboratory tests\(^1\) have shown that cement treatment of granular subbase materials will prevent consolidation. These laboratory results are borne out by the performance of concrete pavements constructed on similar cement-treated subbases. To effectively prevent consolidation under heavy truck traffic, enough cement should be mixed with the granular materials to develop an unconfined compressive strength of 300 to 650 psi at 7 days.

Cement treatment is particularly advantageous where low-cost local materials do not meet grading requirements of subbase which will prevent pumping.

USE OF STEEL IN CONCRETE PAVEMENT

When thickened-edge cross sections are used, tie bars are needed at all longitudinal joints in the thin interiors of the slab, and dowel bars are needed at all transverse expansion joints and under certain conditions at the transverse contraction joints.\(^2\) Uniform-thickness cross sections designed under case I (protected corners) require dowel bars at expansion joints and at some transverse contraction joints, and tie bars at wide spacing are needed across the longitudinal joints. There are divergent views regarding the value of distributed steel ("reinforcing") in concrete pavement slabs.

Design of Distributed Steel

The principal function of bar mats or welded wire fabric is to hold together the fractured faces of slabs after cracks have formed. The infiltration of incompressible material into the crack is thus prevented and adequate load transmission across the crack is provided by the interlocking action of the rough faces of the crack.

Distributed steel is designed to withstand tensile stresses only. The maximum tension in the steel members across any crack is equal to the force required to overcome friction between pavement and subgrade from the crack to the nearest free joint\(^3\) or edge. This force will be the greatest when the crack occurs at the middle of a slab. Distributed steel is designed to be adequate for a crack at this location. For practical reasons the steel is usually made the same weight throughout the length of short slabs, but on the longer slabs (over 50 ft) considerable saving can be realized by using lighter steel near the ends of the slabs.

The amount of longitudinal and transverse steel required per 1-ft width or length of slab is computed by the following formula:

\[
A_s = \frac{L w}{28} \quad (11-6)
\]

\(^1\) Performance of Subbases for Concrete Pavements under Repetitive Loading, Highway Research Board Bull. 202, pp. 32–79.

\(^2\) See the later discussion of pavement joints and joint spacings.

\(^3\) The term "free joint" is applied to any type of pavement joint that has no tie bars or bonded reinforcement crossing it.
in which \( A_s \) = area of steel required per ft width or length of slab, sq in.

\( L \) = distance between free transverse joints when the equation is used to calculate longitudinal steel or between free longitudinal joints or edges when figuring transverse steel, ft

\( f \) = coefficient of friction between slab and subgrade

\( W \) = weight of 1 sq ft of slab, in lb (12.5 psf per in. of slab thickness)

\( S \) = allowable working stress in the steel, psi

When computing the value of \( W \) for use in this formula, concrete may be assumed to weigh 150 lb per cu ft. The value of \( f \) may range from 1 to 2, with 1.5 most commonly used. The value of \( S \) will depend on the type of steel used. It need never be less than half the yield point of the steel (Table 11-8). Some safety factor should be provided,

<table>
<thead>
<tr>
<th>Type and grade of steel</th>
<th>Yield point (min), psi</th>
<th>Min* working stress, psi</th>
</tr>
</thead>
<tbody>
<tr>
<td>Billet and axle steel, structural grade†</td>
<td>33,000</td>
<td>16,500</td>
</tr>
<tr>
<td>Billet and axle steel, intermediate grade</td>
<td>40,000</td>
<td>20,000</td>
</tr>
<tr>
<td>Billet and axle steel, hard grade</td>
<td>50,000</td>
<td>25,000</td>
</tr>
<tr>
<td>Rail steel</td>
<td>50,000</td>
<td>25,000</td>
</tr>
<tr>
<td>Welded steel wire fabric</td>
<td>56,000</td>
<td>28,000</td>
</tr>
</tbody>
</table>

* There is no occasion for using working stresses lower than these for distributed steel or tie bars in concrete pavement. Many engineers make a practice of using higher working stresses in distributed steel, up to 60 per cent or even up to 80 per cent of the specified yield point of the steel. Since the consequences of an occasional failure are not so serious as in a bridge or building this practice seems to be justified.

† Specify this grade of steel for all tie bars that must be bent.

**Fig. 11-15.** Welded wire fabric and bar mats for pavement slabs.
but a safety factor of 2.0 is not justified. A survey of pavements in a group of states shows that a working stress of 45,000 psi can safely be used for welded wire fabric. Failures of steel in pavements with working stresses of 50,000 psi have been negligible. In the sample problem and design chart (Fig. 11-15) working stresses of 45,000 and 35,000 psi have been used for welded wire fabric and bar mats, respectively.

Equation (11-6) or Fig. 11-15 should be used for determining the amount of steel needed in the longitudinal direction. Since pavements are usually divided into slab widths of 12.5 ft or less by longitudinal joints, no longitudinal cracks should develop. For this reason, transverse steel need not be so heavy as required by formula. Normally, transverse wires serve only as spacer wires, and they should be the minimum

Table 11-9. Standard Pavement Styles of Welded Steel Wire Fabric

<table>
<thead>
<tr>
<th>Style</th>
<th>Weight of fabric based on net width of 60 in.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Lb/100 sq ft</td>
</tr>
<tr>
<td></td>
<td>Longitudinal</td>
</tr>
<tr>
<td>66-88</td>
<td>30</td>
</tr>
<tr>
<td>66-77</td>
<td>36</td>
</tr>
<tr>
<td>66-66</td>
<td>42</td>
</tr>
<tr>
<td>66-55</td>
<td>49</td>
</tr>
<tr>
<td>66-44</td>
<td>58</td>
</tr>
<tr>
<td>66-33</td>
<td>68</td>
</tr>
<tr>
<td>66-22</td>
<td>78</td>
</tr>
<tr>
<td>66-11</td>
<td>91</td>
</tr>
<tr>
<td>66-00</td>
<td>107</td>
</tr>
<tr>
<td>612-77</td>
<td>27</td>
</tr>
<tr>
<td>612-66</td>
<td>32</td>
</tr>
<tr>
<td>612-55</td>
<td>37</td>
</tr>
<tr>
<td>612-44</td>
<td>44</td>
</tr>
<tr>
<td>612-33</td>
<td>51</td>
</tr>
<tr>
<td>612-24</td>
<td>54</td>
</tr>
<tr>
<td>612-22</td>
<td>59</td>
</tr>
<tr>
<td>612-11</td>
<td>69</td>
</tr>
<tr>
<td>612-00</td>
<td>81</td>
</tr>
<tr>
<td>612-2/04</td>
<td>78</td>
</tr>
<tr>
<td>612-3/04</td>
<td>91</td>
</tr>
</tbody>
</table>

*Courtesy of American Steel & Wire Co.
For a complete list of standard styles refer to any steel company's catalogue.

size in fabric styles available from manufacturers. The only exception would be in a pavement in which the distributed steel extends across longitudinal center joints in lieu of deformed tie bars. In this case, the transverse wires should be designed by formula, using the total slab width as L, the distance between free joints.

The following example illustrates the design of distributed steel (welded wire fabric) in pavements, assuming an 8-in. uniform-thickness pavement, 24 ft wide (two 12-ft lanes with a tied longitudinal center joint) and having contraction joints spaced 40 ft apart:

1. The $A_f$ for the longitudinal steel is computed from Eq. (11-6) or read direct from Fig. 11-15. For a 40-ft joint spacing and an 8-in. pavement thickness, $A_f$ is 0.067 sq in. per ft of pavement width.

2. The $A_f$ of transverse steel is likewise obtained from Eq. (11-6) or read direct from Fig. 11-15. For a 24-ft width of slab and an 8-in. pavement thickness, $A_f$ is 0.04 sq in. per ft of pavement length.

3. Table 11-9 lists some of the more commonly employed styles of welded wire fabric.
From this table, Style 612-55 would be selected. The longitudinal wire has an area of 0.067 sq in. per ft and the transverse wire has an area of 0.034 sq in. per ft. The latter is less than the 0.04 required but is entirely practical for the conditions of design (see footnote to Table 11-8).

4. When the fabric is ordered, it should be detailed in accordance with information given in Fig. 11-16.

The same procedure should be followed in designing distributed steel using bar mats. The right-hand ordinate of Fig. 11-15 gives \( A_s \) for bar mats with a working stress of 35,000 psi.

![Diagram of fabric dimensions](image)

When ordering road fabric:

1 - Indicate spacing of longitudinal wires
2 - Indicate spacing of transverse wires
3 - Indicate gauge of longitudinal wires
4 - Indicate gauge of transverse wires
5 - Indicate width of sheet c to c outside longitudinal wires
6 - Indicate length of sheet tip to tip longitudinal wires
7 - Indicate overhang of transverse wires beyond outside longitudinal wires (normally, approximately 1 inch)
8 - Indicate end-projection of longitudinal wires (normally projection at one end plus projection at other end equals the spacing of transverse wires)

When detailing plans: Indicate all the above plus the following:

9 - Indicate dimension from tip of longitudinal wires to end of slab
10 - Indicate dimension from center of outside longitudinal wires to edge of slab

Fig. 11-16. Data for ordering or detailing welded steel wire pavement fabric. (Courtesy of American Steel & Wire Co.)

Since distributed steel is not intended to act in flexure, its position in the slab depth is not important, except that it should be adequately protected from corrosion. A preferred location is near the neutral axis or mid-point in the slab depth. In no case should the distributed steel be placed at less than 2 in. from the top or bottom of the pavement. The common practice of placement is to strike off the concrete at the level for the prescribed location of the steel, place the mesh or bar mats on it, and then place the concrete for the remaining depth of the pavement (Fig. 11-17).

**Design of Tie Bars**

Tie bars are deformed steel bars. They are used across the joints in concrete pavement wherever it is necessary or desirable to ensure firm contact between slab faces or to prevent abutting slabs from separating. They are not designed to act as load-transfer devices; load transmission is accomplished by "aggregate interlock" in the
crack beneath a dummy groove, or by the tongue and groove of a joint formed with deformed metal plate (Fig. 11-23).

Tie bars are designed to withstand tensile stresses only. The maximum tension in the tie bars across any joint is equal to the force required to overcome friction between

![Diagram of welded steel wire fabric sheets]

**Fig. 11-17. Details showing placement of welded steel wire fabric sheets.**

the pavement and subgrade, from the joint in question to the nearest free joint or edge of pavement.

The size (diameter and length) and spacing of tie bars may be determined by one of the following methods:

The area of steel required per foot length of joint is computed by Eq. (11-7).

\[ A_s = \frac{bfW}{S} \]  \hspace{1cm} (11-7)

in which \( A_s \) = area of steel required per ft length of joints, sq in.
\( b \) = distance between the joint in question and the nearest free joint or edge, ft [\( f, W, \) and \( S \) are as for Eq. (11-6)]

\(^1\) The term "free joint" is applied to any type of pavement joint that has no tie bars or bonded reinforcement across it.
For tie bars of intermediate-grade steel across the longitudinal center joint of a 9, 7\(\frac{1}{2}\), 9 parabolic section pavement 24 ft wide, the cross-sectional area of steel required per foot of joint would be

\[
A_s = \frac{12 \times 1.5 \times 8^{\dagger} \times 12.5}{25,000} = 0.072 \text{ sq in.}
\]

If spaced at 18 in., \(\frac{3}{8}\)-in. bars will provide 0.073 sq in. of steel per ft of joint

\[
\frac{0.11 \times 12}{18} = 0.073
\]

which is ample. If \(\frac{1}{2}\)-in.-diameter bars are selected, their spacing will work out to be 33 in. \(\left(\frac{0.20 \times 12}{33} = 0.073\right)\). The \(\frac{3}{8}\)-in. bars at 18-in. spacing should be used, since they can be distributed more uniformly and produce smaller concentrations of stress than the \(\frac{1}{2}\)-in. bars.

The length of any tie bar should be at least twice that required to develop a bond strength equal to the working stress of the steel. Expressed as a formula this becomes

\[
L = 2 \times \frac{S \times A}{200^{\dagger} \times P}
\]

(11-8)

in which \(L\) = length of tie bar, in.

\(S\) = allowable working stress in steel, psi

\(A\) = cross-sectional area of one tie bar, sq in.

\(P\) = perimeter of tie bar, in.

For the \(\frac{3}{8}\)-in.-diameter tie bars previously selected, their length works out to be 24 in.

\[
L = 2 \times \frac{25,000 \times 0.11}{200 \times 1.178} = 23.4 \text{ in. (use 24 in.)}
\]

The size (diameter and length) and spacing of tie bars across thin-edged longitudinal joints for the commonly used pavement thicknesses and lane widths may be read directly from Fig. 11-18.

Tie bars used across the longitudinal joints in uniform-thickness slab, or across thickened-edge longitudinal joints in thickened-edge slabs, should be designed with working stresses approaching the yield point of the steel.

Tie bars of intermediate-grade steel across the longitudinal center joint of an 8-in. uniform pavement 24 ft wide would be designed as follows: Fig. 11-18 shows that \(\frac{3}{8}\)-by 24-in. bars at 18-in. spacings are required for steel stressed to 25,000 psi. With the steel stressed to 35,000 psi (yield point 40,000 psi), the \(\frac{3}{8}\)-in. tie bars should be \(\left(\frac{35,000 \times 24}{25,000} = 33.6\right)\) 34 in. in length and spaced at \(\left(\frac{35,000 \times 18}{25,000} = 25.2\right)\) 24 in.

When used across thin-edged longitudinal joints, tie bars should be spaced at not more than 30 in. and the first and last tie bars in a panel should be not more than 15 in. from the joint.

Whenever tie bars must be bent and then later straightened out, such as in lane-at-a-time construction, structural-grade steel (Table 11-8) should be used. Under these conditions the working stress of the steel should be taken at not over 20,000 psi.

* See Table 11-8 and its footnote.
\(\dagger\) Average thickness of 9, 7\(\frac{1}{2}\), 9 parabolic section, obtained from Table 11-5.
\(\ddagger\) The maximum working stress for bond in deformed bars is generally taken as 0.05 of the compressive strength of the concrete, up to a maximum of 200 psi. It is permissible to use this maximum value in the design of tie bars as paving concrete should have a compressive strength in excess of 4,000 psi.
Design of Dowels

Dowels are mechanical load-transfer devices installed across joints in a concrete pavement to permit the joint to open and close yet hold the slab ends on each side of the joint as nearly as possible at the same elevation.

Dowels are required on pavements designed under Case I, Protected Corners. On case I pavements dowels must be used in expansion and construction joints (Fig. 11-24) and in contraction joints where the slab lengths are greater than about 20 ft, and with shorter slab lengths where experience indicates that aggregate interlock does not provide adequate load transference. Where required, dowels must be used throughout the entire length of the joint: If they are placed only in the slab corners, the interior edges of the joint will have inadequate load-carrying capacity.

Dowel systems [number, size (diameter and length), and spacing of dowels] are designed so that they will be capable of transferring approximately 45 per cent of the gross controlling wheel load to the adjacent slab when the load is located adjacent to the joint and well away from the edge of the slab. Such dowel systems, installed the full length of the joint, will have a transfer capacity at the slab corners that exceeds the minimum requirements (20 per cent) for case I design.

The load-transfer capacity of any dowel is governed by its capacity in both bending and bearing. These are influenced by the width of the joint opening, tensile or bending stress of the steel, diameter of the dowel, length of dowel embedded in the concrete, and bearing stress on the concrete.
USE OF STEEL IN CONCRETE PAVEMENT

Several methods have been developed for the design of dowels. The most recent method, based on the latest data available, is outlined in the report of Subcommittee III, ACI Committee 325, Structural Design Considerations for Pavement Joints published in the July, 1956, Journal of the American Concrete Institute. A summary of this method with some modification is presented here.

![Graph showing load transfer capacity of single round dowels](image)

**Fig. 11-19**

Figure 11-19 shows the load-transfer capacities of single round dowel bars for joint openings of various widths as computed by this method, using values based on a compressive strength of the pavement concrete of 4,500 psi. For contraction joints it is recommended that the load-transfer capacity be based on an opening of 0.25 in. For expansion joints, the width of opening used in determining the load-transfer capacity should be the thickness of the joint filler plus 0.25 in. (1-in. opening for \( \frac{3}{4} \)-in. expansion-joint filler).

**Table 11-10. Range of Effectiveness of Dowels Point of Load to Point of No Load Transfer (1.8\( \times \) In.**

<table>
<thead>
<tr>
<th>Slab d, in.</th>
<th>Modulus of subgrade reaction ( k ), psi/in.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>50</td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td>5.5</td>
<td>54.7</td>
</tr>
<tr>
<td>6</td>
<td>58.8</td>
</tr>
<tr>
<td>6.5</td>
<td>62.7</td>
</tr>
<tr>
<td>7</td>
<td>66.6</td>
</tr>
<tr>
<td>7.5</td>
<td>70.4</td>
</tr>
<tr>
<td>8</td>
<td>74.1</td>
</tr>
<tr>
<td>8.5</td>
<td>77.8</td>
</tr>
<tr>
<td>9</td>
<td>81.4</td>
</tr>
<tr>
<td>9.5</td>
<td>85.0</td>
</tr>
<tr>
<td>10</td>
<td>88.5</td>
</tr>
<tr>
<td>10.5</td>
<td>92.0</td>
</tr>
<tr>
<td>11</td>
<td>95.4</td>
</tr>
<tr>
<td>11.5</td>
<td>98.8</td>
</tr>
<tr>
<td>12</td>
<td>102.1</td>
</tr>
<tr>
<td></td>
<td>105.5</td>
</tr>
</tbody>
</table>

*Computed by Eq. (11-3) using values of 4,000,000 psi for \( E \) and 0.15 for \( \mu \).*
The length of embedment of dowels on each side of the joint (exclusive of joint opening and length of expansion cap needed at expansion joints) should be eight times the diameter, or 8 in. for a 1.0-in. dowel. Some recent tests by the Bureau of Public Roads indicate that an increase in embedment beyond 8 in. does not add to the load-transfer capacity of any size dowel. Therefore, in general, dowels can be approximately 16 in. long at contraction joints.

A dowel directly under the center of a wheel load is most effective and will transfer its full capacity, thus having a capacity factor of 1.0. The capacity factor of dowels decreases with distance from the wheel load until it reaches 0 for dowels 1.8l or more distant from the point of load. Values of 1.8l (1.8 times the radius of stiffness) for various pavement thicknesses d and values of k, the modulus of subgrade reaction, are shown in Table 11-10. The capacity factor of any dowel in a system may be determined by

\[
\text{Capacity factor} = \frac{1.8l - \text{distance of dowel from center of load}}{1.8l}
\]

The capacity factor of a dowel system for edge loading is the sum of the factors for successive dowels computed on one side of the load. The total capacity factor of any system can be determined if the spacing of dowels and l (values in Table 11-10 divided by 1.8) of the pavement are known. These relationships are shown graphically in Fig. 11-20.

If the center-to-center spacing of the wheel loads on an axle (tread) is less than 1.8l, there will be overlapping of loads from the two wheels on all dowels between the wheels and for a short distance outside. If this is not taken into account, these dowels will be overstressed. To prevent this, the combined effect of both wheels should not exceed the capacity of the dowels as determined from Fig. 11-19. Correction factors that should be applied to the load-transfer capacity of the dowel system for various ratios of center to center of wheels to l can be read directly in Fig. 11-21.

If the wheel load could be concentrated on a point at the transverse edge and the dowels were 100 per cent efficient (both sides of the joint deflecting equally) a maximum of 50 per cent of the load would be transferred across the joint. Since neither condition can be obtained in the field, the dowels will not be overstressed if they are designed to transfer about 45 per cent of the load. This is well within the requirements (20 per cent load transference) for design by Eq. (11-1) and corresponding design chart (Fig. 11-5).

The following example illustrates the method of determining the diameter, length, and spacing of dowels:

1. Assume an unlimited number of gross wheel loads of 17,400 lb are expected on a pavement placed on a subgrade having a k value of 200 psi per in. and the pavement is to be constructed of concrete which will develop a flexural strength of 650 psi.

2. From Fig. 11-5, with a working stress of 325 psi (65% of), it is found the required pavement thickness is 9 in.

3. With 45 per cent of the wheel load to be transferred across the joints, the amount to be transferred is 7,830 lb (0.45 X 17,400).

4. Assume a dowel spacing of 12 in. and determine the effectiveness of the dowel system. The radius of stiffness l for a 9-in. slab on a subgrade with a k of 200 psi per in. is 33.39 in. and 1.8l would be 60.1 in. (Table 11-10). Any dowel this far or farther
from the load would have an effectiveness of zero in transferring load and a dowel directly under the load would have an effectiveness of 1.0. Intermediate dowels would be effective in inverse proportion to their distance from the load. The effectiveness of the dowel system would be determined as follows:

<table>
<thead>
<tr>
<th>Distance of dowel from load, in.</th>
<th>Computation</th>
<th>Effectiveness of dowels</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.00</td>
<td>1.0</td>
</tr>
<tr>
<td>12</td>
<td>0.00</td>
<td>0.8</td>
</tr>
<tr>
<td>24</td>
<td>0.00</td>
<td>0.6</td>
</tr>
<tr>
<td>36</td>
<td>0.00</td>
<td>0.4</td>
</tr>
<tr>
<td>48</td>
<td>0.00</td>
<td>0.2</td>
</tr>
<tr>
<td>60</td>
<td>0.00</td>
<td>0.0</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>3.0</td>
</tr>
</tbody>
</table>

The effectiveness of the dowel system is the equivalent of three single dowels. This can be read directly from Fig. 11-20. Entering graph for an \( l \) of 33.39 and 12-in. spacing, the capacity factor of the system is found to be 3.0.

5. For a capacity factor of 3.0 and 7,830 lb to be transferred by the system, the required capacity of an individual dowel would be \( \frac{7,830}{3} = 2,610 \) lb.

6. The capacity of individual dowels can be determined from Fig. 11-19. For a 0.25-in. joint opening, the capacity of a 1-in. round dowel is found to be 2,590 lb, so this size dowel 16 in. long (embedment each side of joint eight times the diameter) will be satisfactory at contraction joints. At expansion joints (0.75-in. opening) a 1\( \frac{1}{8} \)-in. dowel which would transfer 2,610 lb would be required. These dowels should be 2 by 8 by 1\( \frac{1}{8} \) in. (18 in.) long for embedment, plus \( \frac{3}{4} \) in. for joint space, plus 1\( \frac{1}{4} \) in. for expansion cap, or 20 in. long.

7. Check dowel system to make sure there is no overlapping of dowel stress from the other wheel load on an axle. The average spacing center to center of wheel loads on an
Table 11-11. Size of Round Dowels Required at 12-in. Spacing for 1.0-in. Openings (Expansion Joints)

<table>
<thead>
<tr>
<th>Pavement thickness, in.</th>
<th>Values of $k$, psi/in.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>50</td>
</tr>
<tr>
<td>7</td>
<td>$\frac{3}{4}$</td>
</tr>
<tr>
<td>8</td>
<td>$\frac{3}{4}$</td>
</tr>
<tr>
<td>9</td>
<td>1</td>
</tr>
<tr>
<td>10</td>
<td>$\frac{3}{4}$</td>
</tr>
<tr>
<td>11</td>
<td>$\frac{3}{4}$</td>
</tr>
</tbody>
</table>

Table 11-12. Size of Round Dowels Required at 12-in. Spacing for 0.25-in. Openings (Contraction Joints)

<table>
<thead>
<tr>
<th>Pavement thickness, in.</th>
<th>Values of $k$, psi/in.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>50</td>
</tr>
<tr>
<td>7</td>
<td>$\frac{3}{4}$</td>
</tr>
<tr>
<td>8</td>
<td>$\frac{3}{4}$</td>
</tr>
<tr>
<td>9</td>
<td>$\frac{3}{8}$</td>
</tr>
<tr>
<td>10</td>
<td>1</td>
</tr>
<tr>
<td>11</td>
<td>$\frac{3}{8}$</td>
</tr>
</tbody>
</table>

axle is 4 ft 11 in., or 59 in. There will be no overlapping of stress unless the spacing of wheels is less than 1.8t. This value for the 9-in. pavement on a subgrade having a $k$ value of 200 (Table 11-10) is 60.1 in.; so there will be only insignificant overlapping of stress in the dowels.

8. For the 9-in. pavement required for the loading, subgrade, and concrete strength, 1- by 16-in. dowels will be satisfactory at transverse contraction joints and 1½- by 20-in. dowels will be satisfactory at transverse expansion joints having a ¾-in. opening and an expansion cap on one end of each dowel. For the 12-in. spacing of intermediate dowels, outside dowels should be half the normal spacing, or 6 in. from each edge of a traffic lane.

When dowel sizes are selected, the diameter should not exceed one-sixth the thickness of the pavement in which they are used. The dowel sizes finally selected should be such that the center-to-center spacing will not be less than 10 or more than 18 in. The spacing most commonly used is 12 in.

The diameter of smooth round dowels, having a 12-in. spacing, for various thicknesses of pavement on subgrades of a wide range of $k$ values, is given in Table 11-11 for expansion joints (1-in. opening) and in Table 11-12 for contraction joints (0.25-in. opening). In determining these dowel sizes, the possible overlapping of dowel stresses was taken into account where necessary, the spacing center to center of wheels being assumed to be 59 in.

**JOINTING OF CONCRETE PAVEMENT**

Longitudinal and transverse joints are installed in concrete pavements to keep volume-change stresses within safe limits and thus prevent the formation of irregular cracks (Fig. 11-22).

Longitudinal joints are used to control longitudinal cracking. The more common methods of forming longitudinal joints are shown in Fig. 11-23. Of these, the dummy groove, either sawed in the hardened concrete or formed in the plastic concrete, and deformed metal plate are most widely used. Pavements more than four lanes in
Fig. 11-22. Principal types of joints, in plan.

Fig. 11-23. Longitudinal joint details.

Width should be provided with one or more free (without tie bars or bonded reinforcement) longitudinal joints. The pavement edges at these free joints must be thickened if the slab has been designed as a thickened-edge section.

Transverse joints fall in two general classes—expansion joints and contraction joints. Details covering the more common types of transverse joints are shown in Fig. 11-24. Expansion joints are formed as indicated in Fig. 11-24 by embedding in the concrete a vertical layer of nonextruding compressible material, ¼ to 1½ in. in thickness, so shaped that a complete separation of the slab is obtained.
Surveys made of pavements in service show that no expansion space need be provided when (1) the pavement is constructed of materials that have normal expansion characteristics, (2) the pavement is constructed during those periods of the year when normal construction temperatures prevail, (3) the pavement is divided into relatively short panels by satisfactory contraction joints, and (4) the contraction joints are properly maintained to prevent the infiltration of relatively incompressible material, such as soil fines.

Concrete pavements built during cold weather should be provided 3/4-in. expansion joints at 600- to 800-ft intervals. Expansion joints should be used where pavements join structures, such as bridges and railroad tracks, or at the intersection with other pavements. The width of these expansion joints need not be excessive; a 3/4-in. width is the most common.

Of the contraction joints shown in Fig. 11-24, the dummy groove, either sawed in hardened concrete or formed in plastic concrete, and the ribbon or premolded-strip contraction joints are the most widely used.

For unreinforced-concrete pavement, in event no experience record is available, the following tabulation may be used as a guide in selecting a spacing for the contraction joints.

---

Complete elimination of cracking by adequate jointing cannot be attained on all projects or ensured on any project. However, if the above recommended spacings are followed and the joints are properly designed, cracking can be reduced to a point where the few cracks that occur will be of little significance and will require no special attention.

Dowels are required across all contraction joints when the joint spacing exceeds about 20 ft or when expansion joints are spaced at intervals less than about 500 ft.

<table>
<thead>
<tr>
<th>Kind of coarse aggregate</th>
<th>Joint spacing, ft</th>
</tr>
</thead>
<tbody>
<tr>
<td>Granite</td>
<td>25</td>
</tr>
<tr>
<td>Limestone</td>
<td>20</td>
</tr>
<tr>
<td>Flinty limestone</td>
<td>20</td>
</tr>
<tr>
<td>Gravel:</td>
<td></td>
</tr>
<tr>
<td>Calcareous</td>
<td>20</td>
</tr>
<tr>
<td>Siliceous</td>
<td>15</td>
</tr>
<tr>
<td>Less than 3/4 in. in size</td>
<td>15</td>
</tr>
<tr>
<td>Slag</td>
<td>15</td>
</tr>
</tbody>
</table>

![Graph](image1.png)  
![Graph](image2.png)

**Fig. 11-25. Typical examples of average seasonal changes in contraction-joint widths. Relationship between joint widths and spacings of expansion and contraction joints.**

Dowels are required across the first 8 to 10 contraction joints either side of an expansion joint when the expansion-joint spacing is greater than about 500 ft and the contraction-joint spacing does not exceed about 20 ft. The amount of opening decreases inversely with the distance away from the expansion joint and at about 10 joints back from the expansion joint is essentially the same as in a pavement without expansion joints.

If distributed steel is used, the spacing of transverse contraction joints should not exceed about 40 ft. If joints are spaced at longer intervals, the joint movement will probably be of such magnitude (greater than 0.20 in.) that it will be difficult to maintain an adequate seal. Figure 11-25 may be used to estimate the probable width of joint opening.

The weight and cost of distributed steel increase and the cost of transverse joints and dowels decreases with greater slab length. The most economical spacings of joints for pavements using distributed steel can be determined by computing, for several joint spacings, the total cost of transverse joints, dowels, and distributed steel per unit length (100 ft) of pavement. It will usually be found to be about 40 ft.

Because of the longer joint spacing used on reinforced pavements and the greater joint openings that result (Fig. 11-25), all contraction joints in reinforced pavements should be doweled. Figure 11-25 may also be used to estimate the average width of...
the joint that must be spanned by a dowel bar. This joint width is needed to determine load-transfer capacity of a single dowel from Fig. 11-19.

The groove above all longitudinal and transverse joints should be sealed with plastic material after the concrete hardens and before the pavement is opened to traffic. If bituminous material is used for sealing, it should be as soft as can be used, without flowing from the joint in warm weather. Rubber-bituminous sealing compounds, developed and marketed by several manufacturers, are now being quite extensively used and are giving good results if properly heated and handled during placement.

**CONCRETE PAVEMENTS FOR MUNICIPAL STREETS**

The structural design of a pavement for a city street differs little from that of a rural pavement, but they are usually multilaned and have curbs to confine surface water.

![Diagram of pavement cross sections](image)

**Note:**
If combined curb and gutter is used in place of the integral curb shown above, the gutter is tied to the pavement by one of the following methods:

- Combined curb and gutter
- \( \frac{3}{8} \times 24^\circ \) tie bar spaced
- \( 30^\circ \) c-c if thickened-edge design used
- \( 60^\circ \) c-c if uniform thickness design used

**Fig. 11-26. Typical cross sections used on city street pavements.**

and provide for its disposal within the limits of the pavement, and intersections are more complicated and occur at more frequent intervals.

For most streets, except in industrial or warehouse districts, the larger slower-moving vehicles tend to travel on the outside lane on each half of the pavement, thereby influencing the computation of the controlling wheel load. Only in those areas where the traffic is almost entirely trucks is it safe to assume that they will be evenly distributed in all lanes. The central areas of some intersections must be thickened where the combined effect of the heavy wheel loads on each street demands it.

Where the thickened-edge type of cross section is used the slab must be thickened along all free longitudinal joints as shown in Fig. 11-26. The edges of the pavement need not be thickened if integral or straight curbs are used, or if combined curb and gutter is adequately keyed and tied to the pavement edge.
Subbases

Subbases should be used where they are required to correct or counteract an unsatisfactory or an unstable soil condition. Subbases to prevent pumping are required under street pavements only when the subgrade is highly plastic. They are not required on fine-grained soils of medium to low plasticity as such soils are not potential pumpers under the conditions of traffic and moisture that exist on street pavements.

Jointing of Street Pavements

Longitudinal joints in city pavements are usually laid out so that they coincide with the lane markings. The 8- to 12-ft spacing that results is adequate to control longitudinal cracking, provided not more than four lanes are tied together.

![Diagram of municipal pavement joint layout](image)

Fig. 11-27. Municipal pavement, typical joint layout for 64- and 36-ft streets and intersection.

Transverse joints, regardless of type, must be continuous back of curb to back of curb. Transverse construction joints should be placed only at locations designated for expansion or contraction joints.

Expansion joints should be placed at the outer limits of all intersections as shown in Fig. 11-27. No expansion joints are required within the block when the pavement is jointed so as to control cracking.

Figure 11-27 shows how an intersection of the common 90° type would be jointed. It is recommended that this same general scheme be followed except where the streets intersect at rather sharp angles. The interior longitudinal joints of each street are continued through the intersection unchanged in type (tied or free). One or more of these joints may serve as a construction joint. A free keyed joint is placed across each
entering street on line with the longitudinal joint between pavement and gutter pan if combined curb and gutter is used or on line with the face of the curb if separate or integral curb is used. From the junction of these joints a short free keyed joint is placed through the center of the return. This joint must be carried through to the back of the curb. If the pavement has been designed as a thickened-edge slab, all free joints within any intersection must be thickened to prevent the formation of a structurally inadequate joint.

CONCRETE BASES

Any concrete base must have the same structural strength and durability as a concrete pavement that would be used under the same traffic and subgrade conditions.

The structural design of a concrete base is identical to that of a concrete pavement, be it for a rural highway or a city street. Subbases should be used under concrete bases where they will correct or counteract an unsatisfactory soil or an unstable soil condition. The spacing of longitudinal and traverse joints should be the same as those used in concrete pavement if the elimination of irregular surface cracks is desired. A nonextruding type of expansion-joint filler must be used; otherwise a bump will be formed in the surface course.

Concrete bases are finished true to line, grade, and cross section, but they do not require the refinements of surface finish that are needed on concrete pavements. To increase the bond between the base and bituminous surface, most designers prefer a surface texture rougher than that obtained with a belt or a burlap drag. The rough texture may be obtained in numerous ways, the most common being to score the surface of the base with a heavy coarse-fiber broom. To produce the desired rough texture, the brooming should be done shortly after the water sheen has disappeared from the concrete.
Section 12

FOUNDATIONS

By

HAL W. HUNT, Editor, Civil Engineering, New York, N.Y. (Formerly Engineer for Western Foundation Corporation.)

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FOUNDATIONS

Foundations are a far more important part of a satisfactory structure than sometimes is recognized. Many a well-planned structure has cost more than anticipated or has had to be redesigned or even abandoned because underlying materials have not been adequate to support the proposed loads.

Knowing what is beneath the surface is an economic prerequisite to selection or purchase of a site for a heavy structure. An experienced foundation man can give valuable money-saving advice in selection of site and in type of structure. And it is essen-
tial for design of any structure to know what it is to rest on and what can reasonably be expected to happen to the underlying soils from the added load. (Discussion here presupposes a general knowledge of soil and of pressure bulbs. It is limited to common soils and simple structures. For conditions out of the ordinary, specialized engineering services and comprehensive study of the problem are necessary.)

The permissible foundation settlement also varies widely with the type of structure. For example, stresses in concrete arches or rigid frames would become dangerously high with minor differential movement while timber structures might deform to follow some settlement without serious danger of failure.

Test pits dug by hand or machine may give sufficient subsurface information to depths of about 20 ft but seldom can be dug deeper without expensive bracing. Frequently the basement and footings go almost this deep so that pits seldom give more information than just the type of soil on which footings will directly bear. Undisturbed samples can be secured from such pits, but despite load tests on small areas, little can be learned of possible compressible strata below. Rod soundings, in which a small-diameter rod is driven into the ground, give some idea of how far it may be to underground obstruction but tell nothing of strata passed.

Subsurface borings to almost any depth can be used to determine underlying material and secure samples of the strata penetrated. Not all borings need go full depth but enough should to establish an area pattern. Samples should be taken as nearly undisturbed as is economically practical. Borings are made by driving a pipe casing into the ground and, at intervals of perhaps 5 ft and at strata changes, taking samples ahead of the casing of the strata penetrated.

Samples are preferably taken by means of a so-called standard penetration test in which a 1½ in. inside diameter (2 in. outside diameter) sampling “spoon” with an open barrel 22 in. long is driven out of the lower end of a cleaned-out 2½-in. pipe casing. The sampling spoon is driven with a 140-lb weight falling 30 in. As a standard basis the spoon is driven 6 in.; then the blows required to drive it the next foot are recorded. There are several other recognized spoon sizes and methods of driving.

“Bucket samples” are taken by collecting solids returned by water used to wash out the hole. These are not dependable because fines are lost in the wash water and the true nature of material is not indicated. Wash samples are useful in indicating the changes of strata while drilling. However, the Building Code of New York City calls it “unlawful” to use wash or bucket samples.

For many simple structures, the material taken in a sample spoon, with information on the energy required to drive the sampler, are adequate for experienced foundation engineers to determine whether simple spread footings are adequate or more expensive foundations are required.

For important structures, and frequently for any structures over deep layers of undependable materials, a complete soil analysis of carefully taken and preserved soil samples is desirable to give reliable indications of probable settlement. Such investigation usually will save many times its cost. Soil analysis for heavy structures is a development of the last few years. Some early errors discredited the theoretical approach but means are now available for dependable analysis that will assure saving with safety.

Samples for such analysis are made with larger apparatus than the 2 in. outside diameter spoon and their taking is supervised by qualified soil engineers who appreciate the importance of careful taking and handling of samples.

All information developed or found on subsurface conditions should be made available to bidders on foundations. Data should include the blows to drive the casing and the sample spoon, obstructions encountered, holes abandoned, and the like. Withholding any information may be the basis for legal action if a contractor is injured by lack of such available knowledge.

SPREAD FOOTINGS, MATS, OR PILES

Spread footings usually are the most economical foundations when they can be placed immediately below the required excavation. Pressure on the soil should be
about the same at all footings so that settlement will be as nearly uniform as possible. Settlement itself in moderate amounts is not necessarily harmful; it is differential settlement that causes trouble.

Allowable loads on soils is fixed by building codes in many cities and some states. The presumptive surface bearing values of foundation materials adopted by the Building Officials Conference of America (BOCA) for nationwide use is shown in Table 12-1 and that for New York City in Table 12-2. The terms used in the New York City Code are defined in Tables 12-3 and 12-4, by relation to spoon blows and soil analysis. This is a larger spoon and different blow from that described for the "standard" penetration test.

### Table 12-1. Building Officials of America Code, Presumptive Surface Bearing Values of Foundation Materials

<table>
<thead>
<tr>
<th>Class of material</th>
<th>Tons/sq ft</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Massive crystalline bedrock including granite, diorite, gneiss, traprock, hard limestone, and dolomite</td>
<td>100</td>
</tr>
<tr>
<td>2. Foliated rock including bedded limestone, schist, and slate in sound condition</td>
<td>40</td>
</tr>
<tr>
<td>3. Sedimentary rock including hard shales, sandstones, and thoroughly cemented conglomerates</td>
<td>25</td>
</tr>
<tr>
<td>4. Soft or broken bedrock (excluding shale) and soft limestone</td>
<td>10</td>
</tr>
<tr>
<td>5. Compacted, partially cemented gravels, sand, and hardpan overlying rock</td>
<td>10</td>
</tr>
<tr>
<td>6. Gravel and sand-gravel mixtures</td>
<td>6</td>
</tr>
<tr>
<td>7. Loose gravel, hard dry clay, compact coarse sand, and soft shales</td>
<td>4</td>
</tr>
<tr>
<td>8. Loose coarse sand and sand-gravel mixtures and compact fine sand (confined)</td>
<td>3</td>
</tr>
<tr>
<td>9. Loose medium sand (confined), stiff clay</td>
<td>2</td>
</tr>
<tr>
<td>10. Soft broken shale, soft clay</td>
<td>1.5</td>
</tr>
</tbody>
</table>

### Table 12-2. Classification of Supporting Soils

As specified in the New York City Code

<table>
<thead>
<tr>
<th>Class</th>
<th>Material</th>
<th>Max allowable presumptive bearing values, tons/sq ft</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Hard sound rock</td>
<td>60</td>
</tr>
<tr>
<td>2</td>
<td>Medium hard rock</td>
<td>40</td>
</tr>
<tr>
<td>3</td>
<td>Hardpan overlying rock</td>
<td>12</td>
</tr>
<tr>
<td>4</td>
<td>Compact gravel and boulder-gravel formations; very compact sandy gravel</td>
<td>10</td>
</tr>
<tr>
<td>5</td>
<td>Soft rock</td>
<td>8</td>
</tr>
<tr>
<td>6</td>
<td>Loose gravel and sandy gravel; compact sand and gravelly sand; very compact sand, inorganic silt soils</td>
<td>6</td>
</tr>
<tr>
<td>7</td>
<td>Hard dry consolidated clay</td>
<td>5</td>
</tr>
<tr>
<td>8</td>
<td>Loose coarse to medium sand; medium compact fine sand</td>
<td>4</td>
</tr>
<tr>
<td>9</td>
<td>Compact sand-clay soils</td>
<td>3</td>
</tr>
<tr>
<td>10</td>
<td>Loose fine sand; medium compact sand-inorganic silt soils</td>
<td>2</td>
</tr>
<tr>
<td>11</td>
<td>Firm or stiff clay</td>
<td>1.5</td>
</tr>
<tr>
<td>12</td>
<td>Loose saturated sand-clay soils; medium soft clay</td>
<td>1</td>
</tr>
</tbody>
</table>

Fordham and Ravenswood gneiss and trap are classed as hard rock. Inwood limestone, Manhattan schist, and massive serpentine are medium hard. Shale, decomposed serpentine, schist, and gneiss are classed as soft rock.

Mat footings over an entire area are expensive and sometimes have been in difficulty as they concentrate the soil load under the center of the structure much more than is done by individual footings. Mat footings have been used as floating foundations after sufficient weight of earth is excavated to about balance the weight of the structure to be built. This should be done only with very competent foundation engineering studies as many factors contributing to equilibrium are disturbed.

Pile or caisson foundations become economical and then necessary for a given site as depth to adequate bearing material increases. Soil strata, ground water, speed desired for construction, and other factors affect the depth at which the change is economically justified.
FOUNDATIONS

Table 12-3. Explanation of Terms
As specified in the New York City Code

<table>
<thead>
<tr>
<th>Descriptive term</th>
<th>Blows/ft</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Loose</td>
<td>15 or less</td>
<td>These figures approximate for medium sand, 2½-in. spoon, 300-lb hammer, 18-in. fall. Coarser soil requires more blows, finer material, fewer blows</td>
</tr>
<tr>
<td>Compact</td>
<td>16 to 50</td>
<td></td>
</tr>
<tr>
<td>Very compact</td>
<td>50 or more</td>
<td></td>
</tr>
</tbody>
</table>

Consistency Related to Spoon Blows; Mud, Clay, Etc.

<table>
<thead>
<tr>
<th>Descriptive term</th>
<th>Blows/ft</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Very soft</td>
<td>Push to 2</td>
<td>Molded with relatively slight finger pressure</td>
</tr>
<tr>
<td>Soft</td>
<td>3 to 10</td>
<td></td>
</tr>
<tr>
<td>Stiff</td>
<td>11 to 30</td>
<td>Molded with substantial finger pressure; might be removed by spading</td>
</tr>
<tr>
<td>Hard</td>
<td>30 or more</td>
<td>Not molded by fingers, or with extreme difficulty; might require picking for removal</td>
</tr>
</tbody>
</table>

Table 12-4. Soil Sizes
As specified in the New York City Code

<table>
<thead>
<tr>
<th>Descriptive term</th>
<th>Pass sieve No.</th>
<th>Retained sieve No.</th>
<th>Size range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clay</td>
<td>200</td>
<td>hydrometer analysis</td>
<td>0.006 mm</td>
</tr>
<tr>
<td>Silt</td>
<td>200</td>
<td></td>
<td>0.006 to 0.074 mm</td>
</tr>
<tr>
<td>Fine sand</td>
<td>65</td>
<td>200</td>
<td>0.074 to 0.208 mm</td>
</tr>
<tr>
<td>Medium sand</td>
<td>28</td>
<td>65</td>
<td>0.208 to 0.589 mm</td>
</tr>
<tr>
<td>Coarse sand</td>
<td>8</td>
<td>28</td>
<td>0.589 to 2.362 mm</td>
</tr>
<tr>
<td>Gravel</td>
<td></td>
<td>8</td>
<td>2.362 mm</td>
</tr>
<tr>
<td>Pebble</td>
<td></td>
<td></td>
<td>2.362 mm to 2½ in.</td>
</tr>
<tr>
<td>Cobble</td>
<td></td>
<td></td>
<td>2½ to 6 in.</td>
</tr>
<tr>
<td>Boulder</td>
<td></td>
<td></td>
<td>6 in.</td>
</tr>
</tbody>
</table>

TYPES OF PILES

Piles are of several types and materials. Best known and simplest are timber piles where a tree trunk is driven into the ground. For convenience and ease of driving the small end usually is driven down. It is coming more and more to be recognized that for some soil conditions it is desirable to have large circumferential area and large bottom for contact with the dependable underlying strata to which the pile is driven. In some cases timber piles have been quite successfully driven butt down (Engineering News-Record, June 21, 1951, p. 30).

Timber piles should be placed so they are below permanent ground water; should be treated with a preservative to add to their dependable life; or as described later, can have concrete tops above ground water. Individual timber piles probably are the cheapest of all such units but are not satisfactory for many uses and seldom are the economic solution for permanently carrying heavy load concentrations.

Next simplest of piles is the structural H, with web and flange made the same thickness and about the same dimension. Heavy sections can be used to penetrate quite hard stratum to reach bearing on rock. Disadvantages of the H pile are high material cost and some uncertainty, where unusual obstructions may be encountered, that the tip reaches the desired stratum. Driving against boulders has been known to rip flanges from the web and perhaps start each of them to curl, resulting in uncertain bearing value. If care is taken when driving in boulder strata H piles are quite satisfactory.

Ordinary pipe may be driven open-end or with a closed bottom to serve as a pile. The open-end pile is frequently used when it can be driven to rock, cleaned out, and then filled with concrete. This pile can be inspected after completion of driving. If
completely cleaned out and redriven for firm seat in the rock it is one of the best and most dependable of piles and is allowed high load value. Limitations are high cost of material and difficulty of cleaning small pipe to considerable depth (12 in. diameter is a desirable minimum but smaller has been used). A minimum wall thickness of 0.3 in. is usual for open-end pipe to prevent collapse or tearing on boulders. Recently, piles 18 in. diameter and up have been driven open end into sand and gravel and concrete added only above the material compacted inside the pile (Engineering News-Record, Jan. 29, 1953, p. 32). In some cases quite heavy test loads have been successfully carried, 300 tons on an 18-in. pipe pile at Portland, Ore.; here the piles were in 60 ft of water when tested.

Pipe can be closed at the lower end with a flat plate, or convex or concave point and driven to form a pile that can be inspected to the bottom and then filled with concrete. For dependable driving without a mandrel, pipe usually must have a wall thickness of \( \frac{3}{16} \) in. or more. Some 8 gage (0.164 in.) and 10 gage (0.134 in.) is now being driven successfully. Most codes do not allow stress value on pipe walls less than \( \frac{3}{8} \) in. thick.

A variation of pipe is the fluted Monotube pile, with a tapered point, designed to be driven without a mandrel. The flutes give it added strength longitudinally and a nice appearance for exposed lengths above the ground in bridge approaches and the like. Weight of the steel is about that of comparable-load-value pipe so that the materials cost is high.

**THIN-SHELL CAST-IN-PLACE PILES**

A variation of the pipe pile is the thin-shell cast-in-place concrete pile in which a thin metal shell is installed in the ground as a form and filled with concrete. It is possible that the shell will rust out; so it is given no structural value in computing the bearing capacity of the pile.

Piles cast in place in thin shells require special equipment for installation and usually are driven by specialist companies. All these piles have been developed under patents, but most of the basic patents have expired. Installation continues by the original patentees who have developed know-how, clientele, and some detail patents that give them a competitive advantage.

In the United States the best known of the cast-in-place piles is Raymond Concrete Pile Company's tapered pile. There are two major types of Raymond piles; both use a light-gage steel shell left permanently in the ground. This is installed by driving with an inside mandrel of heavy steel; the bottom of the pile is closed by a convex steel shell. Where long piles are required, the shell for the next pile can be lowered inside a driven pile shell and then drawn up over the mandrel, permitting installation of a pile as long as the leads, except for the space required for the hammer.

The Raymond straight-taper pile (Fig. 12-1a) increases in diameter uniformly at the rate of 1 in. for each 2.5 ft of length. The shell is as light as 24 gage and is held open by a stiffening spiral wire inside the shell. It is driven with a tapered mandrel that is in contact with the shell for the full depth. The sharp taper rapidly increases the size of the pile; starting from an

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**FIG. 12-1.** Raymond Concrete Pile Company types of piles and caissons. (a) Standard (straight-taper) concrete pile; (b) step-taper concrete pile; (c) pipe—step-taper concrete pile; (d) wood composite pile; (e) Gow caissons.
8-in. tip the pile reaches 23 in. diameter in 37 ft, about the longest pile of this type it has been found practical to drive.

This straight-taper pile is good where it is desired to carry the load on the upper stratum of soil and where soils such as ocean-deposited sands require consolidation to increase the bearing capacity. It should be used with caution where firm material near the surface must be penetrated and then soft strata passed to reach good bearing.

Need for longer piles and to penetrate deeper to good bearing strata brought the development of the step-taper pile (Fig. 12-1b). Corrugated shells in about 8 or 12 ft length and increasing in size 1 in. diameter for each section are screwed together and driven by means of an inside mandrel, which bears at an enlarged “plow ring” at each joint of the pile. The plow ring opens up a hole larger than the shell to ease the driving. These piles have been driven to depths of 80 ft or more.

PILES OF CONSTANT DIAMETER

Piles of constant diameter for their full depth are advocated by the Western Foundation Corporation and the Pneumatic Pile Company. They use an 11- to 14-in.-diameter light corrugated shell similar to culvert material. Each company has its own method of installation.

Western’s standard is known as the button-bottom pile because a precast-concrete “button” larger than the pile is used as the bottom closure. The “button” is driven to the required resistance to penetration with a heavy-walled drive-tube (Fig. 12-2a); then a length of corrugated shell is lowered inside the drive tube and fastened to the precast button (Fig. 12-2b). The shell is filled with concrete and the drive tube is withdrawn (Fig. 12-2c), or the shell may be filled later. Normal maximum length is about 75 ft; using a special extension of the driving apparatus, these economical thin-shell concrete piles have been installed to depths of more than 110 ft.

The Pneumatic or Cobi pile uses a circular shell that is held while installed by an inside mandrel of nonslip steel plates expanded against the inside of the thin shell by pneumatic means to support and grip the shell while it is driven in contact with the soil. The Pneumatic Pile Company leases its process and special equipment for installation by others. Their pile can be readily driven as a batter pile. A mechanical disadvantage for long piles is that leads must be twice the length of the shell to be used plus the space needed for the hammer.

Variations of these piles, all cylindrical cast-in-place concrete for their full depth, are the compressed, pedestal, and caisson types. There also are several composite piles combining two or more materials or methods.

Compressed piles are so named because the concrete is forced out of the drive tube, as the latter is withdrawn, and compressed against the surrounding soil. The pile is economical as no enclosing shell is required, which is also an advantage when steel is not readily available. A heavy-wall drive tube with close-fitting core (Fig. 12-3a) is driven to the strata or resistance to penetration required; the core is withdrawn and the drive tube filled with a very dry (1-in. slump) concrete made with large aggregate; the weight of the core and hammer is then placed on top of the concrete and the drive-tube withdrawn (Fig. 12-3b and c). Western Foundation Corporation’s method of installing compressed piles prevents separation of the concrete in the shaft by rigging the pulling for the drive tube so that the core must be forced down in a fixed ratio to withdrawal of the drive tube. Thus the shaft must be continuous and of constant diameter larger than the withdrawn drive tube.

The pedestal pile is so named because a bulb of concrete is hammered out at a
desired stratum below the surface to compact the soil and provide essentially a spread footing. The large bulk of concrete reduces the unit load on the soil and improves the bearing capacity by tightening the soil.

The pedestal pile is installed by driving a heavy-walled drive tube (Fig. 12-4a) with close-fitting core to the desired depth; the core is withdrawn and a charge of concrete is placed in the drive tube (Fig. 12-4b) and driven out by use of the core and pile hammer as the casing is pulled up a little (Fig. 12-4c, d). The shaft is then formed by lowering a thin metal shell to the pedestal and filling (Fig. 12-4e) or by the method described for the compressed pile. The cased-type shaft can be reinforced against bending or to provide for tension in uplift. The pedestal pile with its large base and with its shaft reinforced is especially valuable and economical to resist uplift forces.

The Franki displacement caisson is an uncased column of cast-in-place concrete literally pounded into the ground. Outside the United States it is one of the most used of all piles. It is installed by (1) setting a steel-pipe drive tube, usually about 18 to 21 in. diameter, at the pile location; (2) very dry concrete is placed to fill the tube to a depth of 2 ft; (3) a 3-ton drop hammer about 15 in. diameter with rounded base is dropped on the concrete, first wedging the concrete plug against the drive tube and then driving the plug and pulling the drive tube into the earth; (4) when the drive tube reaches the acceptable bearing stratum the tube is anchored to the driving frame and the concrete plug partially driven out; (5) small quantities of very dry concrete are added and driven out to form a bulb; (6) concrete is added and pounded out to form a compacted shaft as the drive tube is slowly pulled up.

Franki piles or caissons installed in this way are used to carry 75- to 100-ton loads. Many building codes in the United States do not permit loads as high as this, so that in United States cities less load is allowed on this caisson than is accepted elsewhere. The method of installation used requires a large amount of labor. Where high loading is allowed, labor is cheap, and steel encasement material expensive, the Franki pile is less costly than others. In the United States where piles generally are driven by men working at high-skilled-labor rates, the Franki caisson has an economic handicap unless it is allowed higher load-carrying value. The Franki caisson is typical of European installation practice, where materials are more costly than labor.

Caisson piles are large-diameter concrete columns installed by driving what is essentially an open-end pipe to the rock or hardpan bearing stratum, cleaning out the soil by sand pumps or bailers, and then filling the open pipe with concrete and withdrawing the pipe. Size may range to 36 in. diameter and may be belled out to spread the load on hardpan or soft rock. A caisson pile can be reinforced to have quite high carrying capacity. Building codes generally class this type as a caisson and allow it much higher load than is acceptable for piles.

In dry silty soils and some others in which holes will stand open, earth augers are used very successfully to make a cored hole that can be filled with concrete to form a shaft in the ground. Methods have been developed to bell out the bottom mechani
cally to give enlarged bearing area. Where water is a problem, augers can be made practical by use of an additive to make a heavy slurry of water and soil in the drill hole. This keeps the walls of the hole from caving. A pipe or thin shell casing can be lowered into the hole, displacing the slurry, and concrete placed in the shell. Use of the auger method is increasing rapidly.

Gow Caissons (Fig. 12-1e) are cylindrical steel shafts in sections about 5 ft high with diameters starting at 5 to 7 ft. Method of installation is to hand-dig the shaft, placing successively smaller cylinders one inside another as excavation proceeds, to prevent the earth from caving. Considerable depth can be reached by use of the successively smaller sections but with a corresponding decrease in shaft size. The shaft can be belled out at the bottom to provide greater soil-bearing area. As concrete is placed, the steel sections are withdrawn for reuse.

**COMPOSITE PILES**

The wood-composite pile combines the economy of untreated timber for the length below ground-water level with the permanency of concrete for the upper section exposed to changing moisture conditions. It is important that the wood be kept below the level of any sewer or pumpable level to assure that it does not become dry and deteriorate to a point where it permits failure of a structure. The joint between sections is of utmost importance. It should be designed to take bending or uplift as required and the concrete should be placed in the dry.

For long or moderate-length concrete sections of wood-composite piles (Fig. 12-1d), the light corrugated shell used for the Raymond, Western, or Cobi type of cast-in-place piles is most economical. For short concrete sections, a piece of 12-in.-diameter pipe fitted over and spiked to the top of the timber, then driven with a follower, has been used successfully. Usually, the wood section is driven first to a little above the ground surface, then followed down by apparatus used for installation of the thin shell for cast-in-place concrete piles. The thin shell is connected to the timber and the joint reinforced, if desired, before the concrete is placed.

Pipe-composite piles (Fig. 12-1c) are formed in much the same way as wood composites, but primarily are used where bearing strata are deeper than can be reached by the thin-shelled-type pile. Again, where an outside drive tube is available it frequently is advantageous to drive it first through overlying material and then drive the pipe section out as a projectile without the friction drag of a long length of pile. To prevent water or other foreign material from getting into the long pile, where it is almost impossible to be certain that it is removed, the desirable practice is to fill the pile immediately with concrete.

**DRILLED-IN CAISSONS**

For heavy column loads, where bearing can be obtained on rock, the drilled-in caisson offers many advantages. An 18- to 36-in. pipe is driven open-ended to rock, being cleaned out as required to expedite driving (Fig. 12-5a). If boulders or other obstructions are encountered, they are cut out by a heavy bit on a churn drill (Fig. 12-5b). When dependable bearing stratum is reached, the drill is used to cut into rock and the pipe is redriven to seat it firmly in the rock. Then drilling is continued into the rock for some distance, depending on the load to be carried (Fig. 12-5c). A steel H section is set down into the rock and concrete is placed to bond the H to the rock section and make a unit of the H, the concrete, and the pipe (Fig. 12-5d, e). This “locked in the rock” foundation has been used to carry loads up to 1,800 tons on a single unit and can be designed to carry heavier loads. It has value for tension as well as compression and can be installed on a batter where it gives special economies in bridge pier and similar construction.

The Montee caisson is installed by attaching a serrated cutting edge to a pipe of desired size and turning the pipe to cut its way down while washing and bailing out the material from inside the pipe. A powerful motor is required to turn the pipe against ground friction despite water jets on the exterior. The pipe is left in place to encase the concrete. The Montee caisson has given phenomenal progress on some holes
while adjacent ones have been in considerable difficulty. The greatest fault with this method is that rock and other hard material inside the caisson work their way to the bottom as the soft soil is washed and bailed out. Great difficulty has been experienced in removing the boulders to get a satisfactory seat on the bearing stratum. Only minor use has been made of this system since 1940.

Additional information on the proprietary systems of piling will be found in brochures in Sweet's Engineering and Architectural Catalog files.
PRECAST-CONCRETE PILES

Concrete frequently is precast into a square, round, or octagon shape for bearing piles and in tongue-and-groove type for sheet piling. Casting is done at a central point in horizontal forms and the piles are moved to the point of use as they attain adequate strength.

Precast piles are reinforced primarily to resist handling stresses. The reinforcement also may be figured in computing material stresses in the pile but for permanent bearing is held to about 4 per cent of the cross-sectional area of the pile, with the allowable load limited by a ratio to the concrete strength.

Reinforced precast piles are used extensively for waterfront work in piers, wharfs, and the like and for bridge approaches where long sections of the piles are to be left exposed above ground. The piles have considerable column strength and can be made attractive for permanent exposure. Precast piles have been made in lengths as great as 200 ft but their great weight and cumbersome handling limit use of the long sections to delivery and driving on water with large floating equipment.

Care is necessary when installing precast piles in sea water as sometimes minute cracks open up in handling and driving and permit salt to come in contact with the reinforcing, with resulting progressive spalling from rusting. To reduce handling stresses quite elaborate pickup arrangements are sometimes specified.

Most precast piling going into sea water, and many for ordinary use, are now longitudinally prestressed. High-tensile wires are tensioned along a bed perhaps 400 ft long and bulkheads set in to give the desired pile length. Bearing piles frequently have a void in the center, formed by a heavy paper tube, to reduce weight. For unusual conditions of heavy loads and long lengths or need for column strength, mesh reinforced sections about 16 ft long are cast centrifugally in desired sizes—3-ft to 5-ft diameters are common. After curing, sections are assembled into desired lengths (200 ft is not unusual) and high-tensile wire pulled through them and tensioned by jacks to make a unit. The prestressing greatly simplifies handling as well as keeping the concretes always in compression for greater efficiency.

Considerable information on precast piles is available in a booklet Concrete Piles, which can be obtained from the Portland Cement Association.

An example of what can be done with precasting of piles is the waterfront yard of Ben Gerwick, Inc., on San Francisco Bay. A complete range of sizes of piles are cast stockpiled, moved to barges, delivered anywhere in the Bay area by water, and driven at prices competitive with cast-in-place piles. In most areas, however, cast-in-place piles are more economical for support of average land structures than are precast piles.

Concrete is used extensively for protection of wood and steel piles and sometimes for additional protection or repair of concrete piles at the mud line in sea water.

Pneumatically applied mortar (gunite) has been successfully put on wood piles by notching the pile and then wrapping with a nailed-on mesh. Handling or hard driving does not seem to affect the mortar. The same method has been used to some extent with H piles and has been reported successful.

For waterfront structures precast-concrete shells have been lowered or driven around wood, steel, and even concrete piles and filled with cast-in-place concrete to protect the pile from below the mud line to above high tide. Repairs to New Jersey piling (Civil Engineering, June, 1960) were made without dewatering by pumping portland cement (beaten into a colloidal grout in a special pump) into a simple underwater form.

In moderately firm ground H piles have been encased in concrete by driving a temporary enclosing shell and filling with concrete. For permanent protection a method should be specified that thoroughly cleans the steel and provides a uniform concrete encasement of the pile.

PILE LOADING AND PILE DESIGN

The load that can be safely and permanently carried is the most important question about piles. The load depends not only upon the strength of material in the pile but
also on the strength of the material into which it is driven. So the limiting load is not only that of the pile but that of the soil as well.

Best practice in pile-foundation design is to determine the load a pile will carry by tests, preferably in advance of final design. Usual procedure is to drive a few piles of the types considered and then load one or more to from one and one-half times to twice the load contemplated. Working load and the amount of the test load frequently are fixed by local building code limitations. Early testing is a money-saving procedure for moderate-sized or large jobs but may not be economical for small jobs.

If tests ever are to be made, the best time is in advance of design, as advantage can be taken of the information gained in making the most economical plan for the work. It must be remembered, however, that the load tests on single piles or even a group of piles will not indicate all that may happen under area loading. Adequate soil borings must be made to determine that the bearing stratum reached has sufficient depth to distribute the load over possible poorer soils below. Also it must be ascertained that soils below are not sufficiently compressible to permit damaging settlement from the general area loading.

Piles may obtain their bearing by friction from materials through which they are driven, they may be end bearing, or they may be a combination of both. With the exception of piles installed in water on rock swept clear of overburden there is some combination of end bearing and friction on nearly all piles.

Piles through material firmer than fluid can be designed as short columns. Portions of a pile free-standing in air or water are, of course, subject to the usual stresses of columns. Practice is to assume that the pile is “fixed” at 5 ft below the surface in firm material and 10 ft in soft material.

Piles with sufficient area to prevent overstress where they enter the bearing strata usually are of adequate minimum size but it is desirable to make the smallest dimension at least 8 in. for piles to carry 30 tons or more; 6 in. may be adequate for light loads.

Good practice in cast-in-place pile design allows concrete $\frac{3}{4} f' c$ (one-fourth the ultimate compressive strength of concrete at the age of 28 days) but not to exceed 1,000 to 1,200 psi.

Where pipe or thin corrugated shells are used for encasement of cast-in-place concrete piles metal less than $\frac{3}{8}$ in. thick usually is not allowed any stress, as it may rust away.

If the metal is $\frac{1}{6}$ in. or more thick preferred procedure is to deduct $\frac{1}{6}$ in. from faces subject to corrosion and then allow perhaps 12,000 psi on the reduced net section in addition to the load on the concrete. Some codes allow 9,000 psi on the entire area of steel, if the wall section is $\frac{3}{8}$ in. or more. Deducting the amount the steel is likely to corrode and allowing the higher stress seems more equitable.

For cast-in-place piles or caissons in which a metal shell or form for the concrete is not left in the ground, only 80 per cent of the theoretical area of the concrete should be used in computing the stress. This allows for minor variations of uncased concrete.

Where composite piles are used (combinations of concrete with wood or steel) the limiting loads of the weaker material or section should prevail. For example, Douglas fir, oak, Southern pine, and similar woods may be allowed a stress of 800 psi on the net section but frequently are limited to 20 tons on piles with a 6-in. tip and 25 tons on an 8-in. tip. This would be the limiting value for the composite pile despite the size or strength of the concrete top.

Bethlehem Steel Company recommends 9,000 psi on the entire section of H piles. It will be noted that all the steel-bearing-pile sections have webs and flanges of the same thickness and that all are greater than $\frac{3}{8}$ in.

Drilled-in caisson is a special composite caisson in which the steel H-section core is placed deeply into a rock socket and carefully concreted so that it is not subject to corrosion and is fully supported for its entire length. This H core can be allowed the same stress as the structural steel of the structure it is to support. To this can be added the full working load of the concrete plus the value of the encasing pipe, less perhaps $\frac{3}{16}$ in. for corrosion.
DRIVING PILES

Piles generally are driven with a drop, steam, or diesel hammer, sometimes assisted by water jets. Frequently air takes the place of steam to actuate the ram. Piles also may be installed by jacking against an existing load, the usual procedure for underpinning existing structures.

Use of drop hammers is limited now to smaller jobs and to timber piles. Piles supporting quite important structures have been driven with drop hammers, but steam hammers, which can use air as well, have been developed to do a more economical job. Diesel hammers are gaining widespread acceptance.

Steam hammers are divided into two major classes, single-acting and double-acting. In single-acting hammers the steam works against a piston to raise a heavy weight; the power stroke is a gravity fall of the weight, which strikes the pile to drive it. In the double-acting hammer steam raises the weight or ram, then is applied to the other side of the piston to accelerate and power the fall of the ram. The double-acting hammer hits roughly twice the number of blows of the single-acting.

For heavy piles or driving equipment the single-acting hammer is preferred by many construction men. The heavier ram seems more successful in moving heavy weights. But on one long-pipe-pile job a double-acting hammer drove the piles almost twice as fast as a higher-energy single-acting hammer made by the same manufacturer.

Conversely, the double-acting hammer is preferred for sheet piling and frequently for light shell piles driven without a mandrel.

It is essential that the double-acting hammers be operated at full rated steam pressure, speed, and length of stroke to get the specified energy. Single-acting hammers should be checked for full height of fall. They also operate best at full steam pressure and speed.

Diesel hammers are a development of military engineers seeking greater mobility for pile-driving machines. The hammers are single-acting; the ram is first raised by outside means and dropped on a piston that compresses the fuel to combustion pressure and temperature. Explosion raises the ram and the cycle continues. Diesel hammers are now in successful use. There is some question of the driving force of the diesel hammer as the energy dissipated by the compression is not known.

Vibration driving of piling has been developed by Russian engineers. Rotating weights on several shafts provide the driving force. By having half the shafts rotate at half the speed of the others the downward driving force is larger than the upward component. One unit weighs 12 tons and creates a maximum downward exciting force of 132 tons when operated at 1,450 rpm by a 220-kw motor. Experiments reported from China (Civil Engineering, December, 1958, p. 54) indicate that for open-end piles the rate of sinking is somewhat independent of diameter and is quite rapid in clays as well as in sand. Penetration of 54 ft in clay in 3 hr is reported for a 16-ft-5-in. diameter thin-wall concrete cylinder. In fine sand a depth of 75 ft was reportedly reached in 14 min with a 9-ft 10-in. cylinder. No jetting was used.

Piles are driven to a resistance to penetration determined by pile-driving formula, or by driving and subsequent test loading, to be adequate for the loads to be carried. When the test method is used, formulas may be utilized for correlation of differing conditions.

Complete dependence on resistance to driving as a criterion for the carrying strength of a pile is dangerous and can be disastrous. It cannot be repeated too often that borings to determine subsurface conditions along the pile and for some distance below the pile tip are absolutely essential.

Resistance to driving in granular or noncohesive soils such as sand has in some areas been found to be more or less dependable as a guide and sometimes is accepted by codes, under conservative load limitations.

Formulas have little value for piles obtaining their load-bearing ability by friction in clay. Experienced soils engineers contend that undisturbed samples analyzed in the laboratory will show the depths to which piles need to be driven to support various loads. All foundation men are agreed that load test of piles or pile groups is desirable
in areas to be loaded in a new or more severe way and that for large projects and heavy structures tests are essential for safety as well as economy.

Best known of the pile-driving formulas is the so-called Engineering News formula, promulgated by Wellington, editor of Engineering News, in 1888. It was developed for wood piles driven with a drop hammer. Since inception it has had many additions and modifications to bring it in line with current materials, equipment, and practice in pile driving. The formula is said by many engineers to be misleading; tests have proved beyond doubt that it has too high a factor of safety; just as many tests have proved that it has too low a factor of safety—depending on the type of pile, hammer, and soil. The formula is not supported or defended by the editors of Engineering News-Record; it is by long odds the most-used pile formula, probably because it is the simplest.

The most common objection to the Engineering News formula is that it does not take into account the weight of the pile and driving apparatus. It may indicate inordinately high values on short piles in end bearing. Empirical attempts have been made to correct these defects by modifications, particularly the Navy-McKay variation, but they do not fit all conditions. The Engineering News formula, given in Table 12-5,

<table>
<thead>
<tr>
<th>Table 12-5, Engineering News Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>As specified in the New York City Code</td>
</tr>
<tr>
<td>For drop hammers: ( R = \frac{2WH}{S + 1} )</td>
</tr>
<tr>
<td>For single-acting hammers: ( R = \frac{2WH}{S + 0.1} )</td>
</tr>
<tr>
<td>For double-acting hammers: ( R = \frac{2E}{S + 0.1} )</td>
</tr>
</tbody>
</table>

where \( R \) = allowable pile load, lb
\( W \) = weight of striking part of hammer, lb
\( H \) = effective height of fall, ft
\( E \) = actual energy delivered by hammer per blow, ft-lb
\( S \) = average net penetration, in. per blow for the last five blows after the pile has been driven to a depth where successive blows produce approximately equal net penetration

presumably has a factor of safety of six, but this varies with different soils, length and type of pile, as well as kind and condition of driving apparatus. Tests under the actual conditions of the site are necessary to determine the actual load-carrying ability of any soil under any pile.

Dynamic formulas take into account the weight of the pile and of the driving apparatus as well as efficiency of the hammer and compression of the pile and apparatus under the impact of driving. Theoretically they should give dependable values but piles driven in accord with even the best recognized of these have proved erratic under test.


**NEW YORK CITY CODE**

Recently a number of codes relating to foundations have been completely rewritten to put pile-driving and other foundation specifications on a performance basis. Foremost of these codes is that of New York City, which was adopted late in 1948, followed by the Building Officials Conference of America Code in 1950.

Concerning the New York City code, J. H. Thornley, president of Western Foundation Corporation and a member of the New York City Foundation Code Committee,
wrote as follows about its making in briefing the code provisions for *Engineering News-Record* (May 26, 1949, p. 56):

New York is the first large city of the country to adopt a pile-bearing code based on load tests. . . . Test loading of piles is not in any sense a new idea. It has been required by most codes for as long as codes have existed. The difference is that in the past test-loading was used as a check on the sufficiency of the pile to carry a preconceived, code-stipulated load, whereas the new idea is to use the tests in advance of the determination of the working load. Thus the maximum safe load may be known and used. The old idea was purely negative—the load test might reduce the working load to be allowed, it could not increase it. Such tests verify safety, but do not aid economy. . . .

It has been accepted that the load-by-test basis cannot be economically applied to the smaller, low-cost structures, nor even to larger buildings where the time for starting the work is limited and where the delay of two to three weeks (which would be a minimum for the test program) cannot be permitted. To meet such situations the code sets up certain maximum loads based on use of the *Engineering News* formula and known to be safe for the New York territory because of extensive experience during the past 40 years or more. It would be well to underline "for the New York territory." The writers of the code had no faith in the *Engineering News* formula as a criterion to be applied under all soil conditions but they did believe that it has proved to be a safe rule of thumb when applied to most New York City soil conditions and when rigidly held to conservative top limits. . . .

With development of the load-by-test provision of the code, definite requirements were established for soil investigation by test pits or borings for any building area. Borings or test pits carried sufficiently into good bearing materials to establish its character and thickness are required in at least one location in every 2,500 sq. ft. of building area. For structures of more than one story (except two-story dwellings or for loading greater than 1,000 psf) at least one boring in each 10,000 sq. ft. must be carried to 100 ft. below the curb or 25 ft. into continuous material as good as loose sand [Class 10 or better in Table 12-2] or 5 ft. into ledge rock.

**The Test Program**

With this soil data in hand the test program is set up as follows: "Areas of similar soil conditions" are to be blocked out, covering the entire site. This "similarity" applies principally to the anticipated bearing stratum. For example, if it is expected that the piles will bring up by a combination of friction and end-bearing in a sand stratum overlying gravel to rock, "similarity" is not affected by the depths of the strata overlying this sand. The depth and continuity and uniformity of the sand as well as the underlying gravel strata are points to be studied as a guide to "similarity."

In each "area of similar conditions" three piles of the proposed type must be driven and a careful record kept of each. For piles with an average diameter or side of 8 in. or less, a hammer delivering a blow of at least 7,000 ft. lbs. is required, except that the minimum blow for a pile to carry 25 tons or more is 15,000 ft. lbs. For 8 to 18 in. piles a 15,000 ft. lb. hammer is required and for larger piles 22,000 ft. lb. is specified. The usual requirements of full-rated pressure, speed and stroke govern the use of double-acting hammers.

The driving record of three piles is taken as an indication of the correctness of the assumption made as to the area similarity of the soil conditions. If the results of the test driving are too irregular or are for any reason unsatisfactory, the Building Department may require additional core borings or piles or both.

**One Pile Load-tested**

Assuming that the indications of similar conditions from borings and test driving are satisfactory, then one of the depth test piles in each "area of similar conditions" is to be load-tested. A test must be made on at least one pile for each 15,000 sq. ft. of building area and on a minimum of two piles for any project.

The test must be to twice the proposed load value of the pile, applied in seven increments equal to $\frac{1}{2}, \frac{1}{4}, 1, 1\frac{1}{4}, 1\frac{1}{2}, 1\frac{3}{4}$, and 2 times the proposed working load. Readings of settlement and rebound must be recorded to 0.001 ft. for each change of load. Increment of load may be placed after there has been no settlement for two hours. The test load must remain in place until settlement does not exceed 0.001 ft. in 48 hours; then the load may be removed in decrements not exceeding one-quarter the total load, at intervals of one hour. The maximum working load allowable is one-half the load that causes a net settlement
of not more than 0.01 in. per ton of total test load or one-half the load that causes a gross settlement of 1 in., whichever is less. Net settlement is defined as the total settlement under load minus the recovery or rebound of the pile after load is removed. Loads are further limited by specific provisions for different types of piles.

If the bearing material shown by the borings, test driving or load test indicates, in the opinion of the Building Department's engineers, that there is uncertainty that a group of piles will carry the group load indicated by the multiplication of the single pile test load allowance by the number of piles in the group the Department may order a load test up to 150 per cent of the working load of the group.

Evaluation of Bearing Capacity

The test-load procedures outlined also are applicable to verification of the load-carrying ability of piles evaluated by driving formula [Table 12-5]. Within the limits noted below for specific materials, friction piles may be allowed a load of 30 tons if shown adequate by driving formula or up to 60 tons if proved by tests.

End-bearing piles of an acceptable type driven to rock, hardpan, or gravel-boulder formations directly overlying rock may be used for loads up to 40 tons on the basis of driving to formula requirements. If tested to double the design load they may be loaded to the permissible stress on the material but not to exceed 120 tons for piles bearing on rock or 80 tons for bearing on hardpan or gravel-boulder formations directly over rock. Higher loads are allowed on pipe piles driven open-ended to rock and on a concrete encased H-pile under special conditions.

With this opportunity to utilize the structural value of the pile, permissible stresses on materials and limiting sizes of types of piles becomes of utmost importance. Such limits are outlined as follows:

A few structural regulations affect all piles. These are: Piles may not be founded in material poorer than Class 12 [Table 12-2] and must penetrate at least 10 ft. below cut-off. For friction piles the full load is to be assumed as distributed by the bottom two-thirds of the pile length so that the full bearing area required to support the load must be available above the upper third point. For end-bearing piles passing through material of Class 12 or better only 75 percent of the load need be assumed as carried at the tip for piles more than 40 ft. long. For shorter piles the full load is considered as carried at the tip. The portion of a pile that is free-standing in air or water is considered as a column fixed at a point 5 ft. below the soil contact level in Class 9 material or better, and 10 ft. below in poorer material.

Timber Piles

These piles are allowed 600 psi. on the net area of cedar, western hemlock, Norway pine, spruce and similar woods; 800 psi. is allowed on cypress, Douglas fir, hickory, oak, Southern pine and woods of comparable strength. But, the maximum load on a wood pile with a 6-in. tip is 20 tons and for an 8-in. tip is 25 tons. For temporary structures of a minor character over marsh lands, untreated wood piles having a minimum diameter of 4 in. at the point and 8 in. at the butt may be permitted under special conditions.

Untreated wood piles may be used if the top of the pile is below the permanent water table. Timber piles may extend above ground water if pressure treated Grade I coal tar creosote oil meeting U.S. specification No. TT-W-571-b to a final retention of 12 lbs. of creosote per cubic foot of wood is used. Tops of the piles must be cut off below finished ground level and treated with three coats of hot creosote oil and capped with concrete.

Rolled Structural-steel Piles

These may be H-sections with flange projections not exceeding 14 times the minimum thickness of metal in web or flanges and with a total flange width at least 85 percent of the depth of the section. No metal may be less than 5/8 in. thick. Other structural sections or combinations having flange width and depth of not less than 10 in. and thickness of metal not less than 1/2 in. are permissible. All steel must conform to ASTM designation A7 or A9.

Allowable stress on structural steel sections is 9,000 psi. Under special conditions of bearing on rock and being protected by two inches of concrete a compressive stress of 12,000 psi. may be allowed. To use the 12,000 psi. stress the code provides that the pile shall be installed by driving a casing containing a close-fitting temporary core in such a manner as to exclude foreign matter or by driving an open-ended casing that is cleaned to the bottom. The casing must be driven to rock or hardpan overlying rock, to a final
penetration of not less than eight blows to the inch with a 22,000 ft. lb. hammer. Either the casing or a thin shell lowered inside the casing must be left permanently in the ground to contain the concrete used to encase the H pile.

The H section is placed and the shell or casing is filled with concrete and the H section is driven immediately to refusal, on rock, before the concrete has set. Refusal is indicated by a rate of penetration of \( \frac{1}{4} \) in. or less under the last five blows of the 22,000 ft. lb. hammer. Load value is allowable on the concrete but not on the encasing steel. Top loading for piles made under this section of the code is 100 tons without tests or 200 tons on the basis of tests.

Concrete Piles

Controlled concrete (where a specific compressive strength is maintained) is allowed one fourth of the 28 day compressive stress but not to exceed 1,000 psi. Concrete consisting of one part cement to \( 5\frac{1}{2} \) parts of aggregates and not more than \( 7\frac{1}{2} \) gal. of water per sack of cement is allowed 500 psi. Concrete 1:4\( \frac{1}{2} \) with 6\( \frac{1}{2} \) gal. is allowed 626 psi. Encasing steel is allowed 9,000 psi. load if it is at least \( \frac{1}{6} \) in. thick.

Cast-in-place concrete piles may be of uniform section or tapered and may be cased or uncased. If of uniform section they must be at least 8 in. diameter or, if not circular, have a minimum dimension of \( 7\frac{1}{2} \) in. Tapered piles must be at least 6 in. diameter in any section and have 8 in. diameter cutoff. Tapered shoes or points may be used.

When placing concrete, the tube, shell or bore must be free of foreign matter and methods used that assure that the entire shell or bore is filled. The concrete cap cannot be placed until at least one hour after all piles within the cap group are completely filled.

Concrete-filled steel pipe or shells installed open-ended to rock, cleaned to the bottom and redriven until the pile bears securely on Class 1 or Class 2 rock, may be loaded to as much as 200 tons. Steel thickness of 0.3 in. or more is required for the pipe or shell and stress is limited to 9,000 psi. for steel and to 1,000 psi. for concrete. The working load is limited to 80 percent of the value determined by material stresses, unless greater safe loads are proved by testing to double the proposed working load.

Precast concrete piles are required to have longitudinal reinforcing equal to at least 2 percent of the volume of concrete. Lateral reinforcing, in the form of hooks or spirals is required to be at least \( \frac{1}{4} \) in. round rods or wires spaced 12 in. on centers throughout the length of the pile except at the bottom and top 3 ft. where this spacing must be reduced to not more than 3 in. Reinforcing steel must be covered with at least 2 in. of concrete. For precast concrete piles only, reinforcing up to 4 percent of the concrete area is allowed a modular ratio of 30,000 divided by the compressive strength (not the working load) of the concrete. For 500 psi. this ratio is 15 and for 626 psi. the ratio is 12.

Combination or composite piles may consist of two types joined for greater length or for better resistance to deterioration of the upper section and so constructed as to prevent separation of the sections during construction and thereafter. The maximum permissible load on such a pile is that allowed for the weaker section only.

In addition to driving, piles may be installed by jacking or other methods without impact. Within the limits imposed on materials and types, the carrying capacity of piles jacked to position may not be more than half the force required to install them. The actual capacity must be proved by load test.

For underpinning piles, the temporary working load may not exceed the force required to install the pile. The working load of permanent underpinning piles may be as much as two-thirds of the jacking pressure required to secure the penetration if the load is held constant for ten hours; otherwise the load may be one-half of the total load for penetration, provided the limitations stipulated on materials and types are not exceeded.

Minimum Spacing

Piles in groups must be spaced a minimum of twice the diameter or 1.75 times the diagonal of the pile. This cannot be less than 24 in. for piles bearing on rock and must be at least 30 in. for other piles. Piles in groups, or abutting groups, founded on materials below Class 6 (Table 12-2) must have their spacing increased above the minimum values by 10 percent for each interior pile up to a maximum of 40 percent.

A few additional important points of the code can be briefly as follows: Piles left in place from demolished structures may be used only if evidence of their length, driving conditions and load carrying ability can be produced. . . . Where additional piles are required, new piles must be similar to the old and each will be allowed only 75 percent of the usual load. . . . Tops of piles are to be embedded not less than 3 in. and the cap must extend 4 in.
outside the piles. . . . Piles must be laterally braced, three or more piles in a cap qualifies.
. . . Lateral loads in excess of 1,000 lb. per pile are not permitted unless it has been proved
by test to twice the proposed load that movement will not exceed ½ in. and that under the
working load movement will not be more than 3/16 in. . . . Soil under the pile cap is not
allowed vertical load. . . . A tolerance of 3 in. from design location is allowed. . . . If
piles are out of plumb more than two percent of the pile length the design must be modified.
. . . Jetting may be done only with special permission, and jetted pile must be driven for
final 3 ft. . . . Splices must be avoided as far as practicable. . . . Splices that cannot be
inspected after driving must develop at least 50 percent of pile value in bending.
Section 13

EARTH PRESSURES AND RETAINING WALLS

By

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13-1
PART 1

EARTH PRESSURES AND RETAINING WALLS ON SPREAD FOOTINGS*

RETAINING WALLS

A retaining wall is a masonry or concrete structure which is built to provide lateral support for a mass of earth or other material. A gravity wall is constructed of stone masonry or plain concrete and requires a massive section which is depended upon to provide stability against the thrust exerted by an earth backfill, as shown in Fig. 13-1a, b. Reinforced-concrete walls of various types, shown in Fig. 13-1c, d, e, f, are designed to develop more nearly the full strength of concrete, and depend upon the weight of a considerable part of the supported earth above the footing at the back of the wall for stability.

The design of retaining walls involves first of all a consideration of the lateral pressure or thrust exerted by an earth or other backfill, and requires the determination of the dimensions of a stable wall section to resist this thrust. Failure can occur by overturning or excessive tilting and settlement or by sliding. In the second part of the design the structural elements of the wall are designed with sufficient strength to resist safely the stresses produced by the forces acting on the wall. Failure can occur by a structural collapse of a part of the wall. In order to achieve a proper balance between a satisfactory and at the same time an economical design, it is necessary to make reliable and rather close estimates of these forces, which the wall must resist. Design requires an application of the theory of earth pressure, a knowledge of the procedure involved in construction, and the exercise of good engineering judgment.

* By Donald M. Burmister.
LATERAL EARTH-PRESSURE PHENOMENA PERTAINING TO RETAINING WALLS

The first part of the design problem deals with the lateral earth pressure exerted by an earth backfill on the back of a wall and with the foundation pressures mobilized on the base of the wall to resist the earth thrust. In the past the classical theories of earth pressure have been used to obtain estimates of the magnitude of these forces acting on the wall, but with little regard for the actual nature of the phenomena. Since practically nothing was known as to the accuracy of the results, large factors of safety necessarily had to be used. These are really problems in applied soil mechanics. In order to design safe and economical retaining structures, it is essential to have a clear understanding of the nature of the earth pressure and soil-bearing phenomena.

Important advances in the practical applications of soil mechanics have been made possible because the forces and physical factors with which the engineer has to deal can be more accurately estimated. Although recent researches have greatly increased the knowledge and understanding of such problems, it will nevertheless be many years before the design of retaining structures is a fully rationalized process so that full advantage can safely be taken of increased economies of design and construction. In no field of engineering are a common-sense approach to the problem of design and a wide practical experience and knowledge more essential than in handling earth-pressure and foundation problems.

It must be expected that soils will vary considerably over a site. Furthermore, soils do not have simple, well-established, and generally uniform physical properties. Their behavior under any given circumstances represents the combined influence and interaction of a great number of physical factors, many of which are not susceptible of direct quantitative measurement. Although soils in their physical behavior follow the same natural laws as do all other materials, the relationships are more complex and obscure. Real difficulties are experienced in correctly applying the principles of mechanics to a material which is neither homogeneous nor uniform, which is only imperfectly elastic, and for which the stress-strain relationships are not clearly defined. Engineers have been trained, in general, to deal with materials having quite simple and well-defined physical properties and fairly definite physical relationships, and thus frequently they fail to appreciate the difficulties and uncertainties encountered in dealing with earth-pressure and foundation problems. There is always the chance of oversimplification of the problem and a failure to take into account some vital factors. Because of the importance of the uncertainties involved in such problems the accuracy of the computed results, in the present state of knowledge, seldom exceeds a rough approximation.

It must be emphasized that all theories deal with ideal materials and conditions and that simplifying assumptions have to be made in order to apply the principles of mechanics, which are seldom realized in nature. Although the knowledge of soil behavior has been growing steadily as a result of the investigations of the pioneers in the field of soil mechanics and through experience, there still remains a gap between theory and practice to be bridged, primarily by systematic accurate observations of the actual behavior of retaining structures over a period of years after completion, in order to build up a body of authoritative knowledge as a rational basis for design. In the scientific approach to the problem of design it is necessary at each stage in design to inquire into the nature of the uncertainties involved and into the nature of the assumptions in order to define with reasonable certainty the realm of validity of the simplified theory in relation to the actual phenomena.

Experience has shown that when a retaining wall fails a break occurs in the backfill and a wedge-shaped mass of earth slides down along a rupture surface $bc$ and pushes the wall outward, as shown in Fig. 13-2a. The mass of earth in the wedge $abc$ exerts an "active earth pressure" $P_a$ against the wall. The forces, which must resist the sliding out of this mass of earth and produce a stable condition, are the resultant frictional force $F$ mobilized along the sliding surface $bc$ and the resultant stabilized force or
reaction on the base $R$ mobilized by the mass of the retaining wall to resist the active earth pressure $P_A$.

The methods commonly used for estimating the magnitude of the lateral earth pressure which a backfill exerts against a retaining wall are derived from the well-known earth-pressure theories of Coulomb and Rankine. These theories are based upon certain assumptions regarding the behavior of an ideal homogeneous cohesionless backfill. Rankine investigated theoretically the earth-pressure phenomena from the standpoint of an ideal state of stress at failure in the backfill behind a lateral support. Coulomb, on the other hand, investigated the equilibrium conditions of a wedge of earth which had ruptured and was on the verge of sliding downward along the sliding surface. One or the other of these theories has been used as the basis for earth-pressure computations. There has, however, been a growing doubt as to their validity, primarily because of experiences in the sheeting and bracing of cuts and trenches in earth.

The investigations by Dr. Karl Terzaghi on large-scale earth-pressure tests at the Massachusetts Institute of Technology from 1932 to 1934 are a most important contribution to the knowledge of the real nature of earth-pressure phenomena. Since they were made the development from the original Coulomb theory has followed dealing with ideal materials and conditions to the broader concepts of the "general wedge theory of earth pressure" proposed by Dr. Terzaghi, in which he has defined more clearly the realm of validity of theory for the common earth-pressure problems, and the accuracy of the results of computations to be expected.

**References on Earth-pressure Phenomena by Dr. Karl Terzaghi**


The results of Dr. Terzaghi’s investigations, which apply to the retaining-wall problem, are considered briefly, because it is essential to have a clear understanding of the nature of the real earth-pressure phenomena.

The large-scale earth-pressure tests at MIT have demonstrated that Coulomb’s theory is in reasonable agreement with experience, provided that certain conditions are satisfied. Coulomb’s theory of earth pressure is based upon two sets of assumptions:

1. The first deals with the total magnitude of the lateral earth pressure and involves the equilibrium conditions of a wedge of earth on the verge of sliding when
   
   a. It was assumed that the frictional resistance was fully mobilized along the sliding surface.
   
   b. The surface of rupture was assumed plane, for simplicity.

2. The second has to do with the distribution of the lateral earth pressure on the back of the wall when

![Fig. 13-2. Forces acting on a retaining wall and the type of yield.](image-url)
a. The assumption was made that the lateral earth pressure $p_A$ increased like a hydrostatic pressure in simple proportion to depth.

b. It was therefore assumed that the resultant active earth pressure $P_A$ was applied at one-third the height of the wall above the base.

On the basis of this assumption of a hydrostatic pressure distribution, the intensity and total magnitude of the active pressure are given by the following equations, respectively:

Intensity of active earth pressure

$$p_A = \gamma_s K_A H$$  \hspace{1cm} (13-1a)

Total magnitude of active earth pressure

$$P_A = \frac{1}{2} \gamma_s K_A H^2$$  \hspace{1cm} (13-1b)

where the coefficient of active earth pressure $K_A$ is expressed as the ratio of the lateral to the vertical earth pressure at a point, and $\gamma_s$ is the unit weight of the backfill material.

![Graphs showing the influence of wall yield by tilting on the earth-pressure phenomena.](image)

**(a)** Magnitude of $K$  \hspace{1cm} **(b)** Angles of friction  \hspace{1cm} **(c)** Center of pressure

Fig. 13-3. Influence of wall yield-by-tilting on the earth-pressure phenomena. \((After \ MIT\ tests \ by \ Terzaghi.\)

The basic concept in the new development of the theory of earth pressure, as demonstrated by Terzaghi, is that the lateral earth pressure is a function of the type and amount of yield of a lateral support. Experience shows that a retaining wall practically always yields by tilting outward above its base, as illustrated in Fig. 13-2. As the back of the wall tilts outward from the original position $ab$ to the position $a'b$, the entire mass of earth within the sliding wedge $abc$ is caused to expand laterally an amount which is a constant proportion of the width of every element of the wedge. The lateral expansion is accompanied by a slight subsidence of the earth within the wedge.

The consequences of yield-by-tilting upon the earth-pressure phenomena are of great practical importance, as shown by the results of the MIT tests reproduced in Fig. 13-3. First of all, wall yield-by-tilting has a marked pressure-relieving effect, as shown by the fact that the value of the earth-pressure coefficient $K$ drops to about its minimum value $K_A$ when an almost insignificant yield has occurred of about 0.001 $H$ at the top of the wall for dense sand and about twice this value for loose sand, or a critical yield-by-tilting of a little over $\frac{1}{4}$ and $\frac{1}{2}$ in., respectively, at the top of a 25-ft wall.

The fact that the value of $K$ drops to its minimum value and that the angle of friction $\phi$ and the angle of wall friction $\delta$ increase to their maximum values at the critical yield is evidence that the major part of the frictional resistance has been mobilized
along the rupture surface \( bc \) and also along the back of the wall \( ab \). The value of the earth-pressure coefficient \( K_A \) at the critical yield is identical with Coulomb's value, since assumption (1a) is satisfied.

Experience indicates that the lateral yield-by-tilting of retaining walls founded on earth is almost always greater than the critical values noted above, and this yield is therefore sufficient to mobilize at least the major part of the frictional resistance along the rupture surface and along the back of the wall. Coulomb's assumption (1a) is therefore satisfied. This magnitude of yield is practically unavoidable for retaining walls founded on earth and is in reality an essential factor in design because of its important pressure-relieving effects. On the other hand, for retaining walls founded on rock, particularly if anchored, Coulomb's assumption (1a) is not satisfied and the lateral earth-pressure coefficient approaches the coefficient of earth pressure "at rest," \( K_a \) in Fig. 13-3a, which depends primarily on the method of backfilling and the density of the backfill.

The assumption of a plane sliding surface with wall friction active leads to the incompatibility inherent in the Coulomb theory that with a hydrostatic pressure distribution

![Diagram](image)

Fig. 13-4. Wall friction acting on earth wedge.

Fig. 13-5. Condition of plastic equilibrium.

the forces fail to be concurrent in Fig. 13-4a. However, the curvature of the rupture surface required for equilibrium for sand backfills is slight, and the pressure computed by assuming a plane sliding surface has been shown to be only 6 per cent or so too low. It may therefore be concluded that, so far as the computation of the magnitude of the active earth pressure \( P_A \) is concerned, Coulomb's theory may be expected to give pressures in reasonable agreement with actual values for sandy backfills.

The second group of assumptions, however, concerning the distribution of the lateral earth pressure and the position of the resultant on the back of the wall, are not necessarily satisfied unless the yield is sufficient to cause the entire mass of earth within the wedge to pass into a state of plastic equilibrium, as indicated by the slip-plane pattern of Fig. 13-5a. It is evident that this condition obtains in the case of loose sand, since the resultant pressure on the back of the wall remains practically as \( H/3 \) during the entire process of yielding, in Fig. 13-3c. The distribution of the lateral earth pressure is therefore hydrostatic. Both Eqs. (13-1a) and (13-1b) are directly applicable, and Coulomb's theory is reasonably valid for loose sand, as to both the magnitude and the distribution of the lateral earth pressure.

On the other hand, the distribution of earth pressure for dense sand cannot be hydrostatic, because the center of pressure rises to a value of about 0.40\( H \) in Fig. 13-3c, which is evidence of a considerable degree of "arching action" in the backfill. Equation (13-1b), giving the total magnitude of pressure, regardless of its distribution, is valid and directly applicable, because assumption (1a) is satisfied, whereas Eq. (13-1a) is not valid. The minimum yield at the top of the wall required to produce a hydrostatic distribution of pressure with the center of pressure falling to \( H/3 \) varies from about 0.003\( H \) to 0.005\( H \) for dense sand, when the first slip occurs in the backfill, which is approximately 1\( \frac{1}{2} \) in. or more at the top of a 25-ft wall. This magnitude of yield, however, may be large enough to affect the appearance of the wall somewhat.

The MIT earth-pressure tests showed that important time effects occurred in all earth-pressure phenomena, the condition with the wall stationary for a time at any
given stage in the process of yielding being temporary and unstable. The time effect always involves a slight readjustment of the grain structure of the backfill before a more or less permanent equilibrium condition under stress can be attained. Vibrations from any cause also disturb the equilibrium conditions, the degree of disturbance depending upon the intensity and frequency of the vibrations. As a consequence the angle of friction $\phi$ and to a somewhat greater extent apparently the angle of wall friction $\delta$ tend to decrease with time or under vibrational effects, and to approach lower limiting values. Hence the value of $K$ and the lateral earth pressure tend to increase somewhat to values higher than the minimum or Coulomb values, unless relieved by further yield of the wall.

**PRACTICAL CONSIDERATIONS**

Greater uncertainties are involved in the selection of the appropriate values of the angle of friction $\phi$, of the angle of wall friction $\delta$ of earth on concrete or masonry, and of the unit weight $\gamma$, of the backfill to be used in the computations of the lateral earth pressure than are due to any inherent defects in the Coulomb theory itself. These values must be representative and safe, and at the same time result in economical construction. Such values can be determined only by carefully conducted shearing tests upon representative samples of the different classes of backfill materials to be used, and covering the possible range of densities (unit weights) in which the materials are likely to be placed by the common methods of backfilling. In the past, in the absence of exact information, tabulations of so-called "average" values of the angle of friction and angle of wall friction have been used more or less indiscriminately. Very conservative and therefore less economical values necessarily had to be adopted. In view of the recent increased knowledge of shearing phenomena and of the development of shear-testing methods it is the duty of the engineer to avail himself of the best information for design purposes in order to safeguard the owner's investment. The expense of such investigations represents a very small fraction of the total cost of the construction but pays for itself many times over as an insurance of satisfactory construction.

It has been commonly assumed that the angle of internal friction is identical with the angle of repose. Actually the angle of repose is merely a surface phenomena of a dry sand slope and is independent of the density of the sand and of the height of the mass of sand. On the other hand, the angle of friction, which governs the lateral earth-pressure phenomena, involves a mass action of the sand, varies markedly with the density or degree of compactness of the sand, and is affected by the predominating size of the material and the angularity of the grains. The effective angle of friction is primarily a function of wall yield, as shown in Fig. 13-3b. Numerically the angle of repose of dry sand may under some conditions be about equal to the angle of friction for the loosest dry condition.

In order to give some idea of the range of the angle of friction in the dry loose and moist loose (about maximum bulking) conditions for some of the common classes of earth backfill materials of satisfactory character and also of the angle of wall friction, some test data are shown in Table 13-1. While there are relatively unimportant differences in the angle of friction for the dry loose condition, there are very important decreases in the angle of friction for the loose moist condition, which is also more variable. There is also a slight decrease in the angle of wall friction. The loose condition represents about that obtained by loose dumping from a clamshell bucket. Although the grain-size composition of these materials has been accurately described for reference and comparative purposes, such values should be used with considerable caution in the absence of exact information.

The method of backfilling the wall also may have a very important influence upon the lateral earth pressure. The backfill of ordinary retaining walls is more likely to be placed in a quite loose condition by the common methods employed, such as dumping from a clamshell bucket, particularly if the earth is slightly moist, so as to cause a considerable bulking effect. Mechanical compaction of the backfill in the immediate vicinity of the wall is usually undesirable, unless done with great care, because of the excessive lateral pressure which may be built up, tending to shove the wall progres-
sively outward. The manner in which the backfilling progresses may also affect the wall yield conditions and the magnitude of the earth pressure during and immediately after backfilling. The pressure may be somewhat greater where the backfilling progresses so that the earth slides toward the wall than in the case where earth dumped first immediately adjacent to the wall becomes stabilized and in a sense acts with the wall to support the remainder of the backfill above, as it is placed.

Giving consideration to the nature of the earth-pressure phenomena for retaining walls and to all these factors affecting conditions, certain recommendations may be

<table>
<thead>
<tr>
<th>No.</th>
<th>Description of soil</th>
<th>Angle of friction $\phi$</th>
<th>Wall friction $\delta$</th>
<th>Dry wt $\gamma_s$</th>
<th>Moisture, %</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Dry</td>
<td>Moist</td>
<td>Dry</td>
<td>Moist</td>
</tr>
<tr>
<td>1</td>
<td>Coarse to medium SAND, trace fine Gravel</td>
<td>36°00'</td>
<td>27°30'</td>
<td>27°30'</td>
<td>26°10'</td>
</tr>
<tr>
<td>2</td>
<td>Course to fine SAND, trace + Silt (7.5 %)</td>
<td>37°40'</td>
<td>27°50'</td>
<td>32°10'</td>
<td>26°20'</td>
</tr>
<tr>
<td>3</td>
<td>Course to fine SAND, trace + (7.5 %) fine Gravel</td>
<td>38°40'</td>
<td>30°00'</td>
<td>27°10'</td>
<td>26°20'</td>
</tr>
<tr>
<td>4</td>
<td>Coarse to fine SAND</td>
<td>36°30'</td>
<td>30°00'</td>
<td>28°50'</td>
<td>27°10'</td>
</tr>
<tr>
<td>5</td>
<td>Medium to fine SAND, some Silt (29 %) trace fine Gravel</td>
<td>35°10'</td>
<td>29°10'</td>
<td>25°10'</td>
<td>21°30'</td>
</tr>
<tr>
<td>6</td>
<td>Fine SAND, trace Silt</td>
<td>37°50'</td>
<td>29°20'</td>
<td>29°40'</td>
<td>26°20'</td>
</tr>
<tr>
<td>7</td>
<td>Fine SAND, some Silt</td>
<td>35°00'</td>
<td>30°20'</td>
<td>28°00'</td>
<td>28°00'</td>
</tr>
<tr>
<td>8</td>
<td>Coarse SILT, and fine Sand (45%)</td>
<td>34°50'</td>
<td>26°10'</td>
<td>27°50'</td>
<td>25°40'</td>
</tr>
<tr>
<td>9</td>
<td>SILT, some coarse to fine Sand, trace + Clay (7%)</td>
<td>...</td>
<td>31°20'</td>
<td>...</td>
<td>28°50'</td>
</tr>
</tbody>
</table>

Density conditions: Dry loose and moist loose, initially compacted under 2 psi, approximately equivalent to 2 ft of earth surcharge. Moisture content approximately maximum bulk point.

Normal pressures for direct-shear tests: To maximum of 1 ton per sq ft.

Concrete surface for wall-friction tests: Average condition of smoothness obtained by formwork. Description of soils: Predominating soil fraction in capitals, more than 50 per cent. Other soil fractions: "and" 35 to 50 per cent, "some" 20 to 35 per cent, "little" 10 to 20 per cent, "trace" 0 to 10 per cent.

made as a guide for general design practice, which may be used at the discretion of the designing engineer:

1. The total magnitude of the active lateral earth pressure $P_A$ may be computed by Coulomb's theory and may be expected to be reasonably accurate, because the unavoidable yield-by-titting of retaining walls founded on earth is practically always sufficient to mobilize the major part of the frictional resistance of the backfill.

2. Unit-weight determinations of backfill materials should be made to furnish reliable design information on the possible range of conditions as to the different classes of materials, moisture content, and bulking effects, and as to probable methods of backfilling. Since the earth pressure [Eq. (13-1a)] is directly proportional to the unit weight of backfill, a reasonably high upper limit of the loose (dry or moist) condition should be used in order to be on the safe side.

3. The angle of friction should be determined by carefully conducted shearing tests at normal pressures greater than 1 ton per sq ft upon representative samples of the different classes of backfill materials, covering the possible range of densities (unit weights) and moisture contents in which the material is likely to be placed by the common methods of backfilling. Where vibrational effects are likely to be important or where any unusual conditions are likely to develop, conservative values should be used,
possibly 80 to 90 per cent or less of the value obtained by tests, as a reasonably safe basis for design.

4. The angle of wall friction should also be determined by tests upon specimens of the type of surface of concrete or stone to be used on the back of the wall. The angle of wall friction appears to be a more uncertain quantity, tending to decrease more with time and being adversely affected by vibrations. However, an assumption of the angle of wall friction equal to zero is unnecessarily conservative, because wall yield is always accompanied by a subsidence of the mass of soil within the wedge, which mobilizes at least some part of the wall friction. As a reasonably safe assumption of the wall friction that can be developed, particularly under vibrational effects adjacent to the wall, the value of the angle of wall friction is taken as some fraction of the test value, possibly as low as 50 per cent, depending upon circumstances.

5. For loose backfills of sandy soils, which is the more usual case, the distribution of the earth pressure on the back of the wall may be assumed to be hydrostatic with the center of pressure always at \( H/3 \).

6. For medium compact to compact backfills, if the earth backfill is actually placed in this manner, the distribution of lateral earth pressure is not hydrostatic, unless the wall yields more than about 0.005\( H \) at the top of the wall, as shown in Fig. 13-3c, and

![Figure 13-6. Equivalent nonhydrostatic earth-pressure distribution.](image)

![Figure 13-7. Equilibrium conditions for Coulomb's wedge of maximum thrust.](image)

the center of pressure should be assumed to be as high as 0.40\( H \) for the stability analysis. An equivalent pressure distribution, giving the same total magnitude as the Coulomb hydrostatic distribution, but with the center of pressure at 0.40\( H \), is given in Fig. 13-6 for the stability analysis and for the design of the vertical stem of the wall.

**COMPUTATIONS OF ACTIVE EARTH PRESSURE**

Coulomb assumed that a wedge of earth had ruptured and was on the verge of sliding down a certain plane \( bc \) in Fig. 13-7a, the angle \( \beta \) of the sliding plane being such as to produce the maximum thrust against the wall, as shown in the polygon of forces. The total magnitude of the active earth pressure may be computed from Coulomb's solution for the wedge of maximum thrust:

\[
P_A = \frac{1}{2} \gamma_b K_A H^2
\]

where the lateral active earth-pressure coefficient is

\[
K_A = \frac{\sin^2 (\alpha - \phi)}{\sin^2 \alpha \sin (\alpha + \delta) \left[ 1 + \frac{\sin (\phi - \theta) \sin (\phi + \delta)}{\sin (\alpha - \theta) \sin (\alpha + \delta)} \right]^2}
\]

where
- \( \phi = \) angle of friction of the backfill
- \( \delta = \) angle of wall friction between backfill and wall
- \( \gamma_b = \) unit weight of the backfill
- \( \theta = \) angle of a sloping backfill
- \( \alpha = \) angle of the back of the wall
Because this formula is not always readily applied, except in the simpler cases, certain graphical solutions are preferable, such as those of Poncelet, Rebmann, Culman, and Engesser. One of the most useful of these graphical methods is that of Culman, which is based on simple principles of the equilibrium of forces. This method lends itself readily to the determination of the active earth pressure for various conditions of loading and of surcharges, of which a number of typical cases are given.

Level or Sloping Backfill

Culman's graphical method for determining the active earth pressure for a level or a sloping backfill at any angle is illustrated in Fig. 13-8. The method involves the following steps:

1. The $\phi$ line $bd$ is drawn making an angle $\phi$ with the horizontal.
2. The $\psi$ line $bf$ is drawn making an angle $\psi = (180^\circ - \alpha - \delta)$ with respect to the $\phi$ line.
3. A number of trial sliding planes $bc_1, bc_2, bc_3$, etc., are traced, making equal intercepts on the surface slope $ac$.
4. The weights of these wedges $abc_1, abc_2$, etc., are computed per linear foot of wall (scaling off the bases and altitudes, respectively) and are laid off to a convenient scale on the $\phi$ line, giving the points $bd_1, bd_2$, etc.
5. The lines $ed_1, ed_2$, etc., are then drawn parallel to the $\psi$ line, the points $e_1, e_2$, etc., being located on the respective trial sliding planes.

![Diagram](image)

Fig. 13-8. Culman's graphical method for determining the active lateral earth pressure.

It is evident that the triangles so formed are exactly similar to the force polygon, because of the angle relations. These $e$ points lie on a curve, which is called the Culman line. The maximum value of the intercept $ed$ determined by drawing a tangent to the Culman line parallel to the $\phi$ line gives the magnitude to scale of the total active earth pressure $P_a$. The corresponding surface of sliding passes through this point, as shown by the line $ceb$ in Fig. 13-8a.
Surcharge on a Level Backfill

The effect of a surcharge load of \( p \) lb per sq ft, such as a highway loading, is taken into account by superposing an equivalent depth of fill \( h' = \frac{p}{\gamma_s} \) to each trial wedge. (AASHTO Standard Specifications for Highway Bridges, 1953. Highway loadings, Secs. 3.2.5, 3.2.7, 3.2.8, 3.5.4, 3.2.18, and Appendix C.) The Culman line is then determined, as shown in Fig. 13-9.

The effect of this surcharge is to raise the theoretical point of application of the resultant active earth pressure \( P_A \) above \( H/3 \), since the total earth pressure is made up of the triangular hydrostatic pressure distribution due to the earth backfill, plus the uniform pressure distribution due to the surcharge loading. The position of the resultant can be readily determined by moments, or by the equation

\[
\tilde{y} = \frac{H}{3} \left( \frac{H + 3h'}{H + 2h'} \right)
\]

(13-3)

Concentrated Line Loads on a Level Backfill

Concentrated loads are treated by the Coulomb-Culman method, rather than by the theory of elasticity, because the wall yield conditions control the lateral earth-pressure phenomena. The concentrated line load per linear foot of wall is added to each wedge, and the Culman construction is carried out as before. It will be found that concentrated loads beyond a certain distance from the wall have no influence on the earth pressure on the wall, because the rupture line for maximum pressure cuts off its effect. There is a sharp break in the Culman line, equal in magnitude to the concentrated load (to scale), on the sliding surface through the point of its application. The magnitude of the total lateral pressure depends both on the magnitude of the surcharge loading and on the distance of the surcharge from the crest of the wall, as shown in Fig. 13-10.

The surcharge loading due to a railroad track has been commonly computed by assuming the load distributed uniformly over a width of 14 ft, where the center of the track is 7 ft from the crest of the wall and where double tracks are 14 ft center to cen-
EARTH PRESSURES AND RETAINING WALLS

The solution then corresponds to that for a uniform surcharge on a level backfill. The equivalent uniform surcharge for the Cooper E loading is spread over a length of 5 ft of track and a strip 14 ft wide to midway between tracks. Fifteen per cent is usually added to take care of vibration and impact effects. For walls less than a certain height the second track is cut off by the rupture line for maximum pressure. Comparative analyses show that the total magnitude of the earth pressure obtained by this method does not differ greatly from that obtained by assuming the load distributed uniformly over the width of the ties only. The point of application of the resultant earth pressure will, however, be slightly different. The simpler method in most cases is preferred.

Earth Pressure on Reinforced-concrete Retaining Walls

Reinforced-concrete retaining walls are always constructed in the form of a T wall or an L wall of the cantilever type, or in the form of a counterfort wall with heavy footings extending back beneath the backfill, as illustrated in Figs. 13-1 and 13-11.

It has been common practice to consider the mass of earth above the heel of the wall as inert and, together with any surcharge on the surface of the backfill above, as acting with the wall to provide stability, as shown in Fig. 13-11a. The boundary of the inert mass of earth is commonly taken as the vertical line \( a'b \) at the heel of the wall. The height \( H \) for computing the total earth pressure is now \( a'b \) and the resultant earth pressure \( P_A \) acts at the angle of friction \( \phi \) for earth on earth at a height of \( H/3 \) on this assumed interface. It has been questioned whether the vertical component of the earth pressure \( P_{AV} \) should be considered as fully effective in determining the stability of the wall, or in computing the pressure on the heel of the wall. A more reasonable assumption, and one which is borne out by experience, is that a second rupture plane \( ab \) tends to form in the backfill approximately from the crest of the wall to the heel \( b \) which limits the inert mass of earth to the wedge indicated in Fig. 13-11b. In this case the vertical component of the earth pressure includes the effect of any surcharge on the backfill above the heel, and it is evident that the vertical component of the earth pressure is fully effective. The earth pressure for either case can be readily determined by the Culman method. Because of possible vibrational effects, which may be concen-
trated adjacent to the wall, it has been believed by some that the frictional resistance of earth on earth cannot be fully mobilized on this second rupture plane. As a conservative assumption the value of the angle \( \phi \) (earth on earth) should be taken as some percentage of that used on the rupture plane \( bc \) on the far side of the wedge, possibly as low as 50 per cent, depending on circumstances.

For obtaining the moments and shears for the design of the reinforced-concrete vertical stem of the wall, the earth pressure acting against the stem has been assumed to be equal to the horizontal component acting on the planes \( a''b \) of Fig. 13-11a or \( ab \) of Fig. 13-11b, transposed to the back of the wall along lines parallel to the surface slope. This is probably conservative, since the restraints offered by the heel to earth movement within the inert wedge would tend to reduce the lateral earth pressure against the lower portion of the stem of the wall.

Where reinforced-concrete walls of a considerable height have to be designed and built, the principle of relieving platforms may be used to advantage to reduce the dimensions of the section required for stability, as illustrated in Fig. 13-lf. The Culman graphical method can be readily adapted to the determination of the earth pressure for this type of section, determining the earth pressure in steps. The earth pressure is determined in the usual way above the platform. The earth above the platform is then considered as a simple surcharge for determining the pressure against the section below the platform.

**STABILITY AGAINST OVERTURNING**

The first part of the design problem involves the determination of the dimensions of a stable wall section to resist safely the active earth pressure and provide stability against overturning. The first decision to make is the depth at which to place the base of the wall. In order to prevent objectional movements by frost heaving the base should be located below the frost line. In the northern part of the United States a depth of 3 to 5 ft is usually found to be necessary. The depth must also be sufficient to preclude any possibility of scour under the toe of the wall by water flowing along the wall during heavy storms. There is usually an important increase in the bearing capacity of the foundation soil when the base is placed below the surface of the immediately adjacent ground.

According to the concepts of the middle-third theory, a retaining wall is stable, and there is a satisfactory factor of safety against overturning, if the base of the wall has a width such that the resultant pressure \( R \) on the base lies just within the outer third point, as shown in Fig. 13-12. The magnitude and position of the resultant \( R \) on the base are determined by applying the principles of equilibrium to the forces acting on the wall: \( \Sigma V = 0, \Sigma H = 0, \Sigma M = 0 \). This may be done by graphic statics or by taking moments about the heel of the wall at point \( b \) in Fig. 13-12a. If the resultant falls outside the middle third, the base width may be revised readily by extending a footing at the toe of the wall (point \( a \)) to satisfy this criterion, as shown in Fig. 13-12a. This revision usually has a negligible effect on the magnitude and position of the resultant \( R \) if only a foot or so is added.

The distribution of the earth pressure or reaction on the base of the wall is then computed, as shown by the conventional distribution of Fig. 13-12b, by taking moments or by the following equations:
\[ p_{\text{max}} \text{ (at toe)} = \frac{R_v}{B} \left( 1 + \frac{6e}{B} \right) \] (13-4a)

\[ p_{\text{min}} \text{ (at heel)} = \frac{R_v}{B} \left( 1 - \frac{6e}{B} \right) \] (13-4b)

where \( e \) = eccentricity of the resultant from the middle of the base \( B \)
\( R_v \) = vertical component of the resultant \( R \).

Actually the conditions of stability are not so simple as assumed in the conventional middle-third theory and are indeterminate when considered without regard for the possible yielding of the wall. A yielding and settlement of the earth foundation are a necessary consequence of mobilizing the pressure reaction \( R \) on the base of the wall to resist the thrust of the lateral earth pressure. In order to decide whether the maximum pressure at the toe of the wall is within satisfactory limits, consideration must be given to the bearing value of the soil and to the probable settlement and tilting of the wall. Because the pressure at the toe is greater, it will settle more than the heel, and the wall will tilt outward. The real criterion of stability of the wall must be the limitation of the settlement and tilting of the wall to values which are not objectionable from the standpoint of both safety and appearance. It does not necessarily follow that these conditions will be satisfied for all types of earth foundations when the resultant \( R \) on the base falls just inside the outer-third point. This phenomenon depends primarily on the load-settlement properties of the foundation soils, and secondly on the eccentricity of the loading, expressed by

\[ \frac{p_{\text{max}} - p_{\text{avg}}}{p_{\text{avg}}} = \frac{6e}{B} \]

The bearing value of the foundation soils must be defined in terms of the permissible settlement and tilting. The base pressures, particularly for poor earth foundations, must be reduced to such values that the differential settlement between the toe and the heel of the wall, and hence the tilting, does not exceed some maximum value which would be considered objectionable, for example, greater than 2 in. or so at the top of a 25-ft wall. This may be accomplished by extending the footing at the toe of the wall so as to bring the resultant \( R \) nearer to the center of the base, as shown in Fig. 13-13, thereby reducing the intensity of the maximum pressure at the toe and making the pressure distribution more uniform.

In order to reach a reliable decision as to whether the maximum pressure \( p_{\text{max}} \) at the toe is within satisfactory limits, information is required on the allowable bearing pressure of the particular foundation soil, and on the probable magnitude of settlement and tilting of the wall. Good practice requires that the foundation conditions for all important retaining walls be explored by making borings and load tests at intervals along the line of the wall, depending on the conditions encountered, in order to establish the range of conditions and of the load-settlement properties of the foundation soils. It is very important to found large retaining walls on undisturbed soils. In no case should a wall be placed upon newly deposited fill, because of the objectionable settlement and tilting that are likely to occur. For poor soil conditions and/or where vibrations occur, good practice requires that the resultant on the base fall nearer its center, as shown in Fig. 13-13, so that the pressure is more nearly uniform. The necessary increase in base width thus limits both settlement and tilting.
The Boston Building Code, in general, allows quite conservative values for bearing pressures, which should, however, be used with discretion and only in the absence of actual load test data. Excerpts from Section 2904 on Foundation Bearing Values of the Boston Building Code (1941) are given for reference in Table 13-2.

<table>
<thead>
<tr>
<th>Class</th>
<th>Material</th>
<th>Allowable bearing value, tons/sq ft</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Massive bedrock without laminations</td>
<td>100</td>
</tr>
<tr>
<td>2</td>
<td>Laminated rocks, such as slate and schist in sound condition, some cracks allowed</td>
<td>35</td>
</tr>
<tr>
<td>3</td>
<td>Shale in sound condition, some cracks allowed</td>
<td>10</td>
</tr>
<tr>
<td>4</td>
<td>Residual deposits of shattered or broken bedrock of any kind except shale</td>
<td>10</td>
</tr>
<tr>
<td>5</td>
<td>Hardpan</td>
<td>10</td>
</tr>
<tr>
<td>6</td>
<td>Gravel, sand-gravel mixtures, compact</td>
<td>5</td>
</tr>
<tr>
<td>7</td>
<td>Gravel, sand-gravel mixtures, loose; sand, coarse, compact</td>
<td>4</td>
</tr>
<tr>
<td>8</td>
<td>Sand, coarse, loose; sand, fine, compact</td>
<td>3</td>
</tr>
<tr>
<td>9</td>
<td>Sand, fine, loose</td>
<td>1</td>
</tr>
<tr>
<td>10</td>
<td>Hard clay</td>
<td>6</td>
</tr>
<tr>
<td>11</td>
<td>Medium clay</td>
<td>4</td>
</tr>
<tr>
<td>12</td>
<td>Soft clay</td>
<td>1</td>
</tr>
<tr>
<td>13</td>
<td>Rock flour or any deposit of unusual character not provided for herein</td>
<td>Value to be fixed by the commissioner</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 13-3. Approximate Range of Settlement*</th>
</tr>
</thead>
</table>

Order of magnitude of settlement, in.

**Immediately**

- 3/4-3/4
- 3/8-3/8
- 3/8-1
- 1-1.5

**In a year or two**

- 3/4-1
- 1-1.5
- 1.5-2
- 1.5-2
- 6 plus

*To be expected under average conditions (data compiled from engineering and construction periodicals for the common sizes of column footings for buildings).

In order to have a rough idea of the probable order of magnitude of settlement which may be expected for the different classes of soils under the allowable bearing pressures, a range of corresponding values is given in parallel to Table 13-2 in Table 13-3, which has been compiled from a study of available design and construction data in engineering and construction journals.

This part of the design problem is really a problem in applied soil mechanics. Experience and the theory of elasticity indicate that the settlement of a footing is directly proportional to the average bearing pressure and is also some function of the size of the footing. For clayey soils the settlement is relatively large and is directly proportional to the width of the footing. But in passing from the clayey soils to compact gravelly soils the settlement is relatively small and may become almost independ-
ent of the width or size of the footing. In the absence of exact knowledge on this important relationship, it has been common practice in many engineering offices, as a conservative assumption, to proportion or design column footings for buildings for approximately equal or uniform settlement by the following relation:

$$w_1 = w_2 \frac{p_1 B_1}{p_2 B_2}$$

(13-5)

assuming that $p_1 \sqrt{A_1} = p_2 \sqrt{A_2}$ and $p_1 B_1 = p_2 B_2$

where $p$ = average bearing pressure on footing
$B$ = least width of footing
$A$ = area of footing
$w$ = settlement, in.

For small retaining walls it is sufficient to accept the experience that under ordinary conditions the Coulomb lateral earth pressure obtains, and that the base pressures are satisfactory, if less than the allowable bearing values of Table 13-2, because the wall yield conditions are satisfied for ordinary earth foundations. However, for important retaining walls of considerable height, estimates should be made as to the probable order of magnitude of settlement and tilting of the wall, in order to reach a reliable decision as to whether the design is satisfactory. At present such estimates may be very approximate, because of the uncertainties involved and because of the limited knowledge of the actual behavior of large walls. When sufficient data have been accumulated by actual observations of wall settlement and tilting, reliable information will be available as a basis for a more rational design of retaining walls. Theoretical analyses, employing the theory of elasticity and influence methods, provide an approximate basis for determining the effect of eccentric loadings on the settlement and tilting relationships for retaining walls with base footings long compared with their width. The base pressures under eccentric loadings may have nonuniform distributions of the types shown in Fig. 13-14, depending upon the type and density of the foundation soil, the depth below the immediately adjacent surface of the ground, and the relative flexibility of the footing. Heavy reinforced-concrete footings, such as used for retaining walls, may be considered rigid for all practical purposes. The settlement and tilting of a wall are not a simple function of the base pressure, as if the wall were founded on springs which act entirely independent of each other. The analysis shows that the tilting of the base is a load-settlement phenomenon of earth and is approximately directly proportional both to the magnitude of the average settlement and to the eccentricity of the loading

$$\frac{p_{\text{max}} - p_{\text{avg}}}{p_{\text{avg}}} = \frac{6e}{B}$$

and that the angle of tilt of the base may be expressed by the following approximate relation:

\[\]
tan angle of tilt of base

\[
\frac{\Delta_w}{12B} = \left( \frac{cw_{avg}}{12B} \right) \left( \frac{p_{max} - p_{avg}}{p_{avg}} \right) = \tan T
\]  

(13-6)

where \( \Delta_w \) = differential settlement, in., in the length between the toe and the heel of the base

\( w_{avg} \) = average settlement, in., of the base

\( B \) = base width, ft

\( \frac{p_{max} - p_{avg}}{p_{avg}} \) = eccentricity of the loading (conventional pressure distribution)

\( c \) = a coefficient, which varies from about 0.40 to 0.80, depending on the distribution of base pressure in Fig. 13-14

Substituting data from the results of load tests and using the relationship given in Eq. (13-5), the tangent of the angle of tilt becomes

\[
\frac{\Delta_w}{12B} = \left( \frac{cw_0}{12B} \right) \left( \frac{p_{avg}}{p_0} \right) \left( \frac{B}{b_0} \right) \left( \frac{p_{max} - p_{avg}}{p_{avg}} \right) = \tan T
\]  

(13-7)

where \( w_0 \) and \( p_0 \) (taken equal to \( p_{avg} \) for convenience) are the corresponding values from the load test curve for a test plate of width \( b_0 \) as shown in Fig. 13-15.

The probable order of magnitude of yield-by-tilting of the top of the wall may be estimated approximately by the following equation:

\[
\text{Yield at top of wall} = \frac{\Delta_w}{12B} H > 0.002H < 2 \text{ to } 3 \text{ in.}
\]  

(13-8)

This estimate of wall yield should at least exceed the critical value of 0.002\( H \) for a loose sand backfill to satisfy the Coulomb condition as to the magnitude of the lateral earth pressure. It is evident from Tables 13-2 and 13-3 that Coulomb's condition is satisfied for most earth foundations of class 7 and higher. For classes 1 to 6, the conditions may not be satisfied, and as a result the earth pressure may be as great as twice the Coulomb value or more as shown in Fig. 13-3a, depending on the actual yield conditions. Wall yields much in excess of 2 to 3 in. would be considered objectionable and the toe pressure \( p_{max} \) therefore excessive, regardless of the so-called allowable bearing pressures commonly permitted. The maximum base pressure may be reduced to satisfactory values by extending the toe of the base, as indicated in Fig. 13-13.

**STABILITY AGAINST SLIDING**

A retaining wall founded on sandy soil, which has been designed for stability against excessive settlement and tilting, usually provides a satisfactory factor of safety against actual sliding outward on the base due to the earth thrust. However, it is often difficult to obtain the desired factor of safety, particularly where clayey subgrades are encountered, because of the decrease in the angle of base friction \( \phi_B \) with saturation. Specifications usually require that, for stability, resistance against sliding should be at least twice the computed active horizontal thrust of earth pressure on the wall. This is very conservative. In view of the wall yield phenomena discussed previously, whereby such a pressure would be relieved by a tilting of the wall, a factor of safety against sliding of 1.5 should provide a reasonably safe basis for design. However,
where hydrostatic pressure of water in the backfill occurs, which is not relieved at all by wall yield, an ample factor of safety should be used. For stability, sufficient sliding resistance \( R_H \) must be mobilized on the base of the wall in Fig. 13-16 to resist the thrust of the horizontal component of the earth pressure with a suitable factor of safety. The maximum inclination that the resultant \( R \) on the base of the wall can have is equal to the angle of base friction \( \delta_B \). The criterion against sliding is expressed as follows:

\[
R_H = R_V \tan \delta_B \geq P_{AH} \text{ (factor of safety)}
\]  

(13-9)

where \( R_V = \Sigma V = \Sigma W + P_{AV} \).

Where only a fraction of the angle of wall friction on the back of the wall has been adopted as a safe basis for the determination of the lateral earth pressure, it is more difficult to obtain the desired factor of safety. Where insufficient sliding resistance can be mobilized it may be necessary to design a cutoff or anchorage, as shown in Fig. 13-17, and take into account the passive resistance of the earth in front of the wall to being shoved forward. However, the full depth of earth fill in front of the toe of the wall may be absent from some cause at the critical time of sliding, such as from possible scour by storm floods, future excavation, etc. The wall should be stable without depending upon this force. The frictional force which can be developed on the base is frequently uncertain. For some soils the angle of friction of earth on earth is greater than the angle of friction of concrete on earth. In order to develop this greater frictional resistance a cutoff or anchorage may be designed. The sliding resistance can also be increased by sloping the base of the toe upward, as shown in Fig. 13-17. In computing the passive earth pressure, it is customary to assume that it acts normal to the abutting surface. Coulomb’s equation for passive earth pressure \( P_P \) then reduces to the form

\[
P_P = \frac{1}{2} \gamma_s h^2 \tan^2 \left( 45^\circ + \frac{\phi}{2} \right)
\]  

(13-10)

The criterion for stability against sliding then becomes

\[
(R_V \tan \delta_B + P_P) \geq P_{AH} \text{ (factor of safety)}
\]  

(13-11)

The depth of anchorage is determined to provide the desired factor of safety against sliding, assuming that the earth above the level of the top of the base at the toe is absent, as a conservative basis.
DRAINAGE PROVISIONS

Experience shows that retaining-wall failures occur most frequently during heavy rainstorms, if the backfill becomes saturated. In the case where a retaining wall must be designed for both earth (weight in water) and water pressure, it is necessary to ascertain the probable height of the ground water in the backfill before the total pressure can be computed. Retaining walls are, however, seldom designed to withstand both earth pressure and full hydrostatic pressure of water in the backfill, because of the excessive cost of construction. It is therefore essential to design and install a system of drains, which is adequate and which will function permanently without clogging to carry off surface water and to drain the backfill. If the backfill material is nondrainable, that is, contains more than 10 per cent of material passing the No. 200 sieve (silt and clay), selected sand, gravel, or broken stone should first be placed adjacent to the back of the wall to a sufficient depth so that the drainage system can function properly. The type, size, spacing, etc., of drains and weep holes will depend upon the intensity, duration, and frequency of heavy rainstorms in the given locality. Specifications usually require weep holes at least 4 in. in diameter, spaced about 15 ft apart, and at elevations where water can be readily disposed of. (Recommended Practice and Standard Specifications for Concrete and Reinforced Concrete, Proc. ASCE, June, 1940, part 2, Joint Committee Report, Sec. 877 on drainage of retaining walls, pp. 77-79.) All drain tile or pipe should be embedded in a filter medium of suitable coarse sand or fine gravel, which will permit free entrance of water to the drains but will effectively prevent the eventual clogging of the drains with fine material washed out of the earth. In general, good drainage is one of the cheapest forms of insurance against the ultimate failure of retaining walls from water pressure in the backfill during heavy storms.

DESIGN OF REINFORCED-CONCRETE RETAINING WALL

The conditions, type of loading, and design data are given in Design Sheet 13-1 for the design of a reinforced-concrete cantilever-type retaining wall for a location in central New York State. Tentative dimensions are assumed for the wall section, as shown, in order to make the stability analysis. In this locality the base of the wall should be placed at least 4 ft below the surface of the ground in order to prevent objectionable movements by frost heaving, making the total height of the wall equal to 20 ft. A minimum thickness of 12 in. at the top of the vertical stem is required for pouring concrete. A batter is given to the back of the wall to provide sufficient thickness for movements and shears, assuming a trial width at the bottom of the stem of 24 in. The appearance of the wall may be improved by giving a batter to the front of the stem of about \( \frac{1}{4} \) in. per ft.

Comparative designs have shown that a base width of about 0.5 to 0.7 times the height of the wall is required. The center of the stem is placed over the point where it is desired that the resultant should strike the base, which as a trial design is taken as 0.6B in order to limit settlement and tilting. A trial thickness of 24 in. is assumed for the base of the wall.

The magnitude and position of the resultant active earth pressure \( P_A \) for the given conditions of loading are determined in Design Sheet 13-1b by the Culman graphical method, with the wedge of inert earth assumed to extend from the base of the pavement to the heel of the wall. The position of the resultant \( R \) on the base of the wall is computed in Design Sheet 13-2c by taking moments about the heel of the wall at \( b \). Two possibilities as to the choice of base width are investigated: (1) where the resultant strikes the base at about 0.6B, for which a 12-ft base is required, and (2) where the resultant strikes the base at \( \frac{3}{5}B \), for which an 11-ft base is required. The choice will depend primarily on whether settlement and tilting are limiting factors with respect to the pressure at the toe of the base. The conventional distribution of base pressure is computed for both cases.

In order to reach a reliable decision as to whether the maximum base pressure \( p_{max} \) at
(a) Design data and assumed wall section.
Surcharge—highway loading.
A.A.S.H.O. 1941 Sections 3.2.5-8.
H-20 Equivalent uniform loading
640 lb per linear foot of 10-ft lane.
Pavement—6-in. of concrete 150 pcf.
6-in. of stone base 140 pcf.
Backfill—brown coarse to fine sand,
little silt (15%).
Unit dry weight, \( \gamma_s \).
Loose dry, 95+ pcf.
Loose moist, 80+ pcf.
Angle of friction, \( \phi \).
Loose dry, 36° 30'
Loose moist, 30° 0'
Angle of wall friction, \( \beta \).
Earth on concrete.
Loose dry, 28° 50'
Loose moist, 27° 10'
Concrete—standard specifications 1940.
Unit weight, 150 pcf.
\( f_c \) 2000 psi
\( n \) 15
\( f_c \) 0.45 \( f_c \) 900 psi
\( v \) 0.02 \( f_c \) 40 psi
\( \mu \) 0.03 \( f_c \) 60 psi Special anchorage
\( f_s \) 18,000 psi Structural grade steel

(b) Determination of lateral earth pressure.

Design factors adopted
\( \gamma_s \) 80 pcf.

(c) Determination of required base width and distribution of pressure on the base

<table>
<thead>
<tr>
<th>Item</th>
<th>Force</th>
<th>Arm</th>
<th>Moment</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W_1$</td>
<td>$\frac{1}{2}(1+2) \times 18 \times 50$</td>
<td>4050</td>
<td>7.5</td>
</tr>
<tr>
<td>$W_2$</td>
<td>$2 \times 12 \times 150$</td>
<td>3600</td>
<td>6.0</td>
</tr>
<tr>
<td>$W_3$</td>
<td>$\frac{1}{2} \times 5.7 \times 17 \times 60$</td>
<td>3880</td>
<td>4.85</td>
</tr>
<tr>
<td>$P_{AV}$</td>
<td>6200</td>
<td>1.85</td>
<td>11480</td>
</tr>
<tr>
<td>$P_{AH}$</td>
<td>6900</td>
<td>7.02</td>
<td>48400</td>
</tr>
<tr>
<td></td>
<td>17730</td>
<td></td>
<td>130630</td>
</tr>
</tbody>
</table>

$\bar{X} = \frac{130630}{17730} = 7.36$

To limit settlement and tilting assume:
- Base $B = 12$ ft
- $\bar{X} = 0.612$
- For $\bar{X} = 0.667$ cut off 1-ft

Pressure distribution on the base $B = 12$ ft
- Eccentricity, $e = (7.36-6.00) \times 1.36$

$p_{\text{max}} = \frac{17730}{12} \left[ 1 + \frac{6 \times 1.36}{12} \right] = 2480 \text{ psf}$

$p_{\text{min}} = \frac{17730}{12} \left[ 1 - \frac{6 \times 1.36}{12} \right] = 473 \text{ psf}$

$p_{\text{aver}} = \frac{1475 \text{ psf}}{1475} \left[ \frac{2480-1475}{1475} \right] = 0.68$

(d) Estimates of settlement and tilting of wall

Estimate of probable average settlement Eq.13-5

$W_a = W_o \frac{P_{\text{aver}}}{B_o} \frac{B}{B_0}$

For $B = 12' \quad W_a = 0.78 \text{ in.}$

For $B = 11' \quad W_a = 0.83 \text{ in.}$

Estimate of probable angle of tilt and yield Eq.13-6

$\Delta W = 0.003 \text{ to } 0.006 W_a \frac{p_{\text{max}}-p_{\text{aver}}}{p_{\text{aver}}} = \tan T$

For $B = 12' \quad \tan T = 0.0016 \text{ to } 0.0032$

For $B = 11' \quad \tan T = 0.0025 \text{ to } 0.0050$

Yield at top of wall = 12$H\times\tan T$ inches

For $B = 12' \quad \text{Yield} = 0.38 \text{ to } 0.77 \text{ inches}$

For $B = 11' \quad \text{Yield} = 0.60 \text{ to } 1.2 \text{ inches}$

Minimum yield to satisfy Coulomb's conditions Fig.13-3

Loose backfill - 0.002$\times$20$\times$12 = 0.48-in.

Maximum yield, less than 2-in. Not objectionable.

Base pressures are satisfactory 11-ft base adopted

Natural density of foundation soil, 104pcf

Angle of base friction, $\delta_b = \text{not less than } 27^\circ 10'$ (loose)

Stability Eq.13-9 $R_p \tan \delta_b \geq P_{\text{HF}} (FS)$

$FS = 177300 \times 0.513/6900 \times 1.32 \text{ (Factor of safety)}$

To make $FS = 1.5$ a cut-off or anchorage is required

Passive pressure, $P_p$ required

$[9500 \tan 30^\circ + 8230 \tan 27^\circ 10'] + P_p = 6900 \times 1.5$

$P_p = 650 \text{ lbs } \frac{1}{2} W H_b \tan \left( \frac{45 + \delta_b}{2} \right)$

Required $H_b = 2$ ft Minimum depth of cut-off, $h = 1$ ft to develop resistance of earth on earth along plane a-a

DESIGN SHEET 13-2, Design of reinforced-concrete retaining wall.
EARTH PRESSURES AND RETAINING WALLS

(f) Design of vertical stem of wall

Distribution of lateral earth pressure against vertical stem of wall - horizontal component, \( P_{\text{AH}} \) from Plate 13-1 (lb)

\[ P_{\text{AH}} = \frac{2 \times 6900}{[1 + 2.61/(19 + 2.61)]} \text{ lb} \]

\[ \text{Steel area, } A_s = \frac{600 \text{ lb per linear foot of wall}}{V_{\text{bld}}} \]

<table>
<thead>
<tr>
<th>Vertical stem - shears, moments, required thickness and steel area</th>
<th>( h, \text{ ft} )</th>
<th>( V, \text{ lb} )</th>
<th>( d, \text{ in.} )</th>
<th>( v \cdot \frac{V_{\text{bld}}}{M} )</th>
<th>( M, \text{ ft-lb} )</th>
<th>( R, \text{ lb} )</th>
<th>( \frac{M}{bd} )</th>
<th>( p )</th>
<th>( A_s )</th>
<th>Steel</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.25</td>
<td>559</td>
<td>12</td>
<td>4.5</td>
<td>1004</td>
<td>70</td>
<td>0.001</td>
<td>0.012</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8.50</td>
<td>1621</td>
<td>15</td>
<td>10.3</td>
<td>5460</td>
<td>24.3</td>
<td>0.0015</td>
<td>0.023</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>12.75</td>
<td>3185</td>
<td>18</td>
<td>16.9</td>
<td>15490</td>
<td>47.7</td>
<td>0.0028</td>
<td>0.050</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>17.00</td>
<td>5250</td>
<td>21</td>
<td>23.8</td>
<td>33150</td>
<td>75.1</td>
<td>0.0048</td>
<td>0.100</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Thickness at section a-a required for shear

\[ d = \frac{V}{b(j)} = \frac{5250}{12 \times 0.60 \times 40} = 12.5 \text{ in.} \]

Length of key required at the construction joint, a-a

Thickness at section a-a required for moment

\[ d = \sqrt{\frac{M}{Rb}} = \sqrt{\frac{33150}{165 \times 40}} = 14.2 \text{ in.} \]

Practical thickness adapted - (21-in. + 3-in. cover) x 24 in.

Top of vertical stem - 12-in. for pouring concrete

Embedment - \( L = \frac{f_s}{4u} \)

Bond stress - \( u = \frac{Vb}{\sum 0} = \frac{23.8 \times 6}{2.75} = 51.7 \text{ psi} \) (allowed - 100 psi)

Temperature steel. Longitudinal reinforcement to prevent cracks due to shrinkage and to temperature stresses - not less than 0.50 sq in. area per ft of height with maximum spacing of 12-in. cts and placed 2-in. from the exposed face - 5/8 φ 7½ in. cts horizontal, 1/2 φ 30 in. cts vertical

Moment and shear at section b-b per ft of wall

<table>
<thead>
<tr>
<th>Item</th>
<th>Force</th>
<th>Arm</th>
<th>Moment</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W_2$</td>
<td>+975</td>
<td>3.25</td>
<td>3170</td>
</tr>
<tr>
<td>$W_3$</td>
<td>+3880</td>
<td>1.65</td>
<td>6400</td>
</tr>
<tr>
<td>$P_{av}$</td>
<td>+6200</td>
<td>4.65</td>
<td>28800</td>
</tr>
<tr>
<td>$R_V$</td>
<td>-6170</td>
<td>2.17</td>
<td>-13400</td>
</tr>
<tr>
<td>Shear</td>
<td>+4885</td>
<td></td>
<td>+24970</td>
</tr>
</tbody>
</table>

Thickness required at section b-b

Shear $d = \frac{V}{bJ_V} = \frac{4885}{12 \times 78 \times 40} = 11.5$ in.

Moment $d = \sqrt{\frac{M}{Rb}} = \sqrt{\frac{24970}{165 \times 1}} = 12.3$ in.

Thickness adopted - (21 in. + 3 in. cover) = 24 in.

Steel reinforcement

$R = \frac{M}{bd^2} = \frac{24970}{1 \times 21^2} = 56.5$

$p = 0.0035$, $A_s = 0.0735$ sq in. per in.

Use $\frac{7}{8}^\#$ bars on 8 in. cts (Deformed)

Bond $u = \frac{vb}{2a} = \frac{5.75 \times 8}{2.75} = 64.3$ psi

Embedment $L = \frac{f_cD}{4u} = \frac{18000}{4 \times 100} \times \frac{7}{8} = 40$ in.

Length of bar - 9' 8"

**Moment and Shear at section c-c**

$V = 6400$ lbs, $v = 29.0$ psi

$M = 8370$ ft lbs per ft of wall, $R = 19.0$

Steel reinforcement

$p = 0.0013$, $A_s = 0.0273$ sq in. per in.

Use $\frac{7}{8}^\#$ bars on 6 in. cts (Deformed)

Bond $u = \frac{290 \times 6}{1.96} = 89$ psi

Embedment $L = \frac{18000}{4 \times 100} = 28$ in.

Length of bar - 5' 0"

Resistance required for $FS = 1.5$ against sliding

$P_p < 650$ lbs per ft of wall

Practical dimensions of anchorage as shown

Shear stress $v = 2.5$ psi

**Design Sheet 13-4. Design of reinforced-concrete retaining wall.**
the toe is within satisfactory limits, an estimate is made as to the probable order of magnitude of the settlement and tilting of the wall, using the load-test data on Design Sheet 13-2d and Eqs. (13-5) and (13-6). Since estimates of probable tilting and settlement are within satisfactory limits for the assumed 11-ft base, with the resultant at \( \frac{3}{2}B \) and Coulomb's conditions satisfied (and also the toe pressure \( p_{\text{max}} \) is less than the tabular allowable bearing values of Table 13-2), this base width is adopted as final. The stability against sliding is checked in Design Sheet 13-2e. Assuming that the earth above the base of the wall is absent, a minimum depth of anchorage of 1 ft is required to develop the resistance of earth to earth under the toe section of the base and to provide sufficient additional lateral-pressure resistance in front of the wall to give a factor of safety of at least 1.5.

The design of the stem is made in Design Sheet 13-3, assuming that the horizontal component of the lateral earth pressure of Design Sheet 13-2e is transferred to the back of the stem. A thickness of 24 in. at the base of the stem is adopted as a practical value for construction of a key and for pouring concrete. It is not practicable to design the stem and base for the relatively thin section satisfying the requirements of moments and shears only, and a lower-strength more economical concrete is therefore used. The area, spacing, and lengths of reinforcing steel in the stem are determined, using the moment diagram to locate the cutoff points. The base of the wall is designed in Design Sheet 13-4. The forces acting on the heel and the sections of the
base were obtained from Design Sheet 13-2c. The steel reinforcement is determined for a practical base thickness of 24 in. All steel reinforcement, where the concrete is in contact with the earth, is placed 3 in. from the surface of the concrete. The Joint Committee Report on Recommended Practice and Standard Specifications for Concrete and Reinforced Concrete (Proc. ASCE, June, 1940) has been used in the design (Sec. 878, Table 7 on Unit Stresses, p. 80, and Secs. 872 to 877 on Retaining Walls). If the base pressure should have the probable distribution indicated in Fig. 13-14a, the steel reinforcing in the toe would be overstressed. It is therefore decided in view of this possibility to use \( \frac{3}{8} \) in. round bars in the toe on 6-in. centers, thus using this size of bar throughout for the reinforcing of the stem and the base. The final design is shown in Fig. 13-18 with recommendations for the drainage installation.

The wall is constructed in sections with expansion or contraction joints spaced about 30 ft apart. This joint is made watertight to prevent unsightly stains from seepage. A keyway is frequently designed or short dowel bars with one end encased in a greased tube to permit expansion or contraction are used to prevent relative lateral displacement of the sections, as a result of unequal settlement and tilting.

**DESIGN OF COUNTERFORT WALLS**

For walls greater than about 20 ft in height, a great saving is effected in the amount of material used by placing counterforts at intervals on the back side of the wall, tying them to the base slab, as shown in Fig. 13-1e. The vertical wall is therefore a slab continuous over the counterforts and is loaded by horizontal loads, thus saving much material in the wall itself.

The width of the base and the stability of the wall as a whole are determined in the same manner as for the cantilever-type wall. The counterforts should have a thickness equal to about one-twentieth of the height of the wall, and preferably not less than 12 in. Their spacing should be such as to give a minimum of material required for the wall. This spacing should not be less than 7 ft for walls less than 20 ft in height, and should increase approximately linearly to about 11 ft for walls 40 ft in height.

The thickness of the vertical wall and the reinforcing steel at any height above the base is commonly computed by considering this wall to be made up of horizontally loaded beams continuous over the counterforts, as indicated in Fig. 13-1e. The back floor slab must be designed to resist the upward reaction of the soil against it and the weight of the earth above. The whole floor slab is designed either for shear at the counterforts or for moments in the slab treated as a continuous beam over the counterforts. The toe of the base is designed as for the cantilever wall.

The counterfort must be designed to take the tension developed as a result of the action of the lateral earth pressure against the vertical wall and the net downward earth pressure on the base of the back slab of the base. The counterforts are designed as wedge-shaped cantilever beams fixed to the base slab. Such an assumption requires that the base slab be rigidly attached to the bottom of the counterfort over the entire length of the base. The principal reinforcing in the counterfort is the steel along the inclined face, tying the upper end of the vertical wall to the back edge of the base slab. Both ends of this steel must be thoroughly anchored. Horizontal and vertical steel should extend across the counterfort into the vertical wall and into the base slab and should be anchored to tie them thoroughly together.
PART 2

DESIGN OF REINFORCED-CONCRETE RETAINING WALLS ON PILES*

RETAINING WALLS REQUIRING PILE FOUNDATIONS

The depth at which to place the base of the wall is determined by either the depth of the frost line below the ground level or by the required or desired depth of the top of the base below the ground level, pavement surface, or other construction in front of the wall. Borings or other subsurface investigations may indicate that the foundation material at the depth so determined is wholly unsuitable for a spread-footing foundation. If such unsuitable materials extend for an additional depth of 10 ft or more, the practicalities of construction as well as economy will in general require a pile foundation for the wall. Differences and variations in costs of excavation, concrete construction, reinforcing steel, and the furnishing and driving of piles for various localities and times of construction should always be given full consideration in determining whether to design the wall for a pile foundation or for a spread footing at a depth greater than normally required in order to establish the base on a stable foundation material.

For a given height of wall and given design data, for both spread footings and pile foundations, a most economical section can be determined. The minimum base width requires the minimum amount of excavation, concrete, and in general, reinforcing steel in the base. The projection of the base in front of the front face of the wall, usually called the toe of the footing, is an important factor. For a wall 25 ft high with a spread footing, on foundation material capable of supporting not more than 1½ tons per sq ft, and backfilled with material weighing 120 lb per cu ft, it is obvious that a toeless base would have to be infinitely wide. With a toe of 4 ft or more, a design with a practical base width is possible.

The determination of the loads on a retaining wall on a pile foundation and the design of the vertical stem are the same as for a wall on a spread footing. In the design of the base, however, the similarity ceases, and decision must be made regarding the type of pile to be used, its load capacity, and the maximum and minimum spacing of the piles. The number of rows of piles, the transverse spacing of the rows of piles, and the longitudinal spacing of the piles in each row will affect the location of the center of gravity of the pile group. The relation of the location of the resultant pressure to the location of the center of gravity of the pile group will affect the maximum and minimum loads on the piles.

DESIGN STANDARDS FOR RETAINING WALLS SUPPORTED ON PILES

The purpose of this part of Sec. 13 is to illustrate the design of a reinforced-concrete retaining wall supported on piles and varying in height, in conformity with certain design standards established for a project. A typical section of the wall is shown in Fig. 13-19.

For any given height $h$ the known dimensions are:

- $t =$ assumed (a variation of 6 in. or less will have a negligible effect on the magnitude and position of the resultant load on the pile foundation)
- $h_1 = h - t$
- $d = h_1/12$

* By R. Edward Kuhn.
The unknown dimensions are $b$ and $f$ (or $b_1$ and $f_1$, each 1 ft 6 in. less than $b$ and $f$, respectively).

The design is to be based on the following standards:
1. Factor of safety against overturning. The ratio of resisting moment to overturning moment shall be not less than 2 under normal lateral earth pressure plus live-load surcharge. Moments shall be taken at bottom of footing and at the center line of the front piles.
2. Weights of materials. Fill and backfill, 120 lb per cu ft (compacted). Concrete, 150 lb per cu ft.
3. Lateral earth pressure shall be determined on the basis of an assumed hydrostatic pressure distribution where $\gamma_s K_A = 120 \times 0.29 = 35$.
4. Live-load surcharge, 2 ft.
5. Depth of earth on front of toe, 2 ft minimum.
6. Resultant pressure line at bottom of footing; avoid any uplift on piles.
7. Types of piles, cast-in-place concrete.
8. Allowable load on piles, 30 tons.
9. Pile spacing, minimum 2 ft 9 in., preferably 3 ft 0 in., both transversely and longitudinally; maximum longitudinally 10 ft for lower walls and 7 ft 6 in. for higher walls.
10. Minimum distance from edge of footing to center line of pile, 1 ft 4 in. and preferably 1 ft 6 in.
11. Pile embedment in footing, 6 in. for concrete piles with reinforcing projecting into the footing.
12. Provision for horizontal loads. Piles shall be battered to care for hydrostatic earth pressure only (no live-load surcharge) without bending in the piles. Preferably one row of piles shall be vertical. So far as practicable, avoid batters greater than 12 vertical to 4 horizontal.
13. Design of heel of footing. In computing stresses in the heel, it shall be considered as a cantilever supported by the rear reinforcing steel of the stem and loaded with the weight of the heel, the fill material on it, and the upward pressures on it from the piles.
DESIGN OF REINFORCED-CONCRETE RETAINING WALLS

In addition to the above requirements, the final section, pile group, and structural elements should be checked for conformity to the following requirements:

14. When in Eq. (13-1b) for the total magnitude of active earth pressure

\[ P_A = \frac{1}{2} \gamma h K_A H^2 \]

and \( K_A = 0.1 \), with the center of pressure 0.4\( h \) above the footing,

a. The loads on the piles under the heel shall not exceed 1.33 times the allowable load.

b. The design of the heel of the footing shall be adequate at 33 per cent increase in allowable unit stresses.

15. When \( 1.5 \gamma h K_A = 1.5 \times 35 = 52.5 \) (for a water-contained fill), with the center of lateral earth pressure \( \frac{1}{2} h \) above the bottom of the footing,

a. The maximum load on the piles shall not exceed 1.50 times the allowable load.

b. There shall be no uplift on any of the piles.

With respect to the design standards for lateral earth pressure adopted above, reference is made to the discussion on this in Part I of this section. For want of better knowledge and understanding, these design standards are based on the classical theories of earth pressures. It is conceded that these standards may not result in a rational design but only in a rough approximation. As continued research increases the knowledge and understanding of the application of soil mechanics and establishes a body of authoritative knowledge to be used as a basis of design, the intensity and effect of lateral earth pressure can be more rationally determined. Variations in design criteria will not change the method used in the following example to determine the most economical dimensions of the structure designed for such criteria or standards. The adoption of the above standards for the ensuing example under no circumstances should influence the reader in ignoring the importance of research and investigations of earth-pressure phenomena.

NOMENCLATURE USED IN RETAINING-WALL DESIGN

The vertical and lateral or horizontal loads, and their moments about the center line of the front row of piles at the bottom of the base, for a wall of any height \( h \), base thickness \( t \), base width \( b \), and toe projection \( f \), are in accordance with Table 13-4. These values, and all other calculations in this design example, are based on a 1-ft unit length of wall. For simplicity, the weight of the stem within height \( h_1 \), is broken into two parts, first at 120 lb per cu ft, which is included with the earth back of the stem, and over the base as one unit, and second at 30 lb per cu ft, to make the total weight of 150 lb per cu ft. The term \( s \) is the distance from the front face of the wall to the center of gravity of the stem \( A_s \), which is a known value for a given height \( h_1 \), a given batter on the back face of the stem, and a given thickness at the top of section \( A_s \).

To establish a standard design for walls ranging in height \( h \) from 10 to 30 ft, a simple and logical method consists of constructing curves based upon a design analysis for several different heights of walls, the heights varying by a uniform difference between the lowest and highest wall in the total range, for example, 10, 15, 20, 25, and 30 ft.

Using these five heights of wall, viz., 10, 15, 20, 25, and 30 ft, in this design illustration, the known physical dimensions for each of these heights, based upon an assumed thickness of base \( t \), will be as given in Table 13-5.

Substituting the known values given in Table 13-5 in the summations given in Table 13-4, we get, for each of the five heights of wall for which a design analysis is to be made, values of the vertical loads \( V \) and the resisting moment \( RM \) in terms of the unknowns \( b_1 \) and \( f_1 \), and definite numerical values for the overturning moments \( OM \) and the horizontal loads \( H \) as given in Table 13-6.
<table>
<thead>
<tr>
<th>Load items</th>
<th>Vertical loads</th>
<th>Horizontal loads</th>
<th>Moment arm, ft</th>
<th>Moments, M, ft-lb</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Parapet A₁</td>
<td>150A₁ = 450</td>
<td>f₁ + 0.5</td>
<td>450f₁ + 225</td>
<td></td>
</tr>
<tr>
<td>2. Stem at 120 lb and earth</td>
<td>120h₁(b₁ - f₁)</td>
<td>3½(b₁ + f₁)</td>
<td>60h₁b₁ + 60hf₁</td>
<td></td>
</tr>
<tr>
<td>back of stem</td>
<td>30A₁</td>
<td>f₁ + s</td>
<td>30A₁s + 30A₁s²</td>
<td></td>
</tr>
<tr>
<td>3. Stem A₂ at 30 lb</td>
<td>240f₁</td>
<td>3½f₁</td>
<td>120f₁²</td>
<td></td>
</tr>
<tr>
<td>4. Earth on toe back of B</td>
<td>150b₁</td>
<td>3½b₁</td>
<td>75b₁²</td>
<td></td>
</tr>
<tr>
<td>5. Footing back of B</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6. Earth on toe front of B</td>
<td>2 × 120 × 1.5 = 360</td>
<td></td>
<td>-0.75</td>
<td>-270</td>
</tr>
<tr>
<td>7. Footing front of B</td>
<td>150 × 1.5 × t = 225t</td>
<td></td>
<td>-0.75</td>
<td>-168.75t</td>
</tr>
<tr>
<td>8. Earth pressure (normal)</td>
<td></td>
<td>3½ × 35h₁² = 17.5h₁²</td>
<td>5.83h₁³</td>
<td></td>
</tr>
<tr>
<td>9. Live-load surcharge</td>
<td>2 × 35 × h = 70h</td>
<td>3½h</td>
<td>35h³</td>
<td></td>
</tr>
<tr>
<td>10. Earth pressure (reduced)</td>
<td>3½ × 12 × h² = 6h²</td>
<td>0.4h</td>
<td>2.4h³</td>
<td></td>
</tr>
<tr>
<td>11. Earth pressure (increased)</td>
<td>3½ × 52.5 × h³ = 26.25h³</td>
<td>3½h</td>
<td>8.75h³</td>
<td></td>
</tr>
</tbody>
</table>

\[ \Sigma V = 30b₁(4h₁ + 5t) - 120f₁(h₁ - 2) + 30A₁ + 225t + 810 \]

\[ \Sigma M = - (168.75t + 270 + 5.83h₁³) \]

\[ \Sigma M = - (168.75t + 270 + 5.83h₁³ + 35h₁³) \]

\[ \Sigma M = - (168.75t + 270 + 2.4h³) \]

\[ \Sigma M = - (168.75t + 270 + 8.75h³) \]
RETIWING-WALL LOADING CONDITIONS

The four cases I, II, III, and IV listed in Tables 13-4 and 13-6 indicate the overturning moments and horizontal loads for the particular standards or criteria to which the design of the wall shall conform.

Case I. The loading condition under which the sum of the horizontal components of the loads on the battered piles shall be not less than the horizontal load (normal lateral earth pressure) on the wall.

Case II. The loading condition under which \( RM = 2 \times OM \), and (2) the allowable pile load = 30 tons.

Case III. The loading condition under which (1) the allowable load on the rear row of piles = 40 tons, and (2) no uplift is permitted on the piles in the front row.

Case IV. The loading condition under which (1) the allowable load on the front row of piles = 45 tons, and (2) no uplift is permitted on the piles in the rear row.

### Table 13-5. Properties of Wall Stem

<table>
<thead>
<tr>
<th>( h ), ft</th>
<th>( t ), ft and in.</th>
<th>( h_s ), ft and in.</th>
<th>( A_5 ), sq ft</th>
<th>( a ), ft</th>
<th>( A_{5a} ), ft²</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>2 ft 3 in.</td>
<td>7 ft 9 in.</td>
<td>10.25</td>
<td>0.675</td>
<td>6.92</td>
</tr>
<tr>
<td>15</td>
<td>2 ft 5 in.</td>
<td>12 ft 7 in.</td>
<td>19.18</td>
<td>0.792</td>
<td>15.2</td>
</tr>
<tr>
<td>20</td>
<td>2 ft 10 in.</td>
<td>17 ft 2 in.</td>
<td>29.45</td>
<td>0.907</td>
<td>26.72</td>
</tr>
<tr>
<td>25</td>
<td>3 ft 3 in.</td>
<td>21 ft 9 in.</td>
<td>41.46</td>
<td>1.025</td>
<td>42.5</td>
</tr>
<tr>
<td>30</td>
<td>3 ft 8 in.</td>
<td>26 ft 4 in.</td>
<td>55.23</td>
<td>1.144</td>
<td>63.2</td>
</tr>
</tbody>
</table>

### Table 13-6. Formulas for \( V \), \( RM \), and Values of \( OM \) and \( H \)

<table>
<thead>
<tr>
<th>( h ), ft</th>
<th>( V )</th>
<th>( RM )</th>
<th>Case</th>
<th>( OM )</th>
<th>( H )</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>1.268b₁ - 690t₁ + 1.625</td>
<td>634b₁² - 345t₁² + 758t₁ + 430</td>
<td>I</td>
<td>6.480</td>
<td>1.750</td>
</tr>
<tr>
<td>II</td>
<td>9.980</td>
<td>2.450</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>III</td>
<td>3.050</td>
<td>600</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>IV</td>
<td>9.400</td>
<td>2.625</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>1.873b₁ - 1.270t₁ + 1.940</td>
<td>936b₁² - 1025t₁² + 1.025t₁ + 680</td>
<td>I</td>
<td>20.370</td>
<td>3.940</td>
</tr>
<tr>
<td>II</td>
<td>28.240</td>
<td>4.990</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>III</td>
<td>8.780</td>
<td>1.350</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>IV</td>
<td>30.210</td>
<td>5.900</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>2.485b₁ - 1.820t₁ + 2.330</td>
<td>1243b₁² - 1.135t₁² + 1.335t₁ + 1.030</td>
<td>I</td>
<td>47.410</td>
<td>7.000</td>
</tr>
<tr>
<td>II</td>
<td>61.410</td>
<td>8.400</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>III</td>
<td>19.950</td>
<td>2.400</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>IV</td>
<td>70.750</td>
<td>10.500</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>25</td>
<td>3.098b₁ - 2.370t₁ + 2.785</td>
<td>1.549b₁² - 1.85t₁² + 1.695t₁ + 1.500</td>
<td>I</td>
<td>91.960</td>
<td>10.940</td>
</tr>
<tr>
<td>II</td>
<td>113.840</td>
<td>12.690</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>III</td>
<td>38.320</td>
<td>3.750</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>IV</td>
<td>137.540</td>
<td>16.400</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>3.710b₁ - 2.920t₁ + 3.290</td>
<td>1.855b₁² - 1.460t₁² + 2.107t₁ + 2.120</td>
<td>I</td>
<td>158.400</td>
<td>15.750</td>
</tr>
<tr>
<td>II</td>
<td>189.900</td>
<td>17.850</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>III</td>
<td>65.700</td>
<td>5.400</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>IV</td>
<td>237.140</td>
<td>23.630</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Since the design standards for the horizontal loads are based on the classical theories of earth pressures, resulting in only a rough approximation for the design of the walls, pile loads in excess of the allowable loads stated above by 2 or 3 per cent and a slight deficiency in the sum of the horizontal components of the loads on the battered piles under the Case I loading conditions will be deemed permissible.
STEPS IN THE PRELIMINARY DESIGN OF A RETAINING WALL

The first step in the design of the walls is the determination of the minimum values of $b_1$ for various values of $f_1$ to meet the requirement that $RM = 2 \times OM$ under the Case II loading condition. Using the values of $RM$ and $OM$ for Case II given in Table 13-6, we get minimum values of $b_1$ as given in Table 13-7. It should be noted that for each height of wall the variations in $b_1$, for reasonably possible ranges of $f_1$, are negligible. Under the circumstances it is safe to assume a constant minimum value of $b_1$, as given in Table 13-7, in making trial analyses to determine the proper value of $f_1$.

### Table 13-7. Values of $b_1$

<table>
<thead>
<tr>
<th>$h_1$ ft</th>
<th>$f_1 = 0$ ft</th>
<th>$f_1 = 0.5$ ft</th>
<th>$f_1 = 1.0$ ft</th>
<th>$f_1 = 1.5$ ft</th>
<th>$f_1 = 2.0$ ft</th>
<th>$f_1 = 2.5$ ft</th>
<th>$b_1$ use</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>5.55 ft</td>
<td>5.51 ft</td>
<td>5.49 ft</td>
<td>7.71 ft</td>
<td>9.94 ft</td>
<td>12.17 ft</td>
<td>5 ft 7 in.</td>
</tr>
<tr>
<td>15</td>
<td>7.72 ft</td>
<td>7.69 ft</td>
<td>7.68 ft</td>
<td>9.90 ft</td>
<td>12.12 ft</td>
<td>14.34 ft</td>
<td>7 ft 9 in.</td>
</tr>
<tr>
<td>25</td>
<td>12.08 ft</td>
<td>12.07 ft</td>
<td>12.07 ft</td>
<td>14.27 ft</td>
<td>14.30 ft</td>
<td>14.34 ft</td>
<td>12 ft 1 in.</td>
</tr>
</tbody>
</table>

The second step consists of trial analyses of the two extreme heights of wall, using various values of $f_1$, to determine that value of $f_1$ which will most satisfactorily meet all the design standards or criteria. Such trial analyses, in this design example, resulted in values of $f_1$ equal to zero and 1 ft 4 in. for the 10- and 30-ft-high walls, respectively.

The third step consists of a trial analysis of the 15-ft-high wall to determine whether or not the variation in $f_1$ between the 10- and the 30-ft-high walls is uniform. Such a trial analysis resulted in a value of $f_1$ that most satisfactorily meets all the design standards, equal to zero.

The next step consists of similar trial analyses of the 20- and the 25-ft-high walls. Such trial analyses indicated a reasonably uniform variation in the values of $f_1$ between the 15- and the 30-ft-high walls.

### FINAL DESIGN ANALYSES FOR RETAINING WALLS

For the final design analyses, the values of $f_1$ were assumed as follows: $h = 10, 15, 20, 25,$ and 30 ft; $f_1 = 0, 0, 5, 10,$ and 15 in. This results in a straight-line variation in $f_1$ between the 15- and the 30-ft-high walls of 1 in. for each foot increment in height of wall above 15 ft.

The several trial analyses also indicated that the assumed minimum values of $b_1$ were satisfactory. It should be noted that $b_1$ varies uniformly by 2 ft 2 in. for each 5-ft increment in height of wall.

Using the values of $b_1$ and $f_1$ as determined above, the next step is to determine the vertical loads $V$, the resisting moment $RM$, and for each of the four cases I, II, III, and IV the net moment $MN$ and the location of the resultant pressure line from the center line of the front row of piles, represented by the distance $x$. These are given in Table 13-8 and constitute the necessary data for the final design analyses.

For this design example it was also assumed that the wall would be constructed in sections having a standard length of 30 ft, with either a contraction or an expansion joint between the sections. Because of joint material, keyed joints were assumed to be ineffective in transferring horizontal loads from one section to another, and longitudinal pile spacing was based upon obtaining equal spaces in a row within the 30-ft length with definite or fixed values of piles per linear foot, in accordance with Table 13-9.
## Table 13-8. Values of $\phi$

<table>
<thead>
<tr>
<th>$h =$</th>
<th>10 ft</th>
<th>15 ft</th>
<th>20 ft</th>
<th>25 ft</th>
<th>30 ft</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b_1 =$</td>
<td>5 ft 7 in.</td>
<td>7 ft 9 in.</td>
<td>9 ft 11 in.</td>
<td>12 ft 1 in.</td>
<td>14 ft 3 in.</td>
</tr>
<tr>
<td>$f_1 =$</td>
<td>0</td>
<td>0</td>
<td>5 in.</td>
<td>10 in.</td>
<td>1 ft 3 in.</td>
</tr>
<tr>
<td>$V =$</td>
<td>8,700 lb</td>
<td>16,470 lb</td>
<td>26,210 lb</td>
<td>38,240 lb</td>
<td>52,510 lb</td>
</tr>
<tr>
<td>$RM =$</td>
<td>20,190 ft-lb</td>
<td>56,900 ft-lb</td>
<td>123,620 ft-lb</td>
<td>228,250 ft-lb</td>
<td>379,150 ft-lb</td>
</tr>
</tbody>
</table>

Case I $M_N$ $\phi$: 13,710, 1.58 ft
Case II $M_N$ $\phi$: 10,210, 1.17 ft
Case III $M_N$ $\phi$: 17,140, 1.97 ft
Case IV $M_N$ $\phi$: 10,790, 1.29 ft

## Table 13-9. Number, Spacing, and (Area of) Piles per 30-ft Length of Wall

<table>
<thead>
<tr>
<th>No. of piles in a row</th>
<th>Longitudinal spacing</th>
<th>Piles per lin ft</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>3 ft 0 in.</td>
<td>0.333</td>
</tr>
<tr>
<td>9</td>
<td>3 ft 4 in.</td>
<td>0.300</td>
</tr>
<tr>
<td>8</td>
<td>3 ft 8 in.</td>
<td>0.267</td>
</tr>
<tr>
<td>7</td>
<td>4 ft 3 1/2 in.</td>
<td>0.233</td>
</tr>
<tr>
<td>6</td>
<td>5 ft 0 in.</td>
<td>0.200</td>
</tr>
<tr>
<td>5</td>
<td>6 ft 0 in.</td>
<td>0.167</td>
</tr>
<tr>
<td>4</td>
<td>7 ft 6 in.</td>
<td>0.133</td>
</tr>
<tr>
<td>3</td>
<td>10 ft 0 in.</td>
<td>0.100</td>
</tr>
</tbody>
</table>

Fig. 13-20. Graphical relationship of various wall factors plotted against wall height.
Before proceeding with the final design analyses for each of the five heights of wall, the values of \( V \) and \( \bar{z} \) were plotted in Fig. 13-20. On the basis of the straight-line variation for \( b_t \), the partial straight-line variation for \( f_t \), the almost straight-line variations for \( \bar{z} \), and the smooth-curve variations for \( V \), it is natural to assume that smooth curves, if not straight-line variations, are possible for the number of piles to be used, the location of the center of gravity and the moment of inertia \( I \) of the pile group, and the section moduli of the front and rear rows of piles. The final analyses in this design example were made with this in mind.

It is obvious that a number of trials, involving various combinations of transverse and longitudinal pile spacings, are necessary for each of the five heights of wall, especially for the higher walls where a greater number of combinations of pile spacings are possible. In this design example it was found that only one of the several possible combinations of pile spacing best met all the design standards or criteria, consistent with using the least number of piles.

The results of the final design analyses for each of the five heights of wall are given in Table 13-10. This table includes the number of piles per linear foot of wall, the transverse and longitudinal spacing of piles, the location of the center of gravity of the pile group from the center line of the front row of piles (represented by the dimension \( y \)), the moment of inertia \( I \) of the pile group, the section modulus of the front and rear row of piles, the maximum load on the piles for each of the four cases, the batter of the piles, and the sum of the horizontal components of the loads on the battered piles under Case I and Case II loading conditions.

For the purpose of determining the pile-group design for each 1-ft increment in height of wall between the five heights analyzed, the number of piles per linear foot and the values of \( \bar{y}, I \), and the section moduli of the front and rear rows of piles, \( SM_{f} \) and \( SM_{r} \), respectively, for each of the five heights of wall that were analyzed were plotted and the plotted points connected with straight lines as shown in Fig. 13-20.

A study of the line representing the number of piles per linear foot shows that, between the 15- and 20-ft-high walls, the number of piles varies uniformly by 0.033 piles per linear foot per foot of height of wall, and a similar variation of 0.067 piles between the 25- and 30-ft-high walls. The study further indicates that, since the variation in the number of piles between the 10- and 15-ft-high walls and between the 20- and 25-ft-high walls is not in even multiples of 0.033 piles per linear foot, complete analyses of the 11-, 12-, 13-, 14-, 21-, 22-, 23-, and 24-ft-high walls will be necessary to determine the most economical pile group consistent with the restrictions imposed by the possible longitudinal pile spacings in a 30-ft-long section of wall.

A study of the line representing the values of \( \bar{y} \) shows a distinct break at the 15-ft wall height from what would otherwise be an almost straight line comparable to the plotted lines representing the values of \( \bar{z} \) for Cases II and I. It is obvious that, for a uniform variation in the number of piles per linear foot between 15- and 20-ft-high walls, the variation of \( \bar{y} \) might be either a straight-line or a curved-line variation between the values of \( \bar{y} \) determined for the 15- and 20-ft-high walls. In order to determine the best pile-group design for the 16-, 17-, 18-, and 19-ft-high walls, it was deemed desirable to make complete analyses for these four wall heights also.

Accordingly, tables similar to Tables 13-6, 13-7, and 13-8 were made for the 12 additional wall heights to be analyzed, and various pile groups for each wall height tried.

For the wall heights between 15 and 20 ft, it was found that either a straight- or curved-line variation in the value of \( \bar{y} \) was satisfactory but that in general the curved-line variation resulted in lower maximum pile loads for Case III.

For the wall heights between 20 and 25 ft, it was found that, for the more economical of the two possible pile groups for each wall height, the maximum pile load for Case II was 4 per cent or more above the allowable, and the deficiency in the sum of the horizontal components of the loads in the battered piles under Case I was greater than deemed desirable.

For the wall heights between 10 and 15 ft, the analyses for the 12-, 13-, and 14-ft-high walls indicated that one row only of battered piles was inadequate to satisfy the criteria established under Case I. It was possible to put three rows of piles with two rows battered, in the minimum width of base adopted for the 13- and 14-ft-high walls, with
<table>
<thead>
<tr>
<th>$h$, ft</th>
<th>No. of piles per lin ft</th>
<th>Transverse pile spacing</th>
<th>Longitudinal pile spacing</th>
<th>$g$</th>
<th>$I$</th>
<th>$E$</th>
<th>Section modulus</th>
<th>Case</th>
<th>Max pile load, kips</th>
<th>Sum of horizontal components of loads on battered piles, lb</th>
<th>$H$, lb</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>1.133</td>
<td>3 ft 1 in.</td>
<td>7 ft 13(\frac{1}{2}) in.</td>
<td>12 ft 9 in.</td>
<td>3 ft</td>
<td>3 ft</td>
<td>3 ft</td>
<td>7 ft 6 in.</td>
<td>4.50</td>
<td>18.80</td>
<td>4.18</td>
</tr>
<tr>
<td>25</td>
<td>0.800</td>
<td>3 ft 4 in.</td>
<td>6 ft 10 in.</td>
<td>10 ft 7 in.</td>
<td>3 ft</td>
<td>6 ft</td>
<td>6 ft</td>
<td>7 ft 6 in.</td>
<td>3.88</td>
<td>12.48</td>
<td>3.22</td>
</tr>
<tr>
<td>20</td>
<td>0.533</td>
<td>3 ft 7 in.</td>
<td>8 ft 5 in.</td>
<td>4 ft 3(\frac{3}{4}) in.</td>
<td>5 ft</td>
<td>10 ft</td>
<td>2.92</td>
<td>5.10</td>
<td>1.75</td>
<td>0.93</td>
<td>I II III IV</td>
</tr>
<tr>
<td>15</td>
<td>0.367</td>
<td>3 ft 0 in.</td>
<td>6 ft 2 in.</td>
<td>12 ft 12 in.</td>
<td>6 ft</td>
<td>10 ft</td>
<td>10 ft</td>
<td>2.50</td>
<td>2.41</td>
<td>0.965</td>
<td>0.66</td>
</tr>
<tr>
<td>10</td>
<td>0.233</td>
<td>4 ft 1 in.</td>
<td>7 ft 6 in.</td>
<td>10 ft</td>
<td>12</td>
<td>12</td>
<td>1.75</td>
<td>0.95</td>
<td>0.544</td>
<td>0.408</td>
<td>I II III IV</td>
</tr>
</tbody>
</table>
### Table 13-11. Design Dimensions for Retaining Walls

<table>
<thead>
<tr>
<th>No. of piles per lin ft</th>
<th>$\bar{S}_{fn}$</th>
<th>$S_{Mr}$</th>
<th>Transverse spacing of piles</th>
<th>Longitudinal spacing of piles</th>
<th>Required batter of piles, z horizontal to 12 vertical</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>$h$, ft</th>
<th>$b_1$, in.</th>
<th>$b$, ft</th>
<th>$f$, in.</th>
<th>$t$, ft</th>
<th>3 ft 1½ in.</th>
<th>7 ft 1 in.</th>
<th>12 ft 9 in.</th>
<th>3 ft</th>
<th>3 ft</th>
<th>7 ft 6 in.</th>
<th>4 ft</th>
<th>4 ft</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>14 ft 3 in.</td>
<td>15 ft 9 in.</td>
<td>15</td>
<td>2 ft 9 in.</td>
<td>3 ft 8 in.</td>
<td>1.133 4.5</td>
<td>18.80 4.18</td>
<td>2.28</td>
<td>3 ft</td>
<td>7 ft 1 in.</td>
<td>4 ft</td>
<td>4 ft</td>
</tr>
<tr>
<td>29</td>
<td>13 ft 10 in.</td>
<td>15 ft 4 in.</td>
<td>14</td>
<td>2 ft 8 in.</td>
<td>3 ft 7 in.</td>
<td>1.067 4.38</td>
<td>17.50 4.00</td>
<td>2.20</td>
<td>3 ft 3 in.</td>
<td>7 ft 3½ in.</td>
<td>4 ft</td>
<td>4 ft</td>
</tr>
<tr>
<td>28</td>
<td>13 ft 5 in.</td>
<td>14 ft 11 in.</td>
<td>13</td>
<td>2 ft 7 in.</td>
<td>3 ft 6 in.</td>
<td>1.000 4.26</td>
<td>16.26 3.82</td>
<td>2.12</td>
<td>3 ft 3 in.</td>
<td>7 ft 3 in.</td>
<td>4 ft</td>
<td>4 ft</td>
</tr>
<tr>
<td>27</td>
<td>13 ft 0 in.</td>
<td>14 ft 6 in.</td>
<td>12</td>
<td>2 ft 6 in.</td>
<td>3 ft 5 in.</td>
<td>0.933 4.13</td>
<td>15.02 3.64</td>
<td>2.04</td>
<td>3 ft 3 in.</td>
<td>7 ft 2½ in.</td>
<td>4 ft</td>
<td>4 ft</td>
</tr>
<tr>
<td>26</td>
<td>12 ft 7 in.</td>
<td>14 ft 1 in.</td>
<td>11</td>
<td>2 ft 5 in.</td>
<td>3 ft 4 in.</td>
<td>0.867 4.01</td>
<td>13.76 3.43</td>
<td>1.95</td>
<td>3 ft 3 in.</td>
<td>7 ft 2 in.</td>
<td>4 ft</td>
<td>4 ft</td>
</tr>
<tr>
<td>25</td>
<td>12 ft 1 in.</td>
<td>13 ft 7 in.</td>
<td>10</td>
<td>2 ft 4 in.</td>
<td>3 ft 3 in.</td>
<td>0.800 3.88</td>
<td>12.48 3.22</td>
<td>1.86</td>
<td>3 ft 4 in.</td>
<td>6 ft 10 in.</td>
<td>4 ft</td>
<td>4 ft</td>
</tr>
<tr>
<td>24</td>
<td>11 ft 8 in.</td>
<td>13 ft 2 in.</td>
<td>9</td>
<td>2 ft 3 in.</td>
<td>3 ft 2 in.</td>
<td>0.767 3.75</td>
<td>11.25 3.00</td>
<td>1.75</td>
<td>3 ft 2 in.</td>
<td>6 ft 2 in.</td>
<td>4 ft</td>
<td>4 ft</td>
</tr>
<tr>
<td>23</td>
<td>11 ft 3 in.</td>
<td>12 ft 9 in.</td>
<td>8</td>
<td>2 ft 2 in.</td>
<td>3 ft 1 in.</td>
<td>0.700 3.61</td>
<td>10.00 2.77</td>
<td>1.63</td>
<td>3 ft 2 in.</td>
<td>6 ft 2 in.</td>
<td>4 ft</td>
<td>4 ft</td>
</tr>
<tr>
<td>22</td>
<td>10 ft 10 in.</td>
<td>12 ft 4 in.</td>
<td>7</td>
<td>2 ft 1 in.</td>
<td>3 ft 0 in.</td>
<td>0.667 3.54</td>
<td>8.57 2.42</td>
<td>1.48</td>
<td>3 ft 2 in.</td>
<td>6 ft 2 in.</td>
<td>4 ft</td>
<td>4 ft</td>
</tr>
<tr>
<td>21</td>
<td>10 ft 5 in.</td>
<td>11 ft 11 in.</td>
<td>6</td>
<td>2 ft 0 in.</td>
<td>2 ft 11 in.</td>
<td>0.600 3.34</td>
<td>7.18 2.15</td>
<td>1.29</td>
<td>3 ft 1½ in.</td>
<td>6 ft 10 in.</td>
<td>4 ft</td>
<td>4 ft</td>
</tr>
<tr>
<td>20</td>
<td>9 ft 11 in.</td>
<td>11 ft 5 in.</td>
<td>5</td>
<td>1 ft 11 in.</td>
<td>2 ft 10 in.</td>
<td>0.533 2.92</td>
<td>5.10 1.75</td>
<td>0.93</td>
<td>3 ft 7 in.</td>
<td>8 ft 3 in.</td>
<td>4 ft 3½ in.</td>
<td>5 ft</td>
</tr>
<tr>
<td>19</td>
<td>9 ft 6 in.</td>
<td>11 ft 0 in.</td>
<td>4</td>
<td>1 ft 10 in.</td>
<td>2 ft 9 in.</td>
<td>0.500 2.89</td>
<td>4.72 1.63</td>
<td>0.925</td>
<td>3 ft 7 in.</td>
<td>8 ft 0 in.</td>
<td>4 ft 3½ in.</td>
<td>6 ft</td>
</tr>
<tr>
<td>18</td>
<td>9 ft 1 in.</td>
<td>10 ft 7 in.</td>
<td>3</td>
<td>1 ft 9 in.</td>
<td>2 ft 8 in.</td>
<td>0.467 2.83</td>
<td>3.92 1.38</td>
<td>0.825</td>
<td>3 ft 4½ in.</td>
<td>7 ft 2 in.</td>
<td>4 ft 3½ in.</td>
<td>5 ft</td>
</tr>
<tr>
<td>17</td>
<td>8 ft 8 in.</td>
<td>10 ft 2 in.</td>
<td>2</td>
<td>1 ft 8 in.</td>
<td>2 ft 7 in.</td>
<td>0.433 2.74</td>
<td>3.55 1.29</td>
<td>0.805</td>
<td>3 ft 7 in.</td>
<td>7 ft 0 in.</td>
<td>4 ft 3½ in.</td>
<td>5 ft</td>
</tr>
<tr>
<td>16</td>
<td>8 ft 3 in.</td>
<td>9 ft 9 in.</td>
<td>1</td>
<td>1 ft 7 in.</td>
<td>2 ft 6 in.</td>
<td>0.400 2.63</td>
<td>2.86 1.09</td>
<td>0.695</td>
<td>2 ft 10½ in.</td>
<td>6 ft 9 in.</td>
<td>4 ft 3½ in.</td>
<td>5 ft</td>
</tr>
</tbody>
</table>

| 15      | 7 ft 9 in.  | 9 ft 3 in. | 0       | 1 ft 6 in. | 2 ft 5 in. | 0.367 2.50 | 2.41 0.965 | 0.66 | 3 ft 0 in. | 6 ft 2 in. | 4 ft 3½ in. | 6 ft | 4 ft |
| 14      | 7 ft 4 in.  | 8 ft 10 in.| 0       | 1 ft 6 in. | 2 ft 4 in. | 0.367 2.41 | 2.18 0.90 | 0.635| 3 ft 0 in. | 6 ft 2 in. | 4 ft 3½ in. | 6 ft | 4 ft |
| 13      | 6 ft 11 in.| 8 ft 5 in. | 0       | 1 ft 6 in. | 2 ft 3 in. | 0.333 2.37 | 1.68 0.705 | 0.55 | 2 ft 6 in. | 5 ft 5 in. | 4 ft 3½ in. | 6 ft | 4 ft |
| 12      | 6 ft 11 in.| 8 ft 1 in. | 0       | 1 ft 6 in. | 2 ft 2 in. | 0.267 2.09 | 1.55 0.555 | 0.50 | 2 ft 6 in. | 5 ft 5 in. | 4 ft 3½ in. | 6 ft | 4 ft |
| 11      | 6 ft 1 in. | 7 ft 7 in. | 0       | 1 ft 6 in. | 2 ft 1 in. | 0.233 2.00 | 1.24 0.462 | 0.465| 4 ft 8 in. | 7 ft 6 in. | 4 ft 3½ in. | 6 ft | 4 ft |
| 10      | 5 ft 7 in. | 7 ft 1 in. | 0       | 1 ft 6 in. | 2 ft 0 in. | 0.233 1.75 | 0.95 0.345 | 0.41 | 4 ft 1 in. | 7 ft 6 in. | 4 ft 3½ in. | 6 ft | 4 ft |
satisfactory results for all design criteria, but not for the 12-ft-high wall. The best solution for the 12-ft-high wall was to increase the base width so as to increase the load on the piles and the resultant horizontal component of the battered piles in the front row.

For the wall heights between 25 and 30 ft, no complete design analyses were considered necessary. In these cases it was deemed sufficient to determine pile groups which would have values of $\hat{y}$, $I$, $SM_f$, and $SM_e$ which would conform to a straight-line or a slightly curved line variation between the corresponding values determined for the 25- and 30-ft-high walls. The results of the analyses made are given in Table 13-11.
Section 14

SLAB AND GIRDER BRIDGES

By

L. G. SUMNER, Formerly Bridge Engineer and Chief Engineer, Connecticut State Highway Department (Formerly Chief Engineer, Savin Construction Corp. and Presently Engineer Consultant to Savin Bros., Inc., General Contractors), Hartford, Connecticut.

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The methods of designing slabs, beams, and girders are explained at length in Sec. 9. These methods require no further elaboration in applying them to problems of bridge design, except as regards the treatment of the loads which the structures are designed to carry.
SLAB AND GIRDER BRIDGES

Since bridges are usually intended for the use of vehicular traffic the "live" load to be considered consists of one or more moving concentrated loads representing the wheel loads of the vehicles. In addition to the usual problems of stress determination introduced by these moving concentrated loads, the homogeneous character of the slab or girder bridge presents a further complication in the need to determine the extent to which these loads affect or are distributed to the several parts of the structure. The following text contains data relative to several types of live load most usually employed and also methods of determining its distribution.

In addition to the problems of stress determination, a satisfactory bridge design requires consideration of various other features. These include roadway widths and types of surface, curbs and railing, drainage, expansion, and substructure details.

Construction of slab and girder bridges involves details of forms and falsework which are essentially the same as for similar work required in the construction of buildings. Therefore such construction details receive only incidental consideration in this section.

As a general rule, simple-span slab bridges should be limited to a span length of about 25 ft and girder bridges to a span length of about 60 ft. Above these approximate limits, the dead load of the structure requires such a large part of the total load-supporting capacity as to make the construction uneconomical. These approximate limits may be increased somewhat for continuous construction.

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Preliminary Considerations

Although somewhat beyond the scope of this section, it is worth repeating that bridge designs should be based on accurate information as to conditions at the site and the character of the service to be rendered by the completed structure. Data should be available to determine the requirements of waterway and foundation conditions to be encountered as well as the character and volume of traffic to be carried. Preliminary decisions taken in regard to such features will affect all later work of design and construction as well as the ultimate usefulness of the structure. Therefore, a little study of these general features before starting the design will be time well spent.

Roadway Widths

Roadway widths tend increasingly to become a matter of standardization for which most bridge-building authorities have established at least minimum requirements. For railroad structures such widths must conform to clearance diagrams in use by the particular road. While these vary somewhat among the several railroads and exact information is essential before completing plans, the general requirements will usually not be materially different from those shown in Fig. 14-1. When two or more tracks are involved, they are usually located 13 ft center to center. The clearances shown are for straight track. Where curvature exists, the clearance must be increased to allow for the overhanging and tilting of the cars. For determining these increments, and in the absence of more specific instructions, a typical car 85 ft long, 60 ft between centers of trucks, and 14 ft high may be used.

![Fig. 14-1. AREA minimum railroad clearance diagram on straight track.](image)

For highway bridges, except in very unusual cases, a minimum width of two traffic lanes should be provided. The present trend is to establish the width of a traffic lane
GENERAL

at 12 ft. If there are to be curbs on the roadways approaching the structure, the bridge curbs should be aligned with these; otherwise the bridge roadway should have a minimum width of 6 ft greater than the width of approaching pavements.

Figure 14-2 gives minimum desirable roadway widths for bridges to carry vehicular traffic only.

Loads

The dead load consists of the weight of the structure to be supported, determined from preliminary investigations and assumptions and later checked and adjusted to agree with the details finally decided on.

The following weights may be used in computing the dead loads:

<table>
<thead>
<tr>
<th>Material</th>
<th>Lb/cu ft</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel</td>
<td>490</td>
</tr>
<tr>
<td>Concrete</td>
<td>150</td>
</tr>
<tr>
<td>Timber (treated or untreated)</td>
<td>60</td>
</tr>
<tr>
<td>Compacted earth, gravel, or ballast</td>
<td>120</td>
</tr>
<tr>
<td>Cinder fillings</td>
<td>60</td>
</tr>
<tr>
<td>Macadam or gravel rolled</td>
<td>140</td>
</tr>
<tr>
<td>Other pavements</td>
<td>150</td>
</tr>
<tr>
<td>Asphalt plank</td>
<td>110</td>
</tr>
</tbody>
</table>

For railroad structures, the track rails, inside guardrails, and fastenings may be assumed to weigh 200 lb per line ft for each track.

The live load for railroad bridges varies somewhat depending upon the practice of the different railroads but is usually the familiar Cooper loading. Figure 14-3 gives details of the Cooper E-72 loading.

Highway bridges are usually designed for one of the types of truck loads or lane loads described in the standard bridge specifications of the American Association of State Highway Officials. The truck loads are of two types designated as "H trucks" and "H-S trucks," and each provides for a variation in weight of vehicle depending upon the character of the anticipated traffic. They are shown in detail in Figs. 14-4 and 14-5.

Lane loads are equivalent to trains of trucks and are illustrated in Fig. 14-6. Truck or lane loads are assumed to occupy a roadway width of 10 ft. Within the curb-to-curb distance, the loads are assumed to occupy any position which will produce maximum stress but which will not involve overlapping of adjacent lanes or place the center of the lane less than 5 ft from the roadway face of the curb.
In view of present trends in motor-truck transportation, it seems unwise to employ a live load lighter than H-15 in planning future work.

Should the problem involve provision for the passage of electric-railway cars or freight cars, the loadings illustrated in Fig. 14-7 may be used.

Live load for sidewalk areas may be taken as 85 psf, but consideration should be given to the possibility that truck traffic may use such areas because of accident or temporary conditions or that future roadway widening may include original sidewalk areas.
Fig. 14-6. AASHO lane loads equivalent to trains of trucks.

<table>
<thead>
<tr>
<th>Class</th>
<th>Concentrated load-lbs</th>
<th>Uniform load lbs/lin ft of lane</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Moment</td>
<td>Shear</td>
</tr>
<tr>
<td>H 20</td>
<td>18000</td>
<td>26000</td>
</tr>
<tr>
<td>H 15</td>
<td>13500</td>
<td>19500</td>
</tr>
<tr>
<td>H 10</td>
<td>9000</td>
<td>13000</td>
</tr>
<tr>
<td>H20-S16</td>
<td>32000</td>
<td>40000</td>
</tr>
<tr>
<td>H15-S12</td>
<td>24000</td>
<td>30000</td>
</tr>
</tbody>
</table>

Electric railway loading (1944)

Total loaded weight per car including 10 percent overload:
- 40 ton capacity-132,000 lbs
- 70 ton capacity-212,000 lbs

Freight car loading (1944)

Fig. 14-7. AASHO electric-railway and freight-car loadings.
Curbs and Railings

The inadequate character of the curbs and railings on many bridges constructed in the past has been recognized, and in an attempt to overcome this fault in future designs the Bridge Committee of the AASHO has included the following requirements in the 1953 edition of its Standard Specifications of Highway Bridges:

Substantial railings along each side of the bridge shall be provided for the protection of traffic. Consideration shall be given to the aesthetic features of the railing to obtain proper proportioning of its various members and harmony with the structure as a whole. Consideration shall also be given to avoiding as far as consistent with safety and appearance, obstruction of the view from passing motor cars.

In general, railings shall be of two classes, as follows:
1. Roadway railings.
2. Sidewalk railings.

Roadway railings shall have a minimum height of 2'3" above the roadway adjacent to the curb. They shall be designed to resist a lateral, horizontal force of 150 pounds per linear foot together with a simultaneous vertical force of 100 pounds per linear foot applied at the top of the railing. When curbs are 10 inches or less in height, lower rails shall be designed to resist a lateral horizontal force of 300 pounds per linear foot; or if there is no lower rail, the web members shall be designed to resist a horizontal force of 300 pounds per linear foot applied not less than 21 inches above the roadway. The horizontal forces shall be applied simultaneously. Railings without webs and with single rails shall be designed for the forces specified above for lower rails. For each inch of height of curb above 10 inches, this lateral horizontal force may be reduced 15 pounds per linear foot but this force shall not be less than 150 pounds per linear foot.

Sidewalk railings shall have a minimum height above the surface of the sidewalk of 3 feet less one-half the horizontal width of the top rail. Clear openings shall be proportioned with due regard for safety of persons using the structure. Sidewalk railings shall be designed to resist the same forces as those specified for roadway railings, subject to the same restrictions concerning curb heights. Where through trusses, girders or arches separate the sidewalk and roadway, or when sidewalks are protected by curb railings, the sidewalk railing shall be designed only for the forces specified for the top rail. When the top of a curb is more than 2 feet in width the provisions for sidewalk railing shall apply.

Provisions shall be made for the expansion and contraction of railings consistent with the design.

Impact

Stresses produced by live loads are usually further increased to provide for dynamic, vibratory, and impact effects. This increase, known as "impact," is expressed as a percentage of the live-load stress or load.

For railroad structures the American Railway Engineering Association 1958 edition gives the following:

To the axle loads there shall be added impact forces, applied at the top of rail and distributed thence to the supporting members, comprising:

1. The rolling effect: Vertical forces due to the rolling train from side to side, acting downward on one rail and upward on the other, the forces on each rail being equal to 10 percent of the axle loads.
2. The direct vertical effect: Downward forces, distributed equally to the two rails and acting normal to the top-of-rail plane, due,
   (a) in the case of steam locomotives, to hammer blow, track irregularities, speed effect and car impact, and equalling the following percentage of the axle loads for beam spans, stringers, girders, floorbeams:
   
   \[
   \text{For } L \text{ less than 100 feet } \quad 60 - \frac{L^2}{500} \\
   \text{For } L \text{ 100 feet or more } \quad \frac{1800}{L - 40} + 10
   \]
   
   (b) in the case of rolling equipment without hammer blow (diesels, electric locomotives, tenders alone, etc.) to track irregularities, speed effect and car impact, and equalling the
following percentage of axle loads:

For $L$ less than 80 feet \[ 40 - \frac{3L^2}{1600} \]

For $L$ 80 feet or more \[ \frac{600}{L - 30} + 16 \]

where $L =$ length in feet, center to center of supports for stringers, transverse floorbeams without stringers, and longitudinal girders.

Impact values for highway loads and electric railway or freight car loads may be obtained from the formula

\[ I = \frac{50}{L + 125} \]

where $I =$ impact fraction (maximum 30 per cent)

$L =$ length, ft, of the portion of the span which is loaded to produce the maximum stress

Other forces sometimes considered in designing structures include wind load and longitudinal, centrifugal, and thermal forces. However, it is only in unusual cases that consideration of such forces is necessary in the design of slab or girder bridges.

**Reduction in Load Intensity**

Because it is extremely unlikely that, in the case of a structure carrying more than two tracks of traffic lanes, all tracks or lanes will be loaded at the same time with the maximum loads considered in the design, it is customary to allow a reduction in live-load stress produced by loading more than two tracks or lanes simultaneously.

The following percentages of the specified live load may be used in such cases:

<table>
<thead>
<tr>
<th></th>
<th>Highway</th>
<th>Railroad</th>
</tr>
</thead>
<tbody>
<tr>
<td>For two tracks or lanes</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>For three tracks or lanes</td>
<td>90</td>
<td>83.3</td>
</tr>
<tr>
<td>For four tracks or lanes</td>
<td>75*</td>
<td>68.8*</td>
</tr>
</tbody>
</table>

* For more lanes, no reduction; for more tracks, as specified by the engineer.

**Unit Stresses**

The following unit stresses, expressed in pounds per square inch, are based upon the use of concrete having an ultimate compressive strength at 28 days of 3,000 psi. For concrete having less strength the unit stresses are to be reduced proportionately. If concrete having a higher ultimate strength can be produced, these unit stresses, except the bond on piling, may be increased proportionately, with the extreme upper limit which may be used in computing working stresses at 4,500 psi.

Compression in extreme fiber

Extreme fiber stress in flexure........................................ 1,200

Tension:

In reinforced-concrete members........................................ None

Plain concrete, primarily footings.................................... 90

Shearing stress

Beams without web reinforcement:

Longitudinal bars not anchored........................................ 60

Longitudinal bars anchored........................................... 90

Beams with web reinforcement........................................ 225

Punching shear....................................................... 160

Bond on piles (in seals)

Timber, steel, or concrete piles, 10 psi (provided the pile has the resistance to the pull thereby induced)
SLAB AND GIRDER BRIDGES

Coefficients:
Thermal 0.000006, shrinkage 0.0002
Steel reinforcement:

\[
\begin{array}{ccc}
\text{Structural grade} & \text{Intermediate hard grade} & \text{rail steel grade} \\
\text{Tension in flexural members} & 18,000 & 20,000 \\
\text{Tension in web reinforcement} & 18,000 & 20,000 \\
\text{Compression: } \times \text{times the compression in the surrounding concrete} & & \\
\text{Bond, deformed bars:} \\
\text{Straight or hooked ends, exclusive of top bars} & 350 & 350 \\
\text{In beams, slabs, one-way footings} & 350 & 350 \\
\text{In two-way footings} & 280 & 280 \\
\text{Top bars, bars near top of beams and girders having more than 12 in. of concrete under the bars} & 210 & 210 \\
\end{array}
\]

The value of the modulus of elasticity of concrete in compression shall be assumed as one-tenth that of steel in computation of strength, and shall be assumed as one-eighth that of steel in computing the deflection of the reinforced-concrete beams, which are free to move longitudinally at the supports.

SLAB BRIDGES

General

The dictionary defines a slab as "a comparatively thick plate or slice of anything." As the term is customarily used in bridge work, a slab bridge may be defined as a comparatively thick, flat plate of reinforced concrete, spanning between piers or abutments, with its main reinforcement parallel to the direction of traffic and in which the slab is the sole load-carrying member. Concrete slabs are used in other ways in bridge construction, notably as the flooring over concrete or steel beams or girders or as elements of box culverts or frames, but such structures are not customarily referred to as slab bridges and are not treated as such in this section. The slab bridge may be of one or more spans which either may be "simple" spans or made continuous as seems to offer the best solution for each particular problem.

Distribution of Live Load

The use of the above-described live loads in the design of concrete slabs introduces the problem of determining how much of the slab is effective in resisting the stresses produced or over what area of slab a concentrated load may be considered to be distributed. This area, or width of cross section, is usually called the "effective" width.

Several series of full-sized tests of slabs under concentrated loads have been made to obtain an answer to this problem, and these data have proved extremely useful to designers. In general, these tests indicated that a single concentrated load applied at the center of a slab whose width is at least 1\(\frac{3}{4}\) times its span will be distributed over a width, at right angles to the span, of about two-thirds of the span. To this width may be added the width of the load itself plus an allowance for any fill, ballast, or pavement between the load and the surface of the slab. In the interests of simplicity and uniformity of practice, most design specifications contain requirements relative to such distribution.

The AREA 1958 specifications state:

1. Where a ballasted track is carried on transverse steel beams without stringers, the portion of an axle load on a single beam shall be as follows:

\[
P = \frac{KAd}{S}
\]

where \(A\) = Axle load
\(P\) = Load on a beam from one track
\(d\) = Beam spacing
\(S\) = Axle spacing
\(K\) = 1\(\frac{3}{4}\) for single track and 1\(\frac{1}{4}\) for double track

The load \(P\) shall be assumed distributed as concentrated loads on the beam under each rail. The effects of eccentricity of track and centrifugal force shall be included.
(2) Where the track is carried on longitudinal beams or girders with a ballast floor, the live load shall be considered as uniformly distributed over those beams within a width of 14 feet for single track, but not to exceed the distance between track centers for multiple tracks.

The 1957 edition of the Standard Specifications for Bridges of the AASHO contain the following requirements for the distribution of loads and design of concrete slabs:

**Bending Moment.** Bending moment per foot width of slab shall be calculated according to methods given under cases A, B and C.

**Case A**—Main Reinforcement Perpendicular to Traffic. Slabs shall be designed for standard H or H-S truck loadings.

\[
\text{Distribution of Wheel Loads}
\]

<table>
<thead>
<tr>
<th>Freely Supported</th>
<th>Continuous</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Spans</strong></td>
<td><strong>Spans</strong></td>
</tr>
<tr>
<td>2 to 7 feet, (E = .6S + 2.5)</td>
<td>(+.25 \frac{P}{E} S)</td>
</tr>
<tr>
<td>7 feet and over, (E = .4S + 3.75)</td>
<td>(+.25 \frac{P}{E} S)</td>
</tr>
</tbody>
</table>

[Note: Case A does not apply to slab bridges, according to the above definition, except in unusual cases, but is included here to preserve the continuity of the text.]

**Case B**—Main Reinforcement Parallel to Traffic

Spans 2 to 12 feet, standard H or H-S truck loadings.

Distribution, \(E = .175S + 3.2\)

Moments, freely supported spans = \(+.25 \frac{P}{E} S\)

Continuous spans = \(\pm .2 \frac{P}{E} S\)

The formulas for distribution and moment, Cases A and B, include the effect of all wheel loads placed in positions to produce maximum moments. Continuous spans shall be designed in accordance with the above formula unless moments are calculated by more exact methods which may permit a greater reduction.

**Case C**—Main Reinforcement Parallel to Traffic—H loading. Span over 12 feet.

The slabs shall be designed for the loading, truck or lane which produces maximum moment. Loads shall be distributed as follows:

(a) Wheel loads: \(E = \frac{10N + W}{4N}\)  
Load per foot of slab = \(\frac{P}{E}\)

(b) Lane loads:

Uniform load = \(\frac{NQ}{0.5W + 5N}\) per square foot of slab

Concentrated load = \(\frac{NP}{0.5W + 5N}\) per foot width of slab

In Cases A, B and C:

\(S\) = effective span length  
\(E\) = width of slab over which a wheel load is distributed  
\(N\) = maximum number of lanes of traffic permissible on bridge  
\(W\) = width of roadway between curbs on bridge  
\(W\) = width of graded roadway across culverts  
\(Q\) = uniform lane load per linear foot of lane  
\(P\) = load on one wheel\(^{1}\)  
\(P\) = concentrated lane load per lane

**Edge Beams, Longitudinal.** Edge beams shall be provided for all slabs having main reinforcement parallel to traffic. The beams may consist of the curb section reinforced, of a

\(^{1}\) For H20 or H20-S16 loads, \(P = 24,000\) pounds for single axle for spans under 10.5 feet, and two 16,000 pound tandem axles (spaced 4 feet apart) for spans of or over 10.5 feet.
Fig. 14-9a. Superstructure, plan, elevation, and section for standard slab bridges, Ohio Department of Highways.

beam support or of additional slab width. It shall be designed to resist a live load moment of 0.10 PS where P = the wheel load and S = span length.

The moment as stated is for a freely supported span. It may be reduced 20 per cent for continuous spans unless a greater reduction results from an exact analysis.

Distribution Reinforcement. Reinforcement shall be placed in all slabs transverse to the main steel reinforcement, to provide for lateral distribution of the concentrated live loads.

... The amount shall be the percentage of the main reinforcing steel required for positive moment as given by the following formula:
Fig. 14-9b. Abutment elevations and all tabular data for standard slab bridges, Ohio Department of Highways.

Percent Distribution Steel = 100/$\sqrt{S}$ (max. value 50%) where "S" is the effective span in feet.

Shear and Bond Stress. Slabs designed for bending moment in accordance with the foregoing rules shall be considered satisfactory in bond and shear.

Distribution of Electric Railway Wheel Loads (1944). Electric railway wheel loads shall be assumed to be uniformly distributed longitudinally over a length of 3 feet. In the case of ballasted floors, a lateral distribution of 10 feet for an axle load shall be assumed.
Distribution of Wheel Loads Through Earth Fills. When the depth of fill is 2 feet or more, concentrated loads shall be considered as uniformly distributed over a square, the sides of which are equal to $1\frac{3}{4}$ times the depth of fill. When such areas from several concentrations overlap, the total load shall be considered as uniformly distributed over the area defined by the outside limits of the individual areas, but the total width of distribution shall not exceed the total width of the supporting slab. For single spans, the effect of live load may be neglected when the depth of fill is more than 8 feet and exceeds the span length; for multiple spans, it may be neglected when the depth of fill exceeds the distance between faces of end supports or abutments. When the depth of fill is less than 2 feet, the wheel load shall be distributed as in slabs with concentrated loads.

In the foregoing formulas, the span length for simple spans shall be taken as the distance center to center of supports but not more than the clear span plus the depth of slab. For slabs continuous over more than two supports, the span is to be taken as the clear distance between faces of adjacent supports.

Note: The slab distribution as set forth above is based substantially upon the “Westergaard Theory.” The following references are furnished concerning the subject of slab design:


Details of Superstructures

The completion of the design requires consideration of such details as road surface, curbs, railings, sidewalks if any, joints, and drainage. Various methods of developing these details may be employed, as will be seen from the accompanying illustrations.

Where the span is small, it is often advisable to carry the approach pavements over the structure without interruption. This requires an appreciable amount of fill as a cushion between the bottom of the pavement and the top of the structural slab, preferably about 1 ft, and this adds to the dead load to be carried. However, for short spans, this will usually not be serious and will be offset by the resulting general simplification of detail.

For longer spans and where headroom is not available, the structural slab may become the wearing surface of the roadway, or some additional depth of concrete or bituminous surfacing may be added. Figure 14-8 shows an additional 2 in. of concrete placed monolithically with the structural slab, to serve as a wearing surface. However, experience has shown that, for the weather and traffic conditions prevailing in Connecticut, this additional surfacing is unnecessary, and it has been omitted from more recent designs.

Roadway cross slope is provided to conform with that of the approach pavements or at a rate of about $\frac{1}{36}$ in. per ft. To obtain this the slab may be thickened at the center or the depth kept uniform by using a corresponding crown on the bottom surface. In this case, special treatment is necessary at the bearing areas on piers and abutments. In Fig. 14-9 the standard cross slope of approach pavements is maintained for a corresponding width on the bridge, with the remainder of the bridge slab sloping at the rate of $\frac{1}{36}$ in. per ft to form a gutter section. Although somewhat more difficult to construct, this detail provides for the maximum concentration of water in the gutter section and will provide a drier and therefore a safer travel surface. The gutter portion is also set off from the area of the travel path by a $\frac{1}{3}$-in. depressed joint and a painted line.

Curb heights vary from about 9 to 12 in., with a general tendency to standardize on about 10 in. Because of the low underclearance of modern cars, high curbs are not advisable as they tend to foul running boards and fenders. Even on the lower curbs there is some danger of this, unless the front face of the curb is given an appreciable slope away from the roadway.

Adequate clearance between the curb and the face of railing should be provided as a safety measure. Experience has shown that a motorist will not drive close to a high
Fig. 14-10. Standard continuous slab bridge, Texas State Highway Department.
curb and railing, but this tendency can be largely overcome if there is a generous offset between the curb and rail. The space thus provided may serve as an emergency walkway on structures not provided with sidewalks, as is shown in Fig. 14-10.

The accompanying drawings illustrate a few of the many possibilities in railing design. Figure 14-10 shows a low but sturdy rail suitable for a structure in open coun-
try and where there is little pedestrian crossing. Its open characteristics are welcomed by the passing motorist whose view is not impaired as sometimes results when a higher, more solid rail is used.

A satisfactory detail of sidewalk construction is illustrated in Fig. 14-11. An alternate arrangement, sometimes employed, is to increase the width of slab, using the same detail as for the roadway portion, sufficient to accommodate the walk. The curb is then poured on the slab and the sidewalk constructed on a cinder or gravel fill in much the same way as on the approaches. This has the advantage of ease in construction and also provides an area which may be easily transformed to roadway use should future widening become necessary. However, it is essential for the future maintenance of the structure that the fill on which the sidewalk rests be well drained; otherwise disintegration of the concrete may occur, particularly in regions subject to severe frost action.

Because of the restrictions in span length imposed by their type, it is usually not necessary to provide for large movements due to expansion or contraction in slab bridges, except in the case of multiple continuous spans. It is customary to provide dowels between the slab and abutment at one end and to allow the other simply to rest
SLAB BRIDGES

Fig. 14-12b. Elevation and details for a concrete beam bridge, Missouri State Highway Department.
Fig. 14-13. Precast slab bridge for the New York, New Haven, and Hartford Railroad.
Fig. 14-14. Cast-in-place continuous slab bridge, Grand Trunk Western Railroad.
Fig. 14-15a. Plan and elevation of abutments for a 14-ft standard slab bridge, Connecticut State Highway Department.
on its bearings, taking care to prevent bond between slab and abutment by means of two layers of tarpaper or similar device. A variation of this practice occurs in case the slab is designed to support the top of the abutment against the horizontal earth pressure from the fill or where the slab is made integral with the abutments. The former is discussed under slab bridges of single span; in the latter case, the structure becomes a rigid frame or box. Where the substructure is of pile-bent or thin column-bent construction, both ends may be doweled, expansion being provided by bending the columns. Where joints occur between spans, as at piers, every effort must be made to exclude water and dirt from the joint. Better joint fillers and sealing materials than those now available are urgently needed for this purpose. At present, the best type of filler appears to be the premolded type, although this is lacking in resiliency and recovers very little of its original thickness after being compressed. Exposed edges of joints filled with this material should be sealed with a resilient material which will adhere to the concrete and will not harden in cold weather or flow under summer temperature. As this ideal cannot be fully attained, it is necessary to give such details periodic inspection and maintenance.

Drainage of the gutters may be ignored in the case of a short span on a sufficient grade to discharge the water to the end of the span. Where drainage of the structure is necessary this may be accomplished by scuppers through the slab as shown in Fig. 14-10 or by openings through the curb as is shown in Fig. 14-12. These scuppers and openings should be of generous size as small openings tend to plug with dirt and become useless.

Typical examples of railroad slab-bridge construction are given in Figs. 14-13 and 14-14. Of these, Fig. 14-13 illustrates the more usual type of construction with rails and ballast while Fig. 14-14 shows a detail in which the rails are attached directly to the slab. In Fig. 14-13, the slabs were precast off the site and were then set in position by locomotive crane. The slabs are made in pairs, each designed to carry half the track load. Joints between slabs are sealed by calking.

The structure illustrated in Fig. 14-14 forms the principal feature of a highway-railroad grade separation project at a location having limited headroom. The device of attaching the rails directly to the slab resulted in a reduction of 14 to 18 in. in deck thickness, simplified the drainage problem, and gave a deck which is neat in appearance and easily kept clean.

**Slab Bridges of Single Span**

Because of the limitations inherent in this type of structure, the maximum span for a simply supported concrete slab bridge should not exceed 25 ft. Within this limit, its simplicity, general ruggedness, and ease of construction make the single-span slab
bridge an acceptable solution for many minor drainage problems as well as general
miscellaneous use. Figure 14-9 illustrates a good example of such construction.
Several different solutions of the wing-wall problem are indicated which are capable of
further modification to meet the needs of the particular problem. Slab and abutment
are keyed together at the top of the abutment wall so that this wall may be designed
as a simple slab to resist the horizontal pressure of the retained material. The footing
must be secure against lateral displacement and must be wide enough to provide
foundation pressures consistent with the character of the material on which the footing
is to rest. Such an arrangement will usually prove more economical than the more
conventional type of abutment indicated in Fig. 14-15 in which the abutment resists
the entire thrust of the retained fill without help from the slab. In either case, the
wing walls are designed by means of the customary retaining-wall theory. An expan-
sion joint should be provided at the junction of the wing wall and abutment, or else
this point should be substantially reinforced to guard against cracking due to varia-
tions in deflection between the two parts of the structure.

Moisture getting into the fill behind the abutments and wing walls will greatly
increase the pressure against these walls unless allowed to escape. This is best accom-
plished by installing "weep holes" of about 3 in. diameter extending through the walls
at a height slightly above the normal water level and at intervals of 8 to 10 ft. Selected
material, as impervious to water as can be obtained readily, should be used in the back-
fill up to the level of the bottom of the weep holes and in the area immediately behind
the walls. Above this level and extending upward to the subgrade or surface of the
slopes should be placed a layer of coarse sand or fine gravel about 2 ft thick. This
should be in contact with the back of the walls and should extend for their full length.
The material may be bagged for placing or may be installed by the use of removable
bulkheads. To prevent escape of the fine material through the weep holes, their inner
ends should be surrounded with about a cubic foot of coarse broken stone or gravel.

Slab Bridges of Multiple Span

Multiple-span slab bridges are customarily of the pile-trestle type, although piers
may be substituted for the pile bents where conditions are favorable or for aesthetic
reasons as on grade-separation structures. The spans may be simple or continuous
as best adapted to local conditions. Continuous spans permit some simplification of
detail at piers and also allow the bents to be made integral with the slabs, which will
result in a generally stiffer structure.

Unless the height is small, about 12 ft or less, the use of solid piers will generally not
prove economical for this type of structure because their greater cost indicates the need
of employing spans beyond the practicable limits of slab construction. Continuous
spans will serve to modify this somewhat by permitting greater span lengths than
would be possible otherwise.

This is illustrated by Fig. 14-14 where four continuous spans of 33 ft 0 in., 38 ft 3 in.,
38 ft 3 in., and 33 ft 0 in. were used. Foundation conditions at this site were particu-
larly good so that the continuous-slab structure was a feasible and economical solution.

A common application of multispans slab construction is illustrated by Figs. 14-10
and Fig. 14-12. The three-span construction eliminates the need for expensive abut-
ments and wing walls at the stream bank as the additional spans permit the fill to spill
around the end bents which need only short wings to keep the fill from burying the
ends of the slabs. In each case the spans are supported on the pile bents.

The structure illustrated in Fig. 14-16 has several unusual features which make it of
interest, aside from the special purpose for which it was constructed. The 13-span
structure is built in two continuous units and with pile caps which are integral with the
slab. A novel type of expansion joint located near the center of the structure permits
movement of the two continuous units. Note that this joint is placed 5 ft away from
the center line of a bent so that the continuity of construction is not interrupted at that
point. The parabolic shape of the bottom slab surface is also an unusual detail which
brings the slab dimensions in close conformity with the stress requirements of the
structure.
FIG. 14-15a. Plan, elevation, and typical cross section for Russian River Sidehill Viaduct, Bridge Department, Division of Highways, State of California.
Fig. 14-16c. Edge beam and cap details for Russian River Sidehill Viaduct, Bridge Department, Division of Highways, State of California.
Fig. 14-10d. Railing and drain details for Russian River Sidehill Viaduct, Bridge Department, Division of Highways, State of California.
Fig. 14-16e. Expansion hinge details for Russian River Sidehill Viaduct, Bridge Department, Division of Highways, State of California.
Fig. 14-17. Precast-slab trestle bridges for Missouri Pacific Railroad.
The use of concrete-slab and pile-bent construction in railroad structures is illustrated by Fig. 14-17. This structure is designed for Cooper's E-72 loading using 50 per cent impact in the superstructure and 25 per cent in the piles. Design unit stresses were 1,200 psi compression in concrete and 20,000 psi tension in reinforcement. Piles, which are located in salt water, were produced of concrete having a compressive strength of 4,000 psi at 28 days, with 7 bags of portland cement per cu yd and 5 gal of
water per bag of cement. Slabs were cast at a convenient location off the site and were set in place on the bents by locomotive crane. Note in Fig. 14-13 the four steel loops cast in each slab for lifting and placing the units. Outer slabs weigh 30 tons and inner slabs 22 tons each.

Flat-slab Construction

This type of structure represents a modification of the more conventional slab and bent or girder construction. Methods of design and other related data are given in Sec. 9 and are essentially unchanged for application to bridge structures. Flat slabs have been extensively used by several railroads, notably in conjunction with building construction as at freight or passenger terminals, and also for highway grade-separation structures. Appreciable reductions in floor depth as well as improved appearance and reduced construction costs are among the advantages claimed for this type of structure.

The live loading used in the design of flat-slab bridges is customarily a uniform load producing moments and shears equivalent to those given by the specified concentrated loads. These loads are considered to be distributed according to the rules previously given, and the slab is treated as a continuous beam, supported along lines joining the centers of columns.

Figure 14-18 shows general details of a four-track railroad viaduct using flat-slab construction. The structure has a length of 2,462 ft. The type was adopted because of its economy in construction costs and also because it lent itself more readily to the development of the space beneath the structure for purposes of freight unloading and storage.

### GIRDER BRIDGES

**General**

By concentrating the main load-carrying elements into a few relatively narrow and deep sections made integral with the floor slab, it is possible to attain greater span lengths with girder construction than would be economically justified with solid slabs. The transition point between the slab and girder types is at about 25-ft span but is not sharply defined, and the choice for spans ranging from 20 to 30 ft will be largely controlled by local conditions and prices. A series of comparative structures having the same roadway widths and curb and railing details and designed under the same specifications gave the following results:

<table>
<thead>
<tr>
<th>Span, ft</th>
<th>Slab bridges</th>
<th>Girder bridges</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Depth of structure</td>
<td>Steel, lb</td>
</tr>
<tr>
<td>20</td>
<td>2 ft 1(\frac{3}{4}) in.</td>
<td>5.396</td>
</tr>
<tr>
<td>22</td>
<td>2 ft 2(\frac{3}{4}) in.</td>
<td>6.736</td>
</tr>
<tr>
<td>24</td>
<td>2 ft 4(\frac{3}{4}) in.</td>
<td>7.836</td>
</tr>
<tr>
<td>26</td>
<td>2 ft 6(\frac{3}{4}) in.</td>
<td>9.660</td>
</tr>
</tbody>
</table>

These comparisons show that a girder bridge will contain more reinforcing steel and less concrete than a slab bridge of equal span. Translated into dollars and cents, however, the differences between the two types are so small that minor changes in the unit prices of materials tend to throw the advantage from one type to the other, although the slight advantage in favor of the girder type increases with the span length.

The upper limit of span length for simply supported girders is generally taken at about 60 ft, but this may be increased somewhat under favorable conditions and may be appreciably greater for the continuous spans of either conventional or hollow type.
Concrete girder bridges are almost invariably of the deck type. Some examples of through construction exist, but as a rule they are occasioned by the peculiar conditions presented by the site. With the tendency toward greater roadway widths, the problem becomes increasingly complicated because of the depth and weight of the road slab and the difficulty of attaching it properly to the main girders. Also in case of future widening, the through superstructure will have to be entirely scrapped, whereas with deck construction the work is relatively simple and much of the original structure may be preserved.

Distribution of Loads

Vehicular loads are distributed to the floor slabs between girders according to the rules given under Distribution of Live Load earlier. Distribution of these loads to the girders and to the floor beams and stringers, if used, should conform to the following specification requirements contained in the Standard Bridge Specifications of the AASHO:

a. Distribution of Wheel Loads to Stringers and Floor Beams.

In calculating end shears and end reactions in transverse floor beams and longitudinal beams and stringers, no lateral or longitudinal distribution of the wheel load shall be assumed for the wheel or axle load adjacent to the end at which the stress is being determined. For loads in other positions on the span, the distribution for shear shall be determined by the method prescribed for moment.

b. Bending Moment in Stringers.

In calculating the bending moments in longitudinal beams or stringers, no longitudinal distribution of the wheel loads shall be assumed.

Stringers for H-S loadings, which have a loaded length greater than 140 feet, shall be designed for the lane loading described under the AASHO design specifications. Each lane load shall consist of a uniform load per linear foot of traffic lane combined with a single concentrated load so placed on the span as to produce maximum stress. The concentrated load shall be considered as uniformly distributed across the lane on a line normal to the center line of the lane. For the computation of moments and shears, different concentrated loads shall be used as indicated in Fig. 14-6. The lighter concentrated load shall be used when the stresses are primarily bending stresses and the heavier concentrated loads shall be used when the stresses are primarily shearing stresses.

In computing stresses, each 10-foot traffic lane loading or a single standard truck per lane shall be considered as a unit. The number and position of loaded lanes, and the type of loading, whether truck loading or lane loading, shall be such as to produce a maximum stress subject to reductions in load intensity due to loading more than two lanes simultaneously. Fractional lane widths are not to be considered. The H-S lane loading shall be used for loaded lengths over 140 feet, and the H-S truck loading for loaded lengths of 140 feet or less. For H loading, either the lane loading or the truck loading shall be used depending upon which gives the larger stress.

On any series of continuous spans, discontinuous lengths of lane loading shall be used where necessary for maximum stress, but only one concentrated load shall be used.

For H-loadings, the lane loading shall consist of the load shown in Fig. 14-6 and in addition thereto another concentrated load of equal weight shall be placed in one other span of the same series, in such position as to produce maximum stress.

For H-loadings and H-S truck loadings on concrete floors, the lateral distribution shall be determined as follows:

Interior Stringers.

One traffic lane, fraction of a wheel load to each stringer = $S/7.0$. If $S$ exceeds 10 feet, see note.

Two or more traffic lanes, fraction of wheel load to each stringer = $S/5.5$. If $S$ exceeds 14 feet, see note:

$S$ = average spacing of stringers in feet.

Outside Stringers.

The live load supported by outside stringers shall be the reaction of the truck wheels, assuming the flooring to act as a simple beam between stringers.

Total Capacity of Stringers.

* In this case, the load on each stringer shall be the reaction of the wheel loads, assuming the flooring between stringers to act as a simple beam.
SLAB AND GIRDER BRIDGES

The combined load capacity of the beams in a panel shall not be less than the total live and dead load in the panel.

c. Bending Moment in Floor Beams (Transverse).

In calculating the bending moments in floor beams no transverse distribution of the wheel loads shall be assumed.

If longitudinal stringers are omitted and the concrete floor is supported directly on floor beams, the beams shall be designed for loads determined as follows:

\[ \text{Fraction of wheel load to each floor beam} = \frac{S}{6} \]

\( S \) = spacing of beams in feet.

Note: If \( S \) exceeds the denominator, the load on the beam shall be the reaction of the wheel loads, assuming the flooring between beams to act as a simple beam.

d. Railroad Structures.

Railroad girder structures are customarily planned in separate units for each track carried so that the track load is distributed to the whole structure, subject only to such structural limitations as may be imposed by the details of the structure.

Details of Superstructures

Much of the material given earlier for slab bridges is equally applicable to girder spans. Because of the greater span lengths of the latter, details of expansion joints and drainage take on added importance. Varied treatment of these details is shown in the accompanying illustrations.

Figure 14-19 shows a type of cylindrical bearing under the girders designed to furnish uniform support under varying conditions of expansion and deflection. Freedom of movement between masonry and sole plates is ensured by the insertion of sheets of lead \( \frac{3}{4} \) in. thick. Note also the open roadway expansion joint and the substantial end diaphragms required for the support of the roadway slab at this point.

The bridge illustrated by Fig. 14-20 has a dual roadway with a physical separation between opposing traffic streams, a type of construction now widely adopted for more heavily traveled roads. Girder bearing details for this structure are simpler than in the preceding illustration, but note that the top plate is of bronze while the bottom is of cast iron. As a further precaution against corrosion, a layer of lubricant separates the two parts of the joint. Often a similar type of joint is used, but with both plates of phosphor bronze.

The most elaborate type of girder bearing is that indicated for the railroad bridge in Fig. 14-21 which is due to the greater loads to be carried by the girders of this structure.

Roadway drainage devices vary among the many structures, from the simple 2-by 4-in.-diameter pipes open at their lower end, as shown in Figs. 14-21 and 14-22, to elaborate gratings and downspouts. Requirements for these drains are that they should be large enough to handle maximum quantities of water, that they should not include details likely to clog with the dirt and debris which may be washed into them, and that they should be accessible for easy cleaning in the event they should become obstructed.

Generally water from such drains may be discharged freely just below the bridge structure. On occasion, however, as at highway grade separations, the water must be carefully conducted to some required point of discharge. Fill extending in front of "open" abutments is particularly vulnerable to the action of water falling from open bridge scuppers immediately above it and may sustain serious damage unless such water action is prevented by the use of suitable gutters and leaders.

It will be noted that the depth of the floor slab between girders varies from 6\( \frac{1}{2} \) to 8 in. for highway bridges. This represents about the practical limits in variation for the thickness of this part of the structure and has considerable influence upon the spacing of the supporting beams or girders. A depth of less than 6 in. is considered impracticable for the type of service required of a bridge floor slab under usual conditions, and a girder spacing so wide as to require a slab depth of more than about 8 in. will generally prove uneconomical.

It should also be noted that there is usually no provision for a separate wearing sur-
face on the roadway of most of the highway structures, although the structural slab
may be thickened slightly, above theoretical requirements in some cases. There
appears to be a definite trend away from the former practice of providing such separate
wearing surfaces, not only because the structure is lightened and construction made
simpler, but also because experience has shown that the damage to the surface of the
structural slab, due to the action of traffic, is usually so small as to be negligible. In
case membrane waterproofing is placed as an insurance against leakage through the
bridge deck at overpasses, a separate wearing surface is necessary and performs the
double function of roadway slab and protection for the waterproofing. If a separate
surface is used, care should be taken to prevent the seepage of water between the sur-
face course and the structural slab; otherwise cracking and disintegration of one or
both are apt to occur.

Girder Bridges of Single Span

The single-span girder bridge offers a satisfactory solution to the problem of the
minor stream crossing and for similar use where the span is beyond the economic limit
of the slab bridge. A reasonably low underclearance is desirable; otherwise the cost
of the full abutments and wing wall will represent an unreasonably high proportion of
the total cost of the structure, and consideration should be given to the possible
economies of multispans pile or pier bent construction.

An example of a single-span girder bridge is given by Fig. 14-22. Here the abut-
ment and wing walls are designed as an integral unit so that the restraining action of
the latter is effective in assisting the abutment section to resist the pressure of the
embankment. This requires substantial connections between the abutment and the
wing walls, which is provided by the heavy fillets and reinforcement shown on the
plan. The resulting abutment section is much lighter than would have been possible
had the wing walls and abutment been separated by expansion joints.

Girder Bridges of Multiple Span

The use of multiple spans for concrete girder bridges, thereby admitting of continu-
ous as well as simple beam construction, permits considerable flexibility in matters of
design and makes this type of construction well suited for the solution of many bridge
problems. The accompanying illustrations describing some of the more conventional
designs as well as others containing an element of the unusual indicate some of the possi-
bilities of which the type is capable.

In passing from the single to the multispans structure increased attention must be
paid to expansion details, particularly for continuous construction, where greater
movements must be provided for. Greater freedom is usually permissible in planning
abutments or end bents which may often be of the open type, and the character of the
intermediate piers or bents will require careful consideration.

A structure of three simple spans is illustrated in Fig. 14-12. Expansion occurs at
each abutment for the end spans and at one intermediate bent for the center span.
Note that the center span is slightly longer than the others, thus improving the appear-
ance of the structure at the slight sacrifice of some duplications in formwork. The
spans are comparatively short so that girder bearings and roadway expansion joints
are simple in type. Roadway drainage is provided by a series of 4-ft by 5-in. slots
extending through the curb. End bents, although essentially open in character, have
a curtain wall between columns extending to the ground line and thus completely
retain the approach fills. This wall is extended beyond the outer columns and serves
as wing walls in protecting the bridge seats and ends of the girders. The intermediate
bents are simple but rugged and effective. The presence of rock foundations through-
out the site made it possible to found the several columns on small independent footings.

An application of concrete girder construction to a railroad structure is shown in
Fig. 14-21. This structure has a length of over 3½ miles through low flat country and
was built as part of a major flood-control project. Its great length and the generally
uniform character of the country offered opportunity for developing the design for the
Fig. 14-19a. Cross section and elevation for 50-ft concrete girder spans, Texas State Highway Department.
Fig. 14-19b. Details for 50-ft concrete girder spans, Texas State Highway Department.
Fig. 14-20b. Half elevation and longitudinal sections of beams, with details, for a 38-ft deck concrete girder bridge, Louisiana Highway Commission,
Fig. 14-21. Concrete trestle bent and 30-ft T-beam girder bridge for Norfolk and Western Railroad.
Fig. 14-22c. Plan, elevation, and details for a 40-ft concrete beam bridge, Iowa State Highway Commission.
use of 600 duplicate spans in the trestle. The generally simple but rugged detail of the structure is in keeping with its utilitarian character and also emphasizes the magnitude of the loads which a railroad must carry.

The structure was designed for E-60 loading and an impact factor of (live load) to (live load plus dead load), reduced to 0.25 live load for footings and to zero for the piles. A 3,000-lb concrete was assumed with a working stress of 1,050 psi. Reinforcement was designed for 18,000 psi except for the footings, where 20,000 psi was permitted.

The footings are supported on both cast-in-place concrete and timber piles, the former type being used at the anchor bents and where longitudinal forces are resisted while timber piles are used at the remaining bents.
Section 15

CONCRETE FLOORS AND PIERS AND ABUTMENTS
FOR STEEL BRIDGES

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CONCRETE FLOORS

Concrete Floors for Steel Bridges

In the majority of cases the flooring of modern steel bridges is made of reinforced-concrete slabs supported on the steel beams of the floor system. The effectiveness and economy of this type of construction are well established for both railroad and highway
structures. For railroad bridges, the solid floor, which permits the customary construction of ties and ballast to be carried over the structure without interruption, offers advantages in both riding qualities and simplified maintenance. It also makes possible the placing of adequate waterproofing, which is a desirable feature where the tracks are carried over a highway, pedestrian passageway, or another railroad. On highway bridges the concrete slab is well suited to the needs of modern traffic in that it provides a hard durable pavement surface of high tractive value.

The design of concrete floor slabs for steel bridges is based on the same general theories as those outlined in Sec. 14 for slab and girder bridges and much of the comment contained in that section is equally pertinent here.

**Railroad-bridge Floors**

Figures 15-1 to 15-4 show cross-sectional views of typical concrete floors for railroad bridges of the more usual types. For deck-girder construction the floor slabs may be

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**Fig. 15-1.** Typical concrete floor for single-track through railroad bridge.

**Fig. 15-2.** Typical concrete floor for multiple-track through railroad bridge.

either cast in place or precast off the site and set in place on the girders by locomotive crane. The latter procedure will usually expedite the completion and opening of the structure to traffic, often an important consideration in building railroad structures.

For through bridges, the stringers are usually omitted and the slab is supported on closely spaced transverse floor beams framing to the main girders. In Fig. 15-3 the small longitudinal beams between floor beams are for stiffness only and do not participate in the load-carrying action. The concrete floor slab is carried up on the girder webs sufficiently to retain the ballast and protect the webs, and this protection is carried around all gusset plates and braces also, unless a solid curb is used as in Fig. 15-3.
Emphasis is placed on waterproofing these structures, as is indicated by the plans. The waterproofing is of the "membrane" type involving several layers of cotton fabric with moppings of asphalt or pitch. The membrane is protected against damage by covering it with a layer of cement mortar, brick, asphalt plank, or similar material. Care must be taken to prevent leakage at the sides and ends of the waterproofing, and details of flashing or other sealing device must be carefully worked out. Drainage of

Fig. 15-3. Alternate detail of concrete floor for through railroad bridge.

the deck is provided by installing pipes or scuppers with grates, and these two must be carefully installed to prevent water from getting under the protective covering.

Highway-bridge Floors

Typical examples of concrete floor slabs for highway bridges are illustrated by Figs. 15-5 and 15-7. These are identical in main features but have certain variations in detail which are representative of differences in current practice. The bridge, of
Fig. 15-5a. Roadway cross section and some expansion joint details for a deck beam highway bridge, Missouri State Highway Department.
which Fig. 15-5 shows a cross section, is located on a curve which requires a 51/4-in. bank in the width of the roadway. Main transverse reinforcement in Fig. 15-5 consists of both straight and bent-up bars while in Fig. 15-7 only straight bars are used. As negative moment over the support about equals positive moment at mid-span, it is theoretically desirable to vary the amounts of top and bottom reinforcing accordingly. However, it is argued that the economy is slight and is largely offset by the need to bend some of the bars and that a more practicable detail will result from the use of

![Concrete Floors Diagram](image)

**Fig. 15-5b.** Finger plate layout and details and expansion shoe and bearing details for a deck beam highway bridge, Missouri State Highway Department.

equal amounts of straight bars in the top and bottom of the slab as shown in Fig. 15-7. In either case, the slab may be designed as a double-reinforced beam.

In Fig. 15-5 the concrete is filleted down to a bearing on the top surface of the supporting beams while the detail of Fig. 15-7 shows the full depth of the top flange embedded in the concrete. The latter detail is preferred by some designers on the grounds of improved bond between beam and slab and increased stiffness of the whole floor.

Figure 15-6 shows details of approach spans and main river crossing for the large Mississippi River cantilever bridge at Baton Rouge, La. The approach-span detail illustrates a type of construction in which stringers are omitted and the floor slab spans directly between rather closely spaced floor beams. On the main river bridge,
Fig. 15-6a. Concrete roadway carried on closely spaced floor beams for highway approach structures on each side of railroad viaduct leading to the Mississippi River cantilever bridge at Baton Rouge, La., Louisiana Highway Commission.
Fig. 15-6b. Concrete roadways carried on floor beam brackets and connecting stringers on the outside of the main trusses for the Mississippi River cantilever bridge at Baton Rouge, La., Louisiana Highway Commission.
the roadways are separated and occupy a rather unusual position on brackets outside the trusses, the space between trusses being required for railroad tracks.

A dual-type roadway with central separating curb is illustrated by Fig. 15-8. This construction is used on the 150-ft continuous girder spans approaching the main river crossing of the Connecticut River bridge at Hartford. The curbs are of steel and the trussed-bar type of reinforcement is used; otherwise the design is along conventional lines.

A variation from the conventional type of concrete floor slab supported by steel stringers is offered by the construction in which, by the use of shear-developing devices, the concrete floor and the steel beams are made to act as a composite T-beam section. This type of floor develops certain advantages over the more conventional type, notably in the depth required for comparable conditions.

The design of composite beams follows the same general procedures as for concrete T beams, except that the design of the "shear connectors" required to ensure composite action makes it necessary to determine the horizontal shears throughout the beam. The detail and reinforcement of the slab necessary to its functioning as a bridge floor do not enter into the calculations for composite action.

The design of the steel beams or girders will be appreciably affected by the composite action since the combined steel and concrete section is to be considered. This may be done by translating all steel areas to equivalent concrete areas by multiplying by the ratio of their moduli of elasticity \( n \) and calculating the moment of inertia of the transformed section. As the top flange of the steel beam will carry but little stress, it will usually prove economical to make this as light as specification limitations will permit and to reinforce the bottom flange by adding plates as necessary.

In order to develop complete composite action, it is necessary to provide for the horizontal shear between the top flange of the steel member and the concrete slab which rests upon it. This is accomplished by fastening spiraled or bent bars, flat plates, or other steel shapes to the top flange of the beam in such a way that sufficient area will be embedded in the concrete slab to develop the required strength.

The horizontal shear is obtained from the formula

\[
H = \frac{VQ}{I}
\]

where \( H \) = horizontal shear
\( V \) = vertical shear
\( Q \) = static moment of concrete section about neutral axis
\( I \) = moment of inertia of composite section

These values should be obtained at approximately 2-ft intervals throughout the length of the beam.

The spacing of shear connectors is obtained from the formula

\[
D = \frac{C}{H}
\]

where \( D \) = spacing of connectors, in.
\( C \) = value of a shear connector, lb
\( H \) = horizontal shear, psi, in the element of the girder being considered

The value of a shear connector depends upon its type, size, and method of attachment to the steel beam. Assuming some form of reinforcing bar is to be used, its cross section and length will determine the value of \( C \) based on the tension and bond values of the bar, after which a method of attachment, consistent with this value, must be adopted. Customarily such attachment is made by welding.

Composite action with respect to both dead- and live-load stresses may be considered only where it is possible to support the structure on temporary bents until the concrete has set sufficiently to participate in the beam action. Where this cannot be done, the composite section may be applied only to the resistance of live-load stresses.

The various types of shear connectors and details necessary to develop composite action may be covered by United States patents, and prospective users should obtain proper authorization when necessary before employing this type of construction.
ABUTMENTS AND PIERS

General Considerations

The problems of abutment and pier design begin with the determination of their proper location. This will have a decided influence on the type of superstructure and should be based on obtaining the maximum economy in cost of structure consistent with the general requirements of the project and the conditions imposed by the site. Factors to be considered will include area of waterway to be provided, subsoil conditions at the site, and the character and requirements of any facilities passing beneath the bridge, including those pertaining to the navigation of the stream. Each of these requires study in conjunction with the others so that the first step toward solution of the problem becomes the accumulation of all controlling data.

**Waterway Requirements.** When the grade of the proposed structure is established at a considerable height above the water, either because of local topography or to provide clearance over adjacent highways or railroads or the navigable channel of the stream, the waterway provided will usually be greatly in excess of that required for stream flow. For the more usual conditions, in which it is often desirable to keep the grade line as low as possible, the required area of waterway must be determined.

The presence of abutments and piers within the normal waterway of a stream constitutes an impediment to its free flow. Under flood conditions, this will cause some damming back of the water above the bridge and an increase in velocity of flow through the constricted area. The damming effect will result in raising the water level above the structure with possible damage to adjoining property while the increased velocity of the current may cause erosion to stream bed and banks with the possibility of undermining the piers and abutments, causing damage to approach embankments and the formation of bars and shoals below the structure. The determination of the required waterway for a structure seeks to predict the minimum area necessary to prevent such current action from reaching damaging proportions.

The size of waterway opening required may be determined by the use of formulas, runoff tables, actual stream gagings, or observations as to the adequacy of existing bridges over the same stream and reasonably close to the site of the proposed work.

There are numerous formulas which give the area of opening directly in terms of the drainage area. Perhaps the most widely used is that proposed by Prof. A. N. Talbot, which is

$$A = CD^{0.64}$$

where $A$ = area of waterway, sq ft

$D$ = drainage area, acres

$C$ = constant

The following values of $C$ are commonly employed:

<table>
<thead>
<tr>
<th>Topography</th>
<th>Value of $C$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mountainous</td>
<td>1.00</td>
</tr>
<tr>
<td>Very hilly</td>
<td>0.66</td>
</tr>
<tr>
<td>Hilly</td>
<td>0.50</td>
</tr>
<tr>
<td>Rolling</td>
<td>0.33</td>
</tr>
<tr>
<td>Gently rolling</td>
<td>0.25</td>
</tr>
<tr>
<td>Flat</td>
<td>0.20</td>
</tr>
</tbody>
</table>

As the results obtained with this and similar formulas are affected very greatly by the choice of the constant $C$, their value is open to some question and the values obtained should be compared with the actual behavior of existing waterway openings before acceptance.

Drainage tables give, directly, the size of waterway openings required for various areas of watershed and require a different table for each variation in climate, rainfall, and topography. They are based on actual observations of runoff and stream flow and are very useful when applied to regions having similar characteristics.

Stream gaging consists of taking actual field measurement of the stream cross section
and current velocity during times of maximum flood. From these data, the volume of
discharge may be obtained from the formula

$$Q = AV_m$$

where $Q =$ volume of discharge, cu ft per sec, generally termed "second-feet"
$A =$ the cross-sectional area of stream at the point of observation, sq ft
$V_m =$ mean velocity of the stream, fps

The area of waterway provided must be of such size and shape that it will pass this
volume without causing unreasonably high current velocity or back-water head due to
the construction.

Subsoil Conditions. Subsoil investigations by means of borings and test holes are
made to discover the character of the materials underlying the site of a proposed
structure. Such information is of value in enabling the designer to plan his over-all
layout so as to take advantage of any favorable conditions revealed and also in supplying
the data as to soil resistance necessary to the detailed design of abutment and pier
footings.

For general discussion, foundation materials may be classified as "ledge rock or
hardpan," "gravel and boulders," "sand," and "clay and silt." In nature these
materials will usually be somewhat mixed, so that any statement of empirical bearing
values for arbitrarily classified materials is apt to be misleading. Ledge rock or hard-
pan offers the best possible foundation and will generally support any load which it is
practicable to put upon it with the usual types of construction. Gravel, either with
or without boulders, is also an excellent foundation material when protected from
scour, although foundation pressures must be appreciably lower than for structures
founded on rock.

The acceptability of sand as a foundation material is largely influenced by the
amount of water present in the material. This will be partly displaced by the application
of additional load, with resulting settlement of the structure. The settlement
usually occurs shortly after the load is applied and terminates when a balance for the
new condition of pressure and water content has been reached.

Clay and silt are generally poor materials for foundations because they are liable to
both flow and compress when subjected to additional pressure. The resulting settle-
ments are apt to be slow but of long duration and hence are more troublesome and
difficult to correct. However, if sand or gravel occurs in combination with the clay in
sufficient quantity to reduce its plasticity, a satisfactory foundation may be obtained.

The foregoing merely indicates the general character and scope of the problem. For
a more detailed discussion of the subject of soil mechanics as well as the various
methods of subsoil exploration and sampling, the reader is referred to Foundation of
Bridge and Buildings by Jacoby and Davis (3d ed., McGraw-Hill Book Company,

Types of Foundation Support. When the character of the materials encountered
at convenient depth below the ground surface is suitable for the support of the structure,
the problem involves only the proportioning of the footings to such dimensions
that the allowable unit pressures will not be exceeded. Where this fortunate condition
does not exist, other methods of overcoming the difficulty must be employed. Usually
these will consist of either providing piles or of carrying the foundations to greater
depths in order that they may be placed on more dependable material. This latter
will involve consideration of the methods by which the work is to be done.

Piles. Piles used for the support of foundations are of two general types, those
driven through overlying material to a firm bearing on ledge rock or other solid ma-
terial, and those which penetrate sufficiently into yielding material to develop the
required load-carrying capacity by means of friction between the surface of the pile
and the material penetrated. The piles may be of either timber, concrete, or steel
and these may be further divided into treated and untreated timber, cast-in-place or
precast concrete, and steel with or without protective coating. Each has certain
advantages as to availability, cost, driving characteristics, capacity, etc., which must
be considered in relation to the requirements of the problem in hand.
ABUTMENTS AND PIERS

The load-bearing capacity of piles is often difficult to determine in advance of driving and design assumptions should be compared with driving records and modified if necessary. The 1953 Bridge Specifications of the American Association of State Highway Officials contain the following recommendations as to the design loads on piles:

<table>
<thead>
<tr>
<th>Size or diam of butt, * in.</th>
<th>Timber, tons</th>
<th>Concrete, tons</th>
<th>Steel (friction), tons</th>
<th>Steel point bearing</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>18</td>
<td>20</td>
<td>20</td>
<td>6,000 psi of point area</td>
</tr>
<tr>
<td>12</td>
<td>20</td>
<td>24</td>
<td>24</td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>24</td>
<td>28</td>
<td>28</td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>28</td>
<td>32</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>24</td>
<td>50</td>
<td>50</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* Timber piles, diameter to be measured 3 ft from butt.

These values may be increased 25 percent when the load includes longitudinal forces, wind loads, shrinkage stresses, rib shortening, centrifugal force, backfill to original ground line, erection stresses, ice, current, earthquake and thermal stresses.

Piles should not be driven closer than $2\frac{1}{2}$ ft or preferably 3 ft on centers, and where horizontal forces are involved, the group should include a certain number of batter piles.

**Cofferdams.** The work of excavation and foundation construction for bridge abutments and piers is usually carried out by the use of cofferdams. When the depths are small and the amount of water negligible, these may take the form of earth or sandbag dikes. For more severe conditions sheet piling, usually of steel with interior frames and bracing, is employed. The depths to which it is practicable to carry excavation by means of open cofferdams cannot be stated in general terms, but on several recent structures, depths of approximately 100 ft below the water level have been reached. However, at these great depths the excavation is usually completed and a concrete seal placed under water before the cofferdam is pumped out.

**Caissons.** Where depths beyond the limits of cofferdam construction must be reached, recourse is had to caissons. A caisson is a hollow boxlike structure, usually built of steel and concrete. It is sunk in position by dredging and weighting and remains a part of the permanent structure. An “open” caisson is open at both top and bottom and excavation is carried on through wells left in its interior. A “pneumatic” caisson is an open caisson to which has been added a working chamber at its lower end which is equipped with air locks so that the water may be excluded by introducing compressed air and men may enter to carry on the work of excavation and preparing the foundation.

When the caisson has reached its final position, all hollow spaces within it are filled, usually with concrete. As its top is ordinarily below the water surface when sinking is completed, some form of cofferdam must be added to permit the completion of the work. This is removed when construction has progressed sufficiently above high-water level.

Pneumatic caissons are usually, but not always, the most expensive type of foundation structure. Their advantages are that they permit of greater control during sinking, in that logs or large boulders encountered may be more easily removed, and the foundation material finally reached may be readily examined and cleaned and prepared to receive the masonry. Because of the high air pressures involved, which prevent the men from working to advantage, the depth to which pneumatic caissons may be sunk is limited to about 110 ft. Open caissons are not subject to this limitation and have been the invariable choice where great depths must be reached. Those sunk for the construction of the piers of the San Francisco–Oakland bridge, built in 1936, reached a maximum depth of 242 ft below the water surface.
Bridge Abutments

Bridge abutments and their wing walls provide the transition between approach embankments and bridge structure and act as both retaining walls and bridge piers. Thus they combine two equally important functions, one to provide a support for the end of the superstructure, the other to retain the approach fill as to permit the smooth passage of trains or vehicles.

![Diagram of bridge abutments](image)

**Fig. 15-9.** Low-gravity-type abutment for right-angle crossing with skewed wing walls for a highway bridge.

The extent to which the approach embankments are retained determines the length of the wing walls and often the type of the whole abutment. Where these embankments are fully retained so that none of the material encroaches beyond the front of the abutments, both wings and abutment breast walls must be solid, and where the height is large, the wings must be of considerable length. When the conditions at the site and the general character of the structure permit, these fills may be allowed to "spill around" the front face of the abutments. In this case, the wing walls need only be long enough to protect the bridge seats and the abutments may be a series of columns supporting the bridge seat and back wall. This type, generally called an "open" abutment, will be cheaper than the solid type, but it can be used only when the fill is not liable to erosion or is adequately protected against it by placing riprap or slope paving. In the case of a high solid abutment, it will often be worthwhile to try the effect of adding another short span to the structure, if by doing so the open-type
Fig. 15-10. Low-gravity-type abutment for both a skewed crossing and skewed wing walls for a railroad bridge.
Fig. 15-11a. Plan and elevations of a cantilever abutment
abutment may be used. The resulting saving in cost of abutment and wing walls will
often more than equal the additional cost of the superstructure.

The methods of design for abutments and wing walls are the same as outlined in a
previous section for retaining walls. The type may be a full gravity section of plain
concrete or reinforcement may be used. If reinforced, the section may be either
cantilever, counterforted, buttressed, or hollow. The selection of type is a matter of
economy and suitability for the particular problem.
and wing walls, Connecticut State Highway Department.

Generally, gravity sections (Figs. 15-9 and 15-10) are favored for small height because of their simplicity and ease of construction. A useful modification of the full gravity section is sometimes employed in which the breast wall is made the width of the bridge seat and back wall with both front and rear faces vertical. Sufficient reinforcement is added to keep the tensile stresses within specified limits.

For greater heights the cantilever type (Fig. 15-11) is generally the most satisfactory changing to the counterforted wall (Fig. 15-12) when the height reaches about 20 to
**Fig. 15-11b. Sections and details of a cantilever abutment and wing walls, Connecticut State Highway Department.**

25 ft. The buttressed wall can seldom be used with full abutments because clearance requirements and stream flow will not permit, although this type may be used with open abutments and in other cases when the fill will bury the buttress construction.

A good application of the hollow or cellular abutment is at grade-separation structures where a roadway with sidewalks is to be carried under a railroad or another road.
The superstructure may be supported on columns at the curb line and these columns and their footings, together with the side walls and roof necessary to protect the sidewalk space, when taken as an integral unit, may be formed into an attractive and economical abutment.

Open abutments are of various forms and details but they are essentially reinforced abutments from which most of the breast walls have been removed and the wings shortened. Typical details are shown in Figs. 15-13, 15-14, and 15-15. The separate columns, counterforts, or buttresses are often supported on a common large footing unless the foundation material is sufficiently firm to make differential settlements unlikely, in which case small separate footings may be used. Design procedures are essentially the same as for other types of wall. It must be remembered that all horizontal earth pressures against the wing walls and back walls are brought to the supporting columns and must be transferred by them to the footings. Although any fill in front of the columns might be effective in resisting such pressures, it is customary to ignore this factor because its development cannot be counted on with safety.

As the approach fill immediately behind the abutment will consist of newly placed material, some settlement is to be anticipated. This should be reduced as much as possible by using filling which compacts readily, by placing it in thin layers, and by tamping and compacting thoroughly. In the case of open abutments, a liberal amount of material should be placed in front of the abutment and graded to a flat slope so as to ensure a stable embankment. The use of heavy approach slabs of reinforced concrete about 20 ft long that rest on the abutment at one end and on the fill at the other has been found to improve the riding qualities of the approaches.

The remarks contained in Sec. 14 relative to weep holes and the drainage of approach fills are equally applicable to the construction discussed in this section. The several drawings included illustrate typical examples of the various types of abutments described.
Fig. 15-12b. Elevation of abutment and cross sections of wing walls for a counterfort abutment and wing walls, Connecticut State Highway Department.
Bridge Piers

The principal function of bridge piers is to provide points of intermediate support for the superstructures of multiple-span bridges. Therefore, the first concern in determining their dimensions and details is that they provide sufficient bearing areas and are suitably proportioned for the loads thus brought to them. These loads must of course include all lateral and transverse forces applicable. In addition, when located within the area of stream flow, they form an obstruction to the free passage of the water and must also resist the force produced by current, ice, and drift. The amount of such forces and the effect of the piers on the flow of the stream may be somewhat modified by their location and shape.

For the problem of the ordinary bridge pier, it will usually be found that the requirements of the superstructure in the matter of loads, bearing areas, and other details will be the controlling factors in determining dimensions and that the forces resulting from the action of current, ice, and drift are so small as to be negligible. Also, the amount of back-water head due to the presence of the piers and the effect produced by variations in shape of the pier will not usually be important. For larger structures over major streams and for others where unusual conditions of stream flow exist, these factors will require consideration.

Stream Current, Floating Ice, and Drift. The 1953 Standard Bridge Specifications of the AASHO contain the following requirements:

All piers and other portions of structures which are subject to the force of flowing water, floating ice, or drift shall be designed to resist the maximum stresses induced thereby. Pressure of ice on piers shall be calculated at 400 pounds per square inch. The thickness of ice and height at which it applies shall be determined by investigation at the site of the structure.

Effect of flowing water on pier: \[ P = KV^2 \]
where \( P \) = Pressure in pounds per square foot (of vertical projection of pier)
\( V \) = Velocity of water in feet per second
\( K \) = A constant, being \( \frac{1}{2} \) for square ends, \( \frac{1}{2} \) for angle ends where the angle is 30 degrees or less, and \( \frac{3}{4} \) for circular piers.

The velocity of stream flow varies with the depth, the maximum occurring somewhat beneath the surface. It is customary to assume the center of pressure at one-third the distance from the surface of the water to the stream bed.

The major axis of the piers should be aligned with the direction of the current in order to minimize the formation of eddies or cross currents.

Shape of Piers. The typical bridge pier consists of a rectangle to which is frequently added a curved or angular cutwater at the upstream end or symmetry at both ends. The dimensions of the pier top will be controlled by the requirements of the superstructure to be supported. Conventionally, the sides of the shaft are battered from \( \frac{1}{2} \) to 1 in. per ft. The shaft terminates in a footing which is dimensioned in accordance with the conditions imposed by the foundation. Such a pier is illustrated by Fig. 15-16. The shape of the cutwater will influence the amount of resistance to stream flow caused by the pier. This is discussed by F. A. Nagler (Trans. ASCE, vol. 82, p. 334, 1918) and by D. L. Yarnel (U.S. Dept. Agr. Tech. Bull. 442, 1934) but the effect is negligible in the case of the average pier. In general, the greatest resistance to current is offered by the square-ended pier, with the triangular and curved cutwaters next in order.

Types of Pier. The type of pier illustrated by Fig. 15-16 is often used for small structures and for others where the height of the pier is small. As the principal loads are applied at the superstructure bearing areas, it is apparent that much of the material in this type of pier is but lightly stressed. This indicates possible economies in both cost and weight by substituting columns at the concentrated load points for the solid pier shaft. Where the pier is small, these savings will not be great and may not justify the more complicated formwork involved, but for large high piers, the possibilities of such construction should be investigated. For piers located within the area of stream flow, such open construction may be objectionable on the grounds of increased resistance to current and the possibility that drift and debris may lodge between the columns. However, this difficulty may be overcome by using hollow piers or thin curtain
Fig. 15-13. Open abutment (back wall extended straight) carried on two columns and spread footings, Oregon State Highway Commission.
Fig. 15-14a. Plan, elevation, and cross section for an open abutment (back wall extended skewed) carried on three buttressed columns and spread footings, North Carolina State Highway and Public Works Commission.
Fig. 15-14b. Details and bill of material for an open abutment (back wall extended skewed) carried on three buttressed columns and spread footings, North Carolina State Highway and Public Works Commission.
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Fig. 15-15b. Cross sections and details for an open abutment carried on vertical and battered reinforced-concrete piles, Missouri State Highway Department.
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<td>Weight</td>
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<td>6&quot;</td>
<td>15'-6&quot;</td>
<td>2498</td>
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<td>PH</td>
<td>28</td>
<td>6&quot;</td>
<td>15'-6&quot;</td>
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<td></td>
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Elev top of concrete block: 606.100
Height: 25'-0"

Top of pier at 605.898

Pier #1 Bearing block elevations given at outer edge of block

Top of pier at 606.162

Pier #2

Fig. 15-16b. Plan and bearing plate settings for a skewed beam bridge crossing over Farmington River, Connecticut State Highway Department.
walls between the columns. For very high piers, the solid shaft and cutwater may be extended to the high-water elevation and open column construction employed above that point. Solid piers may be of mass concrete; for the other types, reinforcement must be added.

Where it is desirable, for aesthetic or other reasons, to retain the appearance of a solid pier, the construction may be lightened, and probably cheapened, by leaving
FIG. 15-17b. Cross sections and details for hollow bridge piers carried on deep caissons.
Connecticut State Highway Department.
hollow spaces within the pier shaft as shown in Fig. 15-17. Here the superstructure bearing areas are at each end of the pier. The loads are carried on solid shafts extending from the bridge seat to the top of the footing. These shafts are connected and stiffened by thin reinforced-concrete walls, leaving a large hollow space in the center of the pier. As water will inevitably find its way into such a space, it is advisable, in order to prevent its accumulation there, to provide vents at about the elevation of low water.

The use of a thin curtain wall between separate shafts is illustrated by Fig. 15-18.
Fig. 15-18b. Pier details, columns and webs, and rectangular footings on piles for stream crossing, Missouri State Highway Department.
Fig. 15-19a. Plan and elevation for a pier with solid base and cutwater carried on steel H piling and with open-column construction above high water, Connecticut State Highway Department.
This curtain wall functions only to stiffen the pier and to prevent the accumulation of drift so that it may be stopped off at a convenient distance below the water level. The loads on the pier are carried by the individual column footings, which are located appreciably below the bottom of the curtain wall.

A type of pier in which a solid base and cutwater are employed to a point above high water with open column construction above that point is illustrated by Fig. 15-19. For very large and heavy construction, the solid base may be relieved and the whole
Fig. 15-20a. Elevation and side view for a solitary column bent with collision walls, Louisiana Highway Commission.
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Fig. 15-20b. Plan, footing pile arrangements, and details for a solitary column bent with collision walls, Louisiana Highway Commission.
Fig. 15-21a. Cross sections for a pneumatic caisson pier, Connecticut State Highway Department.
pier further lightened by using hollow construction for the base as illustrated in Fig. 15-20.

The open column type of pier illustrated by Figs. 15-21 and 15-22 may be used to advantage for viaducts over dry land and for structures over water when the action of current, ice, and drift is not an important factor. Removal of the solid base and cut-
Fig. 15-22a. Half horizontal sections of pier and caisson and details of caisson cutting edge for a large caisson pier, Mississippi River cantilever bridge at Baton Rouge, La., Louisiana Highway Commission.
FIG. 15-22b. Plan of caisson cap and half elevation and section of caisson for a large caisson pier, Mississippi River cantilever bridge at Baton Rouge, La., Louisiana Highway Commission.
Fig. 15-22c. Elevation and half section of pier superstructure for a large caisson pier, Mississippi River cantilever bridge at Baton Rouge, La., Louisiana Highway Commission.

Water both lightens and cheapens the work, and where these features are not required for the protection of the pier, their elimination is wholly justified.

Piers of more unusual form and detail are sometimes required because of the peculiar character of the superstructures they support or unusual conditions imposed by the site. The main dimensions of the pivot pier of a swing drawbridge or the piers and counterweight pits of a deck bascule will be largely controlled by the requirements peculiar to these movable spans.
Section 16

RIGID-FRAME AND ARCH BRIDGES

By

DAN H. PLETTA, Professor and Head, Department of Engineering Mechanics, Virginia Polytechnic Institute, Blacksburg, Va.

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Part 2

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PART 1

THE RIGID-FRAME BRIDGE

DEFINITION

Although any monolithic concrete structure is essentially a rigid frame, the type of bridge to which this term has been applied so frequently is one in which the axis of the deck is very nearly straight, in which the deck is connected rigidly to the piers or abutments, and in which these supports are of sufficient size to decrease the bending moment at the knee. The practice of throwing the dead load toward the supports reduces the stress at the crown, proportions the structure for more efficient utilization of material, emphasizes functional characteristics for aesthetic treatment, increases the clearance height at the center, and reduces the height of fill or depth of cut required. These marked economies of material, excavation, and fill over simple-span structures have enhanced the popularity of the rigid frame so that they are now almost always selected for highway grade separations, foundation conditions permitting. The fact that rigid frames may be fashioned along such pleasing architectural lines serves only to further their popularity.

The structural action of the rigid-frame bridge differs from that of the fixed arch in that the deck of the rigid frame is essentially a flexural member whereas the arch rib is designed to support the load mainly by direct compressive stress. The rigid frame is really poorly shaped to carry load but its shape is determined by reasons not primarily analytical. Bending stresses in an arch are reduced to a minimum by shaping the rib so it will hug the equilibrium polygon of the dead load, but in the rigid frame the size of the deck and abutment is adjusted to withstand the greater bending moments without overstress. Both types of bridge require unyielding foundations and are affected by temperature changes and shrinkage, these comprising about 10 per cent of the total stress for the arch and up to 30 per cent for the rigid frame at the crown. For multiple spans it is usually necessary to provide expansion joints in the deck of rigid frames to prevent excessive temperature and shrinkage stresses at the knees of the end spans. It is not feasible or necessary to provide expansion joints for multiple-arch spans, but usually some articulation in the superstructure is required to prevent overstress there. However, the arch is probably affected more by plastic flow than is the rigid frame.

PRELIMINARY DESIGN

Preliminary design involves an estimate of the essential dimensions as well as a choice of the type of structure. With respect to rigid frames, a designer may select a bridge with the deck either of ribbed or of slab construction, and with the end conditions at the foundations assumed as fixed or hinged. Bridges have been built with spans varying from 16 to 85 ft for slab decks, from 23 to 103 ft for ribbed decks, and from 40 to 190 ft for box-girder decks. Up to 50 ft for railway bridges and up to 70 ft for highway bridges the slab will probably prove more economical for single spans, but it should be considered in any case. Ribbed rather than slab structures have generally been used in the past where skewed bridges were required because the analysis is simpler. However, analytical techniques developed recently for skewed bridges have made the slab structures more popular. In ribbed designs the floor slabs between ribs should be kept comparatively thin relative to the rib spacing so that interaction between adjacent ribs is negligible and so that the ribs may be treated individually as for bridges without skew. The decks of ribbed rigid frames are about 50 per cent.
higher at the crown than slab decks, but the added cost of the extra fill for the
approach is offset to some extent by saving of material, especially in the longer spans.

**ASSUMPTIONS IN DESIGN**

Most rigid frames are designed assuming the column bases hinged. Unless the foot-
ings rest on rock or are very wide they are liable to rotate somewhat, thus simulating a

![Diagram of footing details for rigid frames.](image)

**Fig. 16-1. Footing details for rigid frames.**

hinge. Furthermore, experimental laboratory tests indicate that column bases will
rack and act as quasi hinges if the footing is prevented from rotating. It is reasonable
then to assume the bases to be hinged. Where the abutments or piers are well
anchored to the footings the hinge can best be considered as located at the center of the
bottom of the footing (Figs. 16-1a and 16-1b).

Frequently, however, definite quasi articulation is provided between the top of the
footing and bottom of the abutment. In such instances the hinge should be considered
as located on the top of the footing. Figures 16-1c to 16-1f illustrate various details of
existing types.

The analysis required for a complete design requires the use of moments of inertia of
the cross section of the deck and abutments or piers. Fortunately only the relative values are required. For rectangular sections the gross uncracked area should be used, especially for the preliminary analysis. The effect of reinforcing may be either neglected or included. For ribbed structures the full transformed area of the cross section including the flange width to center lines between adjacent ribs is recommended.

Fig. 16-2. Longitudinal section, rigid frame.

Shears and moments due to temperature or shrinkage effects and those due to assumed footing displacements are affected by the modulus of elasticity $E$ of the material. In concrete structures the value of $E$ changes with age, moisture, etc., and may vary throughout the structure also. Such variations have relatively little effect on the final design, and a value of 3,000,000 psi is recommended.

**FRAME PROPORTION**

Preliminary design of an indeterminate structure involves the selection of the size of the various members. The following suggestions will prove helpful for proportioning single-span rigid-frame bridges of either the slab or ribbed type.

1. Sketch the frame to scale on a profile of the site. Allow a thickness of about 2 ft above the top of piles or bottom excavation (Fig. 16-2).
2. Assume dimensions as follows:

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<td>$L/19^\ast$</td>
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<tr>
<td>Depth at knee</td>
<td>$L/15$</td>
<td>$L/10^\ast$</td>
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<tr>
<td>Top of column</td>
<td>$L/15$</td>
<td>$L/10$</td>
</tr>
<tr>
<td>Bottom of column</td>
<td>$\sqrt{L/10}$</td>
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* Width of rib usually varies from 1 ft 4 in. to 2 ft 0 in. Sometimes the width is increased up to 25 per cent near the knee to reduce depth of haunch.

In multiple spans when the pier is of the solid-wall type, make the moment of inertia for the pier top equal the moment of inertia for one girder at the knee, or make the thickness of the pier wall equal 50 to 80 per cent of the depth at the knee. The wall sometimes has a slight batter so that the width decreases toward bottom.

**DESIGN CONSTANTS**

In analyzing single- and multiple-span rigid frames the method of moment distribution will be illustrated because of its inherent adaptability to such structures. The basic principles of this method, originated by Prof. Hardy Cross, have been described elsewhere in engineering literature and will not be developed here. Application of the method requires certain design constants like stiffness and carry-over factors for each of the structural members of the frame, and fixed-end moments on the loaded members. These constants are readily obtained from Charts 16-I to 16-IV, and their use will be illustrated.

**ILLUSTRATIVE EXAMPLE**

Symmetrical Rigid Frame with Slab Deck

Assume a grade separation is to be effected where two highways cross at essentially 90°. Allowance is to be made for 5-ft sidewalks on each side of the lower roadway. The combined clearance diagram is shown in Fig. 16-3. The overhead roadway is to be a two-lane 24-ft

* Superior numbers refer to the references at the end of this section.
The determination of the stiffness and carry-over factors for the three structural members is facilitated by dimensioning their essential proportions (see solid lines of Fig. 16-4).

The necessary factors are: For the beam (or deck) $I_c/I_B = (1.50/3.70)^2 = 0.0666$. Referring to Chart 16-II, the stiffness for a section 1 ft wide is

$$\frac{S \times E \times I_A}{L} = 16.75 \times \frac{1.50^2}{54.5 \times 12} = 0.0865$$

using $E = 1$, and the carry-over factor is $rS/S = 12.45/16.75 = 0.743$. 

---

**Diagram Description:**

- **Fig. 16-3.** Longitudinal section of rigid frame checked for clearance diagram.
- **Fig. 16-4.** Adopted proportions and dimensions of rigid frame.
ILLUSTRATIVE EXAMPLE

Likewise, for the abutment, $I_A/I_B = (2.25/3.67)^2 = 0.231$, and from Chart 16-II, the stiffness, assuming both ends fixed, is

$$\frac{S_A \times E \times I_A}{L} = 5.88 \times \frac{2.25^3}{12 \times 17.9} = 0.312$$

and $S_B \times E \times I_A/L = 12.2 \times 2.25^4/12 \times 17.9 = 0.648$ for a 1-ft width and $E = 1$. But $A$ is hinged and hence the stiffness at $B$ is

$$S_B \frac{EI_B}{L} (1 - r_{AB}) = 0.648 \left(1 - \frac{4.22}{5.88} \times \frac{4.22}{12.2}\right) = 0.493$$

Note that the carry-over factors are obtained from Chart 16-II by reading $rS = 4.22$ and dividing this by the value for $S$.

Fig. 16-5. Stiffness and carry-over factors and moment distribution for unit moments.

The relative stiffness of the deck and abutment at the joint is

$$\frac{0.0865}{0.0865 + 0.493} = 0.1492$$

for the deck and $0.493/0.5795 = 0.8508$ for the column.

The analysis of the frame by moment distribution is now accomplished by assuming an unbalanced unit moment to exist at one end of the deck (Fig. 16-5a) and at the top of the abutment (Fig. 16-5b). In each case these unbalanced unit moments are balanced and distributed until successive convergence yields a final moment within the required accuracy. With these final moments established, final moments due to any loading may be obtained by proportioning the fixed-end moments accompanying the loading. In this way, one analysis by moment distribution suffices for all loadings. It should be noted, however, that the frame is assumed to be held against sidesway and no correction made therefor. This will be included later when unsymmetrical live loads and lateral earth pressures are discussed. Fixed-end moments are assumed as positive when they act clockwise on the joint. The carry-over factor is positive.
Now that the preliminary design and analysis are complete, the actual moments and shears due to the various loadings and distortions must be computed. The following loadings will be used:

1. Dead load, includes weight of structure plus weight of a future wearing surface. Assume that the location of the bridge is such that an extra allowance of \( \frac{3}{4} \) in. of concrete to resist the action of tire chains and frost should be included initially and future allowance of 25 psf for subsequent resurfacing be contemplated. Ice and snow loads are considered offset by reduced traffic and are not to be included.

2. Live load, highway loading plus impact.

3. Thermal and shrinkage distortion loadings.

4. Earth pressure on the abutments.

5. Longitudinal traction.

**Fixed-end Moments, Dead Load**

One-half inch allowance for wear will be added to the extrados to the dimensions of Fig. 16-4. The fixed-end moments for a section of the bridge 1 ft wide may be computed by considering the entire dead load distributed uniformly and parabolically, and by using the coefficients of Chart 16-IV as follows:

\[
\frac{I_C}{I_A} = 0.0666, \quad \frac{w_L}{12} = \frac{18.5}{12} = 1.54 \quad \frac{w_p}{12} = (3.74 - 1.54)150 = 330 \text{ lb/ft}
\]

\[
M = 0.1065 \times 231 \times 54.5^2 + 0.01956 \times 330 \times 54.5^2 = 92,300 \text{ ft-lb}
\]

![Diagram](image)

Fig. 16-6. Variable weight of deck of rigid frame assumed equal to a series of concentrated loads.

An alternate method is to divide the deck into 10 sections of equal horizontal length, compute their weights as in Fig. 16-6, and determine the fixed-end moments using the coefficients for concentrated loads from Chart 16-I. The following table illustrates the computations. These results, which are in close agreement with those above, will be used.

<table>
<thead>
<tr>
<th>Point</th>
<th>Load</th>
<th>Fixed-end* M coef.</th>
<th>Fixed-end moment at knee, load ( \times ) coef. ( \times ) deck length</th>
<th>Point</th>
<th>Simple-beam moment</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05</td>
<td>2,720</td>
<td>0.047</td>
<td>( 0.047 \times 2,720 \times 54.5 = 6,970 \text{ ft-lb} )</td>
<td>0.1</td>
<td>43,200</td>
</tr>
<tr>
<td>0.15</td>
<td>2,150</td>
<td>0.131</td>
<td>15,350</td>
<td>0.2</td>
<td>73,200</td>
</tr>
<tr>
<td>0.25</td>
<td>1,710</td>
<td>0.193</td>
<td>18,000</td>
<td>0.3</td>
<td>92,700</td>
</tr>
<tr>
<td>0.35</td>
<td>1,430</td>
<td>0.217</td>
<td>16,900</td>
<td>0.4</td>
<td>102,700</td>
</tr>
<tr>
<td>0.45</td>
<td>1,280</td>
<td>0.197</td>
<td>13,750</td>
<td>0.5</td>
<td>107,100</td>
</tr>
<tr>
<td>0.55</td>
<td>1,280</td>
<td>0.144</td>
<td>10,000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.65</td>
<td>1,430</td>
<td>0.084</td>
<td>6,550</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.75</td>
<td>1,710</td>
<td>0.059</td>
<td>3,640</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.85</td>
<td>2,150</td>
<td>0.012</td>
<td>1,400</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.95</td>
<td>2,720</td>
<td>0.001</td>
<td>300</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Total \( M = 92,860 \text{ ft-lb} \)

*From Chart 16-I.

The fixed-end moment due to the future wearing surface \( = 0.1065 \times 25 \times 54.5^2 = 7,900 \text{ ft-lb} \) and the simple-beam moment at the crown \( = \frac{wL^3}{8} = 9,290 \text{ ft-lb} \).

**Fixed-end Moments, Live Load**

Since the span is over 40 ft, a lane loading of 640 lb per lin ft of lane and concentrated loads of 32,000 lb for moment and 40,000 lb for shear distributed over a 10-ft
width will be used. These values will first be increased for impact by

\[
\frac{50}{(L + 125)} = \frac{50}{(54.5 + 125)} = 0.278
\]

Hence the uniform live load plus impact is 1.278 \times 64 = 81.8 \text{ lb-ft}, the concentrated load for moment is 1.278 \times 3,200 = 4,090 \text{ lb}, and the concentrated load for shear is 1.278 \times 4,000 = 5,110 \text{ lb}. All values represent loadings on a strip one foot wide.

The fixed-end moment due to the uniform live load being applied over the entire span is 0.1065 \times 81.8 \times 54.5^2 = 25,950 \text{ ft-lb}. This application will produce maximum moment both at the crown and at the knee. The simple-beam moment is \(wL^3/8 = 30,450 \text{ ft-lb}\).

The maximum fixed-end moment at the knee and crown due to the 4,090-lb load is indicated as follows:

<table>
<thead>
<tr>
<th>Point</th>
<th>Fixed-end M coef.,* left end</th>
<th>Fixed-end M</th>
<th>Simple-beam M (Pab/L)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.36</td>
<td>0.217</td>
<td>0.217 \times 4,090 \times 54.5 = 48,400</td>
<td>51,400</td>
</tr>
<tr>
<td>0.5</td>
<td>0.173</td>
<td>38,600</td>
<td>55,700</td>
</tr>
<tr>
<td>0.64</td>
<td>0.089</td>
<td>19,850</td>
<td>51,400</td>
</tr>
</tbody>
</table>

* From Chart 16-I.

Fixed-end Moments, Thermal and Shrinkage Distortion

Changes in moisture content of the concrete and variations in temperature will either shorten or lengthen the deck and abutments. No moment would be produced in this symmetrical rigid frame by a change in the column length unless the two abutments deformed unequally, a condition which may be dismissed as rather unlikely. The changes in deck length will be assumed to be due to a temperature rise of 35°F and a drop of 45°F, and to a shrinkage of 0.002. An allowance equal to the shrinkage will also be made for possible lateral movement of the footings. This distortion of the frame will correspond to another longitudinal deformation of the deck. The effective shortening of the deck is then,

\[
\Delta' = (0.000006 \times 45 + 0.0002 + 0.0002)54.5^2 = 0.0365 \text{ ft}
\]

The increase in length is \(\Delta' = (0.000006 \times 35 - 0.0002 + 0.0002)54.5 = 0.0114 \text{ ft}\). Assuming that the joints at the ends of the deck do not rotate, such change in deck length will produce a bending in the columns. If the columns are fixed at each end (Fig. 16-7a), the end moments accompanying a lateral displacement are

\[
M_A = \frac{\Delta}{L} S_A \frac{EI_A}{L} (1 + r_{AB}) \quad \text{and} \quad M_B = \frac{\Delta}{L} S_B \frac{EI_A}{L} (1 + r_{BA})
\]

If one end is hinged (Fig. 16-7b), as is the case in this illustrative design, the end moment becomes

\[
M_B = \frac{\Delta}{L} S_B \frac{EI_A}{L} (1 - r_{AB}r_{BA})
\]
Note: The $r$ values are positive. Hence

$$M_B = \frac{0.0365}{2 \times 17.9} \times 0.493 \times (3,000,000 \times 144) = +217,000 \text{ ft-lb}$$

for a section 1 ft wide when the deck shortens and the footings spread. Similarly,

$$M_B = \frac{0.0114}{2 \times 17.9} \times 0.493 \times 3 \times 10^4 \times 144 = -68,200 \text{ ft-lb}$$

when the deck lengthens or the footings approach each other. Note that $2\Delta = \Delta'$. 

**Fixed-end Moments, Earth Pressure**

As a general rule the moments developed in a rigid frame at the knee by earth pressure on the abutments constitute a very small part of the total. This is especially true when the columns are short. If they are included in such cases the analysis by an elastic theory is likely to be in serious disagreement with actual moments, because of the relative inflexibility of the abutment with respect to the deck. In such cases it is probably more logical to assume that the end columns are hinged at both the top and the bottom and to find the moment at the crown by multiplying the horizontal reaction at $B$ (Fig. 16-8) by the rise $R$ of the elastic axis. Then

$$H_B = \frac{P_2}{2} + \frac{P_1}{3} = \frac{17.9 \times 400}{2} + \frac{515 \times 17.9}{2 \times 3} = 3,580 + 1,540 = 5,120 \text{ lb}$$

and $M_C = 5,120 \times 1.10 = -5,632 \text{ ft-lb}$. Note that this can be a negative moment only.

When the end columns are long and relatively slender, say at least 0.4 of the deck length, analysis by the elastic theory gives results which are more nearly correct. The illustrated design will be so analyzed here to indicate the procedure. The determination of the fixed-end moments due to the earth pressure follows the procedure outlined above for similar calculations on the deck. In calculating the final moments, as will be explained later for all loadings, it may be well to consider the possibility of earth pressure existing on one abutment only. This condition may exist during construction when only one approach fill has been completed. Live load may exist simultaneously, for the dump trucks may pass over the bridge to discharge their loads at the far end.
Fig. 16-9. Moments, thrusts, and shears on rigid frame.
The fixed-end moments due to lateral earth pressure (Fig. 16-8) are calculated as follows:

<table>
<thead>
<tr>
<th>Value of ( k )</th>
<th>( M ) coef. at ( A^* )</th>
<th>( M ) coef. at ( B^* )</th>
<th>Load</th>
<th>Fixed-end ( M ) at ( A = \text{coef. at } A \times PL )</th>
<th>Fixed-end ( M ) at ( B = \text{coef. at } B \times PL )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05</td>
<td>0.001</td>
<td>0.048</td>
<td>762</td>
<td>0 ft-lb</td>
<td>0 ft-lb</td>
</tr>
<tr>
<td>0.15</td>
<td>0.010</td>
<td>0.120</td>
<td>855</td>
<td>13</td>
<td>665</td>
</tr>
<tr>
<td>0.25</td>
<td>0.026</td>
<td>0.164</td>
<td>947</td>
<td>153</td>
<td>1,835</td>
</tr>
<tr>
<td>0.35</td>
<td>0.048</td>
<td>0.186</td>
<td>1,040</td>
<td>440</td>
<td>2,780</td>
</tr>
<tr>
<td>0.45</td>
<td>0.071</td>
<td>0.184</td>
<td>1,131</td>
<td>894</td>
<td>3,460</td>
</tr>
<tr>
<td>0.55</td>
<td>0.093</td>
<td>0.163</td>
<td>1,222</td>
<td>1,437</td>
<td>3,720</td>
</tr>
<tr>
<td>0.65</td>
<td>0.109</td>
<td>0.125</td>
<td>1,318</td>
<td>2,030</td>
<td>3,560</td>
</tr>
<tr>
<td>0.75</td>
<td>0.113</td>
<td>0.080</td>
<td>1,409</td>
<td>2,570</td>
<td>2,950</td>
</tr>
<tr>
<td>0.85</td>
<td>0.098</td>
<td>0.033</td>
<td>1,500</td>
<td>2,850</td>
<td>2,020</td>
</tr>
<tr>
<td>0.95</td>
<td>0.044</td>
<td>0.006</td>
<td>1,592</td>
<td>2,630</td>
<td>886</td>
</tr>
<tr>
<td>Bottom of pier</td>
<td></td>
<td></td>
<td></td>
<td>1,250</td>
<td>170</td>
</tr>
</tbody>
</table>

* From Chart 16-III.

Final Moments, Thrusts and Shear

The fixed-end moments calculated in the previous paragraphs must now be balanced and distributed according to the analysis illustrated in Fig. 16-5. These fixed-end moments, together with the various moment coefficients and final moments, are summarized in Table 16-1. Most of the loadings are symmetrical, and hence no correction need be made for side way. In the case of the concentrated live load applied off center, and that of the earth pressure on one end only, side way will exist. The conception of the effect of side way and also of the interrelation of the moments, shears, and thrusts for the various loadings is best illustrated by free-body diagrams of either half or all of the rigid frame. These are shown in Fig. 16-9g and 16-9h together with bending-moment diagrams. The concentric load of 4,090 lb is placed off center where the influence line for moment at the knee (Chart 16-1) is maximum. The calculations for the horizontal and vertical components of the reaction at the hinge may best be determined by applying the laws of statics to free-body diagrams of the deck and the end columns. For example, for the case of maximum live-load moment at the knee (Fig. 16-10), the analysis would be as follows:

For the deck:

\[
M_L = 46.65 - 68.43 + 19.6 \times 4.09 + 0.0818 \times \frac{54.5^2}{2} - 54.5V_R = 0
\]

\( V_R = 3.30 \) kips

and from \( \Sigma F_y = 0 \):

\( V_L = 5.25 \) kips

For the right column:

\( M_R = 46.65 - 17.9R_H = 0 \)

\( R_H = 2.606 \) kips
<table>
<thead>
<tr>
<th>Type of loading</th>
<th>Fixed-end moments at knee</th>
<th>Deck $M$ coef.</th>
<th>Final deck $M$</th>
<th>Simple-beam $M$ at crown</th>
<th>Final $M$ at crown</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$M_{BA}$</td>
<td>$M_{BB'}$</td>
<td>$M_{B'B}$</td>
<td>$M_{B'B'}$</td>
<td></td>
</tr>
<tr>
<td>Dead load</td>
<td>0</td>
<td>+92.860</td>
<td>-92.860</td>
<td>0</td>
<td>+0.9569</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>-0.9569</td>
</tr>
<tr>
<td>Future surface</td>
<td>0</td>
<td>+7,900</td>
<td>-7,900</td>
<td>0</td>
<td>+0.9569†</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>-0.9569</td>
</tr>
<tr>
<td>Uniform live load</td>
<td>0</td>
<td>+25,950</td>
<td>-25,950</td>
<td>0</td>
<td>+0.9569†</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>-0.9569</td>
</tr>
<tr>
<td>Cone. live load at 0.5$L$</td>
<td>0</td>
<td>+38,600</td>
<td>-38,600</td>
<td>0</td>
<td>+0.9569†</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>-0.9569</td>
</tr>
<tr>
<td>Cone. live load at 0.36$L$</td>
<td>+48,400</td>
<td>-19,850</td>
<td>0</td>
<td>+217,000</td>
<td>+0.8613†</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>-0.0956</td>
</tr>
<tr>
<td>Deck shortening</td>
<td>-68,200</td>
<td>0</td>
<td>0</td>
<td>-217,000</td>
<td>+36,930†</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>-36,930</td>
</tr>
<tr>
<td>Deck lengthening</td>
<td>-32,290</td>
<td>0</td>
<td>0</td>
<td>+68,200</td>
<td>-9,350</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>+9,350</td>
</tr>
<tr>
<td>Symmetrical earth pressure</td>
<td>-32,290</td>
<td>0</td>
<td>0</td>
<td>+32,290</td>
<td>-2,940</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>-2,940</td>
</tr>
</tbody>
</table>

* Final moments are greater without sidesway.
† $0.9569 = +1.00(+0.8613) + (-1.00)(-0.0956)$. See Fig. 16-5a.
‡ $43,600 = 48,400 \times 0.8613 + (-19,850)(-0.0956)$.
§ $-0.0431 = +1.00(-0.1387) + (-1.00)(-0.0956)$. See Fig. 16-5b.
Similarly, for the left column:

\[ M_L = 68.43 - 17.9L_H = 0 \]
\[ L_H = 3.823 \text{ kips} \]

It should be noted now that \( L_H - R_H \neq 0 \), as should be the case for a free-body diagram of the assembled frame. The reason is that the frame is held against sideways at the deck, the assumption being that the adjoining earth and road slab develop this resistance. Hence \( P = L_H - R_H = 3.823 - 2.606 = 1.217 \text{ kips} \). Then from \( 2F_Z = 0 \) for the right column,

\[ N_R = P + R_H = 3.823 \]
and from \( 2F_Z = 0 \) for the left column,

\[ N_L = L_H = 3.823 \text{ kips} \]

When sideways is permitted

\[ L_H = R_H \text{ and } P = 0 \]

The sideways correction is made as follows:

1. Load a frame with a pull \( Q \) opposite to \( P \) such that a unit moment is developed at the top of the column when the joints are held against rotation as in Fig. 16-11. (Note that these are equal because of the symmetry of the frame. If the columns were unsymmetrical the fixed-end moments could be determined from the relationships stated under Temperature and Shrinkage Distortion, keeping in mind that \( \Delta_L = \Delta_R \).

2. Balance these two unit fixed-end moments according to the analysis of Fig. 16-5b. Then

\[ M_L = M_R = 1.00(0.1387 + 0.0956) = +0.2343 \]

3. Find \( L_H = R_H = +0.2343 + 17.9 = 0.01309 \) and note that

\[ L_H + R_H = Q = 0.02618 \]

4. Since, in our problem, \( Q \) must finally equal \( P \) to counteract it completely, \( M_L \) and \( M_R \) would be \( 0.2343 \times 1.217 + 0.02618 = +10.89 \text{ ft-kips} \) in Fig. 16-11 when \( Q = 1.217 \text{ kips} \).

5. Superimpose this loading on that of Fig. 16-10. Then

\[ M_L = 68.43 - 10.89 = 57.54 \quad M_R = -46.65 - 10.89 = -57.54 \text{ ft-kips} \]

The thrusts and shears accompanying this change are illustrated in Fig. 16-9h.

Live Load, Longitudinal Force

AASHO Specifications require that provision be made for a longitudinal force of 5 per cent of the live load in all lanes, using lane loads, with the concentrated load for moment, but no impact. This force is considered to act 4 ft above the floor and all traffic is assumed headed in the same direction. This force, then, equals \( 0.05(64 \times 54.5 + 3,200) = 335 \) lb. The final bending moment at each knee is, then, \( 335 \times 17.9 = 3,000 \text{ lb-ft} \). The complete statical analysis is illustrated in Fig. 16-9j.

Curves for Maximum and Minimum Moment

The various bending-moment curves of Fig. 16-9 may be combined to get the maximum and minimum (maximum positive and maximum negative) moments at various points along the column and deck axes. Practically the same results, however, are obtained by plotting the maximum and minimum moments for the knee and crown, as determined in Table 16-2, and joining the appropriate pairs with a parabolic curve as in Fig. 16-9m. The thrusts are plotted in Fig. 16-9n, assuming straight-line variations between terminal values.
<table>
<thead>
<tr>
<th>Type of loading</th>
<th>Fig. 16-9a</th>
<th>Crown</th>
<th>Left knee</th>
<th>Deck</th>
<th>Left hinge</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Moment, ft-lb</td>
<td>Thrust, lb</td>
<td>Shear, lb</td>
<td>Moment, ft-lb</td>
</tr>
<tr>
<td>Dead load</td>
<td></td>
<td>+18,240</td>
<td>-4,960</td>
<td>0</td>
<td>-88,860</td>
</tr>
<tr>
<td>Live load, symmetrical</td>
<td>9b</td>
<td>+1,730</td>
<td>-422</td>
<td>0</td>
<td>-7,560</td>
</tr>
<tr>
<td>Live load, unsymmetrical</td>
<td>9c</td>
<td>+24,390</td>
<td>-3,450</td>
<td>0</td>
<td>-68,430</td>
</tr>
<tr>
<td>Live load, unsymmetrical</td>
<td>9d</td>
<td>+9,350</td>
<td>+522</td>
<td>0</td>
<td>+9,350</td>
</tr>
<tr>
<td>Deck length</td>
<td>9e</td>
<td>-2,940</td>
<td>-164</td>
<td>0</td>
<td>-2,940</td>
</tr>
<tr>
<td>Symmetrical earth pressure</td>
<td>9f</td>
<td>-1,390</td>
<td>-5,200</td>
<td>0</td>
<td>-1,390</td>
</tr>
<tr>
<td>Live load, long F</td>
<td>9g</td>
<td>-5,632*</td>
<td>-5,120</td>
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<td>3,000</td>
</tr>
<tr>
<td>Max (+)</td>
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<td>+53,710</td>
<td>-8,310</td>
<td>+3,113</td>
<td>+76,510</td>
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<tr>
<td>Max (-)</td>
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<td>+9,968</td>
<td>-10,244</td>
<td>-3,113</td>
<td>-172,280</td>
</tr>
</tbody>
</table>

* See discussion on earth pressure.
<table>
<thead>
<tr>
<th>Point</th>
<th>Moment ( M, \text{ ft} \cdot \text{lb} )</th>
<th>Thrust ( N, \text{ lb} )</th>
<th>( e' = \frac{M}{N_e} ) in.</th>
<th>Thickness ( t, \text{ in.} )</th>
<th>Depth ( d, \text{ in.} )</th>
<th>( e = e' - \frac{t}{2} + d )</th>
<th>( K = \frac{Ne}{bd^2} )</th>
<th>( f_r ), psi</th>
<th>( f_p ), psi</th>
<th>( p )</th>
<th>( A_n ), sq in.</th>
<th>( A_{r'} ), sq in.</th>
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</thead>
<tbody>
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<td>Hinge</td>
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<td>27,010</td>
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<td>29.2</td>
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<td>19,500</td>
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<td>1.17</td>
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<td>35.5</td>
<td>33.5</td>
<td>102.7</td>
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<td>109</td>
<td>19,500</td>
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<td>1.77</td>
<td>0.500</td>
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<td>18,280</td>
<td>84.8</td>
<td>39.8</td>
<td>37.8</td>
<td>153.6</td>
<td>3.84</td>
<td>123</td>
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<td>40.0</td>
<td>219.5</td>
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<td>3.01</td>
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<tr>
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<td>0.500</td>
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<td>2.36</td>
<td>98</td>
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<td>0.95</td>
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<td>0.0008</td>
<td>0.19</td>
<td>0.500</td>
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<td>9,490</td>
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<td>18.9</td>
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<td>1.29</td>
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<td>0.0008</td>
<td>0.19</td>
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</tr>
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<td>-9,968</td>
<td>8,310</td>
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<td>25.2</td>
<td>1.29</td>
<td>58</td>
<td>19,500</td>
<td>0.0008</td>
<td>0.19</td>
<td>0.500</td>
</tr>
<tr>
<td>Dead Load + Live Load + Impact, ( f_r = 18,000, f_p = 1,000, n = 10 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>5 (col.)</td>
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<td>14,540</td>
<td>129.8</td>
<td>42.0</td>
<td>40.0</td>
<td>148.8</td>
<td>3.72</td>
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<td>2.53</td>
<td>0.500</td>
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<tr>
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<td>60.9</td>
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<td>67.9</td>
<td>2.24</td>
<td>186</td>
<td>14,500</td>
<td>0.0118</td>
<td>2.265</td>
<td>0.633</td>
</tr>
<tr>
<td>Dead Load + 2(Live Load + Impact), ( f_r = 27,000, f_p = 1,500, n = 10 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
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<td>5 (col.)</td>
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<td>40.0</td>
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<td>3.90</td>
<td>161</td>
<td>25,000</td>
<td>0.0053</td>
<td>2.53</td>
<td>0.500</td>
</tr>
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<td>11,860</td>
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<td>289</td>
<td>24,000</td>
<td>0.0118</td>
<td>2.265</td>
<td>0.633</td>
</tr>
</tbody>
</table>

* Actual \( f_r \) is reduced by the addition of the compression steel \( A_{r'} \).
ILLUSTRATIVE EXAMPLE

In plotting the bending-moment diagrams of Fig. 16-9 the sign of the moment notation is changed from that used for the moment distribution to the usually adopted convention where positive moment bends a beam so that the top fiber is in compression.

Determination of the Necessary Reinforcement

(1) AASHO Specifications allow stresses of 18,000 psi in the steel and 0.4f'_c in the concrete when the applied forces are due to dead load, live load, impact, buoyancy, and earth pressure. (2) These allowable stresses may be increased by 25 per cent when thermal stresses, shrinkage, longitudinal forces, etc., are considered. (3) The stresses may also be exceeded by 50 per cent for checking against occasion overload when dead load plus twice the live load and impact constitute the applied forces.

In determining the amount of steel required in the illustrated design, the maximum positive- and negative-moment curves of Fig. 16-9m will be used for a preliminary estimate. In this case the allowable stresses are f'_s = 22,500 psi and f'_c = 1,250 psi (if f''_c = 2,500 psi). However, it will be found necessary to keep f'_s = 19,000 at the crown in order to avoid overstressing when using just dead load + live load + impact. When the first determination is completed, an analysis for stress, which includes the effect of compressive reinforcement at the knee and crown, will be made for the three cases of loading listed in the preceding paragraph.

The preliminary calculations are made most conveniently for the various points along the column and deck by tabulating the moments and thrusts of Fig. 16-9m and n, together with other pertinent data, in Table 16-3. In these calculations it is assumed that no compressive steel exists. Actually a small amount will be provided, as will be seen later. The omission here greatly simplifies the work and is on the safe side. The table is self-explanatory; the final percentages of steel are obtained from Chart 16-XXV, using n = 10 for the concrete.

The stress in the concrete is within the allowable for all cases of loading, but it is further reduced by inclusion of the slight amount of compressive reinforcing. A more exact analysis for stress follows. The method is based on the transformed-area method. At the crown (point 10) t = 18 in., d = 16 in., A_s = 2.265 sq in. (use one 1 in. square and one 1 1/2 in. square bar at 12 in. on centers), A'_s = 0.633 sq in. (use one 1 1/2 in. square bar at 24 in. on centers). For the case of the total combined loading of Table 16-3, the approximate analysis at the crown indicated values of f'_s = 1,300 and f'_c = 19,000 psi. Hence kd = \frac{1,300}{1,300 + 19,000/10} \frac{16}{16} = 6.40 in. approximately (see Fig. 16-12). Then for the transformed section of Fig. 16-12, the area

A = bkd + (n - 1)A'_s + nA_s = 12 \times 6.4 + 9 \times 0.633 + 10 \times 2.265 = 105.1 sq in

The statical moment of this area about the top of the section is

S = 12 \times \frac{6.4^2}{2} + 9 \times 0.633 \times 2 + 10 \times 2.265 \times 16 = 620 \text{ in}^2

The moment of inertia about the top is

I = \frac{1}{12} \times 12 \times 6.4^3 + 9 \times 0.633 \times 2^3 + 10 \times 2.265 \times 16^3 = 6,900 \text{ in}^4

Hence \vartheta = S/A = 620/105.1 = 5.90 \text{ in. and}

\bar{I} = I - A\vartheta^2 = 6,900 - 105.1 \times 5.90^2 = 3,250 \text{ in}^4
Fig. 16-13. Arrangement of reinforcing steel.

Now the stress at the neutral axis is \( f_e = \frac{N}{A} - \frac{M'}{I} \cdot g = 0 \). And, since \( N \) and \( M' \), or \( N \) and \( M' \) of Fig. 16-12, may be resolved into a single force \( N \) acting \( e' = \frac{M}{N} \) above the center of the beam,

\[
\frac{N}{A} = \frac{M'}{I} = \frac{N(e' - 0.5t + \vartheta)(kd - \vartheta)}{I}
\]

Hence

\[
k d = \vartheta + \frac{I}{A(e' - 0.5t + \vartheta)} = 5.90 + \frac{3.250}{105.1(77.7 - 9 + 5.90)} = 6.31 \text{ in.}
\]

This value for \( kd \) does not quite check the value of 6.40 in. assumed originally. Recalculating, using \( kd = 6.30 \text{ in.} \), then \( A = 103.9 \text{ sq in.} \), \( S = 612 \text{ in.}^4 \), \( I = 6,820 \text{ in.}^4 \), \( I = 3,170 \text{ in.}^4 \), \( \vartheta = 5.90 \text{ in.} \), and \( kd = 6.31 \text{ in.} \), which checks.

Then

\[
f_e = \frac{N}{A} + \frac{M'}{I} = \frac{N}{A} + \frac{N(e' - 0.5t + \vartheta)\vartheta}{I} = \frac{8,310}{103.9} + \frac{8,310 \times 74.6 \times 5.90}{3,170} = 1,233 \text{ psi}
\]
and 
\[ f_c = \frac{n f_e d - kd}{kd} = 10 \times 1,233 \times \frac{9.69}{6.31} = 18,950 \text{ psi} \]

These are within the allowable values of \( f_c = 1,250 \) and \( f_s = 22,500 \) psi.

When only dead load, live load, and impact are considered, \( M = 42,630 \) ft-lb and \( N = 8,410 \) lb at the crown (see Table 16-2). Using these in an analysis similar to that indicated above, the stresses become \( f_c = 956 \) and \( f_s = 14,200 \) psi, which are within the allowable values of \( f_c = 1,000 \) and \( f_s = 18,000 \) psi for this loading.

Similarly, for the overload provision, \( M = 67,020 \) ft-lb and \( N = 11,860 \) lb, the stress analysis yields \( f_c = 1,513 \) and \( f_s = 22,900 \) psi which are acceptable for the allowable values of \( f_c = 1,500 \) and \( f_s = 27,000 \) psi respectively for this loading condition.

A similar investigation at the knee with \( M = 172,300 \) ft-lb and \( N = 14,730 \) lb yields \( f_c = 781 \) and \( f_s = 17,900 \) psi. These are less than the allowable values of \( f_c = 1,000 \) and \( f_s = 18,000 \) psi, respectively. When the overload and the combined load provision is checked the stresses are also well within the allowable values.

It should be noted that the moment diagram cannot be used in the usual way to determine where to cut off or bend up the reinforcing steel. This is due to the fact that the thrust must be considered also. However, the area of the steel determined in Table 16-3 can be plotted and so used. The method is illustrated in Fig. 16-13.

Investigation for Shear

For maximum shear at the crown the bridge should be loaded as is indicated in Fig. 16-9! and for maximum shear at the knee as is indicated in Fig. 16-9k. These shears, together with those produced by the other loadings, are summarized in Table 16-2. At the crown \( V = 3,113 \) lb and \( v_0 = V/bjd = 3,113/12 \times 0.875 \times 16 = 18.5 \) psi. Similarly at the end of the deck \( v_0 = 17,446/12 \times 0.875 \times 40 = 41.5 \) psi and at the top of the footing \( v_0 = 9,534/12 \times 0.875 \times 25 = 36.3 \) psi. Since the maximum allowable shear is \( v_0 = 0.03/f_c' = 75 \) psi, no shear reinforcing is necessary but \( \frac{3}{8} \)-in. \( \phi \) spacer bars at 3 ft 0 in. on centers will be used to hold the extrados and intrados steel in place while the concrete is being poured.

Investigation for Bond

At the crown \( u = V/\Sigma qjd = 3,113/8.50 \times 0.875 \times 16 = 26.3 \) psi, at the end of the deck \( u = 17,446/9 \times 0.875 \times 40 = 55.5 \) psi, and at the top of the footing \( u = 9,534/4.5 \times 0.875 \times 25 = 96.7 \) psi. All these are below the allowable value of \( u = 150 \) psi. In splicing, all bars should be lapped so as to develop their full tensile strength.

Transverse Reinforcement

Transversely \( \frac{1}{8} \) sq in. of reinforcing per ft of exposed surface to resist temperature change and shrinkage will be used. Hence \( \frac{3}{4} \) in. \( \phi \) at 3 ft 0 in. center to center on top and bottom of the deck and inside and outside the columns will suffice.

Footing Reinforcement

The maximum thrust on the footing is 27,000 lb. Assuming uniform distribution over the footing this amounts to 4,500 psi. The moment at the center is approximately \( wL^2/2 = 4,500 \times 3^2/2 = 20,300 \) ft-lb or 244,000 in.-lb. Hence the tensile steel required is \( A_s = 244,000/18,000 \times 0.875 \times 28 = 0.554 \) sq in. Use \( \frac{3}{8} \) in. \( \phi \) at 12 in. center to center.
cations) of from 100 to 300 lb per lineal ft. They are best poured after the main bridge centering has been struck so as to avoid the fine cracking which is otherwise usually found where the negative moments are large.

**REDUCTION FACTORS FOR CURVED-SLAB DECK**

The final moments, as determined in Table 16-1, represent the values that exist on frames whose members are straight. These moments are slightly larger than those on frames whose slab decks are curved, for the columns together with the curved deck now approach an arch somewhat more closely in their structural action. The Portland Cement Association analyzed a series of single-span rigid-frame bridges using moment distribution for the straight decked frames and column analogy for similar structures with curved decks. They found that, within the limits of height \( H \) and span \( S \) as indicated in the shaded area of Fig. 16-14, the correction factors indicated in Table 16-4 should be applied to the final moments determined by moment distribution assuming the deck as straight. The factors are not applicable to moments produced by earth pressure.

For the illustrated design these reduction factors would have been 0.972 for moments due to vertical loadings, and for moments due to changes in deck length they become 0.943 at the crown and 0.889 at the knee. Had they been used, somewhat less steel or thinner sections might have been used.

**Table 16-4**

<table>
<thead>
<tr>
<th>Loading</th>
<th>Reduction factor at Crown</th>
<th>Reduction factor at Knee</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vertical loads on the deck</td>
<td>( H + R/2 )</td>
<td>( H + R/2 )</td>
</tr>
<tr>
<td>Change in deck length</td>
<td>( H )</td>
<td>( H + R )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( (H + R)^3 )</td>
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</table>

**ANALYSIS OF AN UNSYMMETRICAL SINGLE-SPAN RIGID FRAME**

The illustrated design just concluded represented only the simplest structural indeterminate analysis possible, because of its symmetry and the hinged condition. However, the problem is only slightly more complicated when conditions at the site require an unsymmetrical bent. One will be analyzed here merely to suggest the procedure.
Usually the deck is symmetrical about its center even though the two abutments may not be. If this is not the case Charts 16-I and 16-II, referred to in the preceding design, cannot be used and the stiffness factors, carry-over factors, and fixed-end moments for unit loads at various points along the span would have to be calculated separately by some method like the column analogy, which is explained later in the discussion on arch design. Aesthetic considerations, however, usually favor the symmetrical deck.

It will be assumed in this example that the deck and one abutment of the unsymmetrical bent are identical with those of the preceding example but that the other bent is somewhat longer and that its lower end is fixed rather than hinged. The dimensions of this longer column will be taken as 3 ft 8 in. wide at the top, 2 ft 3 in. wide at the bottom, and 22 ft 0 in. long up to the axis of the deck. For a ratio of

\[
\frac{I_A}{I_B} = \left(\frac{2.25}{3.67}\right)^3 = 0.231
\]

as before, the values of the stiffness and carry-over factors can be read from Chart 16-II. Then the stiffness at \(A\) is \(5.88 \times 2.25^1/12 \times 22 = 0.254\) and at \(B\) it is \(12.2 \times 2.25^1/12 \times 22 = 0.526\). The carry-over factor from \(B\) to \(A\) (top and bottom) is \(4.22/12.2 = 0.346\). The stiffness of the deck relative to the fixed abutment is \(0.0865/(0.0865 + 0.526) = 0.1416\). The relative stiffness at the other end of the deck remains the same as in the preceding example. Figure 16-15a and b illustrates the calculations for final moments by moment distribution assuming that the frame is held against sidesway.

When the frame is free to sway sideways a correction must be made. Let it be assumed that the deck is pulled to the right by a force \(Q\) (Fig. 16-16a). Then the moment at the top of the hinged column is

\[
M_B = \frac{\Delta}{L} S_B \frac{E I_A}{L} (1 - r_A r_B) = \frac{\Delta}{17.9} 0.648 \left(1 - \frac{4.22}{12.2 \times 5.88}\right) = \frac{\Delta}{17.9} 0.493 = 0.0275\Delta
\]

The moment at the top of the fixed column is

\[
M'_B = \frac{\Delta}{L} S_B' \frac{E I_A'}{L} (1 + r_B) = \frac{\Delta}{22} 0.526(1 + 0.346) = 0.0322\Delta
\]

and at the bottom it is

\[
M_A' = \frac{\Delta}{L} S_A' \frac{E I_A}{L} (1 + r_A) = \frac{\Delta}{22} 0.254 \left(1 + \frac{4.22}{5.88}\right) = 0.0198\Delta
\]

These moments will be applied simultaneously to the frame and balanced. For simplicity \(\Delta\) will be taken as 1,000. The distribution is illustrated in Fig. 16-15c. The force required to cause the sidesway is \(Q = L_H + R_H\) (see Fig. 16-16a).

\[
L_H = \frac{5.73}{17.9} = 0.340 \quad \text{and} \quad R_H = \frac{(6.11 + 9.76)}{22} = 0.720
\]

Hence \(Q = 1.06\).

Suppose now that the final moments are required for the concentrated live load of 4,090 lb applied at the 0.36 point of the deck. From the preceding example (Table 16-1) the fixed-end moments are \(M_B = +48,400\) and \(M_B' = 19,850\) ft-lb. The final deck moment at the left knee is \(M_B = 48,400(+0.8609) - 19,850(-0.0904) = +43,400\) and at the right knee \(M_B' = 48,400(-0.0963) - 19,850(+0.8685) = -21,900\) ft-lb.

At the base of the fixed column, it is

\[
M_A' = 48,400(+0.0336) - 19,850(-0.303) = +7,630\ \text{ft-lb}
\]
Fig. 16-15. Moment distribution applied to frame, no sidesway.

In Fig. 16-16b, the force required to restrain the frame against sidesway is

\[ P = L_H - R_H = \frac{43.4}{17.9} - \frac{(21.9 + 7.63)}{22} = 2.42 - 1.34 = 1.08 \text{ kips} \]

Since \( aQ - P = 0 \), \( a = 1.08/1.06 = +1.02 \). Hence the moments in the deck, after
Fig. 16-16. Frame affected by sidesway.

Correcting for sidesway, are

\[ M_B = +43.4 + 1.02(-5.73) = 43.4 - 5.75 = +37.65 \text{ ft-kips} \]

\[ M_B' = -21.9 + 1.02(-6.11) = -21.9 - 6.23 = -28.1 \text{ ft-kips}, \text{ and at the base of} \]

the fixed column, it is \( M_A' = +7.63 + 1.02(+9.76) = 7.63 + 9.89 = +17.52 \text{ ft-kips.} \)

It should be noted here that sidesway reduces the maximum moment at the knee but increases it at the base of the right abutment. It is on the safe side, therefore, to disregard the effect of sidesway unless it increases the bending moment, and to use
these maximum values to determine the total moments due to the worst combination of loadings.

For unsymmetrical bents the sideways correction must be applied to all loadings, symmetrical or otherwise, because an unsymmetrical bent will sway sideways even under symmetrical deck loads, unless held against lateral movement.

**RIBBED DECKS**

Rigid frames with ribbed decks are generally used for somewhat longer spans than frames with slab decks. When they are selected, the analysis can be carried out just as for the illustrated example described previously. Charts 16-I and 16-II can be used for ribs without serious error although they were intended originally for decks with parabolic haunches. It will usually be found necessary, however, to provide shear reinforcement by bending up rods and including stirrups in the ribs not only of the decks but in the columns as well.

In the design it is usually desirable to design the slab or floor between ribs for the various loadings first. This will be illustrated later in the example on arch design.

**BOX-GIRDER DESIGN**

Reinforced-concrete box-girder construction has been applied, within the past few years, to rigid-frame bridge design.\(^{11}\) The cellular type of structure is well adapted to the longer spans where a saving in weight over the slab type, and a decrease in depth over the ribbed type, is noticeable. Details for box-girder rigid frame are illustrated in Fig. 16-26.

**ARCHED DECKS**

The foregoing analysis and comments should be considered only with reference to rigid-frame bridges whose decks are essentially straight. When the extrados also has considerable curvature the deck approaches an arch in its structural action. In such instances it is advisable to analyze the deck and abutments by some other method than moment distribution, lest errors be introduced because the deck length increases under load and allows the column tops to spread. The column analogy is well adapted to such single-span structures and is illustrated in Part 2 of this section.

**MULTIPLE-SPAN RIGID FRAMES**

About one-third of all rigid frames built to date have been multiple-span structures and roughly half of these have slab decks. Their substitution for simple-span structures is due mainly to the simplification of analysis by methods like moment distribution and to the marked economies in material which usually accompany continuity. In the long multiple-span bridges provision must be made for expansion and contraction due to changes in temperature and moisture content. The expansion joints provide discontinuities in the structure but need not complicate the analysis in the least. (See reference 12.)

The various types of expansion joints in use are illustrated schematically in Fig. 16-17 and some of these are shown in more detail in Fig. 16-22. All have certain advantages and disadvantages. Type I requires a somewhat thicker deck to compensate for the larger bending moments resulting from the discontinuity and inability of the pier to participate. Unless the adjoining decks are superimposed as shown, an increase in width of pier is necessary and this may disturb the architectural proportions of the bridge. Type II is of the split-column type. If the end columns are kept slender so as to avoid an excessive over-all thickness, the columns lack the rigidity to offer much relief from bending to the deck. However, this type is simpler to construct than some of the others and is certainly not so likely to “freeze up” or rust out. Type III illustrates the suspended span. Here the discontinuities are placed at about the quarter points where the moments are practically zero and where
the shears are smaller than at the ends of the deck. Hence the continuity is not
interrupted as much as is the case with type I and all piers can be made identical and
cast integrally with the decks. Type III also has the advantage of accessibility by
jacking up the suspended span should replacement or repair of the joint become
necessary. Type IV is an improvement of type III. The elimination of one of the
hinges effects a considerable saving in cost, but it does not complicate the analysis by
moment distribution when beam constants for hinged decks are used. The analysis
of this type will be illustrated later. The single intermediate hinge has been used
extensively in California, where it was first developed, and has given excellent service.
The hinge should be located at 0.2L in beams where the moment of inertia remains
constant, and its position should move toward the quarter point of the deck as
the haunching increases.

ILLUSTRATED EXAMPLE

A multiple-span rigid frame, regardless of the type of expansion joint incorporated
in the structure, is as well adapted for analysis by moment distribution as was the
single-span structure. A typical analysis will be illustrated here merely for the pur-
pose of indicating the method. Space does not permit inclusion of a complete design.
The initial estimate for size of members, calculation of fixed-end moments due to
various loadings, selection of the beam coefficients from the charts, and final deter-
mination of the required reinforcing are identical with those described in the illustrated
design of the single-span bridge. In this example, then, only the analysis will be
indicated.

The bridge is shown in Fig. 16-18. The beam coefficients for the solid decks are
taken from Charts 16-I to 16-IV, whereas those for the hinged decks are taken from
Charts 16-V to 16-XII.

For beams $AB$ and $EF$:

$$\frac{I_C}{I_A} = \left(\frac{2.50}{5.33}\right)^3 = 0.103$$

Then from Chart 16-II and 16-IV

End stiffness $= 13.2 \times 1 \times 2.50^2/48 \times 12 = 0.358$ for a width of 1 ft

and

Carry-over factor $= \frac{9.35}{13.2} = 0.709$

Fixed-end $M$ due to uniform load $= 0.1035 \times 1 \times 48^2 = 239$ ft-kips

For beam $CD$:

$$\frac{I_C}{I_A} = 0.103$ as before

End stiffness $= 13.2 \times 1 \times 2.50^2/60 \times 12 = 0.2865$

Carry-over factor $= 0.709$
**THE RIGID-FRAME BRIDGE**

Note: Fixed-end moments are plus when they act clockwise on the joint. Sign of carry-over is therefore also plus. The piers are considered pinned at the top of the footings.

Values of $r$ are:

<table>
<thead>
<tr>
<th>Values of $r$</th>
<th>$-.709$</th>
<th>$2$</th>
<th>$-.5$</th>
<th>$-.709$</th>
<th>$2$</th>
<th>$-.709$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fix—E.M.</td>
<td>+239.0</td>
<td>-239.0</td>
<td>+425.0</td>
<td>-358.0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Col. M</td>
<td>-40.3</td>
<td>-432.7</td>
<td>+483.4</td>
<td>-241.0</td>
<td>+965.</td>
<td>-179.</td>
</tr>
<tr>
<td>Bent Shear</td>
<td>-4.595</td>
<td>+2.130</td>
<td>+2.392</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Sidesway Correction—Bent AB**

- Piers A and F $l/L_s = 1.385/27.33 = 0.0799$ Column sidesway $M = 100.0$
- Piers B and E $l/L_s = 1.385/27.33 = 0.0798$ Column sidesway $M = 117.4$
- Piers C and D $l/L_s = 1.385/27.33 = 0.1083$ Column sidesway $M = 223.5$

Values of $r$ are:

<table>
<thead>
<tr>
<th>Values of $r$</th>
<th>$-.709$</th>
<th>$2$</th>
<th>$-.5$</th>
<th>$-.709$</th>
<th>$2$</th>
<th>$-.709$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Col. M</td>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Beam M</td>
<td>-84.7</td>
<td>-86.1</td>
<td>-6.4</td>
<td>-12.5</td>
<td>6.1</td>
<td>3.0</td>
</tr>
<tr>
<td>Col. M</td>
<td>84.7</td>
<td>92.5</td>
<td>6.4</td>
<td>1.5</td>
<td>0.3</td>
<td>0</td>
</tr>
<tr>
<td>Col. Shear</td>
<td>4.89</td>
<td>4.14</td>
<td>0.234</td>
<td>-0.055</td>
<td>0.013</td>
<td>0</td>
</tr>
<tr>
<td>Bent Shear</td>
<td>9.03</td>
<td>0.179</td>
<td>0.013</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Fig. 16-18. A five-span rigid frame.**
**ILLUSTRATED EXAMPLE**

**Similarly, for Bent EF**

<table>
<thead>
<tr>
<th>Col. M.</th>
<th>0</th>
<th>0</th>
<th>0</th>
<th>0</th>
<th>117.4</th>
<th>100.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam M.</td>
<td>0.06</td>
<td>0.59</td>
<td>-0.9</td>
<td>-1.5</td>
<td>3.0</td>
<td>6.1</td>
</tr>
<tr>
<td>Col. M.</td>
<td>0</td>
<td>0.3</td>
<td>-1.5</td>
<td>6.4</td>
<td>92.5</td>
<td>84.7</td>
</tr>
<tr>
<td>Col. Shear</td>
<td>0</td>
<td>0.013</td>
<td>-0.055</td>
<td>0.234</td>
<td>4.14</td>
<td>4.89</td>
</tr>
<tr>
<td>Bent Shear</td>
<td>0.013</td>
<td>0.179</td>
<td>9.03</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Bent CD**

<table>
<thead>
<tr>
<th>Col. M.</th>
<th>0</th>
<th>0</th>
<th>0</th>
<th>0</th>
<th>135.5</th>
<th>135.5</th>
<th>0</th>
<th>0</th>
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</thead>
<tbody>
<tr>
<td>Beam M.</td>
<td>0.60</td>
<td>0</td>
<td>-22.6</td>
<td>0</td>
<td>-34.4</td>
<td>0</td>
<td>-19.6</td>
<td>4.7</td>
</tr>
<tr>
<td>Col. M.</td>
<td>13.5</td>
<td>2.5</td>
<td>11.5</td>
<td>12.5</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Col. Shear</td>
<td>0.13</td>
<td>0</td>
<td>-0.8</td>
<td>-2.2</td>
<td>-1.5</td>
<td>-1.7</td>
<td>-2.7</td>
<td>-4.0</td>
</tr>
<tr>
<td>Bent Shear</td>
<td>0.134</td>
<td>0</td>
<td>0.179</td>
<td>0</td>
<td>7.380</td>
<td>0.174</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

**Proportioning sideways moments**

<table>
<thead>
<tr>
<th>( AB \cdot a )</th>
<th>+40.3</th>
<th>-432.7</th>
<th>+483.4</th>
<th>-241.0</th>
<th>+96.5</th>
<th>+17.9</th>
<th>+68.4</th>
<th>-225.5</th>
<th>+127.0</th>
<th>-35.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>( CD \cdot b )</td>
<td>-43.6</td>
<td>-44.3</td>
<td>-3.3</td>
<td>-6.4</td>
<td>+3.1</td>
<td>+1.5</td>
<td>-0.8</td>
<td>-0.5</td>
<td>+0.3</td>
<td>0</td>
</tr>
<tr>
<td>( EF \cdot c )</td>
<td>-0.7</td>
<td>-2.7</td>
<td>+0.2</td>
<td>+9.5</td>
<td>+20.3</td>
<td>+20.3</td>
<td>+9.5</td>
<td>+4.7</td>
<td>-2.7</td>
<td>-0.7</td>
</tr>
</tbody>
</table>

**Final**

<table>
<thead>
<tr>
<th>Beam M.</th>
<th>-0.4</th>
<th>-479.9</th>
<th>+485.0</th>
<th>-237.5</th>
<th>+119.1</th>
<th>+38.1</th>
<th>+80.4</th>
<th>-219.6</th>
<th>+147.2</th>
<th>+56.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Col. M.</td>
<td>+4.0</td>
<td>-5.1</td>
<td>+118.4</td>
<td>-118.5</td>
<td>+72.4</td>
<td>-56.5</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Col. Shear</td>
<td>+0.23</td>
<td>-0.23</td>
<td>+4.33</td>
<td>-4.33</td>
<td>+3.24</td>
<td>-3.26</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Fig. 16-18 (Continued)**

For beam BC and DE, hinge at \( nL = 1/3L \):

Stiffness at B and E = \( S_B = S_E = 3.05 \times 1 \times 2.50^4/60 \times 12 = 0.0661 \) (See Chart 16-V)

\[
S_C = S_D = 12.3 \times 1 \times 2.50^4/60 \times 12 = 0.267
\]  
(See Chart 16-V)

**Carry-over factor** = \( r_B = r_E = 2 \) (See Chart 16-V)

**Fixed-end moment** = \( M_B = 0.1181 \times 1 \times 60^2 = 425 \) ft-kips (See Chart 16-VI)

\[
M_C = 0.0995 \times 1 \times 60^2 = 358 \text{ ft-kips}
\]  
(See Chart 16-VI)

\[
M_D = 0.0420 \times 10 \times 60 + 0.2071 \times 5 \times 60
\]  
(See Chart 16-XIII)

\[
= 87.3 \text{ ft-kips}
\]

\[
M_E = 0.3124 \times 10 \times 60 + 0.09646 \times 5 \times 60
\]  
(See Chart 16-XIII)

\[
= 216.4 \text{ ft-kips}
\]

**Note:** The moment coefficients for the 10-kip load of beam DE can be determined most conveniently by using the transformation equations of Chart 16-XIII rather than by interpolating on Charts 16-IX and 16-X. The Chart values for \( k = \frac{1}{2} \) fall
on the left or right branches of the curves for \( n = 0.3 \) and 0.4, making interpolation for \( n = \frac{3}{5} \) virtually meaningless. Usually such is not the case and Charts 16-VII to 16-XII may be used directly.

The following example will serve to illustrate the use of the transformation equations. Given \( n = \frac{3}{5}, k = \frac{3}{5} \); from Chart 16-I for a beam with \( I_c/I_A = 0.103 \) and \( k = \frac{3}{5}, M'_A = -0.077PL \) and \( M'_B = -0.2040PL \). Similarly, Chart 16-II indicates that \( S' = 13.2EI_c/L \) and \( r' = r'S'/S' = 9.35/13.2 = 0.709 \). Then, from Chart 16-XIII, and letting \( L = 1.0 \),

\[
A = \frac{2}{S'(1 + r')} = \frac{2}{13.2[1 + (-0.709)]} = +0.5207
\]

\[
i_e = \frac{2}{2S'(1 - r')} = \frac{1}{2(13.2)[1 - (-0.709)]} = +0.02216
\]

\[
i_s = i_e + AL^2(0.5 - n)^2 = +0.02216 + 0.5207(1)^2(0.5 - 0.667)^2
\]

\[
= +0.03663
\]

Considering the load to lie to the left of the hinge,

\[
q = A \left[ -M'_B - \left( \frac{M_{SA} + M'_A - M'_B}{2} \right) \right]
\]

\[
= +0.5207 \left[ -(-0.204) - \frac{(-0.667) + (-0.077) - (-0.204)}{2} \right]
\]
ILLUSTRATED EXAMPLE

\[
\begin{align*}
\frac{1}{2} &= +0.5207 \left( +0.204 - \frac{0.794}{2} \right) = -0.1004 \\
M_a &= \frac{i_c}{L} (\frac{-M_{SA} + M'_{A} - M'_{B}}{i_n}) Firstly, \quad M_a = M_{SA} + \left( \frac{m_n L}{i_n} \right) = +0.01759 - 0.01004(1)(0.5 - 0.667) = +0.03432 \\
M_B &= - \left[ \frac{+0.03432(-0.333)(1.0)}{+0.03663} \right] = -0.3124 \\
\end{align*}
\]

Hence \( M_B = -0.3124PL \) for the 10-kip load on beam \( DE \).

Similarly, \( M_A = M_{SA} + \left( \frac{m_n L}{i_n} \right) = -0.667 + \frac{+0.03432(+0.667)1.0}{+0.03663} = -0.0420 \)

Hence \( H_D = -0.0420PL \) for the 10-kip load on beam \( DE \).

For the piers \( A \) and \( F \) (hinged at the bottom):

Stiffness at the top \( = S_B \frac{E I_A}{L} (1 - r_{AB}) = 4 \times \frac{1 \times 2^4}{12 \times 17.33} \left( 1 - \frac{2}{4} \times \frac{2}{4} \right) = 0.1153 \)

(See Chart 16-II)

**Chart 16-VII.** Hinged member, concentrated load—fluence lines for end moments, \( I_{center}/I_{end} = 1.0 \).

**Chart 16-VIII.** Hinged member, concentrated load—fluence lines for end moments, \( I_{center}/I_{end} = 0.5 \).
For the piers B and E (hinged at the bottom):

\[
\text{Stiffness at the top } = 4 \times \frac{1 \times 2.5^4}{12 \times 22.33} \left(1 - \frac{1}{4}\right) = 0.1746
\]

For piers C and D (hinged at the bottom):

\[
\text{Stiffness at the top } = 4 \times \frac{1 \times 3^4}{12 \times 27.33} \left(1 - \frac{1}{4}\right) = 0.247 \quad (\text{See Chart 16-II})
\]

The relative stiffness of any of the members at a joint must now be calculated. For example, at B, the relative stiffness of the end of B beam BC

\[
\frac{0.0661}{(0.0661 + 0.358 + 0.174)} = 0.0110
\]

All such relative-stiffness factors are indicated at the head of their appropriate columns. The carry-over factors for the beams are shown directly above these stiffness factors.

The moments distributed in the example represent the moments due to the actual loadings rather than the unit moments used previously. The purpose here is merely to illustrate the method. Actually, in design, it will be found convenient to start with a unit moment at each end of each span and at the top of each column (eight in all because of symmetry) and distribute these as was done for the two single-span
structures illustrated previously in Figs. 16-5 and 16-15. Final moments for any loading may then be obtained by proportion, in a manner identical with that used in the previous design (see Table 16-1).

The analysis of the fixed-end deck moments follows the usual procedure of balancing and distributing followed in moment distribution. The unsymmetrical loadings and bents produce unbalanced shears in the columns and also in the three bents as indicated. It seems reasonable to assume these multiple-span structures are free to sway sideways. The sideways corrections are indicated in Fig. 16-18 also. If each bent (and column) is deflected an amount $\Delta$ with the joints held against rotation, so that the moment in the end columns is 100 ft-kips, then the other column moments will be proportioned according to $I/L^3$ since

$$M = \frac{\Delta}{L} S_B \frac{E I_A}{L} \left(1 - r_A r_B\right) = 4E \frac{3I}{4L^3} \Delta$$

The other column moments are indicated in Fig. 16-18 together with the three distributions, one for each bent. The new unbalanced shears accompanying these moment distributions are found for each bent as before. The proportional part of the final moments which must be combined with those due to the original loadings is determined by setting the unbalanced shears in each bent, due to the applied load.
Symmetrically Haunched Beam

\[ i_n = i_e + AL^2(0.5 - n)^2 \]

where \[ A = \frac{S'(1 + r')}{L^3} \]
and \[ i_e = \frac{2S'(1 - r')}{L^3} \]

\[ S_A = nL^2/i_e \quad S_B = cL^2/i_e \quad r_{AB} = c/n \quad r_{BA} = n/c \]

**Concentrated Load to Left of Hinge**

\[ M_{SA} = -P_kL \quad M_{SB} = 0 \]
\[ q = A \left( -M'' + \frac{M_A - M''}{2} \right) \]
\[ m_e = \frac{i_n}{L} (M_{SA} + M' - M''b) \]
\[ m_n = m_e + qL(0.5 - n) \]
\[ M_A = M_{SA} + \frac{m_nL}{i_n} \]
\[ M_B = \frac{m_nL}{i_n} \]

Moments are (+) when bottom fiber is in tension. Carry-over factors are negative.

**Notation of Symbols**

\[ L = \text{length of beam} \]
\[ d = \text{center (or minimum) depth of beam} \]
\[ d' = \text{haunching factor} \frac{\text{end depth} - \text{min. depth}}{\text{min. depth}} \]
\[ A = \text{elastic area} \]
\[ i_n, i_e, i = \text{moment of inertia of the elastic area about the beam center, the hinge point, and the elastic centroid respectively} \]
\[ I_e, I_A = \text{moment of inertia of beam cross section at center and end A of span} \]
\[ M_{SA}, M_{SB} = \text{fixed-end moments} \]
\[ M_{SA}, M_{SB} = \text{statical end moments—hinged beam made statically determinate and treated as a double cantilever by removal of the hinge} \]
\[ q = \text{area of the statical moment diagram} \]
\[ r = \text{carry-over factor---} r_{AB} \text{ is the carry-over from end A to B} \]
\[ S = \text{stiffness factor} \]
\[ P, Q, \text{w} = \text{concentrated and uniform loads respectively} \]
\[ a, b, \text{etc.} = \text{constants} \]

**Note**—Primed symbols refer to solid beams; unprimed symbols to hinged beam.

Unsymmetrically Haunched Beam

\[ z = \frac{-r''(1 - r'A)}{2r'x'b - r'a - r'b} \]
\[ A = \frac{S'\alpha - zL^2}{L^2(2\dot{z} - 1)} \]
\[ i = \frac{S'\alpha - S'b}{S'\alpha - S'b} \]
\[ i_n = i + AL^2(\dot{z} - n)^2 \]
\[ S_A = nL^2/i_n \quad S_B = cL^2/i_n \quad r_{AB} = c/n \quad r_{BA} = n/c \]

**Concentrated Load to Left of Hinge**

\[ M_{SA} = -P_kL \quad M_{SB} = 0 \]
\[ q = A \left( -M'' + \frac{M_A - M''}{2} \right) \]
\[ m = \frac{i_n}{L} (M_{SA} + M' - M''b) \]
\[ m_n = m + qL(\dot{z} - n) \]
\[ M_A = M_{SA} + \frac{m_nL}{i_n} \]
\[ M_B = \frac{m_nL}{i_n} \]

Moments are (+) when bottom fiber is in tension. Carry-over factors are negative.

**Chart** 16-XIII. Symmetrically and unsymmetrically haunched beams.
<table>
<thead>
<tr>
<th>Joint</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Relative stiffness x carry-over factor</td>
<td>-0.424</td>
<td>-0.167</td>
<td>-0.254</td>
<td>-0.220</td>
<td>-0.536</td>
<td>-0.424</td>
</tr>
<tr>
<td>Fixed-end M</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>239.0</td>
<td>358.0</td>
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<td>67.3</td>
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<td>0</td>
</tr>
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<td>-22.2</td>
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<td>+ 0.8</td>
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<td>- 44</td>
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<td>Practical stopping point</td>
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<td>- 0.2</td>
<td>- 0.1</td>
<td>+ 0.1</td>
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<td>+ 1.3</td>
<td>+ 0.2</td>
<td>- 3.6</td>
<td>- 3.6</td>
</tr>
<tr>
<td></td>
<td>+ 0.2</td>
<td>+ 0.1</td>
<td>+ 0.4</td>
<td>- 0.1</td>
<td>- 0.3</td>
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<td></td>
<td>+ 0.1</td>
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<td>+ 0.8</td>
<td>+ 0.2</td>
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<td>+503.1</td>
<td>-71.5</td>
<td>+162.2</td>
<td>-77.2</td>
<td>+144.3</td>
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<td>-105.3</td>
<td>-19.4</td>
<td>+167.7</td>
<td>+203.6</td>
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<tr>
<td>Beam M</td>
<td>+40.3</td>
<td>+483.7</td>
<td>+96.2</td>
<td>+68.6</td>
<td>+126.4</td>
<td>+35.5</td>
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<tr>
<td>ΣM</td>
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<td>-69.1</td>
<td>+160.4</td>
<td>-70.4</td>
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<td>-19.2</td>
<td>+166.5</td>
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<tr>
<td>x relative stiffness</td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Beam M</td>
<td>+40.6</td>
<td>+483.4</td>
<td>+97.4</td>
<td>+67.6</td>
<td>+128.3</td>
<td>+34.6</td>
</tr>
</tbody>
</table>

Fig. 16-19. Moment distribution for five-span frame shown in Fig. 16-18.
and to the induced sidesway, to zero. Then:

\[
\begin{align*}
\text{Bent } AB & \quad -4.595 + 9.030a + 0.174b + 0.013c = 0 \\
\text{Bent } CD & \quad 2.130 + 0.179a + 7.380b + 0.179c = 0 \\
\text{Bent } EF & \quad 2.392 + 0.013a + 0.174b + 9.030c = 0 \\
\text{Hence} & \quad a = 0.5149 \quad b = -0.2948 \quad c = -0.2620
\end{align*}
\]

The final moment in BC at B is then

\[
M_{BC} = 4.834 + 0.5149(-6.4) + (-0.2620)(-0.9) + (-0.2948)(-16.1) = 485.0
\]

The calculations for the entire process appear somewhat lengthy but probably require as little time as would be necessary with any method. Nevertheless, they can be shortened tremendously by balancing the joints alternately (in pairs) and writing only moments carried over. The short cut is illustrated in Fig. 16-19 for the same fixed-end moments due to loadings of Fig. 16-18. The relative-stiffness carry-over factor is used and indicated. For instance, the proportional unbalanced moment at A carried over to B when joint A is unlocked is 0.756 X 0.709 = 0.536. Since only the moments carried over (distributed) are entered, the summation for each column will be in error by the amount of the moment normally entered in the balancing operation at each joint. This step is now accomplished in one stroke by multiplying the summations at each joint by the relative-stiffness number and subtracting this value from each column summation. For instance, at joint C, the final

\[
\begin{align*}
M_{CB} &= -396.8 - (-396.8 - 71.5) \times 0.333 = -396.8 + 155.9 = -240.9 \text{ ft-kips} \\
M_{CD} &= -71.5 - (-396.8 - 71.5) \times 0.358 = -71.5 + 167.7 = +96.2 \text{ ft-kips} \\
M_{col} &= 0 - (-396.8 - 71.5) \times 0.309 = 0 + 144.7 = +144.7 \text{ ft-kips}
\end{align*}
\]

\[
M_{CB} + M_{CD} + M_{col} = 0.
\]

It should be noted in Fig. 16-19 that, although the work has been cut at least in half, the convergence was carried out to smaller increments, thus making the final results perhaps closer to their mathematically correct values. The convergence here has been carried much further than is necessary in practice; it should always be terminated whenever the required accuracy has been attained, say within 5 per cent. For instance, if the process had been stopped after three operations and the terms above the dotted line added, the final moments would have been almost exactly the same. They are indicated at the bottom of Fig. 16-19.

**BEAM COEFFICIENTS FOR HINGED BEAMS, GENERAL CASE**

At times it may be desirable to incorporate hinges in beams for which the haunching is not parabolic or symmetrical. The beam coefficients cannot then be determined from Charts 16-V to 16-XII, but they can be found from the transformation equations of Chart 16-XIII if beam coefficients can be obtained from available charts for solid beams. (See reference 14.)

**INFLUENCE LINES FOR MULTIPLE-SPAN RIGID-FRAME BRIDGES**

It is frequently necessary to know which spans to load so that maximum live-load moments may be determined. Quantitative diagrams of this nature, while useful after completion, are very tedious to construct. Usually qualitative diagrams are all that is required, for they indicate which spans to load. Although these qualitative diagrams do not furnish quantitative results directly, such results may be obtained by loading the critical spans, finding the fixed-end moments, and multiplying these by the final moments produced by the unit fixed-end moment method described and illustrated previously.

Qualitative influence lines are most easily sketched by (1) breaking the structure at the desired section, (2) applying unit reactions (moments, thrusts, or shears) so as to displace the axes of the structural members, and (3) sketching this deflected structure. The structure in its deflected position is identical in shape with the influence line for the particular reaction producing the deflection. Figure 16-20
Influence Lines for Multiple-Span Bridges

Fig. 16-20. Qualitative influence lines for multispans frames. Note: In Fig. 16-20a load span BC for maximum +M at E and spans AB and CD for maximum -M at E. Treat other figures in a similar manner to get maximum (+) and (-) values.

Illustrates several of these and indicates which spans to load to effect the maximum stresses.

When it is advisable to draw quantitative influence lines for multiple-span structures, as, for instance, that for moment at G in Fig. 16-20d, the ordinates to such curves may be determined as follows:

1. Load the bridge with a unit moment at A in span AB. Determine the final moments at each end of every member by moment distribution, including the effect of sidesway whenever such lateral movement is possible.

2. Repeat the operations of step 1 for unit moments at B in span AB, at B in span BC, etc., across the bridge.

3. Apply a unit load at point H in span AB and determine the fixed-end moments at A and B ($M_{FA}$ and $M_{FB}$).

4. Multiply $M_{FA}$ by the final moments in span CD at C and at D due to the unit moment loading at A (step 1). Repeat for $M_{FB}$ and the final moments in span CD at C and at D due to the unit moment loading at B (step 2). Add these two values for the final total moment at C and at D.

5. Draw a free-body diagram of span CD and determine the moment at G due to the final end moments at C and D. This moment is the ordinate of the influence line for moment at G due to a unit load at H. It should be plotted at H.

6. Repeat steps 3, 4, and 5 moving point H to as many positions along the entire structure as may be necessary to draw a smooth curve for the influence line. Usually five positions (at the fifth points) in each span will suffice.
CURVES FOR MAXIMUM POSITIVE AND MAXIMUM NEGATIVE MOMENT AND SHEAR FOR MULTIPLE SPANS

As stated in the preceding paragraph the positions of the loads for maximum moment, etc., at some chosen point may easily be determined by means of the qualitative influence line. The quantitative influence line, while useful for the study of conditions at one point, is of no use for any other point in the structure. A much more useful guide in design is a curve for maximum positive and maximum negative moment for the entire structure, similar to the one of Fig. 16-9m for the single-span bridge. These curves (together with the corresponding ones for shear, thrust, etc.) may be obtained as follows (refer to Fig. 16-21):

1. Draw a qualitative influence line for moment at $E$.
2. Load spans $AB$ and $CD$ with live loads so placed as to give maximum positive final moment at $E$. Note that some trial and error is involved for wheel loads to determine their exact location for maximum values. For uniform loads distribute the load over as much of the span as the influence line indicates, which in this case covers the entire spans $AB$ and $CD$.
3. Determine the fixed-end moments in these spans and multiply these by the final moments resulting from the unit moment loading method described previously.
4. Apply the laws of statics to span $AB$ and get the moment at $E$.
5. Repeat steps 2 and 4 for loads in span $BC$ to get the maximum negative moment at $E$. 

![Diagram](image-url)
6. Repeat steps 1 to 5 for a series of points across the entire bridge. The final result will appear as in Fig. 16-21b when plotted.

7. Combine the curves of Fig. 16-21b with the dead-load moments and plot the results as in Fig. 16-21d.

8. Draw similar maximum curves for shear, thrust, or other types of reaction which may have a bearing on the stress analysis at the cross section of the structural member.

9. Use the values for maximum moment, etc., to check the design and original estimate of the size of the members.

DETAILS OF RIGID-FRAME AND ARCH BRIDGES

Figures 16-22 to 16-28 illustrate various details and types of modern highway bridges. The drawings need no further explanation.

Fig. 16-22. Expansion hinge details.
Fig. 16-23. Single-span slab rigid-frame bridge over the Charter Oak Wash, Los Angeles County. (Courtesy of California Division of Highways.)
Plan and profile.
Fig. 16-24. Multiple-span ribbed rigid-frame bridge with expansion hinges, Cache Creek, Yolo County. (Courtesy of California Division of Highways.)
Layout of reinforcing steel.
FIG. 16-26. Hollow-girder continuous bridge. (Taken from "Continuous Hollow Girder Concrete Bridges" through the courtesy of the Portland Cement Association.13)
Diaphragms and expansion joints
Bearing and diaphragm details.
Fig. 16-26 (Continued)
Part plan, elevation and longitudinal section of arch.

Fig. 16-27. Tucker arch bridge over the Hood River. (Courtesy of Oregon State Highway Commission.)
Part plan and elevation of approach spans.

Fig. 16-27 (Continued)
Approach span details.
Fig. 16-27 (Continued)
Section details.

Fig. 16-27 (Continued)
Half plan and section of roadway.

Fig. 16-28 (Continued)
Half section of arch rib and half cross section of arch rib and roadway.

Fig. 16-28 (Continued)
Details of arch ribs, floor, and railings.

Fig. 16-28 (Continued)
Front elevation of abutment and section details.

Fig. 16-28 (Continued)
Plan and section of abutment.

**Fig. 16-28 (Continued)**
Details of Rigid-Frame and Arch Bridges

Side elevation showing front face reinforcing.

Forward tie beam 17'-0"
Rear tie beam 16'-5½"
5'-0"
6'-0"
8'-6"
E. roadway
2 2½ bars T 2 2½ bars T 2 2½ bars T
6 ½ bars in center 6 ½ bars in center 6 ½ bars in center
2 ½ bars in center 2 ½ bars in center 2 ½ bars in center
3 ½ bars in center 3 ½ bars in center 3 ½ bars in center
4 3½ bars in center 4 3½ bars in center 4 3½ bars in center
5 3½ bars in center 5 3½ bars in center 5 3½ bars in center
6 3½ bars in center 6 3½ bars in center 6 3½ bars in center
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100 3½ bars in center 100 3½ bars in center 100 3½ bars in center

General notes:
Abutment footings shall be carried at least 6' into solid rock. Footings shall be carried deeper than shown if necessary. All exposed corners of 90° or sharper shall be filleted with 1' dressed beveled strips except handrail, curb and coping. All reinforcing steel shall be securely wired in place before concrete is placed. All hooked reinforcing bars shall have standard 16" diameter hooks. Abutment fill shall be deposited in layers of about two feet in thickness and each layer thoroughly compacted.

Side elevation and transverse section of abutment.
Fig. 16-28 (Continued)
PART 2
REINFORCED-CONCRETE ARCH BRIDGES

DEFINITION OF TERMS

The nomenclature of terms pertaining to arches is illustrated in Fig. 16-29. In general the terms are self-explanatory, but they are amplified somewhat in the following list for greater clarity.

Crown. The highest part of the arch ring.
Haunch. The mid-point of the arch ring between the crown and the springing line.
Springing line. The intersection of the face of the pier and the soffit.

Fig. 16-29. Nomenclature of terms pertaining to arches.

16-66
SHAPE OF THE ARCH RING

Soffit. The undersurface of the arch.

Back. The top surface of the arch.

Intrados. The line of intersection of the soffit with a vertical plane taken parallel to the roadway.

Extrados. The line of intersection of the back with a vertical plane taken parallel to the roadway.

Rise. The height of the intrados at the crown above the level of the springing lines.

Skewback. The inclined surface perpendicular to the arch axis on which the end of the arch rests.

Clear span. The horizontal distance between the springing lines parallel to the roadway.

Spandrel. The space between the back of the arch and the roadway.

TYPES

Arch bridges in reinforced concrete have been constructed in several types. These are illustrated in Fig. 16-30, and their particular form at any bridge site is determined by such local factors as foundation conditions, clearance requirements, span, and type of loading. The most common type for short spans is the deck-type filled spandrel (Fig. 16-30a). It is most useful where the rise ratio is not too great, say less than 1:10, and the span is less than 100 ft, i.e., when the dead weight of the fill is not excessive. The fill does offer some advantage in reducing vibration, especially for railroad loadings, but the analysis of this type is quite uncertain because of the highly indeterminate reaction between the side retaining walls and the barrel arch. The filled spandrel does, however, offer decided aesthetic possibilities. The depth of the fill, including the pavement, at the crown varies from 1 to 2 ft for highway bridges but 2 to 3 ft below the ties is essential for railway bridges.

The open-spandrel arches illustrated in Fig. 16-30b, c, and d are of the deck, half-through, and through types, respectively. Usually the arch ring is divided into two (sometimes more) ribs, although it is possible to retain the barrel arch when this is desirable. On very long spans hollow ribs of box-girder construction have been used, but for spans up to 200 ft the solid ribs are probably more economical. In the open-spandrel type the road is supported on beams and girders and this load is transferred to the arch ring by means of the columns or hangers. In this type, too, there is some uncertainty regarding the interaction between the superstructure and the arch ring unless definite steps are taken to provide some articulation. The effect of superstructure is discussed at greater length in a later paragraph.

SHAPE OF THE ARCH RING

The selection of the form or curvature of the arch axis is one of the most important considerations in the design of arches. Usually certain assumptions regarding its shape are made in the preliminary design but the curvature of the arch axis is later made to conform as closely as is practicable to the string polygon for the dead load or dead load plus half live load. The shape or alignment of the arch ring depends
upon conditions at the site like high and low water, foundation conditions, and elevation of the roadway. When none of these restrictions is a governing factor a rise ratio of 1:4 will prove economical for open-spandrel arches.

In construction the form of the arch axis is of course governed by the shape of the intrados and extrados. If the intrados is a segment of a circle it is called a segmental arch. Usually the intrados curvature is not circular but is either parabolic, elliptical, or multicentered. In the multicentered arch the intrados is composed of several circular segments of different radii all tangent to one another. Figure 16-31 illustrates the various types. Where the curvatures are complete semicircles or semi-ellipses the arches are full centered and the skewbacks are horizontal planes.

**FINAL SHAPE OF THE ARCH AXIS**

When the final size of the arch ring is determined the true dead weights will be known. A string polygon may then be passed through the springing and crown, using only the dead load or dead load plus one-half of the equivalent uniform live load, whichever is desired. This assumes that the moments here are zero or that the arch is of the three-hinged type. The arch axis is made to correspond to this polygon. The problem is simply one of graphical statics wherein a string polygon for a series of parallel loads is passed through two points. In order to accomplish this the load line $AB$ should first be laid off (Fig. 16-32) and any convenient pole $O'$ displaced horizontally from $A$ selected. The rays of the force polygon are then drawn from $O'$ and the equilibrium polygon drawn beginning at the skewback $S$. This string polygon
will probably not pass through the crown C but the intersection of the first and last strings (parallel to \(O'A\) and \(O'B\) respectively) indicates the position of the resultant \(R\). Obviously the two terminal rays of any other string polygon must also intersect on the resultant. Hence a horizontal line drawn through \(C\) to \(D\) and the new ray \(SD\) will locate the new pole \(O\) if the rays are drawn so that \(OB \parallel SD\) and \(OA \parallel CD\). The new string polygon is now drawn, passing through \(S\) and \(C\).

The thickness at the crown, haunch, and skewback can now be drawn to scale and suitable curves for the intrados and extrados sketched in. Their centers may be determined best by trial.

**PRELIMINARY DESIGN**

**Estimating the Thickness of the Arch Ring**

Various empirical equations have been developed for determining trial dimensions of the arch ring at the crown for barrel arches. Most of them are not entirely satisfactory for they are applicable over only narrow ranges of live load and unit stress, and none of them is useful for ribbed arches. They do, however, furnish estimates based on some reason but were never intended to yield final dimensions. Some of the more common formulas are:

The F. F. Weld formula:

\[
h = \sqrt{L} + \frac{L}{10} + \frac{w}{200} + \frac{w'}{400}
\]

where \(h\) = crown thickness, in.
\(L\) = clear span, ft
\(w\) = live load, psf
\(w'\) = dead load, psf above the arch at the crown

The W. J. Douglas formula:

<table>
<thead>
<tr>
<th>Span</th>
<th>Crown thickness, ft</th>
<th>Increase in thickness for railroad loading, %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Under 20</td>
<td>0.03(6 + L)</td>
<td>25</td>
</tr>
<tr>
<td>20-50</td>
<td>0.015(30 + L)</td>
<td>25</td>
</tr>
<tr>
<td>50-150</td>
<td>0.0001(11,000 + Lt)</td>
<td>20</td>
</tr>
<tr>
<td>Over 150</td>
<td>0.016(75 + L)</td>
<td>15</td>
</tr>
</tbody>
</table>

Estimates based on judgment and on past experience are, in general, better guides than formulas. Table 16-6 lists various sizes covering a series of spans for both filled- and open-spandrel arches. These were taken from several hundred existing bridges. The thickness of the arch at the skewback is generally from 1.5 to 2.5 times the crown thickness.

<table>
<thead>
<tr>
<th>Span, ft</th>
<th>Type</th>
<th>Crown thickness</th>
<th>Skewback thickness, ft</th>
<th>Width of arch ring</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-50</td>
<td>Filled</td>
<td>8-12 in.</td>
<td>1.5-2.5</td>
<td>Bridge</td>
</tr>
<tr>
<td>50-75</td>
<td>Filled</td>
<td>1-1.5 ft</td>
<td>2-3</td>
<td>Bridge</td>
</tr>
<tr>
<td>75-100</td>
<td>Filled</td>
<td>1.25-1.5 ft</td>
<td>2.5-3.5</td>
<td>Bridge</td>
</tr>
<tr>
<td>100-150</td>
<td>Filled</td>
<td>1.5-2 ft</td>
<td>3-4.5</td>
<td>Bridge</td>
</tr>
<tr>
<td>50-75</td>
<td>Open</td>
<td>1.5-2.5 ft</td>
<td>2.5-4.5</td>
<td>2-3.5 ft</td>
</tr>
<tr>
<td>75-100</td>
<td>Open</td>
<td>1.5-2.5 ft</td>
<td>3-4.5</td>
<td>2.5-5 ft</td>
</tr>
<tr>
<td>100-125</td>
<td>Open</td>
<td>2-3 ft</td>
<td>3.5-5</td>
<td>2.5-6 ft</td>
</tr>
</tbody>
</table>
The data of Table 16-6 are useful mainly for an estimate of the dead weight of the arch ring.

The variation in thickness of the arch ring along the arch axis is governed more by appearance than by stress conditions. Usually an arch which presents a pleasing appearance is from 5 to 15 per cent thicker at the haunch (quarter point) than at the crown, these values corresponding to arches with thickness ratios at springing to crown of from 1.5 to 2.5, respectively. The variation in thickness at other sections should be such as will produce a gradual transition. Actually such proportioning will usually keep the section at the quarter point understressed but the extra thickness is slight and is required for aesthetic reasons.

**DESIGN CHARTS AND TABLES**

The foregoing remarks enable a designer to estimate the trial dimensions of an arch ring approximately. The next step in the design is to analyze the arch ring and determine whether any overstress exists. This may, of course, be accomplished by use of the elastic theory. One such method will be described later when an arch rib is analyzed by the column analogy. In most cases, however, it is wise first to analyze the trial arch by means of design charts and to make an exact final elastic analysis only after some further refinement in the design and mainly to determine the final amount of reinforcing steel.

All design charts for arches are based on certain assumed shapes for the neutral axis of the arch ring, and upon established variations in thickness along the arch. If a given arch fits the shape and form of the arches on which the charts are based there would be no need of any final analysis by the elastic theory. However, when the fit is approximate only, that is, when the string polygon for dead load (or dead load plus one-half live load) fits the arch axis but does not coincide well with the theoretical shape and form, results obtained from charts will be only approximate and a final analysis is recommended.

All design charts are useful, even when they may not fit specific applications, because they enable an engineer to study the economical aspects of a design more fully by enabling him to analyze a whole series of structures in a short time. To fulfill this purpose, they should cover a wide enough range to be practical. Such a series of charts was developed by Charles S. Whitney, consulting engineer of Milwaukee, Wis. They include influence lines for thrust and moment, coefficients for maximum live-load moment and the accompanying thrust, and a few tables and equations for a series of arches of certain shape and forms. The notation for the equations is as follows:

- \( x, y_0 \) = coordinates of the arch axis with the origin at the crown
- \( w_e, w_c \) = dead load per foot at the springing and crown, respectively
- \( r \) = rise
- \( L \) = span of neutral axis
- \( d \) = depth of the arch rib
- \( I_x, I_y \) = moment of inertia of the cross-sectional area of the arch rib at the crown and springing, respectively
- \( N \) = shape factor = ratio of \( y_0 \) at \( \frac{1}{4} \) point to \( r \)
- \( m \) = form factor = \( I_x/I_y \cos \phi \)
- \( \phi \) = angle between the horizontal and neutral axis at any point

\[
A = \text{cross-sectional area of the rib} \quad A_m = \frac{L}{\int_L^R \cos \phi \, dx} \quad A'_m = \frac{L}{\int_L^R \frac{dx}{A \cos \phi}}
\]

\[
k = \cosh^{-1} \frac{w_c}{w_e} = \ln \left[ \frac{w_c}{w_e} + \sqrt{\left(\frac{w_c}{w_e}\right)^2 - 1} \right]
\]

\[
t = \text{temperature change, degrees}
\]
\[ \alpha = \text{thermal coefficient of expansion} \]
\[ y_e = \text{vertical distance from the crown to the centroid of the elastic weight} \]
\[ I_y = \text{moment of inertia of the elastic weight about } X \text{ axis through the centroid of the elastic weight} \]
\[ H = \text{horizontal thrust} \]
\[ V = \text{vertical shear} \]
\[ T = \text{axial thrust} = \sqrt{H^2 + V^2} \]
\[ d, s, t, RS = \text{subscripts meaning dead load, at the springing, temperature, and rib shortening, respectively} \]

The shape of the arch axis is assumed to follow the pressure line of the dead loads shown in Fig. 16-33 and is given by the equation

\[ y_0 = \frac{r}{w_e/w_e} \left( \cosh \frac{2xk}{L} - 1 \right) \]  \hspace{1cm} (16-1)

The thickness along this ring is assumed to vary as follows:

\[ d_\phi = \frac{d_e}{\left\{ 1 - (1 - m) \frac{2x}{L} \cos \phi \right\}^\frac{1}{2}} = \frac{d_\phi (1 + \tan^2 \phi)^1_6}{\left[ 1 - (1 - m) \frac{2x}{L} \right]^\frac{1}{2}} = d_\phi (1 + \tan^2 \phi)^1_6 \]  \hspace{1cm} (16-2)

Solution of these two equations would be rather tedious. However, Eq. (16-1) is worked out completely in Table 16-8, which lists values of \( y_0/r \) at 10 points along the arch axis for a series of shape factors \( N \). Also, Tables 16-7, 16-9, and 16-10 list values of \( \tan \phi \) and \( \tan^2 \phi \) in terms of \( L/r \) and also of the coefficient \( c \) so that a solution of Eq. (16-2) is greatly facilitated.

Although Eq. (16-1) is based upon a ratio of dead load distributed at the springing and at the crown, it is possible to include part of the live load also when it seems desirable to do so. AASHO Specifications require the axis to fit the polygon for dead load only, or for dead load plus one-half live load, whichever produces the smallest bending stresses under combined loads. Hence half the live load \( w \) must be added to \( w_e \) and \( w_c \), and a ratio \((w_e + w/2)/(w_e + w/2)\) used instead of \( w_e/w_c \) if a suitable shape factor \( N \) is to be selected (Table 16-7) under such circumstances.

Coefficients for the \( H_d \) and \( V_d \) components of the dead-load thrust at the springing are listed in Table 16-7. There will be no moment in the arch ring due to this loading because the arch axis coincides with the pressure line of the load. When part of the live load is included with the dead load (like \( w_e + w/2 \)) the thrusts of Table 16-7 are those for this total loading. However, the dead-load effect may be separated from the total by subtracting the thrust due to the live load \( w/2 \). The live-load thrust may be obtained from Chart 16-XX or 16-XXI by adding the horizontal thrust accompanying maximum positive and maximum negative moment either at the crown or at the springing. Since for maximum \(+M_e\) the central one-fourth of the arch is loaded, whereas for maximum \(-M_e\) the outer three-eighths is loaded, the sum of the horizontal thrusts represents \( H \) for a uniform load distributed across the
### Table 16-7. Values of $N$, $w_s/w_e$, $H_d$, $V_d$, $\tan \phi_s$

<table>
<thead>
<tr>
<th>$N$</th>
<th>$\sigma = \frac{w_s}{w_e}$</th>
<th>$C_d = \frac{rH_d}{w_eL_e}$</th>
<th>$\frac{V_d}{w_eL_e}$</th>
<th>$\frac{L}{r \tan \phi_s}$</th>
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### Table 16-8. Arch Axis Coordinates

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### Table 16-9. Determination of Rib Thickness

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### Table 16-10. Determination of Rib Thickness

\( \begin{align*}
\text{Values of } c = \frac{1}{\sqrt{1 - (1 - m) \frac{2\pi}{L}}} \text{ for point}
\end{align*} \)

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<td>1.000</td>
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<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
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<td>0.919</td>
<td>0.855</td>
<td>0.784</td>
<td>0.706</td>
<td>0.625</td>
<td>0.538</td>
<td>0.447</td>
<td>0.351</td>
<td>0.256</td>
</tr>
<tr>
<td>( z = 0.1 )</td>
<td>0.976</td>
<td>0.953</td>
<td>0.907</td>
<td>0.830</td>
<td>0.748</td>
<td>0.665</td>
<td>0.579</td>
<td>0.489</td>
<td>0.393</td>
<td>0.292</td>
<td>0.189</td>
</tr>
<tr>
<td>( z = 0 )</td>
<td>0.972</td>
<td>0.947</td>
<td>0.896</td>
<td>0.815</td>
<td>0.722</td>
<td>0.635</td>
<td>0.545</td>
<td>0.453</td>
<td>0.354</td>
<td>0.250</td>
<td>0.143</td>
</tr>
</tbody>
</table>

The moments at the springing and crown for a load of \( w/2 \) can also be found from Charts 16-XX and 16-XXI in a like manner. The moments due to the dead load will be equal but opposite in sign to these since the moment due to \( (w_x + w/2) \) is zero. Stating this case of superposition another way, we have

\[
H \text{ due to } (w_x + w/2) = H \text{ due to } \frac{w}{2} + H \text{ due to } w_d
\]

Similar relationships holds for \( M \) and \( V \). Knowing two of the terms in the equation, the third is readily computed.

**Influence Lines**

The influence lines for horizontal thrust and for moment at the crown, quarter point, and springing, as shown in Charts 16-XIV to 16-XIX, inclusive, are taken from...
Mr. Whitney's paper. Since ordinates to the influence lines represent the coefficients for unit concentrated loads of thrust or of moment at some point along the axis, they are most useful for open-spandrel arches. However, the areas under the influence lines represent the thrust (or moment) due to unit distributed loads. It should be noted that the influence lines for moment cross the zero axis at approximately the three-eighths point of the span for arches of all shapes and form. Hence the maximum live-load moments at the crown, quarter point, and springing

**Chart 16-XIV.** Influence lines for horizontal thrust and for moment at crown, \( m = 0.15 \).

**Chart 16-XV.** Influence lines for horizontal thrust and for moment at crown, \( m = 0.25 \).

are determined by loading the arch with the six arrangements of live loading as designated in Fig. 16-34.

There is no influence line for the vertical component of the reaction \( V \). However, with \( H \), \( M_n \), and \( M_s \) known, the unknown \( V \) can be found by applying the laws of statics to a free-body diagram of the arch as is indicated in Fig. 16-35.

**Charts for Maximum Moment Due to Uniform Live Load**

Although maximum moments at the crown and springing can be obtained for a uniformly distributed unit live load by measuring the area under the influence diagrams, direct-reading charts indicating coefficients for these reactions are more convenient. Charts 16-XX and 16-XXI include such coefficients for moment resulting from the six arrangements of live load indicated in Fig. 16-34, together with the accompanying horizontal thrusts. Again they represent the work of Mr. Whitney but in a somewhat more consolidated form, the simplification being a contribution of Professors Cross and Morgan.

**Effects of Temperature, Rib Shortening, and Shrinkage**

Variations in temperature produce distortions in arch rings. The horizontal thrust (Fig. 16-36) is given by

\[
H_t = t \alpha \frac{L}{I_y}
\]

(16-3)
This should be multiplied by $1/(1 + u)$ for extremely flat arches, where
\[
u = \frac{L}{EA_m I_v}\]
The moment at any point $x$ along the ring due to the thrust $H$ is
\[
M = H_i(y_v - y_c)
\]
(16-4)
Coefficients $y_v/r$ and $I_v$ are plotted in Chart 16-XXII. $A_m$ can be found by multiplying the cross-sectional area of the arch rib at the crown by the coefficient plotted

![Chart 16-XVI. Influence lines for horizontal thrust and for moment at crown, $m = 0.40$.](chart16-xvi)

![Chart 16-XVII. Influence lines for moment at springing line and at quarter point, $m = 0.15$.](chart16-xvii)

in Chart 16-XXIII. The data in the charts are also from Mr. Whitney's paper, rearranged slightly by the author.

Distortion, in the form of a shortening of the arch axis, also accompanies any loading of the arch. This shortening is the result of the compressive thrust along the arch axis. That portion due to dead load accompanies decentering of the arch during construction and is permanent, whereas that due to live load is of a more intermittent nature. The constant compressive stress in the arch due to dead load produces plastic flow in the concrete which, in turn, increases the dead-load rib shortening. The plastic flow, however, also provides enough relief from the stress so that concrete stresses are no greater than if the concrete remained elastic.\(^8\) Hence thrust and moment due to dead-load rib shortening can be calculated as though the concrete remains elastic. Plastic flow does not affect live-load rib shortening.
The horizontal thrust due to dead-load (Fig. 16-36) rib shortening is given by

\[ H_{R.S.} = -H_d \frac{u'}{1 + u} \] (16-5)

where \( u' = \frac{L}{EA'_mI_y} \) and \( A'_m \) is taken from Chart 16-XXIII as for \( A_m \). The effect

\[ H_{R.S.} = -H_d u' \] (16-6)

For both dead-load and live-load rib shortening the moment is, as for temperature,

\[ M_{R.S.} = H_{R.S.} (y_0 - y_c) \] (16-7)

Note that for distortion due to temperature and rib shortening the \( H \) of Eqs. (16-3), (16-5), and (16-6) acts through the elastic centroid (Fig. 16-36).

Rib shortening due to shrinkage is most easily treated as an equivalent drop in temperature of 15° or replacing \( t_o \) in Eq. (16-3) with some shrinkage factor such as 0.0002.
Elastic Weight

The elastic weight for arches of the shape and form expressed by Eqs. (16-1) and (16-2) is

\[ \int \frac{ds}{EI} = \frac{L}{EI_z} \times \frac{1 + \frac{m}{2}}{2} \]

Also

\[ I_z = \int \frac{\partial^2 ds}{EI} = \frac{L^2}{EI_z} \times \frac{1 + 3m}{48} \]

**Chart 16-XX.** Maximum moment at crown and value of thrust with maximum \( M_c \) for uniform load.

**Chart 16-XXI.** Maximum moment at springing line and value of thrust with maximum \( M_s \) for uniform load.

**ILLUSTRATED EXAMPLE**

A highway bridge is to be built over a stream, the profile of which is shown in Fig. 16-37. Such factors as bedrock foundation and high elevation of the roadway above the stream favor an arch bridge. For the 100-ft span it is probable that a two-ribbed open-spandrel arch will prove most economical. The preliminary layout of the arch in Fig. 16-37 indicates a rise of 30 ft and a selection of 10 spandrel openings. The roadway will be taken as 24 ft wide, and an H-15-44 loading used. Other design specifications will be as follows: \( E_c = 3,000,000 \) psi for the concrete in the superstructure; \( E_c = 3,750,000 \) psi for the concrete in the ribs; the coefficient of expansion \( = 0.000006 \); the temperature variation \( = \pm 40^\circ F \); and the shrinkage equivalent to a \( 15^F \) drop in temperature, bond \( u = 150 \) psi; shearing stress \( v = 180 \) psi for anchored bars; \( f_s = 1,000 \) psi; and \( f_s = 18,000 \) psi for stresses due to dead load, live
**Chart 16-XXII.** Coefficients \( y_c/r \) and \( I_s \) for values of \( N \) and \( m \).

**Chart 16-XXIII.** Ratios of \( A_m/A \) and \( A_m'/A \) for values of \( r/L \) and \( m \).

**Chart 16-XXIV.** Values of shear at springing line for partial uniform load.
Bending and direct stress—steel in tension face only. Tension over part of section. Based on $n = 15$

**Chart 16-XXVb.** Percentage of steel reinforcement for section under direct stress and bending, $n = 15$. (Courtesy of Hayden.)
load, and impact but these allowable stresses may be increased 25 per cent when the effects of shrinkage, temperature, rib shortening, etc., are added.

**Fig. 16-34.** Maximum positive and negative moments due to partial uniform load at crown, \( \frac{1}{4} \) point, and springing line.

**Fig. 16-35.** Applying statics to free-body diagram to find \( V \).

**Fig. 16-36.** Horizontal thrust due to temperature.

**Fig. 16-37.** Two-ribbed open-spandrel arch.

**Preliminary Design, Superstructure**

**Road Slab.** Estimate the floor beam to be 15 in. wide. Then the span of the floor = 10 - 1.25 = 8.75 ft. The wheel-load distribution (see Specifications of AASHO, 1953 edition) = \( E = 0.175S + 3.2 = 0.175 \times 8.75 + 3.2 = 4.73 \) ft.

Impact factor \( I = \frac{50}{S + 125} = 0.373 \) Use 0.30, max required

H-15 wheel load \( P = 12,000 \times 1.3 = 15,600 \) lb
REINFORCED-CONCRETE ARCH BRIDGES

Live load

\[ M = \pm 0.2 \frac{P}{E} S = 0.2 \times \frac{15,600}{4.73} \times 8.75 = 5,770 \text{ ft-lb per 1-ft width} \]

Dead load, 10-in. slab \(- M = \frac{wL^2}{12} \)

\[ = \frac{1}{4} \times \frac{1}{12} \times 150 \times 8.75^2 = 798 \text{ ft-lb} \]

With \( d = 10 - 1\frac{3}{4} = 8\frac{3}{4} \) in.

\[ A_s = \frac{(5,770 + 798)12}{18,000 \times 0.875 \times 8.62} = 0.581 \text{ sq in.} \]

For 100 per cent overload

\[ A_s = \frac{(2 \times 5,770 + 798)12}{27,000 \times 0.875 \times 8.62} = 0.728 \text{ sq in.} \]

Use \( \frac{3}{4} \) in. \( \phi \) bars at 7 in. on centers.

Then

\[ f_c = \frac{6,570 \times 12 \times 2}{\frac{3}{8} \times 8.62 \times 12 \times \frac{3}{8} \times 8.62} = 540 \text{ psi O.K.} \]

Floor Beam. Assume a section through the bridge as shown in Fig. 16-38.

![Cross section of bridge showing deck, floor beam, column, and rib](image)

Fig. 16-38. Cross section of bridge showing deck, floor beam, column, and rib

Dead load slab \( 1\frac{1}{4} \times 10 \times 150 = 1,250 \text{ lb/ft} \)

Beam \( \frac{15 \times 28}{144} \times 150 = 438 \text{ lb/ft} \)

Assume 21 ft 0 in. simple span.

Then the end shear \( V = 10.5 \times 1.688 + 31.2 = 48.92 \text{ kips} \)

and

\[ V = \frac{48,920}{15 \times 33.8 \times 0.9} = 107 \text{ psi O.K. in shear} \]

\[ M = 1.688 \times 21\frac{1}{8} + 15.6(2 \times 10.6 - 2 - 8) = 93.0 + 174.7 \]

\[ = 267.7 \text{ ft-kips} \]

\[ A_s = \frac{267.7 \times 12}{18 \times 0.9 \times 33.8} = 5.86 \text{ sq in.} \]
ILLUSTRATED EXAMPLE

For

$$v = \frac{80,120}{15 \times 33.8 \times 0.9} = 176 \text{ psi}$$

Allowable $$v = 180 \times 1.5 = 270 \text{ psi}$$ O.K.

$$A_s = \frac{442.4 \times 12}{27 \times 0.9 \times 33.8} = 6.47 \text{ sq in.}$$

Use 5 - 1¼ in. square bars (Fig. 16-39).

Floor beam

Fig. 16-39. Reinforcing steel in floor beam.

Loads on Columns

Road slab $1\frac{1}{2}_2 \times 13.5 \times 10 \times 150 = 16,880 \text{ lb}$
Curb $1\frac{1}{2}_2 \times 1.5 \times 10 \times 150 = 2,060$
Arch spandrel 1,200
Beam $1.25 \times 2.33 \times 11.5 \times 150 = 5,030$
Bracket 230
Bailing $156 \text{ lb/ft} \times 10 = 1,560$

$26,960 \text{ lb}$

Size of Columns. For shape of the arch axis assume $N = 0.23$, length of column $y_o$, width $= 18.45/12 = 1.5 \text{ ft}$, and depth $= 2 \text{ ft}$.

<table>
<thead>
<tr>
<th></th>
<th>Springing</th>
<th>Point 2</th>
<th>Point 4</th>
<th>Point 6</th>
<th>Point 8</th>
<th>Crown</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y_o/r$</td>
<td>1.000</td>
<td>0.6151</td>
<td>0.3353</td>
<td>0.1458</td>
<td>0.0360</td>
<td>0</td>
</tr>
<tr>
<td>$y_o \text{ ft}$</td>
<td>30.00</td>
<td>18.45</td>
<td>10.05</td>
<td>4.37</td>
<td>1.08</td>
<td>0</td>
</tr>
<tr>
<td>Wt of col., lb</td>
<td>8,300</td>
<td>4,520</td>
<td>1,970</td>
<td>486</td>
<td>26,960</td>
<td>0</td>
</tr>
<tr>
<td>Load on col., lb</td>
<td>26,960</td>
<td>26,960</td>
<td>26,960</td>
<td>26,960</td>
<td>26,960</td>
<td>26,960</td>
</tr>
<tr>
<td>Total on arch, lb</td>
<td>33,260</td>
<td>31,480</td>
<td>28,930</td>
<td>27,450</td>
<td>26,960</td>
<td>26,960</td>
</tr>
</tbody>
</table>

Check on the Shape (Value of $N$)

Assume the arch is 2 ft thick by 4 ft wide at the crown and 4 by 4 ft at the springing. Then from Table 16-7, $(L/r) \tan \phi_s = 4.442$ for $N = 0.23$. Hence

$$\phi_s = \tan^{-1} \left( 4.442 \times \frac{30}{100} \right) = 53^\circ 07'$$

Then $\frac{w_s}{w_c} = \frac{4 \times 4 \times 150}{\cos 53^\circ} \times \frac{1}{2 \times 4 \times 150} = 3.33$. The reactions at the springing due to the weight of the arch rib can then be obtained from Table 16-7 for $w_s/w_c = 3.33$.

$$H_s = 0.1661 \times 2 \times 4 \times 150 \times \frac{100^2}{30} = 66,440 \text{ lb}$$

$$V_s = 0.8481 \times 1,200 \times 100 = 101,900 \text{ lb}$$
The springing reactions for the column loads are:

\[ V_s = 35.26 + 31.48 + 28.93 + 27.45 + 13.48 = 136.60 \text{ kips} \]

\[ H_s = (35.26 \times 10 + 31.48 \times 20 + 28.93 \times 30 + 27.45 \times 40 + 13.48 \times 50)/30 = 3,622.1/30 = 120.74 \text{ kips} \]

Then

\[ \phi_s = \tan^{-1} \frac{136.6 + 101.9}{120.74 + 66.44} = 51^\circ 51' \]

and

\[ m = \frac{I_s}{I_s \cos \phi_s} = \frac{2^2}{4 \times 0.618} = 0.202 \]

For a check or the true value of \( N \) the weight of the several arch sections must be found. The lengths of these segments are shown in Fig. 16-40. Then, from Tables 16-9 and 16-10:

<table>
<thead>
<tr>
<th>Point</th>
<th>( \frac{L_s}{r_2} \tan^2 \phi )</th>
<th>( 1 + \tan^2 \phi )</th>
<th>( (1 + \tan^2 \phi)^{\frac{1}{2}} )</th>
<th>( e )</th>
<th>( d_a/d_e )</th>
<th>( d_a )</th>
<th>Wt. arch segment, kips</th>
</tr>
</thead>
<tbody>
<tr>
<td>Springing</td>
<td>19.722</td>
<td>2.773</td>
<td>1.185</td>
<td>1.705</td>
<td>2.02</td>
<td>4.04</td>
<td>33.00</td>
</tr>
<tr>
<td>2</td>
<td>10.852</td>
<td>1.976</td>
<td>1.120</td>
<td>1.403</td>
<td>1.57</td>
<td>3.15</td>
<td>22.80</td>
</tr>
<tr>
<td>4</td>
<td>5.403</td>
<td>1.486</td>
<td>1.168</td>
<td>1.243</td>
<td>1.33</td>
<td>2.66</td>
<td>9.09</td>
</tr>
<tr>
<td>5</td>
<td>3.578</td>
<td>1.322</td>
<td>1.104</td>
<td>1.185</td>
<td>1.24</td>
<td>2.48</td>
<td>8.10</td>
</tr>
<tr>
<td>6</td>
<td>2.200</td>
<td>1.198</td>
<td>1.031</td>
<td>1.137</td>
<td>1.17</td>
<td>2.34</td>
<td>14.13</td>
</tr>
<tr>
<td>8</td>
<td>0.523</td>
<td>1.047</td>
<td>1.008</td>
<td>1.060</td>
<td>1.07</td>
<td>2.14</td>
<td>12.50</td>
</tr>
<tr>
<td>Crown</td>
<td>0</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.00</td>
<td>2.00</td>
<td>99.65</td>
</tr>
<tr>
<td>Total</td>
<td>99.65</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Then, for the arch ring,

\[ \Sigma M_x = 12.50 \times 45 + 14.16 \times 35 + 17.19 \times 25 + 22.8 \times 15 + 33.0 \times 5 + 30H = 0 \]

\[ H = \frac{1,995}{30} = 66.47 \text{ kips} \]

Similarly from Fig. 16-41

<table>
<thead>
<tr>
<th>( P ), kips</th>
<th>( x )</th>
<th>( M_x ), ft-kips</th>
</tr>
</thead>
<tbody>
<tr>
<td>13.48</td>
<td>25</td>
<td>337</td>
</tr>
<tr>
<td>12.50</td>
<td>20</td>
<td>250</td>
</tr>
<tr>
<td>27.45</td>
<td>15</td>
<td>412</td>
</tr>
<tr>
<td>14.16</td>
<td>10</td>
<td>142</td>
</tr>
<tr>
<td>28.93</td>
<td>5</td>
<td>145</td>
</tr>
<tr>
<td>8.10</td>
<td>2.5</td>
<td>20</td>
</tr>
<tr>
<td>104.62</td>
<td></td>
<td>1306</td>
</tr>
</tbody>
</table>

\[ y_0 = \frac{1,306}{(120.74 + 66.47)} = 6.97 \text{ ft.} \]

Hence \( N = 0.232 \), which is a close check if the value of \( m \) checks out also. Now

\[ \phi_x = \frac{\tan^{-1} (136.6 + 99.65)}{(120.74 + 66.47)} = 51^\circ 36' \]

Hence \( m = 1/(2.02)^2 \times 0.621 = 0.196 \), which is also a good check.

**Live Load**

AASHO Specifications require the use of either wheel loading or lane loading for type H-15, whichever produces the maximum. Try lane loading first.

\[ \text{Impact} = \frac{50}{100 + 125} = 0.222 \]

Uniform load = \( 480 \times 1.222 = 587 \text{ lb/ft of lane} \)

Concentrated load (for max moment) \( P = 13,500 \times 1.222 = 16,500 \text{ lb} \)

At the crown, for max \( +M \), uniform load, \( N = 0.232 \), \( m = 0.196 \):

\[ M_e = 0.0045 \times 0.587 \times 100^2 = +26.4 \text{ ft-kips} \]

\[ H = 0.0632 \times 0.587 \times 100^2/30 = 12.35 \text{ kips} \]

\[ V_5 = 0.127 \times 0.587 \times 100 = 7.45 \text{ kips} \]

Charts 16-XIV and 16-XV. Load center \( 0.254L \) of span
REINFORCED-CONCRETE ARCH BRIDGES

For max $-M$, uniform load:

$M_s = -0.0038 \times 5,870 = -22.3$ ft-kips
$H = 0.064 \times 5,870/30 = 12.5$ kips
$V_s = 0.373 \times 0.587 \times 100 = 21.9$ kips

At the springing, for max $+M$, uniform load:

$M_s = 0.0254 \times 5,870 = +149$ ft-kips
$H = 0.0913 \times 5,870/30 = 17.9$ kips
$V_s = (0.627 - 0.465) \times 0.587 \times 100 = 9.50$ kips

Charts 16-XXI
Chart 16-XXI
Chart 16-XXIV.
Load right
$0.627L$ of span

For max $-M$, uniform load:

$M_s = -0.0215 \times 5,870 = -126$ ft-kips
$H = 0.0370 \times 5,870/30 = 7.25$ kips
$V_s = 0.338 \times 0.587 \times 100 = 19.8$ kips

Load left $0.373L$ of span

At the crown, for max $+M$, concentrated lane load:

$P$ at $0.5L$ $M_s = 0.0414 \times 100 \times 16.5 = +68.3$ ft-kips

$P$ at $0.5L$ $H = 0.261 \times 1,650/30 = 14.37$ kips
$P$ at $0.5L$ $V_s = 0.5 \times 16.50 = 8.25$ kips

Charts 16-XIV and 16-XV

For max $-M$, concentrated lane load:

$P$ at $0.26L$ and $0.74L$ $M_s = -0.0003 \times 1,650 \times 2 = -30.7$ ft-kips
$H = 0.1333 \times 2 \times 1,650/30 = 14.66$ kips
$V_s = 16.5$ kips

At the springing, for max $+M$, concentrated lane load (Fig. 16-42):

$P$ at $0.61L$ $M_s = 0.075 \times 1,650 = +124$ ft-kips
$H = 0.229 \times 1,650/30 = 12.6$ kips

Fig. 16-42. Concentrated lane load placement for maximum positive moment at the springing line.

At the right springing with $P$ at $0.61L(-0.39L)$, $M = 0 \times 1,650 = 0$:

$V_s$ at left end $= (39 \times 16.5 - 124) / 100 = 5.19$ kips

Similarly, for max $-M$, concentrated lane load, $P$ at $0.18L$:

$M_s = -140$ ft-kips $H = 3.82$ kips $V_s$ at left $= 15.5$ kips

$H$-15 Wheel Load, Truck Train (Fig. 16-43)

$P = 1.222 \times 0.2 \times 15 \times 2,000 = 7,330$ lb (20 per cent of truck on front wheels)

Fig. 16-43. $H$-15 wheel loads, truck train.
Crown Wheel, at Point

\[ +M_e = (0.0414 \times 4 - 0.0024 \times 1)733 = 120 \text{ ft-kips} \]

Charts 16-XIV and 16-XV

\[ H = (0.261 \times 4 + 0.21 \times 1)7.33 \times 10^{-\%} = 30.6 \text{ kips} \]

Charts 16-XIV and 16-XV

\[ -M_e = (-0.0092 \times 4 - 0.0042 \times 1)733 \times 2 = -60.1 \text{ ft-kips} \]

\[ H = (0.142 \times 4 + 0.037 \times 1)7.33 \times 10^{-\%} \times 2 = 29.5 \text{ kips} \]

Springing (Fig. 16-44) Wheel, at Point

\[ -M_e = (-0.082 \times 4 - 0.056 \times 1)733 = -282 \text{ ft-kips} \]

Charts 16-XVII and 16-XVIII

\[ H = (0.092 \times 4 + 0.011 \times 1)7.33 \times 10^{-\%} = 9.25 \text{ kips} \]

Charts 16-XIV and 16-XV

\[ M_R = (0.0425 \times 4 + 0.006 \times 1)733 = +129 \text{ ft-kips} \]

\[ V_S = (0.93 \times 1 + 0.79 \times 4)7.33 + \frac{282 + 129}{100} = 34.0 \text{ kips} \]

Fig. 16-44. Truck placement for maximum negative moment at the springing line.

Similarly, with \( P \) at 0.51\( L \), 4\( P \) at 0.65\( L \), \( \frac{3}{4} \)\( P \) at 0.95\( L \):

\[ +M_e = (0.058 \times 1 + 0.074 \times 4 + 0.004 \times \frac{3}{4})733 = +262 \text{ ft-kips} \]

\[ H = (0.261 \times 1 + 0.204 \times 4 + 0.006 \times \frac{3}{4})7.33 \times 10^{-\%} = 26.4 \text{ kips} \]

\[ M_R = (-0.043 \times \frac{3}{4} - 0.024 \times 4 + 0.050 \times 1)733 = -57.2 \text{ ft-kips} \]

\[ V_S = (0.49 \times 1 + 0.35 \times 4 + 0.05 \times \frac{3}{4})7.33 - \frac{262 + 57}{100} = 10.9 \text{ kips} \]

Temperature

Assume 0.75 per cent steel at each face of the arch ring with 2-in. covering of concrete.

<table>
<thead>
<tr>
<th></th>
<th>( A_s )</th>
<th>( (8 - 1)A_s )</th>
<th>( A )</th>
<th>( I_s )</th>
<th>( I_s )</th>
<th>( I )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Crown</td>
<td>8.00</td>
<td>0.84</td>
<td>8.84</td>
<td>2.67</td>
<td>0.53</td>
<td>3.20</td>
</tr>
<tr>
<td>Springing</td>
<td>16.16</td>
<td>0.84</td>
<td>17.00</td>
<td>21.95</td>
<td>2.76</td>
<td>24.7</td>
</tr>
</tbody>
</table>

Then, from Chart 16-XXII,

\[ I_y = 0.0352 \times Lr^3 / I_{crown} = 0.0352 \times 100 \times 30^3 / 3.20 = 990 \]

and for \( \pm 40^\circ \text{F} \):

\[ H = \frac{Etail}{I_y} = \frac{3,750,000 \times 144 \times 40 \times 0.000006 \times 100}{990 \times 1,000} = 13.1 \text{ kips} \]
REINFORCED-CONCRETE ARCH BRIDGES

Note: Recommended values of $E_s$ vary from 3,000,000 to 4,000,000 psi for the purpose of computing reactions, but values of one-third to one-fourth of this amount should be used to calculate the dead-load deflection.

From Chart 16-XXII, $y_e/r = 0.2106$, $y_e = 6.32$ ft.

$$M_e = \pm 13.1 \times 6.32 = \pm 82.7 \text{ ft-kips}$$
$$M_s = \pm 13.1(30 - 6.32) = \pm 310.3 \text{ ft-kips}$$

Shrinkage

Assume equivalent to 15°F drop. Do not include with temperature rise because of the possibility that shrinkage may be offset by moisture conditions.

Rib Shortening

$$u = \frac{L}{A_m I_y} = \frac{100}{8.84 \times 1.437 \times 990} = 0.00795$$

$u' = \frac{L}{A_m I_y} = \frac{100}{8.84 \times 1.085 \times 990} = 0.01054$

$H_{R.S.} = H \frac{0.01054}{1.008} = 0.01046H$

Summary

<table>
<thead>
<tr>
<th>Loading</th>
<th>Crown</th>
<th></th>
<th>Springing</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$Max + M$</td>
<td>$Max - M$</td>
<td>$Max + M$</td>
</tr>
<tr>
<td>$M_e$</td>
<td>$H$</td>
<td>$M_e$</td>
<td>$H$</td>
</tr>
<tr>
<td>Dead load</td>
<td>0</td>
<td>187.2</td>
<td>0</td>
</tr>
<tr>
<td>Lane live load uniform</td>
<td>+26.4</td>
<td>12.3</td>
<td>-22.3</td>
</tr>
<tr>
<td>Concentrated</td>
<td>+68.3</td>
<td>14.4</td>
<td>-30.7</td>
</tr>
<tr>
<td>Truck live load</td>
<td>+120.0</td>
<td>30.6</td>
<td>-60.1</td>
</tr>
<tr>
<td>Rib shortening</td>
<td>+14.4</td>
<td>-2.3</td>
<td>14.3</td>
</tr>
<tr>
<td>Subtotal</td>
<td>+134.4</td>
<td>215.5</td>
<td>-45.8</td>
</tr>
<tr>
<td>Temperature</td>
<td>+82.7</td>
<td>-13.1</td>
<td>-82.7</td>
</tr>
<tr>
<td>Shrinkage</td>
<td>+31.0</td>
<td>-4.9</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>+248</td>
<td>197.5</td>
<td>-128</td>
</tr>
</tbody>
</table>

*Includes dead load, truck live load, and rib shortening. Note: Truck live load was used instead of lane live load because truck live load produces larger stresses in the arch rib.

For rib shortening

$$M_s = H \frac{u'}{1 + u} y_e = (187.2 + 30.6) \times 0.01046 \times 6.32 = 14.4 \text{ ft-kips}$$

Also (Fig. 16-45),

$$T = T_H + T_V = \frac{187.2 \times 224.5}{\sqrt{187.2^2 + 236.3^2}} + \frac{236.2 \times 247.2}{\sqrt{187.2^2 + 236.3^2}}$$

$$= 139 + 193 = 332 \text{ kips}$$

Note that $H_s = 187.2$ and $V_s = 236.3$ for dead load.
Stress Calculation

Using the standard method of analysis for the determination of stress in reinforced-concrete members subjected to both moment and thrust, the following results were obtained:

At the crown:

\[
M_e = 248 \text{ ft-kips} \quad n = 8 \quad b = 48 \text{ in.} \quad d = 21.5 \text{ in.} \\
N = 198 \text{ kips} \quad p = 0.015 \quad t = 24 \text{ in.} \quad d' = 2.5 \text{ in.} \quad f_e = 903 \text{ psi}
\]

At the springing:

\[
M_e = 756 \text{ ft-kips} \quad n = 8 \quad b = 48 \text{ in.} \quad d = 46.0 \text{ in.} \\
N = 321 \text{ kips} \quad p = 0.0075 \quad t = 48.5 \text{ in.} \quad d' = 2.5 \text{ in.} \quad f_e = 758 \text{ psi}
\]

Both stresses are too low. Thickness at the springing must be reduced. It will also be advisable to reduce the width somewhat to, say, 39 in. to avoid making the

thickness at the crown too small for aesthetic reasons. In the redesign, then, a series of arch ribs with the following characteristics will be considered.

<table>
<thead>
<tr>
<th>Width of rib, in.</th>
<th>Crown thickness, in.</th>
<th>Shape factor</th>
<th>Form factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>39</td>
<td>22</td>
<td>( N = 0.23 )</td>
<td>( m = {0.2, 0.3, 0.4})</td>
</tr>
<tr>
<td>39</td>
<td>24</td>
<td>( N = 0.23 )</td>
<td>( m = {0.2, 0.3, 0.4})</td>
</tr>
<tr>
<td>39</td>
<td>26</td>
<td>( N = 0.23 )</td>
<td>( m = {0.2, 0.3, 0.4})</td>
</tr>
</tbody>
</table>

In the redesign, the dead-load thrusts due to the superstructure will be altered slightly to include the effect of two struts tying the two arch ribs together at points 2 and 4. Such lateral bracing is usually recommended in arch design whenever the ratio of the unsupported length of the rib axis to the width of the ribs exceeds 30. The bracing is included here merely to provide added resistance against the impact of debris at high water now that the rib has been reduced in width. For the size of the strut a square section with one side equal in length to one-twelfth of the clear span will be used. Hence the depth = \((21 - 3.25)/12 = 1.5\) ft. The weight of the strut = 17.75 \times 1.5^2 \times 15\% = 3,000 lb. The vertical and horizontal reactions at the springing, due to the superstructure and the two struts, will now be

\[
V_s = 136.6 + 3 \times 2 = 142.6 \text{ kips} \\
H = (3622.1 + 3 \times 10 + 3 \times 20)/30 = 3,712.1/30 = 123.7 \text{ kips}
\]
REINFORCED-CONCRETE ARCH BRIDGES

Assuming \( \cos \phi_s = 0.600 \) as before, the depth ratio \( d_e/d_s \) and the weight ratios \( w_s/w_e \) will be

\[
\frac{d_e}{d_s} = \left( \frac{I_e}{I_s} \right)^{\frac{3}{8}} \quad \text{and} \quad \frac{w_s}{w_e} = \frac{t_e}{t_s} \cos \phi_s
\]

Hence, for the arch rib:

<table>
<thead>
<tr>
<th>( m )</th>
<th>( d_e/d_s )</th>
<th>( w_s/w_e )</th>
<th>( H )</th>
<th>( V )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>1/2.02</td>
<td>3.37</td>
<td>0.1663</td>
<td>0.852 \times 100w_e = 85.2w_e</td>
</tr>
<tr>
<td>0.3</td>
<td>1/1.77</td>
<td>2.95</td>
<td>0.1600</td>
<td>0.795 \times 100w_e = 79.5w_e</td>
</tr>
<tr>
<td>0.4</td>
<td>1/1.61</td>
<td>2.68</td>
<td>0.1555</td>
<td>0.756 \times 100w_e = 75.6w_e</td>
</tr>
</tbody>
</table>

Depth at the crown \( d_e \), in:

<table>
<thead>
<tr>
<th>Depth at the crown ( d_e ), in.</th>
<th>22</th>
<th>24</th>
<th>26</th>
</tr>
</thead>
<tbody>
<tr>
<td>( w_e = 150d_e )</td>
<td>895</td>
<td>975</td>
<td>1,056 lb/ft</td>
</tr>
<tr>
<td>( I_e = \frac{1}{12} b d_e^3 + \left( \frac{d_e - 5}{2} \right)^2 (8 - 1) d_e b \times 0.015 )</td>
<td>1.98</td>
<td>2.60</td>
<td>3.32 ft^4</td>
</tr>
<tr>
<td>( A = b d_e + (8 - 1) \times 0.015 b d_e )</td>
<td>6.58</td>
<td>7.18</td>
<td>7.78 ft^4</td>
</tr>
</tbody>
</table>

Thrusts and moments for the dead and live loads may now be determined from the charts in the same manner as were those of the first trial arch. These are summarized in Table 16-13. Moments and thrusts due to temperature and shrinkage are also included together with ribshortening effects. The moment of inertia of the elastic weight \( I_y \) is needed to calculate the moment and thrust due to temperature, and the determination of \( I_y \) for the series of nine arches under consideration is indicated in Table 16-11. Note that

\[
I_y = C \times 100 \times 30^2/I_s = CLr^2/I_s
\]

\[
y_e = \frac{y_e}{r} \times 30
\]

\[
H = \frac{3,750,000 \times 144 \times 40 \times 0.0000006 \times 100}{1,000 I_y} = \frac{E t_o L}{I_y}
\]

\[
M_e = y_e H
\]

\[
M_s = (r - y_e) H
\]

**Table 16-11**

<table>
<thead>
<tr>
<th>( m )</th>
<th>( C )</th>
<th>( I_y )</th>
<th>( y_e/r )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( d_e = 22 \text{ in.} ) &amp; ( d_e = 24 \text{ in.} ) &amp; ( d_e = 26 \text{ in.} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.2</td>
<td>0.0352</td>
<td>1,600</td>
<td>1,220</td>
</tr>
<tr>
<td>0.3</td>
<td>0.0428</td>
<td>1,930</td>
<td>1,480</td>
</tr>
<tr>
<td>0.4</td>
<td>0.0498</td>
<td>2,260</td>
<td>1,724</td>
</tr>
</tbody>
</table>
ILLUSTRATED EXAMPLE

Similarly, the correction for rib shortening is summarized in Table 16-12. Note that

\[
\begin{align*}
u &= \frac{L}{A_m I_y} = \frac{L}{(A_m/A) A I_y} \\
u' &= \frac{L}{A'_m I_y} = \frac{L}{(A'_m/A) A I_y}
\end{align*}
\]

<table>
<thead>
<tr>
<th>(m)</th>
<th>(d_o, \text{in.})</th>
<th>(A)</th>
<th>(\frac{A_m}{A})</th>
<th>(I_y)</th>
<th>(u)</th>
<th>(\frac{A'_m}{A})</th>
<th>(u')</th>
<th>(\frac{u'}{1+u})</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>22</td>
<td>6.58</td>
<td>1.433</td>
<td>1.600</td>
<td>0.00663</td>
<td>1.080</td>
<td>0.00880</td>
<td>0.00875</td>
</tr>
<tr>
<td>0.2</td>
<td>24</td>
<td>7.18</td>
<td>1.433</td>
<td>1.220</td>
<td>0.00798</td>
<td>1.080</td>
<td>0.01051</td>
<td>0.01050</td>
</tr>
<tr>
<td>0.2</td>
<td>26</td>
<td>7.78</td>
<td>1.433</td>
<td>0.953</td>
<td>0.00940</td>
<td>1.080</td>
<td>0.0125</td>
<td>0.0124</td>
</tr>
<tr>
<td>0.3</td>
<td>22</td>
<td>6.58</td>
<td>1.395</td>
<td>1.950</td>
<td>0.00559</td>
<td>1.042</td>
<td>0.00748</td>
<td>0.00745</td>
</tr>
<tr>
<td>0.3</td>
<td>24</td>
<td>7.18</td>
<td>1.395</td>
<td>1.480</td>
<td>0.00674</td>
<td>1.042</td>
<td>0.00903</td>
<td>0.00896</td>
</tr>
<tr>
<td>0.3</td>
<td>26</td>
<td>7.78</td>
<td>1.395</td>
<td>1.160</td>
<td>0.00795</td>
<td>1.042</td>
<td>0.01064</td>
<td>0.01056</td>
</tr>
<tr>
<td>0.4</td>
<td>22</td>
<td>6.58</td>
<td>1.362</td>
<td>2.260</td>
<td>0.00494</td>
<td>1.010</td>
<td>0.00666</td>
<td>0.00664</td>
</tr>
<tr>
<td>0.4</td>
<td>24</td>
<td>7.18</td>
<td>1.362</td>
<td>1.724</td>
<td>0.00593</td>
<td>1.010</td>
<td>0.00800</td>
<td>0.00796</td>
</tr>
<tr>
<td>0.4</td>
<td>26</td>
<td>7.78</td>
<td>1.362</td>
<td>1.350</td>
<td>0.00699</td>
<td>1.010</td>
<td>0.00943</td>
<td>0.00938</td>
</tr>
</tbody>
</table>

The results summarized in Table 16-13 indicate that the thrusts in the rib vary only slightly as the dimensions of the rib are altered but that the moments vary considerably. It should be noted in Table 16-13 that moments due to dead load are not taken equal to zero as should be the case for arch axes which coincide perfectly with the dead-load string polygon. This perfect coincidence is possible in spandrel-filled arches but impractical in open-spandrel arches. As a consequence, some dead-load moment may be expected for the open-spandrel arch. This moment will vary from zero to possibly 20 per cent of the total moment due to live load plus impact, temperature, rib shortening, and shrinkage. An extra 12.5 per cent of this total will arbitrarily be used as the dead-load moment.

The unit concrete stresses at the crown and springing, produced by the combined thrusts and moments, were calculated for the cracked transformed section of the rib and are plotted in Fig. 16-46. Note that the crown stress decreases whereas that at the springing increases as the thickness at the springing decreases (as \(m\) increases). The point on the graph where the two curves for the same crown thickness intersect represents a balanced design for which the stress at the crown and springing are equal. For the final design a rib with \(d_c = 24\) in. and \(m = 0.36\) will be chosen so that \(f_c = 1,090\) at both crown and springing. Then \(d_c/d_s = 1/1.67\) and \(d_s = 40\) in.

![Fig. 16-46. Unit concrete stresses at the crown and springing line due to thrusts and moments for variable values of \(m\) and \(d_o\).](image-url)
<table>
<thead>
<tr>
<th>d_e = 22 in.</th>
<th>( m = 0.2 )</th>
<th>( m = 0.3 )</th>
<th>( m = 0.4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( +M_e )</td>
<td>( H_e )</td>
<td>(-M_e)</td>
<td>( H_e )</td>
</tr>
<tr>
<td>Dead load:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S.S.</td>
<td>25.3</td>
<td>123.7</td>
<td>-72.7</td>
</tr>
<tr>
<td>Arch.</td>
<td>0</td>
<td>49.5</td>
<td>0</td>
</tr>
<tr>
<td>Live load</td>
<td>120.9</td>
<td>30.7</td>
<td>-280</td>
</tr>
<tr>
<td>Rib shortening</td>
<td>11.2</td>
<td>-1.8</td>
<td>-37.8</td>
</tr>
<tr>
<td>Temp plus shrinkage</td>
<td>70.2</td>
<td>-11.1</td>
<td>-264</td>
</tr>
<tr>
<td>Total</td>
<td>227.6</td>
<td>191.0</td>
<td>-654.5</td>
</tr>
<tr>
<td>( T = 304 )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>d_e = 24 in.</td>
<td>( m = 0.2 )</td>
<td>( m = 0.3 )</td>
<td>( m = 0.4 )</td>
</tr>
<tr>
<td>( +M_e )</td>
<td>( H_e )</td>
<td>(-M_e)</td>
<td>( H_e )</td>
</tr>
<tr>
<td>Dead load:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S.S.</td>
<td>28.4</td>
<td>123.7</td>
<td>-84.0</td>
</tr>
<tr>
<td>Arch.</td>
<td>0</td>
<td>54.0</td>
<td>0</td>
</tr>
<tr>
<td>Live load</td>
<td>120.9</td>
<td>30.7</td>
<td>-280</td>
</tr>
<tr>
<td>Rib shortening</td>
<td>13.8</td>
<td>-2.2</td>
<td>-46.5</td>
</tr>
<tr>
<td>Temp plus shrinkage</td>
<td>92.1</td>
<td>-14.6</td>
<td>-346</td>
</tr>
<tr>
<td>Total</td>
<td>255.3</td>
<td>191.6</td>
<td>-756.5</td>
</tr>
<tr>
<td>( T = 310 )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>d_e = 26 in.</td>
<td>( m = 0.2 )</td>
<td>( m = 0.3 )</td>
<td>( m = 0.4 )</td>
</tr>
<tr>
<td>( +M_e )</td>
<td>( H_e )</td>
<td>(-M_e)</td>
<td>( H_e )</td>
</tr>
<tr>
<td>Dead load:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S.S.</td>
<td>31.9</td>
<td>123.7</td>
<td>-96.3</td>
</tr>
<tr>
<td>Arch.</td>
<td>0</td>
<td>56.6</td>
<td>0</td>
</tr>
<tr>
<td>Live load</td>
<td>120.9</td>
<td>30.7</td>
<td>-280</td>
</tr>
<tr>
<td>Rib shortening</td>
<td>16.7</td>
<td>-2.6</td>
<td>-56.3</td>
</tr>
<tr>
<td>Temp plus shrinkage</td>
<td>118.0</td>
<td>-18.7</td>
<td>-443</td>
</tr>
<tr>
<td>Total</td>
<td>287.5</td>
<td>191.7</td>
<td>-875.6</td>
</tr>
<tr>
<td>( T = 315 )</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
ILLUSTRATED EXAMPLE

The per cent of steel at the crown used in the foregoing analysis was 1.5. If four-teen 1-in.-square bars are used in the final design \( p = 1.49 \) per cent at the crown, and 0.897 per cent at the springing. A 2-in. covering of concrete will be assumed for this main steel throughout the arch.

Final Analysis

The variation in thickness of the rib along its length must now be found for points 0 to 10, inclusive, from the values in Tables 16-9 and 16-10. Assuming \( N = 0.232 \) (this was the actual value in the first trial arch), the ordinates along the axis can also

![Diagram](image)

Crosses indicate position of arch axis—lines represent dead load polygon

**Fig. 16-47. Total dead-load string polygon drawn to check shape of arch axis.**

Values of \( y_0 \)

<table>
<thead>
<tr>
<th>Point</th>
<th>Arch</th>
<th>D L</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sprg</td>
<td>30.00'</td>
<td>30.00'</td>
</tr>
<tr>
<td>1</td>
<td>26.82</td>
<td>26.89</td>
</tr>
<tr>
<td>2</td>
<td>21.10</td>
<td>21.04</td>
</tr>
<tr>
<td>3</td>
<td>16.18</td>
<td>16.04</td>
</tr>
<tr>
<td>4</td>
<td>11.97</td>
<td>11.84</td>
</tr>
<tr>
<td>5</td>
<td>8.47</td>
<td>8.36</td>
</tr>
<tr>
<td>6</td>
<td>5.62</td>
<td>5.58</td>
</tr>
<tr>
<td>7</td>
<td>3.37</td>
<td>3.39</td>
</tr>
<tr>
<td>8</td>
<td>1.71</td>
<td>1.78</td>
</tr>
<tr>
<td>9</td>
<td>0.61</td>
<td>0.72</td>
</tr>
<tr>
<td>10</td>
<td>0.07</td>
<td>0.19</td>
</tr>
<tr>
<td>Crown</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

be found from the values of Table 16-8, and the increments in length along the axis for equal increments in span can then be calculated. The weights of the 10 segments of the rib follow from the foregoing data, the entire calculations being indicated in Table 16-14. The weights of these segments must now be combined with the super-structure loading to draw the string polygon as a check on the shape of the axis. Figure 16-47 illustrates this step and indicates that an almost perfect agreement was realized between the assumed shape (determined from Table 16-8 for \( N = 0.232 \)) and the string polygon. No further changes will therefore be necessary and the rib will now be analyzed by the elastic theory to determine the actual indeterminate moments, thrusts, and shears.

The column analogy, as developed by Prof. Hardy Cross,\(^4\) will be used for this
analysis. The structure will be made statically determinate by breaking the rib at the crown. Hence the static moment \( m_s \) will be zero at the crown and at the springing it will be that on the cantilevered half of the rib. Calculations for the moment of inertia of the arch rib must be made for the gross section or for the transformed section.

<table>
<thead>
<tr>
<th>Point</th>
<th>((1 + \tan^2 \phi)^{1/6})</th>
<th>(c)</th>
<th>(d_\alpha/d\alpha)</th>
<th>(t_\alpha), ft</th>
<th>(y_\alpha/r)</th>
<th>(y_\alpha), ft</th>
<th>(\Delta s), ft</th>
<th>(w), kips</th>
<th>(I), ft^4</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.183</td>
<td>1.407</td>
<td>1.667</td>
<td>3.334</td>
<td>1.0000</td>
<td>30.00</td>
<td>7.93</td>
<td>12.24</td>
<td>11.48</td>
</tr>
<tr>
<td>1</td>
<td>1.211</td>
<td>1.331</td>
<td>1.532</td>
<td>3.064</td>
<td>0.7951</td>
<td>23.85</td>
<td>7.70</td>
<td>9.08</td>
<td>5.99</td>
</tr>
<tr>
<td>2</td>
<td>1.240</td>
<td>1.270</td>
<td>1.423</td>
<td>2.846</td>
<td>0.6176</td>
<td>18.53</td>
<td>6.77</td>
<td>7.25</td>
<td>5.05</td>
</tr>
<tr>
<td>3</td>
<td>1.269</td>
<td>1.219</td>
<td>1.333</td>
<td>2.666</td>
<td>0.4660</td>
<td>13.98</td>
<td>5.62</td>
<td>6.32</td>
<td>3.76</td>
</tr>
<tr>
<td>4</td>
<td>1.297</td>
<td>1.175</td>
<td>1.257</td>
<td>2.514</td>
<td>0.3378</td>
<td>10.13</td>
<td>5.20</td>
<td>5.83</td>
<td>4.32</td>
</tr>
<tr>
<td>5</td>
<td>1.325</td>
<td>1.137</td>
<td>1.191</td>
<td>2.382</td>
<td>0.2320</td>
<td>6.96</td>
<td>4.42</td>
<td>5.19</td>
<td>3.76</td>
</tr>
<tr>
<td>6</td>
<td>1.353</td>
<td>1.104</td>
<td>1.139</td>
<td>2.278</td>
<td>0.1472</td>
<td>4.42</td>
<td>5.07</td>
<td>5.13</td>
<td>3.03</td>
</tr>
<tr>
<td>7</td>
<td>1.380</td>
<td>1.074</td>
<td>1.093</td>
<td>2.186</td>
<td>0.0822</td>
<td>2.47</td>
<td>5.01</td>
<td>4.93</td>
<td>2.78</td>
</tr>
<tr>
<td>8</td>
<td>1.397</td>
<td>1.047</td>
<td>1.055</td>
<td>2.110</td>
<td>0.0364</td>
<td>1.09</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>1.402</td>
<td>1.022</td>
<td>1.024</td>
<td>2.048</td>
<td>0.0090</td>
<td>0.27</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>2.000</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

In this problem the transformed section will be used. With fourteen 1-in.-square bars embedded with a 2-in. covering, the effective depth of the steel \( d = d_\alpha - \frac{5}{12} \) ft. Hence the value for the moment of inertia \( I = b(d_\alpha)^2/12 + \frac{14}{244}(n - 1)d^2/A \) ft^4. The results for \( I \) at the 10 arch points are listed in the foregoing tabulation. In the column analogy, however, values midway between the arch points (at the centers of the arch segments) will be used, and these values for \( x, y_\alpha, I, \) etc., will be determined by interpolating between the calculated values for the 10 arch points.

The calculations for the elastic area \( f = ds/EI; \) for the moment of inertia of this area about the \( X \) axis, \( I_y = \int y^2 ds/EI, \) and also about the \( Y \) axis, \( I_x = \int x^2 ds/EI; \) and for the determination of the vertical distance from the original axes through the crown
### Table 16-15. Analysis of Arch Rib by Column Analogy

<table>
<thead>
<tr>
<th>Section</th>
<th>α(−)</th>
<th>y(−)</th>
<th>Δy</th>
<th>Δs</th>
<th>I</th>
<th>α</th>
<th>ay(−)</th>
<th>ay²</th>
<th>i_y*</th>
<th>az²</th>
<th>i_z*</th>
<th>m_(s)(−)</th>
<th>P(−)</th>
<th>M_(y)(+)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>47.5</td>
<td>26.82</td>
<td>6.15</td>
<td>7.93</td>
<td>10.06</td>
<td>0.788</td>
<td>21.13</td>
<td>567</td>
<td>2.5</td>
<td>1.778</td>
<td>1.6</td>
<td>4,710</td>
<td>3,711</td>
<td>99,529</td>
</tr>
<tr>
<td>2</td>
<td>42.5</td>
<td>21.10</td>
<td>5.32</td>
<td>7.30</td>
<td>8.03</td>
<td>0.909</td>
<td>19.18</td>
<td>405</td>
<td>2.1</td>
<td>1.642</td>
<td>1.9</td>
<td>3,686</td>
<td>3,351</td>
<td>70,706</td>
</tr>
<tr>
<td>3</td>
<td>37.5</td>
<td>16.18</td>
<td>4.55</td>
<td>6.77</td>
<td>6.54</td>
<td>1.035</td>
<td>16.75</td>
<td>271</td>
<td>1.8</td>
<td>1.455</td>
<td>2.1</td>
<td>2,809</td>
<td>2,907</td>
<td>42,035</td>
</tr>
<tr>
<td>4</td>
<td>32.5</td>
<td>11.97</td>
<td>3.85</td>
<td>6.32</td>
<td>5.48</td>
<td>1.153</td>
<td>13.80</td>
<td>165</td>
<td>1.4</td>
<td>1.218</td>
<td>2.4</td>
<td>2,074</td>
<td>2,391</td>
<td>26,620</td>
</tr>
<tr>
<td>5</td>
<td>27.5</td>
<td>8.47</td>
<td>3.17</td>
<td>5.92</td>
<td>4.66</td>
<td>1.270</td>
<td>10.76</td>
<td>91</td>
<td>1.1</td>
<td>960</td>
<td>2.6</td>
<td>1,465</td>
<td>1,861</td>
<td>15,763</td>
</tr>
<tr>
<td>6</td>
<td>22.5</td>
<td>5.62</td>
<td>2.54</td>
<td>5.60</td>
<td>4.04</td>
<td>1.386</td>
<td>7.79</td>
<td>44</td>
<td>0.7</td>
<td>702</td>
<td>2.9</td>
<td>978</td>
<td>1,356</td>
<td>7,621</td>
</tr>
<tr>
<td>7</td>
<td>17.5</td>
<td>3.37</td>
<td>1.95</td>
<td>5.37</td>
<td>3.56</td>
<td>1.508</td>
<td>5.08</td>
<td>17</td>
<td>0.5</td>
<td>462</td>
<td>3.2</td>
<td>594</td>
<td>896</td>
<td>3,020</td>
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<td>1.38</td>
<td>5.19</td>
<td>3.19</td>
<td>1.627</td>
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<td>5</td>
<td>0.3</td>
<td>254</td>
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<td>312</td>
<td>508</td>
<td>869</td>
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<tr>
<td>9</td>
<td>7.5</td>
<td>0.61</td>
<td>0.81</td>
<td>5.07</td>
<td>2.90</td>
<td>1.748</td>
<td>1.07</td>
<td>1</td>
<td>0.1</td>
<td>98</td>
<td>3.6</td>
<td>126</td>
<td>220</td>
<td>134</td>
</tr>
<tr>
<td>10</td>
<td>2.5</td>
<td>0.07</td>
<td>0.28</td>
<td>5.01</td>
<td>2.68</td>
<td>1.869</td>
<td>0.13</td>
<td>0</td>
<td>0</td>
<td>12</td>
<td>3.9</td>
<td>34</td>
<td>64</td>
<td>4</td>
</tr>
</tbody>
</table>

\[ \bar{y} = -7.408 \]

Correct to centroid

<table>
<thead>
<tr>
<th>13.29 = ( A )</th>
<th>848 = ( I_y )</th>
<th>8,609 = ( I_z )</th>
</tr>
</thead>
<tbody>
<tr>
<td>13.29 = ( A )</td>
<td>848 = ( I_y )</td>
<td>8,609 = ( I_z )</td>
</tr>
<tr>
<td>17,265</td>
<td>273,301</td>
<td></td>
</tr>
</tbody>
</table>

See (*) and Note in Fig. 16-48.
### Table 16-16a. Analysis of Arch Rib by Column Analogy

<table>
<thead>
<tr>
<th>Unit load at col. 1</th>
<th>Unit load at col. 2</th>
<th>Unit load at col. 3</th>
<th>Unit load at col. 4</th>
<th>Unit load at col. 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_0(-)$</td>
<td>$P(-)$</td>
<td>$M_x(+)$</td>
<td>$M_y(+)$</td>
<td>$m_0(-)$</td>
</tr>
<tr>
<td>7.5</td>
<td>5.91</td>
<td>281</td>
<td>159</td>
<td>27.5</td>
</tr>
<tr>
<td>2.5</td>
<td>2.27</td>
<td>96</td>
<td>48</td>
<td>12.5</td>
</tr>
<tr>
<td>17.5</td>
<td>7.5</td>
<td>7.76</td>
<td>291</td>
<td>126</td>
</tr>
<tr>
<td>2.5</td>
<td>2.88</td>
<td>94</td>
<td>34</td>
<td>12.5</td>
</tr>
<tr>
<td>7.5</td>
<td>9.52</td>
<td>262</td>
<td>81</td>
<td>12.5</td>
</tr>
<tr>
<td>2.5</td>
<td>4.07</td>
<td>51</td>
<td>7</td>
<td>22.5</td>
</tr>
<tr>
<td>-16.96</td>
<td>-81.8</td>
<td>+377</td>
<td>+207</td>
<td>-35.79</td>
</tr>
</tbody>
</table>

### Table 16-16b

<table>
<thead>
<tr>
<th>Load at col. No.</th>
<th>Crown</th>
<th>Springing</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>$M_{x}/I_x$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$m_0$</td>
<td>+0.331</td>
<td>+0.860</td>
</tr>
<tr>
<td>$m_x$</td>
<td>+0.331</td>
<td>+0.860</td>
</tr>
</tbody>
</table>

### Table 16-16c

<table>
<thead>
<tr>
<th>Load at col. No.</th>
<th>$V_L$</th>
<th>$H$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.9781</td>
<td>0.0861</td>
</tr>
<tr>
<td>2</td>
<td>0.9116</td>
<td>0.298</td>
</tr>
<tr>
<td>3</td>
<td>0.8034</td>
<td>0.547</td>
</tr>
<tr>
<td>4</td>
<td>0.6620</td>
<td>0.748</td>
</tr>
<tr>
<td>5</td>
<td>0.5000</td>
<td>0.825</td>
</tr>
<tr>
<td>4'</td>
<td>0.3380</td>
<td>0.748</td>
</tr>
<tr>
<td>3'</td>
<td>0.1966</td>
<td>0.547</td>
</tr>
<tr>
<td>2'</td>
<td>0.0884</td>
<td>0.298</td>
</tr>
<tr>
<td>1'</td>
<td>0.0219</td>
<td>0.0861</td>
</tr>
</tbody>
</table>
to the elastic centroid, \( y_e \), are shown in Table 16-15. Corrections are made to transfer the moments of inertia, as determined originally about the \( X \) and \( Y \) axes through the crown, to parallel axes through the elastic centroid.

Computations for the dead-load moments are made in Table 16-15 merely to illustrate the method.

If the arch axis coincided perfectly with the dead-load string polygon (Fig. 16-47) the dead-load moments would necessarily be zero. For the live loads, unit loads are placed at the five arch points where the columns join the rib, and the moments at the crown and springing are determined as shown. It should be remembered that the moments determined from the basic formula of the column analogy, i.e.,

\[
m_i = \frac{P}{A} \pm \frac{M_i y}{I_y} \pm \frac{M_i z}{I_z}
\]

are the indeterminate moments required to restore the statically determinate arch rib (it was cut at the crown) to the shape assumed by the continuous uncut structure.

Hence final moments are obtained from \( M = m_e - m_i \). Once these final moments are determined at the springing and at the crown, the values for \( H \) and \( V_L \) follow from the laws of statics. These results, together with sample calculations, are also shown in Tables 16-16a, b and c. The graph constructed in Fig. 16-50 shows these values of \( M \), \( H \), and \( V \) plotted along the span. Hence they represent influence lines for these reactions.

The use of these influence lines, for the determination of live-load thrusts and moments due to an H-15 wheel loading, follows. The unit wheel load

\[
P = 1.222 \times 6,000 = 7,330 \text{ lb}
\]

as in the preliminary analysis.

At the Crown

Max +\( M \), place wheel 4\( P \) at 0.5\( L \) and wheel 1\( P \) at 0.36\( L \).

\[
\begin{align*}
M_e &= (4.514 \times 4 + 0.170 \times 1)7.33 = 133.6 \text{ ft-kips} \\
H &= (0.825 \times 4 + 0.668 \times 1)7.33 = 29.09 \text{ kips} \\
V_L &= (0.500 \times 4 + 0.719 \times 1)7.33 = 19.93 \text{ kips}
\end{align*}
\]

Max -\( M \), place wheel 4\( P \) at 0.30\( L \) and wheel 1\( P \) at 0.16\( L \).

\[
\begin{align*}
M_e &= (-0.762 \times 4 - 0.648 \times 1)7.33 \times 2 = -54.18 \text{ ft-kips} \\
H &= (0.547 \times 4 + 0.213 \times 1)7.33 \times 2 = 35.20 \text{ kips}
\end{align*}
\]
At the Springing
Max $+M$, place wheel 1P at 0.50$L$, wheel 4P at 0.64$L$, and wheel $\frac{3}{4}P$ at 0.94$L$.

$$M_* = (4.25 \times 1 + 6.13 \times 4 + 0.694 \times \frac{3}{4})7.33 = 214.7 \text{ ft-kips}$$
$$H = (0.825 \times 1 + 0.668 \times 4 + 0.0517 \times \frac{3}{4})7.33 = 25.66 \text{ kips}$$
$$V_L = (0.500 \times 1 + 0.2814 \times 4 + 0.0131 \times \frac{3}{4})7.33 = 11.99 \text{ kips}$$

Max $-M$, place wheel 1P at 0.06$L$ and wheel 4P at 0.20$L$.

$$M_* = (3.99 \times 1 + 7.50 \times 4)7.33 = 249.2 \text{ ft-kips}$$
$$H = (0.0517 \times 1 + 0.298 \times 4)7.33 = 12.53 \text{ kips}$$
$$V_L = (0.9912 \times 1 + 0.9116 \times 4)7.33 = 33.99 \text{ kips}$$

The effects of rib shortening must now be determined. Since these effects are a small part of the total, and since the dead load produces most of the thrust, it alone might be used to calculate the rib shortening. In this case, however, both dead load and live load will be used. The approximate average stress in the rib will then be the average of that at the crown and that at the springing. Hence,

$$f_* = \left[ \frac{175.1 + 29.1}{7.18} + \frac{\sqrt{(171.5 + 25.7)^2 + (217 + 12)^2}}{11.51} \right] \frac{0.5}{144} = 203 \text{ psi}$$

The values of 7.18 and 11.51 represent the cross-sectional area in square feet at the crown and springing, respectively.

Then

$$\epsilon L = \frac{203 \times 100}{3,750,000} = 0.00541 \text{ ft}$$

and

$$H = \frac{0.00541 \times 3,750,000 \times 144}{2 \times 848 \times 1,000} = 1.72 \text{ kips}$$

$$M_* = 7.408H = 12.76 \text{ ft-kips}$$

$$M_* = 22.59H = 38.90 \text{ ft-kips}$$
A summary of the thrusts and moments at the crown and springing due to the various loadings is shown in Table 16-17.

<table>
<thead>
<tr>
<th>Loading</th>
<th>Max + M</th>
<th>Max - M</th>
<th>Max + M</th>
<th>Max - M</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$M_s$</td>
<td>$H$</td>
<td>$M_s$</td>
<td>$H$</td>
</tr>
<tr>
<td>Dead load</td>
<td>+29.0</td>
<td>171.5</td>
<td>+29.0</td>
<td>171.5</td>
</tr>
<tr>
<td>Live load</td>
<td>+133.6</td>
<td>29.1</td>
<td>-54.2</td>
<td>35.2</td>
</tr>
<tr>
<td>Rib shortening</td>
<td>+12.8</td>
<td>-1.7</td>
<td>+12.8</td>
<td>-1.7</td>
</tr>
<tr>
<td>Temperature</td>
<td>+56.6</td>
<td>-7.7</td>
<td>-56.6</td>
<td>7.7</td>
</tr>
<tr>
<td>Shrinkage</td>
<td>+21.2</td>
<td>-2.9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>+253</td>
<td>+187</td>
<td>-73.6</td>
<td>195</td>
</tr>
</tbody>
</table>

The unit stresses in the concrete may now be found in the usual way for a reinforced-concrete member loaded with thrust and moment. The maximum values are $f_c = 1,120$ psi at the crown and $f_c = 1,090$ psi at the springing. They are close to the estimates of 1,090 psi determined previously and below the allowable of 1,250 psi. In the final design the stresses at the two sections are almost equal and represent a well-balanced stress distribution. If now it were desirable either to increase or to decrease these stresses, the change could be accomplished simply by altering the width of the rib.

The shear stress in the rib must now be investigated. The dead-load effect will be nil for the arch axis was made to conform very closely to the dead-load polygon. At the crown only the live load will be effective and half the bridge should be loaded for the maximum shear. This occurs with wheel 4P at 0.5L, 1P at 0.64L, and 3P at 0.94L. Hence $V_L = \frac{4 \times 0.5 + 1 \times 0.2814 + 3 \times 0.0131}{7.33} = 17.0$ kips. Then $v = \frac{17,000}{24 \times 39 \times 0.875} = 20.8$ psi. For maximum shear at the springing the entire span should be loaded, placing wheel 4P at 0L, 1P at 0.14L, 3P at 0.44L, $\frac{3}{4}$P at 0.58L, and 3P at 0.88L. Then $V_L = 53.9$ kips and $H = 25.4$ kips. However, rib shortening, temperature, and shrinkage will reduce $H$ by $1.7 - 7.7 + 2.9$, making the final value = 22.3 kips. The net shear will then be

$17.279 \times 22.3 - 21.729 \times 53.9 = 28.3$ kips

The unit shear stress is then $28,300/40 \times 39 \times 0.875 = 20.7$ psi. Although these stresses are very small and require no stirrup reinforcing, modern design practice always provides hoop steel in some form to parallel column design and to provide for spacing bars during erection and as a measure of safety later. The detailed drawings of Fig. 16-28 indicate this feature.

### PLASTIC FLOW

The preceding arch design took no account of the plastic flow of the concrete. This flow occurs whenever concrete is subjected to continued loads such as dead load and the effects of shrinkage. The distortions due to live load and temperature, however, are practically elastic. The plastic distortion progresses at a rather rapid rate just after the stress is applied, but this rate decreases rapidly and ceases entirely after about 5 years. The unit plastic strain per unit stress, $c$, accumulated after 5 years, varies as follows: For 1:2:4 concrete

<table>
<thead>
<tr>
<th>Age of concrete when loaded, months</th>
<th>$c$ (Values of $c$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$10.0 \times 10^{-7}$</td>
</tr>
<tr>
<td>3</td>
<td>$6.6 \times 10^{-7}$</td>
</tr>
<tr>
<td>12</td>
<td>$2.9 \times 10^{-7}$</td>
</tr>
</tbody>
</table>
Plastic flow causes a decrease in the compressive stress in the concrete but increases that in the steel materially. In an arch this flow is equivalent to extra rib shortening. Since the concrete is much more capable of plastic action at early ages, arches should be decentered as soon as possible, provided, of course, that provision has been made to prevent overstress in the steel. According to Mr. Whitney, concretes are plastic enough after decentering to exclude the necessity of any arch adjustment such as the Freyssinet method for stress compensation described later. A brief description is included here in an approximate form, but a thorough study of the ACI Report and its excellent bibliography is recommended.

The following relationships governing the plastic flow of concrete were developed by Glanville and Whitney.

For dead load:

\[ f_{ce} = \frac{f_{oc}}{\varepsilon_0 + \varepsilon_0} \quad \text{and} \quad A_s(f_{ce} - f_{oc}) = A_s(f_{oc} - f_{oc}) \]

where \( f_{ce} \) = final concrete stress after plastic flow

\[ f_{oc} = \text{original concrete stress} = \frac{P}{A_c + nA_s} \]

\[ c = \text{plastic-flow coefficient} = 0.000001 \text{ usually} \]

\[ b = \frac{A_s}{A_c E_c} + \frac{1}{E_c} \]

\[ f_{ce} = \text{final steel stress after plastic flow} \]

\[ f_{oc} = \text{original steel stress} = n f_{oc} \]

Professor Davis of the University of California suggests an alternate method using a "modulus of resistance" \( R = 1/(\varepsilon + c) \) where \( c \) = plastic-flow coefficient and \( \varepsilon = 1/E_c \). The ratio \( E_s/R = n_r \) is thus a modified \( n \) factor and may be used as such in the usual way. For instance, if \( f_{ce} \) is determined for the arch using \( E_s' = E_s/n_r \) in the arch analysis, and \( f_{oc} \) is found using \( E_s = E_s/n \) in the usual way, their difference would represent the effect of plastic flow. Results agree fairly well with those of the preceding method.

The rib shortening \( \Delta x = f_{f_t} L_s/E_s \) for the effect of dead load and plastic flow combined. The arch thrust is then \( H = \Delta x/I_{y'} \) and \( M_z = y_c H \) as for elastic action. Note, however, that \( I_{y'} \) is based on \( n_r \) and not \( n \).

**Shrinkage**

\[ \varepsilon_s = \frac{s}{pn' + 1} \quad f_s = E_s \varepsilon_s \quad A_s f_s = A_s f_s \]

where \( s \) = unit shrinkage

\[ n = E_s/E_s' \text{ and } E_s' \text{ is a modified value of } E_s \text{ such that } n \text{' varies between 10 and } 20 \text{ and is usually taken as } 15^{15} \]

Again \( \Delta x = \varepsilon_s L_s \) and \( H = \Delta x/I_{y'} \) where \( I_{y'} \) is based on \( n_r \).

**Temperature**

Whenever the thermal coefficients of expansion of the concrete and the reinforcement are not identical, differential stress will accompany a change in temperature. Some authorities take this added stress into account but usually the coefficients are assumed equal. When they vary

\[ f_s = \frac{E_s \Delta t (\alpha_s - \alpha_c)}{1 + pn} \quad \text{and} \quad \Delta t (\alpha_s - \alpha_c) = \frac{f_s}{E_s} + \frac{f_c}{E_c} \]

As before, \( \Delta x = \Delta t \alpha_s L_s \). Here \( \Delta t \) is usually taken as 0.45 of the yearly range in
air temperature, and \( \Delta t' \) as \( \pm 0.3 \) of the yearly range in air temperature. The thrust is again \( H = \Delta x/I' \) but \( I' \) is now based on \( n \).

**Unit Stresses**

These are usually calculated assuming the concrete cracks and takes no tension. This method will be used in the following example. Experimental evidence indicates that measured steel stresses are less than those assuming the concrete takes no tension but greater than those computed using the gross transformed area, and that they are closer to the latter.\(^{16}\)

**EXAMPLE OF PLASTIC FLOW**

**Dead Load, Direct Stress**

Plastic flow for the arch, analyzed in the preceding pages by the elastic theory, will now be considered. The average plastic flow will be approximated by considering the separate affects at the crown and at the springing and averaging these values. The small dead-load moment determined in the elastic analysis will be neglected in these calculations and only the direct thrust included. The following data apply:

\[
\begin{align*}
E_c &= 3.75 \times 10^4 \text{ psi} \\
E_s &= 30 \times 10^4 \text{ psi} \\
c &= 0.0000001 \\
s &= 0.0003 \\
A_s &= 14 \text{ sq in.} \\
d_s &= 24 \text{ in.} \\
H_s &= 171.5 \text{ kips} \\
b &= 39 \text{ in.} \\
d &= 40 \text{ in.} \\
T_s &= 276.5 \text{ kips} \\
n = 8 \\
d' &= 2.5 \text{ in.}
\end{align*}
\]

**At the Crown**

\[
\begin{align*}
H &= \frac{A_c}{A_c + nA_s} = \frac{171,500}{39 \times 24 + 8 \times 14} = 165.9 \text{ psi} \\
f_{oc} &= \frac{H}{E_c} = \frac{165.9}{3.75 \times 10^4} = 0.0000001 \\
m_{oc} &= 8 \times 165.9 = 1,327 \text{ psi} \\
b &= \frac{A_s}{A_c} + \frac{1}{E_c} = \left( \frac{922}{14 \times 30} + \frac{1}{3.75} \right) \times 10^{-4} = (2.17 + 0.27) \times 10^{-4} = 2.44 \times 10^{-4} \\
c &= \frac{1}{b} = 2.44 = 0.41 \\
f_{ts} &= \frac{f_{oc}}{e^{c/b}} = \frac{165.9}{2.718^{0.41}} = \frac{165.9}{1.507} = 110.0 \text{ psi} \\
A_s(f_{ts} - f_{oc}) &= A_s(f_{ts} - f_{oc}) \\
14(f_{ts} - 1,327) &= 922(165.9 - 110) \\
f_{ts} &= 3,615 + 1,327 = 4,942 \text{ psi}
\end{align*}
\]

**At the Springing**

\[
\begin{align*}
H &= \frac{276,500}{39 \times 40 + 7 \times 14} = 166.8 \\
f_{oc} &= 8 \times 166.8 = 1,334 \\
b &= \left( \frac{1,546}{14 \times 30} + \frac{1}{3.75} \right) \times 10^{-4} = 3.95 \times 10^{-4} \\
c &= \frac{1}{b} = 0.253 \\
f_{ts} &= \frac{166.8}{e^{0.253}} = \frac{166.8}{1.288} = 129.5 \text{ psi} \\
14(f_{ts} - 1,334) &= 1,546(166.8 - 129.5) \\
f_{ts} &= 4,120 + 1,334 = 5,454 \text{ psi} \\
R.S. &= \frac{f_s}{E_s} L = \Delta x = \frac{4,942 + 5,454}{2 \times 30 \times 10^4} \times 100 = 0.0173 \text{ ft}
\end{align*}
\]
Dead Load, Rib Shortening

It is now necessary to find the moment of inertia $I'_{y'}$ of the elastic area of the arch ring using a modified $n_r$. Then

$$n_r = \frac{E_r}{R} = \frac{E_r}{1/(\epsilon + c)} = 30 \times 10^6 \left( \frac{1}{3.75} + 1 \right) \times 10^{-6} = 38.0$$

Using $n_r = 38$ in place of $n = 8$ in calculations, gives values of the moment of inertia $I$ at each arch cross section of 15.65, 12.70, 10.51, 8.91, 7.66, 6.70, 5.96, 5.38, 4.91, and 4.57 for the 10 sections, inclusive. The column analogy then yields $A/2 = 8.039$, $\bar{y} = 7.624$ ft, $I'_{y'}/2 = 526$. Hence

$$H = \frac{\Delta x}{I'_{y'}} = \frac{0.0173}{2 \times 526 \times 38 \times 1,000}{30 \times 10^6 \times 144} = 1.87 \text{kips}$$

$$M_e = 7.62 \times 1.87 = 14.25 \text{ ft-kips} \quad M_s = 22.38 \times 1.87 = 41.9 \text{ ft-kips}$$

The unit stresses, based on the cracked cross section, are then calculated in the usual way for the case of bending and direct thrust, using $n_r = 38$. They are:

At the crown

- $f_c = 26.4 \text{ psi}$
- $f' = 515 \text{ psi}$
- $f_s = 1,190 \text{ psi}$

At the springing

- $f_c = 32.2 \text{ psi}$
- $f' = 1,080 \text{ psi}$
- $f_s = 2,060 \text{ psi}$

Shrinkage, Direct Stress

At the Crown

$$\epsilon_s = \frac{s}{n'p + 1} = \frac{0.0003}{15 \times 0.015 + 1} = 0.000245$$

$$f_{oc} = \epsilon_s E_r = 0.000245 \times 30 \times 10^6 = 7,350 \text{ psi}$$

$$A_{j\sigma} = A_{f_{oc}} \quad f_{oc} = \frac{14}{1,546} \times 7,350 = 112 \text{ psi}$$

At the Springing

$$\epsilon_s = \frac{0.0003}{15 \times 0.00897 + 1} = 0.000265$$

$$f_{os} = 0.000265 \times 30 \times 10^6 = 7,950 \text{ psi} \quad f_{oc} = \frac{14}{1,546} \times 7,950 = 72.0 \text{ psi}$$

Shrinkage, Rib Shortening

The average $\epsilon_s = 0.000255$ and $\Delta x = \epsilon_s L = 0.000255 \times 100 = 0.0255 \text{ ft}$.

$$H = \frac{\Delta x}{I'_{y'}} = \frac{0.0255}{1,052 \times 38 \times 1,000}{30 \times 10^6 \times 144} = 2.75 \text{kips}$$

$$M_e = 2.75 \times 7.62 = 21.0 \text{ ft-kips} \quad M_s = 2.75 \times 22.38 = 61.6 \text{ ft-kips}$$

Note the close check here with the reactions of $H = 2.87$, $M_e = 21.2$, and $M_s = 64.5$ calculated in the elastic analysis using an equivalent temperature drop of 15°F for shrinkage (Table 16-17).

Using $n_r = 38$, $H = 2.75$, and $M_e = 21.2$ in calculating for unit stresses, we find $f_c = 37.4$ psi, $f' = 758$ psi, and $f_s = 1,750$ psi at the crown. From Fig. 16-51, at the springing

$$T_s = H \cos \phi_2 = 2.75 \times 1752.79 = 1.72 \text{kips}$$

Using $n_r = 38$ and $M_s = 61.5$, we get $f_c = 47.5$ psi, $f' = 1,590$ psi, and $f_s = 3,030$ psi.
Live Load and Temperature

Unit stresses due to live loads plus impact and temperature will be calculated assuming elastic action (n = 8). These thrusts and moments are shown in Table 16-17. The effect of rib shortening for the live load was figured separately, i.e., for live load alone, and the accompanying unit stresses are listed in Table 16-18. No rib shortening accompanies temperature variations. Since the thermal coefficients of expansion for steel and concrete are assumed equal, no direct differential stress exists and the direct temperature stress is zero.

### Table 16-18. Elastic Action Plus Plastic Flow*

<table>
<thead>
<tr>
<th></th>
<th>D.L. + flow</th>
<th>L.L. + imp.</th>
<th>Temp.</th>
<th>Shrinkage + flow</th>
<th>Max Σf</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Concrete</td>
<td>Steel</td>
<td>Concrete</td>
<td>Steel</td>
<td>Concrete</td>
</tr>
<tr>
<td>Crown extrados:</td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Direct............</td>
<td>-110</td>
<td>-4,942</td>
<td>-577</td>
<td>-2,960</td>
<td>0</td>
</tr>
<tr>
<td>Rib shortening...</td>
<td>-26</td>
<td>-515</td>
<td>-5</td>
<td>-26</td>
<td>Φ242</td>
</tr>
<tr>
<td>Crown intrados:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Direct............</td>
<td>-110</td>
<td>-4,942</td>
<td>0</td>
<td>+9,650</td>
<td>0</td>
</tr>
<tr>
<td>Rib shortening...</td>
<td>0</td>
<td>+1,190</td>
<td>0</td>
<td>+95</td>
<td>Φ4,370</td>
</tr>
<tr>
<td>Springing extrados:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Direct............</td>
<td>-130</td>
<td>-5,454</td>
<td>0</td>
<td>+9,890</td>
<td>0</td>
</tr>
<tr>
<td>Rib shortening...</td>
<td>0</td>
<td>+2,060</td>
<td>0</td>
<td>+151</td>
<td>Φ8,320</td>
</tr>
<tr>
<td>Springing intrados:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Direct............</td>
<td>-130</td>
<td>-5,454</td>
<td>-435</td>
<td>-2,590</td>
<td>0</td>
</tr>
<tr>
<td>Rib shortening...</td>
<td>-32</td>
<td>-1,080</td>
<td>-5</td>
<td>-32</td>
<td>Φ310</td>
</tr>
</tbody>
</table>

* All stresses are computed assuming that the concrete takes no tension.

Stresses calculated assuming elastic action for all loadings are given in Table 16-19. Note that the final summations for f, of 1,100 psi at the crown and 1,060 psi at the springing are in close agreement with those of 1,120 and 1,000, respectively, computed for the combined loads and appearing shortly before the discussion on Plastic Flow. They differ slightly because of the difficulty of reading design charts for concrete compression members subjected to bending and thrust.

### Table 16-19. Elastic Action Only*

<table>
<thead>
<tr>
<th></th>
<th>D.L. + L.L. + imp.</th>
<th>Temp.</th>
<th>Shrinkage</th>
<th>Max Σf</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Concrete</td>
<td>Steel</td>
<td>Concrete</td>
<td>Steel</td>
</tr>
<tr>
<td>Crown extrados:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Direct............</td>
<td>-712</td>
<td>-1,710</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Rib shortening...</td>
<td>-55</td>
<td>-270</td>
<td>Φ242</td>
<td>Φ1,200</td>
</tr>
<tr>
<td>Crown intrados:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Direct............</td>
<td>0</td>
<td>+3,000</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Rib shortening...</td>
<td>0</td>
<td>+982</td>
<td>0</td>
<td>Φ4,370</td>
</tr>
<tr>
<td>Springing extrados:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Direct............</td>
<td>0</td>
<td>+4,100</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Rib shortening...</td>
<td>0</td>
<td>+1,800</td>
<td>0</td>
<td>Φ8,920</td>
</tr>
<tr>
<td>Springing intrados:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Direct............</td>
<td>-562</td>
<td>-1,750</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Rib shortening...</td>
<td>-70</td>
<td>-396</td>
<td>Φ310</td>
<td>Φ1,760</td>
</tr>
</tbody>
</table>

* All stresses are computed assuming that the concrete takes no tension.
REINFORCED-CONCRETE ARCH BRIDGES

To illustrate the effect of using the gross transformed cross section the unit stresses were recomputed and are compared in Table 16-20. These calculations were based on the formula

\[ f_c = \frac{N}{A_t} \pm \frac{Mc}{I_t} \quad \text{and} \quad f_s = n f_c \left( 1 - \frac{2d'}{d} \right) \]

where \( N \) = normal thrust, lb
\( M \) = bending moment, lb-in.
\( I_t = \frac{1}{12} bd'^2 + (n - 1) A_s \left( \frac{d}{2} - d' \right)^2 \), in.4
\( A_t = bd + (n - 1) A_s \), sq in.
\( n = \frac{E_s}{E_c} \) for elastic action or \( n = n_p = \frac{E_s}{E_c} \frac{R}{1/(\varepsilon + c)} \) for plastic action

<table>
<thead>
<tr>
<th>Table 16-20</th>
</tr>
</thead>
<tbody>
<tr>
<td>Loadings + plastic action</td>
</tr>
<tr>
<td>Net area</td>
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<tr>
<td>Concrete</td>
</tr>
<tr>
<td>Crown:</td>
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<tr>
<td>Extrados:</td>
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<td>Intrados:</td>
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<tr>
<td>Springing:</td>
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<tr>
<td>Extrados:</td>
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<tr>
<td>Intrados:</td>
</tr>
</tbody>
</table>

Note that in all cases the allowable \( f_c < 1,250 \) psi and \( f_s < 22,500 \) psi.

ANALYSIS OF AN UNSYMMETRICAL ARCH

Had the arch of the preceding illustrated design been an unsymmetrical one, the analysis could still have been carried out by the method of the column analogy, but certain modifications would have been necessary. These changes relate to the skew terms for moment of inertia of the elastic area and for the first moment of the moment-area diagram. The skew terms are as follows:

\[ I_{x'} = I_x - I_{xy} \frac{I_{xy}}{I_y} \quad \text{and} \quad I_{y'} = I_y - I_{xy} \frac{I_{xy}}{I_x} \]

\[ M_{x'} = M_x - M_{xy} \frac{I_{xy}}{I_y} \quad \text{and} \quad M_{y'} = M_y - M_{xy} \frac{I_{xy}}{I_x} \]

The final indeterminate moment is then

\[ m_i = \frac{P}{A} + \frac{M_x'x}{I_{x'}} + \frac{M_y'y}{I_{y'}} \]

and the final moment is \( M = m_i - m_i \) as before. Note that \( x \) and \( y \) are still measured along the axes whose origin is at the elastic centroid and whose orientation is usually chosen as horizontal and vertical for convenience. These are not the principal axes. It is possible to use the principal axes to get \( m_i \) but the \( x \) and \( y \) terms must then represent distances measured along the principal axes and the principal moments of inertia must likewise be used instead of the skew terms. The \( M \) terms must also represent
the first moment of the moment area about the principal axes. Since the calculations are more complicated when using the principal axes, the formulas employing the skew terms with horizontal and vertical axes are recommended.

The analysis illustrated in Fig. 16-52 represents the symmetrical arch of Fig. 16-37 with the right 30 ft removed. Table 16-21 shows computations for the skew terms, and final moments for a single concentrated load.
<table>
<thead>
<tr>
<th>Section</th>
<th>Δy</th>
<th>x</th>
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<th>a</th>
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<th>az</th>
<th>αy</th>
<th>αz</th>
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<td>23</td>
<td>24</td>
<td>25</td>
</tr>
</tbody>
</table>

---

Correct to centroid

\[ \frac{\Delta y}{l} = \frac{\Delta s}{s} \]

Correct for asymmetry

\[ \frac{\Delta y}{l} = \frac{\Delta s}{s} \]

---

16-106
CARRY-OVER AND STIFFNESS FACTORS

In the discussion on the design of rigid-frame bridges mention was made of the fact that it is at times necessary to calculate the stiffness and carry-over factors and fixed-end moments for the haunched or arched members of continuous structures. These computations are required whenever the structure is analyzed by the method of moment distribution and when no charts exist from which the factors can be determined.

In the preceding paragraph for the unsymmetrical arch (Fig. 16-52) the computations for fixed-end moments due to a single concentrated load were indicated. As for the stiffness factor it should be remembered that the stiffness of a structural member is defined as the moment at end A necessary to produce unit rotation at end A when end B is fixed. Hence the stiffness factor can be readily determined from the column analogy by loading the elastic area with a unit angular twist at ends A and B separately. Then the stiffness at A is

$$\frac{S_A E I_{cr}}{L} = \frac{1}{A} + \frac{M_s'z}{I_z'} + \frac{M_y'y}{I_y'}$$

where $S_A$ is the stiffness factor in terms of $EI_{cr}/L$. $M_s'$ and $M_y'$ are determined most conveniently by setting up Table 16-22 for $\phi_A = 1$ and $\phi_B = 1$.

<table>
<thead>
<tr>
<th>$\phi_A = 1$</th>
<th>$\phi_B = 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P$</td>
<td>$z$</td>
</tr>
<tr>
<td>+1.0</td>
<td>-39.10</td>
</tr>
<tr>
<td>Correct for dissymmetry</td>
<td>-53.1</td>
</tr>
<tr>
<td>$M_s' = +14.0$</td>
<td>-12.4 = $M_y'$</td>
</tr>
</tbody>
</table>

Hence $S_A \frac{(1)(2.59)}{70} = 1 \frac{1}{20.05} + \frac{(14.0)(-39.1)}{2,318} + \frac{(-12.4)(-24.64)}{335} = 0.726$

$S_A = 19.6$

Likewise $S_B \frac{(1)(2.59)}{70} = 1 \frac{1}{20.05} + \frac{(28.9)(+30.9)}{2,318} + \frac{(-8.72)(+0.945)}{335} = 0.411$

$S_B = 11.1$

The carry-over factor $r_A = \frac{M_B}{M_A}$ due to $\phi = 1$ at A

$$\frac{1}{20.05} + \frac{(14.0)(+30.90)}{2,318} + \frac{(-12.4)(+0.945)}{335} = 0.202 \quad +0.278$$

Similarly, $r_B = \frac{0.202}{0.411} = +0.491$

For straight haunched beams the $y$ terms vanish and the carry-over factors are negative using this column-analogy sign convention.

MULTIPLE ARCHES

Multiple arches may be classified into two types: (1) those whose intermediate supports rest on bedrock or on very short pile-bearing piers and (2) those supported on
intermediate elastic piers. These are illustrated in Fig. 16-53. The former have been much more popular and give the appearance of greater strength. Bridge sites suitable for those on high elastic piers are also well adapted to arches whose springings are close to their foundations, as the dotted lines of Fig. 16-53b indicate.

Multiple-arch analysis would involve no complications if the interior supports or piers were perfectly rigid, i.e., incapable of lateral or angular displacement, for the design would then be restricted to single spans and these could be treated individually. But even arches on bedrock (joint B, Fig. 16-53a) may rotate jointly if cast monolithically, and those on piles (joint C, Fig. 16-53a) may settle and deflect horizontally as well. Short piers on piles may be transformed for purposes of analysis into “equivalent” elastic supports capable of the same movement. For this pseudo member the lower end may be considered as fixed if the piles are driven into suitably hard material or hinged there if not. The soil surrounding the piles may also be assumed as contributing toward the elastic vertical and horizontal resistance of the piles, but such assumptions should be based on data obtained from comprehensive field tests of driven piles. It should be noted here that present specifications for pile foundations usually prohibit inclusion of the added resistance of the soil.

It appears from the foregoing discussion that interaction between arches and displacement of their intermediate supports, whether on actual or transformed elastic piers, should be investigated. Analysis of multiple arches on elastic piers can be carried out conveniently by Professor Cross’s method of moment distribution, but the method must be modified so as to include the balancing of thrusts as well as of moments and of their mutual effects on each other or of the sidewise effects. Another algebraic method is based on generalized slope-deflection equations. An ingenious arrangement of this method proposed by Hrennikoff avoids the solution of more than one pair of simultaneous equations at a time. Both of these methods will be illustrated here and may be used as independent checks. Various other methods based on graphical concepts, on the ellipse of elasticity, etc., may also be used, and these are fully described by McCullough and Thayer in their excellent book on Elastic Arch Bridges published by John Wiley & Sons, Inc.* But no matter which method is used the designer should remember that only the arches and piers are being considered in this analysis and that the superstructure has been neglected. Its effect is discussed in a paragraph which follows.

The example which follows includes horizontal displacement and rotation of the pier tops, but not vertical settlement due to compression of the piers. This effect would be small but could be included in the analysis. Rib shortening has also been omitted but this is most conveniently considered separately as was the case for the single span illustrated previously.

**ANALYSIS BY MOMENT-THRUST DISTRIBUTION**

When thrusts and moments are balanced about the joint, as is usual with this method, the convergence is not particularly rapid and the method somewhat tedious. In order to accelerate the convergence Professor Cross suggests balancing the thrusts and moments about a *neutral point* so located that changes in thrusts accompanying displacement of the joint (pier top) produce no total unbalanced moment about this point and, likewise, changes in thrusts accompanying rotation of the joint about the neutral point produce no total unbalanced thrust. This neutral point may then be defined as the intersection with the vertical through a joint of the resultant of the forces acting through the elastic centroids of the connecting members and which
produce pure translation of the joint. The original position of the three connecting members is shown dotted in Fig. 16-54a. A horizontal force $\Sigma H$ is then applied at the neutral point displacing it $\Delta_{\text{N.PT.}} = 1$ but preventing rotation of this point. Free-body diagrams of each member and the isolated joint showing the thrusts acting are illustrated in Fig. 16-54b.

If no unbalanced moment about the joint is produced by $\Sigma H$,

$$\Sigma M_0 = 0 = H_a y_a - H_b y_b - H_p y_p + y \Sigma H \quad \text{or} \quad y \Sigma H = \Sigma y H \quad (16-8)$$

From the column analogy the moment at the end of a member undergoing pure joint translation is $M_0 = \Delta y / I y = H \Delta$. Hence Eq. (16-8) becomes $y \Sigma (\Delta / I y) = \Sigma y (\Delta / I y)$. The $\Delta$'s cancel, and letting $1/I y = J$, we have $y \Sigma J = \Sigma y J$. Had we taken $\Sigma M_{\text{N.PT.}} = 0$ the equation would have reduced to $\Sigma d J = 0$.

![Diagram](image)

**Fig. 16-54.** The use of the neutral point in moment-thrust distribution.

![Diagram](image)

**Fig. 16-55.** Sign conventions for moment distribution and column analogy.

The proportional part of $\Sigma H$ absorbed by arch $a$ is

$$\frac{H_a}{\Sigma H} = \frac{H_a}{H_a + H_b + H_p} = \frac{\Delta I_{v_a}}{\Delta I_{v_a} + \Delta I_{v_b} + \Delta I_{v_p}} = \frac{J_a}{\Sigma J} = \text{thrust-distribution factor}$$

Figure 16-54c illustrates the effect of applying a moment at the neutral point. Note that a rotation but no translation is produced at this point and hence no unbalanced thrust is created. The proportional moment absorbed by arch $a$ will be $k_a/2k$ where $k =$ stiffness factor for each member $= \text{moment at the neutral point necessary to produce unit rotation at the neutral point, the far end of the member being fixed (Fig. 16-54d).}$

The necessary factors involved in the moment and thrust distribution may be obtained from the column analogy or from Chart 16-II for piers and Chart 16-XXII for symmetrical arches. The sign conventions used in the column analogy and in the charts differ from those used in moment distribution. Figure 16-55 illustrates this difference. Note that for moment distribution:

1. $+M$ acts clockwise on the joint.
2. $+H$ acts to the right on the joint.
3. Moment-distribution factor \( k/\Sigma k \) and thrust-distribution factors \( J/\Sigma J \) are negative.

4. Carry-over factors for moment in arches are usually negative. They become positive for very flat arches, i.e., for straight members like beams and piers.

5. Carry-over factors for thrust are negative and equal to unity.

The factors are obtained as follows (Fig. 16-56):

For the Symmetrical Arch

\[
A = \int \frac{ds}{EI} = \frac{L}{EI_e} \times \frac{1 + m}{2}
\]

\[
I_z = \int x^2 \frac{ds}{EI} = \frac{L}{EI_e} \frac{1 + 3m}{48}
\]

\[
I_y = \int y^2 \frac{ds}{EI} = \frac{L}{EI_e} \frac{r^2}{C}
\]

(get \( C \) from Chart 16-XXII)

\[
\bar{y} = r - y_c \quad \text{(get} \ y_c/r \text{from Chart 16-XXII)}
\]

Locate the neutral point of the joint so that \( y_2 J - \Sigma \bar{y} J = 0 = \Sigma d J \). Thrust stiffness \( J = 1/\bar{I}_y \). Carry-over factor (for thrust) \( = -1 \).

\[
\text{Moment stiffness} = \frac{1}{A} + \frac{L}{4I_z} \frac{d_a^2}{\bar{I}_y} = \frac{d_a(d_a + d_a')J}{J}
\]

\[
\frac{d_a'}{d_a} = \frac{(1/A) - (L^2/4I_z)}{(1/A) + (L^2/4I_z)}
\]

The carry-over factor for moment \( r = \frac{(1/A) - (L^2/4I_z) + (d_a d_b/\bar{I}_y)}{(1/A) + (L^2/4I_z) + (d_a^2/\bar{I}_y)} \)

For the Unsymmetrical Arch

The factors listed above must be altered somewhat for the unsymmetrical case (Fig. 16-57). The thrust line for the distributed thrusts is now inclined at an angle \( \tan^{-1} I_{xy}/I_x \) and \( d' \) and \( d'' \) are measured from it. From the column analogy, then

\[
A = \int \frac{ds}{EI}
\]

\[
I_z = \int x^2 \frac{ds}{EI}
\]

\[
I_y = \int y^2 \frac{ds}{EI}
\]

\[
I_{xy} = \int xy \frac{ds}{EI}
\]

\[
J' = \frac{1}{\bar{I}_y'}
\]

where \( I_y' = I_y - I_{xy}^2/I_x \). Also \( I_z' = I_z - I_{xy}^2/I_y \).

\[
d_a d_a' = (1/A) \left( x_a^2/I_x' \right)
\]

\[
d_a'' = (1/A) \left( x_a x_b/I_x' \right) - \left( x_a^2/I_x' \right)
\]

\[
k = d_a(d_a + d_a')J'
\]

and the carry-over factor is identical with the formula for the symmetrical case. The neutral point should be located so that \( \Sigma d J' = 0 \).
For the Pier

Values for \( A \) and \( I_y \) (Fig. 16-58) may be determined from the column analogy as for arches, but it will be more convenient to use the coefficients of Chart 16-II. Then, taking \( r_A \) and \( r_B \) as positive,

\[
\begin{align*}
\dot{y} &= \frac{L}{S_B(1 + r_B)} + \frac{1}{S_A(1 + r_A)} \\
\frac{1}{A} &= \frac{E I_A}{L} - \frac{\dot{y}^2}{I_y} \\
I_y &= \frac{S_B(1 + r_B) + S_A(1 + r_A) EI_A}{L^2} \\
I_y' &= 0
\end{align*}
\]

Fig. 16-58. Notation for pier.

Procedure

The procedure followed in the moment-thrust distribution is as follows:

Step 1. Determine the fixed-end moments \( M_F \) and thrusts \( H_F \) in the arch due to the applied loads or deformations. These reactions exist at the springing and may be found from Charts 16-XIV to 16-XIX.

Step 2. Distribute the unbalanced thrusts at each joint to the connecting members in proportion to their resistance to horizontal displacement \( J/2J \) and write down that carried over to the far end of the member.

The thrust carried over = \((-1)(-J/2J) = +J/2J\)

Continue until all joints are balanced and find the total thrust distributed.

Step 3. Convert these distributed thrusts into moments about the neutral point by multiplying by \( d_a \). Transfer the original \( M_F \) to the neutral point and find the moment at the neutral point due to \( H \) acting at the springing. Add these three moments.

Step 4. Distribute the unbalanced moments about each neutral point to each of the connecting members in proportion to their resistance to rotation \( k/2k \), multiply this by the carry-over factor \( r \), and write down the moment carried over at the far end of the member. Continue the operation until all points are balanced and find the total moment distributed.

Step 5. Convert these distributed moments to thrusts by dividing by \((d_a - d_a')\).

Step 6. Repeat steps 2 to 5 until the desired accuracy is attained.

Step 7. Determine the total change in thrust due to distributing thrusts. Multiply this by \( \dot{y} \) to get the moment at the springing. Determine the total thrust converted from the moments. Multiply this by \((\dot{y} - d_a')\) and by \((\dot{y} + d_a')\) to get the moments at the springing. Add these moments to \( M_F \) to get the final moments at the springing. Add the thrust due to distributing thrust \( (2\Delta H) \) and that converted from the moments \( (2H) \) to \( H_F \) to get the final thrusts.

The necessary factors involved in the foregoing procedure may be obtained from the column analogy or from Chart 16-II for piers and from Chart 16-XXII for symmetrical arches.

Illustrated Example

A four-span structure (Fig. 16-59) will be chosen so as to illustrate the method of analysis without making the problem either too simple, as for a three-span structure, or too involved, as for five or more spans. Usually continuous-arch spans are odd in number, principally for aesthetic reasons. The problem illustrated here indicates all the calculations (Figs. 16-59 and 16-60) for a single load applied in the second span. Actually the design of a complete bridge would entail inclusions of loadings in all spans. These can be handled most conveniently by first determining the final
**Fig. 16-59. Computation of moment and thrust distribution factors for a four-span arch structure.**
<table>
<thead>
<tr>
<th>Line</th>
<th>Moment and thrust distribution</th>
<th>Outer span</th>
<th>Loaded inner span</th>
<th>Inner span</th>
<th>Outer span</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>d</td>
<td>9.56</td>
<td>16.67</td>
<td>20.50</td>
<td>20.16</td>
</tr>
<tr>
<td>2</td>
<td>d+d'</td>
<td>-19.42</td>
<td>2.489</td>
<td>24.62</td>
<td>24.62</td>
</tr>
<tr>
<td>3</td>
<td>d-d'</td>
<td>7.88</td>
<td>17.79</td>
<td>17.50</td>
<td>17.50</td>
</tr>
<tr>
<td>4</td>
<td>y</td>
<td>9.56</td>
<td>7.11</td>
<td>13.39</td>
<td>6.77</td>
</tr>
<tr>
<td>5</td>
<td>y+d'</td>
<td>7.88</td>
<td>17.78</td>
<td>17.85</td>
<td>17.85</td>
</tr>
<tr>
<td>6</td>
<td>y-d'</td>
<td>-12.31</td>
<td>0.0477</td>
<td>+0.0477</td>
<td>+0.1176</td>
</tr>
<tr>
<td>7</td>
<td>( \gamma ) J/\Sigma J</td>
<td>0.0477</td>
<td>0.009</td>
<td>+0.0378</td>
<td>-0.0378</td>
</tr>
<tr>
<td>8</td>
<td>( \gamma ) J/k\k</td>
<td>-0.009</td>
<td>-0.0300</td>
<td>-0.300</td>
<td>-0.300</td>
</tr>
<tr>
<td>9</td>
<td>( \gamma ) J/k\k</td>
<td>0.0378</td>
<td>0.009</td>
<td>+0.0378</td>
<td>-0.0378</td>
</tr>
<tr>
<td>10</td>
<td>Fixed end ( H_F )</td>
<td>-0.338</td>
<td>-7.08</td>
<td>+7.08</td>
<td>-7.08</td>
</tr>
<tr>
<td>11</td>
<td>Total ( \Delta H )</td>
<td>-0.338</td>
<td>-7.08</td>
<td>+7.08</td>
<td>-7.08</td>
</tr>
<tr>
<td>12</td>
<td>Convert to M at Npt ( \Delta H \times d )</td>
<td>-2.686</td>
<td>+16.44</td>
<td>-6.048</td>
<td>+0.762</td>
</tr>
<tr>
<td>13</td>
<td>Fixed end M at sprg ( H_F ) ( x y )</td>
<td>-49.56</td>
<td>-25.76</td>
<td>+47.92</td>
<td>+47.92</td>
</tr>
<tr>
<td>14</td>
<td>M at Npt due to ( H_F ) ( x y )</td>
<td>-50.4</td>
<td>447.92</td>
<td>+16.44</td>
<td>-2.686</td>
</tr>
<tr>
<td>15</td>
<td>Total M at Npt ( M_F )</td>
<td>-2.686</td>
<td>-83.52</td>
<td>+16.112</td>
<td>+0.762</td>
</tr>
<tr>
<td>16</td>
<td>Total ( \Delta M )</td>
<td>-7.102</td>
<td>-19.40</td>
<td>+4.83</td>
<td>+0.01</td>
</tr>
<tr>
<td>17</td>
<td>Convert to H ( \Delta M \times (d-d') )</td>
<td>-0.9013</td>
<td>+0.0007</td>
<td>-0.1041</td>
<td>-0.0050</td>
</tr>
<tr>
<td>18</td>
<td>Convert to M at Npt ( \Delta H \times d )</td>
<td>-0.3181</td>
<td>-0.2452</td>
<td>-0.0290</td>
<td>-0.0002</td>
</tr>
<tr>
<td>19</td>
<td>Convert to H ( \Delta M \times (d-d') )</td>
<td>-0.0004</td>
<td>-0.0004</td>
<td>+0.0004</td>
<td>+0.0004</td>
</tr>
<tr>
<td>20</td>
<td>H due to translation of the N pt</td>
<td>-0.0004</td>
<td>-0.0004</td>
<td>+0.0004</td>
<td>+0.0004</td>
</tr>
<tr>
<td>21</td>
<td>( \Sigma H ) due to H carried over ( \Sigma (11) )</td>
<td>-0.3054</td>
<td>+0.3054</td>
<td>-0.3054</td>
<td>+0.0324</td>
</tr>
<tr>
<td>22</td>
<td>( \Sigma H ) due to H distributed</td>
<td>0.0000</td>
<td>-0.0000</td>
<td>+0.0000</td>
<td>+0.0000</td>
</tr>
<tr>
<td>23</td>
<td>( \Sigma H ) due to rotation of the N pt</td>
<td>-0.3054</td>
<td>+0.3054</td>
<td>-0.3054</td>
<td>+0.0324</td>
</tr>
<tr>
<td>24</td>
<td>( \Sigma H ) due to M carried over ( \Sigma (17) )</td>
<td>-0.3054</td>
<td>+0.3054</td>
<td>-0.3054</td>
<td>+0.0324</td>
</tr>
<tr>
<td>25</td>
<td>( \Sigma H ) due to M carried over ( \Sigma (25) )</td>
<td>-0.3054</td>
<td>+0.3054</td>
<td>-0.3054</td>
<td>+0.0324</td>
</tr>
<tr>
<td>26</td>
<td>( \Sigma H ) due to M distributed ( \Sigma (25) )</td>
<td>-0.3054</td>
<td>+0.3054</td>
<td>-0.3054</td>
<td>+0.0324</td>
</tr>
<tr>
<td>27</td>
<td>( \Sigma H ) due to M distributed ( \Sigma (25) )</td>
<td>-0.3054</td>
<td>+0.3054</td>
<td>-0.3054</td>
<td>+0.0324</td>
</tr>
<tr>
<td>28</td>
<td>Fixed end H ( \Sigma (23)+\Sigma (27)+\Sigma (28) )</td>
<td>-1.2392</td>
<td>+1.2392</td>
<td>-4.9445</td>
<td>+4.9445</td>
</tr>
<tr>
<td>29</td>
<td>Final H ( \Sigma (23)+\Sigma (27)+\Sigma (28) )</td>
<td>-1.2392</td>
<td>+1.2392</td>
<td>-4.9445</td>
<td>+4.9445</td>
</tr>
<tr>
<td>30</td>
<td>M at springing due to ( \Sigma (23)+\Sigma (27)+\Sigma (28) )</td>
<td>-1.2392</td>
<td>+1.2392</td>
<td>-4.9445</td>
<td>+4.9445</td>
</tr>
<tr>
<td>31</td>
<td>( \Sigma H ) due to M carried over ( \Sigma (31) )</td>
<td>-0.3054</td>
<td>+0.3054</td>
<td>-0.3054</td>
<td>+0.0324</td>
</tr>
<tr>
<td>32</td>
<td>( \Sigma H ) due to M carried over ( \Sigma (31) )</td>
<td>-0.3054</td>
<td>+0.3054</td>
<td>-0.3054</td>
<td>+0.0324</td>
</tr>
<tr>
<td>33</td>
<td>( \Sigma H ) due to M distributed ( \Sigma (31) )</td>
<td>-0.3054</td>
<td>+0.3054</td>
<td>-0.3054</td>
<td>+0.0324</td>
</tr>
<tr>
<td>34</td>
<td>Fixed end H ( \Sigma (23)+\Sigma (27)+\Sigma (28) )</td>
<td>-1.2392</td>
<td>+1.2392</td>
<td>-4.9445</td>
<td>+4.9445</td>
</tr>
<tr>
<td>35</td>
<td>Final M at springing ( \Sigma (23)+\Sigma (27)+\Sigma (28) )</td>
<td>-1.2392</td>
<td>+1.2392</td>
<td>-4.9445</td>
<td>+4.9445</td>
</tr>
</tbody>
</table>

**Fig. 10-60. Distribution of moments and thrusts for a four-span arch structure.**
moments and thrusts due to unit moments applied at the ends of the various individual members in turn. The process must then be repeated for the application of unit thrusts at the ends of the various members in turn. This method was explained previously for continuous rigid-frame bridges, and the pattern is similar when the method is applied to arches except that thrusts as well as moments must be included.

**ANALYSIS BY MOMENT DISTRIBUTION**

The analysis of fixed-end arch moments follows the usual procedure of balancing and distributing followed in moment distribution. As was the case for the multiple-span rigid frame corrections must be made for sidesway of the piers. This is accomplished by first assuming the joints held against lateral motion but free to rotate, balancing and distributing the fixed-end moments due to loads or distortions such as temperature and shrinkage effects about the joints, and calculating the unbalanced horizontal forces at each pier top. Each joint capable of translation is then displaced a unit amount, and the fixed-end moments at each end of every connecting member at the joint in question are determined with the far ends fixed. The fixed-end moments resulting from the displacement of one joint are then balanced over the entire structure by moment distribution, and the unbalanced forces at each joint are determined as before. This step is repeated for each joint capable of moving laterally. Simultaneous equations are then written at each such joint equating the unbalanced forces to zero as follows:

Let \( H_{ip} \) = the unbalanced force at joint \( i \) due to the applied loads with all joints fixed against translation

\[ H_{ij} \] = the unbalanced force at joint \( i \) due to a unit displacement at joint \( j \), with the other joints fixed against translation

Then for joint \( i \), \( \Sigma F_x = 0 = H_{ip} + aH_{ii} + bH_{ij} + cH_{ik} + \cdots \)
Then for joint \( j \), \( \Sigma F_x = 0 = H_{ip} + aH_{ji} + bH_{jj} + cH_{jk} + \cdots \)
Then for joint \( k \), \( \Sigma F_x = 0 = H_{ip} + aH_{ki} + bH_{kj} + cH_{kk} + \cdots \)

The final moments are then determined by proportion after the coefficients \( a, b, c \), etc., are calculated. Hence,

\[ M_i = M_{ip} + aM_{ii} + bM_{ij} + cM_{ik} + \cdots \text{ etc.} \]

The method will now be illustrated by reworking the four-span series analyzed by moment and thrust distribution. Column analogy will be used to determine the stiffness factors, carry-over factors, fixed-end moments, and thrusts accompanying horizontal displacement, etc.

**Outer Arch**

From the left-hand tabulation of Fig. 16-59, \( A = 66.6, \ I_z = 10,131, \ I_y = 598. \) Applying a clockwise rotation \( \phi \) at the right end with the left end fixed requires a moment of

\[ M_R = \phi \left( \frac{1}{A} + \frac{L_z}{4I_z} + \frac{\phi^2}{I_y} \right) = \phi_R \left( \frac{1}{66.6} + \frac{25^2}{10,130} + \frac{9.56^2}{598} \right) = -0.2295\phi_R \]

Note that a \(+\phi_R\) in moment-distribution sign convention and a \(+\phi_R\) in column-analogy sign convention are both clockwise, and that the moments and thrusts have the same sign only at the right end (Fig. 16-55). Signs are best determined by inspection.

\[ M_L = \phi \left( \frac{1}{A} - \frac{L_z}{4I_z} + \frac{\phi^2}{I_y} \right) = \phi_R \left( \frac{1}{66.6} - \frac{25^2}{10,130} + \frac{9.56^2}{598} \right) = -0.1061\phi_R \]

for column analogy but \(+0.1061\phi_R\) for moment distribution and slope deflection.
Hence the stiffness is \( k = 0.2295 \) and the carry-over factor is
\[
r = \frac{M_L}{M_R} = \frac{0.1061}{0.2295} = 0.4628
\]
The sign of \( r \) will be negative in the moment-distribution sign convention. Similarly,
\[
d' = \left( \frac{1}{A} + \frac{L^2}{4I_z} \right) \frac{I_y}{g} = 4.80 \text{ ft} \\
H_R = \frac{M_R}{\bar{y} + d'} = \frac{-0.2295\phi_R}{14.36} = -0.01598\phi_R \\
H_L = -H_R = +0.01598\phi_R
\]
Similarly, for \( +\Delta_R \),
\[
M_R = \frac{\Delta \bar{y}}{I_y} = \frac{9.56}{5.98} \Delta = -0.01598\Delta \\
H_R = \frac{M_R}{\bar{y}} = \frac{\Delta}{I_y} = -0.001672\Delta \\
M_L = +0.01598\Delta \\
H_L = +0.001672\Delta
\]
Outer Pier

From Fig. 16-59, \( \frac{1}{A} = 1.058 \), \( \bar{I}_y = 31.1 \), \( \bar{y} = 9.05' \).

For \( +\phi_T \),
\[
M_{\text{top}} = \phi \left( \frac{1.058 + \frac{9.05}{31.1}}{31.1} \right) = -3.693\phi \\
M_{\text{bot.}} = \phi \left[ \frac{1.058 - \frac{9.05(10.95)}{31.1}}{31.1} \right] = -2.129\phi \\
H_{\text{top}} = \frac{M_T + M_B}{h} = \frac{5.822}{20} = 0.2910\phi = -12.686 \text{ ft}
\]
For \( +\Delta_T \),
\[
M_T = \frac{9.05}{31.1} \Delta = +0.2910\Delta \\
M_B = \frac{10.95}{31.1} \Delta = +0.3521\Delta \\
H_T = \frac{0.6431}{20} = -0.03215\Delta
\]
Similar factors for all the other members are listed in lines 2, 4, 6, 8, and 16 of the solution shown in Fig. 16-61, but those of lines 8 and 16 were multiplied by 100 to avoid using an excessive number of decimal places.

**ANALYSIS BY SLOPE DEFLECTION**

The method of slope deflection requires the solution of as many simultaneous equations as there are unknown joint rotations and translations. The same sign convention as was used in moment distribution will be continued. The procedure is as follows:

**Determination of rotation and translation factors.**
1. Determine the moment \( M_\phi \) and the thrust \( H_\phi \) at each end of every member due only to a rotation of \( +\phi \) radians of the right end (or top).
2. Repeat step 1 for a rotation of \( +\phi \) radians of the left end.
3. Determine the moment \( M_\Delta \) and the thrust \( H_\Delta \) at each end of every member due only to a translation of \( +\Delta \) ft of the right end (or top).
4. Repeat step 3 for a translation of \( +\Delta \) ft of the left end.
5. Determine the fixed-end thrusts \( H_F \) and moments \( M_F \) on the loaded members.
### REINFORCED-CONCRETE ARCH BRIDGES

<table>
<thead>
<tr>
<th>$P$</th>
<th>Outer span</th>
<th>Outer pier</th>
<th>Inner span</th>
<th>Inner pier</th>
<th>Inner span</th>
<th>Outer span</th>
<th>Outer pier</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$y$</td>
<td>9.56'</td>
<td>9.05'</td>
<td>13.39'</td>
<td>12.97'</td>
<td>13.39'</td>
<td>9.05'</td>
</tr>
<tr>
<td>2</td>
<td>$d'$</td>
<td>4.80'</td>
<td>-3.64'</td>
<td>6.73'</td>
<td>6.73'</td>
<td>5.63'</td>
<td>6.73'</td>
</tr>
<tr>
<td>3</td>
<td>$d'$+d'</td>
<td>14.36'</td>
<td>-12.69'</td>
<td>20.12'</td>
<td>20.12'</td>
<td>-18.60'</td>
<td>20.12'</td>
</tr>
<tr>
<td>4</td>
<td>$k$</td>
<td>0.2295</td>
<td>0.2295</td>
<td>3.693</td>
<td>0.4502</td>
<td>0.4502</td>
<td>3.693</td>
</tr>
<tr>
<td>5</td>
<td>$\sqrt{a}$</td>
<td>0</td>
<td>0.0025</td>
<td>0.8445</td>
<td>0.1030</td>
<td>0.1277</td>
<td>0.7446</td>
</tr>
<tr>
<td>6</td>
<td>$\sqrt{e}$</td>
<td>-0.4628</td>
<td>0.0576</td>
<td>-0.4628</td>
<td>-0.4628</td>
<td>0.4618</td>
<td>-0.5758</td>
</tr>
<tr>
<td>7</td>
<td>$\sqrt{d}$</td>
<td>0</td>
<td>0.0043</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- **Fixed and $M_y$**
  - CO Moment
  - $0$  $0$  $0$  $-4.56'$  $-25.76'$  $0$  $0$  $0$

- **$M_y$+d'$**
  - $-1.245$  $+2.690$  $+43.264$  $+257.77$  $+3.611$  $+21.056$  $+3.611$  $+21.056$

- **Final M at joint**
  - $-1.245$  $+2.690$  $+43.264$  $-45.953$  $-24.508$  $-21.056$  $+3.532$  $-1.498$

- **COM to pier bottom**
  - $-24.911$  $-12.682$  $-12.682$

- **Final M at $+$**
  - $0$  $+1.979$  $+3.007$  $+0.007$

- **Final H at joint**
  - $-10.219$  $+6.035$

- **Fixed and $M_y$**
  - COM
  - $1.5098$  $-1.598$  $-29.100$  $-2.239$  $+22.39$  $0$  $0$  $0$

- **$M_y$+$\Delta M$**
  - $-1.596$  $-29.100$  $-2.035$  $-3.452$  $0$  $+0.010$  $+0.001$

- **COM to pier bottom**
  - $+2.217$  $-2.935$  $+7.591$  $+4.565$  $+3.010$  $+2.578$  $+0.432$  $+0.163$

- **Final M at $+$**
  - $0$  $+33.510$  $0$

- **Final M at $-$**
  - $0$  $-22.926$  $-1.577$

- **Final H at joint**
  - $0$  $+0.057$  $+0.393$  $+0.0271$

- **Fixed and $M_y$**
  - COM
  - $0$  $+2.339$  $-2.239$  $+14.097$  $-2.239$  $+22.39$  $0$  $0$

- **$M_y$+$\Delta M$**
  - $0$  $+3.056$  $-3.452$  $0$  $+0.016$  $+0.134$

- **COM to pier bottom**
  - $0$  $+18.512$  $0$

- **Final M at $+$**
  - $0$  $+1.373$

- **Final M at $-$**
  - $0$  $-1.373$

- **Final H at joint**
  - $0$  $+0.0043$  $+0.001$

- **Fixed and $M_y$**
  - COM
  - $0$  $+0.1873$  $+0.1873$  $+0.044$  $+0.0628$  $+0.3959$  $+0.0628$  $+0.1873$

- **$M_y$+$\Delta M$**
  - $0$  $+0.100$  $-0.007$  $-0.0394$  $-0.0271$

- **Final M at $+$**
  - $0$  $-1.123$

- **Final M at $-$**
  - $0$  $-1.123$

- **Final H at joint**
  - $0$  $-1.123$

- **Line 11**
  - $-1.245$  $+2.690$  $+43.264$  $-45.953$  $-24.508$  $-21.056$  $+3.532$  $-1.498$

- **Line 12**
  - $-0.293$  $-0.611$  $-9.844$  $-10.499$  $-13.903$  $-27.806$  $-13.903$

- **Line 13**
  - $-0.293$  $-0.611$  $-9.844$  $-10.499$  $-13.903$  $-27.806$  $-13.903$

- **Line 14**
  - $-0.293$  $-0.611$  $-9.844$  $-10.499$  $-13.903$  $-27.806$  $-13.903$

- **Left pier joint**
  - $+10.286$  $-2.0652$  $+0.3931$  $+0.0271c$  $+0.3737$

- **Right**
  - $+0.2792$  $+0.0271c$  $+0.3931$  $-0.0271c$  $+0.3737$

- **Line 15**
  - $+0.293$  $-0.611$  $-9.844$  $-10.499$  $-13.903$  $-27.806$  $-13.903$

- **Line 16**
  - $+0.293$  $-0.611$  $-9.844$  $-10.499$  $-13.903$  $-27.806$  $-13.903$

- **Line 17**
  - $+0.293$  $-0.611$  $-9.844$  $-10.499$  $-13.903$  $-27.806$  $-13.903$

**By similar proportioning the M of the pier bottom and H at the joints can be found.**

**Fig. 16-61.** Moment distribution and sideways for a four-span arch structure.
The values of these moments and thrusts are shown on free-body diagrams of the arches and joints in Fig. 16-62. After the structure responds to the applied load, rotations and horizontal displacements will occur at each joint, but the joints will be in equilibrium. Hence $\Sigma M = 0$ and $\Sigma H = 0$ for each joint. These summations, the resulting six equations, the values of $\phi$ and $\Delta$ which satisfy them, and the final moments and thrusts are shown in Fig. 16-63. The six equations were solved by a relaxation technique. The final moments (and thrusts) were determined by multiplying the proper values of $\phi$ and $\Delta$ by the coefficients for $M$ (and $H$) in Fig. 16-62 at a given end of any member and adding these effects.

**COMPARISON OF THE ANALYTICAL METHODS**

A comparison of the final moments and thrusts as determined by the three methods illustrated indicates excellent agreement for the major values, though some deviation
### End M and H at joint

<table>
<thead>
<tr>
<th>Load</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>M</td>
<td>H</td>
<td>M</td>
</tr>
<tr>
<td>$\phi_B$</td>
<td>+0.2295</td>
<td>+0.01598</td>
<td>-0.2084</td>
</tr>
<tr>
<td></td>
<td>+0.4502</td>
<td>+0.02239</td>
<td></td>
</tr>
<tr>
<td></td>
<td>+3.693</td>
<td>-0.2910</td>
<td></td>
</tr>
<tr>
<td></td>
<td>+4.3727</td>
<td>-0.2526</td>
<td></td>
</tr>
<tr>
<td>$\Delta_B$</td>
<td>-0.01598</td>
<td>-0.001672</td>
<td>+0.02239</td>
</tr>
<tr>
<td></td>
<td>-0.02239</td>
<td>-0.001672</td>
<td></td>
</tr>
<tr>
<td></td>
<td>+0.2910</td>
<td>-0.03215</td>
<td></td>
</tr>
<tr>
<td></td>
<td>+0.2526</td>
<td>-0.03549</td>
<td></td>
</tr>
<tr>
<td>$\phi_C$</td>
<td>-0.2084</td>
<td>-0.02239</td>
<td>+0.4502</td>
</tr>
<tr>
<td></td>
<td>+0.4502</td>
<td>+0.02239</td>
<td></td>
</tr>
<tr>
<td></td>
<td>+2.624</td>
<td>-0.1410</td>
<td></td>
</tr>
<tr>
<td></td>
<td>+3.5244</td>
<td>-0.09622</td>
<td></td>
</tr>
<tr>
<td>$\Delta_C$</td>
<td>+0.02239</td>
<td>+0.001672</td>
<td>-0.02239</td>
</tr>
<tr>
<td></td>
<td>-0.02239</td>
<td>-0.001672</td>
<td></td>
</tr>
<tr>
<td></td>
<td>+0.1410</td>
<td>-0.010870</td>
<td></td>
</tr>
<tr>
<td></td>
<td>+0.09622</td>
<td>+0.014241</td>
<td></td>
</tr>
<tr>
<td>$\phi_D$</td>
<td>-0.2084</td>
<td>-0.02239</td>
<td>+0.4502</td>
</tr>
<tr>
<td></td>
<td>+0.2295</td>
<td>+0.01598</td>
<td></td>
</tr>
<tr>
<td></td>
<td>+3.693</td>
<td>-0.2910</td>
<td></td>
</tr>
<tr>
<td></td>
<td>+4.3727</td>
<td>-0.2526</td>
<td></td>
</tr>
<tr>
<td>$\Delta_D$</td>
<td>+0.02239</td>
<td>+0.001672</td>
<td>-0.02239</td>
</tr>
<tr>
<td></td>
<td>-0.01598</td>
<td>-0.001672</td>
<td></td>
</tr>
<tr>
<td></td>
<td>-0.2910</td>
<td>+0.03215</td>
<td></td>
</tr>
<tr>
<td></td>
<td>+0.2526</td>
<td>-0.03549</td>
<td></td>
</tr>
<tr>
<td>$P=10^k$</td>
<td>-49.56</td>
<td>-7.08</td>
<td>-25.76</td>
</tr>
</tbody>
</table>

### Equilibrium equations

\[
\begin{align*}
\sum M_B &= 0 = +4.3727 \phi_B + 0.2526 \Delta_B - 0.2084 \phi_C + 0.02239 \Delta_C - 49.56 \\
\sum H_B &= 0 = -0.2526 \phi_B - 0.03549 \Delta_B - 0.2239 \phi_C + 0.01672 \Delta_C - 7.08 \\
\sum M_C &= 0 = -0.2084 \phi_B + 0.02239 \Delta_B + 3.5244 \phi_C + 0.09622 \Delta_C \\
\sum H_C &= 0 = 0.02239 \phi_B + 0.001672 \Delta_B - 0.2526 \phi_C + 0.014241 \Delta_C \\
\sum M_D &= 0 = -0.2084 \phi_C + 0.02239 \Delta_C + 4.3727 \phi_D + 0.2526 \Delta_D \\
\sum H_D &= 0 = -0.2239 \phi_C + 0.001672 \Delta_C - 0.2526 \phi_D - 0.03549 \Delta_D \\
\end{align*}
\]

Solving the six equations above:

$\phi_B = 33.3239$, $\phi_C = 0.024513$, $\phi_D = -5.49508$

$\Delta_B = -417.233$, $\Delta_C = +412.3546$, $\Delta_D = +58.5535$

### Final moment and thrust

<table>
<thead>
<tr>
<th></th>
<th>M</th>
<th>H</th>
</tr>
</thead>
<tbody>
<tr>
<td>$AB$</td>
<td>-10.205</td>
<td>-1.230</td>
</tr>
<tr>
<td>$BA$</td>
<td>+14.315</td>
<td>+1.230</td>
</tr>
<tr>
<td>$BB'$</td>
<td>+1.650</td>
<td>+3.717</td>
</tr>
<tr>
<td>$BC$</td>
<td>-15.978</td>
<td>-4.946</td>
</tr>
<tr>
<td>$CB$</td>
<td>-51.290</td>
<td>+4.946</td>
</tr>
<tr>
<td>$CC'$</td>
<td>+58.078</td>
<td>-4.479</td>
</tr>
<tr>
<td>$CD$</td>
<td>-6.787</td>
<td>-0.469</td>
</tr>
<tr>
<td>$DC$</td>
<td>+5.453</td>
<td>+0.469</td>
</tr>
<tr>
<td>$DD'$</td>
<td>+3.254</td>
<td>-0.283</td>
</tr>
<tr>
<td>$DE$</td>
<td>-2.197</td>
<td>-0.186</td>
</tr>
<tr>
<td>$ED$</td>
<td>+1.519</td>
<td>+0.186</td>
</tr>
</tbody>
</table>

![Fig. 16-63. Slope-deflection analysis.](image)

is noticeable for the small values. This would not be serious in design, for it must be remembered that this problem covered the case of only one loading.

The method of moment and thrust distribution about a neutral point avoids the solution of any simultaneous equations and yields results directly in terms of moments and thrusts. The author prefers this method over all others.

The method of moment distribution about the joints with subsequent sidesway corrections requires the solution of as many simultaneous equations as there are joints. These could prove tedious of solution if their number were large. Furthermore, they must be solved completely before the final moments can be determined by proportioning the various effects.

The method of slope deflection requires twice as many equations as that of moment distribution plus final determination of the moments, etc., from the final values of $\phi$ and $\Delta$ determined from them. If an electronic computer is available, however, the work would be reduced enormously. Furthermore, only those terms involving the fixed-end moments and thrusts need be changed for any other loading.

Unit moments and unit thrusts can be applied to each end of every member and the final effects resulting therefrom determined by any of the three methods. Final moments can then be determined by proportion as was done in Fig. 16-5a and Table 16-1 for the rigid-frame bridge.

The variation in results will necessarily vary with the shape and form of the arches and of their relative stiffness compared with the piers. Studies indicate, except for relatively thin long piers, that the percentage decrease in fixed-end thrust due to translation of the joints is fairly constant for arch-rise to pier-height ratios greater than one and amounts to less than 10 per cent. Similarly the decrease in per cent of fixed-end thrusts due to rotation is fairly constant and is roughly twice that due to translation.

**SKewed BRIDGES**

If the center line of the roadway and the face of the bridge abutments intersect at anything other than a right angle, the structure is said to be a skewed bridge. Skew crossings are the rule rather than the exception, but skewed bridges account for only about 14 per cent of the total rigid frames built. On single-span structures it is possible to retain a right bridge without altering the alignment of the bridge and the axis of the stream or other highway being crossed, but such design requires a longer span and is seldom economical except for very slight skew angles. On the multiple-span structures the scour around the intermediate piers prohibits such design. In any case the reduction of sight distance by bending the roadway into an S shape so that right bridges of shorter spans might be employed is to be condemned as criminal and economically unwise. Many such designs are now being replaced merely because of their obsolescence.

The mathematical analysis outlined so far in this section is applicable only to right rigid-frame bridges and arches. If skewed rigid-frame slabs or skewed barrel arches are required, more inclusive analyses which incorporate torsional effects should be used. It is possible for a designer to avoid the more complicated analysis required of barrel or slab skewed bridges by using ribbed decks for rigid frames or open-spandrel construction on separate arch ribs. Unless the skew angle is excessive the analysis can then be carried out for the individual T section of the rigid frame or for the rib of the arch just like the calculations for a bridge without skew. Care should be exercised, however, to keep the floor thickness small relative to the rib spacing to avoid interaction and torsional effects.

The pioneering work of J. C. Rathbun on the analysis of skewed arches involved the solution of six rather involved simultaneous equations for the arch reactions. Weiner simplified this analysis by dropping certain terms of secondary importance. Hodges and Barron secured results based on a comparison of skewed with right frames. Baron and Michalos developed an analysis for space structures based on the shear and torsion analogy. Later Michalos applied this method to skewed frames and arches. It yields results identical with Rathbun's and others, and is a
tabular solution similar to the column analogy developed by Prof. Hardy Cross.\(^4\) (Also see references 23, 24, and 25.)

In the shear and torsion analogy the structure is first made statically determinate as in the column analogy. This produces errors in the geometry of the deflected structure. These are then corrected by the addition of some distribution of moments without disturbing the statical equilibrium. Similarly, the final moments in any con-
Correction shears and moments at any section of the structure:

\[ \begin{align*}
\nu_{xi} &= \nu_{xc} \\
\nu_{yi} &= \nu_{yc} \\
\nu_{zi} &= \nu_{zc} \\
m_{xi} &= m_{xc} + \nu_{zc} \nu_{cz} - \nu_{yc} \nu_{cy} \\
m_{yi} &= m_{yc} + \nu_{xc} \nu_{cx} - \nu_{zc} \nu_{cz} \\
m_{zi} &= m_{zc} + \nu_{yc} \nu_{cy} - \nu_{xc} \nu_{cx}
\end{align*} \]

Note:
Terms such as \( \nu_{cz} \) represent centroidal distances to a section of the structure.

(a) Statical relationships

State the relationship with respect to centroidal axes. (Courtesy of Dr. Michalos,\textsuperscript{21})

(b) Geometrical relationships

Rotations and displacements due to corrective moments:

\[ \begin{align*}
\sum d\theta_{x0} &= \sum m_{x0} \delta_{axx} + \sum m_{y0} \delta_{axy} \\
\sum d\theta_{x0} &= \sum m_{y0} \delta_{axy} + \sum m_{x0} \delta_{axy} \\
\sum d\theta_{z0} &= \sum m_{z0} \delta_{azz} \\
\delta_{xc0} &= \sum z_{cx} \delta_{x0} - \sum y_{cx} \delta_{z0} \\
\delta_{yc0} &= \sum x_{cy} \delta_{x0} - \sum z_{cy} \delta_{x0} \\
\delta_{zco} &= \sum z_{z0} \delta_{x0} - \sum x_{z0} \delta_{z0}
\end{align*} \]

Continuous elastic ring in space can be obtained from the following equations:

\[ \begin{align*}
m_s &= m_{x0} - m_{xi} \\
V_s &= V_{x0} - V_{xi} \\
m_y &= m_{y0} - m_{yi} \\
V_y &= V_{y0} - V_{yi} \\
m_z &= m_{z0} - m_{zi} \\
V_z &= V_{z0} - V_{zi}
\end{align*} \] \hspace{1cm} (16-9)

The subscripts refer to the original assumed and to the corrective distributions. The directions of these vectors and sign convention are shown in Fig. 16-64.* Their statical and geometrical relationships are given in Fig. 16-65. In Fig. 16-65b,

\[ \begin{align*}
da_{ss} &= -\frac{ds}{K_{ss}} \\
da_{sy} &= \frac{ds}{K_{sy}} \\
da_{as} &= \frac{ds}{K_{as}} \\
da_{ay} &= \frac{ds}{K_{ay}} \\
da_{sv} &= \frac{ds}{K_{sv}} \\
da_{sv} &= \frac{ds}{K_{sv}} \\
da_{ys} &= \frac{ds}{K_{ys}} \hspace{1cm} (16-10)
\end{align*} \]

* The author is indebted to Dr. James P. Michalos for permission to use Figs. 16-64 to 16-67 taken from one of his papers.\textsuperscript{21}
The terms $da_{xz}$ and $da_{zy}$ represent angle changes in a length $ds$ about the $x$ axis due to unit moments applied about the $x$ and $y$ axes, respectively. The $K$ terms are stiffness factors or the moment required to produce unit angle changes. It will be found that $K_{xy} = K_{yx}$. These factors are related to those about the principal axes as follows:

$$K_{xx} = \frac{K_{n1}K_{l}}{K_{n1} \cos^2 \alpha + K_{l} \sin^2 \alpha}$$

$$K_{yy} = \frac{K_{n1}K_{l}}{K_{n1} \sin^2 \alpha + K_{l} \cos^2 \alpha}$$

$$K_{zz} = K_{n2}$$

$$K_{xy} = \frac{(K_{n1} - K_{l}) \sin \alpha \cos \alpha}{K_{n1}K_{l}}$$

(16-11)

where

$$K_{n1} = EI_1$$

$$K_{n2} = EI_2$$

and where $\alpha$ = slope of the element projected on the $xy$ plane (Fig. 16-64)

$E$ = modulus of elasticity as generally defined

$G$ = modulus of rigidity, or modulus of elasticity in shear

$I_1$ = moment of inertia of a cross section about the section's major principal axis

$I_2$ = moment of inertia of a cross section about the section's minor principal axis

$J$ = a torsional factor such that $GJ$ is the moment, about the axis of twist, necessary to deform a segment a unit angle per unit length about the same axis

The geometry of the structure requires that $\Sigma d\theta_z = \Sigma d\theta_x - \Sigma d\theta_y$, $\delta_z = \delta_{xc} - \delta_{xx}$, etc. The solution of these six equations yields the following relationships for the corrective shears and moments:

$$v_{x'i} = M_{x'i}$$

$$v_{y'i} = M_{y'i}$$

$$v_{z'i} = M_{z'i}$$

$$m_{x'i} = P_{x'i} + \frac{M_{y'i}A_{x'i}}{J_{y'}} + \frac{M_{y'i}z_{xy}}{J_{y'}}$$

$$m_{y'i} = P_{y'i} + \frac{M_{y'i}A_{y'i}}{J_{y'}} + \frac{M_{y'i}z_{ys}}{J_{y'}}$$

$$m_{z'i} = P_{z'i} + \frac{M_{y'i}z_{xy}}{J_{y'}} - \frac{M_{y'i}y_{ca}}{J_{y'}}$$

(16-12)

In these equations the $P'$ terms represent rotations due to assumed distributions of moment, and are the total analogous forces. The $M'$ terms represent displacements along centroidal axes, due to the assumed distributions of moment, and are the total analogous moments. The $A'$ terms and the $J'$ terms represent rotations and displacements, respectively, resulting from unit moments and forces applied along centroidal axes, and may be considered as analogous areas and moments of inertia.

All terms are summarized in Fig. 16-66.

Illustrative Example

A hinged skewed frame of constant cross section supporting a uniform load is illustrated in Fig. 16-67. All calculations are shown except those for $(i_x)n$, etc. These are similar to those used in the column analogy. For instance,

$$(i_x)n + y^2 da_{xz} = \frac{1}{12} da_{xz}(\text{length})^3 + y^2 da_{xz}$$

$$= \frac{1}{12}(0.42 \times 10^{-11})(12)^3 + 6^2(0.42 \times 10^{-11}) = 0.20(10)^{-7}$$

Remarks

The analysis described above is applicable to any continuous elastic ring in space subjected to loads or distortions in any direction. The method will yield results identical with those of any other method based on the same assumptions. However, the conversion of the shears and moments into unit stresses, related as they are to the torsional factor $J$, may not have any simple relationship for wide skewed decks. An "exact" analysis based on the differential equation governing thin-plate theory
**Fig. 16-66. General procedure for analysis of skewed frames and arches.**  
(Courtesy of Dr. Michalos.)
would be required for such a case. It has been developed but has not yet been published.\textsuperscript{9}

**LATERAL LOADS**

The effect of lateral loads on arches has received little attention in the engineering literature until recently.\textsuperscript{18} Seismic loads have been omitted generally in structural design until recently and even now receive consideration only in those geographical regions where earthquakes have been active. Wind loads on arches have been neglected to some extent, also mainly because their lateral effect on short wide arch spans is negligible, especially when the stiffening effect of the superstructure is considered. However, wind stresses amounting to 40 per cent of the total stress were reported for the 426-ft Ammer Arch Bridge near Echelsbach, Germany.\textsuperscript{18} It is quite evident that wind and especially seismic loadings could contribute very materially to the stresses, principally for the longer spans. The AASHO Standard Specifications for Highway Bridges require the application of lateral forces to the structure, with those due to wind and earthquake considered as acting in any horizontal direction, and it is advisable to investigate their effect thoroughly.

Plane structures subjected to lateral loads can be analyzed easily by the shear and torsion analogy described under Skewed Bridges. The general relationships of the various terms shown in Figs. 16-68 and 16-69 are simplified versions of those for skewed rings of Fig. 16-66. Figure 16-70 illustrates the analysis of a parabolic arch subjected to a single unsymmetrical load.*

An analysis of the effects of haunching, rise ratio, variation in bending to torsional stiffness, and connecting struts on arches subjected to lateral loads was made by James P. Michalos.\textsuperscript{18} Figures 16-71 and 16-72† illustrate the effect of bending to torsional stiffness. Haunching increases $m_y$ at the support by about 10 per cent over the value of $0.313L$ for a straight beam of constant cross section. Hence a fairly good approximation for stiffness conditions can be obtained for preliminary design purposes by using the results for fixed-end beams.

The effects of flexible and stiff struts, used for lateral bracing between parallel-arch ribs, on the moments and shears due to lateral loads are shown in Figs. 16-73 and 16-74. Stiff bracing reduces the large moments at the springing and crown materially; it generally also reduces the torsional moments but may increase moments at about the quarter points.

**HINGED AND FIXED ARCHES**

Reinforced-concrete arches should be designed so as to distribute the material in the most efficient manner, and the arch axis will then be confined necessarily to follow the equilibrium polygon of the loading. Generally the axis follows the string polygon of the dead load only, or of the dead plus half live load, whichever produces the smallest bending stresses under combined loads. Longitudinal, seismic, and lateral (wind) forces; and forces due to distortion like thermal changes, shrinkage, plastic flow, and elastic rib shortening all affect the thrusts somewhat and the bending stresses to a considerable extent in plane fixed arches. Usually the stresses produced by these distortions amount to only about 10 to 20 per cent of the total, and these combined with the live-load flexural stresses may total approximately 60 per cent. Were it possible to relieve part of the live-load bending and all the stress due to distortion by the introduction of hinges, some economy would be effected but the total cost of the bridge would probably not be reduced more than a few per cent.\textsuperscript{4} However, when foundations are not rigid or for long spans, especially those above 300 ft with low rise ratios, or where clearance requirements on the through type of arch limit the width


\[ v_z = v_{zo} - v_{zi} \]
\[ m_x = m_{xo} - m_{xi} \]
\[ m_y = m_{yo} - m_{yi} \]
\[ v_{zi} = \frac{M_z}{J_z} \]
\[ m_{xi} = \frac{P_x'}{A_x} + \frac{M_x}{J_z} y_c \]
\[ m_{yi} = \frac{P_y'}{A_y} - \frac{M_y}{J_z} x_c \]
\[ J_z = I_x + I_y - 2I_{xy} \]
\[ I_x = \Sigma y^2 da_{xx} - \overline{y}^2 A_x \]
\[ A_x' = A_x - \frac{A_{xy}A_{xy}}{A_y} \]
\[ A_y' = A_y - \frac{A_{xy}A_{xy}}{A_x} \]
\[ I_y = \Sigma x^2 da_{yy} - \overline{x}^2 A_y \]
\[ I_{xy} = \Sigma xy da_{xy} - \overline{x}\overline{y} A_{xy} \]
\[ da_{xx} = \frac{ds}{K_{xx}} \]
\[ da_{yy} = \frac{ds}{K_{yy}} \]
\[ da_{xy} = \frac{ds}{K_{xy}} \]
\[ K_{xx} = \frac{K_{nt}K_t}{K_{nt} \cos^2 \alpha + K_t \sin^2 \alpha} \]
\[ K_{yy} = \frac{K_{nt}K_t}{K_{nt} \sin^2 \alpha + K_t \cos^2 \alpha} \]
\[ K_{xy} = \frac{K_{nt}K_t}{(K_{nt} - K_t) \sin \alpha \cos \alpha} \]
\[ P_x' = P_x - \frac{A_{xy}}{A_y} P_y \]
\[ P_y' = P_y - \frac{A_{xy}}{A_x} P_x \]
\[ P_x = \Sigma m_{xo} da_{xx} + \Sigma m_{yo} da_{xy} \]
\[ P_y = \Sigma m_{yo} da_{yy} + \Sigma m_{xo} da_{xy} \]
\[ M_z = M_{xx} - M_{yy} \]
\[ M_{xx} = \Sigma y (m_{xo} da_{xx} + m_{yo} da_{xy}) - \overline{y} P_x \]
\[ M_{yy} = \Sigma x (m_{yo} da_{yy} + m_{xo} da_{xy}) - \overline{x} P_y \]
\[ x_c = x - \overline{x} \]
\[ \overline{x} = -\frac{C_y'}{A_y} \]
\[ C_y' = C_y - \frac{A_{xy}}{A_x} C_x \]
\[ y_c = y - \overline{y} \]
\[ \overline{y} = -\frac{C_x'}{A_x} \]
\[ C_x' = C_x - \frac{A_{xy}}{A_y} C_y \]

(a) General relationships

Fig. 16-68. Summary for structures lying in a plane and loaded normal to their plane
(Courtesy of Dr. Baron and Dr. Michalos,26)
Elastic area = \( \frac{1}{K_{xx}} \cdot da_{xx} \)

\[ i_x = \frac{1}{12} da_{xx} L_y^2 \]
\[ i_y = 0 \]
\[ i_{xy} = 0 \]

Elastic area = \( \frac{1}{K_{yy}} \cdot da_{yy} \)

\[ i_x = 0 \]
\[ i_y = \frac{1}{12} da_{yy} L_x^2 \]
\[ i_{xy} = 0 \]

Elastic area = \( \frac{1}{K_{xy}} \cdot da_{xy} \)

\[ i_x = 0 \]
\[ i_y = 0 \]
\[ i_{xy} = \frac{1}{12} da_{xy} L_x L_y \]

Note: \( K_{xy} \) is positive if the segment slopes \( \rightarrow \), and negative if it slopes \( \leftarrow \); therefore, \( i_{xy} \) for a symmetrical structure is not equal to zero.

(b) Convenient relationships of finite elastic areas

Fig. 16.68 (Continued)
Fig. 16-69. Tabular form for analysis of plane structures subjected to lateral loads.  *(Courtesy of Dr. Baron and Dr. Michalos,)*
Fig. 16-70. Parabolic arch of varying cross section with single unsymmetrical load. (Courtesy of Dr. Baron and Dr. Michalos,\textsuperscript{16})
and necessitate deep ribs, decided economies might be effected; in fact, savings up to 25 per cent have been reported on some arches built in France.

The elimination of much of the stress due to distortion has been accomplished by using either temporary or permanent construction hinges or by some form of stress compensation. In the early attempts to articulate concrete arches, hinges of the

\[ \frac{P_1}{L/4} = L \]

Distribution of \( m_y \)

\[ \frac{K_n}{K_t} \] varies from 1.0 at the crown to 0.6 at the springing

\[ \frac{P_1}{L/4} = L \]

Distribution of \( m_x \)

\[ \frac{K_n}{K_t} \] varies from 1.0 at the crown to 0.6 at the springing

\[ \frac{P_1}{L/4} = L \]

Distribution of \( m_y \)

\[ \frac{K_n}{K_t} \] varies from 100 at the crown to 25 at the springing

Fig. 16-71. Effects of variations in \( K_n/K_t \) along the length of an arch rib on distribution of moments. (Courtesy of Dr. Michalos.)

Steel-roller or pin type were employed, but their high cost, the difficulty experienced in aligning them during construction or of replacing them afterward, and the probability of corrosion taking place all militated against their use. Two types of quasi hinges were developed about the turn of the century which overcame many of the disadvantages of the early types. They were invented by two Frenchmen, Considère and Mesnager.

The Considère type (Fig. 16-75) consists essentially of straight (or slightly curved) bars closely spaced and embedded in concrete with or without spiral reinforcing. It
is somewhat more flexible and probably a little sturdier when spiral steel is used than the Mesnager type. Because of its weakness to shear, especially that due to unsymmetrical live loads, it is used only as a temporary hinge, the loose ends of the extradosal main reinforcing bars being wrapped with stirrups before the opening is keyed in with concrete and the bridge subjected to live loads. Considère used average working stresses on the reduced cross section of the hinge equal to the ultimate compressive strength of the plain concrete. Tests reported by McCullough and Thayer indicate ultimate strengths for these hinges at about 13,000 psi for concrete whose ultimate strength at the same age of 60 to 90 days ranged from 6,000 to 6,400 psi. These hinges had a 4- by 4-in. core, the diameter of the spiral reinforcing being 3.5 in.

<table>
<thead>
<tr>
<th>Values of $K_n$ (relative)</th>
<th>Values of $K_n/K_T$</th>
<th>Values of $h/L$</th>
<th>Distance of unit load from A</th>
<th>Values of $m_{YA}$ ($-$)</th>
<th>Values of $m_{YA}$ ($+$)</th>
<th>Values of $v_A$ ($+$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Varies from 1.00 at crown to 1.76 at springing</td>
<td>Varies from 1.00 at crown to 0.60 at springing</td>
<td>0.15</td>
<td>0.25 L</td>
<td>0.586 h</td>
<td>0.150 L</td>
<td>0.852</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.50 L</td>
<td>0.500 h</td>
<td>0.136 L</td>
<td>0.500</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.75 L</td>
<td>0.164 h</td>
<td>0.048 L</td>
<td>0.148</td>
</tr>
<tr>
<td>Varies from 1.00 at crown to 1.76 at springing</td>
<td>Varies from 1.00 at crown to 0.60 at springing</td>
<td>0.45</td>
<td>0.25 L</td>
<td>0.540 h</td>
<td>0.124 L</td>
<td>0.806</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.50 L</td>
<td>0.500 h</td>
<td>0.125 L</td>
<td>0.500</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.75 L</td>
<td>0.210 h</td>
<td>0.068 L</td>
<td>0.194</td>
</tr>
<tr>
<td>Varies from 1.00 at crown to 2.00 at springing</td>
<td>Varies from 1.00 at crown to 25 at springing (barrel arch)</td>
<td>0.15</td>
<td>0.25 L</td>
<td>0.611 h</td>
<td>0.177 L</td>
<td>0.877</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.50 L</td>
<td>0.500 h</td>
<td>0.166 L</td>
<td>0.500</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.75 L</td>
<td>0.139 h</td>
<td>0.050 L</td>
<td>0.123</td>
</tr>
</tbody>
</table>

Fig. 16-72. Effects of haunching on moments and shears. (Courtesy of Dr. Michalos.)

Variations of longitudinal steel from 6 to 14 per cent of the cross-sectional area, and of spiral reinforcing from 3.25 to 6.50 per cent of the volume of the core did not affect the ultimate strengths of the hinge noticeably. On the Grant's Pass Bridge over the Rogue River in Oregon, McCullough and Thayer incorporated them at each skewback and crown, using a design stress of 3,900 psi, the dead-load crown thrust being 403,000 lb.

The Mesnager hinge has enjoyed considerable popularity in Europe especially for smaller spans and for arch roofs. One disadvantage in the longer spans seems to be the difficulty of getting the required steel into the allowable space. The Mesnager type is illustrated in Fig. 16-76 and differs from the Considère hinge in the way the longitudinal steel crosses itself at an angle, thus enabling this type to develop considerable resistance against shear and thrust but to offer little resistance to rotation. Usually the Mesnager type is not keyed in but the reinforcing is encased in a small concrete core to prevent corrosion and the recess above the core is covered to prevent ice and dirt from accumulating therein. Analysis of stresses in the Mesnager hinge and results of tests were reported by Parsons and Stang. A solution of their formula in chart form was developed by Admiral Moreell to facilitate design.

The method of stress compensation used to relieve the arch rib of much of the flexural stress was developed by Freyssinet, another French engineer. In this system
<table>
<thead>
<tr>
<th>Case</th>
<th>Unit load at C</th>
<th>Shears and moments at end A</th>
<th>Shears and moments at station 2</th>
<th>Shears and moments at station 4</th>
<th>Shears and moments at station 6</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>v</td>
<td>$m_0$</td>
<td>$m_t$</td>
<td>v</td>
<td>$m_0$</td>
</tr>
<tr>
<td><strong>I</strong></td>
<td>2</td>
<td>0.422</td>
<td>9.490</td>
<td>-0.311</td>
<td>0.422</td>
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<tr>
<td></td>
<td>4</td>
<td>0.338</td>
<td>7.266</td>
<td>-0.149</td>
<td>0.338</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>± 0.250</td>
<td>5.698</td>
<td>0.120</td>
<td>± 0.250</td>
</tr>
<tr>
<td><strong>II</strong></td>
<td>2</td>
<td>0.407</td>
<td>14.797</td>
<td>-1.076</td>
<td>0.407</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>0.331</td>
<td>12.095</td>
<td>-0.349</td>
<td>0.331</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>± 0.250</td>
<td>9.187</td>
<td>0.644</td>
<td>± 0.250</td>
</tr>
<tr>
<td><strong>III</strong></td>
<td>2</td>
<td>0.368</td>
<td>13.299</td>
<td>-0.009</td>
<td>0.368</td>
</tr>
<tr>
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<td>4</td>
<td>0.300</td>
<td>10.611</td>
<td>-0.004</td>
<td>0.300</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>± 0.250</td>
<td>9.034</td>
<td>0.004</td>
<td>± 0.250</td>
</tr>
<tr>
<td><strong>IV</strong></td>
<td>2</td>
<td>0.425</td>
<td>8.312</td>
<td>-0.257</td>
<td>0.425</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>0.342</td>
<td>6.772</td>
<td>0.060</td>
<td>0.342</td>
</tr>
<tr>
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<td>6</td>
<td>± 0.250</td>
<td>4.993</td>
<td>0.407</td>
<td>± 0.250</td>
</tr>
</tbody>
</table>

* Shears and moments in an arch braced with very stiff struts. Values are for a single rib.  
** Shears and moments in an arch braced with very flexible struts.  

Fig. 16-73. Summary of computed shears and moments in arches braced with very stiff struts and with very flexible struts (different ratios of rise to span and bending stiffness are considered). (Courtesy of Dr. Michalos.)
a set of hydraulic jacks is inserted in the rib, usually at the crown, and as pressure is applied to the jacks, they lengthen the arch axis. The actual separation of the two sections of the rib is made equal to the calculated compression due to elastic rib shortening, shrinkage, and early plastic flow, and thus relief from all such bending stresses is effected. When locating the jacks it is well to use four, placing one near each corner of the rib, so that definite moments as well as thrusts may be applied during the centering and later for final adjustment. Where the cross sections of the rib are small it may be necessary to increase the width of the rib to provide a seat for the jacks, but this bracket can be treated architecturally and left for possible future need. As is the case with Considère hinges, recesses should be provided at the top and bottom of the rib so that enough of the main steel may be exposed to facilitate final welding before the opening is keyed in. The Freyssinet method has been used in Europe on single-span and multiple-arch bridges up to spans of 612 ft. In the United States the Oregon Highway Department has used it on a multiple-arch bridge on which each of the seven spans is 230 ft long.
The use of temporary or permanent hinges, or of the Freyssinet method for the relief of stress due to distortion, is in reality a means for relieving strains due to shrinkage, etc. The relationship between these stresses and strains depends upon the elasticity of the concrete only initially, for the stress produced by adjustments to the rib during construction are materially reduced later by plastic flow. The plasticity of the concrete is a function of stress, time, quality of the mix, and temperature, and it decreases as the concrete ages. Hence the sooner arches are decentered, the smaller will be the stress in the concrete due to distortion after several years of service, but the higher will be the stress in the steel. The ability of the concrete to deform plastically obviates the necessity of any artificial stress compensation in most instances. The methods are probably most useful in facilitating decentering and allowing for excessive sagging in very flat arches.

**EFFECTS OF SUPERSTRUCTURE**

Usually arch bridges have been designed neglecting the effect of superstructure and assuming that the rib carries the load. Evidently such analysis is sound in general for there have been no failures of structures so designed of which the author is aware. Cases are on record, however, describing cracks in spandrel walls over piers and at bases of columns in open-spandrel structures, and of openings and general disintegration and spalling around construction and expansion joints, all of which decreased service life although structural failure did not occur. There is little doubt, then, that the superstructure does affect the stresses in the arch rib. For the filled-spandrel type, the retaining walls required to hold the earth fill react in an indeterminate fashion even when piers are taken to incorporate expansion and construction joints in the walls and between the walls and ribs, respectively. An attempt to "control" this interaction is evident in the following extract from the 1953 Standard Specifications for Highway Bridges issued by AASHO.

**Spandrel Walls for Arches:** When the spandrel walls of the filled spandrel arch exceed 8 feet in height above the extrados, they shall be designed as vertical slabs supported by transverse diaphragm walls or deep counterforts. Vertical cantilever walls over 8 feet in height, or counterforts having a back slope of less than 45° with the vertical, shall not be used, on account of the excessive and indeterminate stresses set up in the arch ring by torsion.

**Expansion Joints in Spandrel Walls:** Vertical expansion joints shall be placed in the spandrel walls of the arches to provide for movement due to temperature change and arch deflection. These joints shall be placed at the ends of spans and at intermediate points, generally not more than 50 feet apart.

In the open-spandrel arch the uncertainty regarding interaction between rib and superstructure also exists, but with this type it is easier to provide relief from overstressing by some form of articulation if an analysis indicates this to be a necessity. For single-span structures an analytical treatment is probably less time-consuming than so-called approximate analyzes or than investigations by means of mechanical models. For the multiple-span structure, especially those on elastic piers, the analytical treatment is likely to prove very involved and a study by means of models with instruments like the Beggs deformeter is probably the only feasible way to make a
thorough analysis. McCullough and Thayer report on the results of a very exhaustive study that was made on a model of the Rogue River Bridge in Oregon. This is a three-span open-spandrel arch with the two central supports on elastic piers. They found that sufficient relief from overstress at the base of the columns was afforded by incorporating expansion joints in the deck over each of the two central piers and over each crown and by constructing hinges (quasi) at the footings of two columns in each span, there being eight spandrel openings over each arch ring. Naturally all this articulation reduced the relative stiffness of the bridge, for the only way in which moments induced in one span could now be distributed to an adjoining one was through ribs and piers, but it also relieved highly overstressed columns.

In the design it is well to remember that, if the entire structure is monolithic in nature, any loading favors the rib at the expense of the superstructure, that is, the superstructure serves to stiffen the bridge, if the rib was first designed to carry the load and the superstructure not counted on except to transmit the load to the rib. Pouring schedules may affect the interaction between rib and superstructure considerably. For instance, if the columns and deck are poured after the centering is struck, the superstructure will be unable to relieve the dead-load stresses but usually these comprise only about 35 per cent of the total. It will probably increase temperature stresses.

Generally aesthetic considerations require that all columns have the same thickness. Their cross section thus depends upon the height of the longest column, as well as upon any direct stress, but usually they are not designed to resist moment.

For the through or half-through type of arch bridge the hangars are not likely to prove so troublesome as the columns as far as the effect of superstructure is concerned. The hangars are, in general, not so stiff since no L/r requirements need be met for pure tension members and their smaller cross section necessarily increases their flexibility, unless they are very short.

For the longer spans, the live loads become relatively less important and the deck and columns can be rather flexible relative to the rib. Hence the effect of interaction is considerably reduced and may be neglected.

If, then, the superstructure is counted on to assist the rib, the structure should be designed more like a Vierendeel girder. If the rib is assumed to carry the load, the deck should then be provided with expansion joints so that structural action will follow analysis with some degree of accuracy.

REFERENCES

5. Whitney, C. S.: Plain and Reinforced Concrete Arches, Report of Committee 312, American Concrete Institute, September, 1940.
29. Unpublished Memorandum, Mar. 26, 1954, Committee 314, American Concrete Institute.
Section 17

CONCRETE SKIN STRUCTURES

By

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SKIN STRUCTURES AND CONSTRUCTION

Skin structures may be considered as including any construction having the principal strength and stiffness of the structure furnished by means of forces acting in the plane of the enclosing surface. Examples range from such small-scale structural elements as light-gage corrugated-metal plates, commonly used as siding and decking

17-1
in conventional construction, to such major structures as the familiar long-span Orly hanger of cross section diagrammatically illustrated in Fig. 17-1.

Skin construction can be readily adapted to structures having supports located in the walls because of requirements for clear interior space, such as chimneys, tanks, and cooling towers; structures in which the shape and function of the structure provide enclosing surfaces which may also be readily adapted to carry loads as well as acting as curtain walls, such as the hulls of ships and barges, bins, hoppers, airplanes, floating drydocks, and in many cases of building construction; structures which must resist intense lateral loads, such as blast-resistant buildings; and long-span roof structures.

The design of skin structures consists of arranging available surfaces so that normal load forces acting perpendicular to the skin can be carried by direct stresses acting within the plane of the skin.

While steel, aluminum, and other materials can be used to form skin construction, concrete is simpler to adapt to this purpose than most other materials because of the ease with which the fluid concrete can be cast into complex shapes which then act as rigid integral units when the concrete sets. Curved or folded thin concrete slabs automatically provide rigidity and load-carrying capacity which will result in the use of a minimum quantity of materials and elimination of the necessity for closely spaced columns. The latter feature greatly improves the architectural arrangement of the structure.

Types of Skin Construction

Skin construction may be formed either by joining flat plates, as in cellular and hipped plate construction, or by curving the surfaces as in single- and double-curved shells.

Cellular Construction

Examples of cellular construction occur in concrete ships and floating drydocks (Fig. 17-2) and box-frame building construction (Fig. 17-3).

While cast-in-place concrete docks and ships have been used for many years, their excellent durability characteristics have been offset by a clumsiness and lack of maneuverability caused by their relatively large dead weight. In recent years, however, this disadvantage has been circumvented by the ingenious techniques using precast units developed largely by the United States Navy. In this method, thin sections 1 1/2 to 2 in. in thickness are cast as boxes and slabs using a few standardized forms held in such a manner that the large areas of the boxes or slabs are poured in the flat. This construction method obviates the necessity for pouring concrete between closely spaced form walls and facilitates obtaining dense waterproof sections regardless of the final thickness of the skin. The boxes and slabs are then assembled and joined to form the major components of the ship or dock.

Bridges are another group of structures susceptible to treatment as cellular skin structures.

In bridges, as in floating structures, dead-load weight and forming costs are impor-

* Superior numbers refer to the references at the end of this section.
tant factors in determining the economy of the structure, and in these structures, as well as in floating structures, precasting techniques seem to offer substantial advantages. While precast sections have been largely used for the smaller bridges, there is no reason why larger sections could not be used equally well.

Skin construction of the cellular type has been utilized to a limited extent in this country and to a large extent in Europe as framing for buildings. In these structures flat-plate floors are supported by thin reinforced-concrete walls which replace the conventional masonry partition walls. Box-frame construction of this type, while somewhat inflexible to future changes in room arrangements, becomes surprisingly economical if the formwork can be reused on successive floors. Besides initial construction economy, this construction offers appreciable resistance to lateral loads and is one of the simplest and most economical types of structures to adapt to blast- and shock-resistant construction.

Analysis of Cellular Construction

The analysis of cellular structures is similar to the analysis used for conventional structures. Using the pontoon decks and transverse bulkheads of a floating drydock as a typical example of cellular construction, the exterior skin will distribute local surface loads to the transverse frames and bulkheads by means of typical one-way or two-way slab action. As the frame or bulkhead deflects under these local loads, the skin in the top and bottom pontoon decks will act as flanges in the manner anticipated by simple-beam theory. The resistance to change in curvature of the combined web member and skin flange will depend on the combined moment resistance developed by the frame or bulkhead plus that due to the shearing force occurring between the frame and the top and bottom skins as illustrated in Fig. 17-4 and described as follows:

The resistance of the web or bulkhead strain is

\[ M_B = \frac{2fI}{d} \]  

(17-1)

where \( f \) = extreme fiber stress,
\( I \) = moment of inertia of the web
\( d \) = depth of the web

The resistance due to the shear between the skin and the bulkhead is

\[ M_s = \int_0^x (V_f d) \, dx \]  

(17-2)

The total moment resistance will be

\[ M = \frac{2fI}{d} + \int_0^x (V_f d) \, dx \]  

(17-3)

and the vertical shearing force will be

\[ V = \frac{dM}{dz} = \left( \frac{2I}{d} \right) \frac{df}{dz} + V_f d \]  

(17-4)

In ordinary simple bending theory the assumption is made that plane sections remain plane after bending and that the fiber bending stresses throughout the section will be proportional to the distance from the neutral axis. However, in cellular structures this assumption would require a condition of equal strain over the width of skin acting as the flange of the frame. Several limitations to this assumption must be kept in mind when designing light skin sections.

1. Design of the skin in the vicinity of the stress initiation should conform to standard concrete design practice in which local cracking dictates the selection of the effective width of the skin acting as flange to the web. This width will be either
FIG. 17-2. Concrete floating drydock. (Courtesy of Ammann and Whitney, Consulting Engineers, New York.)
Fig. 17-2 (Continued)
sixteen or six times the slab thicknesses, depending on whether both or one side of the web is flanged.

2. Away from the point of stress initiation the stress will be more uniform across the flange, but shearing strain across the width of the flange skin may cause an appreciable drop in stress at some distance from the supporting webs. The reduction in stress between the supports caused by shear strain is indicated in Fig. 17-5.

To determine the loss in stress over the flange, the sum of the change in shear and change in direct stress on each element of width across the skin is equated to zero. As both the direct and shearing stress are functions of strain, strain relationships may be substituted for the shear and direct stress terms, thus furnishing a differential equation of displacement. Solution of the differential equation and substitution of the boundary conditions will provide the distribution of stress across the width and along the length of the skin. The ratio of total load carried by the straining skin to the total load that would be carried by a rigid skin represents the efficiency of the skin.

3. If the distance between the side or edge supports is large and the skin thickness is small, the skin may buckle before developing the stresses computed by the bending theory. The critical buckling stresses for a flat plate may be written as

\[
f_{cr} = K \frac{x^2 EI^2}{12b^2(1 - \mu^2)}
\]  

(17-5)

where 
- \( t \) = thickness of the plate
- \( a \) = length of the plate between loaded supports parallel to the direction of the edge load
- \( b \) = width of the plate
- \( K \) = function of \( a/b \) and the number of waves in a buckled sheet
- \( E \) = modulus of elasticity
- \( \mu \) = Poisson's ratio

Values of \( K \) for various boundary conditions are listed in standard textbooks on elastic stability. For the most common case of a flat slab simply supported along all edges, the critical buckling stress will be

\[
f_{cr} = \frac{x^2 EI^2}{12b^2(1 - \mu^2)} \left( \frac{a}{bn} + \frac{bn^2}{a} \right)
\]

(17-6)
where \( a \) = width between supports in a direction perpendicular to the direction of the load

\( b \) = distance between loaded supports

and for \( f_{cr} \) = minimum, \( n = a/b \) = an even integer; \( K = 4 \). Though developed for use in the design of metal plates which better suit the design assumptions, this equation may also be used for the design of concrete plates, except that arbitrarily high factors of safety should be used because of the nonhomogeneous nature of the concrete material, the reduced modulus of concrete under long-time loads and/or high stress, and the abrupt failure which might be expected at the buckling load. Test results dealing with critical buckling loads in thin concrete plates are described by Ernst,3 based on tests performed at the University of Nebraska.

Referring to the example of the floating drydock, loads causing curvature of the transverse frames or webs will introduce compressive stress in the skin forming the top and bottom deck of the pontoon due to the shear developed at the juncture of the skin and web. The magnitude of the shearing forces will be such as to force the skin to conform to the curvature of the web member at the line of juncture between the web and the skin. Because of this joint action of the two connecting members, the total vertical displacement will depend partly on the flexural strain of the web and skin and partly on the shearing strain in the web.

The distribution of stress in the skin is indicated in Fig. 17-6a. At line 1, the stresses due to \( P_1 \) are mainly concentrated at the web member or within sixteen times the slab thickness. However, these stresses distribute rapidly across the skin and
at succeeding lines nearer the center line of the dock the total stress will consist of a
fairly uniform stress from previous concentrations plus local concentrations at that
line.

The skin is also effective in resisting torsional loads caused by quartering seas,
unsymmetrical dock loads, or other loading conditions. Torsional forces such as $T$
in Fig. 17-7a will cause a rotation of the cross section as shown in Fig. 17-7c and bend-
ing in the planes of the skin as shown in Fig. 17-7b. This deformation will be resisted
by curvature of each of the four sides in their own plane and by shearing forces due to
elongation or shortening of the connecting planes as shown in Fig. 17-7b. The resis-
tance to bending of each of the four planes is similar to the resistance to flexure

![Diagram of cross section and bending](image)

**Fig. 17-7.** Torsional forces cause bending and deformation in pontoon decks.

of combined webs and flanges as described previously. The torque would thus be
resisted by the curvature of each skin as a web plus shearing forces due to the joining
skins acting as flanges of the straining web members, or

$$T = V_1d + V_2b$$  \hspace{1cm} (17-7)

where $T$ is the applied torque and $V_1$ and $V_2$ are the total shears on the two sides
caused by the applied torque.

The value of $V$ found previously may be substituted in the above equation, giving

$$T = \frac{bd}{6} (t_1b - t_2d) \frac{df}{dx} + 2b dV_f$$  \hspace{1cm} (17-8)

where $t$ is the thickness of the respective sides. When the areas of the two sides are
equal, then the tension and compression term drops out and

$$T = 2b dV_f$$  \hspace{1cm} (17-9a)

$$T = 2A \cdot V_f$$  \hspace{1cm} (17-9b)

where $A$ is the area formed by the median lines.

As

$$V_f = ts$$  \hspace{1cm} (17-10)

then

$$s = \frac{T}{2At}$$  \hspace{1cm} (17-11)

where $s$ is the shear per unit area.

The same action can be utilized to simplify conventional framing throughout the
dock structure. For example, the rigid vertical frames and bulkheads usually spaced
at approximately 6-ft centers along the dock wing walls can be replaced by relatively
small members carrying hydrostatic loads using the over-all cellular action of the wing
wall to distribute the collected loads to the pontoon structure (Fig. 17-2). The cellular resistance prevents motions likely to be large enough to cause appreciable
stress in the relatively flexible frames.

**ROOF CONSTRUCTION**

Roofs utilizing skin construction are formed either by folding flat plates into cor-
rugated shapes or by shaping the skin into single- or double-curved shells. Each of
these methods has been used in a variety of shapes and schemes, the economy of each
depending on the size and arrangement of the structure.
If the section is sufficiently rigid to maintain its original cross section under applied loads, conventional methods of analyzing beams could be used in the design. However, because of distortion of the section, special methods must be adopted. While specific rigorous analytical solutions are available for shell and plate structures, the shell solutions in particular apply only to the simpler shapes and even in this case contain somewhat questionable assumptions. Furthermore, the calculations used in the rigorous solutions of shell analyses are generally so abstruse that it is difficult to judge the action of the structure from the course of the computations. However, skin-type roofs of minimum practical proportions are inherently so strong that great preciseness is not required in most analyses and suitable approximations can be made which will both provide necessary strength and furnish a feel for the action of the structure.

Virtually all the approximate solutions first assume that the skin is without flexural strength and stiffness. With this simplifying assumption the membrane stresses necessary to carry the applied loads are then determined. Next, strains caused by the membrane stresses are computed and local flexural stresses caused by these strains are approximated and superimposed on the membrane stresses previously determined. The flexural stresses usually will have little effect on the magnitude of the membrane stresses or on the over-all load-carrying capacity of the skin, but they may be of sufficient magnitude to cause local failure and unstability if not provided for in the design.

**Hipped Plate Roofs**

The behavior of hipped plate roofs may be illustrated by considering the simple gable roof of Fig. 17-8a supported by end gables \(ABA'\) and \(C'D'\). It is obvious that the roof surfaces will act as flat plates and the local surface loads acting normal to the two surfaces \(ABCD\) and \(A'B'C'D'\) would tend to deflect these plates between the end supports were it not for the interaction of the connecting plates. In this deformation, edge \(BD\) of the plate \(ABCD\) will tend to deflect downward in the direction of \(BD-B_1D_1\) (Fig. 17-8b) and edge \(BD\) of plate \(BDA'C'\) will tend to deflect downward along the line \(BD-B_2D_2\) of the same figure.

However, it is equally obvious that the plates are not free to deflect in this direction and that the two slabs will interact with each other, forcing a final deflected position of \(B_1D_1\). Deflection to the position \(B_1D_1\) necessitates strains \(\varepsilon_1\) and \(\varepsilon_2\) in the strong direction of the connecting slabs, and resistance of the slabs to bending in this strong direction will result in reactions supporting the slab at line \(BD\). The roof slab \(ABCD\) is therefore supported and restrained along edges \(BD\) as well as at ends \(AB\) and \(CD\). The plate is also subjected to an edge force at \(BD\) acting in the plane of the plate (Fig. 17-8d). The magnitude of the edge force in the line common to the two plates will be such as to have a vertical component at each point equal to the vertical reaction of the plate connecting at that point (Fig. 17-8c).

If the connecting plates in the example are identical and the loading is symmetrical about the center line, edge \(BD\) of both slabs will tend to shorten the same amount \(\varepsilon_1 = \varepsilon_2\) on the two sides, the strains in each connecting member will be consistent in sign and magnitude, and longitudinal shearing forces will not be developed along the line of juncture. Each half of the roof will resist the edge load developed along \(BD\) as a simple girder supported at the gable ends (Fig. 17-8d). Such a roof would be
very stiff and the addition of purlins or a beam along $BD$ would add little to the stiffness of the structure.

However, the gable roof described above is not very efficient for carrying local loads acting normal to the roof surface because of the free edge $AC$. The framing can be improved by adding vertical member $AE CF$ along line $AC$ (Fig. 17-9a). In this case the slabs joining at line $AC$ act in the same manner as previously described for the slabs joining at the ridge line $BC$. The roof slab $ABCD$ will now behave essentially as a flat plate supported on four sides (Fig. 17-9b).

At the ridge line the two intersecting roof planes were each independently compressed or less equally under the action of the edge load, and as a result of this asymmetry in action, shearing forces were not developed along the common edge. However, the lower joint along line $AC$ acts as a bottom chord of the top plate and as a top chord of the bottom plate. Therefore, the flexural strains in the plane of the plates tend to extend line $AC$, due to the tension at line $AC$ in plate $ABCD$, and to compress the same line $AC$ in plane $AEFC$ (Fig. 17-9c). As the two plates are integral members, forces will be set up along the common edge $AC$ which will prevent relative motion.

The behavior of the connected members is now similar to that described for the cellular members in which deformation is resisted both by the bending capacity of each member in its own plane ($M_s = 2fI/d$) and by the bending resistance due to the shears developed at the juncture of the slabs ($M_s = \int_0^x V_t \, dx$).

Interaction of adjoining plates in general adds to the inherent stiffness of the individual deep plates in the same manner that flanges add to the stiffness of deep web beams.

The principal load-carrying forces acting in a single-span hipped plate structure may be obtained as follows:

1. The skin is first considered as a continuous plate supported at the end diaphragms and along each line of juncture with adjacent connecting slabs. In the usual case, the distance between diaphragms will be sufficiently great so that the slab can be considered as supported solely along the lines of juncture of the connecting slabs. For short spans with greater dimensions between the lines of juncture, a portion of the load will be transferred in the usual manner of two-way slabs to the diaphragms. The portion of the load transferred directly to the diaphragms will not enter into the treatment described in the following paragraphs.

2. The reactions at the support points or lines of juncture obtained from step 1 are resolved into forces acting parallel to the planes of the connecting plates and these forces are considered as acting as edge loads on the individual plates. Each separate plate will usually be strong enough to carry the edge loads from end support to end support.

3. As described earlier, the edges of the connecting surfaces are not free to strain independently of each other and tension, compression, and shearing forces will be developed between the plates. The forces will be of such magnitude that the strains at the common points will be compatible. If it is assumed that only longitudinal forces are developed along the line of juncture between connecting planes, the forces acting on each plate will be as shown in Fig. 17-10a. The forces acting on any cross section, taken normal to edge $AB$, will then be subject to the forces shown in Fig.
17-10b (notation is for slabs or plates of nonhomogeneous material). The effect of the longitudinal shearing forces will be equivalent to adding a concentrated force \( N \) along the joining edges analogous to a variable prestress force, at the edge of each connecting member of such magnitude that it will be equal to the sum of the longitudinal shearing forces along that line. The internal moment \( M \) in the plane of each plate will be that caused by the curvature of the member in its final deflected position.

The magnitude of the internal stresses in the various members may be approximated by separating the various plates at the points of juncture, thus making each section statically determinate, then solving for the shear forces and moment necessary to make the edge strains compatible in the connecting plates. The solution is as follows:

Resolve the reactions at the edges caused by local loads acting transverse to the plate into edge reactions parallel to the planes of the intersecting plates \( W_c \) in Fig. 17-11b. These forces, acting on the edge of the plate supported at the two ends, will produce the statically determinate moment \( M_0 \). After obtaining \( M_0 \), introduce redundant longitudinal shearing forces \( V' \) at the edges to make the strains in adjoining plates consistent at their common points. As the strains at the common points in the intersecting plates may be considered equal if the stresses at the same points are equal, the fiber stress may be equated at each point of juncture in the connecting slabs. Considering flexural strains only,

\[
f = \frac{M_0}{I}
\]

where

\[
M = M_0 - (N_c c_t + N_s c_b)
\]

The extreme fiber stresses in any member corresponding to these moments will be

\[
f_t = -\frac{N_t}{A} + \frac{N_b}{A} - \frac{(M_0 + N_c c_t + N_s c_b)}{I} c_t
\]

and

\[
f_b = -\frac{N_t}{A} + \frac{N_b}{A} + \frac{(M_0 + N_c c_t + N_s c_b)}{I} c_b
\]

where \( A, I, c_t, c_b \) are, respectively, the cross-sectional area, moment of inertia, and distances to the extreme fibers of the slab in question. In the usual case of rectangular homogeneous plates of constant thickness these equations reduce to

\[
f_t = -\frac{[M_0 + (2N_d h/3) + (N_j h/3)]}{S}
\]

\[
f_b = \frac{[M_0 + (N_d h/3) + (2N_j h/3)]}{S}
\]
At the intersections the stress in the adjoining plates \( n \) and \( n + 1 \) must be equal and hence

\[
N_{n-1} \frac{h_n}{S_n} + 2N_n \left( \frac{h_n}{S_n} + \frac{h_{n+1}}{S_{n+1}} \right) + N_{n+1} \frac{h_{n+1}}{S_{n+1}} = -3 \left( \frac{M_{on}}{S_n} + \frac{M_{on+1}}{S_{n+1}} \right)
\]  

(17-18)

Moment equations of this type can be written for each common joint in the roof.

By differentiating the above equation with respect to \( x \) and substituting the terms \( T \) and \( V_0 \) for \( dN/dx \) and \( dM_0/dx \), respectively, where \( T \) represents the longitudinal shearing force at the connecting edge and \( V_0 \) represents the vertical shearing force at the gable ends, the following equation for shearing forces is obtained:

\[
T_{n-1} \frac{h_n}{S_n} + 2T_n \left( \frac{h_n}{S_n} + \frac{h_{n+1}}{S_{n+1}} \right) + T_{n+1} \frac{h_{n+1}}{S_{n+1}} = -3 \left( \frac{V_{on}}{S_n} + \frac{V_{on+1}}{S_{n+1}} \right)
\]  

(17-19)

and again, in this case, as many equations can be written for the longitudinal shearing stress as there are common edges in the structure.

The equations can be solved directly by conventional means of solving simultaneous equations. If the number of edges is large, however, the numerical work can be reduced by use of the methods described by Winter and Pei² (similar to moment distribution) or by other approximate solutions for simultaneous equations. Since the coefficients for the joint in question will usually be large compared with the coefficients for the other edges, approximate solutions will converge in a few cycles.

The values of \( T \) and \( N \) must be determined at a sufficiently large number of points along the length of the span to permit design of the member. However, the solution is simplified in the usual case of uniform load by considering that \( T \) and \( N \) will vary in the same manner as \( V_0 \) and \( M_0 \), respectively. Making use of this relationship, \( N \) and \( T \) may be determined analytically at the mid-point and by analogy at other points along the edge.

Numerous reinforced-concrete hipped structures have been successfully constructed using the above design assumptions. These structures have given satisfactory service, though many were subject, at one time or another, to overload conditions. The accuracy of the method depends largely on the limitations of the assumption restricting the edge stresses at the line of juncture to longitudinal shear forces. For the case of plates connecting at a 90° angle (box girders, T beams, etc.), the action will be largely as assumed with shear developed along the edge but with tension and compression force components in the plane of the plate but normal to the edge being relieved by the torsional flexibility of the connecting plates. For the case of plates connecting at 180° (an arbitrary division of a deep beam), such force components will not be relieved and a trial calculation will quickly show that analysis of the beam as a full-depth girder will not provide the results obtained by dividing the beam into sections and analyzing the separate sections in the manner described for the hipped plate design. While the errors caused by neglect of forces normal to the line of juncture are not of great practical importance in most hipped plate structures, it would seem advisable to check the stresses by means of conventional beam analysis. If the stress pattern differs greatly from that obtained from conventional beam analysis assuming a rigid cross section, either a careful inspection should be made to determine that the discrepancy is actually due to the characteristics of the structure rather than to the assumptions adopted for the analysis or strength should be provided for the maximum stress determined by either of the two methods.

While the hipped plate units will be relatively stiff as compared with conventional frames, some deflection of the lines of juncture does occur caused by strains \( e_1 \) and \( e_2 \) of Fig. 17-8. These strains change the angle between the connecting plates and alter the moments and reactions originally obtained, assuming the slab continuous over unyielding supports at the points of juncture. The flexural stresses in the skin should be modified to correct for this displacement using the computed deflection at the two edges of each plate to determine the deflection angle. The consequent change in the pattern of reactions, caused by yielding of the supports, will also change the edge reactions and juncture shear applied to each plate. These can be computed by correcting
the original loading or by applying the changes as loads in a similar cycle, and the effects of all motion can be corrected in a few similar cycles. Often the change in plate reactions will be small, and inaccuracies in the design assumptions would hardly justify any more than correcting the original flexural skin stress for the effect of the initial deflection angles.

Shell Construction

Shell roofs consisting of light skin members curved in single or double curvature have been used with increasing frequency in recent years. Because of their light weight and rigidity, shell structures offer a particularly economical method of covering large clear areas, provided the formwork can be reused often enough to offset its relatively high initial cost.

As in hipped plates, the major portion of the dead and live loads acting on the shell surface is carried by internal forces acting in the plane of the shell surface. In contrast to hipped plates, which carry local loads by flexure between plane junctures, shells are capable of carrying load mainly by means of membrane or skin stress, provided the shells are not cut and provided they are shaped to carry the applied load.

Shells of Double Curvature

Shells of double curvature are usually shaped as surfaces of revolution. Spheres and ellipsoids are typical examples of these structures and common applications of the construction occur in containers, holders, tanks, and in the roofs of storage tanks. The following outline follows the discussion presented in detail by Timoshenko.

Concrete shells of double curvature usually consist of domes of the type pictured in Fig. 17-12a where the shell terminates in a ring member and the shell edge at this point is subject to various conditions of restraint due to strain of the ring. The edge may be on either the top or the bottom of the shell, depending on whether the shell is used as a roof or as a holder. In most cases the structure supporting the ring is such as to provide a uniform continuous vertical reaction around the circumference of the shell, while the ring girder takes the horizontal components of the shell stress occurring at the discontinuous edge.

In the design of domes, loads acting normal to the surface are usually resisted solely by radial components of the membrane stresses $N_{\phi}$ and $N_{\theta}$. By considering the equilibrium condition, the normal component is equal to the membrane stress divided by the respective radius of curvature $N/R$, and the membrane forces caused by a load will be equal to the algebraic sum of the components of the membrane stresses, or

$$-Z = \frac{N_{\phi}}{r_1} + \frac{N_{\theta}}{r_2}$$

(17-20)

where $r_1$ and $r_2$ are the principal radii of curvature as shown in Fig. 17-13. In an actual design, the intensity of the external load components acting in the $X$, $Y$, and $Z$ directions will vary at different points in the shell and the equation of equilibrium for each
element may be written
\[
\frac{d}{d\phi} (N_\phi r_0) - N_\phi r_1 \cos \phi + Yr_1r_0 = 0
\]  
(17-21)

and
\[
N_\phi r_0 + N_\phi r_1 \sin \phi + Zr_1r_0 = 0
\]  
(17-22)

in directions tangent to the meridian and normal to the membrane, respectively. \(N_\phi\) and \(N_\theta\) are as shown in Fig. 17-13, and each element of the surface is defined by two meridian planes measured from a reference meridian plane and by two planes perpendicular to the axis of rotation referred by angle to the axis of rotation, as shown in the same figure.

This solution for symmetrical loads may be simplified by considering the equilibrium of the entire surface above a horizontal plane (Fig. 17-14). In this case, the equations for symmetrical loads become
\[
2\pi r_0 N_\phi \sin \phi + R = 0
\]  
(17-23)

\[
N_\phi = \frac{-R}{2\pi r_0 \sin \phi}
\]  
(17-24)

\[
-Z = \frac{N_\phi}{r_1} + \frac{N_\theta}{r_2}
\]  
(17-25)

or
\[
N_\theta = -Zr_2 - \frac{Rr_2}{2\pi r_0 r_1 \sin \phi}
\]  
(17-26)

Fig. 17-14. Forces above a horizontal plane.

where \(N_\phi\) and \(N_\theta\) may be solved for directly.

In the solutions described above, the dome is usually provided with a continuous support capable of carrying the vertical reaction, equal to \(N_\phi \sin \phi\). Per unit length this reaction is equal to the total vertical load above the edge divided by the circumference \((2\pi r_0)\). In addition, the ring provided around the circumference is designed to carry the horizontal component of the load \((N_\phi \cos \phi)\). The tension in this ring will be \(T_{ring} = N_\phi r_0 \cos \phi\).

It is obvious that consideration of membrane stresses alone does not account for the effect of elastic and inelastic flexural strains occurring in the shell and particularly ingores possible differences in strain that may occur at the edge between the shell and the restraining ring under the above design assumptions. Analyses of these flexural effects in double-curved shells were developed and presented by H. Reissner\textsuperscript{4} and E. Meissner\textsuperscript{4} with simplifications by O. Blumenthal\textsuperscript{7} and J. Geckeler.\textsuperscript{8} However, it has also been shown by M. Hetényi\textsuperscript{11} that the general solutions for single- and double-curved shells lead to equations and results similar, respectively, to those obtained in the analysis of curved beams and straight beams supported by an elastic foundation. The latter treatment is a useful tool for approximating the stress in both single- and double-curved shells and provides a consistent design method for designing shells of various shapes, stiffnesses, loading conditions, and means of support.

Treatment of edge conditions as loads or displacements applied to straight beams on elastic foundations is in effect treating the double-curved member as a cylinder with single curvature having the same radius and skin thickness, and the results are reasonably accurate if the skin thickness-to-radius ratio is small. The procedure used in designing by this method is to find direct stress by the membrane theory, to calculate the edge displacements caused by these direct stresses, and then to apply loads compatible with the boundary conditions to the end of a beam on an elastic foundation having characteristics determined by the properties of the analogous cylinder. The equations are similar to those described for the "short" barrel except that the analogous beam will be a beam of unlimited length. Displacement, shears, and moments for members on elastic foundations are given by M. Hetényi\textsuperscript{11} and S. Timoshenko\textsuperscript{10} for a wide variety of loading conditions.

A more accurate treatment of shells treats the edge loads and displacements as loads
applied to a curved beam on an elastic foundation. In this case the beam is a meridional element of a shell of variable width supported by a foundation having a variable modulus, and the stresses for a spherical shell uniformly loaded can be determined as shown by M. Hetényi. By omitting derivatives having little effect on the results, workable design relationships such as shown below are obtained. Again membrane stresses are first calculated, providing \( N_\theta \) and \( N_\phi \). The horizontal displacement at the edge \( U_0 \) is then found from the relationship

\[
U_0 = \frac{r}{Et} \left( N_\theta - \mu N_\phi \right)
\]

The integration constants are found from the boundary conditions, and all other values may be determined from the equations following. The flexural stresses so obtained are superimposed on the membrane stresses for the final solution.

The equations relating forces and displacement at the shell edge are

\[
Q = \frac{C e^{-\lambda \omega}}{\sqrt{\sin (\phi_0 - \omega)}} \sin (\lambda \omega + \psi)
\]

\[
M_1 = \frac{rCe^{-\lambda \omega}}{2\lambda \sqrt{\sin (\phi_0 - \omega)}} \left[ \cos (\lambda \omega + \psi) + \sin (\lambda \omega + \psi) \right]
\]

\[
M_2 = \mu M_1
\]

\[
N_\theta = \frac{C e^{-\lambda \omega}}{\sqrt{\sin (\phi_0 - \omega)}} \left[ \cos (\lambda \omega + \psi) - \sin (\lambda \omega + \psi) \right]
\]

\[
N_\phi = -Q \cot (\phi_0 - \omega)
\]

\[
U = \frac{r[\sin (\phi_0 - \omega)]}{Et} N_\theta
\]

\[
\theta = \frac{2\lambda Ce^{-\lambda \omega}}{Et \sin (\phi_0 - \omega)} \cos (\lambda \omega + \psi)
\]

\[
\phi = \phi_0 - \omega
\]

\[
\lambda = \sqrt{3(1 - \mu^2)}(r^2/t^2)
\]

where \( \mu = \frac{1}{2} \) = Poisson’s ratio

\( t = \) thickness of shell

\( C \) and \( \psi \) are constants of integration

and where \( M_1, M_2, N_\theta, N_\phi, \theta, \phi, \) and \( Q \) are as shown in Fig. 17-15.

If the loading is unsymmetrical rather than symmetrical as previously, shearing stresses as well as the direct stress \( N_\phi \) and \( N_\theta \) will be acting in the plane of the shell surface. These forces, shown by Fig. 17-16, are described by the following equations:

\[
\Sigma X: \quad \frac{\partial (r_0 N_\phi)}{\partial \phi} + \frac{\partial N_\theta}{\partial \theta} r_1 + N_\theta r_1 \cos \phi + Xr_0r_1 = 0
\]

\[
\Sigma Y: \quad \frac{\partial (N_\phi r_0)}{\partial \phi} + \frac{\partial N_\theta}{\partial \theta} r_1 - N_\theta r_1 \cos \phi + Yr_0r_1 = 0
\]

\[
\Sigma Z: \quad \frac{\partial}{\partial \phi} N_\theta = -Zr_2 - \frac{N_\phi r_2}{r_1}
\]

where

\[
N_\theta r_0 = N_\phi r_2
\]

and \( r_0, r_1, \) and \( r_2 \) are the same as shown in Fig. 17-13.
Equation (17-26) may be used to eliminate \( N_\theta \) from Eqs. (17-37) and (17-38), leaving two differential equations for determining \( N_\phi \) and \( N_{\phi\theta} \).

In certain cases the supports of the shell may be restricted to certain locations around the base of the shell and the membrane stresses will be materially affected by the concentrations of load. The same equations can be applied to this case as for the case of the unsymmetrical load described above, though the solution becomes difficult in most practical problems.

Shells of Single Curvature

Shells of single curvature are formed by curving thin skins or membranes into cylindrical shapes. The resulting vaults may be single or compound radii in the form of circles, ellipses, parabolas, or catenaries. Large changes in cross-sectional geometry of the skin are prevented by means of diaphragms or ribs located at the ends of the shell or at intervals along the length of the shell. Thin flexible skins become surprisingly stiff load-carrying members when shaped in this manner.

As for the case of double curvature, shells of single curvature derive their load-carrying capacity almost entirely from membrane forces, the membrane action determining the feasibility and economy of any particular type of shell construction.

The membrane forces acting on elements of the shell are as shown in Fig. 17-17a. They are located with respect to vertical and transverse reference planes passed through the center of the shell as shown in Fig. 17-17b. The magnitude of the membrane forces may be found as follows:

The components of applied load per unit area are \( X \), \( Y \), and \( Z \) in the longitudinal, the tangential, and the normal directions, respectively. The total forces acting on an element of \( dx\cdot dy \) area will be \( Y \ dx \), \( R \ d\phi \), and \( ZR \ dy \ dx \), if the shell is assumed to be of unit thickness.

The internal forces acting on the element may be found by summing the changes in stress per unit length multiplied by the dimensions of the element, thus providing equilibrium of the element in the \( X \), \( Y \), and \( Z \) directions.

\[
\sum X: \quad \frac{\partial N_s R \ d\phi \ dx}{\partial x} + \frac{\partial N_{\phi z} \ d\phi \ dx}{\partial \phi} + XR d\phi \ dx = 0 \quad (17-40)
\]

or

\[
\frac{\partial N_s}{\partial x} = - \frac{\partial N_{\phi z}}{R d\phi} - X \quad (17-41)
\]
\[
\Sigma Y: \quad \frac{\partial N_\phi}{\partial \phi} dx \frac{d\phi}{dx} + \frac{\partial N_{x\phi}}{\partial x} R \frac{d\phi}{dx} + YR \frac{d\phi}{dx} = 0 \quad (17-42)
\]

or
\[
\frac{\partial N_{x\phi}}{\partial x} = -\frac{\partial N_\phi}{R \frac{d\phi}{dx}} - Y \quad (17-43)
\]

\[
\Sigma Z: \quad ZR \frac{d\phi}{dx} + N_\phi \frac{dx}{d\phi} = 0 \quad (17-44)
\]

or
\[
N_\phi = -ZR \quad (17-45)
\]

In the above equations \( R \) is a function of \( \phi \) and the general equations found by integrating the above expressions with respect to \( z \) are
\[
N_z = -\int \frac{\partial N_{\phi z}}{\partial \phi} \frac{dx}{R \frac{d\phi}{dx}} - \int X \frac{dx}{d\phi} + f_2(\phi) \quad (17-46)
\]
\[
N_{\phi z} = -\int \frac{\partial N_\phi}{\partial \phi} \frac{dx}{R \frac{d\phi}{dx}} - \int Y \frac{dx}{d\phi} + f_1(\phi) \quad (17-47)
\]
\[
N_\phi = -ZR \quad (17-45)
\]

where \( f_2(\phi) \) and \( f_1(\phi) \) depend on the edge conditions and the selected coordinate system.

For shell curvatures where \( R, X, Y, \) and \( Z \) can be expressed in terms of \( \phi \), the solution of these values is determinable by analytical procedures. For members having irregular shapes or discontinuities, these direct and shear stresses may be more easily obtained by numerical methods of integration.

The \( N_{z\phi} \) and \( N_{\phi z} \) forces cause the shell to carry the load to the supporting end members with the curved member acting essentially as a deep beam. The variation of \( N_z \) along the length and breadth of the member will depend on the ratio of span between end supports to the radius \( R \) of the shell. For long barrels (with a length approximately five or more times the radius) the distribution of the longitudinal stresses over any transverse section will be essentially a straight line and will be similar to the distribution of forces obtained by use of the section modulus of the undistorted section.

Membrane action assumes the development of membrane forces throughout the surface of the shell. However, most shells consist of only a segment of a complete shell, and consequently the necessary forces are not developed at the free edges and important strains occur at these locations. As the membrane solution does not account for strains in the shell, it does not account for flexural stresses caused by rigid supports, discontinuous edges, or other disturbances. The membrane solution, which does not consider strains, would also lead to the belief that it is solely the degree of curvature that determines the load-carrying capacity of the shell. The fallacy of the latter assumption is made apparent by the catenary-shaped shell where substitution of the equations for the catenary into the membrane equations described above would result in \( N_\phi \) forces only without shear, without \( N_{\phi z} \) forces, and without the development of longitudinal \( N_z \) forces.

This mathematical answer is easily visualized by realizing that the shape of a catenary is defined and derived as such, if the components of force normal to the surface at each point along the shell are maintained in equilibrium by the tangential forces along the arc and no lateral distribution of load is necessary to maintain static equilibrium. It is obvious, however, that the shell will compress in a tangential direction under the \( N_\phi \) forces, and if this tangential strain is considered in the analysis, it will cause radial and tangential motions with respect to the supporting ribs. The elements also will tend to displace at the lower edges unless a rigid support is provided capable of supplying the \( N_\phi \) forces at this point. All the strain displacements mentioned above cause appreciable shearing forces, and some of the major shells now in existence are of catenary shape with analysis and tests demonstrating that these members act as true shells.

The difficulty in determining the effect of elastic strains leads to the complexities involved in shell designs. Yet, though the flexural stresses caused by elastic strains may not contribute appreciably to the overall load-carrying capacity of the shell, they
must be considered as they often determine the critical unit stresses, reinforcement requirements, and the minimum skin thickness at certain local points.

Some of the flexural stresses are caused by restraint at relatively rigid supports and similar points and local yield may relieve the stress caused by the restraint without affecting the load-carrying capacity of the shell, provided local failure does not occur at these points. The flexural forces acting in addition to the relatively more important membrane forces previously discussed are shown in Fig. 17-18.

All told, there are eight unknowns which have been determined for cylindrical shells by using the three equations of equilibrium in the $X$, $Y$, and $Z$ directions, the equilibrium equations for the moment about and $X$ and $Y$ axes, and the equations for the displacements $u$, $v$, and $w$ relative to these axes.

The solution of the general equations for most shell configurations is very complicated, and assumptions may be made for specific solutions which are known to have little effect on the computed behavior but which do result in considerable simplification of the design analysis. To accomplish these simplifications it is convenient to classify shells as "long" or "short" barrels, with the classification depending on the ratio of the span between the stiffening ribs to the radius of the shell.

Knowing that flexural stresses reduce rapidly in intensity away from the point of disturbance, it can be assumed that the $M_x$, $Q_x$, and $M_{x\phi}$ stress components will have little influence on the load-carrying capacity of "long" barrels in which the stiffening ribs are widely spaced. By the same reasoning $M_\phi$, $Q_\phi$, and $M_{x\phi}$ will have little effect on the load-carrying capacity of "short" barrels except near longitudinal edges, at concentrated loads, or at longitudinal line loads.

Further simplifications have been made of many shell types by using analogies which also ignore stress components that have little effect on the design. These particular design methods outlined below permit an intelligent appraisal of the behavior of the structure, regardless of the configuration of the shell, the distribution of the loading, and the stiffness of the supporting ribs. At the same time these approximate solutions provide reasonably accurate results.

**Design of Long Shells**

The stresses in long shells can be approximated by the use of iterative procedures such as described by Lundgren. In this method the shell is first considered as a beam of a fixed or undistorted shape spanning between the end supports. Longitudinal fiber stresses and shearing stresses under this preliminary assumption are found by the same methods as are used to find the shear and fiber stress in conventional beams. The section modulus is first obtained assuming the curved cross section is a rigid element. The load on the beam is the total load per foot of length between end diaphragms.

With the preliminary stresses determined as a simple beam, account is next made of the effect of these forces in bending the relatively flexible cross section of the shell. The flexing of the shell on a section cut normal to the span can be approximated by removing a section of unit width and by applying to this section the external loads acting directly on this cut section and the tangential shear forces along the cut edges as determined from the previous beam analysis. The section is then analyzed as an
elastic arch subject to the applied loads. It is assumed, as mentioned previously, that local bending moments $M_z$ in the skin and shears due to these moments $Q_z$ can be neglected.

The arch section so analyzed must possess sufficient moment capacity $M_\phi$ to resist the moments caused by the applied loads and must be sufficiently rigid essentially to maintain its shape and prevent large buckling stresses.

While the intensity of the longitudinal (beam) stresses will be proportional to the distance from the neutral axis for the lower loads, as was assumed in the "elastic" solution using the section modulus of the original cross-sectional geometry, the lever arm of the longitudinal forces and hence the capacity of the shell to carry load between the supports will continue to increase with increased load in accordance with the ultimate-strength theories, provided there is sufficient flexural strength available to resist the transverse moments $M_\phi$ caused by such stress redistributions. On an ultimate-strength basis, the $N_\phi$ forces will be concentrated near the top and bottom of the cross section, resulting in high shearing forces over a greater part of the arc.

If the shell is relatively thin and flexible, the calculations used to determine the total moments, shears, and thrusts on the arched cross section should be extended in the same manner as is described for obtaining deflection stresses and checking the stability of the arch ribs. The stability of the section can be checked by assuming a reasonably large deflection and seeing whether the calculations show the arch to be returning to the original shape in the first trial. If the section strains greatly, the dimensional change in the lever arms may be sufficient to alter the longitudinal stresses determined from the original beam analysis. In this case the beam analysis should be repeated using the new lever arms. Restraint along the longitudinal edges of the shell caused by beams or other intersecting shells may be accounted for by supplying forces along these edges sufficient in magnitude to return the edges to the position dictated by the boundary conditions. The analysis for these cases will be similar to that of the free arch described above except that motion of the arch under the applied loads will be elastically restrained at the lower edge in one or more directions.

If large concentrated loads are to be applied to the shell, stiffening ribs should be added under the loads to assure a suitable shear distribution to the shell and to maintain the cross-sectional shape of the shell assumed in the analysis. If such ribs are not provided, then the load should be applied to the arch-section analysis (distributing the load in a manner similar to concentrated loads on a plate) and the arch-section analysis must be undertaken to make sure that sufficient transverse bending capacity is furnished in the cross section to stabilize the shape of the shell in the areas where the concentrated loads are being distributed to the shell surface.

**Design of Short Shells**

Approximate but fairly accurate stress patterns can be obtained for short shells by application of the theory of cylindrical shells to the shell design. This method consists of determining the direct stresses in the shell by means of the membrane theory developed for stiffened cylindrical shells and then modifying these membrane stresses by superimposing flexural effects caused by variation in radius of curvature loss in membrane forces near the free longitudinal edges and the effects of bending throughout the shell caused by dead load, live load, and by such differential motions as tend to occur between ribs and shell.

The membrane stresses are found by integrating Eqs. (17-45), (17-46), and (17-47). For the usual case of the shell shaped to follow the catenary of the dead load, continuity may be first assumed at the lower longitudinal edges. The stress components determined assuming the shell a stiffened cylinder are next corrected for the effect of the discontinuity of the shell at the longitudinal edges by applying a load at the lower edge equal in magnitude but reversed in direction to the forces at this point obtained assuming the shell continuous at these edges. The corrective load will reduce the net $N_\phi$ forces to zero at the free edge but will have a rapidly diminishing effect for points at an increasing distance from the support. The effect of the free edge will be relatively unimportant at a distance one to two times the span between ribs away from the
edge. The variation in the distribution of the \( N_{\phi} \) forces caused by the correction load at the edge may be found by shear-lag calculations such as were described for the cellular structures. The portion of the external dead- and live-load forces in excess of that balanced by the reduced \( N_{\phi} \) forces must be carried by the shell in plate action.

Besides the membrane stresses and the shear and flexural stresses near the free edges described above, the loads acting on the shell will cause elastic, plastic, and volumetric strains in the shell which will tend to cause differential movements between the shell and the stiffening ribs. These strains in turn will produce flexural stresses in the shell and reactions of the shell on the rib which will tend to compress and bend the rib.

The flexural stresses in the shell may be found by treating the shell as parts of a continuous cylinder of appropriate radius. When the load on the shell is evenly distributed for a considerable distance on each side of the point being investigated, the action of the shell is similar to that of a ribbed circular cylinder subject to external pressure. The external pressure will be the radial component of the applied load, the tangential component being resisted directly by the membrane stresses. If the load is fairly uniform for a short distance and if the radii of curvature change at a slow rate, the \( N_{\phi} \) forces will be essentially constant along any short length of circumference of the shell and the \( N_{z\phi}, N_{\phi z}, M_{z\phi}, \) and \( M_{\phi z} \) forces will largely vanish. The \( N_{\phi} \) forces will be fairly equal along the circumference of the shell as assumed except near line loads or at edges where the edge treatment previously described must be used to determine the internal stresses.

Under these assumptions the equations describing the action of a longitudinal strip of the shell are the same as those for a beam supported by a continuous elastic foundation. The beam will be subjected to a load equal in intensity to the radial component of the applied loads and subject to the end conditions of rotation and displacement offered by the supporting ribs. The beam is a longitudinal strip of the shell spanning between two or more arch ribs. It may be of constant stiffness or of variable stiffness, depending on whether the shell is of constant thickness or haunched. The modulus of the elastic foundation is determined by the radius of the shell at the point in question. As in all beams on elastic foundations, load and displacement effects die out rapidly and the stresses at any given point are not greatly affected by load displacements or by changes in radii at some distance from the point. This fact allows the use of the cylindrical theory for shells having a variable radius or nonuniform loads.

The stresses in the shell, however, are appreciably affected if the radius of curvature of the shell at the point in question changes appreciably under the applied loads since the modulus of the supporting foundation varies inversely as the square of the radius of curvature. Consequently, the appreciable changes in curvature caused by deflections of the relatively slender ribs should be considered in the design analysis. In the same manner, changes in length of the stiffening rib due to rib shortening and volumetric effects will also have appreciable effect on stresses in the shell. These latter effects may be important enough to add several hundred per cent to the stresses caused by dead and live loads acting on the shell. It is easy to compensate for these effects by means of the ribbed-cylinder analogy and this recognition of important volumetric and strain effects more than compensates for other minor inaccuracies introduced by the general assumptions of the analysis.

The relationship between the action of the shell and the action of the stiffening ribs is as follows:

Compressive radial strains of the shell \( y_r \) will produce compressive hoop forces \( N \) of magnitude \( N = Ety_r/R \), where \( R \) is the initial radius of the shell, \( y_r \) is the unrestricted radial displacement of the shell, and \( N \) is the tangential force in the shell \( (N_{\phi}) \).

The resultant radial forces caused by \( N \) will be

\[
P = \frac{N}{R} = \frac{Ety_r}{R^2} \quad \text{or} \quad y_r = \frac{PR^2}{Et} \quad (17-48)
\]

Because of the load \( P \), the radius of the shell \( R_r \) has changed to \( R_r = R - y_r \).

The stiffening rib also had an initial radius equal to \( R \), but under the load \( P_R \), transferred by the shell to the rib, the radius of the rib will decrease by \( y_r' = P_R R^2/\pi E \), and the new radius of the rib will be \( R_r' = R - y_r' \).
The ribs will act to restrain the compressing shell, causing cross bending in the shell sufficient to relieve the "free" strain of the shell and ribs. The difference in radius made compatible at the juncture of the shell and ribs will be \((R - y_r) - (R - y_s)\), and the deflection causing cross bending in the shell will be \(y_s - y_r\). With the above relationships, any imaginable case of variation in stiffness of the stiffening beam and any variation in beam support, rib displacement, or rib rotation can be considered by combining the appropriate available solutions for finite-length beams supported by a continuous elastic foundation. Special cases of haunched shells, etc., can be readily solved by using the cases for beams having a variable flexural rigidity supported by a foundation having a variable modulus.

![Diagrams](image)

**Fig. 17-19. Loading and support conditions.**

The following are a few of the cases of particular interest in shell design, where, for a prismatical beam supported by a continuous elastic foundation, the properties are

\[
y = \text{radial deflection}
\]

\[
l = \text{span between ribs}
\]

\[
K_{\text{shell}} = \frac{E_t}{R^2}
\]

\[
D_{\text{shell}} = \frac{l^4}{12(1 - \mu^2)}
\]

\[
\lambda_{\text{shell}} = \sqrt{\frac{K}{4ED}} = \sqrt{\frac{3(1 - \mu^2)}{2l^2}}
\]

Differential motion of shells and ribs (Fig. 17-19a)

\[
y_A = y_B = \frac{2P\lambda \cosh \lambda + \cos \lambda}{K \sinh \lambda + \sin \lambda}
\]

\[
M_z = -\frac{2P}{\lambda} \frac{\sinh \lambda/2}{\sin \lambda/2} \frac{\sin \lambda}{\sinh \lambda + \sin \lambda}
\]

where \(\text{sinh and cosh are hyperbolic sine and cosine.}\)
Moments at the ribs (Fig. 17-19b)

\[ y_A = y_B = -\frac{2M_0\lambda^2 \sinh \lambda - \sin \lambda}{K \sinh \lambda + \sin \lambda} \] (17-51)

\[ \theta_A = -\theta_B = \frac{4M_0\lambda^3 \cosh \lambda - \cos \lambda}{K \sinh \lambda + \sin \lambda} \] (17-52)

\[ M_x = 2M_0 \frac{\sinh \lambda/2 \cos \lambda/2 + \cosh \lambda/2 \sin \lambda/2}{\sinh \lambda + \sin \lambda} \] (17-53)

Uniformly distributed load with rigid reactions at ends (Fig. 17-19c)

\[ \theta_A = -\theta_B = \frac{q\lambda \sinh \lambda - \sin \lambda}{K \cosh \lambda + \cos \lambda} \] (17-54)

\[ M_x = \frac{q}{\lambda^2} \frac{(\sinh \lambda/2)(\sin \lambda/2)}{\cosh \lambda + \cos \lambda} \] (17-55)

\[ R_A = R_B = \frac{q}{2\lambda} \frac{\sinh \lambda + \sin \lambda}{\cosh \lambda + \cos \lambda} \] (17-56)

Equal moments at each end with rigidly held supports (Fig. 17-19d)

\[ \theta_A = -\theta_B = \frac{2M_0\lambda^3 \sinh \lambda + \sin \lambda}{K \cosh \lambda + \cos \lambda} \] (17-57)

\[ M_x = 2M_0 \frac{\cosh \lambda/2 \sin \lambda/2}{\cosh \lambda + \cos \lambda} \] (17-58)

\[ R_A = R_B = -M_0\lambda \frac{\sinh \lambda - \sin \lambda}{\cosh \lambda + \cos \lambda} \] (17-59)

Fixed ends, uniformly distributed load over entire span (Fig. 17-19e)

\[ M_A = M_B = \frac{q}{2\lambda^2} \frac{\sinh \lambda - \sin \lambda}{\sinh \lambda + \sin \lambda} \] (17-60)

\[ M_x = \frac{q}{\lambda^2} \frac{\sin \lambda/2 \cos \lambda/2 - \cos \lambda/2 \sin \lambda/2}{\sinh \lambda + \sin \lambda} \] (17-61)

\[ R_A = R_B = \frac{q}{\lambda} \frac{\cosh \lambda - \cos \lambda}{\sinh \lambda + \sin \lambda} \] (17-62)

If the rib is rigid, the moments and shears determined by the loading case described above will apply directly. If the rib is yielding, an elastic correction should be made by adding the effect of the support motions.

If the shell participates in resisting flexure of the rib (the point of juncture of the shell is not at the neutral axis of the rib), then this participating stress in the flange \( f_s \) will have a radial component of force equal to \( f_{jr} \) which will cause further cross bending moments \( M_x \) in the shell. The magnitude of this effect can be approximated by adding the displacement \( y \) at the rib such that \( y = NR/Et \), where \( N \) is the stress in the shell due to the shell participation in rib bending. Several trials will be necessary to determine an \( N \) force which will be consistent with the calculated efficiency of the shell.

Analysis of Arch Ribs

For “short” shell roofs, such as are commonly used to cover long clear spans, the loads acting on the shell will be carried to the supporting buttresses partly by \( N_x \), \( N_\phi \), and \( N_{x\phi} \) forces in the plane of the shell skin and partly by the shell spanning in flexure as a slab supported by the ribs. The portion of the load carried directly to the ribs in
flexure will produce radial reactions on the rib similar to the forces acting on conventional frames. Similar radial rib reactions will be caused by temperature and shrinkage effects, which will be different for the shell and rib, and by displacement of the abutments, which will distort the rib and force strains into the shell. Eventually, regardless of the shape of the shell, all loads, including those carried by the $N_4$ forces in the shell, will be transferred to the rib as tangential or radial shearing forces and all these forces will add to the thrust and moment in the rib.

The manner in which the rib and shell cooperate in carrying the load depends on the relative stiffness of the shell and rib and is greatly complicated by rib shortening and by changes in curvature that occur in the supporting ribs as the load is transferred from the shell to the rib. However, a conservative assumption would assume that all loads acting on the shell will transfer laterally to the rib at the point at which the load is applied to the shell. This assumption will result in ribs having proportions and reinforcement sufficient to accommodate any transfer or distribution of load likely to occur in the structure.

**Loads.** Live loads are usually established for each city or section of the country by the codes governing the design of structures in that area. The Navy Code shown in Table 17-1, as prepared by the Bureau of Yards and Docks, presents load specifications typical of such design criteria.

The magnitude of volume change in concrete has been the subject of many investigations\(^\text{16,16}\) and Table 17-2 gives a summary of the temperature records and data suitable for use in calculating volumetric effects for typical areas in the United States.

### Table 17-1

<table>
<thead>
<tr>
<th>Area by Naval Districts</th>
<th>Vertical live load, psf</th>
<th>Wind, psf, design pressure flat plate on ground</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Maine, N.H., Vt., R.I.</td>
<td>40</td>
<td>30</td>
</tr>
<tr>
<td>3. N.Y., Conn., N.J.</td>
<td>35</td>
<td>30</td>
</tr>
<tr>
<td>4. Pa., Del.</td>
<td>35</td>
<td>30</td>
</tr>
<tr>
<td>5. W.Va., Va., Md.</td>
<td>25</td>
<td>30</td>
</tr>
<tr>
<td>6. N.C., S.C., Ga.</td>
<td>15</td>
<td>40</td>
</tr>
<tr>
<td>7. Fla.</td>
<td>15</td>
<td>50</td>
</tr>
<tr>
<td>8. Tex., Okla., Ark., La., Tenn., Ala., Miss.</td>
<td>15</td>
<td>50</td>
</tr>
<tr>
<td>12. Calif., Nev., Utah, Colo.</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>13. Wash., Ore., Idaho, Mont., Wyo.</td>
<td>25</td>
<td>30</td>
</tr>
</tbody>
</table>

### Table 17-2

<table>
<thead>
<tr>
<th>City</th>
<th>Max</th>
<th>Mean annual</th>
<th>Min</th>
<th>Max range</th>
<th>Recommended range</th>
</tr>
</thead>
</table>
| Buffalo, N.Y.      | 95   | 46.9        | -14  | 109       | 66
| Chicago, Ill.      | 103  | 49.0        | -23  | 126       | 76
| Dallas, Tex.       | 115  | 65.0        | -10  | 125       | 75
| Denver, Colo.      | 105  | 50.0        | -29  | 134       | 81
| Helena, Mont.      | 103  | 43.5        | -42  | 145       | 87
| Kansas City, Mo.   | 108  | 53.2        | -22  | 130       | 78
| Los Angeles, Calif.| 109  | 62.3        | 28   | 81        | 49
| Miami, Fla.        | 96   | 75.1        | 27   | 69        | 42
| Minneapolis, Minn. | 102  | 44.5        | -33  | 135       | 81
| New Orleans, La.   | 102  | 69.2        | 7    | 95        | 57
| New York, N.Y.     | 102  | 52.1        | -13  | 115       | 69
| Omaha, Nebr.       | 110  | 50.5        | -32  | 142       | 85
| Seattle, Wash.     | 96   | 51.3        | 3    | 93        | 56
| Tucson, Ariz.      | 112  | 65.6        | 6    | 106       | 64
| Washington, D.C.   | 106  | 54.9        | -15  | 121       | 73
This range is probably adequate for stiffening ribs provided they are insulated to protect them from sun and low temperatures.

**Factor of Safety.** Usual building codes are not prepared with skin structures in mind and consequently are neither suitable nor always on the conservative side. It is therefore desirable to design skin structures or at least to check the design of these structures by other criteria.

One method proposed for use in the design of stiffening ribs is to select the working loads specified for the area and then to provide member capacity to carry this multiple of the expected load. The multiple of the expected load is also selected so that critical working-load combinations will not cause cracking sufficient to cause deterioration of the members and will not result in stresses likely to cause critical fatigue effects.

The upper limit of the dead load, temperature changes, and shrinkage can be predicted with reasonable certainty, while live loads and wind loads are less susceptible to accurate estimate. Therefore, larger safety factors are reasonably applied to the loads subject to larger variations and to the loads likely to vary and cause fatigue in the concrete.

On the basis of these criteria, the use of the following two sets of ultimate load equations has been recommended:

\[
U = 1.2 \left( B + 2L + \frac{W}{2} \right) + D
\]

or

\[
U = 1.2 \left( B + \frac{L}{2} + 2W \right) + D
\]

where

- \( U \) = ultimate thrust-moment strength of the rib
- \( B \) = thrust-moment strength required by basic load, i.e., by dead load, temperature change, and shrinkage
- \( L \) = thrust-moment strength required by live load
- \( W \) = thrust-moment strength required by wind load
- \( D \) = thrust-moment strength required by rib deformation under ultimate-load conditions

\[
U = 2.0 \left( B + L + \frac{W}{2} + D' \right)
\]

or

\[
U = 2.0 \left( B + \frac{L}{2} + W + D' \right)
\]

where

- \( U, B, L, W \) = thrust-moment strength as in Eq. (17-I)
- \( D' \) = thrust-moment strength required by rib deformation under working stress

The thrusts used in the design should be those calculated for the loading combinations used to determine the moments.

This thrust will be distributed between the shell and rib in a manner dependent on the relative stiffness and efficiency of the shell. However, it is convenient to assume as limiting assumptions that the thrust will be divided in proportion to the areas of the shell and rib for the load condition causing maximum thrust eccentricity [Eq. (17-I)] in the rib and that the thrust will be carried solely by the rib for the load conditions causing maximum thrust in the rib [Eq. (17-II)].

The sectional area of the rib and the steel reinforcement should be determined at each point along the length of the arch by use of the most severe of the two design equations. If the rib is shaped to follow the dead-load pressure line, a case resulting in low dead-load moments, then Eq. (17-1a) or (17-1b) will usually be the most critical loading condition, and this equation will control the minimum proportions of the rib.

Equation (17-II) serves primarily to control the minimum rib proportions when the axial-load and/or dead-load moments are unusually high. This equation also acts to limit the stress in members having appreciable bending due to dead load. The second equation is very similar, both qualitatively and quantitatively, to the design equations specified by conventional codes.
Care must be taken to account for unusual loads, displacements, and load distributions. For example, a rib shaped to follow the dead-load pressure theoretically will be free of dead-load moments. However, deflection of the member and small errors in construction may cause eccentricities of the rib axis from the dead-load pressure line which will result in appreciable increases in the total moment. Each source of variation of this type should be limited by providing and enforcing adequate construction specifications. Furthermore, likely accidental and unavoidable effects should be considered and provided for in the design analysis.

To account for the above-described deflections and construction errors and to assure a strong tough rib, a minimum load, such as a 20 psf live load, should be used as the minimum design live loading regardless of locality of the structure. This load should be considered as a moving load and should be placed to give the most critical effect at each different point along the rib.

**Ultimate-strength Design.** When the working loads have been determined and overload factors necessary to assure the safety and durability of the rib have been established, these loads should be checked against accepted practice for conventional design wherever possible. However, as the "straight-line theory" is particularly inadequate for anticipating the strength of members subject to combined thrust and moment, ultimate-strength design methods such as proposed by Charles S. Whitney are suggested. Reference is made to the original papers for the details of these design methods.

**Rib Analysis.** While some redistribution of moment will occur in the arch ribs because of flow and creep in the concrete, the recommended practice is to proportion the rib in accordance with the familiar "elastic" design procedures. The general equations used in the design of statically indeterminate arch ribs under this assumption are as follows:

\[
H_1 \left( \sum y^2 \frac{ds}{EI} + \sum \cos^2 \phi \frac{ds}{AE} \right) + M_1 \sum y \frac{ds}{EI} = - \left( \sum M_2 y \frac{ds}{EI} + \sum V \sin \phi \cos \phi \frac{ds}{AE} + \sum H \cos^2 \phi \frac{ds}{AE} \right) \tag{17-63}
\]

\[
V_1 \left( \sum x^2 \frac{ds}{EI} + \sum \sin^2 \phi \frac{ds}{AE} \right) = - \left( \sum M_2 x \frac{ds}{EI} + \sum V \sin \phi \cos \phi \frac{ds}{AE} + \sum H \sin \phi \cos \phi \frac{ds}{AE} \right) \tag{17-64}
\]

\[
V_1 \left( \sum x^2 \frac{ds}{EI} + \sum \sin^2 \phi \frac{ds}{AE} \right) - \left( \sum M_2 x \frac{ds}{EI} + \sum V \sin \phi \cos \phi \frac{ds}{AE} + \sum H \sin \phi \cos \phi \frac{ds}{AE} \right) \tag{17-65}
\]

where \( H, V, \) and \( M_0 \) are external forces on the cut member. \( H_1, V_1, \) and \( M_1 \) are redundant reactions. All other terms represent displacement in the \( x \) direction [Eq. (17-63)], the angular change [Eq. (17-64)], and the displacement in the \( y \) direction [Eq. (17-65)] due to the strain under the separate forces.

While Eqs. (17-63), (17-64), and (17-65) can be applied to an arch of any shape and proportions, the work may be simplified and the size and coordinates of ribs shaped to follow the dead-load pressure line can be readily obtained by the methods described by Charles S. Whitney. The tables, charts, and formulas that follow are from ref. 18 by courtesy of Mr. Whitney and the ASCE.

The procedure for using this design method is as shown in the following example:

- **a.** The span and rise \( (l \text{ and } r) \) of the arch will be established by the functional requirements of the building. In most cases the rise will be from 15 to 20 per cent of the span.
- **b.** The rib dimension at the crown and springing line is estimated either from previous experience or by trials. Having these dimensions, the dead load of the rib and shell is computed at the spring line \( (w_s') \) and at the crown \( (w_c) \).
c. The angle of the arch axis at the spring line \( \phi_s \) is estimated and the dead load per foot of horizontal projection is computed \( w_s = w_r \sec \phi_s \) where \( \phi_s \) is the angle of the arch axis to the horizontal at the spring line.

d. Using the assumed value of \( w_s \) and a value of \( g = w_s/w_e \), then the angle at the spring line \( \phi_s \) is checked by solving for \( \phi_s \) from the value \( l/r \tan \phi_s \) obtained from Table 16-7. The value to \( w_s \) used to obtain \( g \) is revised by trial and error until the assumed value of \( \phi_s \) agrees with the value of \( \phi_s \) determined from \( l/r \tan \phi_r \) of Table 16-7. After the corrected

![Diagram](image)

**Fig. 17-21. Notation for arch analysis.**

value for \( w_s/w_e \) is obtained the horizontal and vertical dead-load reactions of the arch are obtained from values obtained from the coefficients in the other columns of the table.

e. The coordinates of the arch rib are determined by use of the equation

\[
y_0 = \frac{r}{g} - 1 \left( \cosh zk - 1 \right)
\]  

(17-66)

where \( r \) is the rise of the arch

\[
g = \frac{w_s}{w_e} = \cosh k
\]  

(17-67)

f. The depth of the rib at any point along the axis may be found from the equation

\[
d_z = \frac{d_z}{\sqrt{1 - (1 - m)x \cos \phi}}
\]  

(17-68)

where

\[
m = \frac{d_z}{d_z \cos \phi_s} = \frac{I_s}{I_s \cos \phi_s}
\]  

(17-69)

and \( \phi \) is the angle of the arch axis to the horizontal (see Fig. 17-21 and Table 17-3).

**Table 17-3. Value of \( y_c \)**

<table>
<thead>
<tr>
<th>( N )</th>
<th>( g )</th>
<th>( m = 0.15 )</th>
<th>( m = 0.20 )</th>
<th>( m = 0.25 )</th>
<th>( m = 0.30 )</th>
<th>( m = 0.40 )</th>
<th>( m = 0.50 )</th>
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<td>0.2333</td>
<td>0.2436</td>
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<td>0.2374</td>
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<td>0.2713</td>
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<tr>
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<td>0.2103</td>
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<td>0.2312</td>
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<td>0.2647</td>
</tr>
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<td>0.2427</td>
<td>0.2580</td>
</tr>
<tr>
<td>0.21</td>
<td>2.814</td>
<td>0.1867</td>
<td>0.1984</td>
<td>0.2080</td>
<td>0.2187</td>
<td>0.2362</td>
<td>0.2513</td>
</tr>
<tr>
<td>0.20</td>
<td>3.500</td>
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<td>0.2027</td>
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<td>0.1861</td>
<td>0.1964</td>
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<tr>
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<tr>
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<td>0.1506</td>
<td>0.1612</td>
<td>0.1709</td>
<td>0.1799</td>
<td>0.1960</td>
<td>0.2099</td>
</tr>
</tbody>
</table>

\( g \). Having the values of \( m \) and \( N \), the maximum positive and negative moments at the crown, quarter point, and spring line may be obtained from Figs. 17-22 to 17-27. The temperature thrust can be obtained from Fig. 17-28.

With a close approximation of the final size, shape coordinates, and angular functions established by these preliminary calculations, the analysis may proceed by substituting
the particular loads and arch properties into the general equations (17-63), (17-64), and (17-65).

**Deflection Moments.** The stiffening ribs of long-span shells will usually be very flexible because of the relatively small live loads acting on building roofs and because of the economic importance of keeping the ribs as light as possible. The slender ribs in turn require a more careful investigation of the buckling and a closer determination of the effect of rib distortions on the stresses than is necessary in heavier ribs.

![Graph showing moment and thrust values](image)

**Fig. 17-22.** Maximum positive live (uniformly distributed) load moment at crown and the thrust it produces.

Each external load, volumetric change, and displacement will strain the arch rib to a new position and with each added load condition the arch axis will diverge further and further from the pressure line. This motion causes added moments which also must be carried by the section. The moments caused by this eccentricity will in turn cause additional deflections and moments, a cycle which continues until either equilibrium is reached if the arch is stable or the arch buckles or exceeds the ultimate strength if the arch is unstable.

While each separate loading will cause a separate deflection, much more critical
effects will be obtained using the full combination loading producing the maximum moments and the total thrust due to this combined loading. Because of this, the most critical loading combinations including the factor of safety must be used in the design.

The following simple and rapid tabular method may be used for checking the stability of the rib and finding the magnitude of the deflection moments, using the arch-property data obtained in the solution of Eqs. (17-63), (17-64), and (17-65):

1. Combine the moments and thrusts obtained from the individual loads in such a manner as to produce the maximum positive and negative bending moments at the crown, at the quarter points, and at the spring line.

![Graph showing maximum negative live load moment and thrust values](image)

**Fig. 17-23.** Maximum negative live (uniformly distributed) load moment at crown and the thrust it produces.

2. For each loading combination compute the elastic vertical deflection of the points along the arch rib.

3. For this computed deflection, calculate the moments $M_s$ caused by the deviation of the arch axis from the original arch axis, assuming that redundant restraints of the arch are removed. This value of $M_s$ is equal to the product of the thrust and the displacement.

4. Applying these $M_s$ moments in the original arch equations (17-63), (17-64), and (17-65) will give the redundant reactions and moments caused by the deflection of the rib due to the original loads.

5. Add the deflection moments and thrusts obtained in step 4 to the previous moments and thrusts determined in step 1. Repeat this series of calculations until the series converges. If the series is divergent the rib will be unstable and must be stiffened. If the rib is of proper stiffness the series will converge within a few tries.

Similar calculations must be performed using the load combinations causing maximum moments at a sufficient number of points along the arch rib to permit a fairly
accurate estimate of the maximum deflection stress occurring at all points. A convenient arrangement for the computations is indicated in Table 17-4 (p. 17-34), which contains calculated moments for an arch rib. \(M\) and \(N\) represent the moment and thrust in foot-kips and kips, respectively, as obtained from the basic elastic analysis.

While consideration of the horizontal as well as the vertical deflections can be incorporated into the same analysis in the same manner to increase the apparent

![Graph](image)

**Fig. 17-24.** Maximum positive live (uniformly distributed) load moment at springing and the \(H\) it produces.

accuracy of the calculation, this refinement is rarely worthwhile because of the low rise-span ratios ordinarily used on these buildings and the consequent small effect of lateral displacements on the total moments.

The original elastic arch equations and summations determined for Eqs. (17-63), (17-64), and (17-65) may be used throughout the analysis, since the error due to the changes in arch coordinates is too small to have an appreciable effect on the behavior of the arch.

The moments added because of the rib deflection can, of course, be reduced by increasing the stiffness of the rib. If the rib is stiffened, however, this will add large moments due to volumetric changes and foundation displacements. The proportions
of the rib should be such as to reduce the sum of all effects to a minimum (see point \( m \), Fig. 17-29).

It is usually advantageous to keep the rib slightly more flexible than the optimum indicated by Fig. 17-29. This advantage occurs because the smaller rib requires higher steel percentages to carry the design moments, thereby helping to control the long-time plastic flow and providing a tougher rib which will accommodate larger foundation displacements.

![Graph showing maximum negative live load moment at springing](image)

**Fig. 17-25.** Maximum negative live (uniformly distributed) load moment at springing and the \( H \) it produces.

The total moment and the stability of the rib depend to a large extent on the modulus of elasticity of the concrete. The modulus of elasticity cannot be evaluated with great accuracy, since the effective modulus of the section will vary with changes in the materials, with the mix, and with the moisture content of the concrete. It will also depend on the duration and intensity of the stresses. Because of the variation in stress alone, the effective modulus of elasticity for a 4,000-psi concrete may vary from 4,500,000 psi at normal working stresses to 1,000,000 psi for stresses near the ultimate strength of the section.

Moreover, stresses of long duration produce a creep in the concrete with an effect on deflection equivalent to a drop in the elastic modulus. Accordingly, lower values of the modulus of elasticity should be chosen for structures having high dead-load
moments than for structures subjected largely to loads of short duration. Until further studies of existent long-span structures are available, conservative values of the modulus should be chosen. For example, 4,000,000 psi may be used to determine the effect of volumetric changes and 2,000,000 psi may be reasonable for the determination of deflection moments.

**FACTORS INFLUENCING THE COST OF SKIN CONSTRUCTION**

The cost of poured-in-place skin construction will depend on the same factors that influence the cost of conventional concrete construction, namely, the size of the structure, the complexity of the formwork needed, and the number of times the formwork can be reused. The costs will depend to a lesser degree on the size and amount of the equipment necessary in the construction procedures.

While savings in concrete and reinforcing steel obtained by efficient use of materials may pay for a single use of the more intricate and expensive formwork often needed for the skin-supported structures, the real economic advantage is obtained if the forms are standardized to permit multiple reuse of the forms.

So planned, the structural walls of box-framed buildings six to seven stories in height, for example, can often be erected at less cost than the block-masonry partition walls they replace, thus eliminating the duplication of the structural system and the space restrictions made necessary by conventional beam and column framing. While the initial cost of wall forms for such buildings is high, this cost is greatly reduced when averaged over a large number of reuses.

Precasting of the concrete members provides cost advantages over conventional concrete construction for a number of structural types as it offers a method of combining the low material costs of concrete construction with the low erection cost of steel construction. To accomplish this saving, precast slabs, channels, or boxes should be standardized to permit 20 to 40 or more reuses of each form, thus reducing the ordinarily high concrete formwork costs to a nominal amount. The arrangement of
the units should permit a continuous placing operation, in order to use the necessary heavy equipment in an efficient manner. Unless the project is well organized, the cost of the equipment needed for the operations of removing the castings from the forms and for the subsequent handling, storing, hauling, and erecting of the units will eat up the savings in materials and formwork. Precasting lends itself best to

mass-production methods and can be utilized in this manner on relatively small projects, whereas similar advantages obtained by repetitive use of the falsework for shell and plate structures can be obtained only in the larger poured-in-place constructions.

The factors pertinent to obtaining economical designs for long-span shell structures may be best illustrated by examining the items making up the total cost. Because of the lightness of the shell, the quantities of materials required in the shell will usually be appreciably less than those required for other types of conventional construction. This saving in materials helps to offset the higher initial costs of the more complex formwork required for such construction. The effect of the higher initial cost of the shell formwork can again be reduced by designs which permit a liberal reuse of the traveler and formwork.

Fig. 17-27. Maximum negative live (uniformly distributed) load moment at quarter point and the \( H \) it produces.
For buildings of limited length, it may be desirable to decrease the bay spacing below that resulting in minimum materials in order to reduce the area and thus increase the number of reuses of the formwork. In certain cases the pouring of a single bay at a time may be necessary to obtain the greatest over-all economies. Studies comparing the increased costs inherent in the shorter bay spacings with the savings obtained by the smaller area of traveling falsework obtained by forming single bays are accordingly in order for the shorter buildings.

In contrast to beams and truss construction, in which the cost increases sharply with increase in span, the span of reinforced-concrete shell roofs has relatively little effect on the unit cost of the portion of the barrel between the arch springing lines. While the radius of curvature of the shell will usually increase for the longer spans, causing somewhat increased stresses and decreased stability, the shell thickness is largely governed by practical construction requirements and is not affected too much.
<table>
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<th>Pt.</th>
<th>$M$</th>
<th>$N$</th>
<th>$W_e$</th>
<th>$V_e$</th>
<th>$M_e$</th>
<th>$\Delta_1$</th>
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<td>$-1,141.0$</td>
<td>$-18,290$</td>
<td>$-0.1270$</td>
<td>$-161.2$</td>
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<td>$+218.2$</td>
<td>$+294.2$</td>
<td>$+231.1$</td>
<td>$1,492.1$</td>
<td>$-1,749$</td>
<td>$-0.0121$</td>
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<tr>
<td>10</td>
<td>$+364.0$</td>
<td>$+1,307.7$</td>
<td>$+852.9$</td>
<td>$-1,081.3$</td>
<td>$-1,747$</td>
<td>$-0.0121$</td>
<td>$-15.8$</td>
<td>$1,404.4$</td>
<td>$+656.5$</td>
<td>$+2,020.5$</td>
<td>$+1,263.4$</td>
<td>$228.7$</td>
<td>$0$</td>
<td>$0$</td>
<td></td>
</tr>
<tr>
<td>S</td>
<td>$+1,918.7$</td>
<td>$+1,330.8$</td>
<td>$-228.4$</td>
<td>$0$</td>
<td>$0$</td>
<td>$0$</td>
<td>$0$</td>
<td>$1,565.6$</td>
<td>$+833.5$</td>
<td>$+2,752.2$</td>
<td>$-228.7$</td>
<td>$0$</td>
<td>$0$</td>
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<td></td>
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</table>
Table 17-4. Deflection Moments, Loading for Maximum Positive Moment at Crown (Continued)

<table>
<thead>
<tr>
<th>Pt.</th>
<th>$M_0$</th>
<th>$M_1$</th>
<th>$+36.119$</th>
<th>$\Delta M_3$</th>
<th>$M_3$</th>
<th>$W_s$</th>
<th>$V_s$</th>
<th>$M_s$</th>
<th>$\Delta_3$</th>
<th>$M_0$</th>
<th>$+38.735$</th>
<th>$\Delta M_4$</th>
<th>$M_4$</th>
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</thead>
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<tr>
<td>E</td>
<td>-866.1</td>
<td>0</td>
<td>0</td>
<td>+4.7</td>
<td>4.7</td>
<td>+748.5</td>
<td>+2,181.1</td>
<td>+2,558.6</td>
<td>0</td>
<td>+225,098</td>
<td>+1.5632</td>
<td>+1,737.0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>+1,609.9</td>
<td>+4.7</td>
<td>+748.5</td>
<td>+2,181.1</td>
<td>+2,558.6</td>
<td>0</td>
<td>+225,098</td>
<td>+1.5632</td>
<td>+1,737.0</td>
<td>+5.0</td>
<td>+826.9</td>
<td>+2,259.5</td>
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<tr>
<td>2</td>
<td>+1,345.8</td>
<td>41.5</td>
<td>+521.2</td>
<td>+1,637.6</td>
<td>+1,844.1</td>
<td>+4.402.7</td>
<td>+118,138</td>
<td>0.8204</td>
<td>+930.7</td>
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<td>+574.4</td>
<td>+1,690.8</td>
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<td>3</td>
<td>+881.8</td>
<td>115.3</td>
<td>+131.0</td>
<td>+608.7</td>
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<td>+139.2</td>
<td>+616.9</td>
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<tr>
<td>4</td>
<td>+329.2</td>
<td>226.6</td>
<td>-310.3</td>
<td>-784.4</td>
<td>-807.8</td>
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<td>-24,292</td>
<td>-0.1687</td>
<td>-197.3</td>
<td>243.0</td>
<td>-346.0</td>
<td>-820.1</td>
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<tr>
<td>5</td>
<td>-152.0</td>
<td>376.9</td>
<td>-641.2</td>
<td>-1,724.0</td>
<td>-1,690.0</td>
<td>+2,561.0</td>
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<td>-524.6</td>
<td>404.2</td>
<td>-708.2</td>
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<tr>
<td>6</td>
<td>-457.6</td>
<td>566.5</td>
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<td>-1,969.8</td>
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<td>-0.5036</td>
<td>-611.0</td>
<td>607.5</td>
<td>-832.2</td>
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</tr>
<tr>
<td>7</td>
<td>-545.9</td>
<td>797.4</td>
<td>-614.6</td>
<td>-1,874.6</td>
<td>-1,660.3</td>
<td>-1,069.1</td>
<td>-56,162</td>
<td>-0.3900</td>
<td>-483.3</td>
<td>855.2</td>
<td>-670.9</td>
<td>-1,930.9</td>
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<tr>
<td>8</td>
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<td>1,071.0</td>
<td>-233.4</td>
<td>-1,030.9</td>
<td>-859.8</td>
<td>-1,928.9</td>
<td>-26,651</td>
<td>-0.1851</td>
<td>-234.9</td>
<td>1,148.5</td>
<td>-249.9</td>
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</tr>
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<td>9</td>
<td>-216.7</td>
<td>1,390.0</td>
<td>+302.7</td>
<td>+383.2</td>
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<td>-0.0121</td>
<td>-15.8</td>
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</tr>
<tr>
<td>10</td>
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<td>1,756.2</td>
<td>+874.3</td>
<td>+2,238.3</td>
<td>+1,399.6</td>
<td>+228.3</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1,883.4</td>
<td>+952.5</td>
<td>+2,316.5</td>
<td>0</td>
</tr>
</tbody>
</table>

1. $M$ and $N$ are the initial elastic moments and thrusts obtained from the loading combination of $1.2(B + 2L)$.
2. $\Delta_1, \Delta_2, \ldots, \Delta_3$ are the successive deflections.
3. $W_s, V_s$, and $M_s$ are the conjugate weights, shears, and moments, respectively.
4. $\Delta M_3, \Delta M_4, \Delta M_5$ are the successive moments caused by the increasing deflection.
5. $M_s, M_3$, and $M_4$ are the total moment after each successive cycle.
by span within the usual sizes of such structures. The ribs for the longer spans must be somewhat heavier to furnish the necessary stability and the required strength, but this also adds little to the unit costs.

The cost of the abutments, supporting frames, and substructure outside the rib springing line, however, does depend on the span length, particularly where the spring line is high and where the supporting framing is restricted by the functional requirements of the building.

The cost study indicated in Fig. 17-30 illustrates a typical variation of unit cost with clear span for barrels having a clear height at the spring line of approximately 18 ft and a total building length of 350 ft.

While many examples of structures having double-curved members have been con-

\[ c = \text{Mom due to t.2 temp} \]
\[ d = \text{Deflection mom due to t.2 (B + 2L)} \]
\[ f = \text{Mom due to temp} \]
\[ g = \text{Deflection mom due to (B + L)} \]
\[ B = \text{dead + temp + shrinkage + abutment motion moment} \]
\[ L = \text{live load moment} \]

**Fig. 17-29. Variation of moments with rib stiffness.**

**Fig. 17-30. Cost-study chart giving variations of unit cost with clear span for type of structure shown.**
REFERENCES

17-37

structured and used successfully to roof planetariums, tanks, churches, etc., general use of this construction has been limited largely by the lack of reuse obtained from the costly formwork. As a result, the structural costs are usually considerably higher for double-curved structures covering large areas than for single-curved structures in which the formwork can be used several times. These costs can be absorbed only by special structures, such as listed above.

FALSEWORK

As the bulk of the cost of the cast-in-place concrete used in skin construction may be spent on formwork, there is an obvious necessity for reducing this cost to a minimum. The largest reduction in cost, as described earlier, is obtained by arranging the concrete structures so that the area formed per pour will be reduced to a minimum with subsequent use of the same formwork for succeeding pours. However, appreciable savings also may be made by care in the design of whatever falsework is provided. Among the more important of the details requiring consideration are the method of carrying the forms on the supporting trusses, the number of posts or frames used to support the trusses, the location and number of jacking points, and the horizontal and vertical bracing systems used to tie the vertical members.

Design

The design loads for the falsework should include the vertical load caused by the dead load of the falsework, the wet weight of the concrete, and a live-load allowance for dynamic loads caused by concentrations of men, equipment, and the local piling of concrete over small areas. Sufficient lateral strength must be provided to resist wind loads since expensive and time-consuming losses have occurred because of failure of the falsework under sudden wind loads.

If reasonable allowance is made for the above loading conditions, the permissible unit stresses of the falsework members may be increased in accordance with recommendations for short-time loads; obviously the size of the members should always be ample to assure elastic stability.

General Arrangement

The most common type of falsework consists of sheathing, purlins, and rafters supported by a series of short-span trusses located immediately below the barrel surface. The ends of each truss are supported by screw jacks which permit the raising and lowering of the upper formwork relative to the braced framework below. The jacks in turn are each supported by a number of braced timber posts which extend down to the ground and bear on footing pads during the pouring operation. At the bottom, the posts are interconnected by horizontal and vertical framing members such that when the wedges between the vertical posts and the supporting footing pads are removed the entire post load is transferred by these frames to a wheel and track arrangement which permits moving the falsework to the succeeding pour position. The movement of the falsework unit may be accomplished by cranes, bulldozers, hand winches, or by car jacks.

Because of the numerous supporting points needed in this arrangement, as many as 150 to 200 jacking points may be required in the longer structures to support the numerous trusses.

These usually consist of trusses located under each arch rib with each truss supported by relatively widely spaced vertical posts. Only jacks located at the floor level are needed to raise the widely spaced vertical posts to the specified position. To reduce these costs and at the same time to reduce the number and improve the location of jacking points, many simplified systems of falsework framing have been developed.

REFERENCES

Section 18
DESIGN OF DEEP BINS AND SILOS

By

RAYMOND L. BRANDES, Chief Structural Engineer, W. J. Barney Corp. (and Formerly Chief Engineer of The Nicholson Company), New York, N. Y.

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CONCRETE BINS

General Characteristics

Reinforced concrete is an ideal structural material for the building of permanent bulk-storage facilities for dry granularlike fillings.

Concrete storage units are economical in design and reasonable in first cost. Later discussion will treat more fully with the several stresses encountered in this type of structure but two, peculiar to bin loading, are of primary importance. The first, a lateral stress, results from the horizontal thrust of the filling; it is considerably smaller than that which might be expected from a hydraulic fill. The second, a vertical stress, results from the drag or vertical frictional force of the filling on the walls; this force does not exist with a hydraulic fill. In a reinforced-concrete structure the first of these two stresses can be provided for, in any magnitude, with the tensile strength of reinforcing steel placed in the exact size and spacing required; the second can be provided for, again in any magnitude, with the compressive strength of concrete in any mix or wall thickness required.

Concrete storage units can be designed and built in any shape or size to fit the site or the process for which they are required. They can be poured monolithically by the use of sliding forms when the walls are high, in single lifts when they are low, and in rapidly following lifts of fixed forms when they are of moderate height.

Being monolithic and rigid, they are not readily affected by impact and local strain. They are heavy (sometimes a great disadvantage) and thus seldom subject to overturning, by reason of wind loads, when empty.

Reinforced concrete requires large proportions of inexpensive and readily available
CONCRETE BINS

materials—sand and gravel, and small proportions of more costly and sometimes less available materials—cement, lumber, and reinforcing steel. Concrete storage units can be built economically and safely using a small supervisory force and, where union conditions permit, a large proportion of unskilled labor.

Poured concrete storage units are fireproof and verminproof, considerations which prompted the first use of concrete bins. They are dust-tight, providing cleanliness and, where necessary, efficient fumigation of the filling. They are moistureproof, allowing the storage of cement, flour, sugar, and other materials readily affected by dampness. They offer some measure of insulation, permitting the heating of the filling where necessary to prevent freezing. They are corrosionproof, an important factor when they must be built in an industrial or dockside area. They are practically maintenance-free, a very major consideration in over-all cost.

That acceptable service is provided by such works is proved by the continuing construction of conventionally reinforced poured-concrete bins in all countries of the world. The first reported poured-concrete deep bin in this country was erected in Minneapolis in 1899. Since that date thousands of such units have been built storing hundreds of different materials.

Concrete storage units are variously termed "bins," "silos," "elevators," "pockets," and "bunkers."

A partial list of materials known to be presently stored in reinforced-concrete bins follows:

<table>
<thead>
<tr>
<th>Materials</th>
<th>Materials</th>
<th>Materials</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alum, pulverized</td>
<td>Coal, bituminous</td>
<td>Oats</td>
</tr>
<tr>
<td>Antimony</td>
<td>Coke, coal</td>
<td>Peanuts</td>
</tr>
<tr>
<td>Apelite, ground</td>
<td>Coke, petroleum</td>
<td>Phosphate, ground</td>
</tr>
<tr>
<td>Arsenic, pulverized</td>
<td>Cork, ground</td>
<td>Phosphate rock, crushed</td>
</tr>
<tr>
<td>Asbestos rock</td>
<td>Corn, shelled</td>
<td>Quartz, broken</td>
</tr>
<tr>
<td>Ashes</td>
<td>Cottonseed</td>
<td>Razorite, pulverized</td>
</tr>
<tr>
<td>Barley</td>
<td>Dolomite</td>
<td>Rice</td>
</tr>
<tr>
<td>Barytes, ground</td>
<td>Emery, ground</td>
<td>Rye</td>
</tr>
<tr>
<td>Bauxite</td>
<td>Feldspar, ground</td>
<td>Salt, rock</td>
</tr>
<tr>
<td>Beans, cocoa</td>
<td>Flaxseed</td>
<td>Salt, table</td>
</tr>
<tr>
<td>Beans, coffee</td>
<td>Flour, wheat</td>
<td>Salt cake</td>
</tr>
<tr>
<td>Beans, navy</td>
<td>Fluspar, pulverized</td>
<td>Sand</td>
</tr>
<tr>
<td>Beans, soya</td>
<td>Glass batch</td>
<td>Sandstone, crushed</td>
</tr>
<tr>
<td>Bone meal</td>
<td>Glass cullet</td>
<td>Sawdust</td>
</tr>
<tr>
<td>Borax, ground</td>
<td>Gravel</td>
<td>Serpentine, crushed</td>
</tr>
<tr>
<td>Boric acid</td>
<td>Gypsum, ground</td>
<td>Shale, crushed</td>
</tr>
<tr>
<td>Brick, crushed</td>
<td>Iron ore</td>
<td>Silage</td>
</tr>
<tr>
<td>Buckwheat</td>
<td>Lignite</td>
<td>Silicon carbide</td>
</tr>
<tr>
<td>Calcium carbonate</td>
<td>Lime, burnt</td>
<td>Slag</td>
</tr>
<tr>
<td>Carbon black</td>
<td>Lime, hydrated</td>
<td>Soap</td>
</tr>
<tr>
<td>Caustic soda</td>
<td>Limestone, crushed</td>
<td>Soda ash</td>
</tr>
<tr>
<td>Cement, natural</td>
<td>Malt</td>
<td>Sugar, granulated</td>
</tr>
<tr>
<td>Cement, portland</td>
<td>Manure</td>
<td>Sulfur ore</td>
</tr>
<tr>
<td>Cement clinker</td>
<td>Marble, crushed</td>
<td>Talc</td>
</tr>
<tr>
<td>Cement raw mix</td>
<td>Mill scale</td>
<td>Traprock, broken</td>
</tr>
<tr>
<td>Cinders, coal</td>
<td>Milo-kaffir</td>
<td>Vinylite</td>
</tr>
<tr>
<td>Clay, dry lump</td>
<td>Mullite</td>
<td>Wheat</td>
</tr>
<tr>
<td>Clay, potter's</td>
<td>Nephaline</td>
<td>Wood chips</td>
</tr>
<tr>
<td>Coal, anthracite</td>
<td>Nitre</td>
<td>Zinc oxide</td>
</tr>
<tr>
<td>Also the following feed materials:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Alfalfa meal</td>
<td>Fish meal</td>
<td>Middlings</td>
</tr>
<tr>
<td>Bran</td>
<td>Gluten feed</td>
<td>Pellet mash</td>
</tr>
<tr>
<td>Corn grits</td>
<td>Gluten meal</td>
<td>Pellets</td>
</tr>
<tr>
<td>Cornmeal</td>
<td>Hominy feed</td>
<td>Poultry mash</td>
</tr>
<tr>
<td>Crumbles</td>
<td>Linseed-oil meal</td>
<td>Scratch feed</td>
</tr>
<tr>
<td>Distiller's grain</td>
<td>Meat meal</td>
<td>Soya meal</td>
</tr>
</tbody>
</table>

A comprehensive treatment of the subject of bin design is not feasible, and possibly not desirable in the limited pages of a handbook. It is the intention of this section
to present ample data to assist the structural engineer in the economical and rapid
design of bins most frequently required in present-day practice and to set down suffi-
cient supporting theory to give him confidence in the use of the procedure presented
and to assist him in the design of bin structures which are beyond the scope of this
work.

Limitations of Basic Data

The first condition of structural design—knowledge of the forces acting upon the
structure—is not well defined in the field of bin design. There is lack of definite
information concerning the values of horizontal and vertical pressures exerted on the
walls and bottoms of bins by most fillings. Considerable effort has been and is pre-
""
the angle of repose of the filling and thus is confused with its angle of natural slope. The angle of internal friction of a material without cohesion is approximately equal to its angle of repose or natural slope; for most materials, although not all, it is slightly larger than the angle of natural slope.

\[ \phi' = \text{angle of friction of filling on the bin wall, deg} \]

\[ \mu = \text{tangent of the angle of internal friction, or the coefficient of friction of filling on filling} \]

\[ \mu' = \text{tangent of the angle of friction of filling on the bin wall, or the coefficient of friction of filling on the wall surface} \]

\[ x = \text{angle of rupture of the filling, deg. More complete definition of this term is made in later discussion.} \]

\[ \theta = \text{angle between the horizontal and the slope of the hopper bottom, measured through the filling from either direction, deg (Fig. 18-1a)} \]

\[ \theta_1 = \text{angle between the horizontal and the slope of the hopper bottom measured from the outside of the hopper, deg (Fig. 18-1b)} \]

\[ \delta = \text{angle of surcharge, or angle between the horizontal taken at the top of the wall, and the upper surface of the filling, deg (Fig. 18-1c)} \]

\[ P = \text{total pressure of filling (for the full height of the filling) per linear foot of wall, applied at an angle of } \phi' \text{ with the normal to the wall, lb (Fig. 18-1d)} \]

\[ P_N = \text{total pressure of filling (for the full height of the filling) per lin ft of wall, applied normally to the wall, lb} \]

\[ P_T = \text{total pressure of filling (for the full height of the filling) per lin ft of wall, applied downward along the face of the wall, lb} \]

\[ p, p_N, p_T = \text{pressures of filling as defined above for } P, P_N, P_T, \text{ but for 1 ft of height of wall, psf} \]

\[ L = \text{lateral unit pressure of filling, psf} \]

\[ V = \text{vertical unit pressure of filling, psf} \]

\[ k = \frac{L}{V} = \text{ratio of lateral to vertical pressure} \]

\[ e = \text{base of natural (hyperbolic or Naperian) logarithms, 2.71828 . . .} \]

**Physical Characteristics of Bin-stored Materials**

Determination of bin pressures by the theories applied in office practice requires the use, for the filling under consideration, of values of:

1. Unit weight \( w \), lb per cu ft
2. Angle of internal friction \( \phi \) or coefficient of friction of filling on filling \( \mu \) (\( = \tan \phi \))
3. Angle of friction of filling on wall surface \( \phi' \) or coefficient of friction of filling on wall surface \( \mu' \) (\( = \tan \phi' \))
4. Ratio of lateral to vertical pressure \( k \)

These values for some materials are tabulated in Table 18-1.
### Table 18-1

<table>
<thead>
<tr>
<th>Material</th>
<th>ω</th>
<th>μ = tan φ</th>
<th>Angle of friction on concrete</th>
<th>μ' = tan φ'</th>
<th>Authority</th>
<th>k</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ashes</td>
<td>40</td>
<td>40°</td>
<td>0.839</td>
<td>40°</td>
<td>0.839</td>
<td>Link-Belt, Ketchum</td>
</tr>
<tr>
<td>Cement, portland</td>
<td>100</td>
<td>24°</td>
<td>0.445</td>
<td>24°</td>
<td>0.445</td>
<td>Link-Belt, Ketchum</td>
</tr>
<tr>
<td>Cement, clinker</td>
<td>95</td>
<td>30°</td>
<td>0.577</td>
<td>27°30'</td>
<td>0.520</td>
<td>Link-Belt, Ketchum</td>
</tr>
<tr>
<td>Cement raw mix</td>
<td>75</td>
<td>24°</td>
<td>0.445</td>
<td>24°</td>
<td>0.445</td>
<td>Link-Belt, Ketchum</td>
</tr>
<tr>
<td>Cinder</td>
<td>45</td>
<td>25°20'</td>
<td>0.473</td>
<td>25°20'</td>
<td>0.473</td>
<td>Link-Belt, Ketchum</td>
</tr>
<tr>
<td>Coal, anthracite</td>
<td>55</td>
<td>27°</td>
<td>0.510</td>
<td>27°</td>
<td>0.510</td>
<td>Link-Belt, Ketchum</td>
</tr>
<tr>
<td>Coal, bituminous(a)</td>
<td>50</td>
<td>35°</td>
<td>0.700</td>
<td>35°</td>
<td>0.700</td>
<td>Link-Belt, Ketchum</td>
</tr>
<tr>
<td>Coal, bituminous(b)</td>
<td>50</td>
<td>35°</td>
<td>0.700</td>
<td>35°</td>
<td>0.700</td>
<td>Link-Belt, Ketchum</td>
</tr>
<tr>
<td>Coke, coal</td>
<td>32</td>
<td>45°</td>
<td>1.000</td>
<td>40°</td>
<td>0.839</td>
<td>Link-Belt, Ketchum</td>
</tr>
<tr>
<td>Gravel, rounded(a)</td>
<td>100</td>
<td>32°30'</td>
<td>0.637</td>
<td>27°30'</td>
<td>0.520</td>
<td>Link-Belt, Ketchum</td>
</tr>
<tr>
<td>Gravel, rounded(b)</td>
<td>100</td>
<td>30°</td>
<td>0.577</td>
<td>27°30'</td>
<td>0.520</td>
<td>Link-Belt, Ketchum</td>
</tr>
<tr>
<td>Sand(a)</td>
<td>95</td>
<td>32°30'</td>
<td>0.637</td>
<td>27°30'</td>
<td>0.520</td>
<td>Link-Belt, Ketchum</td>
</tr>
<tr>
<td>Sand(b)</td>
<td>90-120</td>
<td>34°</td>
<td>0.675</td>
<td>30°</td>
<td>0.577</td>
<td>Link-Belt, Ketchum</td>
</tr>
<tr>
<td>Stone, crushed</td>
<td>100</td>
<td>32°30'</td>
<td>0.637</td>
<td>37°30'</td>
<td>0.767</td>
<td>Link-Belt, Ketchum</td>
</tr>
<tr>
<td>Barley</td>
<td>39</td>
<td>*27°</td>
<td>0.510</td>
<td>24°20'</td>
<td>0.452</td>
<td>Airy</td>
</tr>
<tr>
<td>Corn</td>
<td>44</td>
<td>*27°30'</td>
<td>0.520</td>
<td>23°</td>
<td>0.424</td>
<td>Airy</td>
</tr>
<tr>
<td>Flaxseed</td>
<td>41</td>
<td>*24°30'</td>
<td>0.456</td>
<td>22°30'</td>
<td>0.414</td>
<td>Airy</td>
</tr>
<tr>
<td>Oats</td>
<td>28</td>
<td>*28°</td>
<td>0.532</td>
<td>25°</td>
<td>0.466</td>
<td>Airy</td>
</tr>
<tr>
<td>Pears</td>
<td>50</td>
<td>25°20'</td>
<td>0.473</td>
<td>16°30'</td>
<td>0.296</td>
<td>Airy</td>
</tr>
<tr>
<td>Sugar</td>
<td>50</td>
<td>*35°14'</td>
<td>0.706</td>
<td>29°50'</td>
<td>0.573</td>
<td>Great Western Sugar Co.</td>
</tr>
<tr>
<td>Wheat(a)</td>
<td>50</td>
<td>28°</td>
<td>0.532</td>
<td>22°40'</td>
<td>0.417</td>
<td>Jamieson</td>
</tr>
<tr>
<td>Wheat(b)</td>
<td>50</td>
<td>*25°</td>
<td>0.466</td>
<td>24°</td>
<td>0.445</td>
<td>Airy</td>
</tr>
</tbody>
</table>

Comment on Table 18-1:
1. All the above values have been in continual use in design offices for at least 30 years; practically all of them have been used with 20,000 lb allowable reinforcing-stress stress.
2. The values for which no authority is given are office standards for three of the major United States design organizations; the writer was unable to determine their original source.
3. Values of φ marked * are angles of internal friction; it is not certain whether other values in the table are angles of internal friction or angles of natural slope.
4. For some few materials the value of φ' exceeds that of φ. In such instances values of φ' equaling those of φ must be used for design. As both Cain² and Feld⁴ point out, in such instances where φ' exceeds φ, the vertical frictional force acting in the support of the filling is that exerted between the main body of the filling and a thin layer of the same material that will cling to the wall surface. The apparently greater value of P sin φ' (or P* tan φ') therefore cannot be counted upon to assist in the support of the filling. On this point Jamieson⁴ notes that "The rougher the wall, the higher will be the coefficient of friction until it may reach a maximum of grain on grain."
5. Values tabulated are for dry materials.
6. All values of k except those marked† were determined from \( \frac{1 - \sin \phi}{1 + \sin \phi} \). It has not been possible to determine the origin of the 0.18 figure for bituminous coal. Jamieson determined the 0.6 figure for wheat by his experiments on full-sized bins and this figure seems to have been accepted as a standard in United States practice. However, other early experimenters on full-sized bins with apparently equally reliable apparatus arrived at values of k approaching \( \frac{1 - \sin \phi}{1 + \sin \phi} \). The practical application of k will be discussed in a later paragraph. The tabulated values have proved adequate for structural design. However, these values for each material may vary, depending upon the source of supply, percentage of fines, and moisture content of the material and upon bin-wall texture.
7. Certain finely ground materials such as cement and lime have no fixed angle of internal friction. When contained this angle for cement is about 6°. When aerated, cement will flow at an even flatter angle. When contained, even for a short period of time, this angle approaches the order of 40°. Since the advent of the use of air devices for conveying such materials, with consequent aeration of the filling and faster loading of the bins, some offices lean toward more conservative values in their design of units for such fillings.
Definition of Deep and Shallow Bins

Accepted office design procedure divides bulk-storage containers into two basic groups, shallow bins and deep bins. Different methods of calculation are applied to each group to determine lateral and vertical pressures of the filling. Shallow bins are considered those from which the plane of rupture (Fig. 18-4a) of the filling emerges from the top of the filling before it strikes the opposite wall, deep bins those in which the plane of rupture (Fig. 18-4b) strikes the opposite wall before it emerges from the top of the filling.

The plane of rupture is that surface down which a wedge of material, bounded by one wall face, the free surface, and the plane of rupture, would start to slide if the bounding wall were to move.

The value of the angle which this plane of rupture makes with a horizontal through the lowest part of the bin bottom can be stated

\[
\tan z = \tan \phi + \sqrt{\frac{\tan \phi(1 + \tan^2 \phi)}{\tan \phi + \tan \phi'}}
\]

(18-1)

or

\[
\tan z = \mu + \sqrt{\frac{\mu(1 + \mu^2)}{\mu + \mu'}}
\]

Thus when the dimensions of the bin are such that the value of the ratio \( H/D \) (or \( H/b \)) is smaller than that of the right-hand member of the equation we have a shallow bin; when greater, a deep bin.

Fig. 18-4. Relations of plane of rupture to bin height: (a) shallow bins, (b) deep bins.

Fig. 18-5. Location of plane of rupture between vertical wall and plane of repose.

Airy\(^8\) gives a nice algebraic proof of the above equation. The statement can also be proved graphically, starting with Cain's graphical solution of thrust of granular filling on a retaining wall.

Substitution, in Eq. (18-1), of values of \( \phi \) and \( \phi' \) from Table 18-1 gives values for \( \tan z \) of 1.22 for cement, 1.30 for anthracite coal, 1.38 for wheat(a), and 1.56 for bituminous coal.

When the effect of wall friction on filling is neglected, or \( \phi' \) is taken equal to 0, Eq. (18-1) reduces to

\[
\tan z = \frac{1 + \sin \phi}{\cos \phi}
\]

(18-2)

---

8. Coefficient of friction of the material sliding on like material can be determined by placing a quantity of the material in an open box (Fig. 18-2) on a level surface of like material and applying sufficient lateral pull to just start movement of the box. It may be recalled that (1) friction between two bodies is directly proportional to the pressure and (2) the coefficient and amount of friction for any given pressure are independent of the area of contact. Thus \( \tan \phi = F/W \).

Coefficient of friction of the material sliding on concrete can be found in the same manner by substituting a concrete slab for the filling on which the boxed material is to slide. The surface texture of the slab should approximate that to be used in the structure.

The same end result can be obtained by measuring the angle at which a shallow boxed quantity of the material will just start to slide on either like material or on a concrete slab (Fig. 18-3).
Equation (18-2) can be arrived at through a second approach. Both Cain\(^7\) and Ketchum\(^8\) state that, for a vertical wall without surcharge, and neglecting the effect of friction between the wall and the filling, the plane of rupture bisects the angle made by the wall and the plane of repose (internal friction) (Fig. 18-5).

Substitution in the trigonometric formula for tangent of one-half the sum of two angles also gives Eq. (18-2).

Values for \( \tan x \) for the same materials listed above when the effect of wall friction on filling is neglected are: 1.54 for cement, 1.63 for anthracite coal, 1.66 for wheat\((a)\), and 1.92 for bituminous coal.

These values can be computed by use of Eq. (18-2) or by use of Eq. (18-1) with the quantity \( \tan \phi' \) omitted.

Considering the method of derivation of Eq. (18-1) it is doubtful that this criterion can be used to fix exactly the boundary between deep and shallow bins when a hopper bottom of comparatively large proportions is involved. However, exactness in this step is rarely, if ever, essential. Further, as will be noted later, where the friction of the wall on the filling is taken into account, the transition of the curve of lateral pressures, from those of a shallow to those of a deep bin, is smooth.

Practice among designers concerning the application of Eq. (18-1) to bins with large hoppers varies. Some assume the plane of rupture to start at the hopper gate (Fig. 18-6a); others at the foot of the vertical wall (Fig. 18-6b).

Where the hopper bottom is small or where several outlets are located in a flat bin bottom, the plane of rupture is considered as starting at the foot of the vertical wall (Fig. 18-6c).

It is the practice of many designers to use the foot of the vertical wall as the starting point of the plane of rupture regardless of the size of the hopper.

### SHALLOW BINS

**Pressure Determination**

Determination of pressures in bins of this class is made by one of two methods. The first, which is based on Coulomb's theory, is more widely used than the second, which follows Airy's solution.

**Coulomb's Theory**

The formulas below derive from the theory of the wedge of maximum pressure for granular materials originally advanced by Coulomb\(^3\) and since modified by numerous experimenters and writers. Recent experiment, particularly since 1920, on large-scale models and full-sized walls, has proved this theory as the most nearly accurate approach to the calculation of lateral pressures of granular materials on retaining walls—provided that \( \phi' \), the angle of friction of the filling on the face of the wall, is used as the angle between the normal to the wall and the direction of the thrust, and provided further that \( \phi \) is taken as the angle of internal friction of the filling upon itself.

Both Cain\(^9\) and Ketchum\(^11\) develop this theory mathematically. Cain's development is the clearer of the two. Ketchum arrives at a variable expression for the angle between the normal to the wall and the direction of thrust; Cain uses \( \phi' \) for this angle. The form of Cain's equations differs from Ketchum's but the value of the thrust is the same.

Using \( \phi' \) as the angle between the normal to the wall and the direction of thrust
For 18-7. Wall and loading conditions covered by Eqs. (18-3) to (18-15).

(Fig. 18-7a, b), Ketchum's expression for the general condition becomes

$$P = \frac{wh^2 \sin^2 (\theta - \phi)}{2 \sin^2 \theta \sin (\theta + \phi') \left[ 1 + \frac{\sin (\phi + \phi') \sin (\phi - \delta)}{\sin (\theta + \phi') \sin (\theta - \delta)} \right]^2} \quad (18-3)$$

This general equation (18-3) reduces, for the conditions most frequently encountered in bin and hopper bottom design, to the following:

1. Vertical wall, with level filling, and friction of filling on wall neglected (see Fig. 18-7c).

$$P = P_N = \frac{wh^2(1 - \sin \phi)}{2(1 + \sin \phi)} \quad (18-4)$$

also = \((wh^2/2) \tan^2 (45^\circ - \phi/2)\), which is the same result as derived by Rankine for these conditions.

2. Vertical wall, with maximum slope of filling, and friction of filling on wall neglected (see Fig. 18-7d).

$$P = P_N = \frac{wh^2 \cos^2 \phi}{2} \quad (18-5)$$

which is the same result as derived by Rankine for these conditions.

3. Wall leaning away from filling, with level filling, and friction of filling on wall neglected (see Fig. 18-7e).

$$P = P_N = \frac{wh^2 \sin^2 (\theta - \phi)}{2 \sin \theta (\sin \theta + \sin \phi')^2} \quad (18-6)$$

4. Wall leaning away from filling, with maximum slope of filling, and friction of filling on wall neglected (see Fig. 18-7f).

$$P = P_N = \frac{wh^2 \sin^2 (\theta - \phi)}{2 \sin^2 \theta} \quad (18-7)$$
5. Wall leaning away from filling at angle $\theta_1$ with horizontal, with level filling, and friction of filling on wall neglected (see Fig. 18-7g).

$$ P = P_N = \frac{w h^2 \sin^2 (\theta_1 + \phi)}{2 \sin \theta_1 (\sin \theta_1 + \sin \phi)^2} $$

$$ = \frac{w h^2 \sin^2 (\theta_1 + \phi)}{2 \sin^2 \theta_1 [1 + (\sin \phi / \sin \theta_1)]^2} $$

(18-8)

6. Wall leaning away from filling at angle $\theta_1$ with horizontal, with maximum slope of filling, and friction of filling on wall neglected (see Fig. 18-7h).

$$ P = P_N = \frac{w h^2 \sin^2 (\theta_1 + \phi)}{2 \sin^2 \theta_1} $$

(18-9)

7. Vertical wall, with level filling, and friction of filling on wall included (see Fig. 18-7i). $\phi'$ and $\phi$ are different.

$$ P = \frac{w h^2 \cos \phi \left[ 1 + \sqrt{\frac{\sin (\phi + \phi') \sin \phi}{\cos \phi'}} \right]^2}{2 \cos \phi'} $$

$$ P_N = P \cos \phi' $$

$$ P_T = P \sin \phi' $$

(18-10)

8. Vertical wall, with level filling, and friction of filling on wall included (see Fig. 18-7i). $\phi' = \phi$.

$$ P = \frac{w h^2 \cos \phi}{2(1 + \sqrt{2 \sin \phi})^2} $$

(18-11)

9. Vertical wall, with maximum slope of filling, and friction of filling on wall included (see Fig. 18-7f). $\phi'$ and $\phi$ are different.

$$ P = \frac{w h^2 \cos^2 \phi}{2 \cos \phi'} $$

$$ P_N = P \cos \phi' $$

$$ P_T = P \sin \phi' $$

(18-12)

10. Vertical wall, with maximum slope of filling, and friction of filling on wall included (see Fig. 18-7f). $\phi' = \phi$.

$$ P = \frac{w h^2}{2} \cos \phi $$

$$ P_N = P \cos \phi $$

$$ P_T = P \sin \phi $$

(18-13)

11. Wall leaning away from filling at angle $\theta_1$ with horizontal, with level filling, and friction on wall included (see Fig. 18-7k). $\phi'$ and $\phi$ are different.

$$ P = \frac{w h^2 \sin^2 (\theta_1 + \phi)}{2 \sin^2 \theta_1 \sin (\theta_1 - \phi') \left[ 1 + \sqrt{\frac{\sin (\phi + \phi') \sin \phi}{\sin (\theta_1 - \phi) \sin \theta_1}} \right]^2} $$

$$ P_N = P \cos \phi' $$

$$ P_T = P \sin \phi' $$

(18-14)

12. Wall leaning away from filling at angle $\theta_1$ with horizontal, with level filling, and friction of filling on wall included (see Fig. 18-7k). $\phi' = \phi$.

$$ P = \frac{w h^2 \sin^2 (\theta_1 + \phi)}{2 \sin^2 \theta_1 \sin (\theta_1 - \phi) \left[ 1 + \sqrt{\frac{\sin 2\phi \sin \phi}{\sin (\theta_1 - \phi) \sin \theta_1}} \right]^3} $$

$$ P_N = P \cos \phi $$

$$ P_T = P \sin \phi $$

(18-15)
It should be observed that the equations are written for \( P \), the total pressure for the full height of the loaded wall per linear foot. They are in the form of \( \frac{1}{2}wh^2 \) multiplied by a reduction factor. The unit pressure at any depth \( h \) on the loaded wall, for the same conditions of loading, can be calculated as \( wh \) multiplied by the same factor.

Those equations above pertaining to walls sloping away from the filling are valid only up to a certain limiting condition. Cain points out that this condition, for level filling, is that the angle of slope of the wall or hopper bottom may not be flatter than the angle of rupture of the filling; i.e., the acute angle between the face of the wall and the horizontal may not be less than the angle \( z \) of the plane of rupture as determined by Eq. (18-1). When the wall or hopper bottom lies at a flatter slope than that of the plane of rupture, the force \( p \) is determined by combining the forces \( L \) and \( V \). The forces \( p_n \) and \( p_t \) are then derived from \( p \). Detailed discussion of this subject will be made under Hopper Bottoms.

Whether or not these equations, the development of which is based upon incipient movement of the restraining wall, are strictly applicable to the comparatively rigid walls of a concrete bin is an open question. Their use for the computation of bin pressures can be questioned also on the grounds that they were developed for a straight wall whereas bin walls change direction, resulting in an overlapping of the wedges of maximum pressure. However, the use of these equations has produced safe structures. Until reliable quantitative test data prove them inapplicable, it seems reasonable to continue with this procedure.

**Airy’s Solution**

A second method of computation of the lateral and vertical pressures in shallow bins was originally advanced by Airy. In a later statement by Airy,\(^{13}\) he simplified this approach. The procedure presupposes vertical walls and the friction of the filling on the walls effective.

Airy’s formula reads

\[
P_N = \frac{wh^2}{2} \left[ \frac{1}{\sqrt{\mu (\mu + \mu') + \sqrt{1 + \mu^2}}} \right]^2 \quad (18-16)
\]

and

\[
p_n = wh \left[ \frac{1}{\sqrt{\mu (\mu + \mu') + \sqrt{1 + \mu^2}}} \right]^2 \quad (18-17)
\]

With Eq. (18-16) the vertical pressure at any depth can be found by deducting the weight of the filling supported by the friction of the filling on the walls from the total weight of the filling above the level in question.

Thus total vertical load in the bin at any height is

\[
V_T = whA - P_N U \mu' \quad (18-18)
\]

and unit vertical load

\[
V = wh - \frac{P_N U \mu'}{A} \quad (18-19)
\]

For a circular bin

\[
V = wh - \frac{4P_N \mu'}{D} \quad (18-20)
\]

For a square bin

\[
V = wh - \frac{4P_N \mu'}{b} \quad (18-21)
\]

Both Ketchum and Gray\(^{14}\) restate this solution in detail. Ketchum\(^{14}\) adds the statement that unit vertical pressure \( V \) can be calculated from the relation of \( k = L/V \). Gray, pointing out that one major value of Airy’s solution is its elimination of the uncertain \( k \) quantity, adheres to the procedure indicated above.

For level filling and if the effect of the friction of the wall on the filling is included, Airy’s solution and Coulomb’s theory give practically identical lateral thrust.
Because of space limitation a detailed example of shallow-bin-wall design is not presented here. Both Ketchum\(^4\) and Gray\(^18\) illustrate the design of square- and rectangular-bin walls in which conditions of bottom restraint permit horizontal reinforcing. Where definite restraint exists at the junction of the bin bottom and the walls, analysis of the problem is readily made according to the procedures of the Portland Cement Association paper on Rectangular Concrete Tanks, ST 63, or can be made using as a guide the two-way slab coefficients of the American Concrete Institute Building Code. Reinforcement to take care of axial tension must be placed horizontally regardless of the direction of the moment steel. Circular-bin walls are rarely tied to the bin bottom slab. Determination of the horizontal wall reinforcing is therefore simple and direct. The forces are computed as for any cylindrical vessel with equivalent hydrostatic loading.

Consideration must be given, in fixing wall thickness and detailing the placing of reinforcing steel, to the depositing of the concrete so as to avoid segregation. In no case should the wall thickness be less than 6 in. A rule-of-thumb approach for preliminary determination of wall thickness for both shallow and deep circular bins for moderately heavy filling gives 6 in. for bin diameters up to and including 18 ft, 7 in. for diameters between 18 and 26 ft, and 8 in. for diameters between 26 and 34 ft. For lighter fillings these diameter limits are increased from 2 to 4 ft, and for very heavy fillings they are reduced in the same amount. These thicknesses are generally ample for all shallow bins even when the effect of friction of walls on the filling is included.

For the design of shallow bins with heaped fill, consideration should be given to the bin proportions before applying strictly the equations for sloping fill based on Coulomb’s theory. It will be recalled that the lateral thrust determined by this theory is arrived at by evaluating the forces in a wedge of the filling bounded by the planes of natural slope and rupture of the material and by the restraining wall (Fig. 18-8a). When the opposite bin wall is introduced into this diagram (Fig. 18-8b), it is apparent that a part of this computed filling does not exist and that therefore the equations for sloping fill are not applicable.

For comparatively narrow bins (although still of such proportions that they may be classed as shallow bins) a more reasonable approach is to calculate the weight of the heaped filling as a surcharge on the material contained in the bin level full, and to apply the equations for level filling. When it is desired to take into account the effect of friction between the filling and the wall, the weight of the heaped filling can be introduced into the applicable equations by increasing the unit weight \(w\) in the ratio of average height of the filling to the height of the wall.

For very shallow bin walls, where the volume of heaped filling above the plane of rupture forms a large proportion of the total filling behind the wall, use of the equations for sloping filling appears reasonable. When the loading of the bin is done from a centrally located charging point (Fig. 18-8c), the equations are definitely applicable.

Comment on Friction of Filling on the Bin Wall

Practice in some offices in the design of shallow bins assigns zero value to the frictional effect of the wall on the filling regardless of the conditions of storage. This procedure results in high lateral thrust and zero force downward along the face of the wall. Its application gives amply safe section to the walls against horizontal pressures but it overlooks entirely the added vertical load of the filling carried by the walls through friction. While this force is seldom of sufficient magnitude to affect the design of shallow-bin concrete walls, occasions do arise where its neglect in the calculation of the substructure could result in an unsound design.

It is true that, under certain conditions, the quantitative value of the friction of the
walls on the filling would be reduced. Saturation of the filling, introduction of air to expedite flow, location of unloading outlets at the periphery of the bin, possible vibration of the container, very rapid filling of the bin—any of these conditions would affect to some extent this frictional force. However, such situations are the exception. Where they cannot obtain, it hardly seems logical not to make use of this frictional effect in the calculation of a shallow bin and yet make full use of it in the calculation of a deep bin, particularly as the distinction between these two types is a somewhat arbitrary one. Experiment has shown that the frictional force between the face of wall and the filling is nearly always present. A simple and graphic experiment on this subject is reported by J. L. Campbell.¹⁵

**DEEP BINS**

**Load Carried by Walls**

The greater number, by far, of concrete bulk-storage containers is of the deep-bin class. Their economy lies in the proved phenomena that the major part of the weight of the filling is borne by the bin walls directly and thus need not be supported on the hopper or bin bottom, and that the horizontal thrust of the filling is comparatively small. By reason of the friction between the particles of the filling, the material arches and wedges against the vertical walls where again, by reason of friction, but in this case the friction between filling and wall, the weight of the filling is largely transferred into the walls. This action is illustrated diagrammatically by Fig. 18-9, in which the arched lines indicate the filling supported by the walls and the vertical, and nearly vertical lines indicate the filling carried by the bottom. That this is at least the general nature of the action in deep bins is confirmed by results of tests on several full-size elevators and a great number of model bins, all containing grain, conducted in the early 1900s. Many of these experiments are reported in *Engineering News* of that period and a number of them are summarized in Ketchum.¹⁶

The support of the filling by the walls is in no wise reduced during emptying of the bins. The flow of material is the same as that of the granules in an egg timer or an hourglass (Fig. 18-10). The material on the upper surface falls away from the side walls and moves downward as a column through the surrounding fill to the open gate. There is very little attrition of the side walls during this movement. Hopper bottoms, however, are subject to abrasion during the final emptying of the bin. When the height of filling drops to such level in the hopper that the area of the falling column over the gate becomes large in proportion to the total area, the dome action is broken and the balance of the material slides down the surfaces of the hopper. Hopper bottoms are also subject to abrasion during the initial charging of the bin when the first particles of the stored material, often falling freely from the top of the bin, strike the hopper surfaces.

**Structural Loading Conditions**

Structural design of deep bins requires determination of pressures or loads at four principal sections (Fig. 18-11):

1. Maximum lateral pressure—to compute maximum reinforcing in circular bins and to compute maximum wall thickness and maximum reinforcing in straight-walled bins
2. Lateral pressure at intermediate heights—to determine elevations at which reinforcing may be safely reduced
3. Maximum vertical pressure—to afford data for bin-bottom design
4. Maximum load of filling carried by the walls—to be added to wall dead loads, superstructure live and dead loads, and sometimes wind and earthquake loads in order to:
   a. Check concrete wall thickness
   b. Afford data for footing or substructure design

Calculation of these data in deep bins is generally made from one of two sets of formulas.

**Janssen's Formula**

The first of these, advanced by Janssen\(^{17}\) is the more widely used. Janssen's formula for vertical pressure:

\[
V = \frac{wR}{k\mu'} \left( 1 - \frac{1}{k\mu'h} \right) \quad (18-22)
\]

and for lateral pressure, since \(k = \frac{L}{V}\),

\[
L = \frac{wR}{\mu'} \left( 1 - \frac{1}{k\mu'h} \right) \quad (18-23)
\]

or where only tables of common logarithms are available

\[
V = \frac{wR}{k\mu'} \left( 1 - \frac{1}{N} \right) \quad \text{and} \quad L = \frac{wR}{\mu'} \left( 1 - \frac{1}{N} \right)
\]

where \(N\) is the number whose common logarithm is \(0.4343k\mu'h/R\).

Deviation of these equations is completely restated by both Ketchum\(^{18}\) and Gray.\(^{19}\)

The formulas contain the quantity \(R\), the hydraulic radius, or as noted previously, the ratio of interior bin area to its perimeter. This quantity should not be confused with the dimensional radius of the bin.

For a circular bin

\[
\text{Hydraulic radius } R = \frac{A}{U} = \frac{\pi D^2/4}{\pi D/4} = D
\]

For a square bin

\[
R = \frac{A}{U} = \frac{bb}{4b} = \frac{b}{4}
\]

For a rectangular bin

\[
R = \frac{A}{U} = \frac{bl}{2b + 2l} = \frac{bl}{2(b + l)}
\]

\(R\) can be readily determined geometrically for any regular polygon as \(D/4\) where \(D\) is the diameter of the inscribed circle. For interstice, segmental, or other irregularly shaped bins, the areas and perimeters can usually be calculated; where this procedure becomes difficult or involved, the quantities may be scaled from the layout.

Janssen's formula can be applied in one of four different ways, depending upon the previously prepared data available to the designer.

As an illustrative problem, take a circular bin of 20 ft diameter (or a bin of any regular shape with an inscribed diameter of 20 ft) and a total height of 75 ft and con-
taining anthracite coal (Fig. 18-12). Find the lateral pressure \( L \) immediately above the bin bottom and the vertical pressure \( V \) on the bin bottom.

Effective depth of filling

\[
h = 75 - \frac{3}{4}(2\frac{9}{4} \times \tan 27^\circ) = 71.6 \text{ ft}
\]

**Method I.** The formulas

\[
L = \frac{wR}{\mu'} \left( 1 - \frac{1}{\frac{k\mu'h}{e^R}} \right)
\]

and \( V = L/k \) can be applied directly, computing the value of the fraction within the parentheses by either natural or common logarithms. From Table 18-1, \( w = 55 \text{ lb}, k = 0.375, \mu' = 0.510, \phi = 27^\circ. R = D/4 = 2\frac{9}{4} = 5. \) At the level of the bin bottom

\[
L = \frac{55 \times 5}{0.51} \left( 1 - \frac{1}{\frac{0.375 \times 0.51 \times 71.6}{5}} \right)
\]

\[
= 540 \left( 1 - \frac{1}{e^{2.74}} \right)
\]

\[
= 540 \left( 1 - \frac{1}{15.49} \right)
\]

\[
= 505 \text{ lb}
\]

\[
V = \frac{505}{0.375} = 1,346 \text{ lb}
\]

To calculate the value of \( e^{2.74} \) by common logarithms, multiply

\[
2.74 \times 0.4343 = 1.180982
\]

and find \( N = 15.49 \) in a table of common logarithms.

**Method II.** The formulas can be applied directly, determining the value of the entire quantity within the parentheses from Table 18-2. At the level of the bin bottom, as above,

\[
L = \frac{55 \times 5}{0.51} \left( 1 - \frac{1}{\frac{0.375 \times 0.51 \times 71.6}{5}} \right)
\]

\[
= 540 \left( 1 - \frac{1}{e^{2.74}} \right)
\]

From Table 18-2 the value of \( 1 - \frac{1}{e^{2.74}} = 0.935 \) and

\[
L = 540 \times 0.935 = 505 \text{ lb}
\]

\[
V = \frac{505}{0.375} = 1,346 \text{ lb}
\]

This method is the most rapid one when tables or graphs for specific fillings are not available.

**Method III.** Tables can be prepared for fillings of different values of the product of \( k \times \mu' \). To set up such tables the form of the equations is changed. Accepting the hydraulic radius as a measure of pressure we can evaluate \( A/U = D/4 \) for bins of any
Table 18-2. Values of \( \left( 1 - \frac{1}{e^3} \right) \) Where \( n = \frac{k/a^h}{R} \)

<table>
<thead>
<tr>
<th>n</th>
<th>( \left( 1 - \frac{1}{e^3} \right) )</th>
<th>( n</th>
<th>( \left( 1 - \frac{1}{e^3} \right) )</th>
<th>( n</th>
<th>( \left( 1 - \frac{1}{e^3} \right) )</th>
<th>( n</th>
<th>( \left( 1 - \frac{1}{e^3} \right) )</th>
<th>( n</th>
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*Courtesy of W. S. Gray, ref. 13, p. 115.
shape. Then Eq. (18-22) can be changed to

$$V = \frac{w \times D}{4 \times k \times \mu'} \left(1 - \frac{1}{1 + \frac{4 \times k \times \mu' \times h}{D}}\right)$$  \hspace{1cm} (18-24)

and

$$V = \frac{w}{D} \left(1 - \frac{1}{1 + \frac{4 \times k \times \mu' \times h}{D}}\right)$$  \hspace{1cm} (18-25)

and

$$L = \frac{w}{D} \left(1 - \frac{1}{1 + \frac{4 \times k \times \mu' \times h}{D}}\right) = \frac{V}{D} \times k$$  \hspace{1cm} (18-26)

Table 18-3. Bin Pressures of Anthracite Coal

Calculated by Janssen’s formula

| $w = 55$ lb, $\mu' = 0.5095$, $k = 0.375$ |
| For bituminous coal, multiply $L/D \times 0.66$ and $V/D \times 0.915$ |
| $w = 50$ lb, $\mu' = 0.70$, $k = 0.271$ |

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</table>
Tabular values are arranged for ratios of $h/D$, $L/D$, and $V/D$. The table is entered with the value of $h/D$ at the level of the effective depth in question. $L/D$ and $V/D$ are read directly, and $L$ and $V$ are found by multiplying the values of $L/D$ and $V/D$, respectively, by $D$.

Tabular values of this type for anthracite coal, gravel, crushed stone, portland cement, and wheat are given in Tables 8-3 to 8-7, inclusive.

**Table 18-4. Bin Pressures of Gravel**

Calculated by Janssen’s formula

$w = 100$ lb  \( \mu' = 0.52 \)  \( k = 0.30 \)

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At the level of the bin bottom

$$
\frac{h}{D} = \frac{71.6}{20} = 3.58
$$
Enter Table 8-3 at $h/D = 3.5$.

\[
\frac{L}{D} = 24.99 + \frac{18}{20} \times 0.29 = 25.25
\]

\[
\frac{V}{D} = 66.64 + \frac{18}{20} \times 0.76 = 67.32
\]

$L = 25.25 \times 20 = 505 \text{ lb}$

$V = 67.32 \times 20 = 1,346 \text{ lb}$

**Method IV.** Graphs can be prepared for fillings of different values of the product of $k \times \mu'$. Figure 18-13 is such a graph for anthracite coal. The graph is entered at

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**Table 18-5. Bin Pressures of Crushed Stone**

Calculated by Jansen’s formula

$w = 100 \text{ lb}$  \quad $\mu' = 0.637$  \quad $k = 0.30$
DESIGN OF DEEP BINS AND SILOS

the level of the effective depth in question. $L$ is read directly on the curves for the given bin diameters, and $V$ is found by dividing $L$ by $k$ (or multiplying $L$ by the reciprocal of $k$).

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At the level of the bin bottom

$h = 71.6'$

$L$ on curve for $D = 20$ ft is read as $505$ lb

$V = 505 \times \frac{1}{0.375} = 505 \times \frac{8}{3} = 1,346$ lb

Design from tables or graphs is standard practice in most offices doing considerable bin work. This procedure is more rapid and less subject to error than direct application of the formula.
Lateral pressures at other depths, for use in determining the height at which reinforcing steel may be safely reduced, can be computed, of course, by the same methods used to calculate the lateral pressure at the level of the bin bottom.

**Table 18-7. Bin Pressures of Wheat**

Calculated by Janssen’s formula

\[ w = 50 \text{ lb} \quad \mu' = 0.4167 \quad k = 0.6 \]

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<td>4.4</td>
<td>29.63</td>
<td>0.07</td>
<td>49.39</td>
<td>0.11</td>
</tr>
<tr>
<td>1.5</td>
<td>23.30</td>
<td>0.64</td>
<td>38.84</td>
<td>4.6</td>
<td>29.70</td>
<td>0.05</td>
<td>49.50</td>
<td>0.09</td>
</tr>
<tr>
<td>1.6</td>
<td>23.94</td>
<td>0.58</td>
<td>39.90</td>
<td>4.8</td>
<td>29.75</td>
<td>0.05</td>
<td>49.59</td>
<td>0.07</td>
</tr>
<tr>
<td>1.7</td>
<td>24.52</td>
<td>0.52</td>
<td>40.87</td>
<td>0.97</td>
<td>5.0</td>
<td>29.80</td>
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</tr>
<tr>
<td>1.8</td>
<td>25.04</td>
<td>0.47</td>
<td>41.74</td>
<td>0.87</td>
<td>6.0</td>
<td>29.92</td>
<td>0.05</td>
<td>49.88</td>
</tr>
<tr>
<td>1.9</td>
<td>25.51</td>
<td>0.43</td>
<td>42.52</td>
<td>0.78</td>
<td>7.0</td>
<td>29.97</td>
<td>0.02</td>
<td>49.95</td>
</tr>
<tr>
<td>2.0</td>
<td>25.94</td>
<td>0.39</td>
<td>43.23</td>
<td>0.71</td>
<td>8.0</td>
<td>29.99</td>
<td>0.00</td>
<td>49.98</td>
</tr>
<tr>
<td>2.1</td>
<td>26.33</td>
<td>0.35</td>
<td>43.88</td>
<td>0.65</td>
<td>9.0</td>
<td>29.99</td>
<td>0.01</td>
<td>49.99</td>
</tr>
<tr>
<td>2.2</td>
<td>26.68</td>
<td>0.31</td>
<td>44.46</td>
<td>0.58</td>
<td>10.0</td>
<td>30.00</td>
<td>0.01</td>
<td>50.00</td>
</tr>
<tr>
<td>2.3</td>
<td>26.99</td>
<td>0.29</td>
<td>44.99</td>
<td>0.53</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

To determine the load of filling borne by the walls, the vertical load \( V \) is subtracted from the total load of filling. This load per linear foot of wall for a bin of any regular shape can be reduced to \( P_T = D/4 (wh - V) \). Thus

\[
P_T = \frac{1}{4} (55 \times 71.6 - 1,346) = 12,960 \text{ lb}
\]

For circular bins graphs can be prepared for various fillings and for specific allowable steel stresses from which the reinforcing can be read directly. An example of such a graph is given by Larkin.20

In cases where a table or graph for a specific filling is not available, but where the product of \( k \times \mu' \) of the filling in question equals (or approximately equals) that of a
filling for which a table or graph is available, such prepared data can be used to advantage. The value of the quantity within the parentheses of Eqs. (18-22) to (18-25) changes slowly and for fillings of approximately equal products of $k \times \mu'$ may be safely taken as equal at like values of $h/R$ or $h/D$. $L$ or $L/D$ will then vary directly as $w$ and inversely as $\mu'$ and $V$ or $V/D$ will vary directly as $w$ and inversely as the product of $k \times \mu'$.

**Fig. 18-13.** Lateral pressure chart for anthracite coal bins of variable height and diameter.

Thus for bituminous coal with $w = 50$ lb, $k = 0.271$ and $\mu' = 0.7$, $L$ or $L/D$ may be taken as $50/0.7 + 55/0.5095 = 66$ per cent of anthracite coal and $V$ or $V/D$ as $50/0.271 \times 0.7 + 55/0.375 \times 0.5095 = 91.5$ per cent of anthracite.

With like values of $k$ and $\mu'$, $L$ or $L/D$ and $V$ or $V/D$ for fillings of differing values of $w$ will vary directly as $w$. Thus for cement raw mix, $L$ or $L/D$ and $V$ or $V/D$ may be taken as 75 lb/100 lb = 75 per cent of that of portland cement.

From the form of Eqs. (18-22) and (18-23) it will be noted that the lateral and vertical pressures vary inversely as the roughness of the concrete walls; that is, the rougher the wall, the smaller the pressure. It should be recalled that the limiting value for this coefficient $\mu' = \tan \phi'$ must be that of the friction of the material upon itself, which is $\mu = \tan \phi$.

Use of Janssen's formula has been questioned because of the uncertainty of the value
DEEP BINS

of \( k = (L/V) \). No analytical determination of this factor for use in bins has been proved. The value in general use, \( \frac{1 - \sin \phi}{1 + \sin \phi} \), is derived in Eq. (18-4). This is the same value as proposed by Rankine\(^{33}\) for active pressure of a level filling of an incompressible granular material without cohesion and of indefinite extent. It is an approximation in that it ignores the effect of the friction of the walls on the filling. Accurate determination of this factor can be made only by experiment for various fillings and for varying dimensions and wall surfaces of the bins. Work in this field has been meager\(^*\) and such published quantitative results as have been obtained in full-scale tests, solely in grain, vary widely.

Effect of variation of the \( k \) value on the results obtained by Janssen’s formula may be observed from Table 18-8, which gives governing design pressures for anthracite coal in a circular bin of 20 ft diameter and 90 ft height as computed by the formula. Column \( B \) is based on the usually accepted value of \( k \) of 0.375, column \( A \) on a 20 per cent smaller value, and column \( C \) on a 20 per cent larger value.

**Table 18-8**

<table>
<thead>
<tr>
<th>( k )</th>
<th>( A )</th>
<th>( B )</th>
<th>( C )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( L )</td>
<td>( V )</td>
<td>Carried by wall, lb/lin ft</td>
</tr>
<tr>
<td>0.30</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>248</td>
<td></td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>323</td>
<td></td>
<td></td>
</tr>
<tr>
<td>40</td>
<td>380</td>
<td></td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>422</td>
<td></td>
<td></td>
</tr>
<tr>
<td>60</td>
<td>453</td>
<td>1.512</td>
<td>8,940</td>
</tr>
<tr>
<td>70</td>
<td>476</td>
<td>1.588</td>
<td>11,310</td>
</tr>
<tr>
<td>80</td>
<td>493</td>
<td>1.643</td>
<td>13,780</td>
</tr>
<tr>
<td>90</td>
<td>506</td>
<td>1.685</td>
<td>16,330</td>
</tr>
</tbody>
</table>

Table 18-8 indicates that:

1. Lateral pressures vary as \( k \), although not in direct ratio, and the effect of this variation is most pronounced in the upper portions of the bin and becomes negligible in the lower portions. It might be noted that a reasonable amount of steel reinforcement placed to resist temperature stress is usually ample to take care of load stresses in upper wall areas for light to moderately heavy fillings.

2. Bin bottom pressures vary inversely as \( k \) and generally in the same ratio, and the effect of this variation increases as the \( h/D \) ratio increases.

\* An exception is the development of a device for the measurement of \( k \) for fine-grained materials by William Runo of the Richardson Scale Company as part of a recent investigation of solid-particle flow from bins.

It is called the “\( k \) determinator” and consists of a metal pressure tank in which is supported a cylinder of material to be tested. The material is placed in a very thin rubber sleeve and is supported in such a manner that if the specimen collapsed it could do so only by expanding in a horizontal direction. When air pressure is supplied to the tank it acts over the lateral surface of the rubber-contained specimen and reinforces it.

A piston acting on the top of the specimen is pressed down by means of an air cylinder, tending to cause the specimen to collapse. With the lateral pressure in the tank held constant, a vertical load is applied in increments until the specimen just collapses.

When this occurs, the horizontal component of the vertical pressure applied on the specimen cross section is just equal to or slightly greater than the lateral air pressure acting on the specimen. The ratio of these pressures \( L/V \) is then equal to \( k \).

With this apparatus, the value of \( k \) for portland cement, with an \( h/D \) ratio of 4, was found to be 0.25.
DESIGN OF DEEP BINS AND SILOS

3. The weight of filling carried by the walls through friction varies as \( k \) but in a fractional ratio, and the effect of this variation decreases as the \( h/D \) ratio increases.

4. In all cases a decrease in the value of \( k \) has a greater effect than an increase. From these observations it would seem logical, when doubt exists as to the true value of \( k \) and when a conservative design is required, to use a slightly higher value of this quantity for calculations of lateral pressures and weight of filling carried by the walls, and a slightly lower value for calculation of bin-bottom pressures.

Airy's Solution

A second set of formulas for the determination of pressures in deep bins was presented by Airy in the latter parts of his two papers mentioned previously.

Lateral pressure in pounds per square foot

\[
L = \frac{w \times D}{\mu + \mu'} \left[ 1 - \frac{\sqrt{1 + \mu^2}}{\sqrt{(2h/D) (\mu + \mu')} + (1 - \mu \times \mu')} \right]
\]

(18-27)

Maximum vertical pressure is found at

\[
h_{\text{max}} = \frac{D}{2} \left[ \frac{1 + \mu^2}{\mu + \mu'} \times \left( \frac{4\mu'}{3\mu' - \mu} \right)^2 - \frac{1 - \mu \times \mu'}{\mu + \mu'} \right]
\]

(18-28)

Total lateral pressure in pounds per linear foot for full height of filling above the elevation being considered

\[
P = \frac{w \times D^2}{2(\mu + \mu')^2} \left[ \sqrt{\frac{2h}{D} (\mu + \mu') + (1 - \mu \times \mu')} - \sqrt{1 + \mu^2} \right]^2
\]

(18-29)

Vertical load carried by bin walls in pounds per linear foot

\[
P_T = P \times \mu'
\]

(18-30)

Vertical pressure in pounds per square foot

\[
V = wh - \frac{P_T}{D/4}
\]

(18-31)

It should be emphasized that \( D \) as used in the above equations is either the diameter of a circular bin or the inscribed diameter of a bin of any regular geometrical figure. It may also be the lesser dimension of a rectangular bin, and when so used is more often designated as \( b \). The formulas are applicable to bins of any such shapes.

Derivation of these equations is restated in part and discussed by both Ketchum and Gray.

Airy's formulas can be applied in one of three different ways depending, as in the case of Janssen's formula, upon the previously prepared data available to the designer.

**Method I.** The formulas can be applied directly calculating:

1. Unit lateral pressure \( L \) at the elevation of greatest effective depth and at whatever other depths are deemed necessary to determine safe reduction of reinforcing steel.
2. Effective depth \( h \) at which maximum vertical pressure occurs, and then computing the total lateral pressure \( P \) and the vertical pressure \( V \) at this depth.
3. Total lateral pressure \( P \) at the elevation of greatest effective depth and then computing the vertical load of the filling carried by the wall through friction \( P_T \) at this depth.

As an illustrative problem, take a circular bin of 15 ft diameter (or a bin of any regular shape with an inscribed diameter of 15 ft) with a maximum effective depth of 90' (Fig. 18-14) and containing wheat. \( w = 50 \text{ lb}, \mu = 0.532, \mu' = 0.441 \).
DEEP BINS

Then
\[ 1 + \mu^2 = 1.283024 \]
\[ \sqrt{1 + \mu^2} = 1.1327 \]
\[ \mu + \mu' = 0.973 \]
\[ (\mu + \mu')^2 = 0.946729 \]
\[ 1 - \mu \times \mu' = 0.7654 \]
\[ 4\mu' = 1.764 \]
\[ 3\mu' - \mu = 0.791 \]

1. At the level of the bin bottom

\[ L = \frac{50 \times 15}{0.973} \left[ 1 - \frac{1.133}{\sqrt{18945} \times 0.973 - 0.765} \right] \]
\[ = 771 \left[ 1 - \frac{1.133}{3.527} \right] \]
\[ = 771 \times 0.679 \]
\[ = 523 \text{ lb} \]

2. \( h_{\text{max}} \) = 7.5 \[ \left[ \frac{1.283024}{0.973} \times \left( \frac{1.764}{0.791} \right)^2 - 0.765 \right] \]
\[ = 7.5\left[1.32 \times 4.97 - 0.787 \right] \]
\[ = 7.5 \times 5.773 \]
\[ = 43.29 \text{ ft} \]

\[ P \text{ at } h_{\text{max}} \]
\[ = \frac{50 \times 225}{2 \times 0.9467} \left[ \sqrt{\frac{86.58}{15} \times 0.973 + 0.765388 - 1.1327} \right]^2 \]
\[ = 5941.5(2.5262 - 1.1327)^2 \]
\[ = 5941.5 \times 1.942 \]
\[ = 11,537 \text{ lb} \]

\[ P_T \text{ at } h_{\text{max}} \]
\[ = 11,537 \times 0.441 = 5,088 \text{ lb} \]

\[ V_{\text{max}} \]
\[ = 50 \times 43.29 - 5,088 \]
\[ = 2,165 - 1,357 \]
\[ = 808 \text{ lb} \]

3. \( P \) at bin bottom
\[ = 5,941.5(3.527 - 1.133)^2 \]
\[ = 5,941.5 \times 5.731 \]
\[ = 34,050 \text{ lb} \]

\[ P_T \text{ at bin bottom} \]
\[ = 34,050 \times 0.441 \]
\[ = 15,020 \text{ lb} \]

The above procedure is obviously slow and not susceptible to slide-rule calculation.

**Method II.** Tables can be prepared for fillings with like values of \( \mu \) and \( \mu' \) in bins of any selected \( D \). In the preparation of such tables \( D \) is preferably chosen somewhat smaller than the mean of this quantity generally used for the filling. The essential tabulated data are values of \( h/D, P, L, P_T, \) and \( V \) for each foot of effective depth, and \( \tan x \) for the filling and \( h_{\text{max}} \) for the \( D \) used. Tables with a range of \( h \) of from 10 to 100 ft are generally ample for bins of usual proportions.

Airy, in his original paper, includes a table for wheat, using

\[ \mu = 0.466 \text{ and } \mu' = 0.444 \]

in a 10-ft square bin 100 ft high. Gray has reproduced this table with additional comment.

Table 18-9 is a part table from which the principal design loads for any shaped bin
can be rapidly determined. For practical design, the table should be entirely completed. It will be observed, in the completed portion of the table, that $L$ at any elevation equals the difference between $P$ at that elevation and $P$ at the next greater foot of effective depth.

Table 18-9. Wheat

<table>
<thead>
<tr>
<th>$h$</th>
<th>$\frac{h}{D}$</th>
<th>$P$</th>
<th>$L$</th>
<th>$P_T = P \times \mu'$</th>
<th>$V$</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>1.333</td>
<td>2.914</td>
<td>295</td>
<td>1,285</td>
<td>657</td>
</tr>
<tr>
<td>30</td>
<td>2.000</td>
<td>6.247</td>
<td>366</td>
<td>2,755</td>
<td>765</td>
</tr>
<tr>
<td>40</td>
<td>2.667</td>
<td>10.157</td>
<td>410</td>
<td>4,479</td>
<td>806</td>
</tr>
<tr>
<td>41</td>
<td>2.733</td>
<td>10.572</td>
<td>415</td>
<td>4,662</td>
<td>807</td>
</tr>
<tr>
<td>42</td>
<td>2.800</td>
<td>10.991</td>
<td>419</td>
<td>4,847</td>
<td>807</td>
</tr>
<tr>
<td>43</td>
<td>2.876</td>
<td>11.414</td>
<td>423</td>
<td>5,034</td>
<td>808</td>
</tr>
<tr>
<td>44</td>
<td>2.933</td>
<td>11.840</td>
<td>426</td>
<td>5,221</td>
<td>808</td>
</tr>
<tr>
<td>45</td>
<td>3.000</td>
<td>12.269</td>
<td>429</td>
<td>5,411</td>
<td>807</td>
</tr>
<tr>
<td>46</td>
<td>3.067</td>
<td>12.702</td>
<td>433</td>
<td>5,602</td>
<td>806</td>
</tr>
<tr>
<td>47</td>
<td>3.133</td>
<td>13.138</td>
<td>436</td>
<td>5,794</td>
<td>805</td>
</tr>
<tr>
<td>48</td>
<td>3.200</td>
<td>13.577</td>
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<td>5,987</td>
<td>803</td>
</tr>
<tr>
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<td>3.267</td>
<td>14.020</td>
<td>443</td>
<td>6,182</td>
<td>801</td>
</tr>
<tr>
<td>50</td>
<td>3.333</td>
<td>14.465</td>
<td>445</td>
<td>6,379</td>
<td>799</td>
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<tr>
<td>55</td>
<td>4.000</td>
<td>19.063</td>
<td>472</td>
<td>8,407</td>
<td>758</td>
</tr>
<tr>
<td>70</td>
<td>4.667</td>
<td>23.890</td>
<td>493</td>
<td>10,536</td>
<td>691</td>
</tr>
<tr>
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<td>5.333</td>
<td>28.903</td>
<td>509</td>
<td>12,746</td>
<td>601</td>
</tr>
<tr>
<td>90</td>
<td>6.000</td>
<td>34.066</td>
<td>523</td>
<td>15,203</td>
<td>492</td>
</tr>
<tr>
<td>100</td>
<td>6.667</td>
<td>39.362</td>
<td>535</td>
<td>17,359</td>
<td>371</td>
</tr>
<tr>
<td>120</td>
<td>8.000</td>
<td>50.272</td>
<td>555</td>
<td>22,170</td>
<td>88</td>
</tr>
</tbody>
</table>

For a bin containing wheat with $D = 15$ ft the principal design loads are found from Table 18-9 as follows:

1. Maximum $L$ at effective depth of 90 ft is read directly as 523 lb. $L$ at intermediate heights to determine elevations at which reinforcing may be reduced can also be read directly from a completed table.

2. The value of $h_{max}$ is a basic part of the table and is read as 43.29 ft. $V_{max}$ at 43 ft is read directly as 808 lb.

3. $P_T$ at effective depth of 90 ft is read directly as 15,023 lb.

Table 18-9 can also be used to determine the pressures of wheat in bins of any other dimension of $D$. When the table is so used the values of $\mu$ and $\mu'$ remain constant and:

1. $L$, for the same values of the $h/D$ factor, will vary directly as the ratio of the $D$ used to the $D$ of the table ($D_2/D_1$). Equation (18-27) shows that, when $h/D$ is constant, the only variable is $D$.

2. $h_{max}$ will vary directly as $(D_2/D_1)$. Equation (18-28) shows $D$ to be the only variable.

$P$, for the same values of the $h/D$ factor, will vary as the square of the ratio of the $D$ used to the $D$ of the table ($D_2/D_1$). Equation (18-29) shows that, when $h/D$ is constant, the only variable is $D$.

$P_T$, for the same values of the $h/D$ factor, will also vary as $(D_2/D_1)^2$. Equation (18-30) shows that $P_T$ varies in the same manner as $P$.

$V_{max}$, for the same values of the $h/D$ factor, will vary directly as $(D_2/D_1)$. Observation of the form of Eq. (18-31) shows this to be true. As noted above $h_{max}$ varies directly as $(D_2/D_1)$ and therefore $V_{max}$ will vary in this same ratio. $P_T$, the numerator of the second part of Eq. (18-39), varies as $(D_2/D_1)^2$, as noted above. The denominator varies directly as $D_2/D_1$ and the entire fraction therefore varies directly as $D_2/D_1$. Thus the whole equation, for the same values of the $h/D$ factor, varies directly as $D_2/D_1$. 
3. $P_T$ at greatest effective depth will vary as $P_T$ at any other depth in the bin or, as noted above, as $(D_2/D_1)^2$.

Thus for a bin of 25 ft $D$ and 100 ft effective depth (Fig. 18-15) containing wheat with the same constants as used in the table:

1. $\frac{h}{D}$ at greatest effective depth = $\frac{100}{25} = 4.0$
   
   $L$ for $D = 15$ at $\frac{h}{D}$ of 4.0 from Table 18-9 = 472 lb
   
   $L$ for $D = 25$ at $\frac{h}{D}$ of 4.0 = $472 \times \frac{25}{15} = 787$ lb

1a. $\frac{h}{D}$ at some intermediate depth, say 50 ft = $\frac{50}{25} = 2.0$

   $L$ for $D = 15$ at $\frac{h}{D}$ of 2.0 from Table 18-9 = 366 lb
   
   $L$ for $D = 25$ at $\frac{h}{D}$ of 2.0 = $366 \times \frac{25}{15} = 610$ lb

2. $V_{\text{max}}$ for $D = 15$ from Table 18-9 = 808 lb
   
   $V_{\text{max}}$ for $D = 25 = 808 \times \frac{25}{15} = 1,347$ lb

3. $P_T$ is maximum at greatest effective depth = $10\%_5 = 4.0$

   $P_T$ for $D = 15$ at $\frac{h}{D}$ of 4.0 from Table 18-9 = 8,407 lb
   
   $P_T$ for $D = 25$ at $\frac{h}{D}$ of 4.0 = $8,407 \times \frac{(25)^2}{(15)^2} = 23,350$ lb

At the head of the table are noted the values of $\tan x$ and shallow bin constant. The table for deep bins is not applicable in the upper portions of the bin at effective depths of less than $D \tan x$. At lesser depths Airy's solution for shallow bins must be used. For practical design the only data required at such depths are the value of

$$L = \frac{wh}{\left[\frac{1}{\sqrt{\mu(\mu + \mu')} + \sqrt{1 + \mu'}}\right]^2} \quad (18-17)$$

The shallow-bin constant given in the table is the value of the quantity within the brackets, squared.

Thus to determine $L$ in the 25 ft $D$ bin above, at $h = 30$ ft: $30 < 25 \times 1.37 = 34.25$. Therefore at this effective depth the table is not applicable and $L$ must be calculated:

$$L_{25 \text{ ft}} = 50 \text{ lb} \times 30 \text{ ft} \times 0.2915 = 437 \text{ lb}$$

At elevation $D \times 1.37$, both deep- and shallow-bin calculation procedures give the same results for lateral pressure, indicating continuity of Airy's solution for the two classes of bins. Observation of Fig. 18-16 shows a smooth transition of the curves of lateral pressure at these elevations.

**Method III.** Graphs can be prepared for fillings of constant values of $\mu$ and $\mu'$ and for varying values of $D$. Separate graphs must be drawn for lateral and vertical pressures. Just as in the case of tables, data for the drawing of curves of all values of $D$ can be assembled from the information supplied by any one completed curve. Two such graphs are shown in Figs. 18-16 and 18-17. As noted, the solid lines indicate pressures according to Airy's solution. The graphs are entered at the level of effective depth in question and all design loads except $P_T$ can be read directly. $P_T$ can
be readily calculated, however, from data furnished by the curves as follows: Equation (18-31) can be rearranged to read

$$P_T = \frac{D}{4} (wh - V)$$

(18-32)

Then, to calculate $P_T$ for $D = 15$ at 90 ft effective depth, enter graph at 90 ft and read $V = 492$ lb. At 90 ft $wh = 4,500$ lb and $P_T = \frac{15}{4}(4,500 - 492) = 15,030$ lb.

Fig. 18-16. Lateral-pressure chart for wheat bins of variable height and diameter.

Ketchum shows a graph similar to that of Fig. 18-16 giving lateral pressures of wheat based on the same constants of $w$, $\mu$, and $\mu'$ as used above in bins of 10, 20, and 30 ft diameters.

From observation of the curves of Fig. 18-17 it might be concluded that vertical pressure at the bottoms of bins of greater $h$ or smaller $D$ approaches zero. However, investigation of the derivation of Airy’s formulas shows that this cannot be true but
that the least total pressure is the weight of a volume of filling equal to

\[ \frac{1}{3} \times A \times \frac{D}{2} \times \tan \phi = \frac{w \times A \times D \times \mu}{6} \]

and that the least unit pressure = \[ \frac{w \times A \times D \times \mu}{6A} = \frac{w \times D \times \mu}{6} \]

as shown in Fig. 18-18.

The greatest weight of filling that can be carried by the walls through friction is therefore that of the total filling less that of this volume.

Consideration of this condition is not of major practical importance, however, in bin design. It occurs only in bins of small D and large h; it enters the calculations only by way of Eq. (18-32), and the quantities involved are negligible.*

* Moreover, the bin bottom must be designed to support the maximum load imposed upon it, even though this load may occur only during the filling of the bin.
From the form of Eqs. (18-27) through (18-31) it can be observed that for fillings of like values of \( \mu \) and \( \mu' \), \( L \) and \( V \) vary directly as \( w \).

Differences in pressures as calculated by Airy’s solution and Janssen’s formula are indicated in the graphs of Figs. 18-16 and 18-17. It will be observed that, for the bin sizes and constants used, Airy’s solution gives lateral pressures about one-fifth greater and maximum vertical pressure about one-third less than those calculated by Janssen’s formula.* From the data presented in the graphs it can be determined by Eq. (18-32) that the load carried by friction on the bin walls as calculated by Airy’s solution is about one-fifth greater than that arrived at by Janssen’s formula. For rectangular bins, because of the manner in which \( D \) for Janssen’s formula is determined, the pressures as calculated by the two methods are in closer agreement.

It should be noted that Janssen’s formula is developed for circular bins. The accuracy of its application to bins of any other shape has been questioned by several recent writers. Blanchard\(^{22} \) has pointed out that pressures computed for square or rectangular bins by this formula may be underestimated by 10 per cent in the lower portions of an 80-ft-high bin and by as much as 33 per cent in the upper 10 ft of the container. These conclusions were reached through mathematical treatment and further investigation on model-sized units.

Insufficient full-scale experiment has been conducted on bin pressures to determine the exactness of results deriving from either of these procedures. Plotted results of such limited work as has been done on lateral pressures give curves more nearly approaching those of Janssen’s formula. Experiment on bottom pressures, on the other hand, shows that maximum pressures were reached before the bins were fully loaded, thus giving support to Airy’s solution for vertical pressures.

Airy’s solution does not depend upon the controversial \( k \) factor of Janssen’s formula, and the value of \( w, \mu, \) and \( \mu' \) can be readily and accurately determined.

With prepared tables or graphs the application of both solutions for practical design is equally rapid. Preparation of such tables and graphs is a somewhat shorter operation when based on Janssen’s formula than when based on Airy’s solution. The majority of design offices in the United States use Janssen’s formula.

Both Janssen’s and Airy’s methods give pressures for static loading. Some increase in local lateral pressures, particularly in the area opposite side-wall outlets, has been observed during very rapid bin unloading. However, the pressures computed by these methods have proved adequate for the design of reinforced-concrete bins.

**DESIGN OF CIRCULAR-DEEP-BIN WALLS**

**Outline of Design Steps**

Circular deep bins, either single or multiple, are most often used for simple storage where site space is not limited. Bin dimensions are generally fixed by reasons of economy. The prime object of this type of storage construction is to provide the required capacity at minimum unit cost of the material to be stored. The several factors, some of which are interrelated, affecting the over-all economy of a complete installation will be mentioned later.

After capacity, internal diameter, and height have been fixed, the structural design is made in eight well-defined steps. These are determination of:

1. Unit lateral pressure at the level of the bin bottom and at several elevations on the walls, starting above the bin bottom where \( h/D \) is approximately 2.00.
2. Total lateral tensile stresses for the unit pressures found in step 1.
3. Horizontal reinforcing at the level of the bin bottom and at such other elevations where it appears, from step 2, that reduction, or reductions, of reinforcing steel is safe.

* Critics of this solution point out, with apparent justification, the improbability of a reduction in bin-bottom load with an increase in height of filling.
4. Tentative wall thickness, by the recommendations suggested under circular shallow bin walls.
5. (a) Unit vertical pressure at level of the bin bottom (from step 1), (b) total bin-bottom load, and (c) net portion of the stored material carried through friction by the side walls.
6. Total vertical loads carried by the walls.
   a. Roof live and dead loads.
   b. Head-house dead loads.
   c. Machinery and equipment loads.
   d. Wall dead loads, from step 4.
   e. Portion of the stored material carried through friction by the side walls from step 5c.
   f. Portion of the stored material carried directly on the bin bottom, from step 5b, and bin-bottom dead load. This calculation is required only if a part or all of the bin-bottom load is carried on the walls.
   g. Vertical component of the wind load on the leeward side or vertical component of earthquake forces, if any.

7. Adequacy of the wall thickness assigned in step 4. In making such check, the net wall section is first computed by deducting all openings and recesses. This net area should be sufficient to carry the loads 6a through 6e, or 6a through 6f, above the computed areas at a stress of not more than 0.15f', and the loads 6a through 6b at a stress of not more than 0.20f'. The check for wind-load stress is usually important only for installations of one bin width but the check for earthquake-load stress should always be made.
8. Detailing of reinforcing steel, both horizontal and vertical.

Steel Reinforcement

Detailing of the reinforcing steel is of major importance. Construction of deep bin walls is a rapid 24-hr-a-day operation, carried out in cramped working space and often during inclement weather. Economical and safe construction requires that the reinforcing steel be detailed so that it can be stored and handled easily on the moving forms and be placed quickly and accurately. Length of horizontal bars should range between 15 and 25 ft, depending upon the degree of curvature and the spacing of the yokes.

Assuming a template built above the yokes to guide and steady them, the vertical bars to be placed while the forms are moving should be no longer than 16 ft. Vertical bars resting directly on the base slab and placed before the form has started to move may be as long as 20 ft. In either case, for adequate rigidity, bars should be no smaller in diameter than ½ in. round. Jack rods, to be placed while the forms are moving, should be no longer than 15 ft and where set on the base slab may be 20 ft.

Practice relative to spacing vertical steel varies considerably. For storage of materials at normal temperatures and with construction of the walls by a method which leaves the jack rods in place, reasonably adequate vertical reinforcing of 6- to 8-in.-thick walls is obtained by spacing ½-in. bars approximately 2 ft 0 in. on centers between the jack rods. A ½ or ¾-in. splice bar should be placed adjacent to each joint in the smooth jack rods. When major temperature differential between the stored material and the wall concrete is anticipated, the design procedure should follow that of chimney construction, with consequent larger percentage of vertical (and horizontal) steel. If construction of the walls is by a method which permits extraction of the jack rods, an equivalent area of permanent vertical reinforcing steel should be added.

The functions of vertical reinforcing steel are to (1) guide the placing of the horizontal bars, (2) distribute unequal local load more uniformly to the horizontal bands, (3) reinforce the concrete for differential temperature stress, and (4) reinforce the concrete for bending stress caused by the eccentric loading of the walls brought about
by the frictional load of the filling on the walls. None of these purposes can be properly served by a wider spacing of vertical steel. Some of the horizontal cracks occasionally apparent in bin walls could have been prevented by a more generous detailing of the vertical steel.

Design Example

The following design is an example of straightforward construction of multiple circular bins. The building is an annex to a grain storage plant in Paullina, Iowa. It consists of eight circular bins 22 ft 0 in. in diameter by 96 ft 0 in. high (Fig. 18-19). The walls rest on a concrete mat which serves also as the bin-bottom slab. Reclaiming is accomplished through off-center outlets through the mat, loading to a conveyor tunnel under the center of the unit.

Fig. 18-19. Multiple circular bins.  Fro. 18-20. 92-ft effective-depth bin containing wheat.

Specifications for the design were the following: Design by Janssen’s formula for wheat with $w = 50 \text{ lb}$, $\mu' = 0.4167$, $k = 0.6$, $\phi = 28^\circ$, $f_x' = 2,500$, $f_s = 18,000$.

From Table 18-1, $\tan \phi = 0.532$.

Maximum effective depth $= 96 - \frac{3}{4} (22 \times 0.532) = 96 - 3.9 = 92.1$, say 92 ft

Design Step 1. From Table 18-7, at the level of the bin bottom (Fig. 18-20),

$$h = 92 \quad \frac{h}{D} = \frac{92}{22} = 4.18$$

$$\frac{L}{D} = 29.45 + \frac{0.18}{0.20} \times 0.10 = 29.54$$

$$L = 29.54 \times 22 = 650 \text{ psf}$$

Following the same procedure:

- $L$ where $h = 50 \text{ ft}$ (42 ft above the bin bottom) = 592 psf
- $L$ where $h = 40 \text{ ft}$ (52 ft above the bin bottom) = 553 psf
- $L$ where $h = 30 \text{ ft}$ (62 ft above the bin bottom) = 490 psf
- $L$ where $h = 20 \text{ ft}$ (72 ft above the bin bottom) = 394 psf
- $L$ where $h = 15 \text{ ft}$ (77 ft above the bin bottom) = 325 psf
- $L$ where $h = 10 \text{ ft}$ (82 ft above the bin bottom) = 243 psf

Design Steps 2 and 3. At level of the bin bottom where $h = 92 \text{ ft}$,

$$T = \frac{650 \times 22}{2} = 7,150 \text{ lb} \quad A_s = \frac{7,150}{18,000} = 0.397 \text{ or } \frac{3}{9} \text{ ft} \text{ at } 9/8 \text{ in.}$$
Similarly,

<table>
<thead>
<tr>
<th>$h$ (ft)</th>
<th>$T$ (lb)</th>
<th>$A_s$</th>
<th>$\frac{h}{D}$ at 10 in.</th>
<th>$\frac{h}{D}$ at 11 in.</th>
<th>$\frac{h}{D}$ at 12 in. or $\frac{h}{D}$ at 8 in.</th>
<th>$\frac{h}{D}$ at 10 in.</th>
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<tr>
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</tr>
<tr>
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<tr>
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</tr>
<tr>
<td>10</td>
<td>2,670</td>
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<td>$\frac{5}{8}$</td>
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</tbody>
</table>

**Design Step 4.** Wall thickness was tentatively set at 7 in. in accordance with the recommendations suggested under circular-shallow-bin walls.

**Design Step 5.** Bin loads from the 22-ft-diameter bins carried by the walls:

a. Lateral pressure $L$ at the level of the bin bottom, from step 1, is 650 psf. From Janssen’s formula $k = L/V$ or $V = L/k$ and $V = 650/0.6 = 1,083$ lb. $V$ can also be found directly from Table 18-7. At $h/D = 4.18$,

$$V/D = 49.08 + (0.18/0.20) \times 0.17 = 49.23$$

and $V = 49.23 \times 22 = 1,083$ lb.

b. Total bin-bottom pressure $= 1,083$ (lb/sq ft) $\times 380$ (sq ft) $= 412,000$ lb

c. Filling load carried by walls per bin $= 1,336,000$ lb

Note: Bin area is obtained from Table 18-10.

Bin loads from the interstice bins carried by the walls: Area $A$ of an interstice bin equals approximately 97.0 sq ft. Perimeter $U$ of an interstice bin is found to be by sealing 46.6 lin ft.

$$R = \frac{A}{U} = \frac{97.0}{46.6} = 2.08 \quad D = 2.08 \times 4 = 8.32$$

Effective depth $h$ at bin bottom is approximately 94 ft.

$$\frac{h}{D} = \frac{94.0}{8.32} = 11.3$$

For this ratio Table 18-7 shows $V/D$ at the absolute maximum of 50.0.

a. $V = 50.0 \times 8.32 = 416$ psf

b. Total filling load per bin $= 50$ (lb/cu ft) $\times 97.0$ (sq ft) $\times 94$ (ft) $= 456,000$ lb

c. Filling load carried by the walls per bin $= 416,000$ lb

Each interior 22-in.-diameter bin supports one-half of an interstice-bin wall load. Thus the total filling load carried by the walls of an interior main bin $= 1,336,000 + 208,000 = 1,544,000$ lb.

**Design Step 6.** All loads for one interior bin:

a. Roof live load, dead load, and roofing

$$= (40 \text{ lb} + 50 \text{ lb} + 5 \text{ lb}) \times 24 \text{ ft} \pm = 2,280 \text{ lb/lin ft}$$

b. Head-house floor live load + dead load

$$= (60 \text{ lb} + 50 \text{ lb}) \times 7 \text{ ft} = 770$$

Head-house wall dead load

$$= 40 \text{ lb} \times 8 \text{ ft} = 320$$

c. Conveyor dead load

$$= 120 \text{ lb} = 120$$

$$= 3,490 \text{ lb/lin ft}$$

$$= 80 \text{ kips}$$

d. Wall dead load $= 421.52 \text{ sq ft} - 380.13 \text{ sq ft} \times 0.15 \text{ kips} = 596$

e. Filling load carried by the walls (from step 5) $= 1,554$

Total load on the walls at the level of the bin bottom $= 2,230$ kips



### Table 18-10 (Continued)

<table>
<thead>
<tr>
<th>Diam, ft in.</th>
<th>Area, sq ft</th>
<th>Circum, ft in.</th>
<th>Diam, ft in.</th>
<th>Area, sq ft</th>
<th>Circum, ft in.</th>
<th>Diam, ft in.</th>
<th>Area, sq ft</th>
<th>Circum, ft in.</th>
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<td>48</td>
<td>1809.562</td>
<td>150 93/4</td>
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<td>1963.5</td>
<td>157 3/4</td>
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<td>1</td>
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<td>145 33/4</td>
<td>3</td>
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<td>4</td>
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<td>147 43/4</td>
<td>11</td>
<td>1803.283</td>
<td>150 63/4</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

---

**Fig. 18-21. Typical reinforcing plan for bins.**
**Design Step 7**

Unit vertical wall stress = \[
\frac{2,230,000}{(421.52 - 380.13) \times 144} = 373 \text{ psi} < 375 \text{ O.K.}
\]

**Design Step 8.** Using the results obtained in steps 2 and 3, band arrangement might have been as indicated. As actually built the walls were more conservatively reinforced in the lower area.

Figure 18-21 shows a part reinforcing plan of the structure. Horizontal steel is usually placed near the outside face allowing about 2 in. of cover. At this dimension, 22 ft 10 in., the steel circle from Table 18-10 is 71 ft 8 in. and the net length of each of three bars is 23 ft 11 in. Adding for a 40-diameter lap (20 diameters to develop the bar and 20 diameters for placing variation) gives 26 ft 0 in. for the \( \frac{3}{8} \)-in. bars and 25 ft 6 in. for the \( \frac{1}{2} \)-in. bars (Fig. 18-22). Splices should be staggered around the periphery of the bin. Some field man would find greater economy in the use of four bars of shorter length for this size bin.

**Special Considerations**

One-half-inch vertical bars were placed between the jack rods as indicated. The omission of all vertical reinforcing in the walls adjacent to the interstice bins is a questionable, although common, practice in grain-bin work. It must be admitted that distress in these areas, where such steel is omitted, is seldom apparent.

All vertical reinforcing used was \( \frac{1}{2} \) in. by 15 ft 0 in. long with 1 ft 6 in. lap. The arrangement (Fig. 18-23) whereby splices occur in alternate runs is a better scheme structurally and spreads the work of placing over a greater time interval.

Alternate runs of jack rods were used (Fig. 18-24). Increasing the number of these rods to seven and eight, respectively, would have permitted somewhat shorter and more easily handled bars. The total jack rod run as used was 96 ft 0 in., the same as the height of the bin. This arrangement resulted in the extension of the upper half of the top jack sleeve into the roof slab. Some designers prefer complete separation of wall and roof with the thought that restraint at this point may cause cracking in the upper part of the wall.

A 2,500-lb concrete was used for this work. Some contractors prefer to use a minimum of 9-bag batch yielding 3,500- to 4,000-lb concrete. Use of this richer mix permits faster raising of the forms and may occasionally make completion of the wall construction possible without week-end overtime. While greater shrinkage can result from a rich mix, reduced water content and the use of A305 deformed bars have produced sound concrete, using such mixes, in a wide variety of bin structures.

The allowable vertical compressive wall stress of 0.15 \( f'_c \) is conservative for the determination of concrete section to support centrally applied loads. However, other stresses, which are not usually individually considered, are present in most bin walls.

These stresses arise from:

1. Eccentric vertical loading. The live load of the filling is brought into the wall by friction between the filling and the interior face of the wall. Only under the unusual condition of a wall between two adjacent bins of the same cross section filled to the same heights with the same material can the wall be considered as centrally loaded.

2. Usual and minor temperature differential between the exterior and interior faces of the bin wall.
3. Restraint at the horizontal junction between the base of the wall and the supporting slab.
   Except when there is possibility of overturning due to wind or earthquake, the walls are not dowelled or keyed to the slab. Nevertheless, the friction between the heavily loaded wall and the floated, or even troweled, surface of the slab prevents free movement at this location.

4. Restraint at the vertical junction between walls of adjoining bins.

5. Horizontal radial loads on the walls of an empty bin in a cluster of otherwise loaded bins.

6. Concrete shrinkage.

The 0.15 figure has proved to give sufficient section to provide for these stresses except under unusual requirements of bin proportions and loading. Adequate

![Fig. 18-23. Arrangement of vertical reinforcing for bins.](image)

![Fig. 18-24. Arrangement of jack rods for bins.](image)

investigation of all of them might justify the use of a higher allowable vertical stress. Timoshenko's *Theory of Elastic Stability* and the ASCE Manual No. 31 deal with buckling of walls of similar proportions; the Portland Cement Association's bulletins ST 57 and ST 63 discuss vertical wall moments; and Bulletin ST 57 also treats in detail with shrinkage of concrete walls.

The latter stress may be of major importance in the design of bins for the storage of materials readily affected by dampness and where even hair cracks must be prevented, i.e., cement, lime, flour, and sugar.

**DESIGN OF RECTANGULAR- AND SQUARE-DEEP-BIN WALLS**

**Outline of Design Steps**

Rectangular and square deep bins, usually multiple, are most often used for simple storage on limited sites or for storage as part of a manufacturing process. These conditions generally fix the bin dimensions.

The sequence of design for rectangular- or square-deep-bin walls is the same as that for circular-bin walls but the calculations are more detailed. The steps are determination of:

1. Unit lateral pressure at the level of the bin bottom and at several elevations on the walls, starting above the bin bottom, where \( h/D \) is approximately 2.00.
2. Maximum end and mid-span moments at the level of the bin bottom.
3. Wall thicknesses required for maximum moment at the level of the bin bottom.
4. Tentative horizontal steel areas required by moment only at the level of the bin bottom.
5. Direct tension stresses in the walls at the level of the bin bottom.
6. Horizontal steel areas, required by combined moment and tensile stresses at the level of the bin bottom.
7. Shear and bond stresses.
8. Horizontal steel areas required by combined moment and tensile stresses at such elevations where it appears, from step 1, that steel may be safely reduced. Reinforcing required reduces in direct ratio to the reduction in lateral pressures. It is the practice of some designers to omit the computation of stresses due to direct tension in the bins and to determine horizontal reinforcing areas from moment requirement only, using a reduced allowable steel stress.

For rectangular bins steps 2 through 8 are computed for the walls in both directions.
9. Vertical pressures. The same as step 5 for circular-bin walls.
10. Total vertical loads carried by the walls. The same as step 6 for circular-bin walls.
11. Adequacy of the wall thickness computed in step 3. The same as step 7 for circular-bin walls.
12. Detailing of reinforcing steel, both horizontal and vertical.

The comments made under circular deep bins relative to ease and accuracy of placing the reinforcing are even more pertinent in the case of rectangular and square bins, for here the effectiveness of the steel depends upon its exact positioning from the face of the wall.

Design Example

The following design is an example of the calculation of the walls of multiple rectangular bins. The structure, a blending bin for the manufacture of silicon carbide built on the West Coast, is divided into 44 storage units, each with clear dimensions of 9 by 10 ft and with height of 45 ft 4 in., and containing materials as indicated on Fig. 18-25. Bin dimensions were determined by the arrangement of the processing equipment.

The difference between length and width of each bin is small. In fact, as will be noted later, the variation was too little to require different bar sizes in the two directions. It is the practice of some designers to use the moments computed for a fixed-end span of the longer wall to arrive at wall thickness and reinforcing areas for the walls in both directions. Such procedure would have been quite justified for this design but when the ratio of length to width of bin reaches 1.25 to 1.30, the use of different-
sized bars in the two directions, or of different thickness of walls, gives a more economical design. The procedure as presented here is applicable to bins of any degree of rectangularity.

Specifications called for the design to be based on the ACI Building Code, using 3,000-lb concrete with 20,000 lb allowable steel stress.

Decision was further made, preliminary to the design, to:
1. Use Janssen’s formula for the calculation of pressures.
2. Use moment distribution to compute end moments, and carry the distribution for only two cycles. It is to be noted that a poured bin is an ideal rigid frame. Use clear spans in the computation of fixed-end moments and relative stiffnesses.

Neglect the minor effect on the moment distribution of the small splays at the ends of the walls.

Consider each wall as a series of horizontal prismatic beams, each 1 ft in height.
3. Use the same vertical spacing of steel, for given run of height, throughout the entire structure.
4. Use the same diameter and length of reinforcing bars on both faces of any given section of wall.
5. Use the same reinforcing in the sawdust bins as used in the coke bins.

Had the bins been square and all walls of the same thickness, there would have been no need for the distribution procedure of step 2 immediately above. With spans of the same relative stiffness, there would have been no unbalanced moment to distribute, regardless of the loading pattern in the bins. Negative moments at the ends of the walls would then have been \( pi^2/12 \) and positive moments at mid-span would have been \( pi^2/24 \).

**Design Step 1.** Data used for wall design for the three fillings were:

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<tr>
<th>Filling</th>
<th>( M )</th>
<th>( \phi )</th>
<th>( \phi' )</th>
<th>( \mu' )</th>
<th>( h )</th>
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<tbody>
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</tr>
<tr>
<td>Crust and old mix</td>
<td>70</td>
<td>34°</td>
<td>30°</td>
<td>0.577</td>
<td>0.283</td>
</tr>
<tr>
<td>Coke</td>
<td>45</td>
<td>35°</td>
<td>35°</td>
<td>0.700</td>
<td>0.271</td>
</tr>
</tbody>
</table>

Effective height of filling was taken as 44 ft. By Janssen’s formula, lateral pressure per foot of height

\[
L = \frac{w \times R}{\mu'} \left( 1 - \frac{1}{k \mu'h} \right) \tag{18-23}
\]

For all bins, \( R = \frac{A}{U} = \frac{9 \times 10}{2(9 + 10)} = \frac{90}{38} = 2.37 \).

For sand at \( h = 44 \) ft the value of \( \frac{k \mu'h}{R} = \frac{9.283 \times 0.577 \times 44}{2.37} = 3.03 \).

For this value, Table 18-2 gives the value of 0.952 to the quantity within the parentheses and \( L = \frac{100 \times 2.37}{0.577} \times 0.952 = 390 \) psf.

By the same procedure, lateral pressures for the three fillings at several effective depths were calculated as:

<table>
<thead>
<tr>
<th>( h )</th>
<th>Sand</th>
<th>Crust and old mix</th>
<th>Coke</th>
</tr>
</thead>
<tbody>
<tr>
<td>44</td>
<td>390</td>
<td>280</td>
<td>150</td>
</tr>
<tr>
<td>25</td>
<td>340</td>
<td>240</td>
<td>130</td>
</tr>
<tr>
<td>20</td>
<td>310</td>
<td>215</td>
<td>120</td>
</tr>
<tr>
<td>15</td>
<td>265</td>
<td>185</td>
<td>110</td>
</tr>
<tr>
<td>10</td>
<td>205</td>
<td>140</td>
<td>85</td>
</tr>
</tbody>
</table>
Design Step 2. Fixed-end moments for sand filling at full loading where \( h = 44 \text{ ft} \).

10-ft span
\[
\frac{390 \times 10 \times 10}{12} = 3.25 \text{ ft-kips/ft of height}
\]

9-ft span
\[
\frac{390 \times 9 \times 9}{12} = 2.60 \text{ ft-kips/ft of height}
\]

Moment distribution for bin 3, containing sand, is set down in detail in Fig. 18-26. The relative stiffness of 1.0 and 0.9 is noted on each member. With all walls of the same thickness, the \( EI \)s of all members are equal and the stiffnesses vary only inversely as their respective lengths.

![Diagram](image-url)

Fig. 18-26. Moment distribution for bin 3, Fig. 18-25.

The sign convention adopted gives (+) values to the ends of all those members in the Fig. 18-26 calculation which, when observed as a drawing is usually read (i.e., from the bottom or from the right-hand side of the sheet), are bent upward from the joint by the loading, and give (−) values to those members which are bent downward as in Fig. 18-27.

Moments for the left-hand end of each member are written parallel to and below the member and those for the right-hand end of each member are written parallel to and above the member (Fig. 18-28a, b). Here again, the drawing is read from the bottom and the right-hand side of the sheet.

A joint is balanced when the algebraic sums of the moments on either side of the joint are equal. Interpretation of "either side of the joint" can lead to confusion. An arbitrary procedure which both fixes this term and also eliminates chance of error in determining the amount of unbalanced moment to be distributed and sign values of distributed moment is that of considering the net total moment in the members to the right and above the joint. In this procedure the drawing is read from the bottom of the sheet.
DESIGN OF RECTANGULAR- AND SQUARE-DEEP-BIN WALLS 18-43

Thus at joint $B$, before distribution, there is a moment of $+3.25$ on one side of the joint and of $+2.60$ on the other. The unbalanced moment equals $+3.25$ minus $+2.60 = 0.65$. This moment is distributed to the members meeting at the joint in proportion to their relative stiffnesses or 0.22 to $BA$, 0.21 to $BE$, and 0.22 to $BC$. Remembering that the moments should balance, we assign a minus value to the 0.21 distributed to $BE$ so as to reduce the moment in $BE$ (on one side of the joint) and a plus value to the 0.22 distributed to $BC$ so as to increase the moment in $BC$ (on the other side of the joint). A minus value is assigned to the 0.22 distributed to $BA$ because $BA$ lies on the same side of the joint as does $BE$.

Again at joint $E$, before distribution there is a net moment of 0 on one side of the joint (members $ED$ and $EG$) and of $+3.25$ plus $-2.60 = 0.65$ on the other side of the joint (member $EB$ and $EF$). After distribution, again remembering that the

![Fig. 18-27. Sign convention for moments in bin walls.](image)

Fig. 18-27. Sign convention for moments in bin walls.

![Fig. 18-28. Fixed end moments for bin 3, Fig. 18-25.](image)

Fig. 18-28. Fixed end moments for bin 3, Fig. 18-25.

larger moment should decrease, assign a minus value to the 0.15 distributed to $EB$. The 0.17 in $EF$ also receives a minus value because it lies on the same side of the joint as does $EB$. Plus values are assigned to both the 0.15 in $EG$ and to the 0.17 in $ED$ because those members lie on the opposite side of the joint from $EB$.

The carry-overs to the opposite ends of the members are, in accordance with standard moment-distribution procedure for prismatic members, $-\frac{3}{4}$ times the distributed moment.

In the loaded bins, the fixed-end moments are written immediately adjacent to the walls and the distributed moment above or below them; in the unloaded bins, the distributed moments are written immediately adjacent to the walls. It will be observed that in the loaded bins the distributed moments are larger than the fixed-end moments in the shorter spans, and smaller than the fixed-end moments in the longer spans.

By collecting the distributed moments for various combinations of loading, it is possible to obtain absolute maximum or absolute minimum end-moment values for each wall. Thus on the wall between bins 20 and 21 and at the joint adjacent to bin 9, it is possible to obtain a maximum end moment of 3.39 ft-kips by considering bins 20, 9, and 32 fully loaded with adjacent bins entirely empty; and a minimum end moment of 2.87 ft-kips by considering bins 20, 10, and 31 fully loaded and adjacent bins entirely empty. However, in the usual operation of storage facilities such loading combinations are unlikely, and it is considered safe to design for values obtained for single bin loading. Of course, when severe loading combinations are anticipated, they should be provided for in the design.

For design of the wall spans it is desirable to designate the moments as negative and positive in the customary sense used in structural design. Conversion of the signs of the distributed moments to those used in design can be done readily if the convention adopted to arrive at the signs as explained above is kept in mind.

**Design Step 3.** Wall thickness for the long walls of the sand bins (Fig. 18-29). Moment at the ends of the 6-in. splays from Eq. (18-33):

\[
M = -3.13 + 1.95 \times 0.5 - 0.39 \times 0.5 \times 0.5 = 2.20 \text{ ft-kips}
\]

\[
d = \sqrt{\frac{2.200}{236}} = \sqrt{9.32} = 3.1 \quad \text{Say 3} \frac{1}{4} \text{ in.}
\]
Assuming $3\frac{1}{4}$-in. bars, $\frac{3}{8}$ in. for variation in placing, and 1-in. cover, wall thickness could have been made $3\frac{1}{4} + \frac{3}{8} + \frac{3}{8} + 1 = 5$ in. Thickness was made 7 in. and $d$ taken as $5\frac{1}{2}$ in.

**Design Step 4.** Moment at mid-span of the long walls of the sand bins = $pl^2/8$ — average end moment $= 0.39 \times 10 \times 10/8 = 3.13 = 4.88 - 3.13 = 1.75$ ft-kips (Fig. 18-29).

$$A_s = \frac{1.75}{1.44 \times 5.25} = 0.231 \text{ sq in.}$$

The calculation for reinforcing-steel area is made at mid-span rather than at the ends of the walls because the bars will be spliced at the wall intersections and the laps can be made of sufficient length to provide for the larger moment at these points.

**Design Step 5.** Calculation of tension in the long walls of the sand bins is based on the full loading of the bin on one side of the wall only (Fig. 18-30) and this tension is assumed to be divided equally between the two equal lines of reinforcing steel in the wall.

$$T = \frac{0.39 \times \frac{3}{8}}{2} = 0.8775 \text{ kip}$$

While axial-tension stress would be doubled with both bins adjacent to the wall loaded, such loading would reduce or eliminate the bending stress in the steel.

**Design Step 6.** Two procedures are available for this calculation. An exact method is indicated in the Portland Cement Association’s paper on Rectangular Tanks, ST 63, and in the American Concrete Institute’s *Reinforced Concrete Design Handbook*. However, for concrete sections of the proportions dealt with in bin walls, the following simpler computation is amply accurate:

At mid-span of the longer walls of the sand bins

$$M = 1.75 \text{ ft-kips} \quad \text{and} \quad N = 0.8775 \text{ kip}$$

$$d'' = 5.25 - \frac{7.0}{2} = 1.75 \text{ in.}$$

$$e = \frac{1.75 \text{ ft-kips} \times 12}{0.8775 \text{ kip}} - 1.75 \text{ in.} = 22.18 \text{ in.}$$

$$j = 0.875 \text{ approximately}$$

$$A_s = \frac{877.5}{20,000 \times 0.875 \times 5.25} (22.18 + 0.875 \times 5.25) = 0.256 \text{ sq in.}$$

$\frac{3}{8}$ at 14 in. gives 0.27 sq in. per ft of height. $A_s$ may be determined as above, for from mechanics and Fig. 18-31a, $b$, $c$ we know that $M = NE$.

If $d''$ = distance from the gravity axis to the reinforcing steel and $e$ = distance from the reinforcing steel to the line of action of $N$

$$M = N(d'' + e)$$

and

$$e = \frac{M}{N} + d''$$
then taking moments about the line of compression in the concrete:

\[ A_s f_{jd} = N \left( e + jd \right) \]
\[ A_s = \frac{N \left( e + jd \right)}{f_{jd}} \]

**Design Step 7.** At the ends of the wall

\[ V = 390 \times 1\frac{1}{4} = 1,950 \text{ lb} \]
\[ v = \frac{1,950}{12 \times 9.875 \times 5.25} = 35 \text{ psi} \quad < 90 \text{ psi O.K.} \]
\[ u = \frac{1,950}{1.7 \times 0.875 \times 5.25} = 248 \text{ psi} \quad < 300 \text{ psi O.K.} \]

Note that this computation is based on only a single bar being available at the end of the wall on the loaded face to provide bond. This is the least amount of steel possible at this point.

Step 8 would normally follow at this point. However, in view of the design specification that the same spacing of reinforcing be maintained for any given run of height throughout the structure, and that several different fillings are involved, it is well to consider first the reinforcing requirements for the other loads.

![Fig. 18-30. Tension in walls for bin 3, Fig. 18-25.](image)

Fig. 18-30. Tension in walls for bin 3, Fig. 18-25.

At bin-bottom level in the longer interior bin walls, these requirements, computed in accordance with steps 4 through 6, or by direct proportion of the unit loads, are 0.184 sq in. per ft of height for the crust and old mix bins and 0.092 sq in. for the coke and sawdust bins. Bars \( \frac{5}{8} \) in. at 13 in. and \( \frac{3}{4} \) in. at 14 in. would have fulfilled these requirements. Spacing was fixed at 12 in. for all bars and minimum bar also at \( \frac{1}{2} \) in. Use of \( \frac{3}{4} \)-in. bars would have provided less than minimum reinforcing of 0.0025 bd. The same bar size was used in both long and short walls because the reduction in steel requirements in the shorter walls was considerably less than the reduction of steel area furnished by the next smaller size bar.

**Design Step 8.** The summary of lateral pressures calculated earlier gives, at effective depth of 20 ft, unit load of 310 lb for the sand bins and 215 lb for the crust and old mix bins. By direct proportion the steel requirements at this elevation are

\[ \frac{310}{20} \times 0.256 = 0.204 \text{ sq in. equivalent to } \frac{5}{8} \text{ in. at } 18 \text{ in.} \]

and

\[ \frac{215}{80} \times 0.184 = 0.141 \text{ sq in. equivalent to } \frac{3}{4} \text{ in. at } 17 \text{ in.} \]

Bar spacing at this elevation was increased from 12 to 15 in. While the above figures would justify a slightly greater increase in spacing it was felt desirable to keep the spacing within 3 x d.

At the elevation where effective depth is 10 ft, unit load for the sand bins becomes 205 lb. Again by direct proportion steel area required is

\[ \frac{205}{80} \times 0.256 = 0.135 \text{ sq in. equivalent to } \frac{1}{2} \text{ in. at } 18 \text{ in.} \]
The \( \frac{5}{8} \)-in. bars in the sand bins were reduced to \( \frac{3}{4} \) in. at this elevation, at 15-in. spacing.

**Design Step 9.** As actually built, the interior walls of this structure are supported on columns, built into certain of the wall intersections, and the bin bottom slabs are hung from the underside of the walls. However, for purpose of illustration it will be assumed that all walls are self-supporting.

Computations for steps 9, 10, and 11 are made for the sand bins where the load is the heaviest.

---

**Diagram:**
- **Fig. 18-32.** Pilasters and splay in rectangular bin framing.
- **Fig. 18-33.** Bin bottom supported in wall prints.

---

a. Lateral pressure \( L \) at the level of the bin bottom, from step 1, is 390 psf. From Janssen's formula \( k = L/V \) or \( V = L/k \) and \( V = 390/0.283 = 1,380 \) lb.

b. Total bin bottom pressure \( = 1,380 \times 9 \times 10 = 124,200 \) lb
   Total load per bin \( = 100 \times 9 \times 10 \times 44 = 396,000 \) lb

\[ c. \text{Load carried by walls per bin} = 271,800 \text{ lb} \]

**Design Step 10.** Total load on the walls at the elevation of the top of the bin-bottom slab (Fig. 18-32).

- Roof live load and roofing \( = 45 \) psf
- Roof slab and beams \( = 60 \)
- Head-house floor live load \( = 100 \)
- Head-house floor slab \( = 75 \)

\[ \frac{280 \times 9.58 \times 42.92}{280 \text{ psf}} = 115 \text{ kips} \]

Walls 0.583(42.92 + 9 × 5) \( = 51.26 \) sq ft

Splays \( \frac{\frac{3}{4} \times \frac{3}{4}}{2} \times 4 + \frac{\frac{1}{2} \times \frac{1}{2}}{2} \times 12 = 2.39 \)

Pilasters \( \frac{3}{4} \times 3 \times 2 \)

\[ \frac{55.15 \times 45.33 \times 0.150}{55.15 \text{ sq ft}} = 375 \]

Load carried by the walls (from step 9) \( 272 \times 4 = 1,088 \)

\[ \text{Total} = 1,578 \text{ kips} \]

Total load on the walls at the elevation of the underside of the bin-bottom slab.

- Load from above \( = 1,578 \) kips
- Bin-bottom live load (from step 9) \( 124 \times 4 = 496 \)
- Bin-bottom dead load \( 0.15 \times 0.833 \times 9.58 \times 41.75 = 50 \)

\[ \text{Total} = 2,124 \text{ kips} \]
Design Step 11. The net wall area carrying the load immediately above the bin-bottom slab is the gross area computed in step 10, less the prints or recesses provided to support the bin-bottom slab (Fig. 18-33). These prints usually extend from toe to toe of the splays; they are not placed in the splays.

Net wall area, taking all splays as 6 in. for this calculation,

\[
= 55.15 - \frac{1.375}{12} (9 \times 4 \times 2 + 8 \times 3 \times 2 + 8 \times 2 \times 1) = 39.55 \text{ sq ft}
\]

Unit vertical wall stress at the elevation of the top of the bin bottom slab

\[
= \frac{1,578,000}{39.55 \times 144} = 277 \text{ psi} \quad <450 \text{ O.K.}
\]

Unit vertical wall stress at the underside of the bin-bottom slab

\[
= \frac{2,124,000}{55.15 \times 144} = 267 \text{ psi} \quad <450 \text{ O.K.}
\]

This stress will actually be somewhat greater because of deduction of wall areas for passageways and possibly windows.

Design Step 12. The length of the horizontal steel for a given span is determined (Fig. 18-34a, b), as it is in any continuous structure, by adding to the dimension of the clear span sufficient extension to (1) anchor the bar, by bond, for tensile stresses occurring in the span when the span is loaded, and (2) supplement the bars in the adjoining loaded spans for (a) bond and (b) tensile stresses occurring in these spans.

Following the same procedure as indicated in step 6, the required tensile-steel area for the long walls of the sand bins is found to be 0.203 sq in. at the face of the cross walls with the effect of the splay taken into account. With two \( \frac{5}{6} \)-in. bars available

\[f_s = \frac{0.203}{0.62} \times 20,000 = 6,550 \text{ psi}\]

and \[x_1 = \frac{6,550 \times 0.31}{300 \times 2.0} = 3.5 \text{ in.}\]

Fig. 18-34. Extension of bin wall reinforcement beyond bin dimensions.

Step 7 shows the bond requirements of a loaded span to be satisfied with the bar existing in that span. It is not necessary, therefore, to extend the bar from \( l_1 \) into \( l_2 \) for this purpose.

To find \( x_2 \) for tensile stresses in the adjacent loaded span, it is necessary to determine first the point at which the bar in that span cannot, itself, resist these stresses.

Steps 4 and 6 indicate that 0.256 - 0.231 = 0.025 sq in. are required to resist direct tension stress, leaving 0.310 - 0.025 = 0.285 sq in. as the available steel to resist moment-tension stress. The resisting moment of 0.285 sq in. of steel, at \( d = 5.25 \text{ in.} \), closely approximates 0.285 \times 1.44 \times 5.25 = 2,155 \text{ ft-lb}.

The distance from the face of the support at which this steel is stressed to its allowable maximum is found through the proposition that "the bending moment at any given point in a beam is equal to the bending moment at any other point, plus the shear at the other point multiplied by the distance between the points, plus the alge-
braic sum of the moments of the forces between the points about the given point," as shown in Fig. 18-35. For uniform loading with a given end moment this reduces to

\[ M_x = \pm M_a + V_x x - \frac{Vx^2}{2} \]  \hspace{1cm} (18-33)

Assigning the proper algebraic signs and quantities gives

\[-M = -M_{\text{max}} + V_x x - \frac{Vx^2}{2}\]
\[-2,155 = -3,130 + 1,950x - \frac{390x^2}{2}\]
\[x^2 - 10x + 5 = 0\]
\[x = 9 \text{ ft} \ 5\frac{5}{6} \text{ in.}\]
\[(x = 6\frac{5}{6} \text{ in. not used})\]

The bar from span \( I_1 \) will extend at least to this point and 12 diameters beyond as specified by code. The required length is

\[10 \text{ ft} \ 0 \text{ in.} + 1 \text{ ft} \ 2 \text{ in.} + 1 \text{ ft} \ 1 \text{ in.} + 1 \text{ ft} \ 3 \text{ in.} = 13 \text{ ft} \ 6 \text{ in.}\]

The bars were made 15 ft 0 in. to take care of possible severe bin-loading conditions and variation in placing steel. Bars in the short sand-bin walls were made 14 ft 0 in. By the same reasoning the \( \frac{3}{4} \)-in. bars in the long and short crust and old mix bins were made 15 ft 0 in. and 14 ft 0 in., respectively. The \( \frac{3}{4} \)-in. bars in coke bins were made the same respective lengths for purpose of uniformity.

The outside ends of bars in the exterior bays were carried out as far as possible into the pilasters and hooked, primarily to provide additional ties between walls. Adequate moment steel section is furnished at this junction with single bars through an 8-in. splay. A standard hook is preferable to a right-angle bend because, for equal effectiveness, the hook can be made in a shorter distance from the axis of the bar.

The space through which these hooks must be moved to place the bar is limited to a width of from 6 to 10 in., the distance between the underside of the sliding form yoke and the upper surface of the freshly placed concrete.

Vertical bars (Fig. 18-36) were made up in two alternate runs, one of four bars 12 ft 6 in. long, and the other of two bars 7 ft 0 in. long and three bars 12 ft 6 in. long. Laps were arbitrarily made 1 ft 7 in. An 18-ft-long bar might have been substituted for the lowest 7 ft 0 in. and 12 ft 6 in. bars.
For this height alternate jack-rod runs (Fig. 18-37) might have been 14, 11, 11, and 9 ft and 9, 11, 11, and 14 ft.

A part reinforcing plan of the structure is shown in Fig. 18-38. Jack rods were set near the ends of the walls but in such location as to permit access, between the ends of the sliding form yokes, to the wall intersections. Vertical temperature bars were placed in pairs as an aid to holding the horizontal steel in correct position. The 3 ft 0 in. spacing between certain of the verticals provided clearance for the concrete buggies.

![Diagram showing part reinforcing plan of rectangular bin structure.](image)

**Fig. 18-38.** Part reinforcing plan of rectangular bin structure.

The 3/8-in. horizontal "hairpins," indicated at the corners and at the exterior wall intersections, were placed both as an added precaution to tie the walls at these points and to prevent possible cracking and spalling of the splays. Practice varies on the installation of hairpins. Some offices call for them at exterior wall intersections only where frequent unbalanced loading is probable; others require them regardless of size of bins or probable loading pattern. Hairpins are seldom placed at the intersections of interior walls.

**Hexagonal-deep-bin Walls**

Limitation of space precludes the illustration of design of hexagonal-bin walls. The subject is discussed in a later paragraph dealing with bin patterns. Paul Rogers presents such a design very ably. Alternate schemes of detailing the horizontal reinforcing might be considered, bearing in mind the interference of jack rods, lift rods, and vertical steel.
SLIDING FORMS

The economy of deep concrete bins derives primarily from the use of sliding—or "slip" or "moving" or "draw"—forms. Vertically moving forms for this type of structure were first developed in the early 1900s and until very recently there has been little change in their application.

Basically, the sliding form for a bin consists of two wood forms, generally 4 ft 0 in. high with the face of each built up of vertically placed tongued and grooved straight-grained hardwood strips, usually of 13/4 by 3 in. or 13/4 by 4 in. nominal size. Each of these forms is nailed to two- or three-piece walers, each of 2 in. thickness and of width determined by the shape of the bin and the spacing of the yokes (Fig. 18-39). The walers, in turn, are secured to the legs of the yokes. The yokes consist of the legs, about 7 ft 0 in. long, a crosshead mounted at their tops, and a cross strut set about 2 ft 0 in. below the head. Yokes are built of timber, steel channels, or steel tubular sections. A jack, of either the screw or "pump" type, is mounted on the cross frame and engages the vertical jack rods set at the center of the concrete wall. Jack rods are either solid or pipe sections and are spaced from 6 to 10 ft on centers.

After their initial filling, the forms are raised slowly but continuously until the bin walls are completely poured. Jacking is usually done in 3/4-in. increments and at a rate of from 6 to 10 in. an hour.

The support of the weight of the sliding forms, including that of the working platform, by the slender jack rods is made possible by the stiffening of these rods by the continually following wall concrete.

Jacking is still done manually on many projects with one man tending from 7 to 10 jacks. Several approaches to reducing this labor had been tried, including electric, air, and hydraulic control, but all were found either lacking in some operational detail or not economical. Recently, however, hydraulic systems developed by Swedish firms, with a change in the yoke design, have proved quite satisfactory on a number of major installations.

A working knowledge of sliding forms, including both their possibilities and their limitations, is essential to the economical design of deep bins. The subject is handled very competently in the ACI Committee 608 Report and in their Supplementary Report, and by Broughton. Some additional data are given by Hunter and by Wynn. An interesting descriptive and historical account appears in Grain Elevators of North America.

CONCRETE-STAVE WALLS

Under conditions that will permit the use of circular bins for light to moderately heavy loads and where complete exterior fireproofing is not essential, concrete-stave bins can make an economical installation.

The staves are individual high-strength plain-concrete slabs, generally solid, 23/4 in. thick, 10 in. wide, and 30 in. high and tongued and grooved on both edges and ends. Ends are either square or sloped, depending upon the manufacturer's pattern. A cored slab 35/8 in. thick, made with expanded clay aggregate, is also available for use where better insulation and fire protection are desired.

The staves are laid up with either mastic or mortar joints and are then bound with galvanized-steel rods, generally of 1/4 or 5/8 in. diameter with rolled upset threaded ends. These ends are slipped through galvanized malleable-iron connectors and secured with nuts. The rod pattern and tightening are such as to give reasonably uniform tension on all the rods.
Moistureproofing is obtained by a brush coat of neat cement on the exterior surface and either a similar brush coat or a two- or three-coat troweled mortar finish on the inside face.

Moderate loads of roof, floor, and live-storage shelf beams can be suspended on suitable structural-steel brackets to distribute the end loads over the required bearing area.

Calculation of the lateral and vertical loads of the filling on the walls follows the same procedures as used for both deep and shallow poured bins. Allowable working loads are 18,000 lb per lin ft of wall for vertical compressive stress and 16,000 psi of nominal rod dimension for hoop stress.

**BIN BOTTOMS**

Types and Classification

Design and detailing of bin bottoms are important parts of the over-all bin design. Consideration must be given, in the design of bin bottoms, not only to the safe and economical support of the filling but also to its uninterrupted and economical flow out of the bin. This latter factor can be of critical effect on the successful operation of the storage unit.

Construction of this portion of the work is often a slow and tedious process and added thought spent on detailing the drawings will prove profitable.

Types of bin bottoms are many and varied. Those illustrated have been selected from presently operating installations. Decision as to the type adopted is based on economy. But the factors affecting economy are equally many and varied. Some of these, often interrelated, are:

1. Characteristics of the material to be stored
2. Duration of storage
3. Method of reclaiming (loading out)
4. Speed of operation
5. Elevation of the bin bottom
6. Location of outlets
7. Clearances required for process equipment
8. Size and shape of bin
9. Total weight of stored material to be supported
10. Cost of construction of bin bottom itself
11. Effect of bin-bottom depth on over-all height of the structure
12. Effect of bin-bottom support on wall design and detailing

Bin bottoms are classified according to the slope on which the filling (the stored material) rests: hopped, when this surface pitches to the outlet gate, as in Fig. 18-40; or flat, when it is level, with the fill and topping of Fig. 18-40 omitted.

Bin bottoms are also classified as self-supporting, when they rest on grade as in Fig. 18-40; supported, when they rest on, or form a part of, the substructure as in Figs. 18-41 and 18-42; or suspended, when they hang from the walls as in Figs. 18-43 and 18-44.

**Hoppered Bottoms**

Hoppered bottoms of flat pitch as in Fig. 18-40 are used when it is desired that most, but not necessarily all, of the filling flow to the outlet gate, when the bins are being emptied without mechanical or manual handling. Hoppered bottoms with steeper
pitch, as in Figs. 18-44 and 18-45, are used in self-cleaning bins, those from which it is desired that all the filling flow out by gravity. Complete emptying of the bins may be required for any one of several reasons: material left in the bins might cake, deteriorate in quality, or become subject to spontaneous combustion or attack by vermin; its mixture with a following batch might be undesirable; or the material might be of too great value to allow it to remain unused.

![Fig. 18-41. Supported hoppered bin bottom.](image)

![Fig. 18-42. Supported hoppered bin bottom.](image)

![Fig. 18-43. Suspended hoppered bin bottom.](image)

Flat bottoms are used when some of the filling may remain in the bin to form, at its angle of repose, the envelope of a cone through which the balance of the filling can run to the open gate, as in Figs. 18-41a and 18-46a, or when all or almost all the remaining material can be removed by mechanical devices such as drag scraper or aeration. Flourlike materials lend themselves to the latter form of removal. Flat bottoms are more easily and more economically designed and built than hoppered
bottoms. However, in arriving at comparative over-all costs, consideration must be given, in the case of a flat bottom in which a permanently stored portion of the filling performs the function of a hopper, to the value of this material and also to the added cost of increased footing and possibly stronger substructure required to carry this material in dead storage. Including this material in the calculated bin capacity is not justifiable unless it is anticipated that the material will be removed manually or mechanically. If such removal is contemplated, its cost and risk must be included to arrive at comparative figures.

Hoppering can be accomplished in one of three ways. A full hopper can be supported on the substructure as in Fig. 18-42, or suspended from the substructure as in Fig. 18-47, suspended from the walls as in Figs. 18-44 and 18-48, or suspended from a ring girder resting on pilasters or columns as in Fig. 18-49. A lightweight fill with its surface sloped to the outlet and finished with a wear-resistant topping can be placed on a flat bottom as in Figs. 18-40, 18-41a, 18-43, 18-46b, and 18-54. A combination of both schemes can be used as in Figs. 18-50 and 18-51.

Except where the added burden and material cost of the lightweight fill are deterrents, the combination detail is most generally used where the bin bottom must be elevated. It has the advantages of using a relatively small and inexpensive suspended hopper and of carrying the bin-bottom load on an easily placed level slab. The large clear opening through the slab permits hoisting the materials for the hoppering fill and finish directly into place and gives ample room for convenient installation.

This detail is particularly well adapted to bins of large area with heavy filling where intermediate bin-bottom supports are required. With this arrangement, as in Fig. 18-50, interior columns are located under the bin bottom, with due regard for required clearances of the processing equipment, and topped with a yoke beam or ring girder poured with the bin-bottom slab. The bin-bottom load between the girder and the walls may be carried partly on the walls or may be entirely cantilevered from the girder.

One method of obtaining self-cleaning with a minimum of hoppering is the installation of multiple bin-bottom openings, as in Fig. 18-46. The added number of openings may or may not be a disadvantage, depending upon the arrangement of the collecting equipment.

Suspended hoppers may be concrete as in Fig. 18-44, steel plate as in Figs. 18-47 and 18-48, or a combination of the two as in Fig. 18-45. While relative costs vary according to availability of labor and materials, small suspended hoppers, and under certain conditions of installation, large suspended hoppers, are more economical in steel than in concrete.

Depending upon the shape of the bin, hoppered bottoms are generally in the form of either inverted cones or pyramids with their vertical axes through the outlet gates; and depending upon the location of the outlets, these cones or pyramids may be either regular as in Fig. 18-44 or oblique as in Figs. 18-51 and 18-52.

With fixed minimum bottom pitch, pyramidal hoppers require greater depth than cone hoppers in order to compensate for the longer run of the valleys. A square pyramidal hopper requires approximately 1.4 times the depth of a cone hopper as indicated in Fig. 18-53. For this reason it is not unusual to install a cone hopper in
the bottom of a rectangular or square bin, making the transition inside the bin with hoppering fill. A reverse procedure is occasionally used, as in Fig. 18-45, when it is desired to install a square outlet gate on a cone hopper. Here the transition is made in the steel hopper.

Support for suspended bin bottoms, whether flat or hoppered, can be attained by any one of several details or again by combinations of details. Concrete conical and pyramidal hoppers for lighter materials, and single bin flat bottoms, can be poured into "prints" or recesses formed in the walls, as in Figs. 18-44 and 18-46. When it is undesirable to reduce the wall cross section by prints or to add materially to the wall load, the bottom can be suspended on a ring girder supported on pilasters, or on independent columns adjacent to the walls, as in Fig. 18-49. Where the bin is large or the loads unusually heavy a combination of the two types as in Fig. 18-45, and as previously noted in Fig. 18-50, may be used.

In multiple rectangular or square bins requiring clear floor areas directly under the bin bottom the flat concrete slab can be supported on prints in the exterior walls and hung from the lower edge of the previously poured interior walls as in Fig. 18-43. This detail is particularly well adapted to bins where no interior structural supports are possible immediately under the bin bottom.
**BIN BOTTOMS**

**Fig. 18-47.** Square or circular bins with steel hoppers.

**Fig. 18-48.** Steel hopper bottom supported from welding plates.

**Fig. 18-49.** Bin bottom suspended from ring girder resting on columns.

**Fig. 18-50.** Combination supported and suspended hopper.
Fig. 18-51. Oblique hopped bottom, circular bin.
Steel hoppers can be bolted to the underside of the concrete slab as in Fig. 18-50 when the loads are small. When the loads are heavy they can be seated on the substructure as in Fig. 18-47, welded to plates anchored in the walls as in Fig. 18-48, or supported on the forms and built into the bin-bottom slab as in Fig. 18-51. While it is desirable that field assembly be kept to a minimum, thought must be given to the maximum size and weight of sections that can be economically handled on the site. It is good practice to call for shop preassembly and knock-down to assure perfect alignment before the parts are shipped to the job.

Suspended concrete conical hoppers and the concrete surfacing of conical hopped bottoms on lightweight fill are placed with pneumatically applied concrete. Concrete in these same locations in steep pyramidal hoppers can be similarly placed or can be poured under a top form. Concrete for pitches of 30° or less can be placed with a stiff mix in the same manner as for a level slab or with a top form only at the lower end or start of the pour. Provision for bracing or weighting down of the top forms, if used, is a field operation which should not be overlooked.

Abrasion is an important consideration in the design of hopped bottoms for certain fillings. In fact, it is often the determining factor in deciding upon the over-all arrangement of a storage unit. Bins for crushed stone, ores, sand, and gravel are generally built with flat bottoms. For here the disadvantage of carrying the weight of a portion of the filling as dead storage is offset by the advantage of having the highly abrasive material slide upon itself as it runs out of the bin, thus eliminating the need for installation of expensive wearing surfaces and possible shutdown for replacement. Where self-cleaning bins are desirable for storage of a filling of fairly high abrasion, such as coal, wear-resistant surfaces of tile, paving brick, glass, or alloyed steel are used to line the hopped bottoms as in Figs. 18-46b, 18-50, 18-51, and 18-54. As noted previously, sliding of the filling occurs primarily in the lower portion of the hopper where the dome effect, which carries the weight of the filling to the walls by friction, is broken. economical protection can often be provided by installation of a surface of high wear resistance at the mouth and one of lesser resistance and lesser cost in the upper and larger area of the hopper, as in Fig. 18-51. The fact that embedment of surfacing materials can be made directly on a concrete slab will sometimes determine the use of a concrete rather than a steel hopper.
Arching or bridging of certain fillings must also be taken into account in the design and detailing of hopper bottoms and may also fix the over-all design of the installation. Arching of finely ground and moisture-absorbent materials such as cement, soda ash, and flour can be broken by injection of small blasts of dry compressed air which not only dislodge the packed material but aerate it and increase its flowability. Pipes of small diameter, or tubing, capped and drilled with small holes, can be placed to introduce the air at intervals along the hopper surface as in Fig. 18-52. Arching of coarser fillings, as bituminous coal, can be broken by agitating the filling in the hopper manually with rods or bars through 2- to 4-in.-diameter holes in the hopper shells. These holes may be short lengths of capped pipe set in the concrete or may be holes cut in the steel plate and covered with similar pieces of pipe welded to the underside of the hopper, as in Fig. 18-51. Of course, these openings must be placed in such position that a rod can be introduced, and access to them must be provided. Arching of certain hard fillings can be relieved by vibration, accomplished by attachment of small electrically operated vibrators to the underside of the hoppers, as in Fig. 18-50. This method is limited to fillings which will not consolidate or become compact with vibration, and to installations with steel hoppers.

The shape of the hoppered bottom can sometimes be made so as to minimize arching. Fillings are less likely to arch in a hopper with one side vertical, as in Fig. 18-51, or with two sides vertical, as in Fig. 18-52, than they are in one sloping symmetrically to the outlet. Such arrangement, with a fixed minimum pitch, makes, however, for a deeper hopper and places the outlet off center in the bin. An alternate method of achieving a vertical hopper surface is the introduction of a vertical steel plate or thin wall within a symmetrical hopper, as in Figs. 18-54 and 18-55.

In order to allow free flow of the filling, minimum pitch of concrete surfaces of hoppered bottoms must be at least that for the filling as given in Table 18-1. The following minimum pitches have proved satisfactory for concrete self-cleaning hoppers:

- Portland cement: 50°
- Lime: 50°
- Soda ash: 60°
- Wheat flour: 65°

Ample pitch in the hoppers of self-cleaning bins is of critical importance, but it should be remembered that any excess in pitch makes for uneconomical design. For
Fig. 18-56. Chart for determining hopper bottom pitches. (Courtesy of Power Plant Engineering, July, 1947, Chicago, Ill.)

When a bin or hopper with a wedge-shaped bottom is designed, the angle between the inclined sides and the horizontal is selected for a minimum value depending upon the material to be stored in the bin. However, the valley angle, or corner angle, which is the angle between the intersection of the inclined sides and the horizontal is always less than that of the inclined sides. This causes the material to stick in the corner while it slides freely down the sides. The angle should therefore represent a minimum value, and the sides should be determined accordingly. This chart provides an easy method of finding the relationship of the valley angle to the inclined side angles. Place a straightedge across the two scales representing the adjacent side angles, and the point where it intersects the middle scale gives the valley angle. (By Walter E. Caster.)
the same overall height of bin and hopper, the storage capacity is less with a steep than with a shallow hopper, and the construction of a steep hopper is more difficult and more costly than that of a shallow one. A steep pitch does not always give trouble-free operation of the bin. For some materials, as bituminous coal, an excessively steep pitch in a bin of small cross section may cause arching. The practiced designer need not be reminded that thoroughly complete data on the characteristics of the material and the conditions under which it is to be stored are essential for satisfactory design of each individual bin-bottom installation.

Figure 18-56 is a convenient chart for the determination of hopper-bottom pitches of square and rectangular hoppers when the valley slope is given and it is desired to have the top of the hopper at the same level on all four sides.

The flow of bulk solids from storage is another area of bin design in which knowledge is lacking. Study of the behavior of some materials has been and is being made by several manufacturers of bulk-handling equipment and by A. W. Jenike. The report of the American Society of Mechanical Engineers Materials Handling Division Research Committee on Flow of Bulk Materials from Storage, received by the society in February, 1956, reviews the state of knowledge on this subject and points up the need for scientific analysis of material flow.

For the structural design, bin bottoms fall into two classifications, flat bottoms and hopper (not hoppered) bottoms.

Flat bottoms include those which support the filling directly on the horizontal structural slab, regardless of whether this slab be:

1. Part of the foundation mat as occurs in the example of circular-deep-bin walls
2. An added slab poured at grade between the bin walls (Fig. 18-57)
3. A supported slab as indicated in Fig. 18-41a
4. A suspended slab as indicated in Fig. 18-46

Flat bottoms also include any of the above-mentioned horizontal structural slabs on which a permanent secondary sloping slab has been built to receive the filling (hoppered bottom) as indicated in Figs. 18-40, 18-41b, and 18-46b.

Hopper bottoms are those which support the filling directly on a sloping structural surface. This designation is used regardless of the details of support or suspension of the hoppers (Figs. 18-42, 18-44, 18-45, 18-47, 18-48, and 18-49).

For the structural design, bin bottoms are further classified according to the type of bin in which they are used, namely, shallow bins and deep bins.

As stated previously, exact knowledge of the forces exerted by various fillings on the bin walls and bottoms is lacking. Bin-bottom loads, as calculated by the generally accepted theories and for the several conditions of slope of the bottoms, vary even more widely than lateral loads figured by these same theories. However, again as in the case of bin-wall design, the commonly used procedures have produced structures that are safe and, when the practical details of construction are taken into account, are not necessarily uneconomical.

It should be recalled that bin-bottom loads are those net downward forces within the bin resulting from the stored material, plus the total weight of the hoppering fill and slab, if any, built up on the bin bottom. They do not include wall loads or any loads borne by the walls.

**Flat Bottoms**

Bin-bottom loads for flat bottoms are determined according to the procedures set down and illustrated under the theory of shallow and deep bins. No added example is presented here.

1. When the bin bottom is an integral part of the foundation slab, as in Fig. 18-40, the bin-bottom loads are taken into account in either one of two ways, depending upon the foundation support:
a. If the foundation is a mat resting uniformly on support below, no provision need be made in the mat itself for these loads. They are treated in the same manner as are dead loads of independent column footings. As such, these loads may be deducted from the upward reaction of that portion of the slab which lies between the walls when calculating the moments in the mat foundation.

b. If the foundation is a mat resting primarily on piles set under the walls, then added reinforcing must be placed in the mat to carry the bin-bottom loads to these piles.

2. When the bin bottom is simply an added slab poured at grade between the walls no design for the bin-bottom loads is involved. It is customary to make this slab from 6 to 8 in. thick with either mesh or a light reinforcing bar mat placed about 2 in. from the upper surface. For a 6-in. slab the steel would be either 6 by 6 in., 6 gage or 5/8-in. bars spaced about 12 in. on centers in each direction. For bins up to 24 ft in diameter this concrete would be placed without joints. Expansion-joint material should be placed between the slab and the wall (Fig. 18-58), for while the initial shrinkage probably offsets any future expansion of the covered slab, it is wise to prevent infiltration of fine-grained materials into this area.

3. The design of a supported slab (Fig. 18-41) is carried out in the same manner as that of any horizontal structural slab resting on, and usually poured with, the supporting beams and girders. The weight of any conveying machinery suspended from the slab must be added to the bin-bottom load and to the dead load of the slab to arrive at the total effective load. The size of the bin-bottom opening must be taken into account, and when it becomes too large to permit its deduction from the effective area of the slab, this opening should be framed.

4. Suspended slabs (Figs. 18-43, 18-46, and 18-50) are also designed according to the same procedures used for the design of any similarly supported conventional slab. For single supported slabs the moment coefficients for flat plates as given in the Portland Cement Association Paper ST 63, Rectangular Concrete Tanks, provide an economical design. Caution should be observed in the application of published moment coefficients for floor slabs to bin-bottom slabs continuous under two or more bins. A much wider variation in total loading of adjacent slabs is possible in bin bottoms. Analysis of the slab moments by some method such as moment distribution is a safer procedure. The comments made above in paragraphs 2 and 3 under supported slabs relative to loads and bin-bottom openings apply also to suspended slabs.

A field problem which should have the designer’s attention is that of getting the steel and concrete into place. For bottoms which are built wholly within the walls of each bin (Figs. 18-46 and 18-50), three means of access are possible for placing these materials:

1. The bin-bottom openings
2. The roof, or monitor floor, openings
3. A temporary opening provided in a bin wall

The first of these will generally prove the most economical, but to permit its use it must be possible to make the bin-bottom opening large enough to allow passage of a concrete bucket. Provision must also be made somewhere in the structure for support of the hoisting rig.

**Hopper-bottom Design Procedures**

The procedures used in offices for the design of hopper bottoms vary considerably. However, primarily because of observance of practical construction details, the final
designs as produced by these offices are quite similar. As noted above, bin bottoms are classified as shallow-bin bottoms and deep-bin bottoms. Different methods of design are often applied to hopper bottoms of these two classifications, a procedure which is admittedly arbitrary and occasionally not entirely logical.

Three approaches to the calculation of the forces exerted on hopper bottoms are available to the designer:

1. Coulomb’s theory
2. The prism of stress
3. The ellipse of stress

Coulomb’s Theory. While derivation of this theory is usually demonstrated in standard texts for a sloping wall with the toe of the filling at the top of the wall, its application to hopper bottoms is seen to be valid, for in the derivation no limit is placed on the weight of the sliding wedge (Fig. 18-59). The formula is equally true for weight W and for weight W + W₁. It is usable, therefore, for hopper bottoms of shallow bins in which the friction between the filling and the walls is both included and neglected and also for hopper bottoms of deep bins in which this friction is always included.

As noted previously, Coulomb’s theory is applicable for walls sloping only within certain limits. This limiting condition for bins, where the top slope of the filling may always be considered as level (because of the manner in which bins are usually filled), is that the angle of the hopper bottom with the horizontal (θ₁, Fig. 18-60) be no less than the angle of the plane of rupture. Reference to the discussion of the plane of rupture shows that this limiting angle varies between 50 and 62°, depending upon the characteristics of the material stored and upon consideration of the effect of friction between the material and the hopper bottom. Ketchum states, in effect, that this limiting condition should be the value of the angle of friction between the material and the hopper bottom, measured from the vertical, i.e., θ = 90° + φ' (Fig. 18-61).

The spread between these two recommendations is not wide. Equations (18-6), (18-8), (18-14), and (18-15), based on Coulomb’s theory, are applicable to hopper bottoms. As previously noted, immediately following the statement of these formulas, they may be used to determine unit thrust (Fig. 18-61) on the wall by substituting wh for \( \frac{1}{2} wh^2 \). It should be noted, however, that the formulas then give pressures for 1 ft of vertical height and that to obtain unit pressure on the sloping surface these results must be multiplied by \( \sin \theta_1 \).

The Prism of Stress. When the hopper bottom lies at a flatter slope than the limiting condition for the material to be stored, or when there is some doubt as to
whether the limiting condition is being exceeded, the vertical pressure of the filling on the bin bottom is combined with the lateral pressure, and the thrust normal to the hopper surface finally determined. In this procedure the friction between the stored material and the hopper-bottom surface (but not necessarily the friction between the stored material and the vertical bin-wall surfaces) is always neglected.

The procedure is set down by Dull,\textsuperscript{34} who bases his article on the treatment by Cain.\textsuperscript{35} For hopper bottoms of shallow bins Dull first combines the lateral and vertical loads bearing on the hopper and then resolves this result into components normal and parallel to the hopper surface.

A direct method of combining the forces in the filling to obtain the pressure on the sloping hopper surface is possible by application of the principle of the prism of stress. Rankine\textsuperscript{36} develops this problem for the general case. The following is a statement of the special case where the conjugate stresses are vertical and horizontal.

**Fig. 18-62. Relation of forces, after Rankine.**

**Fig. 18-63. Notation for Table 18-11 shown in relation to bin hopper.**

Prism $ABC$ of unit length perpendicular to the paper (Fig. 18-62).

\[
AC = \text{unit length} \\
V = \text{vertical load} \\
L = \text{lateral load} = k \times V
\]

Then

\[
p_n = L \times \sin \theta_1 + V \times \cos \theta_1
\]

\[
p_n \times AC = L \times \sin \theta_1 \times BC + V \times \cos \theta_1 \times AB
\]

but

\[
BC = AC \times \sin \theta_1 \quad \text{and} \quad AB = AC \times \cos \theta_1
\]

\[
\therefore \quad p_n \times AC = L \times AC \times \sin^2 \theta_1 + V \times AC \times \cos^2 \theta_1
\]

and

\[
p_n = L \times \sin^2 \theta_1 + V \times \cos^2 \theta_1
\]

and

\[
p_n = V \times (k \times \sin^2 \theta_1 + \cos^2 \theta_1)
\]  

(18-34)

In the like manner it can be proved that

\[
p_t = V \times (1 - k) \times \sin \theta_1 \times \cos \theta_1
\]

(18-35)

With $V$ and $L$ expressed in pounds per square foot, $p_n$ and $p_t$ are pressures in pounds per square foot on the sloping surface of the hopper.

Table 18-11 gives directly the coefficient by which the vertical load per square foot at any point on the hopper is multiplied to obtain the unit thrust normal to the hopper surface at the same point (Fig. 18-63). It also gives values of $\cos \theta_1$ to obtain that part of the unit weight of the hopper-bottom slab which acts normal to the hopper surface.
## Table 18-11. Values of \( k \sin^2 \theta_1 + \cos^2 \theta_1 \)

<table>
<thead>
<tr>
<th>(\phi)</th>
<th>(k)</th>
<th>35(^\circ)</th>
<th>40(^\circ)</th>
<th>45(^\circ)</th>
<th>50(^\circ)</th>
<th>55(^\circ)</th>
<th>60(^\circ)</th>
<th>65(^\circ)</th>
<th>70(^\circ)</th>
<th>75(^\circ)</th>
<th>80(^\circ)</th>
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<tr>
<td>Angle of internal friction</td>
<td>(1 - \sin \phi) (1 + \sin \phi)</td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
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<td>10(^\circ)</td>
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<td>0.85</td>
<td>0.83</td>
<td>0.80</td>
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<td>0.74</td>
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<td>0.85</td>
<td>0.82</td>
<td>0.78</td>
<td>0.76</td>
<td>0.73</td>
<td>0.71</td>
<td>0.69</td>
<td>0.67</td>
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<td>15(^\circ)</td>
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<td>0.83</td>
<td>0.80</td>
<td>0.76</td>
<td>0.72</td>
<td>0.69</td>
<td>0.66</td>
<td>0.64</td>
<td>0.62</td>
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<td>17(\frac{1}{2})(^\circ)</td>
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<td>0.85</td>
<td>0.81</td>
<td>0.77</td>
<td>0.73</td>
<td>0.69</td>
<td>0.65</td>
<td>0.62</td>
<td>0.59</td>
<td>0.57</td>
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<td>0.75</td>
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<td>0.58</td>
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<td>0.68</td>
<td>0.63</td>
<td>0.59</td>
<td>0.55</td>
<td>0.51</td>
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<td>0.65</td>
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<td>0.49</td>
<td>0.43</td>
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<td>0.48</td>
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<td>0.53</td>
<td>0.46</td>
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<td>0.34</td>
<td>0.29</td>
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<td>45(^\circ)</td>
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</tbody>
</table>

\[ p_n = \bar{V} (k \sin^2 \theta_1 + \cos^2 \theta_1) + w_r \cos \theta_1 \]

where \(p_n\) = normal pressure  
\(\bar{V}\) = total vertical unit load  
\(w_r\) = either \(V\) or \(\bar{V} + w\) for deep bins

### The Ellipse of Stress

This device for determining graphically the loads normal and parallel to the hopper surface is simply an extension of the prism principle. That an ellipse is involved is seen from the form of Eq. (18-34). Rankine, Cain, and Ketchum all develop the procedure. Cain's treatment is very clear.

For the special case of the conjugate stresses acting vertically and horizontally it is not necessary to complete the ellipse to arrive at the loads required. Its application is illustrated by assuming the following data:

\[ V = 896 \text{ lb} \]

\[ L = k \times V = 0.375 \times 896 = 336 \text{ lb} \]

\[ \theta_1 = 60^\circ \]

Using any convenient scale, lay off \(OA = V\) and \(OB = L\). With \(O\) as the center describe quarter circles through these points. Draw \(OC\) at angle \(\theta_1\) and \(OD\) perpendicular to \(OC\) (Fig. 18-64).

Through the point at which \(OD\) intersects the larger circle draw a horizontal and through the point at which \(OD\) intersects the smaller circle draw a vertical. The intersection of these two lines \(E\) is a point on the ellipse.

![Ellipse of stress for determining loads normal and parallel to hopper surface](image-url)
Draw \( EF \) parallel to \( CO \).

\[
\begin{align*}
EO &= p = \text{total unit pressure on the hopper surface} \\
FO &= p_n = \text{unit thrust normal to the hopper surface} \\
EF &= p_t = \text{unit pressure parallel to the hopper surface}
\end{align*}
\]

At the same scale used to lay off \( OA \) and \( OB \), \( FO \) scales 475 lb.

**Comparison of Design Procedures.** There is a considerable difference of opinion among designers concerning the procedure which gives more nearly the true value of thrust on sloping hopper surfaces of deep bins.

1. Ketchum states that \( V \) on any horizontal plane through the hopper bottom, as \( a-a \) in Fig. 18-65, can be determined by applying Janssen's formula, using a column of material of the same cross section as that of the hopper at the plane being considered. For this approach Ketchum recommends that the value of \( \phi \) be used for \( \phi' \) in the formula because the material in the column will tend to slide on the surrounding filling. With \( V \) determined and \( k \) given, \( p_n \) can be calculated by the principle of the prism of stress or by Coulomb's theory. This method gives decreasing values of \( p_n \) as the sections considered move down the hopper.

2. In a second procedure for calculating the thrust on the hopper bottoms of deep bins, the net bin-bottom load per square foot \( V \) at the top of the hopper is computed by Janssen's formula or Airy's solution and the values thus obtained are combined with the weight of a foot square column of the filling of the height between the top of the hopper and the elevation at which the thrust is sought (Fig. 18-66). With the total \( V \) thus determined and \( k \) given, \( p_n \) can again be calculated by either the principle of the prism of stress or by Coulomb's theory. This method gives increasing values of \( p_n \) as the elevations considered move down the hopper.

Neither method has been proved through full-scale tests.

While the results obtained by these two procedures vary considerably in the lower portions of the hopper this divergence is seldom of great practical importance. Both methods give approximately the same values near the top of the hopper where the spans or the diameters are the greatest and where the concrete or steel-plate thickness is determined.

Mention has been made previously of the total force \( P_t \) and the unit force \( p_n \), act-
ING DOWN THE SLOPE AND PARALLEL TO THE FACE OF THE HOPPER (FIG. 18-67). THE VALUES OF THIS FORCE AS CALCULATED BY EITHER COULOMB'S THEORY OR THE PRISM OF STRESS FOR USE IN DETERMINING THE LONGITUDINAL STRESS IN BIN BOTTOMS ARE NOT CERTAIN.

A METHOD OF OBTAINING MORE RELIABLE VALUES FOR THIS STRESS IN A HOPPER OF REGULAR SHAPE IS TO FIND THE TOTAL OF THE VERTICAL LOADS ACTING ON A HORIZONTAL PLANE THROUGH TO THE BIN BOTTOM AND THEN MULTIPLY THIS TOTAL BY THE COSECANT OF THE HOPPER-BOTTOM ANGLE (FIG. 18-68). UNIT STRESS WILL BE THIS VALUE DIVIDED BY THE PERIMETER OF THE HOPPER BOTTOM AT THE PLANE BEING CONSIDERED.


![Figure 18-68. Relation of vertical weight and force acting down hopper face.]

![Figure 18-69. Prism of loading for section a-a.]

![Figure 18-70. Inward force on hopper at any horizontal section.]

Thus, using method B to arrive at vertical load of the filling on the hopper surface.

\[
W = \left( V + w_1 \right) \times \frac{\pi d_i^2}{4} + w_1 \times \frac{\pi d_i^2}{4} \times \frac{d_i}{6} \times \tan \theta_1 + w_1 \times \frac{\pi d_i^2}{2} \times \frac{d_i}{2} \times \sec \theta_1
\]

(18-36)

\[
T_z = W \times \csc \theta_1
\]

(18-37)

With perimeter = \( \pi d_i \)

\[
t_2 = \left[ \left( V + w_1 \right) + w_1 \times \frac{d_i}{6} \times \tan \theta_1 + w_1 \times \sec \theta_1 \right] \times \frac{d_i}{4} \times \csc \theta_1
\]

(18-38)

For a conical hopper

\[
W = \left( V + w_1 \right) \times d_i^2 + w_1 \times b_i^2 \times \frac{b_i}{6} \times \tan \theta_1 + w_1 \times \frac{4b_i^2}{2} \times \frac{b_i}{2} \times \sec \theta_1
\]

(18-39)

\[
T_z = W \times \csc \theta_1
\]

(18-40)
With perimeter = 4b₁

\[
t_2 = \left[ \left( V + wh_1 \right) + w \times \frac{b_1}{6} \times \tan \theta_1 + w_c \times \sec \theta_1 \right] \times \frac{b_1}{4} \times \cosec \theta_1 \quad (18-41)
\]

Maximum total longitudinal stress \( T_2 \) occurs at the top of the hopper (Fig. 18-70). At the top of the hopper this longitudinal force in the direction of the hopper slope produces an inward horizontal pull on the supporting member. This horizontal component \( t_1 \) is \( t_2 \times \cos \theta_1 \). Under most conditions of loading and hopper construction this force may safely be neglected. In deep bins the hopper-bottom support is often tied in some manner to the side walls. The lateral thrust of the filling acting on a few feet of height of hopper and wall surface is sufficient to offset this pull. In shallow bins, where the lateral thrust of the filling may not be large, the hoppers are most frequently hung from a substantial substructure capable of resisting this pull. In circular bins where the conical hopper is supported on a ring girder, that part of the pull not offset by lateral thrust of the filling is taken by compression in the ring. From above it will be noted that the force causing this compression, without taking into account the offsetting lateral thrust of the filling, is

\[
W \times \cosec \theta_1 \times \cos \theta_1 = W \times \cot \theta_1 \quad (18-42)
\]

However, conditions do arise, as in the case of a single square or rectangular bin with a full hopper built on an independent substructure, where the members supporting the hopper should be investigated for this added force.

**The Ring Girder.** The ring girder is a structural form particularly well adapted to circular-bin work. It is essentially a closed ring of continuous beams, simply supported either on the tops of independent columns or on pilasters pulled up with the walls to the underside of the girder (Fig. 18-71).

The girder supports the full vertical load on the hopper bottom, uniformly distributed. For girders in which the curvature is small (where support is on six or more pilasters) moment coefficients approach those for straight continuous beams. The girder also resists, as noted above, the compression arising from the inward pull of the weight of the hopper bottom. Theoretically, this is a case of combined bending and compression but the forces are too approximate to warrant this refinement in the calculations. It is safe, and not uneconomical, to consider the compression stresses as additive. Some designers prefer to neglect this compression in their direct computations and to size the beam a little more generously than required for stress in bending.

Shear, as a measure of diagonal tension, is the same as for uniformly loaded straight beams.

<table>
<thead>
<tr>
<th>No. of posts</th>
<th>Load on post, lb</th>
<th>Max shear, lb</th>
<th>Bending moment at posts, in-lb</th>
<th>Bending moment midway between posts, in-lb</th>
<th>Angular distance from post to point of max torsion</th>
<th>Max torsional moment, in-lb</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>W + 4</td>
<td>W + 8</td>
<td>-0.03415W&lt;sub&gt;r&lt;/sub&gt;</td>
<td>+0.01762W&lt;sub&gt;r&lt;/sub&gt;</td>
<td>9°12'</td>
<td>0.0053W&lt;sub&gt;r&lt;/sub&gt;</td>
</tr>
<tr>
<td>6</td>
<td>W + 6</td>
<td>W + 12</td>
<td>-0.01482W&lt;sub&gt;r&lt;/sub&gt;</td>
<td>+0.00751W&lt;sub&gt;r&lt;/sub&gt;</td>
<td>12°44'</td>
<td>0.00151W&lt;sub&gt;r&lt;/sub&gt;</td>
</tr>
<tr>
<td>8</td>
<td>W + 8</td>
<td>W + 16</td>
<td>-0.00827W&lt;sub&gt;r&lt;/sub&gt;</td>
<td>+0.00416W&lt;sub&gt;r&lt;/sub&gt;</td>
<td>9°33'</td>
<td>0.00063W&lt;sub&gt;r&lt;/sub&gt;</td>
</tr>
<tr>
<td>12</td>
<td>W + 12</td>
<td>W + 24</td>
<td>-0.00365W&lt;sub&gt;r&lt;/sub&gt;</td>
<td>+0.00190W&lt;sub&gt;r&lt;/sub&gt;</td>
<td>6°21'</td>
<td>0.000185W&lt;sub&gt;r&lt;/sub&gt;</td>
</tr>
</tbody>
</table>

Where \( W \) = total load on girder, lb
\( r \) = mean radius of girder, in.
Because of the curvature, torsion arises in a ring girder. This stress is maximum at the points of contraflexure and zero at the supports and at the centers of the spans. Ketchum treats this subject in detail and gives values of stresses in circular girders with even number of supporting posts, as shown in Table 18-12.

Roark gives the following formulas for a flangeless curved beam uniformly loaded normal to the plane of curvature and not fixed as to roll (Fig. 18-72).

\[
T_0 = 0
\]
\[
V_0 = \frac{1}{2} w_1 r \theta
\]
\[
M_0 = w_1 r^2 \left( \frac{\sin \frac{1}{2} \theta - \frac{1}{2} \theta \cos \frac{1}{2} \theta}{\sin \frac{1}{2} \theta} \right)
\]
\[
M = -M_0 \cos \alpha + \frac{1}{2} w_1 r^2 \theta \sin \alpha - w_1 r^2(1 - \cos \alpha)
\]
\[
T = -M_0 \sin \alpha + \frac{1}{2} w_1 r^2 \theta(1 - \cos \alpha) - w_1 r^2(\alpha - \sin \alpha)
\]

where \( w_1 \) is the uniform load per unit length of beam. Angles \( \theta \) and \( \alpha \) standing independently (not as a part of the expression of the function of these angles) are measured in radians. Roark's formulas as stated here check Ketchum's table.

The shearing stresses due to torsion are combined with those due to bending, and when the total, at any point along the beam, exceeds the allowable concrete stress, the excess is provided for by stirrups. While the ideal beam section for resistance to torsion is square, this shape is not well adapted to ring girders for most hopper bottoms. The beam section is fixed in some measure by the hopper-bottom slope, and because of the heavy loads involved it is found economical to use a beam of considerable depth. The rectangular form is generally used in the computations.

Maximum unit shear stress in a rectangular section where \( d \) is greater than \( b \) occurs at the mid-height and is equal to

\[
v_t = \frac{T}{k_1 b^2 h}
\]

where \( k_1 \) is a numerical term depending upon the ratio of \( h/b \). Peabody gives values for this term in Fig. 18-73. For a square section \( k_1 = 0.208 \) and Eq. (18-48) simplifies to

\[
v_t = \frac{4.8T}{b^3}
\]

\( V_t \) thus obtained is combined with \( v_b \) resulting from bending. Allowable \( v_c(0.03f_c') \) in the concrete is deducted from the total and stirrups are designed for the balance \( v_b \) following the same method as for beams without torsion. Because of the difference in the shapes of the two stress curves, a better visualization of the stress distribution along the length of the beam can be had by plotting the combined curves (Fig. 18-74).

A second torsional force probably exists in bin-bottom ring girders and probably is also present in the supporting beams of square and rectangular hopper bottoms. This force arises from the unbalanced load of filling and slab about the center line of the supporting column, pilaster, or wall print. In the case of the circular girder this force is resisted by a couple consisting of compression in the concrete of the upper part of the girder and of tension in the circumferential steel of the hopper (Fig. 18-75). In the case of the square or rectangular hopper it is resisted largely by horizontal beam action of the hopper slabs. These stresses are highly indeterminate and, judging from the performance of completed structures, probably small. They are generally neglected in the design.
The above procedures are not held to be exact but the results so computed are probably conservative. The integral hopper slab produces a girder shape that is neither flangeless nor free to roll, and the slab undoubtedly adds to the stiffness and strength of the girder. The calculations are fairly rapid, and the designs made according to these procedures have proved adequate.

Example of Circular-hopper-bottom Design

Requirement. 26 ft inside diameter hopper of bin for storage of wheat (Fig. 18-76a). Bin-bottom pitch, 8 in. Hopper opening, 2 ft 6 in. diameter. Hopper-bottom support, six independent columns on 21 ft 8 in. diameter circle (Fig. 18-76b). Hopper-bottom slab, fixed arbitrarily at 6 in. Effective depth of filling to top of hopper, 90 ft. \( w \) for wheat = 50 lb. Design by ACI Building Code. \( f' \) = 3,000 lb. \( f_s = 20,000 \) lb. For dimensions of circular sections use Table 18-10.

Columns. \( V \) load at top of hopper: \( h/D = 99\%\) = 3.46. By Table 18-7, \( V/D = 48.42 \). \( V = 48.42 \times 26 = 1,260 \) lb.

Total \( V \) load at top of hopper, 1,260 lb \( \times 531 \) sq ft = 669,000 lb

Weight of grain in hopper

\[
\frac{531 \times 8.67}{3} \times 50 = 76,700
\]

Weight of 6-in. slab, approximate, 81.67 lin ft \( \times \frac{15.62}{2} \times 75 \) lb = 47,800

Weight of ring girder, approximate, 4.35 ft \( \times 2.9 \) ft \( \times 68.06 \) lin ft \( \times 150 \) lb = 129,000

Total load on columns

\[
\frac{922,500}{6} = 153,750 \text{ lb}
\]

Column weight, approximate, 1.33 \( \times 1.33 \times 12 \times 150 \) lb = 3,200

Design load

\[
\frac{156,950 \text{ lb}}{157 \text{ kips}}
\]
DESIGN OF DEEP BINS AND SILOS

From ACI Handbook. Use 16 in. square column with four 1 in. φ. Good for 138 + 40 = 178 kips. Use ¾-in. ties at 12 in. Use 45° capital.

Ring Girder. By Table 18-12,

\[
M_+ = 0.01482 \times 922,500 \times 10.83 = 148,100 \text{ ft-lb}
\]
\[
M_- = 0.00751 \times 922,500 \times 10.83 = 75,000 \text{ ft-lb}
\]

Using only 2 ft 8 in. width directly over cap,

\[
d = \sqrt{\frac{148,100}{236 \times 2.67}} = \sqrt{235} = 15.33 \text{ in.}
\]

Depth over ĉ of column = 24% (Fig. 18-77).

Using \(d = 22 \text{ in.}\),

\[
A_\sigma^- = \frac{148.1}{1.44 \times 22} = 4.67 \text{ sq in.} \quad \text{Six 1 in. φ = 4.74 sq in.}
\]
\[
A_\sigma^+ = \frac{75.0}{1.44 \times 22} = 2.37 \text{ sq in.} \quad \text{Six ¾ in. φ = 2.64 sq in.}
\]

Distance center to center of columns on 21 ft 8 in. diameter circle = \(\frac{68.06}{6} = 11.34 \text{ ft}\)

Distance center of column to point of maximum torsion:

From Table 18-12, angular distance = 12°44' = 12.73°

\[
\frac{12.73}{360} \times 68.06 \text{ ft} = 2.41 \text{ ft}
\]

From Table 18-12,

\[
T = 0.00151 \times 922,500 \times 10.83 \times 12
= 181,000 \text{ in.-lb}
\]

Using the inscribed circle as a measure of the torsional resistance of the girder,

\(D\) [equivalent to \(b\) in Eq. (18-49)] = 29 in. approximately, and \(v_t\)

\[
\frac{4.8 \times 181,000}{29^2} = 35.5 \text{ psi}
\]

Maximum shear from bending in each span = \(\frac{922,500}{6 \times 2} = 76,875 \text{ lb}\)

Maximum unit shear, assuming section 3 ft 9 in. wide with \(d = 24 \text{ in.}\)

\[
v_b = \frac{76,875}{45 \times 0.875 \times 24} = 81.4 \text{ psi}
\]

At distance 2.41 ft from column center line (Fig. 18-78)

\[
v_b = 81.4 \times \frac{5.67 - 2.41}{5.67} = 46.8 \text{ psi}
\]

Maximum \(v_t + v_b = 35.5 + 46.8 = 82.3 \text{ psi < 90 psi}\)

Stirrups are not required, but for conservative design and to support the longitudinal reinforcing, use three ¾-in. ties each side of each column at alternate lines of radial steel.

Radial Steel. At inside edge of ring girder, 19 ft 0 in. diameter:
From Eq. (18-38),
\[
t_2 = \left(1.260 + 50 \times 2.33 + \frac{50 \times 19 \times 0.666}{6} + 75 \times 1.20\right) \times \frac{19}{4} \times 1.803
\]
\[
= 13,460 \text{ lb}
\]
\[
A_s = \frac{13,460}{20,000} = 0.67 \text{ sq in./lin ft} \quad \text{Use } \frac{3}{4} \text{ in. } \phi \text{ at 8-in. centers.}
\]

At inside face of wall, use a spacing equal to \(8 \times \frac{26}{19} = 11 \text{ in. on centers.}\)
Use 15 equal spaces between column center lines.

At 10 ft 6 in. diameter,
\[
t_2 = \left(1.260 + 50 \times 5.17 + \frac{50 \times 10.5 \times 0.666}{6} + 75 \times 1.20\right) \times \frac{19}{4} \times 1.803
\]
\[
= 7,880 \text{ lb}
\]
\[
A_s = \frac{7,880}{20,000} = 0.39 \text{ sq in./lin ft} \quad \text{Requires } \frac{3}{4} \text{ in. } \phi \text{ at 13.5-in. centers.}
\]

This requirement is filled by continuing one of each three radial bars below this diameter and to the outlet opening.

\[
11 \text{ in. } \times 3 \times \frac{5.25}{13.00} = 13.3 \text{ in. } < 13.5 \text{ in.}
\]

**Circumferential Steel.** The upper 3 ft hopper depth (approximately 5 ft 6 in. down the slope) requires no circumferential steel to carry the vertical component of

![Fig. 18-77. Dimensions of ring girder resting on column.](image)

![Fig. 18-78. Torsional and bending shear in ring girder.](image)

the filling load because this component is carried on the ring girder. Some steel may be required to resist the lateral component. Assume this thrust to be the same as on the vertical wall at the top of the hopper. \(h/D = 9 \frac{9}{46} = 3.46\). By Table 18-7, \(L/D = 29.05\). \(L = 29.05 \times 26 = 754 \text{ lb.}\)

\[
T = 754 \times 2 \frac{9}{4} = 9,800 \text{ lb}
\]
\[
A_s = \frac{9,800}{20,000} = 0.49 \text{ sq in./ft of vertical height}
\]
\[
= \frac{0.49}{1.803} = 0.27 \text{ sq in./ft of slope}
\]
Required ring girder top reinforcing was found to be six 1 in. round. These bars are spaced 9 in. on centers on the slope, giving 1.05 sq in. per lin ft of slope.

1.05 \cdot 0.27 = 1.32 \text{ sq in.} \quad \text{Use six 1-in. square bars for the ring girder top steel, spaced 9 in. on centers, and make these bars continuous.}

Hopper depth from 3 to 6 ft below top:

\[ V + wh = 1,260 + 50 \times 4.5 \quad \text{(avg)} = 1,485 \text{ psf} \]

From Table 18-11, for \( k = 0.6 \) and \( \theta_i = 35^\circ \),

\[ p_s = 0.87 \times 1,485 - 75 \times 0.82 = 1,350 \text{ lb (Fig. 18-79)} \]

\[ F = 1,350 \times \csc 33^\circ 41' = 1,350 \times 1.803 = 2,430 \text{ lb} \]

\[ D = 12 \text{ ft 6 in.} \]

\[ T = 2,430 \times \frac{12.5}{2} = 15,200 \text{ lb} \]

\[ A_s = \frac{15,200}{20,000} = 0.76 \text{ sq in.} \]

Use \( \frac{3}{4} \text{ in. } \phi \) at 5-in. centers.

Lower 2 ft of hopper depth:

\[ V + wh = 1,260 + 50 \times 7.0 = 1,610 \text{ psf} \]

\[ p_s = 0.87 \times 1,610 + 75 \times 0.82 = 1,460 \text{ lb} \]

\[ D = 5 \text{ ft. 0 in.} \]

\[ T = 1,460 \times 1.803 \times \frac{5.0}{2} = 6,570 \text{ lb} \]

\[ A_s = \frac{6,570}{20,000} = 0.33 \text{ sq in.} \]

Use \( \frac{3}{4} \text{ in. } \phi \) at 7-in. centers (Fig. 18-80).

**Example of Square-hopper-bottom Design**

**Requirement.** 13 ft square hopper of bin for storage of corn grits. Bin-bottom slope 50°. Hopper opening, 2 ft 6 in. square. Hopper-bottom support, prints in side walls. Effective depth of filling to top of hopper is 38 ft. \( w \) for corn = 43 lb. Design by ACI Building Code. \( f'c = 3,000 \text{ lb. } f_s = 20,000 \text{ lb. } \) Compute slab for bending in horizontal direction only.

**Prints and Supporting Beams.** \( V \) load at top of hopper: \( h/D = \frac{3}{8} \frac{13}{3} = 2.92 \). By Table 18-7,

\[ \frac{V}{D} = 47.30. \quad V = 47.30 \times 13 = 615 \text{ lb.} \quad V \text{ for corn } = 43\frac{3}{4} \times 615 = 530 \text{ lb} \]

Total \( V \) load at top of hopper, \( 530 \times 169 \text{ sq ft} \)

\[ \text{Weight of grain in hopper (Fig. 18-81) } = 89,570 \text{ lb} \]

\[ \frac{169 \times 7.75}{3} \times 43 \text{ lb } = 18,770 \text{ lb} \]

Weight of slab—assume 5 in. thick

\[ \frac{52 \times 10.11}{2} \times 60 \text{ lb } = 15,770 \text{ lb} \]

Total load on prints

\[ 124,110 \text{ lb} \]

Length of wall between 6-in. corner splays available for prints

\[ (13 - 0.5 \times 2) \times 4 = 48 \text{ ft} \]

Required depth of prints \( \frac{124,110}{48 \times 12 \times 750} = 0.29 \text{ in.} \)

Required height of prints \( \frac{124,110}{48 \times 12 \times 90} = 2.4 \text{ in.} \)
Fig. 18-80. Arrangement of reinforcing steel in hopper.

To make installation of print forms simple and to provide ample bearing in the event of hopper concrete shrinkage or wall deflection, use prints approximately 10 in. high as shown in Fig. 18-82. The hoppers will not be tied to the walls. Therefore, the integral hopper support beam should be investigated for net horizontal inward force.
Inward pull on each side of hopper:

Total vertical load in hopper \( W = 124,110 \text{ lb} \)

Vertical load on each side of hopper \( \frac{\frac{124,110}{4}}{31,030} = 31,030 \text{ lb} \)

From Eq. (18-42), horizontal pull \( 31,030 \times 0.839 \) = \( 26,030 \text{ lb} \)

This force is offset to some extent by the lateral thrust of the filling. Assume the filling in the upper 2 ft 0 in. of the hopper effective for this purpose.

From Table 18-7, where \( h/D = 2.92, L/D = 28.38 \), and \( L \) for corn,

\[ 28.38 \times 13 \times \frac{4\%}{2} = 317 \text{ psf} \]

Total offsetting lateral force (Fig. 18-83) (through restraint of hopper side, bending due to this amount is prevented) in 2-ft height \( 317 \times 13 \times 2 = 8,240 \)

Net lateral force on each support beam

\[ M = \frac{17,790 \times 12}{12} = 17,790 \text{ ft-lb} \]

Assuming the lower 12 in. of concrete of the support beam effective to resist compression caused by bending,

\[ \text{Required } d = \sqrt{\frac{17,790}{236}} = \sqrt{75.5} = 8.7 \text{ in.} \]

Average of 12 in. is available

\[ A_s = \frac{17.79}{1.44 \times 12} = 1.03 \text{ sq in.} \quad \text{Use three } \frac{3}{4} \text{ in. } \phi \text{ continuous on outside face of beam} \]

\[ M^+ = 8,900 \text{ ft-lb} \quad A_s = 0.54 \text{ sq in.} \quad \text{Use two } \frac{5}{8} \text{ in. } \phi \text{ continuous on inside face of hopper} \]

Concrete area available to resist shear caused by lateral bending in the support beam \( \frac{19.5 \times 16.5}{2} = 160 \text{ sq in. (approximate). } p = \frac{17,790}{2 \times 0.875 \times 160} = 63.6 \text{ psi} \).
Stirrups are not required but use \( \frac{3}{4} \)-in. \( \phi \) ties at 3 ft 0 in. to support \( \frac{3}{8} \)-in. \( \phi \) bars at outside face.

**Radial (Sloping) Steel.** At top of hopper,

Total vertical load in hopper \( W = 124,110 \) lb

From Eq. (18-37),

\[
T_2 = W \times \csc \theta_1
\]

and

\[
t_2 = \frac{124,110}{4 \times 13} \times 1.305 = 3,120 \text{ lb/lin ft}
\]

Required \( A_s = \frac{3,120}{20,000} = 0.156 \text{ sq in./lin ft} \)

At 3 ft 0 in. below top of hopper,

\( b_1 \) (horizontal span) = 8 ft 0 in. approximately

From Eq. (18-41),

\[
t_2 = (530 + 43 \times 3 + 43 \times \frac{3}{8} \times 1.192 + 60 \times 1.556) \times \frac{3}{8} \times 1.305 = 2,140 \text{ lb}
\]

Required \( A_s = \frac{2,140}{20,000} \times 8 = 0.856 \text{ sq in.} \) Requires five \( \frac{3}{4} \) in. \( \phi \)

At 5 ft 6 in. below top of hopper,

\( b_1 = 4 \text{ ft 0 in. approximately} \)

\[
t_2 = (530 + 43 \times 5.6 + 43 \times \frac{3}{8} \times 1.192 + 93) \times \frac{3}{8} \times 1.305 = 1,170 \text{ lb}
\]

Required \( A_s = \frac{1,170}{20,000} \times 4 = 0.234 \text{ sq in.} \) Requires two \( \frac{3}{4} \) in. \( \phi \)

Use \( \frac{3}{4} \) in. \( \phi \) at 16-in. centers in bottom of slab. Spacer bars for upper band reinforcing will give added sloping steel.

**Band (Horizontal) Steel.** At 3 ft 0 in. below top of hopper, horizontal clear span = 8 ft 0 in. approximately,

\( V + wh = 530 + 43 \times 3 = 659 \text{ psf} \)

From Table 18-11, for \( k = 0.6 \) and \( \theta_1 = 50^\circ \) (Fig. 18-84),

\[
p_a = 0.77 \times 659 + 60 \times 0.64 = 545 \text{ lb}
\]

\[
M_- = \frac{545 \times 8.0 \times 8.5}{12} = 3,090 \text{ ft-lb}
\]

\[
d = \sqrt{\frac{3,090}{236}} = \sqrt{13.1} = 3.62 \text{ in.} \quad \text{Use 3\( \frac{3}{4} \) in. Make } t = 5 \text{ in.}
\]

\[
A_s = \frac{3,090}{1.44 \times 3.75} = 0.57 \text{ sq in.} \quad \text{Use } \frac{3}{8} \text{ in. } \phi \text{ at 6-in. centers in top at valleys}
\]

\[
M_+ = 1,545 \text{ ft-lb}
\]

Including direct tension:

\[
T = 545 \times \frac{3}{8} \times \csc \theta_1 = 2,180 \times 1.305 = 2,845 \text{ lb (Fig. 18-85)}
\]

\[
d' = 2.5 \times 2.5 = 2.5 \text{ in.}
\]

\[
e = \frac{1,545 \times 12}{2,845} = 1.25 = 5.25 \text{ in.}
\]

\[
A_s = \frac{2,845}{20,000 \times 0.875 \times 3.75} (5.25 + 0.875 \times 3.75)
\]

\[
= 0.0434 \times 8.53 = 0.37 \text{ sq in./lin ft of slope}
\]
Use \( \frac{1}{2} \) in. \( \phi \) at 6-in. centers continuous in bottom.

At 5 ft 6 in. below top, horizontal clear span = 3 ft 9 in. approximately,

\[
V + wh = 530 + 43 \times 5.5 = 767 \text{ psf}
\]

\[
p_a = 0.77 \times 767 + 38 = 629 \text{ psf}
\]

\[
M = \frac{629 \times 3.75 \times 4.25}{12} = 836 \text{ ft-lb}
\]

Because top bars in this portion of the hopper will be short, it will be more economical to make them continuous. Direct tension is therefore divided equally between top and bottom steel.

![Diagram of vertical and normal force against hopper side.](image1)

![Diagram of normal and tension force against hopper side.](image2)

**Fig. 18-84.** Vertical and normal force against hopper side.

**Fig. 18-85.** Normal and tension force against hopper side.

![Diagram of plan and section showing reinforcement for square hopper.](image3)

**Fig. 18-86.** Plan and section showing reinforcement for square hopper.

Including direct tension:

\[
T = \frac{629}{2} \times \frac{3.75}{2} \times 1.305 = 770 \text{ lb}
\]

\[
d'' = 1.25 \text{ in.} \quad e = \frac{836 \times 12}{770} - 1.25 = 11.75 \text{ in.}
\]

\[
A_s = \frac{770}{20,000 \times 0.875 \times 3.75} \times (11.75 + 0.875 \times 3.75)
\]

\[
= 0.0117 \times 15.03 = 0.18 \text{ sq in./lin ft of slope}
\]
BIN PATTERNS

Requires $\frac{1}{2}$ in. φ at 13-in. centers but use 10-in. centers maximum spacing (2 × t).

\[ M_+ = 418 \text{ ft-lb} \quad T = 770 \text{ lb} \]

\[ d'' = 1.25 \text{ in.} \quad e = \frac{418 \times 12}{770} - 1.25 = 5.25 \text{ in.} \]

\[ A_s = 0.0117 \times (5.25 + 3.28) = 0.10 \text{ sq in./lin ft of slope} \]

Requires $\frac{3}{4}$ in. φ at 13-in. centers but use 10-in. centers maximum spacing. Details of hopper are shown in Fig. 18-86.

An alternate detail for the short horizontal bars at the valleys is shown in Fig. 18-87.

The hopper outlet opening was made 2 ft 6 in. square primarily to permit passage of forms, reinforcing, and concrete bucket directly from below. A steel transition piece was later fitted between the opening and the conveyor.

Large square or rectangular concrete hoppers are not often used. Where the area or load becomes such that a slab thicker than 6 in. is required, it will prove more economical to design the slopes, in an approximate approach, as two-way slabs.

![Fig. 18-87. Valley reinforcement for square hopper.](image)

**BIN PATTERNS**

Bulk-storage units are strictly utilitarian structures. As such, the governing considerations in their design are economy and structural adequacy. But as in the case of bin-bottom design, the factors that must be weighed to arrive at maximum over-all economy of an installation are many, and usually interrelated. The more important of these are:

1. Storage capacity required
2. Characteristics of the stored materials, data of the same nature as listed in Table 18-1
3. Purpose of storage, whether simple storage or storage as part of a material-processing operation
4. Limitations of site
5. Location of bins relative to the processing or conveying equipment, whether at grade or elevated
6. Limitations of bin dimensions and structural support because of arrangement of processing or conveying equipment
7. Need for circulating, drying, or fumigating stored materials
8. Required speed of operation
9. Data on conveying equipment to be used to both receive and load out materials, first cost and operating cost per unit of height or length; housing, support, and clearances required in the bin structure; limitations of chuting or spouting
10. Soil conditions, earthquake occurrence
11. Material costs and availability, cement, sand, gravel, reinforcing steel, and labor
12. Labor rates and availability
13. Owner's preferences based on his own previous experience

When storage is part of a material-processing operation, as in the case of a batching plant, maximum works economy is usually attained by providing such storage elevated above the batching equipment. This arrangement generally calls for the
greatest storage in the least possible building area and often with the least possible structural support. Such combination of demands is best met with square or rectangular bins. The size, number, and location of bins will be governed entirely by the kind, quantity, and point of delivery of each material required in the operation. Such layout may be regular, as typified by the arrangement used in the silicon carbide plant (Fig. 18-25) shown in the example of rectangular-bin-wall design, or more irregular, as used in a recent glass-batching building shown in Fig. 18-88.

In both these structures the bins were mounted above three lower work stories.

**Square or Rectangular Bins**

Square or rectangular bins are frequently used when available space is limited. The layout in Fig. 18-89 made storage of corn grits possible in a limited yard area of an East Coast brewery.

As mentioned previously, and in general, circular bins give the greatest economy for simple storage on unlimited site. However, this statement is not universally true.

![Multiple rectangular bins.](image)

![Rectangular bins used in limited yard area.](image)

Multiple square bins for storage of lightweight materials and with spans up to 12 ft may prove more economical than circular bins.

The factors favoring square bins are the need of less concrete, for the interior wall of any bin serves also as the common wall with an adjoining bin; slightly cheaper wall-form construction; easier placing of the horizontal wall steel; simpler foundation mat with more efficient use of the reinforcing steel; simpler roof slab forming and reinforcing; simpler forming of any wall openings or recesses; and possibly shorter runs in the conveying equipment. The adverse factor is their requirement of a comparatively large percentage of horizontal reinforcing steel, as a result not only of the need for reinforcing both faces of the walls but also of the less efficient utilization of this steel in flexure rather than solely in direct tension.

**Circular Bins**

For storage of heavy materials or for bins of diameters upwards of 14 ft 0 in. there is no question concerning the greater economy of the circular section.

Bin patterns for multiple circular bins are many and varied. The most commonly used is the arrangement of regular circular areas in contact, giving storage in both the regular bins and in the intermediate interstice bins (Fig. 18-90a).

One variation of this plan is the spreading of the bins in one direction, thus enlarging the interstice bins, and of adding outside or "blister" bins at the longer sides (Fig. 18-90b).

A second is the spreading of the bins in both directions and the adding of outside bins on all four sides (Fig. 18-90c).

Both these variations result in increased storage capacity with minimum added length of concrete wall. However, there is some danger in carrying this spreading too far, as shown by the occurrence of local failure in some bins of this pattern. With
the interstice bin loaded and the circular bin empty, the stress in the circular arc becomes compression. When the bins are all in contact the resulting thrust is comparatively small and can be resisted by the concrete mass of the contact and of the walls immediately beyond. With the bins spread, the lateral pressure and the thrust increase, a moment is introduced at the end of the straight wall, and the abutment furnished by the contact is removed. Frenzel, Williams, and Vandergrift discuss this subject more fully.

![Multiple circular-bin patterns](image)

**Fig. 18-90.** Multiple circular-bin patterns.

![Variable-size bins within a bin cluster](image)

**Fig. 18-91.** Variable-size bins within a bin cluster.

![Circular bins adapted to irregular site](image)

**Fig. 18-92.** Circular bins adapted to irregular site.

![Circular-bin cluster on irregular site](image)

**Fig. 18-93.** Circular-bin cluster on irregular site.

Circular-bin patterns can be arranged to meet the need for bins of different diameters within the same cluster. Such a layout was used in the design of a cement storage and packing plant in West Winfield, Pa. (Fig. 18-97).

Circular bins can also be adapted to an irregular site. Two examples are indicated, the first for a brewery in Genesee, N.Y. (Fig. 18-92) and the second for a bakery in Toledo, Ohio (Fig. 18-93).
Hexagonal Bins

Multiple hexagonal bins used for simple storage have most of the advantages of multiple square bins. Each interior wall serves two adjacent bins, providing large storage capacity per square foot of concrete wall; the walls are straight, making form construction and steel fabrication relatively simple; all bins are the same size, providing convenient storage, eliminating unbalanced moments for the wall design regardless of bin loading pattern, and making possible considerable duplication in form construction and steel fabrication and placing; the walls of each bin are shorter than those of square bins of comparable capacity, reducing moments and consequent concrete and steel quantities, and shortening lengths of horizontal bars; and the structure as a whole is compact, requiring minimum foundation mat and conveyor run.

Hexagonal bins have the disadvantage of the straight-wall need of double reinforcing and the added disadvantage of a multiplicity of individual pieces of horizontal steel.

Maximum works economy is obtained by a pattern in which the width is in multiples of three bins (Fig. 18-94), and in which the dimension of individual bins between flats (inscribed diameter) falls between 14 and 17 ft. With this arrangement the loading and reclaiming conveyors can be placed through the center of the three bins with minimum structural interference and minimum height requirements for chuting. Erection of the structure is accomplished with only three-way yokes set at the intersection of the 8- to 10-ft-long walls.

Several impressively monumental elevators of this pattern have been built for grain storage in the Middle West: three at Enid, Okla., of 8,000,000 to 16,000,000 bushel capacity; one at Hutchinson, Kans., of 10,000,000 bushels; and a number of others of smaller capacity in these states and in Nebraska and Texas. These structures were built in units 6, 9, and 12 bins wide and from 9 to 15 bins long. The slip forms were constructed of bolted sections and reused in successive units.

No recommendation can be made relative to the cross-sectional dimension, or ratio of height to such dimension to give maximum economy, that would be applicable to all types of bins. These dimensions and ratios vary with the materials to be stored and even for the same materials under apparently slightly different conditions. To arrive at such values, each installation must be analyzed on the basis of at least those
factors noted above. The previously mentioned Grain Elevators of North America provides some indication of the almost limitless combination of patterns and height-to-area ratios used in one industry alone.

SPECIAL STRUCTURAL CONSIDERATIONS

In general, the members of a bin structure, except those already mentioned, are designed in the same manner as comparable members of any concrete structure. Several, however, occasionally require special treatment.

To prevent binding of the sliding forms, it is desirable to make all angles in the forms, both interior and exterior, something greater than right angles. When a storage unit is supported on columns, these members are usually located in plan at the junctions of the bin walls above, they are formed in a shape that will permit easy sliding, and where it becomes economical to do so, they are blocked out to reduce their size in the upper portions of the bin. Columns frequently have the shapes shown in Fig. 18-95, where the shaded areas represent the walls above the columns. Analysis of columns of such shapes with large bending moment is readily made according to the procedure presented by Wessman and by Bakhoum.

The walls spanning between columns carry the full loads of filling, upper floor, and roof live loads and all dead loads as deep girders. Analysis of this problem can be made according to the methods set down in the PCA publication No. 12 and reprinted in the Association’s paper ST 66.

In the design of the foundation for a group of deep bins, consideration should be given to the extreme rigidity of the superstructure. Unbalanced loading of the structure as a whole is possible, but local concentration of wall or column loads is highly improbable.

REFERENCES

7. Cain, William: see ref. 3, p. 64.
11. Ketchum, Milo S.: see ref. 8, pp. 40-46.
18. Ketchum, Milo S.: see ref. 8, pp. 307-309.
25. Portland Cement Association: Circular Concrete Tanks without Prestressing, Structural Bull. ST 57, December, 1942; Rectangular Concrete Tanks, Structural Bull. ST 63, March, 1947; vertical wall moments are treated generally throughout these bulletins; shrinkage is discussed in ST 57, pp. 1 and 2.
27. ACI Committee 608 Report, J. ACI, January, 1933, p. 201.
32. Grain Elevators of North America, 5th ed., March, 1942, Grain and Feed Journals, Consolidated, Chicago, Ill., Moving Forms, pp. 6–10. (Note: The name of C. S. Clark appears at the end of the preface but without designation as author or editor.)
36. Rankine, W. J.: see ref. 21.
45. Portland Cement Association: Modern Developments in Reinforced Concrete, No. 12, pp. 1–11, 1944.
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